Chapter 25
Electric Current and Direct-Current Circuits

Conceptual Problems

***1 •**

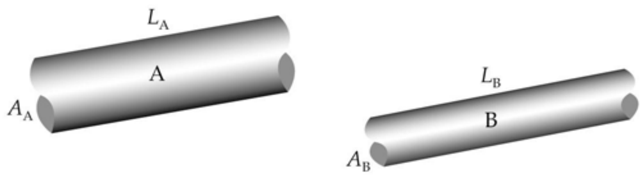
Determine the Concept When current flows, the charges are not in equilibrium. In that case, the electric field provides the force needed for the charge flow.

2 •

Determine the Concept Water, regarded as a viscous liquid flowing from a water tower through a pipe to ground is another mechanical analog of a simple circuit.

3 •

Picture the Problem The resistances of the wires are given by $R = \rho L/A$, where L is the length of the wire and A is its cross-sectional area. We can express the ratio of the resistances and use our knowledge of their lengths and diameters to find the resistance of wire A.



Express the resistance of wire A:

$$R_A = \frac{\rho L_A}{A_A}$$

where ρ is the resistivity of the wire.

Express the resistance of wire B:

$$R = \frac{\rho L_B}{A_B}$$

Divide the first of these equations by the second to obtain:

$$\frac{R_A}{R} = \frac{\frac{\rho L_A}{A_A}}{\frac{\rho L_B}{A_B}} = \frac{L_A}{L_B} \cdot \frac{A_B}{A_A}$$

or, because $L_A = L_B$,

$$R_A = \frac{A_B}{A_A} R \qquad (1)$$

Express the area of wire A in terms of its diameter:

$$A_A = \tfrac{1}{4}\pi d_A^2$$

Express the area of wire B in terms of its diameter:

$$A_B = \tfrac{1}{4}\pi d_B^2$$

Substitute in equation (1) to obtain:

$$R_A = \frac{d_B^2}{d_A^2} R$$

or, because $d_A = 2d_B$,

$$R_A = \frac{d_B^2}{(2d_B)^2} R = \tfrac{1}{4} R$$

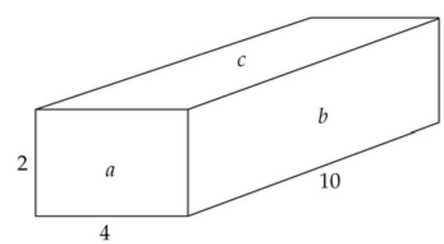
and $\boxed{(e) \text{ is correct.}}$

4 ••

Determine the Concept An emf is a source of energy that gives rise to a potential difference between two points and may result in current flow if there is a conducting path whereas a potential difference is the consequence of two points in space being at different potentials.

***5** ••

Picture the Problem The resistance of the metal bar varies directly with its length and inversely with its cross-sectional area. Hence, to minimize the resistance of the bar, we should connect to the surface for which the ratio of the length to the contact area is least.



Denoting the surfaces as *a*, *b*, and *c*, complete the table to the right:

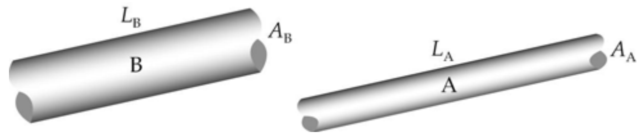
Surface	L	A	L/A
a	10	8	0.8
b	4	20	0.2
c	2	40	0.05

Because connecting to surface *c* minimizes *R*:

$\boxed{(c) \text{ is correct.}}$

6 ••

Picture the Problem The resistances of the wires are given by $R = \rho L/A$, where L is the length of the wire and A is its cross-sectional area. We can express the ratio of the resistances and use the definition of density to eliminate the cross-sectional areas of the wires in favor of the ratio of their lengths.



Express the resistance of wire A:

$$R_A = \frac{\rho L_A}{A_A}$$

where ρ is the resistivity of copper.

Express the resistance of wire B:

$$R_B = \frac{\rho L_B}{A_B}$$

Divide the first of these equations by the second to obtain:

$$\frac{R_A}{R_B} = \frac{\frac{\rho L_A}{A_A}}{\frac{\rho L_B}{A_B}} = \frac{L_A}{L_B}\frac{A_B}{A_A}$$

or, because $L_A = 2L_B$,

$$R_A = 2\frac{A_B}{A_A}R_B \qquad (1)$$

Using the definition of density, express the mass of wire A:

$$m_A = \rho' V_A = \rho' L_A A_A$$

where ρ' is the density of copper.

Express the mass of wire B

$$m_B = \rho' V_B = \rho' L_B A_B$$

Because the masses of the wires are equal:

$$\rho' L_A A_A = \rho' L_B A_B$$

or

$$\frac{A_B}{A_A} = \frac{L_A}{L_B}$$

Substitute in equation (1) to obtain:

$$R_A = 2\frac{L_A}{L_B}R_B = 2(2)R_B = 4R_B$$

and $\boxed{(b) \text{ is correct.}}$

7 •

Picture the Problem The power dissipated in the resistor is given by $P = I^2R$. We can express the power dissipated when the current is $3I$ and, assuming that the resistance does not change, express the ratio of the two rates of energy dissipation to find the power dissipated when the current is $3I$.

Express the power dissipated in the

$$P = I^2R$$

resistor when the current in it is I:

Express the power dissipated in the resistor when the current in it is $3I$:

$$P' = (3I)^2 R = 9I^2 R$$

Divide the second of these equations by the first to obtain:

$$\frac{P'}{P} = \frac{9I^2 R}{I^2 R} = 9$$

or

$$P' = 9P \text{ and } \boxed{(d) \text{ is correct.}}$$

8 •

Picture the Problem Assuming the current (which depends on the resistance) to be constant, the power dissipated in a resistor is directly proportional to the voltage drop across it.

Express the power dissipated in the resistor when the voltage drop across it is V:

$$P = \frac{V^2}{R}$$

Express the power dissipated in the resistor when the voltage drop across it is increased to $2V$:

$$P' = \frac{(2V)^2}{R} = \frac{4V^2}{R}$$

Divide the second of these equations by the first to obtain:

$$\frac{P'}{P} = \frac{\frac{4V^2}{R}}{\frac{V^2}{R}} = 4 \Rightarrow P' = 4P$$

$$\boxed{(c) \text{ is correct.}}$$

9 •

Determine the Concept You should decrease the resistance. Because the voltage across the resistor is constant, the heat out is given by $P = V^2/R$. Hence, decreasing the resistance will increase P.

***10** •

Picture the Problem We can find the equivalent resistance of this two-resistor combination and then apply the condition that $R_1 >> R_2$.

Express the equivalent resistance of R_1 and R_2 in parallel:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Solve for R_{eq} to obtain:

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Factor R_1 from the denominator and simplify to obtain:

$$R_{eq} = \frac{R_1 R_2}{R_1\left(1 + \frac{R_2}{R_1}\right)} = \frac{R_2}{1 + \frac{R_2}{R_1}}$$

If $R_1 >> R_2$, then:

$$R_{eq} = R_{eff} \approx R_2 \text{ and } \boxed{(b) \text{ is correct}}$$

11 •

Picture the Problem We can find the equivalent resistance of this two-resistor combination and then apply the condition that $R_1 >> R_2$.

Express the equivalent resistance of R_1 and R_2 in series:

$$R_{eq} = R_1 + R_2$$

Factor R_1 to obtain:

$$R_{eq} = R_1\left(1 + \frac{R_2}{R_1}\right)$$

If $R_1 >> R_2$, then:

$$R_{eq} = R_{eff} \approx R_1 \text{ and } \boxed{(a) \text{ is correct}}$$

12 •

Picture the Problem Because the potential difference across resistors connected in parallel is the same for each resistor; we can use Ohm's law to relate the currents through the resistors to their resistances.

Using Ohm's law, express the current carried by resistor A:

$$I_A = \frac{V}{R_A} = \frac{V}{2R_B}$$

Using Ohm's law, express the current carried by resistor B:

$$I_B = \frac{V}{R_B}$$

Divide the second of these equations by the first to obtain:

$$\frac{I_B}{I_A} = \frac{\frac{V}{R_B}}{\frac{V}{2R_B}} = 2$$

and

$$I_B = 2I_A \text{ and } \boxed{(b) \text{ is correct.}}$$

***13 •**

Determine the Concept In a series circuit, because there are no alternative pathways, all resistors carry the same current. The potential difference across each resistor, keeping with Ohm's law, is given by the product of the current and the resistance and, hence, is not the same across each resistor unless the resistors are identical. $\boxed{(a) \text{ is correct.}}$

14 ••

Picture the Problem Because the potential difference across the two combinations of resistors is constant, we can use $P = V^2/R$ to relate the power delivered by the battery to the equivalent resistance of each combination of resistors.

Express the power delivered by the battery when the resistors are connected in series:

$$P_s = \frac{V^2}{R_{eq}}$$

Letting R represent the resistance of the identical resistors, express R_{eq}:

$$R_{eq} = R + R = 2R$$

Substitute to obtain:

$$P_s = \frac{V^2}{2R} \quad (1)$$

Express the power delivered by the battery when the resistors are connected in parallel:

$$P_p = \frac{V^2}{R_{eq}}$$

Express the equivalent resistance of the identical resistors connected in parallel:

$$R_{eq} = \frac{(R)(R)}{R + R} = \tfrac{1}{2}R$$

Substitute to obtain:

$$P_p = \frac{V^2}{\tfrac{1}{2}R} = \frac{2V^2}{R} \quad (2)$$

Divide equation (2) by equation (1) to obtain:

$$\frac{P_p}{P_s} = \frac{\dfrac{2V^2}{R}}{\dfrac{V^2}{2R}} = 4$$

Solve for and evaluate P_p:

$$P_p = 4P_s = 4(20\ \text{W}) = 80\ \text{W}$$

and $\boxed{(e) \text{ is correct.}}$

15 •

Determine the Concept While Kirchhoff's loop rule is a statement about potential differences around a closed loop in a circuit, recall that electric potential at a point in space is the work required to bring a charged object from infinity to the given point. Hence, the loop rule is actually a statement that energy is conserved around any closed path in an electric circuit. $\boxed{(b) \text{ is correct.}}$

16 •

Determine the Concept An ideal voltmeter would have infinite resistance. A voltmeter consists of a galvanometer movement connected in series with a large resistance. The large resistor accomplishes two purposes; 1) it protects the galvanometer movement by limiting the current drawn by it, and 2) minimizes the loading of the circuit by the voltmeter by placing a large resistance in parallel with the circuit element across which the potential difference is being measured. $\boxed{(a) \text{ is correct.}}$

***17 •**

Determine the Concept An ideal ammeter would have zero resistance. An ammeter consists of a very small resistance in parallel with a galvanometer movement. The small resistance accomplishes two purposes: 1) It protects the galvanometer movement by shunting most of the current in the circuit around the galvanometer movement, and 2) It minimizes the loading of the circuit by the ammeter by minimizing the resistance of the ammeter. $\boxed{(b) \text{ is correct.}}$

18 •

Determine the Concept An ideal voltage source would have zero internal resistance. The terminal potential difference of a voltage source is given by $V = \mathcal{E} - Ir$, where ε is the emf of the source, I is the current drawn from the source, and r is the internal resistance of the source. $\boxed{(b) \text{ is correct.}}$

19 •

Determine the Concept If we apply Kirchhoff's loop rule with the switch closed, we obtain $\varepsilon - IR - V_C = 0$. Immediately after the switch is closed, $I = 0$ and we have $\varepsilon = V_C$.
$\boxed{(b) \text{ is correct.}}$

20 ••

Determine the Concept The energy stored in the fully charged capacitor is $U = \frac{1}{2}C\mathcal{E}^2$. During the charging process, a total charge $Q_f = \varepsilon C$ flows through the battery. The battery therefore does work $W = Q_f\varepsilon = C\varepsilon^2$. The energy dissipated in the resistor is the difference between W and U. $\boxed{(b) \text{ is correct.}}$

***21 ••**

Determine the Concept Applying Kirchhoff's loop rule to the circuit, we obtain $\mathcal{E} - V_R - V_C = 0$, where V_R is the voltage drop across the resistor. Applying Ohm's law to the resistor, we obtain $V_R = IR$. Because I decreases as the capacitor is charged, V_R decreases with time. $\boxed{(e) \text{ is correct.}}$

22 ••

Picture the Problem We can express the variation of charge on the discharging capacitor as a function of time to find the time T it takes for the charge on the capacitor to drop to half its initial value. We can also express the energy remaining in the electric field of the discharging capacitor as a function of time and find the time t' for the energy to drop to half its initial value in terms of T.

Express the dependence of the charge stored on a capacitor on time:

$$Q(t) = Q_0 e^{-t/\tau}$$

where $\tau = RC$.

For $Q(t) = \frac{1}{2} Q_0$:

$$\tfrac{1}{2} Q_0 = Q_0 e^{-T/\tau}$$

or

$$\tfrac{1}{2} = e^{-T/\tau}$$

Take the natural logarithm of both sides of the equation and solve for T to obtain:

$$T = \tau \ln 2$$

Express the dependence of the energy stored in a capacitor on the potential difference V_C across its terminals:

$$U(t) = \tfrac{1}{2} C V_C^2$$

Express the potential difference across a discharging capacitor as a function of time:

$$V_C = V_0 e^{-t/RC}$$

Substitute to obtain:

$$U(t) = \tfrac{1}{2} C \left(V_0 e^{-t/RC}\right)^2 = \tfrac{1}{2} C V_0^2 e^{-2t/RC} = U_0 e^{-2t/RC}$$

For $U(t) = \frac{1}{2} U_0$:

$$\tfrac{1}{2} U_0 = U_0 e^{-2t'/RC}$$

or

$$\tfrac{1}{2} = e^{-2t'/RC}$$

Take the natural logarithm of both sides of the equation and solve for t' to obtain:

$$t' = \tfrac{1}{2}\tau \ln 2 = \boxed{\tfrac{1}{2}T}$$

23 •

Determine the Concept A small resistance because $P = \varepsilon^2/R$.

***24** •

Determine the Concept The potential difference across an external resistor of resistance R is given by $\dfrac{R}{r+R}V$, where r is the internal resistance and V the voltage supplied by the source. The higher R is, the higher the voltage drop across R. Put differently, the higher the resistance a voltage source sees, the less its own resistance will change the circuit.

25 •

Determine the Concept Yes. Kirchhoff's rules are statements of the conservation of energy and charge and hence apply to all circuits.

26 ••

Determine the Concept All of the current provided by the battery passes through R_1, whereas only half this current passes through R_2 and R_3. Because $P = I^2R$, the power dissipated in R_1 will be four times that dissipated in R_2 and R_3. $\boxed{(c)\text{ is correct.}}$

Estimation and Approximation

27 ••

Picture the Problem We can use Ohm's law and the definition of resistivity to find the maximum voltage that can be applied across 40 m of the 16-gauge copper wire. In part (*b*) we can find the electric field in the wire using $E = V/L$. In part (*c*) we can use $P = I^2R$ to find the power dissipated in the wire when it carries 6 A.

(*a*) Use Ohm's law to relate the potential difference across the wire to its maximum current and its resistance:

$$V_{max} = I_{max}R$$

Use the definition of resistivity to relate the resistance of the wire to its length and cross-sectional area:

$$R = \rho\frac{L}{A}$$

Substitute to obtain:

$$V_{max} = I_{max}\rho\frac{L}{A}$$

Substitute numerical values (see Tables 25-1 and 25-2) for the resistivity of copper and the cross-sectional area of 16-gauge wire:

$$V_{max} = (6\,\text{A})(1.7\times10^{-8}\,\Omega\cdot\text{m}) \times \left(\frac{40\,\text{m}}{1.309\,\text{mm}^2}\right) = \boxed{3.12\,\text{V}}$$

(*b*) Relate the electric in the wire to the potential difference between its ends and the length of the wire:

$$E = \frac{V}{L} = \frac{3.12\,\text{V}}{40\,\text{m}} = \boxed{78.0\,\text{mV/m}}$$

(*c*) Relate the power dissipated in the wire to the current in and the resistance of the wire:

$$P = I^2R$$

Substitute for R to obtain:

$$P = I^2\rho\frac{L}{A}$$

Substitute numerical values and evaluate P:

$$P = (6\,\text{A})^2(1.7\times10^{-8}\,\Omega\cdot\text{m})\left(\frac{40\,\text{m}}{1.309\,\text{mm}^2}\right) = \boxed{18.7\,\text{W}}$$

28 ••

Picture the Problem We can use the definition of resistivity to find the resistance of the jumper cable. In part (*b*), the application of Ohm's law will yield the potential difference across the jumper cable when it is starting a car, and, in part (*c*), we can use the expression for the power dissipated in a conductor to find the power dissipation in the jumper cable.

(*a*) Noting that a jumper cable has two leads, express the resistance of the cable in terms of the wire's resistivity and the cable's length, and cross-sectional area:

$$R = \rho\frac{L}{A}$$

Substitute numerical values (see Table 25-1 for the resistivity of copper) and evaluate R:

$$R = (1.7\times10^{-8}\,\Omega\cdot\text{m})\frac{6\,\text{m}}{10\,\text{mm}^2} = \boxed{0.0102\,\Omega}$$

(*b*) Apply Ohm's law to the cable to obtain:

$$V = IR = (90\,\text{A})(0.0102\,\Omega) = \boxed{0.918\,\text{V}}$$

(*c*) Use the expression for the power dissipated in a conductor to obtain:

$$P = IV = (90\,\text{A})(0.918\,\text{V}) = \boxed{82.6\,\text{W}}$$

29 ••

Picture the Problem We can combine the expression for the rate at which energy is delivered to the water to vaporize it ($P = \varepsilon^2/R$) and the expression for the resistance of a conductor ($R = \rho L/A$) to obtain an expression for the required length L of wire.

Use an expression for the power dissipated in a resistor to relate the required resistance to rate at which energy is delivered to generate the steam:

$$R = \frac{\varepsilon^2}{P}$$

Relate the resistance of the wire to its length, cross-sectional area, and resistivity:

$$R = \rho\frac{L}{A}$$

Equate these two expressions and solve for L to obtain:

$$L = \frac{\varepsilon^2 A}{\rho P}$$

Express the power required to generate the steam in terms of the rate of energy delivery:

$$P = \frac{\Delta E}{\Delta t} = \frac{\Delta(mL_\text{v})}{\Delta t} = L_\text{v}\frac{\Delta m}{\Delta t}$$

Substitute to obtain:

$$L = \frac{\varepsilon^2 A}{\rho L_\text{v}\dfrac{\Delta m}{\Delta t}}$$

Substitute numerical values (see Table 25-1 for the resistivity of Nichrome and Table 18-2 for the latent heat of vaporization of water) and evaluate L:

$$L = \frac{(120\,\text{V})^2\,\frac{\pi}{4}(1.80\,\text{mm})^2}{(10^{-6}\,\Omega\cdot\text{m})(2257\,\text{kJ/kg})(8\,\text{g/s})} = \boxed{2.03\,\text{m}}$$

***30** ••

Picture the Problem We can find the annual savings by taking into account the costs of the two types of bulbs, the rate at which they consume energy and the cost of that energy, and their expected lifetimes.

Express the yearly savings:

$$\Delta\$ = \text{Cost}_\text{incandescent} - \text{Cost}_\text{fluorescent} \qquad (1)$$

Express the annual cost with the incandescent bulbs:

$$\text{Cost}_{\text{incandescent}} = \text{Cost}_{\text{bulbs}} + \text{Cost}_{\text{energy}}$$

Express and evaluate the annual cost of the incandescent bulbs:

$$\begin{aligned}\text{Cost}_{\text{bulbs}} &= \text{number of bulbs in use} \times \text{annual consumption of bulbs} \times \text{cost per bulb}\\ &= (6)\left(\frac{365.24\,\text{d} \times \frac{24\,\text{h}}{\text{d}}}{1200\,\text{h}}\right)(\$1.50) = \$65.74\end{aligned}$$

Find the cost of operating the incandescent bulbs for one year:

$$\begin{aligned}\text{Cost}_{\text{energy}} &= \text{energy consumed} \times \text{cost per unit of energy}\\ &= 6(75\,\text{W})(365.25\,\text{d})(24\,\text{h/d})(\$0.115/\text{kW}\cdot\text{h})\\ &= \$453.64\end{aligned}$$

Express the annual cost with the fluorescent bulbs:

$$\text{Cost}_{\text{fluorescent}} = \text{Cost}_{\text{bulbs}} + \text{Cost}_{\text{energy}}$$

Express and evaluate the annual cost of the fluorescent bulbs:

$$\begin{aligned}\text{Cost}_{\text{bulbs}} &= \text{number of bulbs in use} \times \text{annual consumption of bulbs} \times \text{cost per bulb}\\ &= (6)\left(\frac{365.24\,\text{d} \times \frac{24\,\text{h}}{\text{d}}}{8000\,\text{h}}\right)(\$6) = \$39.45\end{aligned}$$

Find the cost of operating the fluorescent bulbs for one year:

$$\begin{aligned}\text{Cost}_{\text{energy}} &= \text{energy consumed} \times \text{cost per unit of energy}\\ &= 6(20\,\text{W})\left(365.24\,\text{d} \times \frac{24\,\text{h}}{\text{d}}\right)(\$0.115/\text{kW}\cdot\text{h})\\ &= \$120.97\end{aligned}$$

Substitute in equation (1) and evaluate the cost savings $\Delta\$$:

$$\begin{aligned}\Delta\$ &= \text{Cost}_{\text{incandescent}} - \text{Cost}_{\text{fluorescent}} = (\$65.74 + \$453.64) - (\$39.45 + \$120.97)\\ &= \boxed{\$358.96}\end{aligned}$$

31 ••

Picture the Problem We can use an expression for the power dissipated in a resistor to relate the Joule heating in the wire to its resistance and the definition of resistivity to relate the resistance to the length and cross-sectional area of the wire.

Express the power the wires must dissipate in terms of the current they carry and their resistance:

$$P = I^2 R$$

Divide both sides of the equation by L to express the power dissipation per unit length:

$$\frac{P}{L} = \frac{I^2 R}{L}$$

Using the definition of resistivity, relate the resistance of the wire to its resistivity, length and cross-sectional area:

$$R = \rho \frac{L}{A} = \rho \frac{L}{\frac{\pi}{4} d^2} = \frac{4\rho L}{\pi d^2}$$

Substitute to obtain:

$$\frac{P}{L} = \frac{4\rho I^2}{\pi d^2}$$

Solve for d to obtain:

$$d = 2I\sqrt{\frac{\rho}{\pi (P/L)}}$$

Substitute numerical values (see Table 25-1 for the resistivity of copper wire) and evaluate d:

$$d = 2(20\,\text{A})\sqrt{\frac{1.7\times10^{-8}\,\Omega\cdot\text{m}}{\pi(2\,\text{W/m})}} = \boxed{2.08\,\text{mm}}$$

***32** ••

Picture the Problem Let r be the internal resistance of each battery and use Ohm's law to express the current in laser diode as a function of the potential difference across r. We can find the power of the laser diode from the product of the potential difference across the internal resistance of the batteries and the current delivered by them I and the time-to-discharge from the combined capacities of the two batteries and I.

(*a*) Use Ohm's law to express the current in the laser diode:

$$I = \frac{V_{\text{internal resistance}}}{2r}$$

The potential difference across the internal resistance is:

$$V_{\text{internal resistance}} = \mathcal{E} - 2.3\,\text{V}$$

Substitute to obtain:

$$I = \frac{\mathcal{E} - 2.3\,\text{V}}{2r}$$

Assuming that $r = 125\ \Omega$:

$$I = \frac{2(1.55) - 2.3\,\text{V}}{2(125\,\Omega)} = \boxed{3.20\,\text{mA}}$$

(*b*) The power delivered by the batteries is given by:

$$P = IV = (3.2\,\text{mA})(2.3\,\text{V}) = 7.36\,\text{mW}$$

The power of the laser is half this value:

$$P_{\text{laser}} = \tfrac{1}{2}P = \tfrac{1}{2}(7.36\,\text{mW}) = \boxed{3.68\,\text{mW}}$$

Express the ratio of P_{laser} to P_{quoted}:

$$\frac{P_{\text{laser}}}{P_{\text{quoted}}} = \frac{3.68\,\text{mW}}{3\,\text{mW}} = 1.23$$

or

$$P_{\text{laser}} = \boxed{123\% P_{\text{quoted}}}$$

(*c*) Express the time-to-discharge:

$$\Delta t = \frac{\text{Capacity}}{I}$$

Because each battery has a capacity of 20 mA·h, the series combination has a capacity of 40 mA·h and:

$$\Delta t = \frac{40\,\text{mA}\cdot\text{h}}{3.20\,\text{mA}} = \boxed{12.5\,\text{h}}$$

Current and the Motion of Charges

33 •

Picture the Problem We can relate the drift velocity of the electrons to the current density using $I = nev_{\text{d}}A$. We can find the number density of charge carriers n using $n = \rho N_{\text{A}}/M$, where ρ is the mass density, N_{A} Avogadro's number, and M the molar mass. We can find the cross-sectional area of 10-gauge wire in Table 25-2.

Use the relation between current and drift velocity to relate I and n:

$$I = nev_{\text{d}}A$$

Solve for v_{d}:

$$v_{\text{d}} = \frac{I}{neA}$$

The number density of charge carriers n is related to the mass density ρ, Avogadro's number N_A, and the molar mass M:

$$n = \frac{\rho N_A}{M}$$

For copper, $\rho = 8.93\ \text{g/cm}^3$ and $M = 63.5$ g/mol. Substitute and evaluate n:

$$n = \frac{(8.93\,\text{g/cm}^3)(6.02\times10^{23}\ \text{atoms/mol})}{63.5\,\text{g/mol}} = 8.47\times10^{28}\ \text{atoms/m}^3$$

Using Table 25-2, find the cross-sectional area A of 10-gauge wire:

$$A = 5.261\,\text{mm}^2$$

Substitute and evaluate v_d:

$$v_d = \frac{20\,\text{A}}{(8.47\times10^{28}\ \text{m}^{-3})(1.60\times10^{-19}\ \text{C})(5.261\,\text{mm}^2)} = \boxed{0.281\,\text{mm/s}}$$

34 •

Picture the Problem Note that, while the positive and negative charges flow in opposite directions, the total current is their sum.

Express the total current I in the tube as the sum of the electron current and the ion current:

$$I = I_{\text{electron}} + I_{\text{ion}}$$

The electron current is the product of the number of electrons through the cross-sectional area each second and the charge of each electron:

$$I_{\text{electron}} = ne = (2\times10^{18}\ \text{electrons/s})\times(1.60\times10^{-19}\ \text{C/electron}) = 0.320\,\text{A}$$

Proceed in the same manner to find the ion current:

$$I_{\text{ion}} = n_{\text{ion}}q_{\text{ion}} = (0.5\times10^{18}\ \text{electrons/s})\times(1.60\times10^{-19}\ \text{C/electron}) = 0.0800\,\text{A}$$

Substitute to obtain:

$$I = 0.320\,\text{A} + 0.0800\,\text{A} = \boxed{0.400\,\text{A}}$$

35 •

Picture the Problem We can solve $K = \frac{1}{2}m_e v^2$ for the velocity of an electron in the beam and use the relationship between current and drift velocity to find the beam current.

(*a*) Express the kinetic energy of the beam:

$$K = \tfrac{1}{2} m_e v^2$$

Solve for *v*:

$$v = \sqrt{\frac{2K}{m_e}}$$

Substitute numerical values and evaluate *v*:

$$v = \sqrt{\frac{2(10\,\text{keV})(1.60\times10^{-19}\ \text{J/eV})}{9.11\times10^{-31}\ \text{kg}}} = \boxed{5.93\times10^{7}\ \text{m/s}}$$

(*b*) Use the relationship between current and drift velocity (here the velocity of an electron in the beam) to obtain:

$$I = nev_d A$$

Express the cross-sectional area of the beam in terms of its diameter *D*:

$$A = \tfrac{1}{4}\pi D^2$$

Substitute to obtain:

$$I = \tfrac{1}{4}\pi nev_d D^2$$

Substitute numerical values and evaluate *I*:

Substitute numerical values and evaluate *I*:

$$I = \tfrac{1}{4}\pi\left(5\times10^{6}\ \text{cm}^{-3}\right)\left(1.60\times10^{-19}\ \text{C}\right)\left(5.93\times10^{7}\ \text{m/s}\right)\left(10^{-3}\ \text{m}\right)^2 = \boxed{37.3\ \mu\text{A}}$$

36 ••

Picture the Problem We can use the definition of current, the definition of charge density, and the relationship between period and frequency to derive an expression for the current as a function of *a*, λ, and ω.

Use the definition of current to relate the charge ΔQ associated with a segment of the ring to the time Δt it takes the segment to pass a given point:

$$I = \frac{\Delta Q}{\Delta t}$$

Because each segment carries a charge ΔQ and the time for one

$$I = \frac{\Delta Q}{T} = \Delta Q f \qquad (1)$$

revolution is T:

Use the definition of the charge density λ to relate the charge ΔQ to the radius a of the ring:

$$\lambda = \frac{\Delta Q}{2\pi a}$$

Solve for ΔQ to obtain:

$$\Delta Q = 2\pi a\lambda$$

Substitute in equation (1) to obtain:

$$I = 2\pi a\lambda f$$

Because $\omega = 2\pi f$ we have:

$$I = \boxed{a\lambda\omega}$$

***37 ••**

Picture the Problem The current will be the same in the two wires and we can relate the drift velocity of the electrons in each wire to their current densities and the cross-sectional areas of the wires. We can find the number density of charge carriers n using $n = \rho N_A/M$, where ρ is the mass density, N_A Avogadro's number, and M the molar mass. We can find the cross-sectional area of 10- and 14-gauge wires in Table 25-2.

Relate the current density to the drift velocity of the electrons in the 10-gauge wire:

$$\frac{I_{10\,\text{gauge}}}{A_{10\,\text{gauge}}} = nev_d$$

Solve for v_d:

$$v_{d,10} = \frac{I_{10\,\text{gauge}}}{neA_{10\,\text{gauge}}}$$

The number density of charge carriers n is related to the mass density ρ, Avogadro's number N_A, and the molar mass M:

$$n = \frac{\rho N_A}{M}$$

For copper, $\rho = 8.93$ g/cm^3 and $M = 63.5$ g/mol. Substitute and evaluate n:

$$n = \frac{(8.93\,\text{g/cm}^3)(6.02\times10^{23}\,\text{atoms/mol})}{63.5\,\text{g/mol}}$$
$$= 8.47\times10^{28}\,\text{atoms/m}^3$$

Use Table 25-2 to find the cross-sectional area of 10-gauge wire:

$$A_{10} = 5.261\,\text{mm}^2$$

Substitute numerical values and evaluate $v_{d,10}$:

$$v_{\text{d},10} = \frac{15\,\text{A}}{\left(8.47\times10^{28}\,\text{m}^{-3}\right)\left(1.60\times10^{-19}\,\text{C}\right)\left(5.261\,\text{mm}^2\right)} = \boxed{0.210\,\text{mm/s}}$$

Express the continuity of the current in the two wires:

$$I_{10\,\text{gauge}} = I_{14\,\text{gauge}}$$

or

$$nev_{\text{d},10}A_{10\,\text{gauge}} = nev_{\text{d},14}A_{14\,\text{gauge}}$$

Solve for $v_{\text{d},14}$ to obtain:

$$v_{\text{d},14} = v_{\text{d},10}\frac{A_{10\,\text{gauge}}}{A_{14\,\text{gauge}}}$$

Use Table 25-2 to find the cross-sectional area of 14-gauge wire:

$$A_{14} = 2.081\,\text{mm}^2$$

Substitute numerical values and evaluate $v_{\text{d},14}$:

$$v_{\text{d},14} = \left(0.210\,\text{mm/s}\right)\frac{5.261\,\text{mm}^2}{2.081\,\text{mm}^2} = \boxed{0.531\,\text{mm/s}}$$

38 ••

Picture the Problem We can use $I = neAv$ to relate the number n of protons per unit volume in the beam to current I. We can find the speed of the particles in the beam from their kinetic energy. In part (*b*) we can express the number of protons N striking the target per unit time as the product of the number of protons per unit volume n in the beam and the volume of the cylinder containing those protons that will strike the target in an elapsed time Δt and solve for N. Finally, we can use the definition of current to express the charge arriving at the target as a function of time.

(*a*) Use the relation between current and drift velocity to relate I and n:

$$I = neAv$$

Solve for n to obtain:

$$n = \frac{I}{eAv}$$

Express the kinetic energy of the protons and solve for v:

$$K = \tfrac{1}{2}m_\text{p}v^2 \Rightarrow v = \sqrt{\frac{2K}{m_\text{p}}}$$

Relate the cross-sectional area A of the beam to its diameter D:

$$A = \tfrac{1}{4}\pi D^2$$

Substitute for v and A to obtain:

$$n=\frac{I}{\frac{1}{4}\pi eD^2\sqrt{\frac{2K}{m_p}}}=\frac{4I}{\pi eD^2}\sqrt{\frac{m_p}{2K}}$$

Substitute numerical values and evaluate n:

$$n=\frac{4(1\,\text{mA})}{\pi(1.60\times10^{-19}\,\text{C})(2\,\text{mm})^2}\sqrt{\frac{1.67\times10^{-27}\,\text{kg}}{2(20\,\text{MeV})(1.60\times10^{-19}\,\text{J/eV})}}=\boxed{3.21\times10^{13}\,\text{mm}^{-3}}$$

(*b*) Express the number of protons N striking the target per unit time as the product of the number n of protons per unit volume in the beam and the volume of the cylinder containing those protons that will strike the target in an elapsed time Δt and solve for N:

$$\frac{N}{\Delta t}=n(vA)\Rightarrow N=nvA\Delta t$$

Substitute for v and A to obtain:

$$N=\frac{1}{4}\pi D^2 n\Delta t\sqrt{\frac{2K}{m_p}}$$

Substitute numerical values and evaluate N:

$$N=\frac{1}{4}\pi(2\,\text{mm})^2(3.21\times10^{13}\,\text{m}^{-3})(1\,\text{min})\sqrt{\frac{2(20\,\text{MeV})(1.60\times10^{-19}\,\text{J/eV})}{1.67\times10^{-27}\,\text{kg}}}=\boxed{3.75\times10^{17}}$$

(*c*) Using the definition of current, express the charge arriving at the target as a function of time:

$$Q=It=(1\,\text{mA})t=\boxed{(1\,\text{mC/s})t}$$

***39 ••**

Picture the Problem We can relate the number of protons per meter N to the number n of free charge-carrying particles per unit volume in a beam of cross-sectional area A and then use the relation between current and drift velocity to relate n to I.

(*a*) Express the number of protons per meter N in terms of the number n of free charge-carrying particles per unit volume in a beam of cross-

$$N=nA \qquad (1)$$

sectional area A:

Use the relation between current and drift velocity to relate I and n:

$$I = enAv$$

Solve for n to obtain:

$$n = \frac{I}{eAv}$$

Substitute to obtain:

$$N = \frac{IA}{eAv} = \frac{I}{ev}$$

Substitute numerical values and evaluate N:

$$N = \frac{5\,\text{mA}}{(1.60\times10^{-19}\,\text{C})(3\times10^{8}\,\text{m/s})} = \boxed{1.04\times10^{8}\,\text{m}^{-1}}$$

(b) From equation (1) we have:

$$n = \frac{N}{A} = \frac{1.04\times10^{8}\,\text{m}^{-1}}{10^{-6}\,\text{m}^2} = \boxed{1.04\times10^{14}\,\text{m}^{-3}}$$

Resistance and Ohm's Law

40 •

Picture the Problem We can use Ohm's law to find the potential difference between the ends of the wire and $V = EL$ to find the magnitude of the electric field in the wire.

(a) Apply Ohm's law to obtain:

$$V = RI = (0.2\,\Omega)(5\,\text{A}) = \boxed{1.00\,\text{V}}$$

(b) Relate the electric field to the potential difference across the wire and the length of the wire:

$$E = \frac{V}{L} = \frac{1\,\text{V}}{10\,\text{m}} = \boxed{0.100\,\text{V/m}}$$

41 •

Picture the Problem We can apply Ohm's law to both parts of this problem, solving first for R and then for I.

(a) Apply Ohm's law to obtain:

$$R = \frac{V}{I} = \frac{100\,\text{V}}{3\,\text{A}} = \boxed{33.3\,\Omega}$$

(*b*) Apply Ohm's law a second time to obtain:

$$I = \frac{V}{R} = \frac{25\,\text{V}}{33.3\,\Omega} = \boxed{0.751\,\text{A}}$$

42 •

Picture the Problem We can use $R = \rho L/A$ to find the resistance of the block and Ohm's law to find the current in it for the given potential difference across its length.

(*a*) Relate the resistance of the block to its resistivity ρ, cross-sectional area A, and length L:

$$R = \rho\frac{L}{A}$$

Substitute numerical values (see Table 2625-1 for the resistivity of carbon) and evaluate R:

$$R = \left(3500\times10^{-8}\,\Omega\cdot\text{m}\right)\frac{3\,\text{cm}}{(0.5\,\text{cm})^2}$$

$$= \boxed{42.0\,\text{m}\Omega}$$

(*b*) Apply Ohm's law to obtain:

$$I = \frac{V}{R} = \frac{8.4\,\text{V}}{42.0\,\text{m}\Omega} = \boxed{200\,\text{A}}$$

43 •

Picture the Problem We can solve the relation $R = \rho L/A$ for L to find the length of the carbon rod that will have a resistance of 10 Ω.

Relate the resistance of the rod to its resistivity ρ, cross-sectional area A, and length L:

$$R = \rho\frac{L}{A}$$

Solve for L to obtain:

$$L = \frac{AR}{\rho} = \frac{\pi r^2 R}{\rho}$$

Substitute numerical values (see Table 2625-1 for the resistivity of carbon) and evaluate L:

$$L = \frac{\pi(0.1\,\text{mm})^2(10\,\Omega)}{3500\times10^{-8}\,\Omega\cdot\text{m}} = \boxed{8.98\,\text{mm}}$$

***44** •

Picture the Problem We can use $R = \rho L/A$ to find the resistance of the track.

(*a*) Relate the resistance of the track to its resistivity ρ, cross-sectional area A, and length L:

$$R = \rho\frac{L}{A}$$

Substitute numerical values and evaluate R:

$$R = \left(10^{-7}\ \Omega\cdot\text{m}\right)\frac{10\,\text{km}}{55\,\text{cm}^2} = \boxed{0.182\,\Omega}$$

45 •

Picture the Problem We can use Ohm's law in conjunction with $R = \rho L/A$ to find the potential difference across one wire of the extension cord.

Using Ohm's law, express the potential difference across one wire of the extension cord:

$$V = IR$$

Relate the resistance of the wire to its resistivity ρ, cross-sectional area A, and length L:

$$R = \rho\frac{L}{A}$$

Substitute to obtain:

$$V = \rho\frac{LI}{A}$$

Substitute numerical values (see Table 25-1 for the resistivity of copper and Table 25-2 for the cross-sectional area of 16-gauge wire) and evaluate V:

$$V = \left(1.7\times10^{-8}\ \Omega\cdot\text{m}\right)\frac{(30\,\text{m})(5\,\text{A})}{1.309\,\text{mm}^2}$$

$$= \boxed{1.95\,\text{V}}$$

46 •

Picture the Problem We can use $R = \rho L/A$ to find the length of a 14-gauge copper wire that has a resistance of 2 Ω.

(*a*) Relate the resistance of the wire to its resistivity ρ, cross-sectional area A, and length L:

$$R = \rho\frac{L}{A}$$

Solve for L to obtain:

$$L = \frac{RA}{\rho}$$

Substitute numerical values (see Table 2625-1 for the resistivity of copper and Table 2625-2 for the cross-sectional area of 14-gauge wire) and evaluate L:

$$L = \frac{(2\,\Omega)\left(2.081\,\text{mm}^2\right)}{1.7\times10^{-8}\ \Omega\cdot\text{m}} = \boxed{245\,\text{m}}$$

47 ••

Picture the Problem We can use $R = \rho L/A$ to express the resistances of the glass cylinder and the copper wire. Expressing their ratio will eliminate the common cross-sectional areas and leave us with an expression we can solve for the length of the copper wire.

Relate the resistance of the glass cylinder to its resistivity, cross-sectional area, and length:

$$R_{glass} = \rho_{glass}\frac{L_{glass}}{A_{glass}}$$

Relate the resistance of the copper wire to its resistivity, cross-sectional area, and length:

$$R_{Cu} = \rho_{Cu}\frac{L_{Cu}}{A_{Cu}}$$

Divide the second of these equations by the first to obtain:

$$\frac{R_{Cu}}{R_{glass}} = \frac{\rho_{Cu}\frac{L_{Cu}}{A_{Cu}}}{\rho_{glass}\frac{L_{glass}}{A_{glass}}} = \frac{\rho_{Cu}}{\rho_{glass}}\cdot\frac{L_{Cu}}{L_{glass}}$$

or, because $A_{glass} = A_{Cu}$ and $R_{Cu} = R_{glass}$,

$$1 = \frac{\rho_{Cu}}{\rho_{glass}}\cdot\frac{L_{Cu}}{L_{glass}}$$

Solve for L_{Cu} to obtain:

$$L_{Cu} = \frac{\rho_{glass}}{\rho_{Cu}}L_{glass}$$

Substitute numerical values (see Table 25-1 for the resistivities of glass and copper) and evaluate L_{Cu}:

$$L_{Cu} = \frac{10^{12}\,\Omega\cdot\text{m}}{1.7\times10^{-8}\,\Omega\cdot\text{m}}(1\,\text{cm})$$

$$= 5.88\times10^{17}\,\text{m}\times\frac{1\,\text{c}\cdot\text{y}}{9.461\times10^{15}\,\text{m}}$$

$$= \boxed{62.2\,\text{c}\cdot\text{y}}$$

48 ••

Picture the Problem We can use Ohm's law to relate the potential differences across the two wires to their resistances and $R = \rho L/A$ to relate their resistances to their lengths, resistivities, and cross-sectional areas. Once we've found the potential differences across each wire, we can use $E = V/L$ to find the electric field in each wire.

(*b*) Apply Ohm's law to express the potential drop across each wire:

$$V_{Cu} = IR_{Cu}$$
and
$$V_{Fe} = IR_{Fe}$$

Relate the resistances of the wires to their resistivity, cross-sectional area, and length:

$$R_{Cu} = \rho_{Cu}\frac{L_{Cu}}{A_{Cu}}$$
and
$$R_{Fe} = \rho_{Fe}\frac{L_{Fe}}{A_{Fe}}$$

Substitute to obtain:

$$V_{Cu} = \frac{\rho_{Cu}L_{Cu}}{A_{Cu}}I$$
and
$$V_{Fe} = \frac{\rho_{Fe}L_{Fe}}{A_{Fe}}I$$

Substitute numerical values (see Table 2625-1 for the resistivities of copper and iron) and evaluate the potential differences:

$$V_{Cu} = \frac{(1.7\times10^{-8}\,\Omega\cdot\text{m})(80\,\text{m})}{\frac{1}{4}\pi(1\,\text{mm})^2}(2\,\text{A}) = \boxed{3.46\,\text{V}}$$
and
$$V_{Fe} = \frac{(10\times10^{-8}\,\Omega\cdot\text{m})(49\,\text{m})}{\frac{1}{4}\pi(1\,\text{mm})^2}(2\,\text{A}) = \boxed{12.5\,\text{V}}$$

(*a*) Express the electric field in each conductor in terms of its length and the potential difference across it:

$$E_{Cu} = \frac{V_{Cu}}{L_{Cu}}$$
and
$$E_{Fe} = \frac{V_{Fe}}{L_{Fe}}$$

Substitute numerical values and evaluate the electric fields:

$$E_{Cu} = \frac{3.46\,\text{V}}{80\,\text{m}} = \boxed{43.3\,\text{mV/m}}$$

and

$$E_{Fe} = \frac{12.5\,V}{49\,m} = \boxed{255\,mV/m}$$

***49 ••**

Picture the Problem We can use Ohm's law to express the ratio of the potential differences across the two wires and $R = \rho L/A$ to relate the resistances of the wires to their lengths, resistivities, and cross-sectional areas. Once we've found the ratio of the potential differences across the wires, we can use $E = V/L$ to decide which wire has the greater electric field.

(*a*) Apply Ohm's law to express the potential drop across each wire:

$$V_{Cu} = IR_{Cu}$$

and

$$V_{Fe} = IR_{Fe}$$

Divide the first of these equations by the second to express the ratio of the potential drops across the wires:

$$\frac{V_{Cu}}{V_{Fe}} = \frac{IR_{Cu}}{IR_{Fe}} = \frac{R_{Cu}}{R_{Fe}} \qquad (1)$$

Relate the resistances of the wires to their resistivity, cross-sectional area, and length:

$$R_{Cu} = \rho_{Cu}\frac{L_{Cu}}{A_{Cu}}$$

and

$$R_{Fe} = \rho_{Fe}\frac{L_{Fe}}{A_{Fe}}$$

Divide the first of these equations by the second to express the ratio of the resistances of the wires:

$$\frac{R_{Cu}}{R_{Fe}} = \frac{\rho_{Cu}\dfrac{L_{Cu}}{A_{Cu}}}{\rho_{Fe}\dfrac{L_{Fe}}{A_{Fe}}} = \frac{\rho_{Cu}}{\rho_{Fe}}$$

because $L_{Cu} = L_{Fe}$ and $A_{Cu} = A_{Fe}$.

Substitute in equation (1) to obtain:

$$\frac{V_{Cu}}{V_{Fe}} = \frac{\rho_{Cu}}{\rho_{Fe}}$$

Substitute numerical values (see Table 2625-1 for the resistivities of copper and iron) and evaluate the ratio of the potential differences:

$$\frac{V_{Cu}}{V_{Fe}} = \frac{1.7\times10^{-8}\,\Omega\cdot m}{10\times10^{-8}\,\Omega\cdot m} = \boxed{0.170}$$

(*b*) Express the electric field in each conductor in terms of its length and the potential difference across it:

$$E_{\text{Cu}} = \frac{V_{\text{Cu}}}{L_{\text{Cu}}} \text{ and } E_{\text{Fe}} = \frac{V_{\text{Fe}}}{L_{\text{Fe}}}$$

Divide the first of these equations by the second to obtain:

$$\frac{E_{\text{Cu}}}{E_{\text{Fe}}} = \frac{\dfrac{V_{\text{Cu}}}{L_{\text{Cu}}}}{\dfrac{V_{\text{Cu}}}{L_{\text{Cu}}}} = \frac{V_{\text{Cu}}}{V_{\text{Fe}}} = 0.170$$

or

$$E_{\text{Fe}} = \frac{E_{\text{Cu}}}{0.17} = 5.88E_{\text{Cu}}$$

Because $E_{\text{Fe}} = 5.88E_{\text{Cu}}$:

$$\boxed{E \text{ is greater in the iron wire.}}$$

50 ••

Picture the Problem We can use $R = \rho L/A$ to relate the resistance of the salt solution to its length, resistivity, and cross-sectional area. To find the resistance of the filled tube when it is uniformly stretched to a length of 2 m, we can use the fact that the volume of the solution is unchanged to relate the new cross-sectional area of the solution to its original cross-sectional area.

(*a*) Relate the resistance of the filled tube to the resistivity, cross-sectional area, and length of the salt solution:

$$R = \rho\frac{L}{A}$$

Substitute numerical values and evaluate *R*:

$$R = \left(10^{-3}\,\Omega\cdot\text{m}\right)\frac{1\,\text{m}}{\pi(2\,\text{mm})^2} = \boxed{79.6\,\Omega}$$

(*b*) Relate the resistance of the stretched tube to the resistivity, cross-sectional area, and length of the salt solution:

$$R' = \rho\frac{L'}{A'} = \rho\frac{2L}{A'} \qquad (1)$$

Letting *V* represent volume, express the relationship between the before-stretching volume *V* and the after-stretching volume *V*′:

$$V = V'$$

or

$$LA = L'A'$$

Solve for *A*′ to obtain:

$$A' = \frac{L}{L'}A = \tfrac{1}{2}A$$

Substitute in equation (1) to obtain:

$$R' = \rho\frac{2L}{\frac{1}{2}A} = 4\left(\rho\frac{L}{A}\right)$$
$$= 4(79.6\,\Omega) = \boxed{318\,\Omega}$$

51 ••

Picture the Problem We can use $R = \rho L/A$ to relate the resistance of the wires to their lengths, resistivities, and cross-sectional areas. To find the resistance of the stretched wire, we can use the fact that the volume of the wire does not change during the stretching process to relate the new cross-sectional area of the wire to its original cross-sectional area.

Relate the resistance of the unstretched wire to its resistivity, cross-sectional area, and length:

$$R = \rho\frac{L}{A}$$

Relate the resistance of the stretched wire to its resistivity, cross-sectional area, and length:

$$R' = \rho\frac{L'}{A'}$$

Divide the second of these equations by the first to obtain:

$$\frac{R'}{R} = \frac{\rho\frac{L'}{A'}}{\rho\frac{L}{A}} = \frac{L'}{L}\frac{A}{A'}$$

or

$$R' = 2\frac{A}{A'}R \qquad (1)$$

Letting V represent volume, express the relationship between the before-stretching volume V and the after-stretching volume V':

$$V = V'$$

or

$$LA = L'A'$$

Solve for A/A' to obtain:

$$\frac{A}{A'} = \frac{L'}{L} = 2$$

Substitute in equation (1) to obtain:

$$R' = 2(2)R = 4R = 4(0.3\,\Omega) = \boxed{1.20\,\Omega}$$

52 ••

Picture the Problem We can use $R = \rho L/A$ to find the resistance of the wire from its length, resistivity, and cross-sectional area. The electric field can be found using $E = V/L$ and Ohm's law to eliminate V. The time for an electron to travel the length of the wire can be found from $L = v_d\Delta t$, with v_d expressed in term of I using $I = neAv_d$.

(*a*) Relate the resistance of the unstretched wire to its resistivity, cross-sectional area, and length:

$$R = \rho\frac{L}{A}$$

Substitute numerical values (see Table 25-1 for the resistivity of copper and Table 25-2 for the cross-sectional area of 10-gauge wire) and evaluate R:

$$R = \left(1.7\times10^{-8}\,\Omega\cdot\text{m}\right)\frac{100\,\text{m}}{5.261\,\text{mm}^2} = \boxed{0.323\,\Omega}$$

(*b*) Relate the electric field in the wire to the potential difference between its ends:

$$E = \frac{V}{L}$$

Use Ohm's law to obtain:

$$E = \frac{IR}{L}$$

Substitute numerical values and evaluate E:

$$E = \frac{(30\,\text{A})(0.323\,\Omega)}{100\,\text{m}} = \boxed{96.9\,\text{mV/m}}$$

(*c*) Express the time Δt for an electron to travel a distance L in the wire in terms of its drift speed v_d:

$$\Delta t = \frac{L}{v_d}$$

Relate the current in the wire to the drift speed of the charge carriers:

$$I = neAv_d$$

Solve for v_d to obtain:

$$v_d = \frac{I}{neA}$$

Substitute to obtain:

$$\Delta t = \frac{neAL}{I}$$

Substitute numerical values (in Example 25-1 it is shown that $n = 8.47\times10^{28}\,\text{m}^{-3}$) and evaluate Δt:

$$\Delta t = \frac{\left(8.47\times10^{28}\ \text{m}^{-3}\right)\left(1.6\times10^{-19}\ \text{C}\right)\left(5.261\,\text{mm}^2\right)\left(100\,\text{m}\right)}{30\,\text{A}} = \boxed{2.38\times10^{5}\ \text{s}}$$

53 ••

Picture the Problem We can use $R = \rho L/A$ to find express the resistance of the wire from in terms of its length, resistivity, and cross-sectional area. The fact that the volume of the copper does not change as the cube is drawn out to form the wire will allow us to eliminate either the length or the cross-sectional area of the wire and solve for its resistance.

Express the resistance of the wire in terms of its resistivity, cross-sectional area, and length:

$$R = \rho\frac{L}{A}$$

Relate the volume V of the wire to its length and cross-sectional area:

$$V = AL$$

Solve for L to obtain:

$$L = \frac{V}{A}$$

Substitute to obtain:

$$R = \rho\frac{V}{A^2}$$

Substitute numerical values (see Table 25-1 for the resistivity of copper and Table 25-2 for the cross-sectional area of 14-gauge wire) and evaluate R:

$$R = \left(1.7\times10^{-8}\ \Omega\cdot\text{m}\right)\frac{\left(2\,\text{cm}\right)^3}{\left(2.081\,\text{mm}^2\right)^2}$$

$$= \boxed{0.0314\,\Omega}$$

***54** ••

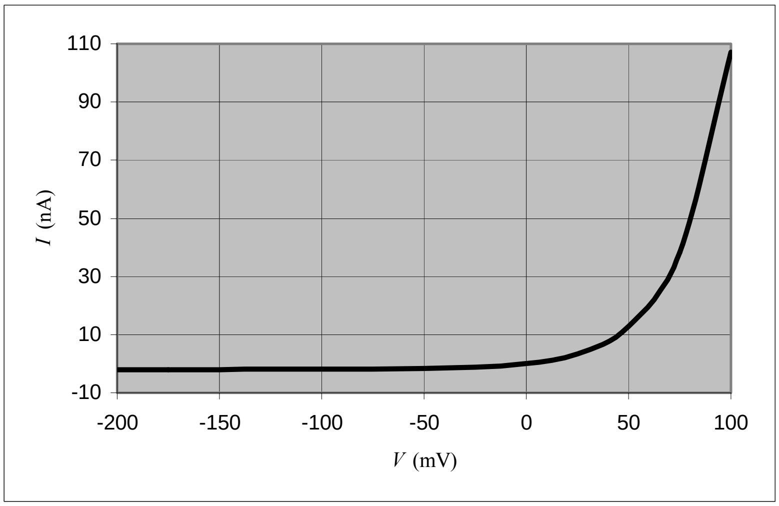
A spreadsheet program to plot I as a function of V is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Content/Formula	Algebraic Form
B1	2	I_0
A5	–200	V (mV)
A6	A5 + 25	$V + \Delta V$
B5	B1*(EXP(A5/25) – 1)	$I_0\left(e^{V/25\,\text{mV}} - 1\right)$

	A	B	C
1	I_0=	2	nA
2			
3	V	I	

4	(mV)	(nA)	
5	–200.0	–2.00	
6	–175.0	–2.00	
7	–150.0	–2.00	
15	50.0	12.78	
16	75.0	38.17	
17	100.0	107.20	

The following graph was plotted using the data in spreadsheet table shown above.



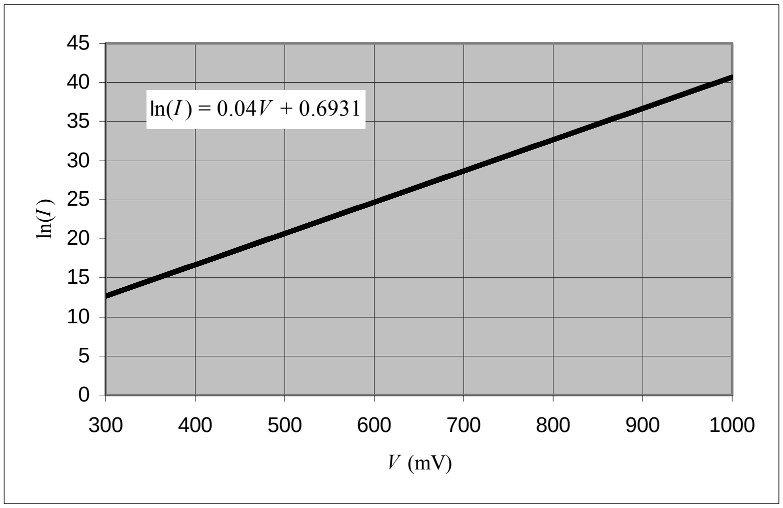
A spreadsheet program to plot ln(I) as a function of V for $V > 0.3$ V follows. The formulas used to calculate the quantities in the columns are as follows:

Cell	Content/Formula	Algebraic Form
B1	2	2 nA
A5	300	V
A6	A5 + 10	$V + \Delta V$
B5	LN(B1*(EXP(A5/25) – 1))	$\ln\left[I_0\left(e^{V/25\text{ mV}} - 1\right)\right]$

A	B	C
I_0=	2	nA
V	ln(I)	
(mV)		
300	12.69	
310	13.09	
320	13.49	
330	13.89	
340	14.29	
350	14.69	
970	39.49	

980	39.89	
990	40.29	
1000	40.69	

A graph of ln(I) as a function of V follows. Microsoft Excel's Trendline feature was used to obtain the equation of the line.



For $V >> 25$ mV:

$$e^{V/25\,\text{mV}} - 1 \approx e^{V/25\,\text{mV}}$$

and

$$I \approx I_0 e^{V/25\,\text{mV}}$$

Take the natural logarithm of both sides of the equation to obtain:

$$\ln(I) = \ln\left(I_0 e^{V/25\,\text{mV}}\right) = \ln(I_0) + \frac{1}{25\,\text{mV}}V$$

which is of the form $y = mx + b$, where

$$m = \frac{1}{25\,\text{mV}} = \boxed{0.04\,(\text{mV})^{-1}}$$

in agreement with our graphical result.

55 ••

Picture the Problem We can use the first graph plotted in Problem 55 to conclude that, if $V < 0.5$ V, then the diode's resistance is effectively infinite. We can use Ohm's law to estimate the current through the diode.

(a) $\boxed{\text{From the first graph plotted in Problem 55 we see that, if } V < 0.5\text{ V, then the current is negligible and the diode has essentially infinite resistance.}}$

(b) Use Ohm's law to express the current flowing through the diode:

$$I = \frac{V_{\text{resistor}}}{R}$$

For a potential difference across the diode of approximately 0.5 V:

$$I = \frac{5\,\text{V} - 0.5\,\text{V}}{50\,\Omega} = \boxed{90.0\,\text{mA}}$$

56 •••

Picture the Problem We can use, as our element of resistance, a semicircular strip of width t, radius r, and thickness dr. Then $dR = (\pi r\rho/t)dr$. Because the strips are in parallel, integrating over them will give us the reciprocal of the resistance of half ring.

Integrate dR from $r = a$ to $r = b$ to obtain:

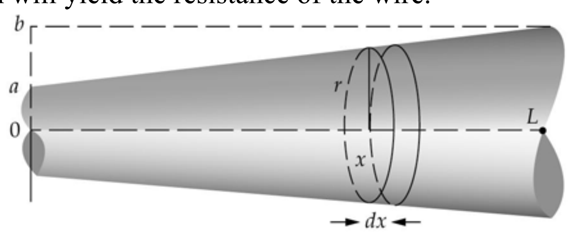
$$\frac{1}{R} = \frac{t}{\pi\rho}\int_a^b \frac{dr}{r} = \frac{t}{\pi\rho}\ln\left(\frac{b}{a}\right)$$

Take the reciprocal of both sides of the equation to obtain:

$$R = \boxed{\frac{\rho\pi}{t\ln\left(\frac{b}{a}\right)}}$$

57 •••

Picture the Problem The element of resistance we use is a segment of length dx and cross-sectional area $\pi[a + (b - a)x/L]^2$. Because these resistance elements are in series, integrating over them will yield the resistance of the wire.



Express the resistance of the chosen element of resistance:

$$dR = \rho\frac{dx}{A} = \frac{\rho}{\pi[a + (b-a)(x/L)]^2}dx$$

Integrate dR from $x = 0$ to $x = L$ and simplify to obtain:

$$R = \frac{\rho}{\pi}\int_0^L \frac{dx}{[a + (b-a)(x/L)]^2}$$

$$= \frac{\rho L}{\pi(b-a)}\left(\frac{1}{a} - \frac{1}{a + (b-a)}\right)$$

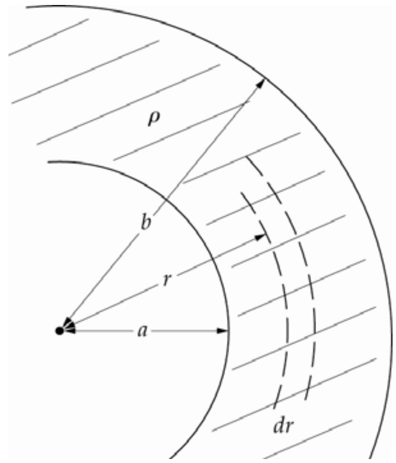
$$= \frac{\rho L}{\pi(b-a)}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$= \frac{\rho L}{\pi(b-a)}\left(\frac{b-a}{ab}\right)$$

$$= \boxed{\frac{\rho L}{\pi ab}}$$

***58** •••

Picture the Problem The diagram shows a cross-sectional view of the concentric spheres of radii *a* and *b* as well as a spherical-shell element of radius *r*. Using the *Hint* we can express the resistance *dR* of the spherical-shell element and then integrate over the volume filled with the material whose resistivity ρ is given to find the resistance between the conductors. Note that the elements of resistance are in series.



Express the element of resistance *dR*:

$$dR = \rho\frac{dr}{A} = \rho\frac{dr}{4\pi r^2}$$

Integrate *dR* from *r* = *a* to *r* = *b* to obtain:

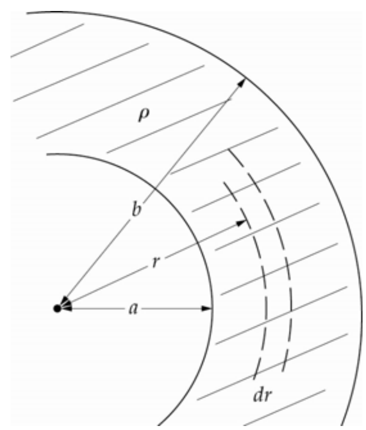
$$R = \frac{\rho}{4\pi}\int_a^b \frac{dr}{r^2} = \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$$

Substitute numerical values and evaluate *R*:

$$R = \frac{10^9\,\Omega\cdot\text{m}}{4\pi}\left(\frac{1}{1.5\,\text{cm}} - \frac{1}{5\,\text{cm}}\right) = \boxed{3.71\times10^9\,\Omega}$$

59 •••

Picture the Problem The diagram shows a cross-sectional view of the coaxial cylinders of radii *a* and *b* as well as a cylindrical-shell element of radius *r*. We can express the resistance *dR* of the cylindrical-shell element and then integrate over the volume filled with the material whose resistivity ρ is given to find the resistance between the two cylinders. Note that the elements of resistance are in series.



Express the element of resistance *dR*:

$$dR = \rho\frac{dr}{A} = \rho\frac{dr}{2\pi rL}$$

Integrate *dR* from *r* = *a* to *r* = *b* to obtain:

$$R = \frac{\rho}{2\pi L}\int_a^b \frac{dr}{r} = \boxed{\frac{\rho}{2\pi L}\ln\left(\frac{b}{a}\right)}$$

(*b*) Apply Ohm's law to obtain:

$$I = \frac{V}{R} = \frac{V}{\frac{\rho}{2\pi L}\ln\left(\frac{b}{a}\right)} = \frac{2\pi LV}{\rho\ln\left(\frac{b}{a}\right)}$$

Substitute numerical values and evaluate *I*:

$$I = \frac{2\pi(50\,\text{cm})(10\,\text{V})}{(30\,\Omega\cdot\text{m})\ln\left(\frac{2.5\,\text{cm}}{1.5\,\text{cm}}\right)} = \boxed{2.05\,\text{A}}$$

Temperature Dependence of Resistance

***60 •**

Picture the Problem We can use $R = \rho L/A$ to find the resistance of the rod at 20°C. Ignoring the effects of thermal expansion, we can we apply the equation defining the temperature coefficient of resistivity, α, to relate the resistance at 40°C to the resistance at 20°C.

(*a*) Express the resistance of the rod at 20°C as a function of its resistivity, length, and cross-sectional area:

$$R_{20} = \rho_{20}\frac{L}{A}$$

Substitute numerical values and evaluate R_{20}:

$$R_{20} = \left(5.5\times10^{-8}\,\Omega\cdot\text{m}\right)\frac{0.5\,\text{m}}{(1\,\text{mm})^2} = \boxed{27.5\,\text{m}\Omega}$$

(*b*) Express the resistance of the rod at 40°C as a function of its resistance at 20°C and the temperature coefficient of resistivity α:

$$\begin{aligned} R_{40} &= \rho_{40}\frac{L}{A} \\ &= \rho_{20}\left[1+\alpha(t_\text{C}-20\text{C}°)\right]\frac{L}{A} \\ &= \rho_{20}\frac{L}{A} + \rho_{20}\frac{L}{A}\alpha(t_\text{C}-20\text{C}°) \\ &= R_{20}\left[1+\alpha(t_\text{C}-20\text{C}°)\right] \end{aligned}$$

Substitute numerical values (see Table 25-1 for the temperature coefficient of resistivity of tungsten) and evaluate R_{40}:

$$R_{40} = (27.5\,\text{m}\Omega)\left[1+\left(4.5\times10^{-3}\,\text{K}^{-1}\right)(20\,\text{C}°)\right] = \boxed{30.0\,\text{m}\Omega}$$

61 •

Picture the Problem The resistance of the copper wire increases with temperature according to $R_{t_C} = R_{20}\left[1+\alpha\left(t_C - 20\text{C}°\right)\right]$. We can replace R_{t_C} by $1.1R_{20}$ and solve for t_C to find the temperature at which the resistance of the wire will be 110% of its value at 20°C.

Express the resistance of the wire at $1.1R_{20}$:

$$1.1R_{20} = R_{20}\left[1+\alpha\left(t_C - 20\text{C}°\right)\right]$$

Simplify this expression to obtain:

$$1.1R_{20} = R_{20} + R_{20}\alpha\left(t_C - 20\text{C}°\right)$$

or

$$0.1 = \alpha\left(t_C - 20\text{C}°\right)$$

Solve to t_C to obtain:

$$t_C = \frac{0.1}{\alpha} + 20\text{C}°$$

Substitute numerical values (see Table 25-1 for the temperature coefficient of resistivity of copper) and evaluate t_C:

$$t_C = \frac{0.1}{3.9\times10^{-3}\ \text{K}^{-1}} + 20\text{C}° = \boxed{45.6°\text{C}}$$

62 ••

Picture the Problem Let the primed quantities denote the current and resistance at the final temperature of the heating element. We can express R' in terms of R_{20} and the final temperature of the wire t_C using $R' = R_{20}\left[1+\alpha\left(t_C - 20\text{C}°\right)\right]$ and relate I', R', I_{20}, and R_{20} using Ohm's law.

Express the resistance of the heating element at its final temperature as a function of its resistance at 20°C and the temperature coefficient of resistivity for Nichrome:

$$R' = R_{20}\left[1+\alpha\left(t_C - 20\text{C}°\right)\right] \qquad (1)$$

Apply Ohm's law to the heating element when it is first turned on:

$$V = I_{20}R_{20}$$

Apply Ohm's law to the heating element when it has reached its final temperature:

$$V = I'R'$$

Because the voltage is constant, we have:

$$I'R' = I_{20}R_{20}$$

or

$$R' = \frac{I_{20}}{I'} R_{20}$$

Substitute in equation (1) to obtain:

$$\frac{I_{20}}{I'} R_{20} = R_{20}\left[1 + \alpha\left(t_C - 20\text{C}°\right)\right]$$

or

$$\frac{I_{20}}{I'} = 1 + \alpha\left(t_C - 20\text{C}°\right)$$

Solve for t_C to obtain:

$$t_C = \frac{\dfrac{I_{20}}{I'} - 1}{\alpha} + 20\text{C}°$$

Substitute numerical values (see Table 25-1 for the temperature coefficient of resistivity of Nichrome) and evaluate t_C:

$$t_C = \frac{\dfrac{1.5\,A}{1.3\,A} - 1}{0.4\times10^{-3}\,\text{K}^{-1}} + 20\text{C}° = \boxed{405°\text{C}}$$

63 ••

Picture the Problem We can apply Ohm's law to find the initial current drawn by the cold heating element. The resistance of the wire at 1000°C can be found using $R_{1000} = R_{20}\left[1 + \alpha\left(t_C - 20\text{C}°\right)\right]$ and the power consumption of the heater at this temperature from $P = V^2/R_{1000}$.

(*a*) Apply Ohm's law to find the initial current I_{20} drawn by the heating element:

$$I = \frac{V}{R_{20}} = \frac{120\,\text{V}}{8\,\Omega} = \boxed{15.0\,\text{A}}$$

(*b*) Express the resistance of the heating element at 1000°C as a function of its resistance at 20°C and the temperature coefficient of resistivity for Nichrome:

$$R_{1000} = R_{20}\left[1 + \alpha\left(t_C - 20\text{C}°\right)\right]$$

Substitute numerical values (see Table 25-1 for the temperature coefficient of resistivity of Nichrome) and evaluate R_{1000}:

$$R_{1000} = (8\,\Omega)\left[1 + \left(0.4\times10^{-3}\,\text{K}^{-1}\right)\times\left(1000°\text{C} - 20\text{C}°\right)\right] = \boxed{11.1\,\Omega}$$

(*c*) Express and evaluate the operating wattage of this heater at 1000°C:

$$P = \frac{V^2}{R_{1000}} = \frac{(120\,\text{V})^2}{11.1\,\Omega} = \boxed{1.30\,\text{kW}}$$

64 ••

Picture the Problem We can find the resistance of the copper leads using $R_{Cu} = \rho_{Cu}L/A$ and express the percentage error in neglecting the resistance of the leads as the ratio of R_{Cu} to $R_{Nichrome}$. In part (*c*) we can express the change in resistance in the Nichrome wire corresponding to a change Δt_C in its temperature and then find Δt_C by substitution of the resistance of the copper wires in this equation.

(*a*) Relate the resistance of the copper leads to their resistivity, length, and cross-sectional area:

$$R_{Cu} = \rho_{Cu}\frac{L}{A}$$

Substitute numerical values (see Table 25-1 for the resistivity of copper) and evaluate R_{Cu}:

$$R_{Cu} = \left(1.7\times10^{-8}\,\Omega\cdot\text{m}\right)\frac{50\,\text{cm}}{\frac{1}{4}\pi(0.6\,\text{mm})^2} = \boxed{30.1\,\text{m}\Omega}$$

(*b*) Express the percentage error as the ratio of R_{Cu} to $R_{Nichrome}$:

$$\%\,\text{error} = \frac{R_{Cu}}{R_{Nichrome}} = \frac{30.1\,\text{m}\Omega}{10\,\Omega} = \boxed{0.301\%}$$

(*c*) Express the change in the resistance of the Nichrome wire as its temperature changes from t_C to t_C':

$$\begin{aligned}\Delta R &= R' - R \\ &= R_{20}\left[1+\alpha\left(t_C' - 20\text{C}^\circ\right)\right] \\ &\quad - R_{20}\left[1+\alpha\left(t_C - 20\text{C}^\circ\right)\right] \\ &= R_{20}\alpha\Delta t_C\end{aligned}$$

Solve for Δt_C to obtain:

$$\Delta t_C = \frac{\Delta R}{R_{20}\alpha}$$

Set ΔR equal to the resistance of the copper wires (see Table 25-1 for the temperature coefficient of resistivity of Nichrome wire) and evaluate Δt_C:

$$\Delta t_C = \frac{30.1\,\text{m}\Omega}{(10\,\Omega)\left(0.4\times10^{-3}\,\text{K}^{-1}\right)} = \boxed{7.53\,\text{C}^\circ}$$

***65** •••

Picture the Problem Expressing the total resistance of the two current-carrying (and hence warming) wires connected in series in terms of their resistivities, temperature coefficients of resistivity, lengths and temperature change will lead us to an expression in which, if $\rho_1L_1\alpha_1 + \rho_2L_2\alpha_2 = 0$, the total resistance is temperature independent. In part (b) we can apply the condition that $\rho_1L_1\alpha_1 + \rho_2L_2\alpha_2 = 0$ to find the ratio of the lengths of the carbon and copper wires.

(*a*) Express the total resistance of these two wires connected in series:

$$R = R_1 + R_2$$
$$= \rho_1 \frac{L_1}{A}(1+\alpha_1 \Delta T) + \rho_2 \frac{L_2}{A}(1+\alpha_2 \Delta T) + \frac{1}{A}\left[\rho_1 L_1 (1+\alpha_1 \Delta T) + \rho_2 L_2 (1+\alpha_2 \Delta T)\right]$$

Expand and simplify this expression to obtain:

$$R = \frac{1}{A}\left[\rho_1 L_1 + \rho_2 L_2 + (\rho_1 L_1 \alpha_1 + \rho_1 L_1 \alpha_2)\Delta T\right]$$

If $\rho_1 L_1 \alpha_1 + \rho_2 L_2 \alpha_2 = 0$, then:

$$R = \boxed{\frac{1}{A}\left[\rho_1 L_1 + \rho_2 L_2\right]}$$ independently of the temperature.

(*b*) Apply the condition for temperature independence obtained in (*a*) to the carbon and copper wires:

$$\rho_{\text{C}} L_{\text{C}} \alpha_{\text{C}} + \rho_{\text{Cu}} L_{\text{Cu}} \alpha_{\text{Cu}} = 0$$

Solve for the ratio of L_{Cu} to L_{C}:

$$\frac{L_{\text{Cu}}}{L_{\text{C}}} = -\frac{\rho_{\text{C}} \alpha_{\text{C}}}{\rho_{\text{Cu}} \alpha_{\text{Cu}}}$$

Substitute numerical values (see Table 25-1 for the temperature coefficient of resistivity of carbon and copper) and evaluate the ratio of L_{Cu} to L_{C}:

$$\frac{L_{\text{Cu}}}{L_{\text{C}}} = -\frac{(3500 \times 10^{-8}\ \Omega \cdot \text{m})(-0.5 \times 10^{-3}\ \text{K}^{-1})}{(1.7 \times 10^{-8}\ \Omega \cdot \text{m})(3.9 \times 10^{-3}\ \text{K}^{-1})} = \boxed{264}$$

66 •••

Picture the Problem We can use the relationship between the rate at which energy is transformed into heat and light in the filament and the resistance of and potential difference across the filament to estimate the resistance of the filament. The linear dependence of the resistivity on temperature will allow us to find the resistivity of the filament at 2500 K. We can then use the relationship between the resistance of the filament, its resistivity, and cross-sectional area to find its diameter.

(*a*) Express the wattage of the lightbulb as a function of its resistance R and the voltage V supplied by the source:

$$P = \frac{V^2}{R}$$

Solve for *R* to obtain:

$$R = \frac{V^2}{P}$$

Substitute numerical values and evaluate *R*:

$$R = \frac{(100\,\text{V})^2}{40\,\text{W}} = \boxed{250\,\Omega}$$

(*b*) Relate the resistance *R* of the filament to its resistivity ρ, radius *r*, and length ℓ:

$$R = \frac{\rho\ell}{\pi r^2}$$

Solve for *r* to obtain:

$$r = \sqrt{\frac{\rho\ell}{\pi R}}$$

and the diameter *d* of the filament is

$$d = 2\sqrt{\frac{\rho\ell}{\pi R}} \qquad (1)$$

Because the resistivity varies linearly with temperature, we can use a proportion to find its value at 2500 K:

$$\frac{\rho_{2500\,\text{K}} - \rho_{293\,\text{K}}}{\rho_{3500\,\text{K}} - \rho_{293\,\text{K}}} = \frac{2500\,\text{K} - 293\,\text{K}}{3500\,\text{K} - 293\,\text{K}} = \frac{2207}{3207}$$

Solve for $\rho_{2500\,\text{K}}$ to obtain:

$$\rho_{2500\,\text{K}} = \frac{2207}{3207}\left(\rho_{3500\,\text{K}} - \rho_{293\,\text{K}}\right) + \rho_{293\,\text{K}}$$

Substitute numerical values and evaluate $\rho_{2500\,\text{K}}$:

$$\rho_{2500\,\text{K}} = \frac{2207}{3207}\left(1.1\,\mu\Omega\cdot\text{m} - 56\,\text{n}\Omega\cdot\text{m}\right) + 56\,\text{n}\Omega\cdot\text{m} = 774.5\,\text{n}\Omega\cdot\text{m}$$

Substitute numerical values in equation (1) and evaluate *d*:

$$d = 2\sqrt{\frac{(774.5\,\text{n}\Omega\cdot\text{m})(0.5\,\text{cm})}{\pi(250\,\Omega)}} = \boxed{4.44\,\mu\text{m}}$$

67 •••

Picture the Problem We can use the relationship between the rate at which an object radiates and its temperature to find the temperature of the bulb.

(*a*) At a temperature *T*, the power emitted by a perfect blackbody is:

$$P = \sigma A T^4$$

where σ = 5.67×10^{-8} $\text{W/m}^2\cdot\text{K}^4$ is the Stefan-Boltzmann constant.

Solve for *T*:

$$T = \sqrt[4]{\frac{P}{\sigma A}} = \sqrt[4]{\frac{P}{\sigma\pi dL}}$$

or, because $P = V^2/R$,

$$T = \sqrt[4]{\frac{V^2}{\sigma\pi dLR}}$$

Relate the resistance R of the filament to its resistivity ρ:

$$R = \frac{\rho L}{A} = \frac{4\rho L}{\pi d^2}$$

Substitute for R in the expression for T to obtain:

$$T = \sqrt[4]{\frac{V^2}{\sigma\pi dL\frac{4\rho L}{\pi d^2}}} = \sqrt[4]{\frac{V^2 d}{4\sigma L^2\rho}}$$

Substitute numerical values and evaluate T:

$$T = \sqrt[4]{\frac{(5\,\text{V})^2(40\times10^{-6}\,\text{m})}{4(5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4)(0.03\,\text{m})^2(3\times10^{-5}\,\Omega\cdot\text{m})}} = \boxed{636\,\text{K}}$$

(*b*) As the filament heats up, its resistance increases, leading to more power being dissipated, leading to further heat, leading to a higher temperature, etc. This thermal runaway can burn out the filament if not controlled.

Energy in Electric Circuits

***68 •**

Picture the Problem We can use $P = V^2/R$ to find the power dissipated by the two resistors.

Express the power dissipated in a resistor as a function of its resistance and the potential difference across it:

$$P = \frac{V^2}{R}$$

(*a*) Evaluate P for V = 120 V and R = 5 Ω:

$$P = \frac{(120\,\text{V})^2}{5\,\Omega} = \boxed{2.88\,\text{kW}}$$

(*b*) Evaluate P for V = 120 V and R = 10 Ω:

$$P = \frac{(120\,\text{V})^2}{10\,\Omega} = \boxed{1.44\,\text{kW}}$$

69 •

Picture the Problem We can solve $P_{max} = I_{max}^2 R$ for the maximum current the resistor can carry and apply Ohm's law to find the maximum voltage that can be placed across the resistor.

(*a*) Express the maximum power the resistor can dissipate in terms of the current flowing through it:

$$P_{max} = I_{max}^2 R$$

Solve for I_{max} to obtain:

$$I_{max} = \sqrt{\frac{P_{max}}{R}}$$

Substitute numerical values and evaluate I_{max}:

$$I_{max} = \sqrt{\frac{0.25\,\text{W}}{10\,\text{k}\Omega}} = \boxed{5.00\,\text{mA}}$$

(*b*) Apply Ohm's law to relate the maximum voltage across this resistor to the maximum current through it:

$$V_{max} = I_{max} R$$

Substitute numerical values and evaluate V_{max}:

$$V_{max} = (5\,\text{mA})(10\,\text{k}\Omega) = \boxed{50.0\,\text{V}}$$

70 •

Picture the Problem We can use $P = V^2/R$ to find the resistance of the heater and Ohm's law to find the current it draws.

(*a*) Express the power output of the heater in terms of its resistance and its operating voltage:

$$P = \frac{V^2}{R}$$

Solve for and evaluate R:

$$R = \frac{V^2}{P} = \frac{(240\,\text{V})^2}{1\,\text{kW}} = \boxed{57.6\,\Omega}$$

Apply Ohm's law to find the current drawn by the heater:

$$I = \frac{V}{R} = \frac{240\,\text{V}}{57.6\,\Omega} = \boxed{4.17\,\text{A}}$$

(*b*) Evaluate the power output of the heater operating at 120 V:

$$P = \frac{(120\,\text{V})^2}{57.6\,\Omega} = \boxed{250\,\text{W}}$$

71 •

Picture the Problem We can use the definition of power and the relationship between the battery's power output and its emf to find the work done by it under the given conditions.

Use the definition of power to relate the work done by the battery to the

$$P = \frac{\Delta W}{\Delta t}$$

time current is drawn from it:

Solve for the work done in time Δt: $\Delta W = P\Delta t$

Express the power output of the battery as a function of the battery's emf: $P = \mathcal{E}I$

Substitute to obtain: $\Delta W = \mathcal{E}I\Delta t$

Substitute numerical values and evaluate ΔW: $\Delta W = (12\text{ V})(3\text{ A})(5\text{ s}) = \boxed{180\text{ J}}$

72 •

Picture the Problem We can relate the terminal voltage of the battery to it emf, internal resistance, and the current delivered by it and then solve this relationship for the internal resistance.

Express the terminal potential difference of the battery in terms of its emf and internal resistance: $V_a - V_b = \mathcal{E} - Ir$

Solve for r: $r = \dfrac{\mathcal{E} - (V_a - V)_b}{I}$

Substitute numerical values and evaluate r: $r = \dfrac{12\text{ V} - 11.4\text{ V}}{20\text{ A}} = \boxed{0.0300\,\Omega}$

***73** •

Picture the Problem We can find the power delivered by the battery from the product of its emf and the current it delivers. The power delivered to the battery can be found from the product of the potential difference across the terminals of the starter (or across the battery when current is being drawn from it) and the current being delivered to it. In part (*c*) we can use the definition of power to relate the decrease in the chemical energy of the battery to the power it is delivering and the time during which current is drawn from it. In part (*d*) we can use conservation of energy to relate the energy delivered by the battery to the heat develope the battery and the energy delivered to the starter

(*a*) Express the power delivered by the battery as a function of its emf: $P = \mathcal{E}I = (12\text{ V})(20\text{ A}) = \boxed{240\text{ W}}$

and the current it delivers:

(*b*) Relate the power delivered to the starter to the potential difference across its terminals:

$$P_{\text{starter}} = V_{\text{starter}} I = (11.4\ \text{V})(20\ \text{A}) = \boxed{228\ \text{W}}$$

(*c*) Use the definition of power to express the decrease in the chemical energy of the battery as it delivers current to the starter:

$$\Delta E = P\Delta t = (240\ \text{W})(3\ \text{min}) = \boxed{43.2\ \text{kJ}}$$

(*d*) Use conservation of energy to relate the energy delivered by the battery to the heat developed in the battery and the energy delivered to the starter:

$$E_{\text{delivered by battery}} = E_{\text{transformed into heat}} + E_{\text{delivered to starter}} = Q + E_{\text{delivered to starter}}$$

Express the energy delivered by the battery and the energy delivered to the starter in terms of the rate at which this energy is delivered:

$$P\Delta t = Q + P_{\text{s}}\Delta t$$

Solve for Q to obtain:

$$Q = (P - P_{\text{s}})\Delta t$$

Substitute numerical values and evaluate Q:

$$Q = (240\ \text{W} - 228\ \text{W})(3\ \text{min}) = \boxed{2.16\ \text{kJ}}$$

74 •

Picture the Problem We can use conservation of energy to relate the emf of the battery to the potential differences across the variable resistor and the energy converted to heat within the battery. Solving this equation for I will allow us to find the current for the four values of R and we can use $P = I^2R$ to find the power delivered the battery for the four values of R.

Apply conservation of energy (Kirchhoff's loop rule) to the circuit to obtain:

$$\mathcal{E} = IR + Ir$$

Solve for I to obtain:

$$I = \frac{\mathcal{E}}{R + r}$$

Express the power delivered by the battery as a function of the current drawn from it:

$$P = I^2 R$$

(*a*) For $R = 0$:

$$I = \frac{\mathcal{E}}{R + r} = \frac{6\,\text{V}}{0.3\,\Omega} = \boxed{20\,\text{A}}$$

and

$$P = (20\,\text{A})^2(0) = \boxed{0}$$

(*b*) For $R = 5\ \Omega$:

$$I = \frac{\mathcal{E}}{R + r} = \frac{6\,\text{V}}{5\,\Omega + 0.3\,\Omega} = \boxed{1.13\,\text{A}}$$

and

$$P = (1.13\,\text{A})^2(5\,\Omega) = \boxed{6.38\,\text{W}}$$

(*c*) For $R = 10\ \Omega$:

$$I = \frac{\mathcal{E}}{R + r} = \frac{6\,\text{V}}{10\,\Omega + 0.3\,\Omega} = \boxed{0.583\,\text{A}}$$

and

$$P = (0.583\text{A})^2(10\,\Omega) = \boxed{3.40\,\text{W}}$$

(*d*) For $R = \infty$:

$$I = \frac{\mathcal{E}}{R + r} = \lim_{R\to\infty} \frac{6\,\text{V}}{R + 0.3\,\Omega} = \boxed{0}$$

and

$$P = \boxed{0}$$

75 ••

Picture the Problem We can express the total stored energy ΔU in the battery in terms of its emf and the product $I\Delta t$ of the current it can deliver for a period of time Δt. We can apply the definition of power to relate the lifetime of the battery to the rate at which it is providing energy to the pair of headlights

(*a*) Express ΔU in terms of ε and the product $I\Delta t$:

$$\Delta U = \mathcal{E} I \Delta t$$

Substitute numerical values and evaluate ΔU:

$$\Delta U = (12\,\text{V})(160\,\text{A}\cdot\text{h}) = 1.92\,\text{kW}\cdot\text{h}$$
$$= 1.92\,\text{kW}\cdot\text{h} \times \frac{3.6\,\text{MJ}}{\text{kW}\cdot\text{h}}$$
$$= \boxed{6.91\,\text{MJ}}$$

(*b*) Use the definition of power to relate the lifetime of the battery to the rate at which it is providing energy to the pair of headlights:

$$\Delta t = \frac{\Delta U}{P}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{1.92\,\text{kW}\cdot\text{h}}{150\,\text{W}} = \boxed{12.8\,\text{h}}$$

76 ••

Picture the Problem We can use conservation of energy (aka Kirchhoff's loop Rule) to relate the emf at the fuse box and the voltage drop in the wires to the voltage at the outlet box (delivered to the space heater). We can find the number of 60-W li bulbs that could be supplied by the this line without blowing the fuse by dividing the current available by the current drawn by each 60-W bulb.

(*a*) Apply Kirchhoff's loop rule to the circuit to obtain:

$$\mathcal{E} - V_{\text{wires}} - V_{\text{outlet}} = 0$$

or

$$\mathcal{E} - IR_{\text{wires}} - V_{\text{outlet}} = 0$$

Solve for V_{outlet} to obtain:

$$V_{\text{outlet}} = \mathcal{E} - IR_{\text{wires}}$$

Relate the resistance of the copper wires to the resistivity of copper, the length of the wires, and the cross-sectional
area of 12-gauge wire:

$$R_{\text{wires}} = \rho_{\text{Cu}}\frac{L}{A}$$

Substitute to obtain:

$$V_{\text{outlet}} = \mathcal{E} - \frac{I\rho_{\text{Cu}}L}{A}$$

Substitute numerical values (see Table 25 -1 for the resistivity of copper and Table 25-2 for the cross-sectional area of
12-gauge wire) and evaluate V_{outlet}:

$$V_{\text{outlet}} = 120\,\text{V} - \frac{(12.5\,\text{A})(1.7\times10^{-8}\,\Omega\cdot\text{m})(60\,\text{m})}{3.309\,\text{mm}^2} = \boxed{116\,\text{V}}$$

(*b*) Relate the number of bulbs N to the maximum current available and the current drawn by each 60-W bulb:

$$N = \frac{I_{\text{max}} - 12.5\,\text{A}}{I_{\text{bulb}}} \qquad (1)$$

Use Ohm's law to relate the current drawn by each bulb to the potential difference across it and it's resistance:

$$I_{bulb} = \frac{V}{R_{bulb}}$$

Express the resistance of each 60-W bulb:

$$R_{bulb} = \frac{\mathcal{E}^2}{P}$$

Substitute to obtain:

$$I_{bulb} = \frac{PV}{\mathcal{E}^2}$$

Substitute in equation (1) to obtain:

$$N = \frac{I_{max} - 12.5\,\text{A}}{\dfrac{PV}{\mathcal{E}^2}} = \frac{\mathcal{E}^2}{PV}\left(I_{max} - 12.5\,\text{A}\right)$$

Substitute numerical values and evaluate N:

$$N = \frac{(120\,\text{V})^2}{(60\,\text{W})(116\,\text{V})}(20\,\text{A} - 12.5\,\text{A}) = \boxed{15\,\text{bulbs}}$$

***77 ••**

Picture the Problem We can use $P = fv$ to find the power the electric motor must develop to move the car at 80 km/h against a frictional force of 1200 N. We can find the total charge that can be delivered by the 10 batteries using $\Delta Q = NI\Delta t$. The total electrical energy delivered by the 10 batteries before recharging can be found using the definition of emf. We can find the distance the car can travel from the definition of work and the cost per kilometer of driving the car this distance by dividing the cost of the required energy by the distance the car has traveled.

(*a*) Express the power the electric motor must develop in terms of the speed of the car and the friction force:

$$P = fv = (1200\,\text{N})(80\,\text{km/h}) = \boxed{26.7\,\text{kW}}$$

(*b*) Use the definition of current to express the total charge that can be delivered before charging:

$$\Delta Q = NI\Delta t = 10(160\,\text{A}\cdot\text{h})\left(\frac{3600\,\text{s}}{\text{h}}\right) = \boxed{5.76\,\text{MC}}$$

where N is the number of batteries.

(*c*) Use the definition of emf to express the total electrical energy available in the batteries:

$$W = Q\mathcal{E} = (5.76\,\text{MC})(12\,\text{V})$$
$$= \boxed{69.1\,\text{MJ}}$$

(*d*) Relate the amount of work the batteries can do to the work required to overcome friction:

$$W = fd$$

Solve for and evaluate *d*:

$$d = \frac{W}{f} = \frac{69.1\,\text{MJ}}{1200\,\text{N}} = \boxed{57.6\,\text{km}}$$

(*e*) Express the cost per kilometer as the ratio of the ratio of the cost of the energy to the distance traveled before recharging:

$$\text{Cost/km} = \frac{(\$0.09/\text{kW}\cdot\text{h})\mathcal{E}It}{d} = \frac{(\$0.09/\text{kW}\cdot\text{h})(120\,\text{V})(160\,\text{A}\cdot\text{h})}{57.6\,\text{km}} = \boxed{\$0.03/\text{km}}$$

78 •••

Picture the Problem We can use the definition of power to find the current drawn by the heater and Ohm's law to find its resistance. In part (*b*) we'll use the hint to show that $\Delta P/P \approx 2\Delta V/V$ and in part (*c*) use this result to find the approximate power dissipated in the heater if the potential difference is decreased to 115 V.

(*a*) Use the definition of power to relate the current *I* drawn by the heater to its power rating *P* and the potential difference across it *V*:

$$I = \frac{P}{V}$$

Substitute numerical values and evaluate *I*:

$$I = \frac{100\,\text{W}}{120\,\text{V}} = \boxed{0.833\,\text{A}}$$

Apply Ohm's law to relate the resistance of the heater to the voltage across it and the current it draws:

$$R = \frac{V}{I} = \frac{120\,\text{V}}{0.833\,\text{A}} = \boxed{144\,\Omega}$$

(*b*) Approximate dP/dV by differentials:

$$\frac{dP}{dV} \approx \frac{\Delta P}{\Delta V} \text{ or } \Delta P \approx \frac{dP}{dV}\Delta V$$

Express the dependence of *P* on *V*:

$$P = \frac{V^2}{R}$$

Assuming R to be constant, evaluate dP/dV:

$$\frac{dP}{dV} = \frac{d}{dV}\left[\frac{V^2}{R}\right] = \frac{2V}{R}$$

Substitute to obtain:

$$\Delta P \approx \frac{2V}{R}\Delta V = 2\frac{V^2}{R}\frac{\Delta V}{V} = 2P\frac{\Delta V}{V}$$

Divide both sides of the equation by P to obtain:

$$\boxed{\frac{\Delta P}{P} = 2\frac{\Delta V}{V}}$$

(c) Express the approximate power dissipated in the heater as the sum of its power consumption and the change in its power dissipation when the voltage is decreased by ΔV:

$$\begin{aligned} P &\approx P_0 + \Delta P \\ &= P_0 + 2P_0\frac{\Delta V}{V} \\ &= P_0\left(1 + 2\frac{\Delta V}{V}\right) \end{aligned}$$

Substitute numerical values and evaluate P:

$$P \approx (100\text{ W})\left(1 + 2\left(\frac{-5\text{ V}}{120\text{ V}}\right)\right) = \boxed{91.7\text{ W}}$$

Combinations of Resistors

***79** •

Picture the Problem We can either solve this problem by using the expression for the equivalent resistance of three resistors connected in parallel and then using Ohm's law to find the current in each resistor, or we can apply Ohm's law first to find the current through each resistor and then use Ohm's law a second time to find the equivalent resistance of the parallel combination. We'll follow the first procedure.

(a) Express the equivalent resistance of the three resistors in parallel and solve for R_{eq}:

$$\frac{1}{R_{eq}} = \frac{1}{4\,\Omega} + \frac{1}{3\,\Omega} + \frac{1}{6\,\Omega}$$

and

$$\boldsymbol{R}_{eq} = \boxed{1.33\,\Omega}$$

(b) Apply Ohm's law to each of the resistors to find the current flowing through each:

$$I_4 = \frac{V}{4\,\Omega} = \frac{12\text{ V}}{4\,\Omega} = \boxed{3.00\text{ A}}$$

$$I_3 = \frac{V}{3\,\Omega} = \frac{12\text{ V}}{3\,\Omega} = \boxed{4.00\text{ A}}$$

and

$$I_6 = \frac{V}{6\,\Omega} = \frac{12\,\text{V}}{6\,\Omega} = \boxed{2.00\,\text{A}}$$

Remarks: You would find it instructive to use Kirchhoff's junction rule (conservation of charge) to confirm our values for the currents through the three resistors.

80 •

Picture the Problem We can simplify the network by first replacing the resistors that are in parallel by their equivalent resistance. We can then add that resistance and the 3-Ω resistance to find the equivalent resistance between points *a* and *b*. In part (*b*) we'll denote the currents through each resistor with subscripts corresponding to the resistance through which the current flows and apply Ohm's law to find those currents.

(*a*) Express the equivalent resistance of the two resistors in parallel and solve for $R_{\text{eq},1}$:

$$\frac{1}{R_{\text{eq},1}} = \frac{1}{R_6} + \frac{1}{R_2} = \frac{1}{6\,\Omega} + \frac{1}{2\,\Omega}$$

and

$$R_{\text{eq},1} = 1.50\,\Omega$$

Because the 3-Ω resistor is in series with $R_{\text{eq},1}$:

$$R_{\text{eq}} = R_3 + R_{\text{eq},1} = 3\,\Omega + 1.5\,\Omega = \boxed{4.50\,\Omega}$$

(*b*) Apply Ohm's law to the network to find I_3:

$$I_3 = \frac{V_{ab}}{R_{\text{eq}}} = \frac{12\,\text{V}}{4.5\,\Omega} = \boxed{2.67\,\text{A}},$$

Find the potential difference across the parallel resistors:

$$V_{6\,\&\,2} = V_{ab} - V_3 = 12\,\text{V} - (2.67\,\text{A})(3\,\Omega) = 4\,\text{V}$$

Use the common potential difference across the resistors in parallel to find the current through each of them:

$$I_6 = \frac{V_6}{R_6} = \frac{4\,\text{V}}{6\Omega} = \boxed{0.667\,\text{A}}$$

and

$$I_2 = \frac{V_{6\,\&\,2}}{R_2} = \frac{4\,\text{V}}{2\Omega} = \boxed{2.00\,\text{A}}$$

81 •

Picture the Problem Note that the resistors between *a* and *c* and between *c* and *b* are in series as are the resistors between *a* and *d* and between *d* and *b*. Hence, we have two branches in parallel, each branch consisting of two resistors *R* in series. In part (b) it will be important to note that the potential difference between point *c* and point *d* is zero.

(*a*) Express the equivalent resistance between points *a* and *b* in terms of the equivalent resistances between *acb* and *adb*:

$$\frac{1}{R_{\text{eq}}}=\frac{1}{R_{acb}}+\frac{1}{R_{adb}}=\frac{1}{2R}+\frac{1}{2R}$$

Solve for R_{eq} to obtain:

$$R_{\text{eq}}=\boxed{R}$$

(*b*) Because the potential difference between points c and d is zero, no current would flow through the resistor connected between these two points, and the addition of that resistor would not change the network.

82 ••

Picture the Problem Note that the 2-Ω resistors are in parallel with each other and with the 4-Ω resistor. We can Apply Kirchhoff's loop rule to relate the current I_3 drawn from the battery to the emf of the battery and equivalent resistance R_{eq} of the resistor network. We can find the current through the resistors connected in parallel by applying Kirchhoff's loop rule a second time. In part (*b*) we can find the power delivered by the battery from the product of its emf and the current it delivers to the circuit.

(*a*) Apply Kirchhoff's loop rule to obtain:

$$\mathcal{E}-I_3R_{\text{eq}}=0$$

Solve for I_3:

$$I_3=\frac{\mathcal{E}}{R_{\text{eq}}}\qquad(1)$$

Find the equivalent resistance of the three resistors in parallel:

$$\frac{1}{R_{\text{eq},1}}=\frac{1}{R_2}+\frac{1}{R_2}+\frac{1}{R_4}=\frac{1}{2\,\Omega}+\frac{1}{2\,\Omega}+\frac{1}{4\,\Omega}$$

and

$$R_{\text{eq},1}=0.8\,\Omega$$

Find the equivalent resistance of $R_{\text{eq},1}$ and R_3 in series:

$$R_{\text{eq}}=R_3+R_{\text{eq},1}=3\,\Omega+0.8\,\Omega=3.8\,\Omega$$

Substitute in equation (1) and evaluate I_3:

$$I_3=\frac{6\text{ V}}{3.8\,\Omega}=\boxed{1.58\text{ A}}$$

Express the current through each of the parallel resistors in terms of their common potential difference V:

$$I_2=\frac{V}{R_2}\text{ and }I_4=\frac{V}{R_4}$$

Apply Kirchhoff's loop rule to obtain:

$$\mathcal{E} - I_3R_3 - V = 0$$

Solve for V:

$$V = \mathcal{E} - I_3R_3 = 6\,\text{V} - (1.58\,\text{A})(3\,\Omega) = 1.26\,\text{V}$$

Substitute numerical values and evaluate I_2 and I_4:

$$I_2 = \frac{1.26\,\text{V}}{2\,\Omega} = \boxed{0.630\,\text{A}}$$

and

$$I_4 = \frac{1.26\,V}{4\,\Omega} = \boxed{0.315\,\text{A}}$$

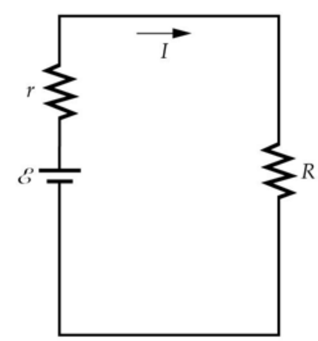
(*b*) Express P in terms of ε and I_3:

$$P = \mathcal{E}I_3 = (6\,\text{V})(1.58\,\text{A}) = \boxed{9.48\,\text{W}}$$

Remarks: Note that the currents I_3, I_2, and I_4 satisfy Kirchhoff's junction rule.

***83 ••**

Picture the Problem Let r represent the resistance of the internal resistance of the power supply, ε the emf of the power supply, R the resistance of the external resistor to be placed in series with the power supply, and I the current drawn from the power supply. We can use Ohm's law to express the potential difference across R and apply Kirchhoff's loop rule to express the current through R in terms of ε, r, and R.



Express the potential difference across the resistor whose resistance is R:

$$V_R = IR \qquad (1)$$

Apply Kirchhoff's loop rule to the circuit to obtain:

$$\mathcal{E} - Ir - IR = 0$$

Solve for I to obtain:

$$I = \frac{\mathcal{E}}{r+R}$$

Substitute in equation (1) to obtain:

$$V_R = \left(\frac{\mathcal{E}}{r+R}\right)R$$

Solve for R to obtain:

$$R = \frac{V_R r}{\varepsilon - V_R}$$

Substitute numerical values and evaluate R:

$$R = \frac{(4.5\,\text{V})(50\,\Omega)}{5\,\text{V} - 4.5\,\text{V}} = \boxed{450\,\Omega}$$

84 ••

Picture the Problem We can apply Kirchhoff's loop rule to the two circuits described in the problem statement and solve the resulting equations simultaneously for r and ε.

(a) and (b) Apply Kirchhoff's loop rule to the two circuits to obtain:

$$\varepsilon - I_1 r - I_1 R_5 = 0$$

and

$$\varepsilon - I_2 r - I_2 R_{11} = 0$$

Substitute numerical values to obtain:

$$\varepsilon - (0.5\,\text{A})r - (0.5\,\text{A})(5\,\Omega) = 0$$

or

$$\varepsilon - (0.5\,\text{A})r = 2.5\,\text{V} \qquad (1)$$

and

$$\varepsilon - (0.25\,\text{A})r - (0.25\,\text{A})(11\,\Omega) = 0$$

or

$$\varepsilon - (0.25\,\text{A})r = 2.75\,\text{V} \qquad (2)$$

Solve equations (1) and (2) simultaneously to obtain:

$$\varepsilon = \boxed{3.00\,\text{V}} \text{ and } r = \boxed{1.00\,\Omega}$$

85 ••

Picture the Problem We can use the formula for the equivalent resistance for two resistors in parallel to show that $R_{eq} = R_1 x/(1 + x)$.

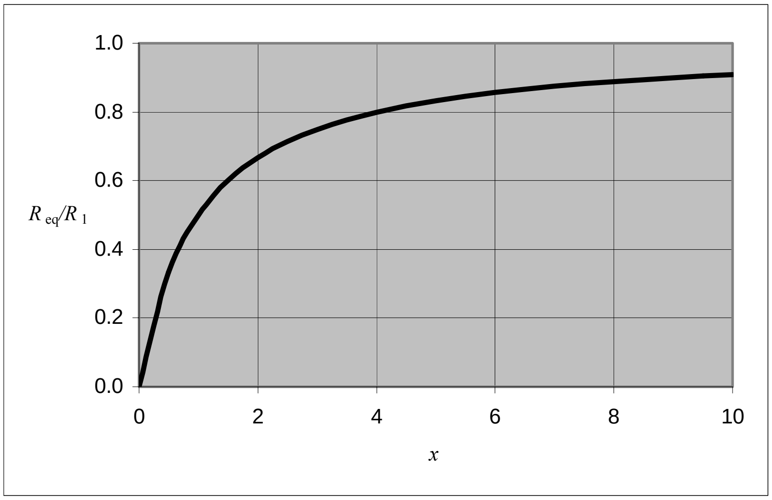
(a) Express the equivalent resistance of R_1 and R_2 in parallel:

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Let $x = R_2/R_1$ to obtain:

$$R_{eq} = \frac{x R_1^2}{R_1 + x R_1} = \boxed{\frac{x}{1+x} R_1}$$

(b) The following graph of R_{eq}/R_1 versus x was plotted using a spreadsheet program.



86 ••

Picture the Problem We can use Kirchhoff's loop rule to relate the required resistance to the emf of the source and the desired current. We can apply Kirchhoff's rule a second time to the circuit that includes the load resistance *r* to establish the largest value it can have if it is to change the current drawn from the source by at most 10 percent.

(*a*) Apply Kirchhoff's loop rule to the circuit that includes the source and the resistance *R* to obtain:

$$\mathcal{E} - IR = 0$$

Solve for *R* to obtain:

$$R = \frac{\mathcal{E}}{I}$$

Substitute numerical values and evaluate *R*:

$$R = \frac{5\,\text{V}}{10\,\text{mA}} = \boxed{500\,\Omega}$$

(*b*) Letting *I*′ represent the current in the loaded circuit, express the condition that the current drops by less than 10%:

$$\frac{I - I'}{I} = 1 - \frac{I'}{I} < 0.1 \qquad (1)$$

Letting *r* represent the load resistance, apply Kirchhoff's loop rule to the circuit to obtain:

$$\mathcal{E} - I'r - I'R = 0$$

Solve for *I*′ to obtain:

$$I' = \frac{\mathcal{E}}{r + R}$$

Substitute for *I* and *I*′ in equation (1):

$$1 - \frac{\dfrac{\mathcal{E}}{r+R}}{\dfrac{\mathcal{E}}{R}} < 0.1 \Rightarrow 1 - \frac{R}{r+R} < 0.1$$

Solve for r to obtain:

$$r < \frac{0.1R}{0.9}$$

Substitute the numerical value for R to obtain:

$$r < \frac{0.1(500\,\Omega)}{0.9} = \boxed{55.6\,\Omega}$$

87 ••

Picture the Problem We can simplify the network by first replacing the resistors that are in parallel by their equivalent resistance. We'll then have a parallel network with two resistors in series in each branch and can use the expressions for resistors in series to simplify the network to two resistors in parallel. The equivalent resistance between points *a* and *b* will be the single resistor equivalent to these two resistors. In part (*b*) we'll use the fact that the potential difference across the upper branch is the same as the potential difference across the lower branch, in conjunction with Ohm's law, to find the currents through each resistor.

(*a*) Express and evaluate the equivalent resistance of the two 6-Ω resistors in parallel and solve for $R_{\text{eq},1}$:

$$R_{\text{eq},1} = \frac{R_6 R_6}{R_6 + R_6} = \frac{(6\,\Omega)^2}{6\,\Omega + 6\,\Omega} = 3\,\Omega$$

Find the equivalent resistance of the 6-Ω resistor is in series with $R_{\text{eq},1}$:

$$R_{\text{eq},2} = R_6 + R_{\text{eq},1} = 6\,\Omega + 3\,\Omega = 9\,\Omega$$

Find the equivalent resistance of the 12-Ω resistor in series with the 6-Ω resistor:

$$R_{\text{eq},3} = R_6 + R_{12} = 6\,\Omega + 12\,\Omega = 18\,\Omega$$

Finally, find the equivalent resistance of $R_{\text{eq},3}$ in parallel with $R_{\text{eq},2}$:

$$R_{\text{eq}} = \frac{R_{\text{eq},2} R_{\text{eq},3}}{R_{\text{eq},2} + R_{\text{eq},3}} = \frac{(9\,\Omega)(18\,\Omega)}{9\,\Omega + 18\,\Omega} = \boxed{6.00\,\Omega}$$

(*b*) Apply Ohm's law to the upper branch to find the current $I_u = I_{12} = I_6$:

$$I_u = I_{12} = I_6 = \frac{V_{ab}}{R_{\text{eq},3}} = \frac{12\text{ V}}{18\,\Omega} = \boxed{0.667\text{ A}},$$

Apply Ohm's law to the lower branch to find the current $I_l = I_{6\text{-}\Omega\text{ resistor in series}}$:

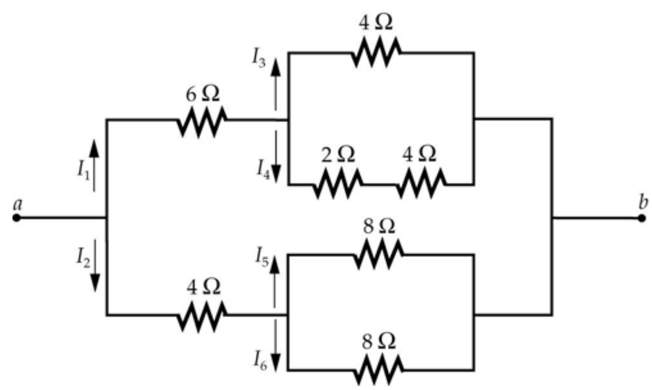
$$I_l = I_{6\text{-}\Omega\text{ resistor in series}} = \frac{V_{ab}}{R_{\text{eq},2}} = \frac{12\text{ V}}{9\,\Omega} = \boxed{1.33\text{ A}}$$

Express the current through the 6-Ω resistors in parallel:

$$I_{6\text{-}\Omega\text{ resistors in parallel}} = \tfrac{1}{2}I_l = \tfrac{1}{2}(1.33\text{ A}) = \boxed{0.667\text{ A}}$$

88 ••

Picture the Problem Assign currents in each of the resistors as shown in the diagram. We can simplify the network by first replacing the resistors that are in parallel by their equivalent resistance. We'll then have a parallel network with two resistors in series in each branch and can use the expressions for resistors in series to simplify the network to two resistors in parallel. The equivalent resistance between points *a* and *b* will be the single resistor equivalent to these two resistors. In part (*b*) we'll use the fact that the potential difference across the upper branch is the same as the potential difference across the lower branch, in conjunction with Ohm's law, to find the currents through each resistor.



(*a*) Express and evaluate the equivalent resistance of the resistors in parallel in the upper branch and solve for $R_{\text{eq},1}$:

$$R_{\text{eq},1} = \frac{(R_2 + R_4)R_4}{(R_2 + R_4) + R_4} = \frac{(6\,\Omega)(4\,\Omega)}{2\,\Omega + 4\,\Omega + 4\,\Omega} = 2.4\,\Omega$$

Find the equivalent resistance of the 6-Ω resistor is in series with $R_{\text{eq},1}$:

$$R_{\text{eq},2} = R_6 + R_{\text{eq},1} = 6\,\Omega + 2.4\,\Omega = 8.4\,\Omega$$

Express and evaluate the equivalent resistance of the resistors in parallel in the lower branch and solve for

$$R_{\text{eq},2} = \frac{R_8 R_8}{R_8 + R_8} = \tfrac{1}{2}R_8 = \tfrac{1}{2}(8\,\Omega) = 4\,\Omega$$

$R_{eq,2}$:

Find the equivalent resistance of the 4-Ω resistor is in series with $R_{eq,2}$:

$$R_{eq,3} = R_4 + R_{eq,2} = 4\,\Omega + 4\,\Omega = 8\,\Omega$$

Finally, find the equivalent resistance of $R_{eq,2}$ in parallel with $R_{eq,3}$:

$$R_{eq} = \frac{R_{eq,2}R_{eq,3}}{R_{eq,2} + R_{eq,3}} = \frac{(8.4\,\Omega)(8\,\Omega)}{8.4\,\Omega + 8\,\Omega} = \boxed{4.10\,\Omega}$$

(*b*) Apply Ohm's law to the upper branch to find the current I_1:

$$I_1 = \frac{V_{ab}}{R_{eq,2}} = \frac{12\,\text{V}}{8.4\,\Omega} = \boxed{1.43\,\text{A}}$$

Find the potential difference across the 4-Ω and 6-Ω parallel combination in the upper branch:

$$V_{4\&6} = 12\,\text{V} - V_6 = 12\,\text{V} - I_u R_6 = 12\,\text{V} - (1.43\,\text{A})(6\,\Omega) = 3.43\,\text{V}$$

Apply Ohm's law to find the current I_4:

$$I_4 = \frac{V_6}{R_6} = \frac{3.43\,\text{V}}{6\,\Omega} = \boxed{0.572\,\text{A}}$$

Apply Ohm's law to find the current I_3:

$$I_3 = \frac{V_4}{R_4} = \frac{3.43\,\text{V}}{4\,\Omega} = \boxed{0.858\,\text{A}}$$

Apply Ohm's law to the lower branch to find the current I_2:

$$I_2 = \frac{V_{ab}}{R_{eq,2}} = \frac{12\,\text{V}}{8\,\Omega} = \boxed{1.50\,\text{A}}$$

Find the potential difference across the 8-Ω and 8-Ω parallel combination in the lower branch:

$$V_{8\&8} = 12\,\text{V} - I_3 R_4 = 12\,\text{V} - (1.5\,\text{A})(4\,\Omega) = 6\,\text{V}$$

Apply Ohm's law to find $I_5 = I_6$:

$$I_5 = I_6 = \frac{V_{8\&8}}{8\,\Omega} = \frac{6\,\text{V}}{8\,\Omega} = \boxed{0.750\,\text{A}}$$

***89 ••**

Picture the Problem We can use the equation for *N* identical resistors connected in parallel to relate *N* to the resistance *R* of each piece of wire and the equivalent resistance

Express the resistance of the *N* pieces connected in parallel:

$$\frac{1}{R_{eq}} = \frac{N}{R}$$

where *R* is the resistance of one of the *N* pieces.

Relate the resistance of one of the N pieces to the resistance of the wire:

$$R = \frac{R_{\text{wire}}}{N}$$

Substitute to obtain:

$$\frac{1}{R_{\text{eq}}} = \frac{N^2}{R_{\text{wire}}}$$

Solve for N:

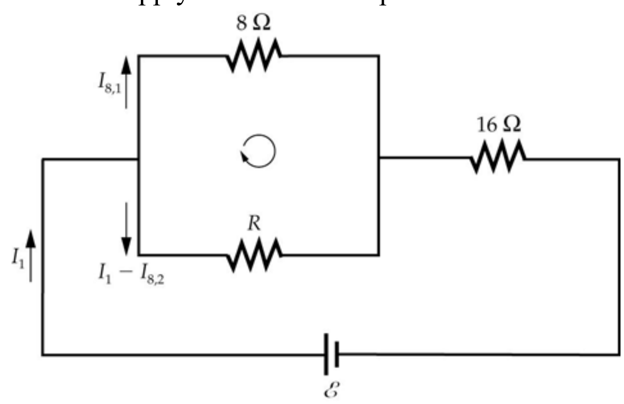
$$N = \sqrt{\frac{R_{\text{wire}}}{R_{\text{eq}}}}$$

Substitute numerical values and evaluate N:

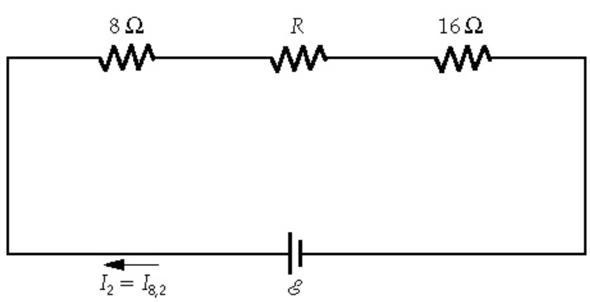
$$N = \sqrt{\frac{120\,\Omega}{1.875\,\Omega}} = \boxed{8}$$

90 ••

Picture the Problem We can assign currents as shown in the diagram of the first arrangement of resistors and apply Kirchhoff's loop rule to obtain an expression for $I_{8,1}$.



Assign currents as shown in the diagram below for the second arrangement of the resistors and apply Kirchhoff's loop rule to obtain an expression for $I_{8,2}$ that we can equate to $I_{8,1}$ and solve for R.



Apply Kirchhoff's loop rule to the first arrangement of the resistors:

$$\mathcal{E} - I_1 R_{\text{eq},1} = 0$$

where I_1 is the current drawn from the battery.

Solve for I_1 to obtain:

$$I_1 = \frac{\mathcal{E}}{R_{\text{eq},1}}$$

Find the equivalent resistance of the first arrangement of the resistors:

$$R_{\text{eq},1} = \frac{(8\,\Omega)R}{8\,\Omega + R} + 16\,\Omega = \frac{(24\,\Omega)R + 128\,\Omega^2}{R + 8\,\Omega}$$

Substitute to obtain:

$$I_1 = \frac{\mathcal{E}}{\dfrac{(24\,\Omega)R + 128\,\Omega^2}{R + 8\,\Omega}} = \frac{\mathcal{E}(R + 8\,\Omega)}{(24\,\Omega)R + 128\,\Omega^2}$$

Apply Kirchhoff's loop rule to the loop containing R and the 8-Ω resistor:

$$-I_8(8\,\Omega) + R(I_1 - I_8) = 0$$

Solve for $I_{8,1}$ to obtain:

$$I_{8,1} = \frac{R}{R + 8\,\Omega} I_1 = \left(\frac{R}{R + 8\,\Omega}\right)\left(\frac{\mathcal{E}(R + 8\,\Omega)}{(24\,\Omega)R + 128\,\Omega^2}\right) = \frac{\mathcal{E}R}{(24\,\Omega)R + 128\,\Omega^2}$$

Express $I_{8,2}$ in terms of I_1 and $I_{8,1}$:

$$I_{8,2} = I_1 - I_{8,1}$$

Substitute for I_1 and $I_{8,1}$ and simplify to obtain:

$$I_{8,2} = \frac{\mathcal{E}(R + 8\,\Omega)}{(24\,\Omega)R + 128\,\Omega^2} - \frac{\mathcal{E}R}{(24\,\Omega)R + 128\,\Omega^2} = \frac{(8\,\Omega)\mathcal{E}}{(24\,\Omega)R + 128\,\Omega^2}$$

Apply Kirchhoff's loop rule to the second arrangement of the resistors:

$$\mathcal{E} - I_2 R_{\text{eq},2} = 0$$

where I_2 is the current drawn from the battery.

Solve for I_2 (= $I_{8,2}$) to obtain:

$$I_2 = I_{8,2} = \frac{\mathcal{E}}{R_{\text{eq},2}}$$

Find the equivalent resistance of the second arrangement of the resistors:

$$R_{\text{eq},2} = R + 24\,\Omega$$

Substitute to obtain:

$$I_{8,2} = \frac{\mathcal{E}}{R + 24\,\Omega}$$

Equate $I_{8,1}$ and $I_{8,2}$:

$$\frac{\mathcal{E}R}{(24\,\Omega)R + 128\,\Omega^2} = \frac{(8\,\Omega)\mathcal{E}}{(24\,\Omega)R + 128\,\Omega^2}$$

Solve for R to obtain:

$$R = \boxed{8.00\,\Omega}$$

91 ••

Picture the Problem We can find the equivalent resistance R_{ab} between points a and b and then set this resistance equal, in turn, to R_1, R_3, and R_1 and solve for R_3, R_2, and R_1, respectively.

(a) Express the equivalent resistance between points a and b:

$$R_{ab} = \frac{R_1R_2}{R_1 + R_2} + R_3$$

Equate this expression to R_1:

$$R_1 = \frac{R_1R_2}{R_1 + R_2} + R_3$$

Solve for R_3 to obtain:

$$R_3 = \boxed{\frac{R_1^2}{R_1 + R_2}}$$

(b) Set R_3 equal to R_{ab}:

$$R_3 = \frac{R_1R_2}{R_1 + R_2} + R_3$$

Solve for R_2 to obtain:

$$R_2 = \boxed{0}$$

(c) Set R_1 equal to R_{ab}:

$$R_1 = \frac{R_1R_2}{R_1 + R_2} + R_3$$

or

$$R_1^2 - R_3R_1 - R_2R_3 = 0$$

Solve the quadratic equation for R_1 to obtain:

$$R_1 = \boxed{\frac{R_3 + \sqrt{R_3^2 + 4R_2R_3}}{2}}$$

where we've used the positive sign because resistance is a non-negative quantity.

92 ••

Picture the Problem We can substitute the given resistances in the equations derived in Problem 91 to check our results from Problem 78.

(*a*) For $R_1 = 4\ \Omega$ and $R_2 = 6\ \Omega$:

$$R_3 = \frac{R_1^2}{R_1 + R_2} = \frac{(4\,\Omega)^2}{4\,\Omega + 6\,\Omega} = \boxed{1.60\,\Omega}$$

and

$$R_{ab} = \frac{R_1R_2}{R_1 + R_2} + R_3$$
$$= \frac{(4\,\Omega)(6\,\Omega)}{4\,\Omega + 6\,\Omega} + 1.6\,\Omega = \boxed{4.00\,\Omega}$$

(*b*) For $R_1 = 4\ \Omega$ and $R_3 = 3\ \Omega$:

$$R_2 = \boxed{0}$$

and

$$R_{ab} = \frac{R_1(0)}{R_1 + 0} + R_3 = 0 + 3\,\Omega$$
$$= \boxed{3.00\,\Omega}$$

(*c*) For $R_2 = 6\ \Omega$ and $R_3 = 3\ \Omega$:

$$R_1 = \frac{3\,\Omega + \sqrt{(3\,\Omega)^2 + 4(6\,\Omega)(3\,\Omega)}}{2}$$
$$= \frac{3\,\Omega + 9\,\Omega}{2} = \boxed{6.00\,\Omega}$$

and

$$R_{ab} = \frac{R_1R_2}{R_1 + R_2} + R_3$$
$$= \frac{(6\,\Omega)(6\,\Omega)}{6\,\Omega + 6\,\Omega} + 3\,\Omega = \boxed{6.00\,\Omega}$$

Kirchhoff's Rules

***93** •

Picture the Problem We can relate the current provided by the source to the rate of Joule heating using $P = I^2R$ and use Ohm's law and Kirchhoff's rules to find the potential difference across R and the value of r.

(*a*) Relate the current I in the circuit to rate at which energy is being dissipated in the form of Joule heat:

$$P = I^2 R \text{ or } I = \sqrt{\frac{P}{R}}$$

Substitute numerical values and evaluate I:

$$I = \sqrt{\frac{8\,\text{W}}{0.5\,\Omega}} = \boxed{4.00\,\text{A}}$$

(*b*) Apply Ohm's law to find V_R:

$$V_R = IR = (4\,\text{A})(0.5\,\Omega) = \boxed{2.00\,\text{V}}$$

(*c*) Apply Kirchhoff's loop rule to obtain:

$$\mathcal{E} - Ir - IR = 0$$

Solve for r:

$$r = \frac{\mathcal{E} - IR}{I} = \frac{\mathcal{E}}{I} - R$$

Substitute numerical values and evaluate r:

$$r = \frac{6\,\text{V}}{4\,\text{A}} - 0.5\,\Omega = \boxed{1.00\,\Omega}$$

94 •

Picture the Problem Assume that the current flows clockwise in the circuit and let $\mathcal{E}_1$ represent the 12-V source and $\mathcal{E}_2$ the 6-V source. We can apply Kirchhoff's loop rule (conservation of energy) to this series circuit to relate the current to the emfs of the sources and the resistance of the circuit. In part (*b*) we can find the power delivered or absorbed by each source using $P = \mathcal{E}I$ and in part (*c*) the rate of Joule heating using $P = I^2R$.

(*a*) Apply Kirchhoff's loop rule to the circuit to obtain:

$$\mathcal{E}_1 - IR_2 - \mathcal{E}_2 - IR_4 = 0$$

Solve for I:

$$I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_2 + R_4}$$

Substitute numerical values and evaluate I:

$$I = \frac{12\,\text{V} - 6\,\text{V}}{2\,\Omega + 4\,\Omega} = \boxed{1.00\,\text{A}}$$

(*b*) Express the power delivered/absorbed by each source in terms of its emf and the current drawn from or forced through it:

$$P_{12} = \mathcal{E}_{12} I = (12\,\text{V})(1\,\text{A}) = \boxed{12\,\text{W}}$$

and

$$P_6 = \mathcal{E}_6 I = (-6\,\text{V})(1\,\text{A}) = \boxed{-6\,\text{W}}$$

where the minus sign means that this source is absorbing power.

(*c*) Express the rate of Joule heating in terms of the current through and the resistance of each resistor:

$$P_2 = I^2R_2 = (1\,\text{A})^2(2\,\Omega) = \boxed{2.00\,\text{W}}$$

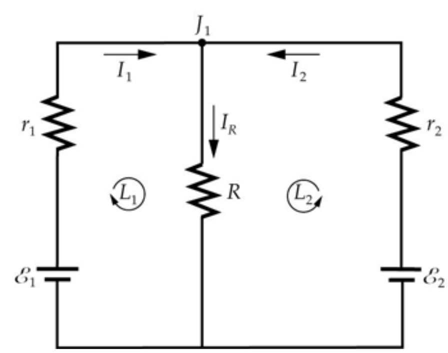
and

$$P_4 = I^2R_4 = (1\,\text{A})^2(4\,\Omega) = \boxed{4.00\,\text{W}}$$

95 ••

Picture the Problem The circuit is shown in the diagram for part (*a*). $\mathcal{E}_1$ and r_1 denote the emf of the "sick" battery and its internal resistance, $\mathcal{E}_2$ and r_2 the emf of the second battery and its internal resistance, and R is the load resistance. Let I_1, I_2, and I_R be the currents. We can apply Kirchhoff's rules to determine the unknown currents. In part (*c*) we can use $P = \mathcal{E}I$ to find the power delivered or absorbed by each battery and $P = I^2R$ to find the power dissipated in the internal and load resistors.

(*a*) The circuit diagram is shown to the right:



(*b*) Apply Kirchhoff's junction rule to junction 1 to obtain:

$$I_1 + I_2 = I_R \quad (1)$$

Apply Kirchhoff's loop rule to loop 1 to obtain:

$$\mathcal{E}_1 - r_1I_1 - RI_R = 0$$

or

$$11.4\,\text{V} - (0.01\,\Omega)I_1 - (2\,\Omega)I_R = 0 \quad (2)$$

Apply Kirchhoff's loop rule to loop 2 to obtain:

$$\mathcal{E}_2 - r_2I_2 - RI_R = 0$$

or

$$12.6\,\text{V} - (0.01\,\Omega)I_2 - (2\,\Omega)I_R = 0 \quad (3)$$

Solve equations (1), (2) and (3) simultaneously to obtain:

$$I_1 = \boxed{-57.0\,\text{A}},$$

$$I_2 = \boxed{63.0\,\text{A}},$$

and

$$I_R = \boxed{6.00\text{ A}}$$

where the minus sign for I_1 means that the current flows in the direction opposite to the direction we arbitrarily chose, i.e., the battery is being charged.

(*c*) Express the power delivered by the second battery in terms of its emf and the current drawn from it:

$$P_2 = \mathcal{E}_2 I_2 = (12.6\text{ V})(63.0\text{ A}) = \boxed{794\text{ W}}$$

Express the power absorbed by the second battery in terms of its emf and the current forced through it:

$$P_1 = \mathcal{E}_1 I_1 = (11.4\text{ V})(57.0\text{ A}) = \boxed{650\text{ W}}$$

Find the power dissipated in the internal resistance r_1:

$$P_{r_1} = I_1^2 r_1 = (57\text{ A})^2(0.01\,\Omega) = \boxed{32.5\text{ W}}$$

Find the power dissipated in the internal resistance r_2:

$$P_{r_2} = I_2^2 r_2 = (63\text{ A})^2(0.01\,\Omega) = \boxed{39.7\text{ W}}$$

Find the power dissipated in the load resistance R:

$$P_R = I_R^2 R = (6\text{ A})^2(2\,\Omega) = \boxed{72.0\text{ W}}$$

Remarks: Note that the sum of the power dissipated in the internal and load resistances and that absorbed by the second battery is the same as that delivered by the first battery … just as we would expect from conservation of energy.

96 ••

Picture the Problem Note that when both switches are closed the 50-Ω resistor is shorted. With both switches open, we can apply Kirchhoff’s loop rule to find the current I in the 100-Ω resistor. With the switches closed, the 100-Ω resistor and R are in parallel. Hence, the potential difference across them is the same and we can express the current I_{100} in terms of the current I_{tot} flowing into the parallel branch whose resistance is R, and the resistance of the 100-Ω resistor. I_{tot}, in turn, depends on the equivalent resistance of the closed-switch circuit, so we can express $I_{100} = I$ in terms of R and solve for R.

Apply Kirchhoff’s loop rule to a loop around the outside of the circuit with both switches open:

$$\mathcal{E} - (300\,\Omega)I - (100\,\Omega)I - (50\,\Omega)I = 0$$

Solve for I to obtain:

$$I = \frac{\mathcal{E}}{450\,\Omega} = \frac{1.5\,\text{V}}{450\,\Omega} = 3.33\,\text{mA}$$

Relate the potential difference across the 100-Ω resistor to the potential difference across R when both switches are closed:

$$(100\,\Omega)I_{100} = RI_R$$

Apply Kirchhoff's junction rule at the junction to the left of the 100-Ω resistor and R:

$$I_{\text{tot}} = I_{100} + I_R$$

or

$$I_R = I_{\text{tot}} - I_{100}$$

where I_{tot} is the current drawn from the source when both switches are closed.

Substitute to obtain:

$$(100\,\Omega)I_{100} = R(I_{\text{tot}} - I_{100})$$

or

$$I_{100} = \frac{RI_{\text{tot}}}{R + 100\,\Omega} \qquad (1)$$

Express the current I_{tot} drawn from the source with both switches closed:

$$I_{\text{tot}} = \frac{\mathcal{E}}{R_{\text{eq}}}$$

Express the equivalent resistance when both switches are closed:

$$R_{\text{eq}} = \frac{(100\,\Omega)R}{R + 100\,\Omega} + 300\,\Omega$$

Substitute to obtain:

$$I_{\text{tot}} = \frac{1.5\,\text{V}}{\dfrac{(100\,\Omega)R}{R + 100\,\Omega} + 300\,\Omega}$$

Substitute in equation (1) to obtain:

$$I_{100} = \frac{R}{R + 100\,\Omega}\left(\frac{1.5\,\text{V}}{\dfrac{(100\,\Omega)R}{R + 100\,\Omega} + 300\,\Omega}\right) = \frac{(1.5\,\text{V})R}{(400\,\Omega)R + 30{,}000\,\Omega^2} = 3.33\,\text{mA}$$

Solve for and evaluate R:

$$R = \boxed{600\,\Omega}$$

Remarks: Note that we can also obtain the result in the third step by applying Kirchhoff's loop rule to the parallel branch of the circuit.

***97 ••**

Picture the Problem Let I_1 be the current delivered by the left battery, I_2 the current delivered by the right battery, and I_3 the current through the 6-Ω resistor, directed down. We can apply Kirchhoff's rules to obtain three equations that we can solve simultaneously for I_1, I_2, and I_3. Knowing the currents in each branch, we can use Ohm's law to find the potential difference between points *a* and *b* and the power delivered by both the sources.

(*a*) Apply Kirchhoff's junction rule at junction *a*:

$$I_1 + I_2 = I_3$$

Apply Kirchhoff's loop rule to a loop around the outside of the circuit to obtain:

$$12\,\text{V} - (4\,\Omega)I_1 + (3\,\Omega)I_2 - 12\,\text{V} = 0$$

or

$$-(4\,\Omega)I_1 + (3\,\Omega)I_2 = 0$$

Apply Kirchhoff's loop rule to a loop around the left-hand branch of the circuit to obtain:

$$12\,\text{V} - (4\,\Omega)I_1 - (6\,\Omega)I_3 = 0$$

Solve these equations simultaneously to obtain:

$$I_1 = \boxed{0.667\,\text{A}},$$

$$I_2 = \boxed{0.889\,\text{A}},$$

and

$$I_3 = \boxed{1.56\,\text{A}}$$

(*b*) Apply Ohm's law to find the potential difference between points *a* and *b*:

$$V_{ab} = (6\,\Omega)I_3 = (6\,\Omega)(1.56\,\text{A}) = \boxed{9.36\,\text{V}}$$

(*c*) Express the power delivered by the 12-V battery in the left-hand branch of the circuit:

$$P_{\text{left}} = \mathcal{E}I_1 = (12\,\text{V})(0.667\,\text{A}) = \boxed{8.00\,\text{W}}$$

Express the power delivered by the 12-V battery in the right-hand branch of the circuit:

$$P_{\text{right}} = \mathcal{E}I_2 = (12\,\text{V})(0.889\,\text{A}) = \boxed{10.7\,\text{W}}$$

98 ••

Picture the Problem Let I_1 be the current delivered by the 7-V battery, I_2 the current delivered by the 5-V battery, and I_3, directed up, the current through the 1-Ω resistor. We can apply Kirchhoff's rules to obtain three equations that we can solve simultaneously for I_1, I_2, and I_3. Knowing the currents in each branch, we can use Ohm's law to find the potential difference between points *a* and *b* and the power delivered by both the sources.

(*a*) Apply Kirchhoff's junction rule at junction *a*:

$$I_1 = I_2 + I_3$$

Apply Kirchhoff's loop rule to a loop around the outside of the circuit to obtain:

$$7\,\text{V} - (2\,\Omega)I_1 - (1\,\Omega)I_3 = 0$$

Apply Kirchhoff's loop rule to a loop around the left-hand branch of the circuit to obtain:

$$7\,\text{V} - (2\,\Omega)I_1 - (3\,\Omega)I_2 + 5\,\text{V} = 0$$

or

$$(2\,\Omega)I_1 + (3\,\Omega)I_2 = 12\,\text{V}$$

Solve these equations simultaneously to obtain:

$$I_1 = \boxed{3.00\,\text{A}}\,,$$

$$I_2 = \boxed{2.00\,\text{A}}\,,$$

and

$$I_3 = \boxed{1.00\,\text{A}}$$

(*b*) Apply Ohm's law to find the potential difference between points *a* and *b*:

$$V_{ab} = -5\,\text{V} + (3\,\Omega)I_2 = -5\,\text{V} + (3\,\Omega)(2\,\text{A}) = \boxed{1.00\,\text{V}}$$

(*c*) Express the power delivered by the 7-V battery:

$$P_7 = \mathcal{E}I_1 = (7\,\text{V})(3\,\text{A}) = \boxed{21.0\,\text{W}}$$

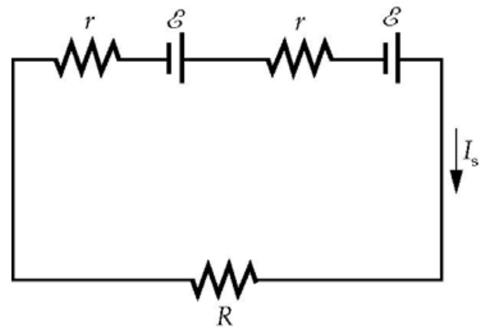
(*c*) Express the power delivered by the 5-V battery:

$$P_5 = \mathcal{E}I_2 = (5\,\text{V})(2\,\text{A}) = \boxed{10.0\,\text{W}}$$

99 ••

Picture the Problem We can apply Kirchhoff's rules to the two circuits to determine the current, and hence the power, supplied to the load resistor *R* for the two connections of the batteries. Differentiation, with respect to the load resistor, of the expressions for the power delivered to the load resistor will allow us to identify the conditions under which the power delivered is a maximum and to decide whether the power supplied to R greater when $R < r$ or when $R > r$.

The series connection of the batteries is shown to the right:



Express the power supplied to R: $P_s = I_s^2 R$

Apply Kirchhoff's loop rule to obtain: $-rI_s + \mathcal{E} - rI_s + \mathcal{E} - RI_s = 0$

Solve for I_s to obtain: $I_s = \dfrac{2\mathcal{E}}{2r + R}$

Substitute to obtain:

$$P_s = \left(\frac{2\mathcal{E}}{2r + R}\right)^2 R = \frac{4\mathcal{E}^2 R}{(2r + R)^2} \qquad (1)$$

Set the derivative, with respect to R, of equation (1) equal to zero for extrema:

$$\frac{dP_s}{dR} = \frac{d}{dR}\left[\frac{4\mathcal{E}^2 R}{(2r + R)^2}\right]$$

$$= \frac{(2r + R)^2 4\varepsilon^2 - 4\varepsilon^2 R(2)(2r + R)}{(2r + R)^4}$$

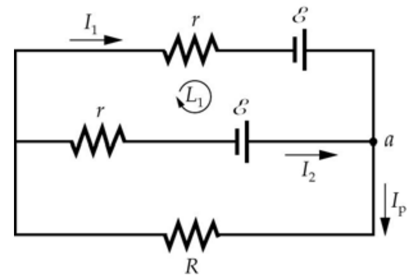
$= 0$ for extrema.

Solve for R to obtain: $R = 2r$

Examination of the second derivative of P_s at $R = 2r$ shows that $R = 2r$ corresponds to a maximum value of P_p and hence, for the series combination,

the power delivered to the load is greater if $R > r$ and is greatest when $R = 2r$.

The parallel connection of the batteries is shown to the right:



Express the power supplied to R:

$$P_p = I_p^2 R$$

Apply Kirchhoff's junction rule to point a to obtain:

$$I_p = I_1 + I_2$$

Apply Kirchhoff's loop rule to loop 1 to obtain:

$$-rI_1 + \mathcal{E} - \mathcal{E} + rI_2 = 0$$

or

$$I_1 = I_2 = \tfrac{1}{2} I_p$$

Apply Kirchhoff's loop rule to the outer loop to obtain:

$$\mathcal{E} - RI_p - rI_1 = 0$$

or

$$\mathcal{E} - RI_p - \tfrac{1}{2} rI_p = 0$$

Solve for I_p to obtain:

$$I_p = \frac{\mathcal{E}}{\frac{1}{2}r + R}$$

Substitute to obtain:

$$P_p = \left(\frac{\mathcal{E}}{\frac{1}{2}r + R}\right)^2 R = \frac{\mathcal{E}^2 R}{\left(\frac{1}{2}r + R\right)^2} \qquad (2)$$

Evaluate equation (1) when $r = R$:

$$P_s(r = R) = \frac{4\mathcal{E}^2 R}{(2R + R)^2} = \frac{4}{9}\frac{\mathcal{E}^2}{R}$$

Evaluate equation (2) when $r = R$:

$$P_p(r = R) = \frac{\mathcal{E}^2 R}{\left(\frac{1}{2}R + R\right)^2} = \frac{4}{9}\frac{\mathcal{E}^2}{R}$$

Thus, we see that if $r = R$, both arrangements provide the same power to the load.

Set the derivative, with respect to R, of equation (2) equal to zero for extrema:

$$\frac{dP_p}{dR} = \frac{d}{dR}\left[\frac{\varepsilon^2 R}{\left(\frac{1}{2}r + R\right)^2}\right] = \frac{\left(\frac{1}{2}r + R\right)^2 \varepsilon^2 - \varepsilon^2 R(2)\left(\frac{1}{2}r + R\right)}{\left(\frac{1}{2}r + R\right)^4} = 0 \text{ for extrema.}$$

Solve for R to obtain:

$$R = \tfrac{1}{2}r$$

Examination of the second derivative of P_p at $R = \frac{1}{2}r$ shows that $R = \frac{1}{2}r$ corresponds to a maximum value of P_p and hence, for the parallel combination, the power delivered to the load is greater if $R < r$ and a maximum when $R = \frac{1}{2}r$.

***100 ••**

Picture the Problem Let the current drawn from the source be I. We can use Ohm's law in conjunction with Kirchhoff's loop rule to express the output voltage as a function of V, R_1, and R_2. In (*b*) we can use the result of (*a*) to express the condition on the output voltages in terms of the effective resistance of the loaded output and the resistances R_1 and R_2.

(*a*) Use Ohm's law to express V_{out} in terms of R_2 and I:

$$V_{out} = IR_2$$

Apply Kirchhoff's loop rule to the circuit to obtain:

$$V - IR_1 - IR_2 = 0$$

Solve for I:

$$I = \frac{V}{R_1 + R_2}$$

Substitute for I in the expression for V_{out} to obtain:

$$V_{out} = \left(\frac{V}{R_1 + R_2}\right)R_2 = \boxed{V\left(\frac{R_2}{R_1 + R_2}\right)}$$

(*b*) Relate the effective resistance of the loaded circuit R_{eff} to R_2 and R_{load}:

$$\frac{1}{R_{eff}} = \frac{1}{R_2} + \frac{1}{R_{load}}$$

Solve for R_{load}:

$$R_{load} = \frac{R_2 R_{eff}}{R_2 - R_{eff}} \quad (1)$$

Letting V'_{out} represent the output voltage under load, express the condition that V_{out} drops by less than 10 percent of its unloaded

$$\frac{V_{out} - V'_{out}}{V_{out}} = 1 - \frac{V'_{out}}{V_{out}} < 0.1 \quad (2)$$

value:

Using the result from (*a*), express V'_{out} in terms of the effective output load R_{eff}:

$$V'_{out} = V\left(\frac{R_{eff}}{R_1 + R_{eff}}\right)$$

Substitute for V_{out} and V'_{out} in equation (2) and simplify to obtain:

$$1 - \frac{\dfrac{R_{eff}}{R_1 + R_{eff}}}{\dfrac{R_2}{R_1 + R_2}} < 0.1$$

or

$$1 - \frac{R_{eff}(R_1 + R_2)}{R_2(R_1 + R_{eff})} < 0.1$$

Solve for R_{eff}:

$$R_{eff} > \frac{0.9R_1R_2}{R_1 + 0.1R_2}$$

Substitute numerical values and evaluate R_{eff}:

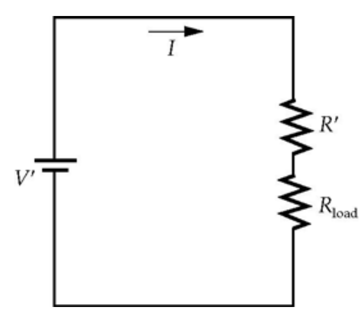
$$R_{eff} > \frac{0.9(10\,\text{k}\Omega)(10\,\text{k}\Omega)}{10\,\text{k}\Omega + 0.1(10\,\text{k}\Omega)} = 8.18\,\text{k}\Omega$$

Finally, substitute numerical values in equation (1) and evaluate R_{load}:

$$R_{load} < \frac{(10\,\text{k}\Omega)(8.18\,\text{k}\Omega)}{10\,\text{k}\Omega - 8.18\,\text{k}\Omega} = \boxed{44.9\,\text{k}\Omega}$$

101 ••

Picture the Problem In the equivalent Thevenin circuit shown to the right, R_2 is in parallel with R_{load}. We can apply Ohm's law to express V_{out} in terms of R_{eff} and I and then use Kirchhoff's loop rule to express I in terms of V, R_1, and R_{eff}. Simplification of the resulting equation will yield both of the indicated results.



(*a*) and (*b*) Use Ohm's law to express V_{out} in terms of R_{eff} and I:

$$V_{out} = IR_{eff}$$

Apply Kirchhoff's loop rule to the circuit to obtain:

$$V - IR_1 - IR_{eff} = 0$$

Solve for I:

$$I = \frac{V}{R_1 + R_{eff}}$$

Substitute for I in the expression for V_{out} to obtain:

$$V_{out} = \left(\frac{V}{R_1 + R_{eff}}\right)R_{eff} = V\left(\frac{R_{eff}}{R_1 + R_{eff}}\right)$$

Express the effective resistance R_{eff} in terms of R_{load} and R_2:

$$R_{eff} = \frac{R_2 R_{load}}{R_2 + R_{load}}$$

Substitute for R_{eff} in the expression for V_{out} to obtain:

$$V_{out} = V\left(\frac{\dfrac{R_2 R_{load}}{R_2 + R_{load}}}{R_1 + \dfrac{R_2 R_{load}}{R_2 + R_{load}}}\right)$$

Simplify to obtain:

$$V_{out} = V\frac{R_2 R_{load}}{R_1 R_{load} + R_2 R_{load} + R_1 R_2}$$

$$= \left(V\frac{R_2}{R_1 + R_2}\right)\left(\frac{R_{load}}{R_{load} + \dfrac{R_1 R_2}{R_1 + R_2}}\right)$$

$$= \left(V\frac{R_2}{R_1 + R_2}\right)\left(\frac{R_{load}}{R_{load} + R'}\right)$$

$$= V'\frac{R_{load}}{R_{load} + R'}$$

where

$$R' = \boxed{\frac{R_1 R_2}{R_1 + R_2}} \text{ and } V' = \boxed{V\frac{R_2}{R_1 + R_2}}$$

102 ••

Picture the Problem Let I_1 be the current in the 1-Ω resistor, directed to the right; let I_2 be the current, directed up, in the middle branch; and let I_3 be the current in the 6-Ω resistor, directed down. We can apply Kirchhoff's rules to find these currents, the power supplied by each source, and the power dissipated in each resistor.

(*a*) Apply Kirchhoff's junction rule at the top junction to obtain:

$$I_1 + I_2 = I_3 \qquad (1)$$

Apply Kirchhoff's loop rule to the outside loop of the circuit to obtain:

$$8\,\text{V} - (1\,\Omega)I_1 + 4\,\text{V} - (2\,\Omega)I_1 - (6\,\Omega)I_3 = 0$$

or

$$(3\,\Omega)I_1 + (6\,\Omega)I_3 = 12\,\text{V} \qquad (2)$$

Apply the loop rule to the inside loop at the left-hand side of the circuit to obtain:

$$8\,\text{V} - (1\,\Omega)I_1 + 4\,\text{V} - (2\,\Omega)I_1 + (2\,\Omega)I_2 - 4\,\text{V} = 0$$

or

$$8\,\text{V} - (3\,\Omega)I_1 + (2\,\Omega)I_2 = 0 \qquad (3)$$

Solve equations (1), (2), and (3) simultaneously to obtain:

$$I_1 = \boxed{2.00\text{ A}},$$

$$I_2 = \boxed{-1.00\text{ A}},$$

and

$$I_3 = \boxed{1.00\text{ A}}$$

where the minus sign indicates that the current flows downward rather than upward as we had assumed.

(*b*) Express the power delivered by the 8-V source:

$$P_8 = \mathcal{E}_8 I_1 = (8\text{ V})(2\text{ A}) = \boxed{16.0\text{ W}}$$

Express the power delivered by the 4-V source:

$$P_4 = \mathcal{E}_4 I_2 = (4\text{ V})(-1\text{ A}) = \boxed{-4.00\text{ W}}$$

where the minus sign indicates that this source is having current forced through it.

(*c*) Express the power dissipated in the 1-Ω resistor:

$$P_{1\Omega} = I_1^2 R_{1\Omega} = (2\text{ A})^2(1\,\Omega) = \boxed{4.00\text{ W}}$$

Express the power dissipated in the 2-Ω resistor in the left branch:

$$P_{2\Omega,\text{left}} = I_1^2 R_{2\Omega} = (2\text{ A})^2(2\,\Omega) = \boxed{8.00\text{ W}}$$

Express the power dissipated in the 2-Ω resistor in the middle branch:

$$P_{2\Omega,\text{middle}} = I_2^2 R_{2\Omega} = (1\text{ A})^2(2\,\Omega) = \boxed{2.00\text{ W}}$$

Express the power dissipated in the 6-Ω resistor:

$$P_{6\Omega} = I_3^2 R_{6\Omega} = (1\text{ A})^2(6\,\Omega) = \boxed{6.00\text{ W}}$$

103 ••

Picture the Problem Let I_1 be the current in the left branch resistor, directed up; let I_3 be the current, directed down, in the middle branch; and let I_2 be the current in the right branch, directed up. We can apply Kirchhoff's rules to find I_3 and then the potential difference between points *a* and *b*.

Relate the potential at *a* to the potential at *b*:

$$V_a - R_4 I_3 - 4\text{ V} = V_b$$

or

$$V_a - V_b = R_4 I_3 + 4\,\text{V}$$

Apply Kirchhoff's junction rule at *a* to obtain:

$$I_1 + I_2 = I_3 \qquad (1)$$

Apply the loop rule to a loop around the outside of the circuit to obtain:

$$2\,\text{V} - (1\,\Omega)I_1 + (1\,\Omega)I_2 - 2\,\text{V} + (1\,\Omega)I_2 - (1\,\Omega)I_1 = 0$$

or

$$I_1 - I_2 = 0 \qquad (2)$$

Apply the loop rule to the left side of the circuit to obtain:

$$2\,\text{V} - (1\,\Omega)I_1 - (4\,\Omega)I_3 - 4\,\text{V} - (1\,\Omega)I_1 = 0$$

or

$$-(1\,\Omega)I_1 - (2\,\Omega)I_3 = 1\,\text{V} \qquad (3)$$

Solve equations (1), (2), and (3) simultaneously to obtain:

$I_1 = -0.200\,\text{A}$,
$I_2 = -0.200\,\text{A}$,
and
$I_3 = -0.400\,\text{A}$
where the minus signs indicate that all the directions we chose for the currents were wrong.

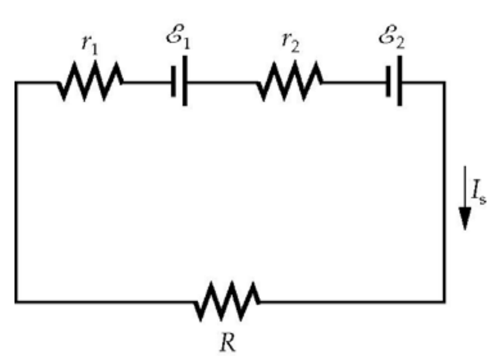
Substitute to obtain:

$$V_a - V_b = (4\,\Omega)(-0.4\,\text{A}) + 4\,\text{V} = \boxed{2.40\,\text{V}}$$

Remarks: Note that point *a* is at the higher potential.

104 ••

Picture the Problem Let ε_1 be the emf of the 9-V battery and r_1 its internal resistance of 0.8 Ω, and ε_2 be the emf of the 3-V battery and r_2 its internal resistance of 0.4 Ω. The series connection is shown to the right. We can apply Kirchhoff's rules to both connections to find the currents I_s and I_p delivered to the load resistor in the series and parallel connections.



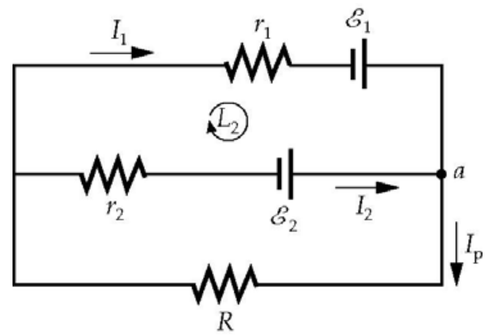
(*a*) Apply Kirchhoff's loop rule to the batteries connected in series:

$$\mathcal{E}_1 - r_2 I_s + \mathcal{E}_2 - R I_s - r_1 I_s = 0$$

Solve for I_s to obtain:

$$I_s = \frac{\mathcal{E}_1 + \mathcal{E}_2}{r_1 + r_2 + R} = \boxed{\frac{12\text{ V}}{1.2\,\Omega + R}}$$

Suppose the two batteries are connected in parallel and their terminals are then connected to *R*. Let I_1 be the current delivered by $\mathcal{E}_1$, I_2 be the current delivered by $\mathcal{E}_2$, and I_p the current through the load resistor *R* in the parallel connection.



Apply the junction rule at *a* to obtain:

$$I_1 + I_2 = I_p \qquad (1)$$

Apply the loop rule to a loop around the outside of the circuit:

$$\mathcal{E}_1 - R I_p - r_1 I_1 = 0$$

or

$$9\text{ V} - R I_p - (0.8\,\Omega) I_1 = 0 \qquad (2)$$

Apply the loop rule to loop 2 to obtain:

$$\mathcal{E}_1 - \mathcal{E}_2 + r_2 I_2 - r_1 I_1 = 0$$

or

$$9\text{ V} - 3\text{ V} + (0.4\,\Omega) I_2 - (0.8\,\Omega) I_1 = 0$$

or

$$6\text{ V} + (0.4\,\Omega) I_2 - (0.8\,\Omega) I_1 = 0 \qquad (3)$$

Eliminate I_2 between equations (1) and (3) to obtain:

$$I_1 = 5\text{ A} + \tfrac{1}{3} I_p \qquad (4)$$

Substitute equation (4) in equation (2) and solve for I_p to obtain:

$$I_p = \boxed{\frac{7.5\text{ V}}{1.5R + 0.4\,\Omega}}$$

(*b*) Evaluate I_s and I_p for $R = 0.2\ \Omega$:

$$I_s(R = 0.2\,\Omega) = \frac{12\text{ V}}{1.2\,\Omega + 0.2\,\Omega} = \boxed{8.57\text{ A}}$$

and

$$I_p(R = 0.2\,\Omega) = \frac{7.5\text{ V}}{1.5(0.2\,\Omega) + 0.4\,\Omega} = \boxed{10.7\text{ A}}$$

(*c*), (*d*), and (*e*) Proceed as in (*b*) to complete the table to the right:

	R	I_s	I_p
	(Ω)	(A)	(A)
(*c*)	0.6	6.67	5.77
(*d*)	1.0	5.45	3.95
(*e*)	1.5	4.44	2.83

Note that for $R = 0.4\,\Omega$, $I_s = I_p = 7.5$ A. When $R > 0.4\,\Omega$, the series connection gives the larger current through R.

Ammeters and Voltmeters

***105 ••**

Picture the Problem Let I be the current drawn from source and R_{eq} the resistance equivalent to R and 10 MΩ connected in parallel and apply Kirchhoff's loop rule to express the measured voltage V across R as a function of R.

The voltage measured by the voltmeter is given by:

$$V = IR_{eq} \qquad (1)$$

Apply Kirchhoff's loop rule to the circuit to obtain:

$$10\text{ V} - IR_{eq} - I(2R) = 0$$

Solve for I:

$$I = \frac{10\text{ V}}{R_{eq} + 2R}$$

Express R_{eq} in terms of R and 10-MΩ resistance in parallel with it:

$$\frac{1}{R_{eq}} = \frac{1}{10\,\text{M}\Omega} + \frac{1}{R}$$

Solve for R_{eq}:

$$R_{eq} = \frac{(10\,\text{M}\Omega)R}{R + 10\,\text{M}\Omega}$$

Substitute for I in equation (1) and simplify to obtain:

$$V = \left(\frac{10\text{ V}}{R_{eq} + 2R}\right)R_{eq} = \frac{10\text{ V}}{1 + \frac{2R}{R_{eq}}}$$

Substitute for R_{eq} and simplify to obtain:

$$V = \frac{(10\text{ V})(5\,\text{M}\Omega)}{R + 15\,\text{M}\Omega} \qquad (2)$$

(*a*) Evaluate equation (2) for R = 1 kΩ:

$$V=\frac{(10\,\text{V})(5\,\text{M}\Omega)}{1\,\text{k}\Omega+15\,\text{M}\Omega}=\boxed{3.33\,\text{V}}$$

(*b*) Evaluate equation (2) for R =10 kΩ:

$$V=\frac{(10\,\text{V})(5\,\text{M}\Omega)}{10\,\text{k}\Omega+15\,\text{M}\Omega}=\boxed{3.33\,\text{V}}$$

(*c*) Evaluate equation (2) for R =1 MΩ:

$$V=\frac{(10\,\text{V})(5\,\text{M}\Omega)}{1\,\text{M}\Omega+15\,\text{M}\Omega}=\boxed{3.13\,\text{V}}$$

(*d*) Evaluate equation (2) for R =10 MΩ:

$$V=\frac{(10\,\text{V})(5\,\text{M}\Omega)}{10\,\text{M}\Omega+15\,\text{M}\Omega}=\boxed{2.00\,\text{V}}$$

(*e*) Evaluate equation (2) for R =100 MΩ:

$$V=\frac{(10\,\text{V})(5\,\text{M}\Omega)}{100\,\text{M}\Omega+15\,\text{M}\Omega}=\boxed{0.435\,\text{V}}$$

(*f*) Express the condition that the measured voltage to be within 10 percent of the *true* voltage V_{true}:

$$\frac{V_{\text{true}}-V}{V_{\text{true}}}=1-\frac{V}{V_{\text{true}}}<0.1$$

Substitute for V and V_{true} to obtain:

$$1-\frac{\dfrac{(10\,\text{V})(5\,\text{M}\Omega)}{R+15\,\text{M}\Omega}}{IR}<0.1$$

or, because I = 10 V/3R,

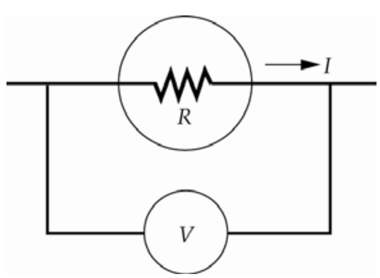
$$1-\frac{\dfrac{(10\,\text{V})(5\,\text{M}\Omega)}{R+15\,\text{M}\Omega}}{\dfrac{10}{3}\text{V}}<0.1$$

Solve for R to obtain:

$$R<\frac{1.5\,\text{M}\Omega}{0.9}=\boxed{1.67\,\text{M}\Omega}$$

106 ••

Picture the Problem The diagram shows a voltmeter connected in parallel with a galvanometer movement whose internal resistance is R. We can apply Kirchhoff's loop rule to express R in terms of I and V.



Apply Kirchhoff's loop rule to the loop that includes the galvanometer movement and the voltmeter:

$$V-IR=0$$

Solve for R:

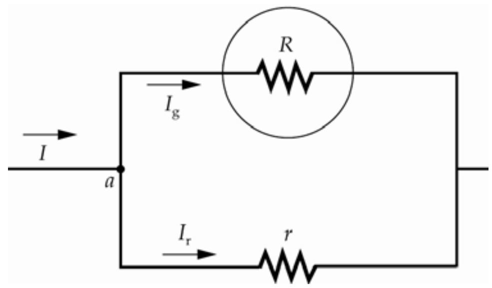
$$R=\frac{V}{I}$$

Substitute numerical values and evaluate R:

$$R = \frac{0.25\,\text{V}}{50\,\mu\text{A}} = \boxed{5.00\,\text{k}\Omega}$$

107 ••

Picture the Problem When there is a voltage drop of 0.25 V across this galvanometer, the meter reads full scale. The diagram shows the galvanometer movement with a resistor of resistance r in parallel. The purpose of this resistor is to limit the current through the movement to $I_g = 50\ \mu\text{A}$. We can apply Kirchhoff's loop rule to the circuit fragment containing the galvanometer movement and the shunt resistor to derive an expression for r.



Apply Kirchhoff's loop rule to the circuit fragment to obtain:

$$-RI_g + rI_r = 0$$

Apply Kirchhoff's junction rule at point a to obtain:

$$I_r = I - I_g$$

Substitute for I_r in the loop equation:

$$-RI_g + r(I - I_g) = 0$$

Solve for r:

$$r = \frac{RI_g}{I - I_g}$$

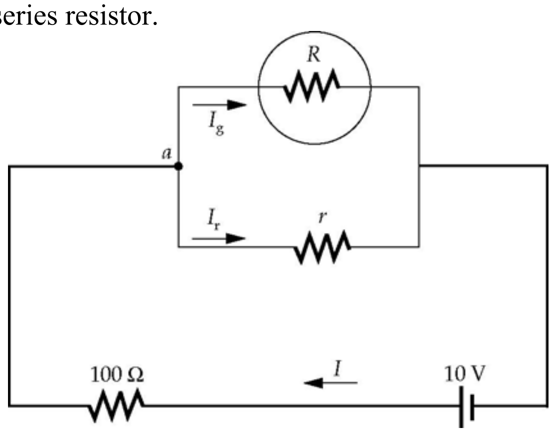
where $RI_g = 0.25$ V

Substitute numerical values and evaluate r:

$$r = \frac{0.25\,\text{V}}{100\,\text{mA} - 50\,\mu\text{A}} = \boxed{2.50\,\Omega}$$

108 ••

Picture the Problem The circuit diagram shows the ammeter connected in series with a 100-Ω resistor and a 10-V power supply. We can apply Kirchhoff's rules to obtain an expression for I as a function of r, I_g, the potential difference provided by the source, and the resistance of the series resistor.



(*a*) Apply Kirchhoff's loop rule to the inner loop of the circuit to obtain:

$$10\,\text{V}-(100\,\Omega)I-rI_{\text{r}}=0$$

Apply Kirchhoff's junction rule at point *a* to obtain:

$$I_{\text{r}}=I-I_{\text{g}}$$

Substitute for I_r in the loop equation:

$$10\,\text{V}-(100\,\Omega)I-r(I-I_{\text{g}})=0$$

Solve for *I*:

$$I=\frac{10\,\text{V}+rI_{\text{g}}}{100\,\Omega+r} \qquad (1)$$

In Problem 107 it was established that $r = 2.50\ \Omega$. Substitute numerical values and evaluate *I*:

$$I=\frac{10\,\text{V}+(2.50\,\Omega)(50\,\mu\text{A})}{100\,\Omega+2.50\,\Omega}=\boxed{97.6\,\text{mA}}$$

(*b*) Under these conditions, equation (1) becomes:

$$I=\frac{1\,\text{V}+rI_{\text{g}}}{10\,\Omega+r}$$

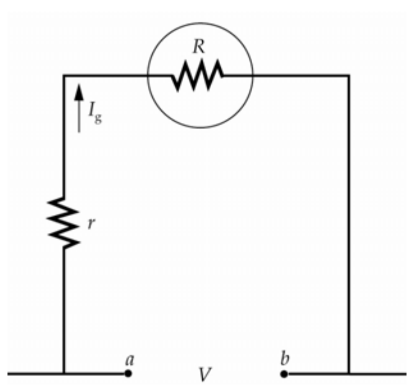
Substitute numerical values and evaluate *I*:

$$I=\frac{1\,\text{V}+(2.50\,\Omega)(50\,\mu\text{A})}{10\,\Omega+2.50\,\Omega}=\boxed{80.0\,\text{mA}}$$

Remarks: Our result in (*b*) differs from that obtained in (*a*) by about 18 percent.

***109 ••**

Picture the Problem The circuit diagram shows a fragment of a circuit in which a resistor of resistance *r* is connected in series with the meter movement of Problem 106. The purpose of this resistor is to limit the current through the galvanometer movement to 50 μA and to produce a deflection of the galvanometer movement that is a measure of the potential difference *V*. We can apply Kirchhoff's loop rule to express *r* in terms of V_g, I_g, and *R*.



Apply Kirchhoff's loop rule to the circuit fragment to obtain:

$$V-rI_{\text{g}}-RI_{\text{g}}=0$$

Solve for *r*:

$$r=\frac{V-RI_{\text{g}}}{I_{\text{g}}}=\frac{V}{I_{\text{g}}}-R \qquad (1)$$

Use Ohm's law to relate the current I_g through the galvanometer movement to the potential difference V_g across it:

$$I_g = \frac{V_g}{R} \Rightarrow R = \frac{V_g}{I_g}$$

Use the values for V_g and I_g given in Problem 106 to evaluate R:

$$R = \frac{0.25\,\text{V}}{50\,\mu\text{A}} = 5000\,\Omega$$

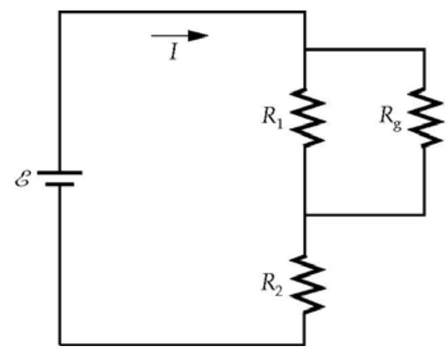
Substitute numerical values in equation (1) and evaluate r:

$$r = \frac{10\,\text{V}}{50\,\mu\text{A}} - 5000\,\Omega = \boxed{195\,\text{k}\Omega}$$

Remarks: The total series resistance is the sum of *r* and *R* or 200 kΩ.

110 ••

Picture the Problem The voltmeter shown in Figure 25-64 is equivalent to a resistor of resistance $R_g = 200$ kΩ as shown in the circuit diagram to the right. The voltage reading across R_1 is given by $V_1 = IR_1$. We can use Kirchhoff's loop rule and the expression for the equivalent resistance of two resistors in parallel to find I.



The voltage reading across R_1 is given by:

$$V_1 = IR_1$$

Apply Kirchhoff's loop rule to the loop including the source, R_1, and R_2:

$$\mathcal{E} - IR_{eq} - IR_2 = 0$$

Solve for I to obtain:

$$I = \frac{\mathcal{E}}{R_{eq} + R_2}$$

Substitute for I in the expression for V_1:

$$V_1 = \frac{\mathcal{E}R_1}{R_{eq} + R_2} \qquad (1)$$

Express R_{eq} in terms of R_1 and R_g:

$$R_{eq} = \frac{R_1 R_g}{R_1 + R_g}$$

Substitute numerical values and evaluate R_{eq}:

$$R_{eq} = \frac{(200\,\text{k}\Omega)(200\,\text{k}\Omega)}{200\,\text{k}\Omega + 200\,\text{k}\Omega} = 100\,\text{k}\Omega$$

Substitute numerical values in equation (1) and evaluate V_1:

$$V_1 = \frac{(10\,\text{V})(200\,\text{k}\Omega)}{200\,\text{k}\Omega + 200\,\text{k}\Omega} = \boxed{5.00\,\text{V}}$$

RC Circuits

111 •

Picture the Problem We can use the definition of capacitance to find the initial charge on the capacitor and Ohm's law to find the initial current in the circuit. We can find the time constant of the circuit using its definition and the charge on the capacitor after 6 ms using $Q(t) = Q_0 e^{-t/\tau}$.

(*a*) Use the definition of capacitance to find the initial charge on the capacitor:

$$Q_0 = CV_0 = (6\,\mu\text{F})(100\,\text{V}) = \boxed{600\,\mu\text{C}}$$

(*b*) Apply Ohm's law to the resistor to obtain:

$$I_0 = \frac{V_0}{R} = \frac{100\,\text{V}}{500\,\Omega} = \boxed{0.200\,\text{A}}$$

(*c*) Use its definition to find the time constant of the circuit:

$$\tau = RC = (500\,\Omega)(6\,\mu\text{F}) = \boxed{3.00\,\text{ms}}$$

(*d*) Express the charge on the capacitor as a function of time:

$$Q(t) = Q_0 e^{-t/\tau}$$

Substitute numerical values and evaluate Q(6 ms):

$$Q(6\,\text{ms}) = (600\,\mu\text{C})e^{-6\,\text{ms}/3\,ms} = \boxed{81.2\,\mu\text{C}}$$

112 •

Picture the Problem We can use $U_0 = \frac{1}{2}CV_0^2$ to find the initial energy stored in the capacitor and $U(t) = \frac{1}{2}C(V_\text{C}(t))^2$ with $V_\text{C}(t) = V_0 e^{-t/\tau}$ to show that $U = U_0 e^{-2t/\tau}$.

(*a*) The initial energy stored in the capacitor is given by:

$$U_0 = \frac{1}{2}CV_0^2$$
$$= \frac{1}{2}(6\,\mu\text{F})(100\,\text{V})^2 = \boxed{30.0\,\text{mJ}}$$

(*b*) Express the energy stored in the discharging capacitor as a function of time:

$$U(t) = \frac{1}{2}C(V_\text{C}(t))^2$$

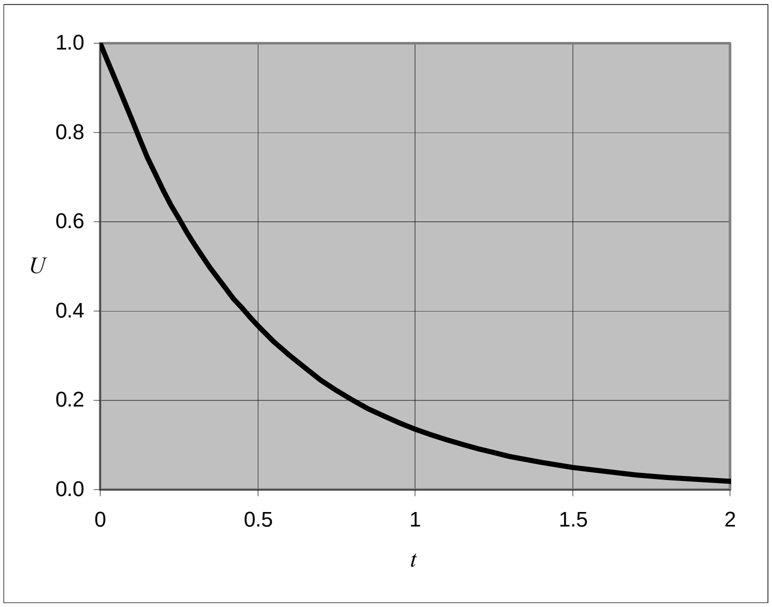
where

$$V_\text{C}(t) = V_0 e^{-t/\tau}$$

Substitute to obtain:

$$U(t)=\tfrac{1}{2}C\left(V_0e^{-t/\tau}\right)^2$$
$$=\tfrac{1}{2}CV_0^2e^{-2t/\tau}=\boxed{U_0e^{-2t/\tau}}$$

(*c*) A graph of *U* versus *t* is shown below. *U* is in units of U_0 and *t* is in units of τ.



***113 ••**

Picture the Problem We can find the resistance of the circuit from its time constant and use Ohm's law and the expression for the current in a charging *RC* circuit to express τ as a function of time, V_0, and $V(t)$.

Express the resistance of the resistor in terms of the time constant of the circuit:

$$R=\frac{\tau}{C} \qquad (1)$$

Using Ohm's law, express the voltage drop across the resistor as a function of time:

$$V(t)=I(t)R$$

Express the current in the circuit as a function of the elapsed time after the switch is closed:

$$I(t)=I_0e^{-t/\tau}$$

Substitute to obtain:

$$V(t)=I_0e^{-t/\tau}R=(I_0R)e^{-t/\tau}=V_0e^{-t/\tau}$$

Take the natural logarithm of both sides of the equation and solve for τ to obtain:

$$\tau = -\frac{t}{\ln\left[\frac{V(t)}{V_0}\right]}$$

Substitute in equation (1) to obtain:

$$R = -\frac{t}{C\ln\left[\frac{V(t)}{V_0}\right]}$$

Substitute numerical values and evaluate R using the data given for t = 4 s:

$$R = -\frac{4\,\text{s}}{(2\,\mu\text{F})\ln\left(\frac{20\,\text{V}}{50\,\text{V}}\right)} = \boxed{2.18\,\text{M}\Omega}$$

***114 ••**

Picture the Problem We can find the resistance of the circuit from its time constant and use the expression for the charge on a discharging capacitor as a function of time to express τ as a function of time, Q_0, and $Q(t)$.

Express the effective resistance across the capacitor in terms of the time constant of the circuit:

$$R = \frac{\tau}{C} \qquad (1)$$

Express the charge on the capacitor as a function of the elapsed time after the switch is closed:

$$Q(t) = Q_0 e^{-t/\tau}$$

Take the natural logarithm of both sides of the equation and solve for τ to obtain:

$$\tau = -\frac{t}{\ln\frac{Q(t)}{Q_0}}$$

Substitute in equation (1) to obtain:

$$R = -\frac{t}{C\ln\frac{Q(t)}{Q_0}}$$

Substitute numerical values and evaluate R:

$$R = -\frac{4\,\text{s}}{(0.12\,\mu\text{F})\ln\frac{\frac{1}{2}Q_0}{Q_0}} = \boxed{48.1\,\text{M}\Omega}$$

115 ••

Picture the Problem We can use the definition of capacitance to find the final charge on the capacitor and $Q(t) = Q_f\left(1 - e^{-t/\tau}\right)$ to express the charge on the capacitor as a function of time. In part (b) we can let $Q(t) = 0.99Q_f$ and solve for t to find the time required for the capacitor to reach 99% of its final charge.

(*a*) After a very long time has elapsed, the capacitor will be fully charged. Use the definition of capacitance to find its charge:

$$Q_f = CV = (1.6\,\mu\text{F})(5\,\text{V}) = \boxed{8.00\,\mu\text{C}}$$

(*b*) Express the charge on the capacitor as a function of time:

$$Q(t) = Q_f\left(1 - e^{-t/\tau}\right)$$

where $\tau = RC$.

When $Q = 0.99Q_f$:

$$0.99Q_f = Q_f\left(1 - e^{-t/\tau}\right)$$

or

$$0.01 = e^{-t/\tau}$$

Take the natural logarithm of both sides of the equation and solve for *t* to obtain:

$$t = -RC\ln(0.01)$$

Substitute numerical values and evaluate *t*:

$$t = -(10\,\text{k}\Omega)(1.6\,\mu\text{F})\ln(0.01) = \boxed{73.7\,\text{ms}}$$

116 ••

Picture the Problem We can use Kirchhoff's loop rule (conservation of energy) to find both the initial and steady-state currents drawn from the battery and Ohm's law to find the maximum voltage across the capacitor.

(*a*) Apply Kirchhoff's loop rule to a loop around the outside of the circuit to obtain:

$$\mathcal{E} - (1.2\,\text{M}\Omega)I_0 - V_{C0} = 0$$

Because the capacitor initially is uncharged:

$$V_{C0} = 0$$

and

$$I_0 = \frac{\mathcal{E}}{1.2\,\text{M}\Omega} = \frac{120\,\text{V}}{1.2\,\text{M}\Omega} = \boxed{0.100\,\text{mA}}$$

(*b*) When a long time has passed:

$$I_{C\infty} = 0$$

Apply Kirchhoff's loop rule to a loop that includes the source and both resistors to obtain:

$$\mathcal{E} - (1.2\,\text{M}\Omega)I_\infty - (600\,\text{k}\Omega)I_\infty = 0$$

Solve for and evaluate I_∞:

$$I_\infty = \frac{\mathcal{E}}{1.2\,\text{M}\Omega + 600\,\text{k}\Omega} = \frac{120\,\text{V}}{1.2\,\text{M}\Omega + 600\,\text{k}\Omega} = \boxed{66.7\,\mu\text{A}}$$

(*c*) The maximum voltage across the capacitor equals the potential difference across the 600-kΩ under steady-state conditions. Apply Ohm's law to obtain:

$$V_{C\infty} = I_\infty R_{600\,\text{k}\Omega} = (66.7\,\mu\text{A})(600\,\text{k}\Omega) = \boxed{40.0\,\text{V}}$$

117 ••

Picture the Problem We can use $Q(t) = Q_f\left(1 - e^{-t/\tau}\right) = C\mathcal{E}\left(1 - e^{-t/\tau}\right)$ to find the charge on the capacitor at $t = \tau$ and differentiate this expression with respect to time to find the rate at which the charge is increasing (the current). The power supplied by the battery is given by $P_\tau = I_\tau \mathcal{E}$ and the power dissipated in the resistor by $P_{R,\tau} = I_\tau^2 R$. In part (f) we can differentiate $U(t) = Q^2(t)/2C$ with respect to time and evaluate the derivative at $t = \tau$ to find the rate at which the energy stored in the capacitor is increasing.

(*a*) Express the charge Q on the capacitor as a function of time:

$$Q(t) = Q_f\left(1 - e^{-t/\tau}\right) = C\mathcal{E}\left(1 - e^{-t/\tau}\right) \quad (1)$$

where $\tau = RC$.

Evaluate $Q(\tau)$ to obtain:

$$Q(\tau) = (1.5\,\mu\text{F})(6\,\text{V})\left(1 - e^{-1}\right) = \boxed{5.69\,\mu\text{C}}$$

(*b*) and (*c*) Differentiate equation (1) with respect to t to obtain:

$$\frac{dQ(t)}{dt} = I = I_0 e^{-t/\tau}$$

Apply Kirchhoff's loop rule to the circuit just after the circuit is completed to obtain:

$$\mathcal{E} - RI_0 - V_{C0} = 0$$

Because $V_{C0} = 0$ we have:

$$I_0 = \frac{\mathcal{E}}{R}$$

Substitute to obtain:

$$\frac{dQ(t)}{dt} = I(t) = \frac{\mathcal{E}}{R} e^{-t/\tau}$$

Substitute numerical values and evaluate $I(\tau)$:

$$I(\tau) = \frac{6\,\text{V}}{2\,\text{M}\Omega} e^{-1} = \boxed{1.10\,\mu\text{A}}$$

(*d*) Express the power supplied by the battery as the product of its emf and the current drawn from it at $t = \tau$.

$$P(\tau) = I(\tau)\mathcal{E} = (1.10\,\mu\text{C/s})(6\,\text{V}) = \boxed{6.60\,\mu\text{W}}$$

(*e*) The power dissipated in the resistor is given by:

$$P_R(\tau) = I^2(\tau)R = (1.10\,\mu\text{A})^2(2\,\text{M}\Omega) = \boxed{2.42\,\mu\text{W}}$$

(*f*) Express the energy stored in the capacitor as a function of time:

$$U(t) = \frac{Q^2(t)}{2C}$$

Differentiate this expression with respect to time to obtain:

$$\frac{dU(t)}{dt} = \frac{1}{2C}\frac{d}{dt}\left[Q^2(t)\right] = \frac{1}{2C}(2Q(t))\frac{dQ(t)}{dt} = \frac{Q(t)}{C}I(t)$$

Evaluate this expression when $t = \tau$ to obtain:

$$\frac{dU(\tau)}{dt} = \frac{Q(\tau)}{C}I(\tau) = \frac{5.69\,\mu\text{C}}{1.5\,\mu\text{F}}(1.10\,\mu\text{A}) = \boxed{4.17\,\mu\text{W}}$$

Remarks: Note that our answer for part (*f*) is the difference between the power delivered by the battery at $t = \tau$ and the rate at which energy is dissipated in the resistor at the same time.

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Picture the Problem We can apply Kirchhoff's junction rule to find the current in each branch of this circuit and then use the loop rule to obtain equations solvable for R_1, R_2, and R_3.

(*a*) Apply Kirchhoff's junction rule at the junction of the 5-μF capacitor and the 10-Ω and 50-Ω resistors under steady-state conditions:

$$I_{\text{bat}} = I_{10\,\Omega} + 5\,\text{A} \qquad (1)$$

Because the potential differences across the 5-μF capacitor and the 10-Ω resistor are the same:

$$I_{10\,\Omega} = \frac{V_{10\,\Omega}}{10\,\Omega} = \frac{V_\text{C}}{10\,\Omega}$$

Express the potential difference across the capacitor to its steady-state charge:

$$V_C = \frac{Q_f}{C}$$

Substitute to obtain:

$$I_{10\,\Omega} = \frac{Q_f}{(10\,\Omega)C}$$

Substitute in equation (1) to obtain:

$$I_{bat} = \frac{Q_f}{(10\,\Omega)C} + 5\,\text{A}$$

Substitute numerical values and evaluate I_{bat}:

$$I_{bat} = \frac{1000\,\mu\text{C}}{(10\,\Omega)(5\,\mu\text{F})} + 5\,\text{A} = \boxed{25.0\,\text{A}}$$

(*b*) Use Kirchhoff's junction rule to find the currents $I_{5\,\Omega}$, I_{R3}, and I_{R1}:

$$I_{5\Omega} = 10\,\text{A}\,,$$

$$I_{R3} = 15\,\text{A}\,,$$

and

$$I_{R1} = I_{bat} = 25\,\text{A}$$

Apply the loop rule to the loop that includes the battery, R_1, and the 50-Ω and 5-Ω resistors:

$$310\,\text{V} - (25\,\text{A})R_1 - (5\,\text{A})(50\,\Omega) - (10\,\text{A})(5\,\Omega) = 0$$

Solve for R_1 to obtain:

$$R_1 = \boxed{0.400\,\Omega}$$

Apply the loop rule to the loop that includes the battery, R_1, the 10-Ω resistor and R_3:

$$310\,\text{V} - (25\,\text{A})(0.4\,\Omega) - (20\,\text{A})(10\,\Omega) - (15\,\text{A})R_3 = 0$$

Solve for R_3 to obtain:

$$R_3 = \boxed{6.67\,\Omega}$$

Apply the loop rule to the loop that includes the 10-Ω and 50-Ω resistors and R_2:

$$-(20\,\text{A})(10\,\Omega) - (5\,\text{A})R_2 + (5\,\text{A})(50\,\Omega) = 0$$

Solve for R_2 to obtain:

$$R_2 = \boxed{10.0\,\Omega}$$

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Picture the Problem We can solve Equation 25-35 for *dQ*/*dt* and separate the variables in order to obtain the equation given above. Integrating this differential equation will yield Equation 25-36.

Solve Equation 25-35 for dQ/dt to obtain:

$$\frac{dQ}{dt}=\frac{\mathcal{E}C-Q}{RC}$$

Separate the variables to obtain:

$$\boxed{\frac{dQ}{\mathcal{E}C-Q}=\frac{dt}{RC}}$$

Integrate dQ' from 0 to Q and dt' from 0 to t:

$$\int_0^Q\frac{dQ'}{\mathcal{E}C-Q'}=\frac{1}{RC}\int_0^t dt'$$

and

$$\ln\left(\frac{\mathcal{E}C}{\mathcal{E}C-Q}\right)=\frac{t}{RC}$$

Transform from logarithmic to exponential form to obtain:

$$\frac{\mathcal{E}C}{\mathcal{E}C-Q}=e^{\frac{t}{RC}}$$

Solve for Q to obtain Equation 25-36:

$$Q=\mathcal{E}C\left(1-e^{-t/RC}\right)=\boxed{Q_f\left(1-e^{-t/RC}\right)}$$

***120 •••**

Picture the Problem We can find the time-to-discharge by expressing the voltage across the capacitor as a function of time and solving for t. We can use $U(t)=\frac{1}{2}CV^2(t)$ to find the energy released/stored in the capacitor when the lamp flashes. In part (c) we can integrate $dU_{bat}=\mathcal{E}dI(t)$ to find the energy supplied by the battery during the charging cycle.

(*a*) Express the voltage across the capacitor as a function of time:

$$V(t)=\frac{Q(t)}{C}=\frac{Q_f}{C}\left(1-e^{-t/RC}\right)=V_f\left(1-e^{-t/RC}\right)$$

Solve for t to obtain:

$$t=-RC\ln\left(1-\frac{V(t)}{V_f}\right)$$

Substitute numerical values and evaluate t:

$$t=-(18\,\text{k}\Omega)(0.15\,\mu\text{F})\ln\left(1-\frac{7\,\text{V}}{9\,\text{V}}\right)=\boxed{4.06\,\text{ms}}$$

(*b*) Express the energy stored in the capacitor as a function of time:

$$U(t)=\tfrac{1}{2}CV^2(t)$$

Substitute for $V(t)$ to obtain:

$$U(t)=\tfrac{1}{2}CV_f^2\left(1-e^{-t/RC}\right)^2$$

Substitute numerical values and evaluate $U(4.06\text{ ms})$:

$$U(4.06\text{ ms})=\tfrac{1}{2}(0.15\,\mu\text{F})(9\text{ V})^2\left(1-e^{-4.06\text{ ms}/(18\text{ k}\Omega)(0.15\,\mu\text{F})}\right)^2=\boxed{3.67\,\mu\text{J}}$$

(c) Relate the energy provided by the battery to its emf and the current it delivers:

$$U_{\text{bat}}(t)=\mathcal{E}\int_0^t I(t')dt'=\frac{\mathcal{E}^2}{R}\int_0^t e^{-t'/RC}dt'$$
$$=\frac{\mathcal{E}^2}{R}\left[RC\left(1-e^{-t/RC}\right)\right]$$
$$=C\mathcal{E}^2\left(1-e^{-t/RC}\right)$$

Substitute numerical values and evaluate $U_{\text{bat}}(4.06\text{ ms})$:

$$U_{\text{bat}}(4.06\text{ ms})=(0.15\,\mu\text{F})(9\text{ V})^2\left(1-e^{-4.06\text{ ms}/(18\text{ k}\Omega)(0.15\,\mu\text{F})}\right)=\boxed{9.45\,\mu\text{J}}$$

Express the fraction f of the energy supplied by the battery during the charging cycle that is dissipated in the resistor:

$$f=\frac{U_R}{U_{\text{bat}}}$$

Use conservation of energy to relate the energy supplied by the battery to the energy dissipated in the resistor and the energy released when the lamp flashes:

$$U_{\text{bat}}=U_R+U_{\text{flash}}$$
or
$$U_R=U_{\text{bat}}-U_{\text{flash}}$$

Substitute to obtain:

$$f=\frac{U_{\text{bat}}-U_{\text{flash}}}{U_{\text{bat}}}=1-\frac{U_{\text{flash}}}{U_{\text{bat}}}$$

Substitute numerical values and evaluate f:

$$f=1-\frac{3.67\,\mu\text{J}}{9.45\,\mu\text{J}}=\boxed{61.2\%}$$

***121** •••

Picture the Problem Let R_1 = 200 Ω, R_2 = 600 Ω, I_1 and I_2 their currents, and I_3 the current into the capacitor. We can apply Kirchhoff's loop rule to find the initial battery current I_0 and the battery current I_∞ a long time after the switch is closed. In part (c) we can apply both the loop and junction rules to obtain equations that we can use to obtain a linear differential equation with constant coefficients describing the current in the 600-Ω resistor as a function of time. We can solve this differential equation by assuming a solution of a given form, differentiating this assumed solution and substituting it and its derivative in the differential equation. Equating coefficients, requiring the solution to hold for all values of the assumed constants, and invoking an initial condition will allow us to find the constants in the assumed solution.

(*a*) Apply Kirchhoff's loop rule to the circuit at the instant the switch is closed:

$$\mathcal{E}-(200\,\Omega)I_0-V_{C0}=0$$

Because the capacitor is initially uncharged:

$$V_{C0}=0$$

Solve for and evaluate I_0:

$$I_0=\frac{\mathcal{E}}{200\,\Omega}=\frac{50\,\text{V}}{200\,\Omega}=\boxed{0.250\,\text{A}}$$

(*b*) Apply Kirchhoff's loop rule to the circuit after a long time has passed:

$$50\,\text{V}-(200\,\Omega)I_\infty-(600\,\Omega)I_\infty=0$$

Solve for I_∞ to obtain:

$$I_\infty=\frac{50\,\text{V}}{800\,\Omega}=\boxed{62.5\,\text{mA}}$$

(*c*) Apply the junction rule at the junction between the 200-Ω resistor and the capacitor to obtain:

$$I_1=I_2+I_3 \qquad (1)$$

Apply the loop rule to the loop containing the source, the 200-Ω resistor and the capacitor to obtain:

$$\mathcal{E}-R_1I_1-\frac{Q}{C}=0 \qquad (2)$$

Apply the loop rule to the loop containing the 600-Ω resistor and the capacitor to obtain:

$$\frac{Q}{C}-R_2I_2=0 \qquad (3)$$

Differentiate equation (2) with respect to time to obtain:

$$\frac{d}{dt}\left[\mathcal{E}-R_1I_1-\frac{Q}{C}\right]=0-R_1\frac{dI_1}{dt}-\frac{1}{C}\frac{dQ}{dt}$$

$$=-R_1\frac{dI_1}{dt}-\frac{1}{C}I_3=0$$

or

$$R_1\frac{dI_1}{dt}=-\frac{1}{C}I_3 \qquad (4)$$

Differentiate equation (3) with respect to time to obtain:

$$\frac{d}{dt}\left[\frac{Q}{C}-R_2I_2\right]=\frac{1}{C}\frac{dQ}{dt}-R_2\frac{dI_2}{dt}=0$$

or

$$R_2\frac{dI_2}{dt}=\frac{1}{C}I_3 \qquad (5)$$

Using equation (1), substitute for I_3 in equation (5) to obtain:

$$\frac{dI_2}{dt}=\frac{1}{R_2C}\left(I_1-I_2\right) \qquad (6)$$

Solve equation (2) for I_1:

$$I_1=\frac{\mathcal{E}-Q/C}{R_1}=\frac{\mathcal{E}-R_2I_2}{R_1}$$

Substitute for I_1 in equation (6) and simplify to obtain the differential equation for I_2:

$$\frac{dI_2}{dt}=\frac{1}{R_2C}\left(\frac{\mathcal{E}-R_2I_2}{R_1}-I_2\right)$$

$$=\frac{\mathcal{E}}{R_1R_2C}-\left(\frac{R_1+R_2}{R_1R_2C}\right)I_2$$

To solve this linear differential equation with constant coefficients we can assume a solution of the form:

$$I_2(t)=a+be^{-t/\tau} \qquad (7)$$

Differentiate $I_2(t)$ with respect to time to obtain:

$$\frac{dI_2}{dt}=\frac{d}{dt}\left[a+be^{-t/\tau}\right]=-\frac{b}{\tau}e^{-t/\tau}$$

Substitute for I_2 and dI_2/dt to obtain:

$$-\frac{b}{\tau}e^{-t/\tau}=\frac{\mathcal{E}}{R_1R_2C}-\left(\frac{R_1+R_2}{R_1R_2C}\right)\left(a+be^{-t/\tau}\right)$$

Equate coefficients of $e^{-t/\tau}$ to obtain:

$$\tau=\frac{R_1R_2C}{R_1+R_2}$$

Requiring the equation to hold for all values of a yields:

$$a=\frac{\mathcal{E}}{R_1+R_2}$$

If I_2 is to be zero when $t = 0$:

$$0=a+b$$

or

$$b=-a=-\frac{\mathcal{E}}{R_1+R_2}$$

Substitute in equation (7) to obtain:

$$I_2(t) = \frac{\mathcal{E}}{R_1 + R_2} - \frac{\mathcal{E}}{R_1 + R_2} e^{-t/\tau}$$
$$= \frac{\mathcal{E}}{R_1 + R_2}\left(1 - e^{-t/\tau}\right)$$

where

$$\tau = \frac{R_1 R_2 C}{R_1 + R_2} = \frac{(200\,\Omega)(600\,\Omega)(5\,\mu\text{F})}{200\,\Omega + 600\,\Omega}$$
$$= 0.750\,\text{ms}$$

Substitute numerical values and evaluate $I_2(t)$:

$$I_2(t) = \frac{50\,\text{V}}{200\,\Omega + 600\,\Omega}\left(1 - e^{-t/0.750\,\text{ms}}\right)$$
$$= \boxed{(62.5\,\text{mA})\left(1 - e^{-t/0.750\,\text{ms}}\right)}$$

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Picture the Problem Let R_1 represent the 1.2-MΩ resistor and R_2 the 600-kΩ resistor. Immediately after switch S is closed, the capacitor has zero charge and so the potential difference across it (and the 600 kΩ-resistor) is zero. A long time after the switch is closed, the capacitor will be fully charged and the potential difference across it will be given by both Q/C and $I_\infty R_2$. When the switch is opened after having been closed for a long time, both the source and the 1.2-MΩ resistor will be out of the circuit and the fully charged capacitor will discharge through R_1. We can use Kirchhoff's loop to find the currents drawn from the source immediately after the switch is closed and a long time after the switch is closed, as well as the current in the *RC* circuit when the switch is again opened and the capacitor discharges through R_2.

(*a*) Apply Kirchhoff's loop rule to the circuit immediately after the switch is closed to obtain:

$$\mathcal{E} - I_0 R_1 - V_{C0} = 0$$

or, because $V_{C0} = 0$,

$$\mathcal{E} - I_0 R_1 = 0$$

Solve for and evaluate I_0:

$$I_0 = \frac{\mathcal{E}}{R_1} = \frac{50\,\text{V}}{1.2\,\text{M}\Omega} = \boxed{41.7\,\mu\text{A}}$$

(*b*) Apply Kirchhoff's loop rule to the circuit a long time after the switch is closed to obtain:

$$\mathcal{E} - I_\infty R_1 - I_\infty R_2 = 0$$

Solve for and evaluate I_∞:

$$I_\infty = \frac{\mathcal{E}}{R_1 + R_2} = \frac{50\,\text{V}}{1.2\,\text{M}\Omega + 600\,\text{k}\Omega} = \boxed{27.8\,\mu\text{A}}$$

(*c*) Apply Kirchhoff's loop rule to the *RC* circuit sometime after the switch is opened and solve for *I*(*t*) to obtain:

$$V_\text{C}(t) - R_2 I(t) = 0$$

or

$$I(t) = \frac{V_\text{C}(t)}{R_2}$$

Substitute for $V_\text{C}(t)$:

$$I(t) = \frac{V_{\text{C}\infty}}{R_2} e^{-t/\tau} = I_\infty e^{-t/\tau}$$

where $\tau = R_2 C$.

Substitute numerical values to obtain:

$$I(t) = (27.8\,\mu\text{A}) e^{-t/(600\,\text{k}\Omega)(2.5\,\mu\text{F})} = \boxed{(27.8\,\mu\text{A}) e^{-t/1.5\,\text{s}}}$$

123 •••

Picture the Problem In part (*a*) we can apply Kirchhoff's loop rule to the circuit immediately after the switch is closed in order to find the initial current I_0. We can find the time at which the voltage across the capacitor is 24 V by again applying Kirchhoff's loop rule to find the voltage across the resistor when this condition is satisfied and then using the expression $I(t) = I_0 e^{-t/\tau}$ for the current through the resistor as a function of time and solving for *t*.

(*a*) Apply Kirchhoff's loop rule to the circuit immediately after the switch is closed:

$$\mathcal{E} - 12\,\text{V} - I_0 R = 0$$

Solve for and evaluate I_0:

$$I_0 = \frac{\mathcal{E} - 12\,\text{V}}{R} = \frac{36\,\text{V} - 12\,\text{V}}{0.5\,\text{M}\Omega} = \boxed{48.0\,\mu\text{A}}$$

(*b*) Apply Kirchhoff's loop rule to the circuit when $V_\text{C} = 24$ V and solve for V_R:

$$36\,\text{V} - 24\,\text{V} - I(t)R = 0$$

and

$$I(t)R = 12\,\text{V}$$

Express the current through the resistor as a function of I_0 and τ:

$$I(t) = I_0 e^{-t/\tau}$$

where $\tau = RC$.

Substitute to obtain:

$$RI_0e^{-t/\tau} = 12\,\text{V}$$

or

$$e^{-t/\tau} = \frac{12\,\text{V}}{RI_0}$$

Take the natural logarithm of both sides of the equation to obtain:

$$-\frac{t}{\tau} = \ln\frac{12\,\text{V}}{RI_0}$$

Solve for t:

$$t = -\tau\ln\left(\frac{12\,\text{V}}{RI_0}\right) = -RC\ln\left(\frac{12\,\text{V}}{RI_0}\right)$$

Substitute numerical values and evaluate t:

$$t = -(0.5\,\text{M}\Omega)(2.5\,\mu\text{F})\ln\left[\frac{12\,\text{V}}{(0.5\,\text{M}\Omega)(48\,\mu\text{A})}\right] = \boxed{0.866\,\text{s}}$$

124 •••

Picture the Problem In part (*a*) we can apply Kirchhoff's loop rule to the circuit immediately after the switch is closed in order to find the initial current I_0. We can find the time at which the voltage across the capacitor is 24 V by again applying Kirchhoff's loop rule to find the voltage across the resistor when this condition is satisfied and then using the expression $I(t) = I_0e^{-t/\tau}$ for the current through the resistor as a function of time and solving for t.

(*a*) Apply Kirchhoff's loop rule to the circuit immediately after the switch is closed:

$$\mathcal{E} + 12\,\text{V} - I_0R = 0$$

Solve for and evaluate I_0:

$$I_0 = \frac{\mathcal{E} + 12\,\text{V}}{R} = \frac{36\,\text{V} + 12\,\text{V}}{0.5\,\text{M}\Omega} = \boxed{96.0\,\mu\text{A}}$$

(*b*) Apply Kirchhoff's loop rule to the circuit when $V_C = 24$ V and solve for V_R:

$$36\,\text{V} - 24\,\text{V} - I(t)R = 0$$

and

$$I(t)R = 12\,\text{V}$$

Express the current through the resistor as a function of I_0 and τ:

$$I(t) = I_0e^{-t/\tau}$$

where $\tau = RC$.

Substitute to obtain:

$$RI_0e^{-t/\tau} = 12\,\text{V}$$

or

$$e^{-t/\tau} = \frac{12\,\text{V}}{RI_0}$$

Take the natural logarithm of both sides of the equation to obtain:

$$-\frac{t}{\tau} = \ln\frac{12\,\text{V}}{RI_0}$$

Solve for t:

$$t = -\tau\ln\left(\frac{12\,\text{V}}{RI_0}\right) = -RC\ln\left(\frac{12\,\text{V}}{RI_0}\right)$$

Substitute numerical values and evaluate t:

$$t = -(0.5\,\text{M}\Omega)(2.5\,\mu\text{F})\ln\left[\frac{12\,\text{V}}{(0.5\,\text{M}\Omega)(96\,\mu\text{A})}\right] = \boxed{1.73\,\text{s}}$$

General Problems

***125 ••**

Determine the Concept Because all of the current drawn from the battery passes through R_1, we know that I_1 is greater than I_2 and I_3. Because $R_2 \neq R_3$, $I_2 \neq I_3$ and so (*b*) is false. Because $R_3 > R_2$, $I_3 < I_2$ and so (*c*) is false. $\boxed{(a)\text{ is correct.}}$

126 •• A 25-W lightbulb is connected in series with a 100-W lightbulb and a voltage V is placed across the combination. Which lightbulb is brighter? Explain.

Determine the Concept The 25-W bulb will be brighter. The brightness of a bulb is proportional to the power it dissipates. The resistance of the 25-W bulb is greater than that of the 100-W bulb, and in the series combination, the same current I flows through the bulbs. Hence, $I^2R_{25} > I^2R_{100}$.

127 •

Picture the Problem We can apply Ohm's law to find the current drawn from the battery and use Kirchhoff's loop rule to find the current in the 6-Ω resistor.

Using Ohm's law, express the current I_1 drawn from the battery:

$$I_1 = \frac{\mathcal{E}}{R_{\text{eq}}}$$

Find R_{eq}:

$$R_{\text{eq}} = 4\,\Omega + \frac{(6\,\Omega)(12\,\Omega)}{6\,\Omega + 12\,\Omega} = 8\,\Omega$$

Substitute and evaluate $I1$:

$$I_1 = \frac{24\,\text{V}}{8\,\Omega} = 3\,\text{A}$$

Apply Kirchhoff's loop rule to a loop that includes the battery and the 4-Ω and 6-Ω resistors:

$$24\,\text{V} - (4\,\Omega)(3\,\text{A}) - (6\,\Omega)I_2 = 0$$

Solve for I_2 to obtain:

$$I_2 = 2\,\text{A} \text{ and } \boxed{(b)\text{ is correct.}}$$

128 •

Picture the Problem We can use $P = I_{max}^2 R$ to find the maximum current the resistor can tolerate and Ohm's law to find the voltage across the resistor that will produce this current.

(*a*) Relate the maximum current the resistor can tolerate to its power and resistance:

$$P = I_{max}^2 R$$

Solve for and evaluate I_{max}:

$$I_{max} = \sqrt{\frac{P}{R}} = \sqrt{\frac{5\,\text{W}}{10\,\Omega}} = \boxed{0.707\,\text{A}}$$

(*b*) Use Ohm's law to relate the voltage across the resistor to this maximum current:

$$V = I_{max}R = (0.707\,\text{A})(10\,\Omega) = \boxed{7.07\,\text{V}}$$

129 •

Picture the Problem We can use Ohm's law to find the short-circuit current drawn from the battery and the relationship between the terminal potential difference, the emf of the battery, and the current being drawn from it to find the terminal voltage when the battery is delivering a current of 20 A.

(*a*) Apply Ohm's law to the shorted battery to find the short-circuit current:

$$I_{sc} = \frac{\mathcal{E}}{r} = \frac{12\,\text{V}}{0.4\,\Omega} = \boxed{30.0\,\text{A}}$$

(*b*) Express the terminal voltage as the difference between the emf of the battery and the current being drawn from it:

$$V_{term} = \mathcal{E} - Ir$$

Substitute numerical values and evaluate V_{term}:

$$V_{term} = 12\,\text{V} - (20\,\text{A})(0.4\,\Omega) = \boxed{4.00\,\text{V}}$$

130 ••

Picture the Problem We can use Kirchhoff's loop rule to obtain two equations relating ε and r that we can solve simultaneously to find these quantities.

Use Kirchhoff's loop rule to relate the emf of the battery to the current drawn from it and the internal and external resistance:

$$\mathcal{E} - IR - Ir = 0 \quad (1)$$

When I = 1.80 A and a 7.0-Ω resistor is connected across the battery terminals equation (1) becomes:

$$\mathcal{E} - (1.8\,\text{A})(7\,\Omega) - (1.8\,\text{A})r = 0$$

or

$$\mathcal{E} - 12.6\,\text{V} - (1.8\,\text{A})r = 0 \quad (2)$$

When I = 2.20 A and a 12-Ω resistor is connected in parallel with the 7.0-Ω resistor:

$$\mathcal{E} - (2.2\,\text{A})R_{eq} - (2.2\,\text{A})r = 0$$

Find the equivalent resistance:

$$R_{eq} = \frac{(7\,\Omega)(12\,\Omega)}{7\,\Omega + 12\,\Omega} = 4.42\,\Omega$$

Substitute to obtain:

$$\mathcal{E} - (2.2\,\text{A})(4.42\,\Omega) - (2.2\,\text{A})r = 0$$

or

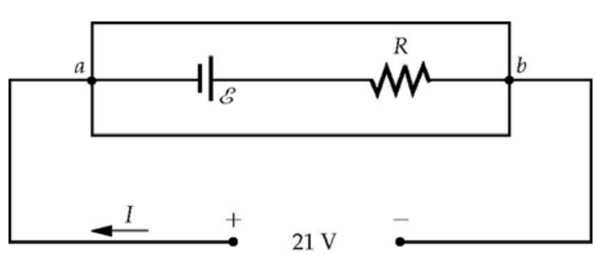
$$\mathcal{E} - 9.72\,\text{V} - (2.2\,\text{A})r = 0 \quad (3)$$

Solve equations (2) and (3) simultaneously to obtain:

$$\mathcal{E} = \boxed{25.5\,\text{V}} \text{ and } r = \boxed{7.19\,\Omega}$$

***131 ••**

Picture the Problem We can apply Kirchhoff's loop rule to the circuit that includes the box and the 21-V source to obtain two equations in the unknowns ε and R that we can solve simultaneously.



Apply Kirchhoff's loop rule to the circuit when the polarity of the 21-V source and the direction of the current are as shown in the diagram:

$$21\,\text{V} + \mathcal{E} - (1\,\text{A})R = 0$$

Apply Kirchhoff's loop rule to the circuit when the polarity of the source is reversed and the current is 2 A in the opposite direction:

$$-21\,\text{V} + \mathcal{E} + (2\,\text{A})R = 0$$

Solve these equations simultaneously to obtain:

$$R = \boxed{14.0\,\Omega} \text{ and } \mathcal{E} = \boxed{-7.00\,\text{V}}$$

132 ••

Picture the Problem When the switch is closed, the initial potential differences across the capacitors are zero (they have no charge) and the resistors in the bridge portion of the circuit are in parallel. When a long time has passed, the current through the capacitors will be zero and the resistors will be in series. In both cases, the application of Kirchhoff's loop rule to the entire circuit will yield the current in the circuit. To find the final charges on the capacitors we can use the definition of capacitance and apply Kirchhoff's loop rule to the loops containing two resistors and a capacitor to find the potential differences across the capacitors.

(*a*) Apply Kirchhoff's loop rule to the circuit immediately after the switch is closed:

$$50\,\text{V} - I_0(10\,\Omega) - I_0 R_{\text{eq}} = 0$$

Solve for I_0:

$$I_0 = \frac{50\,\text{V}}{10\,\Omega + R_{\text{eq}}}$$

Find the equivalent resistance of 15 Ω, 12 Ω, and 15 Ω in parallel:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{15\,\Omega} + \frac{1}{12\,\Omega} + \frac{1}{15\,\Omega}$$

and

$$R_{\text{eq}} = 4.62\,\Omega$$

Substitute for R_{eq} and evaluate I_0:

$$I_0 = \frac{50\,\text{V}}{10\,\Omega + 4.62\,\Omega} = \boxed{3.42\,\text{A}}$$

(*b*) Apply Kirchhoff's loop rule to the circuit a long time after the switch is closed:

$$50\,\text{V} - I_\infty(10\,\Omega) - I_\infty R_{\text{eq}} = 0$$

Solve for I_∞:

$$I_\infty = \frac{50\,\text{V}}{10\,\Omega + R_{\text{eq}}}$$

Find the equivalent resistance of 15 Ω, 12 Ω, and 15 Ω in series:

$$R_{\text{eq}} = 15\,\Omega + 12\,\Omega + 15\,\Omega = 42\,\Omega$$

Substitute for R_{eq} and evaluate I_∞:

$$I_\infty = \frac{50\,\text{V}}{10\,\Omega + 42\,\Omega} = \boxed{0.962\,\text{A}}$$

(*c*) Using the definition of capacitance, express the charge on the capacitors in terms of their final potential differences:

$$Q_{10\,\mu\text{F}} = C_{10\,\mu\text{F}} V_{10\,\mu\text{F}} \quad (1)$$

and

$$Q_{5\,\mu\text{F}} = C_{5\,\mu\text{F}} V_{5\,\mu\text{F}} \quad (2)$$

Apply Kirchhoff's loop rule to the loop containing the 15-Ω and 12-Ω resistors and the 10 μF capacitor to obtain:

$$V_{10\,\mu\text{F}} - (15\,\Omega)I_\infty - (12\,\Omega)I_\infty = 0$$

Solve for $V_{10\,\mu\text{F}}$:

$$V_{10\,\mu\text{F}} = (27\,\Omega)I_\infty$$

Substitute in equation (1) and evaluate $Q_{10\,\mu\text{F}}$:

$$\begin{aligned} Q_{10\,\mu\text{F}} &= C_{10\,\mu\text{F}}(27\,\Omega)I_\infty \\ &= (10\,\mu\text{F})(27\,\Omega)(0.962\,\text{A}) \\ &= \boxed{260\,\mu\text{C}} \end{aligned}$$

Apply Kirchhoff's loop rule to the loop containing the 15-Ω and 12-Ω resistors and the 5 μF capacitor to obtain:

$$V_{5\,\mu\text{F}} - (15\,\Omega)I_\infty - (12\,\Omega)I_\infty = 0$$

Solve for $V_{5\,\mu\text{F}}$:

$$V_{5\,\mu\text{F}} = (27\,\Omega)I_\infty$$

Substitute in equation (2) and evaluate $Q_{5\,\mu\text{F}}$:

$$\begin{aligned} Q_{5\,\mu\text{F}} &= C_{5\,\mu\text{F}}(27\,\Omega)I_\infty \\ &= (5\,\mu\text{F})(27\,\Omega)(0.962\,\text{A}) \\ &= \boxed{130\,\mu\text{C}} \end{aligned}$$

***133** ••

Picture the Problem Let the current flowing through the galvanometer by I_G. By applying Kirchhoff's rules to the loops including 1) R_1, the galvanometer, and R_x, and 2) R_2, the galvanometer, and R_0, we can obtain two equations relating the unknown resistance to R_1, R_2 and R_0. Using $R = \rho L/A$ will allow us to express R_x in terms of the length of wire L_1 that corresponds to R_1 and the length of wire L_2 that corresponds to R_2.

Apply Kirchhoff's loop rule to the loop that includes R_1, the galvanometer, and R_x to obtain:

$$-R_1I_1 + R_xI_2 = 0 \quad (1)$$

Apply Kirchhoff's loop rule to the loop that includes R_2, the galvanometer, and R_0 to obtain:

$$-R_2(I_1 - I_G) + R_0(I_2 + I_G) = 0 \quad (2)$$

When the bridge is balanced, $I_G = 0$ and equations (1) and (2) become:

$$R_1I_1 = R_xI_2 \quad (3)$$

and

$$R_2I_1 = R_0I_2 \quad (4)$$

Divide equation (3) by equation (4) and solve for x to obtain:

$$R_x = R_0\frac{R_1}{R_2} \quad (5)$$

Express R_1 and R_2 in terms of their lengths, cross-sectional areas, and the resistivity of their wire:

$$R_1 = \rho\frac{L_1}{A} \text{ and } R_2 = \rho\frac{L_2}{A}$$

Substitute in equation (5) to obtain:

$$R_x = R_0\frac{L_1}{L_2}$$

(*a*) When the bridge balances at the 18-cm mark, $L_1 = 18$ cm, $L_2 = 82$ cm and:

$$R_x = (200\,\Omega)\frac{18\,\text{cm}}{82\,\text{cm}} = \boxed{43.9\,\Omega}$$

(*b*) When the bridge balances at the 60-cm mark, $L_1 = 60$ cm, $L_2 = 40$ cm and:

$$R_x = (200\,\Omega)\frac{60\,\text{cm}}{40\,\text{cm}} = \boxed{300\,\Omega}$$

(*c*) When the bridge balances at the 95-cm mark, $L_1 = 95$ cm, $L_2 = 5$ cm and:

$$R_x = (200\,\Omega)\frac{95\,\text{cm}}{5\,\text{cm}} = \boxed{3.80\,\text{k}\Omega}$$

134 ••

Picture the Problem Let the current flowing through the galvanometer by I_G. By applying Kirchhoff's rules to the loops including 1) R_1, the galvanometer, and R_x, and 2) R_2, the galvanometer, and R_0, we can obtain two equations relating the unknown resistance to R_1, R_2 and R_0. Using $R = \rho L/A$ will allow us to express R_x in terms of the length of wire L_1 that corresponds to R_1 and the length of wire L_2 that corresponds to R_2. To find the effect of an error of 2 mm in the location of the balance point we can use the relationship $\Delta R_x = (dR_x/dL)\Delta L$ to determine ΔR_x and then divide by $R_x = R_0\,L/(1-L)$ to find the fractional change (error) in R_x resulting from a given error in the determination of the balance point.

Apply Kirchhoff's loop rule to the loop that includes R_1, the galvanometer, and R_x to obtain:

$$-R_1I_1 + R_xI_2 = 0 \quad (1)$$

Apply Kirchhoff's loop rule to the loop that includes R_2, the galvanometer, and R_0 to obtain:

$$-R_2(I_1 - I_G) + R_0(I_2 + I_G) = 0 \quad (2)$$

When the bridge is balanced, $I_G = 0$ and equations (1) and (2) become:

$$R_1I_1 = R_xI_2 \quad (3)$$

and

$$R_2I_1 = R_0I_2 \quad (4)$$

Divide equation (3) by equation (4) and solve for x to obtain:

$$R_x = R_0\frac{R_1}{R_2} \quad (5)$$

Express R_1 and R_2 in terms of their lengths, cross-sectional areas, and the resistivity of their wire:

$$R_1 = \rho\frac{L_1}{A} \text{ and } R_2 = \rho\frac{L_2}{A}$$

Substitute in equation (5) to obtain:

$$R_x = R_0\frac{L_1}{L_2} \quad (6)$$

(*a*) When the bridge balances at the 98-cm mark, $L_1 = 98$ cm, $L_2 = 2$ cm and:

$$R_x = (200\,\Omega)\frac{98\,\text{cm}}{2\,\text{cm}} = \boxed{9.80\,\text{k}\Omega}$$

(*b*) Express R_x in terms of the distance to the balance point:

$$R_x = R_0\frac{L}{1-L}$$

Express the error ΔR_x in R_x resulting from an error ΔL in L:

$$\Delta R_x = \frac{dR_x}{dL}\Delta L = R_0\frac{d}{dL}\left[\frac{L}{1-L}\right]\Delta L$$

$$= R_0\frac{1}{(1-L)^2}\Delta L$$

Divide ΔR_x by R_x to obtain:

$$\frac{\Delta R_x}{R_x}=\frac{R_0\frac{1}{(1-L)^2}\Delta L}{R_0\frac{L}{1-L}}=\frac{1}{1-L}\frac{\Delta L}{L}$$

Evaluate $\Delta R_x/R_x$ for L = 98 cm and ΔL = 2 mm:

$$\frac{\Delta R_x}{R_x}=\frac{1\,\text{m}}{1\,\text{m}-0.98\,\text{m}}\frac{2\,\text{mm}}{1\,\text{m}}=\boxed{10.0\%}$$

(*c*) Solve equation (6) for the ratio of L_1 to L_2:

$$\frac{L_1}{L_2}=\frac{R_x}{R_0}$$

For $L_1=50$ cm, $L_2=50$ cm, and $R_0=R_x=9.80\ \text{k}\Omega$. Hence, a resistor of approximately $10\ \text{k}\Omega$ will cause the bridge to balance near the 50 - cm mark.

135 ••

Picture the Problem Knowing the beam current and charge per proton, we can use $I = ne$ to determine the number of protons striking the target per second. The energy deposited per second is the power delivered to the target and is given by $P = IV$. We can find the elapsed time before the target temperature rises 300C° using $\Delta Q = P\Delta t = mc_{\text{Cu}}\Delta T$.

(*a*) Relate the current to the number of protons per second n arriving at the target:

$$I=ne$$

Solve for and evaluate n:

$$n=\frac{I}{e}=\frac{3.50\ \mu\text{A}}{1.60\times10^{-19}\ \text{C}}=\boxed{2.19\times10^{13}\ /\text{s}}$$

(*b*) Express the power of the beam in terms of the beam current and energy:

$$P=IV=(3.5\ \mu\text{A})(60\ \text{MeV})=\boxed{210\ \text{J/s}}$$

(*c*) Relate the energy delivered to the target to its heat capacity and temperature change:

$$\Delta Q=P\Delta t=C_{\text{Cu}}\Delta T=mc_{\text{Cu}}\Delta T$$

Solve for Δt:

$$\Delta t=\frac{mc_{\text{Cu}}\Delta T}{P}$$

Substitute numerical values (see Table 19-1 for the specific heat of copper)and evaluate Δt:

$$\Delta t = \frac{(50\,\text{g})(0.386\,\text{kJ/kg}\cdot\text{K})(300\text{C}^\circ)}{210\,\text{J/s}} = \boxed{27.6\,\text{s}}$$

136 ••

Picture the Problem We can use the definition of current to express the current delivered by the belt in terms of the surface charge density, width, and speed of the belt. The minimum power needed to drive the belt can be found from $P = IV$.

(*a*) Use its definition to express the current carried by the belt:

$$I = \frac{dQ}{dt} = \sigma w\frac{dx}{dt} = \sigma wv$$

Substitute numerical values and evaluate I:

$$I = (5\,\text{mC/m}^2)(0.5\,\text{m})(20\,\text{m/s}) = \boxed{50.0\,\text{mA}}$$

(*b*) Express the minimum power of the motor in terms of the current delivered and the potential of the charge:

$$P = IV$$

Substitute numerical values and evaluate P:

$$P = (50\,\text{mA})(100\,\text{kV}) = \boxed{5.00\,\text{kW}}$$

137 ••

Picture the Problem We can differentiate the expression relating the amount of heat required to produce a given temperature change with respect to time to express the mass flow-rate required to maintain the temperature of the coils at 50°C. We can then use the definition of density to find the necessary volume flow rate.

Express the heat that must be dissipated in terms of the specific heat and mass of the water and the desired temperature change of the water:

$$Q = mc_{\text{water}}\Delta T$$

Differentiate this expression with respect to time to obtain an expression for the power dissipation:

$$P = \frac{dQ}{dt} = \frac{dm}{dt}c_{\text{water}}\Delta T$$

Solve for dm/dt:

$$\frac{dm}{dt}=\frac{P}{c_{\text{water}}\Delta T}$$

Substitute for the power dissipated to obtain:

$$\frac{dm}{dt}=\frac{IV}{c_{\text{water}}\Delta T}$$

Substitute numerical values and evaluate dm/dt:

$$\frac{dm}{dt}=\frac{(100\,\text{A})(240\,\text{V})}{(4.18\,\text{kJ/kg}\cdot\text{K})(50°\text{C}-15°\text{C})}=0.164\,\text{kg/s}$$

Using the definition of density, express the volume flow rate in terms of the mass flow rate to obtain:

$$\frac{dV}{dt}=\frac{1}{\rho}\frac{dm}{dt}=\frac{0.164\,\text{kg/s}}{10^3\,\text{kg/m}^3}=\left(0.164\times10^{-3}\,\text{m}^3\text{/s}\right)\left(\frac{1\,\text{L}}{10^{-3}\,\text{m}^3}\right)=\boxed{0.164\,\text{L/s}}$$

138 ••

Picture the Problem We can use the expressions for the capacitance of a dielectric-filled parallel-plate capacitor and the resistance of a conductor to show that $RC = \epsilon_0\rho\kappa$.

Express the capacitance of the dielectric-filled parallel-plate capacitor:

$$C=\frac{\kappa\in_0 A}{d}$$

Express the resistance of a conductor with the same dimensions:

$$R=\frac{\rho d}{A}$$

The product of C and R is:

$$RC=\frac{\rho d}{A}\frac{\kappa\in_0 A}{d}=\boxed{\in_0\rho\kappa}$$

139 ••

Picture the Problem We can use the expressions for the capacitance of a dielectric-filled cylindrical capacitor and the resistance of a cylindrical conductor to show that $RC = \epsilon_0\rho\kappa$.

Express the capacitance of the dielectric-filled cylindrical capacitor whose inner and outer radii are r_1 and r_2, respectively:

$$C=\frac{2\pi\ell\kappa\in_0}{\ln\left(\frac{r_2}{r_1}\right)}$$

where ℓ is the length of the capacitor.

Express the resistance of a cylindrical resistor with the same dimensions:

$$R=\frac{\rho\ln\left(\frac{r_2}{r_1}\right)}{2\pi\ell}$$

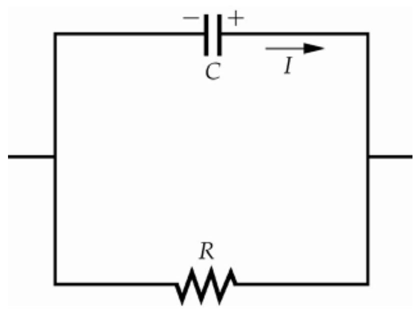
The product of C and R is:

$$RC = \frac{\rho \ln\left(\frac{r_2}{r_1}\right)}{2\pi\ell} \frac{2\pi\ell\kappa \in_0}{\ln\left(\frac{r_2}{r_1}\right)} = \boxed{\in_0 \rho\kappa}$$

This result holds independently of the geometries of the capacitor and the resistor.

***140 ••**

Picture the Problem We'll assume that the capacitor is fully charged initially and apply Kirchhoff's loop rule to the circuit fragment to obtain the differential equation describing the discharge of the leaky capacitor. We'll show that the solution to this equation is the familiar expression for an exponential decay with time constant $\tau = \in_0 \rho\kappa$.



(*a*) If we think of the leaky capacitor as a resistor/capacitor combination, the voltage drop across the resistor must be the same as voltage drop across the capacitor. Hence, they must be in parallel.

(*b*) Assuming that the capacitor is initially fully charged, apply Kirchhoff's loop rule to the circuit fragment to obtain:

$$\frac{Q}{C} - RI = 0$$

or, because $I = -\frac{dQ}{dt}$,

$$\frac{Q}{C} + R\frac{dQ}{dt} = 0$$

Separate variables in this differential equation to obtain:

$$\frac{dQ}{Q} = -\frac{1}{RC}dt$$

From Problems 138 and 139 we have:

$$RC = \in_0 \rho\kappa$$

Substitute for RC in the differential equation to obtain:

$$\frac{dQ}{Q} = -\frac{1}{\in_0 \rho\kappa}dt$$

Integrate this equation from $Q' = Q_0$ to Q to obtain:

$$Q = Q_0 e^{-t/\tau}$$

where

$$\tau = \boxed{\in_0 \rho\kappa}$$

(*c*) Because $Q/Q_0 = 0.1$:

$$e^{-t/\tau} = 0.1$$

Solve for *t* by taking the natural logarithm of both sides of the equation:

$$-\frac{t}{\tau} = \ln 0.1 \Rightarrow t = -\in_0 \rho\kappa \ln 0.1$$

Substitute numerical values and evaluate *t*:

$$t = -\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)\left(9\times10^{13}\ \Omega\cdot\mathrm{m}\right)(5)\ln 0.1 = 9.17\times10^{3}\ \mathrm{s} = \boxed{2.55\ \mathrm{h}}$$

141 •••

Picture the Problem We can use its definition to find the time constant of the charging circuit in part (*a*). In part (*b*) we can use the expression for the potential difference as a function of time across a charging capacitor and utilize the hint given in the problem statement to show that the voltage across the capacitor increases almost linearly over the time required to bring the potential across the switch to its critical value. In part (*c*) we can use the result derived in part (*b*) to find the value of R_1 such that *C* charges from 0.2 V to 4.2 V in 0.1 s. In part (*d*) we can use the expression for the potential difference as a function of time across a discharging capacitor to find the discharge time. Finally, in part (*e*) we can integrate I^2R_1 over the discharge time to find the rate at which energy is dissipated in R_1 during the discharge of the capacitor and use the difference in the energy stored in the capacitor initially and when the switch opens to find the rate of energy dissipation in resistance of the capacitor.

(*a*) When the capacitor is charging, the switch is open and the resistance in the charging circuit is R_1. Hence:

$$\tau = R_1C = (0.5\ \mathrm{M}\Omega)(0.02\ \mu\mathrm{F}) = \boxed{10.0\ \mathrm{ms}}$$

(*b*) Express the voltage across the charging capacitor as a function of time:

$$V(t) = \mathcal{E}\left(1 - e^{-t/\tau}\right)$$

Solve for the exponential term to obtain:

$$e^{-t/\tau} = 1 - \frac{V(t)}{\mathcal{E}} \qquad (1)$$

Noting that $V(t) << \varepsilon$, let $\eta = V(t)/\varepsilon$

$$e^{-t/\tau} = 1 - \eta$$

or

$$e^{t/\tau} = (1-\eta)^{-1} \approx 1 + \eta$$

because $\eta << 1$.

Use the power series for e^x to expand $e^{t/\tau}$:

$$e^{t/\tau} = 1 + \frac{t}{\tau} + \frac{1}{2!}\left(\frac{t}{\tau}\right)^2 + \ldots$$
$$\approx 1 + \frac{1}{\tau}t$$

provided $t/\tau << 1$.

Substitute in equation (1) to obtain:

$$1 + \frac{1}{\tau}t \approx 1 + \eta = 1 + \frac{V(t)}{\mathcal{E}}$$

Solve for t to obtain the linear relationship:

$$\boxed{V(t) = \frac{\mathcal{E}}{\tau}t} \qquad (1)$$

(c) Using the result derived in (b), relate the time Δt required to change the voltage across the capacitor by an amount ΔV to ΔV:

$$\Delta V(t) = \frac{\mathcal{E}}{\tau}\Delta t$$

or

$$\tau = R_1 C = \frac{\mathcal{E}}{\Delta V(t)}\Delta t$$

Solve for R_1:

$$R_1 = \frac{\mathcal{E}}{C\Delta V(t)}\Delta t$$

Substitute numerical values and evaluate R_1:

$$R_1 = \frac{(800\,\text{V})(0.1\,\text{s})}{(0.02\,\mu\text{F})(4.2\,\text{V} - 0.2\,\text{V})} = \boxed{1.00\,\text{G}\Omega}$$

(d) Express the potential difference across the capacitor as a function of time:

$$V_{\text{C}}(t) = V_{\text{C}0}e^{-t/\tau'}$$

where

$\tau' = R_2 C$.

Solve for t to obtain:

$$t = -\tau' \ln\left(\frac{V_{\text{C}}(t)}{V_{\text{C}0}}\right) = -R_2 C \ln\left(\frac{V_{\text{C}}(t)}{V_{\text{C}0}}\right)$$

Substitute numerical values and evaluate t:

$$t = -(0.001\,\Omega)(0.02\,\mu\text{F})\ln\left(\frac{0.2\,\text{V}}{4.2\,\text{V}}\right) = \boxed{60.9\,\text{ps}}$$

(e) Express the rate at which energy is dissipated in R_1 as a function of its

$$P_1 = \frac{\Delta E_1}{\Delta t} = I^2 R_1$$

resistance and the current through it:

Because the current varies with time, we need to integrate over time to find ΔE_1:

$$\Delta E_1 = \int I^2 R_1 dt = \int \left(\frac{V(t)}{R_1}\right)^2 R_1 dt$$
$$= \int \left(\frac{\mathcal{E}\, t}{\tau\, R_1}\right)^2 R_1 dt$$
$$= \left(\frac{\mathcal{E}}{\tau}\right)^2 \frac{1}{R_1} \int_{0.005\,\text{s}}^{0.105\,\text{s}} t^2 dt$$
$$= \left(\frac{800\,\text{V}}{20\,\text{s}}\right)^2 \frac{1}{1\,\text{G}\Omega} \left[\frac{t^3}{3}\right]_{0.005\,\text{s}}^{0.105\,\text{s}}$$
$$= 6.17\times10^{-10}\ \text{J}$$

Substitute and evaluate P_1:

$$P_1 = \frac{6.17\times10^{-10}\ \text{J}}{0.1\,\text{s}} = \boxed{6.17\,\text{nW}}$$

Express the rate at which energy is dissipated in the switch resistance:

$$P_2 = \frac{\Delta U_\text{C}}{\Delta t} = \frac{U_\text{Ci} - U_\text{Cf}}{\Delta t}$$
$$= \frac{\frac{1}{2}CV_\text{i}^2 - \frac{1}{2}CV_\text{f}^2}{\Delta t} = \frac{\frac{1}{2}C\left(V_\text{i}^2 - V_\text{f}^2\right)}{\Delta t}$$

Substitute numerical values and evaluate P_2:

$$P_2 = \frac{\frac{1}{2}(0.02\,\mu\text{F})\left[(4.2\,\text{V})^2 - (0.2\,\text{V})^2\right]}{60.9\,\text{ps}}$$
$$= \boxed{2.89\,\text{kW}}$$

142 •••

Picture the Problem We can apply both the loop and junction rules to obtain equations that we can use to obtain a linear differential equation with constant coefficients describing the current in R_2 as a function of time. We can solve this differential equation by assuming a solution of an appropriate form, differentiating this assumed solution and substituting it and its derivative in the differential equation. Equating coefficients, requiring the solution to hold for all values of the assumed constants, and invoking an initial condition will allow us to find the constants in the assumed solution. Once we know how the current varies with time in R_2, we can express the potential difference across it (as well as across C because they are in parallel). To find the voltage across the capacitor at $t = 8$ s, we can express the dependence of the voltage on time for a discharging capacitor (C is discharging after $t = 2$ s) and evaluate this function, with a time constant differing from that found in (*a*), at $t = 6$ s.

(*a*) Apply the junction rule at the junction between the two resistors to obtain:

$$I_1 = I_2 + I_3 \qquad (1)$$

Apply the loop rule to the loop containing the source, R_1, and the capacitor to obtain:

$$\mathcal{E} - R_1 I_1 - \frac{Q}{C} = 0 \qquad (2)$$

Apply the loop rule to the loop containing R_2 and the capacitor to obtain:

$$\frac{Q}{C} - R_2 I_2 = 0 \qquad (3)$$

Differentiate equation (2) with respect to time to obtain:

$$\frac{d}{dt}\left[\mathcal{E} - R_1 I_1 - \frac{Q}{C}\right] = 0 - R_1 \frac{dI_1}{dt} - \frac{1}{C}\frac{dQ}{dt}$$
$$= -R_1 \frac{dI_1}{dt} - \frac{1}{C} I_3 = 0$$

or

$$R_1 \frac{dI_1}{dt} = -\frac{1}{C} I_3 \qquad (4)$$

Differentiate equation (3) with respect to time to obtain:

$$\frac{d}{dt}\left[\frac{Q}{C} - R_2 I_2\right] = \frac{1}{C}\frac{dQ}{dt} - R_2 \frac{dI_2}{dt} = 0$$

or

$$R_2 \frac{dI_2}{dt} = \frac{1}{C} I_3 \qquad (5)$$

Using equation (1), substitute for I_3 in equation (5) to obtain:

$$\frac{dI_2}{dt} = \frac{1}{R_2 C}\left(I_1 - I_2\right) \qquad (6)$$

Solve equation (2) for I_1:

$$I_1 = \frac{\mathcal{E} - Q/C}{R_1} = \frac{\mathcal{E} - R_2 I_2}{R_1}$$

Substitute for I_1 in equation (6) and simplify to obtain the differential equation for I_2:

$$\frac{dI_2}{dt} = \frac{1}{R_2 C}\left(\frac{\mathcal{E} - R_2 I_2}{R_1} - I_2\right)$$
$$= \frac{\mathcal{E}}{R_1 R_2 C} - \left(\frac{R_1 + R_2}{R_1 R_2 C}\right) I_2$$

To solve this linear differential equation with constant coefficients we can assume a solution of the form:

$$I_2(t) = a + b e^{-t/\tau} \qquad (7)$$

Differentiate $I_2(t)$ with respect to time to obtain:

$$\frac{dI_2}{dt}=\frac{d}{dt}\left[a+be^{-t/\tau}\right]=-\frac{b}{\tau}e^{-t/\tau}$$

Substitute for I_2 and dI_2/dt to obtain:

$$-\frac{b}{\tau}e^{-t/\tau}=\frac{\mathcal{E}}{R_1R_2C}-\left(\frac{R_1+R_2}{R_1R_2C}\right)\left(a+be^{-t/\tau}\right)$$

Equate coefficients of $e^{-t/\tau}$ to obtain:

$$\tau=\frac{R_1R_2C}{R_1+R_2}$$

Requiring the equation to hold for all values of a yields:

$$a=\frac{\mathcal{E}}{R_1+R_2}$$

If I_2 is to be zero when $t = 0$:

$$0=a+b$$

or

$$b=-a=-\frac{\mathcal{E}}{R_1+R_2}$$

Substitute in equation (7) to obtain:

$$I_2(t)=\frac{\mathcal{E}}{R_1+R_2}-\frac{\mathcal{E}}{R_1+R_2}e^{-t/\tau}=\frac{\mathcal{E}}{R_1+R_2}\left(1-e^{-t/\tau}\right)$$

where

$$\tau=\frac{R_1R_2C}{R_1+R_2}$$

Substitute numerical values and evaluate τ:

$$\tau=\frac{(2\,\text{M}\Omega)(5\,\text{M}\Omega)(1\,\mu\text{F})}{2\,\text{M}\Omega+5\,\text{M}\Omega}=1.43\,\text{s}$$

Substitute numerical values and evaluate $I_2(t)$:-

$$I_2(t)=\frac{10\,\text{V}}{2\,\text{M}\Omega+5\,\text{M}\Omega}\left(1-e^{-t/1.43\text{s}}\right)=(1.43\,\mu\text{A})\left(1-e^{-t/1.43\text{s}}\right)$$

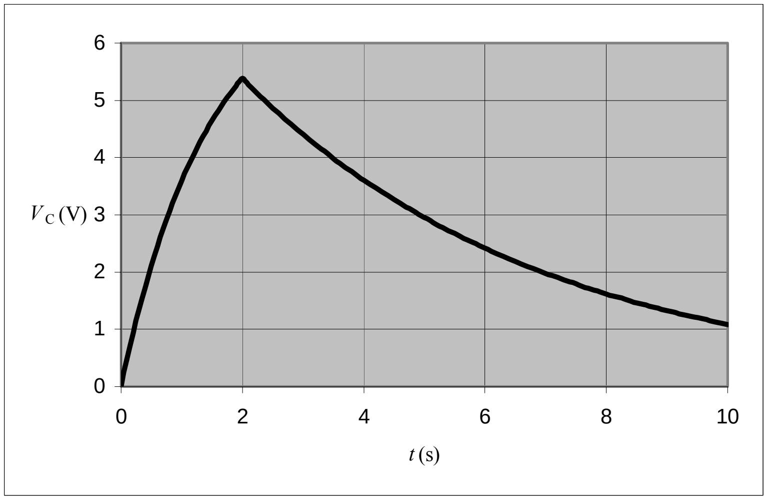
Because C and R_2 are in parallel, they have a common potential difference given by:

$$V_\text{C}(t)=V_2(t)=I_2(t)R_2=(1.43\,\mu\text{A})(5\,\text{M}\Omega)\left(1-e^{-t/1.43\text{s}}\right)=(7.15\,\text{V})\left(1-e^{-t/1.43\text{s}}\right)$$

Evaluate V_C at $t = 2$ s:

$$V_\text{C}(2\,\text{s})=(7.15\,\text{V})\left(1-e^{-2\text{s}/1.43\text{s}}\right)=5.38\,\text{V}$$

The voltage across the capacitor as a function of time is shown in the figure. The current through the 5-MΩ resistor R_2 follows the same time course, its value being $V_C/(5\times10^6)$ A.



(*b*) The value of V_C at t = 2 s has already been determined to be:

$$V_C(2\,\text{s}) = \boxed{5.38\,\text{V}}$$

When S is opened at t = 2 s, C discharges through R_2 with a time constant given by:

$$\tau' = R_2C = (5\,\text{M}\Omega)(1\,\mu\text{F}) = 5\,\text{s}$$

Express the potential difference across C as a function of time:

$$V_C(t) = V_{C0}e^{-t/\tau'} = (5.38\,\text{V})e^{-t/5\text{s}}$$

Evaluate V_C at t = 8 s to obtain:

$$V_C(8\,\text{s}) = (5.38\,\text{V})e^{-6\text{s}/5\text{s}} = \boxed{1.62\,\text{V}}$$

in good agreement with the graph.

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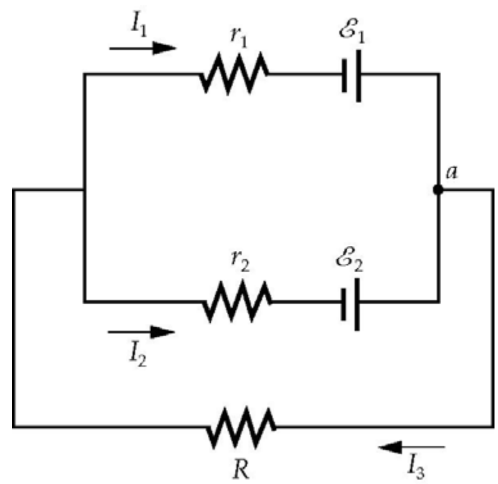
Picture the Problem Let I_1 be the current delivered by ε_1, I_2 the current delivered by ε_2, and I_3 the current through the resistor R. We can apply Kirchhoff's rules to obtain three equations in the unknowns I_1, I_2, and I_3 that we can solve simultaneously to find I_3. We can then express the power delivered by the sources to R. Setting the derivative of this expression equal to zero will allow us to solve for the value of R that maximizes the power delivered by the sources.



Apply Kirchhoff's junction rule at a to obtain:

$$I_1 + I_2 = I_3 \quad (1)$$

Apply the loop rule around the outside
of the circuit to obtain:

$$\mathcal{E}_1 - I_3R - r_1I_1 = 0 \quad (2)$$

Apply the loop rule around the inside of the circuit to obtain:

$$\mathcal{E}_2 - I_3R - r_2I_2 = 0 \quad (3)$$

Eliminate I_1 from equations (1) and (2) to obtain:

$$\mathcal{E}_1 - I_3R - r_1(I_3 - I_2) = 0 \quad (4)$$

Solve equation (3) for I_2 to obtain:

$$I_2 = \frac{\mathcal{E}_2 - I_3R}{r_2}$$

Substitute for I_2 in equation (4) to obtain:

$$\mathcal{E}_1 - I_3R - r_1\left(I_3 - \frac{\mathcal{E}_2 - I_3R}{r_2}\right) = 0$$

Solve for I_3 to obtain:

$$I_3 = \frac{\mathcal{E}_1r_2 + \mathcal{E}_2r_1}{r_1r_2 + R(r_1 + r_2)}$$

Express the power delivered to R:

$$P = I_3^2 R = \left(\frac{\mathcal{E}_1 r_2 + \mathcal{E}_2 r_1}{r_1 r_2 + R(r_1 + r_2)}\right)^2 R$$

$$= \left(\frac{\mathcal{E}_1 r_2 + \mathcal{E}_2 r_1}{r_1 + r_2}\right)^2 \left[\frac{R}{(R + A)^2}\right]$$

where

$$A = \frac{r_1 r_2}{r_1 + r_2}$$

Noting that the quantity in parentheses is independent of R and that therefore we can ignore it, differentiate P with respect to R and set the derivative equal to zero:

$$\frac{dP}{dR} = \frac{d}{dR}\left[\frac{R}{(R + A)^2}\right]$$

$$= \frac{(R + A)^2 - R\frac{d}{dR}(R + A)^2}{(R + A)^4}$$

$$= \frac{(R + A)^2 - 2R(R + A)}{(R + A)^4}$$

$$= 0 \text{ for extrema}$$

Solve for R to obtain:

$$R = A = \frac{r_1 r_2}{r_1 + r_2}$$

To establish that this value for R corresponds to a maximum, we need to evaluate the second derivative of P with respect to R at $R = A$ and show that this quantity is negative, i.e., concave downward:

$$\frac{d^2 P}{dR^2} = \frac{d}{dR}\left[\frac{(R + A)^2 - 2R(R + A)}{(R + A)^4}\right]$$

$$= \frac{2R - 4A}{(R + A)^4}$$

and

$$\left.\frac{d^2 P}{dR^2}\right|_{R=A} = \frac{-2A}{(R + A)^4} < 0$$

We can conclude that:

$$R = \boxed{\frac{r_1 r_2}{r_1 + r_2}} \text{ maximizes the power}$$

delivered by the sources.

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Picture the Problem Let Q_1 and Q_2 represent the final charges on the capacitors C_1 and C_2. Knowing that charge is conserved as it is redistributed to the two capacitors and that the final-state potential differences across the two capacitors will be the same, we can obtain two equations in the unknowns Q_1 and Q_2 that we can solve simultaneously. We can compare the initial and final energies stored in this system by expressing and simplifying their ratio. We can account for any difference between these energies by considering the role of the resistor in the circuit.

(*a*) Relate the total charge stored initially to the final charges Q_1 and Q_2 on C_1 and C_2:

$$Q = C_1V_0 = Q_1 + Q_2 \qquad (1)$$

Because, in their final state, the potential differences across the two capacitors will be the same:

$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2} \qquad (2)$$

Solve equation (2) for $Q2$ and substitute in equation (1) to obtain:

$$\frac{C_1}{C_2}Q_2 + Q_2 = C_1V_0$$

Solve for Q_2 to obtain:

$$Q_2 = \boxed{\frac{C_1C_2}{C_1 + C_2}V_0}$$

Substitute in either (1) or (2) and solve for Q_1 to obtain:

$$Q_1 = \boxed{\frac{C_1^2}{C_1 + C_2}V_0}$$

(*b*) Express the ratio of the initial and final energies of the system:

$$\frac{U_\text{i}}{U_\text{f}} = \frac{\frac{1}{2}C_1V_0^2}{\frac{1}{2}\frac{Q_1^2}{C_1} + \frac{1}{2}\frac{Q_2^2}{C_2}}$$

$$= \frac{C_1V_0^2}{\frac{\left(\frac{C_1^2}{C_1 + C_2}V_0\right)^2}{C_1} + \frac{\left(\frac{C_1C_2}{C_1 + C_2}V_0\right)^2}{C_2}}$$

Simplify this expression further to obtain:

$$\frac{U_\text{i}}{U_\text{f}} = \boxed{1 + \frac{C_2}{C_1}}$$

or U_i is greater than U_f by a factor of $1 + C_2/C_1$.

(*c*) The decrease in energy equals the energy dissipated as Joule heat in the resistor connecting the two capacitors.

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Picture the Problem Let q_1 and q_2 be the time-dependent charges on the two capacitors after the switches are closed. We can use Kirchhoff's loop rule and the conservat charge to a obtain a first-order linear differential equation describing the current I_2 through R after the switches are closed. We can solve this differential equatio assuming a solution of the form $q_2(t) = a + be^{-t/\tau}$ and requiring that the solution satisfy the boundary condition that $q_2(0) = 0$ and the differential equation be satisfied for all values of t. Once we know I_2, we can find the energy dissipated in the resistor as a function of time and the total energy dissipated in the resistor.

(*a*) Apply Kirchhoff's loop rule to the circuit to obtain:

$$\frac{q_1}{C_1} - IR - \frac{q_2}{C_2} = 0$$

or, because $I = dq_2/dt$,

$$\frac{q_1}{C_1} - R\frac{dq_2}{dt} - \frac{q_2}{C_2} = 0$$

Apply conservation of charge during the redistribution of charge to obtain:

$$q_1 = Q - q_2 = C_1V_0 - q_2$$

Substitute for q_1 to obtain:

$$V_0 - \frac{q_2}{C_1} - R\frac{dq_2}{dt} - \frac{q_2}{C_2} = 0$$

Rearrange to obtain the first- order differential equation:

$$R\frac{dq_2}{dt} + \left(\frac{C_1 + C_2}{C_1C_2}\right)q_2 = V_0$$

Assume a solution of the form:

$$q_2(t) = a + be^{-t/\tau} \quad (1)$$

Differentiate the assumed solution with respect to time to obtain:

$$\frac{dq_2(t)}{dt} = \frac{d}{dt}\left[a + be^{-t/\tau}\right] = -\frac{b}{\tau}e^{-t/\tau}$$

Substitute for dq_2/dt and q_2 in the differential equation to obtain:

$$R\left(-\frac{b}{\tau}e^{-t/\tau}\right) + \left(\frac{C_1 + C_2}{C_1C_2}\right)\left(a + be^{-t/\tau}\right) = V_0$$

Rearrange to obtain:

$$\left[-\frac{R}{\tau}e^{-t/\tau}\right]b+\left(\frac{C_1+C_2}{C_1C_2}\right)a+\left[\left(\frac{C_1+C_2}{C_1C_2}\right)e^{-t/\tau}\right]b=V_0$$

If this equation is to be satisfied for all values of t:

$$a=\frac{C_1C_2}{C_1+C_2}V_0=C_{\text{eq}}V_0$$

and

$$\left[-\frac{R}{\tau}e^{-t/\tau}\right]b+\left[\left(\frac{C_1+C_2}{C_1C_2}\right)e^{-t/\tau}\right]b=0$$

or

$$-\frac{R}{\tau}+\frac{C_1+C_2}{C_1C_2}=0$$

Solve for τ to obtain:

$$\tau=R\frac{C_1C_2}{C_1+C_2}=RC_{\text{eq}}$$

Substitute the boundary condition $q_2(0) = 0$ in equation (1):

$$0=a+b$$

or

$$b=-a=-C_{\text{eq}}V_0$$

Substitute for a and b in equation (1) to obtain:

$$q_2(t)=C_{\text{eq}}V_0-C_{\text{eq}}V_0\,e^{-t/\tau}=C_{\text{eq}}V_0\left(1-e^{-t/\tau}\right)$$

Differentiate $q_2(t)$ with respect to time to find the current:

$$I(t)=\frac{dq_2(t)}{dt}=C_{\text{eq}}V_0\frac{d}{dt}\left(1-e^{-t/\tau}\right)=C_{\text{eq}}V_0\left(-e^{-t/\tau}\right)\left(-\frac{1}{\tau}\right)=\frac{C_{\text{eq}}V_0}{\tau}e^{-t/\tau}=\boxed{\frac{V_0}{R}e^{-t/\tau}}$$

(b) Express the energy dissipated in the resistor as a function of time:

$$P(t)=I^2R=\left(\frac{V_0}{R}e^{-t/\tau}\right)^2R=\boxed{\frac{V_0^2}{R}e^{-2t/\tau}}$$

(c) The energy dissipated in the resistor is the integral of $P(t)$

$$E=\frac{V_0^2}{R}\int_0^\infty e^{-2t'/RC_{\text{eq}}}dt'=\boxed{\tfrac{1}{2}V_0^2C_{\text{eq}}}$$

between $t = 0$ and $t = \infty$:

This is exactly the difference between the initial and final stored energies found in the preceding problem, which confirms the statement at the end of that problem that the difference in the stored energies equals the energy dissipated in the resistor.

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Picture the Problem We can apply Kirchhoff's loop rule to find the initial current drawn from the battery and the current drawn from the battery a long time after S_1 is closed. We can also use the loop rule to find the final voltages across the capacitors and the current in the 150-Ω resistor when S_2 is opened after having been closed for a long time.

(*a*) Apply Kirchhoff's loop rule to the loop that includes the source, the 100-Ω resistor, and the capacitor immediately after S_1 is closed to obtain:

$$12\text{ V} - I_{\text{bat}}(0)(100\,\Omega) - V_{\text{C0}} = 0$$

Because the capacitor is initially uncharged:

$$V_{\text{C0}} = 0$$

and

$$12\text{ V} - I_{\text{bat}}(0)(100\,\Omega) = 0$$

Solve for and evaluate $I_{\text{bat}}(0)$:

$$I_{\text{bat}}(0) = \frac{12\,V}{100\,\Omega} = \boxed{0.120\text{ A}}$$

(*b*) Apply Kirchhoff's loop rule to the loop that includes the source, the 100-Ω, 50-Ω, and 150-Ω resistor a long time after S_1 is closed to obtain:

$$12\text{ V} - I_{\infty}(100\,\Omega) - I_{\infty}(50\,\Omega) - I_{\infty}(150\,\Omega) = 0$$

Solve for and evaluate I_{∞}:

$$I_{\infty} = \frac{12\text{ V}}{100\,\Omega + 50\,\Omega + 150\,\Omega} = \boxed{40.0\text{ mA}}$$

(*c*) Apply Kirchhoff's loop rule to the loop that includes the source, the 100-Ω resistor, and C_1 a long time after both switches are closed to obtain:

$$12\text{ V} - I_{\text{bat}}(100\,\Omega) - V_{\text{C1}} = 0$$

Solve for and evaluate V_{C1}:

$$V_{C1} = 12\,\text{V} - (40\,\text{mA})(100\,\Omega) = \boxed{8.00\,\text{V}}$$

(*d*) Apply Kirchhoff's loop rule to the loop that includes the 150-Ω resistor and C_2 a long time after both switches are closed to obtain:

$$-V_{C2} + I_{\text{bat}}(150\,\Omega) = 0$$

Solve for and evaluate V_{C2}:

$$V_{C2} = I_{\text{bat}}(150\,\Omega) = (40\,\text{mA})(150\,\Omega) = \boxed{6.00\,\text{V}}$$

(*e*) Apply Kirchhoff's loop rule to the loop that includes the 150-Ω resistor and C_2 after S_2 is opened to obtain:

$$V_{C_2}(t) - I(t)(150\,\Omega) = 0$$

or

$$V_{C_2}(0)e^{-t/\tau} - I(t)(150\,\Omega) = 0$$

Solve for $I(t)$ to obtain:

$$I(t) = \frac{V_{C_2}(0)}{150\,\Omega}e^{-t/\tau} = \frac{6\,\text{V}}{150\,\Omega}e^{-t/(150\,\Omega)(50\,\mu\text{F})} = \boxed{(40\,\text{mA})e^{-t/7.50\,\text{ms}}}$$

***147** •••

Picture the Problem We can use the definition of differential resistance and the expression for the diode current given in problem 54 to express R_d and establish the required results.

The differential resistance R_d is given by:

$$R_d = \frac{dV}{dI} = \left(\frac{dI}{dV}\right)^{-1}$$

From Problem 54, the current in the diode is given by:

$$I = I_0\left(e^{V/25\,\text{mV}} - 1\right) \qquad (1)$$

Substitute for I to obtain:

$$R_d = \left\{\frac{d}{dV}\left[I_0\left(e^{V/25\,\text{mV}} - 1\right)\right]\right\}^{-1} = \frac{25\,\text{mV}}{I_0}e^{-V/25\,\text{mV}} \qquad (2)$$

For $V > 0.6$ V, equation (1) becomes:

$$I \approx I_0 e^{V/25\,\text{mV}}$$

Solve for the exponential factor to obtain:

$$e^{V/25\,\text{mV}} \approx \frac{I}{I_0} \Rightarrow e^{-V/25\,\text{mV}} \approx \frac{I_0}{I}$$

Substitute in equation (2) to obtain:

$$R_d \approx \frac{25\,\text{mV}}{I_0}\frac{I_0}{I} = \boxed{\frac{25\,\text{mV}}{I}}$$

Examination of equation (2) shows that, for $V < 0$, R_d increases exponentially. This result, together with that for $V > 0.6$ V, justifies the assumptions made in Problem 55.

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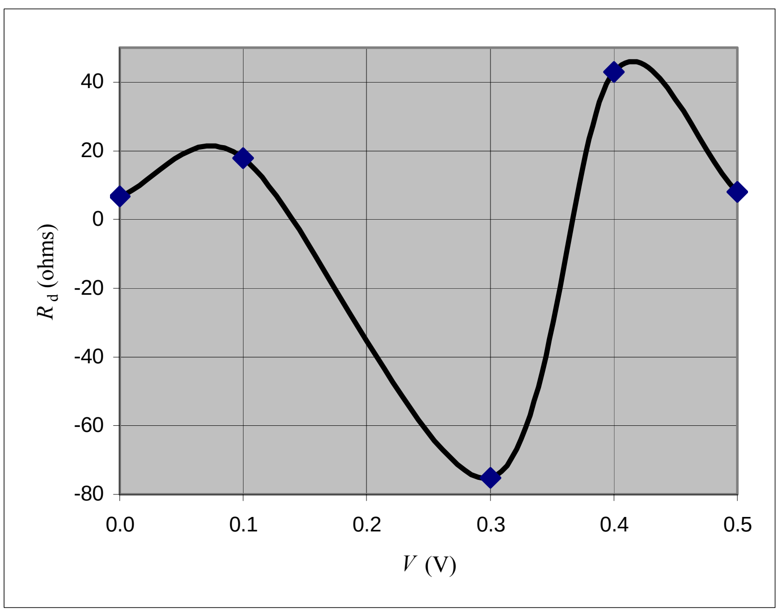
Picture the Problem We can approximate the slope of the graph in Figure 25-77 and take its reciprocal to obtain values for R_d that we can plot as a function of V.

Use the graph in Figure 25-77 to complete the table to the right.

V (V)	R_d (Ω)
0	6.67
0.1	17.9
0.3	−75.2
0.4	42.9
0.5	8

The following graph was plotted using a spreadsheet program.



The differential resistance becomes negative at approximately 0.14 V.

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Picture the Problem We can use the definition of current to find the number of electrons accelerated in each pulse and the average current in the beam. The average and peak power of the accelerator can be found using $P_{av} = I_{av}V$ and

$P_{\text{peak}} = I_{\text{peak}}V$ and the duty factor from its definition.

(*a*) Use the definition of current to relate the number of electrons accelerated in each pulse to the duration of the pulse:

$$I_{\text{pulse}} = \frac{\Delta Q}{\Delta t} = \frac{ne}{\Delta t}$$

where n is the number of electrons in each pulse.

Solve for and evaluate n:

$$n = \frac{I_{\text{pulse}}\Delta t}{e} = \frac{(1.6\,\text{A})(0.1\,\mu\text{s})}{1.602\times10^{-19}\,\text{C}} = 9.99\times10^{11} \approx \boxed{10^{12}}$$

(*b*) Using the definition of current we have:

$$I_{\text{av}} = \frac{Q_{\text{pulse}}}{\Delta t_{\text{between pulses}}} = \frac{ne}{10^{-3}\,\text{s}} = \frac{10^{12}\left(1.60\times10^{-19}\,\text{C}\right)}{10^{-3}\,\text{s}} = \boxed{0.160\,\text{mA}}$$

(*c*) Express the average power output in terms of the average current:

$$P_{\text{av}} = I_{\text{av}}V = (0.160\,\text{mA})(400\,\text{MV}) = \boxed{64.0\,\text{kW}}$$

(*d*) Express the peak power output in terms of the pulse current:

$$P_{\text{peak}} = I_{\text{pulse}}V = (1.6\,\text{A})(400\,\text{MV}) = \boxed{640\,\text{MW}}$$

(*e*) The duty factor is defined to be:

$$\text{duty factor} = \frac{\Delta t}{\text{time between pulses}} = \frac{0.1\,\mu\text{s}}{10^{-3}\,\text{s}} = \boxed{10^{-4}}$$

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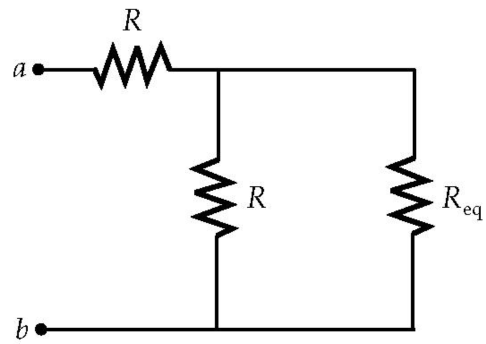
Picture the Problem Let R be the resistance of each resistor in the ladder and let R_{eq} be the equivalent resistance of the infinite ladder. If the resistance is finite and non-zero, then adding one or more stages to the ladder will not change the resistance of the network. We can apply the rules for resistance combination to the diagram shown to the right to obtain a quadratic equation in R_{eq} that we can solve for the equivalent resistance between points *a* and *b*.



The equivalent resistance of the series combination of R and ($R \parallel R_{eq}$) is R_{eq}, so:

$$R_{eq} = R + R \| R_{eq} = R + \frac{RR_{eq}}{R + R_{eq}}$$

Simplify to obtain:

$$R_{eq}^2 - RR_{eq} - R^2 = 0$$

Solve for R_{eq} to obtain:

$$R_{eq} = \left(\frac{1+\sqrt{5}}{2}\right)R$$

For $R = 1\Omega$:

$$R_{eq} = \left(\frac{1+\sqrt{5}}{2}\right)(1\,\Omega) = \boxed{1.62\,\Omega}$$

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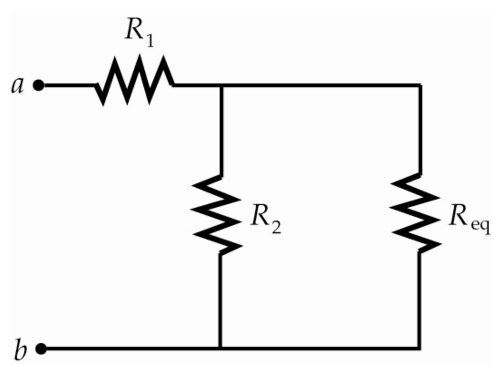
Picture the Problem Let R_{eq} be the equivalent resistance of the infinite ladder. If the resistance is finite and non-zero, then adding one or more stages to the ladder will not change the resistance of the network. We can apply the rules for resistance combination to the diagram shown to the right to obtain a quadratic equation in R_{eq} that we can solve for the equivalent resistance between points *a* and *b*.



The equivalent resistance of the series combination of R_1 and ($R_2 \parallel R_{eq}$) is R_{eq}, so:

$$R_{eq} = R_1 + R_2 \| R_{eq} = R_1 + \frac{R_2 R_{eq}}{R_2 + R_{eq}}$$

Simplify to obtain:

$$R_{eq}^2 - R_1 R_{eq} - R_1 R_2 = 0$$

Solve for the positive value of R_{eq} to obtain:

$$R_{eq} = \boxed{\frac{R_1 + \sqrt{R_1^2 + 4R_1R_2}}{2}}$$