Chapter 21
The Electric Field 1: Discrete Charge Distributions

Conceptual Problems

***1** ••

Similarities:

The force between charges and masses varies as $1/r^2$.

The force is directly proportional to the product of the charges or masses.

Differences:

There are positive and negative charges but only positive masses.

Like charges repel; like masses attract.

The gravitational constant G is many orders of magnitude smaller than the Coulomb constant k.

2 •

Determine the Concept No. In order to charge a body by induction, it must have charges that are free to move about on the body. An insulator does not have such charges.

3 ••

Determine the Concept During this sequence of events, negative charges are attracted from ground to the rectangular metal plate B. When S is opened, these charges are trapped on B and remain there when the charged body is removed. Hence B is negatively charged and $\boxed{(c)\text{ is correct.}}$

4 ••

(*a*) Connect the metal sphere to ground; bring the insulating rod near the metal sphere and disconnect the sphere from ground; then remove the insulating rod. The sphere will be negatively charged.

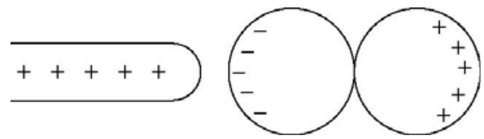
(*b*) Bring the insulating rod in contact with the metal sphere; some of the positive charge on the rod will be transferred to the metal sphere.

(*c*) Yes. First charge one metal sphere negatively by induction as in (*a*). Then use that negatively charged sphere to charge the second metal sphere positively by induction.

***5 ••**

Determine the Concept Because the spheres are conductors, there are free electrons on them that will reposition themselves when the positively charged rod is brought nearby.

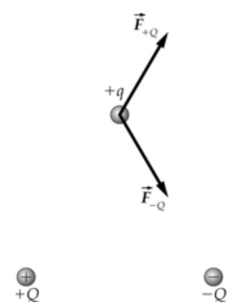
(a) On the sphere near the positively charged rod, the induced charge is negative and near the rod. On the other sphere, the net charge is positive and on the side far from the rod. This is shown in the diagram.



(b) When the spheres are separated and far apart and the rod has been removed, the induced charges are distributed uniformly over each sphere. The charge distributions are shown in the diagram.

6 •

Determine the Concept The forces acting on $+q$ are shown in the diagram. The force acting on $+q$ due to $-Q$ is along the line joining them and directed toward $-Q$. The force acting on $+q$ due to $+Q$ is along the line joining them and directed away from $+Q$.



Because charges $+Q$ and $-Q$ are equal in magnitude, the forces due to these charges are equal and their sum (the net force on $+q$) will be to the right and so $\boxed{(e) \text{ is correct.}}$ Note that the vertical components of these forces add up to zero.

***7 •**

Determine the Concept The acceleration of the positive charge is given by $\vec{a} = \frac{\vec{F}}{m} = \frac{q_0}{m}\vec{E}$. Because q_0 and m are both positive, the acceleration is in the same direction as the electric field. $\boxed{(d) \text{ is correct.}}$

***8 •**

Determine the Concept $\vec{E}$ is zero wherever the net force acting on a test charge is zero. At the center of the square the two positive charges alone would produce a net electric field of zero, and the two negative charges alone would also produce a net electric field of zero. Thus, the net force acting on a test charge at the midpoint of the square will be zero. $\boxed{(b) \text{ is correct.}}$

9 ••

(*a*) The zero net force acting on *Q* could be the consequence of equal collinear charges being equidistant from and on opposite sides of *Q*.

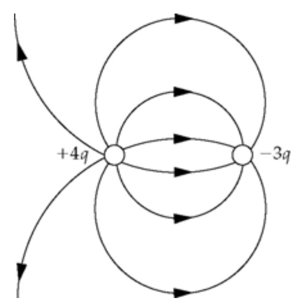
(*b*) The charges described in (*a*) could be either positive or negative and the net force on *Q* would still be zero.

(*c*) Suppose *Q* is positive. Imagine a negative charge situated to its right and a larger positive charge on the same line and the right of the negative charge. Such an arrangement of charges, with the distances properly chosen, would result in a net force of zero acting on *Q*.

(*d*) Because none of the above are correct, $\boxed{(d) \text{ is correct.}}$

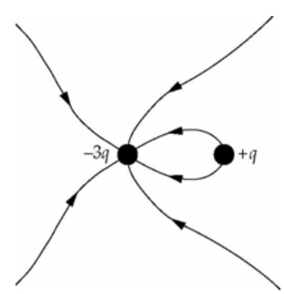
10 •

Determine the Concept We can use the rules for drawing electric field lines to draw the electric field lines for this system. In the sketch to the right we've assigned 2 field lines to each charge *q*.



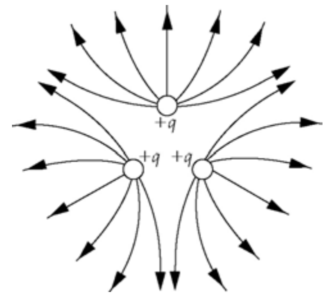
***11** •

Determine the Concept We can use the rules for drawing electric field lines to draw the electric field lines for this system. In the field-line sketch to the right we've assigned 2 field lines to each charge *q*.



***12** •

Determine the Concept We can use the rules for drawing electric field lines to draw the electric field lines for this system. In the field-line sketch to the right we've assigned 7 field lines to each charge q.



13 •

Determine the Concept A positive charge will induce a charge of the opposite sign on the near surface of the nearby neutral conductor. The positive charge and the induced charge on the neutral conductor, being of opposite sign, will always attract one another. $\boxed{(a)\text{ is correct.}}$

***14** •

Determine the Concept Electric field lines around an electric dipole originate at the positive charge and terminate at the negative charge. Only the lines shown in (*d*) satisfy this requirement. $\boxed{(d)\text{ is correct.}}$

***15** ••

Determine the Concept Because $\theta \neq 0$, a dipole in a uniform electric field will experience a restoring torque whose magnitude is $pE_x \sin\theta$. Hence it will oscillate about its equilibrium orientation, $\theta = 0$. If $\theta << 1$, $\sin\theta \approx \theta$, and the motion will be simple harmonic motion. Because the field is nonuniform and is larger in the *x* direction, the force acting on the positive charge of the dipole (in the direction of increasing *x*) will be greater than the force acting on the negative charge of the dipole (in the direction of decreasing *x*) and thus there will be a net electric force on the dipole in the direction of increasing *x*. Hence, the dipole will accelerate in the *x* direction as it oscillates about $\theta = 0$.

16 ••

(*a*) False. The direction of the field is toward a negative charge.

(*b*) True.

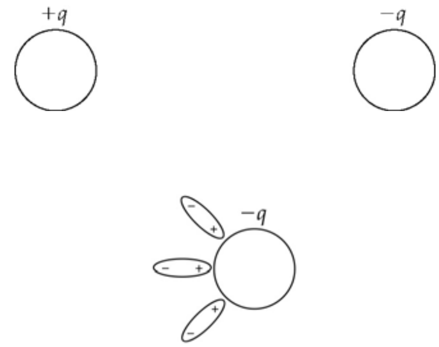
(*c*) False. Electric field lines diverge from any point in space occupied by a positive charge.

(*d*) True

(*e*) True

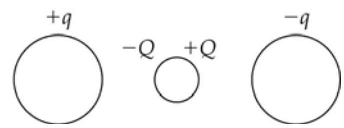
17 ••

Determine the Concept The diagram shows the metal balls before they are placed in the water. In this situation, the net electric field at the location of the sphere on the left is due only to the charge –*q* on the sphere on the right. If the metal balls are placed in water, the water molecules around each ball tend to align themselves with the electric field. This is shown for the ball on the right with charge –*q*.



(*a*) The net electric field $\vec{E}_{net}$ that produces a force on the ball on the left is the field $\vec{E}$ due to the charge –*q* on the ball on the right plus the field due to the layer of positive charge that surrounds the ball on the right. This layer of positive charge is due to the aligning of the water molecules in the electric field, and the amount of positive charge in the layer surrounding the ball on the left will be less than +*q*. Thus, $E_{net} < E$. Because $E_{net} < E$, the force on the ball on the left is less than it would be if the balls had not been placed in water. Hence, the force will **decrease** when the balls are placed in the water.

(*b*) When a third uncharged metal ball is placed between the first two, the net electric field at the location of the sphere on the right is the field due to the charge +*q* on the sphere on the left, plus the field due to the charge –*Q* and +*Q* on the sphere in the middle. This electric field is directed to the right.



The field due to the charge –*Q* and +*Q* on the sphere in the middle at the location of the sphere on the right is to the right. It follows that the net electric field due to the charge +*q* on the sphere on the left, plus the field due to the charge –*Q* and +*Q* on the sphere in the middle is to the right and has a greater magnitude than the field due only to the charge +*q* on the sphere on the left. Hence, the force on either sphere will **increase** if a third uncharged metal ball is placed between them.

Remarks: The reduction of an electric field by the alignment of dipole moments with the field is discussed in further detail in Chapter 24.

***18 ••**

Determine the Concept Yes. A positively charged ball will induce a dipole on the metal ball, and if the two are in close proximity, the net force can be attractive.

***19 ••**

Determine the Concept Assume that the wand has a negative charge. When the charged wand is brought near the tinfoil, the side nearer the wand becomes positively charged by induction, and so it swings toward the wand. When it touches the wand, some of the negative charge is transferred to the foil, which, as a result, acquires a net negative charge and is now repelled by the wand.

Estimation and Approximation

20 ••

Picture the Problem Because it is both very small and repulsive, we can ignore the gravitational force between the spheres. It is also true that we are given no information about the masses of these spheres. We can find the largest possible value of Q by equating the electrostatic force between the charged spheres and the maximum force the cable can withstand.

Using Coulomb's law, express the electrostatic force between the two charged spheres:

$$F = \frac{kQ^2}{\ell^2}$$

Express the tensile strength S_{tensile} of steel in terms of the maximum force $F_{\max}$ in the cable and the cross-sectional area of the cable and solve for F:

$$S_{\text{tensile}} = \frac{F_{\max}}{A} \Rightarrow F_{\max} = AS_{\text{tensile}}$$

Equate these forces to obtain:

$$\frac{kQ^2}{\ell^2} = AS_{\text{tensile}}$$

Solve for Q:

$$Q = \ell\sqrt{\frac{AS_{\text{tensile}}}{k}}$$

Substitute numerical values and evaluate Q:

$$Q = (1\,\text{m})\sqrt{\frac{(1.5\times10^{-4}\ \text{m}^2)(5.2\times10^{8}\ \text{N/m}^2)}{8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2}} = \boxed{2.95\,\text{mC}}$$

21 ••

Picture the Problem We can use Coulomb's law to express the net force acting on the copper cube in terms of the unbalanced charge resulting from the assumed migration of half the charges to opposite sides of the cube. We can, in turn, find the unbalanced charge $Q_{\text{unbalanced}}$ from the number of copper atoms N and the number of electrons per atom.

(*a*) Using Coulomb's law, express the net force acting on the copper rod due to the imbalance in the positive and negative charges:

$$F=\frac{kQ_{\text{unbalanced}}^{2}}{r^{2}} \qquad (1)$$

Relate the number of copper atoms N to the mass m of the rod, the molar mass M of copper, and Avogadro's number N_A:

$$\frac{N}{N_{\text{A}}}=\frac{m}{M}=\frac{\rho_{\text{Cu}}V_{\text{rod}}}{M}$$

Solve for N to obtain:

$$N=\frac{\rho_{\text{Cu}}V_{\text{rod}}}{M}N_{\text{A}}$$

Substitute numerical values and evaluate N:

$$N=\frac{\left(8.93\times10^{3}\text{ kg/m}^{3}\right)\left(0.5\times10^{-2}\text{ m}\right)^{2}\left(4\times10^{-2}\text{ m}\right)\left(6.02\times10^{23}\text{ atoms/mol}\right)}{63.54\times10^{-3}\text{ kg/mol}}$$
$$=8.461\times10^{22}\text{ atoms}$$

Because each atom has 29 electrons and protons, we can express $Q_{\text{unbalanced}}$ as:

$$Q_{\text{unbalanced}}=\tfrac{1}{2}(29)\left(10^{-7}\right)eN$$

Substitute numerical values and evaluate $Q_{\text{unbalanced}}$:

$$Q_{\text{unbalanced}}=\tfrac{1}{2}(29)\left(10^{-7}\right)\left(1.6\times10^{-19}\text{ C}\right)\left(8.461\times10^{22}\right)=1.963\times10^{-2}\text{ C}$$

Substitute for $Q_{\text{unbalanced}}$ in equation (1) to obtain:

$$F=\frac{\left(8.99\times10^{9}\text{ N}\cdot\text{m}^{2}/\text{C}^{2}\right)\left(1.963\times10^{-2}\text{ C}\right)^{2}}{(0.01\text{m})^{2}}=\boxed{3.46\times10^{10}\text{ N}}$$

(*b*) Using Coulomb's law, express the maximum force of repulsion F_{max} in terms of the maximum possible charge Q_{max}:

$$F_{\text{max}}=\frac{kQ_{\text{max}}^{2}}{r^{2}}$$

Solve for Q_{max}:

$$Q_{\text{max}}=\sqrt{\frac{r^{2}F_{\text{max}}}{k}}$$

Express F_{max} in terms of the tensile strength S_{tensile} of copper:

$$F_{\text{max}}=S_{\text{tensile}}A$$

where A is the cross sectional area of the cube.

Substitute to obtain:

$$Q_{max} = \sqrt{\frac{r^2 S_{tensile} A}{k}}$$

Substitute numerical values and evaluate Q_{max}:

$$Q_{max} = \sqrt{\frac{(0.01\,\text{m})^2(2.3\times10^8\ \text{N/m}^2)(10^{-4}\ \text{m}^2)}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}} = 1.60\times10^{-5}\ \text{C}$$

Because $Q_{unbalanced} = 2Q_{max}$:

$$Q_{unbalanced} = 2(1.60\times10^{-5}\ \text{C}) = \boxed{32.0\ \mu\text{C}}$$

Remarks: A net charge of –32 μC means an excess of 2.00×10^{14} electrons, so the net imbalance as a percentage of the total number of charges is $4.06\times10^{-11} = 4\times10^{-9}$ %.

22 •••

Picture the Problem We can use the definition of electric field to express E in terms of the work done on the ionizing electrons and the distance they travel λ between collisions. We can use the ideal-gas law to relate the number density of molecules in the gas ρ and the scattering cross-section σ to the mean free path and, hence, to the electric field.

(*a*) Apply conservation of energy to relate the work done on the electrons by the electric field to the change in their kinetic energy:

$$W = \Delta K = F\Delta s$$

From the definition of electric field we have:

$$F = qE$$

Substitute for F and Δs to obtain:

$W = qE\lambda$, where the mean free path λ is the distance traveled by the electrons between ionizing collisions with nitrogen atoms.

Referring to pages 545-546 for a discussion on the mean-free path, use its definition to relate λ to the scattering cross-section σ and the number density for air molecules n:

$$\lambda = \frac{1}{\sigma n}$$

Substitute for λ and solve for E to obtain:

$$E = \frac{\sigma n W}{q}$$

Use the ideal-gas law to obtain:

$$n = \frac{N}{V} = \frac{P}{kT}$$

Substitute for n to obtain:

$$E = \frac{\sigma PW}{qkT} \qquad (1)$$

Substitute numerical values and evaluate E:

$$E = \frac{(10^{-19}\text{ m}^2)(10^5\text{ N/m}^2)(1\text{eV})(1.6\times10^{-19}\text{ J/eV})}{(1.6\times10^{-19}\text{ C})(1.38\times10^{-23}\text{ J/K})(300\text{ K})} = \boxed{2.41\times10^6\text{ N/C}}$$

(b) From equation (1) we see that:

$\boxed{E \propto P}$ and $\boxed{E \propto T^{-1}}$

i.e., E increases linearly with pressure and varies inversely with temperature.

***23 ••**

Picture the Problem We can use Coulomb's law to express the charge on the rod in terms of the force exerted on it by the soda can and its distance from the can. We can apply Newton's 2nd law in rotational form to the can to relate its acceleration to the electric force exerted on it by the rod. Combining these equations will yield an expression for Q as a function of the mass of the can, its distance from the rod, and its acceleration.

Use Coulomb's law to relate the force on the rod to its charge Q and distance r from the soda can:

$$F = \frac{kQ^2}{r^2}$$

Solve for Q to obtain:

$$Q = \sqrt{\frac{r^2F}{k}} \qquad (1)$$

Apply $\sum \tau_{\text{center of mass}} = I\alpha$ to the can:

$$FR = I\alpha$$

Because the can rolls without slipping, we know that its linear acceleration a and angular acceleration α are related according to:

$$\alpha = \frac{a}{R}$$

where R is the radius of the soda can.

Because the empty can is a hollow cylinder:

$$I = MR^2$$

where M is the mass of the can.

Substitute for I and α and solve for F to obtain:

$$F = \frac{MR^2a}{R^2} = Ma$$

Substitute for F in equation (1):

$$Q = \sqrt{\frac{r^2Ma}{k}}$$

Substitute numerical values and evaluate Q:

$$Q = \sqrt{\frac{(0.1\,\text{m})^2(0.018\,\text{kg})(1\,\text{m/s}^2)}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}} = \boxed{141\,\text{nC}}$$

24 ••

Picture the Problem Because the nucleus is in equilibrium, the binding force must be equal to the electrostatic force of repulsion between the protons.

Apply $\sum\vec{F} = 0$ to a proton:

$$F_{\text{binding}} - F_{\text{electrostatic}} = 0$$

Solve for F_{binding}:

$$F_{\text{binding}} = F_{\text{electrostatic}}$$

Using Coulomb's law, substitute for $F_{\text{electrostatic}}$:

$$F_{\text{binding}} = \frac{kq^2}{r^2}$$

Substitute numerical values and evaluate $F_{\text{electrostatic}}$:

$$F_{\text{binding}} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.6\times10^{-19}\ \text{C})^2}{(10^{-15}\ \text{m})^2} = \boxed{230\,\text{N}}$$

Electric Charge

25 •

Picture the Problem The charge acquired by the plastic rod is an integral number of electronic charges, i.e., $q = n_e(-e)$.

Relate the charge acquired by the plastic rod to the number of electrons transferred from the wool shirt:

$$q = n_e(-e)$$

Solve for and evaluate n_e:

$$n_e = \frac{q}{-e} = \frac{-0.8\,\mu\text{C}}{-1.6\times10^{-19}\ \text{C}} = \boxed{5.00\times10^{12}}$$

26 •

Picture the Problem One faraday = $N_A e$**.** We can use this definition to find the number of coulombs in a faraday.

Use the definition of a faraday to calculate the number of coulombs in a faraday:

$$1\,\text{faraday} = N_A e = (6.02\times10^{23}\ \text{electrons})(1.6\times10^{-19}\ \text{C/electron}) = \boxed{9.63\times10^4\ \text{C}}$$

***27 •**

Picture the Problem We can find the number of coulombs of positive charge there are in 1 kg of carbon from $Q = 6n_C e$, where n_C is the number of atoms in 1 kg of carbon and the factor of 6 is present to account for the presence of 6 protons in each atom. We can find the number of atoms in 1kg of carbon by setting up a proportion relating Avogadro's number, the mass of carbon, and the molecular mass of carbon to n_C.

Express the positive charge in terms of the electronic charge, the number of protons per atom, and the number of atoms in 1 kg of carbon:

$$Q = 6n_C e$$

Using a proportion, relate the number of atoms in 1 kg of carbon n_C, to Avogadro's number and the molecular mass M of carbon:

$$\frac{n_C}{N_A} = \frac{m_C}{M} \Rightarrow n_C = \frac{N_A m_C}{M}$$

Substitute to obtain:

$$Q = \frac{6N_A m_C e}{M}$$

Substitute numerical values and evaluate Q:

$$Q = \frac{6\left(6.02\times10^{23}\text{ atoms/mol}\right)(1\,\text{kg})\left(1.6\times10^{-19}\text{ C}\right)}{0.012\,\text{kg/mol}} = \boxed{4.82\times10^{7}\text{ C}}$$

Coulomb's Law

28 •

Picture the Problem We can find the forces the two charges exert on each by applying Coulomb's law and Newton's 3rd law. Note that $\hat{\boldsymbol{r}}_{1,2} = \hat{\boldsymbol{i}}$ because the vector pointing from q_1 to q_2 is in the positive x direction.

(a) Use Coulomb's law to express the force that q_1 exerts on q_2:

$$\vec{\boldsymbol{F}}_{1,2} = \frac{kq_1q_2}{r_{1,2}^2}\hat{\boldsymbol{r}}_{1,2}$$

Substitute numerical values and evaluate $\vec{\boldsymbol{F}}_{1,2}$:

$$\vec{\boldsymbol{F}}_{1,2} = \frac{\left(8.99\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2\right)(4\,\mu\text{C})(6\,\mu\text{C})}{(3\,\text{m})^2}\boldsymbol{i} = \boxed{(24.0\,\text{mN})\hat{\boldsymbol{i}}}$$

(*b*) Because these are action-and-reaction forces, we can apply Newton's 3rd law to obtain:

$$\vec{F}_{2,1} = -\vec{F}_{1,2} = \boxed{-(24.0\,\text{mN})\hat{\boldsymbol{i}}}$$

(*c*) If q_2 is –6.0 μC:

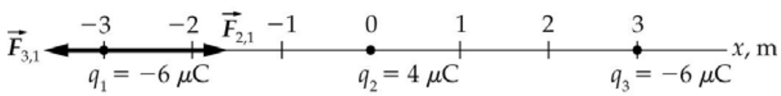
$$\vec{F}_{1,2} = \frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(4\,\mu\text{C})(-6\,\mu\text{C})}{(3\,\text{m})^2}\hat{\boldsymbol{i}} = \boxed{-(24.0\,\text{mN})\hat{\boldsymbol{i}}}$$

and

$$\vec{F}_{2,1} = -\vec{F}_{1,2} = \boxed{(24.0\,\text{mN})\hat{\boldsymbol{i}}}$$

29 •

Picture the Problem q_2 exerts an attractive force $\vec{F}_{2,1}$ on q_1 and q_3 a repulsive force $\vec{F}_{3,1}$. We can find the net force on q_1 by adding these forces.



Express the net force acting on q_1:

$$\vec{F}_1 = \vec{F}_{2,1} + \vec{F}_{3,1}$$

Express the force that q_2 exerts on q_1:

$$\vec{F}_{2,1} = \frac{k|q_1||q_2|}{r_{2,1}^2}\hat{\boldsymbol{i}}$$

Express the force that q_3 exerts on q_1:

$$\vec{F}_{3,1} = \frac{k|q_1||q_3|}{r_{3,1}^2}(-\hat{\boldsymbol{i}})$$

Substitute and simplify to obtain:

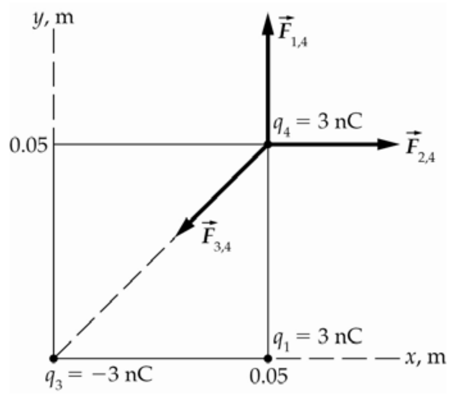
$$\vec{F}_1 = \frac{k|q_1||q_2|}{r_{2,1}^2}\hat{\boldsymbol{i}} - \frac{k|q_1||q_3|}{r_{3,1}^2}\hat{\boldsymbol{i}} = k|q_1|\left(\frac{|q_2|}{r_{2,1}^2} - \frac{|q_3|}{r_{3,1}^2}\right)\hat{\boldsymbol{i}}$$

Substitute numerical values and evaluate $\vec{F}_1$:

$$\vec{F}_1 = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(6\,\mu\text{C})\left(\frac{4\,\mu\text{C}}{(3\,\text{m})^2} - \frac{6\,\mu\text{C}}{(6\,\text{m})^2}\right)\hat{\boldsymbol{i}} = \boxed{(1.50\times10^{-2}\ \text{N})\hat{\boldsymbol{i}}}$$

30 ••

Picture the Problem The configuration of the charges and the forces on the fourth charge are shown in the figure … as is a coordinate system. From the figure it is evident that the net force on q_4 is along the diagonal of the square and directed away from q_3. We can apply Coulomb's law to express $\vec{F}_{1,4}$, $\vec{F}_{2,4}$ and $\vec{F}_{3,4}$ and then add them to find the net force on q_4.



Express the net force acting on q_4:

$$\vec{F}_4 = \vec{F}_{1,4} + \vec{F}_{2,4} + \vec{F}_{3,4}$$

Express the force that q_1 exerts on q_4:

$$\vec{F}_{1,4} = \frac{kq_1q_4}{r_{1,4}^2}\hat{j}$$

Substitute numerical values and evaluate $\vec{F}_{1,4}$:

$$\vec{F}_{1,4} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(3\,\text{nC})\left(\frac{3\,\text{nC}}{(0.05\,\text{m})^2}\right)\hat{j} = \left(3.24\times10^{-5}\ \text{N}\right)\hat{j}$$

Express the force that q_2 exerts on q_4:

$$\vec{F}_{2,4} = \frac{kq_2q_4}{r_{2,4}^2}\hat{i}$$

Substitute numerical values and evaluate $\vec{F}_{2,4}$:

$$\vec{F}_{2,4} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(3\,\text{nC})\left(\frac{3\,\text{nC}}{(0.05\,\text{m})^2}\right)\hat{i} = \left(3.24\times10^{-5}\ \text{N}\right)\hat{i}$$

Express the force that q_3 exerts on q_4:

$$\vec{F}_{3,4} = \frac{kq_3q_4}{r_{3,4}^2}\hat{r}_{3,4}$$, where $\hat{r}_{3,4}$ is a unit vector pointing from q_3 to q_4.

Express $\vec{r}_{3,4}$ in terms of $\vec{r}_{3,1}$ and $\vec{r}_{1,4}$:

$$\vec{r}_{3,4} = \vec{r}_{3,1} + \vec{r}_{1,4} = (0.05\,\text{m})\hat{i} + (0.05\,\text{m})\hat{j}$$

Convert $\vec{r}_{3,4}$ to $\hat{r}_{3,4}$:

$$\hat{r}_{3,4} = \frac{\vec{r}_{3,4}}{\left|\vec{r}_{3,4}\right|} = \frac{(0.05\,\text{m})\hat{i} + (0.05\,\text{m})\hat{j}}{\sqrt{(0.05\,\text{m})^2 + (0.05\,\text{m})^2}} = 0.707\,\hat{i} + 0.707\,\hat{j}$$

Substitute numerical values and evaluate $\vec{F}_{3,4}$:

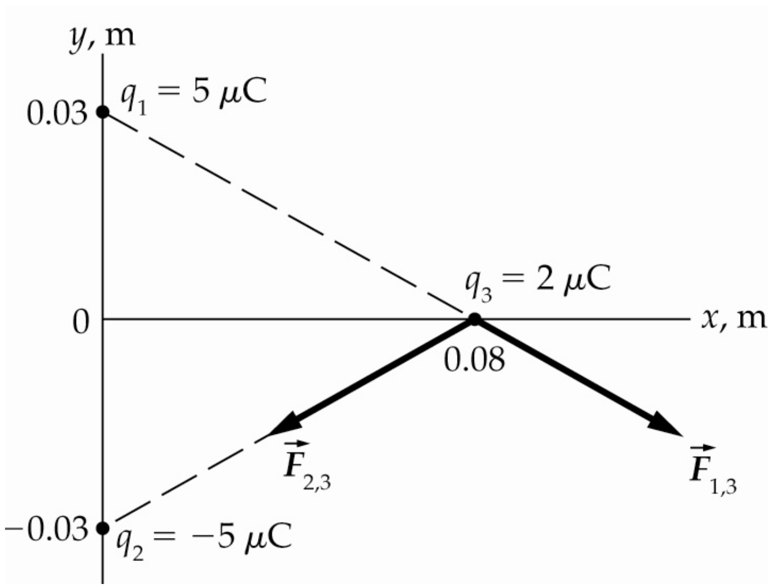
$$\vec{F}_{3,4} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(-3\,\text{nC})\left(\frac{3\,\text{nC}}{\left(0.05\sqrt{2}\,\text{m}\right)^2}\right)\left(0.707\,\hat{i} + 0.707\,\hat{j}\right)$$
$$= -\left(1.14\times10^{-5}\ \text{N}\right)\hat{i} - \left(1.14\times10^{-5}\ \text{N}\right)\hat{j}$$

Substitute and simplify to find $\vec{F}_4$:

$$\vec{F}_4 = \left(3.24\times10^{-5}\ \text{N}\right)\hat{j} + \left(3.24\times10^{-5}\ \text{N}\right)\hat{i} - \left(1.14\times10^{-5}\ \text{N}\right)\hat{i} - \left(1.14\times10^{-5}\ \text{N}\right)\hat{j}$$
$$= \boxed{\left(2.10\times10^{-5}\ \text{N}\right)\hat{i} + \left(2.10\times10^{-5}\ \text{N}\right)\hat{j}}$$

31 ••

Picture the Problem The configuration of the charges and the forces on q_3 are shown in the figure … as is a coordinate system. From the geometry of the charge distribution it is evident that the net force on the 2 μC charge is in the negative y direction. We can apply Coulomb's law to express $\vec{F}_{1,3}$ and $\vec{F}_{2,3}$ and then add them to find the net force on q_3.



The net force acting on q_3 is given by: $\vec{F}_3 = \vec{F}_{1,3} + \vec{F}_{2,3}$

Express the force that q_1 exerts on q_3:

$$\vec{F}_{1,3} = F\cos\theta\,\hat{i} - F\sin\theta\,\hat{j}$$

where

$$F = \frac{kq_1q_3}{r^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(5\,\mu\text{C})(2\,\mu\text{C})}{(0.03\,\text{m})^2 + (0.08\,\text{m})^2} = 12.3\,\text{N}$$

and

$$\theta = \tan^{-1}\left(\frac{3\,\text{cm}}{8\,\text{cm}}\right) = 20.6°$$

Express the force that q_2 exerts on q_3:

$$\vec{F}_{2,3} = -F\cos\theta\,\hat{i} - F\sin\theta\,\hat{j}$$

Substitute for $\vec{F}_{1,3}$ and $\vec{F}_{2,3}$ and simplify to obtain:

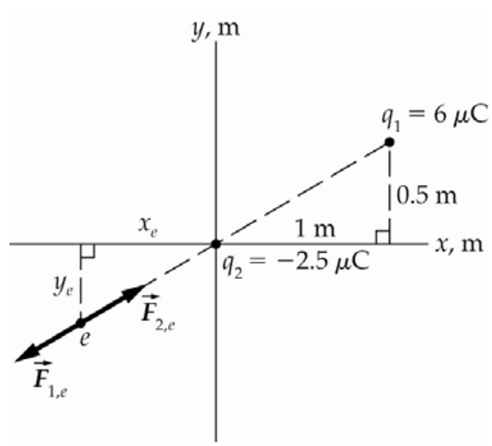
$$\vec{F}_3 = F\cos\theta\,\hat{i} - F\sin\theta\,\hat{j} - F\cos\theta\,\hat{i} - F\sin\theta\,\hat{j} = -2F\sin\theta\,\hat{j}$$

Substitute numerical values and evaluate $\vec{F}_3$:

$$\vec{F}_3 = -2(12.3\,\text{N})\sin 20.6°\,\hat{j} = \boxed{-(8.66\,\text{N})\hat{j}}$$

***32 ••**

Picture the Problem The positions of the charges are shown in the diagram. It is apparent that the electron must be located along the line joining the two charges. Moreover, because it is negatively charged, it must be closer to the –2.5 μC than to the 6.0 μC charge, as is indicated in the figure. We can find the x and y coordinates of the electron's position by equating the two electrostatic forces acting on it and solving for its distance from the origin.



We can use similar triangles to express this radial distance in terms of the x and y coordinates of the electron.

Express the condition that must be

$$F_{1,e} = F_{2,e}$$

satisfied if the electron is to be in equilibrium:

Express the magnitude of the force that q_1 exerts on the electron:

$$F_{1,e}=\frac{kq_1e}{\left(r+\sqrt{1.25\,\text{m}}\right)^2}$$

Express the magnitude of the force that q_2 exerts on the electron:

$$F_{2,e}=\frac{k|q_2|e}{r^2}$$

Substitute and simplify to obtain:

$$\frac{q_1}{\left(r+\sqrt{1.25\,\text{m}}\right)^2}=\frac{|q_2|}{r^2}$$

Substitute for q_1 and q_2 and simplify:

$$\left(-1.4\,\text{m}^{-2}\right)r^2+\left(2.2361\,\text{m}^{-1}\right)r+1.25\,\text{m}=0$$

Solve for r to obtain:

$$r=2.036\,\text{m}$$

and

$$r=-0.4386\,\text{m}$$

Because $r<0$ is unphysical, we'll consider only the positive root.

Use the similar triangles in the diagram to establish the proportion involving the y coordinate of the electron:

$$\frac{y_e}{0.5\,\text{m}}=\frac{2.036\,\text{m}}{1.12\,\text{m}}$$

Solve for y_e:

$$y_e=0.909\,\text{m}$$

Use the similar triangles in the diagram to establish the proportion involving the x coordinate of the electron:

$$\frac{x_e}{1\text{m}}=\frac{2.036\,\text{m}}{1.12\,\text{m}}$$

Solve for x_e:

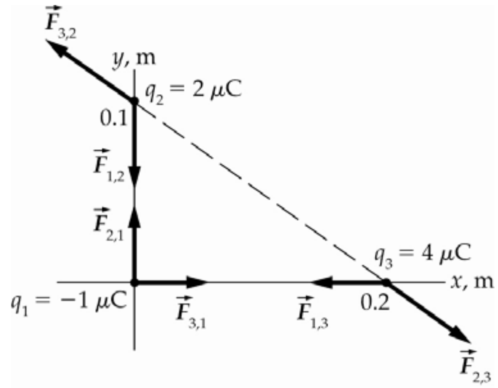
$$x_e=1.82\,\text{m}$$

The coordinates of the electron's position are:

$$\left(x_e,y_e\right)=\boxed{\left(-1.82\,\text{m},-0.909\,\text{m}\right)}$$

***33 ••**

Picture the Problem Let q_1 represent the charge at the origin, q_2 the charge at (0, 0.1 m), and q_3 the charge at (0.2 m, 0). The diagram shows the forces acting on each of the charges. Note the action-and-reaction pairs. We can apply Coulomb's law and the principle of superposition of forces to find the net force acting on each of the charges.



Express the net force acting on q_1:

$$\vec{F}_1 = \vec{F}_{2,1} + \vec{F}_{3,1}$$

Express the force that q_2 exerts on q_1:

$$\vec{F}_{2,1} = \frac{kq_2q_1}{r_{2,1}^2}\hat{r}_{2,1} = \frac{kq_2q_1}{r_{2,1}^2}\frac{\vec{r}_{2,1}}{r_{2,1}} = \frac{kq_2q_1}{r_{2,1}^3}\vec{r}_{2,1}$$

Substitute numerical values and evaluate $\vec{F}_{2,1}$:

$$\vec{F}_{2,1} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2\ \mu\text{C})\frac{(-1\,\mu\text{C})}{(0.1\,\text{m})^3}(-0.1\,\text{m})\hat{j} = (1.80\ \text{N})\hat{j}$$

Express the force that q_3 exerts on q_1:

$$\vec{F}_{3,1} = \frac{kq_3q_1}{r_{3,1}^3}\vec{r}_{3,1}$$

Substitute numerical values and evaluate $\vec{F}_{3,1}$:

$$\vec{F}_{3,1} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(4\ \mu\text{C})\frac{(-1\,\mu\text{C})}{(0.2\,\text{m})^3}(-0.2\,\text{m})\hat{i} = (0.899\ \text{N})\hat{i}$$

Substitute to find $\vec{F}_1$:

$$\vec{F}_1 = \boxed{(0.899\ \text{N})\hat{i} + (1.80\ \text{N})\hat{j}}$$

Express the net force acting on q_2:

$$\vec{F}_2 = \vec{F}_{3,2} + \vec{F}_{1,2} = \vec{F}_{3,2} - \vec{F}_{2,1} = \vec{F}_{3,2} - (1.80\ \text{N})\hat{j}$$

because $\vec{F}_{1,2}$ and $\vec{F}_{2,1}$ are action-and-reaction forces.

Express the force that q_3 exerts on q_2:

$$\vec{F}_{3,2} = \frac{kq_3q_2}{r_{3,2}^3}\vec{r}_{3,2} = \frac{kq_3q_2}{r_{3,2}^3}\left[(-0.2\,\text{m})\hat{i} + (0.1\,\text{m})\hat{j}\right]$$

Substitute numerical values and evaluate $\vec{F}_{3,2}$:

$$\vec{F}_{3,2} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(4\,\mu\text{C})\frac{(2\,\mu\text{C})}{(0.224\,\text{m})^3}\left[(-0.2\,\text{m})\hat{i} + (0.1\,\text{m})\hat{j}\right]$$
$$= (-1.28\,\text{N})\hat{i} + (0.640\,\text{N})\hat{j}$$

Find the net force acting on q_2:

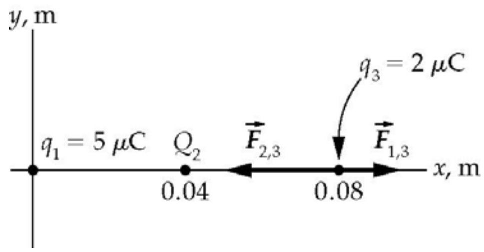
$$\vec{F}_2 = \vec{F}_{3,2} - (1.80\,\text{N})\hat{j} = (-1.28\,\text{N})\hat{i} + (0.640\,\text{N})\hat{j} - (1.80\,\text{N})\hat{j}$$
$$= \boxed{(-1.28\,\text{N})\hat{i} - (1.16\,\text{N})\hat{j}}$$

Noting that $\vec{F}_{1,3}$ and $\vec{F}_{3,1}$ are an action-and-reaction pair, as are $\vec{F}_{2,3}$ and $\vec{F}_{3,2}$, express the net force acting on q_3:

$$\vec{F}_3 = \vec{F}_{1,3} + \vec{F}_{2,3} = -\vec{F}_{3,1} - \vec{F}_{3,2} = -(0.899\,\text{N})\hat{i} - \left[(-1.28\,\text{N})\hat{i} + (0.640\,\text{N})\hat{j}\right]$$
$$= \boxed{(0.381\,\text{N})\hat{i} - (0.640\,\text{N})\hat{j}}$$

34 ••

Picture the Problem Let q_1 represent the charge at the origin and q_3 the charge initially at (8 cm, 0) and later at (17.75 cm, 0). The diagram shows the forces acting on q_3 at (8 cm, 0). We can apply Coulomb's law and the principle of superposition of forces to find the net force acting on each of the charges.



Express the net force on q_2 when it is at (8 cm, 0):

$$\vec{F}_2(8\,\text{cm},0) = \vec{F}_{1,3} + \vec{F}_{2,3} = \frac{kq_1q_3}{r_{1,3}^3}\vec{r}_{1,3} + \frac{kQ_2q_3}{r_{2,3}^3}\vec{r}_{2,3} = kq_3\left(\frac{q_1}{r_{1,3}^3}\vec{r}_{1,3} + \frac{Q_2}{r_{2,3}^3}\vec{r}_{2,3}\right)$$

Substitute numerical values to obtain:

$$(-19.7\,\text{N})\hat{\boldsymbol{i}} = (8.99\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2)(2\,\mu\text{C})\left[\frac{5\,\mu\text{C}}{(0.08\,\text{m})^3}(0.08\,\text{m})\hat{\boldsymbol{i}} + \frac{Q_2}{(0.04\,\text{m})^3}(0.04\,\text{m})\hat{\boldsymbol{i}}\right]$$

Solve for and evaluate Q_2:

$$Q_2 = \boxed{-3.00\,\mu\text{C}}$$

Remarks: An alternative solution is to equate the electrostatic forces acting on q_2 when it is at (17.75 cm, 0).

35 ••

Picture the Problem By considering the symmetry of the array of charges we can see that the y component of the force on q is zero. We can apply Coulomb's law and the principle of superposition of forces to find the net force acting on q.

Express the net force acting on q:

$$\vec{F}_q = \vec{F}_{Q\text{ on }x\text{ axis},q} + 2\vec{F}_{Q\text{ at }45°,q}$$

Express the force on q due to the charge Q on the x axis:

$$\vec{F}_{Q\text{ on }x\text{ axis},q} = \frac{kqQ}{R^2}\hat{\boldsymbol{i}}$$

Express the net force on q due to the charges at 45°:

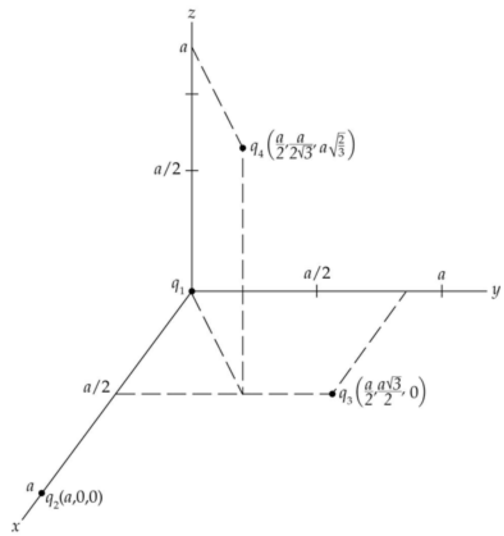
$$2\vec{F}_{Q\text{ at }45°,q} = 2\frac{kqQ}{R^2}\cos 45°\hat{\boldsymbol{i}} = \frac{2}{\sqrt{2}}\frac{kqQ}{R^2}\hat{\boldsymbol{i}}$$

Substitute to obtain:

$$\vec{F}_q = \frac{kqQ}{R^2}\hat{\boldsymbol{i}} + \frac{2}{\sqrt{2}}\frac{kqQ}{R^2}\hat{\boldsymbol{i}} = \boxed{\frac{kqQ}{R^2}\left(1+\frac{\sqrt{2}}{2}\right)\hat{\boldsymbol{i}}}$$

36 •••

Picture the Problem Let the H^+ ions be in the x-y plane with H_1 at (0, 0, 0), H_2 at (a, 0, 0), and H_3 at $\left(a/2, a\sqrt{3}/2, 0\right)$. The N^{-3} ion, q_4 in our notation, is then at $\left(a/2, a/2\sqrt{3}, a\sqrt{2/3}\right)$ where $a = 1.64\times10^{-10}$ m. To simplify our calculations we'll set $ke^2/a^2 = C = 8.56\times10^{-9}\,\text{N}$. We can apply Coulomb's law and the principle of superposition of forces to find the net force acting on each ion.



Express the net force acting on q_1:

$$\vec{F}_1 = \vec{F}_{2,1} + \vec{F}_{3,1} + \vec{F}_{4,1}$$

Find $\vec{F}_{2,1}$:

$$\vec{F}_{2,1} = \frac{kq_1q_2}{r_{2,1}^2}\hat{r}_{2,1} = C\left(-\hat{i}\right) = -C\hat{i}$$

Find $\vec{F}_{3,1}$:

$$\vec{F}_{3,1} = \frac{kq_3q_1}{r_{3,1}^2}\hat{r}_{3,1}$$

$$= C\frac{\left(0-\frac{a}{2}\right)\hat{i} + \left(0-\frac{a\sqrt{3}}{2}\right)\hat{j}}{a}$$

$$= -C\left(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right)$$

Noting that the magnitude of q_4 is three times that of the other charges and that it is negative, express $\vec{F}_{4,1}$:

$$\vec{F}_{4,1} = 3C\hat{r}_{4,1} = -3C\frac{\left(0-\frac{a}{2}\right)\hat{i} + \left(0-\frac{a}{2\sqrt{3}}\right)\hat{j} + \left(0-\frac{a\sqrt{2}}{\sqrt{3}}\right)\hat{k}}{\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2\sqrt{3}}\right)^2 + \left(\frac{a\sqrt{2}}{\sqrt{3}}\right)^2}}$$

$$= 3C\frac{\left(\frac{a}{2}\right)\hat{i} + \left(\frac{a}{2\sqrt{3}}\right)\hat{j} + \left(\frac{a\sqrt{2}}{\sqrt{3}}\right)\hat{k}}{a} = 3C\left[\left(\frac{1}{2}\right)\hat{i} + \left(\frac{1}{2\sqrt{3}}\right)\hat{j} + \left(\sqrt{\frac{2}{3}}\right)\hat{k}\right]$$

Substitute to find $\vec{F}_1$:

$$\vec{F}_1 = -C\hat{i} - C\left(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right) + 3C\left[\left(\frac{1}{2}\right)\hat{i} + \left(\frac{1}{2\sqrt{3}}\right)\hat{j} + \left(\sqrt{\frac{2}{3}}\right)\hat{k}\right] = \boxed{C\sqrt{6}\hat{k}}$$

From symmetry considerations:

$$\vec{F}_2 = \vec{F}_3 = \vec{F}_1 = \boxed{C\sqrt{6}\hat{k}}$$

Express the condition that molecule is in equilibrium:

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 0$$

Solve for and evaluate $\vec{F}_4$:

$$\vec{F}_4 = -\left(\vec{F}_1 + \vec{F}_2 + \vec{F}_3\right) = -3\vec{F}_1 = \boxed{-3C\sqrt{6}\hat{k}}$$

The Electric Field

***37** •

Picture the Problem Let q represent the charge at the origin and use Coulomb's law for $\vec{E}$ due to a point charge to find the electric field at x = 6 m and –10 m.

(a) Express the electric field at a point P located a distance x from a charge q:

$$\vec{E}(x) = \frac{kq}{x^2}\hat{r}_{\text{P},0}$$

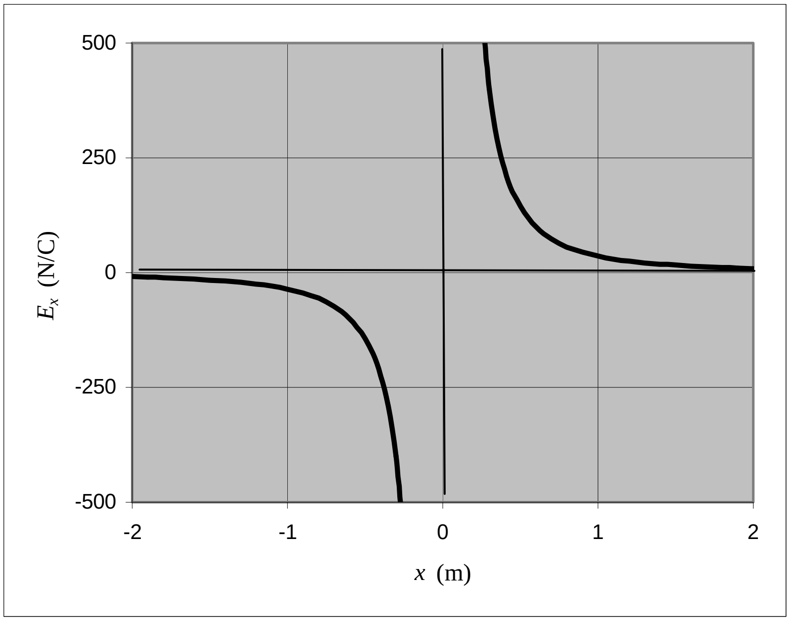
Evaluate this expression for x = 6 m:

$$\vec{E}(6\,\text{m}) = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(4\,\mu\text{C})}{(6\,\text{m})^2}\hat{i} = \boxed{(999\,\text{N/C})\hat{i}}$$

(b) Evaluate $\vec{E}$ at x = –10 m:

$$\vec{E}(-10\,\text{m}) = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(4\,\mu\text{C})}{(10\,\text{m})^2}\left(-\hat{i}\right) = \boxed{(-360\,\text{N/C})\hat{i}}$$

(c) The following graph was plotted using a spreadsheet program:



***38** •

Picture the Problem Let q represent the charges of +4 μC and use Coulomb's law for $\vec{E}$ due to a point charge and the principle of superposition for fields to find the electric field at the locations specified.

Noting that $q_1 = q_2$, use Coulomb's law and the principle of superposition to express the electric field due to the given charges at a point P a distance x from the origin:

$$\vec{E}(x) = \vec{E}_{q_1}(x) + \vec{E}_{q_2}(x) = \frac{kq_1}{x^2}\hat{r}_{q_1,\mathrm{P}} + \frac{kq_2}{(8\,\mathrm{m}-x)^2}\hat{r}_{q_2,\mathrm{P}} = kq_1\left(\frac{1}{x^2}\hat{r}_{q_1,\mathrm{P}} + \frac{1}{(8\,\mathrm{m}-x)^2}\hat{r}_{q_2,\mathrm{P}}\right)$$

$$= (36\,\mathrm{kN\cdot m^2/C})\left(\frac{1}{x^2}\hat{r}_{q_1,\mathrm{P}} + \frac{1}{(8\,\mathrm{m}-x)^2}\hat{r}_{q_2,\mathrm{P}}\right)$$

(a) Apply this equation to the point at x = –2 m:

$$\vec{\mathbf{E}}(-2\,\mathrm{m}) = (36\,\mathrm{kN\cdot m^2/C})\left[\frac{1}{(2\,\mathrm{m})^2}(-\hat{\mathbf{i}}) + \frac{1}{(10\,\mathrm{m})^2}(-\hat{\mathbf{i}})\right] = \boxed{(-9.36\,\mathrm{kN/C})\hat{\mathbf{i}}}$$

(b) Evaluate $\vec{E}$ at x = 2 m:

$$\vec{E}(2\,\mathrm{m}) = (36\,\mathrm{kN\cdot m^2/C})\left[\frac{1}{(2\,\mathrm{m})^2}(\hat{i}) + \frac{1}{(6\,\mathrm{m})^2}(-\hat{i})\right] = \boxed{(8.00\,\mathrm{kN/C})\hat{i}}$$

(c) Evaluate $\vec{E}$ at x = 6 m:

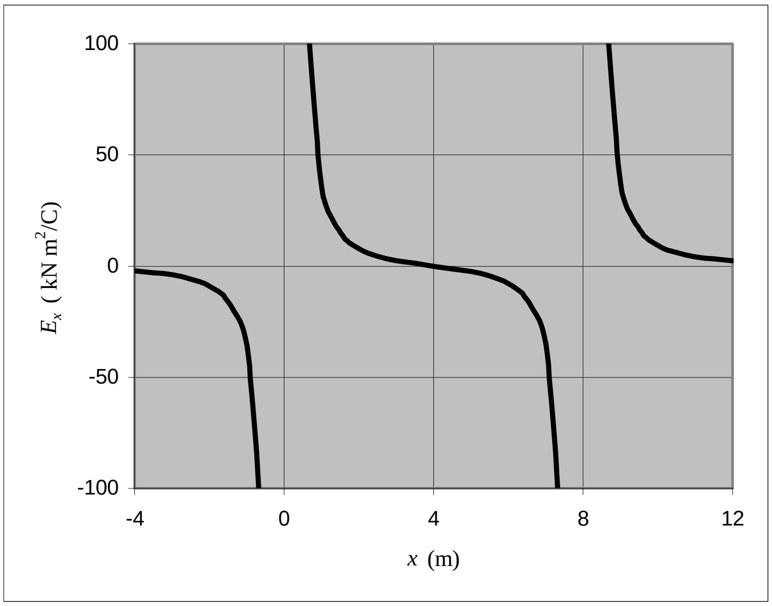
$$\vec{E}(6\,\text{m}) = (36\,\text{kN}\cdot\text{m}^2/\text{C})\left[\frac{1}{(6\,\text{m})^2}(\hat{i}) + \frac{1}{(2\,\text{m})^2}(-\hat{i})\right] = \boxed{(-8.00\,\text{kN/C})\hat{i}}$$

(d) Evaluate $\vec{E}$ at x = 10 m:

$$\vec{E}(10\,\text{m}) = (36\,\text{kN}\cdot\text{m}^2/\text{C})\left[\frac{1}{(10\,\text{m})^2}(\hat{i}) + \frac{1}{(2\,\text{m})^2}(\hat{i})\right] = \boxed{(9.35\,\text{kN/C})\hat{i}}$$

(e) From symmetry considerations: $E(4\,\text{m}) = \boxed{0}$

(f) The following graph was plotted using a spreadsheet program:



39 •

Picture the Problem We can find the electric field at the origin from its definition and the force on a charge placed there from $\vec{F} = q\vec{E}$. We can apply Coulomb's law to find the value of the charge placed at y = 3 cm.

(a) Apply the definition of electric field to obtain:

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{(8\times10^{-4}\,\text{N})\hat{j}}{2\,\text{nC}} = \boxed{(400\,\text{kN/C})\hat{j}}$$

(b) Express and evaluate the force on a charged body in an electric field:

$$\vec{F} = q\vec{E} = (-4\,\text{nC})(400\,\text{kN/C})\hat{j} = \boxed{(-1.60\,\text{mN})\hat{j}}$$

(*c*) Apply Coulomb's law to obtain:

$$\frac{kq(-4\,\text{nC})}{(0.03\,\text{m})^2}(-\hat{\boldsymbol{j}}) = (-1.60\,\text{mN})\hat{\boldsymbol{j}}$$

Solve for and evaluate q:

$$q = -\frac{(1.60\,\text{mN})(0.03\,\text{m})^2}{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(4\,\text{nC})} = \boxed{-40.0\,\text{nC}}$$

40 •

Picture the Problem We can compare the electric and gravitational forces acting on an electron by expressing their ratio. We can equate these forces to find the charge that would have to be placed on a penny in order to balance the earth's gravitational force on it.

(*a*) Express the magnitude of the electric force acting on the electron:

$$F_e = eE$$

Express the magnitude of the gravitational force acting on the electron:

$$F_g = m_e g$$

Express the ratio of these forces to obtain:

$$\frac{F_e}{F_g} = \frac{eE}{mg}$$

Substitute numerical values and evaluate F_e/F_g:

$$\frac{F_e}{F_g} = \frac{(1.6\times10^{-19}\ \text{C})(150\,\text{N/C})}{(9.11\times10^{-31}\ \text{kg})(9.81\,\text{m/s}^2)} = 2.69\times10^{12}$$

or

$F_e = \boxed{(2.69\times10^{12})F_g}$, i.e., the electric force is greater by a factor of 2.69×10^{12}.

(*b*) Equate the electric and gravitational forces acting on the penny and solve for q to obtain:

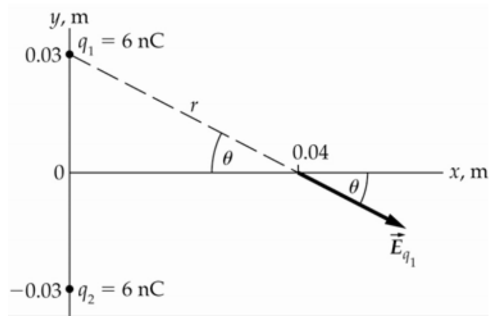
$$q = \frac{mg}{E}$$

Substitute numerical values and evaluate q:

$$q = \frac{(3\times10^{-3}\,\text{kg})(9.81\,\text{m/s}^2)}{150\,\text{N/C}} = \boxed{1.96\times10^{-4}\,\text{C}}$$

41 ••

Picture the Problem The diagram shows the locations of the charges q_1 and q_2 and the point on the x axis at which we are to find $\vec{E}$. From symmetry considerations we can conclude that the y component of $\vec{E}$ at any point on the x axis is zero. We can use Coulomb's law for the electric field due to point charges to find the field at any point on the x axis and $\vec{F} = q\vec{E}$ to find the force on a charge q_0 placed on the x axis at $x = 4$ cm.



(*a*) Letting $q = q_1 = q_2$, express the x-component of the electric field due to one charge as a function of the distance r from either charge to the point of interest:

$$\vec{E}_x = \frac{kq}{r^2}\cos\theta\,\hat{i}$$

Express $\vec{E}_x$ for both charges:

$$\vec{E}_x = 2\frac{kq}{r^2}\cos\theta\,\hat{i}$$

Substitute for $\cos\theta$ and r, substitute numerical values, and evaluate to obtain:

$$\vec{E}_x = 2\frac{kq}{r^2}\frac{0.04\,\text{m}}{r}\hat{i} = \frac{2kq(0.04\,\text{m})}{r^3}\hat{i} = \frac{2(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(6\,\text{nC})(0.04\,\text{m})}{\left[(0.03\,\text{m})^2+(0.04\,\text{m})^2\right]^{3/2}}\hat{i}$$

$$= \boxed{(34.5\,\text{kN/C})\hat{i}}$$

(*b*) Apply $\vec{F} = q\vec{E}$ to find the force on a charge q_0 placed on the x axis at $x = 4$ cm:

$$\vec{F} = (2\,\text{nC})(34.5\,\text{kN/C})\hat{i}$$

$$= \boxed{(69.0\,\mu\text{N})\hat{i}}$$

***42** ••

Picture the Problem If the electric field at $x = 0$ is zero, both its x and y components must be zero. The only way this condition can be satisfied with the point charges of +5.0 μC and –8.0 μC are on the x axis is if the point charge of +6.0 μC is also on the x axis. Let the subscripts 5, –8, and 6 identify the point charges and their fields. We can use Coulomb's law for $\vec{E}$ due to a point charge and the principle of superposition for fields to determine where the +6.0 μC charge should be located so that the electric field at $x = 0$ is zero.

Express the electric field at $x = 0$ in terms of the fields due to the charges of +5.0 μC, –8.0 μC, and +6.0 μC:

$$\vec{E}(0) = \vec{E}_{5\,\mu\text{C}} + \vec{E}_{-8\,\mu\text{C}} + \vec{E}_{6\,\mu\text{C}} = 0$$

Substitute for each of the fields to obtain:

$$\frac{kq_5}{r_5^2}\hat{r}_5 + \frac{kq_6}{r_6^2}\hat{r}_6 + \frac{kq_{-8}}{r_{-8}^2}\hat{r}_{-8} = 0$$

or

$$\frac{kq_5}{r_5^2}\hat{i} + \frac{kq_6}{r_6^2}\left(-\hat{i}\right) + \frac{kq_{-8}}{r_{-8}^2}\left(-\hat{i}\right) = 0$$

Divide out the unit vector $\hat{i}$ to obtain:

$$\frac{q_5}{r_5^2} - \frac{q_6}{r_6^2} - \frac{q_{-8}}{r_{-8}^2} = 0$$

Substitute numerical values to obtain:

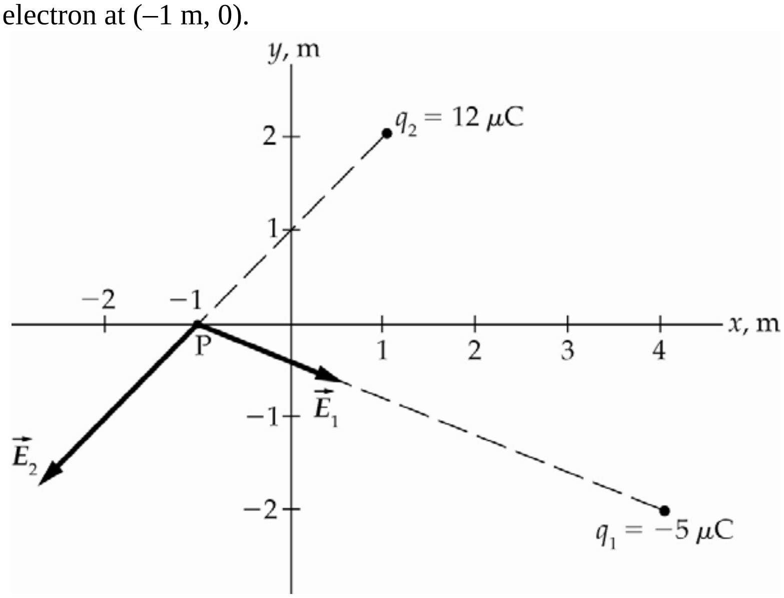
$$\frac{5}{(3\,\text{cm})^2} - \frac{6}{r_6^2} - \frac{-8}{(4\,\text{cm})^2} = 0$$

Solve for r_6:

$$r_6 = \boxed{2.38\,\text{cm}}$$

43 ••

Picture the Problem The diagram shows the electric field vectors at the point of interest P due to the two charges. We can use Coulomb's law for $\vec{E}$ due to point charges and the superposition principle for electric fields to find $\vec{E}_\text{P}$. We can apply $\vec{F} = q\vec{E}$ to find the force on an electron at (–1 m, 0).



(*a*) Express the electric field at (–1 m, 0) due to the charges q_1 and q_2:

$$\vec{E}_\text{P} = \vec{E}_1 + \vec{E}_2$$

Evaluate $\vec{E}_1$:

$$\vec{E}_1 = \frac{kq_1}{r_{1,\text{P}}^2}\hat{r}_{1,\text{P}} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-5\,\mu\text{C})}{(5\,\text{m})^2+(2\,\text{m})^2}\left(\frac{(-5\,\text{m})\hat{i}+(2\,\text{m})\hat{j}}{\sqrt{(5\,\text{m})^2+(2\,\text{m})^2}}\right)$$
$$= (-1.55\times10^3\ \text{N/C})(-0.928\,\hat{i}+0.371\,\hat{j})$$
$$= (1.44\,\text{kN/C})\hat{i} + (-0.575\,\text{kN/C})\hat{j}$$

Evaluate $\vec{E}_2$:

$$\vec{E}_2 = \frac{kq_2}{r_{2,\text{P}}^2}\hat{r}_{2,\text{P}} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(12\,\mu\text{C})}{(2\,\text{m})^2+(2\,\text{m})^2}\left(\frac{(-2\,\text{m})\hat{i}+(-2\,\text{m})\hat{j}}{\sqrt{(2\,\text{m})^2+(2\,\text{m})^2}}\right)$$
$$= (13.5\times10^3\ \text{N/C})(-0.707\,\hat{i}-0.707\,\hat{j})$$
$$= (-9.54\,\text{kN/C})\hat{i} + (-9.54\,\text{kN/C})\hat{j}$$

Substitute for $\vec{E}_1$ and $\vec{E}_2$ and simplify to find $\vec{E}_\text{P}$:

$$\vec{E}_\text{P} = (1.44\,\text{kN/C})\hat{i} + (-0.575\,\text{kN/C})\hat{j} + (-9.54\,\text{kN/C})\hat{i} + (-9.54\,\text{kN/C})\hat{j}$$
$$= (-8.10\,\text{kN/C})\hat{i} + (-10.1\,\text{kN/C})\hat{j}$$

The magnitude of $\vec{E}_\text{P}$ is:

$$E_\text{P} = \sqrt{(-8.10\,\text{kN/C})^2 + (-10.1\,\text{kN/C})^2} = \boxed{12.9\,\text{kN/C}}$$

The direction of $\vec{E}_\text{P}$ is:

$$\theta_E = \tan^{-1}\left(\frac{-10.1\,\text{kN/C}}{-8.10\,\text{kN/C}}\right) = \boxed{231°}$$

Note that the angle returned by your calculator for $\tan^{-1}\left(\frac{-10.1\,\text{kN/C}}{-8.10\,\text{kN/C}}\right)$ is the reference angle and must be increased by 180° to yield θ_E.

(*b*) Express and evaluate the force on an electron at point P:

$$\vec{F} = q\vec{E}_P = (-1.602\times10^{-19}\text{ C})\left[(-8.10\,\text{kN/C})\hat{i} + (-10.1\,\text{kN/C})\hat{j}\right]$$
$$= (1.30\times10^{-15}\text{ N})\hat{i} + (1.62\times10^{-15}\text{ N})\hat{j}$$

Find the magnitude of $\vec{F}$:

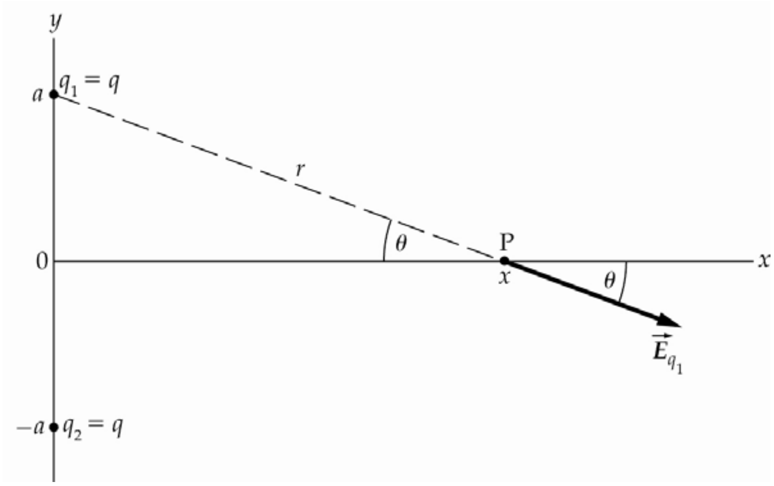
$$F = \sqrt{(1.30\times10^{-15}\text{ N})^2 + (1.62\times10^{-15}\text{ N})^2} = \boxed{2.08\times10^{-15}\text{ N}}$$

Find the direction of $\vec{F}$:

$$\theta_F = \tan^{-1}\left(\frac{1.62\times10^{-15}\text{ N}}{1.3\times10^{-15}\text{ N}}\right) = \boxed{51.3°}$$

44 ••

Picture the Problem The diagram shows the locations of the charges q_1 and q_2 and the point on the x axis at which we are to find $\vec{E}$. From symmetry considerations we can conclude that the y component of $\vec{E}$ at any point on the x axis is zero. We can use Coulomb's law for the electric field due to point charges to find the field at any point on the x axis. We can establish the results called for in parts (*b*) and (*c*) by factoring the radicand and using the approximation $1+\alpha \approx 1$ whenever $\alpha << 1$.



(*a*) Express the *x*-component of the electric field due to the charges at $y = a$ and $y = -a$ as a function of the distance r from either charge to point P:

$$\vec{E}_x = 2\frac{kq}{r^2}\cos\theta\,\hat{i}$$

Substitute for $\cos\theta$ and r to obtain:

$$\vec{E}_x = 2\frac{kq}{r^2}\frac{x}{r}\hat{i} = \frac{2kqx}{r^3}\hat{i} = \frac{2kqx}{(x^2+a^2)^{3/2}}\hat{i}$$
$$= \frac{2kqx}{(x^2+a^2)^{3/2}}\hat{i}$$

and

$$E_x = \boxed{\frac{2kqx}{\left(x^2 + a^2\right)^{3/2}}}$$

(*b*) For $|x| << a, x^2 + a^2 \approx a^2$, so:

$$E_x \approx \frac{2kqx}{\left(a^2\right)^{3/2}} = \boxed{\frac{2kqx}{a^3}}$$

For $|x| >> a, x^2 + a^2 \approx x^2$, so:

$$E_x \approx \frac{2kqx}{\left(x^2\right)^{3/2}} = \boxed{\frac{2kq}{x^2}}$$

(*c*) For $x >> a$, the charges separated by a would appear to be a single charge of magnitude $2q$. Its field would be given by $E_x = \dfrac{2kq}{x^2}$.

Factor the radicand to obtain:

$$E_x = 2kqx\left[x^2\left(1 + \frac{a^2}{x^2}\right)\right]^{-3/2}$$

For $a << x$:

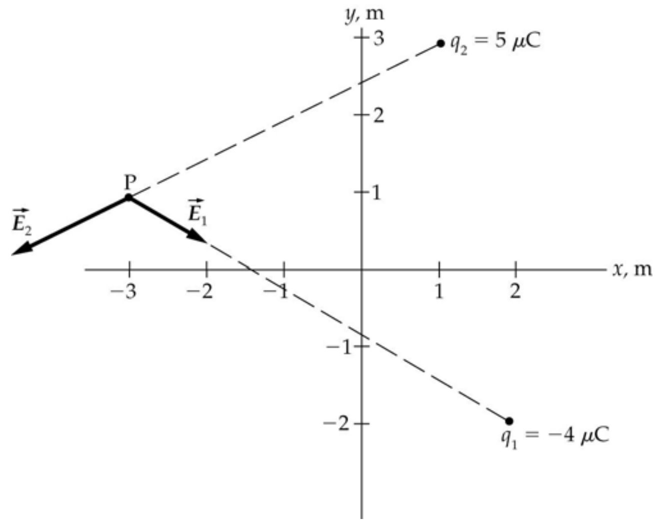
$$1 + \frac{a^2}{x^2} \approx 1$$

and

$$E_x = 2kqx\left[x^2\right]^{-3/2} = \boxed{\frac{2kq}{x^2}}$$

***45 ••**

Picture the Problem The diagram shows the electric field vectors at the point of interest P due to the two charges. We can use Coulomb's law for $\vec{E}$ due to point charges and the superposition principle for electric fields to find $\vec{E}_{\text{P}}$. We can apply $\vec{F} = q\vec{E}$ to find the force on a proton at (–3 m, 1 m).



(*a*) Express the electric field at (–3 m, 1 m) due to the charges q_1 and q_2:

$$\vec{E}_P = \vec{E}_1 + \vec{E}_2$$

Evaluate $\vec{E}_1$:

$$\vec{E}_1 = \frac{kq_1}{r_{1,P}^2}\hat{r}_{1,P} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-4\ \mu\text{C})}{(5\ \text{m})^2 + (3\ \text{m})^2}\left(\frac{(-5\ \text{m})\hat{i} + (3\ \text{m})\hat{j}}{\sqrt{(5\ \text{m})^2 + (3\ \text{m})^2}}\right)$$

$$= (-1.06\ \text{kN/C})(-0.857\,\hat{i} + 0.514\,\hat{j}) = (0.908\ \text{kN/C})\hat{i} + (-0.544\ \text{kN/C})\hat{j}$$

Evaluate $\vec{E}_2$:

$$\vec{E}_2 = \frac{kq_2}{r_{2,P}^2}\hat{r}_{2,P} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(5\ \mu\text{C})}{(4\ \text{m})^2 + (2\ \text{m})^2}\left(\frac{(-4\ \text{m})\hat{i} + (-2\ \text{m})\hat{j}}{\sqrt{(4\ \text{m})^2 + (2\ \text{m})^2}}\right)$$

$$= (2.25\ \text{kN/C})(-0.894\,\hat{i} - 0.447\,\hat{j}) = (-2.01\ \text{kN/C})\hat{i} + (-1.01\ \text{kN/C})\hat{j}$$

Substitute and simplify to find $\vec{E}_P$:

$$\vec{E}_P = (0.908\ \text{kN/C})\hat{i} + (-0.544\ \text{kN/C})\hat{j} + (-2.01\ \text{kN/C})\hat{i} + (-1.01\ \text{kN/C})\hat{j}$$

$$= (-1.10\ \text{kN/C})\hat{i} + (-1.55\ \text{kN/C})\hat{j}$$

The magnitude of $\vec{E}_P$ is:

$$E_P = \sqrt{(1.10\ \text{kN/C})^2 + (1.55\ \text{kN/C})^2} = \boxed{1.90\ \text{kN/C}}$$

The direction of $\vec{E}_P$ is:

$$\theta_E = \tan^{-1}\left(\frac{-1.55\,\text{kN/C}}{-1.10\,\text{kN/C}}\right) = \boxed{235°}$$

Note that the angle returned by your calculator for $\tan^{-1}\left(\frac{-1.55\,\text{kN/C}}{-1.10\,\text{kN/C}}\right)$ is the reference angle and must be increased by 180° to yield θ_E.

(*b*) Express and evaluate the force on a proton at point P:

$$\begin{aligned}\vec{F} &= q\vec{E}_P = \left(1.6\times10^{-19}\,\text{C}\right)\left[(-1.10\,\text{kN/C})\hat{\boldsymbol{i}} + (-1.55\,\text{kN/C})\hat{\boldsymbol{j}}\right]\\ &= \left(-1.76\times10^{-16}\,\text{N}\right)\hat{\boldsymbol{i}} + \left(-2.48\times10^{-16}\,\text{N}\right)\hat{\boldsymbol{j}}\end{aligned}$$

The magnitude of $\vec{F}$ is:

$$F = \sqrt{\left(-1.76\times10^{-16}\,\text{N}\right)^2 + \left(-2.48\times10^{-16}\,\text{N}\right)^2} = \boxed{3.04\times10^{-16}\,\text{N}}$$

The direction of $\vec{F}$ is:

$$\theta_F = \tan^{-1}\left(\frac{-2.48\times10^{-16}\,\text{N}}{-1.76\times10^{-16}\,\text{N}}\right) = \boxed{235°}$$

where, as noted above, the angle returned by your calculator for $\tan^{-1}\left(\frac{-2.48\times10^{-16}\,\text{N}}{-1.76\times10^{-16}\,\text{N}}\right)$ is the reference angle and must be increased by 180° to yield θ_E.

46 ••

Picture the Problem In Problem 44 it is shown that the electric field on the *x* axis, due to equal positive charges located at (0, *a*) and (0,–*a*), is given by $E_x = 2kqx\left(x^2 + a^2\right)^{-3/2}$. We can identify the locations at which E_x has it greatest values by setting dE_x/dx equal to zero.

(*a*) Evaluate $\frac{dE_x}{dx}$:

$$\frac{dE_x}{dx} = \frac{d}{dx}\left[2kqx\left(x^2+a^2\right)^{-3/2}\right] = 2kq\frac{d}{dx}\left[x\left(x^2+a^2\right)^{-3/2}\right]$$
$$= 2kq\left[x\frac{d}{dx}\left(x^2+a^2\right)^{-3/2} + \left(x^2+a^2\right)^{-3/2}\right]$$
$$= 2kq\left[x\left(-\frac{3}{2}\right)\left(x^2+a^2\right)^{-5/2}(2x) + \left(x^2+a^2\right)^{-3/2}\right]$$
$$= 2kq\left[-3x^2\left(x^2+a^2\right)^{-5/2} + \left(x^2+a^2\right)^{-3/2}\right]$$

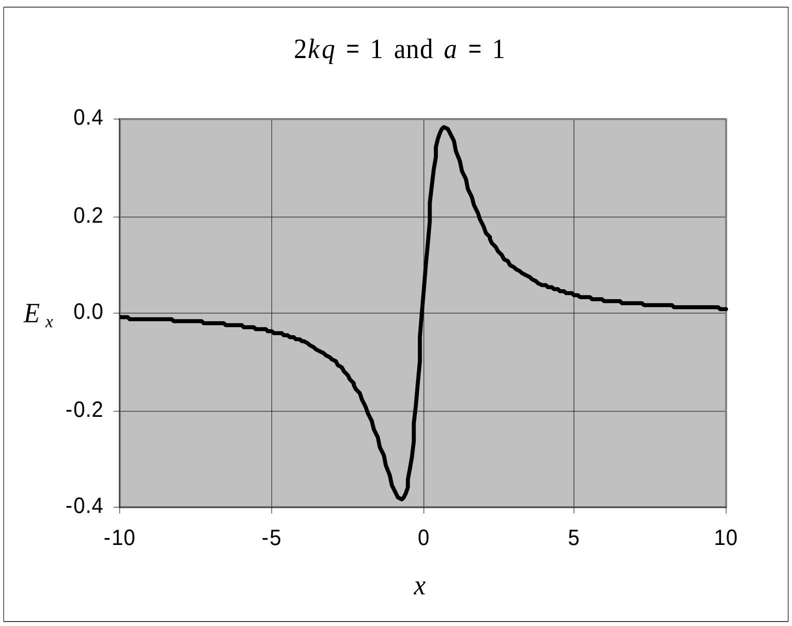
Set this derivative equal to zero:

$$-3x^2\left(x^2+a^2\right)^{-5/2} + \left(x^2+a^2\right)^{-3/2} = 0$$

Solve for x to obtain:

$$x = \boxed{\pm\frac{a}{\sqrt{2}}}$$

(b) The following graph was plotted using a spreadsheet program:



47 •••

Picture the Problem We can determine the stability of the equilibrium in Part (a) and Part (b) by considering the forces the equal charges q at $y = +a$ and $y = -a$ exert on the test charge when it is given a small displacement along either the x or y axis. The application of Coulomb's law in Part (c) will lead to the magnitude and sign of the charge that must be placed at the origin in order that a net force of zero is experienced by each of the three charges.

(a) Because E_x is in the x direction, a positive test charge that is displaced from

(0, 0) in either the $+x$ direction or the $-x$ direction will experience a force pointing away from the origin and accelerate in the direction of the force.

Consequently, the equilibrium at (0,0) is unstable for a small displacement along the x axis.

If the positive test charge is displaced in the direction of increasing y (the positive y direction), the charge at $y = +a$ will exert a greater force than the charge at $y = -a$, and the net force is then in the $-y$ direction; i.e., it is a restoring force. Similarly, if the positive test charge is displaced in the direction of decreasing y (the negative y direction), the charge at $y = -a$ will exert a greater force than the charge at $y = -a$, and the net force is then in the $-y$ direction; i.e., it is a restoring force.

Consequently, the equilibrium at (0,0) is stable for a small displacement along the y axis.

(b)

Following the same arguments as in Part (a), one finds that, for a negative test charge, the equilibrium is stable at (0,0) for displacements along the x axis and unstable for displacements along the y axis.

(c) Express the net force acting on the charge at $y = +a$:

$$\sum F_{q\text{ at }y=+a} = \frac{kqq_0}{a^2} + \frac{kq^2}{(2a)^2} = 0$$

Solve for q_0 to obtain:

$$q_0 = \boxed{-\tfrac{1}{4}q_0}$$

Remarks: In Part (c), we could just as well have expressed the net force acting on the charge at $y = -a$. Due to the symmetric distribution of the charges at $y = -a$ and $y = +a$, summing the forces acting on q_0 at the origin does not lead to a relationship between q_0 and q.

***48** •••

Picture the Problem In Problem 44 it is shown that the electric field on the x axis, due to equal positive charges located at (0, a) and (0,$-a$), is given by $E_x = 2kqx\left(x^2 + a^2\right)^{-3/2}$. We can use $T = 2\pi\sqrt{m/k'}$ to express the period of the motion in terms of the restoring constant k'.

(a) Express the force acting on the on the bead when its displacement from the origin is x:

$$F_x = -qE_x = -\frac{2kq^2x}{\left(x^2 + a^2\right)^{3/2}}$$

Factor a^2 from the denominator to obtain:

$$F_x = -\frac{2kq^2x}{a^2\left(\frac{x^2}{a^2}+1\right)^{3/2}}$$

For $x << a$:

$$F_x = \boxed{-\frac{2kq^2}{a^3}x}$$

i.e., the bead experiences a linear restoring force.

(*b*) Express the period of a simple harmonic oscillator:

$$T = 2\pi\sqrt{\frac{m}{k'}}$$

Obtain k' from our result in part (*a*):

$$k' = \frac{2kq^2}{a^3}$$

Substitute to obtain:

$$T = 2\pi\sqrt{\frac{m}{\frac{2kq^2}{a^3}}} = \boxed{2\pi\sqrt{\frac{ma^3}{2kq^2}}}$$

Motion of Point Charges in Electric Fields

49 •

Picture the Problem We can use Newton's 2nd law of motion to find the acceleration of the electron in the uniform electric field and constant-acceleration equations to find the time required for it to reach a speed of 0.01*c* and the distance it travels while acquiring this speed.

(*a*) Use data found at the back of your text to compute *e*/*m* for an electron:

$$\frac{e}{m_e} = \frac{1.6\times10^{-19}\text{ C}}{9.11\times10^{-31}\text{ kg}} = \boxed{1.76\times10^{11}\text{ C/kg}}$$

(*b*) Apply Newton's 2nd law to relate the acceleration of the electron to the electric field:

$$a = \frac{F_{\text{net}}}{m_e} = \frac{eE}{m_e}$$

Substitute numerical values and evaluate a:

$$a = \frac{(1.6\times10^{-19}\text{ C})(100\text{ N/C})}{9.11\times10^{-31}\text{ kg}} = \boxed{1.76\times10^{13}\text{ m/s}^2}$$

$$\boxed{\text{The direction of the acceleration of an electron is opposite the electric field.}}$$

(*c*) Using the definition of acceleration, relate the time required for an electron to reach 0.01*c* to its acceleration:

$$\Delta t = \frac{v}{a} = \frac{0.01c}{a}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{0.01\left(3\times10^{8}\ \text{m/s}\right)}{1.76\times10^{13}\ \text{m/s}^2} = \boxed{0.170\ \mu\text{s}}$$

(*d*) Find the distance the electron travels from its average speed and the elapsed time:

$$\Delta x = v_{\text{av}}\Delta t$$
$$= \tfrac{1}{2}\left[0 + 0.01\left(3\times10^{8}\ \text{m/s}\right)\right]\left(0.170\ \mu\text{s}\right)$$
$$= \boxed{25.5\ \text{cm}}$$

***50** •

Picture the Problem We can use Newton's 2nd law of motion to find the acceleration of the proton in the uniform electric field and constant-acceleration equations to find the time required for it to reach a speed of 0.01*c* and the distance it travels while acquiring this speed.

(*a*) Use data found at the back of your text to compute *e*/*m* for an electron:

$$\frac{e}{m_p} = \frac{1.6\times10^{-19}\ \text{C}}{1.67\times10^{-27}\ \text{kg}}$$
$$= \boxed{9.58\times10^{7}\ \text{C/kg}}$$

Apply Newton's 2nd law to relate the acceleration of the electron to the electric field:

$$a = \frac{F_{\text{net}}}{m_p} = \frac{eE}{m_p}$$

Substitute numerical values and evaluate *a*:

$$a = \frac{\left(1.6\times10^{-19}\ \text{C}\right)\left(100\ \text{N/C}\right)}{1.67\times10^{-27}\ \text{kg}}$$
$$= \boxed{9.58\times10^{9}\ \text{m/s}^2}$$

$$\boxed{\text{The direction of the acceleration of a proton is in the direction of the electric field.}}$$

(*b*) Using the definition of acceleration, relate the time required for an electron to reach 0.01*c* to its acceleration:

$$\Delta t = \frac{v}{a} = \frac{0.01c}{a}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{0.01\left(3\times10^{8}\text{ m/s}\right)}{9.58\times10^{9}\text{ m/s}^{2}} = \boxed{313\,\mu\text{s}}$$

51 •

Picture the Problem The electric force acting on the electron is opposite the direction of the electric field. We can apply Newton's 2nd law to find the electron's acceleration and use constant acceleration equations to find how long it takes the electron to travel a given distance and its deflection during this interval of time.

(*a*) Use Newton's 2nd law to relate the acceleration of the electron first to the net force acting on it and then the electric field in which it finds itself:

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m_e} = \frac{-e\vec{E}}{m_e}$$

Substitute numerical values and evaluate $\vec{a}$:

$$\vec{a} = -\frac{1.6\times10^{-19}\text{ C}}{9.11\times10^{-31}\text{ kg}}\left(400\text{ N/C}\right)\hat{j}$$
$$= \boxed{\left(-7.03\times10^{13}\text{ m/s}^{2}\right)\hat{j}}$$

(*b*) Relate the time to travel a given distance in the *x* direction to the electron's speed in the *x* direction:

$$\Delta t = \frac{\Delta x}{v_x} = \frac{0.1\text{m}}{2\times10^{6}\text{ m/s}} = \boxed{50.0\text{ns}}$$

(*c*) Using a constant-acceleration equation, relate the displacement of the electron to its acceleration and the elapsed time:

$$\Delta\vec{y} = \tfrac{1}{2}\vec{a}_y\left(\Delta t\right)^2$$
$$= \tfrac{1}{2}\left(-7.03\times10^{13}\text{ m/s}^{2}\right)\left(50\text{ ns}\right)^2\hat{j}$$
$$= \boxed{\left(-8.79\text{cm}\right)\hat{j}}$$

i.e., the electron is deflected 8.79 cm downward.

52 ••

Picture the Problem Because the electric field is uniform, the acceleration of the electron will be constant and we can apply Newton's 2nd law to find its acceleration and use a constant-acceleration equation to find its speed as it leaves the region in which there is a uniform electric field.

Using a constant-acceleration

$$v^2 = v_0^2 + 2a\Delta x$$

equation, relate the speed of the electron as it leaves the region of the electric field to its acceleration and distance of travel:

or, because $v_0 = 0$,

$$v = \sqrt{2a\Delta x}$$

Apply Newton's 2nd law to express the acceleration of the electron in terms of the electric field:

$$a = \frac{F_{\text{net}}}{m_e} = \frac{eE}{m_e}$$

Substitute to obtain:

$$v = \sqrt{\frac{2eE\Delta x}{m_e}}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{\frac{2\left(1.6\times10^{-19}\text{ C}\right)\left(8\times10^{4}\text{ N/C}\right)\left(0.05\text{ m}\right)}{9.11\times10^{-31}\text{ kg}}} = \boxed{3.75\times10^{7}\text{ m/s}}$$

Remarks: Because this result is approximately 13% of the speed of light, it is only an approximation.

53 ••

Picture the Problem We can apply the work-kinetic energy theorem to relate the change in the object's kinetic energy to the net force acting on it. We can express the net force acting on the charged body in terms of its charge and the electric field.

Using the work-kinetic energy theorem, express the kinetic energy of the object in terms of the net force acting on it and its displacement:

$$W = \Delta K = F_{\text{net}}\Delta x$$

Relate the net force acting on the charged object to the electric field:

$$F_{\text{net}} = QE$$

Substitute to obtain:

$$\Delta K = K_{\text{f}} - K_{\text{i}} = QE\Delta x$$

or, because $K_{\text{i}} = 0$,

$$K_{\text{f}} = QE\Delta x$$

Solve for Q:

$$Q = \frac{K_{\text{f}}}{E\Delta x}$$

Substitute numerical values and evaluate Q:

$$Q = \frac{0.12\,\text{J}}{(300\,\text{N/C})(0.50\,\text{m})} = \boxed{800\,\mu\text{C}}$$

***54 ••**

Picture the Problem We can use constant-acceleration equations to express the *x* and *y* coordinates of the particle in terms of the parameter *t* and Newton's 2nd law to express the constant acceleration in terms of the electric field. Eliminating the parameter will yield an equation for *y* as a function of *x*, *q*, and *m* that we can solve for E_y.

Express the *x* and *y* coordinates of the particle as functions of time:

$$x = (v\cos\theta)t$$

and

$$y = (v\sin\theta)t - \tfrac{1}{2}a_y t^2$$

Apply Newton's 2nd law to relate the acceleration of the particle to the net force acting on it:

$$a_y = \frac{F_{\text{net, y}}}{m} = \frac{qE_y}{m}$$

Substitute in the *y*-coordinate equation to obtain:

$$y = (v\sin\theta)t - \frac{qE_y}{2m}t^2$$

Eliminate the parameter *t* between the two equations to obtain:

$$y = (\tan\theta)x - \frac{qE_y}{2mv^2\cos^2\theta}x^2$$

Set $y = 0$ and solve for E_y:

$$E_y = \frac{mv^2\sin 2\theta}{qx}$$

Substitute the non-particle specific data to obtain:

$$E_y = \frac{m\left(3\times10^6\,\text{m/s}\right)^2\sin 70^\circ}{q(0.015\,\text{m})} = \left(5.64\times10^{14}\,\text{m/s}^2\right)\frac{m}{q}$$

(*a*) Substitute for the mass and charge of an electron and evaluate E_y:

$$E_y = \left(5.64\times10^{14}\,\text{m/s}^2\right)\frac{9.11\times10^{-31}\,\text{kg}}{1.6\times10^{-19}\,\text{C}} = \boxed{3.21\,\text{kN/C}}$$

(*b*) Substitute for the mass and charge of a proton and evaluate E_y:

$$E_y = \left(5.64\times10^{14}\,\text{m/s}^2\right)\frac{1.67\times10^{-27}\,\text{kg}}{1.6\times10^{-19}\,\text{C}} = \boxed{5.89\,\text{MN/C}}$$

55 ••

Picture the Problem We can use constant-acceleration equations to express the x and y coordinates of the electron in terms of the parameter t and Newton's 2nd law to express the constant acceleration in terms of the electric field. Eliminating the parameter will yield an equation for y as a function of x, q, and m. We can decide whether the electron will strike the upper plate by finding the maximum value of its y coordinate. Should we find that it does not strike the upper plate, we can determine where it strikes the lower plate by setting $y(x) = 0$.

Express the x and y coordinates of the electron as functions of time:

$$x = (v_0 \cos\theta)t$$

and

$$y = (v_0 \sin\theta)t - \tfrac{1}{2}a_y t^2$$

Apply Newton's 2nd law to relate the acceleration of the electron to the net force acting on it:

$$a_y = \frac{F_{\text{net},y}}{m_e} = \frac{eE_y}{m_e}$$

Substitute in the y-coordinate equation to obtain:

$$y = (v_0 \sin\theta)t - \frac{eE_y}{2m_e}t^2$$

Eliminate the parameter t between the two equations to obtain:

$$y(x) = (\tan\theta)x - \frac{eE_y}{2m_e v_0^2 \cos^2\theta}x^2 \quad (1)$$

To find y_{max}, set $dy/dx = 0$ for extrema:

$$\frac{dy}{dx} = \tan\theta - \frac{eE_y}{m_e v_0^2 \cos^2\theta}x' = 0 \text{ for extrema}$$

Solve for x' to obtain:

$$x' = \frac{m_e v_0^2 \sin 2\theta}{2eE_y} \quad \text{(See remark below.)}$$

Substitute x' in $y(x)$ and simplify to obtain y_{max}:

$$y_{\text{max}} = \frac{m_e v_0^2 \sin^2\theta}{2eE_y}$$

Substitute numerical values and evaluate y_{max}:

$$y_{\text{max}} = \frac{(9.11\times10^{-31}\text{ kg})(5\times10^6\text{ m/s})^2 \sin^2 45^\circ}{2(1.6\times10^{-19}\text{ C})(3.5\times10^3\text{ N/C})} = 1.02\text{ cm}$$

and, because the plates are separated by 2 cm, the electron does not strike the upper plate.

To determine where the electron will strike the lower plate, set $y = 0$ in equation (1) and solve for x to obtain:

$$x = \frac{m_e v_0^2 \sin 2\theta}{eE_y}$$

Substitute numerical values and evaluate x:

$$x = \frac{\left(9.11\times10^{-31}\text{ kg}\right)\left(5\times10^{6}\text{ m/s}\right)^2 \sin 90°}{\left(1.6\times10^{-19}\text{ C}\right)\left(3.5\times10^{3}\text{ N/C}\right)} = \boxed{4.07\,\text{cm}}$$

Remarks: x' is an extremum, i.e., either a maximum or a minimum. To show that it is a maximum we need to show that d^2y/dx^2, evaluated at x', is negative. A simple alternative is to use your graphing calculator to show that the graph of $y(x)$ is a maximum at x'. Yet another alternative is to recognize that, because equation (1) is quadratic and the coefficient of x^2 is negative, its graph is a parabola that opens downward.

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Picture the Problem The trajectory of the electron while it is in the electric field is parabolic (its acceleration is downward and constant) and its trajectory, once it is out of the electric field is, if we ignore the small gravitational force acting on it, linear. We can use constant-acceleration equations and Newton's 2nd law to express the electron's x and y coordinates parametrically and then eliminate the parameter t to express $y(x)$. We can find the angle with the horizontal at which the electron leaves the electric field from the x and y components of its velocity and its total vertical deflection by summing its deflections over the first 4 cm and the final 12 cm of its flight.

(*a*) Using a constant-acceleration equation, express the x and y coordinates of the electron as functions of time:

$$x(t) = v_0 t$$

and

$$y(t) = v_{0,y}t + \tfrac{1}{2}a_y t^2$$

Because $v_{0,y} = 0$:

$$x(t) = v_0 t \qquad (1)$$

and

$$y(t) = \tfrac{1}{2}a_y t^2$$

Using Newton's 2nd law, relate the acceleration of the electron to the electric field:

$$a_y = \frac{F_{\text{net}}}{m_e} = \frac{-eE_y}{m_e}$$

Substitute to obtain:

$$y(t) = -\frac{eE_y}{2m_e}t^2 \qquad (2)$$

Eliminate the parameter t between equations (1) and (2) to obtain:

$$y(x) = -\frac{eE_y}{2m_e v_0^2}x^2 = -\frac{eE_y}{4K}x^2$$

Substitute numerical values and evaluate y(4 cm):

$$y(0.04\,\text{m}) = -\frac{(1.6\times10^{-19}\ \text{C})(2\times10^4\ \text{N/C})(0.04\,\text{m})^2}{4(2\times10^{-16}\ \text{J})} = \boxed{-6.40\,\text{mm}}$$

(*b*) Express the horizontal and vertical components of the electron's speed as it leaves the electric field:

$$v_x = v_0\cos\theta$$
and
$$v_y = v_0\sin\theta$$

Divide the second of these equations by the first to obtain:

$$\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{v_y}{v_0}$$

Using a constant-acceleration equation, express v_y as a function of the electron's acceleration and its time in the electric field:

$$v_y = v_{0,y} + a_y t$$
or, because $v_{0,y} = 0$
$$v_y = a_y t = \frac{F_{\text{net},y}}{m_e}t = -\frac{eE_y}{m_e}\frac{x}{v_0}$$

Substitute to obtain:

$$\theta = \tan^{-1}\left(-\frac{eE_y x}{m_e v_0^2}\right) = \tan^{-1}\left(-\frac{eE_y x}{2K}\right)$$

Substitute numerical values and evaluate θ:

$$\theta = \tan^{-1}\left[-\frac{(1.6\times10^{-19}\ \text{C})(2\times10^4\ \text{N/C})(0.04\,\text{m})}{2(2\times10^{-16}\ \text{J})}\right] = \boxed{-17.7^\circ}$$

(*c*) Express the total vertical displacement of the electron:

$$y_{\text{total}} = y_{4\,\text{cm}} + y_{12\,\text{cm}}$$

Relate the horizontal and vertical distances traveled to the screen to the horizontal and vertical components of its velocity:

$$x = v_x\Delta t$$
and
$$y = v_y\Delta t$$

Eliminate Δt from these equations to obtain:

$$y = \frac{v_y}{v_x} x = (\tan\theta)x$$

Substitute numerical values and evaluate y:

$$y = [\tan(-17.7°)](0.12\,\text{m}) = -3.83\text{cm}$$

Substitute for $y_{4\text{ cm}}$ and $y_{12\text{ cm}}$ and evaluate y_{total}:

$$y_{\text{total}} = -0.640\,\text{cm} - 3.83\,\text{cm}$$
$$= \boxed{-4.47\,\text{cm}}$$

i.e., the electron will strike the fluorescent screen 4.47 cm below the horizontal axis.

57 •

Picture the Problem We can use its definition to find the dipole moment of this pair of charges.

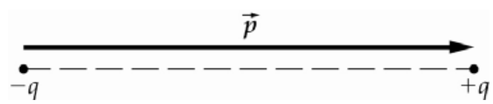
(*a*) Apply the definition of electric dipole moment to obtain:

$$\vec{p} = q\vec{L}$$

and

$$p = (2\,\text{pC})(4\,\mu\text{m}) = \boxed{8.00\times10^{-18}\,\text{C}\cdot\text{m}}$$

(*b*) If we assume that the dipole is oriented as shown to the right, then $\vec{p}$ is to the right; pointing from the negative charge toward the positive charge.



***58** •

Picture the Problem The torque on an electric dipole in an electric field is given by $\vec{\tau} = \vec{p}\times\vec{E}$ and the potential energy of the dipole by $U = -\vec{p}\cdot\vec{E}$.

Using its definition, express the torque on a dipole moment in a uniform electric field:

$$\vec{\tau} = \vec{p}\times\vec{E}$$

and

$$\tau = pE\sin\theta$$

where θ is the angle between the electric dipole moment and the electric field.

(*a*) Evaluate τ for θ = 0°:

$$\tau = pE\sin 0° = \boxed{0}$$

(*b*) Evaluate τ for θ = 90°:

$$\tau = (0.5\,e\cdot\text{nm})(4.0\times10^4\ \text{N/C})\sin 90° = \boxed{3.20\times10^{-24}\ \text{N}\cdot\text{m}}$$

(*c*) Evaluate τ for θ = 30°:

$$\tau = (0.5\,e\cdot\text{nm})(4.0\times10^4\ \text{N/C})\sin 30° = \boxed{1.60\times10^{-24}\ \text{N}\cdot\text{m}}$$

(*d*) Using its definition, express the potential energy of a dipole in an electric field:

$$U = -\vec{\boldsymbol{p}}\cdot\vec{\boldsymbol{E}} = -pE\cos\theta$$

Evaluate U for θ = 0°:

$$U = -(0.5\,e\cdot\text{nm})(4.0\times10^4\ \text{N/C})\cos 0° = \boxed{-3.20\times10^{-24}\ \text{J}}$$

Evaluate U for θ = 90°:

$$U = -(0.5\,e\cdot\text{nm})(4.0\times10^4\ \text{N/C})\cos 90° = \boxed{0}$$

Evaluate U for θ = 30°:

$$U = -(0.5\,e\cdot\text{nm})(4.0\times10^4\ \text{N/C})\cos 30° = \boxed{-2.77\times10^{-24}\ \text{J}}$$

***59 ••**

Picture the Problem We can combine the dimension of an electric field with the dimension of an electric dipole moment to prove that, in any direction, the dimension of the far field is proportional to $1/[L]^3$ and, hence, the electric field far from the dipole falls off as $1/r^3$.

Express the dimension of an electric field:

$$[E] = \frac{[kQ]}{[L]^2}$$

Express the dimension an electric dipole moment:

$$[p] = [Q][L]$$

Write the dimension of charge in terms of the dimension of an electric dipole moment:

$$[Q] = \frac{[p]}{[L]}$$

Substitute to obtain:

$$[E] = \frac{[k][p]}{[L]^2[L]} = \boxed{\frac{[k][p]}{[L]^3}}$$

This shows that the field E due to a dipole

p falls off as $1/r^3$.

60 ••

Picture the Problem We can use its definition to find the molecule's dipole moment. From the symmetry of the system, it is evident that the x component of the dipole moment is zero.

Using its definition, express the molecule's dipole moment:

$$\vec{p} = p_x\hat{i} + p_y\hat{j}$$

From symmetry considerations we have:

$$p_x = 0$$

The y component of the molecule's dipole moment is:

$$\begin{aligned} p_y &= qL = 2eL \\ &= 2\left(1.6\times10^{-19}\text{ C}\right)\left(0.058\text{ nm}\right) \\ &= 1.86\times10^{-29}\text{ C}\cdot\text{m} \end{aligned}$$

Substitute to obtain:

$$\vec{p} = \boxed{\left(1.86\times10^{-29}\text{ C}\cdot\text{m}\right)\hat{j}}$$

61 ••

Picture the Problem We can express the net force on the dipole as the sum of the forces acting on the two charges that constitute the dipole and simplify this expression to show that $\vec{F}_{\text{net}} = Cp\hat{i}$. We can show that, under the given conditions, $\vec{F}_{\text{net}}$ is also given by $\left(dE_x/dx\right)p\hat{i}$ by differentiating the dipole's potential energy function with respect to x.

(a) Express the net force acting on the dipole:

$$\vec{F}_{\text{net}} = \vec{F}_{-q} + \vec{F}_{+q}$$

Apply Coulomb's law to express the forces on the two charges:

$$\vec{F}_{-q} = -q\vec{E} = -qC\left(x_1 - a\right)\hat{i}$$

and

$$\vec{F}_{+q} = +q\vec{E} = qC\left(x_1 + a\right)\hat{i}$$

Substitute to obtain:

$$\begin{aligned} \vec{F}_{\text{net}} &= -qC\left(x_1 - a\right)\hat{i} + qC\left(x_1 + a\right)\hat{i} \\ &= 2aqC\hat{i} = \boxed{Cp\hat{i}} \end{aligned}$$

where $p = 2aq$.

(*b*) Express the net force acting on the dipole as the spatial derivative of *U*:

$$\vec{F}_{net} = -\frac{dU}{dx}\hat{i} = -\frac{d}{dx}\left[-p_x E_x\right]\hat{i} = \boxed{p_x \frac{dE_x}{dx}\hat{i}}$$

62 •••

Picture the Problem We can express the force exerted on the dipole by the electric field as –*dU*/*dr* and the potential energy of the dipole as –*pE*. Because the field is due to a point charge, we can use Coulomb's law to express *E*. In the second part of the problem, we can use Newton's 3rd law to show that the magnitude of the electric field of the dipole along the line of the dipole a distance *r* away is approximately $2kp/r^3$.

(*a*) Express the force exerted by the electric field of the point charge on the dipole:

$$\vec{F} = -\frac{dU}{dr}\hat{r}$$

where $\hat{r}$ is a unit radial vector pointing from *Q* toward the dipole.

Express the potential energy of the dipole in the electric field:

$$U = -pE = -p\frac{kQ}{r^2}$$

Substitute to obtain:

$$\vec{F} = -\frac{d}{dr}\left[-p\frac{kQ}{r^2}\right]\hat{r} = \boxed{-\frac{2kQp}{r^3}\hat{r}}$$

(*b*) Using Newton's 3rd law, express the force that the dipole exerts on the charge *Q* at the origin:

$$\vec{F}_{\text{on }Q} = -\vec{F} \text{ or } F_{\text{on }Q}\hat{r} = -F\hat{r}$$

and

$$F_{\text{on }Q} = F$$

Express $F_{\text{on }Q}$ in terms of the field in which *Q* finds itself:

$$F_{\text{on }Q} = QE$$

Substitute to obtain:

$$QE = \frac{2kQp}{r^3} \Rightarrow E = \boxed{\frac{2kp}{r^3}}$$

General Problems

***63 •**

Picture the Problem We can equate the gravitational force and the electric force acting on a proton to find the mass of the proton under the given condition.

(*a*) Express the condition that must be satisfied if the net force on the

$$F_g = F_e$$

proton is zero:

Use Newton's law of gravity and Coulomb's law to substitute for F_g and F_e:

$$\frac{Gm^2}{r^2} = \frac{ke^2}{r^2}$$

Solve for *m* to obtain:

$$m = e\sqrt{\frac{k}{G}}$$

Substitute numerical values and evaluate *m*:

$$m = \left(1.6\times10^{-19}\ \text{C}\right)\sqrt{\frac{8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2}{6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2}} = \boxed{1.86\times10^{-9}\ \text{kg}}$$

(*b*) Express the ratio of F_e and *F*g:

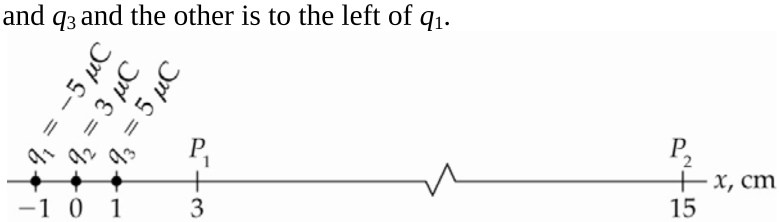
$$\frac{\frac{ke^2}{r^2}}{\frac{Gm_p^2}{r^2}} = \frac{ke^2}{Gm_p^2}$$

Substitute numerical values to obtain:

$$\frac{ke^2}{Gm_p^2} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.6\times10^{-19}\ \text{C}\right)^2}{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.67\times10^{-27}\ \text{kg}\right)^2} = \boxed{1.24\times10^{36}}$$

64 ••

Picture the Problem The locations of the charges q_1, q_2 and q_2 and the points at which we are calculate the field are shown in the diagram. From the diagram it is evident that $\vec{E}$ along the axis has no *y* component. We can use Coulomb's law for $\vec{E}$ due to a point charge and the superposition principle to find $\vec{E}$ at points P_1 and P_2. Examining the distribution of the charges we can see that there are two points where $E = 0$. One is between q_2 and q_3 and the other is to the left of q_1.



Using Coulomb's law, express the electric field at P_1 due to the three charges:

$$\vec{E}_{P_1} = \vec{E}_{q_1} + \vec{E}_{q_2} + \vec{E}_{q_3} = \frac{kq_1}{r_{1,P_1}^2}\hat{i} + \frac{kq_2}{r_{2,P_1}^2}\hat{i} + \frac{kq_3}{r_{3,P_1}^2}\hat{i} = k\left[\frac{q_1}{r_{1,P_1}^2} + \frac{q_2}{r_{2,P_1}^2} + \frac{q_3}{r_{3,P_1}^2}\right]\hat{i}$$

Substitute numerical values and evaluate $\vec{E}_{P_1}$:

$$\vec{E}_{P_1} = \left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{-5\,\mu\text{C}}{(4\,\text{cm})^2} + \frac{3\,\mu\text{C}}{(3\,\text{cm})^2} + \frac{5\,\mu\text{C}}{(2\,\text{cm})^2}\right]\hat{i} = \boxed{\left(1.14\times10^8\ \text{N/C}\right)\hat{i}}$$

Express the electric field at P_2:

$$\vec{E}_{P_2} = \vec{E}_{q_1} + \vec{E}_{q_2} + \vec{E}_{q_3} = k\left[\frac{q_1}{r_{1,P_2}^2} + \frac{q_2}{r_{2,P_2}^2} + \frac{q_3}{r_{3,P_2}^2}\right]\hat{i}$$

Substitute numerical values and evaluate $\vec{E}_{P_2}$:

$$\vec{E}_{P_2} = \left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{-5\,\mu\text{C}}{(16\,\text{cm})^2} + \frac{3\,\mu\text{C}}{(15\,\text{cm})^2} + \frac{5\,\mu\text{C}}{(14\,\text{cm})^2}\right]\hat{i} = \boxed{\left(1.74\times10^6\ \text{N/C}\right)\hat{i}}$$

Letting x represent the x coordinate of a point where the magnitude of the electric field is zero, express the condition that $E = 0$ for the point between $x = 0$ and $x = 1$ cm:

$$E_P = k\left[\frac{q_1}{r_{1,P}^2} + \frac{q_2}{r_{2,P}^2} + \frac{q_3}{r_{3,P}^2}\right] = 0$$

or

$$\frac{-5\,\mu\text{C}}{(x+1\,\text{cm})^2} + \frac{3\,\mu\text{C}}{x^2} - \frac{5\,\mu\text{C}}{(1\,\text{cm}-x)^2} = 0$$

Solve this equation to obtain:

$$x = \boxed{0.417\,\text{cm}}$$

For $x < -1$ cm, let $y = -x$ to obtain:

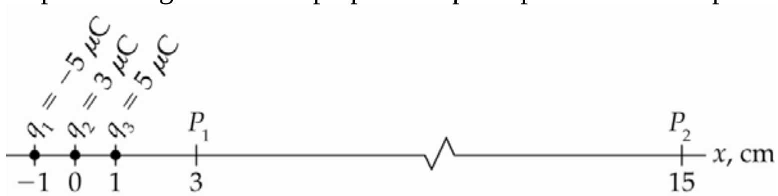
$$\frac{5\,\mu\text{C}}{(y-1\,\text{cm})^2} - \frac{3\,\mu\text{C}}{y^2} - \frac{5\,\mu\text{C}}{(y+1\,\text{cm})^2} = 0$$

Solve this equation to obtain:

$$x = 6.95\,\text{cm} \text{ and } y = \boxed{-6.95\,\text{cm}}$$

65 ••

Picture the Problem The locations of the charges q_1, q_2 and q_2 and the point P_2 at which we are calculate the field are shown in the diagram. The electric field on the x axis due to the dipole is given by $\vec{E}_{\text{dipole}} = 2k\vec{p}/x^3$ where $\vec{p} = 2aq_1\hat{i}$. We can use Coulomb's law for $\vec{E}$ due to a point charge and the superposition principle to find $\vec{E}$ at point P_2.



Express the electric field at P_2 as the sum of the field due to the dipole and the point charge q_2:

$$\vec{E}_{P_2} = \vec{E}_{\text{dipole}} + \vec{E}_{q_2} = \frac{2kp}{x^3}\hat{i} + \frac{kq_2}{x^2}\hat{i} = \frac{2k(2q_1a)}{x^3}\hat{i} + \frac{kq_2}{x^2}\hat{i} = \frac{k}{x^2}\left[\frac{4q_1a}{x} + q_2\right]\hat{i}$$

where a = 1 cm.

Substitute numerical values and evaluate $\vec{E}_{P_2}$:

$$\vec{E}_{P_2} = \frac{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}{(15\times10^{-2}\ \text{m})^2}\left[\frac{4(5\ \mu\text{C})(1\,\text{cm})}{15\,\text{cm}} + 3\mu\text{C}\right]\hat{i} = \boxed{(1.73\times10^6\ \text{N/C})\hat{i}}$$

While the separation of the two charges of the dipole is more than 10% of the distance to the point of interest, i.e., x is not much greater than a, this result is in excellent agreement with that of Problem 64.

***66** ••

Picture the Problem We can find the percentage of the free charge that would have to be removed by finding the ratio of the number of free electrons n_e to be removed to give the penny a charge of 15 μC to the number of free electrons in the penny. Because we're assuming the pennies to be point charges, we can use Coulomb's law to find the force of repulsion between them.

(*a*) Express the fraction f of the free charge to be removed as the quotient of the number of electrons to be removed and the number of free

$$f = \frac{n_e}{N}$$

electrons:

Relate N to Avogadro's number, the mass of the copper penny, and the molecular mass of copper:

$$\frac{N}{N_A} = \frac{m}{M} \Rightarrow N = N_A \frac{m}{M}$$

Relate n_e to the free charge Q to be removed from the penny:

$$Q = n_e[-e] \Rightarrow n_e = \frac{Q}{-e}$$

$$f = \frac{\frac{Q}{-e}}{N_A \frac{m}{M}} = -\frac{QM}{meN_A}$$

Substitute numerical values and evaluate f:

$$f = -\frac{(-15\,\mu\text{C})(63.5\,\text{g/mol})}{(3\,\text{g})(1.6\times10^{-19}\,\text{C})(6.02\times10^{23}\,\text{mol}^{-1})} = 3.29\times10^{-9} = \boxed{3.29\times10^{-7}\%}$$

(*b*) Use Coulomb's law to express the force of repulsion between the two pennies:

$$F = \frac{kq^2}{r^2} = \frac{k(n_e e)^2}{r^2}$$

Substitute numerical values and evaluate F:

$$F = \frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(9.38\times10^{13})^2(1.6\times10^{-19}\,\text{C})^2}{(0.25\,\text{m})^2} = \boxed{32.4\,\text{N}}$$

67 ••

Picture the Problem Knowing the total charge of the two charges, we can use Coulomb's law to find the two combinations of charge that will satisfy the condition that both are positive and hence repel each other. If just one charge is positive, then there is just one distribution of charge that will satisfy the conditions that the force is attractive and the sum of the two charges is 6 μC.

(*a*) Use Coulomb's law to express the repulsive force each charge exerts on the other:

$$F = \frac{kq_1q_2}{r_{1,2}^2}$$

Express q_2 in terms of the total charge and q_1:

$$q_2 = Q - q_1$$

Substitute to obtain:

$$F = \frac{kq_1(Q - q_1)}{r_{1,2}^2}$$

Substitute numerical values to obtain:

$$8\,\text{mN} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)[(6\,\mu\text{C})q_1 - q_1^2]}{(3\,\text{m})^2}$$

Simplify to obtain:

$$q_1^2 + (-6\,\mu\text{C})q_1 + 8.01(\mu\text{C})^2 = 0$$

Solve to obtain:

$$q_1 = \boxed{3.99\,\mu\text{C}}\ \text{and}\ q_2 = \boxed{2.01\,\mu\text{C}}$$

or

$$q_1 = \boxed{2.01\,\mu\text{C}}\ \text{and}\ q_2 = \boxed{3.99\,\mu\text{C}}$$

(*b*) Use Coulomb's law to express the attractive force each charge exerts on the other:

$$F = -\frac{kq_1q_2}{r_{1,2}^2}$$

Proceed as in (*a*) to obtain:

$$q_1^2 + (-6\,\mu\text{C})q_1 - 8.01(\mu\text{C})^2 = 0$$

Solve to obtain:

$$q_1 = \boxed{7.12\,\mu\text{C}}\ \text{and}\ q_2 = \boxed{-1.12\,\mu\text{C}}$$

68 ••

Picture the Problem The electrostatic forces between the charges are responsible for the tensions in the strings. We'll assume that these are point charges and apply Coulomb's law and the principle of the superposition of forces to find the tension in each string.

Use Coulomb's law to express the net force on the charge $+q$:

$$T_1 = F_{2q} + F_{4q}$$

Substitute and simplify to obtain:

$$T_1 = \frac{kq(2q)}{d^2} + \frac{kq(4q)}{(2d)^2} = \boxed{\frac{3kq^2}{d^2}}$$

Use Coulomb's law to express the net force on the charge $+4q$:

$$T_2 = F_q + F_{2q}$$

Substitute and simplify to obtain:

$$T_2 = \frac{k(2q)(4q)}{d^2} + \frac{kq(4q)}{(2d)^2} = \boxed{\frac{9kq^2}{d^2}}$$

***69 ••**

Picture the Problem We can use Coulomb's law to express the force exerted on one charge by the other and then set the derivative of this expression equal to zero to find the distribution of the charge that maximizes this force.

Using Coulomb's law, express the force that either charge exerts on the other:

$$F = \frac{kq_1q_2}{D^2}$$

Express q_2 in terms of Q and q_1:

$$q_2 = Q - q_1$$

Substitute to obtain:

$$F = \frac{kq_1(Q - q_1)}{D^2}$$

Differentiate F with respect to q_1 and set this derivative equal to zero for extreme values:

$$\begin{aligned}\frac{dF}{dq_1} &= \frac{k}{D^2}\frac{d}{dq_1}\left[q_1(Q - q_1)\right] \\ &= \frac{k}{D^2}\left[q_1(-1) + Q - q_1\right] \\ &= 0 \text{ for extrema}\end{aligned}$$

Solve for q_1 to obtain:

$$q_1 = \tfrac{1}{2}Q$$

and

$$q_2 = Q - q_1 = \tfrac{1}{2}Q$$

To determine whether a maximum or a minimum exists at $q_1 = \frac{1}{2}Q$, differentiate F a second time and evaluate this derivative at $q_1 = \frac{1}{2}Q$:

$$\begin{aligned}\frac{d^2F}{dq_1^2} &= \frac{k}{D^2}\frac{d}{dq_1}\left[Q - 2q_1\right] \\ &= \frac{k}{D^2}(-2) \\ &< 0 \text{ independently of } q_1.\end{aligned}$$

$$\boxed{\therefore q_1 = q_2 = \tfrac{1}{2}Q \text{ maximizes } F.}$$

***70 ••**

Picture the Problem We can apply Coulomb's law and the superposition of forces to relate the net force acting on the charge $q = -2\ \mu\text{C}$ to x. Because Q divides out of our equation when $F(x) = 0$, we'll substitute the data given for $x = 8.0$ cm.

Using Coulomb's law, express the net force on q as a function of x:

$$F(x) = -\frac{kqQ}{x^2} + \frac{kq(4Q)}{(12\,\text{cm} - x)^2}$$

Simplify to obtain:

$$\frac{F(x)}{kq}=\left[-\frac{1}{x^2}+\frac{4}{(12\,\text{cm}-x)^2}\right]Q$$

Solve for Q:

$$Q=\frac{F(x)}{kq\left[-\frac{1}{x^2}+\frac{4}{(12\,\text{cm}-x)^2}\right]}$$

Evaluate Q for x = 8 cm:

$$Q=\frac{126.4\,\text{N}}{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2\,\mu\text{C})\left[-\frac{1}{(8\,\text{cm})^2}+\frac{4}{(4\,\text{cm})^2}\right]}=\boxed{3.00\,\mu\text{C}}$$

71 ••

Picture the Problem Knowing the total charge of the two charges, we can use Coulomb's law to find the two combinations of charge that will satisfy the condition that both are positive and hence repel each other. If the spheres attract each other, then there is just one distribution of charge that will satisfy the conditions that the force is attractive and the sum of the two charges is 200 μC.

(*a*) Use Coulomb's law to express the repulsive force each charge exerts on the other:

$$F=\frac{kq_1q_2}{r_{1,2}^2}$$

Express q_2 in terms of the total charge and q_1:

$$q_2=Q-q_1$$

Substitute to obtain:

$$F=\frac{kq_1(Q-q_1)}{r_{1,2}^2}$$

Substitute numerical values to obtain:

$$80\,\text{N}=\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left[(200\,\mu\text{C})q_1-q_1^2\right]}{(0.6\,\text{m})^2}$$

Simplify to obtain the quadratic equation:

$$q_1^2+(-0.2\,\text{mC})q_1+3.20\times10^{-3}(\text{mC})^2=0$$

Solve to obtain:

$$q_1=\boxed{17.5\,\mu\text{C}}\ \text{and}\ q_2=\boxed{183\,\mu\text{C}}$$

or

$$q_1 = \boxed{183\,\mu\text{C}}\ \text{and}\ q_2 = \boxed{17.5\,\mu\text{C}}$$

(*b*) Use Coulomb's law to express the attractive force each charge exerts on the other:

$$F = -\frac{kq_1q_2}{r_{1,2}^2}$$

Proceed as in (*a*) to obtain:

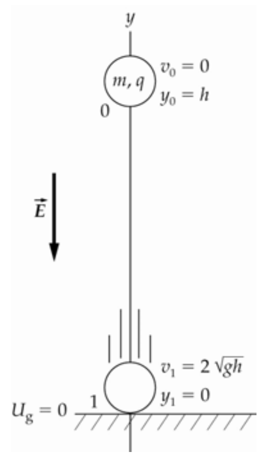
$$q_1^2 + (-0.2\,\text{mC})q_1 - 3.20\times10^{-3}(\text{mC})^2 = 0$$

Solve to obtain:

$$q_1 = \boxed{-15.0\,\mu\text{C}}\ \text{and}\ q_2 = \boxed{215\,\mu\text{C}}$$

72 ••

Picture the Problem Choose the coordinate system shown in the diagram and let $U_g = 0$ where $y = 0$. We'll let our system include the ball and the earth. Then the work done on the ball by the electric field will change the energy of the system. The diagram summarizes what we know about the motion of the ball. We can use the work-energy theorem to our system to relate the work done by the electric field to the change in its energy.



Using the work-energy theorem, relate the work done by the electric field to the change in the energy of the system:

$$W_{\text{electric field}} = \Delta K + \Delta U_g = K_2 - K_1 + U_{g,2} - U_{g,1}$$

or, because $K_1 = U_{g,2} = 0$,

$$W_{\text{electric field}} = K_2 - U_{g,1}$$

Substitute for $W_{\text{electric field}}$, K_2, and $U_{g,0}$ and simplify:

$$qEh = \tfrac{1}{2}mv_1^2 - mgh = \tfrac{1}{2}m\left(2\sqrt{gh}\right)^2 - mgh = mgh$$

Solve for *m*:

$$m = \boxed{\frac{qE}{g}}$$

73 ••

Picture the Problem We can use Coulomb's law, the definition of torque, and the condition for rotational equilibrium to find the electrostatic force between the two charged bodies, the torque this force produces about an axis through the center of the meter stick, and the mass required to maintain equilibrium when it is located either 25 cm to the right or to the left of the mid-point of the rigid stick.

(*a*) Using Coulomb's law, express the electric force between the two charges:

$$F = \frac{kq_1q_2}{d^2}$$

Substitute numerical values and evaluate *F*:

$$F = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(5\times10^{-7}\ \text{C})^2}{(0.1\,\text{m})^2} = \boxed{0.225\,\text{N}}$$

(*b*) Apply the definition of torque to obtain:

$$\tau = F\ell$$

Substitute numerical values and evaluate τ:

$$\tau = (0.225\,\text{N})(0.5\,\text{m}) = \boxed{0.113\,\text{N}\cdot\text{m, counterclockwise}}$$

(*c*) Apply $\sum\tau_{\text{center of the meter stick}} = 0$ to the meterstick:

$$\tau - mg\ell' = 0$$

Solve for *m*:

$$m = \frac{\tau}{g\ell'}$$

Substitute numerical values and evaluate *m*:

$$m = \frac{0.113\,\text{N}}{(9.81\,\text{m/s}^2)(0.25\,\text{m})} = \boxed{0.0461\,\text{kg}}$$

(*d*) Apply $\sum\tau_{\text{center of the meter stick}} = 0$ to the meterstick:

$$-\tau + mg\ell' = 0$$

Substitute for τ:

$$-F\ell + mg\ell' = 0$$

Substitute for *F*:

$$-\frac{kq_1q_2'}{d^2} + mg\ell' = 0$$

where q' is the required charge.

Solve for q_2' to obtain:

$$q_2 = \frac{d^2mg\ell'}{kq_1\ell}$$

Substitute numerical values and evaluate q_2':

$$q_2' = \frac{(0.1\,\text{m})^2(0.0461\,\text{kg})(9.81\,\text{m/s}^2)(0.25\,\text{m})}{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(5\times10^{-7}\ \text{C})(0.5\,\text{m})} = \boxed{5.03\times10^{-7}\ \text{C}}$$

74 ••

Picture the Problem Let the numeral 1 refer to the charge in the 1st quadrant and the numeral 2 to the charge in the 4th quadrant. We can use Coulomb's law for the electric field due to a point charge and the superposition of forces to express the field at the origin and use this equation to solve for *Q*.

Express the electric field at the origin due to the point charges *Q*:

$$\vec{E}(0,0) = \vec{E}_1 + \vec{E}_2 = \frac{kQ}{r_{1,0}^2}\hat{r}_{1,0} + \frac{kQ}{r_{2,0}^2}\hat{r}_{2,0}$$
$$= \frac{kQ}{r^3}\left[(-4\,\text{m})\hat{i} + (-2\,\text{m})\hat{j}\right] + \frac{kQ}{r^3}\left[(-4\,\text{m})\hat{i} + (2\,\text{m})\hat{j}\right] = -\frac{(8\,\text{m})kQ}{r^3}\hat{i}$$
$$= E_x\hat{i}$$

where r is the distance from each charge to the origin and $E_x = -\frac{(8\,\text{m})kQ}{r^3}$.

Express r in terms of the coordinates (x, y) of the point charges:

$$r = \sqrt{x^2 + y^2}$$

Substitute to obtain:

$$E_x = -\frac{(8\,\text{m})kQ}{\left(x^2 + y^2\right)^{3/2}}$$

Solve for Q to obtain:

$$Q = \frac{E_x\left(x^2 + y^2\right)^{3/2}}{k(8\,\text{m})}$$

Substitute numerical values and evaluate Q:

$$Q = -\frac{(4\,\text{kN/C})\left[(4\,\text{m})^2 + (2\,\text{m})^2\right]^{3/2}}{\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)(8\,\text{m})}$$
$$= \boxed{-4.97\,\mu\text{C}}$$

75 ••

Picture the Problem Let the numeral 1 denote one of the spheres and the numeral 2 the other. Knowing the total charge *Q* on the two spheres, we can use Coulomb's law to find the charge on each of them. A second application of Coulomb's law when the spheres carry the same charge and are 0.60 m apart will yield the force each exerts on the other.

(*a*) Use Coulomb's law to express the repulsive force each charge exerts on the other:

$$F = \frac{kq_1q_2}{r_{1,2}^2}$$

Express q_2 in terms of the total charge and q_1:

$$q_2 = Q - q_1$$

Substitute to obtain:

$$F = \frac{kq_1(Q - q_1)}{r_{1,2}^2}$$

Substitute numerical values to obtain:

$$120\ \text{N} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left[(200\ \mu\text{C})q_1 - q_1^2\right]}{(0.6\ \text{m})^2}$$

Simplify to obtain the quadratic equation:

$$q_1^2 + (-200\ \mu\text{C})q_1 + 4810(\mu\text{C})^2 = 0$$

Solve to obtain:

$$q_1 = \boxed{28.0\ \mu\text{C}}\ \text{and}\ q_2 = \boxed{172\ \mu\text{C}}$$

or

$$q_1 = \boxed{172\ \mu\text{C}}\ \text{and}\ q_2 = \boxed{28.0\ \mu\text{C}}$$

(*b*) Use Coulomb's law to express the repulsive force each charge exerts on the other when $q_1 = q_2 = 100\ \mu\text{C}$:

$$F = \frac{kq_1q_2}{r_{1,2}^2}$$

Substitute numerical values and evaluate F:

$$F = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{(100\ \mu\text{C})^2}{(0.6\ \text{m})^2} = \boxed{250\ \text{N}}$$

76 ••

Picture the Problem Let the numeral 1 denote one of the spheres and the numeral 2 the other. Knowing the total charge Q on the two spheres, we can use Coulomb's law to find the charge on each of them. A second application of Coulomb's law when the spheres carry the same charge and are 0.60 m apart will yield the force each exerts on the other.

(*a*) Use Coulomb's law to express the attractive force each charge exerts on the other:

$$F = -\frac{kq_1q_2}{r_{1,2}^2}$$

Express q_2 in terms of the total charge and q_1:

$$q_2 = Q - q_1$$

Substitute to obtain:

$$F = -\frac{kq_1(Q - q_1)}{r_{1,2}^2}$$

Substitute numerical values to obtain:

$$120\,\text{N} = \frac{-\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left[(200\,\mu\text{C})q_1 - q_1^2\right]}{(0.6\,\text{m})^2}$$

Simplify to obtain the quadratic equation:

$$q_1^2 + (-200\,\mu\text{C})q_1 - 4810(\mu\text{C})^2 = 0$$

Solve to obtain:

$$q_1 = \boxed{-21.7\,\mu\text{C}}\ \text{and}\ q_2 = \boxed{222\,\mu\text{C}}$$

or

$$q_1 = \boxed{222\,\mu\text{C}}\ \text{and}\ q_2 = \boxed{-21.7\,\mu\text{C}}$$

(*b*) Use Coulomb's law to express the repulsive force each charge exerts on the other when $q_1 = q_2 = 100\ \mu\text{C}$:

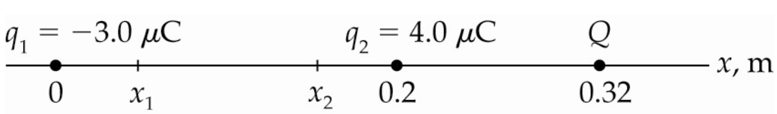
$$F = \frac{kq_1q_2}{r_{1,2}^2}$$

Substitute numerical values and evaluate F:

$$F = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{(100\,\mu\text{C})^2}{(0.6\,\text{m})^2} = \boxed{250\,\text{N}}$$

77 ••

Picture the Problem The charge configuration is shown in the diagram as are the approximate locations, labeled x_1 and x_2, where the electric field is zero. We can determine the charge Q by using Coulomb's law and the superposition of forces to express the net force acting on q_2. In part (b), by inspection, the points where $E = 0$ must be between the $-3\ \mu$C and $+4\ \mu$C charges. We can use Coulomb's law for the field due to point charges and the superposition of electric fields to determine the coordinates x_1 and x_2.



(*a*) Use Coulomb's law to express the force on the 4.0-μC charge:

$$\vec{F}_2 = \vec{F}_{1,2} + \vec{F}_{Q,2} = \frac{kq_1q_2}{r_{1,2}^2}\hat{i} + \frac{kQq_2}{r_{Q,2}^2}\left(-\hat{i}\right) = kq_2\left[\frac{q_1}{r_{1,2}^2} - \frac{Q}{r_{Q,2}^2}\right]\hat{i} = F_2\hat{i}$$

Solve for Q:

$$Q = r_{Q,2}^2\left[\frac{q_1}{r_{1,2}^2} - \frac{F_2}{kq_2}\right]$$

Substitute numerical values and evaluate Q:

$$Q = (0.12\,\text{m})^2\left[\frac{-3\,\mu\text{C}}{(0.2\,\text{m})^2} - \frac{240\,\text{N}}{\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)(4\,\mu\text{C})}\right] = \boxed{-97.2\,\mu\text{C}}$$

(*b*) Use Coulomb's law for electric fields and the superposition of fields to determine the coordinate x at which $E = 0$:

$$\vec{E} = -\frac{kQ}{(0.32\,\text{m}-x)^2}\hat{i} - \frac{kq_2}{(0.2\,\text{m}-x)^2}\hat{i} + \frac{kq_1}{x^2}\hat{i} = 0$$

or

$$-\frac{Q}{(0.32\,\text{m}-x)^2} - \frac{q_2}{(0.2\,\text{m}-x)^2} + \frac{q_1}{x^2} = 0$$

Substitute numerical values to obtain:

$$-\frac{-97.2\,\mu\text{C}}{(0.32\,\text{m}-x)^2} - \frac{4\,\mu\text{C}}{(0.2\,\text{m}-x)^2} + \frac{-3\,\mu\text{C}}{x^2} = 0$$

and

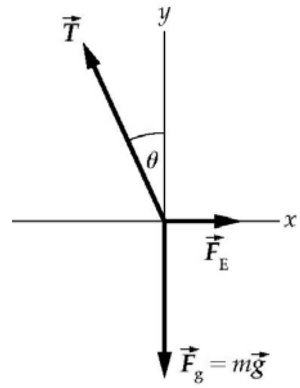
$$\frac{97.2}{(0.32\,\text{m}-x)^2} - \frac{4}{(0.2\,\text{m}-x)^2} - \frac{3}{x^2} = 0$$

Solve (preferably using a graphing calculator!) this equation to obtain:

$$x_1 = \boxed{0.0508\,\text{m}} \text{ and } x_2 = \boxed{0.169\,\text{m}}$$

***78 ••**

Picture the Problem Each sphere is in static equilibrium under the influence of the tension $\vec{T}$, the gravitational force $\vec{F}_g$, and the electric force $\vec{F}_E$. We can use Coulomb's law to relate the electric force to the charge on each sphere and their separation and the conditions for static equilibrium to relate these forces to the charge on each sphere.



(*a*) Apply the conditions for static equilibrium to the charged sphere:

$$\sum F_x = F_E - T\sin\theta = \frac{kq^2}{r^2} - T\sin\theta = 0$$

and

$$\sum F_y = T\cos\theta - mg = 0$$

Eliminate *T* between these equations to obtain:

$$\tan\theta = \frac{kq^2}{mgr^2}$$

Solve for *q*:

$$q = r\sqrt{\frac{mg\tan\theta}{k}}$$

Referring to the figure, relate the separation of the spheres *r* to the length of the pendulum *L*:

$$r = 2L\sin\theta$$

Substitute to obtain:

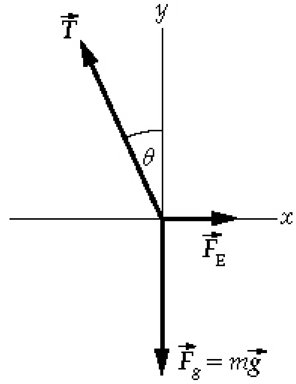
$$q = \boxed{2L\sin\theta\sqrt{\frac{mg\tan\theta}{k}}}$$

(*b*) Evaluate *q* for *m* = 10 g, *L* = 50 cm, and θ = 10°:

$$q = 2(0.5\,\text{m})\sin 10°\sqrt{\frac{(0.01\,\text{kg})(9.81\,\text{m/s}^2)\tan 10°}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}} = \boxed{0.241\,\mu\text{C}}$$

79 ••

Picture the Problem Each sphere is in static equilibrium under the influence of the tension $\vec{T}$, the gravitational force $\vec{F}_g$, and the electric force $\vec{F}_E$. We can use Coulomb's law to relate the electric force to the charge on each sphere and their separation and the conditions for static equilibrium to relate these forces to the charge on each sphere.



(*a*)Apply the conditions for static equilibrium to the charged sphere:

$$\sum F_x = F_E - T\sin\theta = \frac{kq^2}{r^2} - T\sin\theta = 0$$

and

$$\sum F_y = T\cos\theta - mg = 0$$

Eliminate *T* between these equations to obtain:

$$\tan\theta = \frac{kq^2}{mgr^2}$$

Referring to the figure for Problem 80, relate the separation of the spheres *r* to the length of the pendulum *L*:

$$r = 2L\sin\theta$$

Substitute to obtain:

$$\tan\theta = \frac{kq^2}{4mgL^2\sin^2\theta}$$

or

$$\sin^2\theta\tan\theta = \frac{kq^2}{4mgL^2} \quad (1)$$

Substitute numerical values and evaluate $\sin^2\theta\tan\theta$:

$$\sin^2\theta\tan\theta = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(0.75\,\mu\text{C})^2}{4(0.01\text{kg})(9.81\text{m/s}^2)(1.5\text{m})^2} = 5.73\times10^{-3}$$

Because $\sin^2\theta\tan\theta << 1$:

$$\sin\theta \approx \tan\theta \approx \theta$$

and

$$\theta^3 \approx 5.73\times10^{-3}$$

Solve for θ to obtain:

$$\theta = 0.179\,\text{rad} = \boxed{10.3^\circ}$$

(*b*) Evaluate equation (1) with replacing q^2 with q_1q_2:

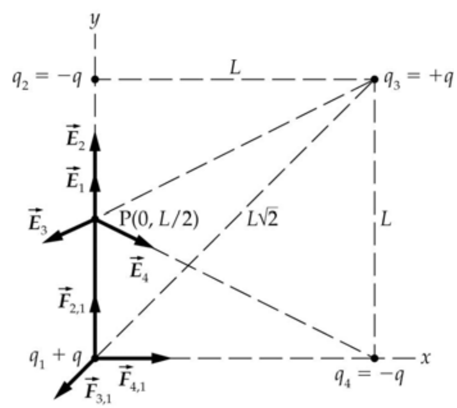
$$\sin^2\theta\tan\theta = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(0.5\,\mu\text{C})(1\,\mu\text{C})}{4(0.01\,\text{kg})\left(9.81\,\text{m/s}^2\right)(1.5\,\text{m})^2} = 5.09\times10^{-3} \approx \theta^3$$

Solve for θ to obtain:

$$\theta = 0.172\,\text{rad} = \boxed{9.86°}$$

80 ••

Picture the Problem Let the origin be at the lower left-hand corner and designate the charges as shown in the diagram. We can apply Coulomb's law for point charges to find the forces exerted on q_1 by q_2, q_3, and q_4 and superimpose these forces to find the net force exerted on q_1. In part (*b*), we'll use Coulomb's law for the electric field due to a point charge and the superposition of fields to find the electric field at point $P(0, L/2)$.



(*a*) Using the superposition of forces, express the net force exerted on q_1:

$$\vec{F}_1 = \vec{F}_{2,1} + \vec{F}_{3,1} + \vec{F}_{4,1}$$

Apply Coulomb's law to express $\vec{F}_{2,1}$:

$$\vec{F}_{2,1} = \frac{kq_2q_1}{r_{2,1}^2}\hat{r}_{2,1} = \frac{kq_2q_1}{r_{2,1}^3}\vec{r}_{2,1} = \frac{k(-q)q}{L^3}\left(-L\,\hat{j}\right) = \frac{kq^2}{L^2}\hat{j}$$

Apply Coulomb's law to express $\vec{F}_{4,1}$:

$$\vec{F}_{4,1} = \frac{kq_4q_1}{r_{4,1}^2}\hat{r}_{4,1} = \frac{kq_4q_1}{r_{4,1}^3}\vec{r}_{4,1} = \frac{k(-q)q}{L^3}\left(-L\,\hat{i}\right) = \frac{kq^2}{L^2}\hat{i}$$

Apply Coulomb's law to express $\vec{F}_{3,1}$:

$$\vec{F}_{3,1} = \frac{kq_3q_1}{r_{3,1}^2}\hat{r}_{3,1} = \frac{kq_3q_1}{r_{3,1}^3}\vec{r}_{3,1} = \frac{kq^2}{2^{3/2}L^3}\left(-L\,\hat{i} - L\,\hat{j}\right) = -\frac{kq^2}{2^{3/2}L^2}\left(\hat{i} + \hat{j}\right)$$

Substitute and simplify to obtain:

$$\vec{F}_1 = \frac{kq^2}{L^2}\hat{j} - \frac{kq^2}{2^{3/2}L^2}\left(\hat{i}+\hat{j}\right) + \frac{kq^2}{L^2}\hat{i}$$
$$= \frac{kq^2}{L^2}\left(\hat{i}+\hat{j}\right) - \frac{kq^2}{2^{3/2}L^2}\left(\hat{i}+\hat{j}\right)$$
$$= \boxed{\frac{kq^2}{L^2}\left(1-\frac{1}{2\sqrt{2}}\right)\left(\hat{i}+\hat{j}\right)}$$

(*b*) Using superposition of fields, express the resultant field at point *P*:

$$\vec{E}_P = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 \qquad (1)$$

Use Coulomb's law to express $\vec{E}_1$:

$$\vec{E}_1 = \frac{kq_1}{r_{1,P}^2}\hat{r}_{1,P} = \frac{kq}{r_{1,P}^3}\left(\frac{L}{2}\hat{j}\right) = \frac{kq}{\left(\frac{L}{2}\right)^3}\left(\frac{L}{2}\hat{j}\right) = \frac{4kq}{L^2}\hat{j}$$

Use Coulomb's law to express $\vec{E}_2$:

$$\vec{E}_2 = \frac{kq_2}{r_{2,P}^2}\hat{r}_{2,P} = \frac{k(-q)}{r_{2,P}^3}\left(\frac{L}{2}\hat{j}\right) = \frac{-kq}{\left(\frac{L}{2}\right)^3}\left(-\frac{L}{2}\hat{j}\right) = \frac{4kq}{L^2}\hat{j}$$

Use Coulomb's law to express $\vec{E}_3$:

$$\vec{E}_3 = \frac{kq_3}{r_{3,P}^2}\hat{r}_{3,P} = \frac{kq}{r_{3,P}^3}\left(-L\hat{i}-\frac{L}{2}\hat{j}\right) = \frac{8kq}{5^{3/2}L^2}\left(-\hat{i}-\frac{1}{2}\hat{j}\right)$$

Use Coulomb's law to express $\vec{E}_4$:

$$\vec{E}_4 = \frac{kq_4}{r_{4,P}^2}\hat{r}_{3,P} = \frac{k(-q)}{r_{4,P}^3}\left(L\hat{i}-\frac{L}{2}\hat{j}\right) = \frac{8kq}{5^{3/2}L^2}\left(\hat{i}-\frac{1}{2}\hat{j}\right)$$

Substitute in equation (1) and simplify to obtain:

$$\vec{E}_P = \frac{4kq}{L^2}\hat{j} + \frac{4kq}{L^2}\hat{j} + \frac{8kq}{5^{3/2}L^2}\left(-\hat{i}-\frac{1}{2}\hat{j}\right) + \frac{8kq}{5^{3/2}L^2}\left(\hat{i}-\frac{1}{2}\hat{j}\right) = \boxed{\frac{8kq}{L^2}\left(1+\frac{\sqrt{5}}{25}\right)\hat{j}}$$

81 ••

Picture the Problem We can apply Newton's 2nd law in rotational form to obtain the differential equation of motion of the dipole and then use the small angle approximation $\sin\theta \approx \theta$ to show that the dipole experiences a linear restoring torque and, hence, will experience simple harmonic motion.

Apply $\sum\tau = I\alpha$ to the dipole:

$$-pE\sin\theta = I\frac{d^2\theta}{dt^2}$$

where τ is negative because acts in such a direction as to decrease θ.

For small values of θ, $\sin\theta \approx \theta$ and:

$$-pE\theta = I\frac{d^2\theta}{dt^2}$$

Express the moment of inertia of the dipole:

$$I = \tfrac{1}{2}ma^2$$

Relate the dipole moment of the dipole to its charge and the charge separation:

$$p = qa$$

Substitute to obtain:

$$\tfrac{1}{2}ma^2\frac{d^2\theta}{dt^2} = -qaE\theta$$

or

$$\boxed{\frac{d^2\theta}{dt^2} = -\frac{2qE}{ma}\theta}$$

the differential equation for a simple harmonic oscillator with angular frequency $\omega = \sqrt{2qE/ma}$.

Express the period of a simple harmonic oscillator:

$$T = \frac{2\pi}{\omega}$$

Substitute to obtain:

$$T = \boxed{2\pi\sqrt{\frac{ma}{2qE}}}$$

82 ••

Picture the Problem We can apply conservation of energy and the definition of the potential energy of a dipole in an electric field to relate q to the kinetic energy of the dumbbell when it is aligned with the field.

Using conservation of energy, relate the initial potential energy of the dumbbell to its kinetic energy when it is momentarily aligned with the electric field:

$\Delta K + \Delta U = 0$

or, because $K_i = 0$,

$K + \Delta U = 0$

where K is the kinetic energy when it is aligned with the field.

Express the change in the potential energy of the dumbbell as it aligns with the electric field in terms of its dipole moment, the electric field, and the angle through which it rotates:

$$\begin{aligned}\Delta U &= U_f - U_i \\ &= -pE\cos\theta_f + pE\cos\theta_f \\ &= qaE(\cos 60° - 1)\end{aligned}$$

Substitute to obtain:

$$K + qaE(\cos 60° - 1) = 0$$

Solve for q:

$$q = \frac{K}{aE(1 - \cos 60°)}$$

Substitute numerical values and evaluate q:

$$q = \frac{5\times10^{-3}\text{ J}}{(0.3\text{ m})(600\text{ N/C})(1-\cos 60°)} = \boxed{55.6\,\mu\text{C}}$$

***83 ••**

Picture the Problem The forces the electron and the proton exert on each other constitute an action-and-reaction pair. Because the magnitudes of their charges are equal and their masses are the same, we find the speed of each particle by finding the speed of either one. We'll apply Coulomb's force law for point charges and Newton's 2nd law to relate v to e, m, k, and r.

Apply Newton's 2nd law to the positron:

$$\frac{ke^2}{r^2} = m\frac{v^2}{\frac{1}{2}r} \Rightarrow \frac{ke^2}{r} = 2mv^2$$

Solve for v to obtain:

$$v = \boxed{\sqrt{\frac{ke^2}{2mr}}}$$

84 ••

Picture the Problem In Problem 81 it was established that the period of an electric dipole in an electric field is given by $T = 2\pi\sqrt{ma/2qE}$. We can use this result to find the frequency of oscillation of a KBr molecule in a uniform electric field of 1000 N/C.

Express the frequency of the KBr oscillator:

$$f = \frac{1}{2\pi}\sqrt{\frac{2qE}{ma}}$$

Substitute numerical values and evaluate f:

$$f = \frac{1}{2\pi}\sqrt{\frac{2\left(1.6\times10^{-19}\ \text{C}\right)\left(1000\ \text{N/C}\right)}{\left(1.4\times10^{-25}\ \text{kg}\right)\left(0.282\ \text{nm}\right)}} = \boxed{4.53\times10^{8}\ \text{Hz}}$$

85 •••

Picture the Problem We can use Coulomb's force law for point masses and the condition for translational equilibrium to express the equilibrium position as a function of k, q, Q, m, and g. In part (b) we'll need to show that the displaced point charge experiences a linear restoring force and, hence, will exhibit simple harmonic motion.

(a) Apply the condition for translational equilibrium to the point mass:

$$\frac{kqQ}{y_0^2} - mg = 0$$

Solve for y_0 to obtain:

$$y_0 = \boxed{\sqrt{\frac{kqQ}{mg}}}$$

(b) Express the restoring force that acts on the point mass when it is displaced a distance Δy from its equilibrium position:

$$F = \frac{kqQ}{\left(y_0 + \Delta y\right)^2} - \frac{kqQ}{y_0^2} \approx \frac{kqQ}{y_0^2 + 2y_0\Delta y} - \frac{kqQ}{y_0^2}$$

because $\Delta y << y_0$.

Simplify this expression further by writing it with a common denominator:

$$F = -\frac{2y_0\Delta y kqQ}{y_0^4 + 2y_0^3\Delta y} = -\frac{2y_0\Delta y kqQ}{y_0^4\left(1 + 2\frac{\Delta y}{y_0}\right)} \approx -\frac{2\Delta y kqQ}{y_0^3}$$

again, because $\Delta y << y_0$.

From the 1st step of our solution:

$$\frac{kqQ}{y_0^2} = mg$$

Substitute to obtain:

$$F = -\frac{2mg}{y_0}\Delta y$$

Apply Newton's 2nd law to the displaced point charge to obtain:

$$m\frac{d^2\Delta y}{dt^2} = -\frac{2mg}{y_0}\Delta y$$

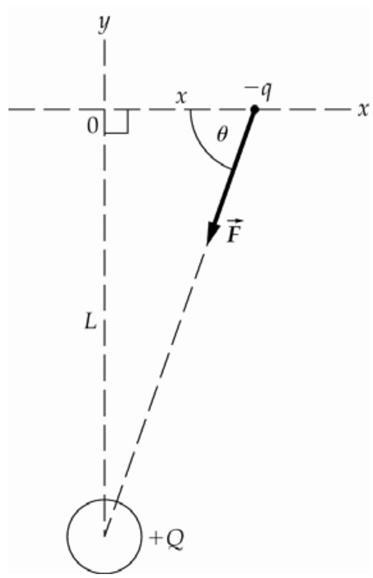
or

$$\boxed{\frac{d^2\Delta y}{dt^2} + \frac{2g}{y_0}\Delta y = 0}$$

the differential equation of simple harmonic motion with $\boxed{\omega = \sqrt{2g/y_0}}$.

86 •••

Picture the Problem The free-body diagram shows the Coulomb force the positive charge Q exerts on the bead that is constrained to move along the x axis. The x component of this force is a restoring force, i.e., it is directed toward the bead's equilibrium position. We can show that, for $x << L$, this restoring force is linear and, hence, that the bead will exhibit simple harmonic motion about its equilibrium position. Once we've obtained the differential equation of SHM we can relate the period of the motion to its angular frequency.



Using Coulomb's law for point charges, express the force F that $+Q$ exerts on $-q$:

$$F = \frac{k(-q)Q}{L^2 + x^2} = -\frac{kqQ}{L^2 + x^2}$$

Express the component of this force along the x axis:

$$F_x = -\frac{kqQ}{L^2 + x^2}\cos\theta$$

$$= -\frac{kqQ}{L^2 + x^2}\frac{x}{\sqrt{L^2 + x^2}}$$

$$= -\frac{kqQ}{\left(L^2 + x^2\right)^{3/2}}x$$

Factor L^2 from the denominator of this equation to obtain:

$$F_x = -\frac{kqQ}{L^3\left(1+\frac{x^2}{L^2}\right)^{3/2}}x \approx -\frac{kqQ}{L^3}x$$

because $x << L$.

Apply $\sum F_x = ma_x$ to the bead to obtain:

$$m\frac{d^2x}{dt^2} = -\frac{kqQ}{L^3}x$$

or

$$\frac{d^2x}{dt^2} + \frac{kqQ}{mL^3}x = 0$$

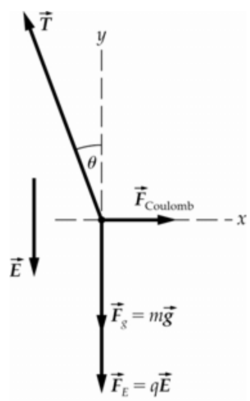
the differential equation of simple harmonic motion with $\omega = \sqrt{kqQ/mL^3}$.

Express the period of the motion of the bead in terms of the angular frequency of the motion:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{mL^3}{kqQ}} = \boxed{2\pi L\sqrt{\frac{mL}{kqQ}}}$$

87 •••

Picture the Problem Each sphere is in static equilibrium under the influence of the tension $\vec{T}$, the gravitational force $\vec{F}_g$, $\vec{F}_{Coulomb}$ and the force $\vec{F}_E$ exerted by the electric field. We can use Coulomb's law to relate the electric force to the charges on the spheres and their separation and the conditions for static equilibrium to relate these forces to the charge on each sphere.



(*a*)Apply the conditions for static equilibrium to the charged sphere:

$$\sum F_x = F_{Coulomb} - T\sin\theta = \frac{kq^2}{r^2} - T\sin\theta = 0$$

and

$$\sum F_y = T\cos\theta - mg - qE = 0$$

Eliminate T between these equations to obtain:

$$\tan\theta = \frac{kq^2}{(mg+qE)r^2}$$

Referring to the figure for Problem 78, relate the separation of the

$$r = 2L\sin\theta$$

spheres r to the length of the pendulum L:

Substitute to obtain:

$$\tan\theta = \frac{kq^2}{4(mg+qE)L^2\sin^2\theta}$$

or

$$\sin^2\theta\tan\theta = \frac{kq^2}{4(mg+qE)L^2} \qquad (1)$$

Substitute numerical values and evaluate $\sin^2\theta\tan\theta$ to obtain:

$$\sin^2\theta\tan\theta = 3.25\times10^{-3}$$

Because $\sin^2\theta\tan\theta << 1$:

$$\sin\theta \approx \tan\theta \approx \theta$$

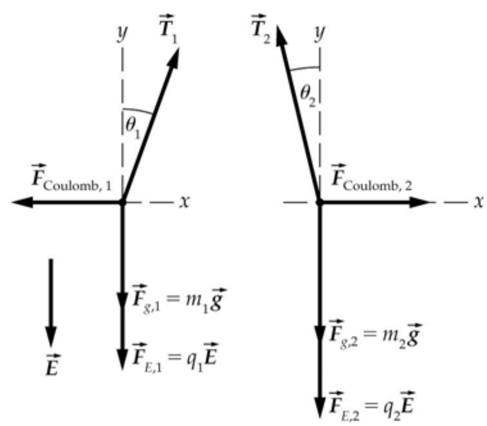
and

$$\theta^3 \approx 3.25\times10^{-3}$$

Solve for θ to obtain:

$$\theta = 0.148\,\text{rad} = \boxed{8.48°}$$

(b) The downward electrical forces acting on the two spheres are no longer equal. Let the mass of the sphere carrying the charge of 0.5 μC be m_1, and that of the sphere carrying the charge of 1.0 μC be m_2. The free-body diagrams show the tension, gravitational, and electrical forces acting on each sphere. Because we already know from part (a) that the angles are small, we can use the small-angle approximation $\sin\theta \approx \tan\theta \approx \theta$.



Apply the conditions for static equilibrium to the charged sphere whose mass is m_1:

$$\begin{aligned}\sum F_{x,1} &= -\frac{kq_1q_2}{r^2} + T_1\sin\theta_1 \\ &= -\frac{kq_1q_2}{(L\sin\theta_1 + L\sin\theta_2)^2} + T_1\sin\theta_1 \\ &\approx -\frac{kq_1q_2}{L^2(\theta_1+\theta_2)^2} + T_1\theta_1 \\ &= 0\end{aligned}$$

and

$$\sum F_{y,1} = T_{1,y} - m_1 g - q_1 E = 0$$

Apply the conditions for static equilibrium to the charged sphere whose mass is m_2:

$$\begin{aligned}\sum F_{x,2} &= \frac{kq_1q_2}{r^2} - T_2 \sin\theta_2 \\ &= \frac{kq_1q_2}{(L\sin\theta_1 + L\sin\theta_2)^2} + T_2 \sin\theta_2 \\ &\approx \frac{kq_1q_2}{L^2(\theta_1+\theta_2)^2} + T_2\theta_2 \\ &= 0\end{aligned}$$

and

$$\sum F_{y,2} = T_{2,y} - m_2 g - q_2 E = 0$$

Express θ_1 and θ_2 in terms of the components of $\vec{T}_1$ and $\vec{T}_2$:

$$\theta_1 = \frac{T_{1,x}}{T_{1,y}} \qquad (1)$$

and

$$\theta_2 = \frac{T_{2,x}}{T_{2,y}} \qquad (2)$$

Divide equation (1) by equation (2) to obtain:

$$\frac{\theta_1}{\theta_2} = \frac{\dfrac{T_{1,x}}{T_{1,y}}}{\dfrac{T_{2,x}}{T_{2,y}}} = \frac{T_{2,y}}{T_{1,y}}$$

because the horizontal components of $\vec{T}_1$ and $\vec{T}_2$ are equal.

Substitute for $T_{2,y}$ and $T_{1,y}$ to obtain:

$$\frac{\theta_1}{\theta_2} = \frac{m_2 g + q_2 E}{m_1 g + q_1 E}$$

Add equations (1) and (2) to obtain:

$$\theta_1 + \theta_2 = \frac{T_{1,x}}{T_{1,y}} + \frac{T_{2,x}}{T_{2,y}} = \frac{kq_1q_2}{L^2(\theta_1+\theta_2)^2}\left[\frac{1}{m_1 g + q_1 E} + \frac{1}{m_2 g + q_2 E}\right]$$

Solve for $\theta_1 + \theta_2$:

$$\theta_1 + \theta_2 = \sqrt[3]{\frac{kq_1q_2}{L^2}\left[\frac{1}{m_1 g + q_1 E} + \frac{1}{m_2 g + q_2 E}\right]}$$

Substitute numerical values and evaluate $\theta_1 + \theta_2$ and θ_1/θ_2:

$$\theta_1 + \theta_2 = 0.287\,\text{rad} = 16.4°$$

and

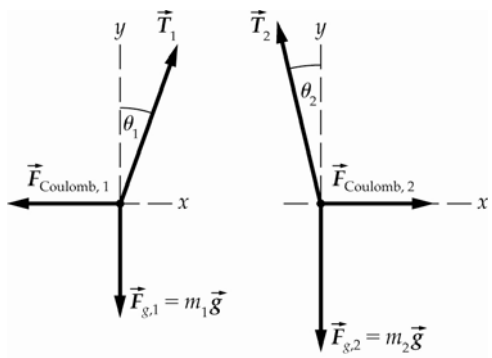
$$\frac{\theta_1}{\theta_2} = 1.34$$

Solve for θ_1 and θ_2 to obtain:

$$\theta_1 = \boxed{9.42°} \text{ and } \theta_1 = \boxed{6.98°}$$

88 •••

Picture the Problem Each sphere is in static equilibrium under the influence of a tension, gravitational and Coulomb force. Let the mass of the sphere carrying the charge of 2.0 μC be m_1 = 0.01 kg, and that of the sphere carrying the charge of 1.0 μC be m_2 = 0.02 kg. We can use Coulomb's law to relate the Coulomb force to the charge on each sphere and their separation and the conditions for static equilibrium to relate these forces to the charges on the spheres.



Apply the conditions for static equilibrium to the charged sphere whose mass is m_1:

$$\begin{aligned}\sum F_{x,1} &= -\frac{kq_1q_2}{r^2} + T_1 \sin\theta_1 \\ &= -\frac{kq_1q_2}{(L\sin\theta_1 + L\sin\theta_2)^2} + T_1\sin\theta_1 \\ &\approx -\frac{kq_1q_2}{L^2(\theta_1+\theta_2)^2} + T_1\theta_1 \\ &= 0\end{aligned}$$

and

$$\sum F_{y,1} = T_{1,y} - m_1 g = 0$$

Apply the conditions for static equilibrium to the charged sphere whose mass is m_2:

$$\begin{aligned}\sum F_{x,2} &= \frac{kq_1q_2}{r^2} - T_2 \sin\theta_2 \\ &= \frac{kq_1q_2}{(L\sin\theta_1 + L\sin\theta_2)^2} + T_2\sin\theta_2 \\ &\approx \frac{kq_1q_2}{L^2(\theta_1+\theta_2)^2} + T_2\theta_2 \\ &= 0\end{aligned}$$

and

$$\sum F_{y,2} = T_{2,y} - m_2 g = 0$$

Using the small-angle approximation $\sin\theta \approx \tan\theta \approx \theta$, express θ_1 and θ_2 in terms of the components of $\vec{T}_1$ and $\vec{T}_2$:

$$\theta_1 = \frac{T_{1,x}}{T_{1,y}} \qquad (1)$$

and

$$\theta_2 = \frac{T_{2,x}}{T_{2,y}} \qquad (2)$$

Divide equation (1) by equation (2) to obtain:

$$\frac{\theta_1}{\theta_2} = \frac{\dfrac{T_{1,x}}{T_{1,y}}}{\dfrac{T_{2,x}}{T_{2,y}}} = \frac{T_{2,y}}{T_{1,y}}$$

because the horizontal components of $\vec{T}_1$ and $\vec{T}_2$ are equal.

Substitute for $T_{2,y}$ and $T_{1,y}$ to obtain:

$$\frac{\theta_1}{\theta_2} = \frac{m_2}{m_1}$$

Add equations (1) and (2) to obtain:

$$\theta_1 + \theta_2 = \frac{T_{1,x}}{T_{1,y}} + \frac{T_{2,x}}{T_{2,y}} = \frac{kq_1q_2}{L^2(\theta_1 + \theta_2)^2}\left[\frac{1}{m_1 g} + \frac{1}{m_2 g}\right]$$

Solve for $\theta_1 + \theta_2$:

$$\theta_1 + \theta_2 = \sqrt[3]{\frac{kq_1q_2}{L^2}\left[\frac{1}{m_1 g} + \frac{1}{m_2 g}\right]}$$

Substitute numerical values and evaluate $\theta_1 + \theta_2$ and θ_1/θ_2:

$$\theta_1 + \theta_2 = 0.496\,\text{rad} = 28.4°$$

and

$$\frac{\theta_1}{\theta_2} = \frac{1}{2}$$

Solve for θ_1 and θ_2 to obtain:

$$\theta_1 = \boxed{9.47°} \text{ and } \theta_1 = \boxed{18.9°}$$

Remarks: While the small angle approximation is not as good here as it was in the preceding problems, the error introduced is less than 3%.

89 •••

Picture the Problem We can find the effective value of the gravitational field by finding the force on the bob due to $\vec{g}$ and $\vec{E}$ and equating this sum to the product of the mass of the bob and $\vec{g}'$. We can then solve this equation for $\vec{E}$ in terms of $\vec{g}$, $\vec{g}'$, q, and M and use the equation for the period of a simple pendulum to find the magnitude of $\vec{g}'$

Express the force on the bob due to $\vec{g}$ and $\vec{E}$:

$$\vec{F} = M\vec{g} + q\vec{E} = M\left(\vec{g} + \frac{q}{M}\vec{E}\right) = M\vec{g}'$$

where

$$\vec{g}' = \vec{g} + \frac{q}{M}\vec{E}$$

Solve for $\vec{E}$ to obtain:

$$\vec{E} = \frac{M}{q}\left(\vec{g}' - \vec{g}\right)$$

Using the expression for the period of a simple pendulum, find the magnitude of g':

$$T' = 2\pi\sqrt{\frac{L}{g'}}$$

and

$$g' = \frac{4\pi^2 L}{T^2} = \frac{4\pi^2(1\,\text{m})}{(1.2\,\text{s})^2} = 27.4\,\text{m/s}^2$$

Substitute numerical values and evaluate $\vec{E}$:

$$\vec{E} = \frac{5\times10^{-3}\,\text{kg}}{-8.0\,\mu\text{C}}\left[\left(27.4\,\text{m/s}^2\right)\hat{j} - \left(9.81\,\text{m/s}^2\right)\hat{j}\right] = \boxed{\left(-1.10\times10^4\,\text{N/C}\right)\hat{j}}$$

***90** •••

Picture the Problem We can relate the force of attraction that each molecule exerts on the other to the potential energy function of either molecule using $F = -dU/dx$. We can relate U to the electric field at either molecule due to the presence of the other through $U = -pE$. Finally, the electric field at either molecule is given by $E = 2kp/x^3$.

Express the force of attraction between the dipoles in terms of the spatial derivative of the potential energy function of p_1:

$$F = -\frac{dU_1}{dx} \qquad (1)$$

Express the potential energy of the dipole p_1:

$$U_1 = -p_1 E_1$$

where E_1 is the field at p_1 due to p_2.

Express the electric field at p_1 due to p_2:

$$E_1 = \frac{2kp_2}{x^3}$$

where x is the separation of the dipoles.

Substitute to obtain:

$$U_1 = -\frac{2kp_1 p_2}{x^3}$$

Substitute in equation (1) and differentiate with respect to x:

$$F = -\frac{d}{dx}\left[-\frac{2kp_1 p_2}{x^3}\right] = \frac{6kp_1 p_2}{x^4}$$

Evaluate F for $p_1 = p_2 = p$ and $x = d$ to obtain:

$$F = \boxed{\frac{6kp^2}{d^4}}$$

91 •••

Picture the Problem We can use Coulomb's law for the electric field due to a point charge and superposition of fields to find the electric field at any point on the y axis. By applying Newton's 2nd law, with the charge on the ring negative, we can show that the ring experiences a linear restoring force and, therefore, will execute simple harmonic motion. We can find ω from the differential equation of motion and use $f = \omega/2\pi$ to find the frequency of the motion.

(*a*) Use Coulomb's law for the electric field due to a point charge and superposition of fields, express the field at point P on the y axis:

$$\begin{aligned}\vec{E}_P &= \vec{E}_1 + \vec{E}_2 = \frac{kq_1}{r_{1,P}^2}\hat{r}_{1,P} + \frac{kq_2}{r_{2,P}^2}\hat{r}_{2,P} = \frac{kQ}{r_{1,P}^3}\vec{r}_{1,P} + \frac{kQ}{r_{2,P}^3}\vec{r}_{2,P} \\ &= \frac{kQ}{\left(a^2+y^2\right)^{3/2}}\left(\frac{L}{2}\hat{i} + y\hat{j}\right) + \frac{kQ}{\left(a^2+y^2\right)^{3/2}}\left(-\frac{L}{2}\hat{i} + y\hat{j}\right) \\ &= \boxed{\frac{2kQy}{\left(a^2+y^2\right)^{3/2}}\hat{j}}\end{aligned}$$

where $a = L/2$.

(*b*) Relate the force on the charged ring to its charge and the electric field:

$$\vec{F}_y = q\vec{E}_y = \boxed{\frac{2kqQy}{\left(a^2+y^2\right)^{3/2}}\hat{j}}$$

where q must be negative if $\vec{F}_y$ is to be a restoring force.

(*c*) Apply Newton's 2nd law to the ring to obtain:

$$m\frac{d^2y}{dt^2}=-\frac{2kqQ}{\left(a^2+y^2\right)^{3/2}}y$$

or

$$\frac{d^2y}{dt^2}=-\frac{2kqQ}{m\left(a^2+y^2\right)^{3/2}}y$$

Factor the radicand to obtain:

$$\frac{d^2y}{dt^2}=-\frac{2kqQ}{ma^3\left(1+\frac{y^2}{a^2}\right)^{3/2}}y$$

$$\approx -\frac{2kqQ}{ma^3}y=-\frac{16kqQ}{mL^3}y$$

provided $y << a = L/2$.

Thus we have:

$$\frac{d^2y}{dt^2}=-\frac{16kqQ}{mL^3}y$$

or

$$\boxed{\frac{d^2y}{dt^2}+\frac{16kqQ}{mL^3}y=0}$$

the differential equation of simple harmonic motion.

Express the frequency of the simple harmonic motion in terms of its angular frequency:

$$f=\frac{\omega}{2\pi}$$

From the differential equation describing the motion we have:

$$\omega^2=\frac{16kqQ}{mL^3}$$

and

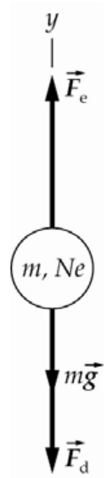
$$f=\frac{1}{2\pi}\sqrt{\frac{16kqQ}{mL^3}}$$

Substitute numerical values and evaluate f:

$$f=\frac{1}{2\pi}\sqrt{\frac{16\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)(5\,\mu\text{C})(2\,\mu\text{C})}{(0.03\,\text{kg})(0.24\,\text{m})^3}}=\boxed{9.37\,\text{Hz}}$$

92 •••

Picture the Problem The free body diagram shows the forces acting on the microsphere of mass m and having an excess charge of $q = Ne$ when the electric field is downward. Under terminal-speed conditions the sphere is in equilibrium under the influence of the electric force $\vec{F}_e$, its weight $m\vec{g}$, and the drag force $\vec{F}_d$. We can apply Newton's 2nd law, under terminal-speed conditions, to relate the number of excess charges N on the sphere to its mass and, using Stokes' law, find its terminal speed.



(*a*) Apply $\sum F_y = ma_y$ to the microsphere:

$$F_e - mg - F_d = ma_y$$

or, because $a_y = 0$,

$$F_e - mg - F_{d,\text{terminal}} = 0$$

Substitute for F_e, m, and $F_{d,\text{terminal}}$ to obtain:

$$qE - \rho Vg - 6\pi\eta r v_t = 0$$

or, because $q = Ne$,

$$NeE - \tfrac{4}{3}\pi r^3 \rho g - 6\pi\eta r v_t = 0$$

Solve for N to obtain:

$$N = \frac{\tfrac{4}{3}\pi r^3 \rho g + 6\pi\eta r v_t}{eE}$$

Substitute numerical values and evaluate $\tfrac{4}{3}\pi r^3 \rho g$:

$$\tfrac{4}{3}\pi r^3 \rho g = \tfrac{4}{3}\pi\left(5.5\times10^{-7}\text{ m}\right)^3 \times\left(1.05\times10^3\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right) = 7.18\times10^{-15}\text{ N}$$

Substitute numerical values and evaluate $6\pi\eta r v_t$:

$$6\pi\eta r v_t = 6\pi\left(1.8\times10^{-5}\text{ Pa}\cdot\text{s}\right)\left(5.5\times10^{-7}\text{ m}\right) \times\left(1.16\times10^{-4}\text{ m/s}\right) = 2.16\times10^{-14}\text{ N}$$

Substitute numerical values in equation (1) and evaluate N:

$$N = \frac{7.18\times10^{-15}\text{ N} + 2.16\times10^{-14}\text{ N}}{\left(1.6\times10^{-19}\text{ C}\right)\left(6\times10^4\text{ V/m}\right)} = \boxed{3}$$

(*b*) With the field pointing upward, the electric force is downward and the application of $\sum F_y = ma_y$ to

$$F_{d,\text{terminal}} - F_e - mg = 0$$

or

$$6\pi\eta r v_t - NeE - \tfrac{4}{3}\pi r^3 \rho g - = 0$$

the bead yields:

Solve for v_t to obtain:

$$v_t = \frac{NeE + \frac{4}{3}\pi r^3 \rho g}{6\pi\eta r}$$

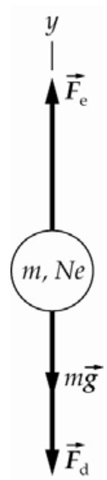
Substitute numerical values and evaluate v_t:

$$v_t = \frac{3\left(1.6\times10^{-19}\text{ C}\right)\left(6\times10^{4}\text{ V/m}\right)+\frac{4}{3}\pi\left(5.5\times10^{-7}\text{ m}\right)^3\left(1.05\times10^{3}\text{ kg/m}^3\right)\left(9.81\text{m/s}^2\right)}{6\pi\left(1.8\times10^{-5}\text{ Pa}\cdot\text{s}\right)\left(5.5\times10^{-7}\text{ m}\right)}$$

$$= \boxed{1.93\times10^{-4}\text{ m/s}}$$

***93 •••**

Picture the Problem The free body diagram shows the forces acting on the microsphere of mass *m* and having an excess charge of $q = Ne$ when the electric field is downward. Under terminal-speed conditions the sphere is in equilibrium under the influence of the electric force $\vec{F}_e$, its weight $m\vec{g}$, and the drag force $\vec{F}_d$. We can apply Newton's 2nd law, under terminal-speed conditions, to relate the number of excess charges *N* on the sphere to its mass and, using Stokes' law, to its terminal speed.



(*a*) Apply $\sum F_y = ma_y$ to the microsphere when the electric field is downward:

$$F_e - mg - F_d = ma_y$$

or, because $a_y = 0$,

$$F_e - mg - F_{d,\text{terminal}} = 0$$

Substitute for F_e and $F_{d,\text{terminal}}$ to obtain:

$$qE - mg - 6\pi\eta r v_u = 0$$

or, because $q = Ne$,

$$NeE - mg - 6\pi\eta r v_u = 0$$

Solve for v_u to obtain:

$$v_u = \frac{NeE - mg}{6\pi\eta r} \qquad (1)$$

With the field pointing upward, the electric force is downward and the application of $\sum F_y = ma_y$ to the microsphere yields:

$$F_{d,\text{terminal}} - F_e - mg = 0$$

or

$$6\pi\eta r v_d - NeE - mg = 0$$

Solve for v_d to obtain:

$$v_d = \frac{NeE + mg}{6\pi\eta r} \qquad (2)$$

Add equations (1) and (2) to obtain:

$$v = v_{\mathrm{u}} + v_{\mathrm{d}} = \frac{NeE - mg}{6\pi\eta r} + \frac{NeE + mg}{6\pi\eta r} = \frac{NeE}{3\pi\eta r} = \boxed{\frac{qE}{3\pi\eta r}}$$

This has the advantage that you don't need to know the mass of the microsphere.

(*b*) Letting Δv represent the change in the terminal speed of the microsphere due to a gain (or loss) of one electron we have:

$$\Delta v = v_{N+1} - v_N$$

Noting that Δv will be the same whether the microsphere is moving upward or downward, express its terminal speed when it is moving upward with *N* electronic charges on it:

$$v_N = \frac{NeE - mg}{6\pi\eta r}$$

Express its terminal speed upward when it has *N* + 1 electronic charges:

$$v_{N+1} = \frac{(N+1)eE - mg}{6\pi\eta r}$$

Substitute and simplify to obtain:

$$\Delta v_{N+1} = \frac{(N+1)eE - mg}{6\pi\eta r} - \frac{NeE - mg}{6\pi\eta r} = \frac{eE}{6\pi\eta r}$$

Substitute numerical values and evaluate Δv:

$$\Delta v = \frac{\left(1.6\times10^{-19}\ \mathrm{C}\right)\left(6\times10^{4}\ \mathrm{V/m}\right)}{6\pi\left(1.8\times10^{-5}\ \mathrm{Pa\cdot m}\right)\left(5.5\times10^{-7}\ \mathrm{m}\right)} = \boxed{5.15\times10^{-5}\ \mathrm{m/s}}$$