Chapter 18
Heat and the First Law of Thermodynamics

Conceptual Problems

1 •

Picture the Problem We can use the relationship $Q = mc\Delta T$ to relate the temperature changes of bodies A and B to their masses, specific heats, and the amount of heat supplied to each.

Express the change in temperature of body A in terms of its mass, specific heat, and the amount of heat supplied to it:

$$\Delta T_{\text{A}} = \frac{Q}{m_{\text{A}} c_{\text{A}}}$$

Express the change in temperature of body B in terms of its mass, specific heat, and the amount of heat supplied to it:

$$\Delta T_{\text{B}} = \frac{Q}{m_{\text{B}} c_{\text{B}}}$$

Divide the second of these equations by the first to obtain:

$$\frac{\Delta T_{\text{B}}}{\Delta T_{\text{A}}} = \frac{m_{\text{A}} c_{\text{A}}}{m_{\text{B}} c_{\text{B}}}$$

Substitute and simplify to obtain:

$$\frac{\Delta T_{\text{B}}}{\Delta T_{\text{A}}} = \frac{(2m_{\text{B}})(2c_{\text{B}})}{m_{\text{B}} c_{\text{B}}} = 4$$

or

$$\Delta T_{\text{B}} = \boxed{4\Delta T_{\text{A}}}$$

***2** •

Picture the Problem We can use the relationship $Q = mc\Delta T$ to relate the temperature changes of bodies A and B to their masses, specific heats, and the amount of heat supplied to each.

Relate the temperature change of block A to its specific heat and mass:

$$\Delta T_{\text{A}} = \frac{Q}{M_{\text{A}} c_{\text{A}}}$$

Relate the temperature change of block B to its specific heat and mass:

$$\Delta T_{\text{B}} = \frac{Q}{M_{\text{B}} c_{\text{B}}}$$

Equate the temperature changes to obtain:

$$\frac{1}{M_{\text{B}}c_{\text{B}}}=\frac{1}{M_{\text{A}}c_{\text{A}}}$$

Solve for c_{A}:

$$c_{\text{A}}=\frac{M_{\text{B}}}{M_{\text{A}}}c_{\text{B}}$$

and $\boxed{(b)\text{ is correct.}}$

3 •

Picture the Problem We can use the relationship $Q = mc\Delta T$ to relate the amount of energy absorbed by the aluminum and copper bodies to their masses, specific heats, and temperature changes.

Express the energy absorbed by the aluminum object:

$$Q_{\text{Al}}=m_{\text{Al}}c_{\text{Al}}\Delta T$$

Express the energy absorbed by the copper object:

$$Q_{\text{Cu}}=m_{\text{Cu}}c_{\text{Cu}}\Delta T$$

Divide the second of these equations by the first to obtain:

$$\frac{Q_{\text{Cu}}}{Q_{\text{Al}}}=\frac{m_{\text{Cu}}c_{\text{Cu}}\Delta T}{m_{\text{Al}}c_{\text{Al}}\Delta T}$$

Because the object's masses are the same and they experience the same change in temperature:

$$\frac{Q_{\text{Cu}}}{Q_{\text{Al}}}=\frac{c_{\text{Cu}}}{c_{\text{Al}}}<1$$

or

$Q_{\text{Cu}}<Q_{\text{Al}}$ and $\boxed{(c)\text{ is correct.}}$

4 •

Determine the Concept Some examples of systems in which internal energy is converted into mechanical energy are: a steam turbine, an internal combustion engine, and a person performing mechanical work, e.g., climbing a hill.

***5** •

Determine the Concept Yes, if the heat absorbed by the system is equal to the work done by the system.

6 •

Determine the Concept According to the first law of thermodynamics, the change in the internal energy of the system is equal to the heat that enters the system plus the work done on the system. $\boxed{(a)\text{ is correct.}}$

7 •

Determine the Concept $\Delta E_{\text{int}} = Q_{\text{in}} + W_{\text{on}}$. For an ideal gas, ΔE_{int} is a function of T only. Because $W_{\text{on}} = 0$ and $Q_{\text{in}} = 0$ in a free expansion, $\Delta E_{\text{int}} = 0$ and T is constant. For a real gas, E_{int} depends on the density of the gas because the molecules exert weak attractive forces on each other. In a free expansion, these forces reduce the average kinetic energy of the molecules and, consequently, the temperature.

8 •

Determine the Concept Because the container is insulated, no energy is exchanged with the surroundings during the expansion of the gas. Neither is any work done on or by the gas during this process. Hence, the internal energy of the gas does not change and we can conclude that the equilibrium temperature will be the same as the initial temperature. Applying the ideal-gas law for a fixed amount of gas we see that the pressure at equilibrium must be half an atmosphere. $\boxed{(c)\text{ is correct.}}$

9 •

Determine the Concept The temperature of the gas increases. The average kinetic energy increases with increasing volume due to the repulsive interaction between the ions.

***10** ••

Determine the Concept The balloon that expands isothermally is larger when it reaches the surface. The balloon that expands adiabatically will be at a lower temperature than the one that expands isothermally. Because each balloon has the same number of gas molecules and are at the same pressure, the one with the higher temperature will be bigger. An analytical argument that leads to the same conclusion is shown below.

Letting the subscript ″a″ denote the adiabatic process and the subscript ″i″ denote the isothermal process, express the equation of state for the adiabatic balloon:

$$P_0V_0^{\gamma} = P_{\text{f}}V_{\text{f,a}}^{\gamma} \Rightarrow V_{\text{f,a}} = V_0\left(\frac{P_0}{P_{\text{f}}}\right)^{1/\gamma}$$

For the isothermal balloon:

$$P_0V_0 = P_{\text{f}}V_{\text{f,i}} \Rightarrow V_{\text{f,i}} = V_0\left(\frac{P_0}{P_{\text{f}}}\right)$$

Divide the second of these equations by the first and simplify to obtain:

$$\frac{V_{\text{f,i}}}{V_{\text{f,a}}} = \frac{V_0\left(\dfrac{P_0}{P_{\text{f}}}\right)}{V_0\left(\dfrac{P_0}{P_{\text{f}}}\right)^{1/\gamma}} = \left(\frac{P_0}{P_{\text{f}}}\right)^{1-1/\lambda}$$

Because $P_0/P_f > 1$ and $\gamma > 1$: $\boxed{V_{f,i} > V_{f,a}}$

11 •

Determine the Concept The work done along each of these paths equals the area under its curve. The area is greatest for the path A→B→C and least for the path A→D→C. $\boxed{(a)\text{ is correct.}}$

12 •

Determine the Concept An adiabatic process is, by definition, one for which no heat enters or leaves the system. $\boxed{(b)\text{ is correct.}}$

13 •

(*a*) False. The heat capacity of a body is the heat needed to raise the temperature of the body by one degree.

(*b*) False. The amount of heat added to a system when it goes from one state to another is path dependent.

(*c*) False. The work done on a system when it goes from one state to another is path dependent.

(*d*) True.

(*e*) True.

(*f*) False. A quasi-static process is one for which the gas is never far from an equilibrium state.

(*g*) True.

***14** •

Determine the Concept For a constant-volume process, no work is done on or by the gas. Applying the first law of thermodynamics, we obtain $Q_{in} = \Delta E_{int}$. Because the temperature must change during such a process, we can conclude that $\Delta E_{int} \neq 0$ and hence $Q_{in} \neq 0$. $\boxed{(b)\text{ and }(d)\text{ are correct.}}$

15 •

Determine the Concept Because the temperature does not change during an isothermal process, the change in the internal energy of the gas is zero. Applying the first law of thermodynamics, we obtain $Q_{\text{in}} = -W_{\text{on}} = W_{\text{by the system}}$. Hence $\boxed{(d)\text{ is correct.}}$

16 ••

Determine the Concept The melting point of propane at 1 atm pressure is 83 K. Hence, at this low temperature and high pressure, C_3H_8 is a solid.

17 ••

Picture the Problem We can use the given dependence of the pressure on the volume and the ideal-gas law to show that if the volume decreases, so does the temperature.

We're given that:

$$P\sqrt{V} = \text{constant}$$

Because the gas is an ideal gas:

$$PV = \left(P\sqrt{V}\right)\sqrt{V} = \text{constant}\sqrt{V} = nRT$$

Solve for T:

$$T = \frac{(\text{constant})\sqrt{V}}{nR}$$

$$\boxed{\begin{array}{l}\text{Because } T \text{ varies with the square root of } V\text{, if the volume decreases,}\\ \text{the temperature decreases.}\end{array}}$$

***18** ••

Determine the Concept At room temperature, most solids have a roughly constant heat capacity per *mole* of 6 cal/mol-K (Dulong-Petit law). Because 1 mole of lead is more massive than 1 mole of copper, the heat capacity of lead should be lower than the heat capacity of copper. This is, in fact, the case.

19 ••

Determine the Concept The heat capacity of a substance is proportional to the number of degrees of freedom per molecule associated with the molecule. Because there are 6 degrees of freedom per molecule in a solid and only 3 per molecule (translational) for a monatomic liquid, you would expect the solid to have the higher heat capacity.

Estimation and Approximation

***20** ••

Picture the Problem The heat capacity of lead is c = 128 J/kg·K. We'll assume that all of the work done in lifting the bag through a vertical distance of 1 m goes into raising the temperature of the lead shot and use conservation of energy to relate the number of drops of the bag and the distance through which it is dropped to the heat capacity and change in temperature of the lead shot.

(*a*) Use conservation of energy to relate the change in the potential energy of the lead shot to the change in its temperature:

$$Nmgh = mc\Delta T$$

where N is the number of times the bag of shot is dropped.

Solve for ΔT to obtain:

$$\Delta T = \frac{Nmgh}{mc} = \frac{Ngh}{c}$$

Substitute numerical values and evaluate ΔT:

$$\Delta T = \frac{50\left(9.81`\mathrm{m/s^2}\right)(1\,\mathrm{m})}{128\,\mathrm{J/kg\cdot K}} = \boxed{3.83\,\mathrm{K}}$$

(*b*) It is better to use a larger mass because the rate at which heat is lost by the lead shot is proportional to its surface area while the rate at which it gains heat is proportional to its mass. The amount of heat lost varies as the surface area of the shot divided by its mass ($\mathrm{L^2/L^3 = L^{-1}}$); which decreases as the mass increases.

21 ••

Picture the Problem Assume that the water is initially at 30°C and that the cup contains 200 g of water. We can use the definition of power to express the required time to bring the water to a boil in terms of its mass, heat capacity, change in temperature, and the rate at which energy is supplied to the water by the microwave oven.

Use the definition of power to relate the energy needed to warm the water to the elapsed time:

$$P = \frac{\Delta W}{\Delta t} = \frac{mc\Delta T}{\Delta t}$$

Solve for Δt to obtain:

$$\Delta t = \frac{mc\Delta T}{P}$$

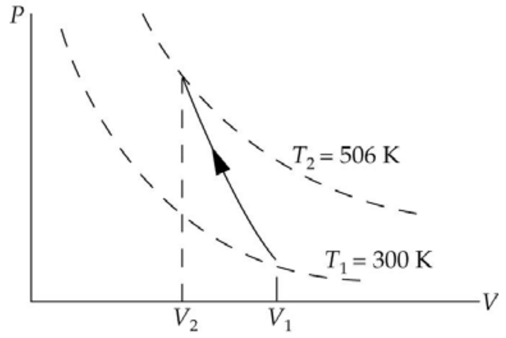
Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{(0.2\,\mathrm{kg})(4.18\,\mathrm{kJ/kg\cdot K})(373\,\mathrm{K} - 303\,\mathrm{K})}{600\,\mathrm{W}} = 97.5\,\mathrm{s} = \boxed{1.63\,\mathrm{min}}$$

, an elapsed time that seems to be consistent with experience.

22 •

Picture the Problem The adiabatic compression from an initial volume V_1 to a final volume V_2 between the isotherms at temperatures T_1 and T_2 is shown to the right. We'll assume a room temperature of 300 K and apply the equation for a quasi-static adiabatic process with $\gamma_{\text{air}} = 1.4$ to solve for the ratio of the initial to the final volume of the air.



Express $TV^{\gamma-1} = \text{constant}$ in terms of the initial and final values of T and V:

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

Solve for V_1/V_2 to obtain:

$$\frac{V_1}{V_2} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}$$

Substitute numerical values and evaluate V_1/V_2:

$$\frac{V_1}{V_2} = \left(\frac{506\,\text{K}}{300\,\text{K}}\right)^{\frac{1}{1.4-1}} = \boxed{3.69}$$

23 ••

Picture the Problem We can use $Q = mc_\text{p}\Delta T$ to express the specific heat of water during heating at constant pressure in terms of the required heat and the resulting change in temperature. Further, we can use the definition of the bulk modulus to express the work done by the water as it expands. Equating the work done by the water during its expansion and the heat gained during this process will allow us to solve for c_p.

Express the heat needed to raise the temperature of a mass m of a substance whose specific heat at constant pressure is c_p by ΔT:

$$Q = mc_\text{p}\Delta T$$

Solve for c_p to obtain:

$$c_\text{p} = \frac{Q}{m\Delta T}$$

Use the definition of the bulk modulus to express the work done by the water as it expands:

$$B = \frac{\Delta P}{\Delta V/V} = \frac{V\Delta P}{\Delta V}$$

or

$$W = V\Delta P = B\Delta V$$

Assuming that the work done by the water in expanding equals the heat gained during the process, substitute to obtain:

$$c_\text{p} = \frac{B\Delta V}{m\Delta T}$$

Using the definition of the coefficient of volume expansion, express ΔV (see Chapter 20, Section 1):

$$\Delta V = \beta V\Delta T$$

Substitute to obtain:

$$c_\text{p} = \frac{B\beta V\Delta T}{m\Delta T} = \frac{B\beta V}{m}$$

Use the data given in the problem statement to find the average volume of 1 kg of water as it warms from 4°C to 100°C:

$$V = \frac{m}{\rho} = \frac{1\,\text{kg}}{\dfrac{1.0000\,\text{g/cm}^3 + 0.9584\,\text{g/cm}^3}{2}} = 1.02\times10^{-3}\,\text{m}^3$$

Substitute numerical values and evaluate c_p:

$$c_\text{p} = \frac{\left(2\times10^{8}\,\text{N/m}^2\right)\left(0.207\times10^{-3}\,\text{K}^{-1}\right)\left(1.02\times10^{-3}\,\text{m}^3\right)}{1\,\text{kg}} = 42.2\,\text{J/kg}\cdot\text{K}$$

Express the ratio of c_p to c_water:

$$\frac{c_\text{p}}{c_\text{water}} = \frac{42.2\,\text{J/kg}\cdot\text{K}}{4184\,\text{J/kg}\cdot\text{K}} = 1.01\times10^{-2}$$

or

$$c_\text{p} = \boxed{(1.01\%)c_\text{water}}$$

***24** ••

Picture the Problem We can apply the condition for the validity of the equipartition theorem, i.e., that the spacing of the energy levels be large compared to *kT*, to find the critical temperature T_c:

Express the failure condition for the equipartition theorem:

$$kT_\text{c} \approx 0.15\,\text{eV}$$

Solve for T_c:

$$T_\text{c} = \frac{0.15\,\text{eV}}{k}$$

Substitute numerical values and evaluate T_c:

$$T_\text{c} = \frac{0.15\,\text{eV}\times\dfrac{1.602\times10^{-19}\,\text{J}}{\text{eV}}}{1.381\times10^{-23}\,\text{J/K}} = \boxed{1740\,\text{K}}$$

Heat Capacity; Specific Heat; Latent Heat

***25** •

Picture the Problem We can use the conversion factor 1 cal = 4.184 J to convert 2500 kcal into joules and the definition of power to find the average output if the consumed energy is dissipated over 24 h.

(*a*) Convert 2500 kcal to joules:

$$2500\,\text{kcal} = 2500\,\text{kcal}\times\frac{4.184\,\text{J}}{\text{cal}} = \boxed{10.5\,\text{MJ}}$$

(*b*) Use the definition of average power to obtain:

$$P_{\text{av}} = \frac{\Delta E}{\Delta t} = \frac{1.05\times 10^{7}\text{ J}}{24\,\text{h}\times\dfrac{3600\,\text{s}}{\text{h}}} = \boxed{121\,\text{W}}$$

Remarks: Note that this average power output is essentially that of a widely used light bulb.

26 •

Picture the Problem We can use the relationship $Q = mc\Delta T$ to calculate the amount of heat given off by the concrete as it cools from 25 to 20°C.

Relate the heat given off by the concrete to its mass, specific heat, and change in temperature:

$$Q = mc\Delta T$$

Substitute numerical values and evaluate *Q*:

$$Q = \left(10^{5}\text{ kg}\right)\left(1\,\text{kJ/kg}\cdot\text{K}\right)\left(298\,\text{K} - 293\,\text{K}\right) = \boxed{500\,\text{MJ}}$$

27 •

Picture the Problem We can find the amount of heat that must be supplied by adding the heat required to warm the ice from –10°C to 0°C, the heat required to melt the ice, and the heat required to warm the water formed from the ice to 40°C.

Express the total heat required:

$$Q = Q_{\text{warm ice}} + Q_{\text{melt ice}} + Q_{\text{warm water}}$$

Substitute for each term to obtain:

$$Q = mc_{\text{ice}}\Delta T_{\text{ice}} + mL_{\text{f}} + mc_{\text{water}}\Delta T_{\text{water}} = m\left(c_{\text{ice}}\Delta T_{\text{ice}} + L_{\text{f}} + c_{\text{water}}\Delta T_{\text{water}}\right)$$

Substitute numerical values and evaluate *Q*:

$$Q = \left(0.06\,\text{kg}\right)\left[\left(0.49\,\text{kcal/kg}\cdot\text{K}\right)\left(273\,\text{K} - 263\,\text{K}\right) + 79.7\,\text{kcal/kg} + \left(1\,\text{kcal/kg}\cdot\text{K}\right)\left(313\,\text{K} - 273\,\text{K}\right)\right] = \boxed{7.48\,\text{kcal}}$$

28 ••

Picture the Problem We can find the amount of heat that must be removed by adding the heat that must be removed to cool the steam from 150°C to 100°C, the heat that must be removed to condense the steam to water, the heat that must be removed to cool the water from 100°C to 0°C, and the heat that must be removed to freeze the water.

Express the total heat that must be removed:

$$Q = Q_{\text{cool steam}} + Q_{\text{condense steam}} + Q_{\text{cool water}} + Q_{\text{freeze water}}$$

Substitute for each term to obtain:

$$\begin{aligned} Q &= mc_{\text{steam}}\Delta T_{\text{steam}} + mL_{\text{v}} + mc_{\text{water}}\Delta T_{\text{water}} + mL_{\text{f}} \\ &= m\left(c_{\text{steam}}\Delta T_{\text{steam}} + L_{\text{v}} + c_{\text{water}}\Delta T_{\text{water}} + L_{\text{f}}\right) \end{aligned}$$

Substitute numerical values and evaluate Q:

$$\begin{aligned} Q &= (0.1\,\text{kg})\big[(2.01\,\text{kJ/kg}\cdot\text{K})(423\,\text{K} - 373\,\text{K}) + 2.26\,\text{MJ/kg} \\ &\qquad + (4.18\,\text{kJ/kg}\cdot\text{K})(373\,\text{K} - 273\,\text{K}) + 333.5\,\text{kJ/kg}\big] \\ &= 311.2\,\text{kJ} \times \frac{1\,\text{kcal}}{4.184\,\text{kJ}} \\ &= \boxed{74.4\,\text{kcal}} \end{aligned}$$

29 ••

Picture the Problem We can find the amount of nitrogen vaporized by equating the heat gained by the liquid nitrogen and the heat lost by the piece of aluminum.

Express the heat gained by the liquid nitrogen as it cools the piece of aluminum:

$$Q_{\text{N}} = m_{\text{N}}L_{\text{vN}}$$

Express the heat lost by the piece of aluminum as it cools:

$$Q_{\text{Al}} = m_{\text{Al}}c_{\text{Al}}\Delta T_{\text{Al}}$$

Equate these two expressions and solve for m_{N}:

$$m_{\text{N}}L_{\text{vN}} = m_{\text{Al}}c_{\text{Al}}\Delta T_{\text{Al}}$$

and

$$m_{\text{N}} = \frac{m_{\text{Al}}c_{\text{Al}}\Delta T_{\text{Al}}}{L_{\text{vN}}}$$

Substitute numerical values and evaluate m_{N}:

$$m_{\text{N}} = \frac{(0.05\,\text{kg})(0.90\,\text{J/kg}\cdot\text{K})(293\,\text{K} - 77\,\text{K})}{199\,\text{kJ/kg}} = 4.88\times10^{-5}\,\text{kg} = \boxed{48.8\,\text{mg}}$$

30 ••

Picture the Problem Because the heat lost by the lead as it cools is gained by the block of ice (we're assuming no heat is lost to the surroundings), we can apply the conservation of energy to determine how much ice melts.

Apply the conservation of energy to this process:

$$\Delta Q = 0$$

or

$$-m_{\text{Pb}}\left(L_{\text{f,Pb}} + c_{\text{Pb}}\Delta T_{\text{Pb}}\right) + m_{\text{w}}L_{\text{f,w}} = 0$$

Solve for m_{w}:

$$m_{\text{w}} = \frac{m_{\text{Pb}}\left(L_{\text{f,Pb}} + c_{\text{Pb}}\Delta T_{\text{Pb}}\right)}{L_{\text{f,w}}}$$

Substitute numerical values and evaluate m_{w}:

$$m_{\text{w}} = \frac{(0.5\,\text{kg})\left(24.7\,\text{kJ/kg} + (0.128\,\text{kJ/kg}\cdot\text{K})(600\,\text{K} - 273\,\text{K})\right)}{333.5\,\text{kJ/kg}} = \boxed{99.8\,\text{g}}$$

***31** ••

Picture the Problem The temperature of the bullet immediately after coming to rest in the block is the sum of its pre-collision temperature and the change in its temperature as a result of being brought to a stop in the block. We can equate the heat gained by the bullet and half its pre-collision kinetic energy to find the change in its temperature.

Express the temperature of the bullet immediately after coming to rest in terms of its initial temperature and the change in its temperature as a result of being stopped in the block:

$$\begin{aligned} T &= T_{\text{i}} + \Delta T \\ &= 293\,\text{K} + \Delta T \end{aligned}$$

Relate the heat absorbed by the bullet as it comes to rest to its kinetic energy before the collision:

$$Q = \tfrac{1}{2}K$$

Substitute for Q and K to obtain:

$$m_{\text{Pb}}c_{\text{Pb}}\Delta T = \tfrac{1}{2}\left(\tfrac{1}{2}m_{\text{Pb}}v^2\right)$$

Solve for ΔT:

$$\Delta T = \frac{v^2}{4c_{\text{Pb}}}$$

Substitute to obtain:

$$T = 293\,\text{K} + \frac{v^2}{4c_{\text{Pb}}}$$

Substitute numerical values and evaluate T:

$$T = 293\,\text{K} + \frac{(420\,\text{m/s})^2}{4(0.128\,\text{kJ/kg}\cdot\text{K})}$$
$$= 638\,\text{K} = \boxed{365°\text{C}}$$

32 ••

Picture the Problem We can find the heat available to warm the brake drums from the initial kinetic energy of the car and the mass of steel contained in the brake drums from $Q = m_{\text{steel}}c_{\text{steel}}\Delta T$.

Express m_{steel} in terms of Q:

$$m_{\text{steel}} = \frac{Q}{c_{\text{steel}}\Delta T}$$

Find the heat available to warm the brake drums from the initial kinetic energy of the car:

$$Q = K = \tfrac{1}{2}m_{\text{car}}v^2$$

Substitute for Q to obtain:

$$m_{\text{steel}} = \frac{\tfrac{1}{2}m_{\text{car}}v^2}{c_{\text{steel}}\Delta T}$$

Substitute numerical values and evaluate m_{steel}:

$$m_{\text{steel}} = \frac{(1400\,\text{kg})\left(80\,\frac{\text{km}}{\text{h}} \times \frac{1\,\text{h}}{3600\,\text{s}}\right)^2}{2\left(0.11\,\frac{\text{kcal}}{\text{kg}\cdot\text{K}} \times \frac{4.186\,\text{kJ}}{\text{kcal}}\right)(120\,\text{K})}$$
$$= \boxed{6.26\,\text{kg}}$$

Calorimetry

33 •

Picture the Problem Let the system consist of the piece of lead, calorimeter, and water. During this process the water will gain energy at the expense of the piece of lead. We can set the heat out of the lead equal to the heat into the water and solve for the final temperature of the lead and water.

Apply conservation of energy to the system to obtain:

$$\Delta Q = 0 \text{ or } Q_{\text{in}} = Q_{\text{out}}$$

Express the heat lost by the lead in terms of its specific heat and temperature change:

$$Q_{\text{out}} = m_{\text{Pb}} c_{\text{Pb}} \Delta T_{\text{Pb}}$$

Express the heat absorbed by the water in terms of its specific heat and temperature change:

$$Q_{\text{in}} = m_{\text{w}} c_{\text{w}} \Delta T_{\text{w}}$$

Substitute to obtain:

$$m_{\text{w}} c_{\text{w}} \Delta T_{\text{w}} = m_{\text{Pb}} c_{\text{Pb}} \Delta T_{\text{Pb}}$$

Substitute numerical values:

$$(0.5\,\text{kg})(4.18\,\text{kJ/kg}\cdot\text{K})(t_{\text{f}} - 293\,\text{K}) = (0.2\,\text{kg})(0.128\,\text{kJ/kg}\cdot\text{K})(363\,\text{K} - t_{\text{f}})$$

Solve for t_{f} to obtain:

$$t_{\text{f}} = 293.8\,\text{K} = \boxed{20.8°\text{C}}$$

***34** •

Picture the Problem During this process the water and the container will gain energy at the expense of the piece of metal. We can set the heat out of the metal equal to the heat into the water and the container and solve for the specific heat of the metal.

Apply conservation of energy to the system to obtain:

$$\Delta Q = 0 \text{ or } Q_{\text{gained}} = Q_{\text{lost}}$$

Express the heat lost by the metal in terms of its specific heat and temperature change:

$$Q_{\text{lost}} = m_{\text{metal}} c_{\text{metal}} \Delta T_{\text{metal}}$$

Express the heat gained by the water and the container in terms of their specific heats and temperature change:

$$Q_{\text{gained}} = m_{\text{w}} c_{\text{w}} \Delta T_{\text{w}} + m_{\text{container}} c_{\text{metal}} \Delta T_{\text{w}}$$

Substitute to obtain:

$$m_{\text{w}} c_{\text{w}} \Delta T_{\text{w}} + m_{\text{container}} c_{\text{metal}} \Delta T_{\text{w}} = m_{\text{metal}} c_{\text{metal}} \Delta T_{\text{metal}}$$

Substitute numerical values:

$$\begin{aligned}(0.5\,\text{kg})(4.18\,\text{kJ/kg}\cdot\text{K})(294.4\,\text{K} - 293\,\text{K}) + (0.2\,\text{kg})(294.4\,\text{K} - 293\,\text{K})c_{\text{metal}} \\ = (0.1\,\text{kg})(373\,\text{K} - 294.4\,\text{K})c_{\text{metal}}\end{aligned}$$

Solve for c_{metal}:

$$c_{\text{metal}} = \boxed{0.386\,\text{kJ/kg}\cdot\text{K}}$$

35 ••

Picture the Problem We can use $Q = mc\Delta T$ to express the mass m of water that can be heated through a temperature interval ΔT by an amount of heat energy Q. We can then find the amount of heat energy expended by Armstrong from the definition of power.

Express the amount of heat energy Q required to raise the temperature of a mass m of water by ΔT:

$$Q = mc\Delta T$$

Solve for m to obtain:

$$m = \frac{Q}{c\Delta T}$$

Use the definition of power to relate the heat energy expended by Armstrong to the rate at which he expended the energy:

$$P = \frac{Q}{\Delta t} \Rightarrow Q = P\Delta t$$

Substitute to obtain:

$$m = \frac{P\Delta t}{c\Delta T}$$

Substitute numerical values and evaluate m:

$$m = \frac{(400\,\text{J/s})(3600\,\text{s/h})(5\,\text{h/d})(20\,\text{d})}{(4.184\,\text{kJ/kg}\cdot\text{K})(373\,\text{K} - 297\,\text{K})} = \boxed{453\,\text{kg}}$$

36 ••

Picture the Problem During this process the ice and the water formed from the melted ice will gain energy at the expense of the glass tumbler and the water in it. We can set the heat out of the tumbler and the water that is initially at 24°C equal to the heat into the ice and ice water and solve for the final temperature of the drink.

Apply conservation of energy to the system to obtain:

$$\Delta Q = 0 \text{ or } Q_{\text{gained}} = Q_{\text{lost}}$$

Express the heat lost by the tumbler and the water in it in terms of their specific heats and common temperature change:

$$Q_{\text{lost}} = m_{\text{glass}}c_{\text{glass}}\Delta T + m_{\text{water}}c_{\text{water}}\Delta T$$

Express the heat gained by the ice and the melted ice in terms of their specific heats and temperature

$$Q_{\text{gained}} = m_{\text{ice}}c_{\text{ice}}\Delta T_{\text{ice}} + m_{\text{ice}}L_{\text{f}} + m_{\text{ice water}}c_{\text{water}}\Delta T_{\text{ice water}}$$

changes:

Substitute to obtain:

$$m_{\text{ice}}c_{\text{ice}}\Delta T_{\text{ice}} + m_{\text{ice}}L_{\text{f}} + m_{\text{ice water}}c_{\text{water}}\Delta T_{\text{ice water}} = m_{\text{glass}}c_{\text{glass}}\Delta T + m_{\text{water}}c_{\text{water}}\Delta T$$

Substitute numerical values:

$$\begin{aligned}&(0.03\,\text{kg})(0.49\,\text{kcal/kg}\cdot\text{K})(273\,\text{K}-270\,\text{K})+(0.03\,\text{kg})(79.7\,\text{kcal/kg})\\&\quad+(0.03\,\text{kg})(1\,\text{kcal/kg}\cdot\text{K})t_{\text{f}} = (0.025\,\text{kg})(0.2\,\text{kcal/kg}\cdot\text{K})(297\,\text{K}-t_{\text{f}})\\&\qquad+(0.2\,\text{kg})(1\,\text{kcal/kg}\cdot\text{K})(297\,\text{K}-t_{\text{f}})\end{aligned}$$

Solve for t_{f}:

$$t_{\text{f}} = 283.6\,\text{K} = \boxed{10.6°\text{C}}$$

37 ••

Picture the Problem Because we can not tell, without performing a couple of calculations, whether there is enough heat available in the 500 g of water to melt all of the ice, we'll need to resolve this question first.

(*a*) Determine the heat required to melt 200 g of ice:

$$\begin{aligned}Q_{\text{melt ice}} &= m_{\text{ice}}L_{\text{f}}\\&= (0.2\,\text{kg})(79.7\,\text{kcal/kg})\\&= 15.94\,\text{kcal}\end{aligned}$$

Determine the heat available from 500 g of water:

$$\begin{aligned}Q_{\text{water}} &= m_{\text{water}}c_{\text{water}}\Delta T_{\text{water}}\\&= (0.5\,\text{kg})(1\,\text{kcal/kg}\cdot\text{K})\\&\quad\times(293\,\text{K}-273\,\text{K})\\&= 10\,\text{kcal}\end{aligned}$$

Because $Q_{\text{water}} < Q_{\text{melt ice}}$:

$$\boxed{\text{The final temperature is }0°\text{C.}}$$

(*b*) Equate the energy available from the water Q_{water} to $m_{\text{ice}}L_{\text{f}}$ and solve for m_{ice}:

$$m_{\text{ice}} = \frac{Q_{\text{water}}}{L_{\text{f}}}$$

Substitute numerical values and evaluate m_{ice}:

$$m_{\text{ice}} = \frac{10\,\text{kcal}}{79.7\,\text{kcal/kg}} = \boxed{125\,\text{g}}$$

38 ••

Picture the Problem Because the bucket contains a mixture of ice and water initially, we know that its temperature must be 0°C. We can equate the heat gained by the mixture of ice and water and the heat lost by the block of copper and solve for the amount of ice initially in the bucket.

Apply conservation of energy to the system to obtain:

$$\Delta Q = 0 \text{ or } Q_{\text{gained}} = Q_{\text{lost}}$$

Express the heat lost by the block of copper:

$$Q_{\text{lost}} = m_{\text{Cu}} c_{\text{Cu}} \Delta T_{\text{Cu}}$$

Express the heat gained by the ice and the melted ice:

$$Q_{\text{gained}} = m_{\text{ice}} L_{\text{f}} + m_{\text{ice water}} c_{\text{water}} \Delta T_{\text{ice water}}$$

Substitute to obtain:

$$m_{\text{ice}} L_{\text{f}} + m_{\text{ice water}} c_{\text{water}} \Delta T_{\text{ice water}} - m_{\text{Cu}} c_{\text{Cu}} \Delta T_{\text{Cu}} = 0$$

Solve for m_{ice}:

$$m_{\text{ice}} = \frac{m_{\text{Cu}} c_{\text{Cu}} \Delta T_{\text{Cu}} - m_{\text{ice water}} c_{\text{water}} \Delta T_{\text{ice water}}}{L_{\text{f}}}$$

Substitute numerical values and evaluate m_{ice}:

$$m_{\text{ice}} = \frac{(3.5\,\text{kg})(0.0923\,\text{kcal/kg}\cdot\text{K})(353\,\text{K} - 281\,\text{K})}{79.7\,\text{kcal/kg}} - \frac{(1.2\,\text{kg})(1\,\text{kcal/kg}\cdot\text{K})(281\,\text{K} - 273\,\text{K})}{79.7\,\text{kcal/kg}} = \boxed{171\,\text{g}}$$

39 ••

Picture the Problem During this process the ice and the water formed from the melted ice will gain energy at the expense of the condensing steam and the water from the condensed steam. We can equate these quantities and solve for the final temperature of the system.

(*a*) Apply conservation of energy to the system to obtain:

$$\Delta Q = 0 \text{ or } Q_{\text{gained}} = Q_{\text{lost}}$$

Express the heat required to melt the ice and raise the temperature of the

$$Q_{\text{gained}} = m_{\text{ice}} L_{\text{f}} + m_{\text{ice water}} c_{\text{water}} \Delta T_{\text{water}}$$

ice water:

Express the heat available from 20 g of steam and the cooling water formed from the condensed steam:

$$Q_{\text{lost}} = m_{\text{steam}} L_{\text{v}} + m_{\text{steam}} c_{\text{water}} \Delta T_{\text{water}}$$

Substitute to obtain:

$$m_{\text{ice}} L_{\text{f}} + m_{\text{ice water}} c_{\text{water}} \Delta T_{\text{water}} = m_{\text{steam}} L_{\text{v}} + m_{\text{steam}} c_{\text{water}} \Delta T_{\text{water}}$$

Substitute numerical values:

$$\begin{aligned}&(0.15\,\text{kg})(79.7\,\text{kcal/kg}) + (0.15\,\text{kg})(1\,\text{kcal/kg}\cdot\text{K})(t_{\text{f}} - 273\,\text{K})\\ &\qquad = (0.02\,\text{kg})(540\,\text{kcal/kg}) + (0.02\,\text{kg})(1\,\text{kcal/kg}\cdot\text{K})(373\,\text{K} - t_{\text{f}})\end{aligned}$$

Solve for t_{f}:

$$t_{\text{f}} = 277.94\,\text{K} = \boxed{4.94^\circ\text{C}}$$

(*b*) $\boxed{\text{Because the final temperture is greater than } 0^\circ\text{C, no ice is left.}}$

40 ••

Picture the Problem During this process the ice will gain heat and the water will lose heat. We can do a preliminary calculation to determine whether there is enough heat available to melt all of the ice and, if there is, equate the heat the heat lost by the water to the heat gained by the ice and resulting ice water as the system achieves thermal equilibrium.

Apply conservation of energy to the system to obtain:

$$\Delta Q = 0 \text{ or } Q_{\text{gained}} = Q_{\text{lost}}$$

Find the heat available to melt the ice:

$$\begin{aligned}Q_{\text{avail}} &= m_{\text{water}} c_{\text{water}} \Delta T_{\text{water}}\\ &= (1\,\text{kg})(1\,\text{kcal/kg}\cdot\text{K})\\ &\quad\times(303\,\text{K} - 273\,\text{K})\\ &= 30\,\text{kcal}\end{aligned}$$

Find the heat required to melt all of the ice:

$$\begin{aligned}Q_{\text{melt ice}} &= m_{\text{ice}} L_{\text{f}}\\ &= (0.05\,\text{kg})(79.7\,\text{kcal/kg})\\ &= 3.985\,\text{kcal}\end{aligned}$$

Because $Q_{\text{avail}} > Q_{\text{melt ice}}$, we know

$$Q_{\text{lost}} = m_{\text{water}} c_{\text{water}} \Delta T_{\text{water}}$$

that the final temperature will be greater than 273 K and we can express Q_{lost} in terms of the change in temperature of the water:

Express Q_{gained}:

$$Q_{\text{gained}} = m_{\text{ice}}L_{\text{f}} + m_{\text{ice water}}c_{\text{water}}\Delta T_{\text{ice water}}$$

Equate the heat gained and the heat lost to obtain:

$$m_{\text{ice}}L_{\text{f}} + m_{\text{ice water}}c_{\text{water}}\Delta T_{\text{ice water}} = m_{\text{water}}c_{\text{water}}\Delta T_{\text{water}}$$

Substitute numerical values to obtain:

$$(0.05\,\text{kg})(79.7\,\text{kcal/kg}) + (0.05\,\text{kg})(1\,\text{kcal/kg}\cdot\text{K})(T_{\text{f}} - 273\,\text{K}) = (1\,\text{kg})(1\,\text{kcal/kg}\cdot\text{K})(303\,\text{K} - T_{\text{f}})$$

Solving for T_{f} yields:

$$T_{\text{f}} = 297.8\,\text{K} = \boxed{24.8°\text{C}}$$

Find the heat required to melt 500 g of ice:

$$\begin{aligned}Q_{\text{melt ice}} &= m_{\text{ice}}L_{\text{f}}\\ &= (0.5\,\text{kg})(79.7\,\text{kcal/kg})\\ &= 39.85\,\text{kcal}\end{aligned}$$

$$\boxed{\begin{array}{l}\text{Because the heat required to melt 500 g of ice is greater than the heat available,}\\ \text{the final temperature will be 0°C.}\end{array}}$$

***41 ••**

Picture the Problem Assume that the calorimeter is in thermal equilibrium with the water it contains. During this process the ice will gain heat in warming to 0°C and melting, as will the water formed from the melted ice. The water in the calorimeter and the calorimeter will lose heat. We can do a preliminary calculation to determine whether there is enough heat available to melt all of the ice and, if there is, equate the heat the heat lost by the water to the heat gained by the ice and resulting ice water as the system achieves thermal equilibrium.

Find the heat available to melt the ice:

$$\begin{aligned}Q_{\text{avail}} &= m_{\text{water}}c_{\text{water}}\Delta T_{\text{water}} + m_{\text{cal}}c_{\text{cal}}\Delta T_{\text{water}}\\ &= [(0.5\,\text{kg})(4.18\,\text{kJ/kg}\cdot\text{K}) + (0.2\,\text{kg})(0.9\,\text{kJ/kg}\cdot\text{K})](293\,\text{K} - 273\,\text{K})\\ &= 45.40\,\text{kJ}\end{aligned}$$

Find the heat required to melt all of the ice:

$$\begin{aligned}Q_{\text{melt ice}} &= m_{\text{ice}}c_{\text{ice}}\Delta T_{\text{ice}} + m_{\text{ice}}L_{\text{f}}\\ &= (0.1\,\text{kg})(2\,\text{kJ/kg}\cdot\text{K})(273\,\text{K}-253\,\text{K}) + (0.1\,\text{kg})(333.5\,\text{kJ/kg})\\ &= 37.35\,\text{kJ}\end{aligned}$$

(*a*) Because $Q_{\text{avail}} > Q_{\text{melt ice}}$, we know that the final temperature will be greater than 0°C. Apply the conservation of energy to the system to obtain:

$$\Delta Q = 0 \text{ or } Q_{\text{gained}} = Q_{\text{lost}}$$

Express Q_{lost} in terms of the final temperature of the system:

$$Q_{\text{lost}} = (m_{\text{water}}c_{\text{water}} + m_{\text{cal}}c_{\text{cal}})\Delta T_{\text{water+calorimeter}}$$

Express Q_{gained} in terms of the final temperature of the system:

$$Q_{\text{gained}} = m_{\text{ice}}c_{\text{ice}}\Delta T_{\text{ice}} + m_{\text{ice}}L_{\text{f}}$$

Substitute to obtain:

$$m_{\text{ice}}c_{\text{ice}}\Delta T_{\text{ice}} + m_{\text{ice}}L_{\text{f}} + m_{\text{ice water}}c_{\text{water}}\Delta T_{\text{ice water}} = (m_{\text{water}}c_{\text{water}} + m_{\text{cal}}c_{\text{cal}})\Delta T_{\text{water+calorimeter}}$$

Substitute numerical values:

$$\begin{aligned}&37.35\,\text{kJ} + (0.1\,\text{kg})(4.18\,\text{kJ/kg}\cdot\text{K})(t_{\text{f}} - 273\,\text{K})\\ &\qquad = [(0.5\,\text{kg})(4.18\,\text{kJ/kg}\cdot\text{K}) + (0.2\,\text{kg})(0.9\,\text{kJ/kg}\cdot\text{K})](293\,\text{K} - t_{\text{f}})\end{aligned}$$

Solving for t_{f} yields:

$$t_{\text{f}} = 276\,\text{K} = \boxed{2.99°\text{C}}$$

(*b*) Find the heat required to raise 200 g of ice to 0°C:

$$Q_{\text{warm ice}} = m_{\text{ice}}c_{\text{ice}}\Delta T_{\text{ice}} = (0.2\,\text{kg})(2\,\text{kJ/kg}\cdot\text{K})(273\,\text{K}-253\,\text{K}) = 8.00\,\text{kJ}$$

Noting that there are now 600 g of water in the calorimeter, find the heat available from cooling the calorimeter and water from 3°C to 0°C:

$$\begin{aligned}Q_{\text{avail}} &= m_{\text{water}}c_{\text{water}}\Delta T_{\text{water}} + m_{\text{cal}}c_{\text{cal}}\Delta T_{\text{water}}\\ &= [(0.6\,\text{kg})(4.18\,\text{kJ/kg}\cdot\text{K}) + (0.2\,\text{kg})(0.9\,\text{kJ/kg}\cdot\text{K})](293\,\text{K}-273\,\text{K})\\ &= 8.064\,\text{kJ}\end{aligned}$$

Express the amount of ice that will melt in terms of the difference between the heat available and the heat required to warm the ice:

$$m_{\text{melted ice}} = \frac{Q_{\text{avail}} - Q_{\text{warm ice}}}{L_{\text{f}}}$$

Substitute numerical values and evaluate $m_{\text{melted ice}}$:

$$m_{\text{melted ice}} = \frac{8.064\,\text{kJ} - 8\,\text{kJ}}{333.5\,\text{kJ/kg}} = 0.1919\,\text{g}$$

Find the ice remaining in the system:

$$m_{\text{remaining ice}} = 200\,\text{g} - 0.1919\,\text{g} = \boxed{199.8\,\text{g}}$$

(*c*) $\boxed{\text{Because the initial and final conditions are the same, the answer would be the same.}}$

42 ••

Picture the Problem Let the subscript B denote the block, w_1 the water initially in the calorimeter, and w_2 the 120 mL of water that is added to the calorimeter vessel. We can equate the heat gained by the calorimeter and its initial contents to the heat lost by the warm water and solve this equation for the specific heat of the block.

Apply conservation of energy to the system to obtain:

$$\Delta Q = 0 \text{ or } Q_{\text{gained}} = Q_{\text{lost}}$$

Express the heat gained by the block, the calorimeter, and the water initially in the calorimeter:

$$\begin{aligned} Q_{\text{gained}} &= m_{\text{B}}c_{\text{B}}\Delta T_{\text{B}} + m_{\text{Cu}}c_{\text{Cu}}\Delta T_{\text{Cu}} \\ &\quad + m_{\text{w}_1}c_{\text{w}_1}\Delta T_{\text{w}_1} \\ &= \left(m_{\text{B}}c_{\text{B}} + m_{\text{Cu}}c_{\text{Cu}} + m_{\text{w}_1}c_{\text{w}_1}\right)\Delta T \end{aligned}$$

because the temperature changes are the same for the block, calorimeter, and the water that is initially at 20°C.

Express the heat lost by the water that is added to the calorimeter:

$$Q_{\text{lost}} = m_{\text{w}_2}c_{\text{w}_2}\Delta T_{\text{w}_2}$$

Substitute to obtain:

$$\left(m_{\text{B}}c_{\text{B}} + m_{\text{Cu}}c_{\text{Cu}} + m_{\text{w}_1}c_{\text{w}_1}\right)\Delta T = m_{\text{w}_2}c_{\text{w}_2}\Delta T_{\text{w}_2}$$

Substitute numerical values:

$$\left[(0.1\,\text{kg})c_B+(0.025\,\text{kg})(0.386\,\text{kJ/kg}\cdot\text{K})+(0.06\,\text{kg})(4.18\,\text{kJ/kg}\cdot\text{K})\right](327\,\text{K}-293\,\text{K})$$
$$=\left(120\times10^{-3}\,\text{kg}\right)(4.18\,\text{kJ/kg}\cdot\text{K})(353\,\text{K}-327\,\text{K})$$

Solve for c_B to obtain:

$$c_B=\boxed{1.23\,\text{kJ/kg}\cdot\text{K}}$$
$$=\boxed{0.294\,\text{cal/g}\cdot\text{K}}$$

43 ••

Picture the Problem We can find the temperature *t* by equating the heat gained by the warming water and calorimeter, and vaporization of some of the water.

Apply conservation of energy to the system to obtain:

$$\Delta Q=0 \text{ or } Q_{\text{gained}}=Q_{\text{lost}}$$

Express the heat gained by the warming and vaporizing water:

$$Q_{\text{gained}}=m_{\text{w, vaporized}}L_{\text{f, w}}+m_{\text{w}}c_{\text{w}}\Delta T_{\text{w}}+m_{\text{cal}}c_{\text{cal}}\Delta T_{\text{w}}$$

Express the heat lost by the 100-g piece of copper as it cools:

$$Q_{\text{lost}}=m_{\text{Cu}}c_{\text{Cu}}\Delta T_{\text{Cu}}$$

Substitute to obtain:

$$m_{\text{w, vaporized}}L_{\text{f, w}}+m_{\text{w}}c_{\text{w}}\Delta T_{\text{w}}+m_{\text{cal}}c_{\text{cal}}\Delta T_{\text{w}}=m_{\text{Cu}}c_{\text{Cu}}\Delta T_{\text{Cu}}$$

Substitute numerical values:

$$(1.2\,\text{g})(540\,\text{cal/g})+(200\,\text{g})(1\,\text{cal/g}\cdot\text{K})(311\,\text{K}-289\,\text{K})$$
$$+(150\,\text{g})(0.0923\,\text{cal/g}\cdot\text{K})(311\,\text{K}-289\,\text{K})=(100\,\text{g})(0.0923\,\text{cal/g}\cdot\text{K})(t-311\,\text{K})$$

Solve for *t* to obtain:

$$t=891\,\text{K}=\boxed{618°\text{C}}$$

44 ••

Picture the Problem We can find the final temperature of the system by equating the heat gained by the calorimeter and the water in it to the heat lost by the cooling aluminum shot. In (*b*) we'll proceed as in (*a*) but with the initial and final temperatures adjusted to minimize heat transfer between the system and its surroundings.

Apply conservation of energy to the system to obtain:

$$\Delta Q=0 \text{ or } Q_{\text{gained}}=Q_{\text{lost}}$$

(*a*) Express the heat gained by the warming water and the calorimeter:

$$Q_{\text{gained}} = m_{\text{w}}c_{\text{w}}\Delta T_{\text{w}} + m_{\text{cal}}c_{\text{Al}}\Delta T_{\text{w}}$$

Express the heat lost by the aluminum shot as it cools:

$$Q_{\text{lost}} = m_{\text{shot}}c_{\text{Al}}\Delta T_{\text{Al}}$$

Substitute to obtain:

$$\left(m_{\text{w}}c_{\text{w}} + m_{\text{cal}}c_{\text{Al}}\right)\Delta T_{\text{w}} = m_{\text{shot}}c_{\text{Al}}\Delta T_{\text{Al}}$$

Substitute numerical values to obtain:

$$\begin{aligned}\left[(500\,\text{g})(1\,\text{cal/g}\cdot\text{K}) + (200\,\text{g})(0.0923\,\text{cal/g}\cdot\text{K})\right]\left(t_{\text{f}} - 293\,\text{K}\right)\\ = (300\,\text{g})(0.215\,\text{cal/g}\cdot\text{K})\left(373\,\text{K} - t_{\text{f}}\right)\end{aligned}$$

Solve for t_{f} to obtain:

$$t_{\text{f}} = 301.9\,\text{K} = \boxed{28.9°\text{C}}$$

(*b*) Let the initial and final temperatures of the calorimeter and its contents be:

$t_{\text{i}} = 20°\text{C} - t_0$ (1)
and
$t_{\text{f}} = 20°\text{C} + t_0$
where t_{i} and t_{f} are the temperatures above and below room temperature and t_0 is the amount t_{i} and t_{f} must be below and above room temperature respectively.

Express and the heat gained by the water and calorimeter:

$$\begin{aligned}Q_{\text{in}} &= m_{\text{w}}c_{\text{w}}\Delta T_{\text{w}} + m_{\text{cal}}c_{\text{Al}}\Delta T_{\text{w}}\\ &= \left(m_{\text{w}}c_{\text{w}} + m_{\text{cal}}c_{\text{Al}}\right)\Delta T_{\text{w}}\end{aligned}$$

Express the heat lost by the aluminum shot as it cools:

$$Q_{\text{out}} = m_{\text{shot}}c_{\text{Al}}\Delta T_{\text{Al}}$$

Equate Q_{in} and Q_{out} to obtain:

$$\left(m_{\text{w}}c_{\text{w}} + m_{\text{cal}}c_{\text{Al}}\right)\Delta T_{\text{w}} = m_{\text{shot}}c_{\text{Al}}\Delta T_{\text{Al}}$$

Substitute numerical values:

$$\begin{aligned}\left[(500\,\text{g})(1\,\text{cal/g}\cdot\text{K}) + (200\,\text{g})(0.215\,\text{cal/g}\cdot\text{K})\right]\left(293\,\text{K} + t_0 - 293\,\text{K} + t_0\right)\\ = (300\,\text{g})(0.215\,\text{cal/g}\cdot\text{K})\left(373\,\text{K} - 293\,\text{K} - t_0\right)\end{aligned}$$

Solve for and evaluate t_0:

$$t_0 = 277.49\,\text{K} = 4.49°\text{C}$$

Substitute in equation (1) to obtain:

$$t_{\text{i}} = 20°\text{C} - 4.49°\text{C} = \boxed{15.5°\text{C}}$$

First Law of Thermodynamics

45 •

Picture the Problem We can apply the first law of thermodynamics to find the change in internal energy of the gas during this process.

Apply the first law of thermodynamics to express the change in internal energy of the gas in terms of the heat added to the system and the work done on the gas:

$$\Delta E_{\text{int}} = Q_{\text{in}} + W_{\text{on}}$$

The work done by the gas equals the negative of the work done on the gas. Substitute numerical values and evaluate ΔE_{int}:

$$\Delta E_{\text{int}} = 600\,\text{cal} \times \frac{4.184\,\text{J}}{\text{cal}} - 300\,\text{J}$$
$$= \boxed{2.21\,\text{kJ}}$$

***46** •

Picture the Problem We can apply the first law of thermodynamics to find the change in internal energy of the gas during this process.

Apply the first law of thermodynamics to express the change in internal energy of the gas in terms of the heat added to the system and the work done on the gas:

$$\Delta E_{\text{int}} = Q_{\text{in}} + W_{\text{on}}$$

The work done by the gas is the negative of the work done on the gas. Substitute numerical values and evaluate ΔE_{int}:

$$\Delta E_{\text{int}} = 400\,\text{kcal} \times \frac{4.184\,\text{J}}{\text{cal}} - 800\,\text{kJ}$$
$$= \boxed{874\,\text{kJ}}$$

47 •

Picture the Problem We can use the first law of thermodynamics to relate the change in the bullet's internal energy to its pre-collision kinetic energy.

Using the first law of thermodynamics, relate the change in the internal energy of the bullet to the work done on it by

$$\Delta E_{\text{int}} = Q_{\text{in}} + W_{\text{on}}$$

or, because $Q_{\text{in}} = 0$,

$$\Delta E_{\text{int}} = W_{\text{on}} = \Delta K = -(K_{\text{f}} - K_i)$$

the block of wood:

Substitute for ΔE_{int}, K_f, and K_i to obtain:

$$mc_{\text{Pb}}(t_f - t_i) = -\left(0 - \tfrac{1}{2}mv^2\right) = \tfrac{1}{2}mv^2$$

Solve for t_f:

$$t_f = t_i + \frac{v^2}{2c_{\text{Pb}}}$$

Substitute numerical values and evaluate t_f:

$$t_f = 293\,\text{K} + \frac{(200\,\text{m/s})^2}{2(0.128\,\text{kJ/kg}\cdot\text{K})} = 449\,\text{K} = \boxed{176°\text{C}}$$

48 •

Picture the Problem What is described above is clearly a limiting case because, as the water falls, it will, for example, collide with rocks and experience air drag; resulting in some of its initial potential energy being converted into internal energy. In this limiting case we can use the first law of thermodynamics to relate the change in the gravitational potential energy (take $U_g = 0$ at the bottom of the waterfalls) to the change in internal energy of the water and solve for the increase in temperature.

(*a*) Using the first law of thermodynamics and noting that, because the gravitational force is conservative, $W_{\text{on}} = -\Delta U$, relate the change in the internal energy of the water to the work done on it by gravity:

$$\Delta E_{\text{int}} = Q_{\text{in}} + W_{\text{on}}$$

or, because $Q_{\text{in}} = 0$,

$$\Delta E_{\text{int}} = W_{\text{on}} = -\Delta U = -(U_f - U_i)$$

Substitute for ΔE_{int}, U_f, and U_i to obtain:

$$mc_w\Delta T = -(0 - mg\Delta h) = mg\Delta h$$

Solve for ΔT:

$$\Delta T = \frac{g\Delta h}{c_w}$$

Substitute numerical values and evaluate ΔT:

$$\Delta T = \frac{(9.81\,\text{m/s}^2)(50\,\text{m})}{4.18\,\text{kJ/kg}\cdot\text{K}} = \boxed{0.117\,\text{K}}$$

(*b*) Proceed as in (*a*) with $\Delta h = 740$ m:

$$\Delta T = \frac{(9.81\,\text{m/s}^2)(740\,\text{m})}{4.18\,\text{kJ/kg}\cdot\text{K}} = \boxed{1.74\,\text{K}}$$

49 •

Picture the Problem We can apply the first law of thermodynamics to find the change in internal energy of the gas during this process.

Apply the first law of thermodynamics to express the change in internal energy of the gas in terms of the heat added to the system and the work done on the gas:

$$\Delta E_{\text{int}} = Q_{\text{in}} + W_{\text{on}}$$

The work done by the gas is the negative of the work done on the gas. Substitute numerical values and evaluate ΔE_{int}:

$$\Delta E_{\text{int}} = 20\,\text{cal} \times \frac{4.184\,\text{J}}{\text{cal}} - 30\,\text{J} = \boxed{53.7\,\text{J}}$$

50 ••

Picture the Problem We can use the definition of kinetic energy to express the speed of the bullet upon impact in terms of its kinetic energy. The heat absorbed by the bullet is the sum of the heat required to warm to bullet from 202 K to its melting temperature of 600 K and the heat required to melt it. We can use the first law of thermodynamics to relate the impact speed of the bullet to the change in its internal energy.

Using the first law of thermodynamics, relate the change in the internal energy of the bullet to the work done on it by the target:

$$\Delta E_{\text{int}} = Q_{\text{in}} + W_{\text{on}}$$

or, because $Q_{\text{in}} = 0$,

$$\Delta E_{\text{int}} = W_{\text{on}} = \Delta K = -(K_{\text{f}} - K_i)$$

Substitute for ΔE_{int}, K_{f}, and K_{i} to obtain:

$$mc_{\text{Pb}}\Delta T_{\text{Pb}} + mL_{\text{f,Pb}} = -\left(0 - \tfrac{1}{2}mv^2\right) = \tfrac{1}{2}mv^2$$

or

$$mc_{\text{Pb}}\left(T_{\text{MP}} - T_{\text{i}}\right) + mL_{\text{f,Pb}} = \tfrac{1}{2}mv^2$$

Solve for v to obtain:

$$v = \sqrt{2\left[c_{\text{Pb}}\left(T_{\text{MP}} - T_{\text{i}}\right) + L_{\text{f,Pb}}\right]}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{2\left\{(0.128\,\text{kJ/kg}\cdot\text{K})(600\,\text{K} - 303\,\text{K}) + 24.7\,\text{kJ/kg}\right\}} = \boxed{354\,\text{m/s}}$$

***51 ••**

Picture the Problem We can find the rate at which heat is generated when you rub your hands together using the definition of power and the rubbing time to produce a 5°C increase in temperature from $\Delta Q = (dQ/dt)\Delta t$ and $Q = mc\Delta T$.

(*a*) Express the rate at which heat is generated as a function of the friction force and the average speed of your hands:

$$\frac{dQ}{dt} = P = f_{\mathrm{k}}v = \mu F_{\mathrm{n}}v$$

Substitute numerical values and evaluate *dQ*/*dt*:

$$\frac{dQ}{dt} = 0.5(35\,\mathrm{N})(0.35\,\mathrm{m/s}) = \boxed{6.13\,\mathrm{W}}$$

(*b*) Relate the heat required to raise the temperature of your hands 5 K to the rate at which it is being generated:

$$\Delta Q = \frac{dQ}{dt}\Delta t = mc\Delta T$$

Solve for Δ*t*:

$$\Delta t = \frac{mc\Delta T}{dQ/dt}$$

Substitute numerical values and evaluate Δ*t*:

$$\Delta t = \frac{(0.35\,\mathrm{kg})(4\,\mathrm{kJ/kg\cdot K})(5\,\mathrm{K})}{6.13\,\mathrm{W}}$$

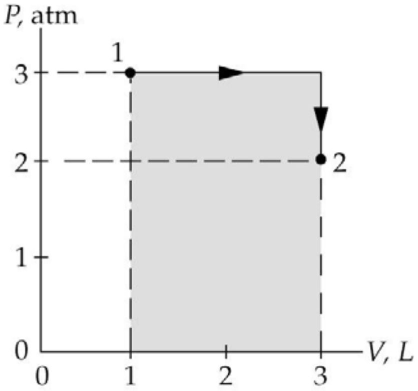
$$= 1143\,\mathrm{s}\times\frac{1\,\mathrm{min}}{60\,\mathrm{s}} = \boxed{19.0\,\mathrm{min}}$$

Work and the *PV* Diagram for a Gas

52 •

Picture the Problem We can find the work done by the gas during this process from the area under the curve. Because no work is done along the constant volume (vertical) part of the path, the work done by the gas is done during its isobaric expansion. We can then use the first law of thermodynamics to find the heat added to the system during this process.

(*a*) The path from the initial state (1) to the final state (2) is shown on the *PV* diagram.



The work done by the gas equals the area under the shaded curve:

$$W_{\text{by gas}} = P\Delta V = (3\,\text{atm})(2\,\text{L}) = \left(3\,\text{atm}\times\frac{101.3\,\text{kPa}}{\text{atm}}\right)\left(2\,\text{L}\times\frac{10^{-3}\,\text{m}^3}{\text{L}}\right) = \boxed{608\,\text{J}}$$

(*b*) The work done by the gas is the negative of the work done on the gas. Apply the first law of thermodynamics to the system to obtain:

$$\begin{aligned} Q_{\text{in}} &= \Delta E_{\text{int}} - W_{\text{on}} \\ &= (E_{\text{int},2} - E_{\text{int},1}) - (-W_{\text{by gas}}) \\ &= (E_{\text{int},2} - E_{\text{int},1}) + W_{\text{by gas}} \end{aligned}$$

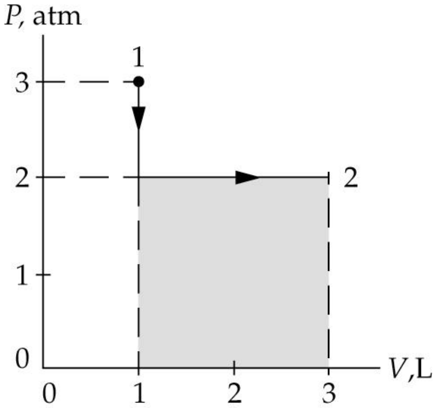
Substitute numerical values and evaluate Q_{in}:

$$Q_{\text{in}} = (912\,\text{J} - 456\,\text{J}) + 608\,\text{J} = \boxed{1.06\,\text{kJ}}$$

53 •

Picture the Problem We can find the work done by the gas during this process from the area under the curve. Because no work is done along the constant volume (vertical) part of the path, the work done by the gas is done during its isobaric expansion. We can then use the first law of thermodynamics to find the heat added to the system during this process

(*a*) The path from the initial state (1) to the final state (2) is shown on the *PV* diagram.



The work done by the gas equals the area under the curve:

$$W_{\text{by gas}} = P\Delta V = (2\,\text{atm})(2\,\text{L}) = \left(2\,\text{atm}\times\frac{101.3\,\text{kPa}}{\text{atm}}\right)\left(2\,\text{L}\times\frac{10^{-3}\,\text{m}^3}{\text{L}}\right) = \boxed{405\,\text{J}}$$

(*b*) The work done by the gas is the negative of the work done on the gas. Apply the first law of thermodynamics to the system to obtain:

$$\begin{aligned} Q_{\text{in}} &= \Delta E_{\text{int}} - W_{\text{on}} \\ &= (E_{\text{int},2} - E_{\text{int},1}) - (-W_{\text{by gas}}) \\ &= (E_{\text{int},2} - E_{\text{int},1}) + W_{\text{by gas}} \end{aligned}$$

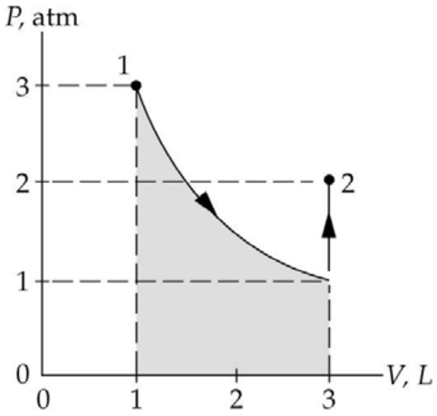
Substitute numerical values and evaluate Q_{in}:

$$Q_{\text{in}} = (912\,\text{J} - 456\,\text{J}) + 405\,\text{J} = \boxed{861\,\text{J}}$$

***54** ••

Picture the Problem We can find the work done by the gas during this process from the area under the curve. Because no work is done along the constant volume (vertical) part of the path, the work done by the gas is done during its isothermal expansion. We can then use the first law of thermodynamics to find the heat added to the system during this process.

(*a*) The path from the initial state (1) to the final state (2) is shown on the *PV* diagram.



The work done by the gas equals the area under the curve:

$$\begin{aligned} W_{\text{by gas}} &= \int_{V_1}^{V_2} P\,dV = nRT_1\int_{1\,\text{L}}^{3\,\text{L}}\frac{dV}{V} \\ &= P_1V_1\int_{1\,\text{L}}^{3\,\text{L}}\frac{dV}{V} = P_1V_1[\ln V]_{1\,\text{L}}^{3\,\text{L}} \\ &= P_1V\ln 3 \end{aligned}$$

Substitute numerical values and evaluate $W_{\text{by gas}}$:

$$W_{\text{by gas}} = \left(3\,\text{atm} \times \frac{101.3\,\text{kPa}}{\text{atm}}\right)\left(1\,\text{L} \times \frac{10^{-3}\,\text{m}^3}{\text{L}}\right)\ln 3 = \boxed{334\,\text{J}}$$

(*b*) The work done by the gas is the negative of the work done on the gas. Apply the first law of thermodynamics to the system to obtain:

$$\begin{aligned} Q_{\text{in}} &= \Delta E_{\text{int}} - W_{\text{on}} \\ &= \left(E_{\text{int},2} - E_{\text{int},1}\right) - \left(-W_{\text{by gas}}\right) \\ &= \left(E_{\text{int},2} - E_{\text{int},1}\right) + W_{\text{by gas}} \end{aligned}$$

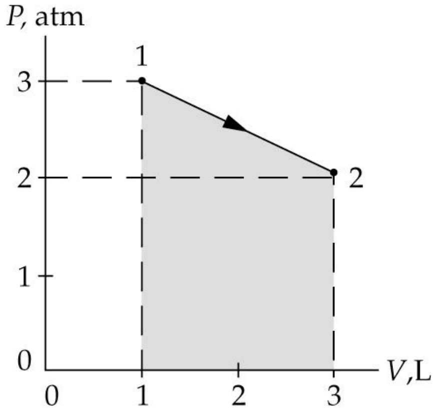
Substitute numerical values and evaluate Q_{in}:

$$Q_{\text{in}} = (912\,\text{J} - 456\,\text{J}) + 334\,\text{J} = \boxed{790\,\text{J}}$$

55 ••

Picture the Problem We can find the work done by the gas during this process from the area under the curve. We can then use the first law of thermodynamics to find the heat added to the system during this process.

(*a*) The path from the initial state (1) to the final state (2) is shown on the *PV* diagram:



The work done by the gas equals the area under the curve:

$$\begin{aligned} W_{\text{by gas}} &= A_{\text{trapezoid}} = \tfrac{1}{2}(3\,\text{atm} + 2\,\text{atm})(2\,\text{L}) \\ &= 5\,\text{atm}\cdot\text{L} \times \frac{101.3\,\text{J}}{\text{atm}\cdot\text{L}} = \boxed{507\,\text{J}} \end{aligned}$$

(*b*) The work done by the gas is the negative of the work done on the gas. Apply the first law of thermodynamics to the system to obtain:

$$\begin{aligned} Q_{\text{in}} &= \Delta E_{\text{int}} - W_{\text{on}} \\ &= \left(E_{\text{int},2} - E_{\text{int},1}\right) - \left(-W_{\text{by gas}}\right) \\ &= \left(E_{\text{int},2} - E_{\text{int},1}\right) + W_{\text{by gas}} \end{aligned}$$

Substitute numerical values and evaluate Q_{in}:

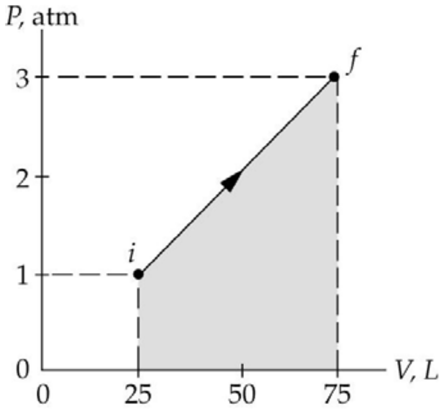
$$Q_{\text{in}} = (912\,\text{J} - 456\,\text{J}) + 507\,\text{J} = \boxed{963\,\text{J}}$$

Remarks: You could use the linearity of the path connecting the initial and final states and the coordinates of the endpoints to express *P* as a function of *V*. You could then integrate this function between 1 and 3 L to find the work done by the gas as it goes from its initial to its final state.

56 ••

Picture the Problem We can find the work done by the gas during this process from the area under the curve.

The path from the initial state *i* to the final state *f* is shown on the *PV* diagram:



The work done by the gas equals the area under the curve:

$$W_{\text{by gas}} = A_{\text{trapezoid}} = \tfrac{1}{2}(1\,\text{atm} + 3\,\text{atm})(50\,\text{L})$$

$$= 100\,\text{atm}\cdot\text{L} \times \frac{101.3\,\text{J}}{\text{atm}\cdot\text{L}} = \boxed{10.1\,\text{kJ}}$$

Remarks: You could use the linearity of the path connecting the initial and final states and the coordinates of the endpoints to express *P* as a function of *V*. You could then integrate this function between 1 and 3 L to find the work done by the gas as it goes from its initial to its final state.

57 ••

Picture the Problem We can find the work done by the gas from the area under the *PV* curve provided we can find the pressure and volume coordinates of the initial and final states. We can find these coordinates by using the ideal gas law and the condition $T = AP^2$.

Apply the ideal-gas law with $n =$ 1 mol and $T = AP^2$ to obtain:

$$PV = RAP^2 \Rightarrow V = RAP \qquad (1)$$

This result tells us that the volume varies linearly with the pressure.

Solve the condition on the temperature for the pressure of the gas:

$$P_0 = \sqrt{\frac{T_0}{A}}$$

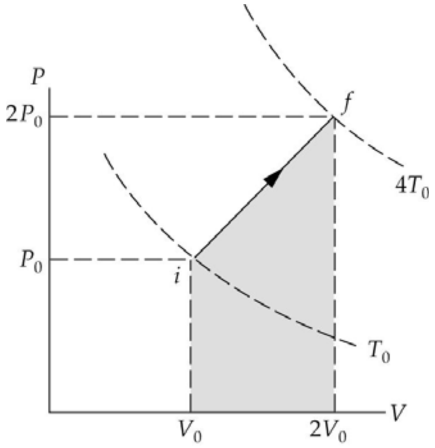
Find the pressure when the temperature is $4T_0$:

$$P=\sqrt{\frac{4T_0}{A}}=2\sqrt{\frac{T_0}{A}}=2P_0$$

Using equation (1), express the coordinates of the final state:

$$(2V_0,2P_0)$$

The *PV* diagram for the process is shown to the right:



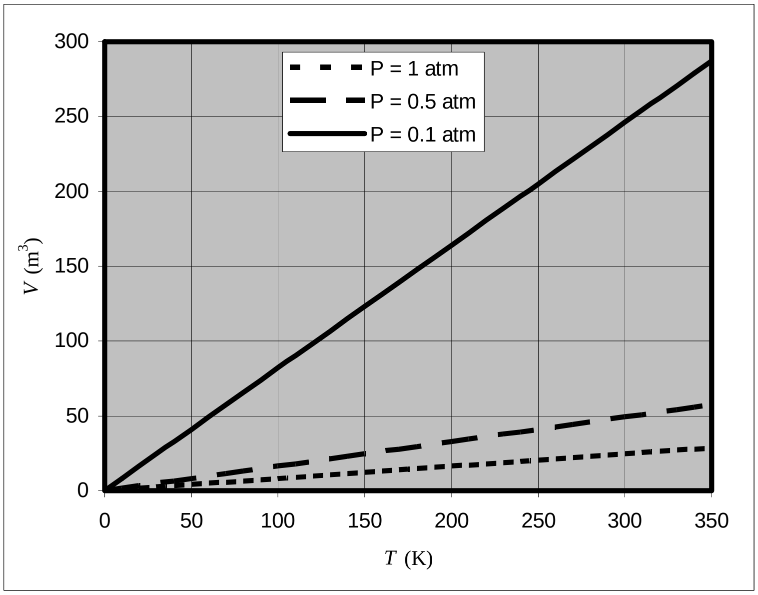
The work done by the gas equals the area under the curve:

$$W_{\text{by gas}}=A_{\text{trapezoid}}=\tfrac{1}{2}(P_0+3P_0)(2V_0-V_0)$$
$$=\boxed{\tfrac{3}{2}P_0V_0}$$

***58** •

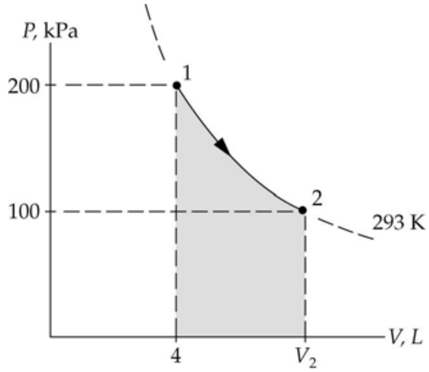
Picture the Problem From the ideal gas law, $PV = NkT$, or $V = NkT/P$. Hence, on a *VT* diagram, isobars will be straight lines with slope $1/P$.

A spreadsheet program was used to plot the following graph. The graph was plotted for 1 mol of gas.



59 ••

Picture the Problem The *PV* diagram shows the isothermal expansion of the ideal gas from its initial state 1 to its final state 2. We can use the ideal-gas law for a fixed amount of gas to find V_2 and then evaluate $\int PdV$ for an isothermal process to find the work done by the gas. In part (b) of the problem we can apply the first law of thermodynamics to find the heat added to the gas during the expansion.



(*a*) Express the work done by a gas during an isothermal process:

$$W_{\text{by gas}} = \int_{V_1}^{V_2} PdV = nRT\int_{V_1}^{V_2}\frac{dV}{V} = P_1V_1\int_{V_1}^{V_2}\frac{dV}{V}$$

Apply the ideal-gas law for a fixed amount of gas undergoing an isothermal process:

$$P_1V_1 = P_2V_2 \text{ or } \frac{V_2}{V_1} = \frac{P_1}{P_1}$$

Solve for and evaluate V_2:

$$V_2 = \frac{P_1}{P_2}V_1 = \frac{200\,\text{kPa}}{100\,\text{kPa}}(4\,\text{L}) = 8\,\text{L}$$

Substitute numerical values and evaluate *W*:

$$\begin{aligned} W_{\text{by gas}} &= (200\,\text{kPa})(4\,\text{L})\int_{4\,\text{L}}^{8\,\text{L}}\frac{dV}{V} \\ &= (800\,\text{kPa}\cdot\text{L})[\ln V]_{4\,\text{L}}^{8\,\text{L}} \\ &= (800\,\text{kPa}\cdot\text{L})\ln\left(\frac{8\,\text{L}}{4\,\text{L}}\right) \\ &= 800\,\text{kPa}\cdot\text{L}\times\frac{10^{-3}\,\text{m}^3}{\text{L}}\boxed{555\,\text{J}} \end{aligned}$$

(*b*) Apply the first law of thermodynamics to the system to obtain:

$$Q_{\text{in}} = \Delta E_{\text{int}} - W_{\text{on}}$$

or, because $\Delta E_{\text{int}} = 0$ for an isothermal process,

$$Q_{\text{in}} = -W_{\text{on}}$$

Because the work done by the gas is the negative of the work done on the gas:

$$Q_{\text{in}} = -(-W_{\text{by gas}}) = W_{\text{by gas}}$$

Substitute numerical values and evaluate Q_{in}:

$$Q_{\text{in}} = \boxed{555\,\text{J}}$$

Heat Capacities of Gases and the Equipartition Theorem

60 •

Picture the Problem We can find the number of moles of the gas from its heat capacity at constant volume using $C_V = \frac{3}{2}nR$. We can find the internal energy of the gas from $E_{\text{int}} = C_V T$ and the heat capacity at constant pressure using $C_P = C_V + nR$.

(*a*) Express C_V in terms of the number of moles in the monatomic gas:

$$C_V = \tfrac{3}{2}nR$$

Solve for *n*:

$$n = \frac{2C_V}{3R}$$

Substitute numerical values and evaluate *n*:

$$n = \frac{2(49.8\,\text{J/K})}{3(8.314\,\text{J/mol}\cdot\text{K})} = \boxed{3.99}$$

(*b*) Relate the internal energy of the gas to its temperature:

$$E_{\text{int}} = C_V T$$

Substitute numerical values and evaluate E_{int}:

$$E_{\text{int}} = (49.8\,\text{J/K})(300\,\text{K}) = \boxed{14.9\,\text{kJ}}$$

(*c*) Relate the heat capacity at constant pressure to the heat capacity at constant volume:

$$C_P = C_V + nR = \tfrac{3}{2}nR + nR = \tfrac{5}{2}nR$$

Substitute numerical values and evaluate C_P:

$$C_P = \tfrac{5}{2}(3.99)(8.314\,\text{J/mol}\cdot\text{K}) = \boxed{82.9\,\text{J/K}}$$

61 •

Picture the Problem The Dulong-Petit law gives the molar specific heat of a solid, c'. The specific heat is defined as $c = c'/M$ where *M* is the molar mass. Hence we can use this definition to find *M* and a periodic table to identify the element.

(*a*) Apply the Dulong-Petit law:

$$c' = 3R \text{ or } c = \frac{3R}{M}$$

Solve for M:

$$M = \frac{3R}{c}$$

Substitute numerical values and evaluate M:

$$M = \frac{24.9\,\text{J/mol}\cdot\text{K}}{0.447\,\text{kJ/kg}\cdot\text{K}} = \boxed{55.7\,\text{g/mol}}$$

Consulting the periodic table of the elements we see that the element is most likely iron.

***62** ••

Picture the Problem The specific heats of air at constant volume and constant pressure are given by $c_V = C_V/m$ and $c_P = C_P/m$ and the heat capacities at constant volume and constant pressure are given by $C_V = \frac{5}{2}nR$ and $C_P = \frac{7}{2}nR$, respectively.

(*a*) Express the specific heats per unit mass of air at constant volume and constant pressure:

$$c_V = \frac{C_V}{m} \quad (1)$$

and

$$c_P = \frac{C_P}{m} \quad (2)$$

Express the heat capacities of a diatomic gas in terms of the gas constant R, the number of moles n, and the number of degrees of freedom:

$$C_V = \tfrac{5}{2}nR$$

and

$$C_P = \tfrac{7}{2}nR$$

Express the mass of 1 mol of air:

$$m = 0.74M_{N_2} + 0.26M_{O_2}$$

Substitute in equation (1) to obtain:

$$c_V = \frac{5nR}{2\left(0.74M_{N_2} + 0.26M_{O_2}\right)}$$

Substitute numerical values and evaluate c_V:

$$c_V = \frac{5(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})}{2\left[0.74\left(28\times10^{-3}\,\text{kg}\right) + 0.26\left(32\times10^{-3}\,\text{kg}\right)\right]} = \boxed{716\,\text{J/kg}\cdot\text{K}}$$

Substitute in equation (2) to obtain:

$$c_P = \frac{7nR}{2\left(0.74M_{N_2} + 0.26M_{O_2}\right)}$$

Substitute numerical values and evaluate c_P:

$$c_P = \frac{7(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})}{2\left[0.74\left(28\times10^{-3}\,\text{kg}\right)+0.26\left(32\times10^{-3}\,\text{kg}\right)\right]} = \boxed{1002\,\text{J/kg}\cdot\text{K}}$$

(*b*) Express the percent difference between the value from the *Handbook of Chemistry and Physics* and the calculated value:

$$\%\,\text{difference} = \frac{1.032\,\text{J/g}\cdot\text{K} - 1.002\text{J/g}\cdot\text{K}}{1.032\,\text{J/g}\cdot\text{K}} = \boxed{2.91\%}$$

63 ••

Picture the Problem We know that, during a constant-volume process, no work is done and that we can calculate the heat added during this expansion from the heat capacity at constant volume and the change in the absolute temperature. We can then use the first law of thermodynamics to find the change in the internal energy of the gas. In part (*b*), we can proceed similarly; using the heat capacity at constant pressure rather than constant volume.

(*a*) For a constant-volume process:

$$W_{\text{on}} = \boxed{0}$$

Relate the heat added to the gas to its heat capacity at constant volume and the change in its temperature:

$$Q_{\text{in}} = C_V\Delta T = \tfrac{5}{2}nR\Delta T$$

Substitute numerical values and evaluate Q_{in}:

$$Q_{\text{in}} = \tfrac{5}{2}(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(300\,\text{K}) = \boxed{6.24\,\text{kJ}}$$

Apply the first law of thermodynamics to find ΔE_{int}:

$$\Delta E_{\text{int}} = Q_{\text{in}} + W_{\text{on}} = 6.24\,\text{kJ} + 0 = \boxed{6.24\,\text{kJ}}$$

(*b*) Relate the heat added to the gas to its heat capacity at constant pressure and the change in its temperature:

$$Q_{\text{in}} = C_P\Delta T = \left(\tfrac{5}{2}nR + nR\right)\Delta T$$

Substitute numerical values and evaluate Q_{in}:

$$Q_{\text{in}} = \tfrac{7}{2}(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(300\,\text{K}) = \boxed{8.73\,\text{kJ}}$$

Because ΔE_{int} depends only on the temperature difference:

$$\Delta E_{\text{int}} = \boxed{6.24\,\text{kJ}}$$

Apply the first law of thermodynamics to find *W*:

$$W_{\text{on}} = \Delta E_{\text{int}} - Q_{\text{in}} = 8.73\,\text{kJ} - 6.24\,\text{kJ}$$
$$= \boxed{2.49\,\text{kJ}}$$

(*c*) Integrate $dW_{\text{on}} = P\,dV$ to obtain:

$$W_{\text{on}} = \int_{V_{\text{i}}}^{V_{\text{f}}} P dV = P\left(V_{\text{f}} - V_{\text{i}}\right) = nR\left(T_{\text{f}} - T_{\text{i}}\right)$$

Substitute numerical values and evaluate W_{on}:

$$W_{\text{on}} = (1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(300\,\text{K})$$
$$= \boxed{2.49\,\text{kJ}}$$

64 ••

Picture the Problem Because this is a constant-volume process, we can use $Q = C_{\text{V}}\Delta T$ to express *Q* in terms of the temperature change and the ideal-gas law for a fixed amount of gas to find ΔT.

Express the amount of heat *Q* that must be transferred to the gas if its pressure is to triple:

$$Q = C_{\text{V}}\Delta T$$
$$= \tfrac{5}{2} nR\left(T_{\text{f}} - T_0\right)$$

Using the ideal-gas law for a fixed amount of gas, relate the initial and final temperatures, pressures and volumes:

$$\frac{P_0 V}{T_0} = \frac{3P_0 V}{T_{\text{f}}}$$

Solve for T_{f}:

$$T_{\text{f}} = 3T_0$$

Substitute and simplify to obtain:

$$Q = \tfrac{5}{2} nR\left(2T_0\right) = 5\left(nRT_0\right) = \boxed{5P_0 V}$$

65 ••

Picture the Problem Let the subscripts i and f refer to the initial and final states of the gas, respectively. We can use the ideal-gas law for a fixed amount of gas to express *V′* in terms of *V* and the change in temperature of the gas when 13,200 J of heat are transferred to it. We can find this change in temperature using $Q = C_{\text{P}}\Delta T$.

Using the ideal-gas law for a fixed amount of gas, relate the initial and final temperatures, volumes, and

$$\frac{P_{\text{i}} V}{T_{\text{i}}} = \frac{P_{\text{f}} V'}{T_{\text{f}}}$$

pressures:

Because the process is isobaric, we can solve for *V′* to obtain:

$$V' = V\frac{T_f}{T_i} = V\frac{T_i + \Delta T}{T_i} = V\left(1 + \frac{\Delta T}{T_i}\right)$$

Relate the heat transferred to the gas to the change in its temperature:

$$Q = C_P\Delta T = \tfrac{7}{2}nR\Delta T$$

Solve Δ*T*:

$$\Delta T = \frac{2Q}{7nR}$$

Substitute to obtain:

$$V' = V\left(1 + \frac{2Q}{7nRT_i}\right)$$

One mol of gas at STP occupies 22.4 L. Substitute numerical values and evaluate *V′*:

$$V' = \left(22.4\times10^{-3}\,\text{m}^3\right)\left(1 + \frac{2(13.2\,\text{kJ})}{7(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(273\,\text{K})}\right) = \boxed{59.6\,\text{L}}$$

66 ••

Picture the Problem We can use the relationship between C_P and C_V ($C_P = C_V + nR$) to find the number of moles of this particular gas. In parts (*b*) and (*c*) we can use the number of degrees of freedom associated with monatomic and diatomic gases, respectively, to find C_P and C_V.

(*a*) Express the heat capacity of the gas at constant pressure to its heat capacity at constant volume:

$$C_P = C_V + nR$$

Solve for *n*:

$$n = \frac{C_P - C_V}{R}$$

Substitute numerical values and evaluate *n*:

$$n = \frac{29.1\,\text{J/K}}{8.314\,\text{J/mol}\cdot\text{K}} = \boxed{3.50\,\text{mol}}$$

(*b*) Express and evaluate C_V for a monatomic gas:

$$C_V = \tfrac{3}{2}nR$$

Substitute numerical values and evaluate C_V:

$$C_V = \tfrac{3}{2}(3.5\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K}) = \boxed{43.6\,\text{J/K}}$$

Express C_P for a monatomic gas:

$$C_P = \tfrac{5}{2}nR$$

Substitute numerical values and evaluate C_P:

$$C_P = \tfrac{5}{2}(3.5\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K}) = \boxed{72.7\,\text{J/K}}$$

(*c*) If the diatomic molecules rotate but do not vibrate they have 5 degrees of freedom:

$$C_V = \tfrac{5}{2}nR = \tfrac{5}{2}(3.5\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K}) = \boxed{72.7\,\text{J/K}}$$

and

$$C_P = \tfrac{7}{2}nR = \tfrac{7}{2}(3.5\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K}) = \boxed{102\,\text{J/K}}$$

***67 ••**

Picture the Problem We can find the change in the heat capacity at constant pressure as CO_2 undergoes sublimation from the energy per molecule of CO_2 in the solid and gaseous states.

Express the change in the heat capacity (at constant pressure) per mole as the CO_2 undergoes sublimation:

$$\Delta C_P = C_{P,\text{gas}} - C_{P,\text{solid}}$$

Express $C_{p,\text{gas}}$ in terms of the number of degrees of freedom per molecule:

$$C_{P,\text{gas}} = f\left(\tfrac{1}{2}Nk\right) = \tfrac{5}{2}Nk$$

because each molecule has three translational and two rotational degrees of freedom in the gaseous state.

We know, from the Dulong-Petit Law, that the molar specific heat of most solids is $3R = 3Nk$. This result is essentially a per-atom result as it was obtained for a monatomic solid with six degrees of freedom. Use this result and the fact CO_2 is triatomic to express $C_{p,\text{solid}}$:

$$C_{P,\text{solid}} = \frac{3Nk}{\text{atom}} \times 3\,\text{atoms} = 9Nk$$

Substitute to obtain:

$$\Delta C_P = \tfrac{5}{2}Nk - \tfrac{18}{2}Nk = \boxed{-\tfrac{13}{2}Nk}$$

68 ••

Picture the Problem We can find the initial internal energy of the gas from $U_i = \tfrac{3}{2}nRT$ and the final internal energy from the change in internal energy resulting from the addition of 500 J of heat. The work done during a constant-volume process is zero and the work done during the constant-pressure process can be found from the first law of thermodynamics.

(*a*) Express the initial internal energy of the gas in terms of its temperature:

$$E_{\text{int,i}} = \tfrac{3}{2}nRT$$

Substitute numerical values and evaluate $E_{\text{int,i}}$:

$$E_{\text{int,i}} = \tfrac{3}{2}(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(273\,\text{K})$$
$$= \boxed{3.40\,\text{kJ}}$$

(*b*) Relate the final internal energy of the gas to its initial internal energy:

$$E_{\text{int,f}} = E_{\text{int,i}} + \Delta E_{\text{int}} = E_{\text{int,i}} + C_{\text{V}}\Delta T$$

Express the change in temperature of the gas resulting from the addition of heat:

$$\Delta T = \frac{Q_{\text{in}}}{C_{\text{P}}}$$

Substitute to obtain:

$$E_{\text{int,f}} = E_{\text{int,i}} + \frac{C_{\text{V}}}{C_{\text{P}}}Q_{\text{in}}$$

Substitute numerical values and evaluate $E_{\text{int,f}}$:

$$E_{\text{int,f}} = 3.40\,\text{kJ} + \frac{\tfrac{3}{2}nR}{\tfrac{5}{2}nR}(500\,\text{J}) = \boxed{3.70\,\text{kJ}}$$

(*c*) Relate the final internal energy of the gas to its initial internal energy:

$$E_{\text{int,f}} = E_{\text{int,i}} + \Delta E_{\text{int}}$$

Apply the first law of thermodynamics to the constant-volume process:

$$\Delta E_{\text{int}} = Q_{\text{in}} + W_{\text{on}}$$

or, because $W_{\text{on}} = 0$,

$$\Delta E_{\text{int}} = Q_{\text{in}} = 500\,\text{J}$$

Substitute numerical values and evaluate $E_{\text{int,f}}$:

$$E_{\text{int,f}} = 3.40\,\text{kJ} + 500\,\text{J} = \boxed{3.90\,\text{kJ}}$$

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Picture the Problem We can use $C_{\text{V,water}} = f\left(\tfrac{1}{2}Nk\right)$ to express $C_{\text{V,water}}$ and then count the number of degrees of freedom associated with a water molecule to determine *f*.

Express $C_{\text{V,water}}$ in terms of the number of degrees of freedom per

$$C_{\text{V,water}} = f\left(\tfrac{1}{2}Nk\right)$$

where *f* is the number of degrees of

molecule: freedom associated with a water molecule.

There are three translational degrees of freedom and three rotational degrees of freedom. In addition, each of the hydrogen atoms can vibrate against the oxygen atom, resulting in an additional 4 degrees of freedom (2 per atom).

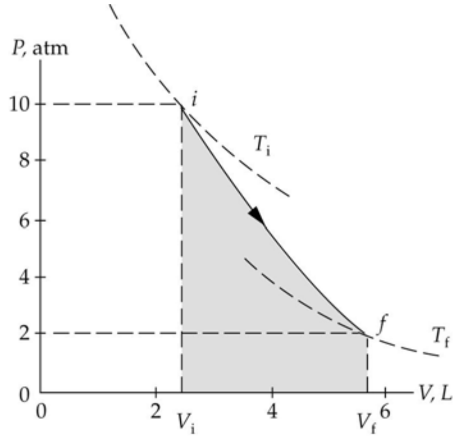
Substitute for *f* to obtain:

$$C_{\mathrm{V,water}} = 10\left(\tfrac{1}{2}Nk\right) = \boxed{5Nk}$$

Quasi-Static Adiabatic Expansion of a Gas

***70 ••**

Picture the Problem The adiabatic expansion is shown in the *PV* diagram. We can use the ideal-gas law to find the initial volume of the gas and the equation for a quasi-static adiabatic process to find the final volume of the gas. A second application of the ideal-gas law, this time at the final state, will yield the final temperature of the gas. In part (*c*) we can use the first law of thermodynamics to find the work done by the gas during this process.



(*a*) Apply the ideal-gas law to express the initial volume of the gas:

$$V_i = \frac{nRT_i}{P_i}$$

Substitute numerical values and evaluate V_i:

$$V_i = \frac{(1\,\mathrm{mol})(8.314\,\mathrm{J/mol\cdot K})(273\,\mathrm{K})}{10\,\mathrm{atm}\times\dfrac{101.3\,\mathrm{kPa}}{\mathrm{atm}}} = 2.24\times10^{-3}\,\mathrm{m}^3 = \boxed{2.24\,\mathrm{L}}$$

Use the relationship between the pressures and volumes for a quasi-static adiabatic process to express V_f:

$$P_iV_i^{\gamma} = P_fV_f^{\gamma} \Rightarrow V_f = V_i\left(\frac{P_i}{P_f}\right)^{1/\gamma}$$

Substitute numerical values and evaluate V_f:

$$V_\text{f} = V_\text{i}\left(\frac{P_\text{i}}{P_\text{f}}\right)^{1/\gamma} = (2.24\,\text{L})\left(\frac{10\,\text{atm}}{2\,\text{atm}}\right)^{3/5} = \boxed{5.88\,\text{L}}$$

(*b*) Apply the ideal-gas law to express the final temperature of the gas:

$$T_\text{f} = \frac{P_\text{f}V_\text{f}}{nR}$$

Substitute numerical values and evaluate T_f:

$$T_\text{f} = \frac{(2\,\text{atm})(5.88\,\text{L})}{8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K}} = \boxed{143\,\text{K}}$$

(*c*) Apply the first law of thermodynamics to express the work done on the gas:

$$W_\text{on} = \Delta E_\text{int} - Q_\text{in}$$

or, because the process is adiabatic,

$$W_\text{on} = \Delta E_\text{int} = C_\text{V}\Delta T = \tfrac{3}{2}nR\Delta T$$

Substitute numerical values and evaluate W_on:

$$W_\text{on} = \tfrac{3}{2}(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(-130\,\text{K}) = -1.62\,\text{kJ}$$

Because $W_\text{by the gas} = -W_\text{on}$:

$$W_\text{by gas} = \boxed{1.62\,\text{kJ}}$$

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Picture the Problem We can use the temperature-volume equation for a quasi-static adiabatic process to express the final temperature of the gas in terms of its initial temperature and the ratio of its heat capacities γ. Because $C_\text{P} = C_\text{V} + nR$, we can determine γ for each of the given heat capacities at constant volume.

Express the temperature-volume relationship for a quasi-static adiabatic process:

$$T_\text{i}V_\text{i}^{\gamma\text{-}1} = T_\text{f}V_\text{f}^{\gamma\text{-}1}$$

Solve for the final temperature:

$$T_\text{f} = T_\text{i}\left(\frac{V_\text{i}}{V_\text{f}}\right)^{\gamma-1} = T_\text{i}\left(\frac{V_\text{i}}{\frac{1}{2}V_\text{i}}\right)^{\gamma-1} = T_\text{i}(2)^{\gamma-1}$$

(*a*) Evaluate γ for $\text{C}_\text{V} = \frac{3}{2}nR$:

$$\gamma = \frac{C_\text{P}}{C_\text{V}} = \frac{\frac{5}{2}nR}{\frac{3}{2}nR} = \frac{5}{3}$$

Evaluate T_f:

$$T_\text{f} = (293\,\text{K})(2)^{\frac{5}{3}-1} = \boxed{465\,\text{K}}$$

(*b*) Evaluate γ for $C_V = \frac{5}{2}nR$:

$$\gamma = \frac{C_P}{C_V} = \frac{\frac{7}{2}nR}{\frac{5}{2}nR} = \frac{7}{5}$$

Evaluate T_f:

$$T_f = (293\,\text{K})(2)^{\frac{7}{5}-1} = \boxed{387\,\text{K}}$$

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Picture the Problem We can use the temperature-volume and pressure-volume equations for a quasi-static adiabatic process to express the final temperature and pressure of the gas in terms of its initial temperature and pressure and the ratio of its heat capacities.

Express the temperature-volume relationship for a quasi-static adiabatic process:

$$T_iV_i^{\gamma-1} = T_fV_f^{\gamma-1}$$

Solve for the final temperature:

$$T_f = T_i\left(\frac{V_i}{V_f}\right)^{\gamma-1} = T_i\left(\frac{V_i}{\frac{1}{4}V_i}\right)^{\gamma-1} = T_i(4)^{\gamma-1}$$

Using γ = 5/3 for neon, evaluate T_f:

$$T_f = (293\,\text{K})(4)^{\frac{5}{3}-1} = \boxed{738\,\text{K}}$$

Express the relationship between the pressures and volumes for a quasi-static adiabatic process:

$$P_iV_i^{\gamma} = P_fV_f^{\gamma}$$

Solve for P_f:

$$P_f = P_i\left(\frac{V_i}{\frac{1}{4}V_i}\right)^{\gamma}$$

Substitute numerical values and evaluate P_f:

$$P_f = (1\,\text{atm})(4)^{5/3} = \boxed{10.1\,\text{atm}}$$

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Picture the Problem We can use the ideal-gas law to find the initial volume of the gas. In part (*a*) we can apply the ideal-gas law for a fixed amount of gas to find the final volume and the expression (Equation 19-16) for the work done in an isothermal process. Application of the first law of thermodynamics will allow us to find the heat absorbed by the gas during this process. In part (*b*) we can use the relationship between the pressures and volumes for a quasi-static adiabatic process to find the final volume of the gas. We can apply the ideal-gas law to find the final temperature and, as in (*a*), apply the first law of thermodynamics, this time to find the work done by the gas.

Use the ideal-gas law to express the initial volume of the gas:

$$V_\mathrm{i} = \frac{nRT_\mathrm{i}}{P_\mathrm{i}}$$

Substitute numerical values and evaluate V_i:

$$V_\mathrm{i} = \frac{(0.5\,\mathrm{mol})(8.314\,\mathrm{J/mol\cdot K})(300\,\mathrm{K})}{400\,\mathrm{kPa}} = 3.12\times10^{-3}\,\mathrm{m}^3 = 3.12\,\mathrm{L}$$

(*a*) Because the process is isothermal:

$$T_\mathrm{f} = T_\mathrm{i} = \boxed{300\,\mathrm{K}}$$

Use the ideal-gas law for a fixed amount of gas to express V_f:

$$\frac{P_\mathrm{i}V_\mathrm{i}}{T_\mathrm{i}} = \frac{P_\mathrm{f}V_\mathrm{f}}{T_\mathrm{f}}$$

or, because T = constant,

$$V_\mathrm{f} = V_\mathrm{i}\frac{P_\mathrm{i}}{P_\mathrm{f}}$$

Substitute numerical values and evaluate V_f:

$$V_\mathrm{f} = (3.12\,\mathrm{L})\left(\frac{400\,\mathrm{kPa}}{160\,\mathrm{kPa}}\right) = \boxed{7.80\,\mathrm{L}}$$

Express the work done by the gas during the isothermal expansion:

$$W_\text{by gas} = nRT\ln\frac{V_\mathrm{f}}{V_\mathrm{i}}$$

Substitute numerical values and evaluate $W_\text{by gas}$:

$$W_\text{by gas} = (0.5\,\mathrm{mol})(8.314\,\mathrm{J/mol\cdot K})\times(300\,\mathrm{K})\ln\left(\frac{7.80\,\mathrm{L}}{3.12\,\mathrm{L}}\right) = \boxed{1.14\,\mathrm{kJ}}$$

Noting that the work done by the gas during the process equals the negative of the work done on the gas, apply the first law of thermodynamics to find the heat absorbed by the gas:

$$Q_\mathrm{in} = \Delta E_\mathrm{int} - W_\mathrm{on} = 0 - (-1.14\,\mathrm{kJ}) = \boxed{1.14\,\mathrm{kJ}}$$

(*b*) Using $\gamma = 5/3$ and the relationship between the pressures and volumes for a quasi-static adiabatic process, express V_f:

$$P_\mathrm{i}V_\mathrm{i}^\gamma = P_\mathrm{f}V_\mathrm{f}^\gamma \Rightarrow V_\mathrm{f} = V_\mathrm{i}\left(\frac{P_\mathrm{i}}{P_\mathrm{f}}\right)^{1/\gamma}$$

Substitute numerical values and evaluate V_f:

$$V_f = (3.12\,\text{L})\left(\frac{400\,\text{kPa}}{160\,\text{kPa}}\right)^{3/5} = \boxed{5.41\,\text{L}}$$

Apply the ideal-gas law to find the final temperature of the gas:

$$T_f = \frac{P_f V_f}{nR}$$

Substitute numerical values and evaluate T_f:

$$T_f = \frac{(160\,\text{kPa})(5.41\times10^{-3}\,\text{m}^3)}{(0.5\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})} = \boxed{208\,\text{K}}$$

For an adiabatic process:

$$Q_{in} = \boxed{0}$$

Apply the first law of thermodynamics to express the work done on the gas during the adiabatic process:

$$W_{on} = \Delta E_{int} - Q_{in} = C_V \Delta T - 0 = \tfrac{3}{2} nR\Delta T$$

Substitute numerical values and evaluate W_{on}:

$$W_{on} = \tfrac{3}{2}(0.5\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K}) \times (208\,\text{K} - 300\,\text{K}) = -574\,\text{J}$$

Because the work done by the gas equals the negative of the work done on the gas:

$$W_{by\ gas} = -(-574\,\text{J}) = \boxed{574\,\text{J}}$$

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Picture the Problem We can use the ideal-gas law to find the initial volume of the gas. In part (*a*) we can apply the ideal-gas law for a fixed amount of gas to find the final volume and the expression (Equation 19-16) for the work done in an isothermal process. Application of the first law of thermodynamics will allow us to find the heat absorbed by the gas during this process. In part (*b*) we can use the relationship between the pressures and volumes for a quasi-static adiabatic process to find the final volume of the gas. We can apply the ideal-gas law to find the final temperature and, as in (*a*), apply the first law of thermodynamics, this time to find the work done by the gas.

Use the ideal-gas law to express the initial volume of the gas:

$$V_i = \frac{nRT_i}{P_i}$$

Substitute numerical values and evaluate V_i:

$$V_i = \frac{(0.5\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(300\,\text{K})}{400\,\text{kPa}}$$
$$= 3.12\times10^{-3}\,\text{m}^3 = 3.12\,\text{L}$$

(*a*) Because the process is isothermal:

$$T_f = T_i = \boxed{300\,\text{K}}$$

Use the ideal-gas law for a fixed amount of gas to express V_f:

$$\frac{P_iV_i}{T_i} = \frac{P_fV_f}{T_f}$$

or, because T = constant,

$$V_f = V_i\frac{P_i}{P_f}$$

Substitute numerical values and evaluate T_f:

$$V_f = (3.12\,\text{L})\left(\frac{400\,\text{kPa}}{160\,\text{kPa}}\right) = \boxed{7.80\,\text{L}}$$

Express the work done by the gas during the isothermal expansion:

$$W_{\text{by gas}} = nRT\ln\frac{V_f}{V_i}$$

Substitute numerical values and evaluate $W_{\text{by gas}}$:

$$W_{\text{by gas}} = (0.5\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})$$
$$\times(300\,\text{K})\ln\left(\frac{7.80\,\text{L}}{3.12\,\text{L}}\right)$$
$$= \boxed{1.14\,\text{kJ}}$$

Noting that the work done by the gas during the isothermal expansion equals the negative of the work done on the gas, apply the first law of thermodynamics to find the heat absorbed by the gas:

$$Q_{\text{in}} = \Delta E_{\text{int}} - W_{\text{on}} = 0 - (-1.14\,\text{kJ})$$
$$= \boxed{1.14\,\text{kJ}}$$

(*b*) Using $\gamma = 1.4$ and the relationship between the pressures and volumes for a quasi-static adiabatic process, express V_f:

$$P_iV_i^{\gamma} = P_fV_f^{\gamma} \Rightarrow V_f = V_i\left(\frac{P_i}{P_f}\right)^{1/\gamma}$$

Substitute numerical values and evaluate V_f:

$$V_f = (3.12\,\text{L})\left(\frac{400\,\text{kPa}}{160\,\text{kPa}}\right)^{1/1.4} = \boxed{6.00\,\text{L}}$$

Apply the ideal-gas law to express the final temperature of the gas:

$$T_f = \frac{P_f V_f}{nR}$$

Substitute numerical values and evaluate T_f:

$$T_f = \frac{(160\,\text{kPa})(6\times 10^{-3}\,\text{m}^3)}{(0.5\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})} = \boxed{231\,\text{K}}$$

For an adiabatic process:

$$Q_{in} = \boxed{0}$$

Apply the first law of thermodynamics to express the work done on the gas during the adiabatic expansion:

$$W_{on} = \Delta E_{int} - Q_{in} = C_V \Delta T - 0 = \tfrac{5}{2} nR\Delta T$$

Substitute numerical values and evaluate W_{on}:

$$W_{on} = \tfrac{5}{2}(0.5\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K}) \times (231\,\text{K} - 300\,\text{K}) = -717\,\text{J}$$

Noting that the work done by the gas during the adiabatic expansion is the negative of the work done on the gas, we have:

$$W_{by\ gas} = -(-717\,\text{J}) = \boxed{717\,\text{J}}$$

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Picture the Problem We can eliminate the volumes from the equations relating the temperatures and volumes and the pressures and volumes for a quasi-static adiabatic process to obtain a relationship between the temperatures and pressures. We can find the initial volume of the gas using the ideal-gas law and the final volume using the pressure-volume relationship. In parts (*d*) and (*c*) we can find the change in the internal energy of the gas from the change in its temperature and use the first law of thermodynamics to find the work done by the gas during its expansion.

(*a*) Express the relationship between temperatures and volumes for a quasi-static adiabatic process:

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$

Express the relationship between pressures and volumes for a quasi-static adiabatic process:

$$P_i V_i^{\gamma} = P_f V_f^{\gamma} \qquad (1)$$

Eliminate the volume between these two equations to obtain:

$$T_f = T_i\left(\frac{P_f}{P_i}\right)^{1-\frac{1}{\gamma}}$$

Substitute numerical values and evaluate T_f:

$$T_f = (500\,\text{K})\left(\frac{1\,\text{atm}}{5\,\text{atm}}\right)^{1-\frac{1}{5/3}} = \boxed{263\,\text{K}}$$

(*b*) Solve equation (1) for V_f:

$$V_f = V_i\left(\frac{P_i}{P_f}\right)^{\frac{1}{\gamma}}$$

Apply the ideal-gas law to express V_i:

$$V_i = \frac{nRT_i}{P_i}$$

Substitute numerical values and evaluate V_i:

$$V_i = \frac{(0.5\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(500\,\text{K})}{5\text{atm}\times\dfrac{101.3\,\text{kPa}}{\text{atm}}} = 4.10\,\text{L}$$

Substitute for V_i and evaluate V_f:

$$V_f = (4.10\,\text{L})\left(\frac{5\,\text{atm}}{1\text{atm}}\right)^{\frac{3}{5}} = \boxed{10.8\,\text{L}}$$

(*d*) Relate the change in the internal energy of the helium gas to the change in its temperature:

$$\Delta E_{\text{int}} = C_V\Delta T = \tfrac{3}{2}nR\Delta T$$

Substitute numerical values and evaluate ΔE_{int}:

$$\Delta E_{\text{int}} = \tfrac{3}{2}(0.5\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})\times(263\,\text{K}-500\,\text{K}) = \boxed{-1.48\,\text{kJ}}$$

(*c*) Use the first law of thermodynamics to express the work done on the gas:

$$W_{\text{on}} = \Delta E_{\text{int}} - Q_{\text{in}} = \Delta E_{\text{int}} - 0 = \Delta E_{\text{int}}$$

Substitute numerical values and evaluate W_{on}:

$$W_{\text{on}} = -1.48\,\text{kJ}$$

Because the work done by the gas equals the negative of W_{on}:

$$W_{\text{by gas}} = -W_{\text{on}} = -(-1.48\,\text{kJ}) = \boxed{1.48\,\text{kJ}}$$

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Picture the Problem Consider the process to be accomplished in a single compression. The initial pressure is 1 atm = 101 kPa. The final pressure is (101 + 482) kPa = 583 kPa, and the final volume is 1 L. Because air is a mixture of diatomic gases, $\gamma_{\text{air}} = 1.4$. We can find the initial volume of the air using $P_\text{i}V_\text{i}^\gamma = P_\text{f}V_\text{f}^\gamma$ and use Equation 19-39 to find the work done by the air.

Express the work done in an adiabatic process:

$$W = \frac{P_\text{i}V_\text{i} - P_\text{f}V_\text{f}}{\gamma - 1} \qquad (1)$$

Use the relationship between pressure and volume for a quasi-static adiabatic process to express the initial volume of the air:

$$P_\text{i}V_\text{i}^\gamma = P_\text{f}V_\text{f}^\gamma \Rightarrow V_\text{i} = V_\text{f}\left(\frac{P_\text{f}}{P_\text{i}}\right)^{\frac{1}{\gamma}}$$

Substitute numerical values and evaluate V_i:

$$V_\text{i} = (1\,\text{L})\left(\frac{583\,\text{kPa}}{101\,\text{kPa}}\right)^{\frac{1}{1.4}} = 3.50\,\text{L}$$

Substitute numerical values in equation (1) and evaluate W:

$$W = \frac{(101\,\text{kPa})(3.5\times10^{-3}\,\text{m}^3) - (583\,\text{kPa})(10^{-3}\,\text{m}^3)}{1.4-1} = \boxed{-574\,\text{J}}$$

where the minus sign tells us that work is done on the gas.

77 •••

Picture the Problem We can integrate PdV using the equation of state for an adiabatic process to obtain Equation 18-39.

Express the work done by the gas during this adiabatic expansion:

$$W_{\text{by gas}} = \int_{V_1}^{V_2} PdV$$

For an adiabatic process:

$$PV^\gamma = \text{constant} = C \qquad (1)$$

and

$$P = CV^{-\gamma}$$

Substitute and evaluate the integral to obtain:

$$W_{\text{by gas}} = C\int_{V_1}^{V_2} V^{-\gamma}dV = \frac{C}{1-\gamma}\left(V_2^{1-\gamma} - V_1^{1-\gamma}\right)$$

From equation (1) we have:

$$CV_2^{1-\gamma} = P_2V_2^\gamma \text{ and } CV_1^{1-\gamma} = P_1V_1^\gamma$$

Substitute to obtain:

$$W_{\text{by gas}} = \frac{P_2V_2^\gamma - P_1V_1^\gamma}{1-\gamma} = \boxed{\frac{P_1V_1^\gamma - P_2V_2^\gamma}{\gamma - 1}},$$

which is Equation 18-39.

Cyclic Processes

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Picture the Problem To construct the *PV* diagram we'll need to determine the volume occupied by the gas at the beginning and ending points for each process. Let these points be A, B, C, and D. We can apply the ideal-gas law to the starting point (A) to find V_A. To find the volume at point B, we can use the relationship between pressure and volume for a quasi-static adiabatic process. We can use the ideal-gas law to find the volume at point C and, because they are equal, the volume at point D. We can apply the first law of thermodynamics to find the amount of heat added to or subtracted from the gas during the complete cycle.

(*a*) Using the ideal-gas law, express the volume of the gas at the starting point A of the cycle:

$$V_A = \frac{nRT_A}{P_A}$$

Substitute numerical values and evaluate V_A:

$$V_A = \frac{(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(293\,\text{K})}{5\,\text{atm}\times\dfrac{101.3\,\text{kPa}}{\text{atm}}} = 4.81\,\text{L}$$

Use the relationship between pressure and volume for a quasi-static adiabatic process to express the volume of the gas at point B; the end point of the adiabatic expansion:

$$V_B = V_A\left(\frac{P_A}{P_B}\right)^{\frac{1}{\gamma}}$$

Substitute numerical values and evaluate V_B:

$$V_B = (4.81\,\text{L})\left(\frac{5\,\text{atm}}{1\,\text{atm}}\right)^{\frac{1}{1.4}} = 15.2\,\text{L}$$

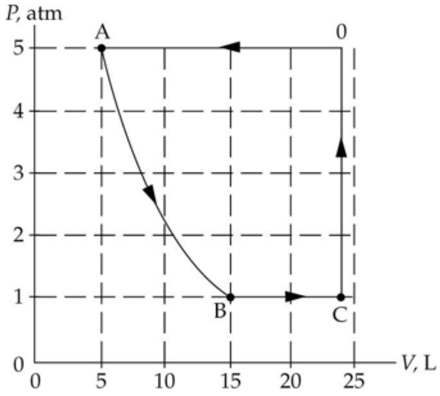
Using the ideal-gas law for a fixed amount of gas, express the volume occupied by the gas at points C and D:

$$V_C = V_D = \frac{nRT_C}{P_C}$$

Substitute numerical values and evaluate V_C:

$$V_C = \frac{(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(293\,\text{K})}{1\,\text{atm}\times\dfrac{101.3\,\text{kPa}}{\text{atm}}} = 24.0\,\text{L}$$

The complete cycle is shown in the diagram.



(*b*) Note that for the paths A→B and B→C, $W_{\text{by gas}}$, the work done by the gas, is positive. For the path D→A, $W_{\text{by gas}}$ is negative, and greater in magnitude than $W_{A\to C}$. Therefore the total work done by the gas is negative. Find the area enclosed by the cycle by noting that each rectangle of dotted lines equals 5 atm·L and counting the rectangles:

$$\begin{aligned} W_{\text{by gas}} &\approx -(13\,\text{rectangles}) \\ &\quad\times(5\,\text{atm}\cdot\text{L/rectangle}) \\ &= (-65\,\text{atm}\cdot\text{L})\left(\frac{101.3\,\text{J}}{\text{atm}\cdot\text{L}}\right) \\ &= \boxed{-6.58\,\text{kJ}} \end{aligned}$$

(*c*) The work done on the gas equals the negative of the work done by the gas. Apply the first law of thermodynamics to find the amount of heat added to or subtracted from the gas during the complete cycle:

$$\begin{aligned} Q_{\text{in}} &= \Delta E_{\text{int}} - W_{\text{on}} = 0-(-6.58\,\text{kJ}) \\ &= \boxed{6.58\,\text{kJ}} \end{aligned}$$

because $\Delta E_{\text{int}} = 0$ for the complete cycle.

(*d*) Express the work done during the complete cycle:

$$W = W_{A\to B} + W_{B\to C} + W_{C\to D} + W_{D\to A}$$

A→B is an adiabatic process:

$$\begin{aligned} W_{\mathrm{A\to B}} &= \frac{P_{\mathrm{A}}V_{\mathrm{A}}^{\gamma}-P_{\mathrm{B}}V_{\mathrm{B}}^{\gamma}}{\gamma-1} \\ &= \frac{(5\,\mathrm{atm})(4.82\,\mathrm{L})-(1\mathrm{atm})(15.2\,\mathrm{L})}{1.4-1} \\ &= (22.3\,\mathrm{atm\cdot L})\left(\frac{101.3\,\mathrm{J}}{\mathrm{atm\cdot L}}\right) \\ &= 2.25\,\mathrm{kJ} \end{aligned}$$

B→C is an isobaric process:

$$\begin{aligned} W_{\mathrm{B\to C}} &= P\Delta V \\ &= (1\mathrm{atm})(24.0\,\mathrm{L}-15.2\,\mathrm{L}) \\ &= (8.80\,\mathrm{atm\cdot L})\left(\frac{101.3\,\mathrm{J}}{\mathrm{atm\cdot L}}\right) \\ &= 0.891\,\mathrm{kJ} \end{aligned}$$

C→D is a constant-volume process:

$$W_{\mathrm{C\to D}} = 0$$

D→A is an isobaric process:

$$\begin{aligned} W_{\mathrm{D\to A}} &= P\Delta V = (5\,\mathrm{atm})(5\,\mathrm{L}-24\,\mathrm{L}) \\ &= (-95.0\,\mathrm{atm\cdot L})\left(\frac{101.3\,\mathrm{J}}{\mathrm{atm\cdot L}}\right) \\ &= -9.62\,\mathrm{kJ} \end{aligned}$$

Substitute to obtain:

$$\begin{aligned} W &= 2.25\mathrm{kJ}+0.891\mathrm{kJ}+0-9.62\,\mathrm{kJ} \\ &= \boxed{-6.48\,\mathrm{kJ}} \end{aligned}$$

Note that our result in part (*b*) agrees with this more accurate value to within 2%.

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Picture the Problem The total work done as the gas is taken through this cycle is the area bounded by the two processes. Because the process from 1→2 is linear, we can use the formula for the area of a trapezoid to find the work done during this expansion. We can use $W_{\text{isothermal process}} = nRT\ln(V_{\mathrm{f}}/V_{\mathrm{i}})$ to find the work done on the gas during the process 2→1. The total work is then the sum of these two terms.

Express the net work done per cycle:

$$W_{\mathrm{total}} = W_{1\to2}+W_{2\to1} \qquad (1)$$

Work is done by the gas during its expansion from 1 to 2 and hence is equal to the negative of the area of the trapezoid defined by this path

$$\begin{aligned} W_{1\to2} &= -A_{\mathrm{trap}} \\ &= -\tfrac{1}{2}(23\,\mathrm{L}-11.5\,\mathrm{L})(2\,\mathrm{atm}+1\mathrm{atm}) \\ &= -17.3\,\mathrm{L\cdot atm} \end{aligned}$$

and the vertical lines at V_1 = 11.5 L and V_2 = 23 L. Use the formula for the area of a trapezoid to express $W_{1\to 2}$:

Work is done on the gas during the isothermal compression from V_2 to V_1 and hence is equal to the area under the curve representing this process. Use the expression for the work done during an isothermal process to express $W_{2\to 1}$:

$$W_{2\to 1} = nRT\ln\frac{V_\mathrm{f}}{V_\mathrm{i}}$$

Apply the ideal-gas law at point 1 to find the temperature along the isotherm 2→1:

$$T = \frac{PV}{nR} = \frac{(2\,\mathrm{atm})(11.5\,\mathrm{L})}{(1\,\mathrm{mol})\left(8.206\times 10^{-2}\,\mathrm{L\cdot atm/mol\cdot K}\right)} = 280\,\mathrm{K}$$

Substitute numerical values and evaluate $W_{2\to 1}$:

$$W_{2\to 1} = \left|(1\,\mathrm{mol})\left(8.206\times 10^{-2}\,\mathrm{L\cdot atm/mol\cdot K}\right)(280\,\mathrm{K})\ln\left(\frac{11.5\,\mathrm{L}}{23\,\mathrm{L}}\right)\right| = 15.9\,\mathrm{L\cdot atm}$$

Substitute in equation (1) and evaluate W_net:

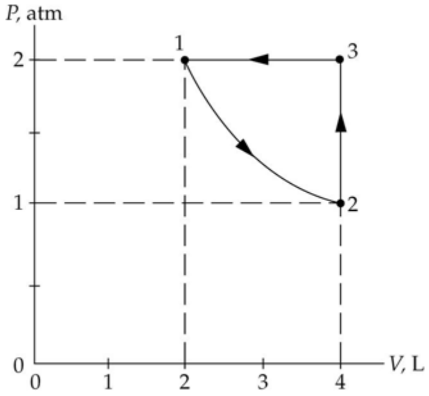
$$\begin{aligned} W_\mathrm{net} &= -17.3\,\mathrm{L\cdot atm} + 15.9\,\mathrm{L\cdot atm} \\ &= -1.40\,\mathrm{L\cdot atm}\times\frac{101.325\,\mathrm{J}}{\mathrm{L\cdot atm}} \\ &= \boxed{-142\,\mathrm{J}} \end{aligned}$$

Remarks: The work done by the gas during each cycle is 142 J.

80 ••

Picture the Problem We can apply the ideal-gas law to find the temperatures T_1, T_2, and T_3. We can use the appropriate work and heat equations to calculate the heat added and the work done by the gas for the isothermal process (1→2), the constant-volume process (2→3), and the isobaric process (3→1).

(*a*) The cycle is shown in the diagram:



(*c*) Use the ideal-gas law to find T_1:

$$T_1 = \frac{P_1V_1}{nR} = \frac{(2\,\text{atm})(2\,\text{L})}{(2\,\text{mol})(8.206\times10^{-2}\ \text{L}\cdot\text{atm/mol}\cdot\text{K})} = \boxed{24.4\,\text{K}}$$

Because the process 1→2 is isothermal:

$$T_2 = \boxed{24.4\,\text{K}}$$

Use the ideal-gas law to find T_3:

$$T_3 = \frac{P_3V_3}{nR} = \frac{(2\,\text{atm})(4\,\text{L})}{(2\,\text{mol})(8.206\times10^{-2}\ \text{L}\cdot\text{atm/mol}\cdot\text{K})} = \boxed{48.7\,\text{K}}$$

(*b*) Because the process 1→2 is isothermal, $Q_{\text{in},1\to2} = W_{\text{by gas},1\to2}$:

$$Q_{\text{in},1\to2} = W_{\text{by gas},1\to2} = nRT\ln\frac{V_2}{V_1} = (2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K}) \times(24.4\,\text{K})\ln\left(\frac{4\,\text{L}}{2\,\text{L}}\right) = \boxed{281\,\text{J}}$$

Because process 2→3 takes place at constant volume:

$$W_{2\to3} = \boxed{0}$$

Because process 2→3 takes place at constant volume, $W_{\text{on},2\to3} = 0$:

$$\begin{aligned} Q_{\text{in},2\to3} &= \Delta E_{\text{int},2\to3} = C_{\text{V}}\Delta T = \tfrac{3}{2}nR\left(T_3 - T_2\right) \\ &= \tfrac{3}{2}\left(2\,\text{mol}\right)\left(8.314\,\text{J/mol}\cdot\text{K}\right) \\ &\quad \times\left(48.7\,\text{K} - 24.4\,\text{K}\right) \\ &= \boxed{606\,\text{J}} \end{aligned}$$

Process 3→1 is isobaric:

$$\begin{aligned} Q_{3\to1} &= C_{\text{P}}\Delta T = \tfrac{5}{2}nR(T_1 - T_3) \\ &= \tfrac{5}{2}\left(2\,\text{mol}\right)\left(8.314\,\text{J/mol}\cdot\text{K}\right) \\ &\quad \times\left(24.4\,\text{K} - 48.7\,\text{K}\right) \\ &= \boxed{-1.01\text{kJ}} \end{aligned}$$

The work done by the gas from 3 to 1 equals the negative of the work done on the gas:

$$\begin{aligned} W_{\text{by gas},3\to1} &= -P_{1,3}\Delta V = P_{1,3}\left(V_1 - V_3\right) \\ &= -\left(2\,\text{atm}\right)\left(2\,\text{L} - 4\,\text{L}\right) \\ &= -\left(-4\,\text{atm}\cdot\text{L}\right)\left(\frac{101.3\,\text{J}}{\text{atm}\cdot\text{L}}\right) \\ &= \boxed{405\,\text{J}} \end{aligned}$$

81 •••

Picture the Problem We can find the temperatures, pressures, and volumes at all points for this ideal monatomic gas (3 degrees of freedom) using the ideal-gas law and the work for each process by finding the areas under each curve. We can find the heat exchanged for each process from the heat capacities and the initial and final temperatures for each process.

Express the total work done by the gas per cycle:

$$W_{\text{by gas,tot}} = W_{\text{D}\to\text{A}} + W_{\text{A}\to\text{B}} + W_{\text{B}\to\text{C}} + W_{\text{C}\to\text{D}}$$

1. Use the ideal-gas law to find the volume of the gas at point D:

$$\begin{aligned} V_{\text{D}} &= \frac{nRT_{\text{D}}}{P_{\text{D}}} \\ &= \frac{\left(2\,\text{mol}\right)\left(8.314\,\text{J/mol}\cdot\text{K}\right)\left(360\,\text{K}\right)}{\left(2\,\text{atm}\right)\left(101.3\,\text{kPa/atm}\right)} \\ &= 29.5\,\text{L} \end{aligned}$$

2. We’re given that the volume of the gas at point B is three times that at point D:

$$\begin{aligned} V_{\text{B}} &= V_{\text{C}} = 3V_{\text{D}} \\ &= 88.6\,\text{L} \end{aligned}$$

Use the ideal-gas law to find the pressure of the gas at point C:

$$P_C = \frac{nRT_C}{V_C} = \frac{(2\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})}{88.6\,\text{L}} \times (360\,\text{K}) = 0.667\,\text{atm}$$

We're given that the pressure at point B is twice that at point C:

$$P_B = 2P_C = 2(0.667\,\text{atm}) = 1.33\,\text{atm}$$

3. Because path DC represents an isothermal process:

$$T_D = T_C = 360\,\text{K}$$

Use the ideal-gas law to find the temperatures at points B and A:

$$T_A = T_B = \frac{P_B V_B}{nR} = \frac{(1.333\,\text{atm})(88.6\,\text{L})}{(2\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})} = 720\,\text{K}$$

Because the temperature at point A is twice that at D and the volumes are the same, we can conclude that:

$$P_A = 2P_D = 4\,\text{atm}$$

The pressure, volume, and temperature at points A, B, C, and D are summarized in the table to the right.

Point	P	V	T
	(atm)	(L)	(K)
A	4	29.5	720
B	1.33	88.6	720
C	0.667	88.6	360
D	2	29.5	360

4. For the path D→A:

$$W_{D\to A} = 0$$

and

$$Q_{D\to A} = \Delta E_{\text{int, }D\to A} = \tfrac{3}{2}nR\Delta T_{D\to A} = \tfrac{3}{2}nR(T_A - T_D) = \tfrac{3}{2}(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K}) \times (720\,\text{K} - 360\,\text{K}) = 8.98\,\text{kJ}$$

For the path A→B:

$$
\begin{aligned}
W_{\mathrm{A}\to\mathrm{B}} &= Q_{\mathrm{A}\to\mathrm{B}} = nRT_{\mathrm{A,B}}\ln\frac{V_{\mathrm{B}}}{V_{\mathrm{A}}} \\
&= (2\,\mathrm{mol})(8.314\,\mathrm{J/mol\cdot K})(720\,\mathrm{K}) \\
&\quad\times\ln\left(\frac{88.6\,\mathrm{L}}{29.5\,\mathrm{L}}\right) \\
&= 13.2\,\mathrm{kJ}
\end{aligned}
$$

and, because process A→B is isothermal, $\Delta E_{\text{int, A}\to\text{B}} = 0$

For the path B→C:

$$W_{\mathrm{B}\to\mathrm{C}} = 0$$

and

$$
\begin{aligned}
Q_{\mathrm{B}\to\mathrm{C}} &= \Delta U_{\mathrm{B}\to\mathrm{C}} = C_{\mathrm{V}}\Delta T = \tfrac{3}{2}nR(T_{\mathrm{C}} - T_{\mathrm{B}}) \\
&= \tfrac{3}{2}(2\,\mathrm{mol})(8.314\,\mathrm{J/mol\cdot K}) \\
&\quad\times(360\,\mathrm{K} - 720\,\mathrm{K}) \\
&= -8.98\,\mathrm{kJ}
\end{aligned}
$$

For the path C→D:

$$
\begin{aligned}
W_{\mathrm{C}\to\mathrm{D}} &= nRT_{\mathrm{C,D}}\ln\frac{V_{\mathrm{D}}}{V_{\mathrm{C}}} \\
&= (2\,\mathrm{mol})(8.314\,\mathrm{J/mol\cdot K})(360\,\mathrm{K}) \\
&\quad\times\ln\left(\frac{29.5\,\mathrm{L}}{88.6\,\mathrm{L}}\right) \\
&= -6.58\,\mathrm{kJ}
\end{aligned}
$$

Also, because process A→B is isothermal, $\Delta E_{\text{int,A}\to\text{B}} = 0$

and $Q_{\mathrm{C}\to\mathrm{D}} = W_{\mathrm{C}\to\mathrm{D}} = -6.58\,\mathrm{kJ}$

Q_{in}, W_{on}, and ΔE_{int} are summarized for each of the processes in the table to the right.

Process	Q_{in}	W_{on}	ΔE_{int}
	(kJ)	(kJ)	(kJ)
D→A	$\boxed{8.98}$	0	8.98
A→B	$\boxed{13.2}$	–13.2	0
B→C	$\boxed{-8.98}$	0	–8.98
C→D	$\boxed{-6.58}$	6.58	0

Referring to the table, find the total work done by the gas per cycle:

$$\begin{aligned} W_{\text{tot}} &= W_{\text{D}\to\text{A}} + W_{\text{A}\to\text{B}} + W_{\text{B}\to\text{C}} + W_{\text{C}\to\text{D}} \\ &= 0 + 13.2\,\text{kJ} + 0 - 6.58\,\text{kJ} \\ &= \boxed{6.62\,\text{kJ}} \end{aligned}$$

Remarks: Note that, as it should be, ΔE_{int} is zero for the complete cycle.

***82** •••

Picture the Problem We can find the temperatures, pressures, and volumes at all points for this ideal diatomic gas (5 degrees of freedom) using the ideal-gas law and the work for each process by finding the areas under each curve. We can find the heat exchanged for each process from the heat capacities and the initial and final temperatures for each process.

Express the total work done by the gas per cycle:

$$W_{\text{by gas,tot}} = W_{\text{D}\to\text{A}} + W_{\text{A}\to\text{B}} + W_{\text{B}\to\text{C}} + W_{\text{C}\to\text{D}}$$

1. Use the ideal-gas law to find the volume of the gas at point D:

$$\begin{aligned} V_{\text{D}} &= \frac{nRT_{\text{D}}}{P_{\text{D}}} \\ &= \frac{(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(360\,\text{K})}{(2\,\text{atm})(101.3\,\text{kPa/atm})} \\ &= 29.5\,\text{L} \end{aligned}$$

2. We're given that the volume of the gas at point B is three times that at point D:

$$\begin{aligned} V_{\text{B}} &= V_{\text{C}} = 3V_{\text{D}} \\ &= 88.6\,\text{L} \end{aligned}$$

Use the ideal-gas law to find the pressure of the gas at point C:

$$\begin{aligned} P_{\text{C}} &= \frac{nRT_{\text{C}}}{V_{\text{C}}} \\ &= \frac{(2\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})}{88.6\,\text{L}} \\ &\quad\times(360\,\text{K}) \\ &= 0.667\,\text{atm} \end{aligned}$$

We're given that the pressure at point B is twice that at point C:

$$P_{\text{B}} = 2P_{\text{C}} = 2(0.667\,\text{atm}) = 1.33\,\text{atm}$$

3. Because path DC represents an isothermal process:

$$T_{\text{D}} = T_{\text{C}} = 360\,\text{K}$$

Use the ideal-gas law to find the temperatures at points B and A:

$$T_A = T_B = \frac{P_B V_B}{nR}$$
$$= \frac{(1.333\,\text{atm})(88.6\,\text{L})}{(2\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})}$$
$$= 720\,\text{K}$$

Because the temperature at point A is twice that at D and the volumes are the same, we can conclude that:

$$P_A = 2P_D = 4\,\text{atm}$$

The pressure, volume, and temperature at points A, B, C, and D are summarized in the table to the right.

Point	*P*	*V*	*T*
	(atm)	(L)	(K)
A	4	29.5	720
B	1.33	88.6	720
C	0.667	88.6	360
D	2	29.5	360

4. For the path D→A:

$$W_{D\to A} = 0$$

and

$$Q_{D\to A} = \Delta U_{D\to A} = \tfrac{5}{2}nR\Delta T_{D\to A}$$
$$= \tfrac{5}{2}nR(T_A - T_D)$$
$$= \tfrac{5}{2}(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})$$
$$\times(720\,\text{K} - 360\,\text{K})$$
$$= 15.0\,\text{kJ}$$

For the path A→B:

$$W_{A\to B} = Q_{A\to B} = nRT_{A,B}\ln\frac{V_B}{V_A}$$
$$= (2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(720\,\text{K})$$
$$\times\ln\left(\frac{88.6\,\text{L}}{29.5\,\text{L}}\right)$$
$$= 13.2\,\text{kJ}$$

and, because process A→B is isothermal, $\Delta E_{\text{int},A\to B} = 0$

For the path B→C:

$$W_{B\to C} = 0$$

and

$$\begin{aligned}Q_{\mathrm{B}\to\mathrm{C}} &= \Delta U_{\mathrm{B}\to\mathrm{C}} = C_{\mathrm{V}}\Delta T\\ &= \tfrac{5}{2}nR\left(T_{\mathrm{C}} - T_{\mathrm{B}}\right)\\ &= \tfrac{5}{2}\left(2\,\mathrm{mol}\right)\left(8.314\,\mathrm{J/mol\cdot K}\right)\\ &\quad\times\left(360\,\mathrm{K} - 720\,\mathrm{K}\right)\\ &= -15.0\,\mathrm{kJ}\end{aligned}$$

For the path C→D:

$$\begin{aligned}W_{\mathrm{C}\to\mathrm{D}} &= nRT_{\mathrm{C,D}}\ln\frac{V_{\mathrm{D}}}{V_{\mathrm{C}}}\\ &= \left(2\,\mathrm{mol}\right)\left(8.314\,\mathrm{J/mol\cdot K}\right)\left(360\,\mathrm{K}\right)\\ &\quad\times\ln\left(\frac{29.5\,\mathrm{L}}{88.6\,\mathrm{L}}\right)\\ &= -6.58\,\mathrm{kJ}\end{aligned}$$

Also, because process A→B is isothermal, $\Delta E_{\mathrm{int,A}\to\mathrm{B}} = 0$ and

$$Q_{\mathrm{C}\to\mathrm{D}} = W_{\mathrm{C}\to\mathrm{D}} = -6.58\,\mathrm{kJ}$$

Q_{in}, W_{on}, and ΔE_{int} are summarized for each of the processes in the table to the right.

Process	Q_{in}	W_{on}	ΔE_{int}
	(kJ)	(kJ)	(kJ)
D→A	$\boxed{15.0}$	0	15.0
A→B	$\boxed{13.2}$	–13.2	0
B→C	$\boxed{-15.0}$	0	–15.0
C→D	$\boxed{-6.58}$	6.58	0

Referring to the table and noting that the work done by the gas equals the negative of the work done on the gas, find the total work done by the gas per cycle:

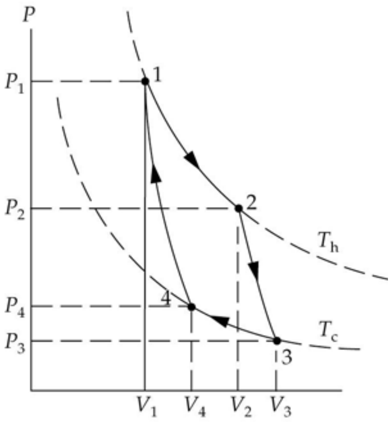
$$\begin{aligned}W_{\text{by gas,tot}} &= W_{\mathrm{D}\to\mathrm{A}} + W_{\mathrm{A}\to\mathrm{B}} + W_{\mathrm{B}\to\mathrm{C}} + W_{\mathrm{C}\to\mathrm{D}}\\ &= 0 + 13.2\,\mathrm{kJ} + 0 - 6.58\,\mathrm{kJ}\\ &= \boxed{6.62\,\mathrm{kJ}}\end{aligned}$$

Remarks: Note that ΔE_{int} for the complete cycle is zero and that the total work done is the same for the diatomic gas of this problem and the monatomic gas of Problem 81.

83 •••

Picture the Problem We can use the equations of state for adiabatic and isothermal processes to express the work done on or by the system, the heat entering or leaving the system, and the change in internal energy for each of the four processes making up the Carnot cycle. We can use the first law of thermodynamics and the definition of the efficiency of a Carnot cycle to show that the efficiency is $1 - Q_c / Q_h$.

(*a*) The cycle is shown on the *PV* diagram to the right:



(*b*) Because the process 1→2 is isothermal:

$$\Delta E_{\text{int},1\to2} = 0$$

Apply the first law of thermodynamics to obtain:

$$Q_h = Q_{1\to2} = W_{1\to2} = \boxed{nRT_h \ln\frac{V_2}{V_1}}$$

(*c*) Because the process 3→4 is isothermal:

$$\Delta U_{3\to4} = 0$$

Apply the first law of thermodynamics to obtain:

$$Q_c = Q_{3\to4} = W_{3\to4} = nRT_c \ln\frac{V_4}{V_3}$$

$$= \boxed{-nRT_c \ln\frac{V_3}{V_4}}$$

where the minus sign tells us that heat is given off by the gas during this process.

(*d*) Apply the equation for a quasi-static adiabatic process at points 4 and 1 to obtain:

$$T_c V_4^{\gamma-1} = T_h V_1^{\gamma-1}$$

Solve for the ratio V_1/V_4:

$$\frac{V_1}{V_4} = \left(\frac{T_c}{T_h}\right)^{\frac{1}{\gamma-1}} \qquad (1)$$

Apply the equation for a quasi-static adiabatic process at points 2 and 3 to

$$T_h V_2^{\gamma-1} = T_c V_3^{\gamma-1}$$

obtain:

Solve for the ratio V_2/V_3:

$$\frac{V_2}{V_3}=\left(\frac{T_c}{T_h}\right)^{\frac{1}{\gamma-1}} \qquad (2)$$

Equate equations (1) and (2) and rearrange to obtain:

$$\boxed{\frac{V_3}{V_4}=\frac{V_2}{V_1}}$$

(*e*) Express the efficiency of the Carnot cycle:

$$\varepsilon=\frac{W}{Q_h}$$

Apply the first law of thermodynamics to obtain:

$$\begin{aligned}W_{on}&=\Delta E_{int,\,cycle}-Q_{in}\\&=0-\left(Q_h-Q_c\right)=-\left(Q_h-Q_c\right)\end{aligned}$$

because E_{int} is a state function and $\Delta E_{int,\,cycle}=0$.

Substitute to obtain:

$$\begin{aligned}\varepsilon&=\frac{W_{\text{by the gas}}}{Q_h}=\frac{-W_{on}}{Q_h}\\&=\frac{Q_h-Q_c}{Q_h}=\boxed{1-\frac{Q_c}{Q_h}}\end{aligned}$$

(*f*) In part (*b*) we established that:

$$Q_h=nRT_h\ln\frac{V_2}{V_1}$$

In part (*c*) we established that the heat leaving the system along the path 3→4 is given by:

$$Q_c=nRT_c\ln\frac{V_3}{V_4}$$

Divide the second of these equations by the first to obtain:

$$\frac{Q_c}{Q_h}=\frac{nRT_c\ln\frac{V_3}{V_4}}{nRT_h\ln\frac{V_2}{V_1}}=\boxed{\frac{T_c}{T_h}}$$

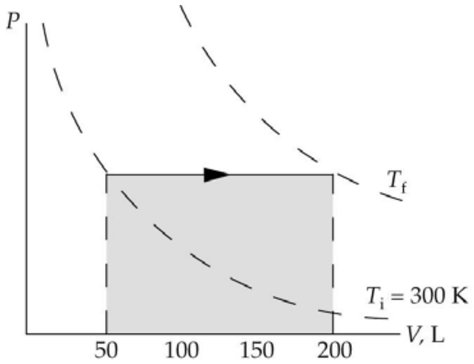
because $\frac{V_3}{V_4}=\frac{V_2}{V_1}$.

Remarks: This last result establishes that the efficiency of a Carnot cycle is also given by $\boldsymbol{\varepsilon_C=1-\frac{T_c}{T_h}}$.

General Problems

84 •

Picture the Problem The isobaric process is shown on the *PV* diagram. We can express the heat that must be supplied to gas in terms of its heat capacity at constant pressure and the change in its temperature and then use the ideal-gas law for a fixed amount of gas to relate the final temperature to the initial temperature.



Relate Q_{in} to C_P and ΔT:

$$Q_{in} = C_P\Delta T = C_P(T_f - T_i) = \tfrac{5}{2}nR(T_f - T_i)$$

Use the ideal-gas law for a fixed amount of gas to relate the initial and final volumes, pressures, and temperatures:

$$\frac{P_iV_i}{T_i} = \frac{P_fV_f}{T_f}$$

or, because the process is isobaric,

$$\frac{V_i}{T_i} = \frac{V_f}{T_f}$$

Solve for T_f:

$$T_f = \frac{V_f}{V_i}T_i = \frac{200\,\text{L}}{50\,\text{L}}T_i = 4T_i$$

Substitute to obtain:

$$Q_{in} = \tfrac{5}{2}nR(4T_i - 3T_i) = \tfrac{15}{2}nRT_i$$

Substitute numerical values and evaluate Q_{in}:

$$Q_{in} = \tfrac{15}{2}(3\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(300\,\text{K}) = \boxed{56.1\,\text{kJ}}$$

85 •

Picture the Problem We can use the first law of thermodynamics to relate the heat removed from the gas to the work done on the gas.

Apply the first law of thermodynamics to this process:

$$Q_{in} = \Delta E_{int} - W_{on} = -W_{on}$$

because $\Delta E_{int} = 0$ for an isothermal process.

Substitute numerical values to obtain:

$$Q_{in} = -180\,\text{kJ}$$

Because $Q_{removed} = -Q_{in}$:

$$Q_{removed} = \boxed{180\,\text{kJ}}$$

*86 •

Picture the Problem We can find the number of moles of the gas from the expression for the work done on or by a gas during an isothermal process.

Express the work done on the gas during the isothermal process:

$$W = nRT\ln\frac{V_\text{f}}{V_\text{i}}$$

Solve for *n*:

$$n = \frac{W}{RT\ln\dfrac{V_\text{f}}{V_\text{i}}}$$

Substitute numerical values and evaluate *n*:

$$n = \frac{-180\,\text{kJ}}{(8.314\,\text{J/mol}\cdot\text{K})(293\,\text{K})\ln\left(\dfrac{1}{5}\right)} = \boxed{45.9\,\text{mol}}$$

87 •

Picture the Problem We can use the ideal-gas law to find the temperatures T_A and T_C. Because the process EDC is isobaric, we can find the area under this line geometrically and the first law of thermodynamics to find Q_AEC.

(*a*) Using the ideal-gas law, find the temperature at point A:

$$T_\text{A} = \frac{P_\text{A}V_\text{A}}{nR} = \frac{(4\,\text{atm})(4.01\,\text{L})}{(3\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})} = \boxed{65.2\,\text{K}}$$

Using the ideal-gas law, find the temperature at point C:

$$T_\text{C} = \frac{P_\text{C}V_\text{C}}{nR} = \frac{(1\,\text{atm})(20\,\text{L})}{(3\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})} = \boxed{81.2\,\text{K}}$$

(*b*) Express the work done by the gas along the path AEC:

$$\begin{aligned} W_{\text{AEC}} &= W_{\text{AE}} + W_{\text{EC}} = 0 + P_{\text{EC}}\Delta V_{\text{EC}} \\ &= (1\,\text{atm})(20\,\text{L} - 4.01\,\text{L}) \\ &= 16.0\,\text{L}\cdot\text{atm} \times \frac{101.3\,\text{J}}{\text{L}\cdot\text{atm}} \\ &= \boxed{1.62\,\text{kJ}} \end{aligned}$$

(*c*) Apply the first law of thermodynamics to express Q_{AEC}:

$$\begin{aligned} Q_{\text{AEC}} &= W_{\text{AEC}} + \Delta E_{\text{int}} = W_{\text{AEC}} + C_{\text{V}}\Delta T \\ &= W_{\text{AEC}} + \tfrac{3}{2}nR\Delta T \\ &= W_{\text{AEC}} + \tfrac{3}{2}nR(T_{\text{C}} - T_A) \end{aligned}$$

Substitute numerical values and evaluate Q_{AEC}:

$$Q_{\text{AEC}} = 1.62\,\text{kJ} + \tfrac{3}{2}(3\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(81.2\,\text{K} - 65.2\,\text{K}) = \boxed{2.22\,\text{kJ}}$$

Remarks The difference between W_{AEC} and Q_{AEC} is the change in the internal energy $\Delta E_{\text{int,AEC}}$ during this process.

88 ••

Picture the Problem We can use the ideal-gas law to find the temperatures T_{A} and T_{C}. Because the process AB is isobaric, we can find the area under this line geometrically. We can use the expression for the work done during an isothermal expansion to find the work done between B and C and the first law of thermodynamics to find Q_{ABC}.

(*a*) Using the ideal-gas law, find the temperature at point A:

$$\begin{aligned} T_{\text{A}} &= \frac{P_{\text{A}}V_{\text{A}}}{nR} \\ &= \frac{(4\,\text{atm})(4.01\,\text{L})}{(3\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})} \\ &= \boxed{65.2\,\text{K}} \end{aligned}$$

Use the ideal-gas law to find the temperature at point C:

$$\begin{aligned} T_{\text{C}} &= \frac{P_{\text{C}}V_{\text{C}}}{nR} \\ &= \frac{(1\,\text{atm})(20\,\text{L})}{(3\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})} \\ &= \boxed{81.2\,\text{K}} \end{aligned}$$

(*b*) Express the work done by the gas along the path ABC:

$$\begin{aligned} W_{\text{ABC}} &= W_{\text{AB}} + W_{\text{BC}} \\ &= P_{\text{AB}}\Delta V_{\text{AB}} + nRT_{\text{B}}\ln\frac{V_{\text{C}}}{V_{\text{B}}} \end{aligned}$$

Use the ideal-gas law to find the volume of the gas at point B:

$$V_{\text{B}} = \frac{nRT_{\text{B}}}{P_{\text{B}}} = \frac{(3\,\text{mol})(8.206\times10^{-2}\ \text{L}\cdot\text{atm/mol}\cdot\text{K})(81.2\,\text{K})}{4\,\text{atm}} = 5.00\,\text{L}$$

Substitute to obtain:

$$\begin{aligned} W_{\text{ABC}} &= (4\,\text{atm})(5\,\text{L} - 4.01\,\text{L}) + (3\,\text{mol})(8.206\times10^{-2}\ \text{L}\cdot\text{atm/mol}\cdot\text{K})(81.2\,\text{K})\ln\left(\frac{20\,\text{L}}{5\,\text{L}}\right) \\ &= 31.7\,\text{L}\cdot\text{atm}\times\frac{101.3\,\text{J}}{\text{atm}} = \boxed{3.21\,\text{kJ}} \end{aligned}$$

(*c*) Apply the first law of thermodynamics to obtain:

$$\begin{aligned} Q_{\text{ABC}} &= W_{\text{ABC}} + \Delta E_{\text{int}} = W_{\text{ABC}} + C_{\text{V}}\Delta T \\ &= W_{\text{AEC}} + \tfrac{3}{2}nR\Delta T \\ &= W_{\text{AEC}} + \tfrac{3}{2}nR(T_{\text{C}} - T_{A}) \end{aligned}$$

Substitute numerical values and evaluate Q_{ABC}:

$$Q_{\text{ABC}} = 3.21\,\text{kJ} + \tfrac{3}{2}(3\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(81.2\,\text{K} - 65.2\,\text{K}) = \boxed{3.81\,\text{kJ}}$$

Remarks: The difference between W_{ABC} and Q_{ABC} is the change in the internal energy $\Delta E_{\text{int,ABC}}$ during this process.

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Picture the Problem We can use the ideal-gas law to find the temperatures T_{A} and T_{C}. Because the process DC is isobaric, we can find the area under this line geometrically. We can use the expression for the work done during an isothermal expansion to find the work done between A and D and the first law of thermodynamics to find Q_{ADC}.

(*a*) Using the ideal-gas law, find the temperature at point A:

$$\begin{aligned} T_{\text{A}} &= \frac{P_{\text{A}}V_{\text{A}}}{nR} \\ &= \frac{(4\,\text{atm})(4.01\,\text{L})}{(3\,\text{mol})(8.206\times10^{-2}\ \text{L}\cdot\text{atm/mol}\cdot\text{K})} \\ &= \boxed{65.2\,\text{K}} \end{aligned}$$

Use the ideal-gas law to find the temperature at point C:

$$T_C = \frac{P_C V_C}{nR} = \frac{(1\,\text{atm})(20\,\text{L})}{(3\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})} = \boxed{81.2\,\text{K}}$$

(*b*) Express the work done by the gas along the path ADC:

$$W_{ADC} = W_{AD} + W_{DC} = nRT_A \ln\frac{V_D}{V_A} + P_{DC}\Delta V_{DC}$$

Use the ideal-gas law to find the volume of the gas at point D:

$$V_D = \frac{nRT_D}{P_D} = \frac{(3\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})(65.2\,\text{K})}{1\,\text{atm}} = 16.1\,\text{L}$$

Substitute numerical values and evaluate W_{ADC}:

$$\begin{aligned} W_{ADC} &= (3\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})(65.2\,\text{K})\ln\left(\frac{16.1\,\text{L}}{4.01\,\text{L}}\right) \\ &\quad + (1\,\text{atm})(20\,\text{L} - 16.1\,\text{L}) \\ &= 26.2\,\text{L}\cdot\text{atm}\times\frac{101.325\,\text{J}}{\text{L}\cdot\text{atm}} = \boxed{2.65\,\text{kJ}} \end{aligned}$$

(*c*) Apply the first law of thermodynamics to obtain:

$$\begin{aligned} Q_{ADC} &= W_{ADC} + \Delta E_{\text{int}} = W_{ADC} + C_V\Delta T \\ &= W_{ADC} + \tfrac{3}{2}nR\Delta T \\ &= W_{ADC} + \tfrac{3}{2}nR(T_C - T_A) \end{aligned}$$

Substitute numerical values and evaluate Q_{ADC}:

$$Q_{ADC} = 2.65\,\text{kJ} + \tfrac{3}{2}(3\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(81.2\,\text{K} - 65.2\,\text{K}) = \boxed{3.25\,\text{kJ}}$$

90 ••

Picture the Problem We can use the ideal-gas law to find the temperatures T_A and T_C. Because the process AB is isobaric, we can find the area under this line geometrically. We can find the work done during the adiabatic expansion between B and C using $W_{BC} = -C_V\Delta T_{BC}$ and the first law of thermodynamics to find Q_{ABC}.

(*a*) Using the ideal-gas law, find the temperature at point A:

$$T_A = \frac{P_A V_A}{nR} = \frac{(4\,\text{atm})(4.01\,\text{L})}{(3\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})} = \boxed{65.2\,\text{K}}$$

Use the ideal-gas law to find the temperature at point C:

$$T_C = \frac{P_C V_C}{nR} = \frac{(1\,\text{atm})(20\,\text{L})}{(3\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})} = \boxed{81.2\,\text{K}}$$

(*b*) Express the work done by the gas along the path ABC:

$$\begin{aligned} W_{ABC} &= W_{AB} + W_{BC} \\ &= P_{AB}\Delta V_{AB} - C_V \Delta T_{BC} \\ &= P_{AB}\Delta V_{AB} - \tfrac{3}{2} nR\Delta T_{BC} \end{aligned}$$

because, with $Q_{in} = 0$, $W_{BC} = -\Delta E_{int,BC}$.

Apply the pressure-volume relationship for a quasi-static adiabatic process to the gas at points B and C to find the volume of the gas at point B:

$$P_B V_B^{\gamma} = P_C V_C^{\gamma}$$

and

$$V_B = \left(\frac{P_C}{P_B}\right)^{\frac{1}{\gamma}} V_C = \left(\frac{1\,\text{atm}}{4\,\text{atm}}\right)^{\frac{3}{5}} (20\,\text{L}) = 8.71\,\text{L}$$

Use the ideal-gas law to find T_B:

$$T_B = \frac{P_B V_B}{nR} = \frac{(4\,\text{atm})(8.71\,\text{L})}{(3\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K})} = 142\,\text{K}$$

Substitute numerical values and evaluate W_{ABC}:

$$\begin{aligned} W_{ABC} &= (4\,\text{atm})(8.71\,\text{L} - 4.01\,\text{L}) - \tfrac{3}{2}(3\,\text{mol})(8.206\times10^{-2}\,\text{L}\cdot\text{atm/mol}\cdot\text{K}) \\ &\qquad \times (81.2\,\text{K} - 142\,\text{K}) \\ &= 41.3\,\text{L}\cdot\text{atm} \times \frac{101.325\,\text{J}}{\text{atm}} = \boxed{4.18\,\text{kJ}} \end{aligned}$$

(*c*) Apply the first law of thermodynamics to obtain:

$$\begin{aligned} Q_{\text{ABC}} &= W_{\text{ABC}} + \Delta E_{\text{int}} = W_{\text{ABC}} + C_{\text{V}}\Delta T \\ &= W_{\text{ABC}} + \tfrac{3}{2}nR\Delta T \\ &= W_{\text{ABC}} + \tfrac{3}{2}nR(T_{\text{C}} - T_A) \end{aligned}$$

Substitute numerical values and evaluate Q_{ABC}:

$$Q_{\text{ABC}} = 4.18\,\text{kJ} + \tfrac{3}{2}(3\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(81.2\,\text{K} - 65.2\,\text{K}) = \boxed{4.78\,\text{kJ}}$$

91 ••

Picture the Problem We can find *c* at *T* = 4 K by direct substitution. Because *c* is a function of *T*, we'll integrate *dQ* over the given temperature interval in order to find the heat required to heat copper from 1 to 3 K.

(*a*) Substitute for *a* and *b* to obtain:

$$c = \left(0.0108\,\text{J/kg}\cdot\text{K}^2\right)T + \left(7.62\times10^{-4}\,\text{J/kg}\cdot\text{K}^4\right)T^3$$

Evaluate *c* at *T* = 4 K:

$$\begin{aligned} c(4\,\text{K}) &= \left(0.0108\,\text{J/kg}\cdot\text{K}^2\right)(4\,\text{K}) \\ &\quad + \left(7.62\times10^{-4}\,\text{J/kg}\cdot\text{K}^4\right)(4\,\text{K})^3 \\ &= \boxed{9.20\times10^{-2}\,\text{J/kg}\cdot\text{K}} \end{aligned}$$

(*b*) Express and evaluate the integral of *Q*:

$$\begin{aligned} Q &= \int_{T_{\text{i}}}^{T_{\text{f}}} c(T)\,dT = \left(0.0108\,\text{J/kg}\cdot\text{K}^2\right)\int_{1\,\text{K}}^{3\,\text{K}} T\,dT + \left(7.62\times10^{-4}\,\text{J/kg}\cdot\text{K}^4\right)\int_{1\,\text{K}}^{3\,\text{K}} T^3\,dT \\ &= \left(0.0108\,\text{J/kg}\cdot\text{K}^2\right)\left[\frac{T^2}{2}\right]_{1\,\text{K}}^{3\,\text{K}} + \left(7.62\times10^{-4}\,\text{J/kg}\cdot\text{K}^4\right)\left[\frac{T^4}{4}\right]_{1\,\text{K}}^{3\,\text{K}} = \boxed{0.0584\,\text{J/kg}} \end{aligned}$$

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Picture the Problem We can use the first law of thermodynamics to relate the heat escaping from the system to the amount of work done by the gas and the change in its internal energy. We can use the expression for the work done during an isothermal process to find the temperature along the isotherm.

Apply the first law of thermodynamics to this isothermal process:

$$Q_{\text{in}} = \Delta E_{\text{int}} - W_{\text{on}}$$

For an isothermal process:

$$\Delta E_{\text{int}} = \boxed{0}$$

Substitute to obtain:

$$W_{\text{on}} = -Q_{\text{in}} = -\left(-170\,\text{cal} \times \frac{4.184\,\text{J}}{\text{cal}}\right) = 711\,\text{J}$$

Because $W_{\text{by gas}} = -W_{\text{on}}$:

$$W_{\text{by gas}} = \boxed{-711\,\text{J}}$$

Express the work done during an isothermal process:

$$W_{\text{by gas}} = nRT\ln\frac{V_2}{V_1}$$

Solve for $T = T_{\text{i}} = T_{\text{f}}$:

$$T = \frac{W_{\text{by gas}}}{nR\ln\frac{V_2}{V_1}}$$

Substitute numerical values and evaluate T:

$$T = \frac{-711\,\text{J}}{(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})\ln\left(\frac{8\,\text{L}}{18\,\text{L}}\right)} = \boxed{52.7\,\text{K}}$$

93 ••

Picture the Problem Let the subscripts 1 and 2 refer to the initial and final values of temperature, pressure, and volume. We can relate the work done on a gas during an adiabatic process to the pressures and volumes of the initial and final points on the path using $W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$ and find P_1 by eliminating P_2 using $P_1V_1^{\gamma} = P_2V_2^{\gamma}$, where, for a diatomic gas, $\gamma = 1.4$. Once we've determined P_1 we can use the ideal-gas law to find T_1 and the first law of thermodynamics to find T_2. Finally, we can apply the ideal-gas law a second time to determine P_2.

Relate the work done on a gas during an adiabatic process to the pressures and volumes of the initial and final points on the path:

$$W_{\text{on}} = \frac{P_1V_1 - P_2V_2}{\gamma - 1} = \frac{P_1\left(V_1 - \frac{P_2}{P_1}V_2\right)}{\gamma - 1}$$

Using the equation for a quasi-static adiabatic process, relate the initial and final pressures and volumes:

$$P_1V_1^{\gamma} = P_2V_2^{\gamma} \Rightarrow \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$$

Substitute to obtain:

$$W_{\text{on}} = \frac{P_1\left(V_1 - \left(\dfrac{V_1}{V_2}\right)^{\gamma} V_2\right)}{\gamma - 1}$$

Solve for P_1:

$$P_1 = \frac{W(\gamma - 1)}{V_1 - \left(\dfrac{V_1}{V_2}\right)^{\gamma} V_2}$$

Substitute numerical values and evaluate P_1:

$$P_1 = \frac{(-820\,\text{J})(1.4 - 1)}{18\,\text{L} - \left(\dfrac{18\,\text{L}}{8\,\text{L}}\right)^{1.4}(8\,\text{L})} = \boxed{47.6\,\text{kPa}}$$

Use the ideal-gas law to find T_1:

$$T_1 = \frac{P_1 V_1}{nR} = \frac{(47.6\,\text{kPa})(18\,\text{L})}{(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})}$$

$$= \boxed{51.5\,\text{K}}$$

Apply the first law of thermodynamics to obtain:

$$\Delta E_{\text{int}} = Q_{\text{in}} + W_{\text{on}}$$

or, because $Q_{\text{in}} = 0$ for an adiabatic process,

$$\Delta E_{\text{int}} = W_{\text{on}} = C_{\text{V}}\Delta T = \tfrac{5}{2} nR(T_2 - T_1)$$

Solve for and evaluate T_2:

$$T_2 = T_1 - \frac{W_{\text{on}}}{\frac{5}{2} nR}$$

$$= 51.5\,\text{K} - \frac{-820\,\text{J}}{\frac{5}{2}(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})}$$

$$= \boxed{71.2\,\text{K}}$$

Use the ideal-gas law to find P_2:

$$P_2 = \frac{nRT_2}{V_2}$$

$$= \frac{(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(71.2\,\text{K})}{8\,\text{L}}$$

$$= \boxed{148\,\text{kPa}}$$

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Picture the Problem Let the subscripts 1 and 2 refer to the initial and final state respectively. Because the gas is initially at STP, we know that V_1 = 22.4 L, P_1 = 1 atm, and T_1 = 273 K. We can use $W = -nRT\ln(V_2/V_1)$ to find the work done on the gas during an isothermal compression. We can relate the work done on a gas during an adiabatic process to the pressures and volumes of the initial and final points on the path using $W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$ and find P_1 by eliminating P_2 using $P_1V_1^\gamma = P_2V_2^\gamma$, where, for a diatomic gas, γ = 1.4.

(*a*) Express the work done on the gas in compressing it isothermally:

$$W_{\text{on}} = -nRT\ln\frac{V_2}{V_1}$$

Find the number of moles in 30 g of CO (M = 28 g/mol):

$$n = \frac{30\,\text{g}}{28\,\text{g/mol}} = 1.07\,\text{mol}$$

Substitute and evaluate W_{on}:

$$\begin{aligned}W_{\text{on}} &= -(1.07\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})\\ &\quad\times(273\,\text{K})\ln\left(\frac{1}{5}\right)\\ &= \boxed{3.91\,\text{kJ}}\end{aligned}$$

(*b*) Express the work done on the gas in compressing it adiabatically:

$$\begin{aligned}W_{\text{on}} &= -\frac{P_1V_1 - P_2V_2}{\gamma - 1}\\ &= -\frac{P_1\left(V_1 - \frac{P_2}{P_1}V_2\right)}{\gamma - 1}\end{aligned}$$

Using the equation for a quasi-static adiabatic process, relate the initial and final pressures and volumes:

$$P_1V_1^\gamma = P_2V_2^\gamma \Rightarrow \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma$$

Substitute and simplify to obtain:

$$\begin{aligned}W_{\text{on}} &= -\frac{P_1\left(V_1 - \left(\frac{V_1}{V_2}\right)^\gamma V_2\right)}{\gamma - 1}\\ &= -\frac{P_1\left(V_1 - \left(\frac{V_1}{V_2}\right)^\gamma \frac{V_1}{5}\right)}{\gamma - 1}\\ &= -\frac{P_1V_1\left(1 - 0.2\left(\frac{V_1}{V_2}\right)^\gamma\right)}{\gamma - 1}\end{aligned}$$

Substitute numerical values and evaluate W_{on}:

$$W = -\frac{(101.3\,\text{kPa})(1.07\,\text{mol})(22.4\,\text{L/mol})\left(1-0.2(5)^{1.4}\right)}{1.4-1} = \boxed{5.49\,\text{kJ}}$$

95 ••

Picture the Problem Let the subscripts 1 and 2 refer to the initial and final state respectively. Because the gas is initially at STP, we know that V_1 = 22.4 L, P_1 = 1 atm, and T_1 = 273 K. We can use $W = -nRT\ln(V_2/V_1)$ to find the work done on the gas during an isothermal compression. We can relate the work done on a gas during an adiabatic process to the pressures and volumes of the initial and final points on the path using $W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$ and find P_1 by eliminating P_2 using $P_1V_1^{\gamma} = P_2V_2^{\gamma}$. We can find γ using the data in Table 19-3.

(*a*) Express the work done on the gas in compressing it isothermally:

$$W_{\text{on}} = -nRT\ln\frac{V_2}{V_1}$$

Find the number of moles in 30 g of CO_2 (M = 44 g/mol):

$$n = \frac{30\,\text{g}}{44\,\text{g/mol}} = 0.682\,\text{mol}$$

Substitute and evaluate W_{on}:

$$\begin{aligned} W_{\text{on}} &= -(0.682\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K}) \\ &\quad\times(273\,\text{K})\ln\left(\frac{1}{5}\right) \\ &= \boxed{2.49\,\text{kJ}} \end{aligned}$$

(*b*) Express the work done on the gas in compressing it adiabatically:

$$\begin{aligned} W_{\text{on}} &= -\frac{P_1V_1 - P_2V_2}{\gamma - 1} \\ &= -\frac{P_1\left(V_1 - \frac{P_2}{P_1}V_2\right)}{\gamma - 1} \end{aligned}$$

Using the equation for a quasi-static adiabatic process, relate the initial and final pressures and volumes:

$$P_1V_1^{\gamma} = P_2V_2^{\gamma} \Rightarrow \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$$

Substitute and simplify to obtain:

$$W_{\text{on}} = -\frac{P_1\left(V_1 - \left(\frac{V_1}{V_2}\right)^{\gamma} V_2\right)}{\gamma - 1} = -\frac{P_1\left(V_1 - \left(\frac{V_1}{V_2}\right)^{\gamma} \frac{V_1}{5}\right)}{\gamma - 1} = -\frac{P_1V_1\left(1 - 0.2\left(\frac{V_1}{V_2}\right)^{\gamma}\right)}{\gamma - 1}$$

From Table 18-3 we have:

$$c_{\text{V}} = 3.39R$$

and

$$c_{\text{P}} = (3.39 + 1.02)R = 4.41R$$

Evaluate γ:

$$\gamma = \frac{c_{\text{P}}}{c_{\text{V}}} = \frac{4.41R}{3.39R} = 1.30$$

Substitute numerical values and evaluate W_{on}:

$$W_{\text{on}} = -\frac{(101.3\,\text{kPa})(0.682\,\text{mol})(22.4\,\text{L/mol})\left(1 - 0.2(5)^{1.3}\right)}{1.3 - 1} = \boxed{3.20\,\text{kJ}}$$

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Picture the Problem Let the subscripts 1 and 2 refer to the initial and final states respectively. Because the gas is initially at STP, we know that V_1 = 22.4 L, P_1 = 1 atm, and T_1 = 273 K. We can use $W = -nRT\ln(V_2/V_1)$ to find the work done on the gas during an isothermal compression. We can relate the work done on a gas during an adiabatic process to the pressures and volumes of the initial and final points on the path using $W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$ and find P_1 by eliminating P_2 using $P_1V_1^{\gamma} = P_2V_2^{\gamma}$, where, for a monatomic gas, γ = 1.67.

(*a*) Express the work done on the gas in compressing it isothermally:

$$W_{\text{on}} = -nRT\ln\frac{V_2}{V_1}$$

Find the number of moles in 30 g of Ar (M = 40 g/mol):

$$n = \frac{30\,\text{g}}{40\,\text{g/mol}} = 0.750\,\text{mol}$$

Substitute and evaluate W_{on}:

$$W_{\text{on}} = -(0.75\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K}) \times (273\,\text{K})\ln\left(\frac{1}{5}\right) = \boxed{2.74\,\text{kJ}}$$

(*b*) Express the work done on the gas in compressing it adiabatically:

$$W_{\text{on}} = -\frac{P_1V_1 - P_2V_2}{\gamma - 1} = -\frac{P_1\left(V_1 - \dfrac{P_2}{P_1}V_2\right)}{\gamma - 1}$$

Using the equation for a quasi-static adiabatic process, relate the initial and final pressures and volumes:

$$P_1V_1^{\gamma} = P_2V_2^{\gamma} \Rightarrow \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$$

Substitute and simplify to obtain:

$$W_{\text{on}} = -\frac{P_1\left(V_1 - \left(\dfrac{V_1}{V_2}\right)^{\gamma} V_2\right)}{\gamma - 1} = -\frac{P_1\left(V_1 - \left(\dfrac{V_1}{V_2}\right)^{\gamma} \dfrac{V_1}{5}\right)}{\gamma - 1} = -\frac{P_1V_1\left(1 - 0.2\left(\dfrac{V_1}{V_2}\right)^{\gamma}\right)}{\gamma - 1}$$

Substitute numerical values and evaluate W_{on}:

$$W_{\text{on}} = -\frac{(101.3\,\text{kPa})(0.75\,\text{mol})(22.4\,\text{L/mol})\left(1 - 0.2(5)^{1.67}\right)}{1.67 - 1} = \boxed{4.93\,\text{kJ}}$$

97 ••

Picture the Problem We can use conservation of energy to relate the final temperature to the heat capacities of the gas and the solid. We can apply the Dulong-Petit law to find the heat capacity of the solid at constant volume and use the fact that the gas is diatomic to find its heat capacity at constant volume.

Apply conservation of energy to this

$$\Delta Q = 0$$

process:

or

$$C_{\text{V,gas}}\left(T_{\text{f}}-100\,\text{K}\right)-C_{\text{V,solid}}\left(200\,\text{K}-T_{\text{f}}\right)=0$$

Solve for T_{f}:

$$T_{\text{f}}=\frac{\left(100\,\text{K}\right)\left(C_{\text{V,gas}}\right)+\left(200\,\text{K}\right)\left(C_{\text{V,solid}}\right)}{C_{\text{V,gas}}+C_{\text{V,solid}}}$$

Using the Dulong-Petit law, determine the heat capacity of the solid at constant volume:

$$\begin{aligned}C_{\text{V,solid}}&=3nR\\&=3\left(2\,\text{mol}\right)\left(8.314\,\text{J/mol}\cdot\text{K}\right)\\&=49.9\,\text{J/K}\end{aligned}$$

Determine the heat capacity of the gas at constant volume:

$$\begin{aligned}C_{\text{V,gas}}&=\tfrac{5}{2}nR\\&=\tfrac{5}{2}\left(1\,\text{mol}\right)\left(8.314\,\text{J/mol}\cdot\text{K}\right)\\&=20.8\,\text{J/K}\end{aligned}$$

Substitute numerical values and evaluate T_{f}:

$$T_{\text{f}}=\frac{\left(100\,\text{K}\right)\left(20.8\,\text{J/K}\right)+\left(200\,\text{K}\right)\left(49.9\,\text{J/K}\right)}{20.8\,\text{J/K}+49.9\,\text{J/K}}=\boxed{171\,\text{K}}$$

***98** ••

Picture the Problem We can express the work done during an isobaric process as the product of the temperature and the change in volume and relate Q to ΔT through the definition of C_{P}. Finally, we can use the first law of thermodynamics to show that $\Delta E_{\text{int}} = C_{\text{v}}\Delta T$.

(*a*) $\boxed{\text{For an ideal gas, the internal energy is the sum of the kinetic energies of the gas molecules, which is proportional to } kT\text{. Consequently, } U \text{ is a function of } T \text{ only and } \Delta E_{\text{int}}=C_{\text{V}}\Delta T.}$

(*b*) Use the first law of thermodynamics to relate the work done on the gas, the heat entering the gas, and the change in the internal energy of the gas:

$$\Delta E_{\text{int}}=Q_{\text{in}}+W_{\text{on}}$$

At constant pressure:

$$W_{\text{by gas}}=P\left(V_{\text{f}}-V_{\text{i}}\right)=nR\left(T_{\text{f}}-T_{\text{i}}\right)=nR\Delta T$$

and

$$W_{\text{on}}=-W_{\text{by gas}}=-nR\Delta T$$

Relate Q_{in} to C_P and ΔT:

$$Q_{in} = C_P \Delta T$$

Substitute to obtain:

$$\Delta E_{int} = C_P \Delta T - nR\Delta T$$
$$= (C_P - nR)\Delta T = \boxed{C_V \Delta T}$$

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Picture the Problem We can use the first law of thermodynamics to relate the work done by the gas, the heat added to the gas, and the change in its internal energy. We can use $\Delta E_{int} = C_V \Delta T = \frac{3}{2} nR\Delta T$ to find the change in the internal energy in both cases and $Q_{in} = C_P \Delta T = \frac{5}{2} nR\Delta T$ to find Q_{in} for the isobaric process.

(*a*) Apply the first law of thermodynamics to obtain:

$$Q_{in} = \Delta E_{int} - W_{on}$$

For a constant-volume process:

$$W_{on} = \boxed{0} \text{ and } \Delta E_{int} = \tfrac{3}{2} nR\Delta T$$

Substitute to obtain:

$$Q_{in} = \tfrac{3}{2} nR\Delta T$$

Substitute numerical values and evaluate Q_{in}:

$$Q_{in} = \tfrac{3}{2}(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(300\,\text{K})$$
$$= \boxed{3.74\,\text{kJ}}$$

(*b*) Express the change in internal energy of an ideal monatomic gas for any process between 300 K and 600 K:

$$\Delta E_{int} = C_V \Delta T$$
$$= \tfrac{3}{2} nR\Delta T$$
$$= \tfrac{3}{2}(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(300\,\text{K})$$
$$= \boxed{3.74\,\text{kJ}}$$

Relate the heat absorbed by the gas to the change in temperature:

$$Q_{in} = C_P \Delta T = \tfrac{5}{2} nR\Delta T$$
$$= \tfrac{5}{2}(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(300\,\text{K})$$
$$= \boxed{6.24\,\text{kJ}}$$

Apply the first law of thermodynamics to find the work done on the gas during this expansion:

$$W_{on} = \Delta E_{int} - Q_{in} = 3.74\,\text{kJ} - 6.24\,\text{kJ}$$
$$= -2.50\,\text{kJ}$$

Because the $W_{by\ gas} = -W_{on}$:

$$W_{by\ gas} = \boxed{2.50\,\text{kJ}}$$

***100** ••

Picture the Problem We can use $Q_{\text{in}} = C_P\Delta T$ to find the change in temperature during this isobaric process and the first law of thermodynamics to relate W, Q, and ΔE_{int}. We can use $\Delta E_{\text{int}} = \frac{5}{2}nR\Delta T$ to find the change in the internal energy of the gas during the isobaric process and the ideal-gas law for a fixed amount of gas to express the ratio of the final and initial volumes.

(*a*) Relate the change in temperature to Q_{in} and C_P and evaluate ΔT:

$$\Delta T = \frac{Q_{\text{in}}}{C_P} = \frac{Q_{\text{in}}}{\frac{7}{2}nR} = \frac{500\,\text{J}}{\frac{7}{2}(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})} = \boxed{8.59\,\text{K}}$$

(*b*) Apply the first law of thermodynamics to relate the work done on the gas to the heat supplied and the change in its internal energy:

$$W_{\text{on}} = \Delta E_{\text{int}} - Q_{\text{in}} = C_V\Delta T - Q_{\text{in}} = \frac{5}{2}nR\Delta T - Q_{\text{in}}$$

Substitute numerical values and evaluate W_{on}:

$$W_{\text{on}} = \frac{5}{2}(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(8.59\,\text{K}) - 500\,\text{J} = -143\,\text{J}$$

Because $W_{\text{by gas}} = -W_{\text{on}}$:

$$W_{\text{by gas}} = \boxed{143\,\text{J}}$$

(*c*) Using the ideal-gas law for a fixed amount of gas, relate the initial and final pressures, volumes and temperatures:

$$\frac{P_iV_i}{T_i} = \frac{P_fV_f}{T_f}$$

or, because the process is isobaric,

$$\frac{V_i}{T_i} = \frac{V_f}{T_f}$$

Solve for and evaluate V_f/V_i:

$$\frac{V_f}{V_i} = \frac{T_f}{T_i} = \frac{T_i + \Delta T}{T_i} = \frac{293.15\,\text{K} + 8.59\,\text{K}}{293.15\,\text{K}} = \boxed{1.03}$$

101 ••

Picture the Problem Knowing the rate at which energy is supplied, we can obtain the data we need to plot this graph by finding the time required to warm the ice to 0°C, melt the ice, warm the water formed from the ice to 100°C, vaporize the water, and warm the water to 110°C.

Find the time required to warm the ice to 0°C:

$$\Delta t_1 = \frac{mc_{\text{ice}}\Delta T}{P} = \frac{(0.1\,\text{kg})(2\,\text{kJ/kg}\cdot\text{K})(10\,\text{K})}{100\,\text{J/s}} = 20.0\,\text{s}$$

Find the time required to melt the ice:

$$\Delta t_2 = \frac{mL_{\text{f}}}{P} = \frac{(0.1\,\text{kg})(333.5\,\text{kJ/kg})}{100\,\text{J/s}} = 333.5\,\text{s}$$

Find the time required to heat the water to 100°C:

$$\Delta t_3 = \frac{mc_{\text{w}}\Delta T}{P} = \frac{(0.1\,\text{kg})(4.18\,\text{kJ/kg}\cdot\text{K})(100\,\text{K})}{100\,\text{J/s}} = 418\,\text{s}$$

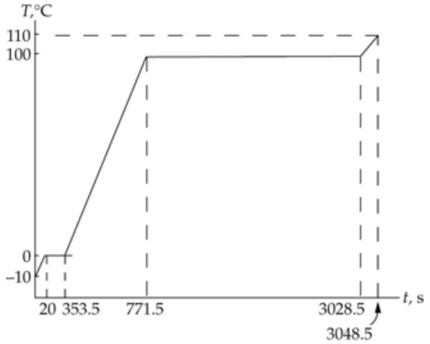
Find the time required to vaporize the water:

$$\Delta t_4 = \frac{mL_{\text{V}}}{P} = \frac{(0.1\,\text{kg})(2257\,\text{kJ/kg})}{100\,\text{J/s}} = 2257\,\text{s}$$

Find the time required to heat the vapor to 110°C:

$$\Delta t_5 = \frac{mc_{\text{steam}}\Delta T}{P} = \frac{(0.1\,\text{kg})(2\,\text{kJ/kg}\cdot\text{K})(10\,\text{K})}{100\,\text{J/s}} = 20\,\text{s}$$

The temperature T as a function of time t is shown to the right:



***102** ••

Picture the Problem We know that, for an adiabatic process, $Q_{\text{in}} = 0$. Hence the work done by the expanding gas equals the change in its internal energy. Because we're given the work done by the gas during the expansion, we can express the change in the temperature of the gas in terms of this work and C_{V}.

Express the final temperature of the gas as a result of its expansion:

$$T_{\text{f}} = T_{\text{i}} + \Delta T$$

Apply the equation for adiabatic work and solve for ΔT:

$$W_{\text{adiabatic}} = -C_{\text{V}}\Delta T$$

and

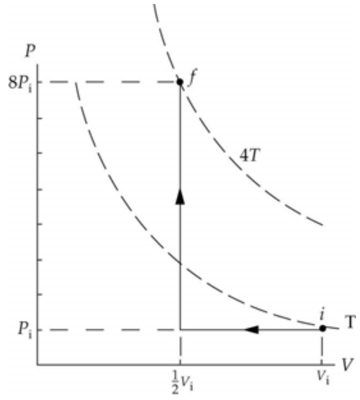
$$\Delta T = -\frac{W_{\text{adiabatic}}}{C_{\text{V}}} = -\frac{W_{\text{adiabatic}}}{\frac{5}{2}nR}$$

Substitute and evaluate T_{f}:

$$\begin{aligned} T_{\text{f}} &= T_{\text{i}} - \frac{W_{\text{adiabatic}}}{\frac{5}{2}nR} \\ &= 300\,\text{K} - \frac{3.5\,\text{kJ}}{\frac{5}{2}(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})} \\ &= \boxed{216\,\text{K}} \end{aligned}$$

103 ••

Picture the Problem Because $P_{\text{f}}V_{\text{f}} = 4P_{\text{i}}V_{\text{i}}$ and $V_{\text{f}} = V_{\text{i}}/2$, the path for which the work done by the gas is a minimum while the pressure never falls below P_{i} is shown on the adjacent PV diagram. We can apply the first law of thermodynamics to relate the heat transferred to the gas to its change in internal energy and the work done on the gas.



Using the first law of thermodynamics, relate the heat transferred to the gas to its change in internal energy and the work done on the gas:

$$\Delta E_{\text{int}} = Q_{\text{in}} + W_{\text{on}}$$

Solve for Q_{in}:

$$Q_{\text{in}} = \Delta E_{\text{int}} - W_{\text{on}}$$

Express the work done during this process:

$$\begin{aligned} W_{\text{on}} &= W_{\text{isobaric process}} + W_{\text{constant volume}} \\ &= P_{\text{i}}\Delta V + 0 = P_{\text{i}}\left(\tfrac{1}{2}V_{\text{i}} - V_{\text{i}}\right) \\ &= \tfrac{1}{2}P_{\text{i}}V_{\text{i}} = \tfrac{1}{2}nRT = \tfrac{1}{2}RT \end{aligned}$$

because $n = 1$ mol.

Express ΔE_{int} for the process:

$$\begin{aligned} \Delta E_{\text{int}} &= C_{\text{V}}\Delta T = \tfrac{3}{2}nR\Delta T = \tfrac{3}{2}nR(3T) \\ &= \tfrac{9}{2}RT \end{aligned}$$

because $n = 1$ mol.

Substitute to obtain:

$$Q_{\text{in}} = \tfrac{9}{2}RT - \tfrac{1}{2}RT = \boxed{4RT}$$

104 ••

Picture the Problem We can solve the ideal-gas law for the dilute solution for the increase in pressure and find the number of solute molecules dissolved in the water from their mass and molecular weight.

Solve the ideal gas law for *P* to obtain:

$$P = \frac{NkT}{V}$$

Express the number of solute molecules *N* in terms of the number of moles *n* and Avogadro's number and then express the number of moles in terms of the mass of the salt and its molecular mass:

$$N = nN_{\text{A}} = \frac{mN_{\text{A}}}{M_{\text{NaCl}}}$$

Substitute to obtain:

$$P = \frac{mN_{\text{A}}kT}{M_{\text{NaCl}}V}$$

Substitute numerical values and evaluate *P*:

$$P = \frac{(30\,\text{g})\left(6.022\times10^{23}\ \text{particles/mol}\right)\left(1.381\times10^{-23}\ \text{J/K}\right)(297\,\text{K})}{(58.4\,\text{g/mol})\left(10^{-3}\ \text{m}^3\right)} = \boxed{1.27\times10^{6}\ \text{N/m}^2}$$

105 ••

Picture the Problem Let the subscripts 1 and 2 refer to the initial and final states in this adiabatic expansion. We can use an equation describing a quasi-static adiabatic process to express the final temperature as a function of the initial temperature and the initial and final volumes.

Using the equation for a quasi-static

$$T_2V_2^{\gamma-1} = T_1V_1^{\gamma-1}$$

adiabatic process, relate the initial and final volumes and temperatures:

Solve for and evaluate T_2:

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = (300\,\text{K})(2)^{1.4-1} = \boxed{396\,\text{K}}$$

106 ••

Picture the Problem We can simplify our calculations by relating Avogadro's number N_A, Boltzmann's constant k, the number of moles n, and the number of molecules N in the gas and solving for N_Ak. We can then calculate $U_{300\text{ K}}$ and $U_{600\text{ K}}$ and their difference.

Express the increase in internal energy per mole resulting from the heating of diamond:

$$\Delta U = U_{600\text{ K}} - U_{300\text{ K}}$$

Express the relationship between Avogadro's number N_A, Boltzmann's constant k, the number of moles n, and the number of molecules N in the gas:

$$nR = Nk \Rightarrow R = \frac{N}{n}k = N_A k$$

Substitute in the given equation to obtain:

$$U = \frac{3RT_E}{e^{T_E/T}-1}$$

Determine $U_{300\text{ K}}$:

$$U_{300\text{ K}} = \frac{3(8.314\,\text{J/mol}\cdot\text{K})(1060\,\text{K})}{e^{1060\,\text{K}/300\,\text{K}}-1} = \boxed{795\,\text{J}}$$

Determine $U_{600\text{ K}}$:

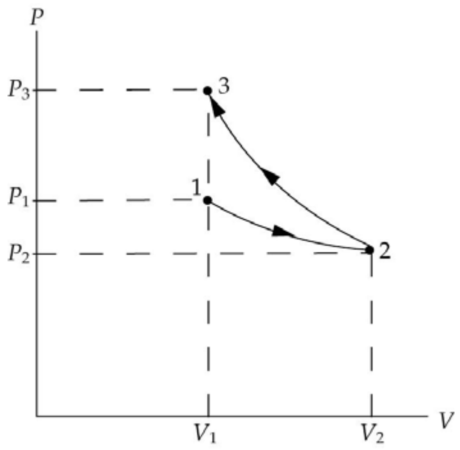
$$U_{600\text{ K}} = \frac{3(8.314\,\text{J/mol}\cdot\text{K})(1060\,\text{K})}{e^{1060\,\text{K}/600\,\text{K}}-1} = \boxed{5.45\,\text{kJ}}$$

Substitute to obtain:

$$\Delta U = U_{600\text{ K}} - U_{300\text{ K}} = 5.45\,\text{kJ} - 795\,\text{J} = \boxed{4.66\,\text{kJ}}$$

***107** •••

Picture the Problem The isothermal expansion followed by an adiabatic compression is shown on the *PV* diagram. The path 1→2 is isothermal and the path 2→3 is adiabatic. We can apply the ideal-gas law for a fixed amount of gas and an isothermal process to find the pressure at point 2 and the pressure-volume relationship for a quasi-static adiabatic process to determine γ.



(*a*) Relate the initial and final pressures and volumes for the isothermal expansion and solve for and evaluate the final pressure:

$$P_1V_1 = P_2V_2$$

and

$$P_2 = P_1\frac{V_1}{V_2} = P_0\frac{V_1}{2V_1} = \boxed{\tfrac{1}{2}P_0}$$

(*b*) Relate the initial and final pressures and volumes for the adiabatic compression:

$$P_2V_2^\gamma = P_3V_3^\gamma$$

or

$$\tfrac{1}{2}P_0\left(2V_0\right)^\gamma = 1.32P_0V_0^\gamma$$

which simplifies to

$$2^\gamma = 2.64$$

Take the natural logarithm of both sides of this equation and solve for and evaluate γ:

$$\gamma\ln 2 = \ln 2.64$$

and

$$\gamma = \frac{\ln 2.64}{\ln 2} = 1.40$$

$$\boxed{\therefore \text{ the gas is diatomic.}}$$

(*c*) $\boxed{\text{In the isothermal process, } T \text{ is constant, and the translational kinetic energy is unchanged.}}$

$\boxed{\text{In the adiabatic process, } T_3 = 1.32T_0\text{, and the translational kinetic energy increases by a factor of 1.32.}}$

108 •••

Picture the Problem In this problem the specific heat of the combustion products depends on the temperature. Although C_P increases gradually from (9/2)*R* per mol to (15/2)*R* per mol at high temperatures, we'll assume that $C_P = 4.5R$ below $T = 2000$ K and

$C_P = 7.5R$ above $T = 2000$ K. We'll also use $R = 2.0$ cal/mol·K. We can find the final temperature following combustion from the heat made available during the combustion and the final pressure by applying the ideal-gas law to the initial and final states of the gases.

(*a*) Relate the heat available in this combustion process to the change in temperature of the triatomic gases:

$$Q_{\text{available}} = nC_P\Delta T = n(7.5R)(T_f - T_i)$$

Solve for T_f to obtain:

$$T_f = \frac{Q_{\text{available}}}{7.5nR} + T_i \qquad (1)$$

Express Q available to heat the gases above 2000 K:

$$Q_{\text{available}} = Q_{\text{released}} - Q_{9\,\text{mol to}\,2000\,\text{K}} - Q_{\text{heat}\,CO_2} - Q_{\text{steam}} \qquad (2)$$

Express the energy released in the combustion of 1 mol of benzene:

$$Q_{\text{released}} = \tfrac{1}{2}(1516\,\text{kcal}) = 758\,\text{kcal}$$

Noting that there are 3 mol of H_2O and 6 mol of CO_2, find the heat required to form the products at 100°C:

$$\begin{aligned}Q_{\text{steam}} &= nM_w c_w\Delta T + nM_w L_V \\ &= (3\,\text{mol})(18\,\text{g/mol}) \\ &\quad\times(1\,\text{cal/g}\cdot\text{K})(373\,\text{K} - 300\,\text{K}) \\ &\quad+(3\,\text{mol})(18\,\text{g/mol})(540\,\text{cal/g}) \\ &= 33.10\,\text{kcal}\end{aligned}$$

and

$$\begin{aligned}Q_{\text{heat}\,CO_2} &= nC_P\Delta T = 4.5nR\Delta T \\ &= 4.5(6\,\text{mol})(2\,\text{cal/mol}\cdot\text{K}) \\ &\quad\times(373\,\text{K} - 300\,\text{K}) \\ &= 3.942\,\text{kcal}\end{aligned}$$

Find Q required to heat 9 mol of gas to 2000 K:

$$\begin{aligned}Q_{9\,\text{mol to}\,2000\,\text{K}} &= nC_P\Delta T = 4.5nR\Delta T \\ &= 4.5(9\,\text{mol})(2\,\text{cal/mol}\cdot\text{K}) \\ &\quad\times(2000\,\text{K} - 373\,\text{K}) \\ &= 43.93\,\text{kcal}\end{aligned}$$

Substitute in equation (2) to obtain:

$$\begin{aligned}Q_{\text{available}} &= 758\,\text{kcal} - 131.79\,\text{kcal} \\ &\quad - 3.94\,\text{kcal} - 33.10\,\text{kcal} \\ &= 589.2\,\text{kcal}\end{aligned}$$

Substitute in equation (1) and evaluate T_f:

$$T_f = \frac{589.2\,\text{kcal}}{7.5(9\,\text{mol})(2\,\text{cal/mol}\cdot\text{K})} + 2000\,\text{K}$$
$$= \boxed{6364\,\text{K}}$$

Apply the ideal-gas law to express the final volume in terms of the final temperature and pressure:

$$V_f = \frac{nRT_f}{P_f}$$
$$= \frac{(9\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(6364\,\text{K})}{101.3\,\text{kPa}}$$
$$= \boxed{4.70\,\text{m}^3}$$

(*b*) Apply the ideal-gas law to relate the final temperature, pressure, and volume to the number of moles in the final state:

$$P_f V_f = n_f R T_f$$

Apply the ideal-gas law to relate the initial temperature, pressure, and volume to the number of moles in the initial state:

$$P_i V_i = n_i R T_i$$

Divide the first of these equations by the second and solve for P_f:

$$\frac{P_f V_f}{P_i V_i} = \frac{n_f R T_f}{n_i R T_i}$$

or, because $T_f = T_i$,

$$P_f = P_i\left(\frac{n_f}{n_i}\right)\left(\frac{V_i}{V_f}\right) \qquad (3)$$

Find the initial volume V_i occupied by 8.5 mol of gas at 300 K and 1 atm:

$$V_i = (22.4\,\text{L/mol})(8.5\,\text{mol})\left(\frac{300\,\text{K}}{273\,\text{K}}\right)$$
$$= 209.2\,\text{L}$$

Substitute numerical values in equation (3) and evaluate V_f:

$$P_f = (101.3\,\text{kPa})\left(\frac{9\,\text{mol}}{8.5\,\text{mol}}\right)\left(\frac{209.2\,\text{L}}{4700\,\text{L}}\right)$$
$$= 4.774\,\text{kPa} \times \frac{1\,\text{atm}}{101.325\,\text{kPa}}$$
$$= \boxed{0.0471\,\text{atm}}$$

***109** •••

Picture the Problem In this problem the specific heat of the combustion products depends on the temperature. Although C_P increases gradually from $(9/2)R$ per mol to $(15/2)R$ per mol at high temperatures, we'll assume that $C_P = 4.5R$ below $T = 2000$ K and $C_P = 7.5R$ above $T = 2000$ K. We can find the final temperature following combustion from the heat made available during the combustion and the final pressure by applying the ideal-gas law to the initial and final states of the gases.

(*a*) Apply the ideal-gas law to find the pressure due to 3 mol at 300 K in the container prior to the reaction:

$$P_\text{i} = \frac{nRT_\text{i}}{V_\text{i}} = \frac{(3\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(300\,\text{K})}{80\,\text{L}} = \boxed{93.5\,\text{kPa}}$$

(*b*) Relate the heat available in this adiabatic process to C_V and the change in temperature of the gases:

$$\Delta E_\text{int} = Q_\text{available} = C_\text{V}(T_\text{f} - T_\text{i})$$

Because $T > 2000$ K:

$$C_\text{V} = C_\text{P} - nR = n(7.5R) - nR = 6.5nR$$

Substitute to obtain:

$$Q_\text{available} = 6.5nR(T_\text{f} - T_\text{i})$$

Solve for T_f to obtain:

$$T_\text{f} = \frac{Q_\text{available}}{6.5nR} + T_\text{i} \qquad (1)$$

Find Q required to raise 2 mol of CO_2 to 2000 K:

$$Q_{\text{heat}\,CO_2} = C_\text{V}\Delta T$$

For $T < 2000$ K:

$$C_\text{V} = C_\text{P} - nR = n(4.5R) - nR = 3.5nR$$

Substitute for C_V and find the heat required to warm to CO_2 to 2000 K:

$$Q_{\text{heat}\,CO_2} = 3.5nR\Delta T = 3.5(2\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K}) \times (2000\,\text{K} - 300\text{K}) = 98.94\,\text{kJ}$$

Find Q available to heat 2 mol of CO_2 above 2000 K:

$$Q_\text{available} = 560\,\text{kJ} - 98.94\,\text{kJ} = 461.1\,\text{kJ}$$

Substitute in equation (1) and evaluate T_f:

$$T_\mathrm{f} = \frac{461.1\,\mathrm{kJ}}{6.5(2\,\mathrm{mol})(8.314\,\mathrm{J/mol\cdot K})} + 2000\,\mathrm{K} = \boxed{6266\,\mathrm{K}}$$

Apply the ideal-gas law to relate the final temperature, pressure, and volume to the number of moles in the final state:

$$P_\mathrm{f} V_\mathrm{f} = n_\mathrm{f} R T_\mathrm{f}$$

Apply the ideal-gas law to relate the initial temperature, pressure, and volume to the number of moles in the initial state:

$$P_\mathrm{i} V_\mathrm{i} = n_\mathrm{i} R T_\mathrm{i}$$

Divide the first of these equations by the second and solve for P_f:

$$\frac{P_\mathrm{f} V_\mathrm{f}}{P_\mathrm{i} V_\mathrm{i}} = \frac{n_\mathrm{f} R T_\mathrm{f}}{n_\mathrm{i} R T_\mathrm{i}}$$

or, because $V_\mathrm{f} = V_\mathrm{i}$,

$$P_\mathrm{f} = P_\mathrm{i}\left(\frac{n_\mathrm{f}}{n_\mathrm{i}}\right)\left(\frac{T_\mathrm{f}}{T_\mathrm{i}}\right) \qquad (2)$$

Substitute numerical values in equation (2) and evaluate P_f:

$$P_\mathrm{f} = (93.53\,\mathrm{kPa})\left(\frac{2\,\mathrm{mol}}{3\,\mathrm{mol}}\right)\left(\frac{6266\,\mathrm{K}}{300\,\mathrm{K}}\right) = \boxed{1.30\,\mathrm{MPa}}$$

(*c*) Substitute numerical values in equation (2) and evaluate P_f for $T_\mathrm{f} = 273$ K:

$$P_\mathrm{f} = (93.53\,\mathrm{kPa})\left(\frac{2\,\mathrm{mol}}{3\,\mathrm{mol}}\right)\left(\frac{273\,\mathrm{K}}{300\,\mathrm{K}}\right) = \boxed{56.7\,\mathrm{kPa}}$$

110 •••

Picture the Problem The molar heat capacity at constant volume is related to the internal energy per mole according to $c'_\mathrm{V} = \frac{1}{n}\frac{dU}{dT}$. We can differentiate U with respect to temperature and use $nR = Nk$ or $R = N_\mathrm{A}k$ to establish the result given in the problem statement.

From Problem 106 we have, for the internal energy per mol:

$$U = \frac{3N_\mathrm{A} k T_\mathrm{E}}{e^{T_\mathrm{E}/T} - 1}$$

Relate the molar heat capacity at constant volume to the internal energy per mol:

$$c'_{\mathrm{V}} = \frac{1}{n}\frac{dU}{dT}$$

Use $c'_{\mathrm{V}} = \frac{1}{n}\frac{dU}{dT}$ to express c'_{V}:

$$c'_{\mathrm{V}} = \frac{1}{n}\frac{d}{dT}\left[\frac{3N_{\mathrm{A}}kT_{\mathrm{E}}}{e^{T_{\mathrm{E}}/T}-1}\right] = 3RT_{\mathrm{E}}\frac{d}{dT}\left[\frac{1}{e^{T_{\mathrm{E}}/T}-1}\right] = 3RT_{\mathrm{E}}\left[\frac{-1}{\left(e^{T_{\mathrm{E}}/T}-1\right)^2}\right]\frac{d}{dT}\left(e^{T_{\mathrm{E}}/T}-1\right)$$

$$= 3RT_{\mathrm{E}}\left[\frac{-1}{\left(e^{T_{\mathrm{E}}/T}-1\right)^2}\right]\left[e^{T_{\mathrm{E}}/T}\left(-\frac{T_E}{T^2}\right)\right] = \boxed{3R\left(\frac{T_E}{T}\right)^2\frac{e^{T_{\mathrm{E}}/T}}{\left(e^{T_{\mathrm{E}}/T}-1\right)^2}}$$

111 •••

Picture the Problem We can rewrite our expression for c'_{V} by dividing its numerator and denominator by $e^{T_{\mathrm{E}}/T}$ and then using the power series for e^x to show that, for $T > T_{\mathrm{E}}$, $c'_{\mathrm{V}} \approx 3R$. In part (*b*), we can use the result of Problem 103 to obtain values for c'_{V} every 100 K between 300 K and 600 K and use this data to find ΔU numerically.

(*a*) From Problem 110 we have:

$$c'_{\mathrm{V}} = 3R\left(\frac{T_{\mathrm{E}}}{T}\right)^2\frac{e^{T_{\mathrm{E}}/T}}{\left(e^{T_{\mathrm{E}}/T}-1\right)^2}$$

Divide the numerator and denominator by $e^{T_{\mathrm{E}}/T}$ to obtain:

$$c'_{\mathrm{V}} = 3R\left(\frac{T_{\mathrm{E}}}{T}\right)^2\frac{1}{\dfrac{e^{2T_{\mathrm{E}}/T}-2e^{T_{\mathrm{E}}/T}+1}{e^{T_{\mathrm{E}}/T}}}$$

$$= 3R\left(\frac{T_{\mathrm{E}}}{T}\right)^2\frac{1}{e^{T_{\mathrm{E}}/T}-2+e^{-T_{\mathrm{E}}/T}}$$

Apply the power series expansion to obtain:

$$e^{T_{\mathrm{E}}/T}-2+e^{-T_{\mathrm{E}}/T} = 1+\frac{T_{\mathrm{E}}}{T}+\frac{1}{2}\left(\frac{T_{\mathrm{E}}}{T}\right)^2$$

$$+\ldots-2+1-\frac{T_{\mathrm{E}}}{T}+\frac{1}{2}\left(\frac{T_{\mathrm{E}}}{T}\right)^2+\ldots$$

$$\approx\left(\frac{T_{\mathrm{E}}}{T}\right)^2 \text{ for } T > T_{\mathrm{E}}$$

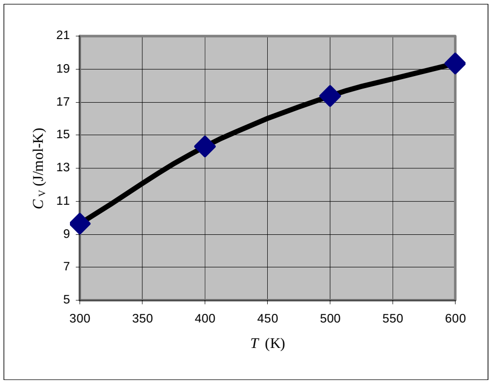
Substitute to obtain:

$$c'_{\mathrm{V}} \approx 3R\left(\frac{T_{\mathrm{E}}}{T}\right)^2\frac{1}{\left(\dfrac{T_{\mathrm{E}}}{T}\right)^2} = \boxed{3R}$$

(*b*) Use the result of Problem 110 to verify the table to the right:

T	c_V
(K)	(J/mol·K)
300	9.65
400	14.33
500	17.38
600	19.35

The graph of specific heat as a function of temperature shown to the right was plotted using a spreadsheet program:



Integrate numerically, using the formula for the area of a trapezoid, to obtain:

$$\Delta U = \tfrac{1}{2}(100\,\text{K})(9.65+14.33)\,\text{J/mol}\cdot\text{K} + \tfrac{1}{2}(100\,\text{K})(14.33+17.38)\,\text{J/mol}\cdot\text{K} + \tfrac{1}{2}(100\,\text{K})(17.38+19.35)\,\text{J/mol}\cdot\text{K} = \boxed{4.62\,\text{kJ}},$$

a result in good agreement (< 1% difference) with the result of Problem 106.

112 •••

Picture the Problem In (*a*) we'll assume that $\tau = f(A/V, T, k, m)$ with the factors dependent on constants *a*, *b*, *c*, and *d* that we'll find using dimensional analysis. In (*b*) we'll use our result from (*a*) and assume that the diameter of the puncture is about 2 mm, that the tire volume is 0.1 m^3, and that the air temperature is 20°C.

(*a*) Express $\tau = f(A/V, T, k, m)$:

$$\tau = \left(\frac{A}{V}\right)^a (T)^b (k)^c (m)^d \qquad (1)$$

Rewrite this equation in terms of the dimensions of the physical quantities to obtain:

$$\text{T}^1 = (\text{L})^{-a}(\text{K})^b\left(\frac{\text{ML}^2}{\text{T}^2\text{K}}\right)^c(\text{M})^d$$

where K represents the dimension of temperature.

Simplify this dimensional equation

$$\text{T}^1 = \text{L}^{-a}\text{K}^b\text{M}^c\text{L}^{2c}\text{K}^{-c}\text{T}^{-2c}\text{M}^d$$

to obtain:

or

$$\mathrm{T}^1 = \mathrm{L}^{2c-a}\mathrm{K}^{b-c}\mathrm{M}^{c+d}\mathrm{T}^{-2c}$$

Equate exponents to obtain:

$$\mathrm{T}:\ -2c = 1,$$

$$\mathrm{L}:\ 2c - a = 0,$$

$$\mathrm{K}:\ b - c = 0,$$

and

$$\mathrm{M}:\ c + d = 0$$

Solve these equations simultaneously to obtain:

$$c = -\tfrac{1}{2},$$

$$a = -1,$$

$$b = -\tfrac{1}{2},$$

and

$$d = \tfrac{1}{2}$$

Substitute in equation (1):

$$\tau = \left(\frac{A}{V}\right)^{-1}(T)^{-\frac{1}{2}}(k)^{-\frac{1}{2}}(m)^{\frac{1}{2}}$$

$$= \boxed{\frac{V}{A}\sqrt{\frac{m}{kT}}}$$

(*b*) Substitute numerical values and evaluate τ:

$$\tau = \frac{0.1\,\mathrm{m}^3}{\frac{\pi}{4}\left(2\times10^{-3}\,\mathrm{m}\right)^2}\sqrt{\frac{\left(1.293\,\mathrm{kg/m}^3\right)\left(0.1\,\mathrm{m}^3\right)}{\left(8.314\,\mathrm{J/mol\cdot K}\right)\left(293\,\mathrm{K}\right)}} = 232\,\mathrm{s} = \boxed{3.87\,\mathrm{min}}$$