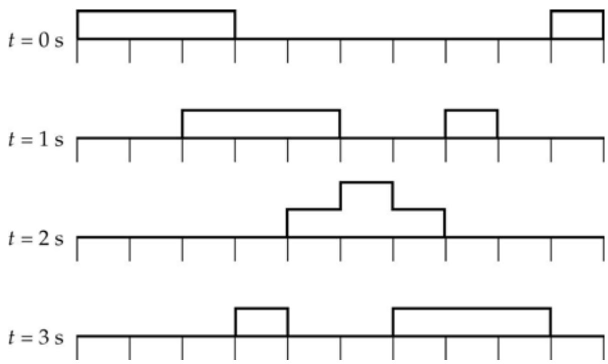
Chapter 16
Superposition and Standing Waves

Conceptual Problems

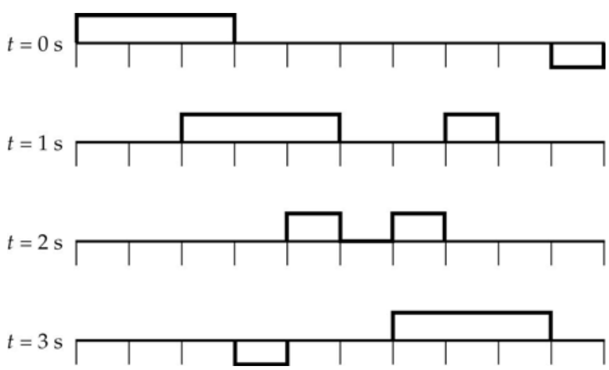
***1** ••

Picture the Problem We can use the speeds of the pulses to determine their positions at the given times.



2 ••

Picture the Problem We can use the speeds of the pulses to determine their positions at the given times.



3 •

Determine the Concept Beats are a consequence of the alternating constructive and destructive interference of waves due to slightly different frequencies. The amplitudes of the waves play no role in producing the beats. $\boxed{(c)\text{ is correct.}}$

4 •

(*a*) True. The harmonics for a string fixed at both ends are integral multiples of the frequency of the fundamental mode (first harmonic).

(*b*) True. The harmonics for a string fixed at both ends are integral multiples of the frequency of the fundamental mode (first harmonic).

(*c*) True. If ℓ is the length of the pipe and *v* the speed of sound, the excited harmonics are given by $f_n = n\dfrac{v}{4\ell}$, where *n* = 1, 3, 5…

5 ••

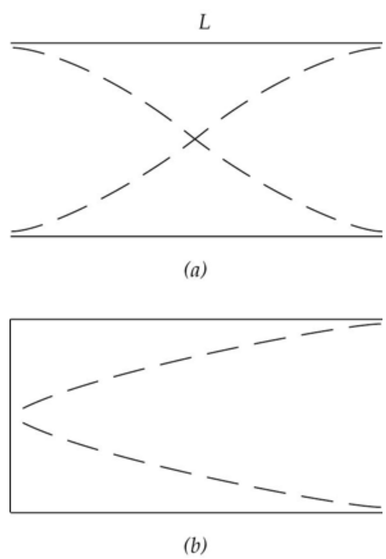
Determine the Concept Standing waves are the consequence of the constructive interference of waves that have the same amplitude and frequency but are traveling in opposite directions. $\boxed{(b)\text{ is correct.}}$

***6** •

Determine the Concept Our ears and brain find frequencies which are small-integer multiples of one another pleasing when played in combination. In particular, the ear hears frequencies related by a factor of 2 (one octave) as identical. Thus, a violin sounds much more "musical" than the sound of a drum.

7 •

Picture the Problem The first harmonic displacement-wave pattern in an organ pipe open at both ends and vibrating in its fundamental mode is represented in part (*a*) of the diagram. Part (*b*) of the diagram shows the wave pattern corresponding to the fundamental frequency for a pipe of the same length *L* that is closed at one end. Letting unprimed quantities refer to the open pipe and primed quantities refer to the closed pipe, we can relate the wavelength and, hence, the frequency of the fundamental modes using $v = f\lambda$.



Express the frequency of the first harmonic in the open pipe in terms of the speed and wavelength of the waves:

$$f_1 = \frac{v}{\lambda_1}$$

Relate the length of the open pipe to the wavelength of the fundamental mode:

$$\lambda_1 = 2L$$

Substitute to obtain:

$$f_1 = \frac{v}{2L}$$

Express the frequency of the first harmonic in the closed pipe in terms of the speed and wavelength of the waves:

$$f_1{}' = \frac{v}{\lambda_1{}'}$$

Relate the length of the closed pipe to the wavelength of the fundamental mode:

$$\lambda_1{}' = 4L$$

Substitute to obtain:

$$f_1{}' = \frac{v}{4L} = \frac{1}{2}\left(\frac{v}{2L}\right) = \frac{1}{2} f_1$$

Substitute numerical values and evaluate $f_1{}'$:

$$f_1{}' = \frac{1}{2}(400\,\text{Hz}) = 200\,\text{Hz}$$

and $\boxed{(a)\text{ is correct.}}$

8 ••

Picture the Problem The frequency of the fundamental mode of vibration is directly proportional to the speed of waves on the string and inversely proportional to the wavelength which, in turn, is directly proportional to the length of the string. By expressing the fundamental frequency in terms of the length *L* of the string and the tension *F* in it we can examine the various changes in lengths and tension to determine which would halve it.

Express the dependence of the frequency of the fundamental mode of vibration of the string on its wavelength:

$$f_1 = \frac{v}{\lambda_1}$$

Relate the length of the string to the wavelength of the fundamental mode:

$$\lambda_1 = 2L$$

Substitute to obtain:

$$f_1 = \frac{v}{2L}$$

Express the dependence of the speed of waves on the string on the tension

$$v = \sqrt{\frac{F}{\mu}}$$

in the string:

Substitute to obtain:

$$f_1 = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$$

(*a*) Doubling the tension and the length would increase the frequency by a factor of $\sqrt{2}/2$.

(*b*) Halving the tension and keeping the length fixed would decrease the frequency by a factor of $1/\sqrt{2}$.

(*c*) Keeping the tension fixed and halving the length would double the frequency.

$\boxed{(c)\text{ is correct.}}$

9 ••

Determine the Concept We can relate the resonant frequencies of an organ pipe to the speed of sound in air and the speed of sound to the absolute temperature.

Express the dependence of the resonant frequencies on the speed of sound:

$$f = \frac{v}{\lambda}$$

Relate the speed of sound to the temperature of the air:

$$v = \sqrt{\frac{\gamma RT}{M}}$$

where γ and R are constants, M is the molar mass of the gas (air), and T is the absolute temperature.

Substitute to obtain:

$$f = \frac{1}{\lambda}\sqrt{\frac{\gamma RT}{M}}$$

$\boxed{\text{Because } v \propto \sqrt{T}\text{, increasing the temperature increases the resonant frequencies.}}$

***10** •

Determine the Concept Because the two waves move independently, neither impedes the progress of the other.

11 •

Determine the Concept No; the wavelength of a wave is related to its frequency and speed of propagation ($\lambda = v/f$). The frequency of the plucked string will be the same as the wave it produces in air, but the speeds of the waves depend on the media in which they are propagating. Because the velocities of propagation differ, the wavelengths will not be the same.

12 •

Determine the Concept No; when averaged over a region in space including one or more wavelengths, the energy is unchanged.

13 •

Determine the Concept When the edges of the glass vibrate, sound waves are produced in the air in the glass. The resonance frequency of the air columns depends on the length of the air column, which depends on how much water is in the glass.

14 ••

Picture the Problem We can use $v = f\lambda$ to relate the frequency of the sound waves in the organ pipes to the speed of sound in air, nitrogen, and helium. We can use $v = \sqrt{\gamma RT/M}$ to relate the speed of sound, and hence its frequency, to the properties of the three gases.

Express the frequency of a given note as a function of its wavelength and the speed of sound:

$$f = \frac{v}{\lambda}$$

Relate the speed of sound to the absolute temperature and the molar mass of the gas used in the organ:

$$v = \sqrt{\frac{\gamma RT}{M}}$$

where γ depends on the kind of gas, R is a constant, T is the absolute temperature, and M is the molar mass.

Substitute to obtain:

$$f = \frac{1}{\lambda}\sqrt{\frac{\gamma RT}{M}}$$

For air in the organ pipes we have:

$$f_{\text{air}} = \frac{1}{\lambda}\sqrt{\frac{\gamma_{\text{air}} RT}{M_{\text{air}}}} \qquad (1)$$

When nitrogen is in the organ pipes:

$$f_{N_2} = \frac{1}{\lambda}\sqrt{\frac{\gamma_{N_2} RT}{M_{N_2}}} \qquad (2)$$

Express the ratio of equation (2) to equation (1) and solve for f_{N_2}:

$$\frac{f_{N_2}}{f_{air}} = \sqrt{\frac{\gamma_{N_2}}{\gamma_{air}}\frac{M_{air}}{M_{N_2}}}$$

and

$$f_{N_2} = f_{air}\sqrt{\frac{\gamma_{N_2}}{\gamma_{air}}\frac{M_{air}}{M_{N_2}}}$$

Because $\gamma_{N_2} = \gamma_{air}$ and $M_{air} > M_{N_2}$:

$$f_{N_2} > f_{air}$$

i.e.,

f will increase for each organ pipe.

If helium were used, we'd have:

$$f_{He} = f_{air}\sqrt{\frac{\gamma_{He}}{\gamma_{air}}\frac{M_{air}}{M_{He}}}$$

Because $\gamma_{He} > \gamma_{air}$ and $M_{air} >> M_{He}$:

$$f_{He} >> f_{air}$$

i.e., the effect will be even more pronounced.

***15 ••**

Determine the Concept Increasing the tension on a piano wire increases the speed of the waves. The wavelength of these waves is determined by the length of the wire. Because the speed of the waves is the product of their wavelength and frequency, the wavelength remains the same and the frequency increases. (b) is correct.

16 ••

Determine the Concept If connected properly, the speakers will oscillate in phase and interfere constructively. If connected incorrectly, they interfere destructively. It would be difficult to detect the interference if the wavelength is short, less than the distance between the ears of the observer. Thus, one should use bass notes of low frequency and long wavelength.

17 ••

Determine the Concept The pitch is determined mostly by the resonant cavity of the mouth; the frequency of sounds he makes is directly proportional to their speed. Because $v_{He} > v_{air}$ (see Equation 15-5), the resonance frequency is higher if helium is the gas in the cavity.

***18 ••**

Determine the Concept The light is being projected up from underneath the silk, so you will see light where there is a gap and darkness where two threads overlap. Because the two weaves have almost the same spatial period but not exactly identical (because the two are stretched unequally), there will be places where, for large sections of the cloth, the two weaves overlap in phase, leading to brightness, and large sections where the two overlap 90° out of phase (i.e., thread on gap and vice versa) leading to darkness. This is exactly the same idea as in the interference of two waves.

Estimation and Approximation

19 ••

Determine the Concept Pianos are tuned by ringing the tuning fork and the piano note simultaneously and tuning the piano string until the beats are far apart; i.e., the time between beats is very long. If we assume that 2 s is the maximum detectable period for the beats, then one should be able to tune the piano string to at least 0.5 Hz.

***20 •**

Picture the Problem We can use $v = f_1\lambda_1$ to express the resonance frequencies in the organ pipes in terms of their wavelengths and $L = n\frac{\lambda_n}{2},\ n = 1, 2, 3, \ldots$ to relate the length of the pipes to the resonance wavelengths.

(*a*) Relate the fundamental frequency of the pipe to its wavelength and the speed of sound:

$$f_1 = \frac{v}{\lambda_1}$$

Express the condition for constructive interference in a pipe that is open at both ends:

$$L = n\frac{\lambda_n}{2},\ n = 1, 2, 3, \ldots \qquad (1)$$

Solve for λ_1:

$$\lambda_1 = 2L$$

Substitute and evaluate f_1:

$$f_1 = \frac{v}{2L} = \frac{340\ \text{m/s}}{2(7.5\times10^{-2}\ \text{m})} = \boxed{2.27\ \text{kHz}}$$

(*b*) Relate the resonance frequencies of the pipe to their wavelengths and the speed of sound:

$$f_n = \frac{v}{\lambda_n}$$

Solve equation (2) for λ_n:

$$\lambda_n = \frac{2L}{n}$$

Substitute to obtain:

$$f_n = n\frac{v}{2L} = n\frac{340\ \text{m/s}}{2\left(7.5\times10^{-2}\ \text{m}\right)} = n(2.27\ \text{kHz})$$

Set f_n = 20 kHz and evaluate n:

$$n = \frac{20\ \text{kHz}}{2.27\ \text{kHz}} = 8.81$$

The eighth harmonic is within the range defined as audible. The ninth harmonic might be heard by a person with very good hearing.

21 ••

Picture the Problem Assume a pipe length of 5 m and apply the standing-wave resonance frequencies condition for a pipe that is open at both ends (the same conditions hold for a string that is fixed at both ends).

Relate the resonance frequencies for a pipe open at both ends to the length of the pipe:

$$f_n = n\frac{v}{2L},\ n = 1, 2, 3, \ldots$$

Evaluate this expression for n = 1:

$$f_1 = \frac{340\ \text{m/s}}{2(5\ \text{m})} = \boxed{34.0\ \text{Hz}}$$

Express the dependence of the speed of sound in a gas on the temperature:

$$v = \sqrt{\frac{\gamma RT}{M}}$$

where γ and R are constants, M is the molar mass, and T is the absolute temperature.

Because $v \propto \sqrt{T}$, the frequency will be somewhat higher in the summer.

Superposition and Interference

22 •

Picture the Problem We can use $A = 2y_0\cos\frac{1}{2}\delta$ to find the amplitude of the resultant wave.

(*a*) Evaluate the amplitude of the resultant wave when $\delta = \pi/6$:

$$A = 2y_0\cos\tfrac{1}{2}\delta = 2(0.02\ \text{m})\cos\frac{1}{2}\left(\frac{\pi}{6}\right) = \boxed{3.86\ \text{cm}}$$

(*b*) Proceed as in (*a*) with $\delta = \pi/3$:

$$A = 2y_0 \cos\tfrac{1}{2}\delta = 2(0.02\,\text{m})\cos\frac{1}{2}\left(\frac{\pi}{3}\right) = \boxed{3.46\,\text{cm}}$$

23 •

Picture the Problem We can use $A = 2y_0 \cos\tfrac{1}{2}\delta$ to find the amplitude of the resultant wave.

Evaluate the amplitude of the resultant wave when $\delta = \pi/2$:

$$A = 2y_0 \cos\tfrac{1}{2}\delta = 2(0.05\,\text{m})\cos\frac{1}{2}\left(\frac{\pi}{2}\right) = \boxed{7.07\,\text{cm}}$$

***24** •

Picture the Problem The phase shift in the waves generated by these two sources is due to their separation of $\lambda/3$. We can find the phase difference due to the path difference from $\delta = 2\pi\frac{\Delta x}{\lambda}$ and then the amplitude of the resultant wave from $A = 2y_0 \cos\tfrac{1}{2}\delta$.

Evaluate the phase difference δ:

$$\delta = 2\pi\frac{\Delta x}{\lambda} = 2\pi\frac{\lambda/3}{\lambda} = \frac{2}{3}\pi$$

Find the amplitude of the resultant wave:

$$A_{\text{res}} = 2y_0 \cos\tfrac{1}{2}\delta = 2A\cos\frac{1}{2}\left(\frac{2}{3}\pi\right) = 2A\cos\frac{\pi}{3} = \boxed{A}$$

25 •

Picture the Problem The phase shift in the waves generated by these two sources is due to a path difference Δx = 5.85 m – 5.00 m = 0.85 m. We can find the phase difference due to this path difference from $\delta = 2\pi\frac{\Delta x}{\lambda}$ and then the amplitude of the resultant wave from $A = 2y_0 \cos\tfrac{1}{2}\delta$.

(*a*) Find the phase difference due to the path difference:

$$\delta = 2\pi\frac{\Delta x}{\lambda}$$

Calculate the wavelength of the sound waves:

$$\lambda = \frac{v}{f} = \frac{340\,\text{m/s}}{100\,\text{s}^{-1}} = 3.4\,\text{m}$$

Substitute and evaluate δ:

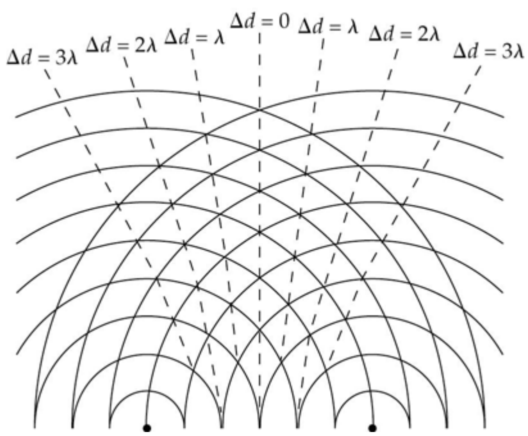
$$\delta = 2\pi\frac{0.85\,\text{m}}{3.4\,\text{m}} = \frac{\pi}{2}\,\text{rad} = \boxed{90.0°}$$

(*b*) Relate the amplitude of the resultant wave to the amplitudes of the interfering waves and the phase difference between them:

$$A = 2y_0\cos\tfrac{1}{2}\delta = 2A\cos\frac{1}{2}\left(\frac{\pi}{2}\right)$$
$$= \boxed{\sqrt{2}A}$$

***26** •

Picture the Problem The diagram is shown below. Lines of constructive interference are shown for path differences of 0, λ, 2λ, and 3λ.



27 •

Picture the Problem The intensity at the point of interest is dependent on whether the speakers are coherent and on the total phase difference in the waves arriving at the given point. We can use $\delta = 2\pi\dfrac{\Delta x}{\lambda}$ to determine the phase difference δ, $A = \left|2p_0\cos\tfrac{1}{2}\delta\right|$ to find the amplitude of the resultant wave, and the fact that the intensity I is proportional to the square of the amplitude to find the intensity at P for the given conditions.

(*a*) Find the phase difference δ:

$$\delta = 2\pi\frac{\tfrac{1}{2}\lambda}{\lambda} = \pi$$

Find the amplitude of the resultant wave:

$$A = \left|2p_0\cos\tfrac{1}{2}\pi\right| = 0$$

Because the intensity is proportional to A^2:

$$I = \boxed{0}$$

(*b*) The sources are incoherent and

$$I = \boxed{2I_0}$$

the intensities add:

(*c*) Express the total phase difference:

$$\delta_{tot} = \delta_{sources} + \delta_{path\ difference} = \pi + 2\pi\frac{\Delta x}{\lambda} = \pi + 2\pi\left(\frac{1}{2}\right) = 2\pi$$

Find the amplitude of the resultant wave:

$$A = \left|2p_0 \cos\tfrac{1}{2}(2\pi)\right| = 2p_0$$

Because the intensity is proportional to A^2:

$$I = \frac{A^2}{p_0^2}I_0 = \frac{(2p_0)^2}{p_0^2}I_0 = \boxed{4I_0}$$

28 •

Picture the Problem The intensity at the point of interest is dependent on whether the speakers are coherent and on the total phase difference in the waves arriving at the given point. We can use $\delta = 2\pi\frac{\Delta x}{\lambda}$ to determine the phase difference δ, $A = \left|2p_0\cos\frac{1}{2}\delta\right|$ to find the amplitude of the resultant wave, and the fact that the intensity *I* is proportional to the square of the amplitude to find the intensity at *P* for the given conditions.

(*a*) Find the phase difference δ:

$$\delta = 2\pi\frac{\lambda}{\lambda} = 2\pi$$

Find the amplitude of the resultant wave:

$$A = \left|2p_0 \cos\tfrac{1}{2}(2\pi)\right| = 2p_0$$

Because the intensity is proportional to A^2:

$$I = \frac{A^2}{p_0^2}I_0 = \frac{(2p_0)^2}{p_0^2}I_0 = \boxed{4I_0}$$

(*b*) The sources are incoherent and the intensities add:

$$I = \boxed{2I_0}$$

(*c*) Express the total phase difference:

$$\delta_{tot} = \delta_{sources} + \delta_{path\ difference} = \pi + 2\pi\frac{\Delta x}{\lambda} = \pi + 2\pi\left(\frac{\lambda}{\lambda}\right) = 3\pi$$

Find the amplitude of the resultant

$$A = \left|2p_0 \cos\tfrac{1}{2}(3\pi)\right| = 0$$

wave:

Because the intensity is proportional to A^2:

$$I = \boxed{0}$$

29 •

Picture the Problem Let P be the point located a distance r_1 from speaker 1 and a distance r_2 from speaker 2. If the sound at point P is to be either a maximum or a minimum, the difference in the distances to the speakers will have to be such that this difference compensates for the 90° out-of-phase condition of the speakers.

(*a*) Express the phase shift due to the speakers in terms of a path difference:

$$\Delta r = \frac{\delta_{\text{sources}}}{360°}\lambda = \frac{90°}{360°}\lambda = \tfrac{1}{4}\lambda$$

Express the condition that $r_2 - r_1$ must satisfy in order to compensate for this path difference:

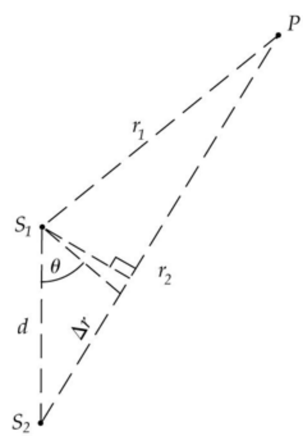
$$r_2 - r_1 = \boxed{\tfrac{1}{4}\lambda}$$

(*b*) In this case, the smallest difference in path is again $\lambda/4$, but now:

$$r_1 - r_2 = \boxed{\tfrac{1}{4}\lambda}$$

***30** ••

Picture the Problem The drawing shows a generic point P located a distance r_1 from source S_1 and a distance r_2 from source S_2. The sources are separated by a distance d and we're given that $d < \lambda/2$. Because the condition for destructive interference is that $\delta = n\pi$ where n = 1, 2, 3,..., we'll show that, with $d < \lambda/2$, this condition cannot be satisfied.



Relate the phase shift to the path difference and the wavelength of the sound:

$$\delta = 2\pi\frac{\Delta r}{\lambda}$$

Relate Δr to d and θ:

$$\Delta r < d\sin\theta \le d$$

Substitute to obtain:

$$\delta < 2\pi\frac{d\sin\theta}{\lambda} \le 2\pi\frac{d}{\lambda}$$

Because $d < \lambda/2$:

$$\delta < 2\pi \frac{\lambda/2}{\lambda} = \pi$$

Express the condition for destructive interference:

$$\delta = n\pi$$
where n = 1, 2, 3,…

$$\boxed{\text{Because } \delta < \pi, \text{there is no complete destructive interference in any direction.}}$$

31 ••

Picture the Problem Let the positive *x* direction be the direction of propagation of the wave. We can express the phase difference in terms of the separation of the two points and the wavelength of the wave and solve for λ. In part (*b*) we can find the phase difference by relating the time between displacements to the period of the wave. I in part (*c*) we can use the relationship between the speed, frequency, and wavelength of a wave to find its velocity.

(*a*) Relate the phase difference to the wavelength of the wave:

$$\delta = 2\pi \frac{\Delta x}{\lambda}$$

Solve for and evaluate λ:

$$\lambda = 2\pi \frac{\Delta x}{\delta} = 2\pi \frac{5\,\text{cm}}{\pi/6} = \boxed{60.0\,\text{cm}}$$

(*b*) Express and evaluate the period of the wave:

$$T = \frac{1}{f} = \frac{1}{40\,\text{s}^{-1}} = 25\,\text{ms}$$

Relate the time between the two displacements to the period of the wave:

$$5\,\text{ms} = \frac{1}{5}T$$

Express the phase difference corresponding to one-fifth of a period:

$$\delta = \boxed{\frac{2\pi}{5}}$$

(*c*) Express the wave speed in terms of its frequency and wavelength:

$$v = f\lambda = \left(40\,\text{s}^{-1}\right)\left(0.6\,\text{m}\right) = \boxed{24.0\,\text{m/s}}$$

32 ••

Picture the Problem Assume a distance of about 20 cm between your ears. When you rotate your head through 90°, you introduce a path difference of 20 cm. We can apply the equation for the phase difference due to a path difference to determine the change in phase between the sounds received by your ears as you rotate your head through 90°.

Express the phase difference due to the rotation of your head through 90°:

$$\delta = 2\pi \frac{20\,\text{cm}}{\lambda}$$

Find the wavelength of the sound waves:

$$\lambda = \frac{v}{f} = \frac{340\,\text{m/s}}{680\,\text{s}^{-1}} = 50\,\text{cm}$$

Substitute to obtain:

$$\delta = 2\pi \frac{20\,\text{cm}}{50\,\text{cm}} = \boxed{0.8\pi\,\text{rad}}$$

33 ••

Picture the Problem Because the sound intensity diminishes as the observer moves, parallel to a line through the sources, away from her initial position, we can conclude that her initial position is one at which there is constructive interference of the sound coming from the two sources. We can apply the condition for constructive interference to relate the wavelength of the sound to the path difference at her initial position and the relationship between the velocity, frequency, and wavelength of the waves to express this path difference in terms of the frequency of the sources.

Express the condition for constructive interference at (40 m, 0):

$$\Delta r = n\lambda,\ n = 1, 2, 3, \ldots \qquad (1)$$

Express the path difference Δr:

$$\Delta r = r_\text{B} - r_\text{A}$$

Using the Pythagorean theorem, find r_B:

$$r_\text{B} = \sqrt{(40\,\text{m})^2 + (2.4\,\text{m})^2}$$

Substitute for r_B and evaluate Δr:

$$\Delta r = \sqrt{(40\,\text{m})^2 + (2.4\,\text{m})^2} - 40\,\text{m} = 0.07194\,\text{m}$$

Substitute in equation (1) and solve for λ:

$$\lambda = \frac{0.07194\,\text{m}}{n}$$

Using $v = f\lambda$, express f in terms of λ and n:

$$f_n = n\frac{v}{0.07194\,\text{m}} = n\frac{340\,\text{m/s}}{0.07194\,\text{m}} = (4726\,\text{Hz})n$$

Evaluate f for n = 1 and 2:

$$f_1 = \boxed{4726\,\text{Hz}} \text{ and } f_2 = \boxed{9452\,\text{Hz}}$$

34 ••

Picture the Problem Because the sound intensity increases as the observer moves, parallel to a line through the sources, away from her initial position, we can conclude that her initial position is one at which there is destructive interference of the sound coming from the two sources. We can apply the condition for destructive interference to relate the wavelength of the sound to the path difference at her initial position and the relationship between the velocity, frequency, and wavelength of the waves to express this path difference in terms of the frequency of the sources.

Express the condition for destructive interference at (40 m, 0):

$$\Delta r = n\frac{\lambda}{2},\ n = 1, 3, 5, \ldots \qquad (1)$$

Express the path difference Δr:

$$\Delta r = r_B - r_A$$

Using the Pythagorean theorem, find r_B:

$$r_B = \sqrt{(40\,\text{m})^2 + (2.4\,\text{m})^2}$$

Substitute for r_B and evaluate Δr:

$$\Delta r = \sqrt{(40\,\text{m})^2 + (2.4\,\text{m})^2} - 40\,\text{m} = 0.07194\,\text{m}$$

Substitute in equation (1) and solve for λ:

$$\lambda = \frac{2(0.07194\,\text{m})}{n} = \frac{0.1439\,\text{m}}{n}$$

Using $v = f\lambda$, express f in terms of λ:

$$f_n = n\frac{v}{0.1439\,\text{m}} = n\frac{340\,\text{m/s}}{0.1439\,\text{m}} = (2363\,\text{Hz})n$$

Evaluate f for n = 1 and 3:

$$f_1 = \boxed{2363\,\text{Hz}}$$

and

$$f_3 = \boxed{7089\,\text{Hz}}$$

***35** ••

Picture the Problem We can use the trigonometric identity $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ to derive the expression given in (*a*) and the speed of the envelope can be found from the second factor in this expression; i.e., from $\cos[(\Delta k/2)x - (\Delta\omega/2)t]$.

(*a*) Express the amplitude of the resultant wave function $y(x,t)$:

$$y(x,t) = A(\cos(k_1x - \omega_1t) + \cos(k_2x - \omega_2t))$$

Use the trigonometric identity $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ to obtain:

$$y(x,t) = 2A\left[\cos\frac{k_1x-\omega_1t+k_2x-\omega_2t}{2}\cos\frac{k_1x-\omega_1t-k_2x+\omega_2t}{2}\right]$$
$$= 2A\left[\cos\left(\frac{k_1+k_2}{2}x-\frac{\omega_1+\omega_2}{2}t\right)\cos\left(\frac{k_1-k_2}{2}x+\frac{\omega_2-\omega_1}{2}t\right)\right]$$

Substitute $\omega_{ave} = (\omega_1 + \omega_2)/2$, $k_{ave} = (k_1 + k_2)/2$, $\Delta\omega = \omega_1 - \omega_2$ and $\Delta k = k_1 - k_2$ to obtain:

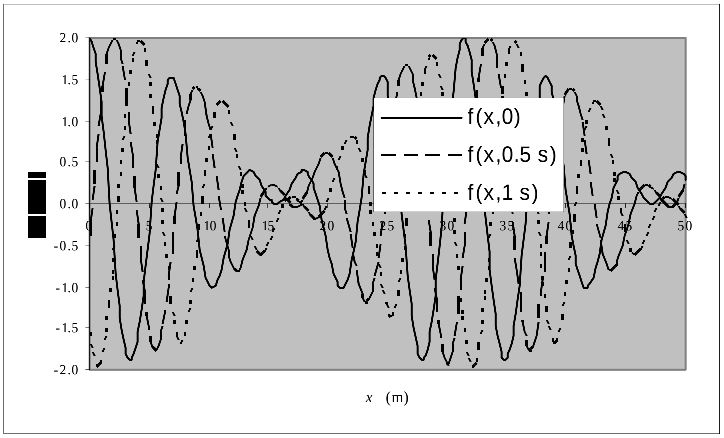
$$y(x,t) = \boxed{2A\left[\cos\left(k_{ave}x-\omega_{ave}t\right)\cos\left(\frac{\Delta k}{2}x-\frac{\Delta\omega}{2}t\right)\right]}$$

(*b*) A spreadsheet program to calculate $y(x,t)$ between 0 m and 50 m at t = 0, 0.5 s, and 1 s follows. The constants and cell formulas used are shown in the table.

Cell	Content/Formula	Algebraic Form
B11	B10+0.25	$x + \Delta x$
C10	COS(B3*B10–B5*C9) + COS(B4*B10–B6*C9)	$y(x,0)$
D10	COS(B3*B10-B5*D9) + COS(B4*B10–B6*D9)	$y(x,0.5\text{ s})$
E10	COS(B3*B10–B5*E9) + COS(B4*B10–B6*E9)	$y(x,1\text{ s})$

	A	B	C	D	E
1					
2					
3	k1=	1	m^{-1}		
4	k2=	0.8	m^{-1}		
5	w1=	1	rad/s		
6	w2=	0.9	rad/s		
7		x	y(x,0)	y(x,0.5 s)	y(x,1 s)
8		(m)			
9			0.000	2.000	4.000
10		0.00	2.000	–0.643	–1.550
11		0.25	1.949	–0.207	–1.787
12		0.50	1.799	0.241	–1.935
13		0.75	1.557	0.678	–1.984
14		1.00	1.237	1.081	–1.932
206		49.00	0.370	–0.037	0.021
207		49.25	0.397	0.003	–0.024
208		49.50	0.397	0.065	–0.075
209		49.75	0.364	0.145	–0.124
210		50.00	0.298	0.237	–0.164

The solid line is the graph of $y(x,0)$, the dashed line that of $y(x,0.5\text{ s})$, and the dotted line is the graph of $y(x,1\text{ s})$.



(*c*) Express the speed of the envelope:

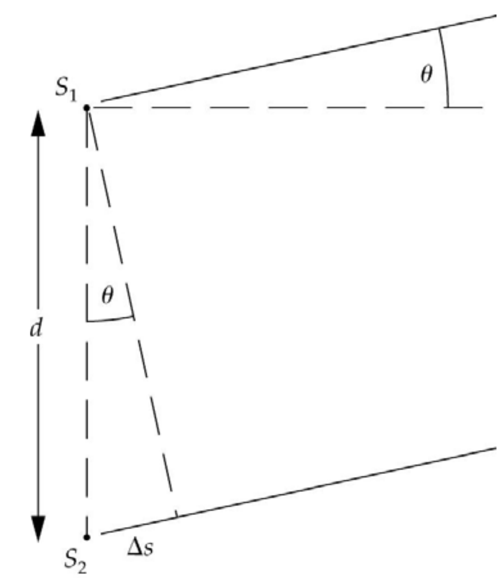
$$v_{\text{envelope}} = \frac{\Delta\omega}{\Delta k} = \frac{\omega_1 - \omega_2}{k_1 - k_2}$$

Substitute numerical values and evaluate v_{envelope}:

$$v_{\text{envelope}} = \frac{1\,\text{rad/s} - 0.9\,\text{rad/s}}{1\,\text{m}^{-1} - 0.8\,\text{m}^{-1}} = \boxed{0.500\,\text{m/s}}$$

36 ••

Picture the Problem The diagram shows the two sources separated by a distance d and the path difference Δs. Because the lines from the sources to the distant point are approximately parallel, the triangle shown in the diagram is approximately a right triangle and we can use trigonometry to express Δs in terms of d and θ. In the second part of the problem, we can apply a small-angle approximation to the larger triangle shown in Figure 16-29 to relate y_m to D and θ and then use the condition for constructive interference to relate y_m to D, λ, and d.



(*a*) Using the diagram, relate Δs to the separation of the sources and the angle θ:

$$\sin\theta \approx \frac{\Delta s}{d} \text{ and } \Delta s \approx \boxed{d\sin\theta}$$

(*b*) For $\theta << 1$, we can approximate

$$\Delta s \approx d\tan\theta$$

$\sin\theta$ with $\tan\theta$ to obtain:

Referring to Figure 16-29, express $\tan\theta$ in terms of y and D:

$$\tan\theta \approx \frac{y_m}{D}$$

Substitute to obtain:

$$\Delta s \approx \frac{dy_m}{D}$$

Express the condition on the phase difference for constructive interference:

$$\delta = 2\pi\frac{\Delta s}{\lambda} = 2\pi m,\ m = 1, 2, 3, \ldots$$

Substitute for Δs:

$$2\pi\frac{dy_m}{D\lambda} = 2\pi m,\ m = 1, 2, 3, \ldots$$

Simplify and solve for y_m:

$$y_m = \boxed{m\frac{D\lambda}{d}}$$

37 ••

Picture the Problem Because a maximum is heard at 0° and the sources are in phase, we can conclude that the path difference is 0. Because the next maximum is heard at 23°, the path difference to that position must be one wavelength. We can use the result of part (*a*) of Problem 36 to relate the separation of the sources to the path difference and the angle θ. We'll apply the condition for constructive interference to determine the angular locations of other points of maximum intensity in the interference pattern.

Using the result of part (*a*) of Problem 36, express the separation of the sources in terms of Δs and θ:

$$d = \frac{\Delta s}{\sin\theta}$$

Evaluate d with $\Delta s = \lambda$ and $\theta = 23°$:

$$d = \frac{\lambda}{\sin 23°} = \frac{v}{f\sin 23°} = \frac{340\ \text{m/s}}{\left(480\ \text{s}^{-1}\right)\sin 23°} = \boxed{1.81\ \text{m}}$$

Express the condition for additional intensity maxima:

$$d\sin\theta_m = m\lambda$$

where m = 1, 2, 3, …, or

$$\theta_m = \sin^{-1}\left[\frac{m\lambda}{d}\right]$$

Evaluate this expression for $m = 2$:

$$\theta_2 = \sin^{-1}\left[\frac{2(340\,\text{m/s})}{(480\,\text{s}^{-1})(1.81\,\text{m})}\right] = \boxed{51.5°}$$

Remarks: It is easy to show that, for *m* > 2, the inverse sine function is undefined and that, therefore, there are no additional relative maxima at angles larger than 51.5°.

***38 •••**

Picture the Problem Because the speakers are driven in phase and the path difference is 0 at her initial position, the listener will hear a maximum at (*D*, 0). As she walks along a line parallel to the *y* axis she will hear a minimum wherever it is true that the path difference is an odd multiple of a half wavelength. She will hear an intensity maximum wherever the path difference is an integral multiple of a wavelength. We'll apply the condition for destructive interference in part (*a*) to determine the angular location of the first minimum and, in part (*b*), the condition for constructive interference find the angle at which she'll hear the first maximum after the one at 0°. In part (*c*), we can apply the condition for constructive interference to determine the number of maxima she can hear as keeps walking parallel to the *y* axis.

(*a*) Express the condition for destructive interference:

$$d\sin\theta_m = m\frac{\lambda}{2}$$

where $m = 1, 3, 5, \ldots$, or

$$\theta_m = \sin^{-1}\left(\frac{m\lambda}{2d}\right)$$

Evaluate this expression for $m = 1$:

$$\theta_1 = \sin^{-1}\left(\frac{v}{2fd}\right) = \sin^{-1}\left[\frac{340\,\text{m/s}}{2(600\,\text{s}^{-1})(2\,\text{m})}\right]$$

$$= \boxed{8.14°}$$

(*b*) Express the condition for additional intensity maxima:

$$d\sin\theta_m = m\lambda$$

where $m = 0, 1, 2, 3, \ldots$, or

$$\theta_m = \sin^{-1}\left(\frac{m\lambda}{d}\right)$$

Evaluate this expression for $m = 1$:

$$\theta_1 = \sin^{-1}\left(\frac{v}{fd}\right) = \sin^{-1}\left[\frac{340\,\text{m/s}}{(600\,\text{s}^{-1})(2\,\text{m})}\right]$$

$$= \boxed{16.5°}$$

(*c*) Express the limiting condition on $\sin\theta$:

$$\sin\theta_m = m\frac{\lambda}{d} \le 1$$

Solve for *m* to obtain:

$$m \le \frac{d}{\lambda} = \frac{fd}{v} = \frac{(600\,\text{s}^{-1})(2\,\text{m})}{340\,\text{m/s}} = 3.53$$

Because *m* must be an integer:

$$m = \boxed{3}$$

39 •••

Picture the Problem Let *d* be the separation of the two sound sources. We can express the wavelength of the sound in terms of the *d* and either of the angles at which intensity maxima are heard. We can find the frequency of the sources from its relationship to the speed of the waves and their wavelengths. Using the condition for constructive interference, we can find the angles at which intensity maxima are heard. Finally, in part (*d*), we'll use the condition for destructive interference to find the smallest angle for which the sound waves cancel.

(*a*) Express the condition for constructive interference:

$$d\sin\theta_m = m\lambda \qquad (1)$$

where $m = 0, 1, 2, 3,\ldots$

Solve for λ:

$$\lambda = \frac{d\sin\theta_m}{m}$$

Evaluate λ for $m = 1$:

$$\lambda = (2\,\text{m})\sin(0.140\,\text{rad}) = \boxed{0.279\,\text{m}}$$

(*b*) Express the frequency of the sound in terms of its wavelength and speed:

$$f = \frac{v}{\lambda} = \frac{340\,\text{m/s}}{0.279\,\text{m}} = \boxed{1.22\,\text{kHz}}$$

(*c*) Solve equation (1) for θ_m:

$$\theta_m = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left[\frac{m(0.279\,\text{m})}{2\,\text{m}}\right] = \sin^{-1}[(0.1395)m]$$

The table shows the values for θ as a function of *m*:

m	θ_m
	(rad)
3	0.432
4	0.592
5	0.772

6	0.992
7	1.354
8	undefined

(*d*) Express the condition for destructive interference:

$$d\sin\theta_m = m\frac{\lambda}{2}$$

where m = 1, 3, 5,…

Solve for θ_m:

$$\theta_m = \sin^{-1}\left(m\frac{\lambda}{2d}\right)$$

Evaluate this expression for m = 1:

$$\theta_1 = \sin^{-1}\left[\frac{0.279\,\text{m}}{2(2\,\text{m})}\right] = \boxed{0.0698\,\text{rad}}$$

40 •••

Picture the Problem The total phase shift in the waves arriving at the points of interest is the sum of the phase shift due to the difference in path lengths from the two sources to a given point and the phase shift due to the sources being out of phase by 90°. From Problem 39 we know that λ = 0.279 m. Using the conditions on the path difference Δx for constructive and destructive interference, we can find the angles at which intensity maxima are heard.

Letting the subscript ″pd ″ denote ″path difference″ and the subscript ″s″ the ″sources″, express the total phase shift δ:

$$\delta = \delta_{\text{pd}} + \delta_{\text{s}} = 2\pi\frac{\Delta x}{\lambda} + \frac{\pi}{4}$$

where Δx is the path difference between the two sources and the points at which constructive or destructive interference is heard.

Express the condition for constructive interference:

$$\delta = 2\pi\frac{\Delta x}{\lambda} + \frac{\pi}{4} = 2\pi, 4\pi, 6\pi, ...$$

Solve for Δx to obtain:

$$\Delta x = \frac{7}{8}\lambda, \frac{15}{8}\lambda, \frac{23}{8}\lambda, ... = \frac{(8m-1)}{8}\lambda$$

where m = 1, 2, 3,…

Relate Δx to d to obtain:

$$\Delta x = \frac{(8m-1)}{8}\lambda = d\sin\theta_{\text{c}}$$

where the ″c″ denotes constructive interference.

Solve for θ_c:

$$\theta_c = \sin^{-1}\left[\frac{(8m-1)\lambda}{8d}\right], m = 1, 2, 3, \ldots$$

The table shows the values for θ_c for m = 1 to 5:

m	θ_c
1	7.01°
2	15.2°
3	23.6°
4	35.1°
5	42.8°

Express the condition for destructive interference:

$$\delta = 2\pi\frac{\Delta x}{\lambda} + \frac{\pi}{4} = \pi, 3\pi, 5\pi, \ldots$$

Solve for Δx to obtain:

$$\Delta x = \frac{3}{8}\lambda, \frac{11}{8}\lambda, \frac{19}{8}\lambda, \ldots = \frac{(8m-5)}{8}\lambda$$

where m = 1, 2, 3,…

Letting "d" denotes destructive interference, relate Δx to d to obtain:

$$\Delta x = \frac{(8m-5)}{8}\lambda = d\sin\theta_d$$

Solve for θ_d:

$$\theta_d = \sin^{-1}\left[\frac{(8m-5)\lambda}{8d}\right], m = 1, 2, 3, \ldots$$

The table shows the values for θ_d for m = 1 to 5:

m	θ_d
1	3.00°
2	11.1°
3	19.3°
4	28.1°
5	37.6°

41 •••

Picture the Problem We can calculate the required phase shift from the path difference and the wavelength of the radio waves using $\delta = 2\pi\frac{\Delta s}{\lambda}$.

Express the phase delay as a function of the path difference and

$$\delta = 2\pi\frac{\Delta s}{\lambda} \qquad (1)$$

the wavelength of the radio waves:

Find the wavelength of the radio waves:

$$\lambda = \frac{v}{f} = \frac{3\times10^{8}\ \text{m/s}}{20\times10^{6}\ \text{s}^{-1}} = 15\,\text{m}$$

Express the path difference for the signals coming from an angle θ with the vertical:

$$\Delta s = d\sin\theta$$

Substitute numerical values and evaluate Δs:

$$\Delta s = (200\,\text{m})\sin10° = 34.73\,\text{m} = 2.315\lambda$$
$$= 2\lambda + 0.315\lambda$$

Substitute in equation (1) and evaluate δ:

$$\delta = 2\pi\frac{0.315\lambda}{\lambda} = 1.98\,\text{rad} = \boxed{113°}$$

Beats

42 •

Picture the Problem The beat frequency is the difference between the frequency of the tuning fork and the frequency of the violin string. Let f_2 = 500 Hz.

(*a*) Express the relationship between the beat frequency of the frequencies of the two tuning forks:

$$f_2 = f_1 \pm \Delta f$$
$$= 500\,\text{Hz} \pm 4\,\text{Hz}$$

Solve for f_2:

$$f_2 = \boxed{504\,\text{Hz or } 496\,\text{Hz}}$$

(*b*) If the beat frequency is increased, then f_2 = 504 Hz; if it is diminished, $f_2 = 496$ Hz.

43 ••

Picture the Problem The Doppler shift of the siren as heard by one of the drivers is given by the formula for source and receiver both moving and approaching each other $f_r = f_s\left[(1+u/v)/(1-u/v)\right]$, where *u* is the speed of the ambulance and *v* is the speed of sound.

(*a*) Express the beat frequency:

$$f_{\text{beat}} = f_r - f_s$$

where f_r is the frequency heard by either driver due to the other's siren, Express f_r:

$$f_r = f_s \frac{1+\frac{u}{v}}{1-\frac{u}{v}}$$

Substitute to obtain:

$$f_{beat} = f_s \frac{1+\frac{u}{v}}{1-\frac{u}{v}} - f_s = f_s\left(\frac{1+\frac{u}{v}}{1-\frac{u}{v}} - 1\right)$$

$$= f_s \frac{2}{\frac{v}{u} - 1}$$

Substitute numerical values and evaluate f_{beat}:

$$f_{beat} = (500\,\text{Hz})\frac{2}{\frac{340\,\text{m/s}}{22.4\,\text{m/s}} - 1} = \boxed{70.5\,\text{Hz}}$$

(*b*) The person on the street hears no beat frequency as the sirens of both ambulances are Doppler shifted up by the same amount (approx. 35 Hz).

Standing Waves

***44 •**

Picture the Problem We can use $v = f\lambda$ to relate the second-harmonic frequency to the wavelength of the standing wave for the second harmonic.

Relate the speed of transverse waves on the string to their frequency and wavelength:

$$v = f_2\lambda_2$$

Express λ_2 in terms of the length L of the string:

$$\lambda_2 = L$$

Substitute for λ_2 and evaluate v:

$$v = f_2 L = \left(60\,\text{s}^{-1}\right)(3\,\text{m}) = \boxed{180\,\text{m/s}}$$

45 •

Picture the Problem We can find the wavelength of this standing wave from the standing-wave condition for a string fixed at both ends and its frequency from $v = f_3\lambda_3$. We can use the wave function for a standing wave on a string fixed at both ends ($y_n(x,t) = A_n \sin k_n x \cos \omega_n t$) to write the wave function for the wave described in this problem.

(*a*) Using the standing-wave condition for a string fixed at both ends, relate the length of the string to the wavelength of the harmonic mode in which it is vibrating:

$$L = n\frac{\lambda_n}{2},\ n = 1, 2, 3, \ldots$$

Solve for λ_3:

$$\lambda_3 = \frac{2}{3}L = \frac{2}{3}(3\,\text{m}) = \boxed{2.00\,\text{m}}$$

Express the frequency of the third harmonic in terms of the speed of transverse waves on the string and their wavelength:

$$f_3 = \frac{v}{\lambda_3} = \frac{50\,\text{m/s}}{2\,\text{m}} = \boxed{25.0\,\text{Hz}}$$

(*b*) Write the equation for a standing wave, fixed at both ends, in its third harmonic:

$$y_3(x,t) = A_3 \sin k_3 x \cos \omega_3 t$$

Evaluate k_3:

$$k_3 = \frac{2\pi}{\lambda_3} = \frac{2\pi}{2\,\text{m}} = \pi\,\text{m}^{-1}$$

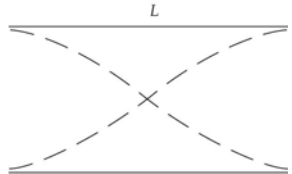
Evaluate ω_3:

$$\omega_3 = 2\pi f_3 = 2\pi\left(25\,\text{s}^{-1}\right) = 50\pi\,\text{s}^{-1}$$

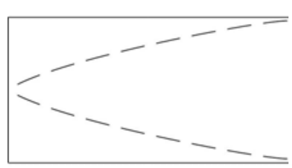
Substitute to obtain: $\boxed{y_3(x,t) = (4\,\text{mm})\sin kx \cos \omega t}$ where $k = \pi\,\text{m}^{-1}$ and $\omega = 50\pi\,\text{s}^{-1}$.

46 •

Picture the Problem The first harmonic displacement-wave pattern in an organ pipe open at both ends and vibrating in its fundamental mode is represented in part (*a*) of the diagram. Part (*b*) of the diagram shows the wave pattern corresponding to the fundamental frequency for a pipe of the same length *L* that is closed at one end. We can relate the wavelength to the frequency of the fundamental modes using $v = f\lambda$.



(a)


(b)

(*a*) Express the dependence of the frequency of the fundamental mode of vibration in the open pipe on its

$$f_{1,\text{open}} = \frac{v}{\lambda_{1,\text{open}}}$$

wavelength:

Relate the length of the open pipe to the wavelength of the fundamental mode:

$$\lambda_{1,\text{open}} = 2L$$

Substitute and evaluate $f_{1,\text{open}}$:

$$f_{1,\text{open}} = \frac{v}{2L} = \frac{340\,\text{m/s}}{2(10\,\text{m})} = \boxed{17.0\,\text{Hz}}$$

(*b*) Express the dependence of the frequency of the fundamental mode of vibration in the closed pipe on its wavelength:

$$f_{1,\text{closed}} = \frac{v}{\lambda_{1,\text{closed}}}$$

Relate the length of the closed pipe to the wavelength of the fundamental mode:

$$\lambda_{1,\text{closed}} = 4L$$

Substitute to obtain:

$$f_{1,\text{closed}} = \frac{v}{4L} = \frac{340\,\text{m/s}}{4(10\,\text{m})} = \boxed{8.50\,\text{Hz}}$$

47 •

Picture the Problem We can find the speed of transverse waves on the wire using $v = \sqrt{F/\mu}$ and the wavelengths of any harmonic from $L = n\frac{\lambda_n}{2}, n = 1, 2, 3, \ldots$. We can use $v = f\lambda$ to find the frequency of the fundamental. For a wire fixed at both ends, the higher harmonics are integer multiples of the first harmonic (fundamental).

(*a*) Relate the speed of transverse waves on the wire to the tension in the wire and its linear density:

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{F}{m/L}}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{\frac{968\,\text{N}}{(0.005\,\text{kg})/(1.4\,\text{m})}} = \boxed{521\,\text{m/s}}$$

(*b*) Using the standing-wave condition for a wire fixed at both ends, relate the length of the wire to the wavelength of the harmonic mode in which it is vibrating:

$$L = n\frac{\lambda_n}{2}, n = 1, 2, 3, \ldots$$

Solve for λ_1:

$$\lambda_1 = 2L = 2(1.4\,\text{m}) = \boxed{2.80\,\text{m}}$$

Express the frequency of the first harmonic in terms of the speed and wavelength of the waves:

$$f_1 = \frac{v}{\lambda_1} = \frac{521\,\text{m/s}}{2.80\,\text{m}} = \boxed{186\,\text{Hz}}$$

(*c*) Because, for a wire fixed at both ends, the higher harmonics are integer multiples of the first harmonic:

$$f_2 = 2f_1 = 2(186\,\text{Hz}) = \boxed{372\,\text{Hz}}$$

and

$$f_3 = 3f_1 = 3(186\,\text{Hz}) = \boxed{558\,\text{Hz}}$$

48 •

Picture the Problem We can use Equation 16-13, $f_n = n\frac{v}{4L} = nf_1, n = 1,3,5,\ldots,$ to find the resonance frequencies for a rope that is fixed at one end.

(*a*) Using the resonance-frequency condition for a rope fixed at one end, relate the resonance frequencies to the speed of the waves and the length of the rope:

$$f_n = n\frac{v}{4L} = nf_1, n = 1,3,5,\ldots$$

Solve for f_1:

$$f_1 = \frac{20\,\text{m/s}}{4(4\,\text{m})} = \boxed{1.25\,\text{Hz}}$$

(*b*) Because this rope is fixed at just one end, the system does not support a second harmonic.

(*c*) For the third harmonic, $n = 3$:

$$f_3 = 3f_1 = 3(1.25\,\text{Hz}) = \boxed{3.75\,\text{Hz}}$$

49 •

Picture the Problem We can find the fundamental frequency of the piano wire using the general expression for the resonance frequencies of a wire fixed at both ends, $f_n = n\frac{v}{2L} = nf_1, n = 1, 2, 3, \ldots,$ with $n = 1$. We can use $v = \sqrt{F/\mu}$ to express the frequencies of the fundamentals of the two wires in terms of their linear densities.

Relate the fundamental frequency of the piano wire to the speed of transverse waves on it and its linear density:

$$f_1 = \frac{v}{2L}$$

Express the dependence of the speed of transverse waves on the tension and linear density:

$$v = \sqrt{\frac{F}{\mu}}$$

Substitute to obtain:

$$f_1 = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$$

Doubling the linear density results in a new fundamental frequency f' given by:

$$f_1' = \frac{1}{2L}\sqrt{\frac{F}{2\mu}} = \frac{1}{\sqrt{2}}\left(\frac{1}{2L}\sqrt{\frac{F}{\mu}}\right) = \frac{1}{\sqrt{2}} f_1$$

Substitute for f_1 to obtain:

$$f_1' = \frac{1}{\sqrt{2}}(200\,\text{Hz}) = \boxed{141\,\text{Hz}}$$

***50** •

Picture the Problem Because the frequency and wavelength of sounds waves are inversely proportional, the greatest length of the organ pipe corresponds to the lowest frequency in the normal hearing range. We can relate wavelengths to the length of the pipes using the expressions for the resonance frequencies for pipes that are open at both ends and open at one end.

Find the wavelength of a 20-Hz note:

$$\lambda_{\text{max}} = \frac{v}{f_{\text{lowest}}} = \frac{340\,\text{m/s}}{20\,\text{s}^{-1}} = 17\,\text{m}$$

(*a*) Relate the length L of a closed-at-one-end organ pipe to the wavelengths of its standing waves:

$$L = n\frac{\lambda_n}{4},\ n = 1, 3, 5, \ldots$$

Solve for and evaluate λ_1:

$$L = \frac{\lambda_{\text{max}}}{4} = \frac{17\,\text{m}}{4} = \boxed{4.25\,\text{m}}$$

(*b*) Relate the length L of an open organ pipe to the wavelengths of its standing waves:

$$L = n\frac{\lambda_n}{2},\ n = 1, 2, 3, \ldots$$

Solve for and evaluate λ_1:

$$L = \frac{\lambda_{\text{max}}}{2} = \frac{17\,\text{m}}{2} = \boxed{8.50\,\text{m}}$$

51 ••

Picture the Problem We can find λ and f by comparing the given wave function to the general wave function for a string fixed at both ends. The speed of the waves can then be found from $v = f\lambda$. We can find the length of the string from its fourth harmonic wavelength.

(*a*) Using the wave function, relate k and λ:

$$k = \frac{2\pi}{\lambda} = 0.20\,\text{cm}^{-1}$$

Solve for λ:

$$\lambda = \frac{2\pi}{0.20\,\text{cm}^{-1}} = 10\pi\,\text{cm} = \boxed{31.4\,\text{cm}}$$

Using the wave function, relate f and ω:

$$\omega = 2\pi f = 300\,\text{s}^{-1}$$

Solve for f:

$$f = \frac{300\,\text{s}^{-1}}{2\pi} = \boxed{47.7\,\text{Hz}}$$

(*b*) Express the speed of transverse waves in terms of their frequency and wavelength:

$$v = f\lambda = (47.7\,\text{Hz})(0.314\,\text{m})$$
$$= \boxed{15.0\,\text{m/s}}$$

(*c*) Relate the length of the string to the wavelengths of its standing-wave patterns:

$$L = n\frac{\lambda_n}{2},\ n = 1, 2, 3, \ldots$$

Solve for L when $n = 4$:

$$L = 2\lambda_4 = 2(31.4\,\text{cm}) = \boxed{62.8\,\text{cm}}$$

52 ••

Picture the Problem We can find λ and f by comparing the given wave function to the general wave function for a string fixed at both ends. The speed of the waves can then be found from $v = f\lambda$. In a standing wave pattern, the nodes are separated by one-half wavelength.

(*a*) Express the speed of the traveling waves in terms of their frequency and wavelength:

$$v = f\lambda$$

Using the wave function, relate k and λ:

$$k = \frac{2\pi}{\lambda} = 2.5\,\text{m}^{-1}$$

Solve for λ:

$$\lambda = \frac{2\pi}{2.5\,\text{m}^{-1}} = 0.8\pi\,\text{m} = 2.51\,\text{m}$$

Using the wave function, relate ω

$$\omega = 2\pi f = 500\,\text{s}^{-1}$$

and f:

Solve for f:

$$f = \frac{500\,\text{s}^{-1}}{2\pi} = 79.6\,\text{Hz}$$

Substitute to find v:

$$v = \left(79.6\,\text{s}^{-1}\right)\left(2.51\,\text{m}\right) = \boxed{200\,\text{m/s}}$$

Express the amplitude of the standing wave in terms of the amplitude of the two traveling waves that result in the standing wave:

$$A_{\text{SW}} = 2A$$

Solve for and evaluate A:

$$A = \frac{A_{\text{SW}}}{2} = \frac{0.05\,\text{m}}{2} = \boxed{2.50\,\text{cm}}$$

(*b*) The distance between nodes is half the wavelength:

$$\frac{\lambda}{2} = \frac{2.51\,\text{m}}{2} = \boxed{1.26\,\text{m}}$$

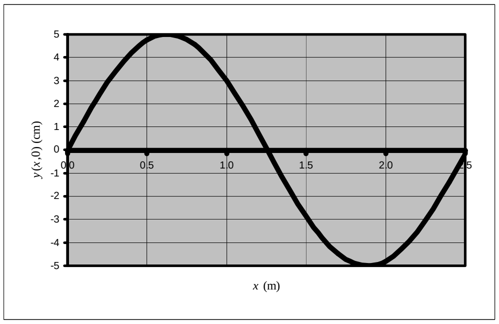
(*c*) Because there is a standing wave on the string, the shortest possible length is:

$$L_{\text{min}} = \frac{\lambda}{2} = \boxed{1.26\,\text{m}}$$

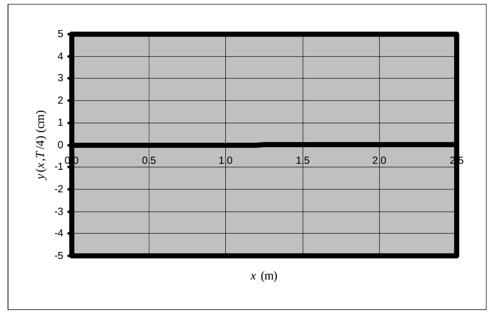
53 ••

Picture the Problem We can evaluate the wave function of Problem 52 at the given times to obtain graphs of position as a function of x. We can find the period of the motion from its frequency f and find f from its angular frequency ω.

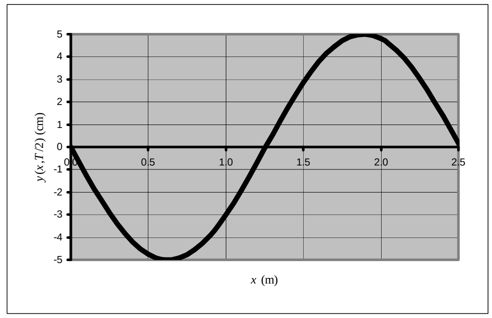
(*a*) The function $y(x,0)$ is shown to the right.



The functions $y(x,T/4)$ and $y(x,3T/4)$ are shown to the right. Because these functions are identical, only one graph is shown.



The function $y(x,T/2)$ is shown to the right.



(*b*) Express the period in terms of the frequency:

$$T = \frac{1}{f}$$

Using the wave function, relate ω and f:

$$\omega = 2\pi f = 500\,\text{s}^{-1}$$

Solve for f:

$$f = \frac{500\,\text{s}^{-1}}{2\pi} = 79.6\,\text{Hz}$$

Substitute for f and evaluate T:

$$T = \frac{1}{79.6\,\text{s}^{-1}} = \boxed{12.6\,\text{ms}}$$

(*c*) $\boxed{\text{Because the string is moving either upward or downward when } y(x) = 0 \text{ for all } x, \text{the energy of the wave is entirely kinetic energy.}}$

***54** ••

Picture the Problem Whether these frequencies are for a string fixed at one end only rather than for a string fixed at both ends can be decided by determining whether they are integral multiples or odd-integral multiples of a fundamental frequency. The length of the string can be found from the wave speed and the wavelength of the fundamental frequency using the standing-wave condition for a string with one end free.

(*a*) Letting the three frequencies be represented by f', f'', and f''', find the ratio of the first two frequencies:

$$\frac{f'}{f''} = \frac{75\,\text{Hz}}{125\,\text{Hz}} = \boxed{\frac{3}{5}}$$

Find the ratio of the second and third frequencies:

$$\frac{f''}{f'''}=\frac{125\text{ Hz}}{175\text{ Hz}}=\boxed{\frac{5}{7}}$$

(*b*) $\boxed{\text{There are no even harmonics, so the string must be fixed at one end only.}}$

(*c*) Express the resonance frequencies in terms of the fundamental frequency:

$$f_n=nf_1,\ n=1,3,5,\ldots$$

Noting that the frequencies are multiples of 25 Hz, we can conclude that:

$$f_1=\frac{f_3}{3}=\frac{75\text{ Hz}}{3}=\boxed{25\text{ Hz}}$$

(*d*) $\boxed{\begin{array}{l}\text{Because the frequencies are 3, 5, and 7 times the fundamental frequency,}\\ \text{they are the third, fifth, and seventh harmonics.}\end{array}}$

(*e*) Express the length of the string in terms of the standing-wave condition for a string fixed at one end:

$$L=n\frac{\lambda_n}{4},\ n=1,3,5,\ldots$$

Using $v = f_1\lambda_1$, find λ_1:

$$\lambda_1=\frac{v}{f_1}=\frac{400\text{ m/s}}{25\text{ s}^{-1}}=16\text{ m}$$

Evaluate L for λ_1 = 16 m and n = 1:

$$L=\frac{\lambda_1}{4}=\frac{16\text{ m}}{4}=\boxed{4.00\text{ m}}$$

55 ••

Picture the Problem The lowest resonant frequency in this closed-at-one-end tube is its fundamental frequency. This frequency is related to its wavelength through $v = f_{min}\lambda_{max}$. We can use the relationship between the nth harmonic and the fundamental frequency, $f_n=(2n+1)f_1,\ n=1,2,3,\ldots$, to find the highest frequency less than or equal to 5000 Hz that will produce resonance.

(*a*) Express the length of the space above the water in terms of the standing-wave condition for a closed pipe:

$$L=n\frac{\lambda_n}{4},\ n=1,3,5,\ldots$$

Solve for λ_n:

$$\lambda_n=\frac{4L}{n},\ n=1,3,5,\ldots$$

λ_{max} corresponds to n = 1:

$$\lambda_{max} = 4L = 4(1.2\,\text{m}) = 4.8\,\text{m}$$

Using $v = f_{min}\lambda_{max}$, find f_{min}:

$$f_{min} = \frac{v}{\lambda_{max}} = \frac{340\,\text{m/s}}{4.8\,\text{m}} = \boxed{70.8\,\text{Hz}}$$

(*b*) Express the nth harmonic in terms of the fundamental frequency (first harmonic):

$$f_n = (2n+1)f_1,\ n = 1, 2, 3, \ldots$$

To find the highest harmonic below 5000 Hz, let f_n = 5000 Hz:

$$5000\,\text{Hz} = (2n+1)(70.8\,\text{Hz})$$

Solve for n (an integer) to obtain:

n = 34

Evaluate f_{34}:

$$f_{34} = 69\,f_1 = 69(70.8\,\text{Hz}) = \boxed{4.89\,\text{kHz}}$$

(*c*) There are 34 harmonics higher than the fundamental frequency so the total number is:

$$\boxed{35}$$

56 ••

Picture the Problem Sound waves of frequency 460 Hz are excited in the tube, whose length L can be adjusted. Resonance occurs when the effective length of the tube $L_{eff} = L + \Delta L$ equals $\frac{1}{4}\lambda, \frac{3}{4}\lambda, \frac{5}{4}\lambda,$ and so on, where λ is the wavelength of the sound. Even though the pressure node is not exactly at the end of the tube, the wavelength can be found from the fact that the distance between water levels for successive resonances is half the wavelength. We can find the speed from $v = f\lambda$ and the end correction from the fact that, for the fundamental, $L_{eff} = \frac{1}{4}\lambda = L_1 + \Delta L,$ where L_1 is the distance from the top of the tube to the location of the first resonance.

(*a*) Relate the speed of sound in air to its wavelength and the frequency of the tuning fork:

$$v = f\lambda$$

Using the fact that nodes are separated by one-half wavelength, find the wavelength of the sound waves:

$$\lambda = 2(55.8\,\text{cm} - 18.3\,\text{cm}) = 75\,\text{cm}$$

Substitute and evaluate v:

$$v = (460\,\text{s}^{-1})(0.75\,\text{m}) = \boxed{345\,\text{m/s}}$$

(*b*) Relate the end correction ΔL to the wavelength of the sound and effective length of the tube:

$$L_{\text{eff}} = \tfrac{1}{4}\lambda = L_1 + \Delta L$$

Solve for and evaluate ΔL:

$$\Delta L = \tfrac{1}{4}\lambda - L_1 = \tfrac{1}{4}(75\,\text{cm}) - 18.3\,\text{cm} = \boxed{0.450\,\text{cm}}$$

***57 ••**

Picture the Problem We can use $v = f\lambda$ to express the fundamental frequency of the organ pipe in terms of the speed of sound and $v = \sqrt{\frac{\gamma RT}{M}}$ to relate the speed of sound and the fundamental frequency to the absolute temperature.

Express the fundamental frequency of the organ pipe in terms of the speed of sound:

$$f = \frac{v}{\lambda}$$

Relate the speed of sound to the temperature:

$$v = \sqrt{\frac{\gamma RT}{M}}$$

where γ and R are constants, M is the molar mass, and T is the absolute temperature.

Substitute to obtain:

$$f = \frac{1}{\lambda}\sqrt{\frac{\gamma RT}{M}}$$

Using primed quantities to represent the higher temperature, express the new frequency as a function of T:

$$f' = \frac{1}{\lambda'}\sqrt{\frac{\gamma RT'}{M}}$$

As we have seen, λ is proportional to the length of the pipe. For the first question, we assume the length of the pipe does not change, so $\lambda = \lambda'$. Then the ratio of f' to f is:

$$\frac{f'}{f} = \sqrt{\frac{T'}{T}}$$

Solve for and evaluate f' with $T' = 305$ K and $T = 289$ K:

$$f' = f_{305\,\text{K}} = f_{289\,\text{K}}\sqrt{\frac{305\,\text{K}}{289\,\text{K}}}$$

$$= (440.0\,\text{Hz})\sqrt{\frac{305\,\text{K}}{289\,\text{K}}}$$

$$= \boxed{452\,\text{Hz}}$$

$$\boxed{\text{It would be better to have the pipe expand so that } v/L, \text{ where } L \text{ is the length of the pipe, is independent of temperature.}}$$

58 ••

Picture the Problem We can express the wavelength of the fundamental in a pipe open at both ends in terms of the effective length of the pipe using $\lambda = 2L_{\text{eff}} = 2(L + \Delta L)$, where L is the physical length of the pipe and $\lambda = v/f$. Solving these equations simultaneously will lead us to an expression for L as a function of D.

Express the wavelength of the fundamental in a pipe open at both ends in terms of the pipe's effective length L_{eff}:

$$\lambda = 2L_{\text{eff}} = 2(L + \Delta L)$$

where L is its physical length.

Solve for L to obtain:

$$L = \frac{\lambda}{2} - \Delta L = \frac{\lambda}{2} - 0.3186D$$

Express the wavelength of middle C in terms of its frequency f and the speed of sound v:

$$\lambda = \frac{v}{f}$$

Substitute to obtain:

$$L = \frac{v}{2f} - 0.3186D$$

Substitute numerical values to express L as a function of D:

$$L = \frac{340\,\text{m/s}}{2(256\,\text{s}^{-1})} - 0.3186D$$

$$= 0.664\,\text{m} - 0.3186D$$

Evaluate L for $D = 1$ cm:

$$L = 0.664\,\text{m} - 0.3186(0.01\,\text{m})$$

$$= \boxed{66.1\,\text{cm}}$$

Evaluate L for $D = 10$ cm:

$$L = 0.664\,\text{m} - 0.3186(0.1\,\text{m})$$

$$= \boxed{63.2\,\text{cm}}$$

Evaluate L for D = 30 cm:

$$L = 0.664\,\text{m} - 0.3186(0.3\,\text{m})$$
$$= \boxed{56.8\,\text{cm}}$$

59 ••

Picture the Problem We know that, when a string is vibrating in its fundamental mode, its ends are one-half wavelength apart. We can use $v = f\lambda$ to express the fundamental frequency of the organ pipe in terms of the speed of sound and $v = \sqrt{F/\mu}$ to relate the speed of sound and the fundamental frequency to the tension in the string. We can use this relationship between f and L, the length of the string, to find the length of string when it vibrates with a frequency of 650 Hz.

(*a*) Express the wavelength of the standing wave, vibrating in its fundamental mode, to the length L of the string:

$$\lambda = 2L = 2(40\,\text{cm}) = \boxed{80\,\text{cm}}$$

(*b*) Relate the speed of the waves combining to form the standing wave to its frequency and wavelength:

$$v = f\lambda$$

Express the speed of transverse waves as a function of the tension in the string:

$$v = \sqrt{\frac{F}{\mu}}$$

Substitute and solve for F to obtain:

$$F = f^2\lambda^2\frac{m}{L}$$

where m is the mass of the string and L is its length.

Substitute numerical values and evaluate F:

$$F = \left(500\,\text{s}^{-1}\right)^2(0.8\,\text{m})^2\frac{1.2\times10^{-3}\,\text{kg}}{0.4\,\text{m}}$$
$$= \boxed{480\,\text{N}}$$

(*c*) Using $v = f\lambda$ and assuming that the string is still vibrating in its fundamental mode, express its frequency in terms of its length:

$$f = \frac{v}{\lambda} = \frac{v}{2L}$$

Solve for L:

$$L = \frac{v}{2f}$$

Letting primed quantities refer to a second length and frequency, express L' in terms of f':

$$L' = \frac{v}{2f'}$$

Express the ratio of L' to L and solve for L':

$$\frac{L'}{L} = \frac{f}{f'} \Rightarrow L' = \frac{f}{f'}L$$

Evaluate $L_{650\ \text{Hz}}$:

$$L_{650\,\text{Hz}} = \frac{500\,\text{Hz}}{650\,\text{Hz}} L_{500\,\text{Hz}}$$

$$= \frac{500\,\text{Hz}}{650\,\text{Hz}}(40\,\text{cm}) = 30.77\,\text{cm}$$

You should place your finger 9.23 cm from the scroll bridge.

60 ••

Picture the Problem Let f' represent the frequencies corresponding to the A, B, C, and D notes and $x(f')$ represent the distances from the end of the string that a finger must be placed to play each of these notes. Then, the distances at which the finger must be placed are given by $x(f') = L(f_G) - L(f')$.

Express the distances at which the finger must be placed in terms of the lengths of the G string and the frequencies f' of the A, B, C, and D notes:

$$x(f') = L(f_G) - L(f') \qquad (1)$$

Assuming that it vibrates in its fundamental mode, express the frequency of the G string in terms of its length:

$$f_G = \frac{v}{\lambda_G} = \frac{v}{2L_G}$$

Solve for L_G:

$$L_G = \frac{v}{2f_G}$$

Letting primed quantities refer to the string lengths and frequencies of

$$L' = \frac{v}{2f'}$$

the A, B, C, and D notes, express L' in terms of $f\,'$:

Express the ratio of L' to L and solve for L':

$$\frac{L'}{L_G} = \frac{f_G}{f\,'} \Rightarrow L' = \frac{f_G}{f\,'} L_G$$

Evaluate $L' = L(f')$ for the notes A, B, C and D to complete the table:

Note	Frequency	$L(f\,')$
	(Hz)	(cm)
A	220	26.73
B	247	23.81
C	262	22.44
D	294	20.00

Use equation (1) to evaluate $x(f\,')$ and complete the table to the right:

Note	Frequency	$L(f\,')$	$x(f\,')$
	(Hz)	(cm)	(cm)
A	220	26.73	3.27
B	247	23.81	6.19
C	262	22.44	7.56
D	294	20.00	10.0

61 ••

Picture the Problem We can use the fact that the resonance frequencies are multiples of the fundamental frequency to find both the fundamental frequency and the harmonic numbers corresponding to 375 Hz and 450 Hz. We can find the length of the string by relating it to the wavelength of the waves on it and the wavelength to the speed and frequency of the waves. The speed of the waves is, in turn, a function of the tension in the string and its linear density, both of which we are given.

(*a*) Express 375 Hz as an integer multiple of the fundamental frequency of the string:

$$nf_1 = 375\,\text{Hz} \qquad (1)$$

Express 450 Hz as an integer multiple of the fundamental frequency of the string:

$$(n+1)f_1 = 450\,\text{Hz} \qquad (2)$$

Solve equations (1) and (2) simultaneously for f_1:

$$f_1 = \boxed{75.0\,\text{Hz}}$$

(*b*) Substitute in equation (1) to obtain: $n=5$

$$\boxed{\text{The harmonics are the fifth and sixth.}}$$

(*c*) Express the length of the string as a function of the speed of transverse waves on it and its fundamental frequency:

$$L=\frac{\lambda}{2}=\frac{v}{2f_1}$$

Express the speed of transverse waves on the string in terms of the tension in the string and its linear density:

$$v=\sqrt{\frac{F}{\mu}}$$

Substitute to obtain:

$$L=\frac{1}{2f_1}\sqrt{\frac{F}{\mu}}$$

Substitute numerical values and evaluate *L*:

$$L=\frac{1}{2\left(75\,\text{s}^{-1}\right)}\sqrt{\frac{360\,\text{N}}{4\times10^{-3}\,\text{kg/m}}}=\boxed{2.00\,\text{m}}$$

62 ••

Picture the Problem We can use the fact that the resonance frequencies are multiples of the fundamental frequency and are expressible in terms of the speed of the waves and their wavelengths to find the harmonic numbers corresponding to wavelengths of 0.54 m and 0.48 m. We can find the length of the string by using the standing-wave condition for a string fixed at both ends.

(*a*) Express the frequency of the *n*th harmonic in terms of its wavelength:

$$nf_1=\frac{v}{\lambda_n}=\frac{v}{0.54\,\text{m}}$$

Express the frequency of the (*n* + 1)th harmonic in terms of its wavelength:

$$(n+1)f_1=\frac{v}{\lambda_{n+1}}=\frac{v}{0.48\,\text{m}}$$

Solve these equations simultaneously for *n*:

$$n=8$$

$$\boxed{\text{The harmonics are the eighth and ninth.}}$$

(*b*) Using the standing-wave condition, both ends fixed, relate the length of the string to the wavelength of its *n*th harmonic:

$$L = n\frac{\lambda_n}{2},\ n = 1, 2, 3, \ldots$$

Evaluate L for the eighth harmonic:

$$L = 8\left(\frac{0.54\ \text{m}}{2}\right) = \boxed{2.16\ \text{m}}$$

63 ••

Picture the Problem The linear densities of the strings are related to the transverse wave speed and tension through $v = \sqrt{F/\mu}$. We can use $v = f\lambda = 2fL$ to relate the frequencies of the violin strings to their lengths and linear densities.

(*a*) Relate the speed of transverse waves on a string to the tension in the string and solve for the string's linear density:

$$v = \sqrt{\frac{F}{\mu}}$$

and

$$\mu = \frac{F}{v^2}$$

Express the dependence of the speed of the transverse waves on their frequency and wavelength:

$$v = f_E\lambda = 2f_E L$$

Substitute to obtain:

$$\mu_E = \frac{F_E}{4f_E^2L^2}$$

Substitute numerical values and evaluate μ_E:

$$\mu_E = \frac{90\ \text{N}}{4\left[1.5\left(440\ \text{s}^{-1}\right)\right]^2\left(0.3\ \text{m}\right)^2} = 5.74\times10^{-4}\ \text{kg/m} = \boxed{0.574\ \text{g/m}}$$

(*b*) Evaluate μ_A:

$$\mu_A = \frac{90\ \text{N}}{4\left(440\ \text{s}^{-1}\right)^2\left(0.3\ \text{m}\right)^2} = 1.29\times10^{-3}\ \text{kg/m} = \boxed{1.29\ \text{g/m}}$$

Evaluate μ_D:

$$\mu_D = \frac{90\,\text{N}}{4\left(293\,\text{s}^{-1}\right)^2(0.3\,\text{m})^2} = 2.91\times10^{-3}\,\text{kg/m} = \boxed{2.91\,\text{g/m}}$$

Evaluate μ_G:

$$\mu_G = \frac{90\,\text{N}}{4\left(195\,\text{s}^{-1}\right)^2(0.3\,\text{m})^2} = 6.57\times10^{-3}\,\text{kg/m} = \boxed{6.57\,\text{g/m}}$$

64 ••

Picture the Problem The spatial period is one-half the wavelength of the standing wave produced by the sound and its reflection. Hence we can solve $c = f'\lambda'$ for λ' and use $f' = f\left[1/(1 - v/c)\right]$ to derive an expression for $\lambda'/2$ in terms of c, v, and f.

(*a*) Express the wavelength of the reflected sound as a function of its frequency and the speed of sound in air:

$$\lambda' = \frac{c}{f'}$$

Use the expression for the Doppler-shift in frequency when to source is in motion to obtain:

$$f' = f\frac{1}{1-\frac{v}{c}}$$

where c is the speed of sound.

Substitute to obtain:

$$\frac{\lambda'}{2} = \frac{c}{2f'} = \frac{c}{2f\dfrac{1}{1-\frac{v}{c}}} = \frac{c}{2f}\left(1-\frac{v}{c}\right) = \frac{c-v}{2f}$$

Substitute numerical values and evaluate the spatial period of the standing wave:

$$\frac{\lambda'}{2} = \frac{340\,\text{m/s} - 22.4\,\text{m/s}}{2\left(500\,\text{s}^{-1}\right)} = \boxed{0.318\,\text{m}}$$

(*b*) As the ambulance moves closer to the wall, the sound waves from its siren will periodically move in and out of resonance (i.e., the reflected waves will sometimes interfere constructively and sometimes partially destructively) so the intensity will periodically get louder and softer.

65 ••

Picture the Problem Beat frequencies are heard when the strings are vibrating with slightly different frequencies. To understand the beat frequency heard when the A and E strings are bowed simultaneously, we need to consider the harmonics of both strings. In part (c) we'll relate the tension in the string to the frequency of its vibration and set up a proportion involving the frequencies corresponding to the two tensions that we can solve for the tension when the E string is perfectly tuned.

(*a*) The two sounds produce a beat because the third harmonic of the A string equals the second harmonic of the E string, and the original frequency of the E string is slightly greater than 660 Hz. If $f_E = (660 + \Delta f)$ Hz, a beat of $2\Delta f$ will be heard.

(*b*) Because f_{beat} increases with increasing tension, the frequency of the E string is greater than 660 Hz. Thus the frequency of the E string is:

$$f_E = 660\,\text{Hz} + \tfrac{1}{2}(3\,\text{Hz}) = \boxed{661.5\,\text{Hz}}$$

(*c*) Express the frequency of a string as a function of its tension:

$$f = \frac{v}{\lambda} = \frac{1}{\lambda}\sqrt{\frac{F}{\mu}}$$

When the frequency of the E string is 660 Hz we have:

$$660\,\text{Hz} = \frac{1}{\lambda}\sqrt{\frac{F_{660\,\text{Hz}}}{\mu}}$$

When the frequency of the E string is 661.5 Hz we have:

$$661.5\,\text{Hz} = \frac{1}{\lambda}\sqrt{\frac{80\,\text{N}}{\mu}}$$

Divide the first of these equations by the second and solve for $F_{660\,\text{Hz}}$ to obtain:

$$F_{660\,\text{Hz}} = \left(\frac{660\,\text{Hz}}{661.5\,\text{Hz}}\right)^2 (80\,\text{N}) = \boxed{79.6\,\text{N}}$$

66 ••

Picture the Problem We can use the condition for constructive interference of the waves reflected from the walls in front of and behind you to relate the path difference to the wavelength of the sound. We can find the wavelength of the sound from its frequency and the speed of sound in air.

Express the total path difference as you walk toward the far wall of the hall:

$$\Delta x = \Delta x_{\text{near wall}} + \Delta x_{\text{far wall}} \qquad (1)$$

Express the condition on the path difference for constructive interference:

$$n\lambda = \Delta x \text{ where } n = 1, 2, 3, \ldots \qquad (2)$$

The reduction in the distance to the nearer wall as you walk a distance d is:

$$\Delta x_{\text{near wall}} = 2d$$

The increase in the distance to the farther wall as you walk a distance d is:

$$\Delta x_{\text{far wall}} = 2d$$

Substitute in equation (1) to find the total path difference as you walk a distance d:

$$\Delta x = 2d + 2d = 4d$$

Relate λ to f and v:

$$\lambda = \frac{v}{f}$$

Substitute in equation (2) to obtain:

$$n\frac{v}{f} = 4d$$

Solve for and evaluate d for n = 1:

$$d = \frac{v}{4f} = \frac{340\text{ m/s}}{4\left(680\text{ s}^{-1}\right)} = \boxed{12.5\text{ cm}}$$

***67 ••**

Picture the Problem Let the wave function for the wave traveling to the right be $y_R(x,t) = A\sin(kx - \omega t - \delta)$ and the wave function for the wave traveling to the left be $y_L(x,t) = A\sin(kx + \omega t + \delta)$ and use the identity $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$ to show that the sum of the wave functions can be written in the form $y(x,t) = A'\sin kx\cos(\omega t + \delta)$.

Express the sum of the traveling waves of equal amplitude moving in opposite directions:

$$y(x,t) = y_R(x,t) + y_L(x,t) = A\sin(kx - \omega t - \delta) \quad + A\sin(kx + \omega t + \delta)$$

Use the trigonometric identity to obtain:

$$y(x,t) = 2A\sin\left(\frac{kx - \omega t - \delta + kx + \omega t + \delta}{2}\right)\cos\left(\frac{kx - \omega t - \delta - kx - \omega t - \delta}{2}\right)$$
$$= 2A\sin kx\cos(-\omega t - \delta)$$

Because the cosine function is even; i.e., $\cos(-\theta) = \cos\theta$:

$$y(x,t) = 2A\sin kx\cos(\omega t + \delta)$$
$$= A'\sin kx\cos(\omega t + \delta)$$

where $A' = 2A$.

Thus we have:

$$y(x,t) = \boxed{A'\sin kx\cos(\omega t + \delta)}$$

provided $A' = 2A$.

68 ••

Picture the Problem We can find ω_3 and k_3 from the given information and substitute to find the wave function for the 3rd harmonic. We can use the time-derivative of this expression (the transverse speed) to express the kinetic energy of a segment of mass *dm* and length *dx* of the string. Integrating this expression will give us the maximum kinetic energy of the string in terms of its mass.

(*a*) Write the general form of the wave function for the 3rd harmonic:

$$y_3(x,t) = A_3\sin k_3 x\cos\omega_3 t$$

Evaluate ω_3:

$$\omega_3 = 2\pi f_3 = 2\pi\left(100\,\text{s}^{-1}\right) = 200\pi\,\text{s}^{-1}$$

Using the standing-wave condition for a string fixed at one end, relate the length of the string to its 3rd harmonic wavelength:

$$L = 3\frac{\lambda_3}{4}$$

and

$$\lambda_3 = \frac{4}{3}L = \frac{4}{3}(2\,\text{m}) = \frac{8}{3}\,\text{m}$$

Evaluate k_3:

$$k_3 = \frac{2\pi}{\lambda_3} = \frac{2\pi}{(8/3)\text{m}} = \frac{3\pi}{4}\,\text{m}^{-1}$$

Substitute numerical values and evaluate K_{max}:

$$K_{max} = \tfrac{1}{4}m\left(200\pi\,\text{s}^{-1}\right)^2(0.03\,\text{m})^2 = \boxed{(88.8\,\text{J/kg})m}$$

Substitute to obtain:

$$y_3(x,t) = \boxed{(0.03\,\text{m})\sin\left[\left(\frac{3\pi}{4}\,\text{m}^{-1}\right)x\right]\cos\left(200\pi\,\text{s}^{-1}\right)t}$$

(*b*) Express the kinetic energy of a segment of string of mass *dm*:

$$dK = \tfrac{1}{2}dmv_y^2$$

Express the mass of the segment in terms of its length *dx* and the linear density of the string:

$$dm = \mu dx$$

Using our result in (*a*), evaluate v_y:

$$\begin{aligned} v_y &= \frac{\partial}{\partial t}\left[(0.03\,\text{m})\sin\left[\left(\frac{3\pi}{4}\,\text{m}^{-1}\right)x\right]\cos\left(200\pi\,\text{s}^{-1}\right)t\right] \\ &= -\left(200\pi\,\text{s}^{-1}\right)(0.03\,\text{m})\sin\left[\left(\frac{3\pi}{4}\,\text{m}^{-1}\right)x\right]\sin\left(200\pi\,\text{s}^{-1}\right)t \\ &= -(6\pi\,\text{m/s})\sin\left[\left(\frac{3\pi}{4}\,\text{m}^{-1}\right)x\right]\sin\left(200\pi\,\text{s}^{-1}\right)t \end{aligned}$$

Substitute to obtain:

$$dK = \boxed{\tfrac{1}{2}\left[(6\pi\,\text{m/s})\sin\left[\left(\frac{3\pi}{4}\,\text{m}^{-1}\right)x\right]\sin\left(200\pi\,\text{s}^{-1}\right)t\right]^2\mu dx}$$

Express the condition on the time that *dK* is a maximum:

$$\sin\left(200\pi\,\text{s}^{-1}\right)t = 1$$

or

$$\left(200\pi\,\text{s}^{-1}\right)t = \frac{\pi}{2}, \frac{3\pi}{2}, ...$$

Solve for and evaluate *t*:

$$t = \frac{1}{200\pi\,\text{s}^{-1}}\frac{\pi}{2}, \frac{1}{200\pi\,\text{s}^{-1}}\frac{3\pi}{2}, ... = \boxed{2.50\,\text{ms}, 7.50\,\text{ms}, ...}$$

Because the string's maximum

kinetic energy occurs when $y(x,t) = 0$:

The string is a straight line.

(c) Integrate dK from (b) over the length of the string to obtain:

$$K_{max} = \int_0^L \tfrac{1}{2}\left[\omega A \sin kx \sin \omega t\right]^2 \mu dx$$
$$= \tfrac{1}{2}\mu\omega^2 A^2 \int_0^L \sin^2 kx dx$$
$$= \tfrac{1}{2}\mu\omega^2 A^2 \frac{1}{k}\left[\tfrac{1}{2}kx - \tfrac{1}{4}\sin 2kx\right]_0^L$$
$$= \tfrac{1}{4}m\omega^2 A^2$$

where m is the mass of the string.

***69 ••**

Picture the Problem We can equate the expression for the velocity of a wave on a string and the expression for the velocity of a wave in terms of its frequency and wavelength to obtain an expression for the weight that must be suspended from the end of the string in order to produce a given standing wave pattern. By using the condition on the wavelength that must be satisfied at resonance, we can express the weight on the end of the string in terms of μ, f, L, and an integer n and then evaluate this expression for $n = 1$, 2, and 3 for the first three standing wave patterns.

Express the velocity of a wave on the string in terms of the tension T in the string and its linear density μ:

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{mg}{\mu}}$$

where mg is the weight of the object suspended from the end of the string.

Express the wave speed in terms of its wavelength λ and frequency f:

$$v = f\lambda$$

Eliminate v to obtain:

$$f\lambda = \sqrt{\frac{mg}{\mu}}$$

Solve for mg:

$$mg = \mu f^2\lambda^2$$

Express the condition on λ that corresponds to resonance:

$$\lambda = \frac{2L}{n}, n = 1, 2, 3, \ldots$$

Substitute to obtain:

$$mg = \mu f^2\left(\frac{2L}{n}\right)^2, n = 1, 2, 3, \ldots$$

or

$$mg = \frac{4\mu f^2 L^2}{n^2}, n = 1, 2, 3, \ldots$$

Evaluate mg for $n = 1$:

$$mg = \frac{4(0.415\,\text{g/m})(80\,\text{s}^{-1})^2(0.2\,\text{m})^2}{(1)^2}$$

$$= \boxed{0.425\,\text{N}}$$

which corresponds, at sea level, to a mass of 43.3 g.

Evaluate mg for $n = 2$:

$$mg = \frac{4(0.415\,\text{g/m})(80\,\text{s}^{-1})^2(0.2\,\text{m})^2}{(2)^2}$$

$$= \boxed{0.106\,\text{N}}$$

which corresponds, at sea level, to a mass of 10.8 g.

Wave Packets

70 •

Picture the Problem We can find the maximum duration of each pulse under the conditions given in the problem from the reciprocal of frequency of the pulses and the range of frequencies from the wave packet condition on $\Delta\omega$ and Δt.

(*a*) The maximum duration of each pulse is its period:

$$T = \frac{1}{f} = \frac{1}{10^7\,\text{s}^{-1}} = 10^{-7}\,\text{s} = \boxed{0.100\,\mu\text{s}}$$

(*b*) Express the wave packet condition on $\Delta\omega$ and Δt:

$$\Delta\omega\Delta t \approx 1 \text{ or } 2\pi\Delta f\Delta t \approx 1$$

Solve for Δf:

$$\Delta f \approx \frac{1}{2\pi\Delta t} = \frac{T}{2\pi}$$

Substitute numerical values and evaluate Δf:

$$\Delta f \approx \frac{10^7\,\text{s}^{-1}}{2\pi} = \boxed{1.59\,\text{MHz}}$$

71 •

Picture the Problem We can approximate the duration of the pulse from the product of the number of cycles in the interval and the period of each cycle and the wavelength from the number of complete wavelengths in Δx. We can use its definition to find the wave number k from the wavelength λ.

(*a*) Relate the duration of the pulse to the number of cycles in the interval and the period of each cycle:

$$\Delta t \approx NT = \boxed{\frac{N}{f_0}}$$

(*b*) There are about *N* complete wavelengths in Δx; hence:

$$\lambda \approx \boxed{\frac{\Delta x}{N}}$$

(*c*) Use its definition to express the wave number *k*:

$$k = \frac{2\pi}{\lambda} = \boxed{\frac{2\pi N}{\Delta x}}$$

(*d*) *N* is uncertain because the waveform dies out gradually rather than stopping abruptly at some time; hence, where the pulse starts and stops is not well defined.

(*e*) Using our result in part (*c*), express the uncertainty in *k*:

$$\Delta k = \frac{2\pi \Delta N}{\Delta x} = \boxed{\frac{2\pi}{\Delta x}}$$

because $\Delta N = \pm 1$.

General Problems

72 •

Picture the Problem We can use $v = f\lambda$ and $v = \sqrt{F/\mu}$ to relate the tension in the piano wire to its fundamental frequency.

Relate the tension in the wire to the speed of transverse waves on it:

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{FL}{m}}$$

Express the speed of the transverse in terms of their wavelength and frequency:

$$v = f\lambda$$

Equate these expressions and solve for *F* to obtain:

$$F = \frac{mf^2\lambda^2}{L}$$

Relate λ for the fundamental mode of vibration to the length of the piano wire:

$$\lambda = 2L$$

Substitute to obtain:

$$F = 4mf^2L$$

Substitute numerical values and evaluate *F*:

$$F = 4\left(7\times10^{-3}\text{ kg}\right)\left(261.63\text{ s}^{-1}\right)^2\left(0.8\text{ m}\right) = \boxed{1.53\text{ kN}}$$

73 •

Picture the Problem We can use $v = f_n\lambda_n$ to express the resonance frequencies of the ear canal in terms of their wavelengths and $L = n\frac{\lambda_n}{4}, n = 1, 3, 5, \ldots$ to relate the length of the ear canal to its resonance wavelengths.

(*a*) Relate the resonance frequencies to the speed of sound and the wavelength of the compressional vibrations:

$$f_n = \frac{v}{\lambda_n}$$

Express the condition for constructive interference in a pipe that is open at one end:

$$L = n\frac{\lambda_n}{4}, n = 1, 3, 5, \ldots$$

Solve for λ_n:

$$\lambda_n = \frac{4L}{n}$$

Substitute to obtain:

$$f_n = n\frac{v}{4L} = n\frac{340\,\text{m/s}}{4\left(2.5\times10^{-2}\,\text{m}\right)} = n(3.40\,\text{kHz})$$

Evaluate f_1, f_2, and f_3:

$$f_1 = \boxed{3.40\,\text{kHz}},$$

$$f_3 = 3\times3.40\,\text{kHz} = \boxed{10.2\,\text{kHz}},$$

and

$$f_5 = 5\times3.40\,\text{kHz} = \boxed{17.0\,\text{kHz}}$$

(*b*)

Frequencies near 3400 Hz will be most readily perceived.

74 •

Picture the Problem We can use $L = n\frac{\lambda_n}{4}, n = 1, 3, 5, \ldots$ to express the wavelengths of the fundamental and next two harmonics in terms of the length of the rope and $v = f_n\lambda_n$ and $v = \sqrt{\frac{F}{\mu}}$ to relate the resonance frequencies to their wavelengths.

(*a*) Express the condition for constructive interference on a rope

$$L = n\frac{\lambda_n}{4}, n = 1, 3, 5, \ldots$$

that is fixed at one end:

Solve for λ_n:

$$\lambda_n = \frac{4L}{n} = \frac{4(4\,\text{m})}{n} = \frac{16\,\text{m}}{n}$$

Evaluate λ_n for n = 1, 3, and 5:

$$\lambda_1 = \boxed{16.0\,\text{m}}$$

$$\lambda_3 = \frac{16\,\text{m}}{3} = \boxed{5.33\,\text{m}}$$

and

$$\lambda_5 = \frac{16\,\text{m}}{5} = \boxed{3.20\,\text{m}}$$

(b) Relate the resonance frequencies to the speed and wavelength of the transverse waves:

$$f_n = \frac{v}{\lambda_n}$$

Express the speed of the transverse waves as a function of the tension in the rope:

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{FL}{m}}$$

where m and L are the mass and length of the rope.

Substitute to obtain:

$$f_n = \frac{1}{\lambda_n}\sqrt{\frac{FL}{m}} = \frac{1}{\lambda_n}\sqrt{\frac{(400\,\text{N})(4\,\text{m})}{0.16\,\text{kg}}} = \frac{100\,\text{m/s}}{\lambda_n}$$

Evaluate f_n for n = 1, 3, and 5:

$$f_1 = \frac{100\,\text{m/s}}{16\,\text{m}} = \boxed{6.25\,\text{Hz}}$$

$$f_3 = \frac{100\,\text{m/s}}{5.33\,\text{m}} = \boxed{18.8\,\text{Hz}}$$

and

$$f_5 = \frac{100\,\text{m/s}}{3.20\,\text{m}} = \boxed{31.3\,\text{Hz}}$$

75 ••

Picture the Problem The path difference at the point where the resultant wave an amplitude A is related to the phase shift between the interfering waves according to $\Delta x/\lambda = \delta/2\pi$. We can use this relationship to find the phase shift and the expression for the amplitude resulting from the superposition of two waves of the same amplitude and frequency to find the phase shift.

Express the relation between the path difference and the phase shift at the point where the resultant wave has an amplitude *A*:

$$\Delta x = \lambda \frac{\delta}{2\pi}$$

Express the amplitude resulting from the superposition of two waves of the same amplitude and frequency:

$$A = 2y_0 \cos\tfrac{1}{2}\delta$$

Solve for and evaluate δ:

$$\delta = 2\cos^{-1}\frac{A}{2y_0} = 2\cos^{-1}\frac{A}{2A} = \frac{2\pi}{3}$$

Substitute and simplify to obtain:

$$\Delta x = \lambda \frac{2\pi/3}{2\pi} = \boxed{\tfrac{1}{3}\lambda}$$

76 ••

Picture the Problem We can use $v = f_n\lambda_n$ to express the resonance frequencies of the string in terms of their wavelengths and $L = n\frac{\lambda_n}{2},\ n = 1, 2, 3, \ldots$ to relate the length of the string to the resonance wavelengths for a string fixed at both ends. Our strategy for part (*b*) will be the same … except that we'll use the standing-wave condition $L = n\frac{\lambda_n}{4},\ n = 1, 3, 5, \ldots$ for strings with one end free.

(*a*) Relate the frequencies of the harmonics to their wavelengths and the speed of transverse waves on the string:

$$f_n = \frac{v}{\lambda_n}$$

Express the standing-wave condition for a string with both ends fixed:

$$L = n\frac{\lambda_n}{2},\ n = 1, 2, 3, \ldots$$

Solve for λ_n:

$$\lambda_n = \frac{2L}{n}$$

Substitute to obtain:

$$f_n = n\frac{v}{2L}$$

Express the speed of the transverse waves as a function of the tension in the string:

$$v = \sqrt{\frac{F}{\mu}}$$

Substitute to obtain:

$$f_n = n\frac{1}{2L}\sqrt{\frac{F}{\mu}} = n\frac{1}{2(35\,\text{m})}\sqrt{\frac{18\,\text{N}}{0.0085\,\text{kg/m}}} = n(0.657\,\text{Hz})$$

Calculate the 1st four harmonics:

$$f_1 = \boxed{0.657\,\text{Hz}}$$

$$f_2 = 2(0.657\,\text{Hz}) = \boxed{1.31\,\text{Hz}}$$

$$f_3 = 3(0.657\,\text{Hz}) = \boxed{1.97\,\text{Hz}}$$

and

$$f_4 = 4(0.657\,\text{Hz}) = \boxed{2.63\,\text{Hz}}$$

(*b*) Express the standing-wave condition for a string fixed at one end:

$$L = n\frac{\lambda_n}{4},\ n = 1, 3, 5, \ldots$$

Solve for λ_n:

$$\lambda_n = \frac{4L}{n}$$

The resonance frequencies equation becomes:

$$f_n = n\frac{1}{4L}\sqrt{\frac{F}{\mu}} = n\frac{1}{4(35\,\text{m})}\sqrt{\frac{18\,\text{N}}{0.0085\,\text{kg/m}}} = n(0.329\,\text{Hz})$$

Calculate the 1st four harmonics:

$$f_1 = \boxed{0.329\,\text{Hz}}$$

$$f_3 = 3(0.329\,\text{Hz}) = \boxed{0.987\,\text{Hz}}$$

$$f_5 = 5(0.329\,\text{Hz}) = \boxed{1.65\,\text{Hz}}$$

and

$$f_7 = 7(0.329\,\text{Hz}) = \boxed{2.30\,\text{Hz}}$$

77 ••

Picture the Problem We'll model the shaft as a pipe of length *L* with one end open. We can relate the frequencies of the harmonics to their wavelengths and the speed of sound using $v = f_n\lambda_n$ and the depth of the mine shaft to the resonance wavelengths using the standing-wave condition for a pipe with one end open; $L = n\frac{\lambda_n}{4}$, $n = 1, 3, 5, \ldots$.

Relate the frequencies of the harmonics to their wavelengths and the speed of sound:

$$f_n = \frac{v}{\lambda_n}$$

Express the standing-wave condition for a pipe with one end open:

$$L = n\frac{\lambda_n}{4},\ n = 1, 3, 5, \ldots$$

Solve for λ_n:

$$\lambda_n = \frac{4L}{n}$$

Substitute to obtain:

$$f_n = n\frac{v}{4L}$$

For f_n = 63.58 Hz:

$$63.58\,\text{Hz} = n\frac{v}{4L}$$

For f_{n+2} = 89.25 Hz:

$$89.25\,\text{Hz} = (n+2)\frac{v}{4L}$$

Divide either of these equations by the other and solve for n to obtain:

$$n = 4.95 \approx 5$$

Substitute in the equation for $f_n = f_5$ = 63.58 Hz:

$$f_5 = \frac{5v}{4L}$$

Solve for and evaluate L:

$$L = \frac{5v}{4f_5} = \frac{5(340\,\text{m/s})}{4(63.58\,\text{s}^{-1})} = \boxed{6.68\,\text{m}}$$

78 ••

Picture the Problem We can use the standing-wave condition for a string with one end free to find the wavelength of the 5th harmonic and the definitions of the wave number and angular frequency to calculate these quantitities. We can then substitute in the wave function for a wave in the *n*th harmonic to find the wave function for this standing wave.

(*a*) Express the standing-wave condition for a string with one end free:

$$L = n\frac{\lambda_n}{4},\ n = 1, 3, 5, \ldots$$

Solve for and evaluate λ_5:

$$\lambda_5 = \frac{4L}{5} = \frac{4(5\,\text{m})}{5} = \boxed{4.00\,\text{m}}$$

(*b*) Use its definition to calculate the wave number:

$$k_5 = \frac{2\pi}{\lambda_5} = \frac{2\pi}{4\,\text{m}} = \boxed{\frac{\pi}{2}\,\text{m}^{-1}}$$

(*c*) Using its definition, calculate the angular frequency:

$$\omega_5 = 2\pi f_5 = 2\pi\left(400\,\text{s}^{-1}\right) = \boxed{800\pi\,\text{s}^{-1}}$$

(*d*) Write the wave function for a standing wave in the *n*th harmonic:

$$y_n(x,t) = A\sin k_n x\cos\omega_n t$$

Substitute to obtain:

$$y_5(x,t) = A\sin(k_5 x)\cos(\omega_5 t) = \boxed{(0.03\,\text{m})\sin\left[\left(\frac{\pi}{2}\,\text{m}^{-1}\right)x\right]\cos\left(800\pi\,\text{s}^{-1}\right)t}$$

79 ••

Picture the Problem The coefficient of the factor containing the time dependence in the wave function is the maximum displacement of any point on the string. The time derivative of the wave function is the instantaneous speed of any point on the string and the coefficient of the factor containing the time dependence is the maximum speed of any point on the string.

Differentiate the wave function with respect to *t* to find the speed of any point on the string:

$$\begin{aligned} v_y &= \frac{\partial}{\partial t}\left[0.02\sin 4\pi x\cos 60\pi t\right] \\ &= -(0.02)(60\pi)\sin 4\pi x\sin 60\pi t \\ &= -1.2\pi\sin 4\pi x\sin 60\pi t \end{aligned}$$

(*a*) Referring to the wave function, express the maximum displacement of the standing wave:

$$y_{\text{max}}(x) = (0.02\,\text{m})\sin\left[\left(4\pi\,\text{m}^{-1}\right)x\right] \quad (1)$$

Evaluate equation (1) at $x = 0.10$ m:

$$\begin{aligned} y_{\text{max}}(0.10\,\text{m}) &= (0.02\,\text{m}) \\ &\quad\times\sin\left[\left(4\pi\,\text{m}^{-1}\right)(0.10\,\text{m})\right] \\ &= \boxed{1.90\,\text{cm}} \end{aligned}$$

Referring to the derivative of the wave function with respect to *t*, express the maximum speed of the

$$v_{y,\text{max}}(x) = (1.2\pi\,\text{m/s})\sin\left[\left(4\pi\,\text{m}^{-1}\right)x\right] \quad (2)$$

standing wave:

Evaluate equation (2) at x = 0.10 m:

$$v_{y,\max}(0.10\,\text{m}) = (1.2\pi\ \text{m/s}) \times \sin\left[\left(4\pi\ \text{m}^{-1}\right)(0.10\,\text{m})\right] = \boxed{3.59\,\text{m/s}}$$

(*b*) Evaluate equation (1) at x = 0.25 m:

$$y_{\max}(0.25\,\text{m}) = (0.02\,\text{m}) \times \sin\left[\left(4\pi\ \text{m}^{-1}\right)(0.25\,\text{m})\right] = \boxed{0}$$

Evaluate equation (2) at x = 0.25 m:

$$v_{y,\max}(0.25\,\text{m}) = (1.2\pi\ \text{m/s}) \times \sin\left[\left(4\pi\ \text{m}^{-1}\right)(0.25\,\text{m})\right] = \boxed{0}$$

(*c*) Evaluate equation (1) at x = 0.30 m:

$$y_{\max}(0.30\,\text{m}) = (0.02\,\text{m}) \times \sin\left[\left(4\pi\ \text{m}^{-1}\right)(0.30\,\text{m})\right] = \boxed{1.18\,\text{cm}}$$

Evaluate equation (2) at x = 0.30 m:

$$v_{y,\max}(0.30\,\text{m}) = (1.2\pi\ \text{m/s}) \times \sin\left[\left(4\pi\ \text{m}^{-1}\right)(0.30\,\text{m})\right] = \boxed{2.22\,\text{m/s}}$$

(*d*) Evaluate equation (1) at x = 0.50 m:

$$y_{\max}(0.50\,\text{m}) = (0.02\,\text{m}) \times \sin\left[\left(4\pi\ \text{m}^{-1}\right)(0.50\,\text{m})\right] = \boxed{0}$$

Evaluate equation (2) at x = 0.50 m:

$$v_{y,\max}(0.50\,\text{m}) = (1.2\pi\ \text{m/s}) \times \sin\left[\left(4\pi\ \text{m}^{-1}\right)(0.50\,\text{m})\right] = \boxed{0}$$

80 ••

Picture the Problem In part (*a*) we can use the standing-wave condition for a wire fixed at both ends and the fact that nodes are separated by one-half wavelength to find the harmonic number. In part (*b*) we can relate the resonance frequencies to their wavelengths and the speed of transverse waves and express the speed of the transverse waves in terms of the tension in the wire and its linear density.

(*a*) Express the standing-wave condition for a wire fixed at both ends:

$$L = n\frac{\lambda_n}{2},\ n = 1, 2, 3, \ldots$$

Solve for *n*:

$$n = \frac{2L}{\lambda_n}$$

Solve for and evaluate λ_1:

$$\lambda_1 = 2L = 2(2.5\,\text{m}) = 5\,\text{m}$$

Relate the distance between nodes to the distance of the node closest to one end and solve for λ_n:

$$\tfrac{1}{2}\lambda_n = 0.5\,\text{m}$$

and

$$\lambda_n = 1\,\text{m}$$

Substitute and evaluate *n*:

$$n = \frac{2(2.5\,\text{m})}{1\,\text{m}} = \boxed{5}$$

(*b*) Express the resonance frequencies in terms of the their wavelengths and the speed of transverse waves on the wire:

$$f_n = \frac{v}{\lambda_n} = n\frac{v}{\lambda_1}$$

Relate the speed of transverse waves on the wire to the tension in the wire:

$$v = \sqrt{\frac{F}{\mu}}$$

Substitute and simplify to obtain:

$$f_n = n\frac{1}{\lambda_1}\sqrt{\frac{FL}{m}} = n\frac{1}{5\,\text{m}}\sqrt{\frac{(30\,\text{N})(2.5\,\text{m})}{0.1\,\text{kg}}} = n(5.48\,\text{Hz})$$

Evaluate f_n for $n = 1$, 2, and 3:

$$f_1 = \boxed{5.48\,\text{Hz}}$$

$$f_2 = 2(5.48\,\text{Hz}) = \boxed{11.0\,\text{Hz}}$$

and

$$f_3 = 3(5.48\,\text{Hz}) = \boxed{16.4\,\text{Hz}}$$

***81** ••

Picture the Problem We can use $v = f\lambda$ to relate the speed of sound in the gas to the distance between the piles of powder in the glass tube.

(*a*) At resonance, standing waves are set up in the tube. At a displacement antinode, the powder is moved about; at a node the powder is stationary, and so it collects at the nodes.

(*b*) Relate the speed of sound to its frequency and wavelength:

$$v = f\lambda$$

Letting *D* = distance between nodes, relate the distance between the nodes to the wavelength of the sound:

$$\lambda = 2D$$

Substitute to obtain:

$$v = \boxed{2fD}$$

(*c*) If we let the length *L* of the tube be 1.2 m and assume that v_{air} = 344 m/s (the speed of sound in air at 20°C), then the 10th harmonic corresponds to *D* = 25.3 cm and a driving frequency of:

$$f_{air} = \frac{v_{air}}{2D} = \frac{344\text{ m/s}}{2(0.253\text{ m})} = \boxed{680\text{ Hz}}$$

(*d*) If $f = 2$ kHz and $v_{He} = 1008$ m/s (the speed of sound in helium at 20°C), then *D* for the 10th harmonic in helium would 25.3 cm and *D* for the 10th harmonic in air would be 8.60 cm. Hence, neglecting end effects at the driven end, a tube whose length is the least common multiple of 8.60 cm and 25.3 cm (218 cm) would work well for the measurement of the speed of sound in either air or helium.

82 ••

Picture the Problem We can use $v = \sqrt{F/\mu}$ to express *F* as a function of *v* and $v = f\lambda$ to relate *v* to the frequency and wavelength of the string's fundamental mode. Because, for a string fixed at both ends, $f_n = nf_1$, we can extend our result in part (*a*) to part (*b*).

(*a*) Relate the speed of the transverse waves on the string to the tension in it:

$$v = \sqrt{\frac{F}{\mu}}$$

Solve for *F*:

$$F = \mu v^2 \qquad (1)$$

Relate the speed of the transverse waves on the string to their frequency and wavelength:

$$v = f_1\lambda_1$$

Express the wavelength of the fundamental mode to the length of the string:

$$\lambda_1 = 2L$$

Substitute to obtain:

$$v = 2fL$$

Substitute in equation (1) to obtain:

$$F = 4f^2L^2\mu \qquad (2)$$

Substitute numerical values and evaluate F:

$$F = 4\left(60\,\text{s}^{-1}\right)^2(2.5\,\text{m})^2\left(8\times10^{-3}\,\text{kg/m}\right) = \boxed{720\,\text{N}}$$

(*b*) For the *n*th harmonic, equation (2) becomes:

$$F_n = f_n^2L^2\mu = n^2f_1^2L^2\mu = n^2(720\,\text{N})$$

Evaluate this expression for n = 2, 3, and 4:

$$F_2 = 4(720\,\text{N}) = \boxed{2.88\,\text{kN}}$$

$$F_3 = 9(720\,\text{N}) = \boxed{6.48\,\text{kN}}$$

and

$$F_4 = 16(720\,\text{N}) = \boxed{11.5\,\text{kN}}$$

83 ••

Picture the Problem We can use the conditions $\Delta f = f_1$ and $f_n = nf_1$, where n is an integer, which must be satisfied if the pipe is open at both ends to decide whether the pipe is closed at one end or open at both ends. Once we have decided this question, we can use the condition relating Δf and the fundamental frequency to determine the latter. In part (*c*) we can use the standing-wave condition for the appropriate pipe to relate its length to its resonance wavelengths.

(*a*) Express the conditions on the frequencies for a pipe that is open at both ends:

$$\Delta f = f_1$$

and

$$f_n = nf_1$$

Evaluate $\Delta f = f_1$:

$$\Delta f = 1834\,\text{Hz} - 1310\,\text{Hz} = 524\,\text{Hz}$$

Using the 2nd condition, find n:

$$n = \frac{f_n}{f_1} = \frac{1310\,\text{Hz}}{524\,\text{Hz}} = 2.5$$

$\boxed{\text{The pipe is closed at one end.}}$

(*b*) Express the condition on the frequencies for a pipe that is open at both ends:

$$\Delta f = 2f_1$$

Solve for and evaluate f_1:

$$f_1 = \tfrac{1}{2}\Delta f = \tfrac{1}{2}(524\,\text{Hz}) = \boxed{262\,\text{Hz}}$$

(*c*) Using the standing-wave condition for a pipe open at one end, relate the length of the pipe to its resonance wavelengths:

$$L = n\frac{\lambda_n}{4},\ n = 1, 3, 5, \ldots$$

For $n = 1$ we have:

$$\lambda_1 = \frac{v}{f_1} \text{ and } L = \frac{\lambda_1}{4} = \frac{v}{4f_1}$$

Substitute numerical values and evaluate L:

$$L = \frac{340\,\text{m/s}}{4(262\,\text{s}^{-1})} = \boxed{32.4\,\text{cm}}$$

84 ••

Picture the Problem We can relate the speed of sound in air to the frequency of the violin string and the wavelength of the sound in the open tube that is closed at one end by water. The wavelength of the sound, in turn, is a function of the length of the air column and so we can derive an expression for the speed of sound as a function of the frequency of the transverse waves on the violin string and the length of the air column above the water. Knowing that the violin string is vibrating in its fundamental mode, we can express this frequency in terms of the tension in the string and its linear density.

Express the speed of sound in the tube in terms of its fundamental frequency and wavelength:

$$v_s = f_1\lambda_1$$

Using the standing-wave condition for a tube open at one end, relate the speed of sound to the length of the air column in the tube:

$$L_{\text{air column}} = n\frac{\lambda_n}{4},\ n = 1, 3, 5, \ldots$$

Solve for λ_1:

$$\lambda_1 = 4L_{\text{air column}}$$

Substitute to obtain:

$$v_s = 4f_1L_{\text{air column}} \qquad (1)$$

Express the frequency of the transverse waves on the violin string in terms of their wavelength and the speed with which they propagate on the string:

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L_{\text{string}}}$$

Relate the speed of the transverse waves on the string to the tension in it:

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{FL_{\text{string}}}{m}}$$

Substitute to obtain:

$$f_1 = \frac{1}{2L_{\text{string}}}\sqrt{\frac{FL_{\text{string}}}{m}} = \sqrt{\frac{F}{4mL_{\text{string}}}}$$

Substitute in equation (1) to obtain:

$$v_{\text{s}} = 4L_{\text{air column}}\sqrt{\frac{F}{4mL_{\text{string}}}} = 2L_{\text{air column}}\sqrt{\frac{F}{mL_{\text{string}}}}$$

Substitute numerical values and evaluate v_{s}:

$$v_{\text{s}} = 2(0.18\,\text{m})\sqrt{\frac{(440\,\text{N})}{(10^{-3}\,\text{kg})(0.5\,\text{m})}} = \boxed{338\,\text{m/s}}$$

The method is not very accurate because it neglects end effects (see Problem 56).

85 ••

Picture the Problem We know that the superimposed traveling waves have the same wave number and angular frequency as the standing-wave function, have equal amplitudes that are half that of the standing-wave function, and travel in opposite directions. From inspection of the standing-wave function we note that $k = \frac{1}{2}\pi\,\text{m}^{-1}$ and $\omega = 40\pi\,\text{s}^{-1}$. We can express the velocity of a segment of the rope by differentiating the standing-wave function with respect to time and the acceleration by differentiating the velocity function with respect to time.

(*a*) Write the wave function for the wave traveling in the positive *x* direction:

$$y_1(x,t) = \boxed{(0.01\text{m})\sin\left[\left(\frac{\pi}{2}\text{m}^{-1}\right)x - \left(40\pi\,\text{s}^{-1}\right)t\right]}$$

Write the wave function for the wave traveling in the negative x direction:

$$y_2(x,t) = \boxed{(0.01\,\text{m})\sin\left[\left(\frac{\pi}{2}\,\text{m}^{-1}\right)x + \left(40\pi\,\text{s}^{-1}\right)t\right]}$$

(b) Express the distance d between nodes in terms of the wavelength of the standing wave:

$$d = \tfrac{1}{2}\lambda$$

Use the wave number to find the wavelength:

$$k = \tfrac{1}{2}\pi\,\text{m}^{-1} = \frac{2\pi}{\lambda}$$

and

$$\lambda = 4\,\text{m}$$

Substitute and evaluate d:

$$d = \tfrac{1}{2}(4\,\text{m}) = \boxed{2.00\,\text{m}}$$

(c) Differentiate the given wave function with respect to t to express the velocity of any segment of the rope:

$$v_y(x,t) = \frac{\partial}{\partial t}\left[(0.02\,\text{m})\sin\left(\frac{\pi}{2}\,\text{m}^{-1}\right)x\cos\left(40\pi\,\text{s}^{-1}\right)t\right]$$

$$= -(0.8\pi\,\text{m/s})\sin\left(\frac{\pi}{2}\,\text{m}^{-1}\right)x\sin\left(40\pi\,\text{s}^{-1}\right)t$$

Evaluate $v_y(1\,\text{m},t)$:

$$v_y(1\,\text{m},t) = -(0.8\pi\,\text{m/s})\sin\left(\frac{\pi}{2}\,\text{m}^{-1}\right)(1\,\text{m})\sin\left(40\pi\,\text{s}^{-1}\right)t$$

$$= -(0.8\pi\,\text{m/s})\sin\left(40\pi\,\text{s}^{-1}\right)t$$

$$= \boxed{-(2.51\,\text{m/s})\sin\left(40\pi\,\text{s}^{-1}\right)t}$$

(d) Differentiate $v_y(x,t)$ with respect to time to obtain $a_y(x,t)$:

$$a_y(x,t) = \frac{\partial}{\partial t}\left[-(0.8\pi\,\text{m/s})\sin\left(\frac{\pi}{2}\,\text{m}^{-1}\right)x\sin\left(40\pi\,\text{s}^{-1}\right)t\right]$$

$$= -\left(32\pi^2\,\text{m/s}^2\right)\sin\left(\frac{\pi}{2}\,\text{m}^{-1}\right)x\cos\left(40\pi\,\text{s}^{-1}\right)t$$

Evaluate $a_y(1\,\text{m},t)$:

$$a_y(1\,\text{m},t) = -\left(32\pi^2\ \text{m/s}^2\right)\sin\left(\frac{\pi}{2}\text{m}^{-1}\right)(1\,\text{m})\cos\left(40\pi\,\text{s}^{-1}\right)t$$
$$= -\left(32\pi^2\ \text{m/s}^2\right)\cos\left(40\pi\,\text{s}^{-1}\right)t$$
$$= \boxed{-\left(316\,\text{m/s}^2\right)\cos\left(40\pi\,\text{s}^{-1}\right)t}$$

86 ••

Picture the Problem We can use the definition of intensity to find the intensity of each speaker, the dependence of intensity on the square of the amplitude of the wave disturbance to express the amplitudes of the waves, and the dependence of the intensity on whether the speakers are coherent and their phase difference to find the intensity at the given point.

(*a*) Express the intensity as a function of the distance of a point from the source:

$$I = \frac{P}{4\pi r^2}$$

Evaluate I_1:

$$I_1 = \frac{1\,\text{mW}}{4\pi(2\,\text{m})^2} = \boxed{19.9\,\mu\text{W/m}^2}$$

Evaluate I_2:

$$I_2 = \frac{1\,\text{mW}}{4\pi(3\,\text{m})^2} = \boxed{8.84\,\mu\text{W/m}^2}$$

(*b*) Using $v = f\lambda$, find the wavelength of the sound:

$$\lambda = \frac{v}{f} = \frac{340\,\text{m/s}}{680\,\text{s}^{-1}} = 0.5\,\text{m}$$

Express the path difference in terms of λ:

$$\Delta x = 2\lambda$$

and so there is constructive interference at point P.

Express the intensity at point P due to the sound from source 1:

$$I_1 = \text{constant}\times A_1^2$$

or

$$A_1 = C\sqrt{I_1}$$

where C is a constant.

Express the intensity at point P due the sound from source 2:

$$I_2 = \text{constant}\times A_2^2$$

or

$$A_2 = C\sqrt{I_2}$$

Express the square of the resultant

$$A^2 = C^2\left(\sqrt{I_1}+\sqrt{I_2}\right)^2 = C^2 I$$

amplitude at point P:

Solve for and evaluate I:

$$I = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = \left(\sqrt{19.9\,\mu\text{W/m}^2} + \sqrt{8.84\,\mu\text{W/m}^2}\right)^2 = \boxed{55.3\,\mu\text{W/m}^2}$$

(*c*) If they are driven coherently but are 180° out of phase we will have destructive interference at point P and the intensity is given by:

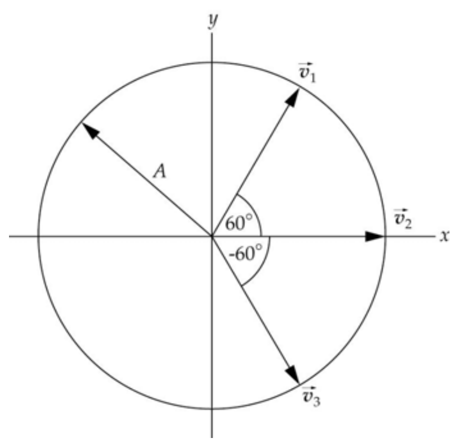
$$I = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = \left(\sqrt{19.9\,\mu\text{W/m}^2} - \sqrt{8.84\,\mu\text{W/m}^2}\right)^2 = \boxed{2.21\,\mu\text{W/m}^2}$$

(*d*) Because the sources are incoherent, the intensities add arithmetically:

$$I = I_1 + I_2 = 19.9\,\mu\text{W/m}^2 + 8.84\,\mu\text{W/m}^2 = \boxed{28.7\,\mu\text{W/m}^2}$$

87 ••

Picture the Problem In Chapter 14, Section 14.1, it was shown that a harmonic function could be represented by a vector rotating at the angular frequency ω. The simplest way to do this problem is to use that representation. The vectors, of equal magnitude, are shown in the diagram. We can find the resultant wave function by finding the magnitude and direction of the resultant vector.



From the diagram it is evident that:

$$\sum v_y = 0$$

Find the sum of the x components of the vectors:

$$\sum v_x = A\cos 60° + A\cos 60° + A = 2A$$

Relate the magnitude of the resultant vector to the sum of its x and y components:

$$v = \sqrt{\left(\sum v_x\right)^2 + \left(\sum v_y\right)^2} = \sqrt{(2A)^2 + (0)^2} = 2A$$

Find the direction of the resultant vector:

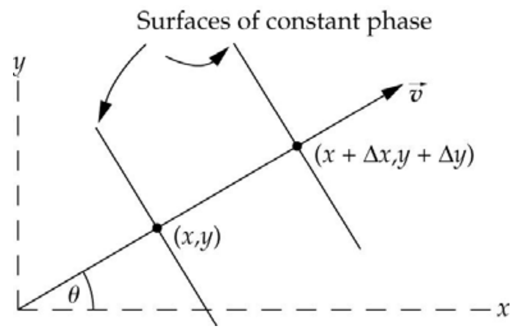
$$\theta = \tan^{-1}\left(\frac{\sum v_y}{\sum v_x}\right) = \tan^{-1}\left(\frac{0}{2A}\right) = 0$$

Express the resultant wave:

$$y_{\text{res}}(x,t) = 2A\sin(kx - \omega t) = \boxed{0.1\sin(kx - \omega t)}$$

88 ••

Picture the Problem The diagram shows a two dimensional plane wave propagating at an angle θ with respect to the x axis. At a given point in time, the surface of constant phase for the wave is the line defined by $k_x x + k_y y = \phi$, or $y = -(k_x/k_y)x + \phi$.



The wave itself moves in a direction perpendicular to the wavefront, i.e., in a direction specified by a line with slope k_y/k_x. Choose two points (x, y) and ($x + \Delta x$, $y + \Delta y$) that have a separation of 1 wavelength along such a line.

Express the phase difference ϕ between the two points that have a separation of 1 wavelength along the line $y = -(k_x/k_y)x + \phi$ in terms of the spatial separation Δr of the points:

$$\frac{\phi}{\Delta r} = \frac{2\pi}{\lambda} \text{ or } \phi = \frac{2\pi}{\lambda}\Delta r$$

where $\Delta r = \sqrt{(\Delta x)^2 + (\Delta y)^2}$

Substitute $\phi = 2\pi$ to obtain:

$$2\pi = \frac{2\pi}{\lambda}\sqrt{(\Delta x)^2 + (\Delta y)^2}$$

or

$$\lambda = \sqrt{(\Delta x)^2 + (\Delta y)^2} \qquad (1)$$

Express ϕ in terms of k_x, k_y, Δx and Δy:

$$\phi = k\Delta r = k_x \Delta x + k_y \Delta y$$

or, because $\phi = 2\pi$,

$$k_x \Delta x + k_y \Delta y = 2\pi$$

Because $\Delta y = \frac{k_y}{k_x}\Delta x$:

$$k_x \Delta x + \frac{k_y^2}{k_x}\Delta x = 2\pi$$

or

$$\Delta x = \frac{2\pi k_x}{k_x^2 + k_y^2}$$

Similarly:

$$\Delta y = \frac{2\pi k_y}{k_x^2 + k_y^2}$$

Substitute in equation (1) to obtain:

$$\lambda = \sqrt{\left(\frac{2\pi k_x}{k_x^2 + k_y^2}\right)^2 + \left(\frac{2\pi k_y}{k_x^2 + k_y^2}\right)^2} = \frac{2\pi}{\sqrt{k_x^2 + k_y^2}}$$

Relate the wave velocity v to its angular frequency ω and wave number k:

$$v = \frac{\omega}{k} = \omega \frac{\lambda}{2\pi}$$

Substitute for λ to obtain:

$$v = \frac{\omega}{2\pi} \frac{2\pi}{\sqrt{k_x^2 + k_y^2}} = \boxed{\frac{\omega}{\sqrt{k_x^2 + k_y^2}}}$$

Express the angle between the wave velocity and the x axis:

$$\theta = \tan^{-1} \frac{\Delta y}{\Delta x} = \tan^{-1} \frac{\frac{2\pi k_y}{k_x^2 + k_y^2}}{\frac{2\pi k_x}{k_x^2 + k_y^2}} = \boxed{\tan^{-1}\left(\frac{k_y}{k_x}\right)}$$

***89 ••**

Picture the Problem We can express the fundamental frequency of the organ pipe as a function of the air temperature and differentiate this expression with respect to the temperature to express the rate at which the frequency changes with respect to temperature. For changes in temperature that are small compared to the temperature, we can approximate the differential changes in frequency and temperature with finite changes to complete the derivation of $\Delta f/f = ½\Delta T/T$. In part (b) we'll use this relationship and the data for the frequency at 20°C to find the frequency of the fundamental at 30°C.

(*a*) Express the fundamental frequency of an organ pipe in terms of its wavelength and the speed of sound:

$$f = \frac{v}{\lambda}$$

Relate the speed of sound in air to the absolute temperature:

$$v = \sqrt{\frac{\gamma RT}{M}} = C\sqrt{T}$$

where

$$C = \sqrt{\frac{\gamma R}{M}} = \text{constant}$$

Defining a new constant C', substitute to obtain:

$$f = \frac{C}{\lambda}\sqrt{T} = C'\sqrt{T}$$

because λ is constant for the fundamental frequency we ignore any change in the length of the pipe.

Differentiate this expression with respect to T:

$$\frac{df}{dT} = \frac{1}{2}C'T^{-1/2} = \frac{f}{2T}$$

Separate the variables to obtain:

$$\frac{df}{f} = \frac{1}{2}\frac{dT}{T}$$

For $\Delta T << T$, we can approximate df by Δf and dT by ΔT to obtain:

$$\boxed{\frac{\Delta f}{f} = \frac{1}{2}\frac{\Delta T}{T}}$$

(*b*) Express the fundamental frequency at 30°C in terms of its frequency at 20°C:

$$f_{30} = f_{20} + \Delta f$$

Solve our result in (*a*) for Δf:

$$\Delta f = \tfrac{1}{2} f \frac{\Delta T}{T}$$

Substitute numerical values and evaluate Δf:

$$f_{30} = 200\,\text{Hz} + \tfrac{1}{2}(200\,\text{Hz})\frac{10\,\text{K}}{293\,\text{K}}$$

$$= \boxed{203\,\text{Hz}}$$

90 ••

Picture the Problem We'll use a spreadsheet program to graph the wave functions individually and their sum as functions of x at $t = 0$ and at $t = 1$ s. In (*c*) and (*d*) we can add the wave functions algebraically to find the result wave function at $t = 0$ and at $t = 1$ s.

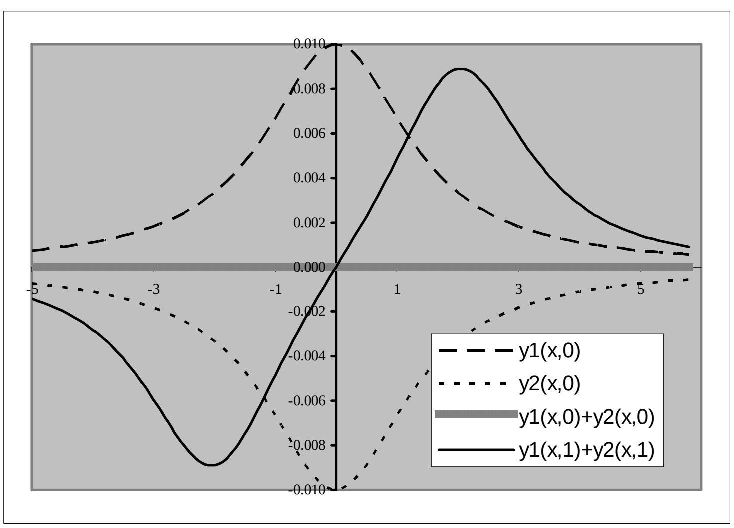
(*a*) and (*d*) A spreadsheet program to calculate values for $y_1(x,t)$ and $y_2(x,t)$ between and plot their graphs is shown below. The constants and cell formulas used are shown in the table.

Cell	Content/Formula	Algebraic Form
A5	–5.0	x
A6	A5+0.1	$x + \Delta x$

B5	0.05/(2+(A5–2*B1)^2)	$y_1(x,0)$
C5	–0.05/(2+(A5+2*B1)^2)	$y_2(x,0)$
D5	0.05/(2+(A5–2*B1)^2) –0.05/(2+(A5+2*B1)^2)	$y_1(x,0)+y_2(x,0)$
E5	0.05/(2+(A5–2*B2)^2) –0.05/(2+(A5+2*B2)^2)	$y_1(x,1)+y_2(x,1)$

	A	B	C	D	E
1	t=	0			
2	t=	1	s		
3					
4	x	y1(x,0)	y2(x,0)	y1(x,0)+y2(x,0)	y1(x,1)+y2(x,1)
5	–5.0	0.001	–0.001	0.000	–0.001
6	–4.9	0.001	–0.001	0.000	–0.002
7	–4.8	0.001	–0.001	0.000	–0.002
8	–4.7	0.001	–0.001	0.000	–0.002
9	–4.6	0.001	–0.001	0.000	–0.002
10	–4.5	0.001	–0.001	0.000	–0.002
110	5.5	0.001	–0.001	0.000	0.001
111	5.6	0.001	–0.001	0.000	0.001
112	5.7	0.001	–0.001	0.000	0.001
113	5.8	0.001	–0.001	0.000	0.001

The four curves on the graph are identified in the legend. y_1 is traveling from left to right and y_2 from right to left. As time increases, y_1 is farther to the right and y_2 is farther to the left.



(*b*) Express the resultant wave function at $t = 0$:

$$y_1(x,0)+y_2(x,0)=\frac{0.02\,\text{m}^3}{2\,\text{m}^2+x^2}+\frac{-0.02\,\text{m}^3}{2\,\text{m}^2+x^2}=\boxed{0}$$

(*c*) Express the resultant wave function at t = 1 s:

$$y_1(x,1\text{s})+y_2(x,1\text{s})=\boxed{\frac{0.02\,\text{m}^3}{2\,\text{m}^2+(x-2\,\text{s})^2}+\frac{-0.02\,\text{m}^3}{2\,\text{m}^2+(x+2\,\text{s})^2}}$$

91 ••

Picture the Problem We can relate the frequency of the standing waves in the open-ended tube to its length and the speed of sound in air.

(*a*) What you hear is the fundamental mode of the tube and its overtones. A more physical explanation is that the echo of the finger snap moves back and forth along the tube with a characteristic time of $2L/c$, leading to a series of clicks from each echo. Because the clicks happen with a frequency of $c/2L$, the ear interprets this as a musical note of that frequency.

(*b*) Express the frequency of the sound in terms of the length of the tube:

$$f=\frac{v}{2L}$$

Solve for L:

$$L=\frac{v}{2f}$$

Substitute numerical values and evaluate L:

$$L=\frac{340\,\text{m/s}}{2\left(440\,\text{s}^{-1}\right)}=\boxed{38.6\,\text{cm}}$$

92 ••

Picture the Problem To find the total kinetic energy of the *n*th mode of vibration, we'll need to differentiate $y_n(x,t)=A_n\sin k_n x\cos\omega_n t$ with respect to time, substitute in the expression for ΔK, and then integrate over the length of the string.

(*a*) Write the wave function for a standing wave on a string fixed at both ends:

$$y_n(x,t)=A_n\sin k_n x\cos\omega_n t$$

where $k_n=\dfrac{2\pi}{\lambda_n}$.

Using the standing-wave condition for a string with both ends fixed, relate the length of the string to the wavelength of the nth harmonic:

$$L=n\frac{\lambda_n}{2},\ n=1,2,3,\ldots$$

Solve for λ_n:

$$\lambda_n = \frac{2L}{n}$$

Substitute in the expression for k_n to obtain:

$$k_n = n\frac{\pi}{L}$$

Differentiate this expression with respect to t:

$$\frac{\partial y}{\partial t} = \frac{\partial}{\partial t}\left[A_n \sin k_n x \cos \omega_n t\right]$$
$$= -\omega_n A_n \sin k_n x \sin \omega_n t$$

Substitute in the given expression and simplify to obtain:

$$\Delta K = \tfrac{1}{2}\mu\left(-\omega_n A_n \sin k_n x \sin \omega_n t\right)^2 \Delta x$$
$$= \tfrac{1}{2}\mu\omega_n^2 A_n^2 \sin^2 k_n x \sin^2 \omega_n t \Delta x$$

Integrate this expression over the length of the string to find its total kinetic energy:

$$K = \tfrac{1}{2}\mu\omega_n^2 A_n^2 \sin^2 \omega_n t \int_0^L \sin^2\left(n\frac{\pi}{L}x\right)dx$$
$$= \boxed{\tfrac{1}{4}m\omega_n^2 A_n^2 \sin^2 \omega_n t}$$

(*b*) Express the condition that $K = K_{max}$:

$$\sin^2 \omega_n t = 1 \qquad (1)$$

Substitute to obtain:

$$K_{max} = \boxed{\tfrac{1}{4}m\omega_n^2 A_n^2}$$

(*c*) From equation (1), for $K = K_{max}$:

$$\sin^2 \omega_n t = 1 \text{ or } \omega_n t = \frac{\pi}{2}$$

Evaluate the wave function in (*a*) when $\omega_n t = \dfrac{\pi}{2}$:

$$y_n\left(x, \frac{\pi}{2\omega_n}\right) = A_n \sin k_n x \cos\frac{\pi}{2} = \boxed{0}$$

(*d*) Using the result from part (*b*), express the maximum kinetic energy:

$$K_{max} = \tfrac{1}{4}m\omega_n^2 A_n^2$$

Relate ω_n to ω_1:

$$\omega_n = n\omega_1$$

Substitute to obtain:

$$K_{max} = n^2\left(\tfrac{1}{4}m\omega_1^2 A_n^2\right)$$

or, because m and ω_1 are constants,

$$K_{max} \propto \boxed{n^2 A_n^2}$$

Remarks: Our result in part (*b*) is exactly the same result obtained in Problem 68 with ω_n and A_n replacing ω and A.

93 ••

Picture the Problem We can use $f_n = n\frac{v}{2L}, n = 1, 2, 3, \ldots$ to relate the resonant frequencies to the length of the string and the speed of transverse waves on the string and $v = \sqrt{F/\mu}$ to express the speed of the transverse waves on the string in terms of the tension in the string. Differentiating of the resulting expression with respect to *F* will lead to $\frac{df_n}{f_n} = \frac{1}{2}\frac{dF}{F}$. For changes in *f* that are small compared to *f*, we can use a differential approximation to obtain $\frac{\Delta f_n}{f_n} = \frac{1}{2}\frac{\Delta F}{F}$.

(*a*) Using the standing-wave condition for a string fixed at both ends, relate the resonant frequencies to the length of the string and the speed of transverse waves on the string:

$$f_n = n\frac{v}{2L}, n = 1, 2, 3, \ldots$$

Express the speed of transverse waves on the string in terms of the tension in the string:

$$v = \sqrt{\frac{F}{\mu}}$$

Substitute to obtain:

$$f_n = \frac{n}{2L}\sqrt{\frac{F}{\mu}} = C\sqrt{F}$$

because *n*, *L*, and μ are constants.

Differentiate f_n with respect to *F* to obtain:

$$\frac{df_n}{dF} = \frac{C}{2}\frac{1}{\sqrt{F}} = \frac{1}{2}\frac{f_n}{F}$$

Separate the variables to obtain:

$$\boxed{\frac{df_n}{f_n} = \frac{1}{2}\frac{dF}{F}}$$

Because no conditions were placed on its derivation, this expression is valid for all harmonics.

(*b*) Because $\Delta f \ll f$, one can approximate the differential quantitities in our result for part (*a*)

$$\frac{\Delta f_n}{f_n} = \frac{1}{2}\frac{\Delta F}{F}$$

to obtain:

Solve for $\Delta F/F$:

$$\frac{\Delta F}{F}=2\frac{\Delta f_n}{f_n}$$

Substitute numerical values and evaluate $\Delta F/F$:

$$\frac{\Delta F}{F}=2\left(\frac{2\,\text{Hz}}{260\,\text{Hz}}\right)=\boxed{1.54\%}$$

94 ••

Picture the Problem Let the sources be denoted by the numerals 1 and 2. The phase difference between the two waves at point P is the sum of the phase difference due to the sources δ_0 and the phase difference due to the path difference δ.

(*a*) Write the wave function due to source 1:

$$f_1(x,t)=\boxed{A_0\cos(kx_1-\omega t)}$$

Write the wave function due to source 2:

$$f_2(x,t)=\boxed{A_0\cos(k(x_1+\Delta x)-\omega t+\delta_s)}$$

(*b*) Express the sum of the two wave functions:

$$\begin{aligned}f(x,t)&=f_1(x,t)+f_2(x,t)=A_0\cos(kx_1-\omega t)+A_0\cos(k(x_1+\Delta x)-\omega t+\delta_s)\\&=A_0[\cos(kx_1-\omega t)\cos(k(x_1+\Delta x)-\omega t+\delta_s)]\end{aligned}$$

Use $\cos\alpha+\cos\beta=2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$ to obtain:

$$f(x,t)=2A_0\left[\cos\left(\frac{k\Delta x}{2}+\frac{\delta_s}{2}\right)\cos\left(k\left(x+\frac{\Delta x}{2}\right)-\omega t+\frac{\delta_s}{2}\right)\right]$$

Express the phase difference δ in terms of the path difference Δx and the wave number k:

$$\frac{\delta}{\Delta x}=\frac{2\pi}{\lambda}=k\ \text{ or }\ k\Delta x=\delta$$

Substitute to obtain:

$$f(x,t)=\boxed{2A_0\left[\cos\left(\frac{\delta+\delta_s}{2}\right)\cos\left(k\left(x+\frac{\Delta x}{2}\right)-\omega t+\frac{\delta_s}{2}\right)\right]}$$

The amplitude of the resultant wave

$$A=\boxed{2A_0\cos\tfrac{1}{2}(\delta+\delta_s)}$$

function is the coefficient of the time-dependent factor:

(*c*) Express the intensity at an arbitrary point *P*:

$$I_P = C'A^2 = C'\left[2A_0\cos\tfrac{1}{2}(\delta+\delta_s)\right]^2 = C'\left[4A_0^2\cos^2\tfrac{1}{2}(\delta+\delta_s)\right]$$

Evaluate *I* for $\delta = 0$ and $\delta_s = Ct$:

$$I = C'\left[4A_0^2\cos^2\tfrac{1}{2}(Ct)\right]$$

Because the average value of $\cos^2\theta$ over a complete period is ½:

$$I_{ave} \propto 2A_0^2 = 2I_0$$

and

$$I \propto \boxed{4I_0\cos^2\tfrac{1}{2}(Ct)}$$

(*d*) Evaluate *I* for $\Delta x = \frac{1}{2}\lambda$ and $\delta_s = Ct$:

$$\Delta x = \tfrac{1}{2}\lambda \Rightarrow \delta = \pi$$

$$\therefore I = C'\left[4A_0^2\cos^2\tfrac{1}{2}(\pi + Ct)\right]$$

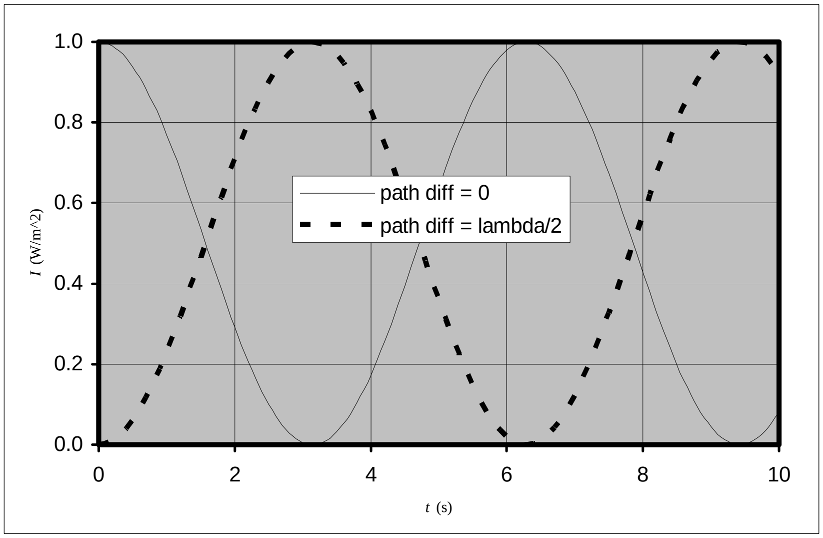
and at $t = 0$, $I = 0$. i.e., the waves interfere destructively.

A spreadsheet program to calculate the intensity at point *P* as a function of time for a zero path difference and a path difference of λ is shown below. The constants and cell formulas used are shown in the table.

Cell	Content/Formula	Algebraic Form
B1	1	C
B7	B6+0.1	$t + \Delta t$
C6	COS(B6*B6/2)^2	$\cos^2\frac{1}{2}(Ct)$
D6	COS(B6*B6/2-PI()/2)^2	$\cos^2\frac{1}{2}(\pi + Ct)$

	A	B	C	D
1	C=	1	s^{-1}	
2				
3				
4		t	I	I
5		(s)	(W/m^2)	(W/m^2)
6		0.00	1.000	0.000
7		0.10	0.998	0.002
8		0.20	0.990	0.010
9		0.30	0.978	0.022
103		9.70	0.019	0.981
104		9.80	0.035	0.965
105		9.90	0.055	0.945
106		10.00	0.080	0.920

The solid curve is the graph of $\cos^2 \frac{1}{2}(Ct)$ and the dashed curve is the graph of $\cos^2 \frac{1}{2}(\pi + Ct)$.



95 •••

Picture the Problem We can differentiate the sum of the two wave functions to find the velocity of a segment *dx* of the string. We can find the kinetic energy of this segment from $dK = \frac{1}{2}v_y^2 dm = \frac{1}{2}\mu v_y^2 dx$ and integrate this expression from 0 to *L* to find the total kinetic energy of the resultant wave.

(*a*) Express the resultant wave function:

$$y_r(x,t) = y_1(x,t) + y_2(x,t) = A_1 \cos\omega_1 t \sin k_1 x + A_2 \cos\omega_2 t \sin k_2 x$$

Differentiate this expression with respect to *t* to find v_y:

$$v_y(x,t) = \frac{\partial}{\partial t}\left[A_1 \cos\omega_1 t \sin k_1 x + A_2 \cos\omega_2 t \sin k_2 x\right]$$
$$= \boxed{-\omega_1 A_1 \sin\omega_1 t \sin k_1 x - \omega_2 A_2 \sin\omega_2 t \sin k_2 x}$$

(*b*) Express the kinetic energy of a segment of the string of length *dx* and mass *dm*:

$$dK = \tfrac{1}{2}v_y^2 dm = \tfrac{1}{2}\mu v_y^2 dx = \tfrac{1}{2}\mu(\omega_1 A_1 \sin\omega_1 t \sin k_1 x + \omega_2 A_2 \sin\omega_2 t \sin k_2 x)^2 dx$$
$$= \boxed{\begin{aligned}&\tfrac{1}{2}\mu\big[\omega_1^2 A_1^2 \sin^2\omega_1 t \sin^2 k_1 x + 2\omega_1\omega_2 A_1 A_2 \sin\omega_1 t \sin k_1 x \sin\omega_2 t \sin k_2 x \\ &\quad + \omega_2^2 A_2^2 \sin^2\omega_2 t \sin^2 k_2 x\big]dx\end{aligned}}$$

(*c*) Integrate *dK* from 0 to *L* to obtain:

$$K=\tfrac{1}{2}\mu\int_0^L \omega_1^2A_1^2\sin^2\omega_1 t\sin^2 k_1x\,dx$$
$$+\tfrac{1}{2}\mu\int_0^L 2\omega_1\omega_2A_1A_2\sin\omega_1 t\sin k_1x\sin\omega_2 t\sin k_2x\,dx$$
$$+\tfrac{1}{2}\mu\int_0^L \omega_2^2A_2^2\sin^2\omega_2 t\sin^2 k_2x\,dx$$
$$=\tfrac{1}{2}\mu\omega_1^2A_1^2\sin^2\omega_1 t\int_0^L\sin^2 n_1\frac{\pi}{L}x\,dx$$
$$+\mu\omega_1\omega_2A_1A_2\sin\omega_1 t\sin\omega_2 t\int_0^L\sin n_1\frac{\pi}{L}x\sin n_2\frac{\pi}{L}x\,dx$$
$$+\tfrac{1}{2}\mu\omega_2^2A_2^2\sin^2\omega_2 t\int_0^L\sin^2 n_2\frac{\pi}{L}x\,dx$$
$$=\left(\tfrac{1}{2}\mu\omega_1^2A_1^2\sin^2\omega_1 t\right)\left(\tfrac{1}{2}L\right)+\left(\mu\omega_1\omega_2A_1A_2\sin\omega_1 t\sin\omega_2 t\right)(0)_{n_1\neq n_2}$$
$$+\left(\tfrac{1}{2}\mu\omega_2^2A_2^2\sin^2\omega_2 t\right)\left(\tfrac{1}{2}L\right)$$
$$=\boxed{\tfrac{1}{4}m\omega_1^2A_1^2\sin^2\omega_1 t+\tfrac{1}{4}m\omega_2^2A_2^2\sin^2\omega_2 t}$$

Note that, from Problem 92: $\tfrac{1}{4}m\omega_1^2A_1^2\sin^2\omega_1 t+\tfrac{1}{4}m\omega_2^2A_2^2\sin^2\omega_2 t=K_1+K_2$

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Picture the Problem We can use the relationship $K_{\max}=\tfrac{1}{4}m\omega^2A^2$ from Problem 92 to express the maximum kinetic energy of the wire and $v=f\lambda$ and $v=\sqrt{F/\mu}$ to find an expression for ω. In part (*d*) we'll use $\frac{\Delta U}{\Delta x}\approx\tfrac{1}{2}F\left(\frac{\partial y}{\partial x}\right)^2$ from Problem 15-120 to determine where the potential energy per unit length has its maximum value.

(*a*) From Problem 92 we have:

$$K_{\max}=\tfrac{1}{4}m\omega^2A^2 \qquad (1)$$

Express ω_1 in terms of f_1:

$$\omega_1=2\pi f_1$$

Relate f_1 to the speed of transverse waves on the wire and the wavelength of the fundamental mode:

$$f_1=\frac{v}{\lambda_1}=\frac{v}{2L}$$

where L is the length of the wire.

Express the speed of the transverse waves on the wire in terms of the tension in the wire:

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{FL}{m}}$$

Substitute and simplify to obtain:

$$f_1 = \frac{1}{2L}\sqrt{\frac{FL}{m}} = \sqrt{\frac{F}{4mL}}$$

Substitute for ω_1 and f_1 in equation (1) to obtain:

$$K_{\max} = \tfrac{1}{4}m\left[2\pi\sqrt{\frac{F}{4mL}}\right]^2 A^2 = \frac{\pi^2 F}{4L}A^2$$

Substitute numerical values and evaluate $K_{\max}$:

$$K_{\max} = \frac{\pi^2(40\,\text{N})}{4(2\,\text{m})}\left(2\times10^{-2}\,\text{m}\right)^2 = \boxed{19.7\,\text{mJ}}$$

(*b*) Express the wave function for a standing wave in its first harmonic:

$$y_1(x,t) = A_1\sin k_1 x\cos\omega_1 t \qquad (2)$$

At the instant the transverse displacement is given by (0.02 m) sin (πx/2):

$$\cos\omega_1 t = 1 \Rightarrow \omega_1 t = 0$$

and

$$K = \boxed{0}$$

(*c*) dK is a maximum where the displacement of the wire is greatest; i.e., at its midpoint:

$$x = \tfrac{1}{2}L = \tfrac{1}{2}(2\,\text{m}) = \boxed{1.00\,\text{m}}$$

(*d*) From Problem 15-120:

$$\frac{\Delta U}{\Delta x} \approx \tfrac{1}{2}F\left(\frac{\partial y}{\partial x}\right)^2$$

Express the condition on $\partial y/\partial x$ that maximizes $\Delta U/\Delta x$:

$$\frac{\partial y}{\partial x} = \left(\frac{\partial y}{\partial x}\right)_{\max}$$

Differentiate $y_1(x,t) = A_1\sin k_1 x\cos\omega_1 t$ with respect to x and set the derivative equal to zero for extrema:

$$\frac{\partial y_1}{\partial x} = \frac{\partial}{\partial x}\left(A_1\sin k_1 x\cos\omega_1 t\right) = k_1 A_1\cos k_1 x\cos\omega_1 t = 0$$

or

$$\cos k_1 x = 0$$

Solve for k_1x and then x:

$$k_1x = \frac{\pi}{2}$$

and

$$x = \frac{\pi}{2k_1} = \frac{\pi\lambda}{2(2\pi)} = \tfrac{1}{4}\lambda = \tfrac{1}{4}(2L)$$

$$= \tfrac{1}{2}(2\,\text{m}) = \boxed{1.00\,\text{m}}$$

i.e., the potential energy per unit length is a maximum at the midpoint of the wire.

Remarks: In part (*d*) we've shown that Δ*U*/Δ*x* has an extreme value at *x* = 1 m. To show that Δ*U*/Δ*x* is a *maximum* at this location, you need to examine the sign of the 2nd derivative of $y_1(x,t)$ at this point.

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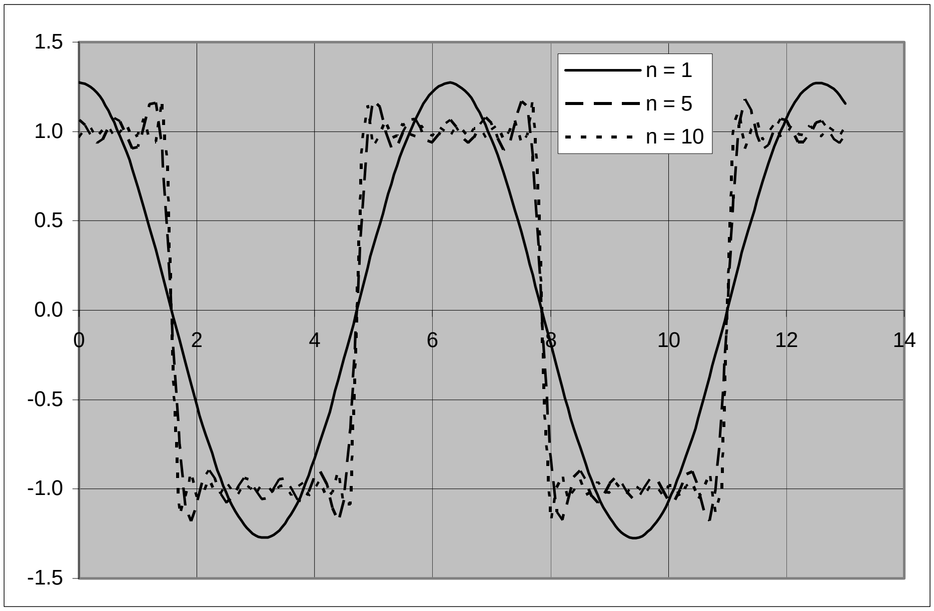
(*a*) A spreadsheet program to evaluate *f*(*x*) is shown below. Typical cell formulas used are shown in the table.

Cell	Content/Formula	Algebraic Form
A6	A5+0.1	$x+\Delta x$
B4	2*B3+1	$2n+1$
B5	(–1)^B$3*COS(B$4*$A5) /B$4*4/PI()	$\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n\cos((2n+1)x)}{2n+1}$
C5	B5+(–1)^C$3*COS(C$4*$A5) /C$4*4/PI()	$\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n\cos((2n+1)x)}{2n+1}$

	A	B	C	D		K	L
1							
2							
3		0	1	2		9	10
4		1	3	5		19	21
5	0.0	1.2732	0.8488	1.1035		0.9682	1.0289
6	0.1	1.2669	0.8614	1.0849		1.0134	0.9828
7	0.2	1.2479	0.8976	1.0352		1.0209	0.9912
8	0.3	1.2164	0.9526	0.9706		0.9680	1.0286
9	0.4	1.1727	1.0189	0.9130		1.0057	0.9742
10	0.5	1.1174	1.0874	0.8833		1.0298	1.0010
130	12.5	1.2704	0.8544	1.0952		0.9924	1.0031
131	12.6	1.2725	0.8503	1.1013		0.9752	1.0213
132	12.7	1.2619	0.8711	1.0710		1.0287	0.9714
133	12.8	1.2386	0.9143	1.0141		1.0009	1.0126
134	12.9	1.2030	0.9740	0.9493		0.9691	1.0146
135	13.0	1.1554	1.0422	0.8990		1.0261	0.9685

The solid curve is plotted from the data in columns A and B and is the graph of *f*(*x*) for 1 term. The dashed curve is plotted from the data in columns A and F and is the graph of $f(x)$ for 5 terms. The dotted curve is plotted from the data in columns A and K and is the graph of $f(x)$ for 10 terms.



(*b*) Evaluate $f(2\pi)$ to obtain:

$$f(2\pi)=\frac{4}{\pi}\left(\frac{\cos 2\pi}{1}-\frac{\cos 3(2\pi)}{3}+\frac{\cos 5(2\pi)}{5}-\ldots\right)$$

$$=\frac{4}{\pi}\left(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots\right)$$

$$=1$$

which is equivalent to the Liebnitz formula.

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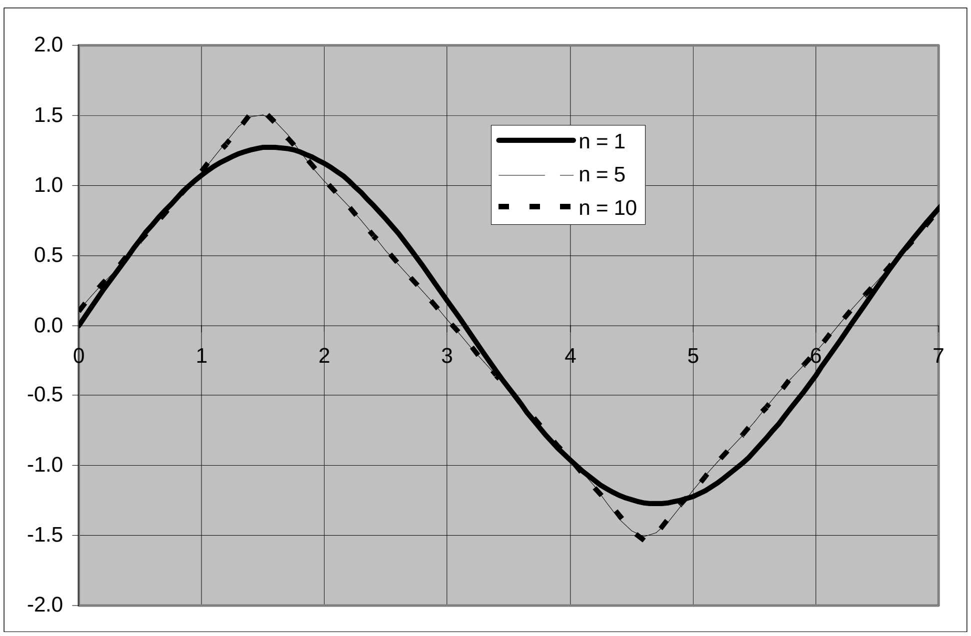
(*a*) A spreadsheet program to evaluate $f(x)$ is shown below. Typical cell formulas used are shown in the table.

Cell	Content/Formula	Algebraic Form
A6	A5+0.1	$x+\Delta x$
B4	2*B3+1	$2n+1$
B5	(–1)^B3*sin(B4*A5)/ (B4)^2*4/PI()	$\frac{4}{\pi}\sum_n\frac{(-1)^n\sin(2n+1)x}{(2n+1)^2}$
C5	B5+((–1)^C3*sin(C4*A5)/ (C4)^2*4/PI()	$\frac{4}{\pi}\sum_n\frac{(-1)^n\sin(2n+1)x}{(2n+1)^2}$

	A	B	C	D		K	L
1							
2							

3		0	1	2		9	10
4		1	3	5		19	21
5	0.0	0.0000	0.0000	0.0000		0.0000	0.0000
6	0.1	0.1271	0.0853	0.1097		0.0986	0.1011
7	0.2	0.2530	0.1731	0.2159		0.2012	0.1987
8	0.3	0.3763	0.2654	0.3163		0.3004	0.3005
9	0.4	0.4958	0.3640	0.4103		0.3983	0.4008
10	0.5	0.6104	0.4693	0.4998		0.5011	0.4985
72	6.7	0.5155	0.3812	0.4256		0.4153	0.4171
73	6.8	0.6291	0.4877	0.5146		0.5183	0.5154
74	6.9	0.7365	0.6005	0.6034		0.6171	0.6182
75	7.0	0.8365	0.7181	0.6963		0.7148	0.7166
76	7.1	0.9282	0.8380	0.7968		0.8183	0.8155

Graphs of $f(x)$ for 1, 5, and 10 terms are shown below. Note that there is little difference between the graphs for 5 terms and 10 terms of this triangular wave function.



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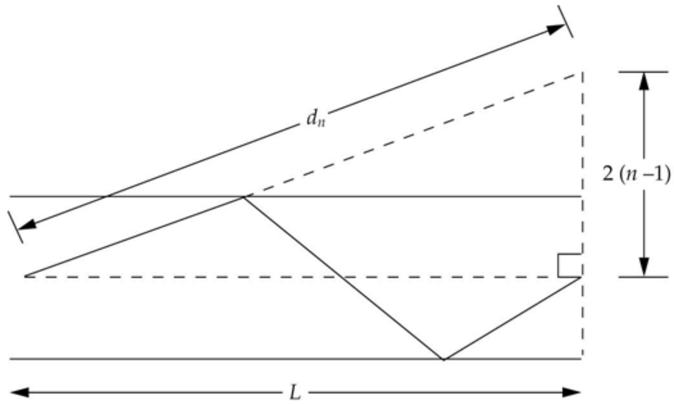
Picture the Problem From the diagram above, the nth echo will reflect $n - 1$ times going out, and the same number of times going back. If we "unfold" the ray into a straight line, we get the representation shown below. Using this figure we can express the distance d_n traveled by the nth echo and then use this result to express the time delay between the n_{th} and $n + 1_{th}$ echoes. The reciprocal of this time delay is the frequency corresponding to the nth echo.



(*a*) Apply the Pythagorean theorem to the right triangle whose base is *L*, whose height is 2(*n* – 1), and whose hypotenuse is d_n to obtain:

$$d_n = 2\sqrt{4(n-1)^2 r^2 + L^2}$$

Express the time delay between the n_{th} and $n + 1_{th}$ echoes:

$$\Delta t_n = \frac{d_n}{v}$$

Substitute to obtain:

$$\Delta t_n = \frac{2}{v}\left(\sqrt{(2n)^2 r^2 + L^2} - \sqrt{[2(n-1)]^2 r^2 + L^2}\right)$$

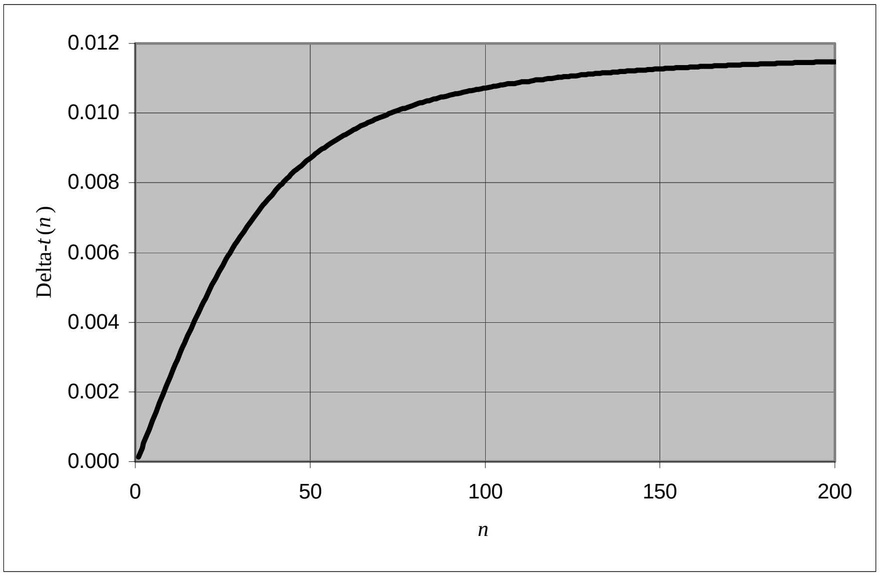
A spreadsheet program to calculate Δt_n as a function of *n* is shown below. The constants and cell formulas used are shown in the table.

Cell	Content/Formula	Algebraic Form
B1	90	L
B2	1	r
B3	340	c
B8	B7+1	$n + 1$
C7	2/\$B\$3*((2*(B7–1) *\$B\$2)^2+\$B\$1^2)^0.5	Δt_n

	A	B	C	D
1	L=	90	m	
2	r=	1	m	
3	c=	340	m/s	
4				
5				
6		n	t(n)	delta t(n)
7		1	0.5294	0.0001
8		2	0.5295	0.0004
9		3	0.5299	0.0007
10		4	0.5306	0.0009
11		5	0.5315	0.0012

202		196	2.3544	0.0115
203		197	2.3659	0.0115
204		198	2.3773	0.0115
205		199	2.3888	0.0115
206		200	2.4003	0.0115

The graph of Δt_n as a function of n shown below was plotted using the data from columns B and D.



(*c*) The frequency heard at any time is $1/\Delta t_n$, so because Δt_n increases over time, the frequency of the culvert whistler decreases.

The highest frequency corresponds to $n = 1$ and is given by:

$$f_{\text{highest}} = \frac{1}{\Delta t_1}$$

Substitute for Δt_1 to obtain:

$$f_{\text{highest}} = \frac{1}{\Delta t_1} = \frac{v}{2\left(\sqrt{(2)^2 r^2 + L^2} - \sqrt{L^2}\right)}$$

Substitute numerical values and evaluate f_{highest}:

$$f_{\text{highest}} = \frac{340\,\text{m/s}}{2\left(\sqrt{4(1\,\text{m})^2 + (90\,\text{m})^2} - 90\,\text{m}\right)}$$

$$= \boxed{7.65\,\text{kHz}}$$

The lowest frequency end can be found by examining the limit of Δt_n as $n \to \infty$:

$$\lim_{n\to\infty}\Delta t_n = \lim_{n\to\infty}\left[\frac{2}{v}\left((2n)\sqrt{r^2+\frac{L^2}{(2n)^2}}-2(n-1)\sqrt{r^2+\frac{L^2}{(2(n-1))^2}}\right)\right]$$

$$= \frac{2r}{v}(2n-2n+2) = \frac{4r}{v}$$

Express f_{lowest} in terms of Δt_∞:

$$f_{\text{lowest}} = \frac{1}{\Delta t_\infty} = \frac{v}{4r}$$

Substitute numerical values and evaluate f_{lowest}:

$$f_{\text{lowest}} = \frac{340\,\text{m/s}}{4(1\,\text{m})} = \boxed{85.0\,\text{Hz}}$$