Chapter 23
Electrical Potential

Conceptual Problems

***1 •**

Determine the Concept A positive charge will move in whatever direction reduces its potential energy. The positive charge will reduce its potential energy if it moves toward a region of lower electric potential.

2 ••

Picture the Problem A charged particle placed in an electric field experiences an accelerating force that does work on the particle. From the work-kinetic energy theorem we know that the work done on the particle by the net force changes its kinetic energy and that the kinetic energy K acquired by such a particle whose charge is q that is accelerated through a potential difference V is given by $K = qV$. Let the numeral 1 refer to the alpha particle and the numeral 2 to the lithium nucleus and equate their kinetic energies after being accelerated through potential differences V_1 and V_2.

Express the kinetic energy of the alpha particle when it has been accelerated through a potential difference V_1:

$$K_1 = q_1V_1 = 2eV_1$$

Express the kinetic energy of the lithium nucleus when it has been accelerated through a potential difference V_2:

$$K_2 = q_2V_2 = 3eV_2$$

Equate the kinetic energies to obtain:

$$2eV_1 = 3eV_2$$

or

$$V_2 = \tfrac{2}{3}V_1 \text{ and } \boxed{(b) \text{ is correct.}}$$

3 •

Determine the Concept If V is constant, its gradient is zero; consequently $\vec{E} = 0$.

4 •

Determine the Concept No. E can be determined from either $E_\ell = -\dfrac{dV}{d\ell}$ provided V is known and differentiable or from $E_\ell = -\dfrac{\Delta V}{\Delta \ell}$ provided V is known at two or more points.

5 •

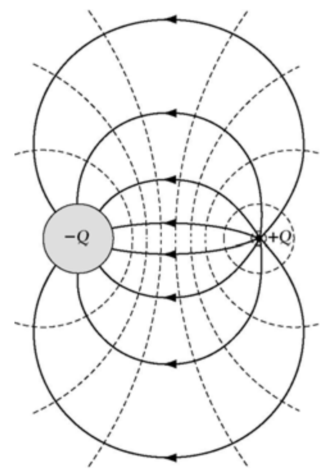
Determine the Concept Because the field lines are always perpendicular to equipotential surfaces, you move always perpendicular to the field.

6 ••

Determine the Concept *V* along the axis of the ring does not depend on the charge distribution. The electric field, however, does depend on the charge distribution, and the result given in Chapter 21 is valid only for a uniform distribution.

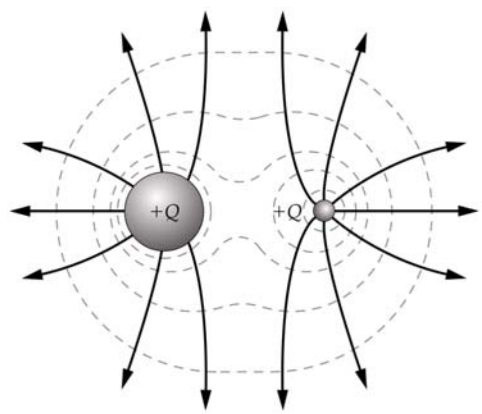
***7** ••

Picture the Problem The electric field lines, shown as solid lines, and the equipotential surfaces (intersecting the plane of the paper), shown as dashed lines, are sketched in the adjacent figure. The point charge +*Q* is the point at the right, and the metal sphere with charge –*Q* is at the left. Near the two charges the equipotential surfaces are spheres, and the field lines are normal to the metal sphere at the sphere's surface.



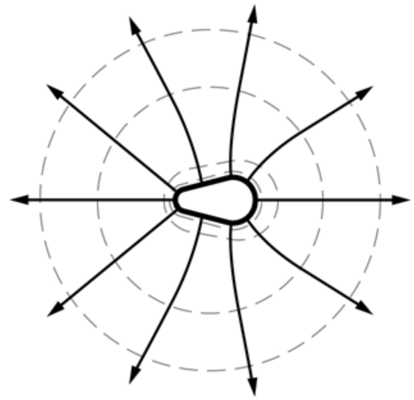
8 ••

Picture the Problem The electric field lines, shown as solid lines, and the equipotential surfaces (intersecting the plane of the paper), shown as dashed lines, are sketched in the adjacent figure. The point charge +*Q* is the point at the right, and the metal sphere with charge +*Q* is at the left. Near the two charges the equipotential surfaces are spheres, and the field lines are normal to the metal sphere at the sphere's surface. Very far from both charges, the equipotential surfaces and field lines approach those of a point charge 2*Q* located at the midpoint.



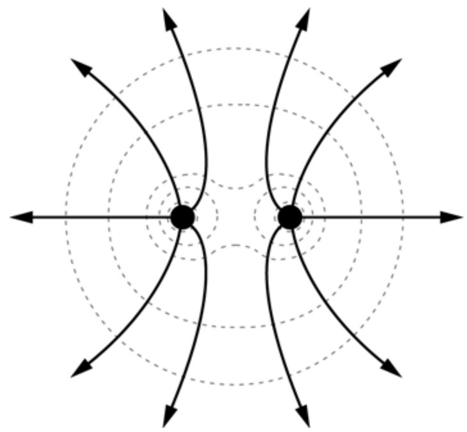
9 ••

Picture the Problem The equipotential surfaces are shown with dashed lines, the field lines are shown in solid lines. It is assumed that the conductor carries a positive charge. Near the conductor the equipotential surfaces follow the conductor's contours; far from the conductor, the equipotential surfaces are spheres centered on the conductor. The electric field lines are perpendicular to the equipotential surfaces.



10 ••

Picture the Problem The equipotential surfaces are shown with dashed lines, the electric field lines are shown with solid lines. Near each charge, the equipotential surfaces are spheres centered on each charge; far from the charges, the equipotential is a sphere centered at the midpoint between the charges. The electric field lines are perpendicular to the equipotential surfaces.



***11** •

Picture the Problem We can use Coulomb's law and the superposition of fields to find E at the origin and the definition of the electric potential due to a point charge to find V at the origin.

Apply Coulomb's law and the superposition of fields to find the electric field E at the origin:

$$\vec{E} = \vec{E}_{+Q\,\mathrm{at}\,-a} + \vec{E}_{+Q\,\mathrm{at}\,a} = \frac{kQ}{a^2}\hat{i} - \frac{kQ}{a^2}\hat{i} = 0$$

Express the potential V at the origin:

$$V = V_{+Q\,\mathrm{at}\,-a} + V_{+Q\,\mathrm{at}\,a} = \frac{kQ}{a} + \frac{kQ}{a} = \frac{2kQ}{a}$$

and $\boxed{(b)\text{ is correct.}}$

12 •

Picture the Problem We can use $\vec{E} = -\frac{\partial V}{\partial x}\hat{i}$ to find the electric field corresponding the given potential and then compare its form to those produced by the four alternatives listed.

Find the electric field corresponding to this potential function:

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} = -\frac{\partial}{\partial x}\left[4|x| + V_0\right]\hat{i}$$
$$= -4\frac{\partial}{\partial x}\left[|x|\right]\hat{i} = -4\begin{bmatrix} 1 \text{ if } x \geq 0 \\ -1 \text{ if } x < 0 \end{bmatrix}\hat{i}$$
$$= \begin{bmatrix} -4 \text{ if } x \geq 0 \\ 4 \text{ if } x < 0 \end{bmatrix}\hat{i}$$

Of the alternatives provided above, only a uniformly charged sheet in the *yz* plane would produce a constant electric field whose direction changes at the origin. $\boxed{(c) \text{ is correct.}}$

13 •

Picture the Problem We can use Coulomb's law and the superposition of fields to find *E* at the origin and the definition of the electric potential due to a point charge to find *V* at the origin.

Apply Coulomb's law and the superposition of fields to find the electric field *E* at the origin:

$$\vec{E} = \vec{E}_{+Q\,\text{at}\,-a} + \vec{E}_{-Q\,\text{at}\,a}$$
$$= \frac{kQ}{a^2}\hat{i} + \frac{kQ}{a^2}\hat{i} = \frac{2kQ}{a^2}\hat{i}$$

Express the potential *V* at the origin:

$$V = V_{+Q\,\text{at}\,-a} + V_{-Q\,\text{at}\,a}$$
$$= \frac{kQ}{a} + \frac{k(-Q)}{a} = 0$$

and $\boxed{(c) \text{ is correct}}$

14 ••

(*a*) False. As a counterexample, consider two equal charges at equal distances from the origin on the *x* axis. The electric field due to such an array is zero at the origin but the electric potential is not zero.

(*b*) True.

(*c*) False. As a counterexample, consider two equal-in-magnitude but opposite-in-sign charges at equal distances from the origin on the *x* axis. The electric potential due to such an array is zero at the origin but the electric field is not zero.

(*d*) True.

(*e*) True.

(*f*) True.

(*g*) False. Dielectric breakdown occurs in air at an *electric field strength* of approximately 3×10^6 V/m.

15 ••

(*a*) No. The potential at the surface of a conductor also depends on the local radius of the surface. Hence r and σ can vary in such a way that V is constant.

(*b*) Yes; yes.

***16** •

Determine the Concept When the two spheres are connected, their charges will redistribute until the two-sphere system is in electrostatic equilibrium. Consequently, the entire system must be an equipotential. $\boxed{(c)\text{ is corrent.}}$

Estimation and Approximation Problems

17 •

Picture the Problem The field of a thundercloud must be of order 3×10^6 V/m just before a lightning strike.

Express the potential difference between the cloud and the earth as a function of their separation d and electric field E between them:

$$V = Ed$$

Assuming that the thundercloud is at a distance of about 1 km above the surface of the earth, the potential difference is approximately:

$$V = \left(3\times10^6\ \text{V/m}\right)\left(10^3\ \text{m}\right) = \boxed{3.00\times10^9\ \text{V}}$$

Note that this is an upper bound, as there will be localized charge distributions on the thundercloud which raise the local electric field above the average value.

***18** •

Picture the Problem The potential difference between the electrodes of the spark plug is the product of the electric field in the gap and the separation of the electrodes. We'll assume that the separation of the electrodes is 1 mm.

Express the potential difference between the electrodes of the spark

$$V = Ed$$

plug as a function of their separation *d* and electric field *E* between them:

Substitute numerical values and evaluate *V*:

$$V = (2\times10^{7}\ \text{V/m})(10^{-3}\ \text{m}) = \boxed{20.0\,\text{kV}}$$

19 ••

Picture the Problem We can use conservation of energy to relate the initial kinetic energy of the protons to their electrostatic potential energy when they have approached each other to the given "radius".

(*a*) Apply conservation of energy to relate the initial kinetic energy of the protons to their electrostatic potential when they are separated by a distance *r*:

$$K_\text{i} + U_\text{i} = K_\text{f} + U_\text{f}$$

or, because $U_\text{i} = K_\text{f} = 0$,

$$K_\text{i} = U_\text{f}$$

Because each proton has kinetic energy *K*:

$$2K = \frac{e^2}{4\pi\in_0 r} \Rightarrow K = \frac{e^2}{8\pi\in_0 r}$$

Substitute numerical values and evaluate *K*:

$$K = \frac{(1.6\times10^{-19}\ \text{C})^2}{8\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(10^{-15}\ \text{m})} = 1.15\times10^{-13}\ \text{J}\times\frac{1\text{eV}}{1.6\times10^{-19}\ \text{J}} = \boxed{0.719\,\text{MeV}}$$

(*b*) Express and evaluate the ratio of the two energies:

$$f = \frac{K}{E_\text{rest}} = \frac{0.719\,\text{MeV}}{938\,\text{MeV}} = \boxed{0.0767\%}$$

20 ••

Picture the Problem The magnitude of the electric field for which dielectric breakdown occurs in air is about 3 MV/m. We can estimate the potential difference between you and your friend from the product of the length of the spark and the dielectric constant of air.

Express the product of the length of the spark and the dielectric constant of air:

$$V = (3\,\text{MV/m})(2\,\text{mm}) = \boxed{6000\,\text{V}}$$

Potential Difference

21 •

Picture the Problem We can use the definition of finite potential difference to find the potential difference $V(4\text{ m}) - V(0)$ and conservation of energy to find the kinetic energy of the charge when it is at $x = 4$ m. We can also find $V(x)$ if $V(x)$ is assigned various values at various positions from the definition of finite potential difference.

(*a*) Apply the definition of finite potential difference to obtain:

$$V(4\,\text{m}) - V(0) = -\int_a^b \vec{E}\cdot d\vec{\ell} = -\int_0^{4\,\text{m}} E\,d\ell = -(2\,\text{kN/C})(4\,\text{m}) = \boxed{-8.00\,\text{kV}}$$

(*b*) By definition, ΔU is given by:

$$\Delta U = q\Delta V = (3\,\mu\text{C})(-8\,\text{kV}) = \boxed{-24.0\,\text{mJ}}$$

(*c*) Use conservation of energy to relate ΔU and ΔK:

$$\Delta K + \Delta U = 0$$

or

$$K_{4\,\text{m}} - K_0 + \Delta U = 0$$

Because $K_0 = 0$:

$$K_{4\,\text{m}} = -\Delta U = \boxed{24.0\,\text{mJ}}$$

Use the definition of finite potential difference to obtain:

$$V(x) - V(x_0) = -E_x(x - x_0) = -(2\,\text{kV/m})(x - x_0)$$

(*d*) For $V(0) = 0$:

$$V(x) - 0 = -(2\,\text{kV/m})(x - 0)$$

or

$$V(x) = \boxed{-(2\,\text{kV/m})x}$$

(*e*) For $V(0) = 4$ kV:

$$V(x) - 4\,\text{kV} = -(2\,\text{kV/m})(x - 0)$$

or

$$V(x) = \boxed{4\,\text{kV} - (2\,\text{kV/m})x}$$

(*f*) For $V(1\text{m}) = 0$:

$$V(x) - 0 = -(2\,\text{kV/m})(x - 1)$$

or

$$V(x) = \boxed{2\,\text{kV} - (2\,\text{kV/m})x}$$

22 •

Picture the Problem Because the electric field is uniform, we can find its magnitude from $E = \Delta V/\Delta x$. We can find the work done by the electric field on the electron from the difference in potential between the plates and the charge of the electron and find the change in potential energy of the electron from the work done on it by the electric field. We can use conservation of energy to find the kinetic energy of the electron when it reaches the positive plate.

(*a*) Express the magnitude of the electric field between the plates in terms of their separation and the potential difference between them:

$$E = \frac{\Delta V}{\Delta x} = \frac{500\,\text{V}}{0.1\,\text{m}} = \boxed{5.00\,\text{kV/m}}$$

Because the electric force on a test charge is away from the positive plate and toward the negative plate, the positive plate is at the higher potential.

(*b*) Relate the work done by the electric field on the electron to the difference in potential between the plates and the charge of the electron:

$$W = q\Delta V = \left(1.6\times10^{-19}\,\text{C}\right)\left(500\,\text{V}\right) = \boxed{8.01\times10^{-17}\,\text{J}}$$

Convert 8.01×10^{-17} J to eV:

$$W = \left(8.01\times10^{-17}\,\text{J}\right)\left(\frac{1\,\text{eV}}{1.6\times10^{-19}\,\text{J}}\right) = \boxed{500\,\text{eV}}$$

(*c*) Relate the change in potential energy of the electron to the work done on it as it moves from the negative plate to the positive plate:

$$\Delta U = -W = \boxed{-500\,\text{eV}}$$

Apply conservation of energy to obtain:

$$\Delta K = -\Delta U = \boxed{500\,\text{eV}}$$

23 •

Picture the Problem The Coulomb potential at a distance r from the origin relative to $V = 0$ at infinity is given by $V = kq/r$ where q is the charge at the origin. The work that must be done by an outside agent to bring a charge from infinity to a position a distance r from the origin is the product of the magnitude of the charge and the potential difference due to the charge at the origin.

(*a*) Express and evaluate the Coulomb potential of the charge:

$$V = \frac{kq}{r} = \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(2\ \mu\text{C})}{4\ \text{m}} = \boxed{4.50\ \text{kV}}$$

(*b*) Relate the work that must be done to the magnitude of the charge and the potential difference through which the charge is moved:

$$W = q\Delta V = (3\ \mu\text{C})(4.50\ \text{kV}) = \boxed{13.5\ \text{mJ}}$$

(*c*) Express the work that must be done by the outside agent in terms of the potential difference through which the 2-μC is to be moved:

$$W = q_2\Delta V_3 = \frac{kq_2q_3}{r}$$

Substitute numerical values and evaluate W:

$$W = \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(2\ \mu\text{C})(3\ \mu\text{C})}{4\ \text{m}} = \boxed{13.5\ \text{mJ}}$$

24 ••

Picture the Problem In general, the work done by an external agent in separating the two ions changes both their kinetic and potential energies. Here we're assuming that they are at rest initially and that they will be at rest when they are infinitely far apart. Because their potential energy is also zero when they are infinitely far apart, the energy W_{ext} required to separate the ions to an infinite distance apart is the negative of their potential energy when they are a distance r apart.

Express the energy required to separate the ions in terms of the work required by an external agent to bring about this separation:

$$W_{ext} = \Delta K + \Delta U = 0 - U_i = -\frac{kq_-q_+}{r} = -\frac{k(-e)e}{r} = \frac{ke^2}{r}$$

Substitute numerical values and evaluate W_{ext}:

$$W_{ext} = \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.6\times10^{-19}\ \text{C})^2}{2.80\times10^{-10}\ \text{m}} = 8.24\times10^{-19}\ \text{J}$$

Convert W_{ext} to eV:

$$W = \left(8.24\times10^{-19}\text{ J}\right)\left(\frac{1\text{eV}}{1.6\times10^{-19}\text{ J}}\right) = \boxed{5.14\text{ eV}}$$

25 ••

Picture the Problem We can find the final speeds of the protons from the potential difference through which they are accelerated and use $E = \Delta V/\Delta x$ to find the accelerating electric field.

(*a*) Apply the work-kinetic energy theorem to the accelerated protons:

$$W = \Delta K = K_f$$

or

$$e\Delta V = \tfrac{1}{2}mv^2$$

Solve for v to obtain:

$$v = \sqrt{\frac{2e\Delta V}{m}}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{\frac{2\left(1.6\times10^{-19}\text{ C}\right)\left(5\text{ MV}\right)}{1.67\times10^{-27}\text{ kg}}} = \boxed{3.10\times10^{7}\text{ m/s}}$$

(*b*) Assuming the same potential change occurred *uniformly* over the distance of 2.0 m, we can use the relationship between E, ΔV, and Δx express and evaluate E:

$$E = \frac{\Delta V}{\Delta x} = \frac{5\text{ MV}}{2\text{ m}} = \boxed{2.50\text{ MV/m}}$$

***26** ••

Picture the Problem The work done on the electrons by the electric field changes their kinetic energy. Hence we can use the work-kinetic energy theorem to find the kinetic energy and the speed of impact of the electrons.

Use the work-kinetic energy theorem to relate the work done by the electric field to the change in the kinetic energy of the electrons:

$$W = \Delta K = K_f$$

or

$$K_f = e\Delta V \qquad (1)$$

(*a*) Substitute numerical values and evaluate K_f:

$$K_f = (1e)(30\text{ kV}) = \boxed{3\times10^{4}\text{ eV}}$$

(*b*) Convert this energy to eV:

$$K_f = (3\times10^4\ \text{eV})\left(\frac{1.6\times10^{-19}\ \text{J}}{\text{eV}}\right) = \boxed{4.80\times10^{-15}\ \text{J}}$$

(*c*) From equation (1) we have:

$$\tfrac{1}{2}mv_f^2 = e\Delta V$$

Solve for v_f to obtain:

$$v_f = \sqrt{\frac{2e\Delta V}{m}}$$

Substitute numerical values and evaluate v_f:

$$v_f = \sqrt{\frac{2(1.6\times10^{-19}\ \text{C})(30\,\text{kV})}{9.11\times10^{-31}\ \text{kg}}} = \boxed{1.03\times10^8\ \text{m/s}}$$

Remarks: Note that this speed is about one-third that of light.

27 ••

Picture the Problem We know that energy is conserved in the interaction between the α particle and the massive nucleus. Under the assumption that the recoil of the massive nucleus is negligible, we know that the initial kinetic energy of the α particle will be transformed into potential energy of the two-body system when the particles are at their distance of closest approach.

(*a*) Apply conservation of energy to the system consisting of the α particle and the massive nucleus:

$$\Delta K + \Delta U = 0$$

or

$$K_f - K_i + U_f - U_i = 0$$

Because $K_f = U_i = 0$ and $K_i = E$:

$$-E + U_f = 0$$

Letting r be the separation of the particles at closest approach, express U_f:

$$U_f = \frac{kq_{\text{nucleus}}q_\alpha}{r} = \frac{k(Ze)(2e)}{r} = \frac{2kZe^2}{r}$$

Substitute to obtain:

$$-E + \frac{2kZe^2}{r} = 0$$

Solve for r to obtain:

$$r = \boxed{\frac{2kZe^2}{E}}$$

(*b*) For a 5.0-MeV α particle and a gold nucleus:

$$r_5 = \frac{2\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)(79)\left(1.6\times10^{-19}\ \mathrm{C}\right)^2}{(5\,\mathrm{MeV})\left(1.6\times10^{-19}\ \mathrm{J/eV}\right)} = 4.55\times10^{-14}\ \mathrm{m} = \boxed{45.4\,\mathrm{fm}}$$

For a 9.0-MeV α particle and a gold nucleus:

$$r_9 = \frac{2\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)(79)\left(1.6\times10^{-19}\ \mathrm{C}\right)^2}{(9\,\mathrm{MeV})\left(1.6\times10^{-19}\ \mathrm{J/eV}\right)} = 2.53\times10^{-14}\ \mathrm{m} = \boxed{25.3\,\mathrm{fm}}$$

Potential Due to a System of Point Charges

28 •

Picture the Problem Let the numerals 1, 2, 3, and 4 denote the charges at the four corners of square and *r* the distance from each charge to the center of the square. The potential at the center of square is the algebraic sum of the potentials due to the four charges.

Express the potential at the center of the square:

$$V = \frac{kq_1}{r} + \frac{kq_2}{r} + \frac{kq_3}{r} + \frac{kq_4}{r} = \frac{k}{r}\left(q_1 + q_2 + q_3 + q_4\right) = \frac{k}{r}\sum_{i=1}^{4} q_i$$

(*a*) If the charges are positive:

$$V = \frac{8.99\times10^9\ \mathrm{N\cdot m^2/C^2}}{2\sqrt{2}\ \mathrm{m}}(4)(2\,\mu\mathrm{C}) = \boxed{25.4\,\mathrm{kV}}$$

(*b*) If three of the charges are positive and one is negative:

$$V = \frac{8.99\times10^9\ \mathrm{N\cdot m^2/C^2}}{2\sqrt{2}\ \mathrm{m}}(2)(2\,\mu\mathrm{C}) = \boxed{12.7\,\mathrm{kV}}$$

(*c*) If two are positive and two are negative:

$$V = \boxed{0}$$

29 •

Picture the Problem The potential at the point whose coordinates are (0, 3 m) is the algebraic sum of the potentials due to the charges at the three locations given.

Express the potential at the point whose coordinates are (0, 3 m):

$$V = k\sum_{i=1}^{3}\frac{q_i}{r_i} = k\left(\frac{q_1}{r_1}+\frac{q_2}{r_2}+\frac{q_3}{r_3}\right)$$

(*a*) For $q_1 = q_2 = q_3 = 2\ \mu$C:

$$V = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2\ \mu\text{C})\left(\frac{1}{3\,\text{m}}+\frac{1}{3\sqrt{2}\ \text{m}}+\frac{1}{3\sqrt{5}\ \text{m}}\right) = \boxed{12.9\,\text{kV}}$$

(*b*) For $q_1 = q_2 = 2\ \mu$C and $q_3 = -2\ \mu$C:

$$V = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2\ \mu\text{C})\left(\frac{1}{3\,\text{m}}+\frac{1}{3\sqrt{2}\ \text{m}}-\frac{1}{3\sqrt{5}\ \text{m}}\right) = \boxed{7.55\,\text{kV}}$$

(*c*) For $q_1 = q_3 = 2\ \mu$C and $q_2 = -2\ \mu$C:

$$V = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2\ \mu\text{C})\left(\frac{1}{3\,\text{m}}-\frac{1}{3\sqrt{2}\ \text{m}}+\frac{1}{3\sqrt{5}\ \text{m}}\right) = \boxed{4.44\,\text{kV}}$$

30 •

Picture the Problem The potential at point C is the algebraic sum of the potentials due to the charges at points A and B and the work required to bring a charge from infinity to point C equals the change in potential energy of the system during this process.

(*a*) Express the potential at point C as the sum of the potentials due to the charges at points A and B:

$$V_C = k\left(\frac{q_A}{r_A}+\frac{q_B}{r_B}\right)$$

Substitute numerical values and evaluate V_C:

$$V_C = kq\left(\frac{1}{r_A}+\frac{1}{r_B}\right) = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2\ \mu\text{C})\left(\frac{1}{3\,\text{m}}+\frac{1}{3\,\text{m}}\right) = \boxed{12.0\,\text{kV}}$$

(*b*) Express the required work in terms of the change in the potential energy of the system:

$$W = \Delta U = q_5V_C = (5\,\mu\text{C})(12.0\,\text{kV}) = \boxed{60.0\,\text{mJ}}$$

(*c*) Proceed as in (*a*) with $q_B = -2\ \mu$C:

$$V_C = k\left(\frac{q_A}{r_A}+\frac{q_B}{r_B}\right) = \left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(\frac{2\,\mu\mathrm{C}}{3\,\mathrm{m}}+\frac{-2\,\mu\mathrm{C}}{3\,\mathrm{m}}\right)=\boxed{0}$$

and $W = \Delta U = q_5V_C = (5\,\mu\mathrm{C})(0) = \boxed{0}$

31 •

Picture the Problem The electric potential at the origin and at the north pole is the algebraic sum of the potentials at those points due to the individual charges distributed along the equator.

(*a*) Express the potential at the origin as the sum of the potentials due to the charges placed at 60° intervals along the equator of the sphere:

$$V = k\sum_{i=1}^{6}\frac{q_i}{r_i} = 6k\frac{q}{r}$$

Substitute numerical values and evaluate *V*:

$$V = 6\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{3\,\mu\mathrm{C}}{0.6\,\mathrm{m}}$$

$$=\boxed{270\,\mathrm{kV}}$$

(*b*) Using geometry, find the distance from each charge to the north pole:

$$r' = 0.6\sqrt{2}\,\mathrm{m}$$

Proceed as in (*a*) with $r' = 0.6\sqrt{2}\,\mathrm{m}$:

$$V = k\sum_{i=1}^{6}\frac{q_i}{r_i'} = 6k\frac{q}{r'}$$

$$= 6\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{3\,\mu\mathrm{C}}{0.6\sqrt{2}\,\mathrm{m}}$$

$$=\boxed{191\,\mathrm{kV}}$$

***32** •

Picture the Problem We can use the fact that the electric potential at the point of interest is the algebraic sum of the potentials at that point due to the charges *q* and *q′* to find the ratio *q/q′*.

Express the potential at the point of interest as the sum of the potentials due to the two charges:

$$\frac{kq}{a/3}+\frac{kq'}{2a/3}=0$$

Simplify to obtain: $$q+\frac{q'}{2}=0$$

Solve for the ratio *q*/*q*′: $$\frac{q}{q'}=\boxed{-\frac{1}{2}}$$

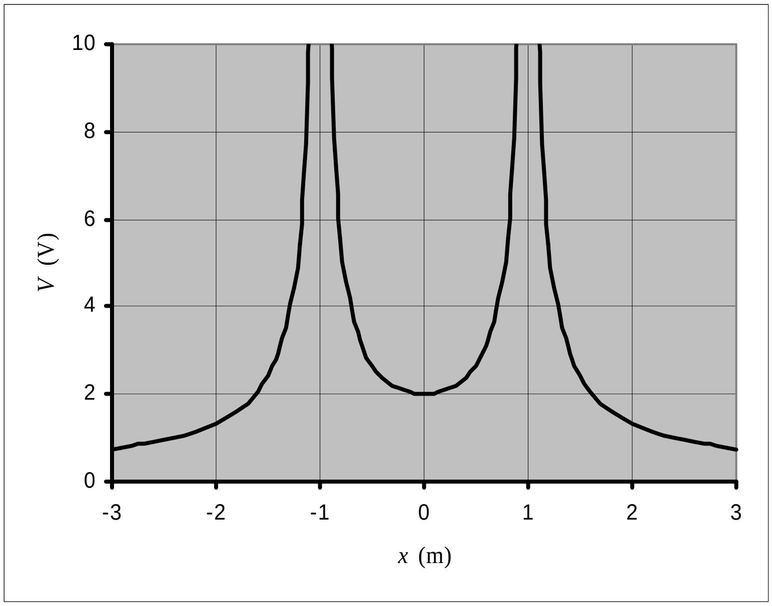
33 ••

Picture the Problem For the two charges, $r=|x-a|$ and $|x+a|$ respectively and the electric potential at *x* is the algebraic sum of the potentials at that point due to the charges at *x* = +*a* and *x* = –*a*.

(*a*) Express *V*(*x*) as the sum of the potentials due to the charges at *x* = +*a* and *x* = –*a*:

$$V=\boxed{kq\left(\frac{1}{|x-a|}+\frac{1}{|x+a|}\right)}$$

(*b*) The following graph of *V*(*x*) versus *x* for *kq* = 1 and *a* = 1 was plotted using a spreadsheet program:



(*c*) At *x* = 0: $$\frac{dV}{dx}=\boxed{0}\text{ and }E_x=-\frac{dV}{dx}=\boxed{0}$$

***34** ••

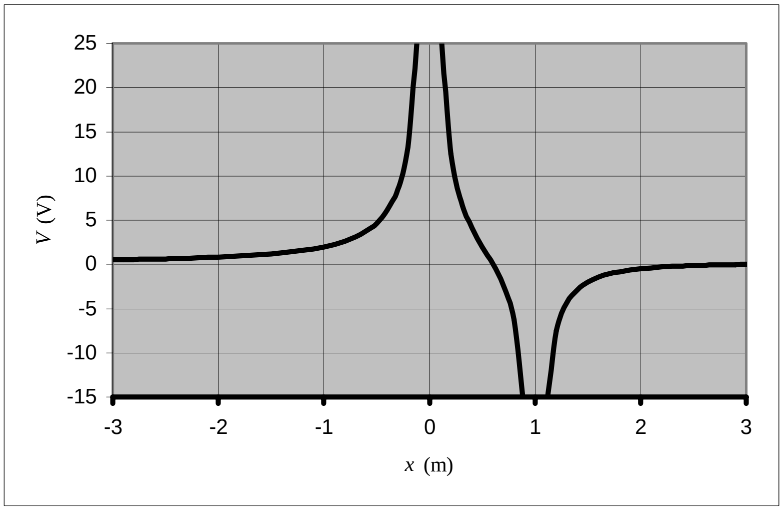
Picture the Problem For the two charges, $r=|x-a|$ and $|x|$ respectively and the electric potential at *x* is the algebraic sum of the potentials at that point due to the charges at *x* = *a* and *x* = 0. We can use the graph and the function found in part (a) to identify the points at which *V*(*x*) = 0. We can find the work needed to bring a third charge +*e* to the point

$x = \frac{1}{2}a$ on the x axis from the change in the potential energy of this third charge.

Express the potential at x:

$$V(x) = \frac{k(3e)}{|x|} + \frac{k(-2e)}{|x-a|}$$

The following graph of $V(x)$ for $ke = 1$ and $a = 1$ was plotted using a spreadsheet program.



(b) From the graph we can see that $V(x) = 0$ when:

$$x = \boxed{\pm\infty}$$

Examining the function, we see that $V(x)$ is also zero provided:

$$\frac{3}{|x|} - \frac{2}{|x-a|} = 0$$

For $x > 0$, $V(x) = 0$ when:

$$x = \boxed{3a}$$

For $0 < x < a$, $V(x) = 0$ when:

$$x = \boxed{0.6a}$$

(c) Express the work that must be done in terms of the change in potential energy of the charge:

$$W = \Delta U = qV\left(\tfrac{1}{2}a\right)$$

Evaluate the potential at $x = \frac{1}{2}a$:

$$V\left(\tfrac{1}{2}a\right) = \frac{k(3e)}{\left|\frac{1}{2}a\right|} + \frac{k(-2e)}{\left|\frac{1}{2}a - a\right|}$$

$$= \frac{6ke}{a} - \frac{4ke}{a} = \frac{2ke}{a}$$

Substitute to obtain:

$$W = e\left(\frac{2ke}{a}\right) = \boxed{\frac{2ke^2}{a}}$$

Computing the Electric Field from the Potential

35 •

Picture the Problem We can use the relationship $E_x = -(dV/dx)$ to decide the sign of $V_b - V_a$ and $E = \Delta V/\Delta x$ to find E.

(*a*) Because $E_x = -(dV/dx)$, V is greater for larger values of x. So:

$$\boxed{V_b - V_a \text{ is positive.}}$$

(*b*) Express E in terms of $V_b - V_a$ and the separation of points a and b:

$$E_x = \frac{\Delta V}{\Delta x} = \frac{V_b - V_a}{\Delta x}$$

Substitute numerical values and evaluate E_x:

$$E_x = \frac{10^5\ \text{V}}{4\,\text{m}} = \boxed{25.0\ \text{kV/m}}$$

***36** •

Picture the Problem Because $E_x = -dV/dx$, we can find the point(s) at which $E_x = 0$ by identifying the values for x for which $dV/dx = 0$.

Examination of the graph indicates that $dV/dx = 0$ at $x = 4.5$ m. Thus $E_x = 0$ at:

$$x = \boxed{4.5\,\text{m}}$$

37 •

Picture the Problem We can use $V(x) = kq/x$ to find the potential V on the x axis at $x = 3.00$ m and at $x = 3.01$ m and $E(x) = kq/r^2$ to find the electric field at $x = 3.00$ m. In part (d) we can express the off-axis potential using $V(x) = kq/r$, where $r = \sqrt{x^2 + y^2}$.

(*a*) Express the potential on the x axis as a function of x and q:

$$V(x) = \frac{kq}{x}$$

Evaluate V at $x = 3$ m:

$$V(3\,\text{m}) = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(3\,\mu\text{C})}{3\,\text{m}} = \boxed{8.99\,\text{kV}}$$

Evaluate V at $x = 3.01$ m:

$$V(3.01\,\text{m}) = \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(3\,\mu\text{C})}{3.01\,\text{m}} = \boxed{8.96\,\text{kV}}$$

(b) The potential decreases as x increases and:

$$-\frac{\Delta V}{\Delta x} = -\frac{8.96\,\text{kV} - 8.99\,\text{kV}}{3.01\,\text{m} - 3.00\,\text{m}} = \boxed{3.00\,\text{kV/m}}$$

(c) Express the Coulomb field as a function of x:

$$E(x) = \frac{kq}{x^2}$$

Evaluate this expression at $x = 3.00$ m to obtain:

$$E(3\,\text{m}) = \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(3\,\mu\text{C})}{(3\,\text{m})^2} = \boxed{3.00\,\text{kV/m}}$$

in agreement with our result in (b).

(d) Express the potential at (x, y) due to a point charge q at the origin:

$$V(x, y) = \frac{kq}{\sqrt{x^2 + y^2}}$$

Evaluate this expression at (3.00 m, 0.01 m):

$$V(3.00\,\text{m}, 0.01\,\text{m}) = \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(3\,\mu\text{C})}{\sqrt{(3.00\,\text{m})^2 + (0.01\,\text{m})^2}} = \boxed{8.99\,\text{kV}}$$

For $y << x$, V is independent of y and the points (x, 0) and (x, y) are at the same potential, i.e., on an equipotential surface.

38 •

Picture the Problem We can find the potential on the x axis at $x = 3.00$ m by expressing it as the sum of the potentials due to the charges at the origin and at $x = 6$ m. We can also express the Coulomb field on the x axis as the sum of the fields due to the charges q_1 and q_2 located at the origin and at $x = 6$ m.

(a) Express the potential on the x axis as the sum of the potentials due to the charges q_1 and q_2 located at the origin and at $x = 6$ m:

$$V(x) = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

Substitute numerical values and evaluate V(3 m):

$$V(x)=\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\times\left(\frac{3\,\mu\text{C}}{3\,\text{m}}+\frac{-3\,\mu\text{C}}{3\,\text{m}}\right)=\boxed{0}$$

(b) Express the Coulomb field on the x axis as the sum of the fields due to the charges q_1 and q_2 located at the origin and at x = 6 m:

$$E_x=\frac{kq_1}{r_1^2}+\frac{kq_2}{r_2^2}=k\left(\frac{q_1}{r_1^2}+\frac{q_2}{r_2^2}\right)$$

Substitute numerical values and evaluate E(3 m):

$$E_x=\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\times\left(\frac{3\,\mu\text{C}}{(3\,\text{m})^2}-\frac{-3\,\mu\text{C}}{(3\,\text{m})^2}\right)=\boxed{5.99\,\text{kV/m}}$$

(c) Express the potential on the x axis as the sum of the potentials due to the charges q_1 and q_2 located at the origin and at x = 6 m:

$$V(x)=k\left(\frac{q_1}{r_1}+\frac{q_2}{r_2}\right)$$

Substitute numerical values and evaluate V(3.01 m):

$$V(3.01\,\text{m})=\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\times\left(\frac{3\,\mu\text{C}}{3.01\,\text{m}}+\frac{-3\,\mu\text{C}}{2.99\,\text{m}}\right)=\boxed{-59.9\,\text{V}}$$

Compute $-\Delta V/\Delta x$:

$$-\frac{\Delta V}{\Delta x}=-\frac{-59.9\,\text{V}-0}{3.01\,\text{m}-3.00\,\text{m}}=\boxed{5.99\,\text{kV/m}}=E_x(3.00\,\text{m})$$

39 •

Picture the Problem We can use the relationship $E_y = -(dV/dy)$ to decide the sign of $V_b - V_a$ and $E = \Delta V/\Delta y$ to find E.

(a) Because $E_x = -(dV/dx)$, V is smaller for larger values of y. So:

$$\boxed{V_b - V_a \text{ is negative.}}$$

(*b*) Express E in terms of $V_b - V_a$ and the separation of points *a* and *b*:

$$E_y = \frac{\Delta V}{\Delta y} = \frac{V_b - V_a}{\Delta y}$$

Substitute numerical values and evaluate E_y:

$$E_y = \frac{2\times 10^4\ \mathrm{V}}{4\ \mathrm{m}} = \boxed{5.00\ \mathrm{kV/m}}$$

40 •

Picture the Problem Given $V(x)$, we can find E_x from $-dV/dx$.

(*a*) Find E_x from $-dV/dx$:

$$E_x = -\frac{d}{dx}[2000 + 3000x] = \boxed{-3.00\ \mathrm{kV/m}}$$

(*b*) Find E_x from $-dV/dx$:

$$E_x = -\frac{d}{dx}[4000 + 3000x] = \boxed{-3.00\ \mathrm{kV/m}}$$

(*c*) Find E_x from $-dV/dx$:

$$E_x = -\frac{d}{dx}[2000 - 3000x] = \boxed{3.00\ \mathrm{kV/m}}$$

(*d*) Find E_x from $-dV/dx$:

$$E_x = -\frac{d}{dx}[-2000] = \boxed{0}$$

41 ••

Picture the Problem We can express the potential at a general point on the x axis as the sum of the potentials due to the charges at $x = 0$ and $x = 1$ m. Setting this expression equal to zero will identify the points at which $V(x) = 0$. We can find the electric field at any point on the x axis from $E_x = -dV/dx$.

(*a*) Express $V(x)$ as the sum of the potentials due to the point charges at $x = 0$ and $x = 1$ m:

$$V(x) = \frac{kq}{|x|} + \frac{k(-3q)}{|x-1|} = \boxed{k\left(\frac{q}{|x|} - \frac{3q}{|x-1|}\right)}$$

(*b*) Set $V(x) = 0$:

$$k\left(\frac{q}{|x|} - \frac{3q}{|x-1|}\right) = 0$$

or

$$\frac{1}{|x|}-\frac{3}{|x-1|}=0$$

For $x < 0$:

$$\frac{1}{-x}-\frac{3}{-(x-1)}=0 \Rightarrow x=\boxed{-0.500\,\mathrm{m}}$$

For $0 < x < 1$:

$$\frac{1}{x}-\frac{3}{-(x-1)}=0 \Rightarrow x=\boxed{0.250\,\mathrm{m}}$$

Note also that:

$$\boxed{V(x)\to 0\,\text{as}\,x\to\pm\infty}$$

(c) Evaluate $V(x)$ for $0 < x < 1$:

$$V(0<x<1)=k\left(\frac{q}{x}+\frac{3q}{x-1}\right)$$

Apply $E_x = -dV/dx$ to find E_x in this region:

$$E_x(0<x<1)=-\frac{d}{dx}\left[k\left(\frac{q}{x}+\frac{3q}{x-1}\right)\right]$$
$$=kq\left[\frac{1}{x^2}+\frac{3}{(x-1)^2}\right]$$

Evaluate this expression at $x = 0.25$ m to obtain:

$$E_x(0.25\,\mathrm{m})=kq\left[\frac{1}{(0.25\,\mathrm{m})^2}+\frac{3}{(0.75\,\mathrm{m})^2}\right]$$
$$=\boxed{\left(21.3\,\mathrm{m}^{-2}\right)kq}$$

Evaluate $V(x)$ for $x < 0$:

$$V(x<0)=-kq\left(\frac{1}{x}+\frac{3}{1-x}\right)$$

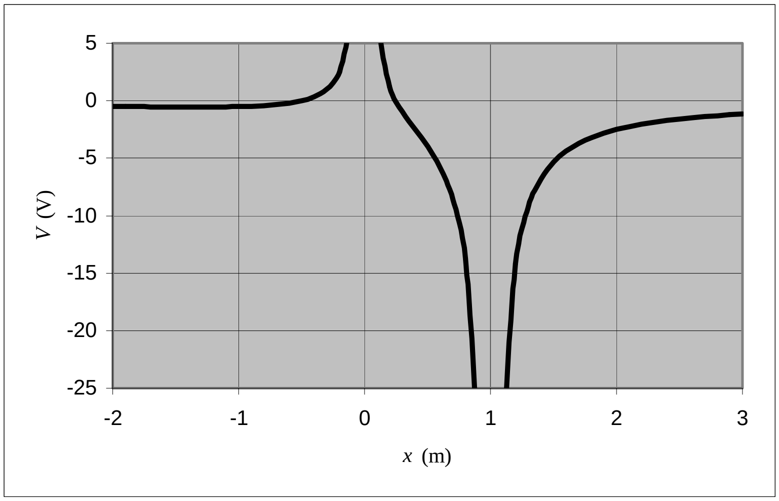
Apply $E_x = -dV/dx$ to find E_x in this region:

$$E_x(x<0)=-kq\frac{d}{dx}\left[\frac{1}{x}+\frac{3}{1-x}\right]$$
$$=kq\left[-\frac{1}{x^2}+\frac{3}{(1-x)^2}\right]$$

Evaluate this expression at $x = -0.5$ m to obtain:

$$E_x(-0.5\,\mathrm{m})=kq\left[-\frac{1}{(-0.5\,\mathrm{m})^2}+\frac{3}{(1.5\,\mathrm{m})^2}\right]=\boxed{\left(-2.67\,\mathrm{m}^{-2}\right)kq}$$

(d) The following graph of $V(x)$ for $kq = 1$ and $a = 1$ was plotted using a spreadsheet program:



***42 ••**

Picture the Problem Because $V(x)$ and E_x are related through $E_x = -\,dV/dx$, we can find V from E by integration.

Separate variables to obtain:

$$dV = -E_x dx = -\left(2.0x^3\ \text{kN/C}\right)dx$$

Integrate V from V_1 to V_2 and x from 1 m to 2 m:

$$\int_{V_1}^{V_2} dV = -\left(2.0\,\text{kN/C}\right)\int_{x_1}^{x_2} x^3 dx$$
$$= -\left(2.0\,\text{kN/C}\right)\left[\tfrac{1}{4}x^4\right]_{1\,\text{m}}^{2\,\text{m}}$$

Simplify to obtain:

$$V_2 - V_1 = \boxed{-7.50\,\text{kV}}$$

43 ••

Picture the Problem Let r_1 be the distance from (0, a) to (x, 0), r_2 the distance from (0, $-a$), and r_3 the distance from (a, 0) to (x, 0). We can express $V(x)$ as the sum of the potentials due to the charges at (0, a), (0, $-a$), and (a, 0) and then find E_x from $-dV/dx$.

(a) Express $V(x)$ as the sum of the potentials due to the charges at (0, a), (0, $-a$), and (a, 0):

$$V(x) = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \frac{kq_3}{r_3}$$

where $q_1 = q_2 = q_3 = q$

At $x = 0$, the fields due to q_1 and q_2 cancel, so $E_x(0) = -kq/a^2$; this is also obtained from (b) if $x = 0$.

As $x \to \infty$, i.e., for $x >> a$, the three charges appear as a point charge $3q$, so $E_x = 3kq/x^2$; this is also the result one obtains from (*b*) for $x >> a$.

Substitute for the r_i to obtain:

$$V(x) = kq\left(\frac{1}{\sqrt{x^2+a^2}} + \frac{1}{\sqrt{x^2+a^2}} + \frac{1}{|x-a|}\right) = \boxed{kq\left(\frac{2}{\sqrt{x^2+a^2}} + \frac{1}{|x-a|}\right)}$$

(*b*) For $x > a$, $x - a > 0$ and: $|x-a| = x-a$

Use $E_x = -dV/dx$ to find E_x:

$$E_x(x>a) = -\frac{d}{dx}\left[kq\left(\frac{2}{\sqrt{x^2+a^2}} + \frac{1}{x-a}\right)\right] = \boxed{\frac{2kqx}{(x^2+a^2)^{3/2}} + \frac{kq}{(x-a)^2}}$$

For $x < a$, $x - a < 0$ and: $|x-a| = -(x-a) = a-x$

Use $E_x = -dV/dx$ to find E_x:

$$E_x(x<a) = -\frac{d}{dx}\left[kq\left(\frac{2}{\sqrt{x^2+a^2}} + \frac{1}{a-x}\right)\right] = \boxed{\frac{2kqx}{(x^2+a^2)^{3/2}} - \frac{kq}{(a-x)^2}}$$

Calculations of V for Continuous Charge Distributions

44 •

Picture the Problem We can construct Gaussian surfaces just inside and just outside the spherical shell and apply Gauss's law to find the electric field at these locations. We can use the expressions for the electric potential inside and outside a spherical shell to find the potential at these locations.

(*a*) Apply Gauss's law to a spherical Gaussian surface of radius $r < 12$ cm:

$$\oint_S \vec{E}\cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0} = 0$$

because the charge resides on the outer surface of the spherical surface. Hence

$$\vec{E}(r < 12\,\text{cm}) = \boxed{0}$$

Apply Gauss's law to a spherical Gaussian surface of radius

$$E\left(4\pi r^2\right) = \frac{q}{\epsilon_0}$$

$r > 12$ cm: and

$$E(r > 12\,\text{cm}) = \frac{q}{4\pi r^2 \epsilon_0} = \frac{kq}{r^2}$$

Substitute numerical values and evaluate $E(r > 12\,\text{cm})$:

$$E(r > 12\,\text{cm}) = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(10^{-8}\ \text{C})}{(0.12\,\text{m})^2} = \boxed{6.24\,\text{kV/m}}$$

(*b*) Express and evaluate the potential just inside the spherical shell:

$$V(r \le R) = \frac{kq}{R} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(10^{-8}\ \text{C})}{0.12\,\text{m}} = \boxed{749\,\text{V}}$$

Express and evaluate the potential just outside the spherical shell:

$$V(r \ge R) = \frac{kq}{r} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(10^{-8}\ \text{C})}{0.12\,\text{m}} = \boxed{749\,\text{V}}$$

(*c*) The electric potential inside a uniformly charged spherical shell is constant and given by:

$$V(r \le R) = \frac{kq}{R} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(10^{-8}\ \text{C})}{0.12\,\text{m}} = \boxed{749\,\text{V}}$$

In part (*a*) we showed that: $\vec{\boldsymbol{E}}(r < 12\,\text{cm}) = \boxed{0}$

45 •

Picture the Problem We can use the expression for the potential due to a line charge $V = -2k\lambda \ln\frac{r}{a}$, where $V = 0$ at some distance $r = a$, to find the potential at these distances from the line.

Express the potential due to a line charge as a function of the distance from the line:

$$V = -2k\lambda \ln\frac{r}{a}$$

Because $V = 0$ at $r = 2.5$ m:

$$0 = -2k\lambda \ln\frac{2.5\,\text{m}}{a},$$

$$0 = \ln\frac{2.5\,\text{m}}{a},$$

and

$$\frac{2.5\,\text{m}}{a} = \ln^{-1} 0 = 1$$

Thus we have a = 2.5 m and:

$$V = -2\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.5\,\mu\text{C/m}\right)\ln\left(\frac{r}{2.5\,\text{m}}\right) = -\left(2.70\times10^{4}\ \text{N}\cdot\text{m/C}\right)\ln\left(\frac{r}{2.5\,\text{m}}\right)$$

(*a*) Evaluate V at r = 2.0 m:

$$V = -\left(2.70\times10^{4}\ \text{N}\cdot\text{m/C}\right)\ln\left(\frac{2\,\text{m}}{2.5\,\text{m}}\right) = \boxed{6.02\,\text{kV}}$$

(*b*) Evaluate V at r = 4.0 m:

$$V = -\left(2.70\times10^{4}\ \text{N}\cdot\text{m/C}\right)\ln\left(\frac{4\,\text{m}}{2.5\,\text{m}}\right) = \boxed{-12.7\,\text{kV}}$$

(*c*) Evaluate V at r = 12.0 m:

$$V = -\left(2.70\times10^{4}\ \text{N}\cdot\text{m/C}\right)\ln\left(\frac{12\,\text{m}}{2.5\,\text{m}}\right) = \boxed{-42.3\,\text{kV}}$$

46 ••

Picture the Problem The electric field on the x axis of a disk charge of radius R is given by $E_x = 2\pi k\sigma\left(1-\frac{x}{\sqrt{x^2+R^2}}\right)$. We'll choose $V(\infty) = 0$ and integrate from $x' = \infty$ to $x' = x$ to obtain Equation 23-21.

Relate the electric potential on the axis of a disk charge to the electric field of the disk:

$$dV = -E_x dx$$

Express the electric field on the x axis of a disk charge:

$$E_x = 2\pi k\sigma\left(1-\frac{x}{\sqrt{x^2+R^2}}\right)$$

Substitute to obtain:

$$dV = -2\pi k\sigma\left(1-\frac{x}{\sqrt{x^2+R^2}}\right)dx$$

Let $V(\infty) = 0$ and integrate from $x' = \infty$ to $x' = x$:

$$V = -2\pi k\sigma\int_{\infty}^{x}\left(1-\frac{x'}{\sqrt{x'^2+R^2}}\right)dx'$$

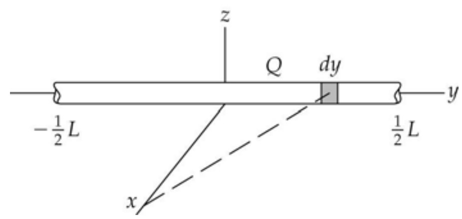
$$= 2\pi k\sigma\left(\sqrt{x^2+R^2}-x\right)$$

$$= \boxed{2\pi k\sigma|x|\left(\sqrt{1+\frac{R^2}{x^2}}-1\right)}$$

which is Equation 23-21.

***47** ••

Picture the Problem Let the charge per unit length be $\lambda = Q/L$ and dy be a line element with charge λdy. We can express the potential dV at any point on the x axis due to λdy and integrate of find $V(x, 0)$.



(*a*) Express the element of potential dV due to the line element dy:

$$dV = \frac{k\lambda}{r}dy$$

where $r = \sqrt{x^2+y^2}$

Integrate dV from $y = -L/2$ to $y = L/2$:

$$V(x,0) = \frac{kQ}{L}\int_{-L/2}^{L/2}\frac{dy}{\sqrt{x^2+y^2}}$$

$$= \boxed{\frac{kQ}{L}\ln\left(\frac{\sqrt{x^2+L^2/4}+L/2}{\sqrt{x^2+L^2/4}-L/2}\right)}$$

(*b*) Factor x from the numerator and denominator within the parentheses to obtain:

$$V(x,0) = \frac{kQ}{L}\ln\left(\frac{\sqrt{1+\frac{L^2}{4x^2}}+\frac{L}{2x}}{\sqrt{1+\frac{L^2}{4x^2}}-\frac{L}{2x}}\right)$$

Use $\ln\frac{a}{b} = \ln a - \ln b$ to obtain:

$$V(x,0) = \frac{kQ}{L}\left\{\ln\left(\sqrt{1+\frac{L^2}{4x^2}}+\frac{L}{2x}\right)-\ln\left(\sqrt{1+\frac{L^2}{4x^2}}-\frac{L}{2x}\right)\right\}$$

Let $\varepsilon = \dfrac{L^2}{4x^2}$ and use $(1+\varepsilon)^{1/2} = 1 + \frac{1}{2}\varepsilon - \frac{1}{8}\varepsilon^2 + \ldots$ to expand $\sqrt{1+\dfrac{L^2}{4x^2}}$:

$$\left(1+\frac{L^2}{4x^2}\right)^{1/2} = 1 + \frac{1}{2}\frac{L^2}{4x^2} - \frac{1}{8}\left(\frac{L^2}{4x^2}\right)^2 + \ldots \approx 1 \text{ for } x >> L.$$

Substitute to obtain:

$$V(x,0) = \frac{kQ}{L}\left\{\ln\left(1+\frac{L}{2x}\right) - \ln\left(1-\frac{L}{2x}\right)\right\}$$

Let $\delta = \dfrac{L}{2x}$ and use $\ln(1+\delta) = \delta - \frac{1}{2}\delta^2 + \ldots$ to expand $\ln\left(1\pm\dfrac{L}{2x}\right)$:

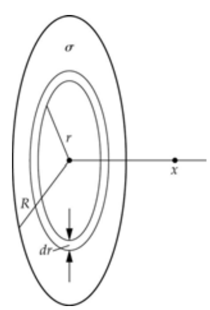
$$\ln\left(1+\frac{L}{2x}\right) \approx \frac{L}{2x} - \frac{L^2}{4x^2} \text{ and } \ln\left(1-\frac{L}{2x}\right) \approx -\frac{L}{2x} - \frac{L^2}{4x^2} \text{ for } x >> L.$$

Substitute and simplify to obtain:

$$V(x,0) = \frac{kQ}{L}\left\{\frac{L}{2x} - \frac{L^2}{4x^2} - \left(-\frac{L}{2x} - \frac{L^2}{4x^2}\right)\right\} = \boxed{\frac{kQ}{x}}$$

48 ••

Picture the Problem We can find Q by integrating the charge on a ring of radius r and thickness dr from $r = 0$ to $r = R$ and the potential on the axis of the disk by integrating the expression for the potential on the axis of a ring of charge between the same limits.



(*a*) Express the charge dq on a ring of radius r and thickness dr:

$$dq = 2\pi r\sigma dr = 2\pi r\left(\sigma_0\frac{R}{r}\right)dr = 2\pi\sigma_0 R dr$$

Integrate from $r = 0$ to $r = R$ to obtain:

$$Q = 2\pi\sigma_0 R\int_0^R dr = \boxed{2\pi\sigma_0 R^2}$$

(*b*) Express the potential on the axis of the disk due to a circular element of charge $dq = 2\pi r\sigma dr$:

$$dV = \frac{kdq}{r'} = \frac{2\pi k\sigma_0 R dr}{\sqrt{x^2 + r^2}}$$

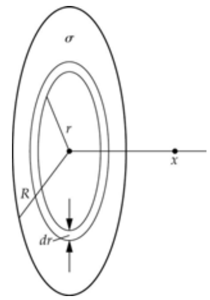
Integrate from $r = 0$ to $r = R$ to obtain:

$$V = 2\pi k\sigma_0 R\int_0^R \frac{dr}{\sqrt{x^2 + r^2}}$$

$$= \boxed{2\pi k\sigma_0 R\ln\left(\frac{R + \sqrt{x^2 + R^2}}{x}\right)}$$

49 ••

Picture the Problem We can find Q by integrating the charge on a ring of radius r and thickness dr from $r = 0$ to $r = R$ and the potential on the axis of the disk by integrating the expression for the potential on the axis of a ring of charge between the same limits.



(*a*) Express the charge dq on a ring of radius r and thickness dr:

$$dq = 2\pi r\sigma dr = 2\pi r\left(\sigma_0 \frac{r^2}{R^2}\right)dr$$

$$= \frac{2\pi\sigma_0}{R^2}r^3 dr$$

Integrate from $r = 0$ to $r = R$ to obtain:

$$Q = \frac{2\pi\sigma_0}{R^2}\int_0^R r^3 dr = \boxed{\tfrac{1}{2}\pi\sigma_0 R^2}$$

(*b*)Express the potential on the axis of the disk due to a circular element of charge $dq = \dfrac{2\pi\sigma_0}{R^2}r^3 dr$:

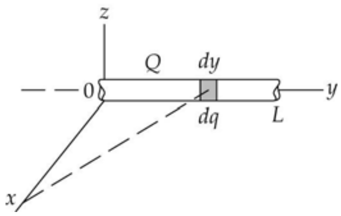
$$dV = \frac{kdq}{r'} = \frac{2\pi k\sigma_0}{R^2}\frac{r^3}{\sqrt{x^2 + r^2}}dr$$

Integrate from $r = 0$ to $r = R$ to obtain:

$$V = \frac{2\pi k\sigma_0}{R^2}\int_0^R \frac{r^3 dr}{\sqrt{x^2 + r^2}} = \boxed{\frac{2\pi k\sigma_0}{R^2}\left(\frac{R^2 - 2x^2}{3}\sqrt{x^2 + R^2} + \frac{2x^3}{3}\right)}$$

50 ••

Picture the Problem Let the charge per unit length be $\lambda = Q/L$ and dy be a line element with charge λdy. We can express the potential dV at any point on the x axis due to λdy and integrate to find $V(x, 0)$.



Express the element of potential dV due to the line element dy:

$$dV = \frac{k\lambda}{r}dy$$

where $r = \sqrt{x^2 + y^2}$

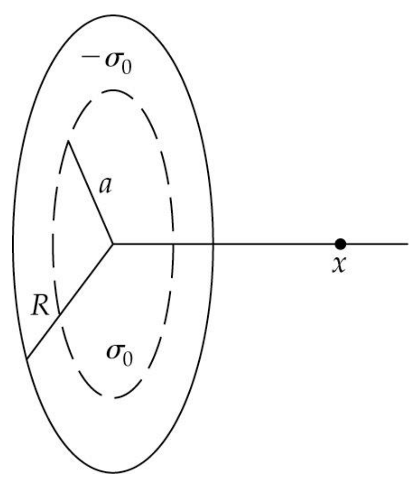
Integrate dV from $y = -L/2$ to $y = L/2$:

$$V(x,0) = \frac{kQ}{L}\int_{-L/2}^{L/2}\frac{dy}{\sqrt{x^2+y^2}}$$

$$= \boxed{\frac{kQ}{L}\ln\left(\frac{\sqrt{x^2+L^2/4}+L/2}{\sqrt{x^2+L^2/4}-L/2}\right)}$$

***51** ••

Picture the Problem The potential at any location on the axis of the disk is the sum of the potentials due to the positive and negative charge distributions on the disk. Knowing that the total charge on the disk is zero and the charge densities are equal in magnitude will allow us to find the radius of the region that is positively charged. We can then use the expression derived in the text to find the potential due to this charge closest to the axis and integrate dV from $r = R/\sqrt{2}$ to $r = R$ to find the potential at x due to the negative charge distribution.



(*a*) Express the potential at a distance x along the axis of the disk as the sum of the potentials due to the positively and negatively charged regions of the disk:

$$V(x) = V_+(x) + V_-(x)$$

We know that the charge densities are equal in magnitude and that the

$$Q_{r<a} = Q_{r>a}$$

or

total charge carried by the disk is zero. Express this condition in terms of the charge in each of two regions of the disk:

$$\sigma_0 \pi a^2 = \sigma_0 \pi R^2 - \sigma_0 \pi a^2$$

Solve for a to obtain:

$$a = \frac{R}{\sqrt{2}}$$

Use this result and the general expression for the potential on the axis of a charged disk to express $V_+(x)$:

$$V_+(x) = 2\pi k\sigma_0\left(\sqrt{x^2 + \frac{R^2}{2}} - x\right)$$

Express the potential on the axis of the disk due to a ring of charge a distance $r > a$ from the axis of the ring:

$$dV_-(x) = -2\pi k\sigma_0 \frac{r}{r'} dr$$

where $r' = \sqrt{x^2 + r^2}$.

Integrate this expression from $r = R/\sqrt{2}$ to $r = R$ to obtain:

$$V_-(x) = -2\pi k\sigma_0 \int_{R/\sqrt{2}}^{R} \frac{r}{\sqrt{x^2 + r^2}} dr$$

$$= -2\pi k\sigma_0\left(\sqrt{x^2 + R^2} - \sqrt{x^2 + \frac{R^2}{2}}\right)$$

Substitute and simplify to obtain:

$$V(x) = 2\pi k\sigma_0\left(\sqrt{x^2 + \frac{R^2}{2}} - x\right) - 2\pi k\sigma_0\left(\sqrt{x^2 + R^2} - \sqrt{x^2 + \frac{R^2}{2}}\right)$$

$$= 2\pi k\sigma_0\left(\sqrt{x^2 + \frac{R^2}{2}} - x - \sqrt{x^2 + R^2} + \sqrt{x^2 + \frac{R^2}{2}}\right)$$

$$= \boxed{2\pi k\sigma_0\left(2\sqrt{x^2 + \frac{R^2}{2}} - \sqrt{x^2 + R^2} - x\right)}$$

(b) To determine V for $x >> R$, factor x from the square roots and expand using the binomial expansion:

$$\sqrt{x^2 + \frac{R^2}{2}} = x\left(1 + \frac{R^2}{2x^2}\right)^{1/2}$$

$$\approx x\left(1 + \frac{R^2}{4x^2} - \frac{R^4}{32x^4}\right)$$

and

$$\sqrt{x^2+R^2}=x\left(1+\frac{R^2}{x^2}\right)^{1/2}$$
$$\approx x\left(1+\frac{R^2}{2x^2}-\frac{R^4}{8x^4}\right)$$

Substitute to obtain:

$$V(x)\approx 2\pi k\sigma_0\left(2x\left(1+\frac{R^2}{4x^2}-\frac{R^4}{32x^4}\right)-x\left(1+\frac{R^2}{2x^2}-\frac{R^4}{8x^4}\right)-x\right)=\boxed{\frac{\pi k\sigma_0 R^4}{8x^3}}$$

52 ••

Picture the Problem Given the potential function $V(x)=2\pi k\sigma_0\left(2\sqrt{x^2+R^2/2}-\sqrt{x^2+R^2}-x\right)$ found in Problem 51(*a*), we can find E_x from $-dV/dx$. In the second part of the problem, we can find the electric field on the axis of the disk by integrating Coulomb's law for the oppositely charged regions of the disk and expressing the sum of the two fields.

Relate E_x to dV/dx:

$$E_x=-\frac{dV}{dx}$$

From Problem 51(*a*) we have:

$$V(x)=2\pi k\sigma_0\left(2\sqrt{x^2+\frac{R^2}{2}}-\sqrt{x^2+R^2}-x\right)$$

Evaluate the negative of the derivative of $V(x)$ to obtain:

$$E_x=-2\pi k\sigma_0\frac{d}{dx}\left(2\sqrt{x^2+\frac{R^2}{2}}-\sqrt{x^2+R^2}-x\right)$$
$$=\boxed{-2\pi k\sigma_0\left(\frac{2x}{\sqrt{x^2+\frac{R^2}{2}}}-\frac{x}{\sqrt{x^2+R^2}}-1\right)}$$

Express the field on the axis of the disk as the sum of the field due to the positive charge on the disk and the field due to the negative charge

$$E_x=E_{x-}+E_{x+}$$

on the disk:

The field due to the positive charge (closest to the axis) is:

$$E_{x+} = 2\pi k\sigma_0\left(1-\frac{x}{\sqrt{x^2+\dfrac{R^2}{2}}}\right)$$

To determine E_{x-} we integrate the field due to a ring charge:

$$E_{x-} = -2\pi k\sigma_0\int_{R/\sqrt{2}}^{R}\frac{rdr}{\left(x^2+r^2\right)^{3/2}}$$

$$= -2\pi k\sigma_0\left(\frac{x}{\sqrt{x^2+\dfrac{R^2}{2}}}-\frac{x}{\sqrt{x^2+R^2}}\right)$$

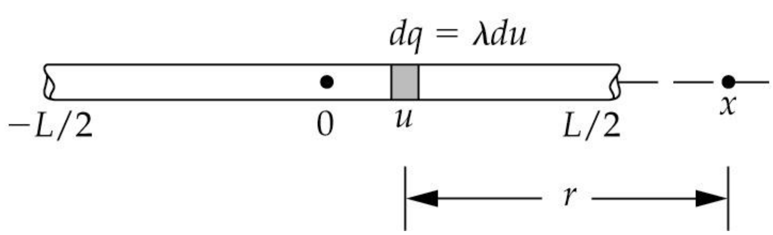
Substitute and simplify to obtain:

$$E_x = -2\pi k\sigma_0\left(\frac{x}{\sqrt{x^2+\dfrac{R^2}{2}}}-\frac{x}{\sqrt{x^2+R^2}}\right)+2\pi k\sigma_0\left(1-\frac{x}{\sqrt{x^2+\dfrac{R^2}{2}}}\right)$$

$$= \boxed{-2\pi k\sigma_0\left(\frac{2x}{\sqrt{x^2+\dfrac{R^2}{2}}}-\frac{x}{\sqrt{x^2+R^2}}-1\right)}$$

53 ••

Picture the Problem We can express the electric potential dV at x due to an elemental charge dq on the rod and then integrate over the length of the rod to find $V(x)$. In the second part of the problem we use a binomial expansion to show that, for $x >> L/2$, our result reduces to that due to a point charge Q.



(*a*) Express the potential at *x* due to the element of charge *dq* located at *u*:

$$dV = \frac{kdq}{r} = \frac{k\lambda du}{x-u}$$

or, because $\lambda = Q/L$,

$$dV = \frac{kQ}{L}\frac{du}{x-u}$$

Integrate *V* from $u = -L/2$ to $L/2$ to obtain:

$$\begin{aligned} V(x) &= \frac{kQ}{L}\int_{-L/2}^{L/2}\frac{du}{x-u} \\ &= \frac{kQ}{L}\ln(x-u)\Big|_{-L/2}^{L/2} \\ &= \left[-\ln\left(x-\frac{L}{2}\right)+\ln\left(x+\frac{L}{2}\right)\right] \\ &= \boxed{\frac{kQ}{L}\ln\left(\frac{x+\frac{L}{2}}{x-\frac{L}{2}}\right)} \end{aligned}$$

(*b*) Divide the numerator and denominator of the argument of the logarithm by *x* to obtain:

$$\ln\left(\frac{x+\frac{L}{2}}{x-\frac{L}{2}}\right) = \ln\left(\frac{1+\frac{L}{2x}}{1-\frac{L}{2x}}\right) = \ln\left(\frac{1+a}{1-a}\right)$$

where $a = L/2x$.

Divide $1 + a$ by $1 - a$ to obtain:

$$\begin{aligned} \ln\left(\frac{1+a}{1-a}\right) &= \ln\left(1+2a+\frac{2a^2}{1-a}\right) \\ &= \ln\left(1+\frac{L}{x}+\frac{\frac{L^2}{x^2}}{2-\frac{L}{x}}\right) \\ &\approx \ln\left(1+\frac{L}{x}\right) \end{aligned}$$

provided $x >> L/2$.

Expand $\ln(1 + L/x)$ binomially to obtain:

$$\ln\left(1+\frac{L}{x}\right) \approx \frac{L}{x}$$

provided $x >> L/2$.

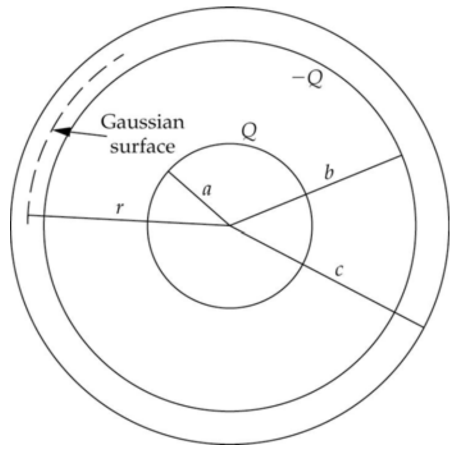
Substitute to express $V(x)$ for $x >> L/2$:

$$V(x) = \frac{kQ}{L}\frac{L}{x} = \boxed{\frac{kQ}{x}}$$, the field due to a

point charge Q.

54 ••

Picture the Problem The diagram is a cross-sectional view showing the charges on the sphere and the spherical conducting shell. A portion of the Gaussian surface over which we'll integrate E in order to find V in the region $r > b$ is also shown. For $a < r < b$, the sphere acts like point charge Q and the potential of the metal sphere is the sum of the potential due to a point charge at its center and the potential at its surface due to the charge on the inner surface of the spherical shell.



(*a*) Express $V_{r>b}$:

$$V_{r>b} = -\int E_{r>b}\,dr$$

Apply Gauss's law for $r > b$:

$$\oint_S \vec{E}_r \cdot \hat{\boldsymbol{n}}\,dA = \frac{Q_{\text{enclosed}}}{\varepsilon_0} = 0$$

and $E_{r>b} = 0$ because $Q_{\text{enclosed}} = 0$ for $r > b$.

Substitute to obtain:

$$V_{r>b} = -\int (0)\,dr = \boxed{0}$$

(*b*) Express the potential of the metal sphere:

$$V_a = V_{Q\text{ at its center}} + V_{\text{surface}}$$

Express the potential at the surface of the metal sphere:

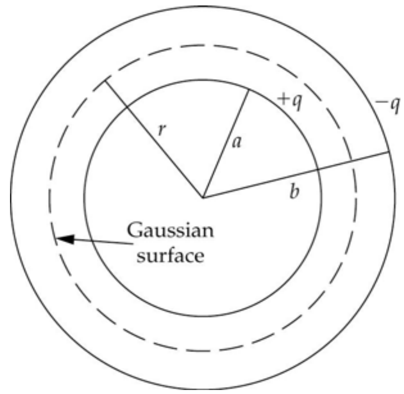
$$V_{\text{surface}} = \frac{k(-Q)}{b} = -\frac{kQ}{b}$$

Substitute and simplify to obtain:

$$V_a = \frac{kQ}{a} - \frac{kQ}{b} = \boxed{kQ\left(\frac{1}{a} - \frac{1}{b}\right)}$$

55 ••

Picture the Problem The diagram is a cross-sectional view showing the charges on the inner and outer conducting shells. A portion of the Gaussian surface over which we'll integrate *E* in order to find *V* in the region $a < r < b$ is also shown. Once we've determined how *E* varies with *r*, we can find $V_b - V_a$ from $V_b - V_a = -\int E_r dr$.



Express the potential difference $V_b - V_a$:

$$V_b - V_a = -\int E_r dr$$

Apply Gauss's law to cylindrical Gaussian surface of radius *r* and length *L*:

$$\oint_S \vec{E} \cdot \hat{n} dA = E_r (2\pi r L) = \frac{q}{\varepsilon_0}$$

Solve for E_r:

$$E_r = \frac{q}{2\pi\varepsilon_0 r L}$$

Substitute for E_r and integrate from $r = a$ to b:

$$V_b - V_a = -\frac{q}{2\pi\varepsilon_0 L}\int_a^b \frac{dr}{r}$$

$$= \boxed{-\frac{2kq}{L}\ln\left(\frac{b}{a}\right)}$$

56 ••

Picture the Problem Let *R* be the radius of the sphere and *Q* its charge. We can express the potential at the two locations given and solve the resulting equations simultaneously for *R* and *Q*.

Relate the potential of the sphere at its surface to its radius:

$$\frac{kQ}{R} = 450\,\text{V} \qquad (1)$$

Express the potential at a distance of 20 cm from its surface:

$$\frac{kQ}{R + 0.2\,\text{m}} = 150\,\text{V} \qquad (2)$$

Divide equation (1) by equation (2) to obtain:

$$\frac{\dfrac{kQ}{R}}{\dfrac{kQ}{R+0.2\,\text{m}}}=\frac{450\,\text{V}}{150\,\text{V}}$$

or

$$\frac{R+0.2\,\text{m}}{R}=3$$

Solve for R to obtain:

$$R=\boxed{0.100\,\text{m}}$$

Solve equation (1) for Q:

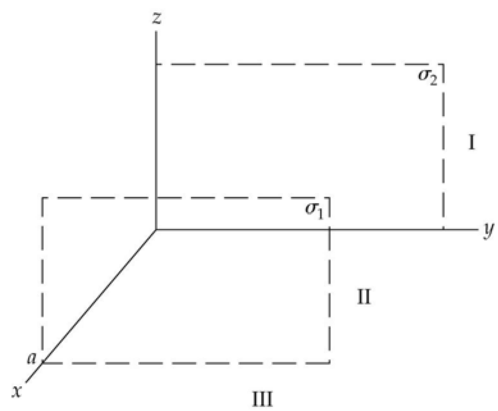
$$Q=(450\,\text{V})\frac{R}{k}$$

Substitute numerical values and evaluate Q:

$$Q=(450\,\text{V})\frac{(0.1\,\text{m})}{(8.99\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2)}=\boxed{5.01\,\text{nC}}$$

57 ••

Picture the Problem Let the charge density on the infinite plane at $x = a$ be σ_1 and that on the infinite plane at $x = 0$ be σ_2. Call that region in space for which $x < 0$, region I, the region for which $0 < x < a$ region II, and the region for which $a < x$ region III. We can integrate E due to the planes of charge to find the electric potential in each of these regions.



(a) Express the potential in region I in terms of the electric field in that region:

$$V_{\text{I}}=-\int_0^x \vec{E}_{\text{I}}\cdot d\vec{x}$$

Express the electric field in region I as the sum of the fields due to the charge densities σ_1 and σ_2:

$$\vec{E}_{\text{I}}=-\frac{\sigma_1}{2\in_0}\hat{i}-\frac{\sigma_2}{2\in_0}\hat{i}=-\frac{\sigma}{2\in_0}\hat{i}-\frac{\sigma}{2\in_0}\hat{i}=-\frac{\sigma}{\in_0}\hat{i}$$

Substitute and evaluate V_I:

$$V_{\mathrm{I}} = -\int_0^x \left(-\frac{\sigma}{\epsilon_0}\right)dx = \frac{\sigma}{\epsilon_0}x + V(0)$$

$$= \frac{\sigma}{\epsilon_0}x + 0 = \boxed{\frac{\sigma}{\epsilon_0}x}$$

Express the potential in region II in terms of the electric field in that region:

$$V_{\mathrm{II}} = -\int \vec{\boldsymbol{E}}_{\mathrm{II}} \cdot d\vec{\boldsymbol{x}} + V(0)$$

Express the electric field in region II as the sum of the fields due to the charge densities σ_1 and σ_2:

$$\vec{\boldsymbol{E}}_{\mathrm{II}} = -\frac{\sigma_1}{2\epsilon_0}\hat{\boldsymbol{i}} + \frac{\sigma_2}{2\epsilon_0}\hat{\boldsymbol{i}} = -\frac{\sigma}{2\epsilon_0}\hat{\boldsymbol{i}} + \frac{\sigma}{2\epsilon_0}\hat{\boldsymbol{i}}$$

$$= 0$$

Substitute and evaluate V_{II}:

$$V_{\mathrm{II}} = -\int_0^x (0)dx = 0 + V(0) = \boxed{0}$$

Express the potential in region III in terms of the electric field in that region:

$$V_{\mathrm{III}} = -\int_a^x \vec{\boldsymbol{E}}_{\mathrm{III}} \cdot d\vec{\boldsymbol{x}}$$

Express the electric field in region III as the sum of the fields due to the charge densities σ_1 and σ_2:

$$\vec{\boldsymbol{E}}_{\mathrm{III}} = \frac{\sigma_1}{2\epsilon_0}\hat{\boldsymbol{i}} + \frac{\sigma_2}{2\epsilon_0}\hat{\boldsymbol{i}} = \frac{\sigma}{2\epsilon_0}\hat{\boldsymbol{i}} + \frac{\sigma}{2\epsilon_0}\hat{\boldsymbol{i}}$$

$$= \frac{\sigma}{\epsilon_0}\hat{\boldsymbol{i}}$$

Substitute and evaluate V_{III}:

$$V_{\mathrm{III}} = -\int_a^x \left(\frac{\sigma}{\epsilon_0}\right)dx = -\frac{\sigma}{\epsilon_0}x + \frac{\sigma}{\epsilon_0}a$$

$$= \boxed{\frac{\sigma}{\epsilon_0}(a - x)}$$

(*b*) Proceed as in (*a*) with $\sigma_1 = -\sigma$ and $\sigma_2 = \sigma$ to obtain:

$$V_{\mathrm{I}} = \boxed{0},$$

$$V_{\mathrm{II}} = \boxed{-\frac{\sigma}{\epsilon_0}x} \text{ and } V_{\mathrm{III}} = \boxed{-\frac{\sigma}{\epsilon_0}a}$$

***58 ••**

Picture the Problem The potential on the axis of a disk charge of radius R and charge density σ is given by $V = 2\pi k\sigma\left[\left(x^2 + R^2\right)^{1/2} - x\right]$.

Express the potential on the axis of the disk charge:

$$V = 2\pi k\sigma\left[\left(x^2 + R^2\right)^{1/2} - x\right]$$

Factor x from the radical and use the binomial expansion to obtain:

$$\left(x^2 + R^2\right)^{1/2} = x\left(1 + \frac{R^2}{x^2}\right)^{1/2} = x\left[1 + \frac{R^2}{2x^2} + \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)\frac{R^4}{x^4} + \ldots\right]$$
$$\approx x\left[1 + \frac{R^2}{2x^2} - \frac{R^4}{8x^4}\right]$$

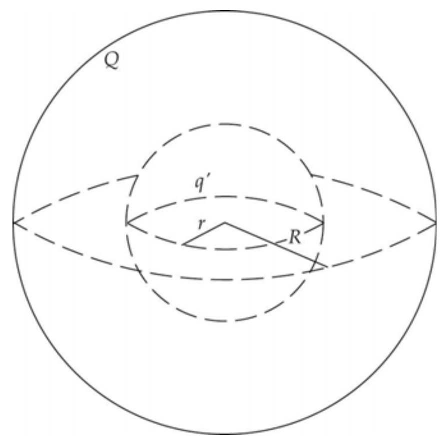
Substitute for the radical term to obtain:

$$V = 2\pi k\sigma\left\{x\left[1 + \frac{R^2}{2x^2} - \frac{R^4}{8x^4}\right] - x\right\}$$
$$= 2\pi k\sigma\left(\frac{R^2}{2x} - \frac{R^4}{8x^3}\right)$$
$$\approx 2\pi k\sigma\left(\frac{R^2}{2x}\right) = \boxed{\frac{kQ}{x}}$$

provided $x >> R$.

59 ••

Picture the Problem The diagram shows a sphere of radius R containing a charge Q uniformly distributed. We can use the definition of density to find the charge q' inside a sphere of radius r and the potential V_1 at r due to this part of the charge. We can express the potential dV_2 at r due to the charge in a shell of radius r' and thickness dr' at $r' > r$ using $dV_2 = kdq'/r$ and then integrate this expression from $r' = r$ to $r' = R$ to find V_2.



(a) Express the potential V_1 at r due to q':

$$V_1 = \frac{kq'}{r}$$

Use the definition of density and the fact that the charge density is uniform to relate q' to Q:

$$\rho = \frac{q'}{\frac{4}{3}\pi r^3} = \frac{Q}{\frac{4}{3}\pi R^3}$$

Solve for q':

$$q' = \frac{r^3}{R^3}Q$$

Substitute to express V_1:

$$V_1 = \frac{k}{r}\left(\frac{r^3}{R^3}Q\right) = \boxed{\frac{kQ}{R^3}r^2}$$

(*b*) Express the potential dV_2 at r due to the charge in a shell of radius r' and thickness dr' at $r' > r$:

$$dV_2 = \frac{kdq'}{r}$$

Express the charge dq' in a shell of radius r' and thickness dr' at $r' > r$:

$$dq' = 4\pi r'^2 \rho dr' = 4\pi r'^2\left(\frac{3Q}{4\pi R^3}\right)dr'$$

$$= \frac{3Q}{R^3}r'^2 dr'$$

Substitute to obtain:

$$dV_2 = \boxed{\frac{3kQ}{R^3}r'dr'}$$

(*c*) Integrate dV_2 from $r' = r$ to $r' = R$ to find V_2:

$$V_2 = \frac{3kQ}{R^3}\int_r^R r'dr' = \boxed{\frac{3kQ}{2R^3}\left(R^2 - r^2\right)}$$

(*d*) Express the potential V at r as the sum of V_1 and V_2:

$$V = V_1 + V_2$$

$$= \frac{kQ}{R^3}r^2 + \frac{3kQ}{2R^3}\left(R^2 - r^2\right)$$

$$= \boxed{\frac{kQ}{2R}\left(3 - \frac{r^2}{R^2}\right)}$$

60 •

Picture the Problem We can equate the expression for the electric field due to an infinite plane of charge and $-\Delta V/\Delta x$ and solve the resulting equation for the separation of the equipotential surfaces.

Express the electric field due to the infinite plane of charge:

$$E = \frac{\sigma}{2\in_0}$$

Relate the electric field to the potential:

$$E = -\frac{\Delta V}{\Delta x}$$

Equate these expressions and solve for Δx to obtain:

$$\Delta x = \frac{2\,\epsilon_0\,\Delta V}{\sigma}$$

Substitute numerical values and evaluate $|\Delta x|$:

$$|\Delta x| = \frac{2\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)(100\ \mathrm{V})}{3.5\,\mu\mathrm{C/m^2}} = \boxed{0.506\ \mathrm{mm}}$$

61 •

Picture the Problem The equipotentials are spheres centered at the origin with radii $r_i = kq/V_i$.

Evaluate r for $V = 20$ V:

$$r_{20\,\mathrm{V}} = \frac{\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(\frac{1}{9}\times10^{-8}\ \mathrm{C}\right)}{20\ \mathrm{V}} = \boxed{0.499\ \mathrm{m}}$$

Evaluate r for $V = 40$ V:

$$r_{40\,\mathrm{V}} = \frac{\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(\frac{1}{9}\times10^{-8}\ \mathrm{C}\right)}{40\ \mathrm{V}} = \boxed{0.250\ \mathrm{m}}$$

Evaluate r for $V = 60$ V:

$$r_{60\,\mathrm{V}} = \frac{\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(\frac{1}{9}\times10^{-8}\ \mathrm{C}\right)}{60\ \mathrm{V}} = \boxed{0.166\ \mathrm{m}}$$

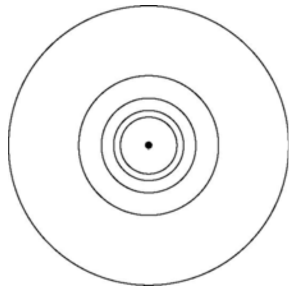
Evaluate r for $V = 80$ V:

$$r_{80\,\mathrm{V}} = \frac{\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(\frac{1}{9}\times10^{-8}\ \mathrm{C}\right)}{80\ \mathrm{V}} = \boxed{0.125\ \mathrm{m}}$$

Evaluate r for $V = 100$ V:

$$r_{100\,\mathrm{V}} = \frac{\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(\frac{1}{9}\times10^{-8}\ \mathrm{C}\right)}{100\ \mathrm{V}} = \boxed{0.0999\ \mathrm{m}}$$

The equipotential surfaces are shown in cross-section to the right:



$$\boxed{\text{The equipotential surfaces are not equally spaced.}}$$

62 •

Picture the Problem We can relate the dielectric strength of air (about 3 MV/m) to the maximum net charge that can be placed on a spherical conductor using the expression for the electric field at its surface. We can find the potential of the sphere when it carries its maximum charge using $V = kQ_{max}/R$.

(*a*) Express the dielectric strength of a spherical conductor in terms of the charge on the sphere:

$$E_{breakdown} = \frac{kQ_{max}}{R^2}$$

Solve for Q_{max}:

$$Q_{max} = \frac{E_{breakdown}R^2}{k}$$

Substitute numerical values and evaluate Q_{max}:

$$Q_{max} = \frac{(3\,\text{MV/m})(0.16\,\text{m})^2}{8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2} = \boxed{8.54\,\mu\text{C}}$$

(*b*) Because the charge carried by the sphere could be either positive or negative:

$$V_{max} = \pm\frac{kQ_{max}}{R} = \pm\frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(8.54\,\mu\text{C})}{0.16\,\text{m}} = \boxed{\pm 480\,\text{kV}}$$

***63** •

Picture the Problem We can solve the equation giving the electric field at the surface of a conductor for the greatest surface charge density that can exist before dielectric breakdown of the air occurs.

Relate the electric field at the surface of a conductor to the surface charge density:

$$E = \frac{\sigma}{\epsilon_0}$$

Solve for σ under dielectric breakdown of the air conditions:

$$\sigma_{\max} = \epsilon_0 E_{\text{breaddown}}$$

Substitute numerical values and evaluate $\sigma_{\max}$:

$$\sigma_{\max} = \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(3\,\text{MV/m}\right) = \boxed{26.6\ \mu\text{C/m}^2}$$

64 ••

Picture the Problem Let L and S refer to the larger and smaller spheres, respectively. We can use the fact that both spheres are at the same potential to find the electric fields near their surfaces. Knowing the electric fields, we can use $\sigma = \epsilon_0 E$ to find the surface charge density of each sphere.

Express the electric fields at the surfaces of the two spheres:

$$E_\text{S} = \frac{kQ_\text{S}}{R_\text{S}^2} \text{ and } E_\text{L} = \frac{kQ_\text{L}}{R_\text{L}^2}$$

Divide the first of these equations by the second to obtain:

$$\frac{E_\text{S}}{E_\text{L}} = \frac{\dfrac{kQ_\text{S}}{R_\text{S}^2}}{\dfrac{kQ_\text{L}}{R_\text{L}^2}} = \frac{Q_\text{S}R_\text{L}^2}{Q_\text{L}R_\text{S}^2}$$

Because the potentials are equal at the surfaces of the spheres:

$$\frac{kQ_\text{L}}{R_\text{L}} = \frac{kQ_\text{S}}{R_\text{S}} \text{ and } \frac{Q_\text{S}}{Q_\text{L}} = \frac{R_\text{S}}{R_\text{L}}$$

Substitute to obtain:

$$\frac{E_\text{S}}{E_\text{L}} = \frac{R_\text{S}R_\text{L}^2}{R_\text{L}R_\text{S}^2} = \frac{R_\text{L}}{R_\text{S}}$$

Solve for E_S:

$$E_\text{S} = \frac{R_\text{L}}{R_\text{S}}E_\text{L} = \frac{12\,\text{cm}}{5\,\text{cm}}\left(200\,\text{kV/m}\right) = 480\,\text{kV/m}$$

Use $\sigma = \epsilon_0 E$ to find the surface charge density of each sphere:

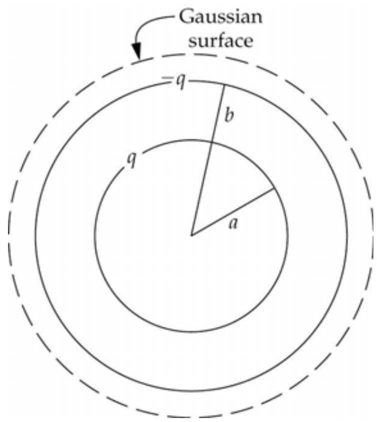
$$\sigma_{12\,\text{cm}} = \epsilon_0 E_{12\,\text{cm}} = \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(200\,\text{kV/m}\right) = \boxed{1.77\ \mu\text{C/m}^2}$$

and

$$\sigma_{5\,\text{cm}} = \epsilon_0 E_{5\,\text{cm}} = \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(480\,\text{kV/m}\right) = \boxed{4.25\ \mu\text{C/m}^2}$$

65 ••

Picture the Problem The diagram is a cross-sectional view showing the charges on the concentric spherical shells. The Gaussian surface over which we'll integrate E in order to find V in the region $r \geq b$ is also shown. We'll also find E in the region for which $a < r < b$. We can then use the relationship $V = -\int E dr$ to find V_a and V_b and their difference.



Express V_b:

$$V_b = -\int_{\infty}^{b} E_{r \geq a} dr$$

Apply Gauss's law for $r \geq b$:

$$\oint_S \vec{E}_r \cdot \hat{n} dA = \frac{Q_{\text{enclosed}}}{\epsilon_0} = 0$$

and $E_{r \geq b} = 0$ because $Q_{\text{enclosed}} = 0$ for $r \geq b$.

Substitute to obtain:

$$V_b = -\int_{\infty}^{b} (0) dr = 0$$

Express V_a:

$$V_a = -\int_{b}^{a} E_{r \geq a} dr$$

Apply Gauss's law for $r \geq a$:

$$E_{r \geq a}\left(4\pi r^2\right) = \frac{q}{\epsilon_0}$$

and

$$E_{r \geq a} = \frac{q}{4\pi \epsilon_0 r^2} = \frac{kq}{r^2}$$

Substitute to obtain:

$$V_a = -kq\int_{b}^{a} \frac{dr}{r^2} = \frac{kq}{a} - \frac{kq}{b}$$

The potential difference between the shells is given by:

$$V_a - V_b = V_a = \boxed{kq\left(\frac{1}{a} - \frac{1}{b}\right)}$$

***66** •••

Picture the Problem We can find the potential relative to infinity at the center of the sphere by integrating the electric field for 0 to ∞. We can apply Gauss's law to find the electric field both inside and outside the spherical shell.

The potential relative to infinity the center of the spherical shell is:

$$V = \int_0^R E_{r<R}dr + \int_R^\infty E_{r>R}dr \qquad (1)$$

Apply Gauss's law to a spherical surface of radius $r < R$ to obtain:

$$\int_S E_n dA = E_{r<R}\left(4\pi r^2\right) = \frac{Q_{\text{inside}}}{\epsilon_0}$$

Using the fact that the sphere is uniformly charged, express Q_{inside} in terms of Q:

$$\frac{Q_{\text{inside}}}{\frac{4}{3}\pi r^3} = \frac{Q}{\frac{4}{3}\pi R^3} \Rightarrow Q_{\text{inside}} = \frac{r^3}{R^3}Q$$

Substitute for Q_{inside} to obtain:

$$E_{r<R}\left(4\pi r^2\right) = \frac{r^3}{\epsilon_0 R^3}Q$$

Solve for $E_{r<R}$:

$$E_{r<R} = \frac{r}{4\pi\epsilon_0 R^3}Q = \frac{kQ}{R^3}r$$

Apply Gauss's law to a spherical surface of radius $r > R$ to obtain:

$$\int_S E_n dA = E_{r>R}\left(4\pi r^2\right) = \frac{Q_{\text{inside}}}{\epsilon_0} = \frac{Q}{\epsilon_0}$$

Solve for $E_{r>R}$ to obtain:

$$E_{r>R} = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{kQ}{r^2}$$

Substitute for $E_{r<R}$ and $E_{r>R}$ in equation (1) and evaluate the resulting integral:

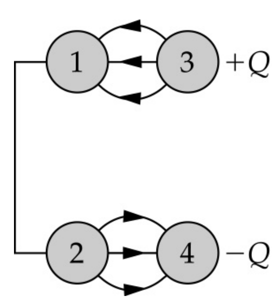
$$V = \frac{kQ}{R^3}\int_0^R r\,dr + kQ\int_R^\infty \frac{dr}{r^2}$$

$$= \frac{kQ}{R^3}\left[\frac{r^2}{2}\right]_0^R + kQ\left[-\frac{1}{r}\right]_R^\infty = \boxed{\frac{3kQ}{2R}}$$

67 ••

Picture the Problem

(*a*) The field lines are shown on the figure. The charged spheres induce charges of opposite sign on the spheres near them so that sphere 1 is negatively charged, and sphere 2 is positively charged. The total charge of the system is zero.



(*b*) $\boxed{\text{$V_1 = V_2$ because the spheres are connected. From the direction of the electric field lines it follows that $V_3 > V_1$.}}$

(*c*) $\boxed{\text{If 3 and 4 are connected, } V_3 = V_4 \text{ and the conditions of part } (b) \text{ can only be satisfied if all potentials are zero. Consequently the charge on each sphere is zero.}}$

General Problems

68 •

Picture the Problem Because the charges at either end of the electric dipole are point charges, we can use the expression for the Coulomb potential to find the field at any distance from the dipole charges.

Using the expression for the potential due to a system of point charges, express the potential at the point 9.2×10^{-10} m from each of the two charges:

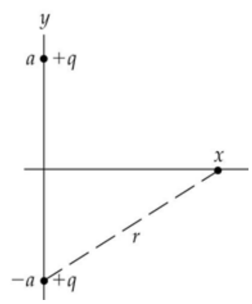
$$V = \frac{kq_+}{d} + \frac{kq_-}{d} = \frac{k}{d}(q_+ + q_-)$$

Because $q_+ = -q_-$:

$q_+ + q_- = 0$, $V = 0$ and $\boxed{(b) \text{ is correct.}}$

69 •

Picture the Problem The potential *V* at any point on the *x* axis is the sum of the Coulomb potentials due to the two point charges. Once we have found *V*, we can use $\vec{E} = -\overrightarrow{\text{grad}}V$ to find the electric field at any point on the *x* axis.



(*a*) Express the potential due to a system of point charges:

$$V = \sum_i \frac{kq_i}{r_i}$$

Substitute to obtain:

$$V(x) = V_{\text{charge at }+a} + V_{\text{charge at -}a} = \frac{kq}{\sqrt{x^2+a^2}} + \frac{kq}{\sqrt{x^2+a^2}} = \boxed{\frac{2kq}{\sqrt{x^2+a^2}}}$$

(b) The electric field at any point on the x axis is given by:

$$\vec{E}(x) = -\overrightarrow{\text{grad}}V = -\frac{d}{dx}\left[\frac{2kq}{\sqrt{x^2+a^2}}\right]\hat{i}$$

$$= \boxed{\frac{2kqx}{\left(x^2+a^2\right)^{3/2}}\hat{i}}$$

70 •

Picture the Problem The radius of the sphere is related to the electric field and the potential at its surface. The dielectric strength of air is about 3 MV/m.

Relate the electric field at the surface of a conducting sphere to the potential at the surface of the sphere:

$$E_r = \frac{V(r)}{r}$$

Solve for r:

$$r = \frac{V(r)}{E_r}$$

When E is a maximum, r is a minimum:

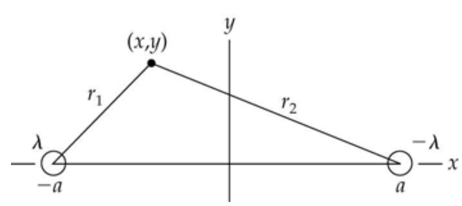
$$r_{\text{min}} = \frac{V(r)}{E_{\text{max}}}$$

Substitute numerical values and evaluate r_{min}:

$$r_{\text{min}} = \frac{10^4\ \text{V}}{3\,\text{MV/m}} = \boxed{3.33\,\text{mm}}$$

***71** ••

Picture the Problem The geometry of the wires is shown to the right. The potential at the point whose coordinates are (x, y) is the sum of the potentials due to the charge distributions on the wires.



(a) Express the potential at the point whose coordinates are (x, y):

$$V(x,y) = V_{\text{wire at }-a} + V_{\text{wire at }a}$$

$$= 2k\lambda\ln\left(\frac{r_{\text{ref}}}{r_1}\right) + 2k(-\lambda)\ln\left(\frac{r_{\text{ref}}}{r_2}\right)$$

$$= 2k\lambda\left[\ln\left(\frac{r_{\text{ref}}}{r_1}\right) - \ln\left(\frac{r_{\text{ref}}}{r_2}\right)\right]$$

$$= \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{r_2}{r_1}\right)$$

where $V(0) = 0$.

Because $r_1 = \sqrt{(x+a)^2 + y^2}$ and $r_2 = \sqrt{(x-a)^2 + y^2}$:

$$V(x,y) = \boxed{\frac{\lambda}{2\pi \in_0} \ln\left(\frac{\sqrt{(x-a)^2 + y^2}}{\sqrt{(x+a)^2 + y^2}}\right)}$$

On the y-axis, $x = 0$ and:

$$V(0,y) = \frac{\lambda}{2\pi \in_0} \ln\left(\frac{\sqrt{a^2 + y^2}}{\sqrt{a^2 + y^2}}\right)$$

$$= \frac{\lambda}{2\pi \in_0} \ln(1) = \boxed{0}$$

(*b*) Evaluate the potential at $\left(\frac{1}{4}a, 0\right) = (1.25\,\text{cm}, 0)$:

$$V\left(\tfrac{1}{4}a, 0\right) = \frac{\lambda}{2\pi \in_0} \ln\left(\frac{\sqrt{\left(\frac{1}{4}a - a\right)^2}}{\sqrt{\left(\frac{1}{4}a + a\right)^2}}\right)$$

$$= \frac{\lambda}{2\pi \in_0} \ln\left(\frac{3}{5}\right)$$

Equate $V(x,y)$ and $V\left(\frac{1}{4}a, 0\right)$:

$$\frac{3}{5} = \frac{\sqrt{(x-5)^2 + y^2}}{\sqrt{(x+5)^2 + y^2}}$$

Solve for y to obtain:

$$y = \pm\sqrt{21.25x - x^2 - 25}$$

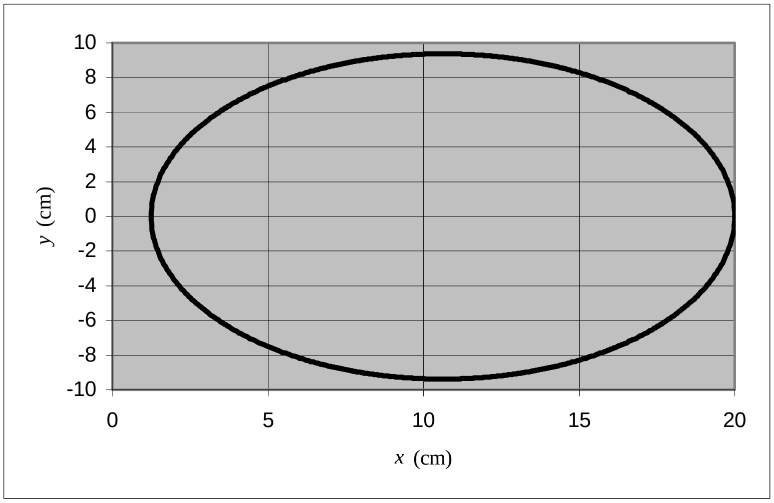
A spreadsheet program to plot $y = \pm\sqrt{21.25x - x^2 - 25}$ is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Content/Formula	Algebraic Form
A2	1.25	$\frac{1}{4}a$
A3	A2 + 0.05	$x + \Delta x$
B2	SQRT(21.25*A2 – A2^2 – 25)	$y = \sqrt{21.25x - x^2 - 25}$
B4	–SQRT(21.25*A2 – A2^2 – 25)	$y = -\sqrt{21.25x - x^2 - 25}$

	A	B	C
1	x	y_pos	y_neg
2	1.25	0.00	0.00
3	1.30	0.97	–0.97
4	1.35	1.37	–1.37
5	1.40	1.67	–1.67
6	1.45	1.93	–1.93
7	1.50	2.15	–2.15
370	19.65	2.54	–2.54
371	19.70	2.35	–2.35
372	19.75	2.15	–2.15

373	19.80	1.93	–1.93
374	19.85	1.67	–1.67
375	19.90	1.37	–1.37
376	19.95	0.97	–0.97

The following graph shows the equipotential curve in the *xy* plane for $V\left(\frac{1}{4}a,0\right)=\dfrac{\lambda}{2\pi\in_0}\ln\left(\dfrac{3}{5}\right)$.



72 ••

Picture the Problem We can use the expression for the potential at any point in the *xy* plane to show that the equipotential curve is a circle.

(*a*) Equipotential surfaces must satisfy the condition:

$$V=\frac{\lambda}{2\pi\in_0}\ln\left(\frac{r_2}{r_1}\right)$$

Solve for r_2/r_1:

$$\frac{r_2}{r_1}=e^{\frac{2\pi\in_0 V}{\lambda}}=C \text{ or } r_2=Cr_1$$

where C is a constant.

Substitute for r_1 and r_2 to obtain:

$$(x-a)^2+y^2=C^2\left[(x+a)^2+y^2\right]$$

Expand this expression, combine like terms, and simplify to obtain:

$$x^2+2a\frac{C^2+1}{C^2-1}x+y^2=-a^2$$

Complete the square by adding $\left[a^2\left(\dfrac{C^2+1}{C^2-1}\right)^2\right]$ to both sides of the equation:

$$x^2 + 2a\frac{C^2+1}{C^2-1}x + \left[a^2\left(\frac{C^2+1}{C^2-1}\right)^2\right] + y^2 = \left[a^2\left(\frac{C^2+1}{C^2-1}\right)^2\right] - a^2 = \frac{4a^2C^2}{\left(C^2-1\right)^2}$$

Let $\alpha = 2a\frac{C^2+1}{C^2-1}$ and $\beta = 2a\frac{C}{C^2-1}$ to obtain:

$\boxed{\left(x+\alpha\right)^2 + y^2 = \beta^2,}$ the equation of circle in the *xy* plane with its center at $(-\alpha, 0)$.

(*b*) $\boxed{\text{The three-dimensional surfaces are cylinders parallel to the wires.}}$

73 ••

Picture the Problem Expressing the charge dq in a spherical shell of volume $4\pi r^2 dr$ within a distance r of the proton and setting the integral of this expression equal to e will allow us to solve for the value of ρ_0 needed for charge neutrality. In part (*b*), we can use the given charge density to express the potential function due to this charge and then integrate this function to find V as a function of r.

Express the charge dq in a spherical shell of volume $4\pi r^2 dr$ within a distance r of the proton:

$$dq = \rho dV = \left(\rho_0 e^{-2r/a}\right)\left(4\pi r^2 dr\right) = 4\pi\rho_0 r^2 e^{-2r/a} dr$$

Express the condition for charge neutrality:

$$e = 4\pi\rho_0 \int_0^\infty r^2 e^{-2r/a} dr$$

Integrate by parts twice to obtain:

$$e = 4\pi\rho_0 \frac{a^3}{4} = \pi\rho_0 a^3$$

Solve for ρ_0:

$$\rho_0 = \boxed{\frac{e}{\pi a^3}}$$

74 •

Picture the Problem Let Q be the sphere's charge, R its radius, and n the number of electrons that have been removed. Then $Q = ne$, where e is the electronic charge. We can use the expression for the Coulomb potential of the sphere to express Q and then $Q = ne$ to find n.

Letting n be the number of electrons that have been removed, express the sphere's charge Q in terms of the electronic charge e:

$$Q = ne$$

Solve for n:

$$n = \frac{Q}{e} \qquad (1)$$

Relate the potential of the sphere to its charge and radius:

$$V=\frac{kQ}{R}$$

Solve for the sphere's charge:

$$Q=\frac{VR}{k}$$

Substitute in equation (1) to obtain:

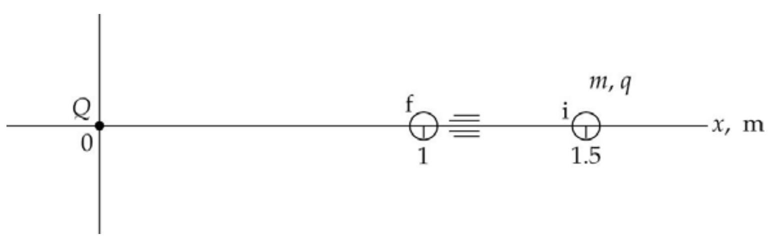
$$n=\frac{VR}{ke}$$

Substitute numerical values and evaluate n:

$$n=\frac{(400\,\mathrm{V})(0.05\,\mathrm{m})}{(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2})(1.6\times10^{-19}\ \mathrm{C})}=\boxed{1.39\times10^{10}}$$

75 •

Picture the Problem We can use conservation of energy to relate the change in the kinetic energy of the particle to the change in potential energy of the charge-and-particle system as the particle moves from x = 1.5 m to x = 1 m. The change in potential energy is, in turn, related to the change in electric potential.



Apply conservation of energy to the point charge Q and particle system:

$$\Delta K+\Delta U=0$$

or, because K_i = 0,

$$K_f+\Delta U_{if}=0$$

Solve for K_f:

$$K_f=-\Delta U_{if}$$

Relate the difference in potential between points i and f to the change in potential energy of the system as the body whose charge is q moves from i to f:

$$\Delta U_{if}=-q\Delta V_{if}=-q(V_f-V_i)$$
$$=-q\left(\frac{kQ}{x_f}-\frac{kQ}{x_i}\right)=-kqQ\left(\frac{1}{x_f}-\frac{1}{x_i}\right)$$

Substitute to obtain:

$$K_f=-kqQ\left(\frac{1}{x_f}-\frac{1}{x_i}\right)$$

Solve for Q:

$$Q = -\frac{K_f}{kq\left(\frac{1}{x_f} - \frac{1}{x_i}\right)}$$

Substitute numerical values and evaluate Q:

$$Q = -\frac{0.24\,\text{J}}{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(4\,\mu\text{C})\left(\frac{1}{1\,\text{m}} - \frac{1}{1.5\,\text{m}}\right)} = \boxed{-20.0\,\mu\text{C}}$$

***76 ••**

Picture the Problem We can use the definition of power and the expression for the work done in moving a charge through a potential difference to find the minimum power needed to drive the moving belt.

Relate the power need to drive the moving belt to the rate at which the generator is doing work:

$$P = \frac{dW}{dt}$$

Express the work done in moving a charge q through a potential difference ΔV:

$$W = q\Delta V$$

Substitute to obtain:

$$P = \frac{d}{dt}[q\Delta V] = \Delta V\,\frac{dq}{dt}$$

Substitute numerical values and evaluate P:

$$P = (1.25\,\text{MV})(200\,\mu\text{C/s}) = \boxed{250\,\text{W}}$$

77 ••

Picture the Problem We can use $W_{q\to\text{final position}} = q\Delta V_{i\to f}$ to find the work required to move these charges between the given points.

(*a*) Express the required work in terms of the charge being moved and the potential due to the charge at $x = +a$:

$$\begin{aligned} W_{+Q\to+a} &= Q\Delta V_{\infty\to+a} \\ &= Q[V(a) - V(\infty)] \\ &= QV(a) = Q\left(\frac{kQ}{2a}\right) = \boxed{\frac{kQ^2}{2a}} \end{aligned}$$

(*b*) Express the required work in terms of the charge being moved and the potentials due to the charges at $x = +a$ and $x = -a$:

$$\begin{aligned} W_{-Q\to 0} &= -Q\Delta V_{\infty\to 0} \\ &= -Q[V(0)-V(\infty)] \\ &= -QV(0) \\ &= -Q[V_{\text{charge at }-a} + V_{\text{charge at }+a}] \\ &= -Q\left(\frac{kQ}{a}+\frac{kQ}{a}\right) = \boxed{\frac{-2kQ^2}{a}} \end{aligned}$$

(*c*) Express the required work in terms of the charge being moved and the potentials due to the charges at $x = +a$ and $x = -a$:

$$\begin{aligned} W_{-Q\to 2a} &= -Q\Delta V_{0\to 2a} \\ &= -Q[V(2a)-V(0)] \\ &= -Q[V_{\text{charge at }-a} + V_{\text{charge at }+a} - V(0)] \\ &= -Q\left(\frac{kQ}{3a}+\frac{kQ}{a}-\frac{2kQ}{a}\right) \\ &= \boxed{\frac{2kQ^2}{3a}} \end{aligned}$$

78 ••

Picture the Problem Let q represent the charge being moved from $x = 50$ cm to the origin, Q the ring charge, and a the radius of the ring. We can use $W_{q\to\text{final position}} = q\Delta V_{i\to f}$, where V is the expression for the axial field due to a ring charge, to find the work required to move q from $x = 50$ cm to the origin.

Express the required work in terms of the charge being moved and the potential due to the ring charge at $x = 50$ cm and $x = 0$:

$$\begin{aligned} W &= q\Delta V \\ &= q[V(0)-V(0.5\,\text{m})] \end{aligned}$$

The potential on the axis of a uniformly charged ring is:

$$V(x) = \frac{kQ}{\sqrt{x^2+a^2}}$$

Evaluate $V(0)$:

$$\begin{aligned} V(0) &= \frac{kQ}{\sqrt{a^2}} \\ &= \frac{(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2\,\text{nC})}{0.1\,\text{m}} \\ &= 180\,\text{V} \end{aligned}$$

Evaluate $V(0.5$ m):

$$\begin{aligned} V(0) &= \frac{(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2\,\text{nC})}{\sqrt{(0.5\,\text{m})^2+(0.1\,\text{m})^2}} \\ &= 35.3\,\text{V} \end{aligned}$$

Substitute in the expression for W to obtain:

$$W = (1\,\text{nC})(180\,\text{V} - 35.3\,\text{V})$$
$$= \boxed{1.45\times10^{-7}\ \text{J}}$$
$$= 1.45\times10^{-7}\ \text{J}\times\frac{1\,\text{eV}}{1.6\times10^{-19}\ \text{J}}$$
$$= \boxed{9.06\times10^{11}\ \text{eV}}$$

79 ••

Picture the Problem We can find the speed of the proton as it strikes the negatively charged sphere from its kinetic energy and, in turn, its kinetic energy from the potential difference through which it is accelerated.

Use the definition of kinetic energy to express the speed of the proton when it strikes the negatively charged sphere:

$$v = \sqrt{\frac{2K_p}{m_p}} \qquad (1)$$

Use the work-kinetic energy theorem to relate the kinetic energy of the proton to the potential difference through which it is accelerated:

$$W = \Delta K = K_f - K_i$$

or, because $K_i = 0$ and $K_f = K_p$,

$$W = \Delta K = K_p$$

Express the work done on the proton in terms of its charge e and the potential difference ΔV between the spheres:

$$W = e\Delta V$$

Substitute to obtain:

$$K_p = e\Delta V$$

Substitute in equation (1) to obtain:

$$v = \sqrt{\frac{2e\Delta V}{m_p}}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{\frac{2(1.6\times10^{-19}\ \text{C})(100\ \text{V})}{1.67\times10^{-27}\ \text{kg}}}$$
$$= \boxed{1.38\times10^{5}\ \text{m/s}}$$

80 ••

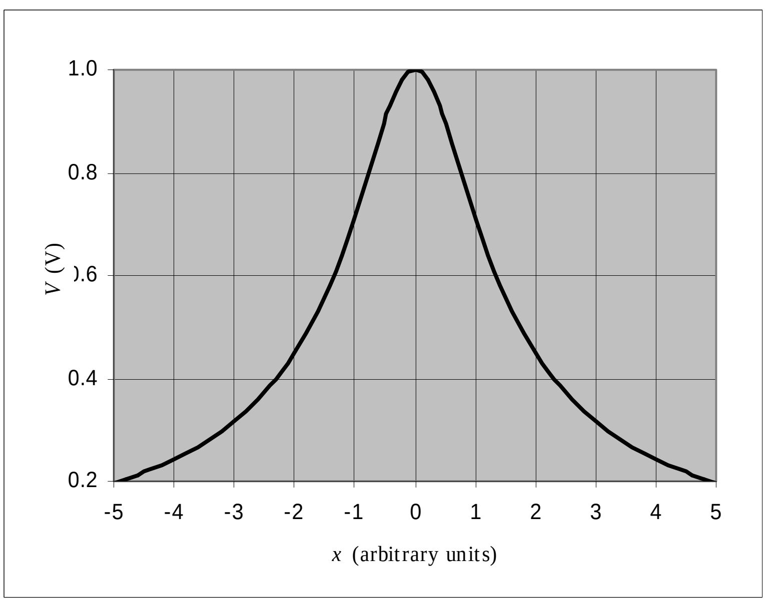
Picture the Problem Equation 23-20 is $V = kQ\big/\sqrt{a^2+x^2}$.

(a) A spreadsheet solution is shown below for $kQ = a = 1$. The formulas used to calculate the quantities in the columns are as follows:

Cell	Content/Formula	Algebraic Form
A4	A3 + 0.1	$x + \Delta x$
B3	1/(1+A3^2)^(1/2)	$\dfrac{kQ}{\sqrt{a^2+x^2}}$

	A	B
1		
2	x	V(x)
3	–5.0	0.196
4	–4.8	0.204
5	–4.6	0.212
6	–4.4	0.222
7	–4.2	0.232
8	–4.0	0.243
9	–3.8	0.254
49	4.2	0.232
50	4.4	0.222
51	4.6	0.212
52	4.8	0.204
53	5.0	0.196

The following graph shows V as a function of x:



(*b*) Examining the graph we see that the maximum value of *V* occurs where:

$$x = \boxed{0}$$

Because $E = -dV/dx$, examination of the graph tells us that:

$$E(0) = \boxed{0}$$

81 ••

Picture the Problem Let R_2 be the radius of the second sphere and Q_1 and Q_2 the charges on the spheres when they have been connected by the wire. When the spheres are connected, the charge initially on the sphere of radius R_1 will redistribute until the spheres are at the same potential.

Express the common potential of the spheres when they are connected:

$$12\,\text{kV} = \frac{kQ_1}{R_1} \qquad (1)$$

and

$$12\,\text{kV} = \frac{kQ_2}{R_2} \qquad (2)$$

Express the potential of the first sphere before it is connected to the second sphere:

$$20\,\text{kV} = \frac{k(Q_1 + Q_2)}{R_1} \qquad (3)$$

Solve equation (1) for Q_1:

$$Q_1 = \frac{(12\,\text{kV})R_1}{k}$$

Solve equation (2) for Q_2:

$$Q_2 = \frac{(12\,\text{kV})R_2}{k}$$

Substitute in equation (3) to obtain:

$$20\,\text{kV} = \frac{k\left(\frac{(12\,\text{kV})R_1}{k} + \frac{(12\,\text{kV})R_2}{k}\right)}{R_1}$$

$$= 12\,\text{kV} + 12\,\text{kV}\left(\frac{R_2}{R_1}\right)$$

or

$$8 = 12\left(\frac{R_2}{R_1}\right)$$

Solve for R_2:

$$R_2 = \boxed{\frac{2}{3}R_1}$$

***82** ••

Picture the Problem We can use the definition of surface charge density to relate the radius R of the sphere to its charge Q and the potential function $V(r)=kQ/r$ to relate Q to the potential at r = 2 m.

Use its definition, relate the surface charge density σ to the charge Q on the sphere and the radius R of the sphere:

$$\sigma = \frac{Q}{4\pi R^2}$$

Solve for R to obtain:

$$R = \sqrt{\frac{Q}{4\pi\sigma}}$$

Relate the potential at r = 2.0 m to the charge on the sphere:

$$V(r) = \frac{kQ}{r}$$

Solve for Q to obtain:

$$Q = \frac{rV(r)}{k}$$

Substitute to obtain:

$$R = \sqrt{\frac{rV(r)}{4\pi k\sigma}} = \sqrt{\frac{4\pi\in_0 rV(r)}{4\pi\sigma}} = \sqrt{\frac{\in_0 rV(r)}{\sigma}}$$

Substitute numerical values and evaluate R:

$$R = \sqrt{\frac{(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})(2\ \mathrm{m})(500\ \mathrm{V})}{24.6\ \mathrm{nC/m^2}}} = \boxed{0.600\ \mathrm{m}}$$

83 ••

Picture the Problem We can use the definition of surface charge density to relate the radius R of the sphere to its charge Q and the potential function $V(r)=kQ/r$ to relate Q to the potential at r = 2 m.

Use its definition, relate the surface charge density σ to the charge Q on the disk and the radius R of the disk:

$$\sigma = \frac{Q}{\pi R^2}$$

Solve for Q to obtain:

$$Q = \pi\sigma R^2 \qquad (1)$$

Relate the potential at r to the charge on the disk:

$$V(r) = 2\pi k\sigma\left(\sqrt{x^2 + R^2} - x\right)$$

Substitute $V(0.6\text{ m}) = 80$ V:

$$80\text{ V} = 2\pi k\sigma\left(\sqrt{(0.6\text{ m})^2 + R^2} - 0.6\text{ m}\right)$$

Substitute $V(1.5\text{ m}) = 40$ V:

$$40\text{ V} = 2\pi k\sigma\left(\sqrt{(1.5\text{ m})^2 + R^2} - 1.5\text{ m}\right)$$

Divide the first of these equations by the second to obtain:

$$2 = \frac{\sqrt{(0.6\text{ m})^2 + R^2} - 0.6\text{ m}}{\sqrt{(1.5\text{ m})^2 + R^2} - 1.5\text{ m}}$$

Solve for R to obtain:

$$R = 0.800\text{ m}$$

Express the electric field on the axis of a disk charge:

$$E_x = 2\pi k\sigma\left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right)$$

Solve for σ to obtain:

$$\sigma = \frac{E_x}{2\pi k\left(1 - \dfrac{x}{\sqrt{x^2 + R^2}}\right)} = \frac{2\in_0 E_x}{1 - \dfrac{x}{\sqrt{x^2 + R^2}}}$$

Evaluate σ using $R = 0.8$ m and $E(1.5\text{ m}) = 23.5$ V/m:

$$\sigma = \frac{2\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)(23.5\text{ V/m})}{1 - \dfrac{1.5\text{ m}}{\sqrt{(1.5\text{ m})^2 + (0.8\text{ m})^2}}} = 3.54\text{ nC/m}^2$$

Substitute in equation (1) and evaluate Q:

$$Q = \pi\left(3.54\text{ nC/m}^2\right)(0.8\text{ m})^2 = \boxed{7.12\text{ nC}}$$

84 ••

Picture the Problem We can use $U = kq_1q_2/R$ to relate the electrostatic potential energy of the particles to their separation.

Express the electrostatic potential energy of the two particles in terms of their charge and separation:

$$U = \frac{kq_1q_2}{R}$$

Solve for R:

$$R = \frac{kq_1q_2}{U}$$

Substitute numerical values and evaluate R:

$$R = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2)(82)\left(1.6\times10^{-19}\ \text{C}\right)^2}{5.30\,\text{MeV}\times\dfrac{1.6\times10^{-19}\ \text{C}}{\text{eV}}} = \boxed{44.6\,\text{fm}}$$

85 ••

Picture the Problem We can use $\Delta V = E\Delta\ell$ and the expression for the electric field due to a plane of charge to find the potential difference between the two planes. The conducting slab introduced between the planes in part (*b*) will have a negative charge induced on its surface closest to the plane with the positive charge density and a positive charge induced on its other surface. We can proceed as in part (*a*) to find the potential difference between the planes with the conducting slab in place.

(*a*) Express the potential difference between the two planes:

$$\Delta V = E\Delta\ell = Ed$$

The electric field due to each plane is:

$$E = \frac{\sigma}{2\in_0}$$

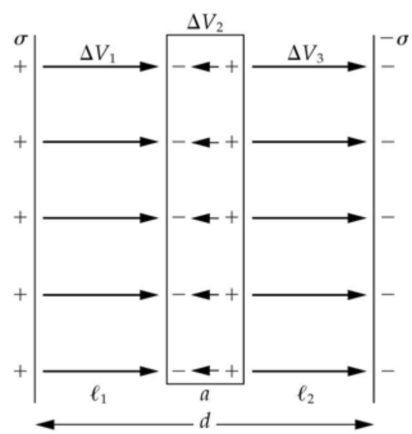
Because the charge densities are of opposite sign, the fields are additive and the resultant electric field between the planes is:

$$E = E_{\text{plane}\,1} + E_{\text{plane}\,2} = \frac{\sigma}{2\in_0} + \frac{\sigma}{2\in_0} = \frac{\sigma}{\in_0}$$

Substitute to obtain:

$$\Delta V = \boxed{\frac{\sigma d}{\in_0}}$$

(*b*) The diagram shows the conducting slab between the two planes and the electric field lines in the region between the original two planes.



Express the new potential difference $\Delta V'$ between the planes in terms of the potential differences ΔV_1, ΔV_2 and ΔV_3:

$$\Delta V' = \Delta V_1 + \Delta V_2 + \Delta V_3 = E_1\ell_1 + E_2 a + E_3\ell_2$$

Express the electric fields in regions 1, 2 and 3:

$$E_1 = E_3 = \frac{\sigma}{\epsilon_0} \text{ and } E_2 = 0$$

Substitute to obtain:

$$\Delta V' = \frac{\sigma}{\epsilon_0}\ell_1 + \frac{\sigma}{\epsilon_0}\ell_2 = \frac{\sigma}{\epsilon_0}(\ell_1 + \ell_2)$$

Express $\ell_1 + \ell_2$ in terms of a and d:

$$\ell_1 + \ell_2 = d - a$$

Substitute to obtain:

$$\Delta V' = \boxed{\frac{\sigma}{\epsilon_0}(d - a)}$$

86 •••

Picture the Problem We need to consider three regions, as in Example 23-5. Region I, $x > a$; region II, $0 < x < a$; and region III, $x < 0$. We can find V in each of these regions and then find E from $E = -dV/d\ell$.

(*a*) Relate E_{I} to V_{I}:

$$E_{\text{I}} = -\frac{dV_{\text{I}}}{dx}$$

In region I we have:

$$V_{\text{I}} = \frac{kq_1}{|x|} + \frac{kq_2}{|x-a|}$$

Substitute and evaluate E_{I}:

$$E_{\text{I}} = -\frac{d}{dx}\left[\frac{kq_1}{|x|} + \frac{kq_2}{|x-a|}\right]$$

Because $x > 0$:

$$|x| = x$$

For $x > a$:

$$|x - a| = x - a$$

Substitute to obtain:

$$E_{\text{I}} = -\frac{d}{dx}\left[\frac{kq_1}{x} + \frac{kq_2}{x-a}\right] = \boxed{\frac{kq_1}{x^2} + \frac{kq_2}{(x-a)^2}}$$

Proceed as above for regions II and III to obtain:

$$E_{\text{II}} = \boxed{\frac{kq_1}{x^2} - \frac{kq_2}{(x-a)^2}}$$

and

$$E_{\text{III}} = \boxed{-\frac{kq_1}{x^2} - \frac{kq_2}{(x-a)^2}}$$

(*b*) The distance between q_1 and a point on y axis is y and the distance between a point on the y axis and q_2 is $\sqrt{y^2+a^2}$. Using these distances, express the potential at a point on the y axis:

$$V(y) = \boxed{\frac{kq_1}{|y|} + \frac{kq_2}{\sqrt{y^2+a^2}}}$$

(*c*) To obtain the y component of $\vec{E}$ at a point on the y axis we take the derivative of $V(y)$. For $y > 0$:

$$E_y = -\frac{d}{dy}\left[\frac{kq_1}{y} + \frac{kq_2}{\sqrt{y^2+a^2}}\right]$$

$$= \boxed{\frac{kq_1}{y^2} + \frac{kq_2 y}{\left(y^2+a^2\right)^{3/2}}}$$

For $y < 0$:

$$E_y = -\frac{d}{dy}\left[-\frac{kq_1}{y} + \frac{kq_2}{\sqrt{y^2+a^2}}\right]$$

$$= \boxed{-\frac{kq_1}{y^2} + \frac{kq_2 y}{\left(y^2+a^2\right)^{3/2}}}$$

$$\boxed{\text{These are the components of the fields due to } q_1 \text{ and } q_2 \text{ that one obtains using Coulomb's law.}}$$

***87** •••

Picture the Problem We can consider the relationship between the potential and the electric field to show that this arrangement is equivalent to replacing the plane by a point charge of magnitude $-q$ located a distance d beneath the plane. In (*b*) we can first find the field at the plane surface and then use $\sigma = \epsilon_0 E$ to find the surface charge density. In (*c*) the work needed to move the charge to a point 2*d* away from the plane is the product of the potential difference between the points at distances 2*d* and 3*d* from $-q$ multiplied by the separation Δx of these points.

(*a*) $\boxed{\text{The potential anywhere on the plane is 0 in either arrangement and the electric field is perpendicular to the plane in both arrangements, so they must give the same potential everywhere in the } xy \text{ plane. Also, because the net charge is zero, the potential at infinity is zero.}}$

(*b*) The surface charge density is given by:

$$\sigma = \in_0 E \qquad (1)$$

At any point on the plane, the electric field points in the negative *x* direction and has magnitude:

$$E = \frac{kq}{d^2 + r^2}\cos\theta$$

where θ is the angle between the horizontal and a vector pointing from the positive charge to the point of interest on the *xz* plane and *r* is the distance along the plane from the origin (i.e., directly to the left of the charge).

Because $\cos\theta = \dfrac{d}{\sqrt{d^2 + r^2}}$:

$$E = \frac{kq}{d^2 + r^2}\frac{d}{\sqrt{d^2 + r^2}} = \frac{kqd}{\left(d^2 + r^2\right)^{3/2}} = \frac{qd}{4\pi \in_0 \left(d^2 + r^2\right)^{3/2}}$$

Substitute for *E* in equation (1) to obtain:

$$\sigma = \boxed{\frac{qd}{4\pi\left(d^2 + r^2\right)^{3/2}}}$$

88 •••

Picture the Problem We can express the potential due to the ring charges as the sum of the potentials due to each of the ring charges. To show that $V(x)$ is a minimum at $x = 0$, we must show that the first derivative of $V(x) = 0$ at $x = 0$ and that the second derivative is positive. In part (*c*) we can use a Taylor expansion to show that, for $x << L$, the potential is of the form $V(x) = V(0) + \alpha x^2$. In part (*d*) we can obtain the potential energy function from the potential function and, noting that it is quadratic in *x*, find the ″spring″ constant and the angular frequency of oscillation of the particle provided its displacement from its equilibrium position is small.

(*a*) Express the potential due to the ring charges as the sum of the

$$V(x) = V_{\text{ring to the left}} + V_{\text{ring to the right}}$$

potentials due to each of their charges:

The potential for a ring of charge is:

$$V(x)=\frac{kQ}{\sqrt{x^2+R^2}}$$

where R is the radius of the ring and Q is the charge of the ring.

For the ring to the left we have:

$$V_{\text{ring to the left}}=\frac{kQ}{\sqrt{(x+L)^2+L^2}}$$

For the ring to the right we have:

$$V_{\text{ring to the right}}=\frac{kQ}{\sqrt{(x-L)^2+L^2}}$$

Substitute to obtain:

$$V(x)=\boxed{\frac{kQ}{\sqrt{(x+L)^2+L^2}}+\frac{kQ}{\sqrt{(x-L)^2+L^2}}}$$

(b) Evaluate dV/dx to obtain:

$$\frac{dV}{dx}=kQ\left\{\frac{L-x}{\left[(L-x)^2+L^2\right]^{3/2}}-\frac{L+x}{\left[(L+x)^2+L^2\right]^{3/2}}\right\}=0\ \text{for extrema}$$

Solve for x to obtain: $x = 0$

Evaluate d^2V/dx^2 to obtain:

$$\frac{d^2V}{dx^2}=kQ\left\{\frac{3(L-x)^2}{\left[(L-x)^2+L^2\right]^{5/2}}-\frac{1}{\left[(L-x)^2+L^2\right]^{3/2}}+\frac{3(L+x)^2}{\left[(L+x)^2+L^2\right]^{5/2}}-\frac{1}{\left[(L-x)^2+L^2\right]^{3/2}}\right\}$$

Evaluate this expression for $x = 0$ to obtain:

$$\frac{d^2V(0)}{dx^2}=\frac{kQ}{2\sqrt{2}L^3}>0$$

$$\boxed{\text{Hence } V(x) \text{ is a maximum at } x=0.}$$

(*c*) The Taylor expansion of $V(x)$ is:

$$V(x)=V(0)+V'(0)x+\tfrac{1}{2}V''(0)x^2 + \text{higher order terms}$$

For $x << L$:

$$V(x)\approx V(0)+V'(0)x+\tfrac{1}{2}V''(0)x^2$$

Substitute our results from part (*b*) to obtain:

$$V(x)=\frac{\sqrt{2}kQ}{L}+(0)x+\tfrac{1}{2}\left(\frac{kQ}{2\sqrt{2}L^3}\right)x^2 = \frac{\sqrt{2}kQ}{L}+\frac{kQ}{4\sqrt{2}L^3}x^2$$

or

$$V(x)=\boxed{V(0)+\alpha x^2}$$

where

$$V(0)=\boxed{\frac{\sqrt{2}kQ}{L}} \text{ and } \alpha=\boxed{\frac{kQ}{4\sqrt{2}L^3}}$$

(*d*) Express the angular frequency of oscillation of a simple harmonic oscillator:

$$\omega=\sqrt{\frac{k'}{m}}$$

where k' is the restoring constant.

From our result for part (*c*) and the definition of electric potential:

$$U(x)=qV(0)+\frac{1}{2}\left(\frac{kqQ}{2\sqrt{2}L^3}\right)x^2 = qV(0)+\tfrac{1}{2}k'x^2$$

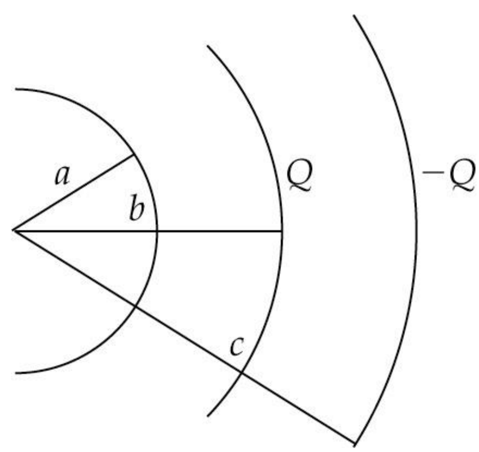
where $k'=\dfrac{kqQ}{2\sqrt{2}L^3}$

Substitute for k' in the expression for ω:

$$\omega=\boxed{\sqrt{\frac{kqQ}{2m\sqrt{2}L^3}}}$$

89 •••

Picture the Problem The diagram shows part of the shells in a cross-sectional view under the conditions of part (*a*) of the problem. We can use Gauss's law to find the electric field in the regions defined by the three surfaces and then find the electric potentials from the electric fields. In part (*b*) we can use the redistributed charges to find the charge on and potentials of the three surfaces.



(*a*) Apply Gauss's law to a spherical Gaussian surface of radius $r \geq c$ to obtain:

$$E_r\left(4\pi r^2\right) = \frac{Q_{\text{enclosed}}}{\epsilon_0} = 0$$

and $E_r = 0$ because the net charge enclosed by the Gaussian surface is zero.

Because $E_r(c) = 0$:

$$V(c) = \boxed{0}$$

Apply Gauss's law to a spherical Gaussian surface of radius $b < r < c$ to obtain:

$$E_r\left(4\pi r^2\right) = \frac{Q}{\epsilon_0}$$

and

$$E_r(b < r < c) = \frac{kQ}{r^2}$$

Use $E_r(b < r < c)$ to find the potential difference between c and b:

$$V(b) - V(c) = -kQ\int_c^b \frac{dr}{r^2} = kQ\left(\frac{1}{b} - \frac{1}{c}\right)$$

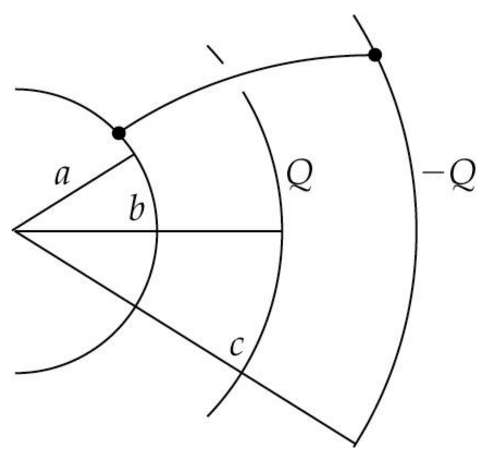
Because $V(c) = 0$:

$$V(b) = \boxed{kQ\left(\frac{1}{b} - \frac{1}{c}\right)}$$

The inner shell carries no charge, so the field between $r = a$ and $r = b$ is zero and:

$$V(a) = V(b) = \boxed{kQ\left(\frac{1}{b} - \frac{1}{c}\right)}$$

(*b*) When the inner and outer shells are connected their potentials become equal as a consequence of the redistribution of charge.



The charges on surfaces a and c are related according to:

$$Q_a + Q_c = -Q \qquad (1)$$

Q_b does not change with the connection of the inner and outer shells:

$$Q_b = \boxed{Q}$$

Express the potentials of shells a and c:

$$V(a) = V(c) = \boxed{0}$$

In the region between the $r = a$ and $r = b$, the field is kQ_a/r^2 and the potential at $r = b$ is then:

$$V(b) = kQ_a\left(\frac{1}{b} - \frac{1}{a}\right) \qquad (2)$$

The enclosed charge for $b < r < c$ is $Q_a + Q$, and by Gauss's law the field in this region is:

$$E_{b<r<c} = \frac{k(Q_a + Q)}{r^2}$$

Express the potential difference between b and c:

$$V(c) - V(b) = k(Q_a + Q)\left(\frac{1}{c} - \frac{1}{b}\right)$$
$$= -V(b)$$

because $V(c) = 0$.

Solve for $V(b)$ to obtain:

$$V(b) = k(Q_a + Q)\left(\frac{1}{b} - \frac{1}{c}\right) \qquad (3)$$

Equate equations (2) and (3) and solve for Q_a to obtain:

$$Q_a = \boxed{-Q\frac{a(c-b)}{b(c-a)}} \qquad (4)$$

Substitute equation (4) in equation (1) and solve for Q_c to obtain:

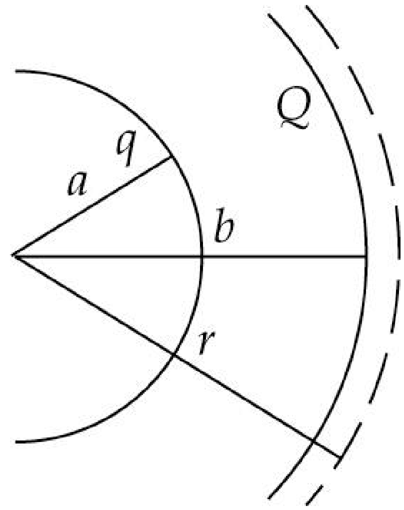
$$Q_c = \boxed{-Q\frac{c(b-a)}{b(c-a)}} \qquad (5)$$

Substitute (4) and (5) in (3) to obtain:

$$V(b) = \boxed{kQ\frac{(c-b)(b-a)}{b^2(c-a)}}$$

***90** •••

Picture the Problem The diagram shows a cross-sectional view of a portion of the concentric spherical shells. Let the charge on the inner shell be q. The dashed line represents a spherical Gaussian surface over which we can integrate $\vec{E}\cdot\hat{\boldsymbol{n}}dA$ in order to find E_r for $r \geq b$. We can find $V(b)$ from the integral of E_r between $r = \infty$ and $r = b$. We can obtain a second expression for $V(b)$ by considering the potential difference between a and b and solving the two equations simultaneously for the charge q on the inner shell.



Apply Gauss's law to a spherical surface of radius $r \geq b$:

$$E_r\left(4\pi r^2\right) = \frac{Q+q}{\varepsilon_0}$$

Solve for E_r to obtain:

$$E_r = \frac{k(Q+q)}{r^2}$$

Use E_r to find $V(b)$:

$$V(b) = -k(Q+q)\int_{\infty}^{b}\frac{dr}{r^2}$$

$$= \frac{k(Q+q)}{b}$$

We can also determine $V(b)$ by considering the potential difference between a, i.e., 0 and b:

$$V(b) = kq\left(\frac{1}{b}-\frac{1}{a}\right)$$

Equate these expressions for $V(b)$ to obtain:

$$\frac{k(Q+q)}{b} = ka\left(\frac{1}{b}-\frac{1}{a}\right)$$

Solve for q to obtain:

$$q = \boxed{-\frac{a}{b}Q}$$

91 •••

Picture the Problem We can use the hint to derive an expression for the electrostatic potential energy dU required to bring in a layer of charge of thickness dr and then integrate this expression from $r = 0$ to R to obtain an expression for the required work.

If we build up the sphere in layers, then at a given radius r the net charge on the sphere will be given by:

$$Q(r) = Q\left(\frac{r}{R}\right)^3$$

When the radius of the sphere is r, the potential relative to infinity is:

$$V(r) = \frac{Q(r)}{4\pi \in_0 r} = \frac{Q}{4\pi \in_0} \frac{r^2}{R^3}$$

Express the work dW required to bring in charge dQ from infinity to the surface of a uniformly charged sphere of radius r:

$$\begin{aligned} dW &= dU = V(r)dQ \\ &= \frac{Q}{4\pi \in_0} \frac{r^2}{R^3}\left(4\pi r^2 \frac{3Q}{4\pi R^3} dr\right) \\ &= \frac{3Q^2}{4\pi \in_0 R^6} r^4 dr \end{aligned}$$

Integrate dW from 0 to R to obtain:

$$\begin{aligned} W = U &= \frac{3Q^2}{4\pi \in_0 R^6} \int_0^R r^4 dr \\ &= \frac{3Q^2}{4\pi \in_0 R^6} \left[\frac{r^5}{5}\right]_0^R = \boxed{\frac{3Q^2}{20\pi \in_0 R}} \end{aligned}$$

92 ••

Picture the Problem We can equate the rest energy of an electron and the result of Problem 91 in order to obtain an expression that we can solve for the classical electron radius.

From Problem 91 we have:

$$U = \frac{3e^2}{20\pi \in_0 R}$$

The rest mass of the electron is given by:

$$E_0 = m_0 c^2$$

Equate these energies to obtain:

$$\frac{3e^2}{20\pi \in_0 R} = m_0 c^2$$

Solve for R:

$$R = \frac{3e^2}{20\pi \in_0 m_0 c^2}$$

Substitute numerical values and evaluate R:

$$\begin{aligned} R &= \frac{3\left(1.6\times10^{-19}\ \text{C}\right)^2}{20\pi\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(5.11\times10^5\ \text{eV}\right)\left(1.6\times10^{-19}\ \text{J/eV}\right)} \\ &= \boxed{1.69\times10^{-15}\ \text{m}} \end{aligned}$$

$\boxed{\text{This model does not explain how the electron holds together against its own mutual repulsion.}}$

93 ••

Picture the Problem Because the post-fission volumes of the fission products are equal, we can express the post-fission radii in terms of the radius of the pre-fission sphere.

(*a*) Relate the initial volume *V* of the uniformly charged sphere to the volumes *V*′ of the fission products:

$$V = 2V'$$

Substitute for *V* and *V* ′:

$$\tfrac{4}{3}\pi R^3 = 2\left(\tfrac{4}{3}\pi R'^3\right)$$

Solve for and evaluate *R*′:

$$R' = \frac{1}{\sqrt[3]{2}}R = \boxed{0.794R}$$

(*b*) Express the difference Δ*E* in the total electrostatic energy as a result of fissioning:

$$\Delta E = E - E'$$

From Problem 91 we have:

$$E = \frac{3Q^2}{20\pi \in_0 R}$$

After fissioning:

$$E' = 2\left(\frac{3Q'^2}{20\pi \in_0 R'}\right) = 2\left[\frac{3\left(\tfrac{1}{2}Q\right)^2}{20\pi \in_0 \frac{1}{\sqrt[3]{2}}R}\right]$$

$$= \frac{\sqrt[3]{2}}{2}\left(\frac{3Q^2}{20\pi \in_0 R}\right) = 0.630E$$

Substitute for *E* and *E*′ to obtain:

$$\Delta E = E - 0.630E = \boxed{0.370E}$$

***94** •••

Picture the Problem We can use the definition of density to express the radius *R* of a nucleus as a function of its atomic mass *N*. We can then use the result derived in Problem 91 to express the electrostatic energies of the ^{235}U nucleus and the nuclei of the fission fragments ^{140}Xe and ^{94}Sr.

The energy released by this fission process is:

$$\Delta E = U_{^{235}\mathrm{U}} - \left(U_{^{140}\mathrm{Xe}} + U_{^{94}\mathrm{Sr}}\right) \qquad (1)$$

Express the mass of a nucleus in terms of its density and volume:

$$Nm = \tfrac{4}{3}\rho\pi R^3$$

where *N* is the nuclear number.

Solve for R to obtain:

$$R = \sqrt[3]{\frac{3Nm}{4\pi\rho}}$$

Substitute numerical values and evaluate R as a function of N:

$$R = \sqrt[3]{\frac{3\left(1.660\times10^{-27}\text{ kg}\right)}{4\pi\left(4\times10^{17}\text{ kg/m}^3\right)}}N^{1/3} = \left(9.97\times10^{-16}\text{ m}\right)N^{1/3}$$

The 'radius' of the ^{235}U nucleus is therefore:

$$R_U = \left(9.97\times10^{-16}\text{ m}\right)(235)^{1/3} = 6.15\times10^{-15}\text{ m}$$

From Problem 91 we have:

$$U = \frac{3Q^2}{20\pi\in_0 R}$$

Substitute numerical values and evaluate the electrostatic energy of the ^{235}U nucleus:

$$U_{^{235}\text{U}} = \frac{3\left(92\times1.6\times10^{-19}\text{ C}\right)^2}{20\pi\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)\left(6.15\times10^{-15}\text{ m}\right)} = 1.91\times10^{-10}\text{ J}\times\frac{1\text{eV}}{1.6\times10^{-19}\text{ J/eV}} = 1189\text{ MeV}$$

Proceed as above to find the electrostatic energy of the fission fragments ^{140}Xe and ^{94}Sr:

$$U_{^{140}\text{Xe}} = \frac{3\left(54\times1.6\times10^{-19}\text{ C}\right)^2}{20\pi\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)\left(6.15\times10^{-15}\text{ m}\right)} = 6.57\times10^{-11}\text{ J}\times\frac{1\text{eV}}{1.6\times10^{-19}\text{ J/eV}} = 410\text{ MeV}$$

and

$$U_{^{94}\text{Sr}} = \frac{3\left(38\times1.6\times10^{-19}\text{ C}\right)^2}{20\pi\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)\left(6.15\times10^{-15}\text{ m}\right)} = 3.25\times10^{-11}\text{ J}\times\frac{1\text{eV}}{1.602\times10^{-19}\text{ J/eV}} = 203\text{ MeV}$$

Substitute for $U_{^{235}\text{U}}$, $U_{^{140}\text{Xe}}$, and $U_{^{94}\text{Sr}}$ in equation (1) and evaluate ΔE:

$$\Delta E = 1189\text{ MeV} - \left(410\text{ MeV} + 203\text{ MeV}\right) = \boxed{576\text{ MeV}}$$

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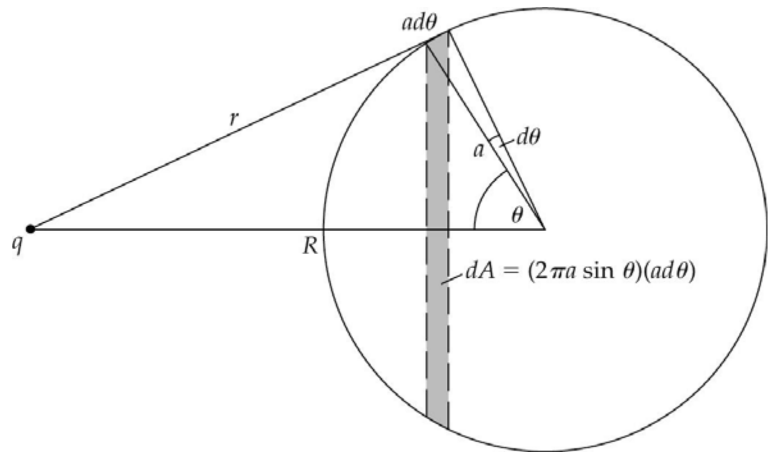
Picture the Problem The geometry of the point charge and the sphere is shown below. The charge is a distance R away from the center of a spherical shell of radius a.



(a) The average potential over the surface of the sphere is given by:

$$V_{\text{av}} = \oint_{\text{sphere}} \frac{k\,dq}{r} = \oint_{\text{sphere}} \frac{k\sigma dA}{r}$$

Substitute for k, σ, and dA to obtain:

$$V_{\text{av}} = \frac{1}{4\pi \in_0} \int_0^{\pi} \frac{q(2\pi a \sin\theta)(a d\theta)}{4\pi a^2 r}$$

Apply the law of cosines to the triangle to obtain:

$$r = \sqrt{R^2 + a^2 - 2aR\cos\theta}$$

Substitute for r and simplify to obtain:

$$V_{\text{av}} = \frac{q}{8\pi \in_0} \int_0^{\pi} \frac{\sin\theta d\theta}{\left(R^2 + a^2 - 2aR\cos\theta\right)^{1/2}}$$

Change variables by letting $u = \cos\theta$. Then:

$$du = -\sin\theta d\theta$$

and

$$V_{\text{av}} = \frac{-q}{8\pi \in_0} \int_1^{-1} \frac{du}{\left(R^2 + a^2 - 2aRu\right)^{1/2}} \qquad (1)$$

To simplify the integrand, let:

$$\alpha = R^2 + a^2,\ \beta = 2aR, \text{ and } v = \alpha - \beta u$$

Then $dv = -\beta du$ and:

$$\int_1^{-1} \frac{du}{\left(R^2 + a^2 - 2aRu\right)^{1/2}} = -\frac{1}{\beta}\int_{\ell_1}^{\ell_2} \frac{dv}{\sqrt{v}} = -\frac{2}{\beta}\sqrt{v}\Big|_{\ell_1}^{\ell_2} = -\frac{1}{aR}\sqrt{\alpha - \beta u}\Big|_1^{-1}$$

$$= -\frac{1}{aR}\left[\sqrt{\alpha + \beta} - \sqrt{\alpha - \beta}\right]$$

Substitute for α and β to obtain:

$$\int_{1}^{-1} \frac{du}{\left(R^2 + a^2 - 2aRu\right)^{1/2}} = -\frac{1}{aR}\left[\sqrt{R^2 + a^2 + 2aR} - \sqrt{R^2 + a^2 - 2aR}\right]$$

$$= -\frac{1}{aR}\left[\sqrt{(R+a)^2} - \sqrt{(R-a)^2}\right]$$

$$= -\frac{1}{aR}\left[(R+a) - (R-a)\right] = -\frac{2}{R}$$

Substitute in equation (1) to obtain:

$$V_{\text{av}} = \frac{-q}{8\pi \in_0}\left(-\frac{2}{R}\right) = \boxed{\frac{q}{4\pi \in_0 R}}$$

Note that this result is the potential at the center of the sphere due to the point charge.

(*b*) The superposition principle tells us that the potential at any point is the sum of the potentials due to any charge distributions in space. Because this result is independent of any properties of the sphere, this result must hold for any sphere and any configuration of charges outside of it.