Chapter 20
Thermal Properties and Processes

Conceptual Problems

***1 •**

Determine the Concept The glass bulb warms and expands first, before the mercury warms and expands.

2 •

Determine the Concept The heating of the sheet causes the average separation of its molecules to increase. The consequence of this increased separation is that the area of the hole always increases. $\boxed{(b) \text{ is correct.}}$

3 •

Determine the Concept Actually, it can be hard boiled, but it does take quite a bit longer than at sea level. $\boxed{(c) \text{ is the best response.}}$

4 •

Determine the Concept Gases that cannot be liquified by applying pressure at 20°C are those for which $T_c < 293$ K. These are He, Ar, Ne, H_2, O_2, NO.

***5 ••**

(*a*) With increasing altitude, P decreases; from curve OF, T of the liquid-gas interface diminishes, so the boiling temperature decreases. Likewise, from curve OH, the melting temperature increases with increasing altitude.

(*b*) Boiling at a lower temperature means that the cooking time will have to be increased.

6 •

Picture the Problem We can apply the Stefan-Boltzmann law to relate the rate at which an object radiates thermal energy to its environment.

Using the Stefan-Boltzmann law, relate the power radiated by a body to its temperature:

$$P_r = e\sigma AT^4$$

where A is the surface area of the body, σ is Stefan's constant, and e is the emissivity of the object.

Because P varies with the fourth power of T, tripling the temperature increases the rate at which it radiates by a factor of 3^4 and $\boxed{(d) \text{ is correct.}}$

***7 •**

Determine the Concept The thermal conductivity of metal and marble is much greater than that of wood; consequently, heat transfer from the hand is more rapid.

8 •

(*a*) True

(*b*) True

(*c*) False. The rate at which an object radiates energy is proportional to the fourth power of its absolute temperature.

(*d*) False. Water contracts on heating between 0°C and 4°C.

(*e*) True

9 •

Determine the Concept Because atoms are few and far between in space, the earth can not lose heat by conduction or convection. Thermal energy is radiated through space in the form of electromagnetic waves that move at the speed of light. $\boxed{(c)\text{ is correct.}}$

10 •

Determine the Concept Because there is little, if any, molecule-to-molecule transportation of energy into a fireplace-heated room, the mechanisms are radiation and convection.

11 •

Determine the Concept In the absence of matter to support conduction and convection, radiation is the only mechanism.

12 ••

Determine the Concept Because the amount of heat lost by the house is proportional to the difference between the house temperature and that of the outside air, the rate at which the house loses heat (that must be replaced by the furnace) is greater at night when the temperature of the house is kept high than when it is allowed to cool down.

13 ••

Picture the Problem The rate at which heat is conducted through a cylinder is given by $I = dQ/dt = kA\Delta T/\Delta x$ where A is the cross-sectional area of the cylinder.

Express the rate at which heat is conducted through cylinder A:

$$I_A = k_A \pi d_A^2 \frac{\Delta T}{\Delta x}$$

Express the rate at which heat is conducted through cylinder B:

$$I_B = k_B \pi d_B^2 \frac{\Delta T}{\Delta x}$$

Equate these expressions to obtain:

$$k_A \pi d_A^2 \frac{\Delta T}{\Delta x} = k_B \pi d_B^2 \frac{\Delta T}{\Delta x}$$

or

$$k_A d_A^2 = k_B d_B^2$$

Because $d_A = 2d_B$:

$$k_A (2d_B)^2 = k_B d_B^2$$

and

$$4k_A = k_B \Rightarrow \boxed{(a) \text{ is correct.}}$$

14 •

Determine the Concept Most objects of everyday experience are at temperatures near the mean temperature of the earth, about 300 K. Their blackbody spectrum therefore has a peak near λ_{max} = 2.898 mm K/ 300 K $\approx$ 0.01 mm = 10 μm = 10,000 nm. These wavelengths are in the infrared region of the spectrum, so the heat which most objects radiate away can be detected most easily in the infrared, which is the spectral region where most night-vision goggles and other types of optical "heat detectors" operate. However, if the temperature of the object increases, the wavelength decreases; so the peak radiation can be found in any spectral region, not just the infrared.

***15** •

Determine the Concept The temperature of an object is inversely proportional to the maximum wavelength at which the object radiates (Wein's displacement law). Because blue light has a shorter wavelength than red light, an object for which the wavelength of the peak of thermal emission is blue is hotter than one that is red.

Estimation and Approximation

16 •••

Picture the Problem We can express the heat current through the insulation in terms of the rate of evaporation of the liquid helium and in terms of the temperature gradient across the superinsulation. Equating these equations will allow us to solve for the thermal conductivity k of the superinsulation.

Express the heat current in terms of the rate of evaporation of the liquid helium:

$$I = L_v \frac{dm}{dt}$$

Express the heat current in terms of the temperature gradient across the superinsulation and the conductivity of the superinsulation:

$$I = kA \frac{\Delta T}{\Delta x}$$

Equate these expressions and solve for k:

$$k = \frac{L_v \Delta x \frac{dm}{dt}}{A\Delta T}$$

Using the definition of density, express the rate of loss of liquid helium:

$$\frac{dm}{dt} = \rho \frac{dV}{dt}$$

Substitute to obtain:

$$k = \frac{L_v \Delta x \rho \frac{dV}{dt}}{A\Delta T}$$

Express the ratio of the area of the spherical container to its volume:

$$\frac{A}{V} = \frac{4\pi r^2}{\frac{4}{3}\pi r^3}$$

Solve for A:

$$A = \sqrt[3]{36\pi V^2}$$

Substitute to obtain:

$$k = \frac{L_v \Delta x \rho \frac{dV}{dt}}{\sqrt[3]{36\pi V^2}\,\Delta T}$$

Substitute numerical values and evaluate k:

$$k = \frac{(21\,\text{kJ/kg})(7\times10^{-2}\,\text{m})(125\,\text{kg/m}^3)\left(\frac{0.7\times10^{-3}\,\text{m}^3}{86400\,\text{s}}\right)}{\sqrt[3]{36\pi(200\times10^{-3}\,\text{m}^3)^2}\,(288\,\text{K})} = \boxed{3.13\times10^{-6}\,\text{W/m}\cdot\text{K}}$$

17 ••

Picture the Problem We can use the thermal current equation for the thermal conductivity of the skin.

Use the thermal current equation to express the rate of conduction of thermal energy:

$$I = kA\frac{\Delta T}{\Delta x}$$

Solve for k to obtain:

$$k = \frac{I}{A\frac{\Delta T}{\Delta x}}$$

Substitute numerical values and evaluate k:

$$k = \frac{130\,\text{W}}{(1.8\,\text{m}^2)\frac{4\,\text{K}}{10^{-3}\,\text{m}}} = \boxed{18.1\,\text{mW/m}\cdot\text{K}}$$

***18 ••**

Picture the Problem The amount of heat radiated by the earth must equal the solar flux from the sun, or else the temperature on earth would continually increase. The emissivity of the earth is related to the rate at which it radiates energy into space by the Stefan-Boltzmann law $P_r = e\sigma A T^4$.

Using the Stefan-Boltzmann law, express the rate at which the earth radiates energy as a function of its emissivity e and temperature T:

$$P_r = e\sigma A' T^4$$

where A' is the surface area of the earth.

Solve for the emissivity of the earth:

$$e = \frac{P_r}{\sigma A' T^4}$$

Use its definition to express the intensity of the radiation received by the earth:

$$I = \frac{P_{absorbed}}{A}$$

where A is the cross-sectional area of the earth.

For 70% absorption of the sun's radiation incident on the earth:

$$I = \frac{0.7 P_r}{A}$$

Substitute for P_r and A and simplify to obtain:

$$e = \frac{0.7AI}{\sigma A T^4} = \frac{0.7\pi R^2 I}{4\pi R^2 \sigma T^4} = \frac{0.7I}{4\sigma T^4}$$

Substitute numerical values and evaluate e:

$$e = \frac{0.7\left(1370\,\text{W/m}^2\right)}{4\left(5.670\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\right)\left(288\,\text{K}\right)^4} = \boxed{0.615}$$

19 ••

Picture the Problem The wavelength at which maximum power is radiated by the gas falling into a black hole is related to its temperature by Wien's displacement law.

Express Wien's displacement law:

$$\lambda_{max} = \frac{2.898\,\text{mm}\cdot\text{K}}{T}$$

Substitute for T and evaluate λ_{max}:

$$\lambda_{max} = \frac{2.898\,\text{mm}\cdot\text{K}}{10^6\,\text{K}} = \boxed{2.90\,\text{nm}}$$

Thermal Expansion

20 •

Picture the Problem We can find the length of the ruler at 100°C by adding its elongation due to the increase in temperature to its length at 20°C. We can find its elongation using the definition of the coefficient of linear expansion $\alpha = (\Delta L/L)/\Delta T$.

Express the length of the ruler at 100°C in terms of its length at 20°C, its coefficient of linear expansion, and the change in its temperature:

$$\begin{aligned} L_{100^\circ\mathrm{C}} &= L_{20^\circ\mathrm{C}} + \Delta L \\ &= L_{20^\circ\mathrm{C}} + \alpha L_{20^\circ\mathrm{C}}\Delta T \\ &= L_{20^\circ\mathrm{C}}(1+\alpha\Delta T) \end{aligned}$$

Substitute numerical values and evaluate $L_{100^\circ\mathrm{C}}$:

$$\begin{aligned} L_{100^\circ\mathrm{C}} &= (30\,\mathrm{cm})\left[1+\left(11\times10^{-6}\,/\mathrm{K}\right)(80\,\mathrm{K})\right] \\ &= \boxed{30.026\,\mathrm{cm}} \end{aligned}$$

21 ••

Picture the Problem We can let the definition of the coefficient of linear expansion $\alpha = (\Delta L/L)/\Delta T$, with ΔA replacing ΔL and A replacing L suggest a definition of the coefficient of area expansion.

(*a*) Letting γ represent the coefficient of area expansion we have:

$$\gamma \equiv \boxed{\frac{\Delta A/A}{\Delta T}} \qquad (1)$$

(*b*) For a square:

$$\begin{aligned} \Delta A &= [L(1+\alpha\Delta T)]^2 - L^2 \\ &= L^2(1+2\alpha\Delta T+\alpha^2\Delta T^2) - L^2 \\ &= A(2\alpha\Delta T+\alpha^2\Delta T^2) \end{aligned}$$

Divide both sides of the equation by A to obtain:

$$\frac{\Delta A}{A} = 2\alpha\Delta T + \alpha^2\Delta T^2$$

Substitute in equation (1) to obtain:

$$\gamma = \frac{2\alpha\Delta T+\alpha^2\Delta T^2}{\Delta T} = 2\alpha + \alpha^2\Delta T$$

Let $\Delta T \to 0$ to obtain:

$$\gamma \approx \boxed{2\alpha\Delta T}$$

For a circle:

$$\begin{aligned} \Delta A &= \pi[R(1+\alpha\Delta T)]^2 - \pi R^2 \\ &= \pi R^2(1+2\alpha\Delta T+\alpha^2\Delta T^2) - \pi R^2 \\ &= A(2\alpha\Delta T+\alpha^2\Delta T^2) \end{aligned}$$

Divide both sides of the equation by A to obtain:

$$\frac{\Delta A}{A} = 2\alpha\Delta T + \alpha^2\Delta T^2$$

Substitute in equation (1) to obtain:

$$\gamma = \frac{2\alpha\Delta T + \alpha^2\Delta T^2}{\Delta T} = 2\alpha + \alpha^2\Delta T$$

Let $\Delta T \to 0$ to obtain:

$$\gamma \approx \boxed{2\alpha\Delta T}$$

22 ••

Picture the Problem While the mass of a sample of aluminum will remain constant with increasing temperature, its volume will increase due to thermal expansion. Consequently, its density will decrease with increasing temperature. We can use the definition of density (mass/unit volume) to express the density when its volume has increased by ΔV and the definition of the coefficient of volume expansion to relate ΔV to the increase in temperature ΔT. The relationship $\beta = 3\alpha$ will allow us to relate the coefficient of volume expansion to the coefficient of linear expansion.

Express the density of aluminum ρ' when its volume has changed by ΔV:

$$\rho' = \frac{m}{V + \Delta V} = \frac{m/V}{1 + \Delta V/V}$$

Using the definition of the coefficient of volume expansion, substitute for $\Delta V/V$ to obtain:

$$\rho' = \frac{\rho}{1 + \beta\Delta T} = \frac{\rho}{1 + 3\alpha\Delta T}$$

because $\beta = 3\alpha$.

Substitute numerical values and evaluate ρ':

$$\rho' = \frac{2.70\times10^3\ \text{kg/m}^3}{1 + 3\left(24\times10^{-6}\ /\text{K}\right)\left(200\ \text{K}\right)} = \boxed{2.66\times10^3\ \text{kg/m}^3}$$

23 ••

Picture the Problem Because the temperature of the steel shaft does not change, we need consider just the expansion of the copper collar. We can express the required temperature in terms of the initial temperature and the change in temperature that will produce the necessary increase in the diameter D of the copper collar. This increase in the diameter is related to the diameter at 20°C and the increase in temperature through the definition of the coefficient of linear expansion.

Express the temperature to which the copper collar must be raised in terms of its initial temperature and

$$T = T_i + \Delta T$$

the increase in its temperature:

Apply the definition of the coefficient of linear expansion to express the change in temperature required for the collar to fit on the shaft:

$$\Delta T = \frac{\left(\dfrac{\Delta D}{D}\right)}{\alpha}$$

Substitute to obtain:

$$T = T_\text{i} + \frac{\Delta D}{\alpha D}$$

Substitute numerical values and evaluate T:

$$T = 293\,\text{K} + \frac{0.02\,\text{cm}}{\left(17\times10^{-6}\,/\text{K}\right)\left(5.98\,\text{cm}\right)}$$

$$= 490\,\text{K} = \boxed{217°\text{C}}$$

***24** ••

Picture the Problem Because the temperatures of both the steel shaft and the copper collar change together, we can find the temperature change required for the collar to fit the shaft by equating their diameters for a temperature increase ΔT. These diameters are related to their diameters at 20°C and the increase in temperature through the definition of the coefficient of linear expansion.

Express the temperature to which the collar and the shaft must be raised in terms of their initial temperature and the increase in their temperature:

$$T = T_\text{i} + \Delta T \qquad (1)$$

Express the diameter of the steel shaft when its temperature has been increased by ΔT:

$$D_\text{steel} = D_\text{steel,20°C}\left(1 + \alpha_\text{steel}\Delta T\right)$$

Express the diameter of the copper collar when its temperature has been increased by ΔT:

$$D_\text{Cu} = D_\text{Cu,20°C}\left(1 + \alpha_\text{Cu}\Delta T\right)$$

If the collar is to fit over the shaft when the temperature of both has been increased by ΔT:

$$D_\text{Cu,20°C}\left(1 + \alpha_\text{Cu}\Delta T\right) = D_\text{steel,20°C}\left(1 + \alpha_\text{steel}\Delta T\right)$$

Solve for ΔT to obtain:

$$\Delta T = \frac{D_\text{steel,20°C} - D_\text{Cu,20°C}}{D_\text{Cu,20°C}\alpha_\text{Cu} - D_\text{steel,20°C}\alpha_\text{steel}}$$

Substitute in equation (1) to obtain:

$$T = T_i + \frac{D_{\text{steel,20°C}} - D_{\text{Cu,20°C}}}{D_{\text{Cu,20°C}}\alpha_{\text{Cu}} - D_{\text{steel,20°C}}\alpha_{\text{steel}}}$$

Substitute numerical values and evaluate T:

$$T = 293\,\text{K} + \frac{6.0000\,\text{cm} - 5.9800\,\text{cm}}{(5.98\,\text{cm})(17\times10^{-6}\,/\text{K}) - (6.00\,\text{cm})(11\times10^{-6}\,/\text{K})} = 854\,\text{K} = \boxed{581°\text{C}}$$

25 ••

Picture the Problem The linear expansion coefficient of the container is one-third its coefficient of volume expansion. We can relate the changes in volume of the mercury and the container to their initial volumes, temperature change, and coefficients of volume expansion, and, because we know the amount of spillage, obtain an equation that we can solve for β_c.

Relate the linear expansion coefficient of the container to its coefficient of volume expansion:

$$\alpha_c = \tfrac{1}{3}\beta_c \qquad (1)$$

Express the difference in the change in the volume of the mercury and the container in terms of the spillage:

$$\Delta V_{\text{Hg}} - \Delta V_c = 7.5\,\text{mL}$$

Express ΔV_{Hg} using the definition of the coefficient of volume expansion:

$$\Delta V_{\text{Hg}} = \beta_{\text{Hg}} V_{\text{Hg}} \Delta T$$

Express ΔV_c using the definition of the coefficient of volume expansion:

$$\Delta V_c = \beta_c V_c \Delta T$$

Substitute to obtain:

$$\beta_{\text{Hg}} V_{\text{Hg}} \Delta T - \beta_c V_c \Delta T = 7.5\,\text{mL}$$

Solve for β_c:

$$\beta_c = \frac{\beta_{\text{Hg}} V_{\text{Hg}} \Delta T - 7.5\,mL}{V_c \Delta T}$$

or, because $V = V_{\text{Hg}} = V_c$,

$$\beta_c = \frac{\beta_{\text{Hg}} V \Delta T - 7.5\,mL}{V \Delta T} = \beta_{\text{Hg}} - \frac{7.5\,mL}{V \Delta T}$$

Substitute in equation (1) to obtain:

$$\alpha_{\text{c}} = \tfrac{1}{3}\beta_{\text{Hg}} - \frac{7.5\,mL}{3V\Delta T} = \alpha_{\text{Hg}} - \frac{7.5\,mL}{3V\Delta T}$$

Substitute numerical values and evaluate α_{c}:

$$\alpha_{\text{c}} = \tfrac{1}{3}\left(0.18\times10^{-3}/\text{K}\right) - \frac{7.5\,\text{mL}}{3(1.4\,\text{L})(40\,\text{K})} = \boxed{15.4\times10^{-6}\ \text{K}^{-1}}$$

26 ••

Picture the Problem We can use $d_{\text{Fe,168°C}} = d_{\text{Fe,20°C}}(1+\alpha_{\text{Fe}}\Delta T)$ to find the diameter of the hole in the aluminum sheet at 168°C and then $d_{\text{Al,20°C}} = d_{\text{Al,168°C}}(1-\alpha_{\text{Al}}\Delta T)$ to find the diameter of the hole when the sheet has cooled to room temperature.

Relate the diameter of the hole/steel drill bit at 168°C to its diameter at 20° C:

$$d_{\text{Fe,168°C}} = d_{\text{Fe,20°C}}(1+\alpha_{\text{Fe}}\Delta T)$$

Substitute numerical values and evaluate $d_{\text{Fe,168°C}}$:

$$d_{\text{Fe,168°C}} = (6.245\,\text{cm})\left[1+11\times10^{-6}\ \text{K}^{-1}(148\,\text{K})\right] = 6.255\,\text{cm}$$

Express the diameter of the hole in the plate at 20° C:

$$d_{\text{Al,20°C}} = d_{\text{Al,168°C}}(1-\alpha_{\text{Al}}\Delta T)$$

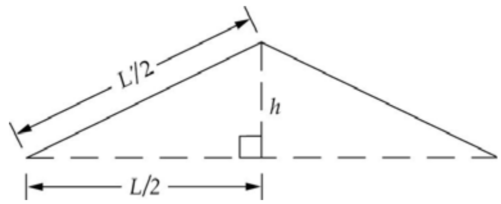
Substitute numerical values and evaluate $d_{\text{Al,20°C}}$:

$$d_{\text{Al,20°C}} = (6.255\,\text{cm})\left[1-\left(24\times10^{-6}\ \text{K}^{-1}\right)(148\,\text{K})\right] = \boxed{6.233\,\text{cm}}$$

Remarks: Note that the diameter of the hole in the plate at 20°C is less than the diameter of the drill bit at 20°C.

***27 ••**

Picture the Problem Let L be the length of the rail at 20°C and L' its length at 25°C. The diagram shows these distances and the height h of the buckle. We can use Pythagorean theorem to relate the height of the buckle to the distances L and L' and the definition of the coefficient of linear expansion to relate L and L'.



Apply the Pythagorean theorem to obtain:

$$h = \sqrt{\left(\frac{L'}{2}\right)^2 - \left(\frac{L}{2}\right)^2} = \tfrac{1}{2}\sqrt{L'^2 - L^2}$$

Use the definition of the coefficient of linear expansion to relate L and L':

$L'^2 = L^2(1 + \alpha_{\text{steel}}\Delta T)^2$

or, because $(\alpha_{\text{steel}}\Delta T)^2 << 2\alpha_{\text{steel}}\Delta T$,

$L'^2 \approx L^2(1 + 2\alpha_{\text{steel}}\Delta T)$

Substitute to obtain:

$$h = \frac{1}{2}\sqrt{L^2(1 + 2\alpha_{\text{steel}}\Delta T) - L^2} = \frac{L}{2}\sqrt{2\alpha_{\text{steel}}\Delta T}$$

Substitute numerical values and evaluate h:

$$h = \frac{1000\,\text{m}}{2}\sqrt{2(11\times10^{-6}\,\text{K}^{-1})(5\,\text{K})} = \boxed{5.24\,\text{m}}$$

28 ••

Picture the Problem The amount of gas that spills is the difference between the change in the volume of the gasoline and the change in volume of the tank. We can find this difference by expressing the changes in volume of the gasoline and the tank in terms of their common volume at 10°C, their coefficients of volume expansion, and the change in the temperature.

Express the spill in terms of the change in volume of the gasoline and the change in volume of the tank:

$V_{\text{spill}} = \Delta V_{\text{gas}} - \Delta V_{\text{tank}}$

Relate ΔV_{gas} to the coefficient of volume expansion for gasoline:

$\Delta V_{\text{gas}} = \beta_{\text{gas}} V \Delta T$

Relate ΔV_{tank} to the coefficient of linear expansion for steel:

$\Delta V_{tank} = \beta_{tank} V\Delta T$
or, because $\beta_{steel} = 3\alpha_{steel}$,
$\Delta V_{tank} = 3\alpha_{steel} V\Delta T$

Substitute to obtain:

$$V_{spill} = \beta_{gas} V\Delta T - 3\alpha_{steel} V\Delta T = V\Delta T\,(\beta_{gas} - 3\alpha_{steel})$$

Substitute numerical values and evaluate V_{spill}:

$$V_{spill} = (60\,\text{L})(15\,\text{K})\left[0.9\times10^{-3}\,\text{K}^{-1} - 3\left(11\times10^{-6}\,\text{K}^{-1}\right)\right] = \boxed{0.780\,\text{L}}$$

29 ••

Picture the Problem We can relate the diameter of the capillary tube to the height the mercury rises for a 1°C increase in temperature and to the difference in the volume changes of the mercury in the bulb and the glass bulb. These volume changes can, in turn, be expressed in terms of the coefficients of volume expansion of mercury and glass.

Express the net change in volume of the mercury in the thermometer and the bulb and tube of the glass thermometer:

$\Delta V = \Delta V_{Hg} - \Delta V_{glass} = A\Delta L$
where $A = \pi d^2/4$ is the cross-sectional area of the capillary tube and d is its diameter.

Relate ΔV_{Hg} to the coefficient of linear expansion for mercury:

$\Delta V_{Hg} = \beta_{Hg} V\Delta T$

Relate ΔV_{glass} to the coefficient of linear expansion for glass:

$\Delta V_{glass} = \beta_{glass} V\Delta T$
or, because $\beta_{glass} = 3\alpha_{glass}$,
$\Delta V_{glass} = 3\alpha_{glass} V\Delta T$

Substitute to obtain:

$$\frac{\pi d^2}{4} = \beta_{Hg} V\Delta T - 3\alpha_{glass} V\Delta T = V\Delta T\left(\beta_{Hg} - 3\alpha_{glass}\right)$$

Solve for d:

$$d = \sqrt{\frac{4V\Delta T}{\pi\Delta L}\left(\beta_{Hg} - 3\alpha_{glass}\right)}$$

Substitute numerical values and evaluate d:

$$d = \sqrt{\frac{4\left(10^{-6}\,\text{m}^3\right)(1\text{K})}{\pi\left(3\times10^{-3}\,\text{m}\right)}\left(0.18\times10^{-3}\,\text{K}^{-1} - 3\left(9\times10^{-6}\,\text{K}^{-1}\right)\right)} = \boxed{0.255\,\text{mm}}$$

30 ••

Picture the Problem We can relate the volume of the thermometer bulb to the height the mercury rises for the 8 C° increase in temperature and to the difference in the volume changes of the mercury in the bulb and the glass bulb. These volume changes can, in turn, be expressed in terms of the coefficients of volume expansion of mercury and glass.

Express the net change in volume of the mercury in the thermometer and the bulb and tube of the glass thermometer:

$\Delta V = \Delta V_{\text{Hg}} - \Delta V_{\text{glass}} = A\Delta L$
where $A = \pi d^2/4$ is the cross-sectional area of the capillary tube and d is its diameter.

Relate ΔV_{Hg} to the coefficient of linear expansion for mercury:

$\Delta V_{\text{Hg}} = \beta_{\text{Hg}} V\Delta T$
or, because $\beta_{\text{Hg}} = 3\alpha_{\text{Hg}}$,
$\Delta V_{\text{Hg}} = 3\alpha_{\text{Hg}} V\Delta T$

Relate ΔV_{glass} to the coefficient of linear expansion for glass:

$\Delta V_{\text{glass}} = \beta_{\text{glass}} V\Delta T$
or, because $\beta_{\text{glass}} = 3\alpha_{\text{glass}}$,
$\Delta V_{\text{glass}} = 3\alpha_{\text{glass}} V\Delta T$

Substitute to obtain:

$$\beta_{\text{Hg}} V\Delta T - 3\alpha_{\text{glass}} V\Delta T = A\Delta L$$

Solve for V and substitute for A:

$$V = \frac{A\Delta L}{\left(\beta_{\text{Hg}} - 3\alpha_{\text{glass}}\right)\Delta T} = \frac{\pi d^2 \Delta L}{4\left(\beta_{\text{Hg}} - 3\alpha_{\text{glass}}\right)\Delta T}$$

Substitute numerical values and evaluate V:

$$V = \frac{\pi\left(0.4\times10^{-3}\text{ m}\right)^2\left(7.5\times10^{-2}\text{ m}\right)}{4\left[0.18\times10^{-3}\text{ K}^{-1} - 3\left(9\times10^{-6}\text{ K}^{-1}\right)\right]\left(8\text{ K}\right)} = \boxed{7.70\text{ mL}}$$

31 •••

Picture the Problem We can determine whether the clock runs fast or slow from the expression for the period of a simple pendulum and the dependence of its length on the temperature. Letting T_{P} represent the period of the pendulum and T the temperature, we can evaluate dT_{P}/dT and use a differential approximation to find the time gained or lost in a 24-h period.

(*a*) Express the period of the pendulum in terms of its length:

$$T_{\text{P}} = 2\pi\sqrt{\frac{L}{g}}$$

Because $T_P \propto \sqrt{L}$ and L is temperature dependent:

$$\boxed{\text{The clock runs slow.}}$$

(*b*) Because the clock runs slow at the higher temperature, we know that it will lose time. Express the loss in terms of the loss each period and the elapsed time Δt:

$$\text{Loss} = \frac{\Delta T_P}{T_P}\Delta t \qquad (1)$$

Write $\frac{dT_P}{dT}$ as the product of $\frac{dT_P}{dL}$ and $\frac{dL}{dT}$:

$$\frac{dT_P}{dT} = \frac{dT_P}{dL}\cdot\frac{dL}{dT}$$

Evaluate $\frac{dT_P}{dL}$ and simplify to obtain:

$$\frac{dT_P}{dL} = \frac{d}{dL}\left[2\pi\sqrt{\frac{L}{g}}\right] = \frac{1}{2}\left(\frac{2\pi}{g}\right)\left(\frac{L}{g}\right)^{-\frac{1}{2}}$$
$$= \frac{1}{2}\left(\frac{2\pi}{g}\right)\sqrt{\frac{g}{L}} = \frac{1}{2L}\left(2\pi\sqrt{\frac{L}{g}}\right)$$
$$= \frac{T_P}{2L}$$

Express the dependence of the length of the pendulum on its calibration length L_0 and the coefficient of linear expansion of brass α:

$$L = L_0(1+\alpha\Delta T)$$

Evaluate $\frac{dL}{dT}$:

$$\frac{dL}{dT} = \frac{d}{dT}\left[L_0(1+\alpha\Delta T)\right] = \alpha L_0$$

Substitute to obtain:

$$\frac{dT_P}{dT} = \left(\frac{T_P}{2L_0}\right)(\alpha L_0) = \frac{\alpha}{2}T_P$$

Use the differential approximation to obtain:

$$\frac{\Delta T_P}{\Delta T} = \frac{\alpha}{2}T_P \text{ or } \frac{\Delta T_P}{T_P} = \frac{\alpha}{2}\Delta T$$

Substitute numerical values and evaluate $\Delta T_P/T_P$:

$$\frac{\Delta T_P}{T_P} = \tfrac{1}{2}\left(19\times10^{-6}\,/\text{K}\right)(10\,\text{K})$$
$$= 9.50\times10^{-5}$$

Substitute in equation (1) to obtain:

$$\text{Loss} = \left(9.50\times10^{-5}\right)\left(24\,\text{h}\times\frac{3600\,\text{s}}{\text{h}}\right)$$
$$= \boxed{8.21\,\text{s}}$$

32 •••

Picture the Problem The steel tube will fit inside the brass tube when its outside diameter equals the inside diameter of the brass tube. We can use the definition of the coefficient of linear expansion to express the diameters of the tubes when they fit in terms of the required temperature change and equate these expressions to find ΔT.

Express the temperature at which the steel tube will fit inside the brass tube in terms of their initial temperature and the change in temperature:

$$T = T_\text{i} + \Delta T = 293\,\text{K} + \Delta T \qquad (1)$$

Express the condition that the steel tube will fit inside the brass tube:

$$d_\text{steel} = d_\text{brass}$$

Relate the diameter of the steel tube to its initial diameter, coefficient of linear expansion, and the change in temperature:

$$d_\text{steel} = d_\text{0,steel}\left(1+\alpha_\text{steel}\Delta T\right)$$

Relate the diameter of the brass tube to its initial diameter, coefficient of linear expansion, and the change in temperature:

$$d_\text{brass} = d_\text{0,brass}\left(1+\alpha_\text{brass}\Delta T\right)$$

Substitute to obtain:

$$d_\text{0,steel}\left(1+\alpha_\text{steel}\Delta T\right) = d_\text{0,brass}\left(1+\alpha_\text{brass}\Delta T\right)$$

Solve for ΔT:

$$\Delta T = \frac{d_\text{0,steel} - d_\text{0,brass}}{d_\text{0,brass}\alpha_\text{brass} - d_\text{0,steel}\alpha_\text{steel}}$$

Substitute numerical values and evaluate ΔT:

$$\Delta T == \frac{3.000\,\text{cm} - 2.997\,\text{cm}}{(3.000\,\text{cm})\left(19\times10^{-6}\,\text{K}^{-1}\right) - (2.997\,\text{cm})\left(11\times10^{-6}\,\text{K}^{-1}\right)} = 125\,\text{K}$$

Substitute in equation (1) to evaluate ΔT:

$$T = 293\,\text{K} + 125\,\text{K} = 418\,\text{K} = \boxed{145^\circ\text{C}}$$

***33 •••**

Picture the Problem We can use the definition of Young's modulus to express the tensile stress in the copper in terms of the strain it undergoes as its temperature returns to 20°C. We can show that $\Delta L/L$ for the circumference of the collar is the same as $\Delta d/d$ for its diameter.

Using Young's modulus, relate the stress in the collar to its strain:

$$\text{Stress} = Y \times \text{Strain} = Y\frac{\Delta L}{L_{20°\text{C}}}$$

where $L_{20°\text{C}}$ is the circumference of the collar at 20°C.

Express the circumference of the collar at the temperature at which it fits over the shaft:

$$L_T = \pi d_T$$

Express the circumference of the collar at 20°C:

$$L_{20°\text{C}} = \pi d_{20°\text{C}}$$

Substitute to obtain:

$$\text{Stress} = Y\frac{\pi d_T - \pi d_{20°\text{C}}}{\pi d_{20°\text{C}}} = Y\frac{d_T - d_{20°\text{C}}}{d_{20°\text{C}}}$$

Substitute numerical values and evaluate the stress:

$$\text{Stress} = \left(11\times 10^{-10}\ \text{N/m}^2\right)\frac{0.02\,\text{cm}}{5.98\,\text{cm}} = \boxed{3.68\times 10^{-12}\ \text{N/m}^2}$$

The van der Waals Equation, Liquid-Vapor Isotherms, and Phase Diagrams

34 •

Picture the Problem We can apply the ideal-gas law to find the volume of 1 mol of steam at 100°C and a pressure of 1 atm and then use the van der Waals equation to find the temperature at which the steam will this volume.

(*a*) Use the ideal-gas law to find the volume:

$$V = \frac{nRT}{P} = \frac{(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(373\,\text{K})}{1\,\text{atm}\times\dfrac{101.325\,\text{kPa}}{\text{atom}}} = 3.06\times10^{-2}\,\text{m}^3\times\frac{1\,\text{L}}{10^{-3}\,\text{m}^3} = \boxed{30.6\,\text{L}}$$

(*b*) Solve van der Waals equation for *T* to obtain:

$$T = \frac{\left(P + \dfrac{an^2}{V^2}\right)(V - bn)}{nR}$$

Substitute numerical values and evaluate *T*:

$$T = \frac{\left(P + \dfrac{an^2}{V^2}\right)(V - bn)}{nR} = \left(101.3\,\text{kPa} + \frac{(0.55\,\text{Pa}\cdot\text{m}^6/\text{mol}^2)(1\,\text{mol})^2}{(3.06\times10^{-2}\,\text{m}^3)^2}\right)$$
$$\times\frac{3.06\times10^{-2}\,\text{m}^3 - (30\times10^{-6}\,\text{m}^3/\text{mol})(1\,\text{mol})}{(1\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})}$$
$$= \boxed{375\,\text{K}}$$

35 ••

Picture the Problem We can find these temperatures and pressure by consulting Figure 20-3.

(*a*) At 70 kPa, water boils at: $t \approx \boxed{90°\text{C}}$

(*b*) At 0.5 atm (about 51 kPa): $t_{\text{boil}} \approx \boxed{82°\text{C}}$

(*c*) For t_{boil} = 115°C: $P \approx \boxed{170\,\text{kPa}}$

***36** ••

Picture the Problem Assume that a helium atom is spherical. Then we can find its radius from $V = \frac{4}{3}\pi r^3$ and its volume from the van der Waals equation.

Express the radius of a spherical atom in terms of its volume:

$$r=\sqrt[3]{\frac{3V}{4\pi}}$$

In the van der Waals equation, b is the volume of 1 mol of molecules. For He, 1 molecule = 1 atom. Use Avogadro's number to express b in cm^3/atom:

$$b=\frac{(0.0237\ \text{L/mol})(10^3\ \text{cm}^3/\text{L})}{6.022\times10^{23}\ \text{atoms/mol}}$$
$$=3.94\times10^{-23}\ \text{cm}^3/\text{atom}$$

Substitute numerical values and evaluate r:

$$r=\sqrt[3]{\frac{3b}{4\pi}}=\sqrt[3]{\frac{3(3.94\times10^{-23}\ \text{cm}^3)}{4\pi}}$$
$$=2.11\times10^{-8}\ \text{cm}=\boxed{0.211\text{nm}}$$

37 •••

Picture the Problem Because, at the critical point, $dP/dV = 0$ and $d^2P/dV^2 = 0$, we can solve the van der Waals equation for P and set its first and second derivatives equal to zero to find V_c. We can then eliminate V_c between these equations to find T_c and then substitute in the van der Waals equation to express P_c. Finally, we can use their definitions to rewrite the van der Waals equation in terms of the reduced variables.

(*a*) Solve the van der Waals equation for P:

$$P=\frac{nRT}{V-bn}-\frac{an^2}{V^2} \qquad (1)$$

Evaluate dP/dV:

$$\frac{dP}{dV}=\frac{d}{dV}\left[\frac{nRT}{V-bn}-\frac{an^2}{V^2}\right]$$
$$=-\frac{nRT}{(V-nb)^2}+\frac{2an^2}{V^3} \qquad (2)$$
$$=0\text{ for extrema}$$

Evaluate $\dfrac{d^2P}{dV^2}$:

$$\frac{d^2P}{dV^2}=\frac{d}{dV}\left[-\frac{nRT}{(V-nb)^2}+\frac{2an^2}{V^3}\right]$$
$$=\frac{2nRT}{(V-nb)^3}-\frac{6an^2}{V^4} \qquad (3)$$
$$=0\text{ for critical points}$$

Solve equation (2) for $\dfrac{2an^2}{V^3}$:

$$\frac{2an^2}{V^3}=\frac{nRT}{(V-nb)^2} \qquad (4)$$

Solve equation (3) for $\frac{6an^2}{V^4}$:

$$\frac{6an^2}{V^4}=\frac{2nRT}{(V-nb)^3} \qquad (5)$$

Divide equation (4) by equation (5) and simplify to obtain:

$$\tfrac{1}{3}V=\tfrac{1}{2}(V-nb)$$

Solve for $V = V_c$:

$$V_c = 3nb$$

Substitute in equation (4):

$$\frac{2an^2}{27n^3b^3}=\frac{nRT_c}{(3nb-nb)^2}$$

Simplify and solve for T_c:

$$T_c=\boxed{\frac{8a}{27Rb}}$$

Substitute for V_c and T_c in equation (1) and simplify to obtain:

$$P_c=\frac{nR\frac{8a}{27Rb}}{3bn-bn}-\frac{an^2}{(3bn)^2}=\boxed{\frac{a}{27b^2}}$$

(*b*) Using the result for V_c from (*a*), express the reduced volume V_r:

$$V_r=\frac{V}{V_c}=\frac{V}{3nb} \text{ and } V=3nbV_r$$

Using the result for T_c from (*a*), express the reduced temperature T_r:

$$T_r=\frac{T}{T_c}=\frac{27RbT}{8a}$$

and

$$T=\frac{8a}{27Rb}T_r$$

Using the result for P_c from (*a*), express the reduced pressure P_r:

$$P_r=\frac{P}{P_c}=\frac{27b^2P}{a}$$

and

$$P=\frac{a}{27b^2}P_r$$

Substitute in the van der Waals equation to obtain:

$$\left(\frac{a}{27b^2}P_r+\frac{an^2}{(3nbV_r)^2}\right)(3nbV_r-bn)=nR\frac{8a}{27Rb}T_r$$

Simplify to obtain:

$$\boxed{\left(P_r + \frac{3}{V_r^2}\right)(3V_r - 1) = 8T_r}$$

Heat Conduction

38 •

Picture the Problem We can use their definitions to find the thermal resistance of the bar, the thermal current in the bar, and the temperature gradient in the bar. Because the temperature varies linearly with distance along the bar, we can express the temperature in terms of the thermal gradient and evaluate this expression 25 cm from the hot end.

(*a*) Using its definition, find the thermal resistance of the bar:

$$R = \frac{\Delta x}{kA} = \frac{\Delta x}{k\pi r^2}$$

$$= \frac{2\,\text{m}}{(401\,\text{W/m}\cdot\text{K})\left[\pi\left(10^{-4}\,\text{m}^2\right)\right]}$$

$$= \boxed{15.9\,\text{K/W}}$$

(*b*) Using its definition, find the thermal current in the bar:

$$I = \frac{\Delta T}{R} = \frac{100\,\text{K}}{15.9\,\text{K/W}} = \boxed{6.29\,\text{W}}$$

(*c*) Substitute numerical values and evaluate the temperature gradient:

$$\frac{\Delta T}{\Delta x} = \frac{100\,\text{K}}{2\,\text{m}} = 50\,\text{K/m} = \boxed{50\,\text{K/m}}$$

(*d*) Express the linear dependence of the temperature in the bar on the distance from the cold end:

$$T = T_0 + \frac{dT}{dx}\Delta x$$

Substitute numerical values and evaluate T(1.75 m):

$$T(1.75\,\text{m}) = 273\,\text{K} + (50\,\text{K/m})(1.75\,\text{m})$$

$$= 360.5\,\text{K} = \boxed{87.5°\text{C}}$$

39 •

Picture the Problem We can use its definition to express the thermal current in the slab in terms of the temperature differential across it and its thermal resistance and use the definition of the R factor to express I as a function of ΔT, the cross-sectional area of the slab, and R_f.

Express the thermal current through the slab in terms of the temperature difference across it and its thermal

$$I = \frac{\Delta T}{R}$$

resistance:

Substitute to express R in terms of the insulation's R factor:

$$I = \frac{\Delta T}{R_f / A} = \frac{A\Delta T}{R_f}$$

Substitute numerical values and evaluate I:

$$I = \frac{(20\,\text{ft})(30\,\text{ft})(68°\text{F} - 30°\text{F})}{11\,\text{h}\cdot\text{ft}^2\cdot\text{F°/Btu}} = \boxed{2.07\,\text{kBtu/h}}$$

40 ••

Picture the Problem We can use $R = \Delta x/kA$ to find the thermal resistance of each cube and the fact that they are in series to find the thermal resistance of the two-cube system. We can use $I = \Delta T/R$ to find the thermal current through the cubes and the temperature at their interface.

(*a*) Using its definition, express the thermal resistance of each cube:

$$R = \frac{\Delta x}{kA}$$

Substitute numerical values and evaluate the thermal resistance of the copper cube:

$$R_{Cu} = \frac{3\,\text{cm}}{(401\,\text{W/m}\cdot\text{K})(3\,\text{cm})^2} = \boxed{0.0831\,\text{K/W}}$$

Substitute numerical values and evaluate the thermal resistance of the aluminum cube:

$$R_{Al} = \frac{3\,\text{cm}}{(237\,\text{W/m}\cdot\text{K})(3\,\text{cm})^2} = \boxed{0.141\,\text{K/W}}$$

(*b*) Because the cubes are in series, their thermal resistances are additive:

$$R = R_{Cu} + R_{Al} = 0.0831\,\text{K/W} + 0.141\,\text{K/W} = \boxed{0.224\,\text{K/W}}$$

(*c*) Using its definition, find the thermal current:

$$I = \frac{\Delta T}{R} = \frac{373\,\text{K} - 293\,\text{K}}{0.224\,\text{K/W}} = \boxed{357\,\text{W}}$$

(*d*) Express the temperature at the interface between the two cubes:

$$T_{\text{interface}} = 373\,\text{K} - \Delta T_{Cu}$$

Express the temperature differential across the copper cube:

$$\Delta T_{Cu} = I_{Cu} R_{Cu} = I R_{Cu}$$

Substitute numerical values and evaluate $T_{\text{interface}}$:

$$T_{\text{interface}} = 373\,\text{K} - IR_{\text{Cu}} = 373\,\text{K} - (357\,\text{W})(0.0831\,\text{K/W}) = 343.3\,\text{K} = \boxed{70.3°\text{C}}$$

41 ••

Picture the Problem We can use $I = \Delta T/R$ and $R = \Delta x/kA$ to find the thermal current in each cube. Because the currents are additive, we can find the equivalent resistance of the two-cube system from $R_{\text{eq}} = \Delta T/I$.

(*a*) Using its definition, express the thermal current through each cube:

$$I = \frac{\Delta T}{R}$$

Using its definition, express the thermal resistance of each cube:

$$R = \frac{\Delta x}{kA}$$

Substitute to obtain:

$$I = \frac{kA\Delta T}{\Delta x}$$

Substitute numerical values and evaluate the thermal current in the copper cube:

$$I_{\text{Cu}} = \frac{(401\,\text{W/m}\cdot\text{K})(3\,\text{cm})^2}{3\,\text{cm}} \times (373\,\text{K} - 293\,\text{K}) = \boxed{962\,\text{W}}$$

Substitute numerical values and evaluate the thermal current in the aluminum cube:

$$I_{\text{Al}} = \frac{(237\,\text{W/m}\cdot\text{K})(3\,\text{cm})^2}{3\,\text{cm}} \times (373\,\text{K} - 293\,\text{K}) = \boxed{569\,\text{W}}$$

(*b*) Because the cubes are in parallel, their total thermal currents are additive:

$$I = I_{\text{Cu}} + I_{\text{Al}} = 962\,\text{W} + 569\,\text{W} = \boxed{1.53\,\text{kW}}$$

(*c*) Use the relationship between the thermal current, temperature differential and thermal resistance to find R_{eq}:

$$R_{\text{eq}} = \frac{\Delta T}{I} = \frac{373\,\text{K} - 293\,\text{K}}{1.53\,\text{kW}} = \boxed{0.0523\,\text{K/W}}$$

42 ••

Picture the Problem The cost of operating the air conditioner is proportional to the energy used in its operation. We can use the definition of the COP to relate the rate at which the air conditioner removes heat from the house to rate at which it must do work to maintain a constant temperature differential between the interior and the exterior of the house. To obtain an expression for the minimum rate at which the air conditioner must do work, we'll assume that it is operating with the maximum efficiency possible. Doing so will allow us to derive an expression for the rate at which energy is used by the air conditioner that we can integrate to obtain the energy (and hence the cost of operation) required.

Relate the cost C of air conditioning the energy W required to operate the air conditioner:

$$C = uW \qquad (1)$$

where u is the unit cost of the energy.

Express the rate dQ/dt at which heat flows into a house provided the house is maintained at a constant temperature:

$$P = \frac{dQ}{dt} = k\Delta T$$

where ΔT is the temperature difference between the interior and exterior of the house.

Use the definition of the COP to relate the rate at which the air conditioner must remove heat dW/dt to maintain a constant temperature:

$$\mathrm{COP} = \frac{dQ/dt}{dW/dt}$$

Solve for dW/dt:

$$dW/dt = \frac{dQ/dt}{\mathrm{COP}}$$

Express the maximum value of the COP:

$$\mathrm{COP}_{\mathrm{max}} = \frac{T_\mathrm{c}}{\Delta T}$$

where T_c is the temperature of the cold reservoir.

Letting COP = $\mathrm{COP}_{\mathrm{max}}$, substitute to obtain an expression for the minimum rate at which the air conditioner must do work in order to maintain a constant temperature:

$$\frac{dW}{dt} = \frac{dQ/dt}{T_\mathrm{c}}\Delta T$$

Substitute for dQ/dt to obtain:

$$\frac{dW}{dt} = \frac{k\Delta T}{T_\mathrm{c}}\Delta T = \frac{k}{T_\mathrm{c}}(\Delta T)^2$$

Separate variables and integrate this equation to obtain:

$$W = \frac{k}{T_\mathrm{c}}(\Delta T)^2\int_0^{\Delta t} dt' = \frac{k}{T_\mathrm{c}}(\Delta T)^2\,\Delta t$$

Substitute in equation (1) to obtain:

$$C = u\frac{k}{T_\mathrm{c}}(\Delta T)^2\,\Delta t \propto \boxed{(\Delta T)^2}$$

43 •••

Picture the Problem We can follow the step-by-step instructions given in the problem statement to obtain the differential equation describing the variation of T with r. Integrating this equation will yield an equation that we can solve for the current I.

(*a*)

$\boxed{\text{Conservation of energy requires that thermal current through each shell be the same.}}$

(*b*) Express the thermal current I through such a shell element in terms of the area $A = 4\pi r^2$, the thickness dr, and the temperature difference dT across the element:

$$I = -kA\frac{dT}{dr} = \boxed{-4\pi kr^2\frac{dT}{dr}}$$

where the minus sign is a consequence of the heat current being opposite the temperature gradient.

(*c*) Separate the variables:

$$dT = -\frac{I}{4\pi k}\frac{dr}{r^2}$$

Integrate from $r = r_1$ to $r = r_2$:

$$\int_{T_1}^{T_2} dT = -\frac{I}{4\pi k}\int_{r_1}^{r_2}\frac{dr}{r^2}$$

and

$$T_2 - T_1 = \frac{I}{4\pi k}\left[\frac{1}{r}\right]_{r_1}^{r_2} = \frac{I}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Solve for I to obtain:

$$I = \boxed{\frac{4\pi kr_1r_2}{r_2 - r_1}(T_2 - T_1)}$$

(*d*) When $r_2 - r_1 << r_1$:

$$r_1 \approx r_2 = r$$

Let $r_2 - r_1 = \Delta r$ and substitute to obtain:

$$I = \frac{4\pi kr^2}{\Delta r}(T_2 - T_1) = \boxed{4\pi kr^2\frac{\Delta T}{\Delta r}}$$

which is Equation 20-7.

***44** ••

Picture the Problem We can use the expression for the thermal current to express the thickness of the walls in terms of the thermal conductivity of copper, the area of the walls, and the temperature difference between the inner and outer surfaces. Letting $\Delta A/\Delta x'$ represent the area per unit length of the pipe and L its length, we can eliminate the surface area and solve for and evaluate L.

Write the expression for the thermal current:

$$I = kA\frac{\Delta T}{\Delta x}$$

Solve for A:

$$A = \frac{I\Delta x}{k\Delta T}$$

Express the total surface area of the pipe:

$$A = \frac{\Delta A}{\Delta x'}L$$

Substitute for A and solve for L to obtain:

$$L = \frac{\frac{I\Delta x}{k\Delta T}}{\Delta A/\Delta x'}$$

Substitute numerical values and evaluate L:

$$L = \frac{\left[\frac{(3\,\text{GW})(4\times10^{-3}\,\text{m})}{(401\,\text{W/m}\cdot\text{K})(873\,\text{K}-498\,\text{K})}\right]}{0.12\,\text{m}}$$

$$= \boxed{665\,\text{m}}$$

45 •••

Picture the Problem Consider an element with a cylindrical area of length L, radius r, and thickness dr. We can relate the heat current through this element to the conductivity of the walls of the pipe, its length and radius, and the temperature gradient across the wall. We can separate the variables in the resulting differential equation and integrate to find the rate of heat transfer.

(*a*) Express the heat current through the cylindrical element:

$$I = -kA\frac{dT}{dr} = -2\pi kLr\frac{dT}{dr}$$

where the minus sign is a consequence of the heat current being opposite the temperature gradient.

Separate the variables:

$$dT = -\frac{I}{2\pi kL}\frac{dr}{r}$$

Integrate from $r = r_1$ to $r = r_2$ and $T = T_1$ to $T = T_2$:

$$\int_{T_1}^{T_2} dT = -\frac{I}{2\pi kL}\int_{r_1}^{r_2}\frac{dr}{r}$$

and

$$T_2 - T_1 = -\frac{I}{2\pi kL}\ln r\Big]_{r_1}^{r_2}$$
$$= -\frac{I}{2\pi kL}\ln\frac{r_2}{r_1}$$
$$= \frac{I}{2\pi kL}\ln\frac{r_1}{r_2}$$

Solve for I to obtain:

$$I = \boxed{\frac{2\pi kL}{\ln(r_1/r_2)}(T_2 - T_1)}$$

Remarks: If we use the above result in Problem 44 (take 0.12 m^2 to be the outside area per unit length of the pipe), then r_1 = 1.91 cm and r_2 = 1.51 cm. Solving for L one obtains 746 m.

Radiation

46 •

Picture the Problem We can apply Wein's displacement law to find the wavelength at which the power is a maximum.

Use Wein's displacement law to relate the wavelength at which the power is a maximum to the surface temperature of the skin:

$$\lambda_{\max} = \frac{2.898\,\text{mm}\cdot\text{K}}{T}$$

Substitute numerical values and evaluate $\lambda_{\max}$:

$$\lambda_{\max} = \frac{2.898\,\text{mm}\cdot\text{K}}{273\,\text{K} + 33\,\text{K}} = \boxed{9.47\,\mu\text{m}}$$

47 •

Picture the Problem We can apply the Stefan-Boltzmann law to find the net power radiated by the wires of its heater to the room.

Relate the net power radiated to the surface area of the heating wires, their temperature, and the room temperature:

$$P_{\text{net}} = e\sigma A\left(T^4 - T_0^4\right)$$

Solve for A:

$$A = \frac{P_{\text{net}}}{e\sigma\left(T^4 - T_0^4\right)}$$

Substitute numerical values and evaluate A:

$$A = \frac{1\,\text{kW}}{(1)\left(5.6703\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)\left[(1173\,\text{K})^4 - (293\,\text{K})^4\right]} = \boxed{9.35\times10^{-3}\ \text{m}^2}$$

48 ••

Picture the Problem The rate at which the sphere absorbs radiant energy is given by $dQ/dt = mcdT/dt$ and, from the Stephan-Boltzmann law, $P_{\text{net}} = e\sigma A\left(T^4 - T_0^4\right)$, where A is the surface area of the sphere, T_0 is its temperature, and T is the temperature of the walls. We can solve the first equation for dT/dt and substitute P_{net} for dQ/dt in order to find the rate at which the temperature of the sphere changes.

Relate the rate at which the sphere absorbs radiant energy to the rate at which its temperature changes:

$$P_{\text{net}} = \frac{dQ}{dt} = mc\frac{dT}{dt}$$

Solve for dT/dt:

$$\frac{dT}{dt} = \frac{P_{\text{net}}}{mc} = \frac{P_{\text{net}}}{\rho Vc} = \frac{P_{\text{net}}}{\frac{4}{3}\pi r^3\rho c}$$

Apply the Stefan-Boltzmann law to relate the net power radiated to the sphere to the difference in temperature of the walls and the blackened copper sphere:

$$\begin{aligned} P_{\text{net}} &= e\sigma A\left(T^4 - T_0^4\right) \\ &= 4\pi r^2 e\sigma\left(T^4 - T_0^4\right) \end{aligned}$$

Substitute to obtain:

$$\begin{aligned} \frac{dT}{dt} &= \frac{4\pi r^2 e\sigma\left(T^4 - T_0^4\right)}{\frac{4}{3}\pi r^3\rho c} \\ &= \frac{3e\sigma\left(T^4 - T_0^4\right)}{r\rho c} \end{aligned}$$

Substitute numerical values and evaluate dT/dt:

$$\frac{dT}{dt} = -\frac{3(1)\left(5.6703\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)\left[(293\,\text{K})^4 - (273\,\text{K})^4\right]}{\left(4\times10^{-2}\ \text{m}\right)\left(8.93\times10^3\ \text{kg/m}^3\right)\left(0.386\,\text{kJ/kg}\cdot\text{K}\right)} = \boxed{2.24\times10^{-3}\ \text{K/s}}$$

49 ••

Picture the Problem We can apply the Stephan-Boltzmann law to express the net power radiated by the incandescent lamp to its surroundings.

Express the rate at which energy is radiated to the surroundings:

$$P_{\text{net}} = e\sigma A\left(T^4 - T_0^4\right) = e\sigma A T^4\left(1 - \left(\frac{T_0}{T}\right)^4\right)$$

Evaluate $\left(T_0/T\right)^4$:

$$\left(\frac{T_0}{T}\right)^4 = \left(\frac{273\,\text{K}}{1573\,\text{K}}\right)^4 \approx 9\times10^{-4}$$

and, because this ratio is so small, we can neglect the temperature of the surroundings.

Substitute to obtain:

$$P_{\text{net}} \approx e\sigma A T^4$$

Solve for T:

$$T = \left(\frac{P_{\text{net}}}{e\sigma A}\right)^{1/4}$$

Express the temperature T' when the electric power input is doubled:

$$T' = \left(\frac{2P_{\text{net}}}{e\sigma A}\right)^{1/4}$$

Divide the second of these equations by the first, simplify, and substitute numerical values and evaluate T':

$$\frac{T'}{T} = (2)^{1/4}$$

and

$$T' = (2)^{1/4}T = (2)^{1/4}(1573\,\text{K}) = 1871\,\text{K} = \boxed{1598°\text{C}}$$

50 ••

Picture the Problem We can differentiate $Q = mL$, where L is the latent heat of boiling for helium, with respect to time to obtain an expression for the rate at which the helium boils away.

Express the rate at which the helium boils away in terms of the rate at which its container absorbs radiant energy:

$$\frac{dm}{dt} = \frac{P_{\text{net}}}{L} = \frac{e\sigma A\left(T^4 - T_0^4\right)}{L} = \frac{e\sigma\pi\, d^2\left(T^4 - T_0^4\right)}{L} = \frac{e\sigma\pi\, d^2}{L}T^4\left(1 - \left(\frac{T_0}{T}\right)^4\right) \approx \frac{e\sigma\pi\, d^2}{L}T^4$$

when $T_0 << T$.

Substitute numerical values and evaluate *dm*/*dt*:

$$\frac{dm}{dt} \approx \frac{(1)\left(5.6703\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)\pi(0.3\,\text{m})^2}{21\,\text{kJ/kg}}(77\,\text{K})^4 = 2.68\times10^{-5}\,\frac{\text{kg}}{\text{s}}\times\frac{3600\,\text{s}}{\text{h}}$$

$$= 9.66\times10^{-2}\ \text{kg/h} = \boxed{96.6\,\text{g/h}}$$

General Problems

***51 •**

Picture the Problem The distance by which the tape clears the ground equals the change in the radius of the circle formed by the tape placed around the earth at the equator.

Express the change in the radius of the circle defined by the steel tape:

$$\Delta R = R\alpha\Delta T$$

where R is the radius of the earth, α is the coefficient of linear expansion of steel, and ΔT is the increase in temperature.

Substitute numerical values and evaluate ΔR.

$$\Delta R = \left(6.37\times10^{6}\ \text{m}\right)\left(11\times10^{-6}\ \text{K}^{-1}\right)(30\,\text{K})$$

$$= 2.10\times10^{3}\ \text{m}$$

$$= \boxed{2.10\,\text{km}}$$

52 ••

Picture the Problem We can differentiate the definition of the density of an isotropic material with respect to T and use the definition of the coefficient of volume expansion to express the rate at which the density of the material changes with respect to temperature. Once we have an expression for $d\rho$ in terms of dT, we can apply a differential approximation to obtain $\Delta\rho$ in terms of ΔT.

Using its definition, relate the density of the material to its mass and volume:

$$\rho = \frac{m}{V}$$

Using its definition, relate the volume of the material to its coefficient of volume expansion:

$$\Delta V = \beta V\Delta T$$

Differentiate ρ with respect to T and simplify to obtain:

$$\frac{d\rho}{dT}=\frac{d\rho}{dV}\frac{dV}{dT}=-\frac{m}{V^2}\beta V$$
$$=-\frac{\rho V}{V^2}\beta V=-\rho\beta$$

or

$$d\rho=-\rho\beta dT$$

Use a differential approximation to obtain:

$$\boxed{\Delta\rho=-\rho\beta\Delta T}$$

53 ••

Picture the Problem We can apply the Stefan-Boltzmann law to express the effective temperature of the sun in terms of the total power it radiates. We can, in turn, use the intensity of the sun's radiation in the upper atmosphere of the earth to approximate the total power it radiates.

Apply the Stefan-Boltzmann law to relate the energy radiated by the sun to its temperature:

$$P_\mathrm{r}=e\sigma AT^4$$

Solve for T:

$$T=\sqrt[4]{\frac{P_\mathrm{r}}{e\sigma A}}$$

Express the area of the sun:

$$A=4\pi R_\mathrm{S}^2$$

Relate the intensity of the sun's radiation in the upper atmosphere to the total power radiated by the sun:

$$I=\frac{P_r}{4\pi R^2}$$

where R is the earth-sun distance.

Solve for P_r:

$$P_r=4\pi R^2 I$$

Substitute for P_r and A and simplify to obtain:

$$T=\sqrt[4]{\frac{4\pi R^2 I}{e\sigma 4\pi R_\mathrm{S}^2}}=\sqrt[4]{\frac{R^2 I}{e\sigma R_\mathrm{S}^2}}$$

Substitute numerical values and evaluate T:

$$T=\sqrt[4]{\frac{\left(1.5\times10^{11}\ \mathrm{m}\right)^2\left(1.35\,\mathrm{kW/m^2}\right)}{(1)\left(5.67\times10^{-8}\ \mathrm{W/m^2\cdot K}\right)\left(6.96\times10^{8}\ \mathrm{m}\right)^2}}=\boxed{5767\,\mathrm{K}}$$

54 ••

Picture the Problem We can solve the thermal-current equation for the R factor of the material.

Use the equation for the thermal current to express I in terms of the temperature gradient across the insulation:

$$I = kA\frac{\Delta T}{\Delta x}$$

Rewrite this expression in terms of the R factor of the material:

$$I = \frac{\Delta T}{\frac{\Delta x}{kA}} = \frac{\Delta T}{\frac{R_f}{A}} = \frac{A\Delta T}{R_f}$$

Solve for the R factor:

$$R_f = \frac{A\Delta T}{I} = \frac{6A_{\text{one side}}\Delta T}{I}$$

Substitute numerical values and evaluate R:

$$R_f = \frac{6\left(12\,\text{in}\times\frac{2.54\times10^{-2}\,\text{m}}{\text{in}}\right)^2}{100\,\text{W}}(363\,\text{K}-293\,\text{K}) = \boxed{0.390\,\frac{\text{K}\cdot\text{m}^2}{\text{W}}}$$

$$= 0.390\frac{\text{K}\cdot\text{m}^2}{\frac{\text{J}}{\text{s}}}\times\frac{9\,\text{F}^\circ}{5\,\text{K}}\times\frac{10.76\,\text{ft}^2}{\text{m}^2}\times\frac{1054\,\text{J}}{\text{Btu}}\times\frac{1\,\text{h}}{3600\,\text{s}} = \boxed{2.21\frac{\text{F}^\circ\cdot\text{ft}^2\cdot\text{h}}{\text{Btu}}}$$

55 ••

Picture the Problem Because the temperature of the copper-aluminum interface is $(T_1 + T_2)/2$, we can conclude that the temperature differences across the two sheets must be the same. We also know, because the sheets are in series, that the heat currents through them are equal.

Express the thermal current through the aluminum sheet:

$$I_{Al} = k_{Al}A_{Al}\frac{\Delta T_{Al}}{\Delta x_{Al}}$$

Express the thermal current through the copper sheet:

$$I_{Cu} = k_{Cu}A_{Cu}\frac{\Delta T_{Cu}}{\Delta x_{Cu}}$$

Equate these currents and solve for Δx_{Al}:

$$k_{Al}A_{Al}\frac{\Delta T_{Al}}{\Delta x_{Al}} = k_{Cu}A_{Cu}\frac{\Delta T_{Cu}}{\Delta x_{Cu}}$$

and

$$\Delta x_{Al} = \Delta x_{Cu}\frac{k_{Al}}{k_{Cu}}$$

Substitute numerical values and evaluate Δx_{Al}:

$$\Delta x_{Al} = (2\,\text{cm})\frac{237\,\text{W/m}\cdot\text{K}}{401\,\text{W/m}\cdot\text{K}} = \boxed{1.18\,\text{cm}}$$

56 ••

Picture the Problem We can relate the stress in the bar to the strain due to its elongation using the definition of Young's modulus and express the strain in terms of the coefficient of linear expansion and the change in temperature of the bar.

Using the definition of Young's modulus, relate the force exerted by the bar on each wall to the strain in the bar due to the change in its length:

$$Y = \frac{\frac{F}{A}}{\frac{\Delta L}{L}}$$

Using the definition of the coefficient of linear expansion, express the strain in the bar:

$$\frac{\Delta L}{L} = \alpha\Delta T$$

Substitute to obtain:

$$Y = \frac{F}{\alpha A\Delta T}$$

Solve for F:

$$F = \alpha AY\Delta T$$

Substitute numerical values and evaluate F:

$$F = \left(11\times10^{-6}\,\text{K}^{-1}\right)\pi(0.022\,\text{m})^2\left(200\,\text{GN/m}^2\right)(40\,\text{K}) = \boxed{1.34\times10^{5}\,\text{N}}$$

57 ••

Picture the Problem We can use the definition of the coefficient of volume expansion with the ideal-gas law to show that $\beta = 1/T$.

(*a*) Use the definition of the coefficient of volume expansion to express β in terms of the rate of change of the volume with temperature:

$$\beta = \frac{1}{V}\frac{dV}{dT}$$

For an ideal gas:

$$V = \frac{nRT}{P} \text{ and } \frac{dV}{dT} = \frac{nR}{P}$$

Substitute to obtain:

$$\beta = \frac{1}{V}\frac{nR}{P} = \boxed{\frac{1}{T}}$$

(*b*) Express the ratio of the experimental value to the theoretical value:

$$\frac{\beta_{\text{exp}} - \beta_{\text{th}}}{\beta_{\text{th}}} = \frac{0.003673\,\text{K}^{-1} - \frac{1}{273}\,\text{K}^{-1}}{\frac{1}{273}\,\text{K}^{-1}} < \boxed{0.3\%}$$

58 ••

Picture the Problem We can express L as the difference between L_B and L_A and express these lengths in terms of the coefficients of linear expansion brass and steel. Requiring that L be constant will lead us to the condition that $L_A/L_B = \alpha_B/\alpha_A$.

(*a*) Express the condition that L does not change when the temperature of the materials changes:

$$L = L_B - L_A = \text{constant}$$

Using the definition of the coefficient of linear expansion, substitute for L_B and L_A:

$$\begin{aligned} L &= (L_B + \alpha_B L_B \Delta T) - (L_A + \alpha_A L_A \Delta T) \\ &= (L_B - L_A) + (\alpha_B L_B - \alpha_A L_A)\Delta T \\ &= L + (\alpha_B L_B - \alpha_A L_A)\Delta T \end{aligned}$$

or

$$(\alpha_B L_B - \alpha_A L_A)\Delta T = 0$$

and

$$\alpha_B L_B - \alpha_A L_A = 0$$

is the condition that L remain constant when the temperature changes by ΔT.

Solve for the ratio of L_A to L_B:

$$\boxed{\frac{L_A}{L_B} = \frac{\alpha_B}{\alpha_A}}$$

(*b*) From (*a*) we have:

$$L_B = L_{steel} = L_A \frac{\alpha_A}{\alpha_B} = L_{brass} \frac{\alpha_{brass}}{\alpha_{steel}}$$

$$= (250\,\text{cm}) \frac{19 \times 10^{-6}\,\text{K}^{-1}}{11 \times 10^{-6}\,\text{K}^{-1}}$$

$$= \boxed{432\,\text{cm}}$$

and

$$L = L_B - L_A = 432\,\text{cm} - 250\,\text{cm}$$

$$= \boxed{182\,\text{cm}}$$

59 ••

Picture the Problem We can apply the thermal-current equation to calculate the heat loss of the earth per second due to conduction from its core. We can also use the thermal-current equation to find the power per unit area radiated from the earth and compare this quantity to the solar constant.

Express the heat loss of the earth per unit time as a function of the thermal conductivity of the earth and its temperature gradient:

$$I = \frac{dQ}{dt} = kA \frac{\Delta T}{\Delta x} \qquad (1)$$

or

$$\frac{dQ}{dt} = 4\pi R_E^2 k \frac{\Delta T}{\Delta x}$$

Substitute numerical values and evaluate dQ/dt:

$$\frac{dQ}{dt} = 4\pi \left(6.37 \times 10^6\,\text{m}\right)^2 \times \left(0.74\,\text{J/m}\cdot\text{s}\cdot\text{K}\right)\left(\frac{1\text{C}^\circ}{30\,\text{m}}\right)$$

$$= \boxed{1.26 \times 10^{10}\,\text{kW}}$$

Rewrite equation (1) to express the thermal current per unit area:

$$\frac{I}{A} = k \frac{\Delta T}{\Delta x}$$

Substitute numerical values and evaluate I/A:

$$\frac{I}{A} = \left(0.74\,\text{J/m}\cdot\text{s}\cdot\text{K}\right)\left(\frac{1\text{C}^\circ}{30\,\text{m}}\right)$$

$$= 0.0247\,\text{W/m}^2$$

Express the ratio of I/A to the solar constant:

$$\frac{I/A}{\text{solar constant}} = \frac{0.0247\,\text{W/m}^2}{1.35\,\text{kW/m}^2}$$

$$< \boxed{0.002\%}$$

60 ••

Picture the Problem We can find the temperature of the outside of the copper bottom by finding the temperature difference between the outside of the saucepan and the boiling water. This temperature difference is related to the rate at which the water is evaporating through the thermal-current equation.

Express the temperature outside the pan in terms of the temperature inside the pan:

$$T_{\text{out}} = T_{\text{in}} + \Delta T = 373\,\text{K} + \Delta T$$

Relate the thermal current through the bottom of the saucepan to its thermal conductivity, area, and the temperature gradient between its surfaces:

$$\frac{\Delta Q}{\Delta t} = kA\frac{\Delta T}{\Delta x}$$

Solve for ΔT:

$$\Delta T = \frac{1}{kA}\frac{\Delta Q}{\Delta t}\Delta x$$

Because the water is boiling:

$$\Delta Q = mL_{\text{v}}$$

Substitute to obtain:

$$\Delta T = \frac{mL_{\text{v}}\Delta x}{kA\Delta t}$$

Substitute numerical values and evaluate ΔT:

$$\Delta T = \frac{(0.8\,\text{kg})(2.26\,\text{MJ/kg})(3\times10^{-3}\,\text{m})}{(401\,\text{W/m}\cdot\text{K})\left[\frac{\pi}{4}(0.15\,\text{m})^2\right](600\,\text{s})} = 1.28\,\text{K}$$

Substitute and evaluate T_{out}:

$$T_{\text{out}} = 373\,\text{K} + 1.28\,\text{K} = 374.3\,\text{K} = \boxed{101.3^\circ\text{C}}$$

***61 ••**

Picture the Problem We'll do this problem twice. First, we'll approximate the answer by disregarding the fact that the surrounding insulation is cylindrical. In the second solution, we'll obtain the exact answer by taking into account the cylindrical insulation surrounding the side of the tank. In both cases, the power required to maintain the temperature of the water in the tank is equal to the rate at which thermal energy is conducted through the insulation.

1st solution:

Using the thermal current equation, relate the rate at which energy is transmitted through the insulation to the temperature gradient, thermal conductivity of the insulation, and the area of the insulation/tank:

$$I = kA\frac{\Delta T}{\Delta x}$$

Letting d represent the inside diameter of the tank and L its inside height, express and evaluate its surface area:

$$\begin{aligned} A &= A_{\text{side}} + A_{\text{bases}} \\ &= \pi dL + 2\left(\frac{\pi d^2}{4}\right) \\ &= \pi\left(dL + \tfrac{1}{2}d^2\right) \\ &= \pi\left[(0.55\,\text{m})(1.2\,\text{m}) + \tfrac{1}{2}(0.55\,\text{m})^2\right] \\ &= 2.55\,\text{m}^2 \end{aligned}$$

Substitute numerical values and evaluate I:

$$\begin{aligned} I &= (0.035\,\text{W/m}\cdot\text{K})(2.55\,\text{m}^2)\left(\frac{74\,\text{K}}{0.05\,\text{m}}\right) \\ &= \boxed{132\,\text{W}} \end{aligned}$$

2nd solution:

Express the total heat loss as the sum of the losses through the top and bottom and the side of the hot-water tank:

$$I = I_{\text{top and bottom}} + I_{\text{side}}$$

Express I through the top and bottom surfaces:

$$\begin{aligned} I_{\text{top and bottom}} &= 2\left(kA\frac{\Delta T}{\Delta x}\right) \\ &= \tfrac{1}{2}\pi d^2 k\frac{\Delta T}{\Delta x} \end{aligned}$$

Substitute numerical values and evaluate $I_{\text{top and bottom}}$:

$$I_{\text{top and bottom}} = \tfrac{1}{2}\pi(0.55\,\text{m})^2 \times \frac{(0.035\,\text{W/m}\cdot\text{K})(74\,\text{K})}{0.05\,\text{m}} = 24.6\,\text{W}$$

Letting r represent the inside radius of the tank, express the heat current through the cylindrical side:

$$I_{\text{side}} = -kA\frac{dT}{dr} = -2\pi kLr\frac{dT}{dr}$$

where the minus sign is a consequence of the heat current being opposite the temperature gradient.

Separate the variables:

$$dT = -\frac{I_{\text{side}}}{2\pi kL}\frac{dr}{r}$$

Integrate from $r = r_1$ to $r = r_2$ and $T = T_1$ to $T = T_2$:

$$\int_{T_1}^{T_2} dT = -\frac{I_{\text{side}}}{2\pi kL}\int_{r_1}^{r_2}\frac{dr}{r}$$

and

$$T_2 - T_1 = -\frac{I_{\text{side}}}{2\pi kL}\ln r\Big]_{r_1}^{r_2} = -\frac{I_{\text{side}}}{2\pi kL}\ln\frac{r_2}{r_1} = \frac{I_{\text{side}}}{2\pi kL}\ln\frac{r_1}{r_2}$$

Solve for I_{side} to obtain:

$$I_{\text{side}} = \frac{2\pi kL}{\ln\frac{r_1}{r_2}}(T_2 - T_1)$$

Substitute numerical values and evaluate I_{side}:

$$I_{\text{side}} = \frac{2\pi(0.035\,\text{W/m}\cdot\text{K})(1.2\,\text{m})}{\ln\left(\frac{0.325\,\text{m}}{0.275\,\text{m}}\right)}(74\,\text{K}) = 117\,\text{W}$$

Substitute for I_{side} and evaluate I:

$$I = 24.6\,\text{W} + 117\,\text{W} = \boxed{142\,\text{W}}$$

62 •••

Picture the Problem We can use $R = \Delta T/I$ and $I = -kAdT/dt$ to express dT in terms of the linearly increasing diameter of the rod. Integrating this expression will allow us to find ΔT and, hence, R.

Express the thermal resistance of the rod in terms of the thermal current in it:

$$R=\frac{\Delta T}{I} \qquad (1)$$

Relate the thermal current in the rod to its thermal conductivity k, cross-sectional area A, and temperature gradient:

$$I=-kA\frac{dT}{dx}$$

where the minus sign is a consequence of the heat current being opposite the temperature gradient.

Using the dependence of the diameter of the rod on x, express the area of the rod:

$$A=\frac{\pi d^2}{4}=\frac{\pi}{4}d_0^2(1+ax)^2$$

Substitute to obtain:

$$I=-k\left[\frac{\pi}{4}d_0^2(1+ax)^2\right]\frac{dT}{dx}$$

Separate variables to obtain:

$$dT=-\frac{Idx}{k\left[\frac{\pi}{4}d_0^2(1+ax)^2\right]}=-\frac{4I}{\pi kd_0^2}\frac{dx}{(1+ax)^2}$$

Integrate T from T_1 to T_2 and x from 0 to L:

$$\int_{T_1}^{T_2}dT=-\frac{4I}{\pi kd_0^2}\int_0^L\frac{dx}{(1+ax)^2}$$

and

$$T_2-T_1=\Delta T=\frac{4IL}{\pi kd_0^2(1+aL)}$$

Substitute for ΔT and I in equation (1) and simplify to obtain:

$$R=\frac{\frac{4IL}{\pi kd_0^2(1+aL)}}{I}=\boxed{\frac{4L}{\pi kd_0^2(1+aL)}}$$

63 •••

Picture the Problem Let $\Delta T = T_2 - T_1$. We can apply Newton's 2nd law to establish the relationship between $L2$ and $L1$ and angular momentum conservation to relate ω_2 and ω_1. We can express both E_2 and E_1 in terms of their angular momenta and rotational inertias and take their ratio to establish their relationship.

Apply $\sum \tau = \frac{\Delta L}{\Delta t}$ to the spinning disk:

Because $\sum \tau = 0$, $\Delta L = 0$ and

$$\boxed{L_2 = L_1}$$

Apply conservation of angular momentum to relate the angular velocity of the disk at T_2 to the angular velocity at T_1:

$$I_2\omega_2 = I_1\omega_1$$

and

$$\omega_2 = \frac{I_1}{I_2}\omega_1$$

Express I_2:

$$\begin{aligned} I_2 &= mr_2^2 = mr_1^2(1+\alpha\Delta T)^2 \\ &= I_1\left(1+2\alpha\Delta T + (\alpha\Delta T)^2\right) \\ &\approx I_1(1+2\alpha\Delta T) \end{aligned}$$

because $(\alpha\Delta T)^2$ is small compared to $\alpha\Delta T$.

Substitute and apply the binomial expansion formula to obtain:

$$\omega_2 = \frac{I_1}{I_1(1+2\alpha\Delta T)}\omega_1$$

and, because $2\alpha\Delta T << 1$,

$$\omega_2 \approx \boxed{(1-2\alpha\Delta T)\omega_1}$$

Express E_2 in terms of L_2 and I_2:

$$E_2 = \frac{L_2^2}{2I_2} = \frac{L_1^2}{2I_2}$$

because $L_2 = L_1$.

Express E_1 in terms of L_1 and I_1:

$$E_1 = \frac{L_1^2}{2I_1}$$

Express the ratio of E_2 to E_1:

$$\frac{E_2}{E_1} = \frac{\frac{L_1^2}{2I_2}}{\frac{L_1^2}{2I_1}} = \frac{I_1}{I_2}$$

Solve for E_2 and substitute for the ratio of I_1 to I_2:

$$E_2 = E_1\frac{I_1}{I_2} = \boxed{E_1(1-2\alpha\Delta T)}$$

64 •••

Picture the Problem The amount of heat radiated by the earth must equal the solar flux from the sun, or else the temperature on earth would continually increase. The emissivity of the earth is related to the rate at which it radiates energy into space by the Stefan-Boltzmann law $P_r = e\sigma AT^4$.

Using the Stefan-Boltzmann law, express the rate at which the earth radiates energy as a function of its emissivity e and temperature T:

$$P_r = e\sigma A' T^4$$

where A' is the surface area of the earth.

Use its definition to express the intensity of the radiation P_a absorbed by the earth:

$$I = \frac{P_a}{A} \text{ or } P_a = AI$$

where A is the cross-sectional area of the earth.

For 70% absorption of the sun's radiation incident on the earth:

$$P_a = 0.7AI$$

Equate P_r and P_a and simplify:

$$0.7AI = e\sigma A' T^4$$

or

$$0.7\pi R^2 I = e\left(4\pi R^2 \sigma T^4\right)$$

Solve for T to obtain:

$$T = \sqrt[4]{\frac{0.7I}{4\sigma e}} = Ce^{-1/4} \qquad (1)$$

Substitute numerical values for I and σ and simplify to obtain:

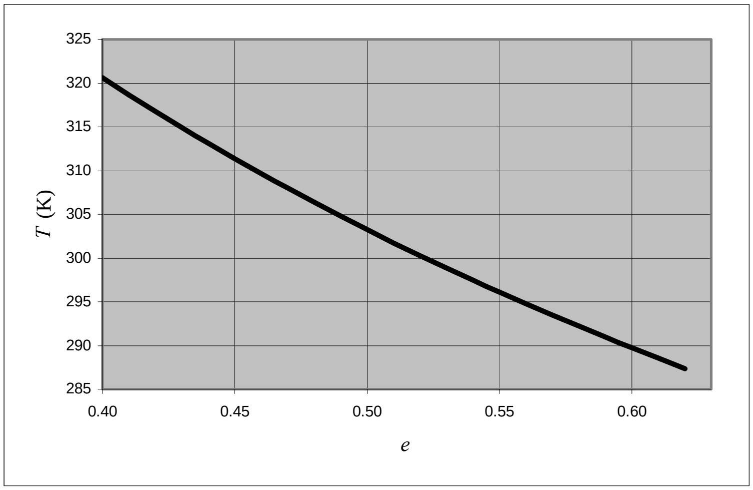
$$T = \sqrt[4]{\frac{0.7\left(1370\ \text{W/m}^2\right)}{4\left(5.670\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)e}} = \left(255\ \text{K}\right)e^{-1/4}$$

A spreadsheet program to evaluate T as a function of e is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
B1	255	
B4	0.4	e
B5	B4+0.01	$e + 0.1$
C4	$B$1/(B4^0.25)	$\left(255\ \text{K}\right)e^{-1/4}$

	A	B	C	D
1	T=	255	K	
2				
3		e	T	
4		0.40	321	
5		0.41	319	
6		0.42	317	
7		0.43	315	
23		0.59	291	
24		0.60	290	
25		0.61	289	
26		0.62	287	

A graph of T as a function of e is shown below.



Treating e as a variable, differentiate equation (1) to obtain:

$$\frac{dT}{de} = -\frac{1}{4}Ce^{-5/4}de \qquad (2)$$

Divide equation (2) by equation (1) to obtain:

$$\frac{dT}{T} = \frac{-\frac{1}{4}Ce^{-5/4}de}{Ce^{-1/4}} = -\frac{1}{4}\frac{de}{e}$$

Use a differential approximation to obtain:

$$\frac{\Delta T}{T} = -\frac{1}{4}\frac{\Delta e}{e}$$

Solve for Δe:

$$\Delta e = -4e\frac{\Delta T}{T}$$

Substitute numerical values ($e \approx 0.615$ for $T_{\text{earth}} = 288$ K) and evaluate Δe:

$$\Delta e = -4(0.615)\frac{1\,\text{K}}{288\,\text{K}} = \boxed{-0.00854}$$

or about a 1.39% change in e.

65 •••

Picture the Problem We can differentiate the expression for the heat that must be removed from water in order to form ice to relate dQ/dt to the rate of ice build-up dm/dt. We can apply the thermal-current equation to express the rate at which heat is removed from the water to the temperature gradient and solve this equation for dm/dt. In part (b) we can separate the variables in the differential equation relating dm/dt and ΔT and integrate to find out how long it takes for a 20-cm layer of ice to be built up.

(*a*) Relate the heat that must be removed from the water to freeze it to its mass and heat of fusion:

$$Q = mL_{\text{f}}$$

Differentiate this expression with respect to time:

$$\frac{dQ}{dt} = L_{\mathrm{f}}\frac{dm}{dt}$$

Using the definition of density, relate the mass of the ice added to the bottom of the layer to its density and volume:

$$m = \rho V = \rho A x$$

Differentiate with respect to time to express the rate of build-up of the ice:

$$\frac{dm}{dt} = \rho A\frac{dx}{dt}$$

Substitute to obtain:

$$\frac{dQ}{dt} = L_{\mathrm{f}}\,\rho A\frac{dx}{dt}$$

Apply the thermal-current equation:

$$\frac{dQ}{dt} = kA\frac{\Delta T}{x}$$

Equate these expressions and solve for dx/dt:

$$L_{\mathrm{f}}\,\rho A\frac{dx}{dt} = kA\frac{\Delta T}{x}$$

and

$$\frac{dx}{dt} = \frac{k}{L_{\mathrm{f}}\,\rho}\frac{\Delta T}{x} \qquad (1)$$

Substitute numerical values and evaluate dx/dt:

$$\frac{dx}{dt} = \frac{(0.592\,\mathrm{W/m\cdot K})(10\,\mathrm{K})}{(333.5\,\mathrm{kJ/kg})(917\,\mathrm{kg/m^3})(0.01\,\mathrm{m})} = 1.94\,\mu\mathrm{m/s} = \boxed{0.698\,\mathrm{cm/h}}$$

(*b*) Separate the variables in equation (1):

$$x\,dx = \frac{k\Delta T}{L_{\mathrm{f}}\,\rho}dt$$

Integrate x from x_{i} to x_{f} and t' from 0 to t:

$$\int_{x_{\mathrm{i}}}^{x_{\mathrm{f}}} x\,dx = \frac{k\Delta T}{L_{\mathrm{f}}\,\rho}\int_0^t dt'$$

and

$$\tfrac{1}{2}\left(x_{\mathrm{f}}^2 - x_{\mathrm{i}}^2\right) = \frac{k\Delta T}{\rho L_{\mathrm{f}}}t$$

Solve for t to obtain:

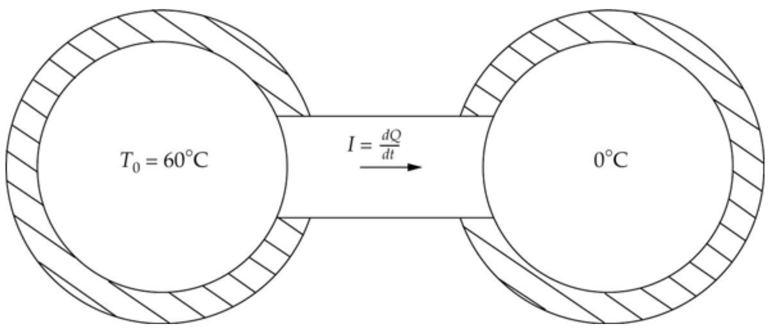
$$t = \frac{\rho L_{\mathrm{f}}\left(x_{\mathrm{f}}^2 - x_{\mathrm{i}}^2\right)}{2k\Delta T}$$

Substitute numerical values and evaluate t:

$$t = \frac{(917\,\mathrm{kg/m^3})(333.5\,\mathrm{kJ/kg})}{2(0.592\,\mathrm{W/m\cdot K})(10\,\mathrm{K})}\left[(0.2\,\mathrm{m})^2 - (0.01\,\mathrm{m})^2\right] = 1.03\times10^6\,\mathrm{s}\times\frac{1\,\mathrm{h}}{3600\,\mathrm{s}}\times\frac{1\,\mathrm{d}}{24\,\mathrm{h}}$$
$$= \boxed{11.9\,\mathrm{d}}$$

***66** •••

Picture the Problem We can use the thermal current equation and the definition of heat capacity to obtain the differential equation describing the rate at which the temperature of the water in the 200-g container is changing. Integrating this equation will yield $T = T_0 e^{-t/RC}$. Substituting for dT/dt in $dQ/dt = -CdT/dt$ and integrating will lead to $Q = CT_0\left(1 - e^{-t/RC}\right)$.



(*a*) Use the thermal current equation to express the rate at which heat is conducted from the water at 60°C by the rod:

$$I = \frac{\Delta T}{R} = \frac{T}{R}$$
because the temperature of the second container is maintained at 0°C.

Using the definition of heat capacity, relate the thermal current to the rate at which the temperature of the water initially at 60°C is changing:

$$I = \frac{dQ}{dt} = -C\frac{dT}{dt} \qquad (1)$$

Equate these two expressions to obtain:

$C\dfrac{dT}{dt} = -\dfrac{1}{R}T$, the differential equation describing the rate at which the temperature of the water in the 200-g container is changing.

Separate variables to obtain:

$$\frac{dT}{T} = -\frac{1}{RC}dt$$

Integrate dT from T_0 to T and dt from 0 to t:

$$\int_{T_0}^{T}\frac{dT'}{T'} = -\frac{1}{RC}\int_0^t dt'$$

or

$$\ln\frac{T}{T_0} = -\frac{1}{RC}t$$

Transform from logarithmic to exponential form and solve for T to obtain:

$$\boxed{T = T_0 e^{-t/RC}} \quad (2)$$

(*b*) Use its definition to express the thermal resistance R:

$$R = \frac{\Delta x}{kA}$$

Substitute numerical values (see Table 20-8 for the thermal conductivity of copper) and evaluate R:

$$R = \frac{0.1\,\mathrm{m}}{(401\,\mathrm{W/m\cdot K})(1.5\times10^{-4}\,\mathrm{m}^2)} = \boxed{1.66\,\mathrm{K/W}}$$

Use its definition to express the heat capacity of the water and the copper container:

$$C = m_\mathrm{c}c_\mathrm{c} + m_\mathrm{w}c_\mathrm{w} = m_\mathrm{c}c_\mathrm{c} + \rho_\mathrm{w}V_\mathrm{w}c_\mathrm{w}$$

Substitute numerical values (see Table 18-1 for the specific heats of water and copper) and evaluate C:

$$C = (0.2\,\mathrm{kg})(386\,\mathrm{kJ/kg\cdot K}) + (10^3\,\mathrm{kg/m^3})(0.7\,\mathrm{L})(4.18\,\mathrm{kJ/kg\cdot K}) = \boxed{3.00\,\mathrm{kJ/K}}$$

Evaluate the product of R and C to find the "time constant" τ:

$$\tau = RC = (1.66\,\mathrm{K/W})(3.00\,\mathrm{kJ/K}) = 4985\,\mathrm{s} = \boxed{1.38\,\mathrm{h}}$$

(*c*) Solve equation (1) for dQ to obtain:

$$dQ = -C\left(\frac{dT}{dt}\right)dt = -CdT$$

Integrate dQ' from $Q = 0$ to Q and dT from T_0 to T:

$$\int_0^Q dQ' = -\int_{T_0}^{T} CdT$$

or

$$Q = C(T_0 - T(t))$$

Substitute (equation (2) for $T(t)$ to obtain:

$$Q = C\left(T_0 - T_0 e^{-t/RC}\right) = \boxed{CT_0\left(1 - e^{-t/RC}\right)}$$

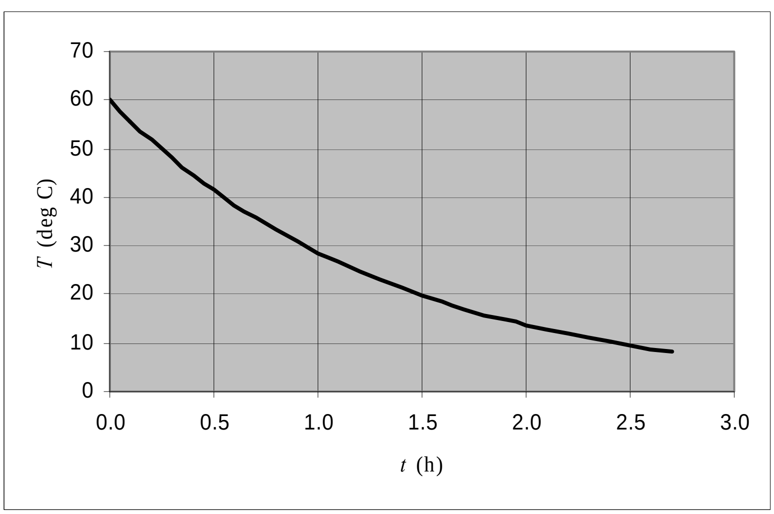
A spreadsheet program to evaluate Q as a function of t is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
D1	1.35	τ
D2	60	T_0

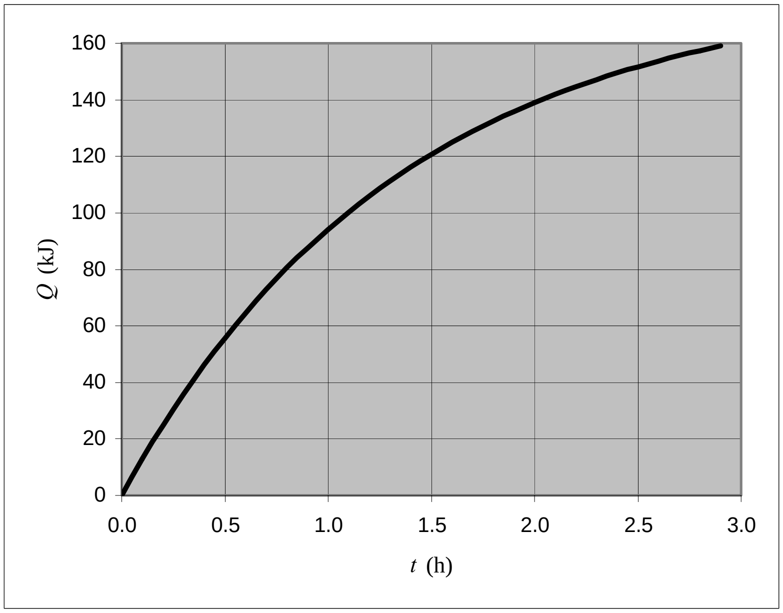
D3	3000	C
A6	0	t
A7	A6+0.1	$t + \Delta t$
B6	$B$2*EXP(–A6/$B$1)	$T_0 e^{-t/RC}$
C7	$B$3*$B$2*(1–EXP(–A6/$B$1))	$CT_0\left(1-e^{-t/RC}\right)$

	A	B	C	D	E
1	tau=	1.35	h		
2	T0=	60	deg-C		
3	C=	3000	J/K		
4					
5	t (hr)	T	Q	Q/1000	
6	0.0	60.00	0.00E+00	0	
7	0.1	55.72	1.29E+04	13	
8	0.2	51.74	2.48E+04	24	
13	0.7	35.72	7.28E+04	71	
14	0.8	33.17	8.05E+04	79	
15	0.9	30.81	8.76E+04	86	
16	1.0	28.61	9.42E+04	92	
33	2.7	8.12	1.56E+05	152	
34	2.8	7.54	1.57E+05	154	
35	2.9	7.00	1.59E+05	155	

From the table we can see that the temperature of the container drops to 30°C in a little more than $\boxed{0.9\,\text{h.}}$ If we wanted to know this time to the nearest hundredth of an hour, we could change the step size in the spreadsheet program to 0.01 h. A graph of T as a function of t is shown in the following graph.



A graph of Q as a function of t follows.



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Picture the Problem We can use the Stefan-Boltzmann equation and the definition of heat capacity to obtain the differential equation expressing the rate at which the temperature of the copper block decreases. We can then approximate the differential equation with a difference equation for the purpose of solving for the temperature of the block as a function of time using Euler's method.

(*a*) Express the rate at which heat is radiated away from the cube:

$$\frac{dQ}{dt} = e\sigma A\left(T^4 - T_0^4\right)$$

Using the definition of heat capacity, relate the thermal current to the rate at which the temperature of the cube is changing:

$$\frac{dQ}{dt} = -C\frac{dT}{dt}$$

Equate these expressions to obtain:

$$\boxed{\frac{dT}{dt} = -\frac{e\sigma A}{C}\left(T^4 - T_0^4\right)}$$

Approximate the differential equation by the difference equation:

$$\frac{\Delta T}{\Delta t} = -\frac{e\sigma A}{C}\left(T^4 - T_0^4\right)$$

Solve for ΔT:

$$\Delta T = -\frac{e\sigma A}{C}\left(T^4 - T_0^4\right)\Delta t$$

or

$$T_{n+1} = T_n - \frac{e\sigma A}{C}\left(T_n^4 - T_0^4\right)\Delta t \qquad (1)$$

Use the definition of heat capacity to obtain:

$$C = mc = \rho V c$$

Substitute numerical values (see Figure 13-1 for ρ_{Cu} and Table 19-1 for c_{Cu}) and evaluate C:

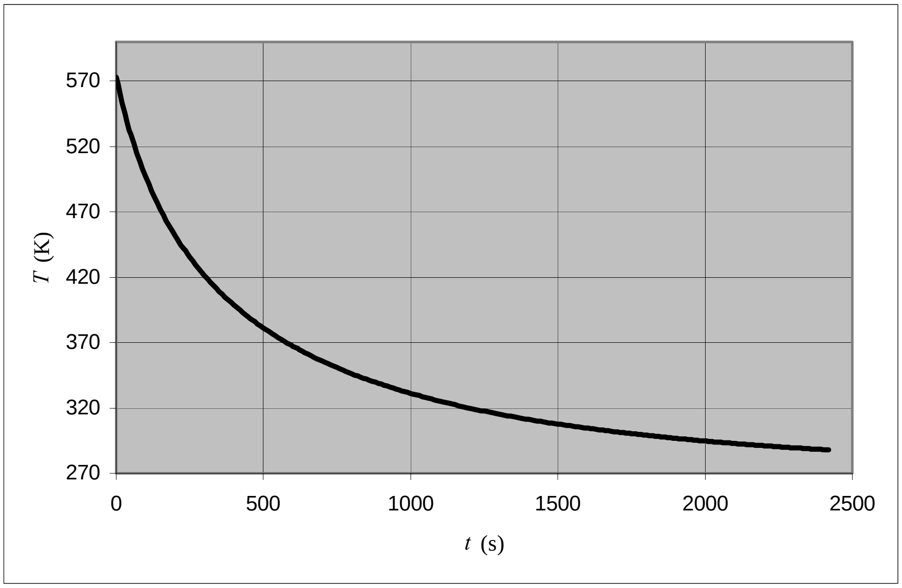
$$C = \left(8.93\times10^{3}\ \mathrm{kg/m^3}\right)\left(10^{-6}\ \mathrm{m^3}\right)\times\left(0.386\,\mathrm{kJ/kg\cdot K}\right) = 3.45\,\mathrm{J/K}$$

(*b*) A spreadsheet program to calculate T as a function of t using equation (1) is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
B1	5.67×10^{-8}	σ
B2	6.00×10^{-4}	A
B3	3.45	C
B4	273	T_0
B5	10	Δt
A9	A8+$B$5	$t+\Delta t$
B9	B8-($B$1*$B$2/$B$3) *(B8^4–$B$4^4)*$B$5	$T_n - \frac{e\sigma A}{C}\left(T_n^4 - T_0^4\right)\Delta t$

	A	B	C
1	sigma=	5.67E–08	W/m^2·K^4
2	A=	6.00E–04	m^2
3	C=	3.45	J/K
4	T0=	273	K
5	dt=	10	s
6			
7	t (s)	T (K)	
8	0	573.00	
9	10	562.92	
10	20	553.56	
11	30	544.85	
248	2400	288.22	
249	2410	288.08	
250	2420	287.95	
251	2430	287.82	

From the spreadsheet solution, the time to cool to 15°C (288 K) is about 2420 s or $\boxed{40.5\,\mathrm{min}.}$ A graph of T as a function of t follows.


570
520
470
420
370
320
270
T (K)
0
500
1000
1500
2000
2500
t (s)