Chapter 22
The Electric Field 2: Continuous Charge Distributions

Conceptual Problems

***1** ••

(*a*) False. Gauss's law states that the net flux through any surface is given by $\phi_{net} = \oint_S E_n dA = 4\pi k Q_{inside}$. While it is true that Gauss's law is easiest to apply to symmetric charge distributions, it holds for *any* surface.

(*b*) True

2 ••

Determine the Concept Gauss's law states that the net flux through any surface is given by $\phi_{net} = \oint_S E_n dA = 4\pi k Q_{inside}$. To use Gauss's law the system must display some symmetry.

3 •••

Determine the Concept The electric field is that due to all the charges, inside and outside the surface. Gauss's law states that the net flux through any surface is given by $\phi_{net} = \oint_S E_n dA = 4\pi k Q_{inside}$. The lines of flux through a Gaussian surface begin on charges on one side of the surface and terminate on charges on the other side of the surface.

4 ••

Picture the Problem We can show that the charge inside a sphere of radius *r* is proportional to r^3 and that the area of a sphere is proportional to r^2. Using Gauss's law, we can show that the field must be proportional to $r^3/r^2 = r$.

Use Gauss's law to express the electric field inside a spherical charge distribution of constant volume charge density:

$$E = \frac{4\pi k Q_{inside}}{A}$$

where $A = 4\pi r^2$.

Express Q_{inside} as a function of ρ and r:

$$Q_{inside} = \rho V = \tfrac{4}{3}\pi \rho r^3$$

Substitute to obtain:

$$E = \frac{4\pi k \tfrac{4}{3}\pi \rho r^3}{4\pi r^2} = \boxed{\frac{4k\pi\rho}{3} r}$$

***5** •

(*a*) False. Consider a spherical shell, in which there is no charge, in the vicinity of an infinite sheet of charge. The electric field due to the infinite sheet would be non-zero everywhere on the spherical surface.

(*b*) True (assuming there are no charges inside the shell).

(*c*) True.

(*d*) False. Consider a spherical conducting shell. Such a surface will have equal charges on its inner and outer surfaces but, because their areas differ, so will their charge densities.

6 •

Determine the Concept Yes. The electric field on a closed surface is related to the net flux through it by Gauss's law: $\phi = \oint_S E\,dA = Q_{\text{inside}} / \epsilon_0$. If the net flux through the closed surface is zero, the net charge inside the surface must be zero by Gauss's law.

7 •

Determine the Concept The negative point charge at the center of the conducting shell induces a charge $+Q$ on the inner surface of the shell. $\boxed{(a)\text{ is correct.}}$

8 •

Determine the Concept The negative point charge at the center of the conducting shell induces a charge $+Q$ on the inner surface of the shell. Because a conductor does not have to be neutral, $\boxed{(d)\text{ is correct.}}$

***9** ••

Determine the Concept We can apply Gauss's law to determine the electric field for $r < R_1$ and $r > R_2$. We also know that the direction of an electric field at any point is determined by the direction of the electric force acting on a positively charged object located at that point.

From the application of Gauss's law we know that the electric field in both of these regions is not zero and is given by:

$$E_n = \frac{kQ}{r^2}$$

A positively charged object placed in either of these regions would experience an attractive force from the charge $-Q$ located at the center of the shell. $\boxed{(b)\text{ is correct.}}$

***10 ••**

Determine the Concept We can decide what will happen when the conducting shell is grounded by thinking about the distribution of charge on the shell before it is grounded and the effect on this distribution of grounding the shell.

The negative point charge at the center of the conducting shell induces a positive charge on the inner surface of the shell and a negative charge on the outer surface. Grounding the shell attracts positive charge from ground; resulting in the outer surface becoming electrically neutral. $\boxed{(b)\text{ is correct.}}$

11 ••

Determine the Concept We can apply Gauss's law to determine the electric field for $r < R_1$ and $r > R_2$. We also know that the direction of an electric field at any point is determined by the direction of the electric force acting on a positively charged object located at that point.

From the application of Gauss's law we know that the electric field in the region $r < R_1$ is given by $E_n = \frac{kQ}{r^2}$. A positively charged object placed in the region $r < R_1$ will experience an attractive force from the charge $-Q$ located at the center of the shell. With the conducting shell grounded, the net charge enclosed by a spherical Gaussian surface of radius $r > R_2$ is zero and hence the electric field in this region is zero.
$\boxed{(c)\text{ is correct.}}$

12 ••

Determine the Concept No. The electric field on a closed surface is related to the net flux through it by Gauss's law: $\phi = \oint_S E dA = Q_{\text{inside}}/\epsilon_0$. ϕ can be zero without E being zero everywhere. If the net flux through the closed surface is zero, the net charge inside the surface must be zero by Gauss's law.

13 ••

False. A physical quantity is discontinuous if its value on one side of a boundary differs from that on the other. We can show that this statement is false by citing a counterexample. Consider the field of a uniformly charged sphere. ρ is discontinuous at the surface, E is not.

Estimation and Approximation

***14 ••**

Picture the Problem We'll assume that the total charge is spread out uniformly (charge density = σ) in a thin layer at the bottom and top of the cloud and that the area of each surface of the cloud is 1 km^2. We can then use the definition of surface charge density and the expression for the electric field at the surface of a charged plane surface to estimate the total charge of the cloud.

Express the total charge Q of a thundercloud in terms of the surface area A of the cloud and the charge density σ:

$$Q = \sigma A$$

Express the electric field just outside the cloud:

$$E = \frac{\sigma}{\epsilon_0}$$

Solve for σ:

$$\sigma = \epsilon_0 E$$

Substitute for σ to obtain:

$$Q = \epsilon_0 EA$$

Substitute numerical values and evaluate Q:

$$Q = \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(3\times10^6\ \text{V/m}\right)\left(1\,\text{km}^2\right) = \boxed{26.6\,\text{C}}$$

Remarks: This charge is in reasonably good agreement with the total charge transferred in a lightning strike of approximately 30 C.

15 ••

Picture the Problem We'll assume that the field is strong enough to produce a spark. Then we know that field must be equal to the dielectric strength of air. We can then use the relationship between the field and the charge density to estimate the latter.

Suppose the field is large enough to produce a spark. Then:

$$E \approx \boxed{3\times10^6\ \text{V/m}}$$

Because rubbing the balloon leaves it with a surface charge density of $+\sigma$ and the hair with a surface charge density of $-\sigma$, the electric field between the balloon and the hair is:

$$E = \frac{\sigma}{2\epsilon_0}$$

Solve for σ:

$$\sigma = 2\epsilon_0 E$$

Substitute numerical values and evaluate σ:

$$\sigma = 2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(3\times10^6\ \text{V/m}\right) = \boxed{5.31\times10^{-5}\ \text{C/m}^2}$$

16 •

Picture the Problem For $x << r$, we can model the disk as an infinite plane. For $x >> r$, we can approximate the ring charge by a point charge.

For $x << r$, express the electric field near an infinite plane of charge:

$$E_x = 2\pi k\sigma$$

(*a*) and (*b*) Because E_x is independent of x for $x << r$:

$$E_x = 2\pi\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(3.6\ \mu\text{C/m}^2\right) = \boxed{2.03\times10^5\ \text{N/C}}$$

For $x >> r$, use Coulomb's law for the electric field due to a point charge to obtain:

$$E_x(x) = \frac{kQ}{x^2} = \frac{k\pi r^2\sigma}{x^2}$$

(*c*) Evaluate E_x at $x = 5$ m:

$$E_x(5\,\text{m}) = \frac{\pi\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2.5\,\text{cm})^2\left(3.6\ \mu\text{C/m}^2\right)}{(5\,\text{m})^2} = \boxed{2.54\,\text{N/C}}$$

(*d*) Evaluate E_x at $x = 5$ cm:

$$E_x(5\,\text{cm}) = \frac{\pi\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2.5\,\text{cm})^2\left(3.6\ \mu\text{C/m}^2\right)}{(0.05\,\text{m})^2} = \boxed{2.54\times10^4\ \text{N/C}}$$

Note that this is a very poor approximation because $x = 2r$ is not much greater than r.

Calculating $\vec{E}$ From Coulomb's Law

***17** •

Picture the Problem We can use the definition of λ to find the total charge of the line of charge and the expression for the electric field on the axis of a finite line of charge to evaluate E_x at the given locations along the x axis. In part (*d*) we can apply Coulomb's law for the electric field due to a point charge to approximate the electric field at $x = 250$ m.

(*a*) Use the definition of linear charge density to express Q in terms of λ:

$$Q = \lambda L = (3.5\,\text{nC/m})(5\,\text{m}) = \boxed{17.5\,\text{nC}}$$

Express the electric field on the axis of a finite line charge:

$$E_x(x_0) = \frac{kQ}{x_0(x_0 - L)}$$

(*b*) Substitute numerical values and evaluate E_x at x = 6 m:

$$E_x(6\,\text{m})=\frac{\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)(17.5\,\text{nC})}{(6\,\text{m})(6\,\text{m}-5\,\text{m})}$$
$$=\boxed{26.2\,\text{N/C}}$$

(*c*) Substitute numerical values and evaluate E_x at x = 9 m:

$$E_x(9\,\text{m})=\frac{\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)(17.5\,\text{nC})}{(9\,\text{m})(9\,\text{m}-5\,\text{m})}$$
$$=\boxed{4.37\,\text{N/C}}$$

(*d*) Substitute numerical values and evaluate E_x at x = 250 m:

$$E_x(250\,\text{m})=\frac{\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)(17.5\,\text{nC})}{(250\,\text{m})(250\,\text{m}-5\,\text{m})}=\boxed{2.57\,\text{mN/C}}$$

(*e*) Use Coulomb's law for the electric field due to a point charge to obtain:

$$E_x(x)=\frac{kQ}{x^2}$$

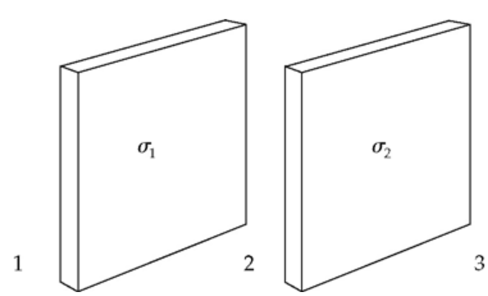
Substitute numerical values and evaluate E_x(250 m):

$$E_x(250\,\text{m})=\frac{\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)(17.5\,\text{nC})}{(250\,\text{m})^2}=\boxed{2.52\,\text{mN/C}}$$

Note that this result agrees to within 2% with the exact value obtained in (*d*).

18 •

Picture the Problem Let the charge densities on the two plates be σ_1 and σ_2 and denote the three regions of interest as 1, 2, and 3. Choose a coordinate system in which the positive x direction is to the right. We can apply the equation for $\vec{E}$ near an infinite plane of charge and the superposition of fields to find the field in each of the three regions.



(*a*) Use the equation for $\vec{E}$ near an infinite plane of charge to express the field in region 1 when $\sigma_1 = \sigma_2 = +3\ \mu\text{C/m}^2$:

$$\vec{E}_1 = \vec{E}_{\sigma_1} + \vec{E}_{\sigma_2} = -2\pi k\sigma_1\hat{i} - 2\pi k\sigma_2\hat{i} = -4\pi k\sigma\,\hat{i}$$

Substitute numerical values and evaluate $\vec{E}_1$:

$$\vec{E}_1 = -4\pi\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(3\ \mu\text{C/m}^2\right)\hat{i} = \boxed{-\left(3.39\times10^5\ \text{N/C}\right)\hat{i}}$$

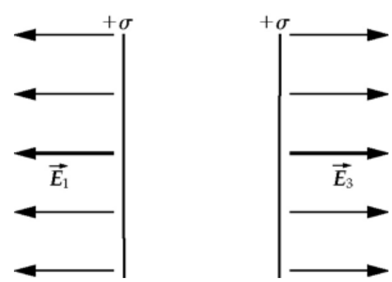
Proceed as above for region 2:

$$\vec{E}_2 = \vec{E}_{\sigma_1} + \vec{E}_{\sigma_2} = 2\pi k\sigma_1\hat{i} - 2\pi k\sigma_2\hat{i} = 2\pi k\sigma\hat{i} - 2\pi k\sigma\hat{i} = \boxed{0}$$

Proceed as above for region 3:

$$\vec{E}_3 = \vec{E}_{\sigma_1} + \vec{E}_{\sigma_2} = 2\pi k\sigma_1\hat{i} + 2\pi k\sigma_2\hat{i} = 4\pi k\sigma\hat{i} = 4\pi\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(3\ \mu\text{C/m}^2\right)\hat{i} = \boxed{\left(3.39\times10^5\ \text{N/C}\right)\hat{i}}$$

The electric field lines are shown to the right:



(*b*) Use the equation for $\vec{E}$ near an infinite plane of charge to express and evaluate the field in region 1 when $\sigma_1 = +3\ \mu\text{C/m}^2$ and $\sigma_2 = -3\ \mu\text{C/m}^2$:

$$\vec{E}_1 = \vec{E}_{\sigma_1} + \vec{E}_{\sigma_2} = 2\pi k\sigma_1\hat{i} - 2\pi k\sigma_2\hat{i} = 2\pi k\sigma\hat{i} - 2\pi k\sigma\hat{i} = \boxed{0}$$

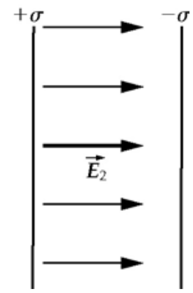
Proceed as above for region 2:

$$\vec{E}_1 = \vec{E}_{\sigma_1} + \vec{E}_{\sigma_2} = 2\pi k\sigma_1\hat{i} + 2\pi k\sigma_2\hat{i} = 4\pi k\sigma\hat{i} = 4\pi\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(3\ \mu\text{C}\right)\hat{i} = \boxed{\left(3.39\times10^5\ \text{N/C}\right)\hat{i}}$$

Proceed as above for region 3:

$$\vec{E}_3 = \vec{E}_{\sigma_1} + \vec{E}_{\sigma_2} = 2\pi k\sigma_1\hat{i} - 2\pi k\sigma_2\hat{i}$$
$$= 2\pi k\sigma\hat{i} - 2\pi k\sigma\hat{i} = \boxed{0}$$

The electric field lines are shown to the right:



19 •

Picture the Problem The magnitude of the electric field on the axis of a ring of charge is given by $E_x(x) = kQx/(x^2 + a^2)^{3/2}$ where Q is the charge on the ring and a is the radius of the ring. We can use this relationship to find the electric field on the x axis at the given distances from the ring.

Express $\vec{E}$ on the axis of a ring charge:

$$E_x(x) = \frac{kQx}{(x^2 + a^2)^{3/2}}$$

(a) Substitute numerical values and evaluate E_x for x = 1.2 cm:

$$E_x(1.2\,\text{cm}) = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.75\,\mu\text{C})(1.2\,\text{cm})}{\left[(1.2\,\text{cm})^2 + (8.5\,\text{cm})^2\right]^{3/2}} = \boxed{4.69\times10^5\ \text{N/C}}$$

(b) Proceed as in (a) with x = 3.6 cm:

$$E_x(3.6\,\text{cm}) = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.75\,\mu\text{C})(3.6\,\text{cm})}{\left[(3.6\,\text{cm})^2 + (8.5\,\text{cm})^2\right]^{3/2}} = \boxed{1.13\times10^6\ \text{N/C}}$$

(c) Proceed as in (a) with x = 4.0 m:

$$E_x(4\,\text{m}) = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.75\,\mu\text{C})(4\,\text{m})}{\left[(4\,\text{m})^2 + (8.5\,\text{cm})^2\right]^{3/2}} = \boxed{1.54\times10^3\ \text{N/C}}$$

(d) Using Coulomb's law for the electric field due to a point charge, express E_x:

$$E_x(x) = \frac{kQ}{x^2}$$

Substitute numerical values and evaluate E_x at x = 4.0 m:

$$E_x(4\,\text{m}) = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.75\,\mu\text{C})}{(4\,\text{m})^2} = \boxed{1.55\times10^3\ \text{N/C}}$$

This result agrees to within 1% with the result obtained in Part (c). It is slightly larger because the point charge is nearer $x = 4$ m than is the ring.

20 •

Picture the Problem We can use $E_x(x) = 2\pi k\sigma\left(1-\frac{x}{\sqrt{x^2+R^2}}\right)$, the expression for the electric field on the axis of a disk charge, to find E_x at x = 0.04 cm and 5 m.

Express the electric field on the axis of a disk charge:

$$E_x(x) = 2\pi k\sigma\left(1-\frac{x}{\sqrt{x^2+R^2}}\right)$$

(a) Evaluate this expression for x = 0.04 cm:

$$E_x = 2\pi(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.6\,\mu\text{C/m}^2)\left(1-\frac{0.04\,\text{cm}}{\sqrt{(0.04\,\text{cm})^2+(2.5\,\text{cm})^2}}\right)$$
$$= \boxed{2.00\times10^5\ \text{N/C}}$$

This value is about 1.5% smaller than the approximate value obtained in Problem 9.

(b) Proceed as in (a) for x = 5 m:

$$E_x = 2\pi(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.6\,\mu\text{C/m}^2)\left(1-\frac{5\,\text{m}}{\sqrt{(5\,\text{m})^2+(2.5\,\text{cm})^2}}\right) = \boxed{2.54\ \text{N/C}}$$

Note that the exact and approximate (from Problem 16) agree to within 1%.

21 •

Picture the Problem We can use the definition of λ to find the total charge of the line of charge and the expression for the electric field on the perpendicular bisector of a finite line of charge to evaluate E_y at the given locations along the y axis. In part (e) we can apply Coulomb's law for the electric field due to a point charge to approximate the electric field at y = 4.5 m.

(a) Use the definition of linear

$$Q = \lambda L = (6\,\text{nC/m})(5\,\text{cm}) = \boxed{0.300\,\text{nC}}$$

charge density to express Q in terms of λ:

Express the electric field on the perpendicular bisector of a finite line charge:

$$E_y(y)=\frac{2k\lambda}{y}\frac{\frac{1}{2}L}{\sqrt{\left(\frac{1}{2}L\right)^2+y^2}}$$

(*b*) Evaluate E_y at y = 4 cm:

$$E_y(4\,\text{cm})=\frac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}{0.04\,\text{m}}\frac{\frac{1}{2}(6\,\text{nC/m})(0.05\,\text{m})}{\sqrt{(0.025\,\text{m})^2+(0.04\,\text{m})^2}}=\boxed{1.43\,\text{kN/C}}$$

(*c*) Evaluate E_y at y = 12 cm:

$$E_y(12\,\text{cm})=\frac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}{0.12\,\text{m}}\frac{\frac{1}{2}(6\,\text{nC/m})(0.05\,\text{m})}{\sqrt{(0.025\,\text{m})^2+(0.12\,\text{m})^2}}=\boxed{183\,\text{N/C}}$$

(*d*) Evaluate E_y at y = 4.5 m:

$$E_y(4.5\,\text{m})=\frac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}{4.5\,\text{m}}\frac{\frac{1}{2}(6\,\text{nC/m})(0.05\,\text{m})}{\sqrt{(0.025\,\text{m})^2+(4.5\,\text{m})^2}}=\boxed{0.133\,\text{N/C}}$$

(*e*) Using Coulomb's law for the electric field due to a point charge, express E_y:

$$E_y(y)=\frac{kQ}{y^2}$$

Substitute numerical values and evaluate E_y at y = 4.5 m:

$$E_y(4.5\,\text{m})=\frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(0.3\,\text{nC})}{(4.5\,\text{m})^2}=\boxed{0.133\,\text{N/C}}$$

This result agrees to three decimal places with the value calculated in Part (*d*).

22 •

Picture the Problem The electric field on the axis of a disk charge is given by $E_x=2\pi kq\left(1-\frac{x}{\sqrt{x^2+a^2}}\right)$. We can equate this expression and $E_x=\frac{1}{2}\sigma/2\epsilon_0$ and solve for x.

Express the electric field on the axis of a disk charge:

$$E_x=2\pi kq\left(1-\frac{x}{\sqrt{x^2+a^2}}\right)$$

We're given that: $E_x = \frac{1}{2}\sigma/2\in_0$

Equate these expressions: $$\frac{\sigma}{4\varepsilon_0} = 2\pi k\sigma\left(1 - \frac{x}{\sqrt{x^2 + a^2}}\right)$$

Simplify to obtain: $$\frac{\sigma}{4\varepsilon_0} = 2\pi k\sigma\left(1 - \frac{x}{\sqrt{x^2 + a^2}}\right)$$

or, because $k = 1/4\pi\varepsilon_0$,

$$\frac{1}{2} = 1 - \frac{x}{\sqrt{x^2 + a^2}}$$

Solve for x to obtain: $$x = \boxed{\frac{a}{\sqrt{3}}}$$

23 •

Picture the Problem We can use $E_x = \dfrac{kQx}{\left(x^2 + a^2\right)^{3/2}}$ to find the electric field at the given distances from the center of the charged ring.

(*a*) Evaluate E_x at $x = 0.2a$:

$$E_x(0.2a) = \frac{kQ(0.2a)}{\left[(0.2a)^2 + a^2\right]^{3/2}} = \boxed{0.189\frac{kQ}{a^2}}$$

(*b*) Evaluate E_x at $x = 0.5a$:

$$E_x(0.5a) = \frac{kQ(0.5a)}{\left[(0.5a)^2 + a^2\right]^{3/2}} = \boxed{0.358\frac{kQ}{a^2}}$$

(*c*) Evaluate E_x at $x = 0.7a$:

$$E_x(0.7a) = \frac{kQ(0.7a)}{\left[(0.7a)^2 + a^2\right]^{3/2}} = \boxed{0.385\frac{kQ}{a^2}}$$

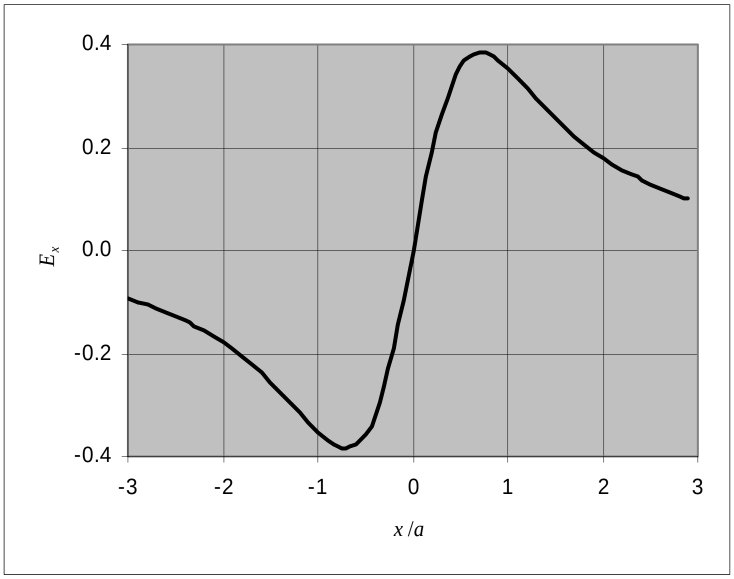
(*d*) Evaluate E_x at $x = a$:

$$E_x(a) = \frac{kQa}{\left[a^2 + a^2\right]^{3/2}} = \boxed{0.354\frac{kQ}{a^2}}$$

(*e*) Evaluate E_x at $x = 2a$:

$$E_x(2a) = \frac{2kQa}{\left[(2a)^2 + a^2\right]^{3/2}} = \boxed{0.179\frac{kQ}{a^2}}$$

The field along the *x* axis is plotted below. The *x* coordinates are in units of x/a and *E* is in units of kQ/a^2.



24 •

Picture the Problem We can use $E_x = 2\pi k\sigma\left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right)$, where *R* is the radius of the disk, to find the electric field on the axis of a disk charge.

Express E_x in terms of ε_0:

$$E_x = \frac{2\pi\sigma}{4\pi\in_0}\left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right) = \frac{\sigma}{2\in_0}\left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right)$$

(*a*) Evaluate E_x at $x = 0.2a$:

$$E_x(0.2a) = \frac{\sigma}{2\in_0}\left(1 - \frac{0.2a}{\sqrt{(0.2a)^2 + a^2}}\right) = \boxed{0.804\frac{\sigma}{2\in_0}}$$

(*b*) Evaluate E_x at $x = 0.5a$:

$$E_x(0.5a) = \frac{\sigma}{2\in_0}\left(1 - \frac{0.5a}{\sqrt{(0.5a)^2 + a^2}}\right)$$

$$= \boxed{0.553\frac{\sigma}{2\in_0}}$$

(*c*) Evaluate E_x at $x = 0.7a$:

$$E_x(0.7a) = \frac{\sigma}{2\in_0}\left(1 - \frac{0.7a}{\sqrt{(0.7a)^2 + a^2}}\right)$$

$$= \boxed{0.427\frac{\sigma}{2\in_0}}$$

(*d*) Evaluate E_x at $x = a$:

$$E_x(a) = \frac{\sigma}{2\in_0}\left(1 - \frac{a}{\sqrt{a^2 + a^2}}\right)$$

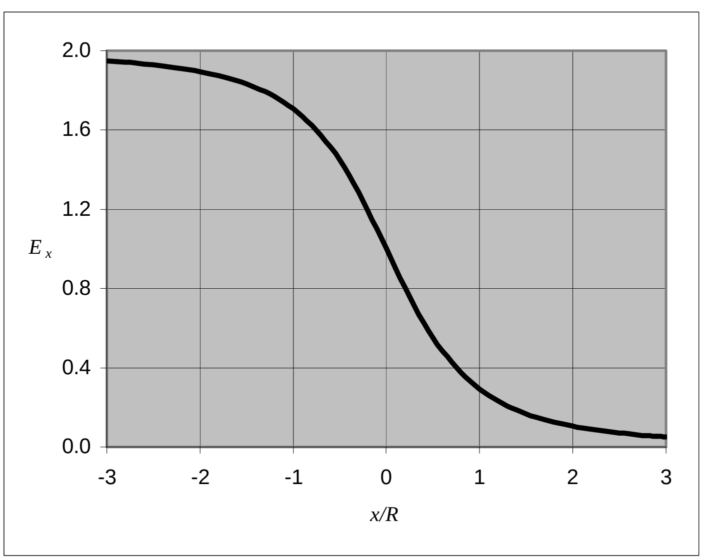
$$= \boxed{0.293\frac{\sigma}{2\in_0}}$$

(*e*) Evaluate E_x at $x = 2a$:

$$E_x(2a) = \frac{\sigma}{2\in_0}\left(1 - \frac{2a}{\sqrt{(2a)^2 + a^2}}\right)$$

$$= \boxed{0.106\frac{\sigma}{2\in_0}}$$

The field along the *x* axis is plotted below. The *x* coordinates are in units of *x*/*a* and *E* is in units of $\sigma/2\in_0$.



***25 ••**
Picture the Problem

(*a*) The electric field on the *x* axis of a disk of radius *r* carrying a surface charge density σ is given by:

$$E_x = 2\pi k\sigma\left(1 - \frac{x}{\sqrt{x^2 + r^2}}\right)$$

(*b*) The electric field due to an infinite sheet of charge density σ is independent of the distance from the plane and is given by:

$$E_{\text{plate}} = 2\pi k\sigma$$

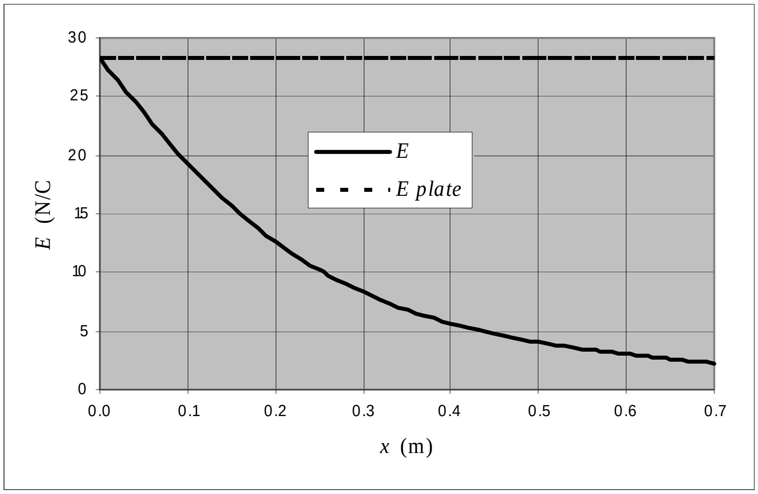
A spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Content/Formula	Algebraic Form
B3	9.00E+09	k
B4	5.00E–10	σ
B5	0.3	r
A8	0	x_0
A9	0.01	$x_0 + 0.01$
B8	2*PI()*B3*B4*(1–A8/(A8^2+B5^2)^2)^0.5)	$2\pi k\sigma\left(1 - \frac{x}{\sqrt{x^2 + r^2}}\right)$
C8	2*PI()*B3*B4	$2\pi k\sigma$

	A	B	C
1			
2			
3	k=	9.00E+09	Nm^2/C^2
4	sigma=	5.00E-10	C/m^2
5	r=	0.3	m
6			
7	x	E(x)	E plate
8	0.00	28.27	28.3
9	0.01	27.33	28.3
10	0.02	26.39	28.3
11	0.03	25.46	28.3
12	0.04	24.54	28.3
13	0.05	23.63	28.3
14	0.06	22.73	28.3
15	0.07	21.85	28.3
73	0.65	2.60	28.3
74	0.66	2.53	28.3
75	0.67	2.47	28.3
76	0.68	2.41	28.3
77	0.69	2.34	28.3

78	0.70	2.29	28.3

The following graph shows E as a function of x. The electric field from an infinite sheet with the same charge density is shown for comparison – the magnitude of the electric fields differ by more than 10 percent for $x = 0.03$ m.



26 ••

Picture the Problem Equation 22-10 expresses the electric field on the axis of a ring charge as a function of distance along the axis from the center of the ring. We can show that it has its maximum and minimum values at $x = +a/\sqrt{2}$ and $x = -a/\sqrt{2}$ by setting its first derivative equal to zero and solving the resulting equation for x. The graph of E_x will confirm that the maximum and minimum occur at these coordinates.

Express the variation of E_x with x on the axis of a ring charge:

$$E_x = \frac{kQx}{\left(x^2+a^2\right)^{3/2}}$$

Differentiate this expression with respect to x to obtain:

$$\frac{dE_x}{dx} = kQ\frac{d}{dx}\left[\frac{x}{\left(x^2+a^2\right)^{3/2}}\right] = kQ\frac{\left(x^2+a^2\right)^{3/2} - x\frac{d}{dx}\left(x^2+a^2\right)^{3/2}}{\left(x^2+a^2\right)^3}$$

$$= kQ\frac{\left(x^2+a^2\right)^{3/2} - x\left(\tfrac{3}{2}\right)\left(x^2+a^2\right)^{1/2}(2x)}{\left(x^2+a^2\right)^3} = kQ\frac{\left(x^2+a^2\right)^{3/2} - 3x^2\left(x^2+a^2\right)^{1/2}}{\left(x^2+a^2\right)^3}$$

Set this expression equal to zero for extrema and simplify:

$$\frac{\left(x^2+a^2\right)^{3/2} - 3x^2\left(x^2+a^2\right)^{1/2}}{\left(x^2+a^2\right)^3} = 0\,,$$

$$\left(x^2+a^2\right)^{3/2} - 3x^2\left(x^2+a^2\right)^{1/2} = 0\,,$$

and

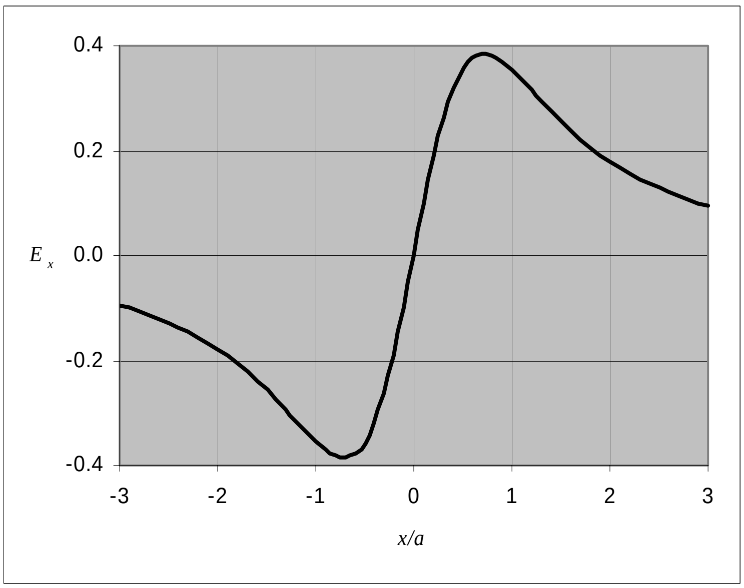
$$x^2 + a^2 - 3x^2 = 0$$

Solve for x to obtain:

$$x = \boxed{\pm\frac{a}{\sqrt{2}}}$$

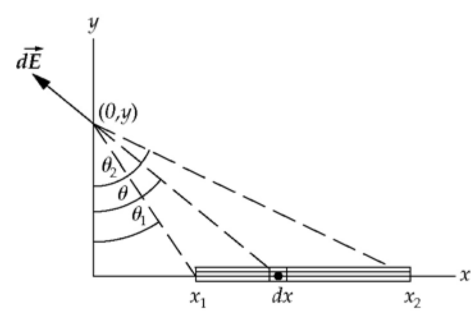
as our candidates for maxima or minima.

A plot of E, in units of kQ/a^2, versus x/a is shown to the right. This graph shows that E is a minimum at $x = -a/\sqrt{2}$ and a maximum at $x = a/\sqrt{2}$.



27 ••

Picture the Problem The line charge and point (0, y) are shown in the diagram. Also shown is a line element of length dx and the field $d\vec{E}$ its charge produces at (0, y). We can find dE_x from $d\vec{E}$ and then integrate from $x = x_1$ to $x = x_2$.



Express the x component of $d\vec{E}$:

$$dE_x = -\frac{k\lambda}{x^2 + y^2}\sin\theta dx = -\frac{k\lambda}{x^2 + y^2}\frac{x}{\sqrt{x^2 + y^2}}dx = -\frac{k\lambda x}{\left(x^2 + y^2\right)^{3/2}}dx$$

Integrate from $x = x_1$ to x_2 to obtain:

$$E_x = -k\lambda\int_{x_1}^{x_2}\frac{x}{\left(x^2+y^2\right)^{3/2}}dx = -k\lambda\left[-\frac{1}{\sqrt{x^2+y^2}}\right]_{x_1}^{x_2} = -k\lambda\left[-\frac{1}{\sqrt{x_2^2+y^2}}+\frac{1}{\sqrt{x_1^2+y^2}}\right] = -\frac{k\lambda}{y}\left[-\frac{y}{\sqrt{x_2^2+y^2}}+\frac{y}{\sqrt{x_1^2+y^2}}\right]$$

From the diagram we see that:

$$\cos\theta_2 = \frac{y}{\sqrt{x_2^2+y^2}} \text{ or } \theta_2 = \tan^{-1}\left(\frac{x_2}{y}\right)$$

and

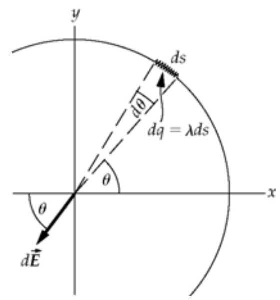
$$\cos\theta_1 = \frac{y}{\sqrt{x_1^2+y^2}} \text{ or } \theta_1 = \tan^{-1}\left(\frac{x_1}{y}\right)$$

Substitute to obtain:

$$E_x = -\frac{k\lambda}{y}\left[-\cos\theta_2+\cos\theta_1\right] = \boxed{\frac{k\lambda}{y}\left[\cos\theta_2-\cos\theta_1\right]}$$

28 ••

Picture the Problem The diagram shows a segment of the ring of length ds that has a charge $dq = \lambda ds$. We can express the electric field $d\vec{E}$ at the center of the ring due to the charge dq and then integrate this expression from $\theta = 0$ to 2π to find the magnitude of the field in the center of the ring.



(*a*) and (*b*) The field $d\vec{E}$ at the center of the ring due to the charge dq is:

$$d\vec{E} = d\vec{E}_x + d\vec{E}_y = -dE\cos\theta\,\hat{i} - dE\sin\theta\,\hat{j} \quad (1)$$

The magnitude dE of the field at the center of the ring is:

$$dE = \frac{kdq}{r^2}$$

Because $dq = \lambda ds$:

$$dE = \frac{k\lambda ds}{r^2}$$

The linear charge density varies with θ according to $\lambda(\theta) = \lambda_0 \sin\theta$:

$$dE = \frac{k\lambda_0 \sin\theta\, ds}{r^2}$$

Substitute $rd\theta$ for ds:

$$dE = \frac{k\lambda_0 \sin\theta\, r d\theta}{r^2} = \frac{k\lambda_0 \sin\theta\, d\theta}{r}$$

Substitute for dE in equation (1) to obtain:

$$d\vec{E} = -\frac{k\lambda_0 \sin\theta \cos\theta\, d\theta}{r}\hat{i} - \frac{k\lambda_0 \sin^2\theta\, d\theta}{r}\hat{j}$$

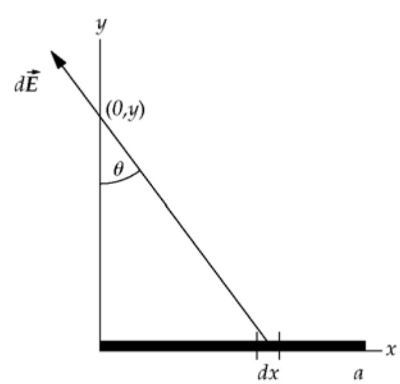
Integrate $d\vec{E}$ from $\theta = 0$ to 2π:

$$\vec{E} = -\frac{k\lambda_0}{2r}\int_0^{2\pi}\sin 2\theta\, d\theta\,\hat{i} - \frac{k\lambda_0}{r}\int_0^{2\pi}\sin^2\theta\, d\theta\,\hat{j} = 0 - \frac{\pi k\lambda_0}{r}\hat{j} = \boxed{-\frac{\pi k\lambda_0}{r}\hat{j}}$$

The field at the origin is in the negative y direction and its magnitude is $\frac{\pi k\lambda_0}{r}$.

29 ••

Picture the Problem The line charge and the point whose coordinates are (0, y) are shown in the diagram. Also shown is a segment of the line of length dx. The field that it produces at (0, y) is $d\vec{E}$. We can find dE_y from $d\vec{E}$ and then integrate from $x = 0$ to $x = a$ to find the y component of the electric field at a point on the y axis.



(a) Express the magnitude of the field $d\vec{E}$ due to charge dq of the

$$dE = \frac{kdq}{r^2}$$

element of length dx: where $r^2 = x^2 + y^2$

Because $dq = \lambda dx$:

$$dE = \frac{k\lambda dx}{x^2 + y^2}$$

Express the y component of dE:

$$dE_y = \frac{k\lambda}{x^2 + y^2}\cos\theta\, dx$$

Refer to the diagram to express $\cos\theta$ in terms of x and y:

$$\cos\theta = \frac{y}{\sqrt{x^2 + y^2}}$$

Substitute for $\cos\theta$ in the expression for dE_y to obtain:

$$dE_y = \frac{k\lambda y}{\left(x^2 + y^2\right)^{3/2}} dx$$

Integrate from $x = 0$ to $x = a$ and simplify to obtain:

$$E_y = k\lambda y \int_0^a \frac{1}{\left(x^2 + y^2\right)^{3/2}} dx$$

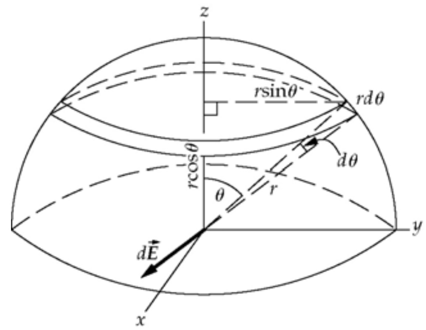
$$= k\lambda y \left[\frac{x}{y^2\sqrt{x^2 + y^2}}\right]_0^a$$

$$= k\lambda \left[\frac{a}{y\sqrt{a^2 + y^2}}\right]$$

$$= \boxed{\frac{k\lambda}{y}\frac{a}{\sqrt{a^2 + y^2}}}$$

***30 •••**

Picture the Problem Consider the ring with its axis along the z direction shown in the diagram. Its radius is $z = r\cos\theta$ and its width is $rd\theta$. We can use the equation for the field on the axis of a ring charge and then integrate to express the field at the center of the hemispherical shell.



Express the field on the axis of the ring charge:

$$dE = \frac{kzdq}{\left(r^2\sin^2\theta + r^2\cos^2\theta\right)^{3/2}}$$

$$= \frac{kzdq}{r^3}$$

where $z = r\cos\theta$

Express the charge dq on the ring:

$$dq = \sigma dA = \sigma(2\pi r\sin\theta)rd\theta = 2\pi\sigma r^2\sin\theta d\theta$$

Substitute to obtain:

$$dE = \frac{k(r\cos\theta)2\pi\sigma r^2\sin\theta d\theta}{r^3} = 2\pi k\sigma\sin\theta\cos\theta d\theta$$

Integrate dE from $\theta = 0$ to $\pi/2$ to obtain:

$$E = 2\pi k\sigma\int_0^{\pi/2}\sin\theta\cos\theta d\theta = 2\pi k\sigma\left[\tfrac{1}{2}\sin^2\theta\right]_0^{\pi/2} = \boxed{\pi k\sigma}$$

Gauss's Law

31 •

Picture the Problem The definition of electric flux is $\phi = \oint_S \vec{E}\cdot\hat{n}dA$. We can apply this definition to find the electric flux through the square in its two orientations.

(*a*) Apply the definition of ϕ to find the flux of the field when the square is parallel to the *yz* plane:

$$\phi = \oint_S(2\,\text{kN/C})\hat{i}\cdot\hat{i}dA = (2\,\text{kN/C})\oint_S dA = (2\,\text{kN/C})(0.1\,\text{m})^2 = \boxed{20.0\,\text{N}\cdot\text{m}^2/\text{C}}$$

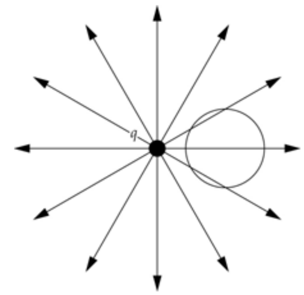
(*b*) Proceed as in (*a*) with $\hat{i}\cdot\hat{n} = \cos 30°$:

$$\phi = \oint_S(2\,\text{kN/C})\cos 30° dA = (2\,\text{kN/C})\cos 30°\oint_S dA = (2\,\text{kN/C})(0.1\,\text{m})^2\cos 30° = \boxed{17.3\,\text{N}\cdot\text{m}^2/\text{C}}$$

***32** •

Determine the Concept While the number of field lines that we choose to draw radially outward from q is arbitrary, we must show them originating at q and, in the absence of other charges, radially symmetric. The number of lines that we draw is, by agreement, in proportion to the magnitude of q.

(*a*) The sketch of the field lines and of the sphere is shown in the diagram to the right.



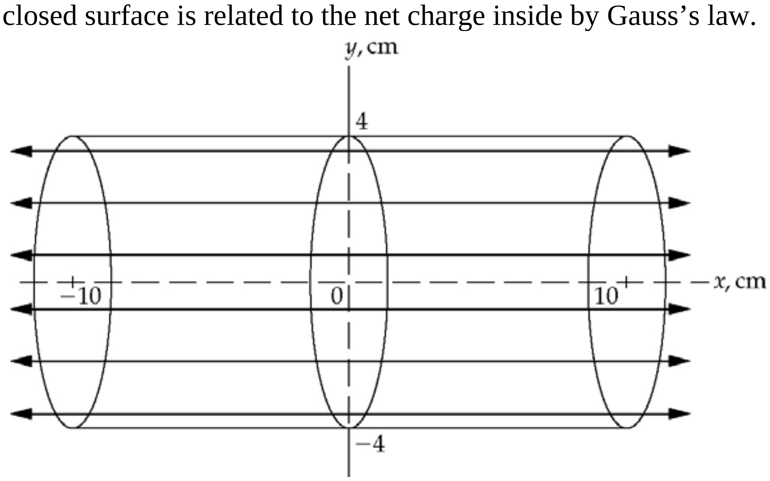
Given the number of field lines drawn from q, 3 lines enter the sphere. Had we chosen to draw 24 field lines, 6 would have entered the spherical surface.

(*b*) The net number of lines crossing the surface is zero.

(*c*) The net flux is zero.

33 •

Picture the Problem The field at both circular faces of the cylinder is parallel to the outward vector normal to the surface, so the flux is just EA. There is no flux through the curved surface because the normal to that surface is perpendicular to $\vec{E}$. The net flux through the closed surface is related to the net charge inside by Gauss's law.



(*a*) Use Gauss's law to calculate the flux through the right circular surface:

$$\phi_{\text{right}} = \vec{E}_{\text{right}} \cdot \hat{n}_{\text{right}} A = (300\text{ N/C})\hat{i} \cdot \hat{i}(\pi)(0.04\text{ m})^2 = \boxed{1.51\text{ N}\cdot\text{m}^2/\text{C}}$$

Apply Gauss's law to left circular surface:

$$\phi_{\text{left}} = \vec{E}_{\text{left}} \cdot \hat{n}_{\text{left}} A = (-300\text{ N/C})\hat{i} \cdot (-\hat{i})(\pi)(0.04\text{ m})^2 = \boxed{1.51\text{ N}\cdot\text{m}^2/\text{C}}$$

(*b*) Because the field lines are parallel to the curved surface of the cylinder:

$$\phi_{\text{curved}} = \boxed{0}$$

(*c*) Express and evaluate the net flux through the entire cylindrical surface:

$$\begin{aligned}\phi_{\text{net}} &= \phi_{\text{right}} + \phi_{\text{left}} + \phi_{\text{curved}} \\ &= 1.51\,\text{N}\cdot\text{m}^2/\text{C} + 1.51\,\text{N}\cdot\text{m}^2/\text{C} + 0 \\ &= \boxed{3.02\,\text{N}\cdot\text{m}^2/\text{C}}\end{aligned}$$

(*d*) Apply Gauss's law to obtain:

$$\phi_{\text{net}} = 4\pi k Q_{\text{inside}}$$

Solve for Q_{inside}:

$$Q_{\text{inside}} = \frac{\phi_{\text{net}}}{4\pi k}$$

Substitute numerical values and evaluate Q_{inside}:

$$\begin{aligned}Q_{\text{inside}} &= \frac{3.02\,\text{N}\cdot\text{m}^2/\text{C}}{4\pi\left(8.99\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2\right)} \\ &= \boxed{2.67\times10^{-11}\,\text{C}}\end{aligned}$$

34 •

Picture the Problem We can use Gauss's law in terms of ε_0 to find the net charge inside the box.

(*a*) Apply Gauss's law in terms of ε_0 to find the net charge inside the box:

$$\phi_{\text{net}} = \frac{1}{\epsilon_0} Q_{\text{inside}}$$

or

$$Q_{\text{inside}} = \epsilon_0\, \phi_{\text{net}}$$

Substitute numerical values and evaluate Q_{inside}:

$$\begin{aligned}Q_{\text{inside}} &= \left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(6\,\text{kN}\cdot\text{m}^2/\text{C}\right) \\ &= \boxed{5.31\times10^{-8}\,\text{C}}\end{aligned}$$

(*b*) You can only conclude that the net charge is zero. There may be an equal number of positive and negative charges present inside the box.

35 •

Picture the Problem We can apply Gauss's law to find the flux of the electric field through the surface of the sphere.

(*a*) Use the formula for the surface area of a sphere to obtain:

$$A = 4\pi r^2 = 4\pi(0.5\,\text{m})^2 = \boxed{3.14\,\text{m}^2}$$

(*b*) Apply Coulomb's law to express and evaluate *E*:

$$E = \frac{1}{4\pi \epsilon_0}\frac{q}{r^2} = \frac{1}{4\pi\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)}\frac{2\ \mu\text{C}}{(0.5\ \text{m})^2} = \boxed{7.19\times10^4\ \text{N/C}}$$

(*c*) Apply Gauss's law to obtain:

$$\phi = \oint_S \vec{E}\cdot\hat{n}dA = \oint_S EdA = \left(7.19\times10^4\ \text{N/C}\right)\left(3.14\ \text{m}^2\right) = \boxed{2.26\times10^5\ \text{N}\cdot\text{m}^2/\text{C}}$$

(*d*) No. The flux through the surface is independent of where the charge is located inside the sphere.

(*e*) Because the cube encloses the sphere, the flux through the surface of the sphere will also be the flux through the cube:

$$\phi_{\text{cube}} = \boxed{2.26\times10^5\ \text{N}\cdot\text{m}^2/\text{C}}$$

***36** •

Picture the Problem We'll define the flux of the gravitational field in a manner that is analogous to the definition of the flux of the electric field and then substitute for the gravitational field and evaluate the integral over the closed spherical surface.

Define the gravitational flux as:

$$\phi_g = \oint_S \vec{g}\cdot\hat{n}dA$$

Substitute for $\vec{g}$ and evaluate the integral to obtain:

$$\phi_g = \oint_S\left(-\frac{Gm}{r^2}\hat{r}\right)\cdot\hat{n}dA = -\frac{Gm}{r^2}\oint_S dA = \left(-\frac{Gm}{r^2}\right)\left(4\pi r^2\right) = \boxed{-4\pi Gm}$$

37 ••

Picture the Problem We'll let the square be one face of a cube whose side is 40 cm. Then the charge is at the center of the cube and we can apply Gauss's law in terms of ε_0 to find the flux through the square.

Apply Gauss's law to the cube to express the net flux:

$$\phi_{\text{net}} = \frac{1}{\epsilon_0}Q_{\text{inside}}$$

Express the flux through one face of the cube:

$$\phi_{\text{square}} = \frac{1}{6\in_0}Q_{\text{inside}}$$

Substitute numerical values and evaluate ϕ_{square}:

$$\phi_{\text{square}} = \frac{2\,\mu\text{C}}{6\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)} = \boxed{3.77\times10^{4}\ \text{N}\cdot\text{m}^2/\text{C}}$$

38 ••

Picture the Problem We can treat this portion of the earth's atmosphere as though it is a cylinder with cross-sectional area *A* and height *h*. Because the electric flux increases with altitude, we can conclude that there is charge inside the cylindrical region and use Gauss's law to find the charge and hence the charge density of the atmosphere in this region.

The definition of volume charge density is:

$$\rho = \frac{Q}{V}$$

Express the charge inside a cylinder of base area *A* and height *h* for a charge density ρ:

$$Q = \rho Ah$$

Taking upward to be the positive direction, apply Gauss's law to the charge in the cylinder:

$$Q = -\left(E_hA - E_0A\right)\in_0 = \left(E_0A - E_hA\right)\in_0$$

where we've taken our zero at 250 m above the surface of a flat earth.

Substitute to obtain:

$$\rho = \frac{\left(E_0A - E_hA\right)\in_0}{Ah} = \frac{\left(E_0 - E_h\right)\in_0}{h}$$

Substitute numerical values and evaluate ρ:

$$\rho = \frac{\left(150\ \text{N/C} - 170\ \text{N/C}\right)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)}{250\ \text{m}} = \boxed{-7.08\times10^{-13}\ \text{C/m}^3}$$

where we've been able to neglect the curvature of the earth because the maximum height of 400 m is approximately 0.006% of the radius of the earth.

Spherical Symmetry

39 •

Picture the Problem To find E_n in these three regions we can choose Gaussian surfaces of appropriate radii and apply Gauss's law. On each of these surfaces, E_r is constant and

Gauss's law relates E_r to the total charge inside the surface.

(*a*) Use Gauss's law to find the electric field in the region $r < R_1$:

$$\oint_S E_n\,dA = \frac{1}{\epsilon_0} Q_{\text{inside}}$$

and

$$E_{r<R_1} = \frac{Q_{\text{inside}}}{\epsilon_0\ A} = \boxed{0}$$

because $Q_{\text{inside}} = 0$.

Apply Gauss's law in the region $R_1 < r < R_2$:

$$E_{R_1<r<R_1} = \frac{q_1}{\epsilon_0\left(4\pi r^2\right)} = \boxed{\frac{kq_1}{r^2}}$$

Using Gauss's law, find the electric field in the region $r > R_2$:

$$E_{r>R_2} = \frac{q_1 + q_2}{\epsilon_0\left(4\pi r^2\right)} = \boxed{\frac{k\left(q_1 + q_2\right)}{r^2}}$$

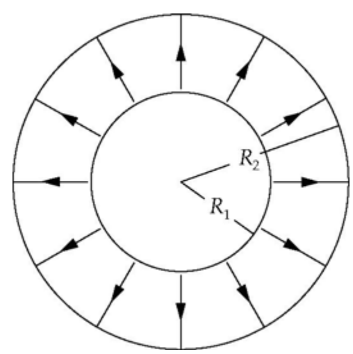
(*b*) Set $E_{r>R_2} = 0$ to obtain:

$$q_1 + q_2 = 0$$

or

$$\frac{q_1}{q_2} = \boxed{-1}$$

(*c*) The electric field lines for the situation in (*b*) with q_1 positive is shown to the right.



40 •

Picture the Problem We can use the definition of surface charge density and the formula for the area of a sphere to find the total charge on the shell. Because the charge is distributed uniformly over a spherical shell, we can choose a spherical Gaussian surface and apply Gauss's law to find the electric field as a function of the distance from the center of the spherical shell.

(*a*) Using the definition of surface charge density, relate the charge on the sphere to its area:

$$Q = \sigma A = 4\pi\sigma\, r^2$$

$$= 4\pi\left(9\,\text{nC/m}^2\right)\left(0.06\,\text{m}\right)^2$$

$$= \boxed{0.407\,\text{nC}}$$

Apply Gauss's law to a spherical surface of radius r that is concentric the spherical shell to obtain:

$$\oint_S E_n dA = \frac{1}{\epsilon_0} Q_{inside}$$

or

$$4\pi r^2 E_n = \frac{Q_{inside}}{\epsilon_0}$$

Solve for E_n:

$$E_n = \frac{Q_{inside}}{4\pi \epsilon_0} \frac{1}{r^2} = \frac{kQ_{inside}}{r^2}$$

(*b*) Q_{inside} a sphere whose radius is 2 cm is zero and hence:

$$E_n(2\,\text{cm}) = \boxed{0}$$

(*c*) Q_{inside} a sphere whose radius is 5.9 cm is zero and hence:

$$E_n(5.9\,\text{cm}) = \boxed{0}$$

(*d*) Q_{inside} a sphere whose radius is 6.1 cm is 0.407 nC and hence:

$$E_n(6.1\,\text{cm}) = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(0.407\,\text{nC})}{(0.061\,\text{m})^2} = \boxed{983\,\text{N/C}}$$

(*e*) Q_{inside} a sphere whose radius is 10 cm is 0.407 nC and hence:

$$E_n(10\,\text{cm}) = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(0.407\,\text{nC})}{(0.1\,\text{m})^2} = \boxed{366\,\text{N/C}}$$

41 ••

Picture the Problem We can use the definition of volume charge density and the formula for the volume of a sphere to find the total charge of the sphere. Because the charge is distributed uniformly throughout the sphere, we can choose a spherical Gaussian surface and apply Gauss's law to find the electric field as a function of the distance from the center of the sphere.

(*a*) Using the definition of volume charge density, relate the charge on the sphere to its volume:

$$Q = \rho V = \tfrac{4}{3}\pi\rho r^3 = \tfrac{4}{3}\pi(450\,\text{nC/m}^3)(0.06\,\text{m})^3 = \boxed{0.407\,\text{nC}}$$

Apply Gauss's law to a spherical surface of radius $r < R$ that is concentric with the spherical shell to obtain:

$$\oint_S E_n dA = \frac{1}{\epsilon_0} Q_{inside}$$

or

$$4\pi r^2 E_n = \frac{Q_{inside}}{\epsilon_0}$$

Solve for E_n:

$$E_n = \frac{Q_{inside}}{4\pi \epsilon_0}\frac{1}{r^2} = \frac{kQ_{inside}}{r^2}$$

Because the charge distribution is uniform, we can find the charge inside the Gaussian surface by using the definition of volume charge density to establish the proportion:

$$\frac{Q}{V} = \frac{Q_{inside}}{V'}$$

where V' is the volume of the Gaussian surface.

Solve for Q_{inside} to obtain:

$$Q_{inside} = Q\frac{V'}{V} = Q\frac{r^3}{R^3}$$

Substitute to obtain:

$$E_n(r < R) = \frac{Q_{inside}}{4\pi \epsilon_0}\frac{1}{r^2} = \frac{kQ}{R^3}r$$

(b) Evaluate E_n at r = 2 cm:

$$E_n(2\,\text{cm}) = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(0.407\,\text{nC})}{(0.06\,\text{m})^3}(0.02\,\text{m}) = \boxed{339\,\text{N/C}}$$

(c) Evaluate E_n at r = 5.9 cm:

$$E_n(5.9\,\text{cm}) = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(0.407\,\text{nC})}{(0.06\,\text{m})^3}(0.059\,\text{m}) = \boxed{999\,\text{N/C}}$$

Apply Gauss's law to the Gaussian surface with $r > R$:

$$4\pi r^2 E_n = \frac{Q_{inside}}{\varepsilon_0}$$

Solve for E_n to obtain:

$$E_n = \frac{kQ_{inside}}{r^2} = \frac{kQ}{r^2}$$

(d) Evaluate E_n at r = 6.1 cm:

$$E_n(6.1\,\text{cm}) = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(0.407\,\text{nC})}{(0.061\,\text{m})^2} = \boxed{983\,\text{N/C}}$$

(e) Evaluate E_n at r = 10 cm:

$$E_n(10\,\text{cm}) = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(0.407\,\text{nC})}{(0.1\,\text{m})^2} = \boxed{366\,\text{N/C}}$$

Note that, for $r > R$, these results are the same as those obtained for in Problem 40 for a uniform charge distribution on a spherical shell. This agreement is a consequence of the choices of σ and ρ so that the total charges on the two spheres is the same.

***42** ••

Determine the Concept The charges on a conducting sphere, in response to the repulsive Coulomb forces each experiences, will separate until electrostatic equilibrium conditions exit. The use of a wire to connect the two spheres or to ground the outer sphere will cause additional redistribution of charge.

(*a*) Because the outer sphere is conducting, the field in the thin shell must vanish. Therefore, $-2Q$, uniformly distributed, resides on the inner surface, and $-5Q$, uniformly distributed, resides on the outer surface.

(*b*) Now there is no charge on the inner surface and $-5Q$ on the outer surface of the spherical shell. The electric field just outside the surface of the inner sphere changes from a finite value to zero.

(*c*) In this case, the $-5Q$ is drained off, leaving no charge on the outer surface and $-2Q$ on the inner surface. The total charge on the outer sphere is then $-2Q$.

43 ••

Picture the Problem By symmetry; the electric field must be radial. To find E_r inside the sphere we choose a spherical Gaussian surface of radius $r < R$. On this surface, E_r is constant. Gauss's law then relates E_r to the total charge inside the surface.

Apply Gauss's law to a spherical surface of radius $r < R$ that is concentric with the nonconducting sphere to obtain:

$$\oint_S E_r\,dA = \frac{1}{\epsilon_0}Q_{\text{inside}}$$

or

$$4\pi r^2 E_r = \frac{Q_{\text{inside}}}{\epsilon_0}$$

Solve for E_r:

$$E_r = \frac{Q_{\text{inside}}}{4\pi\epsilon_0}\frac{1}{r^2} = \frac{kQ_{\text{inside}}}{r^2}$$

Use the definition of charge density to relate Q_{inside} to ρ and the volume defined by the Gaussian surface:

$$Q_{\text{inside}} = \rho V_{\text{Gaussian surface}} = \tfrac{4}{3}\rho\pi r^3$$

Substitute to obtain:

$$E_r(r<R)=\frac{\frac{4}{3}\rho\pi k r^3}{r^2}=\frac{4}{3}\rho\pi k r$$

Substitute numerical values and evaluate E_r at $r = 0.5R = 0.05$ m:

$$E_r(0.05\,\text{m})=\tfrac{4}{3}\pi\left(2\,\text{nC/m}^3\right)\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(0.05\,\text{m})=\boxed{3.77\ \text{N/C}}$$

44 ••

Picture the Problem We can find the total charge on the sphere by expressing the charge dq in a spherical shell and integrating this expression between $r = 0$ and $r = R$. By symmetry, the electric fields must be radial. To find E_r inside the charged sphere we choose a spherical Gaussian surface of radius $r < R$. To find E_r outside the charged sphere we choose a spherical Gaussian surface of radius $r > R$. On each of these surfaces, E_r is constant. Gauss's law then relates E_r to the total charge inside the surface.

(*a*) Express the charge dq in a shell of thickness dr and volume $4\pi r^2\,dr$:

$$dq=4\pi r^2\rho dr=4\pi r^2(Ar)dr$$
$$=4\pi Ar^3dr$$

Integrate this expression from $r = 0$ to R to find the total charge on the sphere:

$$Q=4\pi A\int_0^R r^3dr=\left[\pi Ar^4\right]_0^R=\boxed{\pi AR^4}$$

(*b*) Apply Gauss's law to a spherical surface of radius $r > R$ that is concentric with the nonconducting sphere to obtain:

$$\oint_S E_r dA=\frac{1}{\epsilon_0}Q_{\text{inside}}$$

or

$$4\pi r^2E_r=\frac{Q_{\text{inside}}}{\epsilon_0}$$

Solve for E_r:

$$E_r(r>R)=\frac{Q_{\text{inside}}}{4\pi\epsilon_0}\frac{1}{r^2}=\frac{kQ_{\text{inside}}}{r^2}$$
$$=\frac{kA\pi R^4}{r^2}=\boxed{\frac{AR^4}{4\epsilon_0 r^2}}$$

Apply Gauss's law to a spherical surface of radius $r < R$ that is concentric with the nonconducting sphere to obtain:

$$\oint_S E_r dA=\frac{1}{\epsilon_0}Q_{\text{inside}}$$

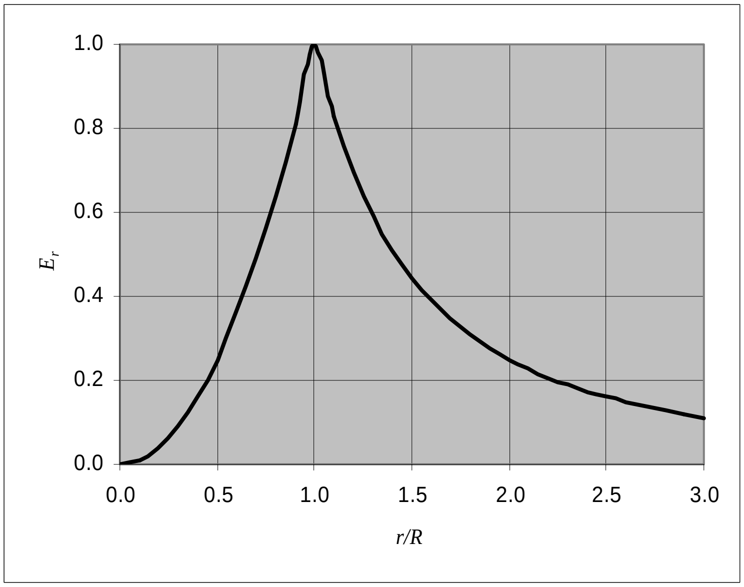
or

$$4\pi r^2E_r=\frac{Q_{\text{inside}}}{\epsilon_0}$$

Solve for E_r:

$$E_r(r<R)=\frac{Q_{\text{inside}}}{4\pi r^2\in_0}=\frac{\pi Ar^4}{4\pi r^2\in_0}=\boxed{\frac{Ar^2}{4\in_0}}$$

The graph of E_r versus r/R, with E_r in units of $A/4\in_0$, was plotted using a spreadsheet program.



Remarks: Note that the results for (*a*) and (*b*) agree at *r* = *R*.

45 ••

Picture the Problem We can find the total charge on the sphere by expressing the charge dq in a spherical shell and integrating this expression between $r = 0$ and $r = R$. By symmetry, the electric fields must be radial. To find E_r inside the charged sphere we choose a spherical Gaussian surface of radius $r < R$. To find E_r outside the charged sphere we choose a spherical Gaussian surface of radius $r > R$. On each of these surfaces, E_r is constant. Gauss's law then relates E_r to the total charge inside the surface.

(*a*) Express the charge dq in a shell of thickness dr and volume $4\pi r^2\,dr$:

$$dq=4\pi r^2\rho dr=4\pi r^2\frac{B}{r}dr$$
$$=4\pi Brdr$$

Integrate this expression from $r = 0$ to R to find the total charge on the sphere:

$$Q==4\pi B\int_0^R rdr=\left[2\pi Br^2\right]_0^R$$
$$=\boxed{2\pi BR^2}$$

(*b*) Apply Gauss's law to a spherical surface of radius $r > R$ that is concentric with the nonconducting sphere to obtain:

$$\oint_S E_r dA = \frac{1}{\in_0} Q_{\text{inside}}$$

or

$$4\pi r^2 E_r = \frac{Q_{\text{inside}}}{\in_0}$$

Solve for E_r:

$$E_r(r > R) = \frac{Q_{\text{inside}}}{4\pi \in_0} \frac{1}{r^2} = \frac{kQ_{\text{inside}}}{r^2}$$

$$= \frac{k2\pi BR^2}{r^2} = \boxed{\frac{BR^2}{2\in_0 r^2}}$$

Apply Gauss's law to a spherical surface of radius $r < R$ that is concentric with the nonconducting sphere to obtain:

$$\oint_S E_r dA = \frac{1}{\in_0} Q_{\text{inside}}$$

or

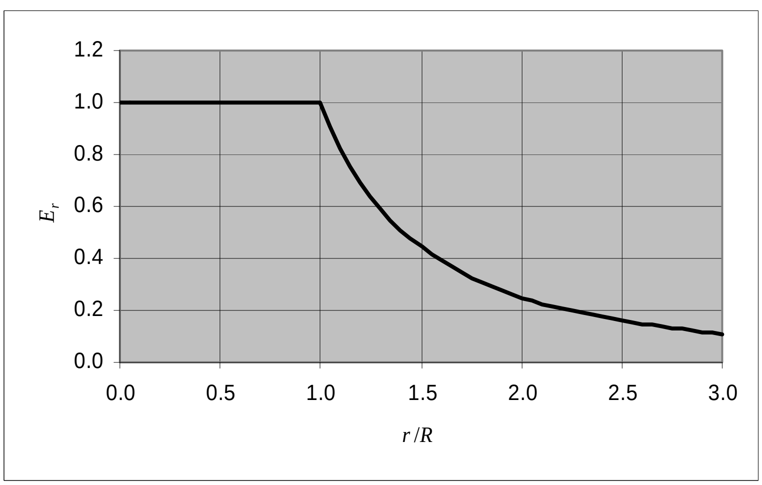
$$4\pi r^2 E_r = \frac{Q_{\text{inside}}}{\in_0}$$

Solve for E_r:

$$E_r(r < R) = \frac{Q_{\text{inside}}}{4\pi r^2 \in_0} = \frac{2\pi B r^2}{4\pi r^2 \in_0}$$

$$= \boxed{\frac{B}{2\in_0}}$$

The graph of E_r versus r/R, with E_r in units of $B/2\in_0$, was plotted using a spreadsheet program.



Remarks: Note that our results for (*a*) and (*b*) agree at $r = R$.

***46** ••

Picture the Problem We can find the total charge on the sphere by expressing the charge dq in a spherical shell and integrating this expression between $r = 0$ and $r = R$. By symmetry, the electric fields must be radial. To find E_r inside the charged sphere we choose a spherical Gaussian surface of radius $r < R$. To find E_r outside the charged sphere we choose a spherical Gaussian surface of radius $r > R$. On each of these surfaces, E_r is constant. Gauss's law then relates E_r to the total charge inside the surface.

(*a*) Express the charge dq in a shell of thickness dr and volume $4\pi r^2\,dr$:

$$dq = 4\pi r^2 \rho dr = 4\pi r^2 \frac{C}{r^2} dr = 4\pi C dr$$

Integrate this expression from $r = 0$ to R to find the total charge on the sphere:

$$Q = 4\pi C \int_0^R dr = \left[4\pi C r\right]_0^R = \boxed{4\pi CR}$$

(*b*) Apply Gauss's law to a spherical surface of radius $r > R$ that is concentric with the nonconducting sphere to obtain:

$$\oint_S E_r dA = \frac{1}{\epsilon_0} Q_{\text{inside}}$$

or

$$4\pi r^2 E_r = \frac{Q_{\text{inside}}}{\epsilon_0}$$

Solve for E_r:

$$E_r(r > R) = \frac{Q_{\text{inside}}}{4\pi \epsilon_0} \frac{1}{r^2} = \frac{kQ_{\text{inside}}}{r^2} = \frac{k 4\pi CR}{r^2} = \boxed{\frac{CR}{\epsilon_0 r^2}}$$

Apply Gauss's law to a spherical surface of radius $r < R$ that is concentric with the nonconducting sphere to obtain:

$$\oint_S E_r dA = \frac{1}{\epsilon_0} Q_{\text{inside}}$$

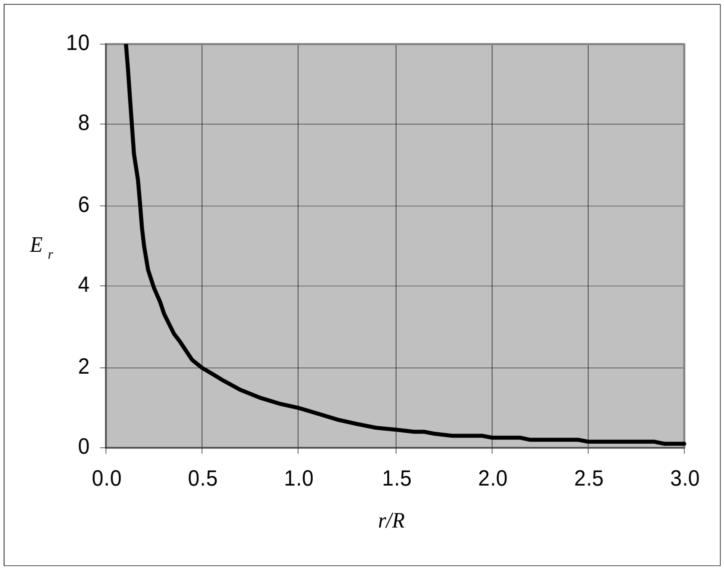
or

$$4\pi r^2 E_r = \frac{Q_{\text{inside}}}{\epsilon_0}$$

Solve for E_r:

$$E_r(r < R) = \frac{Q_{\text{inside}}}{4\pi r^2 \epsilon_0} = \frac{4\pi C r}{4\pi r^2 \epsilon_0} = \boxed{\frac{C}{\epsilon_0 r}}$$

The graph of E_r versus r/R, with E_r in units of $C/\epsilon_0 R$, was plotted using a spreadsheet program.



47 •••

Picture the Problem By symmetry, the electric fields resulting from this charge distribution must be radial. To find E_r for $r < a$ we choose a spherical Gaussian surface of radius $r < a$. To find E_r for $a < r < b$ we choose a spherical Gaussian surface of radius $a < r < b$. To find E_r for $r > b$ we choose a spherical Gaussian surface of radius $r > b$. On each of these surfaces, E_r is constant. Gauss's law then relates E_r to the total charge inside the surface.

(*a*), (*b*) Apply Gauss's law to a spherical surface of radius *r* that is concentric with the nonconducting spherical shell to obtain:

$$\oint_S E_r dA = \frac{1}{\epsilon_0} Q_{\text{inside}}$$

or

$$4\pi r^2 E_r = \frac{Q_{\text{inside}}}{\epsilon_0}$$

Solve for E_r:

$$E_r(r) = \frac{Q_{\text{inside}}}{4\pi \epsilon_0} \frac{1}{r^2} = \frac{kQ_{\text{inside}}}{r^2}$$

Evaluate $E_r(r < a)$:

$$E_r(r < a) = \frac{Q_{\text{inside}}}{4\pi \epsilon_0} \frac{1}{r^2} = \frac{kQ_{\text{inside}}}{r^2} = \boxed{0}$$

because $\rho(r < a) = 0$ and, therefore, $Q_{\text{inside}} = 0$.

Integrate dq from $r = a$ to r to find the total charge in the spherical shell in the interval $a < r < b$:

$$Q_{\text{inside}} = 4\pi\rho\int_a^r r'^2\, dr' = \left[\frac{4\pi C r'^3}{3}\right]_a^r$$

$$= \frac{4\pi\rho}{3}\left(r^3 - a^3\right)$$

Evaluate $E_r(a < r < b)$:

$$E_r(a < r < b) = \frac{kQ_{\text{inside}}}{r^2}$$

$$= \frac{4\pi k\rho}{3r^2}\left(r^3 - a^3\right)$$

$$= \boxed{\frac{\rho}{3\in_0 r^2}\left(r^3 - a^3\right)}$$

For $r > b$:

$$Q_{\text{inside}} = \frac{4\pi\rho}{3}\left(b^3 - a^3\right)$$

and

$$E_r(r > b) = \frac{4\pi k\rho}{3r^2}\left(b^3 - a^3\right)$$

$$= \boxed{\frac{\rho}{3\in_0 r^2}\left(b^3 - a^3\right)}$$

Remarks: Note that E is continuous at $r = b$.

Cylindrical Symmetry

48 ••

Picture the Problem From symmetry, the field in the tangential direction must vanish. We can construct a Gaussian surface in the shape of a cylinder of radius r and length L and apply Gauss's law to find the electric field as a function of the distance from the centerline of the infinitely long, uniformly charged cylindrical shell.

Apply Gauss's law to the cylindrical surface of radius r and length L that is concentric with the infinitely long, uniformly charged cylindrical shell:

$$\oint_S E_n\, dA = \frac{1}{\in_0} Q_{\text{inside}}$$

or

$$2\pi r L E_n = \frac{Q_{\text{inside}}}{\in_0}$$

where we've neglected the end areas because no flux crosses them.

Solve for E_n:

$$E_n = \frac{Q_{\text{inside}}}{2\pi r L \in_0} = \frac{2kQ_{\text{inside}}}{Lr}$$

For $r < R$, $Q_{\text{inside}} = 0$ and:

$$E_n(r < R) = \boxed{0}$$

For $r > R$, $Q_{\text{inside}} = \lambda L$ and:

$$E_n(r > R) = \frac{2k\lambda L}{Lr} = \frac{2k\lambda}{r} = \frac{2k(2\pi R\sigma)}{r} = \boxed{\frac{R\sigma}{\epsilon_0 r}}$$

49 ••

Picture the Problem We can use the definition of surface charge density to find the total charge on the shell. From symmetry, the electric field in the tangential direction must vanish. We can construct a Gaussian surface in the shape of a cylinder of radius *r* and length *L* and apply Gauss's law to find the electric field as a function of the distance from the centerline of the uniformly charged cylindrical shell.

(*a*) Using its definition, relate the surface charge density to the total charge on the shell:

$$Q = \sigma A = 2\pi RL\sigma$$

Substitute numerical values and evaluate Q:

$$Q = 2\pi(0.06\,\text{m})(200\,\text{m})(9\,\text{nC/m}^2) = \boxed{679\,\text{nC}}$$

(*b*) From Problem 48 we have, for $r = 2$ cm:

$$E(2\,\text{cm}) = \boxed{0}$$

(*c*) From Problem 48 we have, for $r = 5.9$ cm:

$$E(5.9\,\text{cm}) = \boxed{0}$$

(*d*) From Problem 48 we have, for $r = 6.1$ cm:

$$E_r = \frac{\sigma R}{\epsilon_0 r}$$

and

$$E(6.1\,\text{cm}) = \frac{(9\,\text{nC/m}^2)(0.06\,\text{m})}{(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(0.061\,\text{m})} = \boxed{1.00\,\text{kN/C}}$$

(*e*) From Problem 48 we have, for $r = 10$ cm:

$$E(10\,\text{cm}) = \frac{(9\,\text{nC/m}^2)(0.06\,\text{m})}{(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(0.1\,\text{m})} = \boxed{610\,\text{N/C}}$$

50 ••

Picture the Problem From symmetry, the field tangent to the surface of the cylinder must vanish. We can construct a Gaussian surface in the shape of a cylinder of radius *r* and length *L* and apply Gauss's law to find the electric field as a function of the distance from the centerline of the infinitely long nonconducting cylinder.

Apply Gauss's law to a cylindrical surface of radius *r* and length *L* that is concentric with the infinitely long nonconducting cylinder:

$$\oint_S E_n dA = \frac{1}{\in_0} Q_{\text{inside}}$$

or

$$2\pi r L E_n = \frac{Q_{\text{inside}}}{\in_0}$$

where we've neglected the end areas because no flux crosses them.

Solve for E_n:

$$E_n = \frac{Q_{\text{inside}}}{2\pi r L \varepsilon_0} = \frac{2kQ_{\text{inside}}}{Lr}$$

Express Q_{inside} for $r < R$:

$$Q_{\text{inside}} = \rho(r)V = \rho_0\left(\pi r^2 L\right)$$

Substitute to obtain:

$$E_n(r < R) = \frac{2k\left(\pi\rho_0 L r^2\right)}{Lr} = \boxed{\frac{\rho_0}{2\in_0} r}$$

or, because $\lambda = \rho\pi R^2$

$$E_n(r < R) = \boxed{\frac{\lambda}{2\pi \in_0 R^2} r}$$

Express Q_{inside} for $r > R$:

$$Q_{\text{inside}} = \rho(r)V = \rho_0\left(\pi R^2 L\right)$$

Substitute to obtain:

$$E_n(r > R) = \frac{2k\left(\pi\rho_0 L R^2\right)}{Lr} = \boxed{\frac{\rho_0 R^2}{2\in_0 r}}$$

or, because $\lambda = \rho\pi R^2$

$$E_n(r > R) = \boxed{\frac{\lambda}{2\pi \in_0 r}}$$

51 ••

Picture the Problem We can use the definition of volume charge density to find the total charge on the cylinder. From symmetry, the electric field tangent to the surface of the cylinder must vanish. We can construct a Gaussian surface in the shape of a cylinder of radius *r* and length *L* and apply Gauss's law to find the electric field as a function of the distance from the centerline of the uniformly charged cylinder.

(*a*) Use the definition of volume charge density to express the total charge of the cylinder:

$$Q_{\text{tot}} = \rho V = \rho\left(\pi R^2 L\right)$$

Substitute numerical values to obtain:

$$Q_{\text{tot}} = \pi\left(300\,\text{nC/m}^3\right)(0.06\,\text{m})^2(200\,\text{m}) = \boxed{679\,\text{nC}}$$

From Problem 50, for $r < R$, we have:

$$E_r = \frac{\rho}{2\in_0} r$$

(*b*) For r = 2 cm:

$$E_r(2\,\text{cm}) = \frac{\left(300\,\text{nC/m}^3\right)(0.02\,\text{m})}{2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)} = \boxed{339\,\text{N/C}}$$

(*c*) For r = 5.9 cm:

$$E_r(5.9\,\text{cm}) = \frac{\left(300\,\text{nC/m}^3\right)(0.059\,\text{m})}{2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)} = \boxed{1.00\,\text{kN/C}}$$

From Problem 50, for $r > R$, we have:

$$E_r = \frac{\rho R^2}{2\in_0 r}$$

(*d*) For r = 6.1 cm:

$$E_r(6.1\,\text{cm}) = \frac{\left(300\,\text{nC/m}^3\right)(0.06\,\text{m})^2}{2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)(0.061\,\text{m})} = \boxed{1.00\,\text{kN/C}}$$

(*e*) For r = 10 cm:

$$E_r(10\,\text{cm}) = \frac{\left(300\,\text{nC/m}^3\right)(0.06\,\text{m})^2}{2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)(0.1\,\text{m})} = \boxed{610\,\text{N/C}}$$

Note that, given the choice of charge densities in Problems 49 and 51, the electric fields for $r > R$ are the same.

***52** ••

Picture the Problem From symmetry; the field tangent to the surfaces of the shells must vanish. We can construct a Gaussian surface in the shape of a cylinder of radius r and length L and apply Gauss's law to find the electric field as a function of the distance from the centerline of the infinitely long, uniformly charged cylindrical shells.

(*a*) Apply Gauss's law to the cylindrical surface of radius r and length L that is concentric with the infinitely long, uniformly charged cylindrical shell:

$$\oint_S E_n dA = \frac{1}{\epsilon_0} Q_{\text{inside}}$$

or

$$2\pi r L E_n = \frac{Q_{\text{inside}}}{\epsilon_0}$$

where we've neglected the end areas because no flux crosses them.

Solve for E_n:

$$E_n = \frac{2kQ_{\text{inside}}}{Lr} \qquad (1)$$

For $r < R_1$, $Q_{\text{inside}} = 0$ and:

$$E_n(r < R_1) = \boxed{0}$$

Express Q_{inside} for $R_1 < r < R_2$:

$$Q_{\text{inside}} = \sigma_1 A_1 = 2\pi\sigma_1 R_1 L$$

Substitute in equation (1) to obtain:

$$E_n(R_1 < r < R_2) = \frac{2k(2\pi\sigma_1 R_1 L)}{Lr} = \boxed{\frac{\sigma_1 R_1}{\epsilon_0 r}}$$

Express Q_{inside} for $r > R_2$:

$$Q_{\text{inside}} = \sigma_1 A_1 + \sigma_2 A_2 = 2\pi\sigma_1 R_1 L + 2\pi\sigma_2 R_2 L$$

Substitute in equation (1) to obtain:

$$E_n(r > R_2) = \frac{2k(2\pi\sigma_1 R_1 L + 2\pi\sigma_2 R_2 L)}{Lr} = \boxed{\frac{\sigma_1 R_1 + \sigma_2 R_2}{\epsilon_0 r}}$$

(*b*) Set $E = 0$ for $r > R_2$ to obtain:

$$\frac{\sigma_1 R_1 + \sigma_2 R_2}{\epsilon_0 r} = 0$$

or

$$\sigma_1 R_1 + \sigma_2 R_2 = 0$$

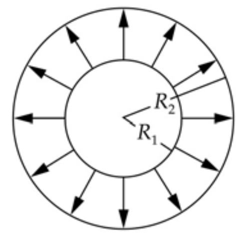
Solve for the ratio of σ_1 to σ_2:

$$\frac{\sigma_1}{\sigma_2} = \boxed{-\frac{R_2}{R_1}}$$

Because the electric field is determined by the charge inside the Gaussian surface, the field under these conditions would be as given above:

$$E_n(R_1 < r < R_2) = \boxed{\frac{\sigma_1 R_1}{\epsilon_0\, r}}$$

(*c*) Assuming that σ_1 is positive, the field lines would be directed as shown to the right.



53 ••

Picture the Problem The electric field is directed radially outward. We can construct a Gaussian surface in the shape of a cylinder of radius *r* and length *L* and apply Gauss's law to find the electric field as a function of the distance from the centerline of the infinitely long, uniformly charged cylindrical shell.

(*a*) Apply Gauss's law to a cylindrical surface of radius *r* and length *L* that is concentric with the inner conductor:

$$\oint_S E_n dA = \frac{1}{\epsilon_0} Q_{\text{inside}}$$

or

$$2\pi r L E_n = \frac{Q_{\text{inside}}}{\epsilon_0}$$

where we've neglected the end areas because no flux crosses them.

Solve for E_n:

$$E_n = \frac{2kQ_{\text{inside}}}{Lr} \quad (1)$$

For $r < 1.5$ cm, $Q_{\text{inside}} = 0$ and:

$$E_n(r < 1.5\,\text{cm}) = \boxed{0}$$

Letting $R = 1.5$ cm, express Q_{inside} for $1.5 \text{ cm} < r < 4.5$ cm:

$$Q_{\text{inside}} = \lambda L = 2\pi\sigma R L$$

Substitute in equation (1) to obtain:

$$E_n(1.5\,\text{cm} < r < 4.5\,\text{cm}) = \frac{2k(\lambda L)}{Lr} = \frac{2k\lambda}{r}$$

Substitute numerical values and evaluate $E_n(1.5 \text{ cm} < r < 4.5 \text{ cm})$:

$$E_n(1.5\,\text{cm} < r < 4.5\,\text{cm}) = 2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(6\,\text{nC/m})}{r} = \boxed{\frac{(108\,\text{N}\cdot\text{m/C})}{r}}$$

Express Q_{inside} for $4.5\text{ cm} < r < 6.5\text{ cm}$:

$$Q_{\text{inside}} = 0$$

and

$$E_n(4.5\,\text{cm} < r < 6.5\,\text{cm}) = \boxed{0}$$

Letting σ_2 represent the charge density on the outer surface, express Q_{inside} for $r > 6.5$ cm:

$$Q_{\text{inside}} = \sigma_2 A_2 = 2\pi\sigma_2 R_2 L$$

where $R_2 = 6.5$ cm.

Substitute in equation (1) to obtain:

$$E_n(r > R_2) = \frac{2k(2\pi\sigma_2 R_2 L)}{Lr} = \frac{\sigma_2 R_2}{\epsilon_0\ r}$$

In (*b*) we show that $\sigma_2 = 21.2\ \text{nC/m}^2$. Substitute numerical values to obtain:

$$E_n(r > 6.5\,\text{cm}) = \frac{(21.2\,\text{nC/m}^2)(6.5\,\text{cm})}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)r} = \boxed{\frac{156\,\text{N}\cdot\text{m/C}}{r}}$$

(*b*) The surface charge densities on the inside and the outside surfaces of the outer conductor are given by:

$$\sigma_1 = \frac{-\lambda}{2\pi R_1} \text{ and } \sigma_2 = -\sigma_1$$

Substitute numerical values and evaluate σ_1 and σ_2:

$$\sigma_1 = \frac{-6\,\text{nC/m}}{2\pi(0.045\,\text{m})} = \boxed{-21.2\,\text{nC/m}^2}$$

and

$$\sigma_2 = \boxed{21.2\,\text{nC/m}^2}$$

54 ••

Picture the Problem From symmetry considerations, we can conclude that the field tangent to the surface of the cylinder must vanish. We can construct a Gaussian surface in the shape of a cylinder of radius *r* and length *L* and apply Gauss's law to find the electric field as a function of the distance from the centerline of the infinitely long nonconducting cylinder.

(*a*) Apply Gauss's law to a cylindrical surface of radius *r* and length *L* that is concentric with the infinitely long nonconducting cylinder:

$$\oint_S E_n\,dA = \frac{1}{\epsilon_0}Q_{\text{inside}}$$

or

$$2\pi r L E_n = \frac{Q_{\text{inside}}}{\epsilon_0}$$

where we've neglected the end areas

because no flux crosses them.

Solve for E_n:

$$E_n = \frac{Q_{inside}}{2\pi rL \in_0} \qquad (1)$$

Express dQ_{inside} for $\rho(r) = ar$:

$$dQ_{inside} = \rho(r)dV = ar(2\pi rL)dr = 2\pi ar^2 Ldr$$

Integrate dQ_{inside} from $r = 0$ to R to obtain:

$$Q_{inside} = 2\pi aL\int_0^R r^2 dr = 2\pi aL\left[\frac{r^3}{3}\right]_0^R = \frac{2\pi aL}{3}R^3$$

Divide both sides of this equation by L to obtain an expression for the charge per unit length λ of the cylinder:

$$\lambda = \frac{Q_{inside}}{L} = \boxed{\frac{2\pi aR^3}{3}}$$

(b) Substitute for Q_{inside} in equation (1) to obtain:

$$E_n(r<R) = \frac{\frac{2\pi aL}{3}r^3}{2\pi \in_0 Lr} = \boxed{\frac{a}{3\in_0}r^2}$$

For $r > R$:

$$Q_{inside} = \frac{2\pi aL}{3}R^3$$

Substitute for Q_{inside} in equation (1) to obtain:

$$E_n(r>R) = \frac{\frac{2\pi aL}{3}R^3}{2\pi rL \in_0} = \boxed{\frac{aR^3}{3r\in_0}}$$

55 ••

Picture the Problem From symmetry; the field tangent to the surface of the cylinder must vanish. We can construct a Gaussian surface in the shape of a cylinder of radius r and length L and apply Gauss's law to find the electric field as a function of the distance from the centerline of the infinitely long nonconducting cylinder.

(a) Apply Gauss's law to a cylindrical surface of radius r and length L that is concentric with the infinitely long nonconducting cylinder:

$$\oint_S E_n dA = \frac{1}{\in_0}Q_{inside}$$

or

$$2\pi rLE_n = \frac{Q_{inside}}{\epsilon_0}$$

where we've neglected the end areas because no flux crosses them.

Solve for E_n:

$$E_n = \frac{Q_{inside}}{2\pi rL\epsilon_0} \qquad (1)$$

Express dQ_{inside} for $\rho(r) = br^2$:

$$dQ_{inside} = \rho(r)dV = br^2(2\pi rL)dr = 2\pi br^3Ldr$$

Integrate dQ_{inside} from $r = 0$ to R to obtain:

$$Q_{inside} = 2\pi bL\int_0^R r^3dr = 2\pi bL\left[\frac{r^4}{4}\right]_0^R = \frac{\pi bL}{2}R^4$$

Divide both sides of this equation by L to obtain an expression for the charge per unit length λ of the cylinder:

$$\lambda = \frac{Q_{inside}}{L} = \boxed{\frac{\pi bR^4}{2}}$$

(*b*) Substitute for Q_{inside} in equation (1) to obtain:

$$E_n(r < R) = \frac{\frac{\pi bL}{2}r^4}{2\pi Lr\epsilon_0} = \boxed{\frac{b}{4\epsilon_0}r^3}$$

For $r > R$:

$$Q_{inside} = \frac{\pi bL}{2}R^4$$

Substitute for Q_{inside} in equation (1) to obtain:

$$E_n(r > R) = \frac{\frac{\pi bL}{2}R^4}{2\pi rL\epsilon_0} = \boxed{\frac{bR^4}{4r\epsilon_0}}$$

56 •••

Picture the Problem From symmetry; the field tangent to the surface of the cylinder must vanish. We can construct a Gaussian surface in the shape of a cylinder of radius r and length L and apply Gauss's law to find the electric field as a function of the distance from the centerline of the infinitely long nonconducting cylindrical shell.

Apply Gauss's law to a cylindrical surface of radius r and length L that

$$\oint_S E_n dA = \frac{1}{\epsilon_0}Q_{inside}$$

is concentric with the infinitely long nonconducting cylindrical shell:

or

$$2\pi rLE_n = \frac{Q_{inside}}{\epsilon_0}$$

where we've neglected the end areas because no flux crosses them.

Solve for E_n:

$$E_n = \frac{Q_{inside}}{2\pi rL\epsilon_0}$$

For $r < a$, $Q_{inside} = 0$:

$$E_n(r < a) = \boxed{0}$$

Express Q_{inside} for $a < r < b$:

$$Q_{inside} = \rho V = \rho\pi r^2 L - \rho\pi a^2 L = \rho\pi L\left(r^2 - a^2\right)$$

Substitute for Q_{inside} to obtain:

$$E_n(a < r < b) = \frac{\rho\pi L\left(r^2 - a^2\right)}{2\pi\epsilon_0 Lr} = \boxed{\frac{\rho\left(r^2 - a^2\right)}{2\epsilon_0 r}}$$

Express Q_{inside} for $r > b$:

$$Q_{inside} = \rho V = \rho\pi b^2 L - \rho\pi a^2 L = \rho\pi L\left(b^2 - a^2\right)$$

Substitute for Q_{inside} to obtain:

$$E_n(r > b) = \frac{\rho\pi L\left(b^2 - a^2\right)}{2\pi\epsilon_0 rL} = \boxed{\frac{\rho\left(b^2 - a^2\right)}{2\epsilon_0 r}}$$

57 •••

Picture the Problem We can integrate the density function over the radius of the inner cylinder to find the charge on it and then calculate the linear charge density from its definition. To find the electric field for all values of *r* we can construct a Gaussian surface in the shape of a cylinder of radius *r* and length *L* and apply Gauss's law to each region of the cable to find the electric field as a function of the distance from its centerline.

(*a*) Find the charge Q_{inner} on the inner cylinder:

$$Q_{inner} = \int_0^R \rho(r)dV = \int_0^R \frac{C}{r}2\pi rLdr = 2\pi CL\int_0^R dr = 2\pi CLR$$

Relate this charge to the linear charge density:

$$\lambda_{\text{inner}} = \frac{Q_{\text{inner}}}{L} = \frac{2\pi CLR}{L} = 2\pi CR$$

Substitute numerical values and evaluate λ_{inner}:

$$\lambda_{\text{inner}} = 2\pi(200\,\text{nC/m})(0.015\,\text{m}) = \boxed{18.8\,\text{nC/m}}$$

(*b*) Apply Gauss's law to a cylindrical surface of radius *r* and length *L* that is concentric with the infinitely long nonconducting cylinder:

$$\oint_S E_n\,dA = \frac{1}{\epsilon_0} Q_{\text{inside}}$$

or

$$2\pi r L E_n = \frac{Q_{\text{inside}}}{\epsilon_0}$$

where we've neglected the end areas because no flux crosses them.

Solve for E_n:

$$E_n = \frac{Q_{\text{inside}}}{2\pi r L \epsilon_0}$$

Substitute to obtain, for $r < 1.5$ cm:

$$E_n(r < 1.5\,\text{cm}) = \frac{2\pi CLr}{2\pi \epsilon_0 Lr} = \frac{C}{\epsilon_0}$$

Substitute numerical values and evaluate $E_n(r < 1.5\text{ cm})$:

$$E_n(r < 1.5\,\text{cm}) = \frac{200\,\text{nC/m}^2}{8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2} = \boxed{22.6\,\text{kN/C}}$$

Express Q_{inside} for $1.5\text{ cm} < r < 4.5\text{ cm}$:

$$Q_{\text{inside}} = 2\pi CLR$$

Substitute to obtain, for $1.5\text{ cm} < r < 4.5\text{ cm}$:

$$E_n(1.5\,\text{cm} < r < 4.5\,\text{cm}) = \frac{2C\pi RL}{2\pi \epsilon_0 rL} = \frac{CR}{\epsilon_0 r}$$

where $R = 1.5$ cm.

Substitute numerical values and evaluate $E_n(1.5\text{ cm} < r < 4.5\text{ cm})$:

$$E_n(1.5\,\text{cm} < r < 4.5\,\text{cm}) = \frac{(200\,\text{nC/m}^2)(0.015\,\text{m})}{(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)r} = \boxed{\frac{339\,\text{N}\cdot\text{m/C}}{r}}$$

Because the outer cylindrical shell is a conductor:

$$E_n(4.5\,\text{cm} < r < 6.5\,\text{cm}) = \boxed{0}$$

For r > 6.5 cm, $Q_{\text{inside}} = 2\pi CLR$ and:

$$E_n(r > 6.5\,\text{cm}) = \boxed{\frac{339\,\text{N}\cdot\text{m/C}}{r}}$$

Charge and Field at Conductor Surfaces

***58** •

Picture the Problem Because the penny is in an external electric field, it will have charges of opposite signs induced on its faces. The induced charge σ is related to the electric field by $E = \sigma/\varepsilon_0$. Once we know σ, we can use the definition of surface charge density to find the total charge on one face of the penny.

(*a*) Relate the electric field to the charge density on each face of the penny:

$$E = \frac{\sigma}{\in_0}$$

Solve for and evaluate σ:

$$\sigma = \in_0 E = \left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(1.6\,\text{kN/C}\right) = \boxed{14.2\,\text{nC/m}^2}$$

(*b*) Use the definition of surface charge density to obtain:

$$\sigma = \frac{Q}{A} = \frac{Q}{\pi r^2}$$

Solve for and evaluate Q:

$$Q = \sigma\pi r^2 = \pi\left(14.2\,\text{nC/m}^2\right)\left(0.01\,\text{m}\right)^2 = \boxed{4.45\,\text{pC}}$$

59 •

Picture the Problem Because the metal slab is in an external electric field, it will have charges of opposite signs induced on its faces. The induced charge σ is related to the electric field by $E = \sigma / \in_0$.

Relate the magnitude of the electric field to the charge density on the metal slab:

$$E = \frac{\sigma}{\in_0}$$

Use its definition to express σ:

$$\sigma = \frac{Q}{A} = \frac{Q}{L^2}$$

Substitute to obtain:

$$E = \frac{Q}{L^2 \in_0}$$

Substitute numerical values and evaluate E:

$$E = \frac{1.2\,\text{nC}}{(0.12\,\text{m})^2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)} = \boxed{9.42\,\text{kN/C}}$$

60 •

Picture the Problem We can apply its definition to find the surface charge density of the nonconducting material and calculate the electric field at either of its surfaces from $\sigma/2\in_0$. When the same charge is placed on a conducting sheet, the charge will distribute itself until half the charge is on each surface.

(a) Use its definition to find σ:

$$\sigma = \frac{Q}{A} = \frac{6\,\text{nC}}{(0.2\,\text{m})^2} = \boxed{150\,\text{nC/m}^2}$$

(b) Relate the electric field on either side of the sheet to the density of charge on its surfaces:

$$E = \frac{\sigma}{2\in_0} = \frac{150\,\text{nC/m}^2}{2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)} = \boxed{8.47\,\text{kN/C}}$$

(c) Because the slab is a conductor the charge will distribute uniformly on its two surfaces so that:

$$\sigma = \frac{Q}{2A} = \frac{6\,\text{nC}}{2(0.2\,\text{m})^2} = \boxed{75.0\,\text{nC/m}^2}$$

(d) The electric field just outside the surface of a conductor is given by:

$$E = \frac{\sigma}{\in_0} = \frac{75\,\text{nC/m}^2}{8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2} = \boxed{8.47\,\text{kN/C}}$$

61 •

Picture the Problem We can construct a Gaussian surface in the shape of a sphere of radius r with the same center as the shell and apply Gauss's law to find the electric field as a function of the distance from this point. The inner and outer surfaces of the shell will have charges induced on them by the charge q at the center of the shell.

(a) Apply Gauss's law to a spherical surface of radius r that is concentric with the point charge:

$$\oint_S E_n\,dA = \frac{1}{\in_0}Q_{\text{inside}}$$

or

$$4\pi r^2 E_n = \frac{Q_{\text{inside}}}{\in_0}$$

Solve for E_n:

$$E_n = \frac{Q_{\text{inside}}}{4\pi r^2\in_0} \qquad (1)$$

For $r < a$, $Q_{\text{inside}} = q$. Substitute in equation (1) and simplify to obtain:

$$E_n(r<a) = \frac{q}{4\pi r^2 \in_0} = \boxed{\frac{kq}{r^2}}$$

Because the spherical shell is a conductor, a charge $-q$ will be induced on its inner surface. Hence, for $a < r < b$:

$$Q_{\text{inside}} = 0$$

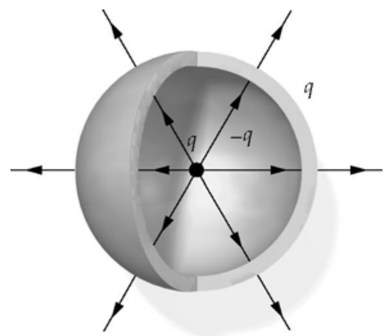
and

$$E_n(a<r<b) = \boxed{0}$$

For $r > b$, $Q_{\text{inside}} = q$. Substitute in equation (1) and simplify to obtain:

$$E_n(r>b) = \frac{q}{4\pi r^2 \in_0} = \boxed{\frac{kq}{r^2}}$$

(*b*) The electric field lines are shown in the diagram to the right:



(*c*) A charge $-q$ is induced on the inner surface. Use the definition of surface charge density to obtain:

$$\sigma_{\text{inner}} = \frac{-q}{4\pi a^2} = \boxed{-\frac{q}{4\pi a^2}}$$

A charge q is induced on the outer surface. Use the definition of surface charge density to obtain:

$$\sigma_{\text{outer}} = \boxed{\frac{q}{4\pi b^2}}$$

62 ••

Picture the Problem We can construct a spherical Gaussian surface at the surface of the earth (we'll assume the Earth is a sphere) and apply Gauss's law to relate the electric field to its total charge.

Apply Gauss's law to a spherical surface of radius R_E that is concentric with the earth:

$$\oint_S E_n dA = \frac{1}{\in_0} Q_{\text{inside}}$$

or

$$4\pi R_E^2 E_n = \frac{Q_{\text{inside}}}{\in_0}$$

Solve for $Q_{\text{inside}} = Q_{\text{earth}}$ to obtain:

$$Q_{\text{earth}} = 4\pi \in_0 R_E^2 E_n = \frac{R_E^2 E_n}{k}$$

Substitute numerical values and evaluate Q_{earth}:

$$Q_{earth} = \frac{(6.37\times10^{6}\text{ m})^{2}(150\text{ N/C})}{8.99\times10^{9}\text{ N}\cdot\text{m}^{2}/\text{C}^{2}} = \boxed{6.77\times10^{5}\text{ C}}$$

***63 ••**

Picture the Problem Let the inner and outer radii of the uncharged spherical conducting shell be *a* and *b* and *q* represent the positive point charge at the center of the shell. The positive point charge at the center will induce a negative charge on the inner surface of the shell and, because the shell is uncharged, an equal positive charge will be induced on its outer surface. To solve part (*b*), we can construct a Gaussian surface in the shape of a sphere of radius *r* with the same center as the shell and apply Gauss's law to find the electric field as a function of the distance from this point. In part (*c*) we can use a similar strategy with the additional charge placed on the shell.

(*a*) Express the charge density on the inner surface:

$$\sigma_{inner} = \frac{q_{inner}}{A}$$

Express the relationship between the positive point charge *q* and the charge induced on the inner surface q_{inner}:

$$q + q_{inner} = 0$$

Substitute for q_{inner} to obtain:

$$\sigma_{inner} = \frac{-q}{4\pi a^{2}}$$

Substitute numerical values and evaluate σ_{inner}:

$$\sigma_{inner} = \frac{-2.5\,\mu\text{C}}{4\pi(0.6\text{ m})^{2}} = \boxed{-0.553\,\mu\text{C/m}^{2}}$$

Express the charge density on the outer surface:

$$\sigma_{outer} = \frac{q_{outer}}{A}$$

Because the spherical shell is uncharged:

$$q_{outer} + q_{inner} = 0$$

Substitute for q_{outer} to obtain:

$$\sigma_{outer} = \frac{-q_{inner}}{4\pi b^{2}}$$

Substitute numerical values and evaluate σ_{outer}:

$$\sigma_{outer} = \frac{2.5\,\mu\text{C}}{4\pi(0.9\text{ m})^{2}} = \boxed{0.246\,\mu\text{C/m}^{2}}$$

(*b*) Apply Gauss's law to a spherical surface of radius *r* that is concentric with the point charge:

$$\oint_S E_n\,dA = \frac{1}{\epsilon_0} Q_{\text{inside}}$$

or

$$4\pi r^2 E_n = \frac{Q_{\text{inside}}}{\epsilon_0}$$

Solve for E_n:

$$E_n = \frac{Q_{\text{inside}}}{4\pi r^2 \epsilon_0} \qquad (1)$$

For $r < a = 0.6$ m, $Q_{\text{inside}} = q$. Substitute in equation (1) and evaluate $E_n(r < 0.6\text{ m})$ to obtain:

$$E_n(r < a) = \frac{q}{4\pi r^2 \epsilon_0} = \frac{kq}{r^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(2.5\,\mu\text{C}\right)}{r^2}$$

$$= \boxed{\left(2.25\times10^4\ \text{N}\cdot\text{m}^2/\text{C}\right)\frac{1}{r^2}}$$

Because the spherical shell is a conductor, a charge –*q* will be induced on its inner surface. Hence, for 0.6 m < *r* < 0.9 m:

$$Q_{\text{inside}} = 0$$

and

$$E_n(0.6\,\text{m} < r < 0.9\,\text{m}) = \boxed{0}$$

For $r > 0.9$ m, the net charge inside the Gaussian surface is *q* and:

$$E_n(r > 0.9\,\text{m}) = \frac{kq}{r^2} = \boxed{\left(2.25\times10^4\ \text{N}\cdot\text{m}^2/\text{C}\right)\frac{1}{r^2}}$$

(*c*) Because $E = 0$ in the conductor:

$$q_{\text{inner}} = -2.5\,\mu\text{C}$$

and

$$\sigma_{\text{inner}} = \boxed{-0.553\,\mu\text{C/m}^2}$$

as before.

Express the relationship between the charges on the inner and outer surfaces of the spherical shell:

$$q_{\text{outer}} + q_{\text{inner}} = 3.5\,\mu\text{C}$$

and

$$q_{\text{outer}} = 3.5\,\mu\text{C} - q_{\text{inner}} = 6.0\,\mu\text{C}$$

σ_{outer} is now given by:

$$\sigma_{\text{outer}} = \frac{6\,\mu\text{C}}{4\pi(0.9\,\text{m})^2} = \boxed{0.589\,\mu\text{C/m}^2}$$

For $r < a = 0.6$ m, $Q_{inside} = q$ and $E_n(r < 0.6 \text{ m})$ is as it was in (*a*):

$$E_n(r < a) = \boxed{\left(2.25\times10^4 \text{ N}\cdot\text{m}^2/\text{C}\right)\frac{1}{r^2}}$$

Because the spherical shell is a conductor, a charge $-q$ will be induced on its inner surface. Hence, for 0.6 m < r < 0.9 m:

$$Q_{inside} = 0$$

and

$$E_n(0.6\,\text{m} < r < 0.9\,\text{m}) = \boxed{0}$$

For $r > 0.9$ m, the net charge inside the Gaussian surface is 6 μC and:

$$E_n(r > 0.9\,\text{m}) = \frac{kq}{r^2} = \left(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)(6\,\mu\text{C})\frac{1}{r^2} = \boxed{\left(5.39\times10^4 \text{ N}\cdot\text{m}^2/\text{C}\right)\frac{1}{r^2}}$$

64 ••

Picture the Problem From Gauss's law we know that the electric field at the surface of the charged sphere is given by $E = kQ/R^2$ where Q is the charge on the sphere and R is its radius. The minimum radius for dielectric breakdown corresponds to the maximum electric field at the surface of the sphere.

Use Gauss's law to express the electric field at the surface of the charged sphere:

$$E = \frac{kQ}{R^2}$$

Express the relationship between E and R for dielectric breakdown:

$$E_{max} = \frac{kQ}{R_{min}^2}$$

Solve for R_{min}:

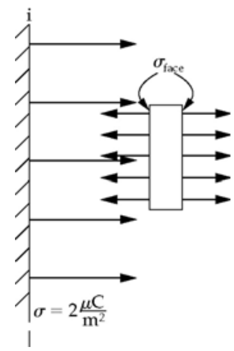
$$R_{min} = \sqrt{\frac{kQ}{E_{max}}}$$

Substitute numerical values and evaluate R_{min}:

$$R_{min} = \sqrt{\frac{\left(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)(18\,\mu\text{C})}{3\times10^6 \text{ N/C}}} = \boxed{23.2\,\text{cm}}$$

65 ••

Picture the Problem We can use its definition to find the surface charge density just outside the face of the slab. The electric field near the surface of the slab is given by $E=\sigma_{\text{face}}/\in_0$. We can find the electric field on each side of the slab by adding the fields due to the slab and the plane of charge.



(*a*) Express the charge density per face in terms of the net charge on the slab:

$$\sigma_{\text{face}}=\frac{q}{2L^2}$$

Substitute numerical values to obtain:

$$\sigma_{\text{face}}=\frac{80\,\mu\text{C}}{2(5\,\text{m})^2}=\boxed{1.60\,\mu\text{C/m}^2}$$

Express the electric field just outside one face of the slab in terms of its surface charge density:

$$E_{\text{slab}}=\frac{\sigma_{\text{face}}}{\in_0}$$

Substitute numerical values and evaluate E_{face}:

$$E_{\text{slab}}=\frac{1.60\,\mu\text{C/m}^2}{8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2}=\boxed{1.81\times10^5\,\text{N/C}}$$

(*b*) Express the total field on the side of the slab closest to the infinite charged plane:

$$\vec{\boldsymbol{E}}_{\text{near}}=\vec{\boldsymbol{E}}_{\text{plane}}+\vec{\boldsymbol{E}}_{\text{slab}}=E_{\text{plane}}\hat{\boldsymbol{r}}-E_{\text{slab}}\hat{\boldsymbol{r}}=\frac{\sigma_{\text{plane}}}{2\in_0}\hat{\boldsymbol{r}}-\frac{\sigma_{\text{face}}}{\in_0}\hat{\boldsymbol{r}}$$

where $\hat{\boldsymbol{r}}$ is a unit vector pointing away from the slab.

Substitute numerical values and evaluate $\vec{\boldsymbol{E}}_{\text{near}}$:

$$\vec{\boldsymbol{E}}_{\text{near}}=\frac{2\,\mu\text{C/m}^2}{2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)}\hat{\boldsymbol{r}}-\left(1.81\times10^5\,\text{N/C}\right)\hat{\boldsymbol{r}}=\boxed{\left(-0.680\times10^5\,\text{N/C}\right)\hat{\boldsymbol{r}}}$$

Express the total field on the side of the slab away from the infinite charged plane:

$$\vec{E}_{\text{far}} = \frac{\sigma_{\text{plane}}}{2\in_0}\hat{r} + \frac{\sigma_{\text{face}}}{\in_0}\hat{r}$$

Substitute numerical values and evaluate $\vec{E}_{\text{far}}$:

$$\vec{E}_{\text{far}} = \frac{2\,\mu\text{C/m}^2}{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)}\hat{r} + \left(1.81\times10^5\ \text{N/C}\right)\hat{r} = \boxed{\left(2.94\times10^5\ \text{N/C}\right)\hat{r}}$$

The charge density on the side of the slab near the plane is:

$$\sigma_{\text{near}} = \in_0 E_{\text{near}} = \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(0.680\times10^5\ \text{N/C}\right) = \boxed{0.602\ \mu\text{C/m}^2}$$

The charge density on the far side of the slab is:

$$\sigma_{\text{near}} = \in_0 E_{\text{near}} = \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(2.94\times10^5\ \text{N/C}\right) = \boxed{2.60\ \mu\text{C/m}^2}$$

General Problems

66 ••

Determine the Concept We can determine the direction of the electric field between spheres I and II by imagining a test charge placed between the spheres and determining the direction of the force acting on it. We can determine the amount and sign of the charge on each sphere by realizing that the charge on a given surface induces a charge of the same magnitude but opposite sign on the next surface of larger radius.

(*a*) The charge placed on sphere III has no bearing on the electric field between spheres I and II. The field in this region will be in the direction of the force exerted on a test charge placed between the spheres. Because the charge at the center is negative,

$\boxed{\text{the field will point toward the center.}}$

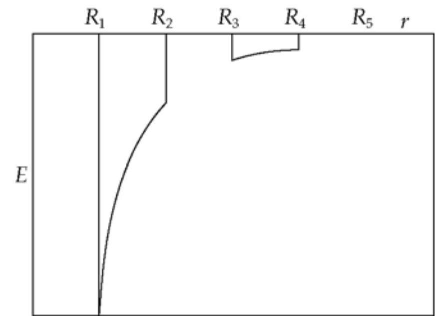
(*b*) The charge on sphere I ($-Q_0$) will induce a charge of the same magnitude but opposite sign on sphere II: $\boxed{+Q_0}$

(*c*) The induction of charge $+Q_0$ on the inner surface of sphere II will leave its outer surface with a charge of the same magnitude but opposite sign: $\boxed{-Q_0}$

(*d*) The presence of charge $-Q_0$ on the outer surface of sphere II will induce a charge of the same magnitude but opposite sign on the inner surface of sphere III: $\boxed{+Q_0}$

(*e*) The presence of charge $+Q_0$ on the inner surface of sphere III will leave the outer surface of sphere III neutral: $\boxed{0}$

(*f*) A graph of *E* as a function of *r* is shown to the right:



67 ••

Picture the Problem Because the difference between the field just to the right of the origin $E_{x,\text{right}}$ and the field just to the left of the origin $E_{x,\text{left}}$ is the field due to the nonuniform surface charge, we can express $E_{x,\text{left}}$ and the difference between $E_{x,\text{right}}$ and σ/ϵ_0.

Express the electric field just to the left of the origin in terms of $E_{x,\text{right}}$ and σ/ϵ_0:

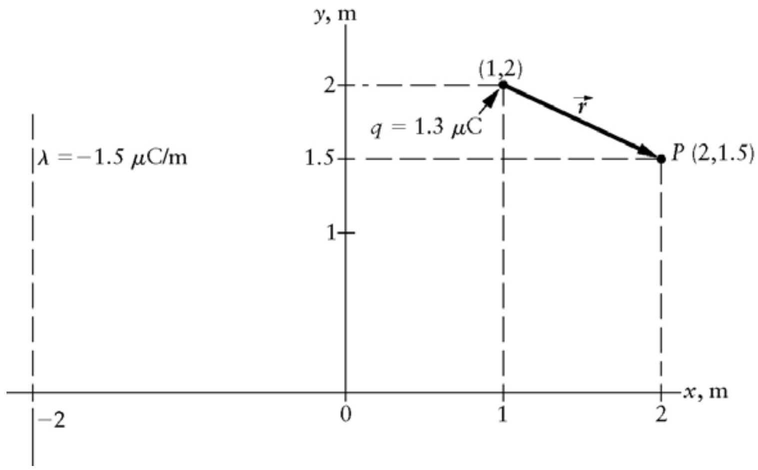
$$E_{x,\text{left}} = E_{x,\text{right}} - \frac{\sigma}{\epsilon_0}$$

Substitute numerical values and evaluate $E_{x,\text{left}}$:

$$E_{x,\text{left}} = 4.65\times10^5\ \text{N/C} - \frac{3.10\ \mu\text{C/m}^2}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = \boxed{1.15\times10^5\ \text{N/C}}$$

68 ••

Picture the Problem Let *P* denote the point of interest at (2 m, 1.5 m). The electric field at *P* is the sum of the electric fields due to the infinite line charge and the point charge.



Express the resultant electric field at *P*: $\vec{E} = \vec{E}_\lambda + \vec{E}_q$

Find the field at *P* due the infinite line charge:

$$\vec{E}_\lambda = \frac{2k\lambda}{r}\hat{r} = \frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-1.5\ \mu\text{C/m})}{4\,\text{m}}\hat{i} = (-6.74\,\text{kN/C})\hat{i}$$

Express the field at *P* due the point charge: $\vec{E}_q = \frac{kq}{r^2}\hat{r}$

Referring to the diagram above, determine *r* and $\hat{r}$: $r = 1.12\,\text{m}$ and $\hat{r} = 0.893\hat{i} - 0.446\hat{j}$

Substitute and evaluate $\vec{E}_q(2\,\text{m}, 1.5\,\text{m})$:

$$\vec{E}_q(2\,\text{m},1.5\,\text{m}) = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.3\ \mu\text{C})}{(1.12\,\text{m})^2}(0.893\hat{i} - 0.446\hat{j})$$
$$= (9.32\,\text{kN/C})(0.893\hat{i} - 0.446\hat{j})$$
$$= (8.32\,\text{kN/C})\hat{i} - (4.16\,\text{kN/C})\hat{j}$$

Substitute to obtain:

$$\vec{E}(2\,\text{m},1.5\,\text{m}) = (-6.74\,\text{kN/C})\hat{i} + (8.35\,\text{kN/C})\hat{i} - (4.17\,\text{kN/C})\hat{j}$$
$$= \boxed{(1.61\,\text{kN/C})\hat{i} - (4.17\,\text{kN/C})\hat{j}}$$

***69 ••**

Picture the Problem If the patch is small enough, the field at the center of the patch comes from two contributions. We can view the field in the hole as the sum of the field from a uniform spherical shell of charge Q plus the field due to a small patch with surface charge density equal but opposite to that of the patch cut out.

(*a*) Express the magnitude of the electric field at the center of the hole:

$$E = E_{\text{spherical shell}} + E_{\text{hole}}$$

Apply Gauss's law to a spherical gaussian surface just outside the given sphere:

$$E_{\text{spherical shell}}\left(4\pi r^2\right) = \frac{Q_{\text{enclosed}}}{\in_0} = \frac{Q}{\in_0}$$

Solve for $E_{\text{spherical shell}}$ to obtain:

$$E_{\text{spherical shell}} = \frac{Q}{4\pi \in_0 r^2}$$

The electric field due to the small hole (small enough so that we can treat it as a plane surface) is:

$$E_{\text{hole}} = \frac{-\sigma}{2\in_0}$$

Substitute and simplify to obtain:

$$E = \frac{Q}{4\pi \in_0 r^2} + \frac{-\sigma}{2\in_0}$$
$$= \frac{Q}{4\pi \in_0 r^2} - \frac{Q}{2\in_0\left(4\pi r^2\right)}$$
$$= \boxed{\frac{Q}{8\pi \in_0 r^2}}$$

(*b*) Express the force on the patch:

$$F = qE$$

where q is the charge on the patch.

Assuming that the patch has radius a, express the proportion between its charge and that of the spherical shell:

$$\frac{q}{\pi a^2} = \frac{Q}{4\pi r^2} \text{ or } q = \frac{a^2}{4r^2}Q$$

Substitute for q and E in the expression for F to obtain:

$$F = \left(\frac{a^2}{4r^2}Q\right)\left(\frac{Q}{8\pi \in_0 r^2}\right) = \boxed{\frac{Q^2a^2}{32\pi \in_0 r^4}}$$

(*c*) The pressure is the force divided by the area of the patch:

$$P = \frac{\dfrac{Q^2a^2}{32\pi \in_0 r^4}}{\pi a^2} = \boxed{\frac{Q^2}{32\pi^2 \in_0 r^4}}$$

70 ••

Picture the Problem The work done by the electrostatic force in expanding the soap bubble is $W = \int P dV$.

From Problem 69:

$$P = \frac{Q^2}{32\pi^2 \in_0 r^4}$$

Express W in terms of dr:

$$W = \int P dV = \int P 4\pi r^2 dr$$

Substitute for P and simplify:

$$W = \frac{Q^2}{8\pi \in_0} \int_{0.1\,\text{m}}^{0.2\,\text{m}} \frac{dr}{r^2}$$

Evaluating the integral yields:

$$W = \frac{Q^2}{8\pi \in_0}\left[-\frac{1}{r}\right]_{0.1\,\text{m}}^{0.2\,\text{m}} = \frac{(3\,\text{nC})^2}{8\pi(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)}\left(\frac{-1}{0.2\,\text{m}} + \frac{1}{0.1\,\text{m}}\right)$$
$$= \boxed{2.02\times10^{-7}\,\text{J}}$$

71 ••

Picture the Problem We can use $E = kq/R^2$, where R is the radius of the droplet, to find the electric field at its surface. We can find R by equating the volume of the bubble at the moment it bursts to the volume of the resulting spherical droplet.

Express the field at the surface of the spherical water droplet:

$$E = \frac{kq}{R^2} \qquad (1)$$

where R is the radius of the droplet and q is its charge.

Express the volume of the bubble just before it pops:

$$V \approx 4\pi r^2 t$$

where t is the thickness of the soap bubble.

Express the volume of the sphere into which the droplet collapses:

$$V = \frac{4}{3}\pi R^3$$

Because the volume of the droplet and the volume of the bubble are equal:

$$4\pi r^2 t = \frac{4}{3}\pi R^3$$

Solve for R:

$$R = \sqrt[3]{3r^2 t}$$

Assume a thickness t of 1 μm and evaluate R:

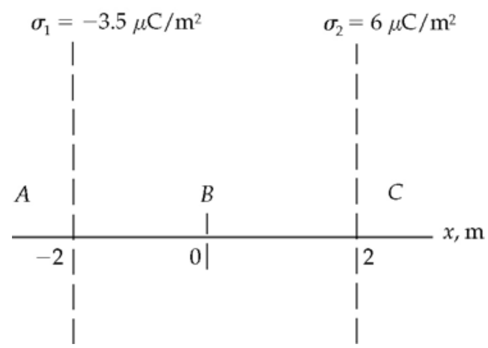
$$R = \sqrt[3]{3(0.2\,\text{m})^2(1\,\mu\text{m})} = 4.93\times10^{-3}\,\text{m}$$

Substitute numerical values in equation (1) and evaluate E:

$$E=\frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(3\,\text{nC})}{\left(4.93\times10^{-3}\ \text{m}\right)^2}=\boxed{1.11\times10^{6}\ \text{N/C}}$$

72 ••

Picture the Problem Let the numeral 1 refer to the infinite plane at $x = -2$ m and the numeral 2 to the plane at $x = 2$ m and let the letter A refer to the region to the left of plane 1, B to the region between the planes, and C to the region to the right of plane 2. We can use the expression for the electric field of in infinite plane of charge to express the electric field due to each plane of charge in each of the three regions. Their sum will be the resultant electric field in each region.



Express the resultant electric field as the sum of the fields due to planes 1 and 2:

$$\vec{E}=\vec{E}_1+\vec{E}_2 \qquad (1)$$

(a) Express and evaluate the field due to plane 1 in region A:

$$\vec{E}_1(A)=\frac{\sigma_1}{2\epsilon_0}\left(-\hat{i}\right)=\frac{-3.5\,\mu\text{C/m}^2}{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)}\left(-\hat{i}\right)=(198\,\text{kN/C})\hat{i}$$

Express and evaluate the field due to plane 2 in region A:

$$\vec{E}_2(A)=\frac{\sigma_2}{2\epsilon_0}\left(-\hat{i}\right)=\frac{6\,\mu\text{C/m}^2}{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)}\left(-\hat{i}\right)=(-339\,\text{kN/C})\hat{i}$$

Substitute in equation (1) to obtain:

$$\vec{E}(A)=(198\ \text{kN/C})\hat{i}+(-339\ \text{kN/C})\hat{i}=\boxed{(-141\,\text{kN/C})\hat{i}}$$

(*b*) Express and evaluate the field due to plane 1 in region *B*:

$$\vec{E}_1(B) = \frac{\sigma_1}{2\in_0}\hat{i} = \frac{-3.5\,\mu\text{C/m}^2}{2(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)}\hat{i} = (-198\,\text{kN/C})\hat{i}$$

Express and evaluate the field due to plane 2 in region *B*:

$$\vec{E}_2(B) = \frac{\sigma_2}{2\in_0}(-\hat{i}) = \frac{6\,\mu\text{C/m}^2}{2(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)}(-\hat{i}) = (-339\,\text{kN/C})\hat{i}$$

Substitute in equation (1) to obtain:

$$\vec{E}(B) = (-198\,\text{kN/C})\hat{i} + (-339\,\text{kN/C})\hat{i} = \boxed{(-537\,\text{kN/C})\hat{i}}$$

(*c*) Express and evaluate the field due to plane 1 in region *C*:

$$\vec{E}_1(C) = \frac{\sigma_1}{2\in_0}\hat{i} = \frac{-3.5\,\mu\text{C/m}^2}{2(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)}\hat{i} = (-198\,\text{kN/C})\hat{i}$$

Express and evaluate the field due to plane 2 in region *C*:

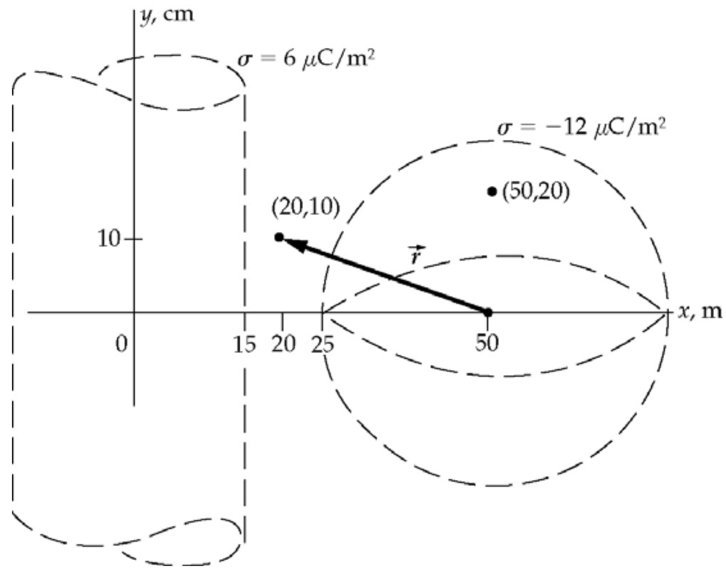
$$\vec{E}_2(C) = \frac{\sigma_2}{2\in_0}\hat{i} = \frac{6\,\mu\text{C/m}^2}{2(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)}\hat{i} = (339\,\text{kN/C})\hat{i}$$

Substitute in equation (1) to obtain:

$$\vec{E}(C) = (-198\,\text{kN/C})\hat{i} + (339\,\text{kN/C})\hat{i} = \boxed{(141\,\text{kN/C})\hat{i}}$$

***73 ••**

Picture the Problem We can find the electric fields at the three points of interest by adding the electric fields due to the infinitely long cylindrical shell and the spherical shell. In Problem 42 it was established that, for an infinitely long cylindrical shell of radius *R*, $E_r(r<R)=0$, and $E_r(r>R)=\sigma R/\in_0 r$. We know that, for a spherical shell of radius *R*, $E_r(r<R)=0$, and $E_r(r>R)=\sigma R^2/\in_0 r^2$.



Express the resultant electric field as the sum of the fields due to the cylinder and sphere:

$$\vec{E} = \vec{E}_{\text{cyl}} + \vec{E}_{\text{sph}} \qquad (1)$$

(*a*) Express and evaluate the electric field due to the cylindrical shell at the origin:

$$\vec{E}_{\text{cyl}}(0,0) = 0$$

because the origin is inside the cylindrical shell.

Express and evaluate the electric field due to the spherical shell at the origin:

$$\vec{E}_{\text{sph}}(0,0) = \frac{\sigma R^2}{\in_0 r^2}\left(-\hat{\boldsymbol{i}}\right) = \frac{\left(-12\,\mu\text{C/m}^2\right)\left(0.25\,\text{m}\right)^2}{\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(0.5\,\text{m}\right)^2}\left(-\hat{\boldsymbol{i}}\right) = \left(339\,\text{kN/C}\right)\hat{\boldsymbol{i}}$$

Substitute in equation (1) to obtain:

$$\vec{E}(0,0) = 0 + \left(339\,\text{kN/C}\right)\hat{\boldsymbol{i}} = \boxed{\left(339\,\text{kN/C}\right)\hat{\boldsymbol{i}}}$$

or

$$E(0,0) = \boxed{339\,\text{kN/C}}$$

and

$$\theta = \boxed{0^\circ}$$

(*b*) Express and evaluate the electric field due to the cylindrical shell at (0.2 m, 0.1 m):

$$\vec{E}_{\text{cyl}}(0.2\,\text{m},0.1\,\text{m}) = \frac{\sigma R}{\epsilon_0\, r}\hat{\boldsymbol{i}} = \frac{(6\,\mu\text{C/m}^2)(0.15\,\text{m})}{(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(0.2\,\text{m})}\hat{\boldsymbol{i}} = (508\,\text{kN/C})\hat{\boldsymbol{i}}$$

Express the electric field due to the charge on the spherical shell as a function of the distance from its center:

$$\vec{E}_{\text{sph}}(r) = \frac{\sigma R^2}{\epsilon_0\, r^2}\hat{\boldsymbol{r}}$$

where $\hat{\boldsymbol{r}}$ is a unit vector pointing from (50 cm, 0) to (20 cm, 10 cm).

Referring to the diagram shown above, find r and $\hat{\boldsymbol{r}}$:

$$r = 0.316\,\text{m}$$

and

$$\vec{r} = -0.949\hat{\boldsymbol{i}} + 0.316\hat{\boldsymbol{j}}$$

Substitute to obtain:

$$\begin{aligned}\vec{E}_{\text{sph}}(0.2\,\text{m},0.1\,\text{m}) &= \frac{(-12\,\mu\text{C/m}^2)(0.25\,\text{m})^2}{(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(0.316\,\text{m})^2}\left(-0.949\hat{\boldsymbol{i}} + 0.316\hat{\boldsymbol{j}}\right)\\ &= (-849\,\text{kN/C})\left(-0.949\hat{\boldsymbol{i}} + 0.316\hat{\boldsymbol{j}}\right)\\ &= (806\,\text{kN/C})\hat{\boldsymbol{i}} + (-268\,\text{kN/C})\hat{\boldsymbol{j}}\end{aligned}$$

Substitute in equation (1) to obtain:

$$\begin{aligned}\vec{E}(0.2\,\text{m},0.1\,\text{m}) &= (508\,\text{kN/C})\hat{\boldsymbol{i}} + (806\,\text{kN/C})\hat{\boldsymbol{i}} + (-268\,\text{kN/C})\hat{\boldsymbol{j}}\\ &= \boxed{(1310\,\text{kN/C})\hat{\boldsymbol{i}} + (-268\,\text{kN/C})\hat{\boldsymbol{j}}}\end{aligned}$$

or

$$E(0.2\,\text{m},0.1\,\text{m}) = \sqrt{(1310\,\text{kN/C})^2 + (-268\,\text{kN/C})^2} = \boxed{1340\,\text{kN/C}}$$

and

$$\theta = \tan^{-1}\left(\frac{-268\ \text{kN/C}}{1310\ \text{kN/C}}\right) = \boxed{348°}$$

(*c*) Express and evaluate the electric field due to the cylindrical shell at (0.5 m, 0.2 m):

$$\vec{E}_{\text{cyl}}(0.5\,\text{m},0.2\,\text{m}) = \frac{(6\,\mu\text{C/m}^2)(0.15\,\text{m})}{(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(0.5\,\text{m})}\hat{\boldsymbol{i}} = (203\,\text{kN/C})\hat{\boldsymbol{i}}$$

Express and evaluate the electric field due to the spherical shell at

$$\vec{E}_{\text{sph}}(0.5\,\text{m},0.2\,\text{m}) = 0$$

(0.5 m, 0.5 m): because (0.5 m, 0.2 m) is inside the spherical shell.

Substitute in equation (1) to obtain:

$$\vec{E}(0.5\,\text{m},0.2\,\text{m}) = (203\,\text{kN/C})\hat{\boldsymbol{i}} + 0 = \boxed{(203\,\text{kN/C})\hat{\boldsymbol{i}}}$$

or

$$E(0.5\,\text{m},0.2\,\text{m}) = \boxed{203\,\text{kN/C}}$$

and

$$\theta = \boxed{0°}$$

74 ••

Picture the Problem Let the numeral 1 refer to the plane with charge density σ_1 and the numeral 2 to the plane with charge density σ_2. We can find the electric field at the two points of interest by adding the electric fields due to the charge distributions of the two infinite planes.

Express the electric field at any point in space due to the charge distributions on the two planes:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 \qquad (1)$$

(*a*) Express the electric field at (6 m, 2 m) due to plane 1:

$$\vec{E}_1(6\,\text{m},2\,\text{m}) = \frac{\sigma_1}{2\in_0}\hat{\boldsymbol{j}} = \frac{65\,\text{nC/m}^2}{2(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)}\hat{\boldsymbol{j}} = (3.67\,\text{kN/C})\hat{\boldsymbol{j}}$$

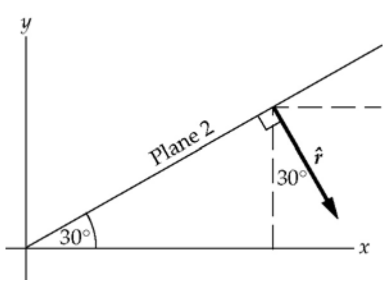
Express the electric field at (6 m, 2 m) due to plane 2:

$$\vec{E}_2(6\,\text{m},2\,\text{m}) = \frac{\sigma_2}{2\in_0}\hat{\boldsymbol{r}} = \frac{45\,\text{nC/m}^2}{2(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)}\hat{\boldsymbol{r}} = (2.54\,\text{kN/C})\hat{\boldsymbol{r}}$$

where $\hat{\boldsymbol{r}}$ is a unit vector pointing from plane 2 toward the point whose coordinates are (6 m, 2 m).

Refer to the diagram below to obtain:

$$\hat{\boldsymbol{r}} = \sin 30°\hat{\boldsymbol{i}} - \cos 30°\hat{\boldsymbol{j}}$$



Substitute to obtain:

$$\vec{E}_2(6\,\text{m},2\,\text{m}) = (2.54\,\text{kN/C})(\sin 30°\hat{i} - \cos 30°\hat{j}) = (1.27\,\text{kN/C})\hat{i} + (-2.20\,\text{kN/C})\hat{j}$$

Substitute in equation (1) to obtain:

$$\vec{E}(6\,\text{m},2\,\text{m}) = (3.67\,\text{kN/C})\hat{j} + (1.27\,\text{kN/C})\hat{i} + (-2.20\,\text{kN/C})\hat{j}$$
$$= \boxed{(1.27\,\text{kN/C})\hat{i} + (1.47\,\text{kN/C})\hat{j}}$$

(*b*) Note that $\vec{E}_1(6\,\text{m},5\,\text{m}) = \vec{E}_1(6\,\text{m},2\,\text{m})$ so that:

$$\vec{E}_1(6\,\text{m},5\,\text{m}) = \frac{\sigma}{2\in_0}\hat{j} = \frac{65\,\text{nC/m}^2}{2(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)}\hat{j} = (3.67\,\text{kN/C})\hat{j}$$

Note also that $\vec{E}_2(6\,\text{m},5\,\text{m}) = -\vec{E}_2(6\,\text{m},2\,\text{m})$ so that:

$$\vec{E}_2(6\,\text{m},5\,\text{m}) = (-1.27\,\text{kN/C})\hat{i} + (2.20\,\text{kN/C})\hat{j}$$

Substitute in equation (1) to obtain:

$$\vec{E}(6\,\text{m},5\,\text{m}) = (3.67\,\text{kN/C})\hat{j} + (-1.27\,\text{kN/C})\hat{i} + (2.20\,\text{kN/C})\hat{j}$$
$$= \boxed{(-1.27\,\text{kN/C})\hat{i} + (5.87\,\text{kN/C})\hat{j}}$$

75 ••

Picture the Problem Because the atom is uncharged, we know that the integral of the electron's charge distribution over all of space must equal its charge *e*. Evaluation of this integral will lead to an expression for ρ_0. In (*b*) we can express the resultant field at any point as the sum of the fields due to the proton and the electron cloud.

(*a*) Because the atom is uncharged:

$$e = \int_0^\infty \rho(r)dV = \int_0^\infty \rho(r)4\pi r^2 dr$$

Substitute for $\rho(r)$:

$$e = \int_0^\infty \rho_0 e^{-2r/a}4\pi r^2 dr = 4\pi\rho_0\int_0^\infty r^2 e^{-2r/a}dr$$

Use integral tables or integration by parts to obtain:

$$\int_0^\infty r^2 e^{-2r/a}dr = \frac{a^3}{4}$$

Substitute to obtain:

$$e = 4\pi\rho_0\left(\frac{a^3}{4}\right) = \pi a^3\rho_0$$

Solve for ρ_0:

$$\rho_0 = \boxed{\frac{e}{\pi a^3}}$$

(*b*) The field will be the sum of the field due to the proton and that of the electron charge cloud:

$$E = E_p + E_{cloud} = \frac{kq}{r^2} + E_{cloud}$$

Express the field due to the electron cloud:

$$E_{cloud}(r) = \frac{kQ(r)}{r^2}$$

where $Q(r)$ is the net negative charge enclosed a distance r from the proton.

Substitute to obtain:

$$E(r) = \frac{ke}{r^2} + \frac{kQ(r)}{r^2}$$

As in (*a*), $Q(r)$ is given by:

$$Q(r) = \int_0^r 4\pi r' \;\rho(r')dr'$$

Integrate to find $Q(r)$ and substitute in the expression for E to obtain:

$$E(r) = \boxed{\frac{ke}{r^2}e^{-2r/a}\left(1 + \frac{2r}{a} + \frac{2r^2}{a^2}\right)}$$

***76 ••**

Picture the Problem We will assume that the radius at which they balance is large enough that only the third term in the expression matters. Apply a condition for equilibrium will yield an equation that we can solve for the distance *r*.

Apply $\sum F = 0$ to the proton:

$$\frac{2ke^2}{a^2}e^{-2r/a} - mg = 0$$

To solve for *r*, isolate the exponential factor and take the natural logarithm of both sides of the equation:

$$r = \frac{a}{2}\ln\left(\frac{2ke^2}{mga^2}\right)$$

Substitute numerical values and evaluate *r*:

$$r = \frac{0.0529\,\text{nm}}{2}\ln\left[\frac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(1.67\times10^{-27}\ \text{kg}\right)\left(9.81\,\text{m/s}^2\right)\left(0.0529\,\text{nm}\right)^2}\right] = \boxed{1.16\,\text{nm}}$$

Thus, even though the unscreened electrostatic force is 40 orders of magnitude larger than the gravitational force, screening reduces it to smaller than the gravitational force within a few nanometers.

Remarks: Note that the argument of the logarithm contains the ratio between the gravitational potential energy of a mass held a distance a_0 above the surface of the earth and the electrostatic potential energy for two unscreened charges a distance a_0 apart.

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Picture the Problem In parts (*a*) and (*b*) we can express the charges on each of the elements as the product of the linear charge density of the ring and the length of the segments. Because the lengths of the segments are the product of the angle subtended at *P* and their distances from *P*, we can express the charges in terms of their distances from *P*. By expressing the ratio of the fields due to the charges on s_1 and s_2 we can determine their dependence on r_1 and r_2 and, hence, the resultant field at *P*. We can proceed similarly in part (*c*) with *E* varying as $1/r$ rather than $1/r^2$. In part (*d*), with s_1 and s_2 representing areas, we'll use the definition of the solid angle subtended by these areas to relate their charges to their distances from point *P*.

(*a*) Express the charge q_1 on the element of length s_1:

$$q_1 = \lambda s_1 = \lambda\theta r_1$$

where θ is the angle subtended by the arcs of length s_1 and s_2.

Express the charge q_2 on the element of length s_2:

$$q_2 = \lambda s_2 = \lambda\theta r_2$$

Divide the first of these equations by the second to obtain:

$$\frac{q_1}{q_2} = \frac{\lambda\theta r_1}{\lambda\theta r_2} = \boxed{\frac{r_1}{r_2}}$$

Express the electric field at *P* due to the charge associated with the element of length s_1:

$$E_1 = \frac{kq_1}{r_1^2} = \frac{k\lambda s_1}{r_1^2} = \frac{k\lambda\theta r_1}{r_1^2} = \frac{k\lambda\theta}{r_1}$$

Express the electric field at *P* due to the charge associated with the element of length s_2:

$$E_2 = \frac{k\lambda\theta}{r_2}$$

Divide the first of these equations by the second to obtain:

$$\frac{E_1}{E_2} = \frac{\dfrac{k\lambda\theta}{r_1}}{\dfrac{k\lambda\theta}{r_2}} = \frac{r_2}{r_1}$$

and, because $r_2 > r_1$,

$$\boxed{E_1 > E_2}$$

(*b*) The two fields point away from their segments of arc.

Because $E_1 > E_2$, the resultant field points toward s_2.

(*c*) If *E* varies as $1/r$:

$$E_1 = \frac{kq_1}{r_1} = \frac{k\lambda s_1}{r_1} = \frac{k\lambda\theta r_1}{r_1} = k\lambda\theta$$

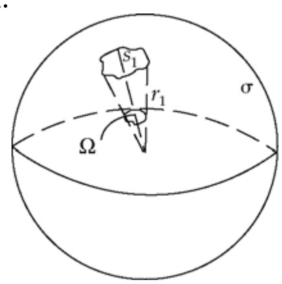
and

$$E_2 = \frac{kq_2}{r_2} = \frac{k\lambda s_2}{r_2} = \frac{k\lambda\theta r_2}{r_2} = k\lambda\theta$$

Therefore:

$$\boxed{E_1 = E_2}$$

(*d*) Use the definition of the solid angle Ω subtended by the area s_1 to obtain:



$$\frac{\Omega}{4\pi} = \frac{s_1}{4\pi r_1^2}$$

or

$$s_1 = \Omega r_1^2$$

Express the charge q_1 of the area s_1:

$$q_1 = \sigma s_1 = \sigma\Omega r_1^2$$

Similarly, for an element of area s_2:

$$s_2 = \Omega r_2^2$$

and

$$q_2 = \sigma\Omega r_2^2$$

Express the ratio of q_1 to q_2 to obtain:

$$\frac{q_1}{q_2} = \frac{\sigma\Omega r_1^2}{\sigma\Omega r_2^2} = \boxed{\frac{r_1^2}{r_2^2}}$$

Proceed as in (*a*) to obtain:

$$\frac{E_1}{E_2} = \frac{\dfrac{kq_1}{r_1^2}}{\dfrac{kq_2}{r_2^2}} = \frac{r_2^2 q_1}{r_1^2 q_2} = \frac{r_2^2\sigma\Omega r_1^2}{r_1^2\sigma\Omega r_2^2} = 1$$

Because the two fields are of equal magnitude and oppositely directed: $\vec{E} = 0$

If $E \propto 1/r$, then s_2 would produce the stronger field at P and $\vec{E}$ would point toward s_1.

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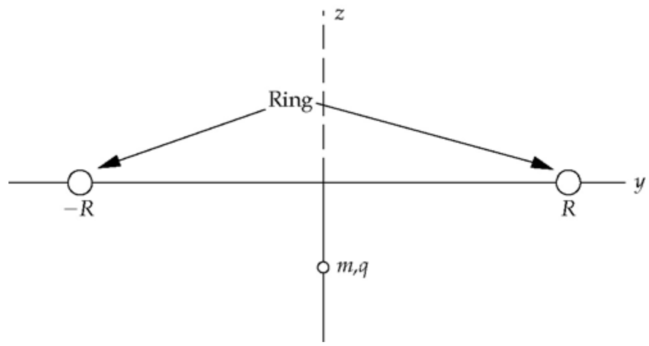
Picture the Problem We can apply the condition for translational equilibrium to the particle and use the expression for the electric field on the axis of a ring charge to obtain an expression for $|q|/m$. Doing so will lead us to the conclusion that $|q|/m$ will be a minimum when E_z is a maximum and so we'll use the result from Problem 26 that $z = -R/\sqrt{2}$ maximizes E_z.



(*a*) Apply $\sum F_z = 0$ to the particle: $|q|E_z - mg = 0$

Solve for $|q|/m$:

$$\frac{|q|}{m} = \frac{g}{E_z} \qquad (1)$$

Note that this result tells us that the minimum value of $|q|/m$ will be where the field due to the ring is greatest.

Express the electric field along the z axis due to the ring of charge:

$$E_z = \frac{kQz}{\left(z^2 + R^2\right)^{3/2}}$$

Differentiate this expression with respect to z to obtain:

$$\frac{dE_x}{dz} = kQ\frac{d}{dz}\left[\frac{x}{\left(z^2+R^2\right)^{3/2}}\right] = kQ\frac{\left(z^2+R^2\right)^{3/2} - z\frac{d}{dx}\left(z^2+R^2\right)^{3/2}}{\left(z^2+R^2\right)^3}$$

$$= kQ\frac{\left(z^2+R^2\right)^{3/2} - z\left(\tfrac{3}{2}\right)\left(z^2+R^2\right)^{1/2}(2z)}{\left(z^2+R^2\right)^3} = kQ\frac{\left(z^2+R^2\right)^{3/2} - 3z^2\left(z^2+R^2\right)^{1/2}}{\left(z^2+R^2\right)^3}$$

Set this expression equal to zero for extrema and simplify:

$$\frac{\left(z^2+R^2\right)^{3/2}-3z^2\left(z^2+R^2\right)^{1/2}}{\left(z^2+R^2\right)^3}=0\,,$$

$$\left(z^2+R^2\right)^{3/2}-3z^2\left(z^2+R^2\right)^{1/2}=0\,,$$

and

$$z^2+R^2-3z^2=0$$

Solve for *x* to obtain:

$$z=\pm\frac{R}{\sqrt{2}}$$

as candidates for maxima or minima.

You can either plot a graph of E_z or evaluate its second derivative at these points to show that it is a maximum at:

$$z=-\frac{R}{\sqrt{2}}$$

Substitute to obtain an expression $E_{z,\max}$:

$$E_{z,\max}=\left|\frac{kQ\left(-\frac{R}{\sqrt{2}}\right)}{\left(\left(-\frac{R}{\sqrt{2}}\right)^2+R^2\right)^{3/2}}\right|=\frac{2kQ}{\sqrt{27}R^2}$$

Substitute in equation (1) to obtain:

$$\frac{|q|}{m}=\boxed{\frac{\sqrt{27}gR^2}{2kQ}}$$

(*b*) If $|q|/m$ is twice as great as in (*a*), then the electric field should be half its value in (*a*), i.e.:

$$\frac{kQ}{\sqrt{27}R^2}=\frac{kQz}{\left(z^2+R^2\right)^{3/2}}$$

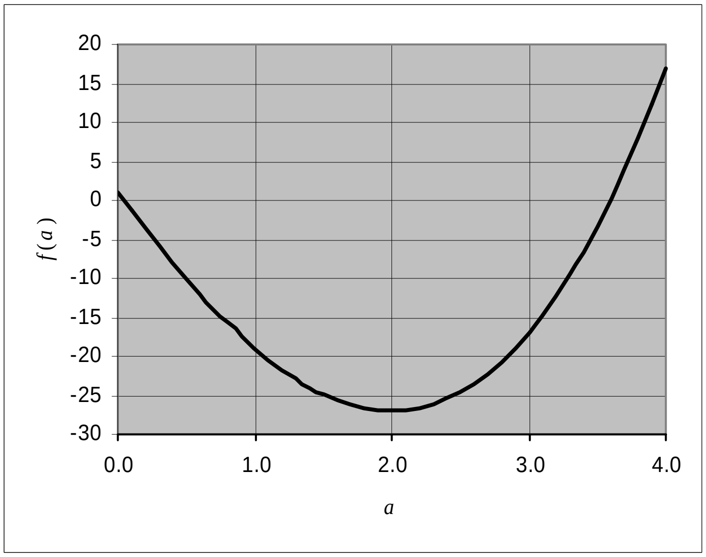
or

$$\frac{1}{27R^4}=\frac{z^2}{R^6\left(1+\frac{z^2}{R^2}\right)^3}$$

Let $a = z^2/R^2$ and simplify to obtain:

$$a^3+3a^2-24a+1=0$$

The graph of $f(a)=a^3+3a^2-24a+1$ shown below was plotted using a spreadsheet program.

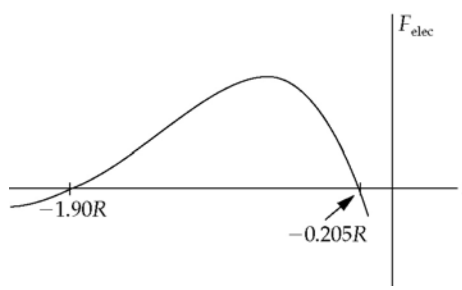


Use your calculator or trial-and-error methods to obtain: $a = 0.04188$ and $a = 3.596$

The corresponding z values are: $z = -0.205R$ and $z = -1.90R$

The condition for a stable equilibrium position is that the particle, when displaced from its equilibrium position, experiences a restoring force, i.e. a force that acts toward the equilibrium position. When the particle in this problem is just above its equilibrium position the net force on it must be downward and when it is just below the equilibrium position the net force on it must be upward. Note that the electric force is zero at the origin, so the net force there is downward and remains downward to the first equilibrium position as the weight force exceeds the electric force in this interval. The net force is upward between the first and second equilibrium positions as the electric force exceeds the weight force. The net force is downward below the second equilibrium position as the weight force exceeds the electric force. Thus, the first (higher) equilibrium position is stable and the second (lower) equilibrium position is unstable.

You might also find it instructive to use your graphing calculator to plot a graph of the electric force (the gravitational force is constant and only shifts the graph of the total force downward). Doing so will produce a graph similar to the one shown in the sketch to the right.



Note that the slope of the graph is negative on both sides of $-0.205R$ whereas it is positive on both sides of $-1.90R$; further evidence that $-0.205R$ is a position of stable equilibrium and $-1.90R$ a position of unstable equilibrium.

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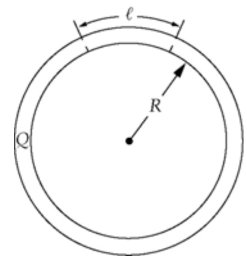
Picture the Problem The loop with the small gap is equivalent to a closed loop and a charge of $-Q\ell/2\pi R$ at the gap. The field at the center of a closed loop of uniform line charge is zero. Thus the field is entirely due to the charge $-Q\ell/2\pi R$.



(*a*) Express the field at the center of the loop:

$$\vec{E}_{\text{center}} = \vec{E}_{\text{loop}} + \vec{E}_{\text{gap}} \qquad (1)$$

Relate the field at the center of the loop to the charge in the gap:

$$\vec{E}_{\text{gap}} = -\frac{kq}{R^2}\hat{r}$$

Use the definition of linear charge density to relate the charge in the gap to the length of the gap:

$$\lambda = \frac{q}{\ell} = \frac{Q}{2\pi R}$$

or

$$q = \frac{Q\ell}{2\pi R}$$

Substitute to obtain:

$$\vec{E}_{\text{gap}} = -\frac{kQ\ell}{2\pi R^3}\hat{r}$$

Substitute in equation (1) to obtain:

$$\vec{E}_{\text{center}} = 0 - \frac{kQ\ell}{2\pi R^3}\hat{r} = -\frac{kQ\ell}{2\pi R^3}\hat{r}$$

If Q is positive, the field at the origin points radially outward.

(*b*) From our result in (*a*) we see that the magnitude of $\vec{E}_{\text{center}}$ is:

$$E_{\text{center}} = \boxed{\frac{kQ\ell}{2\pi R^3}}$$

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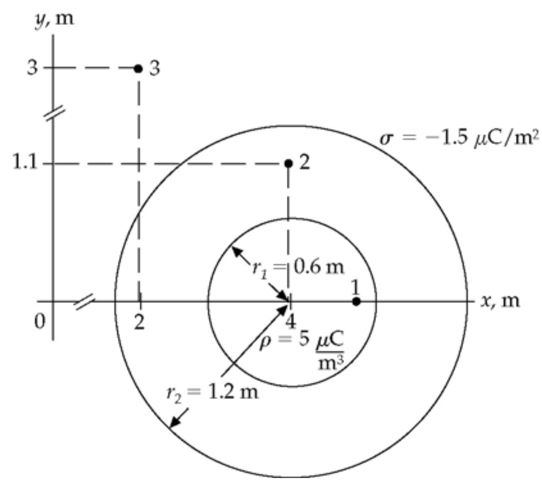
Picture the Problem We can find the electric fields at the three points of interest, labeled 1, 2, and 3 in the diagram, by adding the electric fields due to the charge distributions on the nonconducting sphere and the spherical shell.



Express the electric field due to the nonconducting sphere and the spherical shell at any point in space:

$$\vec{E} = \vec{E}_{\text{sphere}} + \vec{E}_{\text{shell}} \qquad (1)$$

(*a*) Because (4.5 m, 0) is inside the spherical shell:

$$\vec{E}_{\text{shell}}(4.5\,\text{m},0) = 0$$

Apply Gauss's law to a spherical surface inside the nonconducting sphere to obtain:

$$\vec{E}_{\text{sphere}}(r) = \frac{4\pi}{3}k\rho r\hat{\boldsymbol{i}}$$

Evaluate $\vec{E}_{\text{sphere}}(0.5\,\text{m})$:

$$\vec{E}_{\text{sphere}}(0.5\,\text{m}) = \frac{4\pi}{3}\left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(5\,\mu\text{C/m}^2\right)(0.5\,\text{m})\hat{\boldsymbol{i}} = (94.1\,\text{kN/C})\hat{\boldsymbol{i}}$$

Substitute in equation (1) to obtain:

$$\vec{E}(4.5\,\text{m},0) = (94.1\,\text{kN/C})\hat{\boldsymbol{i}} + 0 = \boxed{(94.1\,\text{kN/C})\hat{\boldsymbol{i}}}$$

Find the magnitude and direction of $\vec{E}(4.5\,\text{m},0)$:

$$E(4.5\,\text{m},0) = \boxed{94.1\,\text{kN/C}}$$

and

$$\theta = \boxed{0°}$$

(*b*) Because (4 m, 1.1m) is inside the spherical shell:

$$\vec{E}_{\text{shell}}(4\,\text{m},1.1\,\text{m}) = 0$$

Evaluate $\vec{E}_{\text{sphere}}(1.1\,\text{m})$:

$$\vec{E}_{\text{sphere}}(1.1\,\text{m}) = \frac{4\pi(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(5\,\mu\text{C/m}^2)(0.6\,\text{m})^3}{3(1.1\,\text{m})^2}\,\hat{\boldsymbol{j}} = (33.6\,\text{kN/C})\hat{\boldsymbol{j}}$$

Substitute in equation (1) to obtain:

$$\vec{E}(4.5\,\text{m},0) = (33.6\,\text{kN/C})\hat{\boldsymbol{j}} + 0 = \boxed{(33.6\,\text{kN/C})\hat{\boldsymbol{j}}}$$

Find the magnitude and direction of $\vec{E}(4.5\,\text{m},1.1\,\text{m})$:

$$E(4.5\,\text{m},1.1\,\text{m}) = \boxed{33.6\,\text{kN/C}}$$

and

$$\theta = \boxed{90°}$$

(c) Because (2 m, 3 m) outside the spherical shell:

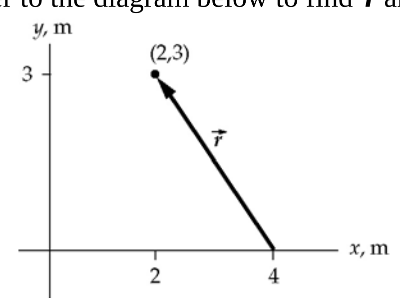
$$\vec{E}_{\text{shell}}(r) = \frac{kQ_{\text{shell}}}{r^2}\hat{\boldsymbol{r}}$$

where $\hat{\boldsymbol{r}}$ is a unit vector pointing from (4 m, 0) to (2 m, 3 m).

Evaluate Q_{shell}:

$$Q_{\text{shell}} = \sigma A_{\text{shell}} = 4\pi(-1.5\,\mu\text{C/m}^2)(1.2\,\text{m})^2 = -27.1\,\mu\text{C}$$

Refer to the diagram below to find $\hat{\boldsymbol{r}}$ and r:



$$r = 3.61\,\text{m}$$

and

$$\hat{\boldsymbol{r}} = -0.555\hat{\boldsymbol{i}} + 0.832\hat{\boldsymbol{j}}$$

Substitute and evaluate $\vec{E}_{\text{shell}}(2\,\text{m},3\,\text{m})$:

$$\vec{E}_{\text{shell}}(3.61\,\text{m}) = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-27.1\,\mu\text{C})}{(3.61\,\text{m})^2}\hat{\boldsymbol{r}}$$
$$= (-18.7\,\text{kN/C})(-0.555\hat{\boldsymbol{i}} + 0.832\hat{\boldsymbol{j}})$$
$$= (10.4\,\text{kN/C})\hat{\boldsymbol{i}} + (-15.6\,\text{kN/C})\hat{\boldsymbol{j}}$$

Express the electric field due to the charged nonconducting sphere at a distance r from its center that is greater than its radius:

$$\vec{E}_{\text{sphere}}(r) = \frac{kQ_{\text{sphere}}}{r^2}\hat{r}$$

Find the charge on the sphere:

$$Q_{\text{sphere}} = \rho V_{\text{sphere}} = \frac{4\pi}{3}\left(5\ \mu\text{C/m}^2\right)\left(0.6\ \text{m}\right)^3 = 4.52\ \mu\text{C}$$

Evaluate $\vec{E}_{\text{sphere}}(3.61\text{m})$:

$$\begin{aligned}\vec{E}_{\text{sphere}}(2\,\text{m},3\,\text{m}) &= \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(4.52\ \mu\text{C}\right)}{\left(3.61\,\text{m}\right)^2}\hat{r} = \left(3.12\,\text{kN/C}\right)\hat{r}\\ &= \left(3.12\,\text{kN/C}\right)\left(-0.555\hat{i} + 0.832\hat{j}\right)\\ &= \left(-1.73\,\text{kN/C}\right)\hat{i} + \left(2.59\,\text{kN/C}\right)\hat{j}\end{aligned}$$

Substitute in equation (1) to obtain:

$$\begin{aligned}\vec{E}(2\,\text{m},3\,\text{m}) &= \left(10.4\,\text{kN/C}\right)\hat{i} + \left(-15.6\,\text{kN/C}\right)\hat{j} + \left(-1.73\,\text{kN/C}\right)\hat{i} + \left(2.59\,\text{kN/C}\right)\hat{j}\\ &= \boxed{\left(8.67\,\text{kN/C}\right)\hat{i} + \left(-13.0\,\text{kN/C}\right)\hat{j}}\end{aligned}$$

Find the magnitude and direction of $\vec{E}(2\,\text{m},3\,\text{m})$:

$$E(2\,\text{m},3\,\text{m}) = \sqrt{\left(8.67\,\text{kN/C}\right)^2 + \left(-13.0\,\text{kN/C}\right)^2} = \boxed{15.6\,\text{kN/C}}$$

and

$$\theta = \tan^{-1}\left(\frac{-13.0\,\text{kN/C}}{8.67\,\text{kN/C}}\right) = \boxed{304°}$$

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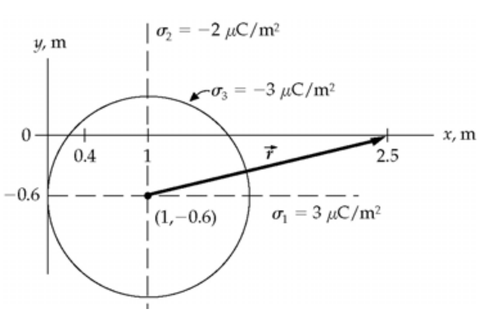
Picture the Problem Let the numeral 1 refer to the infinite plane whose charge density is σ_1 and the numeral 2 to the infinite plane whose charge density is σ_2. We can find the electric fields at the two points of interest by adding the electric fields due to the charge distributions on the infinite planes and the sphere.



Express the electric field due to the infinite planes and the sphere at any point in space:

$$\vec{E} = \vec{E}_{\text{sphere}} + \vec{E}_1 + \vec{E}_2 \qquad (1)$$

(*a*) Because (0.4 m, 0) is inside the sphere:

$$\vec{E}_{\text{sphere}}(0.4\,\text{m},0) = 0$$

Find the field at (0.4 m, 0) due to plane 1:

$$\vec{E}_1(0.4\,\text{m},0) = \frac{\sigma_1}{2\in_0}\hat{\boldsymbol{j}} = \frac{3\,\mu\text{C/m}^2}{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)}\hat{\boldsymbol{j}} = (169\,\text{kN/C})\hat{\boldsymbol{j}}$$

Find the field at (0.4 m, 0) due to plane 2:

$$\vec{E}_2(0.4\,\text{m},0) = \frac{\sigma_2}{2\in_0}\left(-\hat{\boldsymbol{i}}\right) = \frac{-2\,\mu\text{C/m}^2}{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)}\left(-\hat{\boldsymbol{i}}\right) = (113\,\text{kN/C})\hat{\boldsymbol{i}}$$

Substitute in equation (1) to obtain:

$$\vec{E}(0.4\,\text{m},0) = 0 + (169\,\text{kN/C})\hat{\boldsymbol{j}} + (113\,\text{kN/C})\hat{\boldsymbol{i}} = (113\,\text{kN/C})\hat{\boldsymbol{i}} + (169\,\text{kN/C})\hat{\boldsymbol{j}}$$

Find the magnitude and direction of $\vec{E}(0.4\,\text{m},0)$:

$$E(0.4\,\text{m},0) = \sqrt{(113\,\text{kN/C})^2 + (169\,\text{kN/C})^2} = \boxed{203\,\text{kN/C}}$$

and

$$\theta = \tan^{-1}\left(\frac{169\,\text{kN/C}}{113\,\text{kN/C}}\right) = \boxed{56.2^\circ}$$

(*b*) Because (2.5 m, 0) is outside the sphere:

$$\vec{E}_{\text{sphere}}(0.4\,\text{m},0) = \frac{kQ_{\text{sphere}}}{r^2}\hat{\boldsymbol{r}}$$

where $\hat{\boldsymbol{r}}$ is a unit vector pointing from (1 m, –0.6 m) to (2.5 m, 0).

Evaluate Q_{sphere}:

$$Q_{sphere} = \sigma A_{sphere} = 4\pi\sigma R^2 = 4\pi(-3\,\mu\text{C/m}^2)(1\,\text{m})^2 = -37.7\,\mu\text{C}$$

Referring to the diagram above, determine r and $\hat{\boldsymbol{r}}$:

$r = 1.62\,\text{m}$

and

$\hat{\boldsymbol{r}} = 0.928\hat{\boldsymbol{i}} + 0.371\hat{\boldsymbol{j}}$

Substitute and evaluate $\vec{\boldsymbol{E}}_{sphere}(2.5\,\text{m},0)$:

$$\vec{\boldsymbol{E}}_{sphere}(2.5\,\text{m},0) = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-37.7\,\mu C)}{(1.62\,\text{m})^2}\hat{\boldsymbol{r}} = (-129\,\text{kN/C})(0.928\hat{\boldsymbol{i}} + 0.371\hat{\boldsymbol{j}}) = (-120\,\text{kN/C})\hat{\boldsymbol{i}} + (-47.9\,\text{kN/C})\hat{\boldsymbol{j}}$$

Find the field at (2.5 m, 0) due to plane 1:

$$\vec{\boldsymbol{E}}_1(2.5\,\text{m},0) = \frac{\sigma_1}{2\in_0}\hat{\boldsymbol{j}} = \frac{3\,\mu\text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\hat{\boldsymbol{j}} = (169\,\text{kN/C})\hat{\boldsymbol{j}}$$

Find the field at (2.5 m, 0) due to plane 2:

$$\vec{\boldsymbol{E}}_2(2.5\,\text{m},0) = \frac{\sigma_2}{2\in_0}\hat{\boldsymbol{i}} = \frac{-2\,\mu\text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\hat{\boldsymbol{i}} = (-113\,\text{kN/C})\hat{\boldsymbol{i}}$$

Substitute in equation (1) to obtain:

$$\vec{\boldsymbol{E}}(0.4\,\text{m},0) = (-120\,\text{kN/C})\hat{\boldsymbol{i}} + (-47.9\,\text{kN/C})\hat{\boldsymbol{j}} + (169\,\text{kN/C})\hat{\boldsymbol{j}} + (-113\,\text{kN/C})\hat{\boldsymbol{i}} = (-233\,\text{kN/C})\hat{\boldsymbol{i}} + (121\,\text{kN/C})\hat{\boldsymbol{j}}$$

Find the magnitude and direction of $\vec{\boldsymbol{E}}(2.5\,\text{m},0)$:

$$E(2.5\,\text{m},0)=\sqrt{(-233\,\text{kN/C})^2+(121\,\text{kN/C})^2}=\boxed{263\,\text{kN/C}}$$

and

$$\theta=\tan^{-1}\left(\frac{121\,\text{kN/C}}{-233\,\text{kN/C}}\right)=\boxed{153°}$$

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Picture the Problem Let *P* represent the point of interest at (1.5 m, 0.5 m). We can find the electric field at *P* by adding the electric fields due to the infinite plane, the infinite line, and the sphere. Once we've expressed the field at *P* in vector form, we can find its magnitude and direction.

Express the electric field at P:

$$\vec{E}=\vec{E}_{\text{plane}}+\vec{E}_{\text{line}}+\vec{E}_{\text{sphere}}$$

Find $\vec{E}_{\text{plane}}$ at *P*:

$$\vec{E}_{\text{plane}}=-\frac{\sigma}{2\epsilon_0}\hat{i}=-\frac{2\,\mu\text{C/m}^2}{2(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)}\hat{i}=(-113\,\text{kN/C})\hat{i}$$

Express $\vec{E}_{\text{line}}$ at *P*:

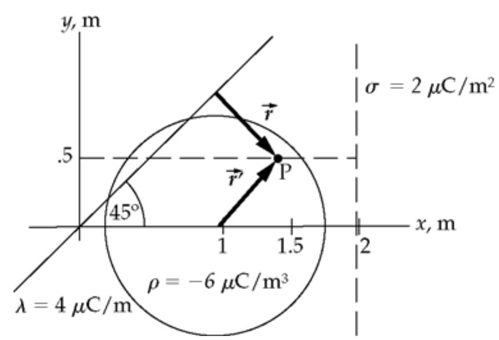
$$\vec{E}_{\text{line}}=\frac{2k\lambda}{r}\hat{r}$$

Refer to the diagram to obtain:

$$\vec{r}=(0.5\,\text{m})\hat{i}-(0.5\,\text{m})\hat{j}$$

and

$$\hat{r}=(0.707)\hat{i}-(0.707)\hat{j}$$



Substitute to obtain:

$$\vec{E}_{\text{line}}=\frac{2(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(4\,\mu\text{C/m})}{0.707\,\text{m}}\left[(0.707)\hat{i}-(0.707)\hat{j}\right]$$
$$=(102\,\text{kN/C})\left[(0.707)\hat{i}-(0.707)\hat{j}\right]=(72.1\,\text{kN/C})\hat{i}+(-72.1\,\text{kN/C})\hat{j}$$

Letting r' represent the distance from the center of the sphere to *P*,

$$\vec{E}_{\text{sphere}}=\frac{4\pi}{3}kr'\rho\hat{r}'$$

apply Gauss's law to a spherical surface of radius r' centered at (1 m, 0) to obtain an expression for $\vec{E}_{\text{sphere}}$ at P:

where $\hat{r}'$ is directed toward the center of the sphere.

Refer to the diagram used above to obtain:

$$\vec{r}' = -(0.5\,\text{m})\hat{i} - (0.5\,\text{m})\hat{j}$$

and

$$\hat{r}' = -(0.707)\hat{i} - (0.707)\hat{j}$$

Substitute to obtain:

$$\begin{aligned}\vec{E}_{\text{sphere}} &= \frac{4\pi}{3}\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(0.707\,\text{m})\left(-6\,\mu\text{C/m}^3\right)\left[(0.707)\hat{i} + (0.707)\hat{j}\right]\\ &= (-113\,\text{kN/C})\left(\hat{i} + \hat{j}\right) = (-113\,\text{kN/C})\hat{i} + (-113\,\text{kN/C})\hat{j}\end{aligned}$$

Substitute and evaluate $\vec{E}$:

$$\begin{aligned}\vec{E} &= (-113\,\text{kN/C})\hat{i} + (72.1\,\text{kN/C})\hat{i} + (-72.1\,\text{kN/C})\hat{j} + (-113\,\text{kN/C})\hat{i}\\ &\quad + (-113\,\text{kN/C})\hat{j}\\ &= (-154\,\text{kN/C})\hat{i} + (-185\,\text{kN/C})\hat{j}\end{aligned}$$

Finally, find the magnitude and direction of $\vec{E}$:

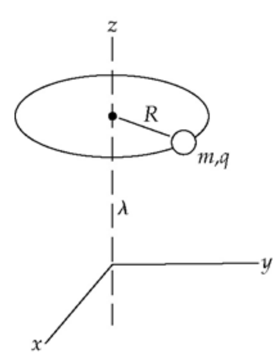
$$E = \sqrt{(-154\,\text{kN/C})^2 + (-185\,\text{kN/C})^2} = \boxed{241\,\text{kN/C}}$$

and

$$\theta = \tan^{-1}\left(\frac{-154\,\text{kN/C}}{-185\,\text{kN/C}}\right) = \boxed{220°}$$

83 ••

Picture the Problem We can find the period of the motion from its angular frequency and apply Newton's 2nd law to relate ω to m, q, R, and the electric field due to the infinite line charge. Because the electric field is given by $E_r = 2k\lambda/r$ we can express ω and, hence, T as a function of m, q, R, and λ.



Relate the period T of the particle to its angular frequency ω:

$$T = \frac{2\pi}{\omega} \qquad (1)$$

Apply Newton's 2nd law to the particle to obtain:

$$\sum F_{\text{radial}} = qE_r = mR\omega^2$$

Solve for ω:

$$\omega = \sqrt{\frac{qE_r}{mR}}$$

Express the electric field at a distance R from the infinite line charge:

$$E_r = 2k\frac{\lambda}{R}$$

Substitute in the expression for ω:

$$\omega = \sqrt{\frac{2k\lambda q}{mR^2}} = \frac{1}{R}\sqrt{\frac{2k\lambda q}{m}}$$

Substitute in equation (1) to obtain:

$$T = \boxed{2\pi R\sqrt{\frac{m}{2k\lambda q}}}$$

***84 ••**

Picture the Problem Starting with the equation for the electric field on the axis of ring charge, we can factor the denominator of the expression to show that, for $x << R$, E_x is proportional to x. We can use $F_x = qE_x$ to express the force acting on the particle and apply Newton's 2nd law to show that, for small displacements from equilibrium, the particle will execute simple harmonic motion. Finally, we can find the period of the motion from its angular frequency, which we can obtain from the differential equation of motion.

(*a*) Express the electric field on the axis of the ring of charge:

$$E_x = \frac{kQx}{\left(x^2 + R^2\right)^{3/2}}$$

Factor R^2 from the denominator of E_x to obtain:

$$E_x = \frac{kQx}{\left[R^2\left(1 + \frac{x^2}{R^2}\right)\right]^{3/2}}$$

$$= \frac{kQx}{R^3\left(1 + \frac{x^2}{R^2}\right)^{3/2}} \approx \boxed{\frac{kQ}{R^3}x}$$

provided $x << R$.

(*b*) Express the force acting on the particle as a function of its charge and the electric field:

$$F_x = qE_x = \boxed{\frac{kqQ}{R^3}x}$$

(c) Because the negatively charged particle experiences a linear restoring force, we know that its motion will be simple harmonic. Apply Newton's 2nd law to the negatively charged particle to obtain:

$$m\frac{d^2x}{dt^2} = -\frac{kqQ}{R^3}x$$

or

$$\boxed{\frac{d^2x}{dt^2} + \frac{kqQ}{mR^3}x = 0}$$

the differential equation of simple harmonic motion.

Relate the period T of the simple harmonic motion to its angular frequency ω:

$$T = \frac{2\pi}{\omega}$$

From the differential equation we have:

$$\omega^2 = \frac{kqQ}{mR^3}$$

Substitute to obtain:

$$T = \boxed{2\pi\sqrt{\frac{mR^3}{kqQ}}}$$

85 ••

Picture the Problem Starting with the equation for the electric field on the axis of a ring charge, we can factor the denominator of the expression to show that, for $x << R$, E_x is proportional to x. We can use $F_x = qE_x$ to express the force acting on the particle and apply Newton's 2nd law to show that, for small displacements from equilibrium, the particle will execute simple harmonic motion. Finally, we can find the angular frequency of the motion from the differential equation and use this expression to find its value when the radius of the ring is doubled and all other parameters remain unchanged.

Express the electric field on the axis of the ring of charge:

$$E_x = \frac{kQx}{\left(x^2 + R^2\right)^{3/2}}$$

Factor R^2 from the denominator of E_x to obtain:

$$E_x = \frac{kQx}{\left[R^2\left(1 + \frac{x^2}{R^2}\right)\right]^{3/2}} = \frac{kQx}{R^3\left(1 + \frac{x^2}{R^2}\right)^{3/2}} \approx \frac{kQ}{R^3}x$$

provided $x << R$.

Express the force acting on the particle as a function of its charge and the electric field:

$$F_x = qE_x = \frac{kqQ}{R^3}x$$

Because the negatively charged particle experiences a linear restoring force, we know that its motion will be simple harmonic. Apply Newton's 2nd law to the negatively charged particle to obtain:

$$m\frac{d^2x}{dt^2} = -\frac{kqQ}{R^3}x$$

or

$$\frac{d^2x}{dt^2} + \frac{kqQ}{mR^3}x = 0$$

the differential equation of simple harmonic motion.

The angular frequency of the simple harmonic motion of the particle is given by:

$$\omega = \sqrt{\frac{kqQ}{mR^3}} \qquad (1)$$

Express the angular frequency of the motion if the radius of the ring is doubled:

$$\omega' = \sqrt{\frac{kqQ}{m(2R)^3}} \qquad (2)$$

Divide equation (2) by equation (1) to obtain:

$$\frac{\omega'}{\omega} = \frac{\sqrt{\dfrac{kqQ}{m(2R)^3}}}{\sqrt{\dfrac{kqQ}{mR^3}}} = \frac{1}{\sqrt{8}}$$

Solve for and evaluate ω':

$$\omega' = \frac{\omega}{\sqrt{8}} = \frac{21\,\text{rad/s}}{\sqrt{8}} = \boxed{7.42\,\text{rad/s}}$$

86 ••

Picture the Problem Starting with the equation for the electric field on the axis of a ring charge, we can factor the denominator of the expression to show that, for $x << R$, E_x is proportional to x. We can use $F_x = qE_x$ to express the force acting on the particle and apply Newton's 2nd law to show that, for small displacements from equilibrium, the particle will execute simple harmonic motion. Finally, we can find the angular frequency of the motion from the differential equation and use this expression to find its value when the radius of the ring is doubled while keeping the linear charge density on the ring constant.

Express the electric field on the axis of the ring of charge:

$$E_x = \frac{kQx}{\left(x^2 + R^2\right)^{3/2}}$$

Factor R^2 from the denominator of E_x to obtain:

$$E_x = \frac{kQx}{\left[R^2\left(1+\frac{x^2}{R^2}\right)\right]^{3/2}}$$

$$= \frac{kQx}{R^3\left(1+\frac{x^2}{R^2}\right)^{3/2}} \approx \boxed{\frac{kQ}{R^3}x}$$

provided $x << R$.

Express the force acting on the particle as a function of its charge and the electric field:

$$F_x = qE_x = \boxed{\frac{kqQ}{R^3}x}$$

Because the negatively charged particle experiences a linear restoring force, we know that its motion will be simple harmonic. Apply Newton's 2nd law to the negatively charged particle to obtain:

$$m\frac{d^2x}{dt^2} = -\frac{kqQ}{R^3}x$$

or

$$\frac{d^2x}{dt^2} + \frac{kqQ}{mR^3}x = 0\,,$$

the differential equation of simple harmonic motion.

The angular frequency of the simple harmonic motion of the particle is given by:

$$\omega = \sqrt{\frac{kqQ}{mR^3}} \qquad (1)$$

Express the angular frequency of the motion if the radius of the ring is doubled while keeping the linear charge density constant (i.e., doubling Q):

$$\omega' = \sqrt{\frac{kq(2Q)}{m(2R)^3}} \qquad (2)$$

Divide equation (2) by equation (1) to obtain:

$$\frac{\omega'}{\omega} = \frac{\sqrt{\frac{kq(2Q)}{m(2R)^3}}}{\sqrt{\frac{kqQ}{mR^3}}} = \frac{1}{2}$$

Solve for and evaluate ω':

$$\omega' = \frac{\omega}{2} = \frac{21\,\text{rad/s}}{2} = \boxed{10.5\,\text{rad/s}}$$

87 ••

Picture the Problem We can apply Gauss's law to express $\vec{E}$ as a function of *r*. We can use the hint to think of the fields at points 1 and 2 as the sum of the fields due to a sphere of radius *a* with a uniform charge distribution ρ and a sphere of radius *b*, centered at *a*/2 with uniform charge distribution $-\rho$.

(*a*) The electric field at a distance *r* from the center of the sphere is given by:

$$\vec{E} = E\hat{r} \qquad (1)$$

where $\hat{r}$ is a unit vector pointing radially outward.

Apply Gauss's law to a spherical surface of radius *r* centered at the origin to obtain:

$$\oint_S E_n dA = E\left(4\pi r^2\right) = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

Relate Q_{enclosed} to the charge density ρ:

$$\rho = \frac{Q_{\text{enclosed}}}{\frac{4}{3}\pi r^3} \Rightarrow Q_{\text{enclosed}} = \tfrac{4}{3}\rho\pi r^3$$

Substitute for Q_{enclosed}:

$$E\left(4\pi r^2\right) = \frac{\frac{4}{3}\rho\pi r^3}{\epsilon_0}$$

Solve for *E* to obtain:

$$E = \frac{\rho r}{3\epsilon_0}$$

Substitute for *E* in equation (1) to obtain:

$$\vec{E} = \boxed{\frac{\rho}{3\epsilon_0} r\,\hat{r}}$$

(*b*) The electric field at point 1 is the sum of the electric fields due to the two charge distributions:

$$\vec{E}_1 = \vec{E}_\rho + \vec{E}_{-\rho} = E_\rho\hat{r} + E_{-\rho}\hat{r} \qquad (2)$$

Apply Gauss's law to relate the magnitude of the field due to the positive charge distribution to the charge enclosed by the sphere:

$$E_\rho\left(4\pi a^2\right) = \frac{q_{\text{encl}}}{\epsilon_0} = \frac{\frac{4}{3}\pi a^3\rho}{\epsilon_0}$$

Solve for E_ρ:

$$E_\rho = \frac{a\rho}{3\epsilon_0} = \frac{2\rho b}{3\epsilon_0}$$

Proceed similarly for the spherical hole to obtain:

$$E_{-\rho}\left(4\pi b^2\right) = \frac{q_{\text{encl}}}{\epsilon_0} = -\frac{\frac{4}{3}\pi b^3\rho}{\epsilon_0}$$

Solve for $E_{-\rho}$:

$$E_{-\rho} = -\frac{\rho b}{3\epsilon_0}$$

Substitute in equation (2) to obtain:

$$\vec{E}_1 = \frac{2\rho b}{3\in_0}\hat{r} - \frac{\rho b}{3\in_0}\hat{r} = \boxed{\frac{\rho b}{3\in_0}\hat{r}}$$

The electric field at point 2 is the sum of the electric fields due to the two charge distributions:

$$\vec{E}_2 = \vec{E}_\rho + \vec{E}_{-\rho} = E_\rho\hat{r} + E_{-\rho}\hat{r} \qquad (3)$$

Because point 2 is at the center of the larger sphere:

$$E_\rho = 0$$

The magnitude of the field at point 2 due to the negative charge distribution is:

$$E_{-\rho} = \frac{\rho b}{3\in_0}$$

Substitute in equation (3) to obtain:

$$\vec{E}_2 = 0 + \frac{\rho b}{3\in_0}\hat{r} = \boxed{\frac{\rho b}{3\in_0}\hat{r}}$$

88 •••

Picture the Problem The electric field in the cavity is the sum of the electric field due to the uniform and positive charge distribution of the sphere whose radius is *a* and the electric field due to any charge in the spherical cavity whose radius is *b*.

The electric field at any point inside the cavity is the sum of the electric fields due to the two charge distributions:

$$\vec{E} = \vec{E}_\rho + \vec{E}_{\text{charge inside}} = E_\rho\hat{r} + E_{\text{charge inside}}\hat{r}$$

where $\hat{r}$ is a unit vector pointing radially outward.

Because there is no charge inside the cavity:

$$E_{\text{charge inside}} = 0$$

The magnitude of the field inside the cavity due to the positive charge distribution is:

$$E_\rho = \frac{\rho b}{3\in_0}$$

Substitute in the expression for $\vec{E}$ to obtain:

$$\vec{E} = 0 + \frac{\rho b}{3\in_0}\hat{r} = \boxed{\frac{\rho}{3\in_0}b\hat{r}}$$

89 ••

Picture the Problem We can use the hint given in Problem 87 to think of the fields at points 1 and 2 as the sum of the fields due to a sphere of radius *a* with a uniform charge distribution ρ and a sphere of radius *b*, centered at *a*/2 with charge *Q* spread uniformly throughout its volume.

The electric field at point 1 is the sum of the electric fields due to the two charge distributions:

$$\vec{E}_1 = \vec{E}_\rho + \vec{E}_Q = E_\rho\hat{r} + E_Q\hat{r} \qquad (1)$$

where $\hat{r}$ is a unit vector pointing radially outward.

Apply Gauss's law to relate the field due to the positive charge distribution to the charge of the sphere:

$$E_\rho\left(4\pi a^2\right)=\frac{q_{\text{encl}}}{\epsilon_0}=\frac{\frac{4}{3}\pi a^3\rho}{\epsilon_0}$$

Solve for E_ρ:

$$E_\rho=\frac{a\rho}{3\epsilon_0}=\frac{2\rho b}{3\epsilon_0}$$

Apply Gauss's law to relate the field due to the negative charge distributed uniformly throughout the volume of the cavity :

$$E_Q\left(4\pi b^2\right)=\frac{q_{\text{encl}}}{\epsilon_0}=\frac{Q}{\epsilon_0}$$

where $Q=\rho' V=\rho'\frac{4}{3}\pi b^3$

Substitute for Q to obtain:

$$E_Q\left(4\pi b^2\right)=\frac{\frac{4}{3}\pi\rho' b^3}{\epsilon_0}$$

Solve for E_Q:

$$E_Q=\frac{\rho' b}{3\epsilon_0}$$

Substitute in equation (1) to obtain:

$$\vec{E}_1=\frac{2\rho b}{3\epsilon_0}\hat{r}+\frac{\rho' b}{3\epsilon_0}\hat{r}=\boxed{\frac{(2\rho+\rho')b}{3\epsilon_0}\hat{r}}$$

The electric field at point 2 is the sum of the electric fields due to the two charge distributions:

$$\vec{E}_2=\vec{E}_\rho+\vec{E}_Q=E_\rho\hat{r}+E_Q\hat{r} \qquad (2)$$

Because point 2 is at the center of the larger sphere:

$$E_\rho=0$$

The magnitude of the field at point 2 due to the uniformly distributed charge Q was shown above to be:

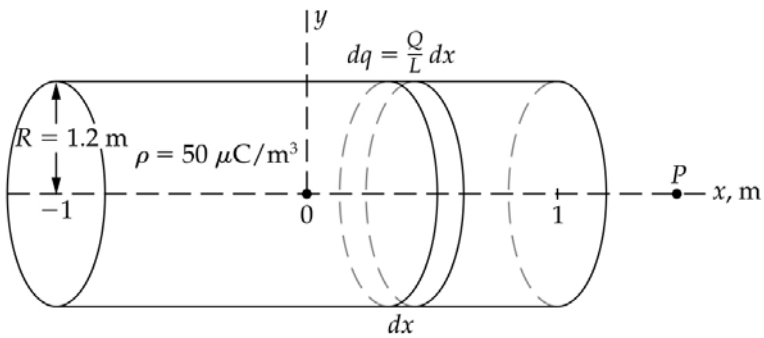
$$E_Q=\frac{\rho' b}{3\epsilon_0}$$

Substitute in equation (2) to obtain:

$$\vec{E}_2=0+\frac{\rho' b}{3\epsilon_0}\hat{r}=\boxed{\frac{\rho'}{3\epsilon_0}b\hat{r}}$$

90 ••

Picture the Problem Let the length of the cylinder be L, its radius R, and charge Q. Let P be a generic point of interest on the x axis. We can find the electric field at P by expressing the field due to an elemental disk of radius R, thickness dx, and charge dq and then integrating $E_x=2\pi k\sigma\left(1-x/\sqrt{x^2+R^2}\right)$.



Express the electric field on the x axis due to the charge carried by the disk of thickness dx:

$$dE_x = 2\pi k\rho\left(1-\frac{x}{\sqrt{x^2+R^2}}\right)dx$$

Integrate dE_x for P beyond the end of the cylinder:

$$\begin{aligned}E_x &= 2\pi k\rho\int_{x-L/2}^{x+L/2}\left(1-\frac{x}{\sqrt{x^2+R^2}}\right)dx\\ &= 2\pi k\rho\left[L-\sqrt{\left(\frac{L}{2}+x\right)^2+R^2}+\sqrt{\left(\frac{L}{2}-x\right)^2+R^2}\right]\end{aligned}$$

Integrate dE_x for P inside the cylinder:

$$\begin{aligned}E_x &= 2\pi k\rho\left[\int_0^{L/2+x}\left(1-\frac{x}{\sqrt{x^2+R^2}}\right)dx-\int_0^{L/2-x}\left(1-\frac{x}{\sqrt{x^2+R^2}}\right)dx\right]\\ &= 2\pi k\rho\left[2x-\sqrt{\left(\frac{L}{2}+x\right)^2+R^2}+\sqrt{\left(\frac{L}{2}-x\right)^2+R^2}\right]\end{aligned}$$

The effective charge density of the disk is given by:

$$\rho = \frac{Q/L}{\pi R^2}$$

Substitute numerical values and evaluate ρ:

$$\rho = \frac{50\ \mu\text{C}}{\pi(1.2\ \text{m})^2(2\ \text{m})} = 5.53\ \mu\text{C/m}^3$$

Evaluate $2\pi k\rho$:

$$2\pi k\rho = 2\pi\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(5.53\ \mu\text{C/m}^3\right) = 3.12\times10^5\ \text{N/C}\cdot\text{m}$$

(*a*) Evaluate E_x(0.5 m):

$$E_x(0.5\,\text{m}) = (3.12\times10^5\ \text{N/C}\cdot\text{m})$$
$$\times\left[2(0.5\,\text{m}) - \sqrt{\left(\frac{2\,\text{m}}{2}+0.5\,\text{m}\right)^2+(1.2\,\text{m})^2}+\sqrt{\left(\frac{2\,\text{m}}{2}-0.5\,\text{m}\right)^2+(1.2\,\text{m})^2}\right]$$
$$= \boxed{118\,\text{kN/C}}$$

(*b*) Evaluate E_x(2 m):

$$E_x(2\,\text{m}) = (3.12\times10^5\ \text{N/C}\cdot\text{m})$$
$$\times\left[2\,\text{m} - \sqrt{\left(\frac{2\,\text{m}}{2}+2\,\text{m}\right)^2+(1.2\,\text{m})^2}+\sqrt{\left(\frac{2\,\text{m}}{2}-2\,\text{m}\right)^2+(1.2\,\text{m})^2}\right]$$
$$= \boxed{103\,\text{kN/C}}$$

(*c*) Evaluate E_x(20 m):

$$E_x(20\,\text{m}) = (3.12\times10^5\ \text{N/C}\cdot\text{m})$$
$$\times\left[2\,\text{m} - \sqrt{\left(\frac{2\,\text{m}}{2}+20\,\text{m}\right)^2+(1.2\,\text{m})^2}+\sqrt{\left(\frac{2\,\text{m}}{2}-20\,\text{m}\right)^2+(1.2\,\text{m})^2}\right]$$
$$= \boxed{1.12\,\text{kN/C}}$$

Remarks: Note that, in (*c*), the distance of 20 m is much greater than the length of the cylinder that we could have used $E_x = kQ/x^2$.

91 ••

Picture the Problem We can use $E_x = kQ/[x_0(x_0 - L)]$ to express the electric fields at $x_0 = 2L$ and $x_0 = 3L$ and take the ratio of these expressions to find the field at $x_0 = 3L$.

Express the electric field along the *x* axis due to a uniform line charge on the *x* axis:

$$E_x(x_0) = \frac{kQ}{x_0(x_0 - L)}$$

Evaluate E_x at $x_0 = 2L$:

$$E_x(2L) = \frac{kQ}{2L(2L-L)} = \frac{kQ}{2L^2} \qquad (1)$$

Evaluate E_x at $x_0 = 3L$:

$$E_x(3L) = \frac{kQ}{3L(3L-L)} = \frac{kQ}{6L^2} \qquad (2)$$

Divide equation (2) by equation (1) to obtain:

$$\frac{E_x(3L)}{E_x(2L)}=\frac{\dfrac{kQ}{6L^2}}{\dfrac{kQ}{2L^2}}=\frac{1}{3}$$

Solve for and evaluate $E_x(3L)$:

$$E_x(3L)=\frac{1}{3}E_x(2L)=\frac{1}{3}(600\text{ N/C})$$
$$=\boxed{200\text{ N/C}}$$

92 •••

Picture the Problem Let the coordinates of one corner of the cube be (x,y,z), and assume that the sides of the cube are Δx, Δy, and Δz and compute the flux through the faces of the cube that are parallel to the yz plane. The net flux of the electric field out of the gaussian surface is the difference between the flux out of the surface and the flux into the surface.

The net flux out of the cube is given by:

$$\phi_{\text{net}}=\phi(x+\Delta x)-\phi(x)$$

Use a Taylor series expansion to express the net flux through faces of the cube that are parallel to the yz plane:

$$\phi_{\text{net}}=\phi(x)+(\Delta x)\phi'(x)+\tfrac{1}{2}(\Delta x)^2\phi''(x)+\ldots-\phi(x)=(\Delta x)\phi'(x)+\tfrac{1}{2}(\Delta x)^2\phi''(x)+\ldots$$

Neglecting terms higher than first order we have:

$$\phi_{\text{net}}=\Delta x\phi'(x)$$

Because the electric field is in the x direction, $\phi(x)$ is:

$$\phi(x)=E_x\Delta y\Delta z$$

and

$$\phi'(x)=\frac{\partial E_x}{\partial x}\Delta y\Delta z$$

Substitute for $\phi'(x)$ to obtain:

$$\phi_{\text{net}}=\Delta x\frac{\partial E_x}{\partial x}(\Delta y\Delta z)$$
$$=\frac{\partial E_x}{\partial x}(\Delta x\Delta y\Delta z)$$
$$=\boxed{\frac{\partial E_x}{\partial x}\Delta V}$$

93 ••

Picture the Problem We can use the definition of electric flux in conjunction with the result derived in Problem 92 to show that $\nabla\cdot\vec{E} = \rho/\epsilon_0$.

From Gauss's law, the net flux through any surface is:

$$\phi_{net} = \frac{q_{encl}}{\epsilon_0} = \frac{\rho}{\epsilon_0}V$$

Generalizing our result from Problem 92 (see the remark following Problem 92):

$$\phi_{net} = \left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right)V = \left(\nabla\cdot\vec{E}\right)V$$

Equate these two expressions to obtain:

$$\left(\nabla\cdot\vec{E}\right)V = \frac{\rho}{\epsilon_0}V \text{ or } \nabla\cdot\vec{E} = \boxed{\frac{\rho}{\epsilon_0}}$$

***94** •••

Picture the Problem We can find the field due to the infinitely long line charge from $E = 2k\lambda/r$ and the force that acts on the dipole using $F = p\,dE/dr$.

Express the force acting on the dipole:

$$F = p\frac{dE}{dr}$$

The electric field at the location of the dipole is given by:

$$E = \frac{2k\lambda}{r}$$

Substitute to obtain:

$$F = p\frac{d}{dr}\left[\frac{2k\lambda}{r}\right] = \boxed{-\frac{2k\lambda p}{r^2}}$$

where the minus sign indicates that the dipole is attracted to the line charge.

95 ••

Picture the Problem We can find the distance from the center where the net force on either charge is zero by setting the sum of the forces acting on either point charge equal to zero. Each point charge experiences two forces; one a Coulomb force of repulsion due to the other point charge, and the second due to that fraction of the sphere's charge that is between the point charge and the center of the sphere that creates an electric field at the location of the point charge.

Apply $\sum F = 0$ to either of the point charges:

$$F_{Coulomb} - F_{field} = 0 \qquad (1)$$

Express the Coulomb force on the proton:

$$F_{Coulomb} = \frac{ke^2}{(2a)^2} = \frac{ke^2}{4a^2}$$

The force exerted by the field E is:

$$F_{field} = eE$$

Apply Gauss's law to a spherical surface of radius *a* centered at the origin:

$$E\left(4\pi a^2\right)=\frac{Q_{\text{enclosed}}}{\in_0}$$

Relate the charge density of the electron sphere to Q_{enclosed}:

$$\frac{2e}{\frac{4}{3}\pi R^3}=\frac{Q_{\text{enclosed}}}{\frac{4}{3}\pi a^3}\Rightarrow Q_{\text{enclosed}}=\frac{2ea^3}{R^3}$$

Substitute for Q_{enclosed}:

$$E\left(4\pi a^2\right)=\frac{2ea^3}{\in_0 R^3}$$

Solve for E to obtain:

$$E=\frac{ea}{2\pi\in_0 R^3}\Rightarrow F_{\text{field}}=\frac{e^2a}{2\pi\in_0 R^3}$$

Substitute for F_{Coulomb} and F_{field} in equation (1):

$$\frac{ke^2}{4a^2}-\frac{e^2a}{2\pi\in_0 R^3}=0$$

or

$$\frac{ke^2}{4a^2}-\frac{2ke^2a}{R^3}=0$$

Solve for a to obtain:

$$a=\sqrt[3]{\frac{1}{8}}R=\boxed{0.5R}$$

96 •••

Picture the Problem We can use the result of Problem 96 to express the force acting on both point charges when they are separated by 2*a*. We can then use this expression to write the force function when the point charges are each displaced a small distance *x* from their equilibrium positions and then expand this function binomially to show that each point charge experiences a linear restoring force.

From Problem 95, the force function at the equilibrium position is:

$$F(a)=\frac{ke^2}{4a^2}-\frac{2ke^2a}{R^3}=0$$

When the charges are displaced a distance *x* symmetrically from their equilibrium positions, the force function becomes:

$$F(a+x)=\frac{ke^2}{4}(a+x)^{-2}-\frac{2ke^2}{R^3}(a+x)$$

Expand this function binomially to obtain:

$$\begin{aligned}F(a+x)&=\frac{ke^2}{4}\left(a^{-2}-2a^{-3}x+\ldots\right)-\frac{2ke^2}{R^3}a-\frac{2ke^2}{R^3}x\\&\approx\frac{ke^2}{4a^2}-\frac{ke^2}{2a^3}x-\frac{2ke^2}{R^3}a-\frac{2ke^2}{R^3}x\end{aligned}$$

Substitute for R using the result obtained in Problem 96 and simplify to obtain:

$$F_{\text{restoring}} = \boxed{-\left(\frac{3ke^2}{4a^3}\right)x}$$

Hence, we've shown that, for a small displacement from equilibrium, the point charges experience a linear restoring force.

Remarks: An alternative approach that you might find instructive is to expand the force function using the Taylor series.

97 •••

Picture the Problem Because the restoring force found in Problem 96 is linear, we can express the differential equation of the proton's motion and then identify ω^2 from this equation.

Apply $\sum F_x = ma$ to the displaced proton to obtain:

$$-\frac{3ke^2}{4r^3}x = m\frac{d^2x}{dt^2}$$

or

$$\frac{d^2x}{dt^2} = -\frac{3ke^2}{4mr^3}x = -\omega^2 x$$

where $\omega^2 = \dfrac{3ke^2}{4mr^3}$

Solve for ω:

$$\omega = \sqrt{\frac{3ke^2}{4mr^3}}$$

Substitute numerical values and evaluate ω:

$$\omega = \sqrt{\frac{3\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.6\times10^{-19}\ \text{C}\right)^2}{4\left(1.67\times10^{-27}\ \text{kg}\right)\left(0.08\,\text{nm}\right)^3}} = \boxed{4.49\times10^{14}\ \text{s}^{-1}}$$