Chapter 17
Temperature and the Kinetic Theory of Gases

Conceptual Problems

***1** •

(*a*) False. If two objects are in thermal equilibrium with a third, then they are in thermal equilibrium with each other.

(*b*) False. The Fahrenheit and Celsius temperature scales differ in the number of intervals between the ice-point temperature and the steam-point temperature.

(*c*) True.

(*d*) False. The result one obtains for the temperature of a given system is thermometer-dependent.

2 •

Determine the Concept Put each in thermal equilibrium with a third body; e.g., a thermometer. If each body is in thermal equilibrium with the third, then they are in thermal equilibrium with each other.

3 •

Picture the Problem We can decide which room was colder by converting 20°F to the equivalent Celsius temperature.

Using the Fahrenheit-Celsius conversion, convert 20°F to the equivalent Celsius temperature:

$$t_{\mathrm{C}} = \tfrac{5}{9}\left(t_{\mathrm{F}} - 32°\right) = \tfrac{5}{9}\left(20° - 32°\right)$$
$$= -6.67°\mathrm{C}$$
$$\text{so } \boxed{\text{Mert's room was colder.}}$$

4 ••

Picture the Problem We can apply the ideal-gas law to the two vessels to decide which of these statements is correct.

Apply the ideal-gas law to the particles in vessel 1:

$$P_1V_1 = N_1kT_1$$

Apply the ideal-gas law to the particles in vessel 2:

$$P_2V_2 = N_2kT_2$$

Divide the equation for vessel 1 by the equation for vessel 2:

$$\frac{P_1V_1}{P_2V_2} = \frac{N_1kT_1}{N_2kT_2}$$

Because the vessels are identical and are at the same temperature and pressure:

$$1=\frac{N_1}{N_2} \text{ and } N_1=N_2$$

$$\boxed{(a)\text{ is correct.}}$$

5 ••

Determine the Concept From the ideal-gas law, we have $P=nRT/V$. In the process depicted, both the temperature and the volume increase, but the temperature increases faster than does the volume. Hence, the pressure increases.

***6** ••

Determine the Concept From the ideal-gas law, we have $V=nRT/P$. In the process depicted, both the temperature and the pressure increase, but the pressure increases faster than does the temperature. Hence, the volume decreases.

7 •

True. The kinetic energy of translation K for n moles of gas is directly proportional to the absolute temperature T of the gas $\left(K=\frac{3}{2}nkT\right)$.

8 •

Determine the Concept We can use $v_{\text{rms}}=\sqrt{3RT/M}$ to relate the temperature of a gas to the rms speed of its molecules.

Express the dependence of the rms speed of the molecules of a gas on their absolute temperature:

$$v_{\text{rms}}=\sqrt{\frac{3RT}{M}}$$

where R is the gas constant, M is the molar mass, and T is the absolute temperature.

$$\boxed{\begin{array}{l}\text{Because } v_{\text{rms}}\propto\sqrt{T}\text{, the temperature must be quadrupled in order}\\ \text{to double the rms speed of the molecules.}\end{array}}$$

9 •

Picture the Problem The average kinetic energy of a molecule, as a function of the temperature, is given by $K_{\text{av}}=\frac{3}{2}kT$ and the pressure, volume, and temperature of an ideal gas are related according to $PV=NkT$.

Express the average kinetic energy of a molecule in terms of its temperature:

$$K_{\text{av}}=\tfrac{3}{2}kT$$

From the ideal-gas law we have: $PV = NkT$

Eliminate kT between these equations to obtain: $K_{\text{av}} = \frac{3PV}{2N}$

$$\boxed{\text{If } P \text{ is doubled at constant } V,\ K_{\text{av}} \text{ increases by a factor of } 2.}$$

$$\boxed{\text{If } V \text{ is doubled at constant } P,\ K_{\text{av}} \text{ increases by a factor of } 2.}$$

10 ••

Picture the Problem We can express the rms speeds of the helium atoms and the methane molecules using $v_{\text{rms}} = \sqrt{3RT/M}$.

Express the rms speed of the helium atoms: $v_{\text{rms}}(\text{He}) = \sqrt{\frac{3RT}{M_{\text{He}}}}$

Express the rms speed of the methane molecules: $v_{\text{rms}}(\text{CH}_4) = \sqrt{\frac{3RT}{M_{\text{CH}_4}}}$

Divide the first of these equations by the second to obtain: $\frac{v_{\text{rms}}(\text{He})}{v_{\text{rms}}(\text{CH}_4)} = \sqrt{\frac{M_{\text{CH}_4}}{M_{\text{He}}}}$

Use Appendix C to find the molar masses of helium and methane: $\frac{v_{\text{rms}}(\text{He})}{v_{\text{rms}}(\text{CH}_4)} = \sqrt{\frac{16\text{ g/mol}}{4\text{ g/mol}}} = 2$

and $\boxed{(b)\text{ is correct.}}$

11 •

False. Whether the pressure changes also depends on whether and how the volume changes. In an isothermal process, the pressure can increase while the volume decreases and the temperature is constant.

12 •

Determine the Concept For the Celsius scale, the ice point (0°C) and the boiling point of water at 1 atm (100°C) are more convenient than 273 K and 373 K; temperatures in roughly this range are normally encountered. On the Fahrenheit scale, the temperature of warm-blooded animals is roughly 100°F; this may be a more convenient reference than approximately 300 K. Throughout most of the world, the Celsius scale is the standard for nonscientific purposes.

***13 •**

Determine the Concept Because $10^7 >> 273$, it does not matter.

14 •

Determine the Concept The average speed of the molecules in an ideal gas depends on the square root of the kelvin temperature. Because $v_{\text{av}} \propto \sqrt{T}$, doubling the temperature while maintaining constant pressure increases the average speed by a factor of $\sqrt{2}$.

$\boxed{(d)\text{ is correct.}}$

15 •

Determine the Concept From the ideal-gas law, we have $PV = nRT$. Halving both the temperature and volume of the gas leaves the pressure unchanged. $\boxed{(b)\text{ is correct.}}$

16 •

Determine the Concept The average translational kinetic energy of the molecules of an ideal gas is given by $K = \frac{3}{2}NkT = \frac{3}{2}nRT$. The temperature of the ideal gas is related to the pressure of the gas. $\boxed{(d)\text{ is correct.}}$

17 •

Determine the Concept The only conclusion we can draw from the information that the vessel contains equal amounts, by weight, of helium and argon is that the temperatures of the helium and argon molecules are the same. $\boxed{(d)\text{ is correct.}}$

18 ••

Determine the Concept The two rooms are in thermal equilibrium and, because they are connected, the air in each is at the same pressure. Because $P = NkT/V$, and the volume of each room is identical, $N_AT_A = N_BT_B$, so the cooler room (A) has more air in it.

19 •

Picture the Problem The rms speed of an ideal gas is given by $v_{\text{rms}} = \sqrt{3RT/M}$ and its average kinetic energy by $K_{\text{av}} = \frac{3}{2}kT$.

Because $v_{\text{rms}} = \sqrt{3RT/M}$:

$\boxed{\text{The rms speeds are inversely proportional to the square root of the molecular masses.}}$

Because $K_{\text{av}} = \frac{3}{2}kT$ and the gases are at the same temperature:

$$\boxed{\text{The kinetic energies of the molecules are the same.}}$$

20 ••

Determine the Concept The pressure is a measure of the change in momentum per second of a gas molecule on collision with the wall of the container. When the gas is heated, the average velocity, the average momentum, and pressure of the molecules increase.

***21 ••**

Determine the Concept Because the temperature remains constant, the average speed of the molecules remains constant. When the volume decreases, the molecules travel less distance between collisions, so the pressure increases because the frequency of collisions increases.

22 ••

Picture the Problem The average kinetic energies of the molecules are given by $K_{\text{av}} = \left(\frac{1}{2}mv^2\right)_{\text{av}} = \frac{3}{2}kT.$ Assuming that the room's temperature distribution is uniform, we can conclude that the oxygen and nitrogen molecules have equal average kinetic energies. Because the oxygen molecules are more massive, they must be moving slower than the nitrogen molecules. $\boxed{(b)\text{ is correct.}}$

23 ••

Determine the Concept The average molecular speed of He gas at 300 K is about 1.4 km/s, so a significant fraction of He molecules have speeds in excess of earth's escape velocity (11.2 km/s). Thus, they "leak" away into space. Over time, the He content of the atmosphere decreases to almost nothing.

Estimation and Approximation

***24 ••**

Picture the Problem Assuming the steam to be an ideal gas at a temperature of 373 K, we can use the ideal-gas law to estimate the pressure inside the test tube when the water is completely boiled away.

Using the ideal-gas law, relate the pressure inside the test tube to its volume and the temperature:

$$P = \frac{NkT}{V}$$

Relate the number of particles N to the mass of water, its molar mass M, and Avogadro's number N_{A}:

$$\frac{m}{N} = \frac{M}{N_{\text{A}}}$$

Solve for *N*:

$$N = m\frac{N_{\rm A}}{M}$$

Relate the mass of 1 mL of water to its density:

$$m = \rho V = \left(10^{3}\ \text{kg/m}^{3}\right)\left(10^{-6}\ \text{m}^{3}\right) = 1\text{g}$$

Substitute for *m, N*$_{\rm A}$, and *M* (18 g/mol) and evaluate *N*:

$$N = (1\text{g})\frac{6.022\times10^{23}\ \text{particles/mol}}{18\,\text{g/mol}}$$

$$= 3.35\times10^{22}\ \text{particles}$$

Substitute numerical values and evaluate *P*:

$$P = \frac{\left(3.35\times10^{22}\ \text{particles}\right)\left(1.381\times10^{-23}\ \text{J/K}\right)(373\text{K})}{10\times10^{-6}\ \text{m}^{3}}$$

$$= 172\times10^{5}\ \text{N/m}^{2}\times\frac{1\text{atm}}{1.01\times10^{5}\ \text{N/m}^{2}}$$

$$= \boxed{171\text{atm}}$$

25 •••

Picture the Problem We can find the escape temperatures for the earth and the moon by equating, in turn, 0.15$v_{\rm e}$ and $v_{\rm rms}$ of O_2 and H_2. We can compare these temperatures to explain the absence from the earth's upper atmosphere and from the surface of the moon.

(*a*) Express $v_{\rm rms}$ for O_2:

$$v_{\rm rms} = \sqrt{\frac{3RT}{M}}$$

where *R* is the gas constant, *T* is the absolute temperature, and *M* is the molar mass of oxygen.

Equate 0.15$v_{\rm e}$ and $v_{\rm rms}$:

$$0.15\sqrt{2gR_{\rm earth}} = \sqrt{\frac{3RT}{M}}$$

Solve for *T* to obtain:

$$T = \frac{0.045gR_{\rm earth}M}{3R} \tag{1}$$

Evaluate *T* for O_2:

$$T = \frac{0.045\left(9.81\text{m/s}^{2}\right)\left(6.37\times10^{6}\ \text{m}\right)}{3(8.314\,\text{J/mol}\cdot\text{K})}$$

$$\times\left(32\times10^{-3}\ \text{kg/mol}\right)$$

$$= \boxed{3.61\times10^{3}\ \text{K}}$$

(*b*) Substitute numerical values and evaluate T for H_2:

$$T=\frac{0.045\left(9.81\,\text{m/s}^2\right)\left(6.37\times10^6\,\text{m}\right)}{3\left(8.314\,\text{J/mol}\cdot\text{K}\right)}\times\left(2\times10^{-3}\,\text{kg/mol}\right)=\boxed{225\,\text{K}}$$

(*c*) $\boxed{\text{If } v_{\text{rms}}>\tfrac{1}{5}v_{\text{e}} \text{ or } T\geq 25T_{\text{atm}}\text{, } H_2 \text{ molecules escape. Therefore, the more energetic } H_2 \text{ molecules escape from the upper atmosphere.}}$

(*d*) Express equation (1) at the surface of the moon:

$$T=\frac{0.045g_{\text{moon}}R_{\text{moon}}M}{3R}=\frac{0.045\left(\frac{1}{6}g_{\text{earth}}\right)R_{\text{moon}}M}{3R}=\frac{0.0025g_{\text{earth}}R_{\text{moon}}M}{R}$$

Substitute numerical values and evaluate T for O_2:

$$T=\frac{0.0025\left(9.81\,\text{m/s}^2\right)\left(1.738\times10^6\,\text{m}\right)\left(32\times10^{-3}\,\text{kg/mol}\right)}{8.314\,\text{J/mol}\cdot\text{K}}=\boxed{164\,\text{K}}$$

Substitute numerical values and evaluate T for H_2:

$$T=\frac{0.0025\left(9.81\,\text{m/s}^2\right)\left(1.738\times10^6\,\text{m}\right)\left(2\times10^{-3}\,\text{kg/mol}\right)}{8.314\,\text{J/mol}\cdot\text{K}}=\boxed{10.3\,\text{K}}$$

$\boxed{\text{If we assume that the temperature on the moon \textit{with an atmosphere} would have been approximately 1000 K, then all } O_2 \text{ and } H_2 \text{ would have escaped during the time since the formation of the moon to the present.}}$

26 ••

Picture the Problem We can use $v_{\text{rms}}=\sqrt{3RT/M}$ to calculate the rms speeds of H_2, O_2, and CO_2 at 273 K and then compare these speeds to 20% of the escape velocity on Mars to decide the likelihood of finding these gases in the atmosphere of Mars.

Express the rms speed of an atom as a function of the temperature:

$$v_{\text{rms}}=\sqrt{\frac{3RT}{M}}$$

(*a*) Substitute numerical values and evaluate v_{rms} for H_2:

$$v_{\text{rms},\text{H}_2} = \sqrt{\frac{3(8.314\,\text{J/mol}\cdot\text{K})(273\,\text{K})}{2\times10^{-3}\,\text{kg/mol}}} = \boxed{1.85\,\text{km/s}}$$

(*b*) Evaluate v_{rms} for O_2:

$$v_{\text{rms},\text{O}_2} = \sqrt{\frac{3(8.314\,\text{J/mol}\cdot\text{K})(273\,\text{K})}{32\times10^{-3}\,\text{kg/mol}}} = \boxed{461\,\text{m/s}}$$

(*c*) Evaluate v_{rms} for CO_2:

$$v_{\text{rms},\text{CO}_2} = \sqrt{\frac{3(8.314\,\text{J/mol}\cdot\text{K})(273\,\text{K})}{44\times10^{-3}\,\text{kg/mol}}} = \boxed{393\,\text{m/s}}$$

(*d*) Calculate 20% of v_{esc} for Mars:

$$v = \tfrac{1}{5}v_{\text{esc}} = \tfrac{1}{5}(5\,\text{km/s}) = 1\,\text{km/s}$$

$$\boxed{\begin{array}{l}\text{Because } v \text{ is greater than } v_{\text{rms}} \text{ for CO}_2 \text{ and O}_2 \text{ but less than } v_{\text{rms}} \text{ for H}_2\\ \text{O}_2 \text{ and CO}_2\text{, but not H}_2\text{, should be present.}\end{array}}$$

***27** ••

Picture the Problem We can use $v_{\text{rms}} = \sqrt{3RT/M}$ to calculate the rms speeds of H_2, O_2, and CO_2 at 123 K and then compare these speeds to 20% of the escape velocity on Jupiter to decide the likelihood of finding these gases in the atmosphere of Jupiter.

Express the rms speed of an atom as a function of the temperature:

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

(*a*) Substitute numerical values and evaluate v_{rms} for H_2:

$$v_{\text{rms},\text{H}_2} = \sqrt{\frac{3(8.314\,\text{J/mol}\cdot\text{K})(123\,\text{K})}{2\times10^{-3}\,\text{kg/mol}}} = \boxed{1.24\,\text{km/s}}$$

(*b*) Evaluate v_{rms} for O_2:

$$v_{\text{rms},\text{O}_2} = \sqrt{\frac{3(8.314\,\text{J/mol}\cdot\text{K})(123\,\text{K})}{32\times10^{-3}\,\text{kg/mol}}} = \boxed{310\,\text{m/s}}$$

(*c*) Evaluate v_{rms} for CO_2:

$$v_{\text{rms},\text{CO}_2} = \sqrt{\frac{3(8.314\,\text{J/mol}\cdot\text{K})(123\,\text{K})}{44\times10^{-3}\,\text{kg/mol}}} = \boxed{264\,\text{m/s}}$$

(*d*) Calculate 20% of v_{esc} for Jupiter:

$$v=\tfrac{1}{5}v_{\text{esc}}=\tfrac{1}{5}(60\text{ km/s})=12\text{ km/s}$$

$$\boxed{\text{Because } v \text{ is greater than } v_{\text{rms}} \text{ for } O_2, CO_2, \text{ and } H_2,\ O_2, CO_2, \text{ and } H_2 \text{ should be found on Jupiter.}}$$

Temperature Scales

28 •

Picture the Problem We can convert both of these temperatures to the Fahrenheit scale and then express their difference to find the range of temperatures.

Solve the Fahrenheit-Celsius conversion equation for the Fahrenheit temperature:

$$t_{\text{F}}=\tfrac{9}{5}t_{\text{C}}+32°$$

Find the Fahrenheit equivalent of −12°C:

$$t_{\text{F}}=\tfrac{9}{5}(-12°)+32°=10.4°$$

Find the Fahrenheit equivalent of −7°C:

$$t_{\text{F}}=\tfrac{9}{5}(-7°)+32°=19.4°\text{F}$$

The difference between these two temperatures is the range on the Fahrenheit scale:

$$\text{Range}=19.4°\text{F}-10.4°\text{F}$$
$$=\boxed{9.00\text{F}°}$$

Remarks: We could take advantage of the fact that 9 F° = 5 C° to arrive at the aforementioned result in which the range of Celsius temperatures happens to be 5C°. If the temperature difference were other than 5C°, we could set up a proportion to quickly find the range on the Fahrenheit scale.

29 •

Picture the Problem We can use the Fahrenheit-Celsius conversion equation to find this temperature on the Celsius scale.

Convert 1945.4°F to the equivalent Celsius temperature:

$$t_{\text{C}}=\tfrac{5}{9}(t_{\text{F}}-32°)=\tfrac{5}{9}(1945.4°-32°)$$
$$=\boxed{1063°\text{C}}$$

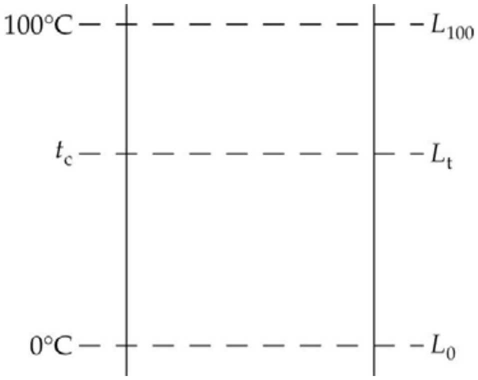
***30** •

Picture the Problem We can use the Fahrenheit-Celsius conversion equation to express the temperature of the human body on the Celsius scale.

Convert 98.6°F to the equivalent Celsius temperature:

$$t_{\text{C}}=\tfrac{5}{9}(t_{\text{F}}-32°)=\tfrac{5}{9}(98.6°-32°)$$
$$=\boxed{37.0°\text{C}}$$

31 •



Picture the Problem While we could use Equation 17-1 to relate the Celsius temperature to the length of the column of mercury in the thermometer, an alternative solution is to use the diagram to the right to set up a proportion that will relate the Celsius temperature to the calibration temperatures and to the lengths of the mercury column.

Using the diagram, set up a proportion relating the temperatures to the lengths of the column of mercury:

$$\frac{t_c - 0°\text{C}}{100°\text{C} - 0°\text{C}} = \frac{L_t - L_0}{L_{100} - L_0}$$

Solve for and evaluate L_t:

$$\begin{aligned} L_t &= \frac{t_c(L_{100} - L_0)}{100°} + L_0 \\ &= \frac{t_c(24.0\,\text{cm} - 4.0\,\text{cm})}{100°} + 4.0\,\text{cm} \\ &= \frac{t_c(20.0\,\text{cm})}{100°} + 4.0\,\text{cm} \end{aligned}$$

(*a*) Substitute 22.0°C for t_c and evaluate L_t:

$$\begin{aligned} L_t &= \frac{(22.0°\text{C})(20.0\,\text{cm})}{100°} + 4.0\,\text{cm} \\ &= \boxed{8.40\,\text{cm}} \end{aligned}$$

(*b*) Substitute 25.4 cm for L_t and evaluate t_c:

$$\begin{aligned} t_c &= \frac{25.4\,\text{cm} - 4.0\,\text{cm}}{20\,\text{cm}} \times 100° \\ &= \boxed{107°\text{C}} \end{aligned}$$

32 •

Picture the Problem We can use the temperature conversion equations $t_F = \frac{9}{5}t_C + 32°$ and $t_C = T - 273.15\,\text{K}$ to convert 10^7 K to the Fahrenheit and Celsius temperatures.

Express the kelvin temperature in terms of the Celsius temperature:

$$T = t_C + 273.15\,\text{K}$$

(*a*) Solve for and evaluate t_C:

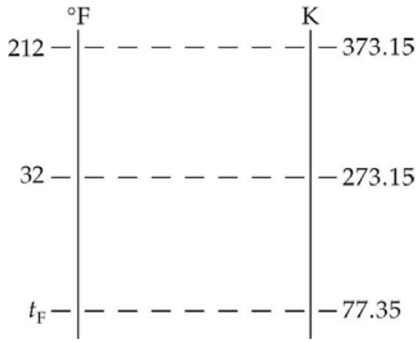
$$t_C = T - 273.15\,\text{K} = 10^7\,\text{K} - 273.15\,\text{K} \approx \boxed{10^7\,\text{K}}$$

(*b*) Use the Celsius to Fahrenheit conversion equation to evaluate t_F:

$$t_F = \tfrac{9}{5}\left(10^7\,°\text{C}\right) + 32° \approx \boxed{1.80\times10^7\,°\text{F}}$$

33 •

Picture the Problem While we could convert 77.35 K to a Celsius temperature and then convert the Celsius temperature to a Fahrenheit temperature, an alternative solution is to use the diagram to the right to set up a proportion for the direct conversion of the kelvin temperature to its Fahrenheit equivalent.



Use the diagram to set up the proportion:

$$\frac{32°\text{F} - t_F}{212°\text{F} - 32°\text{F}} = \frac{273.15\,\text{K} - 77.35\,\text{K}}{373.15\,\text{K} - 273.15\,\text{K}}$$

or

$$\frac{32°\text{F} - t_F}{180\,\text{F}°} = \frac{195}{100}$$

Solve for and evaluate t_F:

$$t_F = 32°\text{F} - \frac{195}{100}\times 180\,\text{F}° = \boxed{-319\,°\text{F}}$$

34 •

Picture the Problem We can use the fact that, for a constant-volume gas thermometer, the pressure and absolute temperature are directly proportional to calibrate the given thermometer; i.e., to find the constant of proportionality relating P and T. We can then use this equation to find the temperature corresponding to a given pressure or the pressure corresponding to a given temperature.

Express the direct proportionality between the pressure and the temperature:

$$P = CT$$

where C is a constant.

Use numerical values to evaluate C:

$$C = \frac{P}{T} = \frac{0.400\,\text{atm}}{273.15\,\text{K}}$$

$$= 1.464\times10^{-3}\,\text{atm/K}$$

Substitute to obtain:

$$P = \left(1.464\times10^{-3}\ \text{atm/K}\right)T \qquad (1)$$

or

$$T = \left(682.9\ \text{K/atm}\right)P \qquad (2)$$

(*a*) Use equation (2) to find the temperature:

$$T = \left(682.9\ \text{K/atm}\right)\left(0.1\text{atm}\right) = \boxed{68.3\text{K}}$$

(*b*) Use equation (1) to find the boiling point of sulfur:

$$P = \left(1.464\times10^{-3}\ \text{atm/K}\right) \times\left(444.6+273.15\right)\text{K} = \boxed{1.05\,\text{atm}}$$

***35** •

Picture the Problem We can use the information that the thermometer reads 50 torr at the triple point of water to calibrate it. We can then use the direct proportionality between the absolute temperature and the pressure to either the pressure at a given temperature or the temperature for a given pressure.

Using the ideal-gas temperature scale, relate the temperature to the pressure:

$$T = \frac{273.16\,\text{K}}{P_3}P = \frac{273.16\,\text{K}}{50\,\text{torr}}P = \left(5.463\text{K/torr}\right)P$$

(*a*) Solve for and evaluate *P* when *T* = 300 K:

$$P = \left(0.1830\,\text{torr/K}\right)T = \left(0.1830\,\text{torr/K}\right)\left(300\,\text{K}\right) = \boxed{54.9\,\text{torr}}$$

(*b*) Find *T* when the pressure is 678 torr:

$$T = \left(5.463\,\text{K/torr}\right)\left(678\,\text{torr}\right) = \boxed{3704\,\text{K}}$$

36 •

Picture the Problem We can use the equation for the ideal-gas temperature scale to express the temperature measured by this thermometer in terms of its pressure and the given data to calibrate the thermometer.

Write the equation for the ideal-gas temperature scale:

$$T = \frac{273.16\,\text{K}}{P_3}P \qquad \text{17-4}$$

(*a*) Solve for and evaluate the thermometer's triple-point pressure:

$$P_3 = \frac{273.16\,\text{K}}{T}P = \frac{273.16\,\text{K}}{373\,\text{K}}(30\,\text{torr})$$
$$= \boxed{22.0\,\text{torr}}$$

(*b*) Substitute for P_3 in Equation 17-4:

$$T = \frac{273.16\,\text{K}}{22.0\,\text{torr}}P = \frac{273.16\,\text{K}}{22.0\,\text{torr}}(0.175\,\text{torr})$$
$$= \boxed{2.17\,\text{K}}$$

37 •

Picture the Problem We can find the temperature at which the Fahrenheit and Celsius scales give the same reading by setting $t_F = t_C$ in the temperature-conversion equation.

Set $t_F = t_C$ in $t_C = \frac{5}{9}(t_F - 32°)$:

$$t_F = \tfrac{5}{9}(t_F - 32°)$$

Solve for and evaluate t_F:

$$t_C = t_F = \boxed{-40.0°\text{C}} = \boxed{-40.0°\text{F}}$$

Remarks: If you've not already thought of doing so, you might use your graphing calculator to plot t_C versus t_F and $t_F = t_C$ (a straight line at 45°) on the same graph. Their intersection is at (–40, –40).

38 •

Picture the Problem We can use the Celsius-to-absolute conversion equation to find 371 K on the Celsius scale and the Celsius-to-Fahrenheit conversion equation to find the Fahrenheit temperature corresponding to 371 K.

Express the absolute temperature as a function of the Celsius temperature:

$$T = t_C + 273.15\,\text{K}$$

Solve for and evaluate t_C:

$$t_C = T - 273.15\,\text{K}$$
$$= 371\,\text{K} - 273.15\,\text{K} = \boxed{97.9°\text{C}}$$

Use the Celsius-to-Fahrenheit conversion equation to find t_F:

$$t_F = \tfrac{9}{5}t_C + 32° = \tfrac{9}{5}(97.9°) + 32°$$
$$= \boxed{208°\text{F}}$$

39 •

Picture the Problem We can use the Celsius-to-absolute conversion equation to find 90.2 K on the Celsius scale and the Celsius-to-Fahrenheit conversion equation to find the

Fahrenheit temperature corresponding to 90.2 K.

Express the absolute temperature as a function of the Celsius temperature:

$$T = t_{\text{C}} + 273.15\,\text{K}$$

Solve for and evaluate t_{C}:

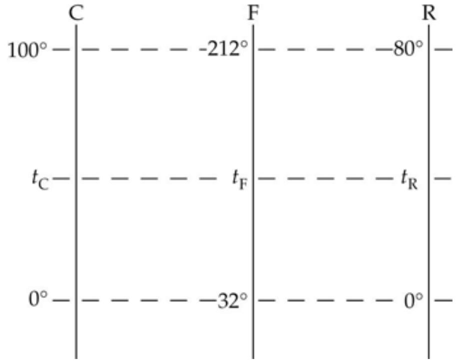
$$t_{\text{C}} = T - 273.15\,\text{K}$$
$$= 90.2\,\text{K} - 273.15\,\text{K} = \boxed{-183°\text{C}}$$

Use the Celsius-to-Fahrenheit conversion equation to find t_{F}:

$$t_{\text{F}} = \tfrac{9}{5}t_{\text{C}} + 32° = \tfrac{9}{5}\left(-183°\right) + 32°$$
$$= \boxed{-297°\text{F}}$$

40 ••

Picture the Problem We can use the diagram to the right to set up proportions that will allow us to convert temperatures on the Réaumur scale to Celsius and Fahrenheit temperatures.



Referring to the diagram, set up a proportion to convert temperatures on the Réaumur scale to Celsius temperatures:

$$\frac{t_{\text{C}} - 0°\text{C}}{100°\text{C} - 0°\text{C}} = \frac{t_{\text{R}} - 0°\text{R}}{80°\text{R} - 0°\text{R}}$$

Simplify to obtain:

$$\frac{t_{\text{C}}}{100} = \frac{t_{\text{R}}}{80} \text{ or } t_{\text{C}} = \boxed{1.25t_{\text{R}}}$$

Referring to the diagram, set up a proportion to convert temperatures on the Réaumur scale to Fahrenheit temperatures:

$$\frac{t_{\text{F}} - 32°\text{F}}{212°\text{F} - 32°\text{F}} = \frac{t_{\text{R}} - 0°\text{R}}{80°\text{R} - 0°\text{R}}$$

Simplify to obtain:

$$\frac{t_{\text{F}} - 32}{180} = \frac{t_{\text{R}}}{80} \text{ or } t_{\text{F}} = \boxed{\tfrac{9}{4}t_{\text{R}} + 32}$$

***41** •••

Picture the Problem We can use the temperature dependence of the resistance of the thermistor and the given data to determine R_0 and B. Once we know these quantities, we can use the temperature-dependence equation to find the resistance at any temperature in the calibration range. Differentiation of R with respect to T will allow us to express the rate of change of resistance with temperature at both the ice point and the steam point temperatures.

(*a*) Express the resistance at the ice point as a function of temperature of the ice point:

$$7360\,\Omega = R_0 e^{B/273\,\text{K}} \qquad (1)$$

Express the resistance at the steam point as a function of temperature of the steam point:

$$153\Omega = R_0 e^{B/373\,\text{K}} \qquad (2)$$

Divide equation (1) by equation (2) to obtain:

$$\frac{7360\,\Omega}{153\Omega} = 48.10 = e^{B/273\,\text{K} - B/373\,\text{K}}$$

Solve for B by taking the logarithm of both sides of the equation:

$$\ln 48.1 = B\left(\frac{1}{273} - \frac{1}{373}\right)\text{K}^{-1}$$

and

$$B = \frac{\ln 48.1}{\left(\frac{1}{273} - \frac{1}{373}\right)\text{K}^{-1}} = \boxed{3.94\times10^{3}\,\text{K}}$$

Solve equation (1) for R_0 and substitute for B:

$$R_0 = \frac{7360\,\Omega}{e^{B/273\,\text{K}}} = (7360\,\Omega)e^{-B/273\,\text{K}}$$
$$= (7360\,\Omega)e^{-3.94\times10^{3}\,\text{K}/273\,\text{K}}$$
$$= \boxed{3.97\times10^{-3}\,\Omega}$$

(*b*) From (*a*) we have:

$$R = \left(3.97\times10^{-3}\,\Omega\right)e^{3.94\times10^{3}\,\text{K}/T}$$

Convert 98.6°F to kelvins to obtain:

$$T = 310\,\text{K}$$

Substitute to obtain:

$$R = \left(3.97\times10^{-3}\,\Omega\right)e^{3.94\times10^{3}\,\text{K}/310\,\text{K}}$$
$$= \boxed{1.31\text{k}\Omega}$$

(*c*) Differentiate R with respect to T to obtain:

$$\frac{dR}{dT} = \frac{d}{dT}\left(R_0 e^{B/T}\right) = R_0 e^{B/T}\,\frac{d}{dT}\left(\frac{B}{T}\right)$$
$$= \frac{-B}{T^2}R_0 e^{B/T} = -\frac{RB}{T^2}$$

Evaluate *dR/dT* at the ice point:

$$\left(\frac{dR}{dT}\right)_{\text{ice point}} = -\frac{(7360\,\Omega)(3.94\times10^3\,\text{K})}{(273.16\,\text{K})^2} = \boxed{-389\,\Omega/\text{K}}$$

Evaluate *dR/dT* at the steam point:

$$\left(\frac{dR}{dT}\right)_{\text{steam point}} = -\frac{(153\,\Omega)(3.94\times10^3\,\text{K})}{(373.16\,\text{K})^2} = \boxed{-4.33\,\Omega/\text{K}}$$

(*d*) $\boxed{\text{The thermistor is more sensitive; i.e., it has greater sensitivity at lower temperatures.}}$

The Ideal-Gas Law

42 •

Picture the Problem Let the subscript 1 refer to the gas at 50°C and the subscript 2 to the gas at 100°C. We can apply the ideal-gas law for a fixed amount of gas to find the ratio of the final and initial volumes.

Apply the ideal-gas law for a fixed amount of gas:

$$\frac{P_2V_2}{T_2} = \frac{P_1V_1}{T_1}$$

or, because $P_2 = P_1$,

$$\frac{V_2}{V_1} = \frac{T_2}{T_1}$$

Substitute numerical values and evaluate V_2/V_1:

$$\frac{V_2}{V_1} = \frac{(273.15+100)\text{K}}{(273.15+50)\text{K}} = \boxed{1.15}$$

43 •

Picture the Problem We can use the ideal-gas law to find the number of moles of gas in the vessel and the definition of Avogadro's number to find the number of molecules.

Apply the ideal-gas law to the gas:

$$PV = nRT$$

Solve for the number of moles of gas in the vessel:

$$n = \frac{PV}{RT}$$

Substitute numerical values and evaluate n:

$$n=\frac{(4\,\text{atm})(10\,\text{L})}{(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})(273\,\text{K})}=\boxed{1.79\,\text{mol}}$$

Relate the number of molecules N in the gas in terms of the number of moles n:

$$N=nN_\text{A}$$

Substitute numerical values and evaluate N:

$$N=(1.79\,\text{mol})(6.022\times10^{23}\,\text{molecules/mol})=\boxed{1.08\times10^{24}\,\text{molecules}}$$

44 ••

Picture the Problem We can use the ideal-gas law to relate the number of molecules in the gas to its pressure, volume, and temperature.

Solve the ideal-gas law for the number of molecules in a gas as a function of its pressure, volume, and temperature:

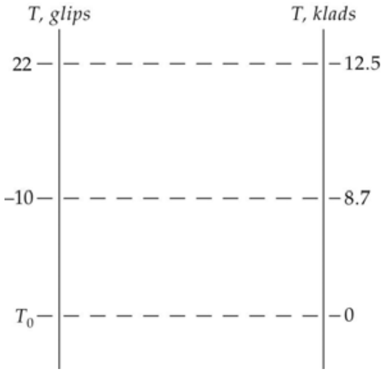
$$N=\frac{PV}{kT}$$

Substitute numerical values and evaluate N:

$$N=\frac{(10^{-8}\,\text{torr})(133.32\,\text{Pa/torr})(10^{-6}\,\text{m}^3)}{(1.381\times10^{-23}\,\text{J/K})(300\,\text{K})}=\boxed{3.22\times10^{8}}$$

45 ••

Picture the Problem The pictorial representation to the right, in which T_0 represents absolute zero, summarizes the information concerning the temperatures and pressures we are given. We know, from the ideal-gas law, that the pressure of a fixed volume of gas is proportional to its absolute temperature. We can use the diagram to set up a proportion relating the temperatures and pressures that we can solve for T_0.



Apply the ideal-gas law to obtain:

$$\frac{22\,glips-T_0}{12.5\,klads}=\frac{-10\,glips-T_0}{8.7\,klads}$$

Solve for T_0 to obtain:

$$T_0=\boxed{-83.2\,glips}$$

Remarks: Because the gas is ideal, its pressure is directly proportional to its temperature. Hence, a graph of *P* versus *T* will be linear and the linear equation relating *P* and *T* can be solved for the temperature corresponding to zero pressure.

46 ••

Picture the Problem Let the subscript 1 refer to the tires when their pressure is 180 kPa and the subscript 2 to conditions when their pressure is 245 kPa. Assume that the air in the tires behaves as an ideal gas. Then, we can apply the ideal-gas law for a fixed amount of gas to relate the temperatures to the pressures and volumes of the tires.

(*a*) Apply the ideal-gas law for a fixed amount of gas to the air in the tires:

$$\frac{P_2V_2}{T_2}=\frac{P_1V_1}{T_1} \qquad (1)$$

Solve for T_2:

$$T_2=T_1\frac{P_2}{P_1}$$ because $V_1 = V_2$.

Substitute numerical values to obtain:

$$T_2=(265\,\text{K})\frac{245\,\text{kPa}}{180\,\text{kPa}}=360.7\,\text{K}$$

$$=\boxed{87.7°\text{C}}$$

(*b*) Use equation (1) with $V_2 = 1.07\ V_1$. Solve for T_2:

$$T_2=\frac{P_2V_2}{P_1V_1}T_1=1.07\left(\frac{P_2}{P_1}T_1\right)$$

Substitute numerical values and evaluate T_2:

$$T_2=1.07(360.7\,\text{K})=385.9\,\text{K}=\boxed{113°\text{C}}$$

47 ••

Picture the Problem We can apply the ideal-gas law to find the number of moles of air in the room as a function of the temperature.

(*a*) Use the ideal-gas law to relate the number of moles of air in the room to the pressure, volume, and temperature of the air:

$$n=\frac{PV}{RT} \qquad (1)$$

Substitute numerical values and evaluate *n*:

$$n=\frac{(101.3\,\text{kPa})(90\,\text{m}^3)}{(8.314\,\text{J/mol}\cdot\text{K})(300\,\text{K})}$$

$$=\boxed{3.66\times10^3\,\text{mol}}$$

(*b*) Letting n' represent the number of moles in the room when the temperature rises by 5 K, express the number of moles of air that leave the room:

$$\Delta n = n - n'$$

Apply the ideal-gas law to obtain:

$$n' = \frac{PV}{RT'} \qquad (2)$$

Divide equation (2) by equation (1) to obtain:

$$\frac{n'}{n} = \frac{T}{T'} \text{ and } n' = n\frac{T}{T'}$$

Substitute for n' to obtain:

$$\Delta n = n - n\frac{T}{T'} = n\left(1 - \frac{T}{T'}\right)$$

Substitute numerical values and evaluate Δn:

$$\Delta n = \left(3.66\times10^{3}\,\text{mol}\right)\left(1 - \frac{300\,\text{K}}{305\,\text{K}}\right)$$
$$= \boxed{60.0\,\text{mol}}$$

***48 ••**

Picture the Problem Let the subscript 1 refer to helium gas at 4.2 K and the subscript 2 to the gas at 293 K. We can apply the ideal-gas law to find the volume of the gas at 4.2 K and a fixed amount of gas to find its volume at 293 K.

(*a*) Apply the ideal-gas law to the helium gas and solve for its volume:

$$V_1 = \frac{nRT_1}{P_1}$$

Substitute numerical values to obtain:

$$V_1 = n\frac{\left(0.08206\,\text{L}\cdot\text{atm/mol}\cdot\text{K}\right)\left(4.2\,\text{K}\right)}{1\,\text{atm}}$$
$$= \left(0.3447\,\text{L/mol}\right)n$$

Find the number of moles in 10 g of helium:

$$n = \frac{10\,\text{g}}{4\,\text{g/mol}} = 2.5\,\text{mol}$$

Substitute for n to obtain:

$$V_1 = \left(0.3447\,\text{L/mol}\right)\left(2.5\,\text{mol}\right)$$
$$= \boxed{0.862\,\text{L}}$$

(*b*) Apply the ideal-gas law for a fixed amount of gas and solve for

$$\frac{P_2V_2}{T_2} = \frac{P_1V_1}{T_1}$$

the volume of the helium gas at 293 K:

and, because $P_1 = P_2$,

$$V_2 = \frac{T_2}{T_1} V_1$$

Substitute numerical values and evaluate V_2:

$$V_2 = \frac{293\,\text{K}}{4.2\,\text{K}}(0.862\,\text{L}) = \boxed{60.1\,\text{L}}$$

49 ••

Picture the Problem Because the helium is initially in the liquid state, its temperature must be 4.2 K. Let the subscript 1 refer to helium gas at 4.2 K and the subscript 2 to the gas at 293 K. We can apply the ideal-gas law for a fixed volume of gas to relate the pressure at 293 K to the pressure at 4.2 K and use the ideal-gas law to find the pressure at 4.2 K.

Apply the ideal-gas law for a fixed amount of gas:

$$\frac{P_2V_2}{T_2} = \frac{P_1V_1}{T_1}$$

Solve for its pressure at 293 K:

$$P_2 = \frac{P_1V_1T_2}{V_2T_1} = P_1\frac{T_2}{T_1} \qquad (1)$$

because $V_1 = V_2$

Apply the ideal-gas law to the helium gas at 4.2 K and solve for its pressure:

$$P_1 = \frac{nRT_1}{V_1}$$

Substitute numerical values to obtain:

$$P_1 = n\frac{(0.08206\,\text{L}\cdot\text{atm/mol}\cdot\text{K})(4.2\,\text{K})}{6\,\text{L}}$$
$$= (0.05744\,\text{atm/mol})n$$

Find the number of moles in 10 g of helium:

$$n = \frac{10\,\text{g}}{4\,\text{g/mol}} = 2.5\,\text{mol}$$

Substitute for n to obtain:

$$P_1 = (0.05744\,\text{atm/mol})(2.5\,\text{mol})$$
$$= 0.1436\,\text{atm}$$

Substitute in equation (1) and evaluate P_2:

$$P_2 = (0.1436\,\text{atm})\frac{293\,\text{K}}{4.2\,\text{K}} = \boxed{10.0\,\text{atm}}$$

***50** ••

Picture the Problem Let the subscript 1 refer to the tire when its temperature is 20°C and the subscript 2 to conditions when its temperature is 50°C. We can apply the ideal-gas law for a fixed amount of gas to relate the temperatures to the pressures of the air in the tire.

(*a*) Apply the ideal-gas law for a fixed amount of gas and solve for pressure at the higher temperature:

$$\frac{P_2V_2}{T_2}=\frac{P_1V_1}{T_1}\qquad(1)$$

and

$$P_2=\frac{T_2}{T_1}P_1$$

because $V_1 = V_2$

Substitute numerical values to obtain:

$$P_2=\frac{323\,\text{K}}{293\,\text{K}}\left(200\,\text{kPa}+101\,\text{kPa}\right)$$
$$=332\,\text{kPa}$$

and

$$P_{2,\text{gauge}}=332\,\text{kPa}-101\,\text{kPa}$$
$$=\boxed{231\,\text{kPa}}$$

(*b*) Solve equation (1) for P_2 with $V_2 = 1.1\ V_1$ and evaluate P_2:

$$P_2=\frac{V_1T_2}{V_2T_1}P_1$$
$$=\frac{323\,\text{K}}{1.1(293\,\text{K})}\left(200\,\text{kPa}+101\,\text{kPa}\right)$$
$$=302\,\text{kPa}$$

and

$$P_{2,\text{gauge}}=302\,\text{kPa}-101\,\text{kPa}=\boxed{201\,\text{kPa}}$$

51 ••

Picture the Problem Let ρ_{N_2} and ρ_{0_2} be the number densities (i.e., the number of particles per unit volume) of N_2 and O_2, respectively. We can express the density of air in terms of the densities of nitrogen and oxygen and their number densities as $\rho_{\text{air}}=m_{N_2}\rho_{N_2}+m_{0_2}\rho_{0_2}$. By applying the ideal-gas law, we can find the number density of air and, using the given composition of air, calculate the number densities of nitrogen and oxygen. Finally, we can find the masses of nitrogen and oxygen molecules from their atomic masses. Knowing ρ_{N_2}, ρ_{O_2}, m_{N_2}, and m_{O_2}, we can calculate ρ_{air}.

Express the density of air in terms of the densities of nitrogen and oxygen:

$$\rho_{\text{air}}=m_{N_2}\rho_{N_2}+m_{0_2}\rho_{0_2}\qquad(1)$$

Using the ideal-gas law, relate the number density of air *N*/*V* to its temperature and pressure:

$$PV=NkT\ \text{ and }\ \frac{N}{V}=\frac{P}{kT}$$

Substitute numerical values and evaluate the number density of air:

$$\frac{N}{V} = \frac{1.01\times10^{5}\ \text{N/m}^2}{\left(1.381\times10^{-23}\ \text{J/K}\right)\left(297\ \text{K}\right)} = 2.46\times10^{25}\ \text{m}^{-3}$$

Because air is approximately 74% N_2 and 26% O_2:

$$\rho_{\text{N}_2} = 0.74\frac{N}{V} = 0.74\left(2.46\times10^{25}\ \text{m}^{-3}\right) = 1.82\times10^{25}\ \text{m}^{-3}$$

and

$$\rho_{\text{O}_2} = 0.26\frac{N}{V} = 0.26\left(2.46\times10^{25}\ \text{m}^{-3}\right) = 6.40\times10^{24}\ \text{m}^{-3}$$

Calculate the masses of N_2 and O_2 molecules:

$$m_{\text{N}_2} = \left(28\ \text{u}\right)\left(1.660\times10^{-27}\ \text{kg/u}\right) = 4.65\times10^{-26}\ \text{kg}$$

and

$$m_{\text{O}_2} = \left(32\ \text{u}\right)\left(1.660\times10^{-27}\ \text{kg/u}\right) = 5.31\times10^{-26}\ \text{kg}$$

Substitute in equation (1) and evaluate ρ_{air}:

$$\rho_{\text{air}} = \left(4.65\times10^{-26}\ \text{kg}\right)\left(1.82\times10^{25}\ \text{m}^{-3}\right) + \left(5.31\times10^{-26}\ \text{kg}\right)\left(6.40\times10^{24}\ \text{m}^{-3}\right) = \boxed{1.19\ \text{kg/m}^3}$$

52 ••

Picture the Problem Let the subscript 1 refer to the conditions at the bottom of the lake and the subscript 2 to the surface of the lake and apply the ideal-gas law for a fixed amount of gas.

Apply the ideal-gas law for a fixed amount of gas:

$$\frac{P_2V_2}{T_2} = \frac{P_1V_1}{T_1}$$

Solve for the volume of the bubble just before it breaks the surface:

$$V_2 = V_1\frac{T_2P_1}{T_1P_2}$$

Find the pressure at the bottom of the lake:

$$P_1 = P_{\text{atm}} + \rho gh = 101.3\ \text{kPa} + \left(10^3\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left(40\ \text{m}\right) = 493.7\ \text{kPa}$$

Substitute numerical values and evaluate V_2:

$$V_2 = \left(15\,\text{cm}^3\right)\frac{(298\,\text{K})(493.7\,\text{kPa})}{(278\,\text{K})(101.3\,\text{kPa})} = \boxed{78.4\,\text{cm}^3}$$

53 ••

Picture the Problem Assume that the volume of the balloon is not changing. Then the air inside and outside the balloon must be at the same pressure of about 1 atm. The contents of the balloon are the air molecules inside it. We can use Archimedes principle to express the buoyant force on the balloon and we can find the weight of the air molecules inside the balloon

Express the net force on the balloon and its contents:

$$F_{\text{net}} = B - w_{\text{air inside the balloon}} \qquad (1)$$

Using Archimedes principle, express the buoyant force on the balloon:

$$B = w_{\text{displaced fluid}} = m_{\text{displaced fluid}}\,g$$

or

$$B = \rho_{\text{o}} V_{\text{balloon}}\, g$$

where ρ_{o} is the density of the air outside the balloon.

Express the weight of the air inside the balloon:

$$w_{\text{air inside the balloon}} = \rho_{\text{i}} V_{\text{balloon}}\, g$$

where ρ_{i} is the density of the air inside the balloon.

Substitute in equation (1) for B and $w_{\text{air inside the balloon}}$ to obtain:

$$F_{\text{net}} = \rho_{\text{o}} V_{\text{balloon}}\, g - \rho_{\text{i}} V_{\text{balloon}}\, g = (\rho_{\text{o}} - \rho_{\text{i}}) V_{\text{balloon}}\, g \qquad (2)$$

Express the densities of the air molecules in terms of their number densities, molecular mass, and Avogadro’s number:

$$\rho = \frac{M}{N_{\text{A}}}\left(\frac{N}{V}\right)$$

Using the ideal-gas law, relate the number density of air N/V to its temperature and pressure:

$$PV = NkT \text{ and } \frac{N}{V} = \frac{P}{kT}$$

Substitute to obtain:

$$\rho = \frac{M}{N_{\text{A}}}\left(\frac{P}{kT}\right)$$

Substitute in equation (2) and simplify to obtain:

$$F_{\text{net}} = \frac{MP}{N_{\text{A}}k}\left(\frac{1}{T_{\text{o}}} - \frac{1}{T_{\text{i}}}\right) V_{\text{balloon}}\, g$$

Assuming that the average molecular weight of air is 29 g/mol, substitute numerical values and evaluate F_{net}:

$$F_{\text{net}}=\frac{(29\,\text{g/mol})(1.01\times10^{5}\,\text{N/m}^2)}{(6.022\times10^{23}\,\text{particles/mol})(1.381\times10^{-23}\,\text{J/K})}\left(\frac{1}{297\,\text{K}}-\frac{1}{348\,\text{K}}\right)$$
$$\times(1.5\,\text{m}^3)(9.81\,\text{m/s}^2)$$
$$=\boxed{2.56\,\text{N}}$$

54 •••

Picture the Problem We can find the number of moles of helium gas in the balloon by applying the ideal-gas law to relate *n* to the pressure, volume, and temperature of the helium and Archimedes principle to find the volume of the helium. In part (*b*), we can apply the result of Problem 13-95 to relate atmospheric pressure to altitude and use the ideal-gas law to determine the pressure of the gas when the balloon is fully inflated. In part (c), we'll find the net force acting on the balloon at the altitude at which it is fully inflated in order to decide whether it can rise to that altitude.

(*a*) Apply the ideal-gas law to the helium in the balloon and solve for *n*:

$$n=\frac{PV}{RT}\qquad(1)$$

Relate the net force on the balloon to its weight:

$$F_{\text{B}}-w_{\text{skin}}-w_{\text{load}}-w_{\text{He}}=30\,\text{N}$$

Use Archimedes principle to express the buoyant force on the balloon in terms of the volume of the balloon:

$$F_{\text{B}}=w_{\text{displaced air}}$$
$$=\rho_{\text{air}}Vg$$

Substitute to obtain:

$$\rho_{\text{air}}Vg-w_{\text{skin}}-w_{\text{load}}-\rho_{\text{He}}Vg=30\,\text{N}$$

Solve for the volume of the helium:

$$V=\frac{30\,\text{N}+w_{\text{skin}}+w_{\text{load}}}{(\rho_{\text{air}}-\rho_{\text{He}})g}$$

Substitute numerical values and evaluate *V*:

$$V=\frac{30\,\text{N}+50\,\text{N}+110\,\text{N}}{(1.293\,\text{kg/m}^3-0.179\,\text{kg/m}^3)}$$
$$\times\frac{1}{(9.81\,\text{m/s}^2)}$$
$$=17.39\,\text{m}^3$$

Substitute numerical values in equation (1) and evaluate n:

$$n=\frac{(1\,\text{atm})(17.39\,\text{m}^3)\left(\dfrac{1\,\text{L}}{10^{-3}\,\text{m}^3}\right)}{(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})(273\,\text{K})}=\boxed{776\,\text{mol}}$$

(b) Using the result of Problem 13-95, express the variation in atmospheric pressure with altitude:

$$P(h)=P_0e^{-h/h_0}$$

where h_0 = 7.93 km

Solve for h:

$$h=h_0\ln\left[\frac{P_0}{P(h)}\right] \qquad (2)$$

Neglecting changes in temperature with elevation, apply the ideal-gas law to find the pressure at which the balloon's volume will be 32 m^3:

$$P=\frac{nRT}{V}=\frac{(776\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})(273\,\text{K})}{32\,\text{m}^3\times\dfrac{1\,\text{L}}{10^{-3}\,\text{m}^3}}=0.543\,\text{atm}$$

Substitute in equation (2) and evaluate h:

$$h=(7.93\,\text{km})\ln\left(\frac{1\,\text{atm}}{0.543\,\text{atm}}\right)=\boxed{4.84\,\text{km}}$$

(c) Express the condition that must be satisfied if the balloon is to reach its fully inflated altitude:

$$F_{\text{net}}=F_{\text{B}}-w_{\text{tot}}\geq0 \qquad (3)$$

Express w_{tot}:

$$\begin{aligned}w_{\text{tot}}&=w_{\text{load}}+w_{\text{skin}}+w_{\text{He}}\\&=110\,N+50\,N+w_{\text{He}}\\&=160\,\text{N}+w_{\text{He}}\end{aligned}$$

Express the weight of the helium:

$$w_{\text{He}}=\rho_{\text{He}}Vg$$

Substitute for w_{He} and evaluate w_{tot}:

$$\begin{aligned}w_{\text{tot}}&=160\,\text{N}+\rho_{\text{He}}Vg\\&=160\,\text{N}+(0.179\,\text{kg/m}^3)(17.38\,\text{m}^3)\\&\quad\times(9.81\,\text{m/s}^2)\\&=190.5\,\text{N}\end{aligned}$$

Determine the buoyant force on the

$$F_{\text{B}}=\rho_{\text{air},h}Vg \qquad (4)$$

balloon at h = 4.84 km:

Express the dependence of the density of the air on atmospheric pressure:

$$\frac{P}{P_0}=\frac{\rho_{\text{air},h}}{\rho_{\text{air}}} \qquad (5)$$

or

$$\rho_{\text{air},h}=\frac{P}{P_0}\rho_{\text{air}}$$

Substitute and evaluate F_B:

$$\begin{aligned}F_\text{B}&=\frac{P}{P_0}\rho_{\text{air}}Vg\\&=0.543\left(1.293\,\text{kg/m}^3\right)\left(32\,\text{m}^3\right)\\&\qquad\times\left(9.81\,\text{m/s}^2\right)\\&=219.9\,\text{N}\end{aligned}$$

Substitute in equation (3) and evaluate F_net:

$$F_\text{net}=219.9\,\text{N}-190.5\,\text{N}=29.4\,\text{N}\geq 0$$

$$\boxed{\text{Because } F_\text{net}>0\text{, the balloon will rise higher than the altitude at which it is fully inflated.}}$$

(*d*) The balloon will rise until the net force acting on it is zero. Because the buoyant force depends on the density of the air, the balloon will rise until the density of the air has decreased sufficiently for the buoyant force to just equal the total weight of the balloon.

Substitute equation (5) in equation (2) to obtain:

$$h=h_0\ln\frac{\rho_{\text{air}}}{\rho_{\text{air},h}}$$

Using equation (4), find the density of the air such that F_B = 190.5 N:

$$\begin{aligned}\rho_{\text{air},h}&=\frac{F_\text{B}}{Vg}=\frac{190.5\,\text{N}}{\left(32\,\text{m}^3\right)\left(9.81\,\text{m/s}^2\right)}\\&=0.6068\,\text{kg/m}^3\end{aligned}$$

Substitute numerical values and evaluate h:

$$\begin{aligned}h&=\left(7.93\,\text{km}\right)\ln\left(\frac{1.293\,\text{kg/m}^3}{0.6068\,\text{kg/m}^3}\right)\\&=\boxed{6.00\,\text{km}}\end{aligned}$$

Kinetic Theory of Gases

***55** •

Picture the Problem We can express the rms speeds of argon and helium atoms by combining $PV = nRT$ and $v_{\text{rms}} = \sqrt{3RT/M}$ to obtain an expression for v_{rms} in terms of P, V, and M.

Express the rms speed of an atom as a function of the temperature:

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

From the ideal-gas law we have:

$$RT = \frac{PV}{n}$$

Substitute to obtain:

$$v_{\text{rms}} = \sqrt{\frac{3PV}{nM}}$$

(*a*) Substitute numerical values and evaluate v_{rms} for an argon atom:

$$v_{\text{rms}}(\text{Ar}) = \sqrt{\frac{3(10\,\text{atm})(101.3\,\text{kPa/atm})(10^{-3}\,\text{m}^3)}{(1\,\text{mol})(40\times10^{-3}\,\text{kg/mol})}} = \boxed{276\,\text{m/s}}$$

(*b*) Substitute numerical values and evaluate v_{rms} for a helium atom:

$$v_{\text{rms}}(\text{He}) = \sqrt{\frac{3(10\,\text{atm})(101.3\,\text{kPa/atm})(10^{-3}\,\text{m}^3)}{(1\,\text{mol})(4\times10^{-3}\,\text{kg/mol})}} = \boxed{872\,\text{m/s}}$$

56 •

Picture the Problem We can express the total translational kinetic energy of the oxygen gas by combining $K = \frac{3}{2}nRT$ and the ideal-gas law to obtain an expression for K in terms of the pressure and volume of the gas.

Relate the total translational kinetic energy of translation to the temperature of the gas:

$$K = \tfrac{3}{2}nRT$$

Using the ideal-gas law, substitute for nRT to obtain:

$$K = \tfrac{3}{2}PV$$

Substitute numerical values and evaluate K:

$$K = \tfrac{3}{2}(101.3\,\text{kPa})(10^{-3}\,\text{m}^3) = \boxed{152\,\text{J}}$$

57 •

Picture the Problem Because we're given the temperature of the hydrogen atom and know its molar mass, we can find its rms speed using $v_{\text{rms}} = \sqrt{3RT/M}$ and its average kinetic energy from $K_{\text{av}} = \frac{3}{2}kT$.

Relate the rms speed of a hydrogen atom to its temperature and molar mass:

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

Substitute numerical values and evaluate v_{rms}:

$$v_{\text{rms}} = \sqrt{\frac{3(8.314\,\text{J/mol}\cdot\text{K})(10^7\,\text{K})}{10^{-3}\,\text{kg/mol}}} = \boxed{499\,\text{km/s}}$$

Express the average kinetic energy of the hydrogen atom as a function of its temperature:

$$K_{\text{av}} = \tfrac{3}{2}kT$$

Substitute numerical values and evaluate K_{av}:

$$K_{\text{av}} = \tfrac{3}{2}(1.381\times10^{-23}\,\text{J/K})(10^7\,\text{K}) = \boxed{2.07\times10^{-16}\,\text{J}}$$

***58** •

Picture the Problem Because there are 6 squared terms in the expression for the total energy of an atom in this model, we can conclude that there are 6 degrees of freedom. Because the system is in equilibrium, we can conclude that there is energy of $\frac{1}{2}kT$ per molecule or $\frac{1}{2}RT$ per mole associated with each degree of freedom.

Express the average energy per atom in the solid in terms of its temperature and the number of degrees of freedom:

$$\frac{E_{\text{av}}}{\text{atom}} = N\left(\tfrac{1}{2}kT\right) = 6\left(\tfrac{1}{2}kT\right) = \boxed{3kT}$$

Relate the total energy of one mole to its temperature and the number of degrees of freedom:

$$\frac{E_{\text{tot}}}{\text{mole}} = N\left(\tfrac{1}{2}RT\right) = 6\left(\tfrac{1}{2}RT\right) = \boxed{3RT}$$

59 •

Picture the Problem We can combine $\lambda = \dfrac{1}{\sqrt{2}n_v\pi d^2}$ and $PV = nRT$ to express the mean free path for a molecule in an ideal gas in terms of the pressure and temperature.

Express the mean free path of a molecule in an ideal gas:

$$\lambda = \frac{1}{\sqrt{2}n_v \pi d^2}$$

where

$$n_v = N/V = nN_A/V$$

Solve the ideal-gas law for the volume of the gas:

$$V = \frac{nRT}{P}$$

Substitute in our expression for n_v to obtain:

$$n_v = \frac{nN_A}{nRT}P = \frac{P}{kT}$$

Substitute in the expression for the mean free path to obtain:

$$\lambda = \boxed{\frac{kT}{\sqrt{2}P\pi d^2}}$$

60 ••

Picture the Problem We can find the collision time from the mean free path and the average (rms) speed of the helium molecules. We can use the result of Problem 43 to find the mean free path of the molecules and $v_{\text{rms}} = \sqrt{3RT/M}$ to find the average speed of the molecules.

Express the collision time in terms of the mean free path for and the average speed of a helium molecule:

$$\tau = \frac{\lambda}{v_{\text{av}}} \qquad (1)$$

Use the result of Problem 43 to express the mean free path of the gas:

$$\lambda = \frac{kT}{\sqrt{2}P\pi d^2}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{(1.381\times10^{-23}\text{ J/K})(300\text{ K})}{\sqrt{2}(7\times10^{-11}\text{ Pa})\pi(10^{-10}\text{ m})^2} = 1.332\times10^{9}\text{ m}$$

Express the average speed of the molecules:

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

Substitute numerical values and evaluate v_{rms}:

$$v_{\text{rms}} = \sqrt{\frac{3(8.314\text{ J/mol}\cdot\text{K})(300\text{ K})}{4\times10^{-3}\text{ kg/mol}}} = 1.368\times10^{3}\text{ m/s}$$

Substitute in equation (1) and evaluate τ:

$$\tau = \frac{1.332\times10^{9}\ \text{m}}{1.368\times10^{3}\ \text{m/s}} = \boxed{9.74\times10^{5}\ \text{s}}$$

***61 ••**

Picture the Problem We can use $K=\frac{3}{2}kT$ and $\Delta U = mgh = Mgh/N_\text{A}$ to express the ratio of the average kinetic energy of a molecule of the gas to the change in its gravitational potential energy if it falls from the top of the container to the bottom.

Express the average kinetic energy of a molecule of the gas as a function of its temperature:

$$K=\tfrac{3}{2}kT$$

Letting *h* represent the height of the container, express the change in the potential energy of a molecule as it falls from the top of the container to the bottom:

$$\Delta U = mgh = \frac{Mgh}{N_\text{A}}$$

Express the ratio of *K* to Δ*U* and simplify to obtain:

$$\frac{K}{\Delta U} = \frac{\frac{3}{2}kT}{\frac{Mgh}{N_\text{A}}} = \frac{3N_\text{A}kT}{2Mgh}$$

Substitute numerical values and evaluate $K/\Delta U$:

$$\frac{K}{\Delta U} = \frac{3\left(6.022\times10^{23}\right)\left(1.381\times10^{-23}\ \text{J/K}\right)\left(300\ \text{K}\right)}{2\left(32\times10^{-3}\ \text{kg}\right)\left(9.81\ \text{m/s}^2\right)\left(0.15\ \text{m}\right)} = \boxed{7.95\times10^{4}}$$

The Distribution of Molecular Speeds

62 ••

Picture the Problem Equation 17-37 gives the Maxwell-Boltzmann speed distribution. Setting its derivative with respect to *v* equal to zero will tell us where the function’s extreme values lie.

Differentiate Equation 17-37 with respect to *v*:

$$\begin{aligned}\frac{df}{dv} &= \frac{d}{dv}\left[\frac{4}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{3/2} v^2 e^{-mv^2/2kT}\right]\\ &= \frac{4}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{3/2}\left(2v-\frac{mv^3}{kT}\right)e^{-mv^2/2kT}\end{aligned}$$

Set $df/dv = 0$ for extrema and solve for v:

$$2v - \frac{mv^3}{kT} = 0 \Rightarrow v = \boxed{\sqrt{\frac{2kT}{m}}}$$

Examination of the graph of $f(v)$ makes it clear that this extreme value is, in fact, a maximum. See Figure 17-16 and note that it is concave downward at $v = \sqrt{2kT/m}$.

Remarks: An alternative to the examination of $f(v)$ in order to conclude that $v = \sqrt{2kT/m}$ maximizes the Maxwell-Boltzmann speed distribution function is to show that $d^2f/dv^2 < 0$ at $v = \sqrt{2kT/m}$.

***63 ••**

Picture the Problem We can show that $f(v)$ is normalized by using the given integral to integrate it over all possible speeds.

Express the integral of Equation 17-37:

$$\int_0^\infty f(v)dv = \frac{4}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{3/2}\int_0^\infty v^2 e^{-mv^2/2kT}dv$$

Let $a = m/2kT$ to obtain:

$$\int_0^\infty f(v)dv = \frac{4}{\sqrt{\pi}}a^{3/2}\int_0^\infty v^2 e^{-av^2}dv$$

Use the given integral to obtain:

$$\int_0^\infty f(v)dv = \frac{4}{\sqrt{\pi}}a^{3/2}\left(\frac{\sqrt{\pi}}{4}a^{-3/2}\right) = \boxed{1}$$

i.e., $f(v)$ is normalized.

64 ••

Picture the Problem In Problem 63 we showed that $f(v)$ is normalized. Hence we can evaluate v_{av} using $\int_0^\infty vf(v)dv$.

Express the average speed of the molecules in the gas:

$$v_{\text{av}} = \int_0^\infty vf(v)dv$$

$$= \frac{4}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{3/2}\int_0^\infty v^3 e^{-mv^2/2kT}dv$$

Substitute $a = m/2kT$:

$$v_{\text{av}} = \frac{4}{\sqrt{\pi}} a^{3/2} \int_0^\infty v^3 e^{-av^2}\,dv$$

Use the given integral to obtain:

$$v_{\text{av}} = \frac{4}{\sqrt{\pi}} a^{3/2}\left(\frac{a^{-2}}{2}\right) = \frac{2}{\sqrt{\pi}}\frac{1}{\sqrt{a}}$$

$$= \boxed{\frac{2}{\sqrt{\pi}}\sqrt{\frac{2kT}{m}}}$$

***65 ••**

Picture the Problem Choose a coordinate system in which downward is the positive direction. We can use a constant-acceleration equation to relate the fall distance to the initial velocity of the molecule, the acceleration due to gravity, the fall time, and $v_{\text{rms}} = \sqrt{3kT/m}$ to find the initial velocity of the molecules.

(*a*) Using a constant-acceleration equation, relate the fall distance to the initial velocity of a molecule, the acceleration due to gravity, and the fall time:

$$y = v_0 t + \tfrac{1}{2} g t^2 \qquad (1)$$

Express the rms speed of the atom to its temperature and mass:

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

Substitute numerical values and evaluate v_{rms}:

$$v_{\text{rms}} = \sqrt{\frac{3\left(1.381\times10^{-23}\ \text{J/K}\right)\left(120\,\text{nK}\right)}{\left(85.47\,\text{u}\right)\left(1.660\times10^{-27}\ \text{kg/u}\right)}}$$

$$= 5.92\times10^{-3}\ \text{m/s}$$

Letting $v_{\text{rms}} = v_0$, substitute in equation (1) to obtain:

$$0.1\text{m} = \left(5.92\times10^{-3}\ \text{m/s}\right)t + \tfrac{1}{2}\left(9.81\text{m/s}^2\right)t^2$$

Solve this equation to obtain:

$$t = \boxed{0.142\,\text{s}}$$

(*b*) If the atom is initially moving upward:

$$v_{\text{rms}} = v_0 = -5.92\times10^{-3}\ \text{m/s}$$

Substitute in equation (1) to obtain:

$$0.1\text{m} = \left(-5.92\times10^{-3}\ \text{m/s}\right)t + \tfrac{1}{2}\left(9.81\text{m/s}^2\right)t^2$$

Solve this equation to obtain:

$$t = \boxed{0.143\,\text{s}}$$

General Problems

66 •

Picture the Problem We can use $v_{\text{rms}} = \sqrt{3RT/M}$ to relate the temperature of the H_2 molecule to its rms speed.

Relate the rms speed of the molecule to its temperature:

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

Solve for the temperature:

$$T = \frac{Mv_{\text{rms}}^2}{3R}$$

Substitute numerical values and evaluate *T*:

$$T = \frac{\left(2\times10^{-3}\,\text{kg/mol}\right)\left(331\,\text{m/s}\right)^2}{3\left(8.314\,\text{J/mol}\cdot\text{K}\right)} = \boxed{8.79\,\text{K}}$$

67 ••

Picture the Problem We can use the ideal-gas law to find the initial temperature of the gas and the ideal-gas law for a fixed amount of gas to relate the volumes, pressures, and temperatures resulting from the given processes.

(*a*) Apply the ideal-gas law to express the temperature of the gas:

$$T = \frac{PV}{nR}$$

Substitute numerical values and evaluate *T*:

$$T = \frac{\left(101.3\,\text{kPa}\right)\left(10\times10^{-3}\,\text{m}^3\right)}{\left(1\,\text{mol}\right)\left(8.314\,\text{J/mol}\cdot\text{K}\right)} = \boxed{122\,\text{K}}$$

(*b*) Use the ideal-gas law for a fixed amount of gas to relate the temperatures and volumes:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

or, because $P_1 = P_2$,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Solve for and evaluate T_2:

$$T_2 = \frac{V_2}{V_1}T_1 = 2\left(122\,\text{K}\right) = \boxed{244\,\text{K}}$$

(*c*) Use the ideal-gas law for a fixed amount of gas to relate the

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

temperatures and pressures:

or, because $V_1 = V_2$,

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Solve for T_2:

$$P_2 = \frac{T_2}{T_1}P_1$$

Substitute numerical values and evaluate P_2:

$$P_2 = \frac{350\,\text{K}}{244\,\text{K}}(1\,\text{atm}) = \boxed{1.43\,\text{atm}}$$

68 ••

Picture the Problem We can use the definition of pressure to express the net force on each wall of the box in terms of its area and the pressure differential between the inside and the outside of the box. We can apply the ideal-gas law for a fixed amount of gas to find the pressure inside the box.

Using the definition of pressure, express the net force on each wall of the box:

$$F = A\Delta P$$
$$= A\left(P_{\text{inside}} - P_{\text{outside}}\right)$$

Use the ideal-gas law for a fixed amount of gas to relate the initial and final pressures of the gas:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

or, because $V_1 = V_2$,

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Solve for and evaluate P_{inside}:

$$P_2 = P_{\text{inside}} = \frac{T_2}{T_1}P_1 = \frac{400\,\text{K}}{300\,\text{K}}(101.3\,\text{kPa})$$
$$= 135.1\,\text{kPa}$$

Substitute and evaluate F:

$$F = (0.2\,\text{m})^2(135.1\,\text{kPa} - 101.3\,\text{kPa})$$
$$= \boxed{1.35\,\text{kN}}$$

***69** ••

Picture the Problem We can use the molar mass of water to find the number of moles in 2 L of water. Because there are two hydrogen atoms in each molecule of water, there must be as many hydrogen molecules in the gas formed by electrolysis as there were molecules of water and, because there is one oxygen atom in each molecule of water, there must be half as many oxygen molecules in the gas formed by electrolysis as there were molecules of water.

Express the electrolysis of water into H_2 and O_2:

$$n(\mathrm{H_2O}) \rightarrow n(\mathrm{H_2}) + \tfrac{1}{2}n(\mathrm{O_2})$$

Express the number of moles in 2 L of water:

$$n(\mathrm{H_2O}) = \frac{2000\,\mathrm{g}}{18\,\mathrm{g/mol}} = 111\,\mathrm{mol}$$

Because there is one hydrogen atom for each water molecule:

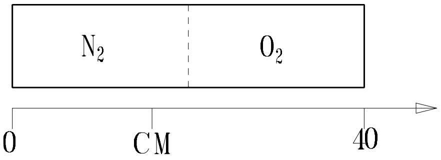
$$n(\mathrm{H_2}) = \boxed{111\,\mathrm{mol}}$$

Because there are two oxygen atoms for each water molecule:

$$n(\mathrm{O_2}) = \tfrac{1}{2}n(\mathrm{H_2O}) = \tfrac{1}{2}(111\,\mathrm{mol}) = \boxed{55.5\,\mathrm{mol}}$$

70 ••

Picture the Problem The diagram shows the cylinder before removal of the membrane. We'll assume that the gases are at the same temperature. The approximate location of the center of mass (CM) is indicated. We can find the distance the cylinder moves by finding the location of the CM after the membrane is removed.



Express the distance the cylinder will move in terms of the movement of the center of mass when the membrane is removed:

$$\Delta x = x_{\mathrm{cm,after}} - x_{\mathrm{cm,before}}$$

Apply the ideal-gas law to both collections of molecules to obtain:

$$P_{\mathrm{N_2}}V_{\mathrm{N_2}} = n(\mathrm{N_2})kT$$

and

$$P_{\mathrm{O_2}}V_{\mathrm{O_2}} = n(\mathrm{O_2})kT$$

Divide the first of these equations by the second to obtain:

$$\frac{P_{\mathrm{N_2}}}{P_{\mathrm{O_2}}} = \frac{n(\mathrm{N_2})}{n(\mathrm{O_2})}$$

or, because $P_{\mathrm{N_2}} = 2P_{\mathrm{O_2}}$,

$$\frac{2P_{\mathrm{O_2}}}{P_{\mathrm{O_2}}} = \frac{n(\mathrm{N_2})}{n(\mathrm{O_2})} \Rightarrow n(\mathrm{N_2}) = 2n(\mathrm{O_2})$$

Express the mass of O_2 in terms of its molar mass and the number of moles of oxygen:

$$m(\mathrm{O_2}) = n(\mathrm{O_2})M(\mathrm{O_2})$$

Express the mass of N_2 in terms of its molar mass and the number of moles of nitrogen:

$$m(N_2) = 2n(O_2)M(N_2).$$

Using its definition, express the center of mass before the membrane is removed:

$$\begin{aligned}x_{\text{cm,before}} &= \frac{\sum_i x_i m_i}{\sum_i m_i} = \frac{n(N_2)M(N_2)x_{\text{cm},N_2} + n(O_2)M(O_2)x_{\text{cm},O_2}}{n(N_2)M(N_2) + n(O_2)M(O_2)} \\ &= \frac{2n(O_2)M(N_2)x_{\text{cm},N_2} + n(O_2)M(O_2)x_{\text{cm},O_2}}{2n(O_2)M(N_2) + n(O_2)M(O_2)} \\ &= \frac{2M(N_2)x_{\text{cm},N_2} + M(O_2)x_{\text{cm},O_2}}{2M(N_2) + M(O_2)}\end{aligned}$$

Substitute numerical values and evaluate $x_{\text{cm,before}}$:

$$x_{\text{cm,before}} = \frac{2(10\,\text{cm})(28\,\text{g}) + (30\,\text{cm})(32\,\text{g})}{2(28\,\text{g}) + 32\,\text{g}} = 17.27\,\text{cm}$$

Locate the center of mass after the membrane is removed:

$$\begin{aligned}x_{\text{cm,after}} &= \frac{2(20\,\text{cm})(28\,\text{g}) + (20\,\text{cm})(32\,\text{g})}{2(28\,\text{g}) + 32\,\text{g}} \\ &= 20.0\,\text{cm}\end{aligned}$$

Substitute to obtain:

$$\begin{aligned}\Delta x &= 20.00\,\text{cm} - 17.27\,\text{cm} \\ &= \boxed{2.73\,\text{cm}}\end{aligned}$$

$$\boxed{\text{Because momentum must be conserved during this process and the center of mass moved to the right, the cylinder moved 2.73 cm to the left.}}$$

71 ••

Picture the Problem We can apply the ideal-gas law to the two processes to find the number of moles of hydrogen in terms of the number of moles of nitrogen in the gas. Using the definition of molar mass, we can relate the mass of each gas to the number of moles of each gas and their molar masses.

Apply the ideal-gas law to the first case:

$$P_1V = [2n(N_2) + n(H_2)]RT_1$$

Apply the ideal-gas law to the

$$3P_1V = [2n(N_2) + 2n(H_2)]2RT_1$$

second case:

Divide the second of these equations by the first and simplify to express $n(H_2)$ in terms of $n(N_2)$:

$$n(H_2) = 2n(N_2) \qquad (1)$$

Relate the m_N to $n(N_2)$:

$$m_N = n(N_2)M(N_2) = n(N_2)(28\,\text{g/mol})$$

and

$$n(N_2) = \frac{m_N}{28\,\text{g/mol}}$$

Relate the m_H to $n(H_2)$:

$$m_H = n(H_2)M(H_2) = n(H_2)(2\,\text{g/mol})$$

and

$$n(H_2) = \frac{m_H}{2\,\text{g/mol}}$$

Substitute in equation (1) and solve for m_N:

$$\frac{m_H}{2\,\text{g/mol}} = \frac{2m_N}{28\,\text{g/mol}} \Rightarrow m_N = \boxed{7m_H}$$

***72** ••

Picture the Problem Initially, we have $3P_0V = n_0RT_0$. Later, the pressures in the three vessels, each of volume V, are still equal, but the number of moles is not. The total number of moles, however, is constant and equal to the number of moles in the three vessels initially. Applying the ideal-gas law to each of the vessels will allow us to relate the number of moles in each to the final pressure and temperature. Equating this sum n_0 will leave us with an equation in P' and P_0 that we can solve for P'.

Relate the number of moles of gas in the system in the three vessels initially to the number in each vessel when the pressure is P':

$$n_0 = n_1 + n_2 + n_3$$

Relate the final pressure in the first vessel to its temperature and solve for n_1:

$$P' = \frac{n_1R(2T_0)}{V} \Rightarrow n_1 = \frac{P'V}{2RT_0}$$

Relate the final pressure in the second vessel to its temperature and solve for n_2:

$$P' = \frac{n_2R(3T_0)}{V} \Rightarrow n_2 = \frac{P'V}{3RT_0}$$

Relate the final pressure in the third vessel to its temperature and solve for n_3:

$$P' = \frac{n_3RT_0}{V} \Rightarrow n_3 = \frac{P'V}{RT_0}$$

Substitute to obtain:

$$n_0 = \frac{P'V}{2RT_0} + \frac{P'V}{3RT_0} + \frac{P'V}{RT_0}$$

$$= \left(\frac{1}{2} + \frac{1}{3} + 1\right)\frac{P'V}{RT_0} = \frac{11}{6}\left(\frac{P'V}{RT_0}\right)$$

Express the number of moles in the three vessels initially in terms of the initial pressure and total volume:

$$n_0 = \frac{P_0(3V)}{RT_0}$$

Equate the two expressions for n_0 and solve for P' to obtain:

$$P' = \boxed{\frac{18}{11}P_0}$$

73 ••

Picture the Problem We can use the ideal-gas temperature scale to relate the temperature of the boiling substance to its pressure and the pressure at the triple point. If we assume a linear relationship between P/P_3 and P_3, we can calibrate this equation using the data from any two (or all) of the temperature measurements and then extrapolate this equation to zero gas pressure to find the ideal-gas temperature of the boiling substance.

Using the ideal-gas temperature scale, relate the temperature of the boiling substance to its pressure and the pressure at the triple point:

$$T = 273.16\,\text{K}\left(\frac{P}{P_3}\right) \qquad (1)$$

Find the temperature of the first measurement:

$$T_1 = 273.16\,\text{K}\left(\frac{734\,\text{torr}}{500\,\text{torr}}\right)$$

$$= 273.16\,\text{K}(1.4680)$$

$$= 401.00\,\text{K}$$

Find the temperature of the third measurement:

$$T_3 = 273.16\,\text{K}\left(\frac{146.65\,\text{torr}}{100\,\text{torr}}\right)$$

$$= 273.16\,\text{K}(1.4655)$$

$$= 400.59\,\text{K}$$

Assume a linear relationship between P/P_3 and P_3:

$$\frac{P}{P_3} = a + bP_3$$

where a is the pressure ratio for $P_3 = 0$.

Substitute using the data from the first measurement:

$$\frac{734\,\text{torr}}{500\,\text{torr}} = a + b(500\,\text{torr})$$

or

$$1.4680 = a + b(500\,\text{torr})$$

Substitute using the data from the third measurement:

$$\frac{146.65\,\text{torr}}{100\,\text{torr}} = a + b(100\,\text{torr})$$

or

$$1.4665 = a + b(100\,\text{torr})$$

Solve these equations simultaneously for a:

$$a = 1.46613$$

Substitute in equation (1) to obtain:

$$T = 273.16\,\text{K}(1.46613) = \boxed{400.49\,\text{K}}$$

***74 ••**

Picture the Problem Because the O_2 molecule resembles 2 spheres stuck together, which in cross section look something like two circles, we can estimate the radius of the molecule from the formula for the area of a circle. We can express the area, and hence the radius, of the circle in terms of the mean free path and the number density of the molecules and use the ideal-gas law to express the number density.

Express the area of two circles of diameter d that touch each other:

$$A = 2\left(\frac{\pi d^2}{4}\right) = \frac{\pi d^2}{2}$$

Solve for d to obtain:

$$d = \sqrt{\frac{2A}{\pi}} \qquad (1)$$

Relate the mean free path of the molecules to their number density and cross-sectional area:

$$\lambda = \frac{1}{n_v A}$$

Solve for A to obtain:

$$A = \frac{1}{n_v \lambda}$$

Substitute in equation (1) to obtain:

$$d = \sqrt{\frac{2}{\pi n_v \lambda}}$$

Use the ideal-gas law to relate the number density of the O_2 molecules to their temperature and pressure:

$$PV = NkT \text{ or } n_v = \frac{N}{V} = \frac{P}{kT}$$

Substitute to obtain:

$$d = \sqrt{\frac{2kT}{\pi P\lambda}}$$

Substitute numerical values and evaluate d:

$$d = \sqrt{\frac{2\left(1.381\times10^{-23}\text{ J/K}\right)\left(300\text{ K}\right)}{\pi\left(1.01\times10^{5}\text{ Pa}\right)\left(7.1\times10^{-8}\text{ m}\right)}}$$

$$= 6.06\times10^{-10}\text{ m} = \boxed{0.606\text{ nm}}$$

75 ••

Picture the Problem We can use its definition to express the mean free path of the molecules and the ideal-gas law to obtain an expression for the number density of the hydrogen gas molecules.

(*a*) Relate the mean free path of the molecules to their number density and cross-sectional area:

$$\lambda = \frac{1}{n_v A}$$

Use the ideal-gas law to relate the number density of the H_2 molecules to their temperature and pressure:

$$PV = NkT \text{ or } n_v = \frac{N}{V} = \frac{P}{kT}$$

Express the effective cross-sectional area of a H_2 molecule:

$$A = \tfrac{1}{4}\pi d^2$$

Substitute for n_v and A to obtain:

$$\lambda = \frac{4kT}{P\pi d^2}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{4\left(1.381\times10^{-23}\text{ J/K}\right)\left(300\text{ K}\right)}{\pi\left(1.01\times10^{5}\text{ N/m}^2\right)\left(1.6\times10^{-10}\text{ m}\right)^2}$$

$$= \boxed{2.04\times10^{-6}\text{ m}}$$

(*b*) Relate the available volume per molecule to the number density n_v:

$$\frac{V}{N} = \frac{1}{n_v} = \frac{kT}{P}$$

Substitute numerical values and evaluate V/N:

$$\frac{V}{N} = \frac{\left(1.381\times10^{-23}\text{ J/K}\right)\left(300\text{ K}\right)}{1.01\times10^{5}\text{ N/m}^2}$$

$$= \boxed{4.10\times10^{-26}\text{ m}^3}$$

Express the volume of a spherical molecule:

$$V = \tfrac{4}{3}\pi r^3 = \tfrac{1}{6}\pi d^3$$

Solve for *d*:

$$d=\sqrt[3]{\frac{6V}{\pi}}$$

Substitute numerical values and evaluate *d*:

$$d=\sqrt[3]{\frac{6\left(4.10\times10^{-26}\ \text{m}^3\right)}{\pi}}=\boxed{4.28\,\text{nm}}$$

$$\boxed{\begin{array}{l}\text{The mean free path is larger by}\\ \text{approximately a factor of 1000.}\end{array}}$$

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Picture the Problem Let *A* be the cross-sectional area of the cylinder. We can use the ideal-gas law to find the height of the piston under equilibrium conditions. In (*b*), we can apply Newton's 2nd law and the ideal-gas law for a fixed amount of gas to the show that, for small displacements from its equilibrium position, the piston executes simple harmonic motion.

(*a*) Express the pressure inside the cylinder:

$$P_{\text{in}}=P_{\text{atm}}+\frac{Mg}{A}$$

Apply the ideal-gas law to obtain a second expression for the pressure of the gas in the cylinder:

$$P_{\text{in}}=\frac{nRT}{V}=\frac{nRT}{hA}\qquad(1)$$

Equate these two expressions:

$$P_{\text{atm}}+\frac{Mg}{A}=\frac{nRT}{hA}$$

Solve for *h* to obtain:

$$h=\frac{nRT}{AP_{\text{atm}}+Mg}=\frac{(2.4\,\text{m})AP_{\text{atm}}}{AP_{\text{atm}}+Mg}=\frac{2.4\,\text{m}}{1+\frac{Mg}{AP_{\text{atm}}}}$$

At STP, 0.1 mol of gas occupies 2.24 L. Therefore:

$$(2.4\,\text{m})A=2.24\times10^{-3}\ \text{m}^3$$

and

$A=9.33\times10^{-4}\ \text{m}^2$

Substitute numerical values and evaluate *h*:

$$h=\frac{2.4\,\text{m}}{1+\frac{(1.4\,\text{kg})\left(9.81\,\text{m/s}^2\right)}{\left(9.333\times10^{-4}\ \text{m}^2\right)(101.3\,\text{kPa})}}=\boxed{2.096\,\text{m}}$$

(*b*) Relate the frequency of vibration of the piston to its mass and a ″stiffness″ constant:

$$f=\frac{1}{2\pi}\sqrt{\frac{k}{M}} \qquad (2)$$

where M is the mass of the piston and k is a constant of proportionality.

Letting y be the displacement from equilibrium, apply $\sum F_y = ma_y$ to the piston in its equilibrium position:

$$P_{\text{in}}A-mg-P_{\text{atm}}A=0$$

For a small displacement y above equilibrium:

$$P_{\text{in}}'A-mg-P_{\text{atm}}A=ma_y$$

or

$$P_{\text{in}}'A-P_{\text{in}}A=ma_y \qquad (3)$$

Using the ideal-gas law for a fixed amount of gas and constant temperature, relate P_{in}' to P_{in}:

$$P_{\text{in}}'V'=P_{\text{in}}V$$

or

$$P_{\text{in}}'(V+Ay)=P_{\text{in}}V$$

Solve for P_{in}':

$$P_{\text{in}}'=P_{\text{in}}\frac{V}{V+Ay}$$

and

$$P_{\text{in}}'A=P_{\text{in}}A\frac{Ah}{Ah+Ay}=P_{\text{in}}A\frac{1}{1+\frac{y}{h}}$$

Substitute in equation (3) to obtain:

$$P_{\text{in}}A\left(1+\frac{y}{h}\right)^{-1}-P_{\text{in}}A=ma_y$$

or, for $y << h$,

$$P_{\text{in}}A\left(1-\frac{y}{h}\right)-P_{\text{in}}A\approx ma_y \qquad (4)$$

Simplify equation (4):

$$-P_{\text{in}}A\frac{y}{h}\approx ma_y$$

Substitute in equation (1) to obtain:

$$-\left(\frac{nRT}{Ah}\right)A\frac{y}{h}\approx ma_y$$

or

$$-\left(\frac{nRT}{h^2}\right)y\approx ma_y$$

Solve for a_y:

$$a_y = -\frac{nRT}{mh^2}y$$

or

$$a_y = -\frac{k}{m}y$$, the condition for SHM

where $\frac{k}{m} = \frac{nRT}{mh^2}$

Substitute in equation (2) to obtain:

$$f = \frac{1}{2\pi}\sqrt{\frac{nRT}{mh^2}}$$

Substitute numerical values and evaluate f:

$$f = \frac{1}{2\pi}\sqrt{\frac{(0.1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(300\,\text{K})}{(1.4\,\text{kg})(2.096\,\text{m})^2}} = \boxed{1.01\,\text{Hz}}$$

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Picture the Problem We can show that $\int_0^V f(v)dv = I(x)$, where $f(v)$ is the Maxwell-Boltzmann distribution function, $x = mV^2/2kT$, and $I(x)$ is the integral whose values are tabulated in the problem statement. Then, we can use this table to find the value of x corresponding to the fraction of the gas molecules with speeds less than v by evaluating $I(x)$.

(*a*) The Maxwell-Boltzmann speed distribution $f(x)$ is given by:

$$f(v) = \frac{4}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{3/2} v^2 e^{-mv^2/2kT}$$

which means that the fraction of particles with speeds between v and $v + dv$ is $f(v)dv$.

Express the fraction $F(V)$ of particles with speeds less than V = 400 m/s:

$$F(V) = \int_0^V f(v)dv$$

$$= \frac{4}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{3/2}\int_0^V v^2 e^{-mv^2/2kT}dv$$

Change integration variables by letting $z = v\sqrt{m/2kT}$ so we can use the table of values to evaluate the integral. Then:

$$v = \sqrt{\frac{2kT}{m}}z \Rightarrow dv = \sqrt{\frac{2kT}{m}}dz$$

Substitute in the integrand of $F(V)$ to obtain:

$$v^2 e^{-mv^2/2kT}dv = z^2\,\frac{2kT}{m}e^{-z^2}\left(\frac{2kT}{m}\right)^{1/2}dz$$

$$= \left(\frac{2kT}{m}\right)^{3/2} z^2 e^{-z^2}dz$$

Transform the integration limits to correspond to the new integration variable $z = v\sqrt{m/2kT}$:

When $v = 0$, $z = 0$,
and
when $v = V$, $z = V\sqrt{m/2kT}$

The new lower integration limit is 0. Evaluate $z = V\sqrt{m/2kT}$ to find the upper limit:

$$z = (400\,\text{m/s})\sqrt{\frac{(32\,\text{u})(1.661\times10^{-27}\,\text{kg})}{2(1.381\times10^{-23}\,\text{J/K})(273\,\text{K})}} = 1.06$$

Evaluate $F(400$ m/s$)$ to obtain:

$$F(400\,\text{m/s}) = \int_0^{400\,\text{m/s}} f(v)dv = \frac{4}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{3/2}\int_0^{400\,\text{m/s}} v^2 e^{-mv^2/2kT}dv = \frac{4}{\sqrt{\pi}}\int_0^{1.06} z^2 e^{-z^2}dz$$

$$= I(1.06)$$

where $I(x) = \frac{4}{\sqrt{\pi}}\int_0^x z^2 e^{-z^2}dz$

Letting r represent the fraction of the molecules with speeds less than 400 m/s, interpolate from the table to obtain:

$$\frac{r - 0.438}{1.06 - 1} = \frac{0.788 - 0.438}{1.5 - 1}$$

and

$$r = \boxed{48.0\%}$$

(*b*) Express the fraction r of the molecules with speeds between $V_1 = 190$ m/s and $V_2 = 565$ m/s:

$$r = F(V_2) - F(V_1) = I(x_2) - I(x_1)$$

where

$x_1 = V_1\sqrt{m/2kT}$ and $x_2 = V_2\sqrt{m/2kT}$

Evaluate x_1 and x_2 to obtain:

$$x_1 = (190\,\text{m/s})\sqrt{\frac{(32\,\text{u})(1.661\times10^{-27}\,\text{kg})}{2(1.381\times10^{-23}\,\text{J/K})(273\,\text{K})}} = 0.504$$

and

$$x_2 = (565\,\text{m/s})\sqrt{\frac{(32\,\text{u})(1.661\times10^{-27}\,\text{kg})}{2(1.381\times10^{-23}\,\text{J/K})(273\,\text{K})}} = 1.50$$

Substitute to obtain:

$$r = I(1.50) - I(0.504) \qquad (1)$$

Using the table, evaluate $I(1.50)$:

$$I(1.50) = 0.788$$

Letting r represent the fraction of the molecules with speeds less than 190 m/s, interpolate from the table to obtain:

$$\frac{r - 0.081}{0.504 - 0.5} = \frac{0.132 - 0.081}{0.6 - 0.5}$$

and

$$r = 0.083$$

Substitute in equation (1) to obtain:

$$r = 0.788 - 0.083 = \boxed{70.5\%}$$