Chapter 13
Fluids

Conceptual Problems

1 •

Determine the Concept The absolute pressure is related to the gauge pressure according to $P = P_{\text{gauge}} + P_{\text{at}}$. While doubling the gauge pressure will increase the absolute pressure, we do not have enough information to say what the resulting absolute pressure will be. $\boxed{(e)\text{ is correct.}}$

***2** •

Determine the Concept No. In an environment where $g_{\text{eff}} = F_{\text{g}} - m\frac{v^2}{r} = 0$, there is no buoyant force; there is no "up" or "down."

3 ••

Determine the Concept As you lower the rock into the water, the upward force you exert on the rock plus the upward buoyant force on the rock balance its weight. When the thread breaks, there will be an additional downward force on the scale equal to the buoyant force on the rock (the water exerts the upward buoyant force on the rock and the reaction force is the force the rock exerts on the water … and hence on the scale). Let ρ represent the density of the water, V the volume of the rock, and w_{f} the weight of the displaced water. Then the density of the rock is 3ρ. We can use Archimedes' principle to find the additional force on the scale.

Apply Archimedes' principle to the rock: $B = w_{\text{f}} = m_{\text{f}}g = \rho_{\text{f}}V_{\text{f}}g$

Because $V_{\text{f}} = V_{\text{rock}}$: $B = \rho\frac{M}{\rho_{\text{rock}}}g = \rho\frac{M}{3\rho}g = \frac{1}{3}Mg$

and $\boxed{(d)\text{ is correct.}}$

4 ••

Determine the Concept The density of water increases with depth and the buoyant force on the rock equals the weight of the displaced water. Because the weight of the displaced water depends on the density of the water, it follows that the buoyant force on the rock increases as it sinks. $\boxed{(b)\text{ is correct.}}$

5 ••

Determine the Concept Nothing. The fish is in neutral buoyancy (that is, its density equals that water), so the upward acceleration of the fish is balanced by the downward

acceleration of the displaced water.

***6** ••

Determine the Concept Yes. Because the volumes of the two objects are equal, the downward force on each side is reduced by the same amount when they are submerged, not in proportion to their masses. That is, if $m_1L_1 = m_2L_2$ and $L_1 \neq L_2$, then $(m_1 - c)L_1 \neq (m_2 - c)L_2$.

7 ••

Determine the Concept The buoyant forces acting on these submerged objects are equal to the weight of the water each displaces. The weight of the displaced water, in turn, is directly proportional to the volume of the submerged object. Because $\rho_{Pb} > \rho_{Cu}$, the volume of the copper must be greater than that of the lead and, hence, the buoyant force on the copper is greater than that on the lead. $\boxed{(b)\text{ is correct.}}$

8 ••

Determine the Concept The buoyant forces acting on these submerged objects are equal to the weight of the water each displaces. The weight of the displaced water, in turn, is directly proportional to the volume of the submerged object. Because their volumes are the same, the buoyant forces on them must be the same. $\boxed{(c)\text{ is correct.}}$

9 •

Determine the Concept It blows over the ball, reducing the pressure above the ball to below atmospheric pressure.

10 •

Determine the Concept From the equation of continuity ($I_V = Av$ = constant), we can conclude that, as the pipe narrows, the velocity of the fluid must increase. Using Bernoulli's equation for constant elevation ($P + \frac{1}{2}\rho v^2 = \text{constant}$), we can conclude that as the velocity of the fluid increases, the pressure must decrease. $\boxed{(b)\text{ is correct.}}$

***11** •

Determine the Concept False. The buoyant force on a submerged object depends on the weight of the displaced fluid which, in turn, depends on the volume of the displaced fluid.

12 •

Determine the Concept When the bottle is squeezed, the force is transmitted equally through the fluid, leading to a pressure increase on the air bubble in the diver. The air bubble shrinks, and the loss in buoyancy is enough to sink the diver.

13 •

Determine the Concept The buoyant force acting on the ice cubes equals the weight of the water they displace, i.e., $B = w_f = \rho_f V_f g$. When the ice melts the volume of water displaced by the ice cubes will occupy the space previously occupied by the submerged part of the ice cubes. Therefore, the water level remains constant.

14 •

Determine the Concept The density of salt water is greater than that of fresh water and so the buoyant force exerted on one in salt water is greater than in fresh water.

15 ••

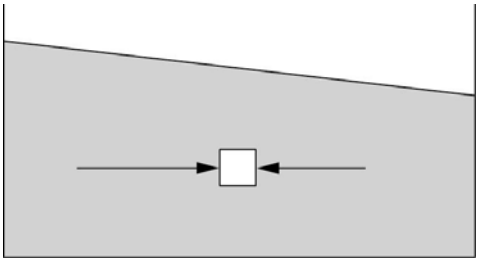
Determine the Concept Because the pressure increases with depth, the object will be compressed and its density will increase. Its volume will decrease. Thus, it will sink to the bottom.

16 ••

Determine the Concept The force acting on the fluid is the difference in pressure between the wide and narrow parts times the area of the narrow part.

17 ••

Determine the Concept The drawing shows the beaker and a strip within the water. As is readily established by a simple demonstration, the surface of the water is not level while the beaker is accelerated, showing that there is a pressure gradient. That pressure gradient results in a net force on the small element shown in the figure.



***18** ••

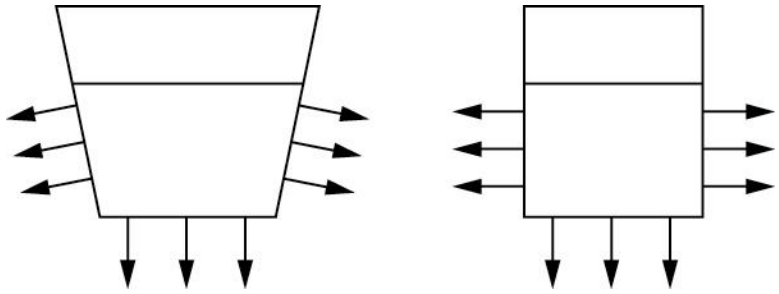
Determine the Concept The water level in the pond will drop slightly. When the anchor is in the boat, the boat displaces enough water so that the buoyant force on it equals the sum of the weight of the boat, your weight, and the weight of the anchor. When you put the anchor overboard, it will displace its volume and the volume of water displaced by the boat will decrease.

19 ••

Determine the Concept From Bernoulli's principle, the opening above which the air flows faster will be at a lower pressure than the other one, which will cause a circulation of air in the tunnel from opening 1 toward opening 2. It has been shown that enough air will circulate inside the tunnel even with the slightest breeze outside.

***20** •

Determine the Concept The diagram that follows shows the forces exerted by the pressure of the liquid on the two cups to the left.



Because the force is normal to the surface of the cup, there is a larger downward component to the net force on the cup on the left. Similarly, there will be less total force exerted by the fluid in the cup on the far right in the diagram in the problem statement.

Density

21 •

Picture the Problem The mass of the cylinder is the product of its density and volume. The density of copper can be found in Figure 13-1.

Using the definition of density, express the mass of the cylinder:

$$m = \rho V = \rho\left(\pi R^2 h\right)$$

Substitute numerical values and evaluate m:

$$\begin{aligned} m &= \pi\left(8.93\times10^3\ \text{kg/m}^3\right)\left(2\times10^{-2}\ \text{m}\right)^2 \\ &\quad\times\left(6\times10^{-2}\ \text{m}\right) \\ &= \boxed{0.673\,\text{kg}} \end{aligned}$$

22 •

Picture the Problem The mass of the sphere is the product of its density and volume. The density of lead can be found in Figure 13-1.

Using the definition of density, express the mass of the sphere:

$$m = \rho V = \rho\left(\tfrac{4}{3}\pi R^3\right)$$

Substitute numerical values and evaluate m:

$$\begin{aligned} m &= \tfrac{4}{3}\pi\left(11.3\times10^3\ \text{kg/m}^3\right)\left(2\times10^{-2}\ \text{m}\right)^3 \\ &= \boxed{0.379\,\text{kg}} \end{aligned}$$

23 •

Picture the Problem The mass of the air in the room is the product of its density and volume. The density of air can be found in Figure 13-1.

Using the definition of density, express the mass of the air:

$$m = \rho V = \rho LWH$$

Substitute numerical values and evaluate m:

$$m = (1.293\,\text{kg/m}^3)(4\,\text{m})(5\,\text{m})(4\,\text{m})$$
$$= \boxed{103\,\text{kg}}$$

***24** •

Picture the Problem Let ρ_0 represent the density of mercury at 0°C and ρ' its density at 80°C, and let m represent the mass of our sample at 0°C and m' its mass at 80°C. We can use the definition of density to relate its value at the higher temperature to its value at the lower temperature and the amount spilled.

Using its definition, express the density of the mercury at 80°C:

$$\rho' = \frac{m'}{V}$$

Express the mass of the mercury at 80°C in terms of its mass at 0°C and the amount spilled at the higher temperature:

$$\rho' = \frac{m - \Delta m}{V} = \frac{m}{V} - \frac{\Delta m}{V}$$
$$= \rho_0 - \frac{\Delta m}{V}$$

Substitute numerical values and evaluate ρ':

$$\rho' = 1.3645\times10^4\,\text{kg/m}^3 - \frac{1.47\times10^{-3}\,\text{kg}}{60\times10^{-6}\,\text{m}^3}$$
$$= \boxed{1.3621\times10^4\,\text{kg/m}^3}$$

Pressure

25 •

Picture the Problem The pressure due to a column of height h of a liquid of density ρ is given by $P = \rho gh$.

Letting h represent the height of the column of mercury, express the pressure at its base:

$$\rho_{\text{Hg}} gh = 101\,\text{kPa}$$

Solve for h:

$$h = \frac{101\,\text{kPa}}{\rho_{\text{Hg}} g}$$

Substitute numerical values and evaluate h:

$$h = \frac{1.01\times10^5\ \text{N/m}^2}{(13.6\times10^3\ \text{kg/m}^3)(9.81\text{m/s}^2)}$$
$$= 0.757\,\text{m}\times\frac{1\,\text{in}}{2.54\times10^{-2}\,\text{m}}$$
$$= \boxed{29.8\,\text{in of Hg}}$$

26 •

Picture the Problem The pressure due to a column of height h of a liquid of density ρ is given by $P = \rho gh$.

(a) Express the pressure as a function of depth in the lake:

$$P = P_{\text{at}} + \rho_{\text{water}}gh$$

Solve for and evaluate h:

$$h = \frac{P - P_{\text{at}}}{\rho_{\text{water}}g} = \frac{2P_{\text{at}} - P_{\text{at}}}{\rho_{\text{water}}g} = \frac{P_{\text{at}}}{\rho_{\text{water}}g}$$

Substitute numerical values and evaluate h:

$$h = \frac{1.01\times10^5\ \text{N/m}^2}{(10^3\ \text{kg/m}^3)(9.81\text{m/s}^2)}$$
$$= \boxed{10.3\,\text{m}}$$

(b) Proceed as in (a) with ρ_{water} replaced by ρ_{Hg} to obtain:

$$h = \frac{2P_{\text{at}} - P_{\text{at}}}{\rho_{\text{Hg}}g} = \frac{P_{\text{at}}}{\rho_{\text{Hg}}g}$$

Substitute numerical values and evaluate h:

$$h = \frac{1.01\times10^5\ \text{N/m}^2}{(13.6\times10^3\ \text{kg/m}^3)(9.81\text{m/s}^2)}$$
$$= \boxed{75.7\,\text{cm}}$$

***27** •

Picture the Problem The pressure applied to an enclosed liquid is transmitted undiminished to every point in the fluid and to the walls of the container. Hence we can equate the pressure produced by the force applied to the piston to the pressure due to the weight of the automobile and solve for F.

Express the pressure the weight of the automobile exerts on the shaft of the lift:

$$P_{\text{auto}} = \frac{w_{\text{auto}}}{A_{\text{shaft}}}$$

Express the pressure the force applied to the piston produces:

$$P = \frac{F}{A_{\text{piston}}}$$

Because the pressures are the same, we can equate them to obtain:

$$\frac{w_{\text{auto}}}{A_{\text{shaft}}} = \frac{F}{A_{\text{piston}}}$$

Solve for F:

$$F = w_{\text{auto}} \frac{A_{\text{piston}}}{A_{\text{shaft}}} = m_{\text{auto}} g \frac{A_{\text{piston}}}{A_{\text{shaft}}}$$

Substitute numerical values and evaluate F:

$$F = (1500\,\text{kg})(9.81\,\text{m/s}^2)\left(\frac{1\,\text{cm}}{8\,\text{cm}}\right)^2$$

$$= \boxed{230\,\text{N}}$$

28 ••

Picture the Problem The pressure exerted by the woman's heel on the floor is her weight divided by the area of her heel.

Using its definition, express the pressure exerted on the floor by the woman's heel:

$$P = \frac{F}{A} = \frac{w}{A} = \frac{mg}{A}$$

Substitute numerical values and evaluate P:

$$P = \frac{(56\,\text{kg})(9.81\,\text{m/s}^2)}{10^{-4}\,\text{m}^2}$$

$$= 5.49 \times 10^6\,\text{N/m}^2 \times \frac{1\,\text{atm}}{101.3\,\text{kPa}}$$

$$= \boxed{54.2\,\text{atm}}$$

***29** •

Picture the Problem The required pressure ΔP is related to the change in volume ΔV and the initial volume V through the definition of the bulk modulus B; $B = -\dfrac{\Delta P}{\Delta V/V}$.

Using the definition of the bulk modulus, relate the change in volume to the initial volume and the required pressure:

$$B = -\frac{\Delta P}{\Delta V/V}$$

Solve for ΔP:

$$\Delta P = -B\frac{\Delta V}{V}$$

Substitute numerical values and evaluate ΔP:

$$\Delta P = -2.0\times10^{9}\ \text{Pa}\times\left(\frac{-0.01\,\text{L}}{1\,\text{L}}\right)$$
$$= 2.00\times10^{7}\ \text{Pa}\times\frac{1\,\text{atm}}{101.325\,\text{kPa}}$$
$$= \boxed{198\,\text{atm}}$$

30 •

Picture the Problem The area of contact of each tire with the road is related to the weight on each tire and the pressure in the tire through the definition of pressure.

Using the definition of gauge pressure, relate the area of contact to the pressure and the weight of the car:

$$A = \frac{\frac{1}{4}w}{P_{\text{gauge}}}$$

Substitute numerical values and evaluate A:

$$A = \frac{\frac{1}{4}(1500\,\text{kg})(9.81\,\text{m/s}^2)}{200\,\text{kPa}}$$
$$= \frac{\frac{1}{4}(1500\,\text{kg})(9.81\,\text{m/s}^2)}{200\times10^{3}\ \text{N/m}^2}$$
$$= 1.84\times10^{-2}\ \text{m}^2 = \boxed{184\,\text{cm}^2}$$

31 ••

Picture the Problem The force on the lid is related to pressure exerted by the water and the cross-sectional area of the column of water through the definition of density. We can find the mass of the water from the product of its density and volume.

(*a*) Using the definition of pressure, express the force exerted on the lid:

$$F = PA$$

Express the pressure due to a column of water of height h:

$$P = \rho_{\text{water}}gh$$

Substitute for P and A to obtain:

$$F = \rho_{\text{water}}gh\pi r^2$$

Substitute numerical values:

$$F = (10^3\ \text{kg/m}^3)(9.81\,\text{m/s}^2)$$
$$\times(12\,\text{m})\pi(0.2\,\text{m})^2$$
$$= \boxed{14.8\,\text{kN}}$$

(*b*) Relate the mass of the water to its density and volume:

$$m = \rho_{\text{water}} V = \rho_{\text{water}} h \pi r^2$$

Substitute numerical values and evaluate *m*:

$$m = \left(10^3\ \text{kg/m}^3\right)(12\,\text{m})\pi\left(3\times10^{-3}\ \text{m}\right)^2$$
$$= \boxed{0.339\,\text{kg}}$$

32 ••

Picture the Problem The minimum elevation of the bag *h* that will produce a pressure of at least 12 mmHg is related to this pressure and the density of the blood plasma through $P = \rho_{\text{blood}} gh$.

Using the definition of the pressure due to a column of liquid, relate the pressure at its base to its height:

$$P = \rho_{\text{blood}} gh$$

Solve for *h*:

$$h = \frac{P}{\rho_{\text{blood}} g}$$

Substitute numerical values and evaluate *h*:

$$h = \frac{12\,\text{mmHg}\times\dfrac{133.32\,\text{Pa}}{1\,\text{mmHg}}}{\left(1.03\times10^3\ \text{kg/m}^3\right)\left(9.81\,\text{m/s}^2\right)}$$
$$= 0.158\,\text{m} = \boxed{15.8\,\text{cm}}$$

33 ••

Picture the Problem The depth *h* below the surface at which you would be able to breath is related to the pressure at that depth and the density of water ρ_{w} through $P = \rho_{\text{w}} gh$.

Express the pressure at a depth *h* and solve for *h*:

$$P = \rho_{\text{w}} gh$$

and

$$h = \frac{P}{\rho_{\text{w}} g}$$

Express the pressure at depth *h* in terms of the weight on your chest:

$$P = \frac{F}{A}$$

Substitute to obtain:

$$h = \frac{F}{A\rho_{\text{w}} g}$$

Substitute numerical values and evaluate h:

$$h = \frac{400\,\text{N}}{(0.09\,\text{m}^2)(10^3\,\text{kg/m}^3)(9.81\,\text{m/s}^2)}$$
$$= \boxed{0.453\,\text{m}}$$

34 ••

Picture the Problem Let A_1 and A_2 represent the cross-sectional areas of the large piston and the small piston, and F_1 and F_2 the forces exerted by the large and on the small piston, respectively. The work done by the large piston is $W_1 = F_1h_1$ and that done on the small piston is $W_2 = F_2h_2$. We'll use Pascal's principle and the equality of the volume of the displaced liquid in both pistons to show that W_1 and W_2 are equal.

Express the work done in lifting the car a distance h:

$$W_1 = F_1h_1$$
where F is the weight of the car.

Using the definition of pressure, relate the forces F_1 (= w) and F_2 to the areas A_1 and A_2:

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Solve for F_1:

$$F_1 = F_2\frac{A_1}{A_2}$$

Equate the volumes of the displaced fluid in the two pistons:

$$h_1A_1 = h_2A_2$$

Solve for h_1:

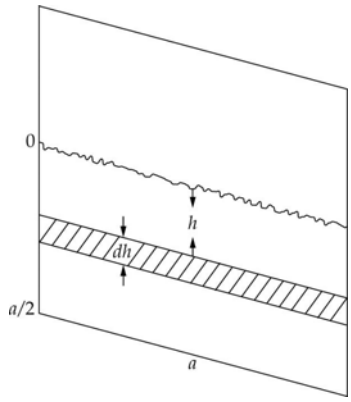
$$h_1 = h_2\frac{A_2}{A_1}$$

Substitute in the expression for W_1 and simplify to obtain:

$$W_1 = F_2\frac{A_1}{A_2}h_2\frac{A_2}{A_1} = F_2h_2 = \boxed{W_2}$$

35 •

Picture the Problem Because the pressure varies with depth, we cannot simply multiply the pressure times the half-area of a side of the cube to find the force exerted by the water. We therefore consider the force exerted on a strip of width a, height dh, and area $dA = adh$ at a depth h and integrate from $h = 0$ to $h = a/2$. The water pressure at depth h is $P_{\text{at}} + \rho gh$. We can



omit the atmospheric pressure because it is exerted on both sides of the wall of the cube.

Express the force dF on the element of length a and height dh in terms of the net pressure ρgh:

$$dF = PdA = \rho ghadh$$

Integrate from $h = 0$ to $h = a/2$:

$$F = \int_0^{a/2} dF = \rho ga \int_0^{a/2} hdh = \frac{1}{2}\rho ga\left(\frac{a^2}{4}\right)$$

$$= \boxed{\frac{\rho ga^3}{8}}$$

***36** •••

Picture the Problem The weight of the water in the vessel is the product of its mass and the gravitational field. Its mass, in turn, is related to its volume through the definition of density. The force the water exerts on the base of the container can be determined from the product of the pressure it creates and the area of the base.

(*a*) Using the definition of density, relate the weight of the water to the volume it occupies:

$$w = mg = \rho Vg$$

Substitute for V to obtain:

$$w = \tfrac{1}{3}\pi\rho r^2 hg$$

Substitute numerical values and evaluate w:

$$w = \tfrac{1}{3}\pi\left(10^3\ \text{kg/m}^3\right)\left(15\times10^{-2}\ \text{m}\right)^2\left(25\times10^{-2}\ \text{m}\right)\left(9.81\text{m/s}^2\right) = \boxed{57.8\,\text{N}}$$

(*b*) Using the definition of pressure, relate the force exerted by the water on the base of the vessel to the pressure it exerts and the area of the base:

$$F = PA = \rho gh\pi r^2$$

Substitute numerical values and evaluate F:

$$F = \left(10^3\ \text{kg/m}^3\right)\left(9.81\text{m/s}^2\right)\left(25\times10^{-2}\ \text{m}\right)\pi\left(15\times10^{-2}\ \text{m}\right)^2 = \boxed{173\,\text{N}}$$

This occurs in the same way that the force on Pascals barrel is much greater than the weight of the water in the tube. The downward force on the base is also the result of the downward component of the force exerted by the slanting walls of the cone on the water.

Buoyancy

***37** •

Picture the Problem The scale's reading will be the difference between the weight of the piece of copper in air and the buoyant force acting on it.

Express the apparent weight w' of the piece of copper:

$$w' = w - B$$

Using the definition of density and Archimedes' principle, substitute for w and B to obtain:

$$\begin{aligned} w' &= \rho_{\text{Cu}} Vg - \rho_{\text{w}} Vg \\ &= (\rho_{\text{Cu}} - \rho_{\text{w}})Vg \end{aligned}$$

Express w in terms of ρ_{Cu} and V and solve for Vg:

$$w = \rho_{\text{Cu}} Vg \Rightarrow Vg = \frac{w}{\rho_{\text{Cu}}}$$

Substitute to obtain:

$$w' = (\rho_{\text{Cu}} - \rho_{\text{w}})\frac{w}{\rho_{\text{Cu}}} = \left(1 - \frac{\rho_{\text{w}}}{\rho_{\text{Cu}}}\right)w$$

Substitute numerical values and evaluate w':

$$\begin{aligned} w' &= \left(1 - \frac{1}{9}\right)(0.5\,\text{kg})(9.81\,\text{m/s}^2) \\ &= \boxed{4.36\,\text{N}} \end{aligned}$$

38 •

Picture the Problem We can use the definition of density and Archimedes' principle to find the density of the stone. The difference between the weight of the stone in air and in water is the buoyant force acting on the stone.

Using its definition, express the density of the stone:

$$\rho_{\text{stone}} = \frac{m_{\text{stone}}}{V_{\text{stone}}} \qquad (1)$$

Apply Archimedes' principle to obtain:

$$B = w_{\text{f}} = m_{\text{f}} g = \rho_{\text{f}} V_{\text{f}} g$$

Solve for V_f:

$$V_f = \frac{B}{\rho_f g}$$

Because $V_f = V_{stone}$ and $\rho_f = \rho_{water}$:

$$V_{stone} = \frac{B}{\rho_{water} g}$$

Substitute in equation (1) and simplify to obtain:

$$\rho_{stone} = \frac{m_{stone} g}{B}\rho_{water} = \frac{w_{stone}}{B}\rho_{water}$$

Substitute numerical values and evaluate ρ_{stone}:

$$\rho_{stone} = \frac{60\text{N}}{60\text{ N} - 20\text{ N}}\left(10^3\text{ kg/m}^3\right)$$
$$= \boxed{3.00\times10^3\text{ kg/m}^3}$$

39 •

Picture the Problem We can use the definition of density and Archimedes' principle to find the density of the unknown object. The difference between the weight of the object in air and in water is the buoyant force acting on the object.

(*a*) Using its definition, express the density of the object:

$$\rho_{object} = \frac{m_{object}}{V_{object}} \qquad (1)$$

Apply Archimedes' principle to obtain:

$$B = w_f = m_f g = \rho_f V_f g$$

Solve for V_f:

$$V_f = \frac{B}{\rho_f g}$$

Because $V_f = V_{object}$ and $\rho_f = \rho_{water}$:

$$V_{object} = \frac{B}{\rho_{water} g}$$

Substitute in equation (1) and simplify to obtain:

$$\rho_{object} = \frac{m_{object} g}{B}\rho_{water} = \frac{w_{object}}{B}\rho_{water}$$

Substitute numerical values and evaluate ρ_{object}:

$$\rho_{object} = \frac{5\text{N}}{5\text{ N} - 4.55\text{ N}}\left(10^3\text{ kg/m}^3\right)$$
$$= \boxed{11.1\times10^3\text{ kg/m}^3}$$

(*b*) From Figure 13 - 1, we see that the unknown material has a density close to that of lead.

40 •

Picture the Problem We can use the definition of density and Archimedes' principle to find the density of the unknown object. The difference between the weight of the object in air and in water is the buoyant force acting on it.

Using its definition, express the density of the metal:

$$\rho_{\text{metal}} = \frac{m_{\text{metal}}}{V_{\text{metal}}} \qquad (1)$$

Apply Archimedes' principle to obtain:

$$B = w_{\text{f}} = m_{\text{f}}g = \rho_{\text{f}}V_{\text{f}}g$$

Solve for V_{f}:

$$V_{\text{f}} = \frac{B}{\rho_{\text{f}}g}$$

Because $V_{\text{f}} = V_{\text{metal}}$ and $\rho_{\text{f}} = \rho_{\text{water}}$:

$$V_{\text{metal}} = \frac{B}{\rho_{\text{water}}g}$$

Substitute in equation (1) and simplify to obtain:

$$\rho_{\text{metal}} = \frac{m_{\text{metal}}g}{B}\rho_{\text{water}} = \frac{w_{\text{metal}}}{B}\rho_{\text{water}}$$

Substitute numerical values and evaluate ρ_{metal}:

$$\rho_{\text{metal}} = \frac{90\text{N}}{90\,\text{N} - 56.6\,\text{N}}\left(10^3\ \text{kg/m}^3\right)$$
$$= \boxed{2.69 \times 10^3\ \text{kg/m}^3}$$

41 ••

Picture the Problem Let V be the volume of the object and V' be the volume that is submerged when it floats. The weight of the object is ρVg and the buoyant force due to the water is $\rho_{\text{w}}V'g$. Because the floating object is translational equilibrium, we can use $\sum F_y = 0$ to relate the buoyant forces acting on the object in the two liquids to its weight.

Apply $\sum F_y = 0$ to the object floating in water:

$$\rho_{\text{w}}V'g - mg = \rho_{\text{w}}V'g - \rho Vg = 0 \qquad (1)$$

Solve for ρ:

$$\rho = \rho_{\text{w}}\frac{V'}{V}$$

Substitute numerical values and evaluate ρ:

$$\rho = \left(10^3\ \text{kg/m}^3\right)\frac{0.8V}{V} = \boxed{800\,\text{kg/m}^3}$$

Apply $\sum F_y = 0$ to the object floating in the second liquid and solve for *mg*:

$$mg = 0.72V\rho_{\text{L}}g$$

Solve equation (1) for *mg*:

$$mg = 0.8\rho_{\text{w}}Vg$$

Equate these two expressions to obtain:

$$0.72\rho_{\text{L}} = 0.8\rho_{\text{w}}$$

Substitute in the definition of specific gravity to obtain:

$$\text{specific gravity} = \frac{\rho_{\text{L}}}{\rho_{\text{w}}} = \frac{0.8}{0.72} = \boxed{1.11}$$

***42** ••

Picture the Problem We can use Archimedes' principle to find the density of the unknown object. The difference between the weight of the block in air and in the fluid is the buoyant force acting on the block.

Apply Archimedes' principle to obtain:

$$B = w_{\text{f}} = m_{\text{f}}g = \rho_{\text{f}}V_{\text{f}}g$$

Solve for ρ_{f}:

$$\rho_{\text{f}} = \frac{B}{V_{\text{f}}g}$$

Because $V_{\text{f}} = V_{\text{Fe block}}$:

$$\rho_{\text{f}} = \frac{B}{V_{\text{Fe block}}g} = \frac{B}{m_{\text{Fe block}}g}\rho_{\text{Fe}}$$

Substitute numerical values and evaluate ρ_{f}:

$$\rho_{\text{f}} = \frac{(5\,\text{kg})(9.81\,\text{m/s}^2) - 6.16\,\text{N}}{(5\,\text{kg})(9.81\,\text{m/s}^2)}\left(7.96\times10^3\,\text{kg/m}^3\right) = \boxed{6.96\times10^3\,\text{kg/m}^3}$$

43 ••

Picture the Problem The forces acting on the cork are *B*, the upward force due to the displacement of water, *mg*, the weight of the piece of cork, and F_{s}, the force exerted by the spring. The piece of cork is in equilibrium under the influence of these forces.

Apply $\sum F_y = 0$ to the piece of cork:

$$B - w - F_{\text{s}} = 0 \qquad (1)$$

or

$$B - \rho_{\text{cork}}Vg - F_{\text{s}} = 0 \qquad (2)$$

Express the buoyant force as a function of the density of water:

$$B = w_{\text{f}} = \rho_{\text{w}}Vg$$

Solve for Vg:

$$Vg = \frac{B}{\rho_{\text{w}}}$$

Substitute for Vg in equation (2):

$$B - \rho_{\text{cork}} \frac{B}{\rho_{\text{w}}} - F_{\text{s}} = 0 \qquad (3)$$

Solve equation (1) for B:

$$B = w + F_{\text{s}}$$

Substitute in equation (3) to obtain:

$$w + F_{\text{s}} - \rho_{\text{cork}} \frac{w + F_{\text{s}}}{\rho_{\text{w}}} - F_{\text{s}} = 0$$

or

$$w - \rho_{\text{cork}} \frac{w + F_{\text{s}}}{\rho_{\text{w}}} = 0$$

Solve for ρ_{cork}:

$$\rho_{\text{cork}} = \rho_{\text{w}} \frac{w}{w + F_{\text{s}}}$$

Substitute numerical values and evaluate ρ_{cork}:

$$\rho_{\text{cork}} = \left(10^3\ \text{kg/m}^3\right)\frac{0.285\ \text{N}}{0.285\ \text{N} + 0.855\ \text{N}}$$

$$= \boxed{250\ \text{kg/m}^3}$$

44 ••

Picture the Problem Under minimum-volume conditions, the balloon will be in equilibrium. Let B represent the buoyant force acting on the balloon, w_{tot} represent its total weight, and V its volume. The total weight is the sum of the weights of its basket, cargo, and helium in its balloon.

Apply $\sum F_y = 0$ to the balloon:

$$B - w_{\text{tot}} = 0$$

Express the total weight of the balloon:

$$w_{\text{tot}} = 2000\ \text{N} + \rho_{\text{He}} Vg$$

Express the buoyant force due to the displaced air:

$$B = w_{\text{f}} = \rho_{\text{air}} Vg$$

Substitute to obtain:

$$\rho_{\text{air}} Vg - 2000\ \text{N} - \rho_{\text{He}} Vg = 0$$

Solve for V:

$$V = \frac{2000\ \text{N}}{(\rho_{\text{air}} - \rho_{\text{He}})g}$$

Substitute numerical values and evaluate V:

$$V = \frac{2000\,\text{N}}{\left(1.29\,\text{kg/m}^3 - 0.178\,\text{kg/m}^3\right)\left(9.81\,\text{m/s}^2\right)} = \boxed{183\,\text{m}^3}$$

***45** ••

Picture the Problem Let V = volume of diver, ρ_D the density of the diver, V_{Pb} the volume of added lead, and m_{Pb} the mass of lead. The diver is in equilibrium under the influence of his weight, the weight of the lead, and the buoyant force of the water.

Apply $\sum F_y = 0$ to the diver:

$$B - w_D - w_{Pb} = 0$$

Substitute to obtain:

$$\rho_w V_{D+Pb} g - \rho_D V_D g - m_{Pb} g = 0$$

or

$$\rho_w V_D + \rho_w V_{Pb} - \rho_D V_D - m_{Pb} = 0$$

Rewrite this expression in terms of masses and densities:

$$\rho_w \frac{m_D}{\rho_D} + \rho_w \frac{m_{Pb}}{\rho_{Pb}} - \rho_D \frac{m_D}{\rho_D} - m_{Pb} = 0$$

Solve for the mass of the lead:

$$m_{Pb} = \frac{\rho_{Pb}\left(\rho_w - \rho_D\right)m_D}{\rho_D\left(\rho_{Pb} - \rho_w\right)}$$

Substitute numerical values and evaluate m_{Pb}:

$$m_{Pb} = \frac{\left(11.3\times10^3\,\text{kg/m}^3\right)\left(10^3\,\text{kg/m}^3 - 0.96\times10^3\,\text{kg/m}^3\right)(85\,\text{kg})}{\left(0.96\times10^3\,\text{kg/m}^3\right)\left(11.3\times10^3\,\text{kg/m}^3 - 10^3\,\text{kg/m}^3\right)} = \boxed{3.89\,\text{kg}}$$

46 ••

Picture the Problem The scale's reading w' is the difference between the weight of the aluminum block in air w and the buoyant force acting on it. The buoyant force is equal to the weight of the displaced fluid, which, in turn, is the product of its density and mass. We can apply a condition for equilibrium to relate the reading of the bottom scale to the weight of the beaker and its contents and the buoyant force acting on the block.

Express the apparent weight w' of the aluminum block:

$$w' = w - B \qquad (1)$$

Letting F be the reading of the bottom scale and choosing upward to be the positive y direction, apply

$$F + w' - M_{tot} g = 0 \qquad (2)$$

$\sum F_y = 0$ to the scale to obtain:

Using the definition of density and Archimedes' principle, substitute for w and B in equation (1) to obtain:

$$\begin{aligned} w' &= \rho_{\text{Al}} Vg - \rho_{\text{w}} Vg \\ &= (\rho_{\text{Al}} - \rho_{\text{w}}) Vg \end{aligned}$$

Express w in terms of ρ_{Al} and V and solve for Vg:

$$w = \rho_{\text{Al}} Vg \Rightarrow Vg = \frac{w}{\rho_{\text{Al}}}$$

Substitute to obtain:

$$w' = (\rho_{\text{Al}} - \rho_{\text{w}}) \frac{w}{\rho_{\text{Al}}} = \left(1 - \frac{\rho_{\text{w}}}{\rho_{\text{Al}}}\right) w$$

Substitute numerical values and evaluate w':

$$\begin{aligned} w' &= \left(1 - \frac{10^3\ \text{kg/m}^3}{2.7 \times 10^3\ \text{kg/m}^3}\right) \\ &\quad \times (2\,\text{kg})(9.81\,\text{m/s}^2) \\ &= \boxed{12.4\,\text{N}} \end{aligned}$$

Solve equation (2) for F:

$$F = M_{\text{tot}} g - w'$$

Substitute numerical values and evaluate the reading of the bottom scale:

$$\begin{aligned} F &= (5\,\text{kg})(9.81\,\text{m/s}^2) - 12.4\,\text{N} \\ &= \boxed{36.7\,\text{N}} \end{aligned}$$

47 •••

Picture the Problem Let V = displacement of ship in the two cases, m be the mass of ship without load, and Δm be the load. The ship is in equilibrium under the influence of the buoyant force exerted by the water and its weight. We'll apply the condition for floating in the two cases and solve the equations simultaneously to determine the loaded mass of the ship.

Apply $\sum F_y = 0$ to the ship in fresh water:

$$\rho_{\text{w}} Vg - mg = 0 \qquad (1)$$

Apply $\sum F_y = 0$ to the ship in salt water:

$$\rho_{\text{sw}} Vg - (m + \Delta m) g = 0 \qquad (2)$$

Solve equation (1) for Vg:

$$Vg = \frac{mg}{\rho_{\text{w}}}$$

Substitute in equation (2) to obtain:

$$\rho_{\text{sw}} \frac{mg}{\rho_{\text{w}}} - (m + \Delta m) g = 0$$

Solve for m:

$$m = \frac{\rho_w \Delta m}{\rho_{sw} - \rho_w}$$

Add Δm to both sides of the equation and simplify to obtain:

$$\begin{aligned} m + \Delta m &= \frac{\rho_w \Delta m}{\rho_{sw} - \rho_w} + \Delta m \\ &= \Delta m\left(\frac{\rho_w}{\rho_{sw} - \rho_w} + 1\right) \\ &= \frac{\Delta m \rho_{sw}}{\rho_{sw} - \rho_w} \end{aligned}$$

Substitute numerical values and evaluate $m + \Delta m$:

$$\begin{aligned} m + \Delta m &= \frac{(6\times10^5\ \text{kg})(1.025\rho_w)}{1.025\rho_w - \rho_w} \\ &= \frac{(6\times10^5\ \text{kg})(1.025)}{1.025 - 1} \\ &= \boxed{2.46\times10^7\ \text{kg}} \end{aligned}$$

***48** •••

Picture the Problem For minimum liquid density, the bulb and its stem will be submerged. For maximum liquid density, only the bulb is submerged. In both cases the hydrometer will be in equilibrium under the influence of its weight and the buoyant force exerted by the liquids.

(*a*) Apply $\sum F_y = 0$ to the hydrometer:

$$B - w = 0$$

Using Archimedes' principle to express B, substitute to obtain:

$$\rho_{min} V g - m_{tot} g = 0$$

or

$$\rho_{min}(V_{bulb} + V_{stem}) = m_{glass} + m_{Pb}$$

Solve for m_{Pb}:

$$m_{Pb} = \rho_{min}(V_{bulb} + V_{stem}) - m_{glass}$$

Substitute numerical values and evaluate m_{Pb}:

$$m_{Pb} = (0.9\,\text{kg/L})\left[0.020\,\text{L} + \frac{\pi}{4}(0.15\,\text{m})(0.005\,\text{m})^2\left(\frac{1\,\text{L}}{10^{-3}\,\text{m}^3}\right)\right] - (6\times10^{-3}\,\text{kg})$$

$$= \boxed{14.7\,\text{g}}$$

(*b*) Apply $\sum F_y = 0$ to the hydrometer:

$$\rho_{max} V g - m_{tot} g = 0$$

or

$$\rho_{\max} V_{\text{bulb}} = m_{\text{glass}} + m_{\text{Pb}}$$

Solve for $\rho_{\max}$:

$$\rho_{\max} = \frac{m_{\text{glass}} + m_{\text{Pb}}}{V_{\text{bulb}}}$$

Substitute numerical values and evaluate $\rho_{\max}$:

$$\rho_{\max} = \frac{6\,\text{g} + 14.7\,\text{g}}{20\,\text{mL}} = \boxed{1.04\,\text{kg/L}}$$

49 •

Picture the Problem We can relate the upward force exerted on the dam wall to the area over which it acts using $F = P_{\text{g}} A$ and express P_{g} in terms of the depth of the water using $P_{\text{g}} = \rho g h$.

Using the definition of pressure, express the upward force exerted on the dam wall:

$$F = P_{\text{g}} A$$

Express the gauge pressure P_{g} of the water 5 m below the surface of the dam:

$$P_{\text{g}} = \rho g h$$

Substitute to obtain:

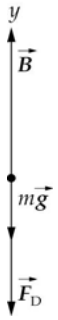
$$F = \rho g h A$$

Substitute numerical values and evaluate F:

$$F = \left(10^3\,\text{kg/m}^3\right)\left(9.81\,\text{m/s}^2\right)\left(5\,\text{m}\right)\left(10\,\text{m}^2\right)$$
$$= \boxed{491\,\text{kN}}$$

50 ••

Picture the Problem The forces acting on the balloon are the buoyant force B, its weight mg, and a drag force F_{D}. We can find the initial upward acceleration of the balloon by applying Newton's 2nd law at the instant it is released. We can find the terminal velocity of the balloon by recognizing that when $a_y = 0$, the net force acting on the balloon will be zero.



(*a*) Apply $\sum F_y = ma_y$ to the balloon at the instant of its release to obtain:

$$B - m_{\text{balloon}} g = m_{\text{balloon}} a_y$$

Solve for a_y:

$$a_y = \frac{B - m_{\text{balloon}} g}{m_{\text{balloon}}} = \frac{B}{m_{\text{balloon}}} - g$$

Using Archimedes principle, express the buoyant force B acting on the balloon:

$$B = w_{\text{f}} = m_{\text{f}}g = \rho_{\text{f}}V_{\text{f}}g$$
$$= \rho_{\text{air}}V_{\text{balloon}}g = \tfrac{4}{3}\pi\rho_{\text{air}}r^3g$$

Substitute to obtain:

$$\tfrac{4}{3}\pi\rho_{\text{air}}r^3g - m_{\text{balloon}}g = m_{\text{balloon}}a_y$$

Solve for a_y:

$$a_y = \left(\frac{\frac{4}{3}\pi\rho_{\text{air}}r^3}{m_{\text{balloon}}} - 1\right)g$$

Substitute numerical values and evaluate a_y:

$$a_y = \left[\frac{\frac{4}{3}\pi\left(1.29\,\text{kg/m}^3\right)\left(2.5\,\text{m}\right)^3}{15\,\text{kg}} - 1\right]$$
$$\times\left(9.81\,\text{m/s}^2\right)$$
$$= \boxed{45.4\,\text{m/s}^2}$$

(*b*) Apply $\sum F_y = ma_y$ to the balloon under terminal-speed conditions to obtain:

$$B - mg - \tfrac{1}{2}\pi r^2\rho v_{\text{t}}^2 = 0$$

Substitute for B:

$$\tfrac{4}{3}\pi\rho_{\text{air}}r^3g - mg - \tfrac{1}{2}\pi r^2\rho v_{\text{t}}^2 = 0$$

Solve for v_{t}:

$$v_{\text{t}} = \sqrt{\frac{2\left(\frac{4}{3}\pi\rho_{\text{air}}r^3 - m\right)g}{\pi r^2\rho}}$$

Substitute numerical values and evaluate v:

$$v_{\text{t}} = \sqrt{\frac{2\left[\frac{4}{3}\pi\left(1.29\,\text{kg/m}^3\right)\left(2.5\,\text{m}\right)^3 - 15\,\text{kg}\right]\left(9.81\,\text{m/s}^2\right)}{\pi\left(2.5\,\text{m}\right)^2\left(1.29\,\text{kg/m}^3\right)}} = \boxed{7.33\,\text{m/s}}$$

(*c*) Relate the time required for the balloon to rise to 10 km to its terminal speed:

$$\Delta t = \frac{h}{v_t} = \frac{10\,\text{km}}{7.33\,\text{m/s}} = 1364\,\text{s}$$
$$= \boxed{22.7\,\text{min}}$$

Continuity and Bernoulli's Equation

***51** ••

Picture the Problem Let J represent the flow rate of the water. Then we can use $J = Av$ to relate the flow rate to the cross-sectional area of the circular tap and the velocity of the water. In (*b*) we can use the equation of continuity to express the diameter of the stream 7.5 cm below the tap and a constant-acceleration equation to find the velocity of the water at this distance. In (*c*) we can use a constant-acceleration equation to express the distance-to-turbulence in terms of the velocity of the water at turbulence v_{t} and the definition of Reynolds number N_{R} to relate v_{t} to N_{R}.

(*a*) Express the flow rate of the water in terms of the cross-sectional area A of the circular tap and the velocity v of the water:

$$J = Av = \pi r^2 v = \tfrac{1}{4}\pi d^2 v \qquad (1)$$

Solve for v:

$$v = \frac{J}{\frac{1}{4}\pi d^2}$$

Substitute numerical values and evaluate v:

$$v = \frac{10.5\,\text{cm}^3/\text{s}}{\frac{1}{4}\pi(1.2\,\text{cm})^2} = \boxed{9.28\,\text{cm/s}}$$

(*b*) Apply the equation of continuity to the stream of water:

$$v_\text{f} A_\text{f} = v_\text{i} A_\text{i} = vA_\text{i}$$

or

$$v_\text{f} \frac{\pi}{4} d_\text{f}^2 = v \frac{\pi}{4} d_\text{i}^2$$

Solve for d_f:

$$d_\text{f} = \sqrt{\frac{v}{v_\text{f}}}\, d_\text{i} \qquad (2)$$

Use a constant-acceleration equation to relate v_f and v to the distance Δh fallen by the water:

$$v_\text{f}^2 = v^2 + 2g\Delta h$$

Solve for v_f to obtain:

$$v_\text{f} = \sqrt{v^2 + 2g\Delta h}$$

Substitute numerical values and evaluate v_f:

$$\begin{aligned} v_\text{f} &= \sqrt{(9.28\,\text{cm/s})^2 + 2(981\,\text{cm/s}^2)(7.5\,\text{cm})} \\ &= 122\,\text{cm/s} \end{aligned}$$

Substitute in equation (2) and evaluate d_f:

$$d_\text{f} = (1.2\,\text{cm})\sqrt{\frac{9.28\,\text{cm/s}}{122\,\text{cm/s}}} = \boxed{0.331\,\text{cm}}$$

(*c*) Using a constant-acceleration equation, relate the fall-distance-to-turbulence Δd to its initial speed v and its speed v_t when its flow becomes turbulent:

$$v_\text{t}^2 = v^2 + 2g\Delta d$$

Solve for Δd to obtain:

$$\Delta d = \frac{v_\text{t}^2 - v^2}{2g} \qquad (3)$$

Express Reynolds number N_R for turbulent flow:

$$N_\text{R} = \frac{2r\rho v_\text{t}}{\eta}$$

From equation (1):

$$r = \sqrt{\frac{J}{\pi v_t}}$$

Substitute to obtain:

$$N_R = \frac{2\rho v_t}{\eta}\sqrt{\frac{J}{\pi v_t}}$$

Solve for v_t:

$$v_t = \frac{\pi N_R^2 \eta^2}{4\rho^2 J}$$

Substitute numerical values (see Figure 13-1 for the density of water and Table 13-1 for the coefficient of viscosity for water) and evaluate v_t:

$$v_t = \frac{\pi(2300)^2\left(1.8\times10^{-3}\ \text{Pa}\cdot\text{s}\right)^2}{4\left(10^3\ \text{kg/m}^3\right)^2\left(10.5\,\text{cm}^3/\text{s}\right)}$$
$$= 1.28\,\text{m/s}$$

Substitute in equation (3) and evaluate the fall-distance-to turbulence:

$$\Delta d = \frac{(128\,\text{cm/s})^2 - (9.28\,\text{cm/s})^2}{2(981\,\text{cm/s}^2)}$$
$$= \boxed{8.31\,\text{cm}}$$

in reasonable agreement with everyday experience.

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Picture the Problem Let A_1 represent the cross-sectional area of the hose, A_2 the cross-sectional area of the nozzle, v_1 the velocity of the water in the hose, and v_2 the velocity of the water as it passes through the nozzle. We can use the continuity equation to find v_2 and Bernoulli's equation for constant elevation to find the pressure at the pump.

(*a*) Using the continuity equation, relate the speeds of the water to the diameter of the hose and the diameter of the nozzle:

$$A_1 v_1 = A_2 v_2$$

or

$$\frac{\pi d_1^2}{4} v_1 = \frac{\pi d_2^2}{4} v_2$$

Solve for v_2:

$$v_2 = \frac{d_1^2}{d_2^2} v_1$$

Substitute numerical values and evaluate v_2:

$$v_2 = \left(\frac{3\,\text{cm}}{0.3\,\text{cm}}\right)^2 (0.65\,\text{m/s}) = \boxed{65.0\,\text{m/s}}$$

(*b*) Using Bernoulli's equation for constant elevation, relate the pressure at the pump P_P to the

$$P_P + \tfrac{1}{2}\rho v_1^2 = P_{at} + \tfrac{1}{2}\rho v_2^2$$

atmospheric pressure and the velocities of the water in the hose and the nozzle:

Solve for the pressure at the pump: $$P_{\text{P}} = P_{\text{at}} + \tfrac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

Substitute numerical values and evaluate P_{P}:

$$\begin{aligned} P_{\text{P}} &= 101\,\text{kPa} + \tfrac{1}{2}\left(10^3\ \text{kg/m}^3\right)\left[(65\,\text{m/s})^2 - (0.65\,\text{m/s})^2\right] \\ &= 2.21\times10^6\,\text{Pa}\times\frac{1\,\text{atm}}{101.325\,\text{kPa}} = \boxed{21.9\,\text{atm}} \end{aligned}$$

53 •

Picture the Problem Let A_1 represent the cross-sectional area of the larger-diameter pipe, A_2 the cross-sectional area of the smaller-diameter pipe, v_1 the velocity of the water in the larger-diameter pipe, and v_2 the velocity of the water in the smaller-diameter pipe. We can use the continuity equation to find v_2 and Bernoulli's equation for constant elevation to find the pressure in the smaller-diameter pipe.

(*a*) Using the continuity equation, relate the velocities of the water to the diameters of the pipe: $$A_1v_1 = A_2v_2$$ or $$\frac{\pi d_1^2}{4}v_1 = \frac{\pi d_2^2}{4}v_2$$

Solve for and evaluate v_2: $$v_2 = \frac{d_1^2}{d_2^2}v_1$$

Substitute numerical values and evaluate v_2: $$v_2 = \left(\frac{d_1}{\frac{1}{2}d_1}\right)^2(3\,\text{m/s}) = \boxed{12.0\,\text{m/s}}$$

(*b*) Using Bernoulli's equation for constant elevation, relate the pressures in the two segments of the pipe to the velocities of the water in these segments: $$P_1 + \tfrac{1}{2}\rho_{\text{w}}v_1^2 = P_2 + \tfrac{1}{2}\rho_{\text{w}}v_2^2$$

Solve for P_2: $$\begin{aligned} P_2 &= P_1 + \tfrac{1}{2}\rho_{\text{w}}v_1^2 - \tfrac{1}{2}\rho_{\text{w}}v_2^2 \\ &= P_1 + \tfrac{1}{2}\rho_{\text{w}}\left(v_1^2 - v_2^2\right) \end{aligned}$$

Substitute numerical values and evaluate P_2:

$$P_2 = 200\,\text{kPa} + \tfrac{1}{2}\left(10^3\ \text{kg/m}^3\right) \times \left[(3\,\text{m/s})^2 - (12\,\text{m/s})^2\right] = \boxed{133\,\text{kPa}}$$

(*c*) Using the continuity equation, evaluate I_{V1}:

$$I_{V1} = A_1 v_1 = \frac{\pi d_1^2}{4} v_1 = \frac{\pi d_1^2}{4}(3\,\text{m/s})$$

Using the continuity equation, express I_{V2}:

$$I_{V2} = A_2 v_2 = \frac{\pi d_2^2}{4} v_2$$

Substitute numerical values and evaluate I_{V2}:

$$I_{V2} = \frac{\pi\left(\frac{d_1}{2}\right)^2}{4}(12\,\text{m/s}) = \frac{\pi d_1^2}{4}(3\,\text{m/s})$$

Thus, as we expected would be the case:

$$\boxed{I_{V1} = I_{V2}}$$

54 •

Picture the Problem Let A_1 represent the cross-sectional area of the 2-cm diameter pipe, A_2 the cross-sectional area of the constricted pipe, v_1 the velocity of the water in the 2-cm diameter pipe, and v_2 the velocity of the water in the constricted pipe. We can use the continuity equation to express d_2 in terms of d_1 and to find v_1 and Bernoulli's equation for constant elevation to find the velocity of the water in the constricted pipe.

Using the continuity equation, relate the volume flow rate in the 2-cm diameter pipe to the volume flow rate in the constricted pipe:

$$A_1 v_1 = A_2 v_2$$

or

$$\frac{\pi d_1^2}{4} v_1 = \frac{\pi d_2^2}{4} v_2$$

Solve for d_2:

$$d_2 = d_1\sqrt{\frac{v_1}{v_2}}$$

Using the continuity equation, relate v_1 to the volume flow rate I_V:

$$v_1 = \frac{I_V}{A_1} = \frac{2.80\,\text{L/s}}{\frac{\pi(0.02\,\text{m})^2}{4}} = 8.91\,\text{m/s}$$

Using Bernoulli's equation for constant elevation, relate the pressures in the two segments of the pipe to the velocities of the water in

$$P_1 + \tfrac{1}{2}\rho_w v_1^2 = P_2 + \tfrac{1}{2}\rho_w v_2^2$$

these segments:

Solve for v_2:

$$v_2 = \sqrt{\frac{2(P_1 - P)}{\rho_w} + v_1^2}$$

Substitute numerical values and evaluate v_2:

$$v_2 = \sqrt{\frac{2(142\,\text{kPa} - 101\,\text{kPa})}{10^3\,\text{kg/m}^3} + (8.91\,\text{m/s})^2}$$
$$= 12.7\,\text{m/s}$$

Substitute and evaluate d_2:

$$d_2 = (2\,\text{cm})\sqrt{\frac{8.91\,\text{m/s}}{12.7\,\text{m/s}}} = \boxed{1.68\,\text{cm}}$$

***55** ••

Picture the Problem We can use the definition of the volume flow rate to find the volume flow rate of blood in an aorta and to find the total cross-sectional area of the capillaries.

(*a*) Use the definition of the volume flow rate to find the volume flow rate through an aorta:

$$I_V = Av$$

Substitute numerical values and evaluate I_V:

$$I_V = \pi(9\times10^{-3}\,\text{m}^3)(0.3\,\text{m/s})$$
$$= 7.63\times10^{-5}\,\frac{\text{m}^3}{\text{s}}\times\frac{60\,\text{s}}{\text{min}}\times\frac{1\,\text{L}}{10^{-3}\,\text{m}^3}$$
$$= \boxed{4.58\,\text{L/min}}$$

(*b*) Use the definition of the volume flow rate to express the volume flow rate through the capillaries:

$$I_V = A_{cap}v_{cap}$$

Solve for the total cross-sectional area of the capillaries:

$$A_{cap} = \frac{I_V}{v_{cap}}$$

Substitute numerical values and evaluate A_{cap}:

$$A_{cap} = \frac{7.63\times10^{-5}\,\text{m}^3/\text{s}}{0.001\,\text{m/s}}$$
$$= 7.63\times10^{-2}\,\text{m}^2 = \boxed{763\,\text{cm}^2}$$

56 ••

Picture the Problem We can apply Bernoulli's equation to points *a* and *b* to determine the rate at which the water exits the tank. Because the diameter of the small pipe is much smaller than the diameter of the tank, we can neglect the velocity of the water at the point *a*. The distance the water travels once it exits the pipe is the product of its speed and the time required to fall the distance $H - h$.

Express the distance x as a function of the exit speed of the water and the time to fall the distance $H - h$:

$$x = v_b \Delta t \qquad (1)$$

Apply Bernoulli's equation to the water at points *a* and *b*:

$$P_a + \rho_w g H + \tfrac{1}{2}\rho_w v_a^2 = P_b + \rho_w g(H - h) + \tfrac{1}{2}\rho_w v_b^2$$

or, because $v_a \approx 0$ and $P_a = P_b = P_{at}$,

$$gH = g(H - h) + \tfrac{1}{2} v_b^2$$

Solve for v_b:

$$v_b = \sqrt{2gh}$$

Using a constant-acceleration equation, relate the time of fall to the distance of fall:

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2} a(\Delta t)^2$$

or, because $v_{0y} = 0$,

$$H - h = \tfrac{1}{2} g(\Delta t)^2$$

Solve for Δt:

$$\Delta t = \sqrt{\frac{2(H - h)}{g}}$$

Substitute in equation (1) to obtain:

$$x = \sqrt{2gh}\sqrt{\frac{2(H - h)}{g}} = \boxed{2\sqrt{h(H - h)}}$$

57 ••

Picture the Problem Let the subscript 60 denote the 60-cm-radius pipe and the subscript 30 denote the 30-cm-radius pipe. We can use Bernoulli's equation for constant elevation to express P_{30} in terms of v_{60} and v_{30}, the definition of volume flow rate to find v_{60} and the continuity equation to find v_{30}.

Using Bernoulli's equation for constant elevation, relate the pressures in the two pipes to the velocities of the oil:

$$P_{60} + \tfrac{1}{2}\rho v_{60}^2 = P_{30} + \tfrac{1}{2}\rho v_{30}^2$$

Solve for P_{30}: $$P_{30} = P_{60} + \tfrac{1}{2}\rho\left(v_{60}^2 - v_{30}^2\right) \qquad (1)$$

Use the definition of volume flow rate to find v_{60}:

$$\begin{aligned} v_{60} &= \frac{I_V}{A_{60}} \\ &= \frac{2.4\times10^5\,\dfrac{\text{m}^3}{\text{day}}\times\dfrac{1\,\text{day}}{24\,\text{h}}\times\dfrac{1\,\text{h}}{3600\,\text{s}}}{\pi(0.6\,\text{m})^2} \\ &= 2.456\,\text{m/s} \end{aligned}$$

Using the continuity equation, relate the velocity of the oil in the half-standard pipe to its velocity in the standard pipe: $$A_{60}v_{60} = A_{30}v_{30}$$

Solve for and evaluate v_{30}:

$$\begin{aligned} v_{30} &= \frac{A_{60}}{A_{30}}v_{60} = \frac{\pi(0.6\,\text{m})^2}{\pi(0.3\,\text{m})^2}(2.456\,\text{m/s}) \\ &= 9.824\,\text{m/s} \end{aligned}$$

Substitute numerical values in equation (1) and evaluate P_{30}:

$$P_{30} = 180\,\text{kPa} + \tfrac{1}{2}\left(800\,\text{kg/m}^3\right)\left[(2.456\,\text{m/s})^2 - (9.824\,\text{m/s})^2\right] = \boxed{144\,\text{kPa}}$$

***58** ••

Picture the Problem We'll use its definition to relate the volume flow rate in the pipe to the velocity of the water and the result of Example 13-9 to find the velocity of the water.

Using its definition, express the volume flow rate: $$I_V = A_1 v_1 = \pi r^2 v_1$$

Using the result of Example 13-9, find the velocity of the water upstream from the Venturi meter: $$v_1 = \sqrt{\frac{2\rho_{\text{Hg}}gh}{\rho_w\left(\dfrac{R_1^2}{R_2^2} - 1\right)}}$$

Substitute numerical values and evaluate v_1:

$$v_1 = \sqrt{\frac{2\left(13.6\times10^3\,\text{kg/m}^3\right)\left(9.81\,\text{m/s}^2\right)(0.024\,\text{m})}{\left(10^3\,\text{kg/m}^3\right)\left[\left(\dfrac{0.095\,\text{m}}{0.056\,\text{m}}\right)^2 - 1\right]}} = 1.847\,\text{m/s}$$

Substitute numerical values and evaluate I_V:

$$I_V = \frac{\pi}{4}(0.095\,\text{m})^2(1.847\,\text{m/s})$$
$$= 1.309\times10^{-2}\ \text{m}^3/\text{s}$$
$$= \boxed{13.1\,\text{L/s}}$$

59 ••

Picture the Problem We can apply the definition of the volume flow rate to find the mass of water emerging from the hose in 1 s and the definition of momentum to find the momentum of the water. The force exerted on the water by the hose can be found from the rate at which the momentum of the water changes.

(*a*) Using its definition, express the volume flow rate of the water emerging from the hose:

$$I_V = \frac{\Delta V}{\Delta t} = \frac{\Delta m}{\rho_w \Delta t} = Av$$

Solve for Δm:

$$\Delta m = Av\rho_w \Delta t$$

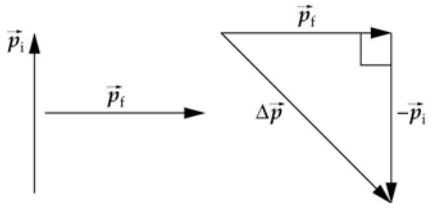
Substitute numerical values and evaluate Δm:

$$\Delta m = \pi(0.015\,\text{m})^2(30\,\text{m/s})(10^3\,\text{kg/m}^3)(1\,\text{s})$$
$$= \boxed{21.2\,\text{kg/s}}$$

(*b*) Using its definition, express and evaluate the momentum of the water:

$$p = \Delta m v = (21.2\,\text{kg/s})(30\,\text{m/s})$$
$$= \boxed{636\,\text{kg}\cdot\text{m/s}}$$

(*c*) The vector diagrams are to the right:



Express the change in momentum of the water:

$$\Delta\vec{\boldsymbol{p}} = \vec{\boldsymbol{p}}_f - \vec{\boldsymbol{p}}_i$$

Substitute numerical values and evaluate Δp:

$$\Delta p = \sqrt{(636\,\text{kg}\cdot\text{m/s})^2 + (636\,\text{kg}\cdot\text{m/s})^2}$$
$$= (636\,\text{kg}\cdot\text{m/s})\sqrt{2}$$
$$= \boxed{899\,\text{kg}\cdot\text{m/s}}$$

Relate the force exerted on the water by the hose to the rate at which the water's momentum changes and evaluate F:

$$F = \frac{\Delta p}{\Delta t} = \frac{899\,\text{kg}\cdot\text{m/s}}{1\,\text{s}} = \boxed{899\,\text{N}}$$

60 ••

Picture the Problem Let the letter P denote the pump and the 2-cm diameter pipe and the letter N the 1-cm diameter nozzle. We'll use Bernoulli's equation to express the necessary pump pressure, the continuity equation to relate the velocity of the water coming out of the pump to its velocity at the nozzle, and a constant-acceleration equation to relate its velocity at the nozzle to the height to which the water rises.

Using Bernoulli's equation, relate the pressures, areas, and velocities in the pipe and nozzle:

$$P_\text{P} + \rho_\text{w} g h_\text{P} + \tfrac{1}{2}\rho_\text{w} v_\text{P}^2 = P_\text{N} + \rho_\text{w} g h_\text{N} + \tfrac{1}{2}\rho_\text{w} v_\text{N}^2$$

or, because $P_\text{N} = P_\text{at}$ and $h_\text{P} = 0$,

$$P_\text{P} + \tfrac{1}{2}\rho_\text{w} v_\text{P}^2 = P_\text{N} + \rho_\text{w} g h_\text{N} + \tfrac{1}{2}\rho_\text{w} v_\text{N}^2$$

Solve for the pump pressure:

$$P_\text{P} = P_\text{at} + \rho_\text{w} g h_\text{N} + \tfrac{1}{2}\rho_\text{w}\left(v_\text{N}^2 - v_\text{P}^2\right) \qquad (1)$$

Use the continuity equation to relate v_P and v_N to the cross-sectional areas of the pipe from the pump and the nozzle:

$$A_\text{P} v_\text{P} = A_\text{N} v_\text{N}$$

and

$$v_\text{P} = \frac{A_\text{N}}{A_\text{P}} v_\text{N} = \frac{\frac{1}{4}\pi d_\text{N}^2}{\frac{1}{4}\pi d_\text{P}^2} v_\text{N} = \left(\frac{1\,\text{cm}}{2\,\text{cm}}\right)^2 v_\text{N} = \tfrac{1}{4} v_\text{N}$$

Using a constant-acceleration equation, express the velocity of the water at the nozzle in terms of the desired height Δh:

$$v^2 = v_\text{N}^2 - 2g\Delta h$$

or, because $v = 0$,

$$v_\text{N}^2 = 2g\Delta h$$

Substitute in equation (1) to obtain:

$$P_\text{P} = P_\text{at} + \rho_\text{w} g h_\text{N} + \tfrac{1}{2}\rho_\text{w}\left[2g\Delta h - \tfrac{1}{16}\left(2g\Delta h\right)\right] = P_\text{at} + \rho_\text{w} g h_\text{N} + \tfrac{1}{2}\rho_\text{w}\left(\tfrac{15}{8} g\Delta h\right)$$
$$= P_\text{at} + \rho_\text{w} g\left(h_\text{N} + \tfrac{15}{16}\Delta h\right)$$

Substitute numerical values and evaluate P_P:

$$P_\text{P} = 101\,\text{kPa} + \left(10^3\,\text{kg/m}^3\right)\left(9.81\,\text{m/s}^2\right)\left[3\,\text{m} + \tfrac{15}{16}\left(12\,\text{m}\right)\right] = \boxed{241\,\text{kPa}}$$

61 •••

Picture the Problem We can apply Bernoulli's equation to points *a* and *b* to determine the rate at which the water exits the tank. Because the diameter of the small pipe is much smaller than the diameter of the tank, we can neglect the velocity of the water at the point *a*. The distance the water travels once it exits the pipe is the product of its velocity and the time required to fall the distance $H - h$. That there are two values of *h* that are equidistant from the point $h = \frac{1}{2}H$ can be shown by solving the quadratic equation that relates *x* to *h* and *H*. That *x* is a maximum for this value of *h* can be established by treating $x = f(h)$ as an extreme-value problem.

(*a*) Express the distance *x* as a function of the exit speed of the water and the time to fall the distance $H - h$:

$$x = v_b \Delta t \qquad (1)$$

Apply Bernoulli's equation to the water at points *a* and *b*:

$$P_a + \rho_w g H + \tfrac{1}{2}\rho_w v_a^2 = P_b + \rho_w g(H - h) + \tfrac{1}{2}\rho_w v_b^2$$

or, because $v_a \approx 0$ and $P_a = P_b = P_{at}$,

$$gH = g(H - h) + \tfrac{1}{2}v_b^2$$

Solve for v_b:

$$v_b = \sqrt{2gh}$$

Using a constant-acceleration equation, relate the time of fall to the distance of fall:

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2}a(\Delta t)^2$$

or, because $v_{0y} = 0$,

$$H - h = \tfrac{1}{2}g(\Delta t)^2$$

Solve for Δt:

$$\Delta t = \sqrt{\frac{2(H - h)}{g}}$$

Substitute in equation (1) to obtain:

$$x = \sqrt{2gh}\sqrt{\frac{2(H - h)}{g}} = \boxed{2\sqrt{h(H - h)}}$$

(*b*) Square both sides of this equation and simplify to obtain:

$$x^2 = 4hH - 4h^2 \text{ or } 4h^2 - 4Hh + x^2 = 0$$

Solve this quadratic equation to obtain:

$$h = \boxed{\tfrac{1}{2}H \pm \tfrac{1}{2}\sqrt{H^2 - x^2}}$$

Find the average of these two values for h:

$$h_{\text{av}} = \frac{\frac{1}{2}H + \sqrt{H^2 - x^2} + \frac{1}{2}H - \sqrt{H^2 - x^2}}{2}$$
$$= \boxed{\tfrac{1}{2}H}$$

(*c*) Differentiate $x = 2\sqrt{h(H-h)}$ with respect to h:

$$\frac{dx}{dh} = 2\left(\frac{1}{2}\right)\left[h(H-h)\right]^{-\frac{1}{2}}(H-2h)$$
$$= \frac{H-2h}{\sqrt{h(H-h)}}$$

Set the derivative equal to zero for extrema:

$$\frac{H-2h}{\sqrt{h(H-h)}} = 0$$

Solve for h to obtain:

$$h = \boxed{\tfrac{1}{2}H}$$

Evaluate $x = 2\sqrt{h(H-h)}$ with $h = \frac{1}{2}H$:

$$x_{\max} = 2\sqrt{\tfrac{1}{2}H\left(H - \tfrac{1}{2}H\right)}$$
$$= \boxed{H}$$

Remarks: To show that this value for *h* corresponds to a *maximum*, one can either show that $\frac{d^2x}{dh^2} < 0$ at $h = \frac{1}{2}H$ or confirm that the graph of *f*(*h*) at $h = \frac{1}{2}H$ is concave downward.

***62** ••

Picture the Problem Let the numeral 1 denote the opening in the end of the inner pipe and the numeral 2 to one of the holes in the outer tube. We can apply Bernoulli's principle at these locations and solve for the pressure difference between them. By equating this pressure difference to the pressure difference due to the height h of the liquid column we can express v as a function of ρ, ρ_g, g, and h.

Apply Bernoulli's principle at locations 1 and 2 to obtain:

$$P_1 + \tfrac{1}{2}\rho_g v_1^2 = P_2 + \tfrac{1}{2}\rho_g v_2^2$$

where we've ignored the difference in elevation between the two openings.

Solve for the pressure difference $\Delta P = P_1 - P_2$:

$$\Delta P = P_1 - P_2 = \tfrac{1}{2}\rho_g v_2^2 - \tfrac{1}{2}\rho_g v_1^2$$

Express the velocity of the gas at 1:

$v_1 = 0$ because the gas is brought to a halt (i.e., is stagnant) at the opening to the inner pipe.

Express the velocity of the gas at 2:

$v_2 = v$ because the gas flows freely past

the holes in the outer ring.

Substitute to obtain: $\Delta P = \frac{1}{2}\rho_g v^2$

Letting A be the cross-sectional area of the tube, express the pressure at a depth h in the column of liquid whose density is ρ_l:

$$P_1 = P_2 + \frac{w_{\text{displaced liquid}}}{A} - \frac{B}{A}$$

where $B = \rho_g Ahg$ is the buoyant force acting on the column of liquid of height h.

Substitute to obtain:

$$P_1 = P_2 + \frac{\rho ghA}{A} - \frac{\rho_g ghA}{A}$$
$$= P_2 + (\rho - \rho_g)gh$$

or

$$\Delta P = P_1 - P_2 = (\rho - \rho_g)gh$$

Equate these two expressions for ΔP:

$$\tfrac{1}{2}\rho_g v^2 = (\rho - \rho_g)gh$$

Solve for v^2 to obtain:

$$\boxed{v^2 = \frac{2gh(\rho - \rho_g)}{\rho_g}}$$

Note that the correction for buoyant force due to the displaced gas is very small and that, to a good approximation,

$$v = \sqrt{\frac{2gh\rho}{\rho_g}}.$$

Remarks: Pitot tubes are used to measure the airspeed of airplanes.

63 ••

Picture the Problem Let the letter "a" denote the entrance to the siphon tube and the letter "b" denote its exit. Assuming streamline flow between these points, we can apply Bernoulli's equation to relate the entrance and exit speeds of the water flowing in the siphon to the pressures at either end, the density of the water, and the difference in elevation between the entrance and exit points. We can use the expression for the pressure as a function of depth in an incompressible fluid to find the pressure at the entrance to the tube in terms of its distance below the surface. We'll also use the equation of continuity to argue that, provided the surface area of the beaker is large compared to the area of the opening of the tube, the entrance speed of the water is approximately zero.

(*a*) Apply Bernoulli's equation at the entrance to the siphon tube (point a) and at its exit (point b):

$$P_a + \tfrac{1}{2}\rho v_a^2 + \rho g(H - h)$$
$$= P_b + \tfrac{1}{2}\rho v_b^2 + \rho g(H - h - d) \quad (1)$$

where H is the height of the containers.

Apply the continuity equation to a point at the surface of the liquid in the container to the left and to point a:

$$v_a A_a = v_{surface} A_{surface}$$

or, because $A_a << A_{surface}$,

$$v_a = v_{surface} = 0$$

Express the pressure at the inlet (point a) and the outlet (point b):

$$P_a = P_{atm} + \rho g(H - h)$$

and

$$P_b = P_{atm} + \rho g(H - h - d)$$

Letting $v_b = v$, substitute in equation (1) to obtain:

$$P_{atm} + \rho g(H - h) + \rho gH = P_{atm} + \rho g(H - h - d) + \tfrac{1}{2}\rho v^2 + \rho g(H - h - d)$$

or, upon simplification,

$$g(H - h) + gH = g(H - h - d) + \tfrac{1}{2}v^2 + g(H - h - d)$$

Solve for v:

$$v = \boxed{\sqrt{2gd}}$$

(*b*) Relate the pressure at the highest part of the tube P_{top} to the pressure at point b:

$$P_{top} + \rho g(H - h) + \tfrac{1}{2}\rho v_h^2 = P_{atm} + \rho g(H - h - d) + \tfrac{1}{2}\rho v_b^2$$

or, because $v_h = v_b$,

$$P_{top} = \boxed{P_{atm} - \rho gd}$$

Remarks: If we let $P_{top} = 0$, we can use this result to find the maximum theoretical height a siphon can lift water.

Viscous Flow

64 •

Picture the Problem The required pressure difference can be found by applying Poiseuille's law to the viscous flow of water through the horizontal tube.

Using Poiseuille's law, relate the pressure difference between the two ends of the tube to its length, radius, and the volume flow rate of the water:

$$\Delta P = \frac{8\eta L}{\pi r^4} I_V$$

Substitute numerical values and evaluate ΔP:

$$\Delta P = \frac{8(1\,\text{mPa}\cdot\text{s})(0.25\,\text{m})}{\pi\left(0.6\times10^{-3}\,\text{m}\right)^4}(0.3\,\text{mL/s}) = \boxed{1.47\,\text{kPa}}$$

65 •

Picture the Problem Because the pressure difference is unchanged, we can equate the expressions of Poiseuille's law for the two tubes and solve for the diameter of the tube that would double the flow rate.

Using Poiseuille's law, express the pressure difference required for the radius and volume flow rate of Problem 64:

$$\Delta P = \frac{8\eta L}{\pi r^4} I_V$$

Express the pressure difference required for the radius r' that would double the volume flow rate of Problem 57:

$$\Delta P = \frac{8\eta L}{\pi r'^4}(2I_V)$$

Equate these equations and simplify to obtain:

$$\frac{8\eta L}{\pi r'^4}(2I_V) = \frac{8\eta L}{\pi r^4} I_V$$

or

$$\frac{2}{r'^4} = \frac{1}{r^4}$$

Solve for r':

$$r' = \sqrt[4]{2}r$$

Express d':

$$d' = 2r' = 2\sqrt[4]{2}r = \sqrt[4]{2}d$$

Substitute numerical values and evaluate d':

$$d' = \sqrt[4]{2}(1.2\,\text{mm}) = \boxed{1.43\,\text{mm}}$$

***66** •

Picture the Problem We can apply Poiseuille's law to relate the pressure drop across the capillary tube to the radius and length of the tube, the rate at which blood is flowing through it, and the viscosity of blood.

Using Poiseuille's law, relate the pressure drop to the length and diameter of the capillary tube, the volume flow rate of the blood, and the viscosity of the blood:

$$\Delta P = \frac{8\eta L}{\pi r^4} I_V$$

Solve for the viscosity of the blood:

$$\eta = \frac{\pi r^4 \Delta P}{8 L I_V}$$

Using its definition, express the volume flow rate of the blood:

$$I_V = A_{cap} v = \pi r^2 v$$

Substitute and simplify:

$$\eta = \frac{r^2 \Delta P}{8Lv}$$

Substitute numerical values to obtain:

$$\eta = \frac{\left(3.5\times10^{-6}\,\text{m}\right)^2\left(2.60\,\text{kPa}\right)}{8\left(10^{-3}\,\text{m}\right)\left(\frac{10^{-3}\,\text{m}}{1\,\text{s}}\right)}$$

$$= \boxed{3.98\,\text{mPa}\cdot\text{s}}$$

***67** •

Picture the Problem We can use the definition of Reynolds number to find the velocity of a baseball at which the drag crisis occurs.

Using its definition, relate Reynolds number to the velocity v of the baseball:

$$N_R = \frac{2r\rho v}{\eta}$$

Solve for v:

$$v = \frac{\eta N_R}{2r\rho}$$

Substitute numerical values (see Figure 13-1 for the density of air and Table 13-1 for the coefficient of viscosity for air) and evaluate v:

$$v = \frac{\left(0.018\,\text{mPa}\cdot\text{s}\right)\left(3\times10^5\right)}{2\left(0.05\,\text{m}\right)\left(1.293\,\text{kg/m}^3\right)}$$

$$= 41.8\,\text{m/s}\times\frac{1\,\text{mi/h}}{0.447\,\text{m/s}}$$

$$= \boxed{93.4\,\text{mi/h}}$$

Because most major league pitchers can throw a fastball in the low - to mid - 90s, this drag crisis may very well play a role in the game.

Remarks: This is a topic which has been fiercely debated by people who study the physics of baseball.

68 •••

Picture the Problem Let the subscripts "f" refer to "displaced fluid", "s" to "soda", and "g" to the "gas" in the bubble. The free-body diagram shows the forces acting on the bubble prior to reaching its terminal velocity. We can apply Newton's 2nd law, Stokes' law, and Archimedes principle to express the terminal velocity of the bubble in terms of its radius, and the viscosity and density of water.

Apply $\sum F_y = ma_y$ to the bubble to obtain:

$$B - m_g g - F_D = ma_y$$

Under terminal speed conditions:

$$B - m_g g - F_D = 0$$

Using Archimedes principle, express the buoyant force B acting on the bubble:

$$B = w_f = m_f g$$
$$= \rho_f V_f g = \rho_s V_{bubble} g$$

Express the mass of the gas bubble:

$$m_g = \rho_g V_g = \rho_g V_{bubble}$$

Substitute to obtain:

$$\rho_w V_{bubble} g - \rho_g V_{bubble} g - 6\pi\eta a v_t = 0$$

Solve for v_t:

$$v_t = \frac{V_{bubble} g(\rho_s - \rho_g)}{6\pi\eta a}$$

Substitute for V_{bubble} and simplify:

$$v_t = \frac{\frac{4}{3}\pi a^3 g(\rho_s - \rho_g)}{6\pi\eta a} = \frac{2a^2 g(\rho_s - \rho_g)}{9\eta}$$
$$\approx \frac{2a^2 g\rho_s}{9\eta}, \text{ since } \rho_s >> \rho_g.$$

Substitute numerical values and evaluate v_t:

$$v_t = \frac{2(0.5\times10^{-3}\text{ m})^2(9.81\text{m/s}^2)}{9(1.8\times10^{-3}\text{ Pa}\cdot\text{s})} \times(1.1\times10^3\text{ kg/m}^3)$$
$$= \boxed{0.333\text{m/s}}$$

Express the rise time Δt in terms of the height of the soda glass h and the terminal speed of the bubble:

$$\Delta t = \frac{h}{v_t}$$

Assuming that a "typical" soda glass has a height of about 15 cm, evaluate Δt:

$$\Delta t = \frac{0.15\,\text{m}}{0.333\,\text{m/s}} = \boxed{0.450\,\text{s}}$$

Remarks: About half a second seems reasonable for the rise time of the bubble.

General Problems

***69** ••

Picture the Problem We can solve the given equation for the coefficient of roundness C and substitute estimates/assumptions of typical masses and heights for adult males and females.

Express the mass of a person as a function of C, ρ, and h:

$$M = C\rho h^3$$

Solve for C:

$$C = \frac{M}{\rho h^3}$$

Assuming that a "typical" adult male stands 5' 10" (1.78 m) and weighs 170 lbs (77 kg), then:

$$C = \frac{77\,\text{kg}}{\left(10^3\,\text{kg/m}^3\right)\left(1.78\,\text{m}\right)^3} = \boxed{0.0137}$$

Assuming that a "typical" adult female stands 5' 4" (1.63 m) and weighs 110 lbs (50 kg), then:

$$C = \frac{50\,\text{kg}}{\left(10^3\,\text{kg/m}^3\right)\left(1.63\,\text{m}\right)^3} = \boxed{0.0115}$$

70 •

Picture the Problem Let the letter "s" denote the shorter of the two men and the letter "t" the taller man. We can find the difference in weight of the two men using the relationship $M = C\rho h^3$ from Problem 69.

Express the difference in weight of the two men:

$$\Delta w = w_\text{t} - w_\text{s} = M_\text{t} g - M_\text{s} g$$
$$= \left(M_\text{t} - M_\text{s}\right)g$$

Express the masses of the two men:

$$M_\text{s} = C\rho h_\text{s}^3$$

and

$$M_\text{t} = C\rho h_\text{t}^3$$

Substitute to obtain:

$$\Delta w = \left(C\rho h_\text{t}^3 - C\rho h_\text{s}^3\right)g$$
$$= \left(h_\text{t}^3 - h_\text{s}^3\right)C\rho g$$

Assuming that a "typical" adult male stands 5' 10" (1.78 m) and weighs 170 lbs (77 kg), then:

$$C = \frac{77\,\text{kg}}{\left(10^3\,\text{kg/m}^3\right)\left(1.78\,\text{m}\right)^3} = 0.0137$$

Express the heights of the two men in SI units:

$$h_\text{t} = 72\,\text{in} \times 2.54\,\text{cm/in} = 1.83\,\text{m}$$

and

$$h_s = 69\,\text{in} \times 2.54\,\text{cm/in} = 1.75\,\text{m}$$

Substitute numerical values (assume that $\rho = 10^3\ \text{kg/m}^3$) and evaluate Δw:

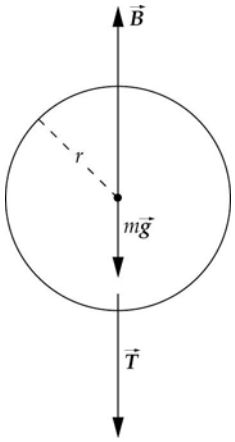
$$\begin{aligned}\Delta w &= \left[(1.83\,\text{m})^3 - (1.75\,\text{m})^3\right] \\ &\quad \times (0.0137)\left(10^3\ \text{kg/m}^3\right)\left(9.81\,\text{m/s}^2\right) \\ &= 103\,\text{N} \times \frac{1\,\text{lb}}{4.4482\,\text{N}} = \boxed{23.2\,\text{lb}}\end{aligned}$$

71 •

Determine the Concept The net force is zero. Neglecting the thickness of the table, the atmospheric pressure is the same above and below the surface of the table.

72 •

Picture the Problem The forces acting on the Ping-Pong ball, shown in the free-body diagram, are the buoyant force, the weight of the ball, and the tension in the string. Because the ball is in equilibrium under the influence of these forces, we can apply the condition for translational equilibrium to establish the relationship between them. We can also apply Archimedes' principle to relate the buoyant force on the ball to its diameter.



Apply $\sum F_y = 0$ to the ball:

$$B - mg - T = 0$$

Using Archimedes' principle, relate the buoyant force on the ball to its diameter:

$$B = w_\text{f} = m_\text{f} g = \rho_\text{w} V_\text{ball} g = \tfrac{1}{6}\pi\rho_\text{w} d^3$$

Substitute to obtain:

$$\tfrac{1}{6}\pi\rho_\text{w} d^3 - mg - T = 0$$

Solve for d:

$$d = \sqrt[3]{\frac{6(T + mg)}{\pi\rho_\text{w}}}$$

Substitute numerical values and evaluate d:

$$d = \sqrt[3]{\frac{6\left[2.8\times10^{-2}\,\text{N} + (0.004\,\text{kg})\left(9.81\,\text{m/s}^2\right)\right]}{\pi\left(10^3\ \text{kg/m}^3\right)}} = \boxed{5.05\,\text{cm}}$$

73 •

Picture the Problem Let ρ_0 represent the density of seawater at the surface. We can use the definition of density and the fact that mass is constant to relate the fractional change in the density of water to its fractional change in volume. We can also use the definition of bulk modulus to relate the fractional change in density to the increase in pressure with depth and solve the resulting equation for the density at the depth at which the pressure is 800 atm.

Using the definition of density, relate the mass of a given volume of seawater to its volume:

$$m = \rho V$$

Noting that the mass does not vary with depth, evaluate its differential:

$$\rho dV + V d\rho = 0$$

Solve for $d\rho/\rho$:

$$\frac{d\rho}{\rho} = -\frac{dV}{V} \text{ or } \frac{\Delta\rho}{\rho} \approx -\frac{\Delta V}{V}$$

Using the definition of the bulk modulus, relate ΔP to $\Delta\rho/\rho_0$:

$$B = -\frac{\Delta P}{\Delta V/V} = \frac{\Delta P}{\Delta\rho/\rho_0}$$

Solve $\Delta\rho$:

$$\Delta\rho = \rho - \rho_0 = \frac{\rho_0 \Delta P}{B}$$

Solve for ρ:

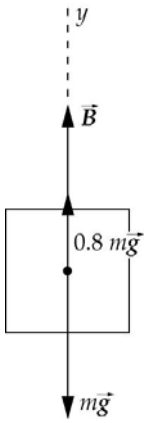
$$\rho = \rho_0 + \frac{\rho_0 \Delta P}{B} = \rho_0\left(1 + \frac{\Delta P}{B}\right)$$

Substitute numerical values and evaluate ρ:

$$\rho = \left(1025\,\text{kg/m}^3\right)\left(1 + \frac{800\,\text{atm} \times \dfrac{1.01\times10^5\,\text{Pa}}{1\,\text{atm}}}{2.3\times10^9\,\text{N/m}^2}\right) = \boxed{1061\,\text{kg/m}^3}$$

74 •

Picture the Problem When it is submerged, the block is in equilibrium under the influence of the buoyant force due to the water, the force exerted by the spring balance, and its weight. We can use the condition for translational equilibrium to relate the buoyant force to the weight of the block and the definition of density to express the weight of the block in terms of its density.



Apply $\sum F_y = 0$ to the block: $$B + 0.8mg - mg = 0 \Rightarrow B = 0.2mg$$

Substitute for B and m to obtain: $$\rho_{\text{w}} V_{\text{block}} g = 0.2\rho_{\text{block}} V_{\text{block}} g$$

Solve for and evaluate ρ_{block}: $$\rho_{\text{block}} = \frac{\rho_{\text{w}}}{0.2} = 5\left(10^3 \text{ kg/m}^3\right)$$
$$= \boxed{5.00\times 10^3 \text{ kg/m}^3}$$

***75** •

Picture the Problem When the copper block is floating on a pool of mercury, it is in equilibrium under the influence of its weight and the buoyant force acting on it. We can apply the condition for translational equilibrium to relate these forces. We can find the fraction of the block that is submerged by applying Archimedes' principle and the definition of density to express the forces in terms of the volume of the block and the volume of the displaced mercury. Let V represent the volume of the copper block, V' the volume of the displaced mercury. Then the fraction submerged when the material is floated on water is V'/V. Choose the upward direction to be the positive y direction.

Apply $\sum F_y = 0$ to the block: $B - w = 0$, where B is the buoyant force and w is the weight of the block.

Apply Archimedes' principle and the definition of density to obtain: $$\rho_{\text{Hg}} V' g - \rho_{\text{Cu}} V g = 0$$

Solve for V'/V: $$\frac{V'}{V} = \frac{\rho_{\text{Cu}}}{\rho_{\text{Hg}}}$$

Substitute numerical values and evaluate V'/V:

$$\frac{V'}{V}=\frac{8.93\times10^3\ \text{kg/m}^3}{13.6\times10^3\ \text{kg/m}^3}=0.657=\boxed{65.7\%}$$

76 •

Picture the Problem When the block is floating on a pool of ethanol, it is in equilibrium under the influence of its weight and the buoyant force acting on it. We can apply the condition for translational equilibrium to relate these forces. We can find the fraction of the block that is submerged by applying Archimedes' principle and the definition of density to express the forces in terms of the volume of the block and the volume of the displaced ethanol. Let V represent the volume of the copper block, V' the volume of the displaced ethanol. Then the fraction of the volume of the block that will be submerged when the material is floated on water is V'/V. Choose the upward direction to be the positive y direction.

Apply $\sum F_y=0$ to the block floating on ethanol:

$B_{\text{eth}}-w=0$, where B_{eth} is the buoyant force due to the ethanol and w is the weight of the block.

Apply Archimedes' principle to obtain:

$$w=\rho_{\text{eth}}(0.9V)g$$

Apply $\sum F_y=0$ to the block floating on water:

$B_{\text{w}}-w=0$, where B_{w} is the buoyant force due to the water and w is the weight of the block.

Apply Archimedes' principle to obtain:

$w=\rho_{\text{w}}V'g$, where V' is the volume of the displaced water.

Equate the two expressions for w and solve for V'/V:

$$\frac{V'}{V}=\frac{0.9\rho_{\text{eth}}}{\rho_{\text{w}}}$$

Substitute numerical values and evaluate V'/V:

$$\frac{V'}{V}=\frac{0.9\left(0.806\times10^3\ \text{kg/m}^3\right)}{10^3\ \text{kg/m}^3}$$

$$=0.725=\boxed{72.5\%}$$

77 •

Determine the Concept If you are floating, the density (or specific gravity) of the liquid in which you are floating is immaterial as you are in translational equilibrium under the influence of your weight and the buoyant force on your body. Thus, the buoyant force on your body is your weight in both (*a*) and (*b*).

78 •

Picture the Problem Let m and V represent the mass and volume of your body. Because you are in translational equilibrium when you are floating, we can apply the condition for translational equilibrium and Archimedes' principle to your body to express the dependence of the volume of water it displaces when it is fully submerged on your weight. Let the upward direction be the positive y direction.

Apply $\sum F_y = 0$ to your floating body: $$B - mg = 0$$

Use Archimedes' principle to relate the density of water to your volume: $$B = w_{\text{f}} = m_{\text{f}}g = \rho_{\text{w}}(0.96V)g$$

Substitute to obtain: $$\rho_{\text{w}}(0.96V)g - mg = 0$$

Solve for V: $$V = \boxed{\frac{m}{0.96\rho_{\text{w}}}}$$

79 ••

Picture the Problem Let m and V represent the mass and volume of the block of wood. Because the block is in translational equilibrium when it is floating, we can apply the condition for translational equilibrium and Archimedes' principle to express the dependence of the volume of water it displaces when it is fully submerged on its weight. We'll repeat this process for the situation in which the lead block is resting on the wood block with the latter fully submerged. Let the upward direction be the positive y direction.

Apply $\sum F_y = 0$ to floating block: $$B - mg = 0$$

Use Archimedes' principle to relate the density of water to the volume of the block of wood: $$B = w_{\text{f}} = m_{\text{f}}g = \rho_{\text{w}}(0.68V)g$$

Using the definition of density, express the weight of the block in terms of its density: $$mg = \rho_{\text{wood}}Vg$$

Substitute to obtain: $$\rho_{\text{w}}(0.68V)g - \rho_{\text{wood}}Vg = 0$$

Solve for and evaluate the density of the wood block:
$$\begin{aligned}\rho_{\text{wood}} &= 0.68\rho_{\text{w}} = 0.68\left(10^3\ \text{kg/m}^3\right)\\ &= 680\ \text{kg/m}^3\end{aligned}$$

Use the definition of density to find the volume of the wood:

$$V = \frac{m_{\text{wood}}}{\rho_{\text{wood}}} = \frac{1.5\,\text{kg}}{680\,\text{kg/m}^3}$$
$$= 2.206\times10^{-3}\,\text{m}^3$$

Apply $\sum F_y = 0$ to the floating block when the lead block is placed on it:

$B' - m'g = 0$, where B' is the new buoyant force on the block and m' is the combined mass of the wood block and the lead block.

Use Archimedes' principle and the definition of density to obtain:

$$\rho_{\text{w}}Vg - (m_{\text{Pb}} + m_{\text{block}})g = 0$$

Solve for the mass of the lead block:

$$m_{\text{Pb}} = \rho_{\text{w}}V - m_{\text{block}}$$

Substitute numerical values and evaluate m_{Pb}:

$$m_{\text{Pb}} = (10^3\,\text{kg/m}^3)(2.206\times10^{-3}\,\text{m}^3) - 1.5\,\text{kg}$$
$$= \boxed{0.706\ \text{kg}}$$

***80** ••

Picture the Problem The true mass of the Styrofoam cube is greater than that indicated by the balance due to the buoyant force acting on it. The balance is in rotational equilibrium under the influence of the buoyant and gravitational forces acting on the Styrofoam cube and the brass masses. Neglect the buoyancy of the brass masses. Let m and V represent the mass and volume of the cube and L the lever arm of the balance.

Apply $\sum \vec{\tau} = 0$ to the balance:

$$(mg - B)L - m_{\text{brass}}gL = 0$$

Use Archimedes' principle to express the buoyant force on the Styrofoam cube as a function of volume and density of the air it displaces:

$$B = \rho_{\text{air}}Vg$$

Substitute and simplify to obtain:

$$m - \rho_{\text{air}}V - m_{\text{brass}} = 0$$

Solve for m:

$$m = \rho_{\text{air}}V + m_{\text{brass}}$$

Substitute numerical values and evaluate m:

$$m = (1.293\,\text{kg/m}^3)(0.25\,\text{m})^3 + 20\times10^{-3}\,\text{kg}$$
$$= 4.02\times10^{-2}\,\text{kg} = \boxed{40.2\,\text{g}}$$

81 ••

Picture the Problem Let d_{in} and d_{out} represent the inner and outer diameters of the copper shell and V' the volume of the sphere that is submerged. Because the spherical shell is floating, it is in translational equilibrium and we can apply a condition for translational equilibrium to relate the buoyant force B due to the displaced water and its weight w.

Apply $\sum F_y = 0$ to the spherical shell: $B - w = 0$

Using Archimedes' principle and the definition of w, substitute to obtain:
$$\rho_w V' g - mg = 0$$
or
$$\rho_w V' - m = 0 \qquad (1)$$

Express V' as a function d_{out}:
$$V' = \frac{1}{2}\frac{\pi}{6}d_{out}^3 = \frac{\pi}{12}d_{out}^3$$

Express m in terms of d_{in} and d_{out}:
$$m = \rho_{Cu}\left(V_{out} - V_{in}\right) = \rho_{Cu}\left(\frac{\pi}{6}d_{out}^3 - \frac{\pi}{6}d_{in}^3\right)$$

Substitute in equation (1) to obtain:
$$\rho_w \frac{\pi}{12}d_{out}^3 - \rho_{Cu}\left(\frac{\pi}{6}d_{out}^3 - \frac{\pi}{6}d_{in}^3\right) = 0$$

Simplify:
$$\frac{1}{2}\rho_w d_{out}^3 - \rho_{Cu}\left(d_{out}^3 - d_{in}^3\right) = 0$$

Solve for d_{in}:
$$d_{in} = d_{out}\sqrt[3]{1 - \frac{\rho_w}{2\rho_{Cu}}}$$

Substitute numerical values and evaluate d_{in}:
$$d_{in} = (12\,\text{cm})\sqrt[3]{1 - \frac{1}{2(8.93)}} = \boxed{11.8\,\text{cm}}$$

82 ••

Determine the Concept The additional weight on the beaker side equals the weight of the displaced water, i.e., 64 g. This is the mass that must be placed on the other cup to maintain balance.

***83** ••

Picture the Problem We can use the definition of Reynolds number and assume a value for N_R of 1000 (well within the laminar flow range) to obtain a trial value for the radius

of the pipe. We'll then use Poiseuille's law to determine the pressure difference between the ends of the pipe that would be required to maintain a volume flow rate of 500 L/s.

Use the definition of Reynolds number to relate N_R to the radius of the pipe:

$$N_R = \frac{2r\rho v}{\eta}$$

Use the definition of I_V to relate the volume flow rate of the pipe to its radius:

$$I_V = Av = \pi r^2 v \Rightarrow v = \frac{I_V}{\pi r^2}$$

Substitute to obtain:

$$N_R = \frac{2\rho I_V}{\eta \pi r}$$

Solve for r:

$$r = \frac{2\rho I_V}{\eta \pi N_R}$$

Substitute numerical values and evaluate r:

$$r = \frac{2\left(700\,\text{kg/m}^3\right)\left(0.500\,\text{m}^3/\text{s}\right)}{\pi\left(0.8\,\text{Pa}\cdot\text{s}\right)(1000)} = 27.9\,\text{cm}$$

Using Poiseuille's law, relate the pressure difference between the ends of the pipe to its radius:

$$\Delta P = \frac{8\eta L}{\pi r^4} I_V$$

Substitute numerical values and evaluate ΔP:

$$\begin{aligned}\Delta P &= \frac{8\left(0.8\,\text{Pa}\cdot\text{s}\right)(50\,\text{km})}{\pi\left(0.279\,\text{m}\right)^4}\left(0.500\text{m}^3/\text{s}\right)\\ &= 8.41\times10^6\ \text{Pa}\\ &= 8.41\times10^6\ \text{Pa}\times\frac{1\,\text{atm}}{1.01325\times10^5\ \text{Pa}}\\ &= 83.0\,\text{atm}\end{aligned}$$

This pressure is too large to maintain in the pipe.

Evaluate ΔP for a pipe of 50 cm radius:

$$\begin{aligned}\Delta P &= \frac{8\left(0.8\,\text{Pa}\cdot\text{s}\right)(50\,\text{km})}{\pi\left(0.50\,\text{m}\right)^4}\left(0.500\,\text{m}^3/\text{s}\right)\\ &= 8.15\times10^5\ \text{Pa}\\ &= 8.15\times10^5\ \text{Pa}\times\frac{1\,\text{atm}}{1.01325\times10^5\ \text{Pa}}\\ &= 8.04\,\text{atm}\end{aligned}$$

1 m is a reasonable diameter for the pipeline.

84 ••

Picture the Problem We'll measure the height of the liquid–air interfaces relative to the centerline of the pipe. We can use the definition of the volume flow rate in a pipe to find the speed of the water at point A and the relationship between the gauge pressures at points A and C to determine the level of the liquid-air interface at A. We can use the continuity equation to express the speed of the water at B in terms of its speed at A and Bernoulli's equation for constant elevation to find the gauge pressure at B. Finally, we can use the relationship between the gauge pressures at points A and B to find the level of the liquid-air interface at B.

Relate the gauge pressure in the pipe at A to the height of the liquid-air interface at A:

$$P_{\text{gauge,A}} = \rho g h_{\text{A}}$$

where h_{A} is measured from the center of the pipe.

Solve for h_{A}:

$$h_{\text{A}} = \frac{P_{\text{gauge,A}}}{\rho g}$$

Substitute numerical values and evaluate h_{A}:

$$h_{\text{A}} = \frac{(1.22\,\text{atm})(1.01\times10^{5}\,\text{Pa/atm})}{(10^{3}\,\text{kg/m}^{3})(9.81\,\text{m/s}^{2})} = \boxed{12.6\,\text{m}}$$

Determine the velocity of the water at A:

$$v_{\text{A}} = \frac{I_{\text{V}}}{A_{\text{A}}} = \frac{0.8\times10^{-3}\,\text{m}^{3}/\text{s}}{\frac{\pi}{4}(0.02\,\text{m})^{2}} = 2.55\,\text{m/s}$$

Apply Bernoulli's equation for constant elevation to relate P_{B} and P_{A}:

$$P_{\text{A}} + \tfrac{1}{2}\rho v_{\text{A}}^{2} = P_{\text{B}} + \tfrac{1}{2}\rho v_{\text{B}}^{2} \qquad (1)$$

Use the continuity equation to relate v_{B} and v_{A}:

$$A_{\text{A}} v_{\text{A}} = A_{\text{B}} v_{\text{B}}$$

Solve for v_{B}:

$$v_{\text{B}} = \frac{A_{\text{A}}}{A_{\text{B}}} v_{\text{A}} = \frac{d_{\text{A}}^{2}}{d_{\text{B}}^{2}} v_{\text{A}} = \frac{(2\,\text{cm})^{2}}{(1\,\text{cm})^{2}} v_{\text{A}} = 4v_{\text{A}}$$

Substitute in equation (1) to obtain: $P_A + \frac{1}{2}\rho v_A^2 = P_B + 8\rho v_A^2$

Solve for P_B: $P_B = P_A - \frac{15}{2}\rho v_A^2$

Substitute numerical values and evaluate P_B:

$$\begin{aligned} P_B &= (2.22\,\text{atm})(1.01\times10^5\ \text{Pa/atm}) \\ &\quad -\tfrac{15}{2}(10^3\ \text{kg/m}^3)(2.55\,\text{m/s})^2 \\ &= 1.75\times10^5\ \text{Pa} \\ &= 1.75\times10^5\ \text{Pa}\times\frac{1\,\text{atm}}{1.01\times10^5\ \text{Pa}} \\ &= 1.733\,\text{atm} \end{aligned}$$

Relate the gauge pressure in the pipe at B to the height of the liquid-air interface at B: $P_{\text{gauge,B}} = \rho g h_B$

Solve for h_B: $h_B = \dfrac{P_{\text{gauge,B}}}{\rho g}$

Substitute numerical values and evaluate h_B:

$$\begin{aligned} h_B &= \frac{[(1.733-1)\text{atm}]\left(1.01\times10^5\ \frac{\text{Pa}}{\text{atm}}\right)}{(10^3\ \text{kg/m}^3)(9.81\,\text{m/s}^2)} \\ &= \boxed{7.55\,\text{m}} \end{aligned}$$

85 ••

Picture the Problem We'll measure the height of the liquid–air interfaces relative to the centerline of the pipe. We can use the definition of the volume flow rate in a pipe to find the speed of the water at point A and the relationship between the gauge pressures at points A and C to determine the level of the liquid-air interface at A. We can use the continuity equation to express the speed of the water at B in terms of its speed at A and Bernoulli's equation for constant elevation to find the gauge pressure at B. Finally, we can use the relationship between the gauge pressures at points A and B to find the level of the liquid-air interface at B.

Relate the gauge pressure in the pipe at A to the height of the liquid-air interface at A: $P_{\text{gauge,A}} = \rho g h_A$ where h_A is measured from the center of the pipe.

Solve for h_A: $h_A = \dfrac{P_{\text{gauge,A}}}{\rho g}$

Substitute numerical values and evaluate h_A:

$$h_A = \frac{(1.22\,\text{atm})(1.01\times10^5\,\text{Pa/atm})}{(10^3\,\text{kg/m}^3)(9.81\,\text{m/s}^2)}$$
$$= \boxed{12.6\,\text{m}}$$

Determine the velocity of the water at A:

$$v_A = \frac{I_V}{A_A} = \frac{0.6\times10^{-3}\,\text{m}^3/\text{s}}{\frac{\pi}{4}(0.02\,\text{m})^2} = 1.91\,\text{m/s}$$

Use the continuity equation to relate v_B and v_A:

$$A_A v_A = A_B v_B$$

Solve for v_B:

$$v_B = \frac{A_A}{A_B} v_A = \frac{d_A^2}{d_B^2} v_A = \frac{(2\,\text{cm})^2}{(1\,\text{cm})^2} v_A$$
$$= 4v_A$$

Apply Bernoulli's equation for constant elevation to relate P_B and P_A:

$$P_A + \tfrac{1}{2}\rho v_A^2 = P_B + \tfrac{1}{2}\rho v_B^2 \qquad (1)$$

Substitute in equation (1) to obtain:

$$P_A + \tfrac{1}{2}\rho v_A^2 = P_B + 8\rho v_A^2$$

Solve for P_B:

$$P_B = P_A - \tfrac{15}{2}\rho v_A^2$$

Substitute numerical values and evaluate P_B:

$$P_B = (2.22\,\text{atm})(1.01\times10^5\,\text{Pa/atm}) - \tfrac{15}{2}(10^3\,\text{kg/m}^3)(1.91\,\text{m/s})^2$$
$$= 1.969\times10^5\,\text{Pa}$$
$$= 1.969\times10^5\,\text{Pa}\times\frac{1\,\text{atm}}{1.01\times10^5\,\text{Pa}}$$
$$= 1.95\,\text{atm}$$

Relate the gauge pressure in the pipe at B to the height of the liquid-air interface at B:

$$P_{\text{gauge,B}} = \rho g h_B$$

Solve for h_B:

$$h_B = \frac{P_{\text{gauge,B}}}{\rho g}$$

Substitute numerical values and evaluate h_B:

$$h_B = \frac{[(1.95-1)\text{atm}]\left(1.01\times10^5\,\frac{\text{Pa}}{\text{atm}}\right)}{(10^3\,\text{kg/m}^3)(9.81\,\text{m/s}^2)}$$
$$= \boxed{9.78\,\text{m}}$$

***86** ••

Picture the Problem Because it is not given, we'll neglect the difference in height between the centers of the pipes at A and B. We can use the definition of the volume flow rate to find the speed of the water at A and Bernoulli's equation for constant elevation to find its speed at B. Once we know the speed of the water at B, we can use the equation of continuity to find the diameter of the constriction at B.

Use the definition of the volume flow rate to find v_A:

$$v_A = \frac{I_V}{A_A} = \frac{0.5\times10^{-3}\,\text{m}^3/\text{s}}{\frac{\pi}{4}(0.02\,\text{m})^2} = 1.59\,\text{m/s}$$

Use Bernoulli's equation for constant elevation to relate the pressures and velocities at A and B:

$$P_B + \tfrac{1}{2}\rho v_B^2 = P_A + \tfrac{1}{2}\rho v_A^2$$

Solve for v_B^2:

$$v_B^2 = \frac{2(P_A - P_B)}{\rho} + v_A^2$$

Substitute numerical values and evaluate v_B^2:

$$v_B^2 = \frac{2[(1.187-0.1)\text{atm}(1.01\times10^5\,\text{Pa/atm})]}{10^3\,\text{kg/m}^3} + (1.59\,\text{m/s})^2 = 222\,\text{m}^2/\text{s}^2$$

Using the continuity equation, relate the volume flow rate to the radius at B:

$$I_V = A_B v_B = \pi r_B^2 v_B$$

Solve for and evaluate r_B:

$$r_B = \sqrt{\frac{I_V}{\pi v_B}} = \sqrt{\frac{0.5\times10^{-3}\,\text{m}^3/\text{s}}{\pi(14.9\,\text{m/s})}} = 3.27\,\text{mm}$$

and

$$d_B = 2r_B = \boxed{6.54\,\text{mm}}$$

87 ••

Picture the Problem Let V' represent the volume of the buoy that is submerged and h' the height of the submerged portion of the cylinder. We can find the fraction of the cylinder's volume that is submerged by applying the condition for translational equilibrium to the buoy and using Archimedes' principle. When the buoy is submerged it is in equilibrium under the influence of the tension T in the cable, the buoyant force due to the displaced water, and its weight. When the cable breaks, the net force acting on the buoy will accelerate it and we can use Newton's 2nd law to find its acceleration.

(*a*) Apply $\sum F_y = 0$ to the cylinder:

$$B - w = 0$$

Using Archimedes' principle and the definition of weight, substitute for B and w:

$$\rho_{sw} V' g - mg = 0$$

or

$$\rho_{sw} h' A g - mg = 0$$

where A is the cross-sectional area of the buoy.

Solve for and evaluate h':

$$h' = \frac{m}{\rho_{sw} A}$$

Substitute numerical values and evaluate h':

$$h' = \frac{600\,\text{kg}}{\left(1.025\times10^3\,\text{kg/m}^3\right)\frac{\pi}{4}(0.9\,\text{m})^2}$$

$$= 0.920\,\text{m}$$

Use h' to find the height h of the buoy:

$$h - h' = 2.6\,\text{m} - 0.920\,\text{m} = \boxed{1.68\,\text{m}}$$

Express the fraction of the volume of the cylinder that is above water:

$$\frac{V - V'}{V} = 1 - \frac{V'}{V} = 1 - \frac{\frac{\pi}{4}d^2 h'}{\frac{\pi}{4}d^2 h}$$

$$= 1 - \frac{h'}{h}$$

Substitute numerical values to obtain:

$$\frac{V - V'}{V} = 1 - \frac{0.920\,\text{m}}{2.6\,\text{m}} = \boxed{64.6\%}$$

(*b*) Apply $\sum F_y = 0$ to the submerged buoy:

$$B - T - w = 0$$

Solve for T and substitute for B and w to obtain:

$$T = B - w = \rho_{sw} Vg - mg$$
$$= (\rho_{sw} V - m)g$$

Substitute numerical values and evaluate T:

$$T = \left[(1.025\times10^3\ \text{kg/m}^3)\frac{\pi}{4}(0.9\,\text{m})^2(2.6\,\text{m}) - 600\,\text{kg}\right](9.81\,\text{m/s}^2)$$
$$= \boxed{10.7\,\text{kN}}$$

(*c*) Apply $\sum F_y = 0$ to the buoy:

$$B - w = ma$$

Substitute for $B - w$ and solve for a to obtain:

$$a = \frac{B-w}{m} = \frac{T}{m}$$

Substitute numerical values and evaluate a:

$$a = \frac{10.75\ \text{kN}}{600\,\text{kg}} = \boxed{17.9\,\text{m/s}^2}$$

88 ••

Picture the Problem Because the floating object is in equilibrium under the influence of the buoyant force acting on it and its weight; we can apply the condition for translational equilibrium to relate B and w. Let Δh represent the change in elevation of the liquid level and V_f the volume of the displaced fluid.

Apply $\sum F_y = 0$ to the floating object:

$$B - w = 0$$

Using Archimedes' principle and the definition of weight, substitute for B and w:

$$\rho_0 g V_f - mg = 0$$

The volume of fluid displaced is the sum of the volume displaced in the two vessels:

$$V_f = \Delta V_A + \Delta V_{3A} = A\Delta h + 3A\Delta h$$
$$= 4A\Delta h$$

Substitute for V_f to obtain:

$$4\rho_0 gA\Delta h - mg = 0$$

Solve for Δh:

$$\Delta h = \boxed{\frac{m}{4A\rho_0}}$$

89 ••

Picture the Problem We can calculate the smallest pressure change ΔP that can be detected from the reading Δh from $\Delta P = \rho g \Delta h$.

Express and evaluate the pressure difference between the two columns of the manometer:

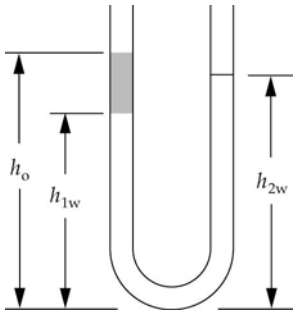
$$\begin{aligned}\Delta P &= \rho g \Delta h \\ &= \left(900\,\text{kg/m}^3\right)\left(9.81\,\text{m/s}^2\right) \\ &\quad \times \left(0.05\times10^{-3}\,\text{m}\right) \\ &= 0.4415\,\text{Pa}\end{aligned}$$

Express this pressure in mmHg and μmHg:

$$\begin{aligned}\Delta P &= 0.4415\,\text{Pa} \times \frac{1\,\text{atm}}{1.01325\times10^5\,\text{Pa}} \\ &\quad \times \frac{760\,\text{mmHg}}{1\,\text{atm}} \\ &= \boxed{3.31\times10^{-3}\,\text{mmHg}} \\ &= \boxed{3.31\,\mu\text{mHg}}\end{aligned}$$

90 ••

Picture the Problem We can use the equality of the pressure at the bottom of the U-tube due to the water on one side and that due to the oil and water on the other to relate the various heights. Let h represent the height of the oil above the water. Then $h_o = h_{1w} + h$.



Using the constancy of the amount of water, express the relationship between h_{1w} and h_{2w}:

$$h_{1w} + h_{2w} = 56\text{ cm}$$

Find the height of the oil-water interface:

$$h_{1w} = 56\,\text{cm} - 34\,\text{cm} = \boxed{22.0\,\text{cm}}$$

Express the equality of the pressure at the bottom of the two arms of the U tube:

$$\rho_w g(34\,\text{cm}) = \rho_w g(22\,\text{cm}) + 0.78\rho_w g h_{oil}$$

Solve for and evaluate h_{oil}:

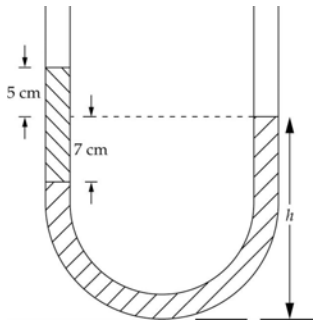
$$h_{oil} = \frac{\rho_w g(34\,\text{cm}) - \rho_w g(22\,\text{cm})}{0.78\rho_w g}$$
$$= \frac{(34\,\text{cm}) - (22\,\text{cm})}{0.78} = 15.4\,\text{cm}$$

Find the height of the air-oil interface h_o:

$$h_o = 22\text{cm} + 15.4\,\text{cm} = \boxed{37.4\,\text{cm}}$$

91 ••

Picture the Problem Let σ_L represent the specific gravity of the liquid. The specific gravity of the oil is $\sigma_o = 0.8$. We can use the equality of the pressure at the bottom of the U-tube due to the water on one side and that due to the oil and water on the other to relate the various heights.



Express the equality of the pressure at the bottom of the two arms of the U tube:

$$\sigma_L g h = \sigma_L g(h - 7\,\text{cm}) + 0.8\sigma_w g(12\,\text{cm})$$

Solve for and evaluate σ_L:

$$\sigma_L = \frac{0.8\sigma_w(12\,\text{cm})}{7\,\text{cm}} = \frac{0.8(12\,\text{cm})}{7\,\text{cm}}$$
$$= \boxed{1.37}$$

92 ••

Picture the Problem The block of wood is in translational equilibrium under the influence of the buoyant force due to the displaced water acting on it and on the lead block, its weight, and the weight of the lead block. We can use a condition for translational equilibrium and Archimedes' principle to obtain a relationship between the mass of the lead block and the densities of water, wood, and lead and the mass of the wood block.

Apply $\sum F_y = 0$ to the block of wood:

$$B_{wood} + B_{Pb} - m_{wood}g - m_{Pb}g = 0$$

Use Archimedes' principle to express the buoyant force on the block of wood:

$$B_{wood} = \rho_w V_{wood} g$$

Use Archimedes' principle to

$$B_{Pb} = \rho_w V_{Pb} g$$

express the buoyant force on the lead block:

Substitute and simplify to obtain: $$\rho_w V_{wood} + \rho_w V_{Pb} - m_{wood} - m_{Pb} = 0$$

Express the volume of the wood block in terms of its density and mass: $$V_{wood} = \frac{m_{wood}}{\rho_{wood}}$$

Express the volume of the lead block in terms of its density and mass: $$V_{Pb} = \frac{m_{Pb}}{\rho_{Pb}}$$

Substitute for V_{wood} and V_{Pb}: $$\rho_w \frac{m_{wood}}{\rho_{wood}} + \rho_w \frac{m_{Pb}}{\rho_{Pb}} - m_{wood} - m_{Pb} = 0$$

Solve for m_{Pb}: $$m_{Pb} = \frac{\left(\frac{\rho_w}{\rho_{wood}} - 1\right) m_{wood}}{1 - \frac{\rho_w}{\rho_{Pb}}}$$

Substitute numerical values and evaluate m_{Pb}: $$m_{Pb} = \frac{\left(\frac{1}{0.7} - 1\right)(0.5\,\text{kg})}{1 - \frac{1}{11.3}} = \boxed{0.235\,\text{kg}}$$

***93** ••

Picture the Problem Because the balloon is in equilibrium under the influence of the buoyant force exerted by the air, the weight of its basket and load w, the weight of the skin of the balloon, and the weight of the helium. Choose upward to be the positive y direction and apply the condition for translational equilibrium to relate these forces. Archimedes' principle relates the buoyant force on the balloon to the density of the air it displaces and the volume of the balloon.

(*a*) Apply $\sum F_y = 0$ to the balloon: $$B - m_{skin}g - m_{He}g - w = 0$$

Letting V represent the volume of the balloon, use Archimedes' principle to express the buoyant force: $$\rho_{air} V g - m_{skin}g - m_{He}g - w = 0$$

Substitute for m_{He}: $$\rho_{air} V g - m_{skin}g - \rho_{He} V g - w = 0$$

Solve for V: $$V = \frac{m_{\text{skin}}g + w}{(\rho_{\text{air}} - \rho_{\text{He}})g}$$

Substitute numerical values and evaluate V: $$V = \frac{(1.5\,\text{kg})(9.81\,\text{m/s}^2) + 750\,\text{N}}{(1.293 - 0.1786)(\text{kg/m}^3)(9.81\,\text{m/s}^2)}$$
$$= \boxed{70.0\,\text{m}^3}$$

(*b*) Apply $\sum F_y = ma$ to the balloon: $$B - m_{\text{tot}}g = m_{\text{tot}}a$$

Solve for a: $$a = \frac{B}{m_{\text{tot}}} - g$$

Assuming that the mass of the skin has not changed and letting V' represent the doubled volume of the balloon, express m_{tot}: $$m_{\text{tot}} = m_{\text{load}} + m_{\text{He}} + m_{\text{skin}}$$
$$= \frac{w_{\text{load}}}{g} + \rho_{\text{He}}V' + m_{\text{skin}}$$

Substitute numerical values and evaluate m_{tot}:

$$m_{\text{tot}} = \frac{900\,\text{N}}{9.81\,\text{m/s}^2} + (0.1786\,\text{kg/m}^3)(140\,\text{m}^3) + 1.5\,\text{kg} = 118\,\text{kg}$$

Express the buoyant force acting on the balloon: $$B = w_{\text{displaced fluid}} = \rho_{\text{air}}V'g$$

Substitute numerical values and evaluate B: $$B = (1.293\,\text{kg/m}^3)(140\,\text{m}^3)(9.81\,\text{m/s}^2)$$
$$= 1.78\,\text{kN}$$

Substitute and evaluate a: $$a = \frac{1.78\,\text{kN}}{118\,\text{kg}} - 9.81\,\text{m/s}^2 = \boxed{5.27\,\text{m/s}^2}$$

94 ••

Picture the Problem When the hollow sphere is completely submerged but floating, it is in translational equilibrium under the influence of a buoyant force and its weight. The buoyant force is given by Archimedes' principle and the weight of the sphere is the sum of the weights of the hollow sphere and the material filling its center.

Apply $\sum F_y = 0$ to the hollow sphere: $$B - w = 0$$

Express the buoyant force acting on the hollow sphere:

$$B = 2\rho_0 V_{\text{sphere}} g = 2\rho_0 \left[\tfrac{4}{3}\pi (2R)^3\right] g$$
$$= \tfrac{64}{3}\rho_0 \pi R^3 g$$

Express the weight of the sphere when it's hollow is filled with a material of density ρ':

$$w = \rho_0 V_{\text{hollow sphere}} g + \rho' V_{\text{hollow}} g$$
$$= \rho_0 \left[\tfrac{4}{3}\pi \left\{(2R)^3 - R^3\right\}\right] g + \rho' \left[\tfrac{4}{3}\pi R^3\right] g$$
$$= \tfrac{28}{3}\rho_0 \pi R^3 g + \tfrac{4}{3}\rho' \pi R^3 g$$

Substitute to obtain:

$$\tfrac{64}{3}\rho_0 \pi R^3 g - \tfrac{28}{3}\rho_0 \pi R^3 g - \tfrac{4}{3}\rho' \pi R^3 g = 0$$

Solve for ρ':

$$\rho' = \boxed{9\rho_0}$$

***95** ••

Picture the Problem We can differentiate the function $P(h)$ to show that it satisfies the differential equation $dP/P = -C\,dh$ and in part (*b*) we can use the approximation $e^{-x} \approx 1 - x$ and $\Delta h << h_0$ to establish the given result.

(*a*) Differentiate $P(h) = P_0 e^{-Ch}$:

$$\frac{dP}{dh} = -CP_0 e^{-Ch}$$
$$= -CP$$

Separate variables to obtain:

$$\boxed{\frac{dP}{P} = -C\,dh}$$

(*b*) Express $P(h + \Delta h)$:

$$P(h + \Delta h) = P_0 e^{-C(h+\Delta h)}$$
$$= P_0 e^{-Ch} e^{-C\Delta h}$$
$$= P(h) e^{-C\Delta h}$$

For $\Delta h << h_0$:

$$\frac{\Delta h}{h_0} << 1$$

Let $h_0 = 1/C$. Then:

$$C\Delta h << 1$$

and

$$e^{-C\Delta h} \approx 1 - C\Delta h = 1 - \frac{\Delta h}{h_0}$$

Substitute to obtain:

$$P(h + \Delta h) = \boxed{P(h)\left(1 - \frac{\Delta h}{h_0}\right)}$$

(*c*) Take the logarithm of both sides of the function $P(h)$:

$$\ln P = \ln P_0 e^{-Ch} = \ln P_0 + \ln e^{-Ch}$$
$$= \ln P_0 - Ch$$

Solve for C:

$$C = \frac{1}{h}\ln\left(\frac{P_0}{P}\right)$$

Substitute numerical values and evaluate C:

$$C = \frac{1}{5.5\,\text{km}}\ln\left(\frac{P_0}{\frac{1}{2}P_0}\right) = \frac{1}{5.5\,\text{km}}\ln 2$$
$$= \boxed{0.126\,\text{km}^{-1}}$$

96 ••

Picture the Problem Let V represent the volume of the submarine and V' the volume of seawater it displaces when it is on the surface. The submarine is in equilibrium in both parts of the problem. Hence we can apply the condition for translational equilibrium (neutral buoyancy) to the submarine to relate its weight to the buoyant force acting on it. We'll also use Archimedes' principle to connect the buoyant forces to the volume of seawater the submarine displaces. Let upward be the positive y direction.

(*a*) Express f, the fraction of the submarine's volume above the surface when the tanks are filled with air:

$$f = \frac{V - V'}{V} = 1 - \frac{V'}{V} \qquad (1)$$

Apply $\sum F_y = 0$ to the submarine when its tanks are full of air:

$$B - w = 0$$

Use Archimedes' principle to express the buoyant force on the submarine in terms of the volume of the displaced water:

$$B = \rho_{\text{sw}} V' g$$

Substitute and solve for V':

$$V' = \frac{m}{\rho_{\text{sw}}}$$

Substitute in equation (1) to obtain:

$$f = 1 - \frac{m}{\rho_{\text{sw}} V}$$

Substitute numerical values and evaluate f:

$$f = 1 - \frac{2.4\times10^{6}\ \text{kg}}{(1.025\times10^{3}\ \text{kg/m}^3)(2.4\times10^{3}\ \text{m}^3)}$$
$$= 2.44\times10^{-2} = \boxed{2.44\%}$$

(b) Express the volume of seawater in terms of its mass and density:

$$V_{\text{sw}} = \frac{m_{\text{sw}}}{\rho_{\text{sw}}} \qquad (2)$$

Apply $\sum F_y = 0$, the condition for neutral buoyancy, to the submarine:

$$B - w_{\text{sub}} - w_{\text{sw}} = 0$$

Use Archimedes' principle to express the buoyant force on the submarine in terms of the volume of the displaced water:

$$B = \rho_{\text{sw}} V g$$

Substitute to obtain:

$$\rho_{\text{sw}} V g - m_{\text{sub}} g - m_{\text{sw}} g = 0$$

Solve for m_{sw}:

$$m_{\text{sw}} = \rho_{\text{sw}} V - m_{\text{sub}}$$

Substitute for V_{sw} in equation (2) to obtain:

$$V_{\text{sw}} = \frac{\rho_{\text{sw}} V - m_{\text{sub}}}{\rho_{\text{sw}}} = V - \frac{m_{\text{sub}}}{\rho_{\text{sw}}}$$

Substitute numerical values and evaluate V_{sw}:

$$V_{\text{sw}} = 2.4\times10^{3}\ \text{m}^3 - \frac{2.4\times10^{6}\ \text{kg}}{1.025\times10^{3}\ \text{kg/m}^3}$$
$$= \boxed{58.5\ \text{m}^3}$$

97 ••

Picture the Problem While the loaded crate is under the surface, it is in equilibrium under the influence of the tension in the cable, the buoyant force acting on the gold, and the gravitational force acting on the gold. The empty crate has neutral buoyancy. When the crate is out of the water, the buoyant force of the air is negligible and the tension in the cable is the sum of the weights of the crate, the gold bullion, and the seawater.

(a) Apply $\sum F_y = 0$ to the crate while it is below the surface:

$$T + B_{\text{Au}} - w_{\text{Au}} = 0$$

Solve for the tension in the cable:

$$T = w_{\text{Au}} - B_{\text{Au}}$$

Using Archimedes' principle, relate the buoyant force acting on the gold

$$B_{\text{Au}} = \rho_{\text{sw}} V_{\text{Au}} g$$

to its density and volume:

Substitute for B_{Au} and simplify to obtain: $T = (\rho_{\text{Au}} - \rho_{\text{sw}})V_{\text{Au}}g$

Substitute numerical values and evaluate T:

$$T = \left[\left(19.3\times10^3\ \text{kg/m}^3\right) - \left(1.025\times10^3\ \text{kg/m}^3\right)\right](0.36)(1.4\,\text{m})(0.75\,\text{m})(0.5\,\text{m})\left(9.81\,\text{m/s}^2\right)$$
$$= \boxed{33.9\,\text{kN}}$$

(*b*) 1. Apply $\sum F_y = 0$ to the crate while it is being lifted to the deck of the ship with none of the seawater leaking out: $T - w_{\text{Au}} - w_{\text{crate}} - w_{\text{sw}} = 0$

Substitute for the weights of the gold, crate, and seawater and solve for the tension in the cable and express:

$$\begin{aligned} T &= w_{\text{Au}} + w_{\text{crate}} + w_{\text{sw}} \\ &= \rho_{\text{Au}}V_{\text{Au}}g + m_{\text{crate}}g + \rho_{\text{sw}}V_{\text{sw}}g \\ &= (\rho_{\text{Au}}V_{\text{Au}} + m_{\text{crate}} + \rho_{\text{sw}}V_{\text{sw}})g \end{aligned}$$

Substitute numerical values and evaluate T:

$$T = \left[\left(19.3\times10^3\ \text{kg/m}^3\right)(0.36)(1.4\,\text{m})(0.75\,\text{m})(0.5\,\text{m}) + 32\,\text{kg} + \left(1.025\times10^3\ \text{kg/m}^3\right)\right.$$
$$\left.\times(0.64)(1.4\,\text{m})(0.75\,\text{m})(0.5\,\text{m})\right]\left(9.81\,\text{m/s}^2\right)$$
$$= \boxed{39.8\,\text{kN}}$$

2. With the seawater term missing, the expression for the tension is:

$$\begin{aligned} T &= w_{\text{Au}} + w_{\text{crate}} \\ &= \rho_{\text{Au}}V_{\text{Au}}g + m_{\text{crate}}g \\ &= (\rho_{\text{Au}}V_{\text{Au}} + m_{\text{crate}})g \end{aligned}$$

Substitute numerical values and evaluate T:

$$T = \left[\left(19.3\times10^3\ \text{kg/m}^3\right)(0.36)(1.4\,\text{m})(0.75\,\text{m})(0.5\,\text{m}) + 32\,\text{kg}\right]\left(9.81\,\text{m/s}^2\right) = \boxed{36.1\,\text{kN}}$$

98 •••

Picture the Problem In the three situations described in the problem the hydrometer will be in equilibrium under the influence of its weight and the buoyant force exerted by the liquids. We can use Archimedes' principle to relate the buoyant force acting on the hydrometer to the density of the liquid in which it is floating and to its weight.

(*a*) Find the volume of the bulb: $V_{\text{bulb}} = \frac{1}{6}\pi d^3 = \frac{1}{6}\pi(2.4\,\text{cm})^3 = 7.238\,\text{cm}^3$

Find the volume of the tube:

$$V_{\text{tube}} = \tfrac{1}{4}\pi d^2 L = \tfrac{1}{4}\pi(0.75\,\text{cm})^2(20\,\text{cm}) = 8.836\,\text{cm}^2$$

Apply $\sum F_y = 0$ to the hydrometer just floating in the liquid:

$$B - w_{\text{hyd}} - m_{\text{Pb}}g = 0$$

Substitute for B and w_{glass}:

$$\rho_{\text{liq}}V_{\text{hyd}}g - m_{\text{glass}}g - m_{\text{Pb}}g = 0$$

Solve for m_{Pb}:

$$m_{\text{Pb}} = \rho_{\text{liq}}V_{\text{hyd}} - m_{\text{hyd}}$$

Substitute numerical values and evaluate m_{Pb}:

$$m_{\text{Pb}} = 0.78(1\,\text{g/cm}^3) \times (7.238\,\text{cm}^3 + 8.836\,\text{cm}^3) - 7.28\,\text{g} = \boxed{5.26\,\text{g}}$$

(*b*) Letting V represent the volume of the hydrometer that is submerged, apply $\sum F_y = 0$ to the hydrometer just floating in the liquid:

$$\rho_{\text{w}}Vg - mg = 0$$

Solve for V:

$$V = \frac{m}{\rho_{\text{w}}} = \frac{m_{\text{hyd}} + m_{\text{Pb}}}{\rho_{\text{w}}}$$

Substitute numerical values and evaluate V:

$$V = \frac{7.28\,\text{g} + 5.26}{1\,\text{g/cm}^3} = 12.54\,\text{cm}^3$$

Relate the volume of the hydrometer that is submerged to the volume of the bulb and the volume of the tube that is submerged:

$$V = \tfrac{1}{4}\pi d_{\text{tube}}^2 h' + V_{\text{bulb}}$$

Solve for h':

$$h' = \frac{V - V_{\text{bulb}}}{\frac{1}{4}\pi d_{\text{tube}}^2}$$

Substitute numerical values and evaluate h':

$$h' = \frac{12.54\,\text{cm}^3 - 7.238\,\text{cm}^3}{\frac{1}{4}\pi(0.75\,\text{cm})^2} = 12.0\,\text{cm}$$

Find the length of the tube that shows above the surface of the water:

$$h = 20\,\text{cm} - h' = 20\,\text{cm} - 12.0\,\text{cm}$$
$$= \boxed{8.00\,\text{cm}}$$

(*c*) Apply $\sum F_y = 0$ to the hydrometer floating in the liquid of unknown specific gravity:

$$\rho_\text{L} V_\text{L} g - m_\text{hyd} g = 0$$

Solve for the density of the liquid:

$$\rho_\text{L} = \frac{m_\text{hyd}}{V_\text{L}}$$

Express the volume of the displaced liquid:

$$V_\text{L} = V_\text{bulb} + \tfrac{1}{4}\pi d_\text{tube}^2 h'$$

Substitute numerical values and evaluate V_L:

$$V_\text{L} = 7.238\,\text{cm}^3 + \tfrac{1}{4}\pi(0.75\,\text{cm})^2 \times (20\,\text{cm} - 12.2\,\text{cm})$$
$$= 10.68\,\text{cm}^3$$

Substitute for V_L and m_hyd and evaluate ρ_L:

$$\rho_\text{L} = \frac{12.54\,\text{g}}{10.68\,\text{cm}^3} = 1.174\,\text{g/cm}^3$$

Express and evaluate the specific gravity of the liquid:

$$\text{specific gravity}_\text{liquid} = \frac{\rho_\text{w}}{\rho_\text{L}} = \boxed{1.17}$$

99 •••

Picture the Problem We can apply Bernoulli's equation to the top of the keg and to the spigot opening to determine the rate at which the root beer exits the tank. Because the area of the spigot is much smaller than that of the keg, we can neglect the velocity of the root beer at the top of the keg. We'll use the continuity equation to obtain an expression for the rate of change of the height of the root beer in the keg as a function of the its height and integrate this function to find *h* as a function of time.

(*a*) Apply Bernoulli's equation to the beer at the top of the keg and at the spigot:

$$P_1 + \rho_\text{beer} g h + \tfrac{1}{2}\rho_\text{beer} v_1^2 = P_2 + \rho_\text{beer} g h_2 + \tfrac{1}{2}\rho_\text{beer} v_2^2$$

or, because $v_1 \approx 0$, $h_2 = 0$, $P_1 = P_2 = P_\text{at}$, and $h_1 = h$,

$$gh = \tfrac{1}{2} v_2^2$$

Solve for v_2:

$$v_2 = \boxed{\sqrt{2gh}}$$

(*b*) Use the continuity equation to relate v_1 and v_2:

$$A_1 v_1 = A_2 v_2$$

Substitute $-dh/dt$ for v_1:

$$-A_1 \frac{dh}{dt} = A_2 v_2$$

Substitute for v_2 and solve for dh/dt to obtain:

$$\frac{dh}{dt} = \boxed{-\frac{A_2}{A_1}\sqrt{2gh}}$$

(*c*) Separate the variables in the differential equation:

$$-\frac{A_1/A_2}{\sqrt{2g}}\frac{dh}{\sqrt{h}} = dt$$

Express the integral from $h' = H$ to h and $t' = 0$ to t:

$$-\frac{A_1/A_2}{\sqrt{2g}}\int_H^h \frac{dh'}{\sqrt{h'}} = \int_0^t dt'$$

Evaluate the integral to obtain:

$$-\frac{A_1/A_2}{\sqrt{2g}}\left(\sqrt{H} - \sqrt{h}\right) = t$$

Solve for h:

$$h = \boxed{\left(\sqrt{H} - \frac{A_2}{2A_1}\sqrt{2g}\,t\right)^2}$$

(*d*) Solve $h(t)$ for the time-to-drain t':

$$t' = \frac{A_1}{A_2}\sqrt{\frac{2H}{g}}$$

Substitute numerical values and evaluate t'

$$t' = \frac{A_1}{10^{-4}A_1}\sqrt{\frac{2(2\,\text{m})}{9.81\,\text{m/s}^2}} = 6.39\times10^3\,\text{s}$$

$$= \boxed{1\text{h } 46\text{ min}}$$