Chapter 15
Wave Motion

Conceptual Problems

***1 •**

Determine the Concept The speed of a transverse wave on a rope is given by $v = \sqrt{F/\mu}$ where F is the tension in the rope and μ is its linear density. The waves on the rope move faster as they move up because the tension increases due to the weight of the rope below.

2 •

Determine the Concept The distance between successive crests is one wavelength and the time between successive crests is the period of the wave motion. Thus, T = 0.2 s and $f = 1/T = 5$ Hz. $\boxed{(b)\text{ is correct.}}$

3 •

Picture the Problem True. The energy per unit volume (the average energy density) is given by $\eta_{\text{av}} = \frac{1}{2}\rho\omega^2 s_0^2$ where s_0 is the displacement amplitude.

4 •

Determine the Concept At every point along the rope the wavelength, speed, and frequency of the wave are related by $\lambda = v/f$. The speed of the wave, in turn, is given by $v = \sqrt{F/\mu}$ where F is the tension in the rope and μ is its linear density. Due to the weight of the rope below, the tension is greater at the top and the speed of the wave is also greater at the top. Because $\lambda \propto v$, the wavelength is greater at the top.

***5 •**

Determine the Concept The speed of the wave v on the bullwhip varies with the tension F in the whip and its linear density μ according to $v = \sqrt{F/\mu}$. As the whip tapers, the wave speed in the tapered end increases due to the decrease in the mass density, so the wave travels faster.

6 •

Picture the Problem The intensity level of a sound wave β, measured in decibels, is given by $\beta = (10\,\text{dB})\log(I/I_0)$ where $I_0 = 10^{-12}$ W/m^2 is defined to be the threshold of hearing.

Express the intensity of the 60-dB sound:

$$60\text{dB} = (10\,\text{dB})\log\frac{I_{60}}{I_0} \Rightarrow I_{60} = 10^6 I_0$$

Express the intensity of the 30-dB sound:

$$30\,\text{dB} = (10\,\text{dB})\log\frac{I_{30}}{I_0} \Rightarrow I_{30} = 10^3 I_0$$

Because $I_{60} = 10^3 I_{30}$:

$$\boxed{\text{The statement is false.}}$$

7 •

Determine the Concept No. Because the source and receiver are at rest relative to each other, there is no relative motion of the source and receiver and there will be no Doppler shift in frequency.

8 •

Determine the Concept Because there is no relative motion of the source and receiver, there will be no Doppler shift and the observer will hear sound of frequency f_0.

$\boxed{(a)\text{ is correct.}}$

***9** ••

Determine the Concept The light from the companion star will be shifted about its mean frequency periodically due to the relative approach to and recession from the earth of the companion star as it revolves about the black hole.

10 •

Determine the Concept In any medium, the wavelength, frequency, and speed of a wave are related through $\lambda = v/f$. Because the frequency of a wave is determined by its source and is independent of the nature of the medium, if v differs in the two media, the wavelengths will also differ. In this situation, the frequencies are the same but the speeds of propagation are different.

11 •

(*a*) True. The particles that make up the string move at right angles to the direction the wave is propagating.

(*b*) False. Sound waves in air are *longitudinal* waves of compression and rarefaction.

(*c*) False. The speed of sound in air varies with the square root of the absolute temperature. The speed of sound at 20°C is 4% greater than that at 5°C.

12 •

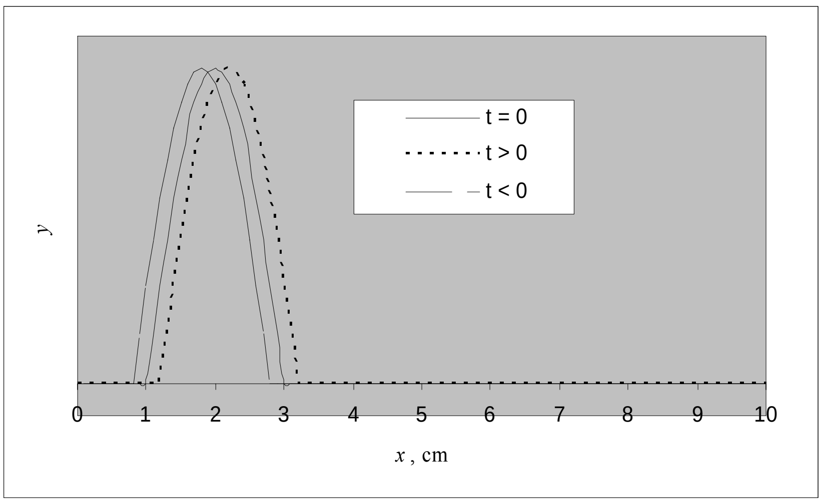
Determine the Concept In any medium, the wavelength, frequency, and speed of a sound wave are related through $\lambda = v/f$. Because the frequency of a wave is determined by its source and is independent of the nature of the medium, if v is greater in water than in air, λ will be greater in water than in air. $\boxed{(a) \text{ is correct.}}$

***13 •**

Determine the Concept There was only one explosion. Sound travels faster in water than air. Abel heard the sound wave in the water first, then, surfacing, heard the sound wave traveling through the air, which took longer to reach him.

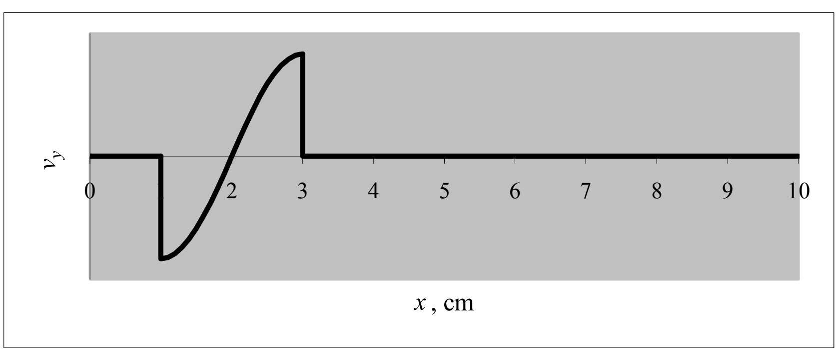
14 ••

Determine the Concept The graph shown below shows the pulse at an earlier time ($-\Delta t$) and later time (Δt). One can see that at $t = 0$, the portion of the string between 1 cm and 2 cm is moving down, the portion between 2 cm and 3 cm is moving up, and the string at $x = 2$ cm is instantaneously at rest.



15 ••

Determine the Concept The velocity of the string at $t = 0$ is shown. Note that the velocity is negative for 1 cm < x < 2 cm and is positive for 2 cm < x < 3 cm.



16 ••

Determine the Concept As the jar is evacuated, the speed of sound inside the jar decreases. Because of the mismatch between the speed of sound inside and outside of the jar, a larger fraction of the sound wave is reflected back into the jar, and a smaller fraction is transmitted through the glass of the bell jar.

***17** ••

Determine the Concept Path C. Because the wave speed is highest in the water, and more of path C is underwater than A or B, the sound wave will spend the least time on path C.

Estimation and Approximation

18 ••

Picture the Problem The rate at which energy is delivered by sound waves is the product of its intensity and the area over which the energy is delivered. We can use the definition of the intensity level of the speech at 1 m to find the intensity of the sound and the formula for the area of a sphere to find the area over which the energy is distributed.

Express the power of human speech as a function of its intensity:

$$P = IA$$

Express the area of a sphere of radius 1 m:

$$A = 4\pi r^2 = 4\pi(1\,\text{m})^2 = 4\pi\,\text{m}^2$$

Use $\beta = (10\,\text{dB})\log(I/I_0)$ to solve for the intensity of the sound at the 65-dB level:

$$65\,\text{dB} = (10\,\text{dB})\log\frac{I}{I_0}$$

and

$$I = 10^{6.5} I_0 = 10^{6.5}\left(10^{-12}\,\text{W/m}^2\right) = 3.16\times10^{-6}\,\text{W/m}^2$$

Substitute and evaluate P:

$$P = \left(3.16\times10^{-6}\,\text{W/m}^2\right)\left(4\pi\,\text{m}^2\right) = \boxed{3.97\times10^{-5}\,\text{W}}$$

19 ••

Picture the Problem Let d represent the distance from the bridge to the water under the assumption that the time for the sound to reach the man is negligible; let t be the elapsed time between dropping the stone and hearing the splash. We'll use a constant-acceleration equation to find the distance to the water in all three parts of the problem, just improving our initial value with corrections taking into account the time required for the sound of the splash to reach the man on the bridge.

(*a*) Using a constant-acceleration equation, relate the distance the stone falls to its time-of-fall:

$$d = v_0 t + \tfrac{1}{2} g t^2$$

or, because $v_0 = 0$,

$$d = \tfrac{1}{2} g t^2$$

Substitute numerical values and evaluate *d*:

$$d = \tfrac{1}{2}\left(9.81\,\text{m/s}^2\right)(4\,\text{s})^2 = \boxed{78.5\,\text{m}}$$

(*b*) Express the actual distance to the water d' in terms of a time correction Δt:

$$d' = \tfrac{1}{2} g (t - \Delta t)^2$$

Express Δt:

$$\Delta t = \frac{d}{v_s}$$

Substitute to obtain:

$$d' = \tfrac{1}{2} g \left(t - \frac{d}{v_s}\right)^2$$

Substitute numerical values and evaluate d':

$$d' = \tfrac{1}{2}\left(9.81\,\text{m/s}^2\right)\left(4\,\text{s} - \frac{78.5\,\text{m}}{340\,\text{m/s}}\right)^2$$

$$= \boxed{69.7\,\text{m}}$$

(*c*) Express the total time for the rock to fall and the sound to return to the man:

$$\Delta t = \Delta t_{\text{falling rock}} + \Delta t_{\text{sound}} = \sqrt{\frac{2d}{g}} + \frac{d}{v_s}$$

Rewrite the equation in quadratic form:

$$d^2 - 2v_s^2\left(\frac{1}{g} + \frac{\Delta t}{v_s}\right)d + v_s^2(\Delta t)^2 = 0$$

Substitute numerical values to obtain:

$$d^2 - \left(2.63\times10^4\,\text{m}\right)d + 1.85\times10^6\,\text{m}^2 = 0$$

Solve for the positive value of *d*:

$$d = \boxed{70.5\,\text{m}}$$ … about 1% larger than our result in part (*b*) and 11% smaller than our first approximation in (*a*).

***20** ••

Picture the Problem You can use a protractor to measure the angle of the shock cone and then estimate the speed of the bullet using $\sin\theta = v/u$. The speed of sound in helium at room temperature (293 K) is 977 m/s.

Relate the speed of the bullet u to the speed of sound v in helium and the angle of the shock cone θ:

$$\sin\theta = \frac{v}{u}$$

Solve for u:

$$u = \frac{v}{\sin\theta}$$

Measure θ to obtain:

$$\theta \approx 70°$$

Substitute numerical values and evaluate u:

$$u = \frac{977\,\text{m/s}}{\sin 70°} = \boxed{1.04\,\text{km/s}}$$

21 ••

Picture the Problem Let d be the distance to the townhouses. We can relate the speed of sound to the distance to the townhouses to the frequency of the clapping for which no echo is heard.

Relate the speed of sound to the distance it travels to the townhouses and back to the elapsed time:

$$v = \frac{2d}{\Delta t}$$

Express d in terms of the number of strides and distance covered per stride:

$$d = (30\,\text{strides})(1.8\,\text{m/stride}) = 54\,\text{m}$$

Relate the elapsed time Δt to the frequency f of the clapping:

$$\Delta t = \frac{1}{f} = \frac{1}{2.5\,\text{claps/s}} = 0.4\,\text{s}$$

Substitute numerical values and evaluate v:

$$v = \frac{2(54\,\text{m})}{0.4\,\text{s}} = \boxed{270\,\text{m/s}}$$

Express the percent difference between this result and 340 m/s:

$$\frac{340\,\text{m/s} - 270\,\text{m/s}}{340\,\text{m/s}} = \boxed{20.6\%}$$

Speed of Waves

22 •

Picture the Problem The speed of sound in a fluid is given by $v = \sqrt{B/\rho}$ where B is the bulk modulus of the fluid and ρ is its density.

(a) Express the speed of sound in water in terms of its bulk modulus:

$$v = \sqrt{\frac{B}{\rho}}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{\frac{2.0\times10^{9}\ \text{N/m}^2}{10^3\ \text{kg/m}^3}} = \boxed{1.41\,\text{km/s}}$$

(*b*) Solve $v = \sqrt{B/\rho}$ for *B*:

$$B = \rho v^2$$

Substitute numerical values and evaluate *B*:

$$B = \left(13.6\times10^3\ \text{kg/m}^3\right)\left(1410\ \text{m/s}\right)^2 = \boxed{2.70\times10^{10}\ \text{N/m}^2}$$

***23** •

Picture the Problem The speed of sound in a gas is given by $v = \sqrt{\gamma RT/M}$ where *R* is the gas constant, *T* is the absolute temperature, *M* is the molecular mass of the gas, and γ is a constant that is characteristic of the particular molecular structure of the gas. Because hydrogen gas is diatomic, γ = 1.4.

Express the dependence of the speed of sound in hydrogen gas on the absolute temperature:

$$v = \sqrt{\frac{\gamma RT}{M}}$$

Substitute numerical values and evaluate *v*:

$$v = \sqrt{\frac{1.4\left(8.314\ \text{J/mol}\cdot\text{K}\right)\left(300\ \text{K}\right)}{2\times10^{-3}\ \text{kg/mol}}} = \boxed{1.32\ \text{km/s}}$$

24 •

Picture the Problem The speed of a transverse wave pulse on a wire is given by $v = \sqrt{F/\mu}$ where *F* is the tension in the wire, *m* is its mass, *L* is its length, and μ is its mass per unit length.

Express the dependence of the speed of the pulse on the tension in the wire:

$$v = \sqrt{\frac{F}{\mu}}$$

where μ is the mass per unit length of the wire.

Substitute numerical values and evaluate *v*:

$$v = \sqrt{\frac{F}{m/L}} = \sqrt{\frac{900\ \text{N}}{0.1\ \text{kg}/7\ \text{m}}} = \boxed{251\ \text{m/s}}$$

25 •

Picture the Problem The speed of transverse waves on a wire is given by $v = \sqrt{F/\mu}$ where *F* is the tension in the wire, *m* is its mass, *L* is its length, and μ is its mass per unit length.

Express the dependence of the speed of the pulse on the tension in the wire and the linear density of the wire:

$$v = \sqrt{\frac{F}{m/L}}$$

Solve for m:

$$m = \frac{FL}{v^2}$$

Substitute numerical values and evaluate m:

$$m = \frac{(550\,\text{N})(0.8\,\text{m})}{(150\,\text{m/s})^2} = \boxed{19.6\,\text{g}}$$

***26** •

Picture the Problem The speed of a wave pulse on a wire is given by $v = \sqrt{F/\mu}$ where F is the tension in the wire, m is its mass, L is its length, and μ is its mass per unit length.

(a) Doubling the length while keeping the mass per unit length constant does not change the linear density:

$$v = \boxed{20\,\text{m/s}}$$

(b) Because v depends of $\sqrt{F}$, doubling the tension increases v by a factor of $\sqrt{2}$:

$$v = \sqrt{2}(20\,\text{m/s}) = \boxed{28.3\,\text{m/s}}$$

(c) Because v depends on $1/\sqrt{\mu}$, doubling μ reduces v by a factor of $\sqrt{2}$:

$$v = \frac{20\,\text{m/s}}{\sqrt{2}} = \boxed{14.1\,\text{m/s}}$$

27 •

Picture the Problem The speed of a transverse wave on the piano wire is given by $v = \sqrt{F/\mu}$ where F is the tension in the wire, m is its mass, L is its length, and μ is its mass per unit length.

(a) The speed of transverse waves on the wire is given by:

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{F}{m/L}}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{\frac{500\,\text{N}}{(0.005\,\text{kg})/(0.7\,\text{m})}} = \boxed{265\,\text{m/s}}$$

(*b*) Letting m' represent the mass of the wire when copper has been wrapped around the steel wire, express Δm, the amount of copper wire required:

$$\Delta m = m' - m$$

Express the new wave speed v':

$$v' = \sqrt{\frac{F}{m'/L}}$$

Express the ratio of the speed of the waves in part (*a*) to the reduced wave speed:

$$\frac{v}{v'} = 2 = \frac{\sqrt{\frac{F}{m/L}}}{\sqrt{\frac{F}{m'/L}}} = \sqrt{\frac{m'}{m}}$$

Solve for and evaluate m':

$$m' = 4m$$

Substitute to obtain:

$$\Delta m = m' - m = 4m - m = 3(5\,\text{g})$$
$$= \boxed{15.0\,\text{g}}$$

28 ••

Picture the Problem We can estimate the accuracy of this procedure by comparing the estimated distance to the actual distance. Whether a correction for the time it takes the light to reach you is important can be decided by comparing the times required for light and sound to travel a given distance.

(*a*) Convert 340 m/s to km/s:

$$v = 340\,\frac{\text{m}}{\text{s}} \times \frac{1\,\text{km}}{1000\,\text{m}} = \boxed{0.340\,\text{km/s}}$$

(*b*) Express the fractional error in the procedure:

$$\frac{\Delta s}{s} = \frac{s - s_{\text{estimated}}}{s} = \frac{0.340t - 0.333t}{0.340t}$$
$$= \frac{0.007}{0.340} = \boxed{2.06\%}$$

(*c*) Compare the time required for light to travel 1 km to the time required for sound to travel the same distance:

$$\frac{\Delta t_{\text{light}}}{\Delta t_{\text{sound}}} = \frac{\frac{1\,\text{km}}{c}}{\frac{1\,\text{km}}{v}} = \frac{v}{c} = \frac{340\,\text{m/s}}{3\times 10^{8}\,\text{m/s}} \approx 10^{-6}$$

$\boxed{\begin{array}{l}\text{Because this fraction is so small, a correction for the time for light to} \\ \text{reach you is not important.}\end{array}}$

***29 ••**

Picture the Problem The speed of a transverse wave on a string is given by $v = \sqrt{F/\mu}$ where F is the tension in the wire and μ is its linear density. We can differentiate this expression with respect to F and then separate the variables to show that the differentials satisfy $dv/v = \frac{1}{2}dF/F$. We'll approximate the differential quantities to determine by how much the tension must be changed to increase the speed of the wave to 312 m/s.

(*a*) Evaluate *dv*/*dF*:

$$\frac{dv}{dF} = \frac{d}{dF}\left[\sqrt{\frac{F}{\mu}}\right] = \frac{1}{2}\sqrt{\frac{1}{F\mu}} = \frac{1}{2}\cdot\frac{v}{F}$$

Separate the variables to obtain:

$$\boxed{\frac{dv}{v} = \frac{1}{2}\frac{dF}{F}}$$

(*b*) Solve for *dF*:

$$dF = 2F\frac{dv}{v}$$

Approximate *dF* with Δ*F* and *dv* with Δ*v* to obtain:

$$\Delta F = 2F\frac{\Delta v}{v}$$

Substitute numerical values and evaluate Δ*F*:

$$\Delta F = 2(500\,\text{N})\frac{12\,\text{m/s}}{300\,\text{m/s}} = \boxed{40.0\,\text{N}}$$

30 ••

Picture the Problem The speed of sound in a gas is given by $v = \sqrt{\gamma RT/M}$ where R is the gas constant, T is the absolute temperature, M is the molecular mass of the gas, and γ is a constant that is characteristic of the particular molecular structure of the gas. We can differentiate this expression with respect to T and then separate the variables to show that the differentials satisfy $dv/v = \frac{1}{2}dT/T$. We'll approximate the differential quantities to determine the percentage change in the velocity of sound when the temperature increases from 0 to 27°C. Lacking information regarding the nature of the gas, we can express the ratio of the speeds of sound at 300 K and 273 K to obtain an expression that involves just the temperatures.

(*a*) Evaluate *dv*/*dT*:

$$\frac{dv}{dT}=\frac{d}{dT}\left[\sqrt{\frac{\gamma RT}{M}}\right]=\frac{1}{2}\sqrt{\frac{M}{\gamma RT}}\left(\frac{\gamma R}{M}\right)$$
$$=\frac{1}{2}\frac{v}{T}$$

Separate the variables to obtain:

$$\boxed{\frac{dv}{v}=\frac{1}{2}\frac{dT}{T}}$$

(*b*) Approximate dT with ΔT and dv with Δv and substitute numerical values to obtain:

$$\frac{\Delta v}{v}=\frac{1}{2}\frac{\Delta T}{T}=\frac{1}{2}\left(\frac{27K}{273\text{K}}\right)=\boxed{4.95\%}$$

(*c*) Using a differential approximation, approximate the speed of sound at 300 K:

$$v_{300\,\text{K}}\approx v_{273\,\text{K}}+v_{273\,\text{K}}\frac{\Delta v}{v}$$
$$=v_{273\,\text{K}}\left(1+\frac{\Delta v}{v}\right)$$

Substitute numerical values and evaluate $v_{300\,\text{K}}$:

$$v_{300\,\text{K}}=(331\,\text{m/s})(1+0.0495)=\boxed{347\,\text{m/s}}$$

Use $v=\sqrt{\gamma RT/M}$ to express the speed of sound at 300 K:

$$v_{300\,\text{K}}=\sqrt{\frac{\gamma T(300\,\text{K})}{M}}$$

Use $v=\sqrt{\gamma RT/M}$ to express the speed of sound at 273 K:

$$v_{273\,\text{K}}=\sqrt{\frac{\gamma T(273\,\text{K})}{M}}$$

Divide the first of these equations by the second and solve for and evaluate $v_{300\,\text{K}}$:

$$\frac{v_{300\,\text{K}}}{v_{273\,\text{K}}}=\frac{\sqrt{\frac{\gamma T(300\,\text{K})}{M}}}{\sqrt{\frac{\gamma T(273\,\text{K})}{M}}}=\sqrt{\frac{300}{273}}$$

and

$$v_{300\,\text{K}}=(331\,\text{m/s})\sqrt{\frac{300}{273}}=\boxed{347\,\text{m/s}}$$

Note that these two results agree to three significant figures.

31 •••

Picture the Problem The speed of sound in a gas is given by $v=\sqrt{\gamma RT/M}$ where R is the gas constant, T is the absolute temperature, M is the molecular mass of the gas, and γ

is a constant that is characteristic of the particular molecular structure of the gas. Because $T = t + 273$ K, we can differentiate v with respect to t to show that $dv/dt = \frac{1}{2}(v/T)$.

Evaluate dv/dt:

$$\frac{dv}{dt} = \frac{d}{dt}\left[\sqrt{\frac{\gamma R(t + 273\,\text{K})}{M}}\right] = \frac{1}{2}\sqrt{\frac{M}{\gamma R(t+273\,\text{K})}}\left(\frac{\gamma R}{M}\right) = \frac{1}{2}\frac{v}{T}$$

Substitute for t to obtain:

$$\frac{dv}{dt} = \frac{1}{2}\left[\frac{v}{t + 273\,\text{K}}\right]$$

Use the approximation

$$v(T) \approx v(T_0) + \left(\frac{dv}{dT}\right)_{T_0} \Delta T$$

to write:

$$v(t) \approx v(0°C) + \frac{1}{2}\left[\frac{v}{t+273\,\text{K}}\right]t = v(0°C) + \Delta v \quad (1)$$

where

$$\Delta v = \frac{1}{2}\left[\frac{331\,\text{m/s}}{t + 273\,\text{K}}\right]t$$

For $t << 273$ K:

$$\Delta v \approx \frac{1}{2}\left[\frac{331\,\text{m/s}}{273\,\text{K}}\right]t = (0.606\,\text{m/s}\cdot\text{K})t$$

Substitute in equation (1) to obtain:

$$v(t) = v(0°C) + (0.606\,\text{m/s}\cdot\text{K})t = \boxed{331\,\text{m/s} + (0.606\,\text{m/s}\cdot\text{K})t}$$

32 ••

Picture the Problem Let d be the distance to the munitions plant, v_1 be the speed of sound in air, v_2 be the speed of sound in rock, and Δt be the difference in the arrival times of the sound at the man's apartment. We can express Δt in terms of $t1$ and $t2$ and then express these travel times in terms of the distance d and the speeds of the sound waves in air and in rock to obtain an equation we can solve for the distance from the man's apartment to the munitions plant.

Express the difference in travel times for the sound wave transmitted through air and the sound wave transmitted through the earth:

$$t_1 - t_2 = \Delta t$$

Express the transmission times in terms of the distance traveled and the speeds in the two media:

$$t_1 = \frac{d}{v_1} \text{ and } t_2 = \frac{d}{v_2}$$

Substitute to obtain:

$$\frac{d}{v_1} - \frac{d}{v_2} = \Delta t$$

or

$$d\left(\frac{1}{v_1} - \frac{1}{v_2}\right) = d\left(\frac{v_2 - v_1}{v_1 v_2}\right) = \Delta t$$

Solve for d:

$$d = \frac{v_1 v_2}{v_2 - v_1}\Delta t$$

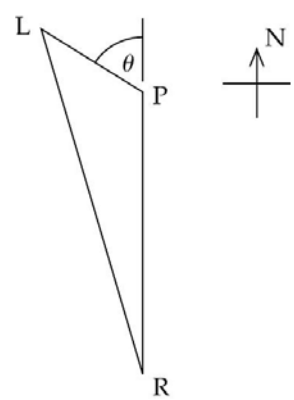
Substitute numerical values and evaluate d:

$$d = \frac{(340\,\text{m/s})(3000\,\text{m/s})}{3000\,\text{m/s} - 340\,\text{m/s}}(3\,\text{s})$$

$$= \boxed{1.15\,\text{km}}$$

33 •••

Picture the Problem The locations of the lightning strike (L), dorm room (R), and baseball park (P) are indicated on the diagram. We can neglect the time required for the electromagnetic pulse to reach the source of the radio transmission, which is the ballpark. The angle θ is related to the sides of the triangle through the law of cosines. We're given the distance d_{PR} and can find the other sides of the triangle using the speed of sound and the elapsed times.



Using the law of cosines, relate the angle θ to the distances that make up the sides of the triangle:

$$d_{\text{LR}}^2 = d_{\text{LP}}^2 + d_{\text{PR}}^2 - 2d_{\text{LP}}d_{\text{PR}}\cos(180° - \theta) = d_{\text{LP}}^2 + d_{\text{PR}}^2 + 2d_{\text{LP}}d_{\text{PR}}\cos\theta$$

Solve for θ:

$$\theta = \cos^{-1}\left(\frac{d_{\text{LR}}^2 - d_{\text{LP}}^2 - d_{\text{PR}}^2}{2d_{\text{LP}}d_{\text{PR}}}\right)$$

Express the distance from the lightning strike to the ball park:

$$d_{\text{LP}} = v_s \Delta t_{\text{LP}} = (340\,\text{m/s})(2\,\text{s}) = 680\,\text{m}$$

Express the distance from the lightning strike to the dorm room:

$$d_{\text{LR}} = v_s \Delta t_{\text{LR}} = (340\,\text{m/s})(6\,\text{s}) = 2040\,\text{m}$$

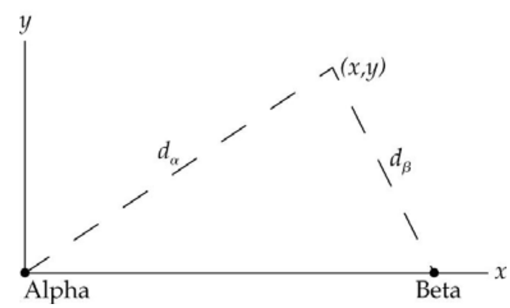
Substitute and evaluate θ:

$$\theta = \cos^{-1}\left(\frac{(2040\,\text{m})^2 - (680\,\text{m})^2 - (1600\,\text{m})^2}{2(680\,\text{m})(1600\,\text{m})}\right) = \pm 58.4°$$

$$\boxed{\text{The lightning struck 680 m from the ballpark, 58.4° W (or E) of north.}}$$

***34 •••**

Picture the Problem Choose a coordinate system in which station Alpha is at the origin and the axes are oriented as shown in the pictorial representation. Because 0.75 mi = 1.21 km, Alpha's coordinates are (0, 0), Beta's are (1.21 km, 0), and those of the lightning strike are (*x, y*). We can relate the distances from the stations to the speed of sound in air and the times required to hear the thunder at the two stations.



Relate the distance d_α to the position coordinates of Alpha and the lightning strike:

$$x^2 + y^2 = d_\alpha^2 \qquad (1)$$

Relate the distance d_β to the position coordinates of Beta and the lightning strike:

$$(x - 1.21\,\text{km})^2 + y^2 = d_\beta^2 \qquad (2)$$

Relate the distance d_α to the speed of sound in air v and the time that elapses between seeing the lightning at Alpha and hearing the thunder:

$$d_\alpha = v\Delta t_\alpha = (340\,\text{m/s})(3.4\,\text{s}) = 1156\,\text{m}$$

Relate the distance d_β to the speed of sound in air v and the time that elapses between seeing the lightning at Beta and hearing the thunder:

$$d_\beta = v\Delta t_\beta = (340\,\text{m/s})(2.5\,\text{s}) = 850\,\text{m}$$

Substitute in equations (1) and (2) to obtain:

$$x^2 + y^2 = (1156\,\text{m})^2 = 1.336\,\text{km}^2 \qquad (3)$$

and

$$(x - 1.21\,\text{km})^2 + y^2 = (850\,\text{m})^2 = 0.7225\,\text{km}^2 \quad (4)$$

Subtract equation (4) from equation (3) to obtain:

$$x^2 - (x - 1.21\,\text{km})^2 = 1.336\,\text{km}^2 - 0.7225\,\text{km}^2$$

or

$$(2.42\,\text{km})x - (1.21\,\text{km})^2 = 0.6135\,\text{km}^2$$

Solve for x to obtain:

$$x = 0.855\,\text{km}$$

Substitute in equation (3) to obtain:

$$(0.855\,\text{km})^2 + y^2 = 1.336\,\text{km}^2$$

Solve for y, keeping the positive root because the lightning strike is to the north of the stations, to obtain:

$$y = 0.778\,\text{km}$$

The coordinates of the lightning strike are:

$$\boxed{(0.855\,\text{km}, 0.778\,\text{km})}$$

or

$$\boxed{(0.531\,\text{mi}, 0.484\,\text{mi})}$$

35 •••

Picture the Problem The velocity of longitudinal waves on the Slinky is given by $v = \sqrt{B/\rho}$ where B is the bulk modulus of the material from which the slinky is constructed and ρ is its mass per unit volume. The velocity of transverse waves on the slinky is given by $v = \sqrt{F/\mu}$ where F is the tension in the slinky and μ is its mass per unit length. Substitution for B and ρ will lead to $v = L\sqrt{k/m}$ in (a) and similar substitutions, together with the assumption that $L_0 << L$ will yield the same result for (b).

(a) Express the velocity of longitudinal waves on the slinky:

$$v = \sqrt{\frac{B}{\rho}} \quad (1)$$

For the slinky:

$$\rho = \frac{m}{V}$$

and

$$B = -\frac{P}{\Delta V/V} \quad (2)$$

Letting A be the cross-sectional area of the slinky:

$$\rho = \frac{m}{AL} \text{ and } P = -k\frac{\Delta L}{A}$$

Substitute in equation (2) and simplify to obtain:

$$B = k\frac{L}{A}$$

Substitute in equation (1):

$$v = \sqrt{\frac{k\dfrac{L}{A}}{\dfrac{m}{AL}}} = \boxed{L\sqrt{\frac{k}{m}}}$$

(*b*) Express the velocity of transverse waves on the slinky:

$$v = \sqrt{\frac{F}{\mu}} \qquad (3)$$

For the slinky:

$$\mu = \frac{m}{L}$$

and

$$F = k\Delta L = k(L - L_0) = kL\left(1 - \frac{L_0}{L}\right)$$
$$\approx kL \text{ if } L_0 << L$$

Substitute in equation (3) to obtain:

$$v = \sqrt{\frac{kL}{\dfrac{m}{L}}} = \boxed{L\sqrt{\frac{k}{m}}}$$

The Wave Equation

36 •

Picture the Problem The general wave equation is $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$. To show that each of the functions satisfies this equation, we'll need to find their first and second derivatives with respect to *x* and *t* and then substitute these derivatives in the wave equation.

(*a*) Find the first two spatial derivatives of $y(x,t) = k(x + vt)^3$:

$$\frac{\partial y}{\partial x} = 3k(x + vt)^2$$

and

$$\frac{\partial^2 y}{\partial x^2} = 6k(x + vt) \qquad (1)$$

Find the first two temporal derivatives of $y(x,t) = k(x + vt)^3$:

$$\frac{\partial y}{\partial t} = 3kv(x + vt)^2$$

and

$$\frac{\partial^2 y}{\partial t^2} = 6kv^2(x+vt) \qquad (2)$$

Express the ratio of equation (1) to equation (2):

$$\frac{\dfrac{\partial^2 y}{\partial x^2}}{\dfrac{\partial^2 y}{\partial t^2}} = \frac{6k(x+vt)}{6kv^2(x+vt)} = \boxed{\frac{1}{v^2}}$$

confirming that $y(x,t) = k(x+vt)^3$ satisfies the general wave equation.

(*b*) Find the first two spatial derivatives of $y(x,t) = Ae^{ik(x-vt)}$:

$$\frac{\partial y}{\partial x} = ikAe^{ik(x-vt)}$$

and

$$\frac{\partial^2 y}{\partial x^2} = i^2k^2Ae^{ik(x-vt)}$$

or

$$\frac{\partial^2 y}{\partial x^2} = -k^2Ae^{ik(x-vt)} \qquad (3)$$

Find the first two temporal derivatives of $y(x,t) = Ae^{ik(x-vt)}$:

$$\frac{\partial y}{\partial t} = -ikvAe^{ik(x-vt)}$$

and

$$\frac{\partial^2 y}{\partial t^2} = i^2k^2v^2Ae^{ik(x-vt)}$$

or

$$\frac{\partial^2 y}{\partial t^2} = -k^2v^2Ae^{ik(x-vt)} \qquad (4)$$

Express the ratio of equation (3) to equation (4):

$$\frac{\dfrac{\partial^2 y}{\partial x^2}}{\dfrac{\partial^2 y}{\partial t^2}} = \frac{-k^2Ae^{ik(x-vt)}}{-k^2v^2Ae^{ik(x-vt)}} = \boxed{\frac{1}{v^2}}$$

confirming that $y(x,t) = Ae^{ik(x-vt)}$ satisfies the general wave equation.

(*c*) Find the first two spatial derivatives of $y(x,\, t) = \ln k(x-vt)$:

$$\frac{\partial y}{\partial x} = \frac{k}{x-vt}$$

and

$$\frac{\partial^2 y}{\partial x^2} = -\frac{k^2}{(x-vt)^2} \qquad (5)$$

Find the first two temporal derivatives of $y(x,t)=\ln k(x-vt)$:

$$\frac{\partial y}{\partial t}=-\frac{vk}{x-vt}$$

and

$$\frac{\partial^2 y}{\partial t^2}=-\frac{v^2k^2}{(x-vt)^2} \qquad (6)$$

Express the ratio of equation (5) to equation (6):

$$\frac{\dfrac{\partial^2 y}{\partial x^2}}{\dfrac{\partial^2 y}{\partial t^2}}=\frac{-\dfrac{k^2}{(x-vt)^2}}{-\dfrac{v^2k^2}{(x-vt)^2}}=\boxed{\frac{1}{v^2}}$$

confirming that $y(x,t)=\ln k(x-vt)$ satisfies the general wave equation.

***37** •

Picture the Problem The general wave equation is $\dfrac{\partial^2 y}{\partial x^2}=\dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2}$. To show that $y=A\sin kx\cos\omega t$ satisfies this equation, we'll need to find the first and second derivatives of y with respect to x and t and then substitute these derivatives in the wave equation.

Find the first two spatial derivatives of $y=A\sin kx\cos\omega t$:

$$\frac{\partial y}{\partial x}=Ak\cos kx\cos\omega t$$

and

$$\frac{\partial^2 y}{\partial x^2}=-Ak^2\sin kx\cos\omega t \qquad (1)$$

Find the first two temporal derivatives of $y=A\sin kx\cos\omega t$:

$$\frac{\partial y}{\partial t}=-\omega A\sin kx\sin\omega t$$

and

$$\frac{\partial^2 y}{\partial t^2}=-\omega^2 A\sin kx\cos\omega t \qquad (2)$$

Express the ratio of equation (1) to equation (2):

$$\frac{\dfrac{\partial^2 y}{\partial x^2}}{\dfrac{\partial^2 y}{\partial t^2}}=\frac{-Ak^2\sin kx\cos\omega t}{-A\omega^2\sin kx\cos\omega t}=\frac{k^2}{\omega^2}$$

$$=\boxed{\frac{1}{v^2}}$$

confirming that $y=A\sin kx\cos\omega t$ satisfies the general wave equation.

Harmonic Waves on a String

38 •

Picture the Problem We can find the velocity of the waves from the definition of velocity and their wavelength from $\lambda = v/f$.

Express the wavelength of the waves:

$$\lambda = \frac{v}{f}$$

Using the definition of velocity, find the wave velocity:

$$v = \frac{\Delta x}{\Delta t} = \frac{6\,\text{m}}{0.5\,\text{s}} = 12\,\text{m/s}$$

Substitute to obtain:

$$\lambda = \frac{12\,\text{m/s}}{60\,\text{s}^{-1}} = \boxed{20.0\,\text{cm}}$$

39 •

Picture the Problem Equation 15-13, $y(x,t) = A\sin(kx - \omega t)$, describes a wave traveling in the positive x direction. For a wave traveling in the negative x direction we have $y(x,t) = A\sin(kx + \omega t)$.

(*a*) Factor k from the argument of the sine function to obtain:

$$y(x,t) = A\sin k\left(x - \frac{\omega}{k}t\right) = \boxed{A\sin k(x - vt)}$$

(*b*) Substitute $k = 2\pi/\lambda$ and $\omega = 2\pi f$ to obtain:

$$y(x,t) = A\sin\left(\frac{2\pi}{\lambda}x - 2\pi ft\right) = \boxed{A\sin 2\pi\left(\frac{x}{\lambda} - ft\right)}$$

(*c*) Substitute $k = 2\pi/\lambda$ and $\omega = 2\pi/T$ to obtain:

$$y(x,t) = A\sin\left(\frac{2\pi}{\lambda} - \frac{2\pi}{T}t\right) = \boxed{A\sin 2\pi\left(\frac{x}{\lambda} - \frac{1}{T}t\right)}$$

(*d*) Substitute $k = 2\pi/\lambda$ to obtain:

$$y(x,t) = A\sin\left(\frac{2\pi}{\lambda}x - \omega t\right)$$
$$= A\sin\frac{2\pi}{\lambda}\left(x - \frac{\lambda\omega}{2\pi}t\right)$$
$$= \boxed{A\sin\frac{2\pi}{\lambda}(x - vt)}$$

(*e*) Substitute $k = 2\pi/\lambda$ and $\omega = 2\pi f$ to obtain:

$$y(x,t) = A\sin(kx - 2\pi ft)$$
$$= A\sin 2\pi f\left(\frac{k}{2\pi f}x - t\right)$$
$$= \boxed{A\sin 2\pi f\left(\frac{x}{v} - t\right)}$$

For waves traveling in the negative x direction, we simply change the – signs to + signs.

***40 •**

Picture the Problem We can use $f = c/\lambda$ to express the frequency of any periodic wave in terms of its wavelength and velocity.

(*a*) Find the frequency of light of wavelength 4×10^{-7} m:

$$f = \frac{c}{\lambda} = \frac{2.998\times10^{8}\ \text{m/s}}{4\times10^{-7}\ \text{m}} = 7.50\times10^{14}\ \text{Hz}$$

Find the frequency of light of wavelength 7×10^{-7} m:

$$f = \frac{c}{\lambda} = \frac{2.998\times10^{8}\ \text{m/s}}{7\times10^{-7}\ \text{m}} = 4.28\times10^{14}\ \text{Hz}$$

Therefore the range of frequencies is:

$$\boxed{4.28\times10^{14}\ \text{Hz} \le f \le 7.50\times10^{14}\ \text{Hz}}$$

(*b*) Use the same relationship to calculate the frequency of these microwaves:

$$f = \frac{c}{\lambda} = \frac{2.998\times10^{8}\ \text{m/s}}{3\times10^{-2}\ \text{m}}$$
$$= \boxed{1.00\times10^{10}\ \text{Hz}}$$

41 •

Picture the Problem The average power propagated along the string by a harmonic wave is $P_{\text{av}} = \frac{1}{2}\mu\omega^2 A^2 v,$ where v is the speed of the wave, and $\mu,$ $\omega,$ and A are the linear density of the string, the angular frequency of the wave, and the amplitude of the wave, respectively.

Express and evaluate the power propagated along the string:

$$P_{av} = \tfrac{1}{2}\mu\omega^2 A^2 v$$

The speed of the wave on the string is given by:

$$v = \sqrt{\frac{F}{\mu}}$$

Substitute for v to obtain:

$$P_{av} = \tfrac{1}{2}\mu\omega^2 A^2\sqrt{\frac{F}{\mu}}$$

Substitute numerical values and evaluate P_{av}:

$$P_{av} = \tfrac{1}{2}\left(4\pi^2\right)(0.05\,\text{kg/m})\left(10\,\text{s}^{-1}\right)^2(0.05\,\text{m})^2\sqrt{\frac{80\,\text{N}}{0.05\,\text{kg/m}}} = \boxed{9.87\,\text{W}}$$

42 •

Picture the Problem The average power propagated along the rope by a harmonic wave is $P_{av} = \tfrac{1}{2}\mu\omega^2 A^2 v,$ where v is the velocity of the wave, and μ, ω, and A are the linear density of the string, the angular frequency of the wave, and the amplitude of the wave, respectively.

Rewrite the power equation in terms of the frequency of the wave:

$$P_{av} = \tfrac{1}{2}\mu\omega^2 A^2 v = 2\pi^2\mu f^2 A^2 v$$

Solve for the frequency:

$$f = \sqrt{\frac{P_{av}}{2\pi^2\mu A^2 v}} = \frac{1}{2\pi A}\sqrt{\frac{2P_{av}}{\mu v}}$$

The wave velocity is given by:

$$v = \sqrt{\frac{F}{\mu}}$$

Substitute for v and simplify to obtain:

$$f = \frac{1}{2\pi A}\sqrt{\frac{2P_{av}}{\mu\sqrt{\frac{F}{\mu}}}} = \frac{1}{2\pi A}\sqrt{\frac{2P_{av}}{\sqrt{\mu F}}}$$

Substitute numerical values and evaluate f:

$$f = \frac{1}{2\pi(0.01\,\text{m})}\sqrt{\frac{2(100\,\text{W})}{\sqrt{\left(\frac{0.1\,\text{kg}}{2\,\text{m}}\right)(60\,\text{N})}}} = \boxed{171\,\text{Hz}}$$

43 ••

Picture the Problem Equation 15-13, $y(x,t) = A\sin(kx - \omega t)$, describes a wave traveling in the positive x direction. For a wave traveling in the negative x direction, we have $y(x,t) = A\sin(kx + \omega t)$. We can determine A, k, and ω by examination of the wave function. The wavelength, frequency, and period of the wave can, in turn, be determined from k and ω.

(*a*) $\boxed{\text{Because the sign between the } kx \text{ and } \omega t \text{ terms is positive, the wave is traveling in the negative } x \text{ direction.}}$

Find the speed of the wave:

$$v = \frac{\omega}{k} = \frac{314\,\text{s}^{-1}}{62.8\,\text{m}^{-1}} = \boxed{5.00\,\text{m/s}}$$

(*b*) The coefficient of x is k and:

$$k = \frac{2\pi}{\lambda}$$

Solve for and evaluate λ:

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{62.8\,\text{m}^{-1}} = \boxed{10.0\,\text{cm}}$$

The coefficient of t is ω and:

$$f = \frac{\omega}{2\pi} = \frac{314\,\text{s}^{-1}}{2\pi} = \boxed{50.0\,\text{Hz}}$$

The period of the wave motion is the reciprocal of its frequency:

$$T = \frac{1}{f} = \frac{1}{50\,\text{s}^{-1}} = \boxed{0.0200\,\text{s}}$$

(*c*) Express and evaluate the maximum speed of any string segment:

$$v_{\text{max}} = A\omega = (0.001\,\text{m})(314\,\text{rad/s}) = \boxed{0.314\,\text{m/s}}$$

44 ••

Picture the Problem Let the positive x direction be to the right. Then equation 15-13, $y(x,t) = A\sin(kx - \omega t)$, describes a wave traveling in the positive x direction. We can find ω and k from the data included in the problem statement and substitute in the general equation. The maximum speed of a point on the string can be found from $v_{\text{max}} = A\omega$ and the maximum acceleration from $a_{\text{max}} = A\omega^2$.

(*a*) Express the general form of the equation of a harmonic wave traveling to the right:

$$y(x,t) = A\sin(kx - \omega t)$$

Evaluate ω:

$$\omega = 2\pi f = 2\pi\left(80\,\text{s}^{-1}\right) = 503\,\text{s}^{-1}$$

Determine k:

$$k = \frac{\omega}{v} = \frac{503\,\text{s}^{-1}}{12\,\text{m/s}} = 41.9\,\text{m}^{-1}$$

Substitute to obtain: $y(x,t) = \boxed{(0.025\,\text{m})\sin\left[\left(41.9\,\text{m}^{-1}\right)x - \left(503\,\text{s}^{-1}\right)t\right]}$

(*b*) Express and evaluate the maximum speed of a point on the string:

$$v_{\text{max}} = A\omega = (0.025\,\text{m})\left(503\,\text{s}^{-1}\right) = \boxed{12.6\,\text{m/s}}$$

(*c*) Express the maximum acceleration of a point on the string:

$$a_{\text{max}} = A\omega^2$$

Substitute numerical values and evaluate a_{max}:

$$a_{\text{max}} = (0.025\,\text{m})\left(503\,\text{s}^{-1}\right)^2 = \boxed{6.33\,\text{km/s}^2}$$

45 ••

Picture the Problem The average total energy of waves on a string is given by $\Delta E_{\text{av}} = \frac{1}{2}\mu\omega^2A^2\Delta x,$ where μ is the linear density of the string, ω is its angular frequency, A the amplitude of the wave motion, and, in this problem, Δx is the length of the string. The average power propagated along the string is $P_{\text{av}} = \frac{1}{2}\mu\omega^2A^2v.$

(*a*) ΔE_{av} is given by:

$$\Delta E_{\text{av}} = \tfrac{1}{2}\mu\omega^2A^2\Delta x = 2\pi^2\mu f^2A^2\Delta x$$

Evaluate $\Delta E_{\text{av}} = \frac{1}{2}\mu\omega^2A^2\Delta x$ with $\Delta x = L = 20$ m:

$$\Delta E_{\text{av}} = 2\pi^2\left(\frac{0.06\,\text{kg}}{20\,\text{m}}\right)\left(200\,\text{s}^{-1}\right)^2(0.012\,\text{m})^2(20\,\text{m}) = \boxed{6.82\,\text{J}}$$

(*b*) Express the power transmitted past a given point on the string:

$$P_{\text{av}} = \tfrac{1}{2}\mu\omega^2A^2v$$

The speed of the wave is given by:

$$v=\sqrt{\frac{F}{\mu}}$$

Substitute for v to obtain:

$$P_{av}=\tfrac{1}{2}\mu\omega^2A^2\sqrt{\frac{F}{\mu}}$$

Substitute numerical values and evaluate P_{av}:

$$P_{av}=2\pi^2\left(\frac{0.06\,\text{kg}}{20\,\text{m}}\right)\left(200\,\text{s}^{-1}\right)^2(0.012\,\text{m})^2\sqrt{\frac{50\,\text{N}}{\frac{0.06\,\text{kg}}{20\,\text{m}}}}=\boxed{44.0\,\text{W}}$$

***46 ••**

Picture the Problem The power propagated along the rope by a harmonic wave is $P=\frac{1}{2}\mu\omega^2A^2v$ where v is the velocity of the wave, and μ, ω, and A are the linear density of the string, the angular frequency of the wave, and the amplitude of the wave, respectively. We can use the wave function $y=\left(A_0e^{-bx}\right)\sin(kx-\omega t)$ to determine the amplitude of the wave at $x = 0$ and at point x.

(*a*) Express the power associated with the wave at the origin:

$$P=\tfrac{1}{2}\mu\omega^2A^2v$$

Evaluate the amplitude at $x = 0$:

$$A(0)=\left(A_0e^0\right)=A_0$$

Substitute to obtain:

$$P(0)=\boxed{\tfrac{1}{2}\mu\omega^2A_0^2v}$$

(*b*) Express the amplitude of the wave at x:

$$A(x)=\left(A_0e^{-bx}\right)$$

Substitute to obtain:

$$P(x)=\tfrac{1}{2}\mu\omega^2\left(A_0e^{-bx}\right)^2v=\boxed{\tfrac{1}{2}\mu\omega^2A_0^2ve^{-2bx}}$$

47 ••

Picture the Problem The average power propagated along the rope by a harmonic wave is $P_{av}=\frac{1}{2}\mu\omega^2A^2v$ where v is the velocity of the wave, and μ, ω, and A are the linear density of the string, the angular frequency of the wave, and the amplitude of the wave, respectively.

(*a*) Express the average power transmitted along the wire:

$$P_{\text{av}} = \tfrac{1}{2}\mu\omega^2 A^2 v = 2\pi^2 \mu f^2 A^2 v$$

Substitute numerical values and evaluate P_{av}:

$$P_{\text{av}} = 2\pi^2 (0.01\,\text{kg/m})(400\,\text{s}^{-1})^2 \times (0.5\times10^{-3}\,\text{m})^2 (10\,\text{m/s}) = \boxed{79.0\,\text{mW}}$$

(*b*) Because $P_{\text{av}} \propto f^2$:

$\boxed{\text{Increasing } f \text{ by a factor of 10 would increase } P_{\text{av}} \text{ by a factor of 100.}}$

Because $P_{\text{av}} \propto A^2$:

$\boxed{\text{Increasing } A \text{ by a factor of 10 would increase } P_{\text{av}} \text{ by a factor of 100.}}$

Because $P_{\text{av}} \propto v$ and $v \propto \sqrt{F}$:

$\boxed{\text{Increasing } F \text{ by a factor of } 10^4 \text{ would increase } v \text{ by a factor of 100 and } P_{\text{av}} \text{ by a factor of 100.}}$

(*c*)

$\boxed{\text{Depending on the adjustability of the power source, increasing } f \text{ or } A \text{ would be the easiest.}}$

***48 •••**

Picture the Problem We can use the assumption that both the wave function and its first spatial derivative are continuous at $x = 0$ to establish equations relating A, B, C, k_1, and k_2. Then, we can solve these simultaneous equations to obtain expressions for B and C in terms of A, v_1, and v_2.

(*a*) Let $y_1(x, t)$ represent the wave function in the region $x < 0$, and $y_2(x, t)$ represent the wave function in the region $x > 0$. Express the continuity of the two wave functions at $x = 0$:

$$y_1(0,t) = y_2(0,t)$$

and

$$A\sin[k_1(0) - \omega t] + B\sin[k_1(0) + \omega t] = C\sin[k_2(0) - \omega t]$$

or

$$A\sin(-\omega t) + B\sin\omega t = C\sin(-\omega t)$$

Because the sine function is odd; i.e., $\sin(-\theta) = -\sin\theta$:

$$-A\sin\omega t + B\sin\omega t = -C\sin\omega t$$

and

$$A - B = C \qquad (1)$$

Differentiate the wave functions with respect to x to obtain:

$$\frac{\partial y_1}{\partial x} = Ak_1 \cos(k_1 x - \omega t) + Bk_1 \cos(k_1 x + \omega t)$$

and

$$\frac{\partial y_2}{\partial x} = Ck_2 \cos(k_2 x - \omega t)$$

Express the continuity of the slopes of the two wave functions at $x = 0$:

$$\left.\frac{\partial y_1}{\partial x}\right|_{x=0} = \left.\frac{\partial y_2}{\partial x}\right|_{x=0}$$

and

$$Ak_1 \cos[k_1(0) - \omega t] + Bk_1 \cos[k_1(0) + \omega t] = Ck_2 \cos[k_2(0) - \omega t]$$

or

$$Ak_1 \cos(-\omega t) + Bk_1 \cos \omega t = Ck_2 \cos(-\omega t)$$

Because the cosine function is even; i.e., $\cos(-\theta) = \cos\theta$:

$$Ak_1 \cos \omega t + Bk_1 \cos \omega t = Ck_2 \cos \omega t$$

and

$$k_1 A + k_1 B = k_2 C \qquad (2)$$

Multiply equation (1) by k_1 and add it to equation (2) to obtain:

$$2k_1 A = (k_1 + k_2)C$$

Solve for C:

$$C = \frac{2k_1}{k_1 + k_2} A = \frac{2}{1 + k_2/k_1} A$$

Solve for C/A and substitute ω/v_1 for k_1 and ω/v_2 for k_2 to obtain:

$$\frac{C}{A} = \frac{2}{1 + k_2/k_1} = \boxed{\frac{2}{1 + v_1/v_2}}$$

Substitute in equation (1) to obtain:

$$A - B = \left(\frac{2}{1 + v_1/v_2}\right) A$$

Solve for B/A:

$$\frac{B}{A} = \boxed{-\frac{1 - v_1/v_2}{1 + v_1/v_2}}$$

(b) We wish to show that

$$B^2 + (v_1/v_2)C^2 = A^2$$

Use the results of (a) to obtain the expressions $B = -[(1-\alpha)/(1+\alpha)]\,A$ and $C = 2A/(1+\alpha)$, where $\alpha = v_1/v_2$.

Substitute these expressions into

$$B^2 + (v_1/v_2)C^2 = A^2$$

and check to see if the resulting equation is an identity:

$$B^2 + \frac{v_1}{v_2}C^2 = A^2$$

$$\left(\frac{1-\alpha}{1+\alpha}\right)^2 A^2 + \alpha\left(\frac{2}{1+\alpha}\right)^2 A^2 = A^2$$

$$\left(\frac{1-\alpha}{1+\alpha}\right)^2 + \alpha\left(\frac{2}{1+\alpha}\right)^2 = 1$$

$$\frac{(1-\alpha)^2 + 4\alpha}{(1+\alpha)^2} = 1$$

$$\frac{1-2\alpha+\alpha^2+4\alpha}{(1+\alpha)^2} = 1$$

$$\frac{1+2\alpha+\alpha^2}{(1+\alpha)^2} = 1$$

$$\frac{(1+\alpha)^2}{(1+\alpha)^2} = 1$$

$$1 = 1$$

The equation is an identity:

Therefore, $\boxed{B^2 + \frac{v_1}{v_2}C^2 = A^2}$

Remarks: Our result in (*a*) can be checked by considering the limit of B/A as $v_2/v_1 \to 0$. This limit gives $B/A = +1$, telling us that the transmitted wave has zero amplitude and the incident and reflected waves superpose to give a standing wave with a node at $x = 0$.

Harmonic Sound Waves

***49** •

Picture the Problem The pressure variation is of the form $p(x,t) = p_0 \cos k(x - vt)$ where $k = \frac{\pi}{2}$ and $v = 340\,\text{m/s}$. We can find λ from k and f from ω and k.

(a) By inspection of the equation: $p_0 = \boxed{0.750\,\text{Pa}}$

(b) Because $k = \frac{2\pi}{\lambda} = \frac{\pi}{2}$: $\lambda = \boxed{4.00\,\text{m}}$

(c) Solve $v = \frac{\omega}{k} = \frac{2\pi f}{k}$ for f to obtain:

$$f = \frac{kv}{2\pi} = \frac{\frac{\pi}{2}(340\,\text{m/s})}{2\pi} = \boxed{85.0\,\text{Hz}}$$

(*d*) By inspection of the equation: $v = \boxed{340\,\text{m/s}}$

50 •

Picture the Problem The frequency, wavelength, and speed of the sound waves are related by $v = f\lambda$.

(*a*) Express and evaluate the wavelength of middle C:

$$\lambda = \frac{v}{f} = \frac{340\,\text{m/s}}{262\,\text{s}^{-1}} = \boxed{1.30\,\text{m}}$$

(*b*) Double the frequency corresponding to middle C; solve for and evaluate λ:

$$\lambda = \frac{v}{f} = \frac{340\,\text{m/s}}{2\left(262\,\text{s}^{-1}\right)} = \boxed{0.649\,\text{m}}$$

51 •

Picture the Problem The pressure amplitude depends on the density of the medium ρ, the angular frequency of the sound wave μ, the speed of the wave v, and the displacement amplitude s_0 according to $p_0 = \rho\omega v s_0$.

(*a*) Solve $p_0 = \rho\omega v s_0$ for s_0:

$$s_0 = \frac{p_0}{\rho\omega v}$$

Substitute numerical values and evaluate s_0:

$$s_0 = \frac{\left(10^{-4}\,\text{atm}\right)\left(1.01325\times10^{5}\,\text{Pa/atm}\right)}{2\pi\left(1.29\,\text{kg/m}^3\right)\left(100\,\text{s}^{-1}\right)\left(340\,\text{m/s}\right)} = \boxed{3.68\times10^{-5}\,\text{m}}$$

(*b*) Use $p_0 = \rho\omega v s_0$ to find p_0:

$$p_0 = 2\pi\left(1.29\,\text{kg/m}^3\right)\left(300\,\text{s}^{-1}\right)\left(340\,\text{m/s}\right)\left(10^{-7}\,\text{m}\right) = \boxed{8.27\times10^{-2}\,\text{Pa}}$$

52 •

Picture the Problem The pressure amplitude depends on the density of the medium ρ, the angular frequency of the sound wave μ, the speed of the wave v, and the displacement amplitude s_0 according to $p_0 = \rho\omega v s_0$.

(*a*) Solve $p_0 = \rho\omega v s_0$ for evaluate s_0:

$$s_0 = \frac{p_0}{\rho\omega v}$$

$$s_0 = \frac{29\,\text{Pa}}{2\pi(1.29\,\text{kg/m}^3)(1000\,\text{s}^{-1})(340\,\text{m/s})} = \boxed{2.10\times10^{-5}\,\text{m}}$$

(*b*) Proceed as in (*a*) with $f = 1$ kHz:

$$s_0 = \frac{29\,\text{Pa}}{2\pi(1.29\,\text{kg/m}^3)(1000\,\text{s}^{-1})(340\,\text{m/s})} = \boxed{1.05\times10^{-5}\,\text{m}}$$

53 •

Picture the Problem The pressure or density wave is 90° out of phase with the displacement wave. When the displacement is zero, the pressure and density changes are either a maximum or a minimum. When the displacement is a maximum or minimum, the pressure and density changes are zero. We can use $p_0 = \rho\omega v s_0$ to find the maximum value of the displacement at any time and place.

(*a*) If the pressure is a maximum at x_1 when $t = 0$:

$\boxed{\text{the displacement } s \text{ is zero.}}$

(*b*) Solve $p_0 = \rho\omega v s_0$ for s_0:

$$s_0 = \frac{p_0}{\rho\omega v}$$

Substitute numerical values and evaluate s_0:

$$s_0 = \frac{(10^{-4}\,\text{atm})(1.01325\times10^5\,\text{Pa/atm})}{2\pi(1.29\,\text{kg/m}^3)(1000\,\text{s}^{-1})(340\,\text{m/s})} = \boxed{3.68\,\mu\text{m}}$$

***54 •**

Picture the Problem A human can hear sounds between roughly 20 Hz and 20 kHz; a factor of 1000. An octave represents a change in frequency by a factor of 2. We can evaluate $2^N = 1000$ to find the number of octaves heard by a person who can hear this range of frequencies.

Relate the number of octaves to the difference between 20 kHz and 20 Hz:

$$2^N = 1000$$

Take the logarithm of both sides of the equation to obtain:

$$\log 2^N = \log 10^3$$

or

$$N\log 2 = 3$$

Solve for and evaluate N:

$$N = \frac{3}{\log 2} = 9.97 \approx \boxed{10}$$

Waves in Three Dimensions: Intensity

55 •

Picture the Problem The pressure amplitude depends on the density of the medium ρ, the angular frequency of the sound wave μ, the speed of the wave v, and the displacement amplitude s_0 according to $p_0 = \rho\omega v s_0$. The intensity of the waves is given by $I = \frac{1}{2}\rho\omega^2 s_0^2 v = \frac{1}{2}\frac{p_0^2}{\rho v}$ and the rate at which energy is delivered (power) is the product of the intensity and the surface area of the piston.

(*a*) Using $p_0 = \rho\omega v s_0$, evaluate p_0:

$$p_0 = 2\pi\left(1.29\,\text{kg/m}^3\right)\left(500\,\text{s}^{-1}\right)\times\left(340\,\text{m/s}\right)\left(0.1\times10^{-3}\,\text{m}\right) = \boxed{138\,\text{Pa}}$$

(*b*) Use $I = \frac{1}{2}\frac{p_0^2}{\rho v}$ to find the intensity of the waves:

$$I = \frac{1}{2}\frac{\left(138\,\text{Pa}\right)^2}{\left(1.29\,\text{kg/m}^3\right)\left(340\,\text{m/s}\right)} = \boxed{21.7\,\text{W/m}^2}$$

(*c*) Using $P_{\text{av}} = IA$ to find the power required to keep the piston oscillating:

$$P_{\text{av}} = \left(21.6\,\text{W/m}^2\right)\left(10^{-2}\,\text{m}^2\right) = \boxed{0.217\,\text{W}}$$

56 •

Picture the Problem The intensity of the sound from the spherical source varies inversely with the square of the distance from the source. The power radiated by the source is the product of the intensity of the radiation and the surface area over which it is distributed.

(*a*) Relate the intensity at 10 m to the distance from the source:

$$I = \frac{P_{\text{av}}}{4\pi r^2}$$

or

$$10^{-4}\,\text{W/m}^2 = \frac{P_{\text{av}}}{4\pi\left(10\,\text{m}\right)^2}$$

Letting r' represent the distance at which the intensity is 10^{-6} W/m^2, express the intensity as in part (*a*):

$$10^{-6}\,\text{W/m}^2 = \frac{P_{\text{av}}}{4\pi r'^2}$$

Divide the first of these equations by the second to obtain:

$$\frac{10^{-4}\ \text{W/m}^2}{10^{-6}\ \text{W/m}^2}=\frac{\dfrac{P_{\text{av}}}{4\pi(10\,\text{m})^2}}{\dfrac{P_{\text{av}}}{4\pi r'^2}}$$

Solve for and evaluate r':

$$r'=\sqrt{(10^2)(10\,\text{m})^2}=\boxed{100\,\text{m}}$$

(*b*) Solve $I=\dfrac{P_{\text{av}}}{4\pi r^2}$ for P_{av}:

$$P_{\text{av}}=4\pi r^2 I$$

Substitute numerical values and evaluate P_{av}:

$$P_{\text{av}}=4\pi(10\,\text{m})^2\left(10^{-4}\ \text{W/m}^2\right)=\boxed{0.126\,\text{W}}$$

***57 •**

Picture the Problem Because the power radiated by the loudspeaker is the product of the intensity of the sound and the surface area over which it is distributed, we can use this relationship to find the average power, the intensity of the radiation, or the distance to the speaker for a given intensity or average power.

(*a*) Use $P_{\text{av}}=4\pi r^2 I$ to find the total acoustic power output of the speaker:

$$P_{\text{av}}=4\pi(20\,\text{m})^2\left(10^{-2}\ \text{W/m}^2\right)=\boxed{50.3\,\text{W}}$$

(*b*) Relate the intensity of the sound at 20 m to the distance from the speaker:

$$10^{-2}\ \text{W/m}^2=\frac{P_{\text{av}}}{4\pi(20\,\text{m})^2}$$

Relate the threshold-of-pain intensity to the distance from the speaker:

$$1\,\text{W/m}^2=\frac{P_{\text{av}}}{4\pi r^2}$$

Divide the first of these equations by the second; solve for and evaluate r:

$$r=\sqrt{10^{-2}(20\,\text{m})^2}=\boxed{2.00\,\text{m}}$$

(*c*) Use $I=\dfrac{P_{\text{av}}}{4\pi r^2}$ to find the intensity at 30 m:

$$I(30\,\text{m})=\frac{50.3\,\text{W}}{4\pi(30\,\text{m})^2}=\boxed{4.45\times10^{-3}\ \text{W/m}^2}$$

58 ••

Picture the Problem We can use conservation of energy to find the acoustical energy resulting from the dropping of the pin. The power developed can then be found from the given time during which the energy was transformed from mechanical to acoustical form. We can find the range at which the dropped pin can be heard from $I = P/4\pi r^2$.

(*a*) Assuming that $I = P/4\pi r^2$, express the distance at which one can hear the dropped pin:

$$r = \sqrt{\frac{P}{4\pi I}}$$

Use conservation of energy to determine the sound energy generated when the pin falls:

$$\begin{aligned} E &= (0.0005)(mgh) \\ &= (0.0005)(0.1\times10^{-3}\text{ kg})(9.81\text{ m/s}^2)\\ &\quad\times(1\text{ m}) \\ &= 4.91\times10^{-7}\text{ J} \end{aligned}$$

Express the power of the sound pulse:

$$\begin{aligned} P &= \frac{E}{\Delta t} = \frac{4.91\times10^{-7}\text{ J}}{0.1\text{ s}} \\ &= 4.91\times10^{-6}\text{ W} \end{aligned}$$

Substitute numerical values and evaluate r:

$$r = \sqrt{\frac{4.91\times10^{-6}\text{ W}}{4\pi(10^{-11}\text{ W/m}^2)}} = \boxed{198\text{ m}}$$

(*b*) Repeat the last step in (*a*) with $I = 10^{-8}$ W/m^2:

$$r = \sqrt{\frac{4.91\times10^{-6}\text{ W}}{4\pi(10^{-8}\text{ W/m}^2)}} = \boxed{6.25\text{ m}}$$

Intensity Level

59 •

Picture the Problem The intensity level of a sound wave β, measured in decibels, is given by $\beta = (10\text{ dB})\log(I/I_0)$ where $I_0 = 10^{-12}$ W/m^2 is defined to be the threshold of hearing.

(*a*) Using its definition, calculate the intensity level of a sound wave whose intensity is 10^{-10} W/m^2:

$$\begin{aligned} \beta &= (10\text{ dB})\log\left(\frac{10^{-10}\text{ W/m}^2}{10^{-12}\text{ W/m}^2}\right) \\ &= 10\log 10^2 = \boxed{20.0\text{ dB}} \end{aligned}$$

(*b*) Proceed as in (*a*) with $I = 10^{-2}$ W/m^2:

$$\beta = (10\,\text{dB})\log\left(\frac{10^{-2}\ \text{W/m}^2}{10^{-12}\ \text{W/m}^2}\right)$$

$$= 10\log 10^{10} = \boxed{100\,\text{dB}}$$

60 •

Picture the Problem The intensity level of a sound wave β, measured in decibels, is given by $\beta = (10\,\text{dB})\log(I/I_0)$ where $I_0 = 10^{-12}$ W/m^2 is defined to be the threshold of hearing.

(*a*) Solve $\beta = (10\,\text{dB})\log(I/I_0)$ for I to obtain:

$$I = 10^{\beta/(10\,\text{dB})} I_0$$

Evaluate I for $\beta = 10$ dB:

$$I = 10^{(10\,\text{dB})/(10\,\text{dB})} I_0 = 10 I_0$$

$$= 10\left(10^{-12}\ \text{W/m}^2\right) = \boxed{10^{-11}\ \text{W/m}^2}$$

(*b*) Proceed as in (*a*) with $\beta = 3$ dB:

$$I = 10^{(3\,\text{dB})/(10\,\text{dB})} I_0 = 2 I_0$$

$$= 2\left(10^{-12}\ \text{W/m}^2\right) = \boxed{2\times 10^{-12}\ \text{W/m}^2}$$

***61** •

Picture the Problem The intensity level of a sound wave β, measured in decibels, is given by $\beta = (10\,\text{dB})\log(I/I_0)$ where $I_0 = 10^{-12}$ W/m^2 is defined to be the threshold of hearing.

Express the sound level of the rock concert:

$$\beta_{\text{concert}} = (10\,\text{dB})\log\left(\frac{I_{\text{concert}}}{I_0}\right) \qquad (1)$$

Express the sound level of the dog's bark:

$$50\,\text{dB} = (10\,\text{dB})\log\left(\frac{I_{\text{dog}}}{I_0}\right)$$

Solve for the intensity of the dog's bark:

$$I_{\text{dog}} = 10^5 I_0 = 10^5\left(10^{-12}\ \text{W/m}^2\right)$$

$$= 10^{-7}\ \text{W/m}^2$$

Express the intensity of the rock concert in terms of the intensity of the dog's bark:

$$I_{\text{concert}} = 10^4 I_{\text{dog}} = 10^4\left(10^{-7}\ \text{W/m}^2\right)$$

$$= 10^{-3}\ \text{W/m}^2$$

Substitute in equation (1) and evaluate β_{concert}:

$$\beta_{\text{concert}} = (10\,\text{dB})\log\left(\frac{10^{-3}\ \text{W/m}^2}{10^{-12}\ \text{W/m}^2}\right) = (10\,\text{dB})\log 10^9 = \boxed{90.0\,\text{dB}}$$

62 •

Picture the Problem The intensity level of a sound wave β, measured in decibels, is given by $\beta = (10\,\text{dB})\log(I/I_0)$ where $I_0 = 10^{-12}$ W/m^2 is defined to be the threshold of hearing.

Express the intensity level of the louder sound:

$$\beta_{\text{L}} = (10\,\text{dB})\log\left(\frac{I_{\text{L}}}{I_0}\right)$$

Express the intensity level of the softer sound:

$$\beta_{\text{S}} = (10\,\text{dB})\log\left(\frac{I_{\text{S}}}{I_0}\right)$$

Express the difference between the intensity levels of the two sounds:

$$\beta_{\text{L}} - \beta_{\text{S}} = 30\,\text{dB} = (10\,\text{dB})\left[\log\left(\frac{I_{\text{L}}}{I_0}\right) - \log\left(\frac{I_{\text{S}}}{I_0}\right)\right] = (10\,\text{dB})\log\left(\frac{I_{\text{L}}/I_0}{I_{\text{S}}/I_0}\right) = (10\,\text{dB})\log\left(\frac{I_{\text{L}}}{I_{\text{S}}}\right)$$

Solve for and evaluate the ratio $I_{\text{L}}/I_{\text{S}}$:

$$\frac{I_{\text{L}}}{I_{\text{S}}} = 10^3 \text{ and } \boxed{(a) \text{ is correct.}}$$

63 •

Picture the Problem We can use the definition of the intensity level to express the difference in the intensity levels of two sounds whose intensities differ by a factor of 2.

Express the intensity level before the intensity is doubled:

$$\beta_1 = (10\,\text{dB})\log\left(\frac{I}{I_0}\right)$$

Express the intensity level with the intensity doubled:

$$\beta_2 = (10\,\text{dB})\log\left(\frac{2I}{I_0}\right)$$

Express and evaluate $\Delta\beta = \beta_2 - \beta_1$:

$$\begin{aligned}\Delta\beta &= \beta_2 - \beta_1\\ &= (10\,\text{dB})\log\left(\frac{2I}{I_0}\right) - (10\,\text{dB})\log\left(\frac{I}{I_0}\right)\\ &= (10\,\text{dB})\log\left(\frac{2I}{I}\right) = (10\,\text{dB})\log 2\\ &= 3.01\,\text{dB} \approx \boxed{3.0\,\text{dB}}\end{aligned}$$

***64** •

Picture the Problem We can express the intensity levels at both 90 dB and 70 dB in terms of the intensities of the sound at those levels. By subtracting the two expressions, we can solve for the ratio of the intensities at the two levels and then find the fractional change in the intensity that corresponds to a decrease in intensity level from 90 dB to 70 dB.

Express the intensity level at 90 dB:

$$90\,\text{dB} = (10\,\text{dB})\log\left(\frac{I_{90}}{I_0}\right)$$

Express the intensity level at 70 dB:

$$70\,\text{dB} = (10\,\text{dB})\log\left(\frac{I_{70}}{I_0}\right)$$

Express $\Delta\beta = \beta_{90} - \beta_{70}$:

$$\begin{aligned}\Delta\beta &= 20\,\text{dB}\\ &= (10\,\text{dB})\log\left(\frac{I_{90}}{I_0}\right) - (10\,\text{dB})\log\left(\frac{I_{70}}{I_0}\right)\\ &= (10\,\text{dB})\log\left(\frac{I_{90}}{I_{70}}\right)\end{aligned}$$

Solve for I_{90}:

$$I_{90} = 100I_{70}$$

Express the fractional change in the intensity from 90 dB to 70 dB:

$$\frac{I_{90} - I_{70}}{I_{90}} = \frac{100I_{70} - I_{70}}{100I_{70}} = \boxed{99\%}$$

65 ••

Picture the Problem The intensity at a distance r from a spherical source varies with distance from the source according to $I = P_{\text{av}}/4\pi r^2$. We can use this relationship to relate the intensities corresponding to an 80-dB intensity level (I_{80}) and the intensity corresponding to a 60-dB intensity level (I_{60}) to their distances from the source. We can relate the intensities to the intensity levels through $\beta = (10\,\text{dB})\log(I/I_0)$.

(*a*) Express the intensity of the sound where the intensity level is 80 dB:

$$I_{10} = \frac{P_{av}}{4\pi r_{10}^2}$$

Express the intensity of the sound where the intensity level is 60 dB:

$$I_{60} = \frac{P_{av}}{4\pi r^2}$$

Divide the first of these equations by the second to obtain:

$$\frac{I_{80}}{I_{60}} = \frac{\dfrac{P_{av}}{4\pi(10\,\text{m})^2}}{\dfrac{P_{av}}{4\pi r^2}} = \frac{r^2}{100\,\text{m}^2}$$

Solve for *r*:

$$r = (10\,\text{m})\sqrt{\frac{I_{80}}{I_{60}}}$$

Find the intensity of the 80-dB sound level radiation:

$$80\,\text{dB} = (10\,\text{dB})\log\left(\frac{I_{80}}{I_0}\right)$$

and

$$I_{80} = 10^8 I_0 = 10^{-4}\ \text{W/m}^2$$

Find the intensity of the 60-dB sound level radiation:

$$60\,\text{dB} = (10\,\text{dB})\log\left(\frac{I_{60}}{I_0}\right)$$

and

$$I_{60} = 10^6 I_0 = 10^{-6}\ \text{W/m}^2$$

Substitute and evaluate *r*:

$$r = (10\,\text{m})\sqrt{\frac{10^{-4}\ \text{W/m}^2}{10^{-6}\ \text{W/m}^2}} = \boxed{100\,\text{m}}$$

(*b*) Using the intensity corresponding to an intensity level of 80 dB, express and evaluate the power radiated by this source:

$$\begin{aligned} P &= I_{80}A \\ &= \left(10^{-4}\ \text{W/m}^2\right)\left[4\pi(10\,\text{m})^2\right] \\ &= \boxed{0.126\,\text{W}} \end{aligned}$$

66 ••

Picture the Problem Let I_1 and I_2 be the intensities of the sound at distances r_1 and r_2. We can relate these intensities to the intensity levels through $\beta = (10\,\text{dB})\log(I/I_0)$ and to the distances through $I = P_{av}/4\pi r^2$.

Using $\beta = (10\,\text{dB})\log(I/I_0)$, express the ratio β_1/β_2:

$$\frac{\beta_2}{\beta_1} = \frac{(10\,\text{dB})\log(I_2/I_0)}{(10\,\text{dB})\log(I_1/I_0)} = \frac{\log I_2 - \log I_0}{\log I_1 - \log I_0}$$

Express the ratio of the intensities at distances r_1 and r_2 from the source and solve for I_2:

$$\frac{I_2}{I_1} = \frac{r_1^2}{r_2^2} \text{ and } I_2 = \frac{r_1^2}{r_2^2} I_1$$

Substitute and simplify to obtain:

$$\frac{\beta_2}{\beta_1} = \frac{10\log\frac{r_1^2}{r_2^2} I_1 - 10\log I_0}{\log I_1 - \log I_0} = \frac{10\log I_1 + 20\log(r_1/r_2) - 10\log I_0}{10\log I_1 - 10\log I_0}$$

$$= \frac{10\log(I_1/I_0) + 20\log(r_1/r_2)}{10\log(I_1/I_0)} = \boxed{\frac{\beta_1 + 20\log(r_1/r_2)}{\beta_1}}$$

67 ••

Picture the Problem We can use $\beta = (10\,\text{dB})\log(I/I_0)$ where $I_0 = 10^{-12}$ W/m^2 is defined to be the threshold of hearing, to find the intensity level at 20 m. Because the power radiated by the loudspeaker is the product of the intensity of the sound and the surface area over which it is distributed, we can use this relationship to find either the average power, the intensity of the radiation, or the distance to the speaker for a given intensity or average power.

(*a*) Relate the intensity level to the intensity at 20 m:

$$\beta = (10\,\text{dB})\log\left(\frac{I}{I_0}\right) = (10\,\text{dB})\log\left(\frac{10^{-2}\ \text{W/m}^2}{10^{-12}\ \text{W/m}^2}\right) = (10\,\text{dB})\log(10^{10}) = \boxed{100\,\text{dB}}$$

(*b*) Use $P_{\text{av}} = 4\pi r^2 I$ to find the total acoustic power output of the speaker:

$$P_{\text{av}} = 4\pi(20\,\text{m})^2(10^{-2}\ \text{W/m}^2) = \boxed{50.3\,\text{W}}$$

(*c*) Relate the intensity of the sound at 20 m to the distance from the speaker:

$$10^{-2}\ \text{W/m}^2 = \frac{P_{\text{av}}}{4\pi(20\,\text{m})^2}$$

Relate the threshold-of-pain intensity to the distance from the speaker:

$$1\,\mathrm{W/m^2} = \frac{P_{\mathrm{av}}}{4\pi r^2}$$

Divide the first of these equations by the second; solve for and evaluate *r*:

$$r = \sqrt{10^{-2}(20\,\mathrm{m})^2} = \boxed{2.00\,\mathrm{m}}$$

(*d*) Use $I = \dfrac{P_{\mathrm{av}}}{4\pi r^2}$ to find the intensity at 30 m:

$$I(30\,\mathrm{m}) = \frac{50.3\,\mathrm{W}}{4\pi(30\,\mathrm{m})^2} = 4.45\times10^{-3}\ \mathrm{W/m^2}$$

Find the intensity level at 30 m:

$$\beta(30\,\mathrm{m}) = (10\,\mathrm{dB})\log\frac{4.45\times10^{-3}\ \mathrm{W/m^2}}{10^{-12}\ \mathrm{W/m^2}} = (10\,\mathrm{dB})\log 4.45\times10^{9} = \boxed{96.5\,\mathrm{dB}}$$

68 ••

Picture the Problem Let *I*′ and *I* represent the sound in consecutive years. Then, we can use $\beta = (10\,\mathrm{dB})\log(I/I_0)$ to express the annual increase in intensity levels.

(*a*) Express the annual change in intensity level:

$$\Delta\beta = \beta' - \beta = 1\,\mathrm{dB} = (10\,\mathrm{dB})\log\frac{I'}{I_0} - (10\,\mathrm{dB})\log\frac{I}{I_0} = (10\,\mathrm{dB})\log\frac{I'}{I}$$

Solve for *I*′/*I*:

$$\frac{I'}{I} = 10^{0.1} = 1.26$$

and the annual increase in intensity is $\boxed{26\%.}$ This is not a plausible annual increase because, if it were true, the intensity level would increase by a factor of 10 in ten years.

(*b*) Because doubling of the intensity corresponds to $\Delta\beta$ = 3 dB and the intensity is increasing 1 dB annually:

$\boxed{\text{The intensity level will double in 3 years.}}$

69 ••

Picture the Problem We can find the intensities of the three sources from their intensity levels and, because their intensities are additive, find the intensity level when all three sources are acting.

(*a*) Express the sound intensity level when the three sources act at the same time:

$$\beta_{3\,\text{sources}} = (10\,\text{dB})\log\frac{I_{3\,\text{sources}}}{I_0}$$
$$= (10\,\text{dB})\log\frac{I_{70} + I_{73} + I_{80}}{I_0}$$

Find the intensities of each of the three sources:

$$70\,\text{dB} = (10\,\text{dB})\log\frac{I_{70}}{I_0} \Rightarrow I_{70} = 10^7 I_0$$

$$73\,\text{dB} = (10\,\text{dB})\log\frac{I_{73}}{I_0} \Rightarrow I_{73} = 10^{7.3} I_0$$

and

$$80\,\text{dB} = (10\,\text{dB})\log\frac{I_{80}}{I_0} \Rightarrow I_{80} = 10^8 I_0$$

Substitute and evaluate $\beta_{3\text{ sources}}$:

$$\beta_{3\,\text{sources}} = (10\,\text{dB})\log\frac{10^7 I_0 + 10^{7.3} I_0 + 10^8 I_0}{I_0} = (10\,\text{dB})\log\left(10^7 + 10^{7.3} + 10^8\right)$$
$$= \boxed{81.1\,\text{dB}}$$

(*b*) Find the intensity level with the two least intense sources eliminated:

$$\beta_{80} = (10\,\text{dB})\log\frac{10^8 I_0}{I_0} = (10\,\text{dB})\log\left(10^8\right)$$
$$= \boxed{80.0\,\text{dB}}$$

$$\boxed{\text{Eliminating the two least intense sources does not reduce the intensity level significantly.}}$$

***70** ••

Picture the Problem Let *P* be the power radiated by the source of sound, and *r* be the initial distance from the source to the receiver. We can use the definition of intensity to find the ratio of the intensities before and after the distance is doubled and then use the definition of the decibel level to find the change in its level.

Relate the change in decibel level to the change in the intensity level:

$$\Delta\beta = 10\log\frac{I}{I'}$$

Using its definition, express the intensity of the sound from the source as a function of P and r:

$$I = \frac{P}{4\pi r^2}$$

Express the intensity when the distance is doubled:

$$I' = \frac{P}{4\pi(2r)^2} = \frac{P}{16\pi r^2}$$

Evaluate the ratio of I to I':

$$\frac{I}{I'} = \frac{\dfrac{P}{4\pi r^2}}{\dfrac{P}{16\pi r^2}} = 4$$

Substitute to obtain:

$$\Delta\beta = 10\log 4 = 6.02\,\text{dB}$$

and $\boxed{(c)\text{ is correct.}}$

71 •••

Picture the Problem The sound level can be found from the intensity of the sound due to the talking people. When 38 people are talking, the intensities add.

Express the sound level when all 38 people are talking:

$$\beta_{38} = (10\,\text{dB})\log\frac{38I_1}{I_0}$$
$$= (10\,\text{dB})\log 38 + (10\,\text{dB})\log\frac{I_1}{I_0}$$
$$= (10\,\text{dB})\log 38 + 72\,\text{dB}$$
$$= \boxed{87.8\,\text{dB}}$$

An equivalent but longer solution:

Express the sound level when all 38 people are talking:

$$\beta_{38} = (10\,\text{dB})\log\frac{38I_1}{I_0}$$

Express the sound level when only one person is talking:

$$\beta_1 = 72\,\text{dB} = (10\,\text{dB})\log\frac{I_1}{I_0}$$

Solve for and evaluate I_1:

$$I_1 = 10^{7.2}I_0 = 10^{7.2}\left(10^{-12}\ \text{W/m}^2\right)$$
$$= 1.58\times10^{-5}\ \text{W/m}^2$$

Express the intensity when all 38 people are talking:

$$I_{38} = 38I_1$$

The decibel level is:

$$\beta_{38} = (10\,\text{dB})\log\frac{38\left(1.58\times10^{-5}\ \text{W/m}^2\right)}{10^{-12}\ \text{W/m}^2}$$
$$= \boxed{87.8\,\text{dB}}$$

***72** •••

Picture the Problem Let η represent the efficiency with which mechanical energy is converted to sound energy. Because we're given information regarding the rate at which mechanical energy is delivered to the string and the rate at which sound energy arrives at the location of the listener, we'll take the efficiency to be the ratio of the sound power delivered to the listener divided by the power delivered to the string. We can calculate the power input directly from the given data. We'll calculate the intensity of the sound at 35 m from its intensity level at that distance and use this result to find the power output.

Express the efficiency of the conversion of mechanical energy to sound energy:

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}}$$

Find the power delivered by the bow to the string:

$$P_{\text{in}} = Fv = (0.6\,\text{N})(0.5\,\text{m/s}) = 0.3\,\text{W}$$

Using $\beta = (10\,\text{dB})\log(I/I_0)$, find the intensity of the sound at 35 m:

$$60\,\text{dB} = (10\,\text{dB})\log\frac{I_{35\,\text{m}}}{I_0}$$

and

$$I_{35\,\text{m}} = 10^6 I_0 = 10^{-6}\ \text{W/m}^2$$

Find the power of the sound emitted:

$$P_{\text{out}} = IA = 4\pi\left(10^{-6}\ \text{W/m}^2\right)(35\,\text{m})^2$$
$$= 0.0154\,\text{W}$$

Substitute numerical values and evaluate η:

$$\eta = \frac{0.0154\,\text{W}}{0.3\,\text{W}} = \boxed{5.13\%}$$

73 •••

Picture the Problem Because the sound intensities are additive, we'll find the intensity due to one student by subtracting the background intensity from the intensity due to the students and dividing by 100. Then, we'll use this result to calculate the intensity level due to 50 students.

Express the intensity level due to 50 students:

$$\beta_{50} = (10\,\text{dB})\log\frac{50I_1}{I_0}$$

Find the sound intensity when 100 students are writing the exam:

$$60\,\text{dB} = (10\,\text{dB})\log\frac{I_{100}}{I_0}$$

and

$$I_{100} = 10^6 I_0 = 10^{-6}\ \text{W/m}^2$$

Find the sound intensity due to the background noise:

$$40\,\text{dB} = (10\,\text{dB})\log\frac{I_{\text{background}}}{I_0}$$

and

$$I_{\text{background}} = 10^4 I_0 = 10^{-8}\ \text{W/m}^2$$

Express the sound intensity due to the 100 students:

$$I_{100} - I_{\text{background}} = 10^{-6}\ \text{W/m}^2 - 10^{-8}\ \text{W/m}^2 \approx 10^{-6}\ \text{W/m}^2$$

Find the sound intensity due to 1 student:

$$\frac{I_{100} - I_{\text{background}}}{100} = 10^{-8}\ \text{W/m}^2$$

Substitute and evaluate the intensity level due to 50 students:

$$\beta_{50} = (10\,\text{dB})\log\frac{50\left(10^{-8}\ \text{W/m}^2\right)}{10^{-12}\ \text{W/m}^2} = \boxed{57.0\,\text{dB}}$$

The Doppler Effect

74 •

Picture the Problem We can use equation 15-32 ($\lambda = \frac{v \pm u}{f_s}$) to find the wavelength of the sound between the source and the listener and 15-35*a* ($f_r = \frac{v \pm u_r}{v \pm u_s} f_s$) to find the frequency heard by the listener.

(*a*) Because the source is approaching the listener, use the minus sign in the numerator of Equation 15-32 to find the wavelength of the sound between the source and the listener:

$$\lambda = \frac{340\,\text{m/s} - 80\,\text{m/s}}{200\,\text{s}^{-1}} = \boxed{1.30\,\text{m}}$$

(*b*) Because the listener is at rest and the source is approaching, $u_r = 0$ and the denominator of Equation 15-35<u>*a*</u> is the difference between the two speeds:

$$f_r = \frac{v}{v - u_s} f_s = \frac{340\,\text{m/s}}{340\,\text{m/s} - 80\,\text{m/s}}\left(200\,\text{s}^{-1}\right) = \boxed{262\,\text{Hz}}$$

75 •

Picture the Problem In the reference frame described, the speed of sound from the source to the listener is reduced by the speed of the wind. We can find the wavelength of the sound in the region between the source and the listener from $v = f\lambda$. Because the sound waves in the region between the source and the listener will be compressed by the motion of the listener, the frequency of the sound heard by the listener will be higher than the frequency emitted by the source and can be calculated using $f_r = \dfrac{v \pm u_r}{v \pm u_s} f_s$.

(*a*) The speed of sound in the reference frame of the source is:

$$v = 340\,\text{m/s} - 80\,\text{m/s} = \boxed{260\,\text{m/s}}$$

(*b*) Noting that the frequency is unchanged, express the wavelength of the sound:

$$\lambda = \frac{v}{f} = \frac{260\,\text{m/s}}{200\,\text{s}^{-1}} = \boxed{1.30\,\text{m}}$$

(*c*) Apply Equation 15-35*a* to obtain:

$$f_r = \left(\frac{v + u_r}{v}\right) f_s = \left(\frac{260\,\text{m/s} + 80\,\text{m/s}}{260\,\text{m/s}}\right)\left(200\,\text{s}^{-1}\right) = \boxed{262\,\text{Hz}}$$

76 •

Picture the Problem We can use $\lambda = (v \pm u)/f_s$ to find the wavelength of the sound in the region between the source and the listener and $f_r = \dfrac{v \pm u_r}{v \pm u_s} f_s$ to find the frequency heard by the listener. Because the sound waves in the region between the source and the listener will be spread out by the motion of the listener, the frequency of the sound heard by the listener will be lower than the frequency emitted by the source.

(*a*) Because the source is moving away from the listener, use the positive sign in the numerator of Equation 15-32 to find the wavelength of the sound between the source and the listener:

$$\lambda = \frac{340\,\text{m/s} + 80\,\text{m/s}}{200\,\text{s}^{-1}} = \boxed{2.10\,\text{m}}$$

(*b*) Because the listener is at rest and the source is receding, $u_r = 0$ and the denominator of Equation 15-35*a* is the sum of the two speeds:

$$f_r = \frac{v}{v + u_s} f_s = \frac{340\,\text{m/s}}{340\,\text{m/s} + 80\,\text{m/s}}\left(200\,\text{s}^{-1}\right) = \boxed{162\,\text{Hz}}$$

77 •

Picture the Problem We can find the wavelength of the sound in the region between the source and the listener from $v = f\lambda$. Because the sound waves in the region between the source and the listener will be compressed by the motion of the listener, the frequency of the sound heard by the listener will be higher than the frequency emitted by the source and can be calculated using $f_r = \frac{v \pm u_r}{v \pm u_s} f_s$.

(*a*) Because the wavelength is unaffected by the motion of the observer:

$$\lambda = \frac{v}{f} = \frac{340\,\text{m/s}}{200\,\text{s}^{-1}} = \boxed{1.70\,\text{m}}$$

(*b*) Apply Equation 15-35*a* to obtain:

$$f_r = \left(\frac{v + u_r}{v}\right) f_0 = \left(\frac{340\,\text{m/s} + 80\,\text{m/s}}{340\,\text{m/s}}\right)\left(200\,\text{s}^{-1}\right) = \boxed{247\,\text{Hz}}$$

78 •

Picture the Problem In this reference frame, the speed of sound will be increased by the speed of the listener. We can find the wavelength of the sound in the region between the source and the listener from $v = f\lambda$. Because the sound waves in the region between the source and the listener will be compressed by the motion of the listener, the frequency of the sound heard by the listener will be higher than the frequency emitted by the source and can be calculated using $v' = f_r\lambda'$.

(*a*) Moving at 80 m/s in still air: $\boxed{\text{The observer experiences a wind velocity of } 80\,\text{m/s.}}$

(*b*) Use the standard Galilean transformation to obtain:

$$v' = v + u_r = 340\,\text{m/s} + 80\,\text{m/s} = \boxed{420\,\text{m/s}}$$

(*c*) Because the distance between the wave crests is unchanged:

$$\lambda' = \frac{v'}{f} = \frac{340\,\text{m/s}}{200\,\text{s}^{-1}} = \boxed{1.70\,\text{m}}$$

(*d*) Using the speed of sound in this reference frame, express and evaluate the frequency heard by the listener:

$$f_r = \frac{v'}{\lambda'} = \frac{420\,\text{m/s}}{1.70\,\text{m}} = \boxed{247\,\text{Hz}}$$

79 •

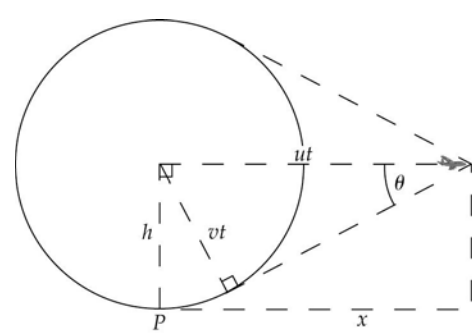
Picture the Problem Because the listener is moving away from the source, we know that the frequency he/she will hear will be less than the frequency emitted by the source. We can use $f_r = \frac{v \pm u_r}{v \pm u_s} f_s$, with $u_s = 0$ and the minus sign in the numerator, to determine its value.

Relate the frequency heard by the listener to that of the source:

$$f_r = \left(\frac{v - u_r}{v}\right) f_s = \left(\frac{340\,\text{m/s} - 80\,\text{m/s}}{340\,\text{m/s}}\right)\left(200\,\text{s}^{-1}\right) = \boxed{153\,\text{Hz}}$$

80 •

Picture the Problem The diagram shows the position of the supersonic plane at time *t* after it was directly over a person located at point *P* 5000 m below it. Let *u* represent the speed of the plane and *v* the speed of sound. We can use trigonometry to determine the angle of the shock wave as well as the location of the jet *x* when the person on the ground hears the shock wave.



(*a*) Referring to the diagram, express θ in terms of *v, u,* and *t*:

$$\theta = \sin^{-1}\left(\frac{vt}{ut}\right) = \sin^{-1}\left(\frac{1}{u/v}\right) = \sin^{-1}\left(\frac{1}{2.5}\right) = \boxed{23.6^\circ}$$

(*b*) Using the diagram, relate the angle θ to the altitude *h* of the plane and the distance *x* and solve for *x*:

$$\tan\theta = \frac{h}{x} \Rightarrow x = \frac{h}{\tan\theta}$$

Substitute numerical values and evaluate *x*:

$$x = \frac{5000\,\text{m}}{\tan 23.6^\circ} = \boxed{11.4\,\text{km}}$$

81 •

Picture the Problem If both u_s and u_r are much smaller than the speed of sound *v,* then the shift in frequency is given approximately by $\frac{\Delta f}{f_r} = \pm\frac{u}{v}$, where $u = u_s \pm u_r$ is the relative speed of the source and receiver. For purposes of this problem, we'll assume that you are an Olympics qualifier and can run at a top speed of approximately 10 m/s.

Express the frequency, f_r, you hear in terms of the frequency of the source f_s and your running speed *u*:

$$f_r = \left(\frac{v+u}{v}\right)f_s$$

Assuming that you can run 10 m/s , substitute numerical values and evaluate f_r:

$$f_r = \left(\frac{340\,\text{m/s} + 10\,\text{m/s}}{340\,\text{m/s}}\right)(1000\,\text{Hz}) = \boxed{1029\,\text{Hz}}$$

Using the positive sign (you are approaching the source), express and evaluate the ratio $\frac{\Delta f}{f_r}$:

$$\frac{\Delta f}{f_r} = \frac{10\,\text{m/s}}{340\,\text{m/s}} = 2.94\%$$

$$\boxed{\begin{array}{l}\text{Because this fractional change in frequency is less than the 3\% criterion for}\\ \text{recognition of a change in frequency, it would be } \textit{impossible} \text{ to use your}\\ \text{sense of pitch to estimate your running speed.}\end{array}}$$

82 ••

Picture the Problem Because the car is moving away from the radar device, the frequency f_r it receives will be less than the frequency emitted by the device. The microwaves reflected from the car, moving away from a stationary detector, will be of a still lower frequency f_r'. We can use the Doppler shift equations to derive an expression for the speed of the car in terms of difference of these frequencies.

Express the frequency f_r received by the moving car in terms of f_s, u, and v:

$$f_r = \left(\frac{c-u}{c}\right)f_s \quad (1)$$

Express the frequency f_r' received by the stationary source in terms of f_r, u, and v:

$$f_r' = \left(\frac{c-u}{c}\right)f_r \quad (2)$$

Substitute equation (1) in equation (2) to eliminate f_r:

$$f_r' = \left(\frac{c-u}{c}\right)^2 f_s \approx \left(1-\frac{2u}{c}\right)f_s$$

provided $u << c$

Express the frequency difference detected at the source:

$$\Delta f = f_s - f_r' = f_s - \left(1-\frac{2u}{c}\right)f_s$$

$$= \frac{2u}{c}f_s$$

Solve for u:

$$u = \frac{c}{2f_s}\Delta f$$

Substitute numerical values and evaluate u:

$$u = \frac{3\times10^8\ \text{m/s}}{2(2\,\text{GHz})}(293\,\text{Hz})$$

$$= 22.0\frac{\text{m}}{\text{s}}\times\frac{1\,\text{km}}{10^3\ \text{m}}\times\frac{3600\,\text{s}}{\text{h}}$$

$$= \boxed{79.1\,\text{km/h}}$$

***83** ••

Picture the Problem Because the radial component u of the velocity of the raindrops is small compared to the speed $v = c$ of the radar pulse, we can approximate the fractional change in the frequency of the reflected radar pulse to find the speed of the winds carrying the raindrops in the storm system.

Express the shift in frequency when the speed of the source (the storm system) u is much smaller than the wave speed $v = c$:

$$\frac{\Delta f}{f_s} \approx \frac{u}{c}$$

Solve for u:

$$u = c\frac{\Delta f}{f_s}$$

Substitute numerical values and evaluate u:

$$u = \left(2.998\times10^8\ \text{m/s}\right)\frac{325\,\text{Hz}}{625\,\text{MHz}}$$
$$= 156\,\text{m/s}\times\frac{0.6215\,\text{mi/h}}{0.2778\,\text{m/s}}$$
$$= \boxed{349\,\text{mi/h}}$$

84 ••

Picture the Problem Let the depth of the submarine be represented by D and its vertical speed by u. The submarine acts as both a receiver and source. We can apply the definition of average speed to determine the depth of the submarine and use the Doppler shift equations to derive an expression for the vertical speed of the submarine in terms of the frequency difference.

(a) Using the definition of average speed, relate the depth of the submarine to the time delay between the transmitted and reflected pulses:

$$2D = v\Delta t \Rightarrow D = \tfrac{1}{2}v\Delta t$$

Substitute numerical values and evaluate D:

$$D = \tfrac{1}{2}\left(1.54\,\text{km/s}\right)\left(80\,\text{ms}\right) = \boxed{61.6\,\text{m}}$$

(b) Express the frequency f_r received by the submarine in terms of f_0, u, and c:

$$f_r = \left(\frac{c\pm u}{c}\right)f_s \qquad (1)$$

Express the frequency f_r' received by the destroyer in terms of f_r, u, and c:

$$f_r' = \left(\frac{c\pm u}{c}\right)f_r \qquad (2)$$

Substitute equation (1) in equation (2) to eliminate f_r:

$$f_r' = \left(\frac{c\pm u}{c}\right)^2 f_s \approx \left(1\pm\frac{2u}{c}\right)f_s$$

provided $u << c$.

Express the frequency difference detected by the destroyer:

$$\Delta f = f_s - f_r' = f_s - \left(1\pm\frac{2u}{c}\right)f_s$$
$$= \frac{2u}{c}f_s$$

Solve for u:

$$u = \frac{c}{2f_s}\Delta f$$

Substitute numerical values and evaluate u:

$$u = \frac{1.54\,\text{km/s}}{2(40\,\text{MHz})}(0.0420\,\text{MHz}) = \boxed{0.809\,\text{m/s}}$$

where the positive speed indicates that the velocity of the submarine is downward.

85 ••

Picture the Problem Because the car is moving away from the radar unit, the frequency f_r it receives will be less than the frequency emitted by the unit. The radar waves reflected from the car, moving away from a stationary detector, will be of a still lower frequency f_r'. Let u represent the relative speed of the police car and the receding car (140 km/h) and use the Doppler shift equations to derive an expression for the difference between f_s, the transmitted signal, and f_r'.

Express the frequency f_r received by the moving car in terms of f_s, u, and c:

$$f_r = \left(\frac{c-u}{c}\right)f_s \qquad (1)$$

Express the frequency f_r' received by the stationary source in terms of f_r, u, and c:

$$f'_r = \left(\frac{c-u}{c}\right)f_r \qquad (2)$$

Substitute equation (1) in equation (2) to eliminate f_r:

$$f_r' = \left(1-\frac{u}{c}\right)^2 f_s \approx \left(1-\frac{2u}{c}\right)f_s$$

provided $u << c$

Express the frequency difference detected at the source:

$$\Delta f = f_s - f'_r = f_s - \left(1-\frac{2u}{c}\right)f_s = \frac{2u}{c}f_s$$

Substitute numerical values and evaluate Δf:

$$\Delta f = \left[\frac{2\left(140\frac{\text{km}}{\text{h}}\times\frac{1\,\text{h}}{3600\,\text{s}}\right)}{3\times10^8\ \text{m/s}}\right](3\times10^{10}\ \text{Hz}) = \boxed{7.78\,\text{kHz}}$$

86 ••

Picture the Problem Because the car is moving away from the radar unit, the frequency f_r it receives will be less than the frequency emitted by the unit. The radar waves reflected from the car, moving away from a stationary detector, will be of a still lower frequency f_r'. Let u represent the relative speed of the police car and the receding car (80 km/h) and use the Doppler shift equations to derive an expression for the difference between f_s, the transmitted signal, and f_r'.

Express the frequency f_r received by the moving car in terms of f_s, u, and c:

$$f_r = \left(\frac{c-u}{c}\right)f_s \qquad (1)$$

Express the frequency f_r' received by the stationary source in terms of f_r, u, and c:

$$f_r' = \left(\frac{c-u}{c}\right)f_r \qquad (2)$$

Substitute equation (1) in equation (2) to eliminate f_r:

$$f_r' = \left(1-\frac{u}{c}\right)^2 f_s \approx \left(1-\frac{2u}{c}\right)f_s$$

provided $u << c$.

Express the frequency difference detected at the source:

$$\Delta f = f_s - f_r' = f_s - \left(1-\frac{2u}{c}\right)fs_0$$

$$= \frac{2u}{c}f_s$$

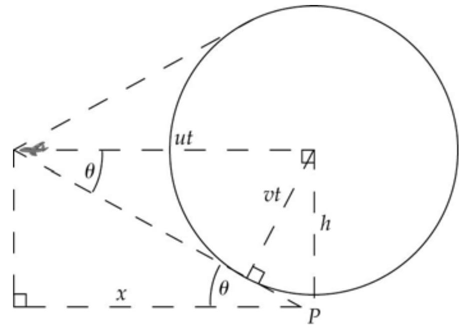
Substitute numerical values and evaluate Δf:

$$\Delta f = \left[\frac{2\left(80\,\frac{\text{km}}{\text{h}}\times\frac{1\,\text{h}}{3600\,\text{s}}\right)}{3\times10^8\ \text{m/s}}\right]\left(3\times10^{10}\ \text{Hz}\right)$$

$$= \boxed{4.44\,\text{kHz}}$$

87 ••

Picture the Problem The diagram shows the position of the supersonic plane flying due west at time t after it was directly over point P 12 km below it. Let x be measured from point P, u represent the speed of the plane, and v be the speed of sound. We can use trigonometry to determine the angle of the shock wave as well as the location of the jet x when the person on the ground hears the shock wave.



Using the diagram to relate the distance x to the shock-wave angle θ and the elevation of the plane:

$$\tan\theta = \frac{h}{x} \text{ and } x = \frac{h}{\tan\theta}$$

Referring to the diagram, express θ in terms of v, u, and t and determine its value:

$$\theta = \sin^{-1}\left(\frac{vt}{ut}\right) = \sin^{-1}\left(\frac{1}{u/v}\right)$$

$$= \sin^{-1}\left(\frac{1}{1.6}\right) = 38.7°$$

Substitute numerical values and evaluate x:

$$x = \frac{12\,\text{km}}{\tan 38.7°} = \boxed{15.0\,\text{km west of } P.}$$

88 ••

Picture the Problem The change in frequency of source as it oscillates on the air track is 3 Hz. We can use $\frac{\Delta f}{f_s} \approx \pm\frac{u}{v}$ to find the maximum speed of the vibrating mass-spring system in terms of this change in frequency and then use this speed to find the energy of the system. Knowing the energy of the system, we can find the amplitude of its motion. We can calculate the period of the motion from knowledge of the mass of the radio and the stiffness constant of the spring.

(*a*) Express the energy of the vibrating mass-spring system in terms of its maximum speed:

$$E = \tfrac{1}{2}mu_{\text{max}}^2 \quad (1)$$

Relate the change in the frequency heard by the listener to the maximum speed of the oscillator:

$$\frac{\Delta f}{f_s} = \pm\frac{u}{v}$$

where $u = u_s \pm u_r$ is the relative speed of the source and receiver and v is the speed of sound.

Solve for and evaluate $u = u_{max}$:

$$u_{max} = \frac{\Delta f}{f_0}v = \frac{3\,\text{Hz}}{800\,\text{Hz}}(340\,\text{m/s}) = 1.275\,\text{m/s}$$

Substitute numerical values in equation (1) and evaluate E:

$$E = \tfrac{1}{2}(0.1\,\text{kg})(1.275\,\text{m/s}^2)^2 = \boxed{81.3\,\text{mJ}}$$

(b) Relate the energy of the oscillator to the amplitude of its motion:

$$E = \tfrac{1}{2}kA^2$$

Solve for A to obtain:

$$A = \sqrt{\frac{2E}{k}}$$

Substitute numerical values and evaluate A:

$$A = \sqrt{\frac{2(81.3\,\text{mJ})}{200\,\text{N/m}}} = \boxed{2.85\,\text{cm}}$$

89 ••

Picture the Problem The received and transmitted frequencies are related through $f_r = \frac{v \pm u_r}{v \pm u_s} f_s$, where the variables have the meanings given in the problem statement. Because the source and receiver are moving in the same direction, we use the minus signs in both the numerator and denominator.

(a) Relate the received frequency f_r to the frequency f_0 of the source:

$$f_r = \frac{1 - u_r/v}{1 - u_s/v} f_0 = \boxed{(1 - u_r/v)(1 - u_s/v)^{-1} f_0}$$

(b) Apply the binomial expansion to $(1 - u_s/v)^{-1}$:

$$(1 - u_s/v)^{-1} \approx 1 + u_s/v$$

Substitute to obtain:

$$\begin{aligned} f_r &= (1 - u_r/v)(1 + u_s/v)f_0 \\ &= [1 + u_s/v - u_r/v - (u_r/v)(u_s/v)]f_0 \\ &\approx \left(1 + \frac{u_s - u_r}{v}\right) \end{aligned}$$

because both u_s and u_r are small compared to v.

Because $u_{rel} = u_s - u_r$

$$f_r' \approx \boxed{\left(1+\frac{u_{rel}}{v}\right)f_0}$$

90 ••

Picture the Problem Because the students are walking away from each other, the frequency f' each receives will be less than the frequency $f_s = 440$ Hz emitted by their tuning forks. Let u represent the speed of each student and use the Doppler shift equations to derive an expression for the difference between f_s, the frequency of the tuning fork each carries, and the frequency heard from the other's tuning fork. Because this equation will contain u, we'll be able to solve for and evaluate each student's walking speed.

Using equation 15-35*a*, express the frequency f_r received by either student, when they are walking away from each other, in terms of f_s, u, and v:

$$f_r = \frac{1-u/v}{1+u/v}f_s = (1-u/v)(1+u/v)^{-1}f_s$$

Expand $(1+u/v)^{-1}$ binomially:

$$(1+u/v)^{-1} \approx 1-u/v \text{ provided } u << v.$$

Substitute and simplify to obtain:

$$f_r = (1-u/v)^2 f_s = \left(1-2u/v+u^2/v^2\right)f_s \approx (1-2u/v)f_s$$

for $u << v$.

Express the frequency difference heard by each student:

$$\Delta f = f_0 - f_r = f_s - \left(1-\frac{2u}{v}\right)f_s = \frac{2u}{v}f_s$$

Solve for u:

$$u = \frac{\Delta f}{2f_s}v$$

Substitute numerical values and evaluate u:

$$u = \frac{2\text{ Hz}}{2(440\text{ Hz})}(340\text{ m/s}) = \boxed{0.773\text{ m/s}}$$

91 ••

Picture the Problem The student serves as a source moving toward the wall, and a moving receiver for the echo. The received and transmitted frequencies are related through $f_r = \dfrac{v \pm u_r}{v \pm u_s}f_s$. We'll express the frequency received by the wall and the frequency it transmits back to the moving student in order to express the difference in the frequency the student hears from the wall and frequency she hears directly from her tuning fork. Because this expression will contain the student's walking speed, we'll be able to solve for and evaluate this speed.

Relate the frequency f_r received by the wall to the frequency of the student's tuning fork f_s:

$$\Delta f = f_\mathrm{s} - f_\mathrm{r}' = f_\mathrm{s} - \left(1 - \frac{2u}{c}\right) f_\mathrm{s}$$
$$= \frac{2u}{c} f_\mathrm{s}$$

Because the source is moving toward a stationary receiver:

$$f_\mathrm{r} = \frac{1}{1 - u_\mathrm{s}/v} f_\mathrm{s} = (1 - u_\mathrm{s}/v)^{-1} f_\mathrm{s}$$

Apply the binomial expansion to $(1 - u_\mathrm{s}/v)^{-1}$:

$$(1 - u_\mathrm{s}/v)^{-1} \approx 1 + u_\mathrm{s}/v$$

because $u_\mathrm{s} << v$.

Substitute to obtain:

$$f_\mathrm{r} = (1 + u_\mathrm{s}/v) f_0$$

Relate the frequency f_r' reflected from the wall and received by the student to the frequency f_r reflected from the wall:

$$f_\mathrm{r}' = \frac{1 \pm u_\mathrm{r}/v}{1 \pm u_\mathrm{s}/v} f_\mathrm{r} = \frac{1}{1 - u_\mathrm{s}/v} f_\mathrm{r}$$

because the source (the wall) is at rest and the receiver is approaching.

Substitute for f_r to obtain:

$$f_\mathrm{r}' = \frac{1}{1 - u_\mathrm{s}/v} (1 + u_\mathrm{s}/v) f_\mathrm{s}$$
$$= (1 + u_\mathrm{s}/v)(1 - u_\mathrm{s}/v)^{-1} f_\mathrm{s}$$

Expand $(1 - u_\mathrm{s}/v)^{-1}$ binomially:

$$(1 - u_\mathrm{s}/v)^{-1} \approx 1 + u_\mathrm{s}/v$$

because $u_\mathrm{s} << v$.

$$f_\mathrm{r}' = (1 + u_\mathrm{s}/v)(1 + u_\mathrm{s}/v) f_\mathrm{s}$$
$$\approx (1 + 2u_\mathrm{s}/v) f_\mathrm{s}$$

because $u_\mathrm{s} << v$

Express the difference between the frequency the student receives from the wall and the frequency of her tuning fork:

$$\Delta f = f_\mathrm{r}' - f_\mathrm{s}$$
$$= (1 + 2u_\mathrm{s}/v) f_\mathrm{s} - f_\mathrm{s}$$
$$= \frac{2u_\mathrm{s}}{v} f_\mathrm{s}$$

Solve for u_s:

$$u_s = \frac{\Delta f}{2f_s}v$$

Substitute numerical values and evaluate u_s:

$$u_s = \frac{4\,\text{Hz}}{2(512\,\text{Hz})}(340\,\text{m/s}) = \boxed{1.33\,\text{m/s}}$$

***92 ••**

Picture the Problem The frequency heard by the stationary observer will vary with time as the speaker rotates on its support arm. We can use a Doppler equation to express the frequency heard by the observer as a function of the velocity of the source and find the velocity of the source from the expression for the tangential velocity of an object moving in a circular path.

Express the frequency f_r heard by a stationary observer:

$$f_r = \frac{1}{1-u_s/v}f_s = (1-u_s/v)^{-1}f_s$$

Expand $(1-u_s/v)^{-1}$ to obtain:

$$(1-u_s/v)^{-1} \approx 1+u_s/v$$

because $u_s/v << 1$

Substitute in the expression for f_r:

$$f_r = (1+u_s/v)f_s \qquad (1)$$

Express the speed of the source as a function of time:

$$\begin{aligned} u_s &= r\omega \sin \omega t \\ &= (0.8\,\text{m})(4\,\text{rad/s})\sin[(4\,\text{rad/s})t] \\ &= (3.2\,\text{m/s})\sin[(4\,\text{rad/s})t] \end{aligned}$$

Substitute in equation (1) to obtain:

$$f_r = \left(1+\frac{3.2\,\text{m/s}}{v}\sin[(4\,\text{rad/s})t]\right)f_s$$

Substitute for v and simplify:

$$\begin{aligned} f_r &= \left(1+\frac{3.2\,\text{m/s}}{340\,\text{m/s}}\sin[(4\,\text{rad/s})t]\right)(1000\,\text{s}^{-1}) \\ &= \boxed{1000\,\text{Hz} + (9.41\,\text{Hz})\sin[(4\,\text{rad/s})t]} \end{aligned}$$

93 ••

Picture the Problem The simplest way to approach this problem is to transform to a reference frame in which the balloon is at rest. In that reference frame, the speed of sound is v = 340 m/s, and u_r = 36 km/h = 10 m/s. Then, we can use the equations for a moving receiver and a moving source to find the frequencies heard at the window and on the balloon.

(*a*) Express the observed frequency in terms of the frequency of the source:

$$f_r = \left(1 + \frac{u_r}{v}\right) f_s$$

Substitute numerical values and evaluate f_r:

$$f_r = \left(1 + \frac{10\,\text{m/s}}{340\,\text{m/s}}\right)(800\,\text{Hz}) = \boxed{824\,\text{Hz}}$$

(*b*) Treating the tall building as a moving source, express the frequency of the reflected sound heard by a person riding in the balloon:

$$f_r' = \left(\frac{1}{1 - \frac{u_s}{v}}\right) f_r$$

Substitute numerical values and evaluate f_r':

$$f_r' = \left(\frac{1}{1 - \frac{10\,\text{m/s}}{340\,\text{m/s}}}\right)(824\,\text{Hz}) = \boxed{849\,\text{Hz}}$$

94 ••

Picture the Problem We can relate the frequencies f_r and f_r' heard by the stationary observer behind the car to the speed of the car u and the frequency of the car's horn f_s. Dividing these equations will eliminate the frequency of the car's horn and allow us to solve for the speed of the car. We can then substitute to find the frequency of the car's horn. We can find the frequency heard by the driver as a moving receiver by relating this frequency to the frequency reflected from the wall.

(*a*) Relate the frequency heard by the observer directly from the car's horn to the speed of the car:

$$f_r = \frac{1}{1 + u/v} f_s \qquad (1)$$

Relate the frequency reflected from the wall to the speed of the car:

$$f_r' = \frac{1}{1 - u/v} f_s \qquad (2)$$

Divide equation (2) by equation (1) to obtain:

$$\frac{f_r'}{f_r} = \frac{1 + u/v}{1 - u/v}$$

Solve for u:

$$u = \frac{f_r' - f_r}{f_r' + f_r'} v$$

Substitute numerical values and evaluate u:

$$u = \frac{863\,\text{Hz} - 745\,\text{Hz}}{863\,\text{Hz} + 745\,\text{Hz}}(340\,\text{m/s})$$
$$= 24.95\frac{\text{m}}{\text{s}} \times \frac{1\,\text{km}}{10^3\,\text{m}} \times \frac{3600\,\text{s}}{\text{h}}$$
$$= \boxed{89.8\,\text{km/h}}$$

(*b*) Solve equation (1) for f_s:

$$f_s = (1 + u/v)f_r$$

Substitute numerical values and evaluate f_s:

$$f_s = \left(1 + \frac{24.95\,\text{m/s}}{340\,\text{m/s}}\right)(745\,\text{Hz})$$
$$= \boxed{800\,\text{Hz}}$$

(*c*) The driver is a moving receiver and so we can relate the frequency heard by the driver to the frequency reflected by the wall (the frequency heard by the stationary observer):

$$f_{\text{driver}} = \left(1 + \frac{u}{v}\right)f_r'$$

Substitute numerical values and evaluate f_{driver}:

$$f_{\text{driver}} = \left(1 + \frac{24.95\,\text{m/s}}{340\,\text{m/s}}\right)(863\,\text{H}z)$$
$$= \boxed{926\,\text{Hz}}$$

95 ••

Picture the Problem Let $t = 0$ when the driver sounds her horn and let the distance to the cliff at that instant be d. The received and transmitted frequencies are related through $f_r = \dfrac{v \pm u_r}{v \pm u_s} f_s$. Solving this equation for f_s will allow us to determine the frequency of the car horn. We can use the total distance the sound travels (car-to-cliff plus cliff back to car … now closer to the cliff) to determine the distance to the cliff when the horn was briefly sounded.

Relate the frequency heard by the driver to her speed and to the frequency of her horn:

$$f_r = \frac{1 + u_r/v}{1 - u_s/v} f_s$$

Solve for f_s:

$$f_s = \frac{1 - u_s/v}{1 + u_r/v} f_r$$

Substitute numerical values and evaluate f_s:

$$f_0=\frac{1-\dfrac{100\dfrac{\text{km}}{\text{h}}\times\dfrac{1\,\text{h}}{3600\,\text{s}}}{340\,\text{m/s}}}{1+\dfrac{100\dfrac{\text{km}}{\text{h}}\times\dfrac{1\,\text{h}}{3600\,\text{s}}}{340\,\text{m/s}}}(840\,\text{Hz})=\frac{1-\dfrac{27.78\,\text{m/s}}{340\,\text{m/s}}}{1+\dfrac{27.78\,\text{m/s}}{340\,\text{m/s}}}(840\,\text{Hz})=713\,\text{Hz}$$

Relate the distance d to the cliff at $t = 0$ to the distance she travels in time $\Delta t = 1$ s, her speed u, and the speed of sound v:

$$d+(d-u\Delta t)=v\Delta t$$

Solve for d:

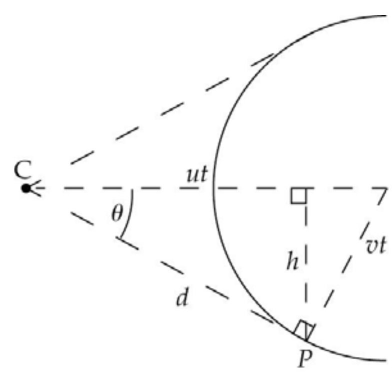
$$d=\tfrac{1}{2}(u+v)\Delta t$$

Substitute numerical values and evaluate d:

$$d=\tfrac{1}{2}(27.78\,\text{m/s}+340\,\text{m/s})(1\,\text{s})$$
$$=\boxed{184\,\text{m}}$$

96 ••

Picture the Problem You'll year the sonic boom when the surface of its cone reaches your plane. In the diagram the Concorde is at C and your plane is at P. The distance $h = 3$ km. The distance between the planes when you hear the sonic boom is d. We can use trigonometry to determine the angle of the shock wave as well as the separation of the planes when you hear the sonic boom.



Using the Pythagorean theorem, relate the separation of the planes d, to the distance h and the angle θ:

$$d^2=h^2+d^2\cos^2\theta$$

Solve for d to obtain:

$$d=h\sqrt{\frac{1}{1-\cos^2\theta}}$$

Express θ in terms of v, u, and t:

$$\theta=\sin^{-1}\left(\frac{vt}{ut}\right)=\sin^{-1}\left(\frac{1}{u/v}\right)$$

Substitute numerical values and evaluate θ:

$$\theta = \sin^{-1}\left(\frac{1}{1.6}\right) = \boxed{38.7°}$$

Substitute numerical values and evaluate *h*:

$$d = (3\,\text{km})\sqrt{\frac{1}{1-\cos^2 38.7°}} = \boxed{4.80\,\text{km}}$$

***97 ••**

Picture the Problem The sun and Jupiter orbit about their effective mass located at their common center of mass. We can apply Newton's 2nd law to the sun to obtain an expression for its orbital speed about the sun-Jupiter center of mass and then use this speed in the Doppler shift equation to estimate the maximum and minimum wavelengths resulting from the Jupiter-induced motion of the sun.

Letting *v* be the orbital speed of the sun about the center of mass of the sun-Jupiter system, express the Doppler shift of the light due to this motion when the sun is approaching the earth:

$$f' = \frac{c}{\lambda'} = f\sqrt{\frac{1+v/c}{1-v/c}} = \frac{c}{\lambda}\sqrt{\frac{1+v/c}{1-v/c}}$$

Solve for λ':

$$\begin{aligned}\lambda' &= \lambda\sqrt{\frac{1-v/c}{1+v/c}}\\ &= \lambda\sqrt{(1-v/c)(1+v/c)^{-1}}\\ &= \lambda(1-v/c)^{1/2}(1+v/c)^{-1/2}\end{aligned}$$

Because $v \ll c$, we can expand $(1-v/c)^{1/2}$ and $(1+v/c)^{-1/2}$ binomially to obtain:

$$(1-v/c)^{1/2} \approx 1-\frac{v}{2c}$$

and

$$(1+v/c)^{-1/2} \approx 1-\frac{v}{2c}$$

Substitute to obtain:

$$\sqrt{\frac{1-v/c}{1+v/c}} = \left(1-\frac{v}{2c}\right)^2 \approx 1-\frac{v}{c}$$

When the sun is receding from the earth:

$$\sqrt{\frac{1+v/c}{1-v/c}} = \left(1+\frac{v}{2c}\right)^2 \approx 1+\frac{v}{c}$$

Hence the motion of the sun will give an observed Doppler shift of:

$$\lambda' \approx \lambda\left(1 \pm \frac{v}{c}\right) \qquad (1)$$

Apply Newton's 2nd law to the sun:

$$\frac{GM_S M_{eff}}{r_{cm}^2} = M_S \frac{v^2}{r_{cm}}$$

Solve for v to obtain:

$$v = \sqrt{\frac{GM_{eff}}{r_{cm}}}$$

Measured from the center of the sun, the distance to the center of mass of the sun-Jupiter system is:

$$r_{cm} = \frac{(0)M_S + r_{s\text{-}J} M_J}{M_s + M_J} = \frac{r_{s\text{-}J} M_J}{M_s + M_J}$$

The effective mass is related to the masses of the sun and Jupiter according to:

$$\frac{1}{M_{eff}} = \frac{1}{M_s} + \frac{1}{M_J}$$

or

$$M_{eff} = \frac{M_s M_J}{M_s + M_J}$$

Substitute for M_{eff} and r_{cm} to obtain:

$$v = \sqrt{\frac{G\dfrac{M_s M_J}{M_s + M_J}}{\dfrac{r_{s\text{-}J} M_J}{M_s + M_J}}} = \sqrt{\frac{GM_s}{r_{s\text{-}J}}}$$

Using $r_{s\text{-}J} = 7.78 \times 10^{11}$ m as the mean orbital radius of Jupiter, substitute numerical values and evaluate v:

$$v = \sqrt{\frac{(6.673 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99 \times 10^{30}\ \text{kg})}{7.78 \times 10^{11}\ \text{m}}} = 1.306 \times 10^4\ \text{m/s}$$

Substitute in equation (1) to obtain:

$$\lambda' \approx (500\ \text{nm})\left(1 \pm \frac{1.306 \times 10^4\ \text{m/s}}{2.998 \times 10^8\ \text{m/s}}\right) = (500\ \text{nm})(1 \pm 4.36 \times 10^{-5})$$

The maximum and minimum wavelengths are:

$$\lambda_{max} = \boxed{(500\ \text{nm})(1 + 4.36 \times 10^{-5})}$$

and

$$\lambda_{min} = \boxed{(500\ \text{nm})(1 - 4.36 \times 10^{-5})}$$

98 •••

Picture the Problem Choose a coordinate system in which downward is the positive y direction. Let d represent the distance the tuning fork has fallen when the student hears a frequency of 400 Hz, t_1 the time for the source to fall that distance, and t_2 the time for the sound to travel back to the student. We can a constant-acceleration equation to express d in terms of the time that elapses between the dropping of the tuning fork and the return of the sound to the student. A Doppler-effect equation will allow us to solve for the speed of the tuning fork when the student hears a frequency of 400 Hz; we can use constant-acceleration equations to find the fall time for the fork and the return time for the sound from the tuning fork.

Using a constant-acceleration equation, relate the distance the source has fallen to the elapsed time:

$$d = v_0 t + \tfrac{1}{2}at^2$$

or, because $v_0 = 0$ and $a = g$,

$$d = \tfrac{1}{2}gt^2 \qquad (1)$$

where $t = t_1 + t_2$.

Relate the frequency f_r heard by the student to the speed of the falling tuning fork:

$$f_r = \frac{1}{1 + u_s/v} f_s$$

Solve for u_s:

$$u_s = \left(\frac{f_s}{f_r} - 1\right)v$$

Substitute numerical values and evaluate u_s:

$$u_s = \left(\frac{440\,\text{Hz}}{400\,\text{Hz}} - 1\right)(340\,\text{m/s}) = 34.0\,\text{m/s}$$

Letting y be the distance the fork has fallen when its speed is u_s, use a constant-acceleration equation to relate y and u_s:

$$u_s^2 = v_0^2 + 2gy$$

or, because $v_0 = 0$,

$$u_s^2 = 2gy$$

Solve for y:

$$y = \frac{u_s^2}{2g}$$

Substitute numerical values and evaluate y:

$$y = \frac{(34\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} = 58.92\,\text{m}$$

Using a constant-acceleration equation, relate the speed of the falling tuning fork to its time of fall t_1:

$$u_s = v_0 + gt_1$$

or, because $v_0 = 0$,

$$u_s = gt_1$$

Solve for and evaluate t_1:

$$t_1 = \frac{u_s}{g} = \frac{34.0\,\text{m/s}}{9.81\,\text{m/s}^2} = 3.466\,\text{s}$$

Using the relationship between distance traveled, time and average speed, find the time t_2 for the sound to travel back to the student:

$$t_2 = \frac{y}{v} = \frac{58.92\,\text{m}}{340\,\text{m/s}} = 0.173\,\text{s}$$

Substitute in equation (1) and evaluate d:

$$d = \tfrac{1}{2}\left(9.81\,\text{m/s}^2\right)\left(3.466\,\text{s} + 0.173\,\text{s}\right)^2 = \boxed{65.0\,\text{m}}$$

99 ••

Picture the Problem The angle θ of the Cerenkov shock wave is related to the speed of light in water v and the speed of light in a vacuum c according to $\sin\theta = v/c$.

Relate the speed of light in water v to the angle of the Cerenkov cone:

$$\sin\theta = \frac{v}{c}$$

Solve for v:

$$v = c\sin\theta$$

Substitute numerical values and evaluate v:

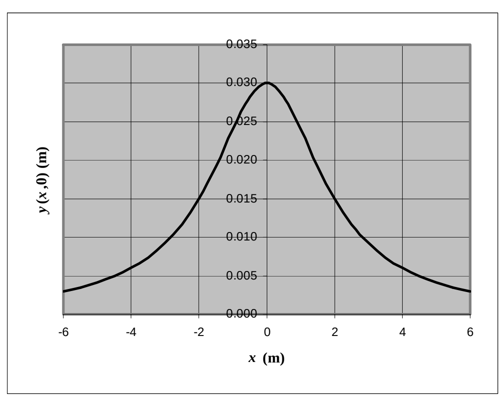
$$v = \left(2.998\times10^8\ \text{m/s}\right)\sin 48.75° = \boxed{2.25\times10^8\ \text{m/s}}$$

General Problems

100 •

Picture the Problem The equation of a wave traveling in the positive x direction is of the form $y(x,t) = f(x - vt)$ and that of a wave traveling in the negative x direction is $y(x,t) = f(x + vt)$.

(a) The pulse at $t = 0$ shown to the right was plotted using a spreadsheet program:



(b) The wave function must be of the form:

$$y(x,t) = f(x - vt) = f\left[x - (10\,\text{m/s})t\right]$$

because $v = 10$ m/s

Replace x with $x-(10\,\text{m/s})t$ to obtain:

$$y(x,t)=\boxed{\frac{0.12\,\text{m}^3}{(2.00\,\text{m})^2+[x-(10\,\text{m/s})t]^2}}$$

(c) The wave function must be of the form:

$$y(x,t)=f(x+vt)=f[x+(10\,\text{m/s})t]$$

because v = 10 m/s

Replace x with $x+(10\,\text{m/s})t$ to obtain:

$$y(x,t)=\boxed{\frac{0.12\,\text{m}^3}{(2.00\,\text{m})^2+[x+(10\,\text{m/s})t]^2}}$$

101 •

Picture the Problem Let the subscript 1 refer to the initial situation–a tension of 800 N and a wavelength of 24 cm. Let the subscript 2 refer to the conditions that the tension is 600 N and the wavelength unknown. We can express the wavelengths of the waves on the wire in terms of the two tensions in the wire and then eliminate the constant frequency by expressing the ratio of the two wavelengths. Finally, we can solve this equation for λ_2.

Express the wavelength in terms of the frequency and speed of the wave:

$$\lambda=\frac{v}{f}$$

Express the speed of the wave as a function of the tension in the wire:

$$v=\sqrt{\frac{T}{\mu}}$$

Substitute to obtain:

$$\lambda=\frac{1}{f}\sqrt{\frac{T}{\mu}}$$

Express the wavelength when the tension in the wire is 600 N:

$$\lambda_2=\frac{1}{f}\sqrt{\frac{T_2}{\mu}}$$

Express the wavelength when the tension in the wire is 800 N:

$$\lambda_1=\frac{1}{f}\sqrt{\frac{T_1}{\mu}}$$

Divide the first of these equations by the second and solve for λ_2:

$$\lambda_2=\lambda_1\sqrt{\frac{T_2}{T_1}}$$

Substitute numerical values and evaluate λ_2:

$$\lambda_2=(24\,\text{cm})\sqrt{\frac{600\,\text{N}}{800\,\text{N}}}=\boxed{20.8\,\text{cm}}$$

102 •

Picture the Problem Let m represent the mass of the rubber tubing whose length is L. We can express the travel time for the pulse t in terms of the separation of the post and the pulley and its speed. The speed of the pulse, in turn, can be found from the tension in the rubber tubing and its linear density.

Express the time required for the pulse to travel the length of the tubing in terms of its speed and the length of the rubber tubing:

$$t = \frac{L}{v}$$

Relate the speed of the pulses to the tension in the tubing:

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{F}{m/L}} = \sqrt{\frac{FL}{m}}$$

Substitute for v and simplify to obtain:

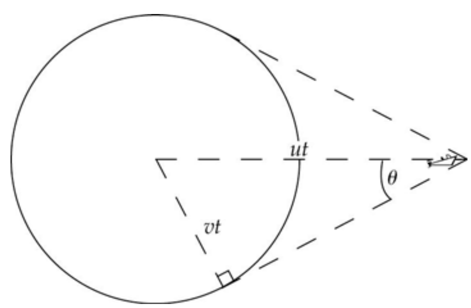
$$t = \sqrt{\frac{Lm}{F}}$$

Substitute numerical values and evaluate t:

$$t = \sqrt{\frac{(10\,\text{m})(0.7\,\text{kg})}{110\,\text{N}}} = \boxed{0.252\,\text{s}}$$

103 •

Picture the Problem The diagram shows the boat traveling on a still lake with a speed v. A bow wave generated a time t earlier is shown at an angle of θ with the direction of the boat's motion. We can use trigonometry to relate the speed of the bow wave to the speed of the boat.



Using the diagram, relate u and v to the angle θ:

$$\sin\theta = \frac{vt}{ut} = \frac{v}{u}$$

Solve for v:

$$v = u\sin\theta$$

Substitute numerical values and evaluate v:

$$v = (10\,\text{m/s})\sin 20° = \boxed{3.42\,\text{m/s}}$$

104 •

Picture the Problem The frequencies and wavelengths of the sound waves are related to the speed of sound through $f = v/\lambda$.

(*a*) Use $f = v/\lambda$ to find f:

$$f = \frac{340\,\text{m/s}}{10(0.3\,\text{m})} = \boxed{113\,\text{Hz}}$$

(*b*) Proceed as in (*a*):

$$f = \frac{340\,\text{m/s}}{0.1(0.3\,\text{m})} = \boxed{11.3\,\text{kHz}}$$

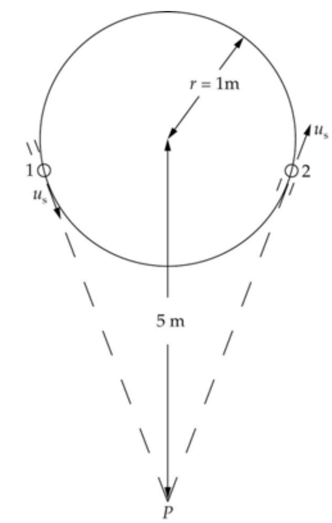
(*c*) Proceed as in (*a*) and (*b*):

$$f = \frac{340\,\text{m/s}}{10(0.06\,\text{m})} = \boxed{567\,\text{Hz}}$$

$$f = \frac{340\,\text{m/s}}{0.1(0.06\,\text{m})} = \boxed{56.7\,\text{kHz}}$$

105 •

Picture the Problem The diagram depicts the whistle traveling in a circular path of radius $r = 1$ m. The stationary listener will hear the maximum frequency when the whistle is at point 1 and the minimum frequency when it is at point 2. These maximum and minimum frequencies are determined by f_0 and the tangential speed $u_s = 2\pi r/T$. We can relate the frequencies heard at point P to the speed of the approaching whistle at point 1 and the speed of the receding whistle at point 2.



Relate the frequency heard at point P to the speed of the approaching whistle at point 1:

$$f_{\max} = \frac{1}{1 - u_s/v} f_s$$

Use the relationship between translational velocity and angular velocity to find the speed u_s of the whistle:

$$u_s = r\omega = (1\,\text{m})\left(3\frac{\text{rev}}{\text{s}} \times \frac{2\pi\,\text{rad}}{\text{rev}}\right)$$
$$= 18.85\,\text{m/s}$$

Substitute numerical values and evaluate $f_{\max}$:

$$f_{\max} = \frac{1}{1 - \dfrac{18.85\,\text{m/s}}{340\,\text{m/s}}}(500\,\text{Hz})$$
$$= \boxed{529\,\text{Hz}}$$

Relate the frequency heard at point P to the speed of the receding whistle at point 2:

$$f_{min} = \frac{1}{1 + u_s/v} f_s$$

Substitute numerical values and evaluate f_{min}:

$$f_{min} = \frac{1}{1 + \dfrac{18.85\,\text{m/s}}{340\,\text{m/s}}}(500\,\text{Hz}) = \boxed{474\,\text{Hz}}$$

106 •

Picture the Problem The crest-to-crest separation of the waves is their wavelength. We can find the frequency of the waves from $v = f\lambda$. When you lift anchor and head out to sea you'll become a moving receiver and we can apply $f_r = \dfrac{v \pm u_r}{v \pm u_s} f_s$ to calculate the frequency you'll observe.

(*a*) Express the frequency of the ocean waves in terms of their speed and wavelength:

$$f_0 = \frac{v}{\lambda}$$

Substitute numerical values and evaluate f_s:

$$f_0 = \frac{8.9\,\text{m/s}}{15\,\text{m}} = \boxed{0.593\,\text{Hz}}$$

(*b*) Express the frequency of the waves in terms of their speed and the speed of a moving receiver:

$$f_r = (1 + u_r/v) f_s$$

Substitute numerical values and evaluate f_r:

$$f_r = \left(1 + \frac{15\,\text{m/s}}{8.9\,\text{m/s}}\right)(0.593\,\text{Hz}) = \boxed{1.59\,\text{Hz}}$$

107 ••

Picture the Problem Let t be the time of travel of the lefthand pulse and the subscripts L and R refer to the pulse coming from the left and right, respectively. Because the pulse traveling from the right starts later than the pulse from the left, its travel time is $t - \Delta t$, where $\Delta t = 25$ ms. Both pulses travel at the same speed and the sum of the distances they travel is 12 m.

Express the total distance the two pulses travel:

$$d = d_L + d_R = vt + v(t - \Delta t)$$

Solve for vt to obtain: $vt = \frac{1}{2}(d + v\Delta t)$

The speed of the pulse is given by: $v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{F}{m/L}}$

Substitute for v to obtain: $vt = \frac{1}{2}\left(d + \sqrt{\frac{F}{m/L}}\Delta t\right)$

Substitute numerical values and evaluate vt:

$$vt = \frac{1}{2}\left[12\,\text{m} + \sqrt{\frac{(180\,\text{N})(12\,\text{m})}{0.085\,\text{kg}}}\left(25\times10^{-3}\,\text{s}\right)\right] = \boxed{7.99\,\text{m}}$$

***108 ••**

Picture the Problem Let the frequency of the car's horn be f_s, the frequency you hear as the car approaches f_r, and the frequency you hear as the car recedes f_r'. We can use $f_r = \frac{v \pm u_r}{v \pm u_s} f_s$ to express the frequencies heard as the car approaches and recedes and then use these frequencies to express the fractional change in frequency as the car passes you.

Express the fractional change in frequency as the car passes you: $\frac{\Delta f}{f_r} = 0.1$

Relate the frequency heard as the car approaches to the speed of the car: $f_r = \frac{1}{1 - u_s/v} f_s$

Express the frequency heard as the car recedes in terms of the speed of the car: $f_r' = \frac{1}{1 + u_s/v} f_s$

Divide the second of these frequency equations by the first to obtain: $\frac{f_r'}{f_r} = \frac{1 - u_s/v}{1 + u_s/v}$

and

$$\frac{f_r}{f_r} - \frac{f_r'}{f_r} = \frac{\Delta f}{f_r} = 1 - \frac{1 - u_s/v}{1 + u_s/v} = 0.1$$

Solve u_s: $u_s = \frac{0.1}{1.9} v$

Substitute numerical values and evaluate u_s:

$$u_s = \frac{0.1}{1.9}(340\,\text{m/s}) = 17.89\frac{\text{m}}{\text{s}} \times \frac{1\,\text{km}}{10^3} \times \frac{3600\,\text{s}}{\text{h}} = \boxed{64.4\,\text{km/h}}$$

109 ••

Picture the Problem The pressure amplitude can be calculated directly from $p_0 = \rho\omega v s_0$, and the intensity from $I = \frac{1}{2}\rho\omega^2 s_0^2 v$. The power radiated is the intensity times the area of the driver.

(*a*) Relate the pressure amplitude to the displacement amplitude, angular frequency, wave velocity, and air density:

$$p_0 = \rho\omega v s_0$$

Substitute numerical values and evaluate p_0:

$$p_0 = (1.29\,\text{kg/m}^3)\left[2\pi(800\,\text{s}^{-1})\right] \times (340\,\text{m/s})(0.025\times10^{-3}\,\text{m}) = \boxed{55.1\,\text{N/m}^2}$$

(*b*) Relate the intensity to these same quantities:

$$I = \tfrac{1}{2}\rho\omega^2 s_0^2 v$$

Substitute numerical values and evaluate I:

$$I = \tfrac{1}{2}(1.29\,\text{kg/m}^3)\left[2\pi(800\,\text{s}^{-1})\right]^2 \times (0.025\times10^{-3}\,\text{m})^2(340\,\text{m/s}) = \boxed{3.46\,\text{W/m}^2}$$

(*c*) Express the power in terms of the intensity and the area of the driver:

$$P = IA = \pi r^2 I$$

Substitute numerical values and evaluate P:

$$P = \pi(0.1\,\text{m})^2(3.46\,\text{W/m}^2) = \boxed{0.109\,\text{W}}$$

110 ••

Picture the Problem The frequency of the sound wave is related to the density of the air, displacement amplitude, and velocity by $I = \frac{1}{2}\rho\omega^2 s_0^2 v$.

Relate the intensity of the sound

$$I = \tfrac{1}{2}\rho\omega^2 s_0^2 v$$

wave to the density of the air, displacement amplitude, velocity, and angular frequency:

Solve for the angular frequency:

$$\omega = \frac{1}{s_0}\sqrt{\frac{2I}{\rho v}}$$

Solve for f:

$$f = \frac{1}{2\pi s_0}\sqrt{\frac{2I}{\rho v}}$$

Substitute numerical values and evaluate f:

$$f = \frac{1}{2\pi(10^{-6}\text{ m})}\sqrt{\frac{2(10^{-2}\text{ W/m}^2)}{(1.29\text{ kg/m}^3)(340\text{ m/s})}} = \boxed{1.07\text{ kHz}}$$

111 ••

Picture the Problem The force exerted on the plate is due to the change in momentum of the water. We can use Newton's 2nd law in the form $F = \Delta p/\Delta t$ to relate F to the mass of water in a length of tube equal to $v_s\Delta t$ and to the speed of the water. This mass of water, in turn, is given by the product of its density and the volume of water in a length of the tube equal to $v_s\Delta t$.

Relate the force exerted on the plate to the change in momentum of the water:

$$F = \frac{\Delta p}{\Delta t} = \frac{\Delta m v_w}{\Delta t}$$

Express Δm in terms of the mass of water in a length of tube equal to $v_s\Delta t$:

$$\Delta m = \rho\Delta V = \rho v_s A\Delta t$$

Substitute to obtain:

$$F = \rho v_s A v_w$$

Substitute numerical values and evaluate F:

$$F = (10^3\text{ kg/m}^3)(1.4\text{ km/s})\left[\pi(0.05\text{ m})^2\right](7\text{ m/s}) = \boxed{77.0\text{ kN}}$$

112 ••

Picture the Problem Let d be the horizontal distance from the soap bubble to the position of the microphone. The angle θ of the shock wave is related to the speed of sound in air u and the speed of the bullet c according to $\sin\theta = u/v$. We can determine θ from the given information and then use this angle to find d.

Express d in terms of the angle of the shock wave and the distance from the soap bubble to the laboratory bench:

$$d = \frac{0.35\,\text{m}}{\tan\theta}$$

Relate the speed of the bullet to the angle of the shock-wave cone:

$$\sin\theta = \frac{u}{v}$$

Solve for θ to obtain:

$$\theta = \sin^{-1}\frac{u}{v}$$

Substitute to obtain:

$$d = \frac{0.35\,\text{m}}{\tan\left[\sin^{-1}\dfrac{u}{v}\right]}$$

Substitute numerical values and evaluate d:

$$d = \frac{0.35\,\text{m}}{\tan\left[\sin^{-1}\left(\dfrac{u}{1.25u}\right)\right]} = \boxed{26.3\,\text{cm}}$$

113 ••

Picture the Problem The source of the problem is that it takes a finite time for the sound to travel from the front of the line of marchers to the back. We can use the given data to determine the time required for the beat to reach the marchers in the back of the column and then use this time and the speed of sound to find the length of the column.

Express the length of the column in terms of the speed of sound and the time required for the beat to travel the length of the column:

$$L = v\Delta t$$

Calculate the time for the sound to travel the length of the column:

$$\Delta t = \frac{1}{100}\,\text{min} = 0.6\,\text{s}$$

Substitute and evaluate L:

$$L = (340\,\text{m/s})(0.6\,\text{s}) = \boxed{204\,\text{m}}$$

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Picture the Problem The interval between the arrival times of the echo pulses heard by the bat is the reciprocal of the frequency of the reflected pulses. We can use $f_\text{r} = \dfrac{v \pm u_\text{r}}{v \pm u_\text{s}} f_\text{s}$ to relate the frequency of the reflected pulses to the speed of the bat and the frequency it emits.

Relate the interval between the arrival times of the echo pulses heard by the bat to frequency of the reflected pulses:

$$\Delta t = \frac{1}{f_{\mathrm{r}}}$$

Relate the frequency of the pulses received by the bat to its speed and the frequency it emits:

$$f_{\mathrm{r}} = \frac{1 + u_{\mathrm{r}}/v}{1 - u_{\mathrm{s}}/v} f_{\mathrm{s}}$$

Substitute to obtain:

$$\Delta t = \frac{1 - u_{\mathrm{s}}/v}{(1 + u_{\mathrm{r}}/v) f_{\mathrm{s}}}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{1 - \dfrac{12\,\mathrm{m/s}}{340\,\mathrm{m/s}}}{\left(1 + \dfrac{12\,\mathrm{m/s}}{340\,\mathrm{m/s}}\right)\left(80\,\mathrm{s}^{-1}\right)} = \boxed{11.6\,\mathrm{ms}}$$

***115** ••

Picture the Problem Let d be the distance to the moon, h be the height of earth's atmosphere, and v be the speed of light in earth's atmosphere. We can express d', the distance measured when the earth's atmosphere is ignored, in terms of the time for a pulse of light to make a round-trip from the earth to the moon and solve this equation for the length of correction $d' - d$.

Express the roundtrip time for a pulse of light to reach the moon and return:

$$t = t_{\text{earth's atmosphere}} + t_{\text{out of earth's atmosphere}}$$
$$= 2\frac{h}{v} + 2\frac{d-h}{c}$$

Express the "measured" distance d' when we do not account for the atmosphere:

$$d' = \frac{1}{2}ct = \frac{1}{2}c\left(2\frac{h}{v} + 2\frac{d-h}{c}\right)$$
$$= \frac{c}{v}h + d - h$$

Solve for the length of correction $d' - d$:

$$d' - d = h\left(\frac{c}{v} - 1\right)$$

Substitute numerical values and evaluate $d' - d$:

$$d' - d = (8\,\mathrm{km})\left(\frac{c}{0.99997c} - 1\right)$$
$$= \boxed{24.0\,\mathrm{cm}}$$

Remarks: This is larger than the accuracy of the measurements, which is about 3 to 4 cm.

116 ••

Picture the Problem The frequency of the waves on the wire is the same as the frequency of the tuning fork and their period is the reciprocal of the frequency. We can find the speed of the waves from the tension in the wire and its linear density. The wavelength can be determined from the frequency and the speed of the waves and the wave number from its definition. The general form of the wave function for waves on a wire is $y(x,t) = A\sin(kx \pm \omega t)$, so, once we know k and ω, because A is given, we can write a suitable wave function for the waves on this wire. The maximum speed and acceleration of a point on the wire can be found from the angular frequency and amplitude of the waves. Finally, we can use $P_{\text{av}} = \frac{1}{2}\mu\omega^2 A^2 v$ to find the average rate at which energy must be supplied to the tuning fork to keep it oscillating with a steady amplitude.

(*a*) The frequency of the waves on the wire is the same as the frequency of the tuning fork:

$$f = \boxed{400\,\text{Hz}}$$

The period of the waves on the wire is the reciprocal of their frequency:

$$T = \frac{1}{f} = \frac{1}{400\,\text{s}^{-1}} = \boxed{2.50\,\text{ms}}$$

(*b*) Relate the speed of the waves to the tension in the wire and its linear density:

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{1\,\text{kN}}{0.01\,\text{kg/m}}} = \boxed{316\,\text{m/s}}$$

(*c*) Use the relationship between the wavelength, speed and frequency of a wave to find λ:

$$\lambda = \frac{v}{f} = \frac{316\,\text{m/s}}{400\,\text{s}^{-1}} = \boxed{79.0\,\text{cm}}$$

Using its definition, express and evaluate the wave number:

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{79\times10^{-2}\,\text{m}} = \boxed{7.95\,\text{m}^{-1}}$$

(*d*) Determine the angular frequency of the waves:

$$\omega = 2\pi f = 2\pi\left(400\,\text{s}^{-1}\right) = 2.51\times10^{3}\,\text{s}^{-1}$$

Substitute for A, k, and ω in the general form of the wave function to obtain:

$$y(x,t) = \boxed{(0.50\,\text{mm})\sin\left[\left(7.95\,\text{m}^{-1}\right)x - \left(2.51\times10^{3}\,\text{s}^{-1}\right)t\right]}$$

(*e*) Relate the maximum speed of a point on the wire to the amplitude of the waves and the angular frequency of the tuning fork:

$$v_{max} = A\omega = (0.5\times10^{-3}\text{ m})(2.51\times10^{3}\text{ s}^{-1}) = \boxed{1.26\text{ m/s}}$$

Express the maximum acceleration of a point on the wire in terms of the amplitude of the waves and the angular frequency of the tuning fork:

$$a_{max} = A\omega^2 = (0.5\times10^{-3}\text{ m})(2.51\times10^{3}\text{ s}^{-1})^2 = \boxed{3.15\times10^{3}\text{ m/s}^2}$$

(*f*) Express the average power required to keep the tuning fork oscillating at a steady amplitude in terms of the linear density of the wire, the amplitude of its vibrations, and the speed of the waves on the wire:

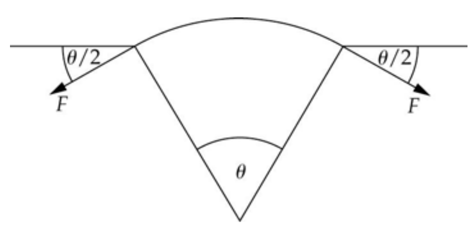
$$P_{av} = \tfrac{1}{2}\mu\omega^2 A^2 v$$

Substitute numerical values and evaluate P_{av}:

$$P_{av} = \tfrac{1}{2}(0.1\text{ kg/m})(2.51\times10^{3}\text{ s}^{-1})^2(0.5\times10^{-3}\text{ m})^2(316\text{ m/s}) = \boxed{24.9\text{ W}}$$

117 •••

Picture the Problem Because the chain is rolling at high speed we can neglect the effect of gravity. The diagram shows a small portion of the chain. We'll assume that the angle θ is small even though it shown as a large angle in the diagram. Let Δm be the mass of the segment of the chain shown. We'll apply Newton's 2nd law to the segment in order to relate the tension in the chain to its linear density and speed.



(*a*) Apply $\sum F_{radial} = m\dfrac{v^2}{R}$ to a segment of the chain whose mass is Δm:

$$F_{net} = \Delta m\frac{v_0^2}{R}$$

Express Δm in terms of μ, θ, and R:

$$\Delta m = \mu d\ell = \mu R\theta$$

Express F_{net} in terms of T and θ:

$$F_{net} = 2F \sin\tfrac{1}{2}\theta$$

Substitute to obtain:

$$2F \sin\tfrac{1}{2}\theta = \mu\theta v_0^2$$

Solve for F:

$$F = \frac{\mu\theta v_0^2}{2\sin\frac{1}{2}\theta}$$

Apply the small angle approximation $\sin\frac{1}{2}\theta \approx \frac{1}{2}\theta$:

$$F = \frac{\mu\theta v_0^2}{2\left(\frac{1}{2}\theta\right)} = \boxed{\mu v_0^2}$$

(*b*) The wave speed is the same as the speed at which the chain is moving:

$$v_0 = \boxed{\sqrt{\frac{F}{\mu}}}$$

(*c*) As seen by an observer at rest, the pulse remains at the same position because its speed along the chain is the same as the speed of the chain. With respect to a fixed point on the chain, the pulse travels through 360°.

118 •••

Picture the Problem Let Δm represent the mass of the segment of length Δx = 1 mm. We can find the wave speed from the given data for the tension in the rope and its linear density. The wavelength can be found from $v = f\lambda$. We'll use the definition of linear momentum to find the maximum transverse linear momentum of the 1-mm segment and apply Newton's 2nd law to the segment to find the maximum net force on it.

(*a*) Find the wave speed from the tension and linear density:

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{10\,\text{N}}{0.1\,\text{kg/m}}} = \boxed{10.0\,\text{m/s}}$$

(*b*) Express the wavelength in terms of the speed and frequency of the wave:

$$\lambda = \frac{v}{f} = \frac{10\,\text{m/s}}{5\,\text{s}^{-1}} = \boxed{2.00\,\text{m}}$$

(*c*) Relate the maximum transverse linear momentum of the 1-mm segment to the maximum transverse speed of the wave:

$$p_{max} = \Delta m v_{max} = \mu\Delta x A\omega = 2\pi f\mu\Delta x A$$

Substitute numerical values and evaluate p_{max}:

$$p_{max} = 2\pi\left(5\,\text{s}^{-1}\right)\left(0.1\,\text{kg/m}\right) \times \left(1\times10^{-3}\,\text{m}\right)\left(0.04\,\text{m}\right) = \boxed{1.26\times10^{-4}\,\text{kg}\cdot\text{m/s}}$$

(*d*) Apply $\sum F_{radial} = m\frac{v^2}{r}$ to the 1-mm segment and simplify to obtain:

$$F_{max} = \Delta m\frac{v^2}{A} = \mu\Delta x\frac{A^2\omega^2}{A} = \mu\Delta x A\omega^2 = \omega p_{max} = 2\pi f p_{max}$$

Substitute numerical values and evaluate F_{max}:

$$F_{max} = 2\pi\left(5\,\text{s}^{-1}\right)\left(1.26\times10^{-4}\,\text{kg}\cdot\text{m/s}\right) = \boxed{3.96\,\text{mN}}$$

***119** •••

Picture the Problem We can relate the speed of the pulse to the tension in the rope and its linear density. Because the rope hangs vertically, the tension in it varies linearly with the distance from its bottom. Once we've established the result in part (*a*), we can integrate the resulting velocity equation to find the time for the pulse to travel the length of the rope and then double this time to get the round-trip time.

(*a*) Relate the speed of transverse waves to tension and linear density:

$$v = \sqrt{\frac{F}{\mu}}$$

Express the force acting on a segment of the rope of length *y*:

$$F = mg = \mu y g$$

Substitute to obtain:

$$v = \sqrt{\frac{\mu y g}{\mu}} = \boxed{\sqrt{gy}}$$

(*b*) Because the speed of the pulse varies with the distance from the bottom of the rope, express *v* as *dy*/*dt* and solve for *dt*:

$$\frac{dy}{dt} = \sqrt{gy} \quad \text{and} \quad dt = \frac{1}{\sqrt{g}}\frac{dy}{\sqrt{y}}$$

Integrate the left side of the equation from 0 to *t* and the right side from 0 to 3 m:

$$\int_0^t dt' = \frac{1}{\sqrt{g}}\int_0^{3\,\text{m}}\frac{dy}{\sqrt{y}}$$

and

$$t = \frac{1}{\sqrt{g}}\left(2\sqrt{y}\right)\Big|_0^{3\,\text{m}} = \frac{2\sqrt{3\,\text{m}}}{\sqrt{9.81\,\text{m/s}^2}} = 1.106\,\text{s}$$

The time for the pulse to make the round-trip is:

$$t_{\text{round-trip}} = 2t = 2(1.106\,\text{s}) = \boxed{2.21\,\text{s}}$$

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Picture the Problem We can follow the step-by-step instructions outlined above to obtain the given expressions for ΔU.

(*a*) Express the potential energy of a segment of the string:

$$\Delta U = F(\Delta\ell - \Delta x)$$

For $\Delta y/\Delta x << 1$:

$$\Delta\ell = \Delta x\left[1 + \tfrac{1}{2}(\Delta y/\Delta x)^2\right]$$

and

$$\begin{aligned}\Delta\ell - \Delta x &= \Delta x\left[1 + \tfrac{1}{2}(\Delta y/\Delta x)^2\right] - \Delta x \\ &= \tfrac{1}{2}(\Delta y/\Delta x)^2\,\Delta x\end{aligned}$$

Substitute to obtain:

$$\begin{aligned}\Delta U &= F\left[\tfrac{1}{2}(\Delta y/\Delta x)^2\right] \\ &= \boxed{\tfrac{1}{2}F(\Delta y/\Delta x)^2\,\Delta x}\end{aligned}$$

(*b*) Differentiate $y(x,t) = A\sin(kx - \omega t)$ to obtain:

$$\frac{dy}{dx} = kA\cos(kx - \omega t)$$

Approximate $\Delta y/\Delta x$ by dy/dx and substitute in our result from part (*a*):

$$\begin{aligned}\Delta U &= \tfrac{1}{2}F\left(kA\cos(kx - \omega t)\right)^2\Delta x \\ &= \boxed{\tfrac{1}{2}FA^2k^2\Delta x\cos^2(kx - \omega t)}\end{aligned}$$