Chapter 26
The Magnetic Field

Conceptual Problems

***1** •

Determine the Concept Because the electrons are initially moving at 90° to the magnetic field, they will be deflected in the direction of the magnetic force acting on them. Use the right-hand rule based on the expression for the magnetic force acting on a moving charge $\vec{F} = q\vec{v} \times \vec{B}$, remembering that, for a negative charge, the force is in the direction opposite that indicated by the right-hand rule, to convince yourself that the particle will follow the path whose terminal point on the screen is 2. $\boxed{(b) \text{ is correct.}}$

2 •

Determine the Concept One cannot define the direction of the force arbitrarily. By experiment, $\vec{F}$ is perpendicular to $\vec{B}$.

3 •

Determine the Concept False. An object experiences acceleration if either its speed changes or the direction it is moving changes. The magnetic force, acting perpendicular to the direction a charged particle is moving, changes the particle's *velocity* by changing the direction it is moving and hence accelerates the particle.

4 •

Determine the Concept Yes; it will be deflected upward. Because the beam passes through undeflected when traveling from left to right, we know that the upward electric force must be balanced by a downward magnetic force. Application of the right-hand rule tells us that the magnetic field must be out of the page. When the beam is reversed, the magnetic force (as well as the electric force) acting on it is upward.

***5** •

Determine the Concept The alternating current running through the filament is changing direction every 1/60 s, so in a magnetic field the filament experiences a force which alternates in direction at that frequency.

6 •

Determine the Concept The magnitude of the torque on a current loop is given by $\tau = \mu B \sin\theta$, where θ is the angle between the magnetic field and a normal to the surface of the loop. To maximize τ, $\sin\theta = 1$ and $\theta = 90°$. Hence the normal to the plane of the loop should be perpendicular to $\vec{B}$.

7 •

(*a*) True. This is an experimental fact and is the basis for the definition of the magnetic force on a moving charged particle being expressed in terms of the cross product of $\vec{v}$ and $\vec{B}$; i.e. $\vec{F} = q\vec{v} \times \vec{B}$.

(*b*) True. This is another experimental fact. The torque on a magnet is a restoring torque, i.e., one that acts in such a direction as to align the magnet with magnetic field.

(*c*) True. We can use a right-hand rule to relate the direction of the magnetic field around the loop to the direction of the current. Doing so indicates that one side of the loop acts like a north pole and the other like a south pole.

(*d*) False. The period of a particle moving in a circular path in a magnetic field is given by $T = 2\pi\sqrt{mr/qvB}$ and, hence, is proportional to the square root of the radius of the circle.

(*e*) True. The drift velocity is related to the Hall voltage according to $v_d = V_H/Bw$ where w is the width of the Hall-effect material.

***8** • .

Determine the Concept The direction in which a particle is deflected by a magnetic field will be unchanged by any change in the definition of the direction of the magnetic field. Since we have reversed the direction of the field, we must define the direction in which particles are deflected by a "left-hand" rule instead of a "right-hand" rule.

9 •

Determine the Concept Choose a right-handed coordinate system in which east is the positive x direction and north is the positive y direction. Then the magnetic force acting on the particle is given by $\vec{F} = qv\hat{i} \times B\hat{j} = qvB(\hat{i} \times \hat{j}) = qvB\hat{k}$. Hence, the magnetic force is upward.

10 •

Determine the Concept Application of the right-hand rule tells us that this positively charged particle would have to be moving in the northwest direction with the magnetic field upward in order for the magnetic force to be toward the northeast. The situation described cannot exist. $\boxed{(e)\text{ is correct.}}$

11 •

Picture the Problem We can use Newton's 2nd law for circular motion to express the radius of curvature R of each particle in terms of its charge, momentum, and the magnetic field. We can then divide the proton's radius of curvature by that of the ^{7}Li nucleus to decide which of these alternatives is correct.

Apply $\sum F_{\text{radial}} = ma_c$ to the lithium nucleus to obtain:

$$qvB = m\frac{v^2}{R}$$

Solve for *r*:

$$R = \frac{mv}{qB}$$

For the ^{7}Li nucleus this becomes:

$$R_{\text{Li}} = \frac{p_{\text{Li}}}{3eB}$$

For the proton we have:

$$R_{\text{p}} = \frac{p_{\text{p}}}{eB}$$

Divide equation (2) by equation (1) and simplify to obtain:

$$\frac{R_{\text{p}}}{R_{\text{Li}}} = \frac{\frac{p_{\text{p}}}{eB}}{\frac{p_{\text{Li}}}{3eB}} = 3\frac{p_{\text{p}}}{p_{\text{Li}}}$$

Because the momenta are equal:

$$\frac{R_{\text{p}}}{R_{\text{Li}}} = 3 \text{ and } \boxed{(a)\text{ is correct.}}$$

***12 •**

Determine the Concept Application of the right-hand rule indicates that a positively charged body would experience a downward force and, in the absence of other forces, be deflected downward. Because the direction of the magnetic force on an electron is opposite that of the force on a positively charged object, an electron will be deflected upward. $\boxed{(c)\text{ is correct.}}$

13 ••

Determine the Concept From relativity; this is equivalent to the electron moving from right to left at velocity *v* with the magnet stationary. When the electron is directly over the magnet, the field points directly up, so there is a force directed out of the page on the electron.

14 •

Similarities	Differences
Magnetic field lines are similar to electric field lines in that their density is a measure of the strength of the field; the lines point in the direction of the field; also, magnetic field lines do not cross.	They differ from electric field lines in that magnetic field lines must close on themselves (there are no isolated magnetic poles), and the force on a charge depends on the velocity of the charge and is perpendicular to the magnetic field lines.

15 •

Determine the Concept If only $\vec{F}$ and I are known, one can only conclude that the magnetic field $\vec{B}$ is in the plane perpendicular to $\vec{F}$. The specific direction of $\vec{B}$ is undetermined.

Estimation and Approximation

***16** ••

Picture the Problem If the electron enters the magnetic field in the coil with speed v, it will travel in a circular path under the influence of the magnetic force acting on. We can apply Newton's 2nd law to the electron in this field to obtain an expression for the magnetic field. We'll assume that the deflection of the electron is small over the distance it travels in the magnetic field, but that, once it is through the region of the magnetic field, it travels at an angle θ with respect to the direction it was originally traveling.

Apply $\sum F = ma_c$ to the electron in the magnetic field to obtain:

$$evB = m\frac{v^2}{r}$$

Solve for B:

$$B = \frac{mv}{er}$$

The kinetic energy of the electron is:

$$K = eV = \tfrac{1}{2}mv^2$$

Solve for v to obtain:

$$v = \sqrt{\frac{2eV}{m}}$$

Substitute for v in the expression for r:

$$B = \frac{m}{er}\sqrt{\frac{2eV}{m}} = \frac{1}{r}\sqrt{\frac{2mV}{e}}$$

Because $\theta << 1$:

$$d \approx r\sin\theta \Rightarrow r \approx \frac{d}{\sin\theta}$$

Substitute for r in the expression for B to obtain:

$$B = \frac{\sin\theta}{d}\sqrt{\frac{2mV}{e}}$$

For maximum deflection, $\theta \approx 45°$. Substitute numerical values and evaluate B:

$$B = \frac{\sin 45°}{0.05\,\text{m}}\sqrt{\frac{2(9.11\times10^{-31}\,\text{kg})(15\,\text{kV})}{1.60\times10^{-19}\,\text{C}}} = \boxed{5.84\,\text{mT}}$$

17 ••

Picture the Problem Let h be the height of the orbit above the surface of the earth, m the mass of the micrometeorite, and v its speed. We can apply Newton's 2nd law to the orbiting micrometeorite with $F_{mag} = qvB$ to derive an expression for the charge-to-mass ratio of the micrometeorite.

(*a*) Apply $\sum F = ma_c$ to the micrometeorite orbiting under the influence of the magnetic force:

$$qvB = m\frac{v^2}{h + R_{\text{earth}}}$$

Solve for *q*/*m* to obtain:

$$\frac{q}{m} = \frac{v}{B(h + R_{\text{earth}})}$$

Substitute numerical values and evaluate *q*/*m*:

$$\frac{q}{m} = \frac{30\,\text{km/s}}{(5\times10^{-5}\,\text{T})(400\,\text{km} + 6370\,\text{km})} = \boxed{88.6\,\text{C/kg}}$$

(*b*) Solve the result for *q*/*m* obtained in (*a*) for *q* to obtain:

$$q = (88.6\,\text{C/kg})m$$

Substitute numerical values and evaluate *q*:

$$q = (88.6\,\text{C/kg})(3\times10^{-10}\,\text{kg}) = \boxed{26.6\,\text{nC}}$$

Force Exerted by a Magnetic Field

18 •

Picture the Problem The magnetic force acting on a charge is given by $\vec{F} = q\vec{v}\times\vec{B}$. We can express $\vec{v}$ and $\vec{B}$, form their vector (a.k.a. ″cross″) product, and multiply by the scalar *q* to find $\vec{F}$.

Express the force acting on the proton:

$$\vec{F} = q\vec{v}\times\vec{B}$$

Express $\vec{v}$:

$$\vec{v} = (4.46\,\text{Mm/s})\hat{i}$$

Express $\vec{B}$:

$$\vec{B} = (1.75\,\text{T})\hat{k}$$

Substitute numerical values and evaluate $\vec{F}$:

$$\vec{F} = (1.60\times10^{-19}\,\text{C})\left[(4.46\,\text{Mm/s})\hat{i}\times(1.75\,\text{T})\hat{k}\right] = \boxed{-(1.25\,\text{pN})\hat{j}}$$

19 •

Picture the Problem The magnetic force acting on the charge is given by $\vec{F} = q\vec{v}\times\vec{B}$. We can express $\vec{v}$ and $\vec{B}$, form their vector (a.k.a. ″cross″) product, and multiply by the scalar q to find $\vec{F}$.

Express the force acting on the charge: $\vec{F} = q\vec{v} \times \vec{B}$

Substitute numerical values to obtain: $\vec{F} = (-3.64\,\text{nC})\left[(2.75\times10^6\ \text{m/s})\hat{i} \times \vec{B}\right]$

(a) Evaluate $\vec{F}$ for $\vec{B}$ = 0.38 T $\hat{j}$:

$$\vec{F} = (-3.64\,\text{nC})\left[(2.75\times10^6\ \text{m/s})\hat{i} \times (0.38\,\text{T})\hat{j}\right] = \boxed{-(3.80\,\text{mN})\hat{k}}$$

(b) Evaluate $\vec{F}$ for $\vec{B}$ = 0.75 T $\hat{i}$ + 0.75 T $\hat{j}$:

$$\vec{F} = (-3.64\,\text{nC})\left[(2.75\times10^6\ \text{m/s})\hat{i} \times \{(0.75\,\text{T})\hat{i} + (0.75\,\text{T})\hat{j}\}\right] = \boxed{-(7.51\,\text{mN})\hat{k}}$$

(c) Evaluate $\vec{F}$ for $\vec{B}$ = 0.65 T $\hat{i}$:

$$\vec{F} = (-3.64\,\text{nC})\left[(2.75\times10^6\ \text{m/s})\hat{i} \times (0.65\,\text{T})\hat{i}\right] = \boxed{0}$$

(d) Evaluate $\vec{F}$ for $\vec{B}$ = 0.75 T $\hat{i}$ + 0.75 T $\hat{k}$:

$$\vec{F} = (-3.64\,\text{nC})\left[(2.75\times10^6\ \text{m/s})\hat{i} \times (0.75\,\text{T})\hat{i} + (0.75\,\text{T})\hat{k}\right] = \boxed{(7.51\,\text{mN})\hat{j}}$$

20 •

Picture the Problem The magnetic force acting on the proton is given by $\vec{F} = q\vec{v} \times \vec{B}$. We can express $\vec{v}$ and $\vec{B}$, form their vector (a.k.a. ″cross″) product, and multiply by the scalar q to find $\vec{F}$.

Express the force acting on the proton: $\vec{F} = q\vec{v} \times \vec{B}$

(a) Evaluate $\vec{F}$ for $\vec{v}$ = 2.7 Mm/s $\hat{i}$:

$$\vec{F} = (1.60\times10^{-19}\ \text{C})\left[(2.7\times10^6\ \text{m/s})\hat{i} \times (1.48\,\text{T})\hat{k}\right] = \boxed{-(0.640\,\text{pN})\hat{j}}$$

(b) Evaluate $\vec{F}$ for $\vec{v}$ = 3.7 Mm/s $\hat{j}$:

$$\vec{F} = (1.60\times10^{-19}\ \text{C})\left[(3.7\times10^6\ \text{m/s})\hat{j} \times (1.48\,\text{T})\hat{k}\right] = \boxed{(0.876\,\text{pN})\hat{i}}$$

(*c*) Evaluate $\vec{F}$ for $\vec{v}$ = 6.8 Mm/s $\hat{k}$:

$$\vec{F} = (1.60\times10^{-19}\text{ C})\left[(6.8\times10^{6}\text{ m/s})\hat{k}\times(1.48\text{ T})\hat{k}\right] = \boxed{0}$$

(*d*) Evaluate $\vec{F}$ for $\vec{v} = 4.0\,\text{Mm/s}\,\hat{i} + 3.0\,\text{Mm/s}\,\hat{j}$:

$$\begin{aligned}\vec{F} &= (1.60\times10^{-19}\text{ C})\left[\{(4.0\,\text{Mm/s})\hat{i} + (3.0\,\text{Mm/s})\hat{j}\}\times(1.48\text{ T})\hat{k}\right]\\ &= \boxed{(0.710\,\text{pN})\hat{i} - (0.947\,\text{pN})\hat{j}}\end{aligned}$$

21 •

Picture the Problem The magnitude of the magnetic force acting on a segment of wire is given by $F = I\ell B\sin\theta$ where ℓ is the length of the segment of wire, B is the magnetic field, and θ is the angle between the segment of wire and the direction of the magnetic field.

Express the magnitude of the magnetic force acting on the segment of wire:

$$F = I\ell B\sin\theta$$

Substitute numerical values and evaluate F:

$$\begin{aligned}F &= (2.6\text{ A})(2\text{ m})(0.37\text{ T})\sin 30^\circ\\ &= \boxed{0.962\text{ N}}\end{aligned}$$

***22** •

Picture the Problem We can use $\vec{F} = I\vec{L}\times\vec{B}$ to find the force acting on the wire segment.

Express the force acting on the wire segment:

$$\vec{F} = I\vec{L}\times\vec{B}$$

Substitute numerical values and evaluate $\vec{F}$:

$$\begin{aligned}\vec{F} &= (2.7\text{ A})\left[(3\text{ cm})\hat{i} + (4\text{ cm})\hat{j}\right]\times(1.3\text{ T})\hat{i}\\ &= \boxed{-(0.140\text{ N})\hat{k}}\end{aligned}$$

23 •

Picture the Problem The magnetic force acting on the electron is given by $\vec{F} = q\vec{v}\times\vec{B}$. We can form the vector product of $\vec{v}$ and $\vec{B}$ and multiply by the charge of the electron to find $\vec{F}$ and obtain its magnitude using $F = \sqrt{F_x^2 + F_y^2 + F_z^2}$. The direction angles are given by $\theta_x = \tan^{-1}(F_x/F)$, $\theta_y = \tan^{-1}(F_y/F)$, and $\theta_z = \tan^{-1}(F_z/F)$.

Express the force acting on the proton: $\vec{F} = q\vec{v} \times \vec{B}$

Express the magnitude of $\vec{F}$ in terms of its components: $F = \sqrt{F_x^2 + F_y^2 + F_z^2}$ (1)

Substitute numerical values and evaluate $\vec{F}$:

$$\begin{aligned}\vec{F} &= \left(-1.60\times10^{-19}\ \text{C}\right)\left[\left\{\left(2\ \text{Mm/s}\right)\hat{\boldsymbol{i}} - \left(3\ \text{Mm/s}\right)\hat{\boldsymbol{j}}\right\}\times\left(0.8\,\hat{\boldsymbol{i}} + 0.6\,\hat{\boldsymbol{j}} - 0.4\,\hat{\boldsymbol{k}}\right)\text{T}\right]\\ &= \left(-0.192\ \text{pN}\right)\hat{\boldsymbol{k}} + \left(-0.128\ \text{pN}\right)\hat{\boldsymbol{j}} + \left(-0.384\ \text{pN}\right)\hat{\boldsymbol{k}} + \left(-0.192\ \text{pN}\right)\hat{\boldsymbol{i}}\\ &= \boxed{-\left(0.192\ \text{pN}\right)\hat{\boldsymbol{i}} - \left(0.128\ \text{pN}\right)\hat{\boldsymbol{j}} - \left(0.576\ \text{pN}\right)\hat{\boldsymbol{k}}}\end{aligned}$$

Substitute in equation (1) to obtain:

$$F = \sqrt{\left(-0.192\ \text{pN}\right)^2 + \left(-0.128\ \text{pN}\right)^2 + \left(-0.576\ \text{pN}\right)^2} = \boxed{0.621\ \text{pN}}$$

Express and evaluate the angle $\vec{F}$ makes with the x axis:

$$\theta_x = \cos^{-1}\left(\frac{F_x}{F}\right) = \cos^{-1}\left(\frac{-0.192\ \text{pN}}{0.621\ \text{pN}}\right) = \boxed{108^\circ}$$

Express and evaluate the angle $\vec{F}$ makes with the y axis:

$$\theta_y = \cos^{-1}\left(\frac{F_y}{F}\right) = \cos^{-1}\left(\frac{-0.128\ \text{pN}}{0.621\ \text{pN}}\right) = \boxed{102^\circ}$$

Express and evaluate the angle $\vec{F}$ makes with the z axis:

$$\theta_z = \cos^{-1}\left(\frac{F_z}{F}\right) = \cos^{-1}\left(\frac{-0.576\ \text{pN}}{0.621\ \text{pN}}\right) = \boxed{158^\circ}$$

24 ••

Picture the Problem We can use $\vec{F} = I\vec{\ell}\times\vec{B}$ to find the force acting on the segments of the wire as well as the magnetic force acting on the wire if it were a straight segment from *a* to *b*.

Express the magnetic force acting on the wire: $\vec{F} = \vec{F}_{3\,\text{cm}} + \vec{F}_{4\,\text{cm}}$

Evaluate $\vec{F}_{3\,\text{cm}}$:

$$\vec{F}_{3\,\text{cm}} = (1.8\,\text{A})\left[(3\,\text{cm})\hat{i} \times (1.2\,\text{T})\hat{k}\right] = (0.0648\,\text{N})\left(-\hat{j}\right) = -(0.0648\,\text{N})\hat{j}$$

Evaluate $\vec{F}_{4\,\text{cm}}$:

$$\vec{F}_{4\,\text{cm}} = (1.8\,\text{A})\left[(4\,\text{cm})\hat{j} \times (1.2\,\text{T})\hat{k}\right] = (0.0864\,\text{N})\hat{i}$$

Substitute to obtain:

$$\vec{F} = -(0.0648\,\text{N})\hat{j} + (0.0864\,\text{N})\hat{i} = \boxed{(0.0864\,\text{N})\hat{i} - (0.0648\,\text{N})\hat{j}}$$

If the wire were straight from *a* to *b*:

$$\vec{\ell} = (3\,\text{cm})\hat{i} + (4\,\text{cm})\hat{j}$$

The magnetic force acting on the wire is:

$$\vec{F} = (1.8\,\text{A})\left[(3\,\text{cm})\hat{i} + (4\,\text{cm})\hat{j}\right] \times (1.2\,\text{T})\hat{k} = -(0.0648\,\text{N})\hat{j} + (0.0864\,\text{N})\hat{i} = \boxed{(0.0864\,\text{N})\hat{i} - (0.0648\,\text{N})\hat{j}}$$

in agreement with the result obtained above when we treated the two straight segments of the wire separately.

25 ••

Picture the Problem Because the magnetic field is horizontal and perpendicular to the wire, the force it exerts on the current-carrying wire will be vertical. Under equilibrium conditions, this upward magnetic force will be equal to the downward gravitational force acting on the wire.

Apply $\sum F_{\text{vertical}} = 0$ to the wire:

$$F_{\text{mag}} - w = 0$$

Express F_{mag}:

$$F_{\text{mag}} = I\ell B$$

because $\theta = 90°$.

Substitute to obtain:

$$I\ell B - mg = 0$$

Solve for I:

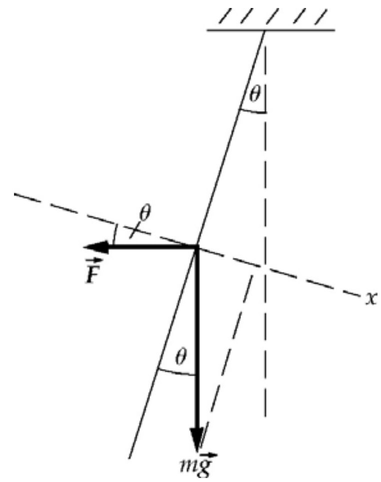
$$I = \frac{mg}{\ell B}$$

Substitute numerical values and evaluate I:

$$I = \frac{(50\,\text{g})(9.81\,\text{m/s}^2)}{(25\,\text{cm})(1.33\,\text{T})} = \boxed{1.48\,\text{A}}$$

***26 ••**

Picture the Problem The diagram shows the gaussmeter displaced from equilibrium under the influence of the gravitational and magnetic forces acting on it. We can apply the condition for translational equilibrium in the x direction to find the equilibrium angular displacement of the wire from the vertical. In part (b) we can solve the equation derived in part (a) for B and evaluate this expression for the given data to find the horizontal magnetic field sensitivity of this gaussmeter.



(*a*) Apply $\sum F_x = 0$ to the wire to obtain:

$$mg\sin\theta - F\cos\theta = 0$$

Substitute for F and solve for θ to obtain:

$$mg\sin\theta - I\ell B\cos\theta = 0 \qquad (1)$$

and

$$\theta = \tan^{-1}\left(\frac{I\ell B}{mg}\right)$$

Substitute numerical values and evaluate θ:

$$\theta = \tan^{-1}\left[\frac{(0.2\,\text{A})(0.5\,\text{m})(0.04\,\text{T})}{(0.005\,\text{kg})(9.81\,\text{m/s}^2)}\right] = \boxed{4.66^\circ}$$

(*b*) Solve equation (1) for B to obtain:

$$B = \frac{mg\tan\theta}{I\ell}$$

For a displacement from vertical of 0.5 mm:

$$\tan\theta \approx \sin\theta = \frac{0.5\,\text{mm}}{0.5\,\text{m}} = 0.001$$

and

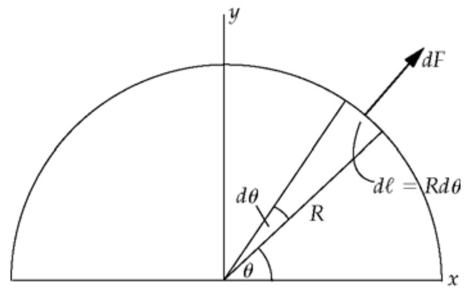
$$\theta = 0.001\,\text{rad}$$

Substitute numerical values and evaluate B:

$$B = \frac{(0.005\,\text{kg})(9.81\,\text{m/s}^2)(0.001\,\text{rad})}{(20\,\text{A})(0.5\,\text{m})} = \boxed{4.91\,\mu\text{T}}$$

27 ••

Picture the Problem With the current in the direction indicated and the magnetic field in the *z* direction, pointing out of the plane of the page, the force is in the radial direction and we can integrate the element of force dF acting on an element of length $d\ell$ between $\theta = 0$ and π to find the force acting on the semicircular portion of the loop and use the expression for the force on a current-carrying wire in a uniform magnetic field to find the force on the straight segment of the loop.



Express the net force acting on the semicircular loop of wire:

$$F = F_{\text{semicircular loop}} + F_{\text{straight segment}} \qquad (1)$$

Express the force acting on the straight segment of the loop:

$$\vec{F}_{\text{straight segment}} = I\vec{\ell} \times \vec{B} = -2RIB$$

Express the force dF acting on the element of the wire of length $d\ell$:

$$dF = Id\ell B = IRBd\theta$$

Express the *x* and *y* components of dF:

$$dF_x = dF \cos\theta$$

and

$$dF_y = dF \sin\theta$$

Because, by symmetry, the *x* component of the force is zero, we can integrate the *y* component to find the force on the wire:

$$dF_y = IRB \sin\theta \, d\theta$$

and

$$F_y = RIB \int_0^{\pi} \sin\theta \, d\theta = 2RIB$$

Substitute in equation (1) to obtain:

$$F = 2RIB - 2RIB = \boxed{0}$$

28 ••

Picture the Problem We can use the information given in the 1st and 2nd sentences to obtain an expression containing the components of the magnetic field $\vec{B}$. We can then use the information in the 1st and 3rd sentences to obtain a second equation in these components that we can solve simultaneously for the components of $\vec{B}$.

Express the magnetic field $\vec{B}$ in terms of its components:

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k} \qquad (1)$$

Express $\vec{F}$ in terms of $\vec{B}$:

$$\begin{aligned}\vec{F} &= I\vec{\ell}\times\vec{B}\\ &= (4\,\text{A})\left[(0.1\,\text{m})\hat{k}\right]\times\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)\\ &= (0.4\,\text{A}\cdot\text{m})\hat{k}\times\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)\\ &= (0.4\,\text{A}\cdot\text{m})B_y\hat{j} - (0.4\,\text{A}\cdot\text{m})B_y\hat{i}\end{aligned}$$

Equate the components of this expression for $\vec{F}$ with those given in the second sentence of the statement of the problem to obtain:

$$(0.4\,\text{A}\cdot\text{m})B_y = 0.2\,\text{N}$$

and

$$(0.4\,\text{A}\cdot\text{m})B_x = 0.2\,\text{N}$$

Noting that B_z is undetermined, solve for B_x and B_y:

$B_x = 0.5\,\text{T}$ and $B_y = 0.5\,\text{T}$

When the wire is rotated so that the current flows in the positive x direction:

$$\begin{aligned}\vec{F} &= I\vec{\ell}\times\vec{B}\\ &= (4\,\text{A})\left[(0.1\,\text{m})\hat{i}\right]\times\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)\\ &= (0.4\,\text{A}\cdot\text{m})\hat{i}\times\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)\\ &= -(0.4\,\text{A}\cdot\text{m})B_z\hat{j} + (0.4\,\text{A}\cdot\text{m})B_y\hat{k}\end{aligned}$$

Equate the components of this expression for $\vec{F}$ with those given in the third sentence of the problem statement to obtain:

$$-(0.4\,\text{A}\cdot\text{m})B_z = 0$$

and

$$(0.4\,\text{A}\cdot\text{m})B_y = 0.2\,\text{N}$$

Solve for B_z and B_y to obtain:

$$B_z = 0$$

and, in agreement with our results above,

$$B_y = 0.5\,\text{T}$$

Substitute in equation (1) to obtain:

$$\vec{B} = \boxed{(0.5\,\text{T})\hat{i} + (0.5\,\text{T})\hat{j}}$$

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Picture the Problem We can use the information given in the 1st and 2nd sentences to obtain an expression containing the components of the magnetic field $\vec{B}$. We can then use the information in the 1st and 3rd sentences to obtain a second equation in these components that we can solve simultaneously for the components of $\vec{B}$.

Express the magnetic field $\vec{B}$ in terms of its components:

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k} \qquad (1)$$

Express $\vec{F}$ in terms of $\vec{B}$:

$$\begin{aligned}\vec{F} &= I\vec{\ell}\times\vec{B}\\ &= (2\,\text{A})\left[(0.1\,\text{m})\hat{i}\right]\times\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)\\ &= (0.2\,\text{A}\cdot\text{m})\hat{i}\times\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)\\ &= -(0.2\,\text{A}\cdot\text{m})B_z\hat{j} + (0.2\,\text{A}\cdot\text{m})B_y\hat{k}\end{aligned}$$

Equate the components of this expression for $\vec{F}$ with those given in the second sentence of the statement of the problem to obtain:

$$-(0.2\,\text{A}\cdot\text{m})B_z = 3\,\text{N}$$

and

$$(0.2\,\text{A}\cdot\text{m})B_y = 2\,\text{N}$$

Noting that B_x is undetermined, solve for B_z and B_y:

$$B_z = -15\,\text{T}$$

and

$$B_y = 10\,\text{T}$$

When the wire is rotated so that the current flows in the positive y direction:

$$\begin{aligned}\vec{F} &= I\vec{\ell}\times\vec{B}\\ &= (2\,\text{A})\left[(0.1\,\text{m})\hat{j}\right]\times\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)\\ &= (0.2\,\text{A}\cdot\text{m})\hat{j}\times\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)\\ &= (0.2\,\text{A}\cdot\text{m})B_z\hat{i} - (0.2\,\text{A}\cdot\text{m})B_x\hat{k}\end{aligned}$$

Equate the components of this expression for $\vec{F}$ with those given in the third sentence of the problem statement to obtain:

$$(0.2\,\text{A}\cdot\text{m})B_x = -3\,\text{N}$$

and

$$-(0.2\,\text{A}\cdot\text{m})B_z = -2\,\text{N}$$

Solve for B_x and B_z to obtain:

$$B_x = -15\,\text{T}$$

and, in agreement with our results above,

$$B_z = 10\,\text{T}$$

Substitute in equation (1) to obtain:

$$\vec{B} = \boxed{(10\,\text{T})\hat{i} + (10\,\text{T})\hat{j} - (15\,\text{T})\hat{k}}$$

30 •••

Picture the Problem We can integrate the expression for the force $d\vec{F}$ acting on an element of the wire of length $d\vec{L}$ from a to b to show that $\vec{F} = I\vec{L} \times \vec{B}$.

Express the force $d\vec{F}$ acting on the element of the wire of length $d\vec{L}$:

$$d\vec{F} = Id\vec{L} \times \vec{B}$$

Integrate this expression to obtain:

$$\vec{F} = \int_a^b Id\vec{L} \times \vec{B}$$

Because $\vec{B}$ and I are constant:

$$\vec{F} = I\left(\int_a^b d\vec{L}\right) \times \vec{B} = \boxed{I\vec{L} \times \vec{B}}$$

where $\vec{L}$ is the vector from a to b.

Motion of a Point Charge in a Magnetic Field

***31** •

Picture the Problem We can apply Newton's 2nd law to the orbiting proton to relate its speed to its radius. We can then use $T = 2\pi r/v$ to find its period. In Part (*b*) we can use the relationship between T and v to determine v. In Part (*c*) we can use its definition to find the kinetic energy of the proton.

(*a*) Relate the period T of the motion of the proton to its orbital speed v:

$$T = \frac{2\pi r}{v} \quad (1)$$

Apply $\sum F_{\text{radial}} = ma_c$ to the proton to obtain:

$$qvB = m\frac{v^2}{r}$$

Solve for v/r to obtain:

$$\frac{v}{r} = \frac{qB}{m}$$

Substitute to obtain:

$$T = \frac{2\pi m}{qB}$$

Substitute numerical values and evaluate T:

$$T = \frac{2\pi(1.67\times10^{-27}\,\text{kg})}{(1.60\times10^{-19}\,\text{C})(0.75\,\text{T})} = \boxed{87.4\,\text{ns}}$$

(*b*) From equation (1) we have:

$$v=\frac{2\pi r}{T}=\frac{2\pi(0.65\,\text{m})}{87.4\,\text{ns}}$$

$$=\boxed{4.67\times10^{7}\ \text{m/s}}$$

(*c*) Using its definition, express and evaluate the kinetic energy of the proton:

$$K=\tfrac{1}{2}mv^2=\tfrac{1}{2}(1.67\times10^{-27}\ \text{kg})(4.67\times10^{7}\ \text{m/s})^2=1.82\times10^{-12}\ \text{J}\times\frac{1\text{eV}}{1.60\times10^{-19}\ \text{J}}$$

$$=\boxed{11.4\,\text{MeV}}$$

32 •

Picture the Problem We can apply Newton's 2nd law to the orbiting electron to obtain an expression for the radius of its orbit as a function of its mass *m*, charge *q*, speed *v*, and the magnetic field *B*. Using the definition of its kinetic energy will allow us to express *r* in terms of *m*, *q*, *B*, and its kinetic energy *K*. We can use $T=2\pi r/v$ to find the period of the motion and calculate the frequency from the reciprocal of the period of the motion.

(*a*) Apply $\sum F_{\text{radial}}=ma_{\text{c}}$ to the proton to obtain:

$$qvB=m\frac{v^2}{r}$$

Solve for *r*:

$$r=\frac{mv}{qB}\qquad(1)$$

Express the kinetic energy of the electron:

$$K=\tfrac{1}{2}mv^2$$

Solve for *v* to obtain:

$$v=\sqrt{\frac{2K}{m}}\qquad(2)$$

Substitute in equation (1) to obtain:

$$r=\frac{m}{qB}\sqrt{\frac{2K}{m}}=\frac{1}{qB}\sqrt{2Km}$$

Find the frequency from the reciprocal of the period:

$$f=\frac{1}{T}=\frac{1}{0.110\,\text{ns}}=\boxed{9.10\,\text{GHz}}$$

Substitute numerical values and evaluate *r*:

$$r = \frac{\sqrt{2(45\,\text{keV})\left(9.11\times10^{-31}\,\text{kg}\times\frac{1.6\times10^{-19}\,\text{J}}{\text{eV}}\right)}}{(1.6\times10^{-19}\,\text{C})(0.325\,\text{T})} = \boxed{2.20\,\text{mm}}$$

(*b*) Relate the period of the electron's motion to the radius of its orbit and its orbital speed:

$$T = \frac{2\pi r}{v}$$

Substitute equation (2) to obtain:

$$T = \frac{2\pi r}{\sqrt{\frac{2K}{m}}} = \pi r\sqrt{\frac{2m}{K}}$$

Substitute numerical values and evaluate T:

$$T = \pi(2.20\,\text{mm})\sqrt{\frac{2(9.11\times10^{-31}\,\text{kg})}{45\,\text{keV}\times\frac{1.6\times10^{-19}\,\text{J}}{\text{eV}}}} = \boxed{0.110\,\text{ns}}$$

33 •

Picture the Problem We can apply Newton's 2nd law to the orbiting electron to obtain an expression for the radius of its orbit as a function of its mass *m*, charge *q*, speed *v*, and the magnetic field *B*.

(*a*) Apply $\sum F_{\text{radial}} = ma_{\text{c}}$ to the proton to obtain:

$$qvB = m\frac{v^2}{r}$$

Solve for *r*:

$$r = \frac{mv}{qB}$$

Substitute numerical values and evaluate *r*:

$$r = \frac{(9.11\times10^{-31}\,\text{kg})(10^{7}\,\text{m/s})}{(1.60\times10^{-19}\,\text{C})(4\times10^{-7}\,\text{T})} = \boxed{142\,\text{m}}$$

(*b*) For $B = 2\times10^{-5}$ T:

$$r = \frac{(9.11\times10^{-31}\,\text{kg})(10^{7}\,\text{m/s})}{(1.60\times10^{-19}\,\text{C})(2\times10^{-5}\,\text{T})} = \boxed{2.84\,\text{m}}$$

34 ••

Picture the Problem We can apply Newton's 2nd law to an orbiting particle to obtain an expression for the radius of its orbit *R* as a function of its mass *m*, charge *q*, speed *v*, and the magnetic field *B*.

Apply $\sum F_{\text{radial}} = ma_{\text{c}}$ to an orbiting particle to obtain:

$$qvB = m\frac{v^2}{r}$$

Solve for *r*:

$$r = \frac{mv}{qB}$$

Express the kinetic energy of the particle:

$$K = \tfrac{1}{2}mv^2$$

Solve for *v* to obtain:

$$v = \sqrt{\frac{2K}{m}}$$

Substitute in equation (1) to obtain:

$$r = \frac{m}{qB}\sqrt{\frac{2K}{m}} = \frac{1}{qB}\sqrt{2Km} \qquad (1)$$

Using equation (1), express the ratio $R_{\text{d}}/R_{\text{p}}$:

$$\frac{R_{\text{d}}}{R_{\text{p}}} = \frac{\frac{1}{q_{\text{d}}B}\sqrt{2Km_{\text{d}}}}{\frac{1}{q_{\text{p}}B}\sqrt{2Km_{\text{p}}}} = \frac{q_{\text{p}}}{q_{\text{d}}}\sqrt{\frac{m_{\text{d}}}{m_{\text{p}}}}$$

$$= \frac{e}{e}\sqrt{\frac{2m_{\text{p}}}{m_{\text{p}}}} = \boxed{\sqrt{2}}$$

Using equation (1), express the ratio R_{α}/R_{p}:

$$\frac{R_{\alpha}}{R_{\text{p}}} = \frac{\frac{1}{q_{\alpha}B}\sqrt{2Km_{\alpha}}}{\frac{1}{q_{\text{p}}B}\sqrt{2Km_{\text{p}}}} = \frac{q_{\text{p}}}{q_{\alpha}}\sqrt{\frac{m_{\alpha}}{m_{\text{p}}}}$$

$$= \frac{e}{2e}\sqrt{\frac{4m_{\text{p}}}{m_{\text{p}}}} = \boxed{1}$$

35 ••

Picture the Problem We can apply Newton's 2nd law to the orbiting particles to derive an expression for their velocities as a function of their charge, their mass, the magnetic field in which they are moving, and the radii of their orbits. We can then compare their velocities by expressing their ratio. In parts (*b*) and (*c*) we can proceed similarly starting with the definitions of kinetic energy and angular momentum.

(*a*) Apply $\sum F_{\text{radial}} = ma_c$ to an orbiting particle to obtain:

$$qvB = m\frac{v^2}{r}$$

Solve for *v*:

$$v = \frac{qBr}{m}$$

Express the velocities of the particles:

$$v_p = \frac{q_p Br}{m_p} \text{ and } v_\alpha = \frac{q_\alpha Br}{m_\alpha}$$

Divide the second of these equations by the first to obtain:

$$\frac{v_\alpha}{v_p} = \frac{\frac{q_\alpha Br}{m_\alpha}}{\frac{q_p Br}{m_p}} = \frac{q_\alpha m_p}{q_p m_\alpha} = \frac{2em_p}{e(4m_p)} = \boxed{\frac{1}{2}}$$

(*b*) Express the kinetic energy of an orbiting particle:

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\left(\frac{qBr}{m}\right)^2 = \tfrac{1}{2}\frac{q^2B^2r^2}{m}$$

Using this relationship, express the ratio of K_α to K_p:

$$\frac{K_\alpha}{K_p} = \frac{\tfrac{1}{2}\frac{q_\alpha^2B^2r^2}{m_\alpha}}{\tfrac{1}{2}\frac{q_p^2B^2r^2}{m_p}} = \frac{q_\alpha^2 m_p}{q_p^2 m_\alpha}$$

$$= \frac{(2e)^2 m_p}{e^2(4m_p)} = \boxed{1}$$

(*c*) Express the angular momenta of the particles:

$$L_\alpha = m_\alpha v_\alpha r \text{ and } L_p = m_p v_p r$$

Express the ratio of L_α to L_p:

$$\frac{L_\alpha}{L_p} = \frac{m_\alpha v_\alpha r}{m_p v_p r} = \frac{(4m_p)\left(\tfrac{1}{2}v_p\right)}{m_p v_p} = \boxed{2}$$

36 ••

Picture the Problem We can use the definition of momentum to express *p* in terms of *v* and apply Newton's 2nd law to the orbiting particle to express *v* in terms of *q*, *B*, *R*, and *m*. In part (*b*) we can express the particle's kinetic energy in terms of its momentum and use our result from part (*a*) to show that $K = \tfrac{1}{2}B^2q^2R^2/m$.

(*a*) Express the momentum of the particle:

$$p = mv \qquad (1)$$

Apply $\sum F_{\text{radial}} = ma_c$ to the orbiting particle to obtain:

$$qvB = m\frac{v^2}{R}$$

Solve for *v*:

$$v = \frac{qBR}{m}$$

Substitute in equation (1) to obtain:

$$p = m\left(\frac{qBR}{m}\right) = \boxed{qBR}$$

(*b*) Express the kinetic energy of the orbiting particle as a function of its momentum:

$$K = \frac{p^2}{2m}$$

Substitute our result from part (*a*) to obtain:

$$K = \frac{(qBR)^2}{2m} = \boxed{\frac{q^2B^2R^2}{2m}}$$

***37 ••**

Picture the Problem The particle's velocity has a component v_1 parallel to $\vec{B}$ and a component v_2 normal to $\vec{B}$. $v_1 = v\cos\theta$ and is constant, whereas $v_2 = v\sin\theta$, being normal to $\vec{B}$, will result in a magnetic force acting on the beam of particles and circular motion perpendicular to $\vec{B}$. We can use the relationship between distance, rate, and time and Newton's 2nd law to express the distance the particle moves in the direction of the field during one period of the motion.

Express the distance moved in the direction of $\vec{B}$ by the particle during one period:

$$x = v_1T \qquad (1)$$

Express the period of the circular motion of the particles in the beam:

$$T = \frac{2\pi r}{v_2}$$

Apply $\sum F_{\text{radial}} = ma_c$ to a particle in the beam to obtain:

$$qv_2B = m\frac{v_2^2}{r}$$

Solve for v_2:

$$v_2 = \frac{qBr}{m}$$

Substitute to obtain:

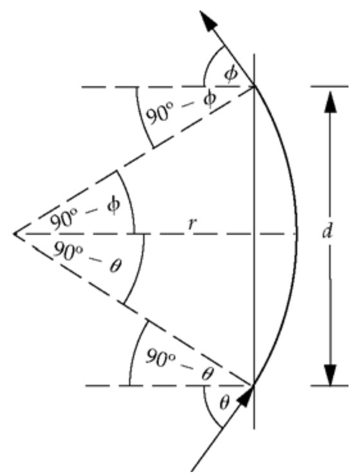
$$T = \frac{2\pi r}{\frac{qBr}{m}} = \frac{2\pi m}{qB}$$

Because $v_1 = v\cos\theta$, equation (1) becomes:

$$x = (v\cos\theta)\left(\frac{2\pi m}{qB}\right) = \boxed{2\pi\left(\frac{m}{qB}\right)v\cos\theta}$$

38 ••

Picture the Problem The trajectory of the proton is shown to the right. We know that, because the proton enters the field perpendicularly to the field, its trajectory while in the field will be circular. We can use symmetry considerations to determine ϕ. The application of Newton's 2nd law to the proton while it is in the magnetic field and of trigonometry will allow us to conclude that $r = d$ and to determine their value.



From symmetry, it is evident that the angle θ in Figure 26-35 equals the angle ϕ:

$$\phi = \boxed{60.0°}$$

Use trigonometry to obtain:

$$\sin(90° - \theta) = \sin 30° = \frac{1}{2} = \frac{d/2}{r}$$

or $r = d$.

Apply $\sum F_{\text{radial}} = ma_c$ to the proton while it is in the magnetic field to obtain:

$$qvB = m\frac{v^2}{r}$$

Solve for r:

$$r = \frac{mv}{qB}$$

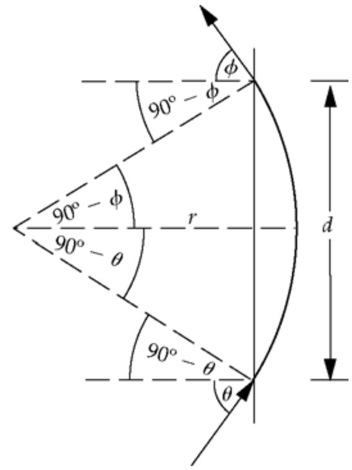
Substitute numerical values and evaluate $r = d$:

$$d = r = \frac{(1.67\times10^{-27}\text{ kg})(10^7\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.8\text{T})}$$

$$= \boxed{0.130\,\text{m}}$$

39 ••

Picture the Problem The trajectory of the proton is shown to the right. We know that, because the proton enters the field perpendicularly to the field, its trajectory while in the field will be circular. We can use symmetry considerations to determine ϕ. The application of Newton's 2nd law to the proton while it is in the magnetic field and of trigonometry will allow us to conclude that $r = d$ and to determine their value.



(*a*) From symmetry, it is evident that the angle θ in Figure 26-33 equals the angle ϕ:

$$\phi = \boxed{24.0°}$$

Use trigonometry to obtain:

$$\sin(90° - \theta) = \sin 24° = \frac{d/2}{r_p}$$

or

$$r_p = \frac{d}{2\sin 24°} = \frac{0.4\,\text{m}}{2\sin 24°} = \boxed{0.492\,\text{m}}$$

Apply $\sum F_{\text{radial}} = ma_c$ to the proton while it is in the magnetic field to obtain:

$$q_p v_p B = m_p \frac{v_p^2}{r_p}$$

Solve for and evaluate v_p:

$$v_p = \frac{q_p r_p B}{m_p}$$

Substitute numerical values and evaluate v_p:

$$v_p = \frac{(1.60\times10^{-19}\,\text{C})(0.492\,\text{m})(0.6\,\text{T})}{1.67\times10^{-27}\,\text{kg}}$$

$$= \boxed{2.83\times10^{7}\,\text{m/s}}$$

(*b*) Express v_d:

$$v_d = \frac{q_d r_d B}{m_d} = \frac{q_p r_p B}{2m_p}$$

Substitute numerical values and evaluate v_d:

$$v_d = \frac{(1.60\times10^{-19}\,\text{C})(0.492\,\text{m})(0.6\,\text{T})}{2(1.67\times10^{-27}\,\text{kg})} = \boxed{1.41\times10^{7}\ \text{m/s}}$$

40 ••

Picture the Problem We can apply Newton's 2nd law of motion to express the orbital speed of the particle and then find the period of the dust particle from this orbital speed.

The period of the dust particle's motion is given by:

$$T = \frac{2\pi r}{v}$$

Apply $\sum F = ma_c$ to the particle:

$$qvB = m\frac{v^2}{r}$$

Solve for v to obtain:

$$v = \frac{qBr}{m}$$

Substitute for v in the expression for T and simplify:

$$T = \frac{2\pi rm}{qBr} = \frac{2\pi m}{qB}$$

Substitute numerical values and evaluate T:

$$T = \frac{2\pi(10\times10^{-6}\,\text{g}\times10^{-3}\,\text{kg/g})}{(0.3\,\text{nC})(10^{-9}\,\text{T})}$$

$$= 2.094\times10^{11}\,\text{s}\times\frac{1\,\text{y}}{31.56\,\text{Ms}}$$

$$= \boxed{6.64\times10^{3}\ \text{y}}$$

The Velocity Selector

***41** •

Picture the Problem Suppose that, for positively charged particles, their motion is from left to right through the velocity selector and the electric field is upward. Then the magnetic force must be downward and the magnetic field out of the page. We can apply the condition for translational equilibrium to relate v to E and B. In (*b*) and (*c*) we can use the definition of kinetic energy to find the energies of protons and electrons that pass through the velocity selector undeflected.

(*a*) Apply $\sum F_y = 0$ to the particle to obtain:

$$F_{\text{elec}} - F_{\text{mag}} = 0$$

or

$$qE - qvB = 0$$

Solve for v to obtain:

$$v = \frac{E}{B}$$

Substitute numerical values and evaluate v:

$$v = \frac{0.46\,\text{MV/m}}{0.28\,\text{T}} = \boxed{1.64\times10^{6}\ \text{m/s}}$$

(b) Express and evaluate the kinetic energy of protons passing through the velocity selector undeflected:

$$\begin{aligned} K_\text{p} &= \tfrac{1}{2} m_\text{p} v^2 \\ &= \tfrac{1}{2}\left(1.67\times10^{-27}\,\text{kg}\right)\left(1.64\times10^{6}\ \text{m/s}\right)^2 \\ &= 2.26\times10^{-15}\ \text{J}\times\frac{1\,\text{eV}}{1.60\times10^{-19}\ \text{J}} \\ &= \boxed{14.0\,\text{keV}} \end{aligned}$$

(c) The kinetic energy of electrons passing through the velocity selector undeflected is given by:

$$\begin{aligned} K_\text{e} &= \tfrac{1}{2} m_\text{e} v^2 \\ &= \tfrac{1}{2}\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.64\times10^{6}\ \text{m/s}\right)^2 \\ &= 1.23\times10^{-18}\ \text{J}\times\frac{1\,\text{eV}}{1.60\times10^{-19}\ \text{J}} \\ &= \boxed{7.66\,\text{eV}} \end{aligned}$$

42 •

Picture the Problem Because the beam of protons is not deflected; we can conclude that the electric force acting on them is balanced by the magnetic force. Hence, we can find the magnetic force from the given data and use its definition to express the electric field.

(a) Use the definition of electric field to relate it to the electric force acting on the beam of protons:

$$\vec{\boldsymbol{E}}_\text{elec} = \frac{\vec{\boldsymbol{F}}_\text{elec}}{q}$$

Express the magnetic force acting on the beam of protons:

$$\vec{\boldsymbol{F}}_\text{mag} = qv\hat{\boldsymbol{i}}\times B\hat{\boldsymbol{j}} = qvB\hat{\boldsymbol{k}}$$

Because the electric force must be equal in magnitude but opposite in direction:

$$\vec{\boldsymbol{F}}_\text{elec} = -qvB\hat{\boldsymbol{k}} = -\left(1.60\times10^{-19}\ \text{C}\right)\left(12.4\,\text{km/s}\right)\left(0.85\,\text{T}\right)\hat{\boldsymbol{k}} = -\left(1.69\times10^{-15}\ \text{N}\right)\hat{\boldsymbol{k}}$$

Substitute in the equation for the electric field to obtain:

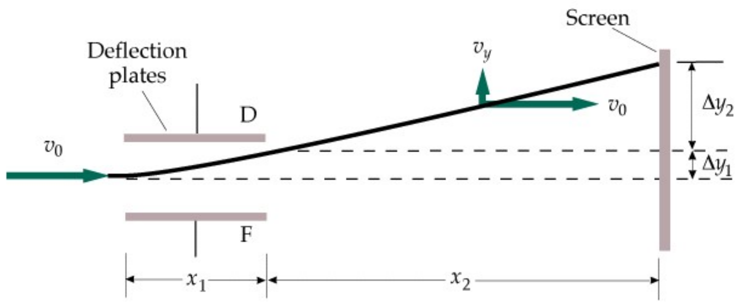
$$\begin{aligned} \vec{\boldsymbol{E}}_\text{elec} &= \frac{-\left(1.69\times10^{-15}\ \text{N}\right)\hat{\boldsymbol{k}}}{1.6\times10^{-19}\ \text{C}} \\ &= \boxed{-\left(10.5\,\text{kV/m}\right)\hat{\boldsymbol{k}}} \end{aligned}$$

(*b*) Because both $\vec{F}_{mag}$ and $\vec{F}_{elec}$ are reversed, electrons are not deflected.

Thomson's Measurement of *q*/*m* for Electrons and the Mass Spectrometer

***43 ••**

Picture the Problem Figure 26-18 is reproduced below. We can express the total deflection of the electron beam as the sum of the deflections while the beam is in the field between the plates and its deflection while it is in the field-free space. We can, in turn, use constant-acceleration equations to express each of these deflections. The resulting equation is in terms of v_0 and E. We can find v_0 from the kinetic energy of the beam and E from the potential difference across the plates and their separation. In part (*b*) we can equate the electric and magnetic forces acting on an electron to express B in terms of E and v_0.



(*a*) Express the total deflection Δy of the electrons:

$$\Delta y = \Delta y_1 + \Delta y_2 \qquad (1)$$

where Δy_1 is the deflection of the beam while it is in the electric field and Δy_2 is the deflection of the beam while it travels along a straight-line path outside the electric field.

Use a constant-acceleration equation to express Δy_1:

$$\Delta y_1 = \tfrac{1}{2} a_y (\Delta t)^2 \qquad (2)$$

where $\Delta t = x_1/v_0$ is the time an electron is in the electric field between the plates.

Apply Newton's 2nd law to an electron between the plates to obtain:

$$qE = ma_y$$

Solve for a_y and substitute into equation (2) to obtain:

$$a_y = \frac{qE}{m}$$

and

$$\Delta y_1 = \tfrac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{x_1}{v_0}\right)^2 \qquad (3)$$

Express the vertical deflection Δy_2 of the electrons once they are out of the electric field:

$$\Delta y_2 = v_y \Delta t_2 \qquad (4)$$

Use a constant-acceleration equation to find the vertical speed of an electron as it leaves the electric field:

$$v_y = v_{0y} + a_y \Delta t_1 = 0 + \frac{qE}{m}\left(\frac{x_1}{v_0}\right)$$

Substitute in equation (4) to obtain:

$$\Delta y_2 = \frac{qE}{m}\left(\frac{x_1}{v_0}\right)\left(\frac{x_2}{v_0}\right) = \frac{qEx_1x_2}{mv_0^2} \qquad (5)$$

Substitute equations (3) and (5) in equation (1) to obtain:

$$\Delta y = \tfrac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{x_1}{v_0}\right)^2 + \frac{qEx_1x_2}{mv_0^2}$$

or

$$\Delta y = \frac{qEx_1}{mv_0^2}\left(\frac{x_1}{2} + x_2\right) \qquad (6)$$

Use the definition of kinetic energy to find the speed of the electrons:

$$K = \tfrac{1}{2} m v_0^2$$

and

$$v_0 = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(2.8\,\text{keV})}{9.11\times10^{-31}\,\text{kg}}} = 3.14\times10^{7}\ \text{m/s}$$

Express the electric field between the plates in terms of their potential difference:

$$E = \frac{V}{d}$$

Substitute numerical values and evaluate E:

$$E = \frac{V}{d} = \frac{25\,\text{V}}{1.2\,\text{cm}} = 2.08\,\text{kV/m}$$

Substitute numerical values in equation (6) and evaluate Δy:

$$\Delta y = \frac{(1.60\times10^{-19}\,\text{C})(2.08\,\text{kV/m})(6\,\text{cm})}{(9.11\times10^{-31}\,\text{kg})(31.4\,\text{Mm/s})^2}\left(\frac{6\,\text{cm}}{2} + 30\,\text{cm}\right) = \boxed{7.34\,\text{mm}}$$

(*b*) Because the electrons are deflected upward, the electric field must be downward and the magnetic field upward. Apply $\sum F_y = 0$ to an electron to obtain:

$$F_{\text{mag}} - F_{\text{elec}} = 0$$

or

$$qvB = qE$$

Solve for *B*:

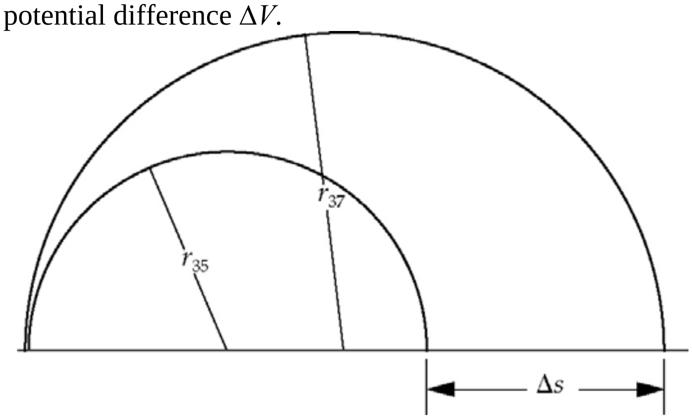
$$B = \frac{E}{v}$$

Substitute numerical values and evaluate *B*:

$$B = \frac{2.08\,\text{kV/m}}{3.14\times10^{7}\ \text{m/s}} = \boxed{66.2\,\mu\text{T}}$$

44 ••

Picture the Problem The diagram below represents the paths of the two ions entering the magnetic field at the left. The magnetic force acting on each causes them to travel in circular paths of differing radii due to their different masses. We can apply Newton's 2nd law to an ion in the magnetic field to obtain an expression for its radius and then express their final separation in terms of these radii that, in turn, depend on the energy with which the ions enter the field. We can connect their energy to the potential through which they are accelerated using the work-kinetic energy theorem and relate their separation Δs to the accelerating potential difference ΔV.



Express the separation Δs of the chlorine ions:

$$\Delta s = 2(r_{37} - r_{35}) \qquad (1)$$

Apply $\sum F_{\text{radial}} = ma_c$ to an ion in the magnetic field of the mass spectrometer:

$$qvB = m\frac{v^2}{r}$$

Solve for *r* to obtain:

$$r = \frac{mv}{qB} \qquad (2)$$

Relate the speed of an ion as it enters the magnetic field to the potential difference through which it has been accelerated:

$$q\Delta V = \tfrac{1}{2}mv^2$$

Solve for v to obtain:

$$v = \sqrt{\frac{2q\Delta V}{m}}$$

Substitute in equation (2) to obtain:

$$r = \frac{m}{qB}\sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{2m\Delta V}{qB^2}}$$

Use this equation to express the radii of the paths of the two chlorine isotopes to obtain:

$$r_{35} = \sqrt{\frac{2m_{35}\Delta V}{qB^2}} \text{ and } r_{37} = \sqrt{\frac{2m_{37}\Delta V}{qB^2}}$$

Substitute in equation (1) to obtain:

$$\Delta s = 2\left(\sqrt{\frac{2m_{35}\Delta V}{qB^2}} - \sqrt{\frac{2m_{35}\Delta V}{qB^2}}\right) = 2\left(\frac{1}{B}\sqrt{\frac{2\Delta V}{q}}\left(\sqrt{m_{37}} - \sqrt{m_{35}}\right)\right)$$

Solve for ΔV:

$$\Delta V = \frac{qB^2(2\Delta s)^2}{2\left(\sqrt{m_{37}} - \sqrt{m_{35}}\right)^2}$$

Substitute numerical values and evaluate ΔV:

$$\Delta V = \frac{\left(1.60\times10^{-19}\,\text{C}\right)(1.2\,\text{T})^2\left(\dfrac{1.4\,\text{cm}}{2}\right)^2}{2\left(\sqrt{37\,\text{u}} - \sqrt{35\,\text{u}}\right)^2} = \frac{5.65\times10^{-24}\,\text{C}\cdot\text{T}^2\cdot\text{m}^2}{\left(\sqrt{37} - \sqrt{35}\right)^2\left(1.66\times10^{-27}\,\text{kg}\right)} = \boxed{122\,\text{kV}}$$

45 ••

Picture the Problem We can apply Newton's 2nd law to an ion in the magnetic field to obtain an expression for r as a function of m, v, q, and B and use the work-kinetic energy theorem to express the kinetic energy in terms of the potential difference through which the ion has been accelerated. Eliminating v between these equations will allow us to express r in terms of m, q, B, and ΔV.

Apply $\sum F_{\text{radial}} = ma_c$ to an ion in the magnetic field of the mass spectrometer:

$$qvB = m\frac{v^2}{r}$$

Solve for r to obtain:

$$r = \frac{mv}{qB} \qquad (1)$$

Apply the work-kinetic energy theorem to relate the speed of an ion as it enters the magnetic field to the potential difference through which it has been accelerated:

$$q\Delta V = \tfrac{1}{2}mv^2$$

Solve for v to obtain:

$$v = \sqrt{\frac{2q\Delta V}{m}}$$

Substitute in equation (1) to obtain:

$$r = \frac{m}{qB}\sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{2m\Delta V}{qB^2}} \qquad (2)$$

(*a*) Substitute numerical values and evaluate equation (2) for ^{24}Mg :

$$r_{24} = \sqrt{\frac{2\left(3.983\times10^{-26}\,\text{kg}\right)\left(2.5\,\text{kV}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(557\times10^{-4}\,\text{T}\right)^2}} = \boxed{63.3\,\text{cm}}$$

(*b*) Express the difference in the radii for ^{24}Mg and ^{26}Mg:

$$\Delta r = r_{26} - r_{24}$$

Substitute numerical values and evaluate equation (2) for ^{26}Mg :

$$r_{26} = \sqrt{\frac{2\left(\frac{26}{24}\right)\left(3.983\times10^{-26}\,\text{kg}\right)\left(2.5\,\text{kV}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(557\times10^{-4}\,\text{T}\right)^2}} = 65.9\,\text{cm}$$

Substitute to obtain:

$$\Delta r = 65.9\,\text{cm} - 63.3\,\text{cm} = \boxed{2.60\,\text{cm}}$$

***46** ••

Picture the Problem We can apply Newton's 2nd law to an ion in the magnetic field of the spectrometer to relate the diameter of its orbit to its charge, mass, velocity, and the magnetic field. If we assume that the velocity is the same for the two ions, we can then express the ratio of the two diameters as the ratio of the masses of the ions and solve for the diameter of the orbit of ^{7}Li.

Apply $\sum F_{\text{radial}} = ma_c$ to an ion in the field of the spectrometer:

$$qvB = m\frac{v^2}{r}$$

Solve for r to obtain:

$$r = \frac{mv}{qB}$$

Express the diameter of the orbit:

$$d = \frac{2mv}{qB}$$

Express the diameters of the orbits for ^{6}Li and ^{7}Li:

$$d_6 = \frac{2m_6v}{qB} \text{ and } d_7 = \frac{2m_7v}{qB}$$

Assume that the velocities of the two ions are the same and divide the 2nd of these diameters by the first to obtain:

$$\frac{d_7}{d_6} = \frac{\frac{2m_7v}{qB}}{\frac{2m_6v}{qB}} = \frac{m_7}{m_6}$$

Solve for and evaluate d_7:

$$d_7 = \frac{m_7}{m_6}d_6 = \frac{7\,\text{u}}{6\,\text{u}}(15\,\text{cm}) = \boxed{17.5\,\text{cm}}$$

The Cyclotron

47 ••

Picture the Problem The time required for each of the ions to complete its semicircular paths is half its period. We can apply Newton's 2nd law to an ion in the magnetic field of the spectrometer to obtain an expression for r as a function of the charge and mass of the ion, its velocity, and the magnetic field.

Express the time for each ion to complete its semicircular path:

$$\Delta t = \tfrac{1}{2}T = \frac{\pi r}{v}$$

Apply $\sum F_{\text{radial}} = ma_c$ to an ion in the field of the spectrometer:

$$qvB = m\frac{v^2}{r}$$

Solve for r to obtain:

$$r = \frac{mv}{qB}$$

Substitute to obtain:

$$\Delta t = \frac{\pi m}{qB}$$

Substitute numerical values and evaluate Δt_{58} and Δt_{60}:

$$\Delta t_{58} = \frac{58\pi\left(1.66\times10^{-27}\text{ kg}\right)}{\left(1.60\times10^{-19}\text{ C}\right)\left(0.12\text{ T}\right)} = \boxed{15.8\,\mu\text{s}}$$

and

$$\Delta t_{60} = \frac{60\pi\left(1.66\times10^{-27}\text{ kg}\right)}{\left(1.60\times10^{-19}\text{ C}\right)\left(0.12\text{ T}\right)} = \boxed{16.3\,\mu\text{s}}$$

48 ••

Picture the Problem We can apply a condition for equilibrium to ions passing through the velocity selector to obtain an expression relating *E*, *B*, and *v* that we can solve for *v*. We can, in turn, express *E* in terms of the potential difference *V* between the plates of the selector and their separation *d*. In (*b*) we can apply Newton's 2nd law to an ion in the bending field of the spectrometer to relate its diameter to its mass, charge, velocity, and the magnetic field.

(*a*) Apply $\sum F_y = 0$ to the ions in the crossed fields of the velocity selector to obtain:

$$F_{\text{elec}} - F_{\text{mag}} = 0$$

or

$$qE - qvB = 0$$

Solve for *v* to obtain:

$$v = \frac{E}{B}$$

Express the electric field between the plates of the velocity selector in terms of their separation and the potential difference across them:

$$E = \frac{V}{d}$$

Substitute to obtain:

$$v = \frac{V}{dB}$$

Substitute numerical values and evaluate *v*:

$$v = \frac{160\text{ V}}{\left(2\text{ mm}\right)\left(0.42\text{ T}\right)} = \boxed{1.90\times10^{5}\text{ m/s}}$$

(*b*) Express the difference in the diameters of the orbits of singly

$$\Delta d = d_{238} - d_{235} \qquad (1)$$

ionized ^{238}U and ^{235}U:

Apply $\sum F_{\text{radial}} = ma_c$ to an ion in the spectrometer's magnetic field:

$$qvB = m\frac{v^2}{r}$$

Solve for the radius of the ion's orbit:

$$r = \frac{mv}{qB}$$

Express the diameter of the orbit:

$$d = \frac{2mv}{qB}$$

Express the diameters of the orbits for ^{238}U and ^{235}U:

$$d_{238} = \frac{2m_{238}v}{qB} \text{ and } d_{235} = \frac{2m_{235}v}{qB}$$

Substitute in equation (1) to obtain:

$$\Delta d = \frac{2m_{238}v}{qB} - \frac{2m_{235}v}{qB} = \frac{2v}{qB}\left(m_{238} - m_{235}\right)$$

Substitute numerical values and evaluate Δd:

$$\Delta d = \frac{2\left(1.90\times10^{5}\text{ m/s}\right)\left(238\,\text{u} - 235\,\text{u}\right)\left(\frac{1.66\times10^{-27}\text{ kg}}{\text{u}}\right)}{\left(1.60\times10^{-19}\text{ C}\right)\left(1.2\,\text{T}\right)} = \boxed{9.86\,\text{mm}}$$

***49 ••**

Picture the Problem We can express the cyclotron frequency in terms of the maximum orbital radius and speed of the protons/deuterons. By applying Newton's 2nd law, we can relate the radius of the particle's orbit to its speed and, hence, express the cyclotron frequency as a function of the particle's mass and charge and the cyclotron's magnetic field. In part (*b*) we can use the definition of kinetic energy and their maximum speed to find the maximum energy of the emerging protons.

(*a*) Express the cyclotron frequency in terms of the proton's orbital speed and radius:

$$f = \frac{1}{T} = \frac{1}{2\pi r/v} = \frac{v}{2\pi r}$$

Apply $\sum F_{\text{radial}} = ma_c$ to a proton in the magnetic field of the cyclotron:

$$qvB = m\frac{v^2}{r} \qquad (1)$$

Solve for r to obtain:

$$r = \frac{mv}{qB}$$

Substitute to obtain:

$$f = \frac{qBv}{2\pi mv} = \frac{qB}{2\pi m} \qquad (2)$$

Substitute numerical values and evaluate f:

$$f = \frac{(1.60\times10^{-19}\text{ C})(1.4\text{ T})}{2\pi(1.67\times10^{-27}\text{ kg})} = \boxed{21.3\text{ MHz}}$$

(*b*) Express the maximum kinetic energy of a proton:

$$K_{max} = \tfrac{1}{2}mv_{max}^2$$

Solve equation (1) for v_{max} to obtain:

$$v_{max} = \frac{qBr_{max}}{m}$$

Substitute to obtain:

$$K = \tfrac{1}{2}m\left(\frac{qBr_{max}}{m}\right)^2 = \tfrac{1}{2}\left(\frac{q^2B^2}{m}\right)r_{max}^2 \qquad (3)$$

Substitute numerical values and evaluate K:

$$K = \tfrac{1}{2}\left(\frac{(1.60\times10^{-19}\text{ C})^2(1.4\text{ T})^2}{1.67\times10^{-27}\text{ kg}}\right)(0.7\text{ m})^2$$

$$= 7.36\times10^{-12}\text{ J}\times\frac{1\text{eV}}{1.60\times10^{-19}\text{ J}}$$

$$= \boxed{46.0\text{ MeV}}$$

(*c*) From equation (2) we see that doubling m halves f:

$$f_{deuterons} = \tfrac{1}{2}f_{protons} = \boxed{10.7\text{ MHz}}$$

From equation (3) we see that doubling m halves K:

$$K_{deuterons} = \tfrac{1}{2}K_{protons} = \boxed{23.0\text{ MeV}}$$

50 ••

Picture the Problem We can express the cyclotron frequency in terms of the maximum orbital radius and speed of the protons be accelerated in the cyclotron. By applying Newton's 2nd law, we can relate the radius of the proton's orbit to its speed and, hence, express the cyclotron frequency as a function of the its mass and charge and the cyclotron's magnetic field. In part (*b*) we can use the definition of kinetic energy express the minimum radius required to achieve the desired emergence energy. In part (*c*) we can find the number of revolutions required to achieve this emergence energy from the energy acquired during each revolution.

(*a*) Express the cyclotron frequency in terms of the proton's orbital speed and radius:

$$f = \frac{1}{T} = \frac{1}{2\pi r/v} = \frac{v}{2\pi r}$$

Apply $\sum F_{\text{radial}} = ma_{\text{c}}$ to a proton in the magnetic field of the cyclotron:

$$qvB = m\frac{v^2}{r}$$

Solve for r to obtain:

$$r = \frac{mv}{qB} \qquad (1)$$

Substitute to obtain:

$$f = \frac{qBv}{2\pi mv} = \frac{qB}{2\pi m}$$

Substitute numerical values and evaluate f:

$$f = \frac{\left(1.60\times10^{-19}\ \text{C}\right)\left(1.8\ \text{T}\right)}{2\pi\left(1.67\times10^{-27}\ \text{kg}\right)} = \boxed{27.4\ \text{MHz}}$$

(*b*) Using the definition of kinetic energy, relate emergence energy of the protons to their velocity:

$$K = \tfrac{1}{2}mv^2$$

Solve for v to obtain:

$$v = \sqrt{\frac{2K}{m}}$$

Substitute in equation (1) and simplify to obtain:

$$r = \frac{m}{qB}\sqrt{\frac{2K}{m}} = \frac{\sqrt{2Km}}{qB}$$

Substitute numerical values and evaluate r_{min}:

$$r = \frac{\sqrt{2\left(25\ \text{MeV}\right)\left(1.67\times10^{-27}\ \text{kg}\right)}}{\left(1.60\times10^{-19}\ \text{C}\right)\left(1.8\ \text{T}\right)} = \boxed{0.401\ \text{m}}$$

(*c*) Express the required number of revolutions N in terms of the energy gained per revolution:

$$N = \frac{25\ \text{MeV}}{E_{\text{rev}}}$$

Because the beam is accelerated through a potential difference of 50

$$E_{\text{rev}} = 2q\Delta V = 100\ \text{keV}$$

kV twice during each revolution:

Substitute and evaluate N:

$$N = \frac{25\,\text{MeV}}{100\,\text{keV/rev}} = \boxed{250\,\text{rev}}$$

51 ••

Picture the Problem We can express the cyclotron frequency in terms of the maximum orbital radius and speed of a particle being accelerated in the cyclotron. By applying Newton's 2nd law, we can relate the radius of the particle's orbit to its speed and, hence, express the cyclotron frequency as a function of its charge-to-mass ratio and the cyclotron's magnetic field. We can then use data for the relative charges and masses of deuterons, alpha particles, and protons to establish the ratios of their cyclotron frequencies.

Express the cyclotron frequency in terms of a particle's orbital speed and radius:

$$f = \frac{1}{T} = \frac{1}{2\pi r/v} = \frac{v}{2\pi r}$$

Apply $\sum F_{\text{radial}} = ma_c$ to a particle in the magnetic field of the cyclotron:

$$qvB = m\frac{v^2}{r}$$

Solve for r to obtain:

$$r = \frac{mv}{qB}$$

Substitute to obtain:

$$f = \frac{qBv}{2\pi mv} = \frac{B}{2\pi}\frac{q}{m} \qquad (1)$$

Evaluate equation (1) for deuterons:

$$f_{\text{d}} = \frac{B}{2\pi}\frac{q_{\text{d}}}{m_{\text{d}}} = \frac{B}{2\pi}\frac{e}{m_{\text{d}}}$$

Evaluate equation (1) for alpha particles:

$$f_\alpha = \frac{B}{2\pi}\frac{q_\alpha}{m_\alpha} = \frac{B}{2\pi}\frac{2e}{2m_{\text{d}}} = \frac{B}{2\pi}\frac{e}{m_{\text{d}}}$$

and

$$\boxed{f_{\text{d}} = f_\alpha}$$

Evaluate equation (1) for protons:

$$f_{\text{p}} = \frac{B}{2\pi}\frac{q_{\text{p}}}{m_{\text{p}}} = \frac{B}{2\pi}\frac{e}{\frac{1}{2}m_{\text{d}}} = 2\left(\frac{B}{2\pi}\frac{e}{m_{\text{d}}}\right)$$

$$= 2f_{\text{d}}$$

and

$$\tfrac{1}{2}f_\text{p} = \boxed{f_\text{d} = f_\alpha}$$

52 ••

Picture the Problem We can apply Newton's 2nd law to the orbiting charged particle to obtain an expression for its radius as a function of its particle's kinetic energy. Because the energy gain per revolution is constant, we can express this kinetic energy as the product of the number of orbits completed and the energy gained per revolution and, hence, show that the radius is proportional to the square root of the number of orbits completed.

Apply $\sum F_\text{radial} = ma_\text{c}$ to a particle in the magnetic field of the cyclotron:

$$qvB = m\frac{v^2}{r}$$

Solve for r to obtain:

$$r = \frac{mv}{qB} \qquad (1)$$

Express the kinetic energy of the particle in terms of its speed and solve for v:

$$K = \tfrac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2K}{m}} \qquad (2)$$

Noting that the energy gain per revolution is constant, express the kinetic energy in terms of the number of orbits N completed by the particle and energy E_rev gained by the particle each revolution:

$$K = NE_\text{rev} \qquad (3)$$

Substitute equations (2) and (3) in equation (1) to obtain:

$$r = \frac{m}{qB}\sqrt{\frac{2K}{m}} = \frac{1}{qB}\sqrt{2mK}$$

$$= \frac{1}{qb}\sqrt{2mNE_\text{rev}} = \frac{\sqrt{2mE_\text{rev}}}{qB}N^{1/2}$$

or $\boxed{r \propto N^{1/2}}$

Torques on Current Loops and Magnets

53 •

Picture the Problem We can use the definition of the magnetic moment of a coil to evaluate μ and the expression for the torque exerted on the coil $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\boldsymbol{B}}$ to find the magnitude of τ.

(*a*) Using its definition, express the magnetic moment of the coil:

$$\mu = NIA = NI\pi r^2$$

Substitute numerical values and evaluate μ:

$$\mu = (20)(3\,\text{A})\pi(0.04\,\text{m})^2 = \boxed{0.302\,\text{A}\cdot\text{m}^2}$$

(*b*) Express the magnitude of the torque exerted on the coil:

$$\tau = \mu B \sin\theta$$

Substitute numerical values and evaluate τ:

$$\tau = \left(0.302\,\text{A}\cdot\text{m}^2\right)(0.5\,\text{T})\sin 60° = \boxed{0.131\,\text{N}\cdot\text{m}}$$

54 •

Picture the Problem The coil will experience the maximum torque when the plane of the coil makes an angle of 90° with the direction of $\vec{B}$. The magnitude of the maximum torque is then given by $\tau_{\max} = \mu B$.

Express the maximum torque acting on the coil:

$$\tau_{\max} = \mu B$$

Use its definition to express the magnetic moment of the coil:

$$\mu = NIA = NI\pi r^2$$

Substitute to obtain:

$$\tau_{\max} = NI\pi r^2 B$$

Substitute numerical values and evaluate τ:

$$\tau_{\max} = (400)(1.6\,\text{mA})\pi(0.75\,\text{cm})^2(0.25\,\text{T}) = \boxed{2.83\times10^{-5}\,\text{N}\cdot\text{m}}$$

***55** •

Picture the Problem We can use $\vec{\tau} = \vec{\mu}\times\vec{B}$ to find the torque on the coil in the two orientations of the magnetic field.

Express the torque acting on the coil:

$$\vec{\tau} = \vec{\mu}\times\vec{B}$$

Express the magnetic moment of the coil:

$$\vec{\mu} = \pm IA\hat{k} = \pm IL^2\hat{k}$$

(*a*) Evaluate $\vec{\tau}$ for $\vec{B}$ in the z direction:

$$\vec{\tau} = \pm IL^2\hat{k} \times B\hat{k} = \pm IL^2 B\left(\hat{k} \times \hat{k}\right) = \boxed{0}$$

(*b*) Evaluate $\vec{\tau}$ for $\vec{B}$ in the x direction:

$$\vec{\tau} = \pm IL^2\hat{k} \times B\hat{i} = \pm IL^2 B\left(\hat{k} \times \hat{i}\right) = \pm(2.5\,\text{A})(0.06\,\text{m})^2(0.3\,\text{T})\hat{j} = \boxed{\pm\left(2.70\times10^{-3}\,\text{N}\cdot\text{m}\right)\hat{j}}$$

56 •

Picture the Problem We can use $\vec{\tau} = \vec{\mu} \times \vec{B}$ to find the torque on the equilateral triangle in the two orientations of the magnetic field.

Express the torque acting on the coil:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Express the magnetic moment of the coil:

$$\vec{\mu} = \pm IA\hat{k}$$

Relate the area of the equilateral triangle to the length of its side:

$$A = \tfrac{1}{2}\text{base} \times \text{altitude} = \frac{1}{2}(L)\left(\frac{\sqrt{3}L}{2}\right) = \frac{\sqrt{3}}{4}L^2$$

Substitute to obtain:

$$\vec{\mu} = \pm\frac{\sqrt{3}L^2 I}{4}\hat{k}$$

(*a*) Evaluate $\vec{\tau}$ for $\vec{B}$ in the z direction:

$$\vec{\tau} = \pm\frac{\sqrt{3}L^2 I}{4}\hat{k} \times B\hat{k} = \pm\frac{\sqrt{3}L^2 IB}{4}\left(\hat{k} \times \hat{k}\right) = \boxed{0}$$

(*b*) Evaluate $\vec{\tau}$ for $\vec{B}$ in the x direction:

$$\vec{\tau} = \pm\frac{\sqrt{3}L^2 I}{4}\hat{k} \times B\hat{i} = \pm\frac{\sqrt{3}L^2 IB}{4}\left(\hat{k} \times \hat{i}\right) = \pm\frac{\sqrt{3}(0.08\,\text{m})^2(2.5\,\text{A})(0.3\,\text{T})}{4}\hat{j} = \boxed{\pm\left(2.08\times10^{-3}\,\text{N}\cdot\text{m}\right)\hat{j}}$$

57 ••

Picture the Problem The loop will start to lift off the table when the magnetic torque equals the gravitational torque.

Express the magnetic torque acting on the loop:

$$\tau_{\text{mag}} = \mu B = I\pi R^2 B$$

Express the gravitational torque acting on the loop:

$$\tau_{\text{grav}} = mgR$$

Because the loop is in equilibrium under the influence of the two torques:

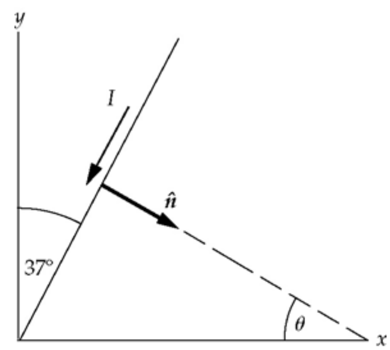
$$I\pi R^2 B = mgR$$

Solve for B to obtain:

$$B = \boxed{\frac{mg}{I\pi R}}$$

58 ••

Picture the Problem The diagram to the right shows the coil as it would appear from along the positive z axis. The right-hand rule for determining the direction of $\hat{\boldsymbol{n}}$ has been used to establish $\hat{\boldsymbol{n}}$ as shown. We can use the geometry of this figure to determine θ and to express the unit normal vector $\hat{\boldsymbol{n}}$. The magnetic moment of the coil is given by $\vec{\boldsymbol{\mu}} = NIA\hat{\boldsymbol{n}}$ and the torque exerted on the coil by $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\boldsymbol{B}}$. Finally, we can find the potential energy of the coil in this field from $U = -\vec{\boldsymbol{\mu}} \cdot \vec{\boldsymbol{B}}$.



(a) Noting that θ and the angle whose measure is 37° have their right and left sides mutually perpendicular, we can conclude that:

$$\theta = \boxed{37°}$$

(b) Use the components of $\hat{\boldsymbol{n}}$ to express $\hat{\boldsymbol{n}}$ in terms of $\hat{\boldsymbol{i}}$ and $\hat{\boldsymbol{j}}$:

$$\hat{\boldsymbol{n}} = n_x\hat{\boldsymbol{i}} + n_y\hat{\boldsymbol{j}} = \cos 37°\hat{\boldsymbol{i}} - \sin 37°\hat{\boldsymbol{j}}$$

$$= \boxed{0.799\hat{\boldsymbol{i}} - 0.602\hat{\boldsymbol{j}}}$$

(c) Express the magnetic moment of

$$\vec{\boldsymbol{\mu}} = NIA\hat{\boldsymbol{n}}$$

the coil:

Substitute numerical values and evaluate $\vec{\mu}$:

$$\vec{\mu} = (50)(1.75\,\text{A})(48\,\text{cm}^2)(0.799\hat{i} - 0.602\hat{j}) = \boxed{(0.336\,\text{A}\cdot\text{m}^2)\hat{i} - (0.253\,\text{A}\cdot\text{m}^2)\hat{j}}$$

(*d*) Express the torque exerted on the coil:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Substitute for $\vec{\mu}$ and $\vec{B}$ to obtain:

$$\begin{aligned}\vec{\tau} &= \left\{(0.336\,\text{A}\cdot\text{m}^2)\hat{i} - (0.253\,\text{A}\cdot\text{m}^2)\hat{j}\right\} \times (1.5\,\text{T})\hat{j} \\ &= (0.504\,\text{N}\cdot\text{m})(\hat{i} \times \hat{j}) - (0.380\,\text{N}\cdot\text{m})(\hat{j} \times \hat{j}) = \boxed{(0.504\,\text{N}\cdot\text{m})\hat{k}}\end{aligned}$$

(*e*) Express the potential energy of the coil in terms of its magnetic moment and the magnetic field:

$$U = -\vec{\mu} \cdot \vec{B}$$

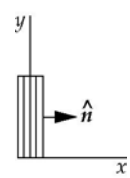
Substitute for $\vec{\mu}$ and $\vec{B}$ and evaluate *U*:

$$\begin{aligned}U &= -\left\{(0.336\,\text{A}\cdot\text{m}^2)\hat{i} - (0.253\,\text{A}\cdot\text{m}^2)\hat{j}\right\} \cdot (1.5\,\text{T})\hat{j} \\ &= -(0.504\,\text{N}\cdot\text{m})(\hat{i} \cdot \hat{j}) + (0.380\,\text{N}\cdot\text{m})(\hat{j} \cdot \hat{j}) = \boxed{0.380\,\text{J}}\end{aligned}$$

59 ••

Picture the Problem We can use the right-hand rule for determining the direction of $\hat{n}$ to establish the orientation of the coil for value of $\hat{n}$ and $\vec{\tau} = \vec{\mu} \times \vec{B}$ to find the torque exerted on the coil in each orientation.

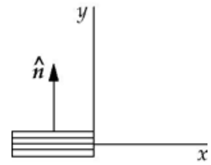
(*a*) The orientation of the coil is shown to the right:



Evaluate $\vec{\tau}$ for $\vec{B}$ = 2.0 T $\hat{j}$ and $\hat{n} = \hat{i}$:

$$\begin{aligned}\vec{\tau} &= \vec{\mu} \times \vec{B} = NIA\hat{n} \times \vec{B} \\ &= (50)(1.75\,\text{A})(48\,\text{cm}^2)\hat{i} \times (2\,\text{T})\hat{j} \\ &= (0.840\,\text{N}\cdot\text{m})(\hat{i} \times \hat{j}) \\ &= \boxed{(0.840\,\text{N}\cdot\text{m})\hat{k}}\end{aligned}$$

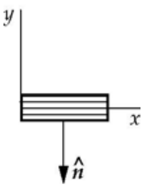
(*b*) The orientation of the coil is shown to the right:



Evaluate $\vec{\tau}$ for $\vec{B} = 2.0\ \text{T}\,\hat{j}$ and $\hat{n} = \hat{j}$:

$$\vec{\tau} = \vec{\mu}\times\vec{B} = NIA\hat{n}\times\vec{B}$$
$$= (50)(1.75\ \text{A})(48\ \text{cm}^2)\hat{j}\times(2\ \text{T})\hat{j}$$
$$= (0.840\ \text{N}\cdot\text{m})(\hat{j}\times\hat{j})$$
$$= \boxed{0}$$

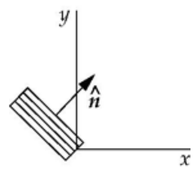
(*c*) The orientation of the coil is shown to the right:



Evaluate $\vec{\tau}$ for $\vec{B} = 2.0\ \text{T}\,\hat{j}$ and $\hat{n} = -\hat{j}$:

$$\vec{\tau} = \vec{\mu}\times\vec{B} = NIA\hat{n}\times\vec{B}$$
$$= -(50)(1.75\ \text{A})(48\ \text{cm}^2)\hat{j}\times(2\ \text{T})\hat{j}$$
$$= (-0.840\ \text{N}\cdot\text{m})(\hat{j}\times\hat{j})$$
$$= \boxed{0}$$

(*d*) The orientation of the coil is shown to the right:



Evaluate $\vec{\tau}$ for $\vec{B} = 2.0\ \text{T}\,\hat{j}$ and $\hat{n} = (\hat{i} + \hat{j})/\sqrt{2}$:

$$\vec{\tau} = \vec{\mu}\times\vec{B} = NIA\hat{n}\times\vec{B}$$
$$= \frac{(50)(1.75\ \text{A})(48\ \text{cm}^2)}{\sqrt{2}}(\hat{i}+\hat{j})\times(2\ \text{T})\hat{j}$$
$$= (0.594\ \text{N}\cdot\text{m})(\hat{i}\times\hat{j})$$
$$+ (0.594\ \text{N}\cdot\text{m})(\hat{j}\times\hat{j})$$
$$= \boxed{(0.594\ \text{N}\cdot\text{m})\hat{k}}$$

Magnetic Moments

***60 ••**

Picture the Problem Because the small magnet can be modeled as a magnetic dipole; we can use the equation for the torque on a current loop to find its magnetic moment.

Express the magnitude of the torque

$$\tau = \mu B\sin\theta$$

acting on the magnet:

Solve for μ to obtain:

$$\mu = \frac{\tau}{B\sin\theta}$$

Substitute numerical values and evaluate μ:

$$\mu = \frac{0.10\ \text{N}\cdot\text{m}}{(0.04\ \text{T})\sin 60^\circ} = \boxed{2.89\ \text{A}\cdot\text{m}^2}$$

61 ••

Picture the Problem We can use the definition of the magnetic moment to find the magnetic moment of the given current loop and a right-hand rule to find its direction.

Using its definition, express the magnetic moment of the current loop:

$$\mu = IA$$

Express the area bounded by the loop:

$$A = \tfrac{1}{2}\left(\pi R_{\text{outer}}^2 - \pi R_{\text{inner}}^2\right) = \frac{\pi}{2}\left(R_{\text{outer}}^2 - R_{\text{inner}}^2\right)$$

Substitute to obtain:

$$\mu = \frac{\pi I}{2}\left(R_{\text{outer}}^2 - R_{\text{inner}}^2\right)$$

Substitute numerical values and evaluate μ:

$$\mu = \frac{\pi(1.5\ \text{A})}{2}\left[(0.5\ \text{m})^2 - (0.3\ \text{m})^2\right] = \boxed{0.377\ \text{A}\cdot\text{m}^2}$$

Apply the right-hand rule for determining the direction of the unit normal vector (the direction of μ) to conclude that $\boxed{\vec{\mu}\ \text{points into the page.}}$

62 ••

Picture the Problem We can use the definition of the magnetic moment of a coil to find the magnetic moment of a wire of length L that is wound into a circular coil of N loops. We can find the area of the coil from its radius R and we can find R by dividing the length of the wire by the number of turns.

Use its definition to express the magnetic moment of the coil:

$$\mu = NIA \qquad (1)$$

Express the circumference of each loop:

$$\frac{L}{N} = 2\pi R$$

where R is the radius of a loop.

Solve for R to obtain:

$$R = \frac{L}{2\pi N}$$

Express the area of the coil:

$$A = \pi R^2 = \pi\left(\frac{L}{2\pi N}\right)^2 = \frac{L^2}{4\pi N^2}$$

Substitute in equation (1) and simplify to obtain:

$$\mu = NI\left(\frac{L^2}{4\pi N^2}\right) = \boxed{\frac{IL^2}{4\pi N}}$$

63 ••

Picture the Problem We can use the definition of current and the relationship between the frequency of the motion and its period to show that $I = q\omega/2\pi$. We can use the definition of angular momentum and the moment of inertia of a point particle to show that the magnetic moment has the magnitude $\mu = \frac{1}{2}q\omega r^2$. Finally, we can express the ratio of μ to L and the fact that $\vec{\boldsymbol{\mu}}$ and $\vec{\boldsymbol{L}}$ are both parallel to $\vec{\boldsymbol{\omega}}$ to conclude that $\vec{\boldsymbol{\mu}} = (q/2m)\vec{\boldsymbol{L}}$.

(*a*) Using its definition, relate the average current to the charge passing a point on the circumference of the circle in a given period of time:

$$I = \frac{\Delta q}{\Delta t} = \frac{q}{T} = qf$$

Relate the frequency of the motion to the angular frequency of the particle:

$$f = \frac{\omega}{2\pi}$$

Substitute to obtain:

$$I = \boxed{\frac{q\omega}{2\pi}}$$

From the definition of the magnetic moment we have:

$$\mu = IA = \left(\frac{q\omega}{2\pi}\right)\left(\pi r^2\right) = \boxed{\tfrac{1}{2}q\omega r^2}$$

(*b*) Express the angular momentum of the particle:

$$L = I\omega$$

The angular momentum of the particle is:

$$I = mr^2$$

Substitute to obtain: $L = (mr^2)\omega = \boxed{mr^2\omega}$

Express the ratio of μ to L and simplify to obtain:

$$\frac{\mu}{L} = \frac{\frac{1}{2}q\omega r^2}{mr^2\omega} = \frac{q}{2m} \Rightarrow \mu = \frac{q}{2m}L$$

Because $\vec{\boldsymbol{\mu}}$ and $\vec{\boldsymbol{L}}$ are both parallel to $\vec{\boldsymbol{\omega}}$:

$$\vec{\boldsymbol{\mu}} = \boxed{\frac{q}{2m}\vec{\boldsymbol{L}}}$$

***64 •••**

Picture the Problem We can express the magnetic moment of an element of charge *dq* in a cylinder of length *L*, radius *r*, and thickness *dr*, relate this charge to the length, radius, and thickness of the cylinder, express the current due to this rotating charge, substitute for *A* and *dI* in our expression for μ and then integrate to complete our derivation for the magnetic moment of the rotating cylinder as a function of its angular velocity.

Express the magnetic moment of an element of charge *dq* in a cylinder of length *L*, radius *r*, and thickness *dr*:

$$d\mu = AdI$$

where $A = \pi r^2$.

Relate the charge *dq* in the cylinder to the length of the cylinder, its radius, and thickness:

$$dq = 2\pi L\rho r dr$$

Express the current due to this rotating charge:

$$dI = \frac{\omega}{2\pi}dq = \frac{\omega}{2\pi}(2\pi L\rho r dr) = L\omega\rho r dr$$

Substitute to obtain:

$$d\mu = \pi r^2(L\omega\rho r dr) = L\omega\rho\pi r^3 dr$$

Integrate *r* from R_i to R_0 to obtain:

$$\mu = L\omega\rho\pi\int_{R_i}^{R_0} r^3 dr = \boxed{\tfrac{1}{4}L\omega\rho\pi\left(R_0^4 - R_i^4\right)}$$

65 •••

Picture the Problem We can follow the step-by-step outline provided in the problem statement to establish the given results.

(*a*) Express the magnetic moment of the rotating element of charge:

$$d\mu = AdI \qquad (1)$$

The area enclosed by the rotating

$$A = \pi x^2$$

element of charge:

$$dI = \frac{dq}{\Delta t} = \frac{\lambda dx}{\Delta t}$$

where Δt is the time required for one revolution.

Express the time Δt required for one revolution:

$$\Delta t = \frac{1}{f} = \frac{2\pi}{\omega}$$

Substitute to obtain:

$$dI = \frac{\lambda\omega}{2\pi} dx$$

Substitute in equation (1) and simplify to obtain:

$$d\mu = \left(\pi x^2\right)\left(\frac{\lambda\omega}{2\pi} dx\right) = \boxed{\tfrac{1}{2}\lambda\omega x^2 dx}$$

(*b*) Integrate $d\mu$ from $x = 0$ to $x = \ell$:

$$\mu = \tfrac{1}{2}\lambda\omega\int_0^\ell x^2 dx = \boxed{\tfrac{1}{6}\lambda\omega\ell^3}$$

(*c*) Express the angular momentum of the rod:

$$L = I\omega$$

where L is the angular momentum of the rod and I is the moment of inertia of the rod with respect to the point about which it is rotating.

Express the moment of inertia of the rod with respect to an axis through its end:

$$I = \tfrac{1}{3} mL^2$$

where L is now the length of the rod.

Substitute to obtain:

$$L = \tfrac{1}{3} mL^2\omega$$

Divide our expression for μ by L to obtain:

$$\frac{\mu}{L} = \frac{\tfrac{1}{6}\lambda\omega L^3}{\tfrac{1}{3} mL^2\omega} = \frac{\lambda L}{2m}$$

or, because $Q = \lambda L$,

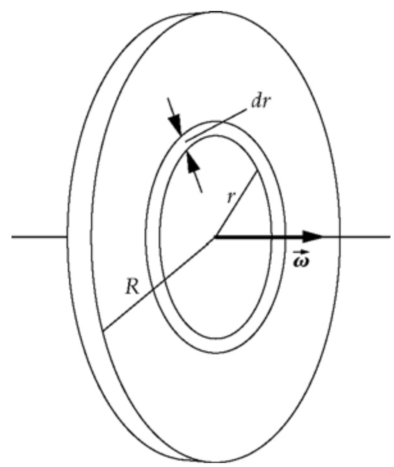
$$\mu = \frac{Q}{2m} L$$

Because $\vec{\boldsymbol{\omega}}$ and $\vec{\boldsymbol{L}} = I\vec{\boldsymbol{\omega}}$ point in the same direction:

$$\vec{\boldsymbol{\mu}} = \boxed{\frac{Q}{2M}\vec{\boldsymbol{L}}}$$

66 •••

Picture the Problem We can express the magnetic moment of an element of current dI due to a ring of radius r, and thickness dr with charge dq. Integrating this expression from $r = 0$ to $r = R$ will give us the magnetic moment of the disk. We can integrate the charge on the ring between these same limits to find the total charge on the disk and divide μ by Q to establish the relationship between them. In part (b) we can find the angular momentum of the disk by first finding the moment of inertia of the disk by integrating r^2dm between the same limits used above.



(a) Express the magnetic moment of an element of the disk:

$$d\mu = AdI$$

The area enclosed by the rotating element of charge is:

$$A = \pi x^2$$

Express the element of current dI:

$$dI = \frac{dq}{\Delta t} = \frac{\sigma dA}{\Delta t} = f\sigma dA$$
$$= \frac{\omega}{2\pi}\left(\sigma_0\frac{r}{R}\right)(2\pi r dr) = \frac{\sigma_0\omega}{R}r^2dr$$

Substitute and simplify to obtain:

$$d\mu = \pi r^2\frac{\sigma_0\omega}{R}r^2dr = \frac{\sigma_0\pi\omega}{R}r^4dr$$

Integrate $d\mu$ from $r = 0$ to $r = R$ to obtain:

$$\mu = \frac{\sigma_0\pi\omega}{R}\int_0^R r^4dr = \boxed{\tfrac{1}{5}\sigma_0\pi\omega R^4} \quad (1)$$

Express the charge dq within a distance r of the center of the disk:

$$dq = 2\pi r\sigma dr = 2\pi r\left(\sigma_0\frac{r}{R}\right)dr$$
$$= \frac{2\pi\sigma_0}{R}r^2dr$$

Integrate dq from $r = 0$ to $r = R$ to obtain:

$$Q = \frac{2\pi\sigma_0}{R}\int_0^R r^2dr = \tfrac{2}{3}\pi\sigma_0R^2 \quad (2)$$

Divide equation (1) by Q to obtain:

$$\frac{\mu}{Q}=\frac{\frac{1}{5}\sigma_0\pi\omega R^4}{\frac{2}{3}\pi\sigma_0 R^2}=\frac{3\omega R^2}{10}$$

and

$$\mu=\boxed{\tfrac{3}{10}Q\omega R^2} \qquad (3)$$

(*b*) Express the moment of inertia of an element of mass *dm* of the disk:

$$dI=r^2dm=r^2\sigma_{\rm m}dA$$
$$=r^2\left(\frac{m}{Q}\sigma\right)(2\pi r dr)$$
$$=\frac{2\pi m\left(\frac{r}{R}\sigma_0\right)}{Q}r^3dr$$
$$=\frac{2\pi m\sigma_0}{QR}r^4dr$$

Integrate dI from $r = 0$ to $r = R$ to obtain:

$$I=\frac{2\pi m\sigma_0}{QR}\int_0^R r^4dr=\frac{2\pi m\sigma_0}{5Q}R^4$$

Divide I by equation (2) and simplify to obtain:

$$\frac{I}{Q}=\frac{\frac{2\pi m\sigma_0}{5Q}R^4}{\frac{2}{3}\pi\sigma_0R^2}=\frac{3m}{5Q}R^2$$

and

$$I=\frac{3m}{5}R^2$$

Express the angular momentum of the disk:

$$L=I\omega=\tfrac{3}{5}mR^2\omega$$

Divide equation (3) by L and simplify to obtain:

$$\frac{\mu}{L}=\frac{\frac{3}{10}Q\omega R^2}{\frac{3}{5}mR^2\omega}=\frac{Q}{2m}$$

and

$$\mu=\frac{Q}{2m}L$$

Because $\vec{\boldsymbol{\mu}}$ is in the same direction as $\vec{\boldsymbol{\omega}}$:

$$\vec{\boldsymbol{\mu}}=\boxed{\frac{Q}{2m}\vec{\boldsymbol{L}}}$$

67 •••

Picture the Problem We can use the general result from Example 26-11 and Problem 63 to express μ as a function of Q, M, and L. We can then use the definitions of surface charge density and angular momentum to substitute for Q and L to obtain the magnetic moment of the rotating sphere.

Express the magnetic moment of the spherical shell in terms of its mass, charge, and angular momentum:

$$\mu = \frac{Q}{2M}L$$

Use the definition of surface charge density to express the charge on the spherical shell is:

$$Q = \sigma A = 4\pi\sigma R^2$$

Express the angular momentum of the spherical shell:

$$L = I\omega = \tfrac{2}{3}MR^2\omega$$

Substitute to obtain:

$$\mu = \left(\frac{4\pi\sigma R^2}{2M}\right)\left(\frac{2}{3}MR^2\omega\right) = \boxed{\tfrac{4}{3}\pi\sigma R^4\omega}$$

68 •••

Picture the Problem We can use the general result from Example 26-11 and Problem 63 to express μ as a function of Q, M, and L. We can then use the definitions of volume charge density and angular momentum to substitute for Q and L to obtain the magnetic moment of the rotating sphere.

Express the magnetic moment of the solid sphere in terms of its mass, charge, and angular momentum:

$$\mu = \frac{Q}{2M}L$$

Use the definition of volume charge density to express the charge of the sphere:

$$Q = \rho V = \tfrac{4}{3}\pi\rho R^3$$

Express the angular momentum of the solid sphere:

$$L = I\omega = \tfrac{2}{5}MR^2\omega$$

Substitute to obtain:

$$\mu = \left(\frac{\tfrac{4}{3}\pi\rho R^3}{2M}\right)\left(\frac{2}{5}MR^2\omega\right) = \boxed{\tfrac{4}{15}\pi\rho R^5\omega}$$

***69** •••

Picture the Problem We can use its definition to express the torque acting on the disk and the definition of the precession frequency to find the precession frequency of the disk.

(*a*) The magnitude of the net torque acting on the disk is:

$$\tau = \mu B\sin\theta$$

where μ is the magnetic moment of the disk.

From example 26-11:

$$\mu = \frac{1}{4}\pi\sigma r^4\omega$$

Substitute for μ in the expression for τ to obtain:

$$\tau = \boxed{\frac{1}{4}\pi\sigma r^4\omega B\sin\theta}$$

(*b*) The precession frequency Ω is equal to the ratio of the torque divided by the spin angular momentum:

$$\Omega = \frac{\tau}{I\omega}$$

For a solid disk, the moment of inertia is given by:

$$I = \frac{1}{2}mr^2$$

Substitute for τ and I to obtain:

$$\Omega = \frac{\frac{1}{4}\pi\sigma r^4\omega B\sin\theta}{\frac{1}{2}mr^2\omega} = \boxed{\frac{\pi\sigma r^2 B}{2m}\sin\theta}$$

Remarks: It's interesting that the precession frequency is independent of ω.

The Hall Effect

70 •

Picture the Problem We can use the Hall effect equation to find the drift velocity of the electrons and the relationship between the current and the number density of charge carriers to find *n*. In (*c*) we can use a right-hand rule to decide whether *a* or *b* is at the higher potential.

(*a*) Express the Hall voltage as a function of the drift velocity of the electrons in the strip:

$$V_H = v_d Bw$$

Solve for v_d:

$$v_d = \frac{V_H}{Bw}$$

Substitute numerical values and evaluate v_d:

$$v_d = \frac{4.27\ \mu\text{V}}{(2\,\text{T})(2\,\text{cm})} = \boxed{0.107\,\text{mm/s}}$$

(*b*) Express the current as a function of the number density of charge carriers:

$$I = nAqv_d$$

Solve for n:

$$n = \frac{I}{Aqv_\text{d}}$$

Substitute numerical values and evaluate n:

$$n = \frac{20\,\text{A}}{(2\,\text{cm})(0.1\,\text{cm})(1.60\times10^{-19}\,\text{C})(0.107\,\text{mm/s})} = \boxed{5.84\times10^{28}\,\text{m}^{-3}}$$

(c) Apply a right-hand rule to $I\vec{\ell}$ and $\vec{\boldsymbol{B}}$ to conclude that positive charge will accumulate at a and negative charge at b and therefore $\boxed{V_a > V_b}$.

71 ••

Picture the Problem We can use $I = nqv_\text{d}A$ to find the drift velocity and $V_\text{H} = v_\text{d}Bw$ to find the Hall voltage.

(a) Express the current in the metal strip in terms of the drift velocity of the electrons:

$$I = nqv_\text{d}A$$

Solve for v_d:

$$v_\text{d} = \frac{I}{nqA}$$

Substitute numerical values and evaluate v_d:

$$v_\text{d} = \frac{10\,\text{A}}{(8.47\times10^{22}\,\text{cm}^{-3})(1.60\times10^{-19}\,\text{C})(2\,\text{cm})(0.1\,\text{cm})} = \boxed{3.69\times10^{-5}\,\text{m/s}}$$

(b) Relate the Hall voltage to the drift velocity and the magnetic field:

$$V_\text{H} = v_\text{d}Bw$$

Substitute numerical values and evaluate V_H:

$$V_\text{H} = (3.69\times10^{-5}\,\text{m/s})(2\,\text{T})(2\,\text{cm}) = \boxed{1.48\,\mu\text{V}}$$

***72** ••

Picture the Problem We can use $V_\text{H} = v_\text{d}Bw$ to express B in terms of V_H and $I = nqv_\text{d}A$ to eliminate the drift velocity v_d and derive an expression for B in terms of V_H, n, and t.

Relate the Hall voltage to the drift velocity and the magnetic field:

$$V_\text{H} = v_\text{d}Bw$$

Solve for B to obtain:

$$B = \frac{V_{\mathrm{H}}}{v_{\mathrm{d}} w}$$

Express the current in the metal strip in terms of the drift velocity of the electrons:

$$I = nqv_{\mathrm{d}}A$$

Solve for v_{d} to obtain:

$$v_{\mathrm{d}} = \frac{I}{nqA}$$

Substitute and simplify to obtain:

$$B = \frac{V_{\mathrm{H}}}{\frac{I}{nqA}w} = \frac{nqAV_{\mathrm{H}}}{Iw} = \frac{nqwtV_{\mathrm{H}}}{Iw} = \frac{nqt}{I}V_{\mathrm{H}}$$

Substitute numerical values and simplify to obtain:

$$B = \frac{\left(8.47\times10^{22}\ \mathrm{cm}^{-3}\right)\left(1.60\times10^{-19}\ \mathrm{C}\right)\left(0.1\,\mathrm{cm}\right)V_{\mathrm{H}}}{20\,\mathrm{A}} = \left(6.78\times10^{5}\ \mathrm{s/m^2}\right)V_{\mathrm{H}}$$

(*a*) Evaluate B for $V_{\mathrm{H}} = 2.00\ \mu\mathrm{V}$:

$$B = \left(6.78\times10^{5}\ \mathrm{s/m^2}\right)\left(2.00\,\mu\mathrm{V}\right) = \boxed{1.36\,\mathrm{T}}$$

(*b*) Evaluate B for $V_{\mathrm{H}} = 5.25\ \mu\mathrm{V}$:

$$B = \left(6.78\times10^{5}\ \mathrm{s/m^2}\right)\left(5.25\,\mu\mathrm{V}\right) = \boxed{3.56\,\mathrm{T}}$$

(*c*) Evaluate B for $V_{\mathrm{H}} = 8.00\ \mu\mathrm{V}$:

$$B = \left(6.78\times10^{5}\ \mathrm{s/m^2}\right)\left(8.00\,\mu\mathrm{V}\right) = \boxed{5.42\,\mathrm{T}}$$

73 ••

Picture the Problem We can use $V_{\mathrm{H}} = v_{\mathrm{d}}Bw$ to find the Hall voltage developed across the diameter of the artery.

Relate the Hall voltage to the flow speed of the blood v_{d}, the diameter of the artery w, and the magnetic field B:

$$V_{\mathrm{H}} = v_{\mathrm{d}}Bw$$

Substitute numerical values and evaluate V_H:

$$V_H = (0.6\,\text{m/s})(0.2\,\text{T})(0.85\,\text{cm}) = \boxed{1.02\,\text{mV}}$$

74 ••

Picture the Problem Let the width of the slab be *w* and its thickness *t*. We can use the definition of the Hall electric field in the slab, the expression for the Hall voltage across it, and the definition of current density to show that the Hall coefficient is also given by 1/(*nq*).

Express the Hall coefficient:

$$R = \frac{E_y}{J_x B_z}$$

Using its definition, express the Hall electric field in the slab:

$$E_y = \frac{V_H}{w}$$

Express the current density in the slab:

$$J_x = \frac{I}{wt} = nqv_d$$

Substitute to obtain:

$$R = \frac{\frac{V_H}{w}}{nqv_d B_z} = \frac{V_H}{nqv_d w B_z}$$

Express the Hall voltage in terms of v_d, B, and w:

$$V_H = v_d B_z w$$

Substitute and simplify to obtain:

$$R = \frac{v_d B_z w}{nqv_d w B_z} = \boxed{\frac{1}{nq}}$$

***75** ••

Picture the Problem We can determine the number of conduction electrons per atom from the quotient of the number density of charge carriers and the number of charge carriers per unit volume. Let the width of a slab of aluminum be *w* and its thickness *t*. We can use the definition of the Hall electric field in the slab, the expression for the Hall voltage across it, and the definition of current density to find *n* in terms of *R* and *q* and $n_a = \rho N_A / M$, to express n_a.

Express the number of electrons per atom N:

$$N = \frac{n}{n_a} \qquad (1)$$

where *n* is the number density of charge carriers and n_a is the number of atoms per unit volume.

From the definition of the Hall coefficient we have:

$$R=\frac{E_y}{J_x B_z}$$

Express the Hall electric field in the slab:

$$E_y=\frac{V_H}{w}$$

Express the current density in the slab:

$$J_x=\frac{I}{wt}=nqv_d$$

Substitute to obtain:

$$R=\frac{\frac{V_H}{w}}{nqv_d B_z}=\frac{V_H}{nqv_d wB_z}$$

Express the Hall voltage in terms of v_d, B, and w:

$$V_H=v_d B_z w$$

Substitute and simplify to obtain:

$$R=\frac{v_d B_z w}{nqv_d wB_z}=\frac{1}{nq}$$

Solve for and evaluate n:

$$n=\frac{1}{Rq} \qquad (2)$$

Express the number of atoms n_a per unit volume:

$$n_a=\rho\frac{N_A}{M} \qquad (3)$$

Substitute equations (2) and (3) in equation (1) to obtain:

$$N=\frac{M}{qR\rho N_A}$$

Substitute numerical values and evaluate N:

$$N=\frac{27\text{ g/mol}}{\left(-1.60\times10^{-19}\text{ C}\right)\left(-0.3\times10^{-10}\text{ m}^3\text{/C}\right)\left(2.7\times10^3\text{ kg/m}^3\right)\left(6.02\times10^{23}\text{ atoms/mol}\right)}=\boxed{3.46}$$

General Problems

76 •

Picture the Problem We can use the expression for the magnetic force acting on a wire ($\vec{F}=I\vec{\ell}\times\vec{B}$) to find the force per unit length on the wire.

Express the magnetic force on the

$$\vec{F}=I\vec{\ell}\times\vec{B}$$

wire:

Substitute for $I\vec{\ell}$ and $\vec{B}$ to obtain:

$$\vec{F} = (6.5\,\text{A})\ell\hat{i} \times (1.35\,\text{T})\hat{j}$$

and

$$\frac{\vec{F}}{\ell} = (6.5\,\text{A})\hat{i} \times (1.35\,\text{T})\hat{j}$$

Simplify to obtain:

$$\frac{\vec{F}}{\ell} = (8.78\,\text{N/m})(\hat{i} \times \hat{j}) = \boxed{(8.78\,\text{N/m})\hat{k}}$$

77 •

Picture the Problem We can express the period of the alpha particle's motion in terms of its orbital speed and use Newton's 2nd law to express its orbital speed in terms of known quantities. Knowing the particle's period and the radius of its motion we can find its speed and kinetic energy.

(*a*) Relate the period of the alpha particle's motion to its orbital speed:

$$T = \frac{2\pi r}{v} \qquad (1)$$

Apply $\sum F_{\text{radial}} = ma_c$ to the alpha particle to obtain:

$$qvB = m\frac{v^2}{r}$$

Solve for v to obtain:

$$v = \frac{qBr}{m}$$

Substitute and simplify to obtain:

$$T = \frac{2\pi r}{\frac{qBr}{m}} = \frac{2\pi m}{qB}$$

Substitute numerical values and evaluate T:

$$T = \frac{2\pi(6.65\times10^{-27}\,\text{kg})}{2(1.60\times10^{-19}\,\text{C})(1\,\text{T})} = \boxed{0.131\,\mu\text{s}}$$

(*b*) Solve equation (1) for v:

$$v = \frac{2\pi r}{T}$$

Substitute numerical values and evaluate v:

$$v = \frac{2\pi(0.5\,\text{m})}{0.131\,\mu\text{s}} = \boxed{2.40\times10^{7}\,\text{m/s}}$$

(*c*) Express the kinetic energy of the alpha particle:

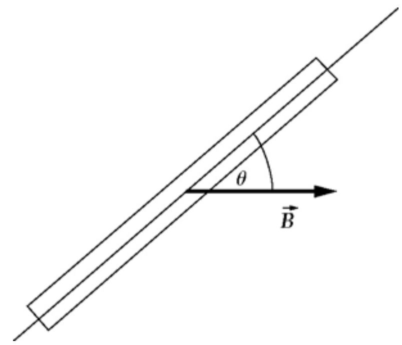
$$K = \tfrac{1}{2}mv^2$$

Substitute numerical values and evaluate K:

$$K = \tfrac{1}{2}\left(6.65\times10^{-27}\text{ kg}\right)\left(2.41\times10^{7}\text{ m/s}\right)^2$$
$$= 1.93\times10^{-12}\text{ J}\times\frac{1\text{eV}}{1.60\times10^{-19}\text{ J}}$$
$$= \boxed{12.0\,\text{MeV}}$$

78 ••

Picture the Problem The configuration of the magnet and field are shown in the figure. We'll assume that a force $+q_\text{m}\vec{\boldsymbol{B}}$ is exerted on the north pole and a force $-q_\text{m}\vec{\boldsymbol{B}}$ is exerted on the south pole and show that this assumption leads to the familiar expression for the torque acting on a magnetic dipole.



Assuming that a force $+q_\text{m}\vec{\boldsymbol{B}}$ is exerted on the north pole and a force $-q_\text{m}\vec{\boldsymbol{B}}$ is exerted on the south pole, express the net torque acting on the bar magnet:

$$\tau = \frac{Bq_\text{m}L}{2}\sin\theta - \frac{-Bq_\text{m}L}{2}\sin\theta$$
$$= Bq_\text{m}L\sin\theta$$

Substitute for q_m to obtain:

$$\tau = B\frac{|\vec{\boldsymbol{\mu}}|}{L}L\sin\theta = \mu B\sin\theta$$

or

$$\vec{\boldsymbol{\tau}} = \boxed{\vec{\boldsymbol{\mu}}\times\vec{\boldsymbol{B}}}$$

***79** ••

Picture the Problem We can use $\vec{\boldsymbol{F}} = q\vec{\boldsymbol{v}}\times\vec{\boldsymbol{B}}$ to show that motion of the particle in the x direction is not affected by the magnetic field. The application of Newton's 2nd law to motion of the particle in yz plane will lead us to the result that $r = mv_{0y}/qB$. By expressing the period of the motion in terms of v_{0y} we can show that the time for one complete orbit around the helix is $t = 2\pi m/qB$.

(*a*) Express the magnetic force acting on the particle:

$$\vec{\boldsymbol{F}} = q\vec{\boldsymbol{v}}\times\vec{\boldsymbol{B}}$$

Substitute for $\vec{v}$ and $\vec{B}$ and simplify to obtain:

$$\vec{F} = q\left(v_{0x}\hat{i} + v_{0y}\hat{j}\right)\times B\hat{i}$$
$$= qv_{0x}B\left(\hat{i}\times\hat{i}\right) + qv_{0y}B\left(\hat{j}\times\hat{i}\right)$$
$$= 0 - qv_{0y}B\hat{k} = -qv_{0y}B\hat{k}$$

i.e., the motion in the direction of the magnetic field (the x direction) is not affected by the field.

Apply $\sum F_{\text{radial}} = ma_{\text{c}}$ to the motion of the particle in the plane perpendicular to $\hat{i}$ (i.e., the yz plane):

$$qv_{0y}B = m\frac{v_{0y}^2}{r} \qquad (1)$$

Solve for r:

$$r = \boxed{\frac{mv_{0y}}{qB}}$$

(b) Relate the time for one orbit around the helix to the particle's orbital speed:

$$t = \frac{2\pi r}{v_{0y}}$$

Solve equation (1) for v_{0y}:

$$v_{0y} = \frac{qBr}{m}$$

Substitute and simplify to obtain:

$$t = \frac{2\pi r}{\frac{qBr}{m}} = \boxed{\frac{2\pi m}{qB}}$$

***80** ••

Picture the Problem We can use a constant-acceleration equation to relate the velocity of the crossbar to its acceleration and Newton's 2nd law to express the acceleration of the crossbar in terms of the magnetic force acting on it. We can determine the direction of motion of the crossbar using a right-hand rule or, equivalently, by applying $\vec{F} = I\vec{\ell}\times\vec{B}$. We can find the minimum field B necessary to start the bar moving by applying a condition for static equilibrium to it.

(a) Using a constant-acceleration equation, express the velocity of the bar as a function of its acceleration and the time it has been in motion:

$$v = v_0 + at$$

or, because $v_0 = 0$,

$$v = at$$

Use Newton's 2nd law to express the acceleration of the rail:

$$a = \frac{F}{m}$$

where F is the magnitude of the magnetic force acting in the direction of the crossbar's motion.

Substitute to obtain:

$$v = \frac{F}{m}t$$

Express the magnetic force acting on the current-carrying crossbar:

$$F = ILB$$

Substitute to obtain:

$$v = \boxed{\frac{ILB}{m}t}$$

(*b*) Apply to conclude that the magnetic force is to the right and so the motion of the crossbar will also be to the right.

(*c*) Apply $\sum F_x = 0$ to the crossbar:

$$ILB_{\text{min}} - f_{\text{s,max}} = 0$$

or

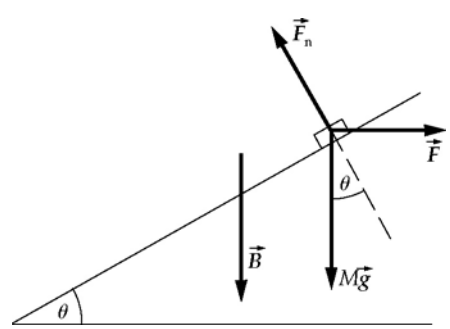
$$ILB_{\text{min}} - \mu_{\text{s}} mg = 0$$

Solve for B_{min} to obtain:

$$B_{\text{min}} = \boxed{\frac{\mu_{\text{s}} mg}{IL}}$$

81 ••

Picture the Problem Note that with the rails tilted, $\vec{F}$ still points horizontally to the right (I, and hence $\vec{\ell}$, is out of the page). Choose a coordinate system in which down the incline is the positive x direction. Then we can apply a condition for translational equilibrium to find the vertical magnetic field $\vec{B}$ is needed to keep the bar from sliding down the rails. In part (b) we can apply Newton's 2nd law to find the acceleration of the crossbar when B is twice its value found in (a).



(*a*) Apply $\sum F_x = 0$ to the crossbar

$$mg\sin\theta - I\ell B\cos\theta = 0$$

to obtain:

Solve for B:

$$B=\frac{mg}{I\ell}\tan\theta \text{ and } \vec{\boldsymbol{B}}=\boxed{-\frac{mg}{I\ell}\tan\theta\,\hat{\boldsymbol{u}}_{\text{v}}}$$

where $\hat{\boldsymbol{u}}_{\text{v}}$ is a unit vector in the vertical direction.

(b) Apply $\sum F_x = ma$ to the crossbar to obtain:

$$I\ell B'\cos\theta - mg\sin\theta = ma$$

Solve for a:

$$a=\frac{I\ell B'}{m}\cos\theta - g\sin\theta$$

Substitute $B' = 2B$ and simplify to obtain:

$$a=\frac{2I\ell\frac{mg}{I\ell}\tan\theta}{m}\cos\theta - g\sin\theta = 2g\sin\theta - g\sin\theta = \boxed{g\sin\theta}$$

Note that the direction of the acceleration is up the slope.

82 ••

Picture the Problem We're being asked to show that, for small displacements from equilibrium, the bar magnet executes simple harmonic motion. To show its motion is SHM we need to show that the bar magnet experiences a linear restoring torque when displaced from equilibrium. We can accomplish this by applying Newton's 2nd law in rotational form and using a small angle approximation to obtain the differential equation for simple harmonic motion. Once we have the DE we can identify ω and express f.

Apply $\sum\tau = I\alpha$ to the bar magnet:

$$-\mu B\sin\theta = I\frac{d^2\theta}{dt^2}$$

where the minus sign indicates that the torque acts in such a manner as to align the magnet with the magnetic field and I is the moment of inertia of the magnet.

For small displacements from equilibrium, $\theta << 1$ and:

$$\sin\theta \approx \theta$$

Hence our differential equation of motion becomes:

$$I\frac{d^2\theta}{dt^2} = -\mu B\theta$$

Thus for small displacements from equilibrium we see that the differential equation describing the motion of the bar magnet is the differential equation of simple harmonic motion. Solve this equation for $d^2\theta/dt^2$ to obtain:

$$\frac{d^2\theta}{dt^2} = -\frac{\mu B}{I}\theta = -\omega^2\theta$$

where $\omega = \sqrt{\frac{\mu B}{I}}$

Relate f to ω to obtain:

$$f = \frac{\omega}{2\pi} = \boxed{\frac{1}{2\pi}\sqrt{\frac{\mu B}{I}}}$$

83 ••

Picture the Problem We can use $\vec{F} = q\vec{v}\times\vec{B}$ to find the magnitude and direction of the magnetic force experienced by an electron in the conducting wire. In (*b*) we can use a condition for translational equilibrium to relate $\vec{E}$ to $\vec{F}$. In (*c*) we can apply the definition of electric field in terms of potential difference to evaluate the difference in potential between the ends of the moving wire.

(*a*) Express the magnetic force on an electron in the conductor:

$$\vec{F} = q\vec{v}\times\vec{B} = qv\hat{i}\times B\hat{k}$$
$$= qvB\left(\hat{i}\times\hat{k}\right) = -qvB\hat{j}$$

Substitute numerical values and evaluate $\vec{F}$:

$$\vec{F} = -\left(-1.60\times10^{-19}\text{ C}\right)(20\text{ m/s})(0.5\text{ T})\hat{j} = \boxed{\left(1.60\times10^{-18}\text{ N}\right)\hat{j}}$$

(*b*) Sum the forces acting on an electron under steady-state conditions to obtain:

$$q\vec{E} + \vec{F} = 0$$

Solve for $\vec{E}$:

$$\vec{E} = -\frac{\vec{F}}{q}$$

Substitute our result in part (*a*) to obtain:

$$\vec{E} = -\frac{\left(1.60\times10^{-18}\text{ N}\right)\hat{j}}{-1.60\times10^{-19}\text{ C}} = \boxed{(10.0\text{ V/m})\hat{j}}$$

(*c*) The potential difference between the ends of the wire is:

$$\Delta V = E\Delta x = (10.0\,\text{V/m})(2\,\text{m}) = \boxed{20.0\,\text{V}}$$

84 •••

Picture the Problem We can use $T = 2\pi\sqrt{I/MgD}$ to find the period of small-displacement oscillations with no current flowing in the frame. With a current flowing, the frame will experience an additional restoring torque that will reduce its period. In part (c) we can apply the condition for rotational equilibrium to find the magnitude of the current that will put the frame in equilibrium.

(*a*) Express the period of a physical pendulum:

$$T = 2\pi\sqrt{\frac{I}{MgD}} \qquad (1)$$

where *D* is the distance from the pivot to the center of mass of the pendulum.

Express the moment of inertia of the frame:

$$I = I_{\text{hor. segment}} + 2I_{\text{vert. segment}} = m_{\text{hor. segment}}h^2 + 2\left(\tfrac{1}{3}m_{\text{ver. segment}}h^2\right)$$

where h = 10 cm.

Using the linear density of the frame, calculate $m_{\text{hor. segment}}$ and $m_{\text{ver. segment}}$:

$$m_{\text{hor. segment}} = \lambda w = (20\,\text{g/cm})(6\,\text{cm}) = 0.12\,\text{kg}$$

and

$$m_{\text{ver. segment}} = \lambda h = (20\,\text{g/cm})(10\,\text{cm}) = 0.2\,\text{kg}$$

Substitute and evaluate *I*:

$$I = (0.12\,\text{kg})(0.1\text{m})^2 + 2\left[\tfrac{1}{3}(0.2\,\text{kg})(0.1\text{m})^2\right] = 2.53\times10^{-3}\,\text{kg}\cdot\text{m}^2$$

Evaluate the distance *D* to the center of mass from the A-A axis:

$$D = \frac{2(0.05\,\text{m})(0.2\,\text{kg}) + (0.1\text{m})(0.12\,\text{kg})}{0.12\,\text{kg} + 0.2\,\text{kg} + 0.2\,\text{kg}} = 6.15\,\text{cm}$$

Substitute in equation (1) and evaluate *T*:

$$T = 2\pi\sqrt{\frac{2.53\times10^{-3}\,\text{kg}\cdot\text{m}^2}{(0.52\,\text{kg})(9.81\,\text{m/s}^2)(6.15\,\text{cm})}} = \boxed{0.564\,\text{s}}$$

(*b*) Express the restoring torque with $\vec{B}$ and *I* as shown:

$$\tau = (MgD + BIA)\theta$$

where *A* is the area of the loop and provided θ << 1 rad.

Rewrite equation (1) with this restoring torque:

$$T' = 2\pi\sqrt{\frac{I}{MgD + BIA}}$$

Evaluate *BIA*:

$$BIA = (0.2\,\text{T})(8\,\text{A})(10\,\text{cm})(6\,\text{cm})$$
$$= 9.60\times10^{-3}\,\text{N}\cdot\text{m}$$

Substitute numerical values and evaluate *T′*:

$$T = 2\pi\sqrt{\frac{2.53\times10^{-3}\,\text{kg}\cdot\text{m}^2}{0.314\,\text{N}\cdot\text{m} + 9.60\times10^{-3}\,\text{N}\cdot\text{m}}}$$
$$= \boxed{0.556\,\text{s}}$$

(*c*) Apply $\sum\tau = 0$ to the frame when it is in equilibrium to obtain:

$$MgD\sin\theta - BIA\sin\theta = 0$$

Solve for *I*:

$$I = \frac{MgD}{BA}$$

Substitute numerical values and evaluate *I*:

$$I = \frac{(0.52\,\text{kg})(9.81\,\text{m/s}^2)(6.15\,\text{cm})}{(0.2\,\text{T})(10\,\text{cm})(6\,\text{cm})}$$
$$= \boxed{262\,\text{A}}$$

***85** •••

Picture the Problem We can use a constant-acceleration equation to express the height to which the wire rises in terms of its initial speed and the acceleration due to gravity. We can then use the impulse-change in momentum equation to express the initial speed of the wire in terms of the impulsive magnetic force acting on it. Finally, we can use the definition of current to relate the charge delivered by the battery to the time during which the impulsive force acts.

Using a constant-acceleration equation, relate the height *h* to the initial and final speeds and the acceleration of the wire:

$$v^2 = v_0^2 + 2a_y h$$

or, because $v = 0$ and $a_y = g$,

$$0 = v_0^2 - 2gh$$

Solve for *h*:

$$h = \frac{v_0^2}{2g} \qquad (1)$$

Use the impulse-momentum equation to relate the change in momentum of the wire to the impulsive force accelerating it:

$\Delta p = F\Delta t$ or $p_f - p_i = F\Delta t$
and, because $p_i = 0$, $mv_0 = F\Delta t$

Express the impulsive (magnetic) force acting on the wire:

$F = I\ell B$

Substitute to obtain:

$mv_0 = I\ell B\Delta t$

Solve for v_0 and substitute in equation (1):

$$h = \frac{\left(\frac{I\ell B\Delta t}{m}\right)^2}{2g} = \frac{(I\ell B\Delta t)^2}{2m^2 g}$$

Use the definition of current to relate the charge delivered by the battery to the time during which it delivers the current:

$\Delta Q = I\Delta t$

Substitute to obtain:

$$h = \frac{(\ell B\Delta Q)^2}{2m^2 g}$$

Substitute numerical values and evaluate h:

$$h = \frac{[(0.25\,\text{m})(0.4\,\text{T})(2\,\text{C})]^2}{2(0.02\,\text{kg})^2(9.81\,\text{m/s}^2)} = \boxed{5.10\,\text{m}}$$

86 •••

Picture the Problem We're being asked to show that, for small displacements from equilibrium, the circular loop executes simple harmonic motion. To show its motion is SHM we must show that the loop experiences a linear restoring torque when displaced from equilibrium. We can accomplish this by applying Newton's 2nd law in rotational form and using a small angle approximation to obtain the differential equation for simple harmonic motion. Once we have the DE we can identify ω and express the period of the motion T.

Apply $\sum\tau = I\alpha$ to the loop:

$$-IAB\sin\theta = I_{\text{inertia}}\frac{d^2\theta}{dt^2}$$

where the minus sign indicates that the torque acts in such a manner as to align the loop with the magnetic field and I_{inertia} is the moment of inertia of the loop.

For small displacements from equilibrium, $\theta << 1$ and:

$$\sin\theta \approx \theta$$

Hence, our differential equation of motion becomes:

$$I_{\text{inertia}} \frac{d^2\theta}{dt^2} = -IAB\theta$$

Thus for small displacements from equilibrium we see that the differential equation describing the motion of the current loop is the differential equation of simple harmonic motion. Solve this equation for $d^2\theta/dt^2$ to obtain:

$$\frac{d^2\theta}{dt^2} = -\frac{IAB}{I_{\text{inertia}}}\theta$$

.

Noting that the moment of inertia of a hoop about its diameter is $\frac{1}{2}mR^2$, substitute for I_{inertia} and simplify to obtain:

$$\frac{d^2\theta}{dt^2} = -\frac{I\pi R^2 B}{\frac{1}{2}mR^2}\theta = -\frac{2I\pi B}{m}\theta = -\omega^2\theta$$

where $\omega = \sqrt{\frac{2\pi IB}{m}}$

Relate the period T of the motion to ω and substitute to obtain:

$$T = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{m}{2\pi IB}}}$$

87 •••

Picture the Problem We can express $\vec{\boldsymbol{\mu}}$ in terms of its components and calculate U from $\vec{\boldsymbol{\mu}}$ and $\vec{\boldsymbol{B}}$ using $U = -\vec{\boldsymbol{\mu}}\cdot\vec{\boldsymbol{B}}$. Knowing U we can calculate the components of $\vec{\boldsymbol{F}}$ using $F_x = -dU/dx$ and $F_y = -dU/dy$.

Express the net force acting on the magnet in terms of its components:

$$\vec{\boldsymbol{F}} = F_x\hat{\boldsymbol{i}} + F_y\hat{\boldsymbol{j}} \qquad (1)$$

Express $\vec{\boldsymbol{\mu}}$ in terms of its components:

$$\vec{\boldsymbol{\mu}} = \mu_x\hat{\boldsymbol{i}} + \mu_y\hat{\boldsymbol{j}} + \mu_z\hat{\boldsymbol{k}}$$

Express the potential energy of the bar magnetic in the nonuniform magnetic field:

$$\begin{aligned} U &= -\vec{\boldsymbol{\mu}}\cdot\vec{\boldsymbol{B}} \\ &= -\left(\mu_x\hat{\boldsymbol{i}} + \mu_y\hat{\boldsymbol{j}} + \mu_z\hat{\boldsymbol{k}}\right)\cdot\left(B_x(x)\hat{\boldsymbol{i}} + B_y(y)\hat{\boldsymbol{j}}\right) \\ &= -\mu_x B_x(x) - \mu_y B_y(y) \end{aligned}$$

Because $\vec{\boldsymbol{\mu}}$ is constant but $\vec{\boldsymbol{B}}$ depends on x and y:

$$F_x = -\frac{dU}{dx} = \mu_x\left(\frac{\partial B_x}{\partial x}\right)$$

and

$$F_y = -\frac{dU}{dy} = \mu_y\left(\frac{\partial B_y}{\partial y}\right)$$

Substitute in equation (1) to obtain:

$$\boxed{\vec{F} = \mu_x \frac{\partial B_x}{\partial x}\hat{\boldsymbol{i}} + \mu_y \frac{\partial B_y}{\partial y}\hat{\boldsymbol{j}}}$$

***88 •••**

Picture the Problem We can apply Newton's 2nd law to the particle to derive an expression for the radius of its orbit and then express its period in terms of its orbital speed and radius.

(*a*) Because $\vec{B}$ is perpendicular to $\vec{v}$, the magnitude of force on the particle is given by:

$$F = qvB$$

Apply $\sum F = ma$ to the orbiting particle to obtain:

$$qvB = m(v)\frac{v^2}{r} = \gamma(v)m\frac{v^2}{r}$$

Solve for *r*:

$$r = \boxed{\frac{\gamma(v)mv}{qB}}$$

The period *T* of the particle's motion is related to the radius *r* of its orbit and its orbital speed *v*:

$$T = \frac{2\pi r}{v}$$

Substitute for *r* and simplify to obtain:

$$T = \boxed{\frac{2\pi\gamma(v)m}{qB}}$$

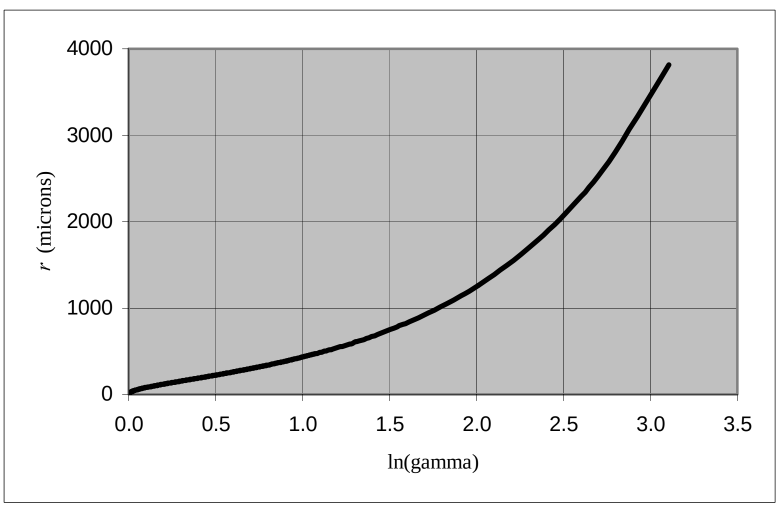
(*b*) A spreadsheet program to calculate *r* and *T* as functions of ln(γ) follows. The formulas used to calculate the quantities in the columns are given in the table.

Cell	Content/Formula	Algebraic Form
B1	9.11E–31	m
B2	1.60E–19	e
B3	10	B
B4	3.00E+08	c
A7	0.100	v/c
A8	0.101	$v/c + 0.001$
B7	1/SQRT(1 – (A7)^2)	γ
C7	LN(B7)	$\ln(\gamma)$
D7	B7*$B$1*A7*$B$4/($B$2*$B$3)	$\frac{\gamma mv}{qB}$
E7	D7*10^8	$10^6 r$

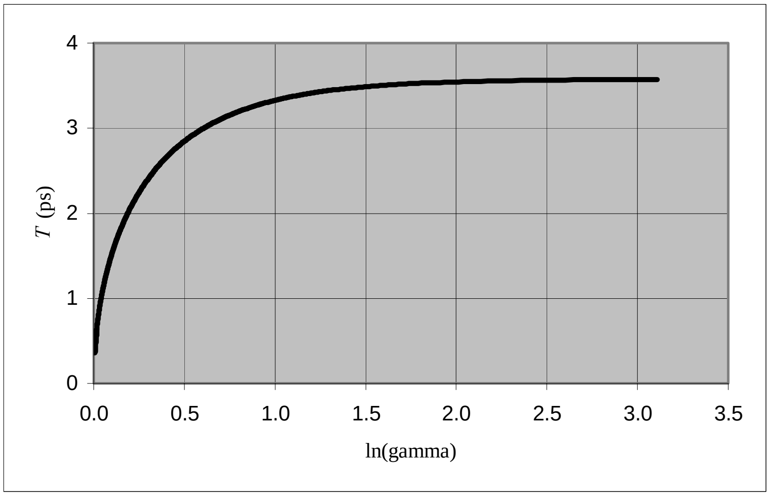
F7	(2*PI()*A7*$B$1/($B$2*$B$3))*10^12	$\frac{2\pi\gamma m}{qB}\times 10^{12}$

	A	B	C	D	E	F
1	m=	9.11E–31	kg			
2	e=	1.60E–19	C			
3	B=	10	T			
4	c=	3.00E+08	m/s			
5						
6	v/c	gamma	ln(gamma)	r	r (microns)	T (ps)
7	0.100	1.0050	0.005	1.72E-05	17.2	0.358
8	0.101	1.0051	0.005	1.73E-05	17.3	0.361
9	0.102	1.0052	0.005	1.75E-05	17.5	0.365
10	0.103	1.0053	0.005	1.77E-05	17.7	0.368
11	0.104	1.0055	0.005	1.79E-05	17.9	0.372
903	0.996	11.1915	2.415	1.90E-03	1904.0	3.563
904	0.997	12.9196	2.559	2.20E-03	2200.2	3.567
905	0.998	15.8193	2.761	2.70E-03	2696.7	3.570
906	0.999	22.3663	3.108	3.82E-03	3816.6	3.574

The following graph of r as a function of $\ln(\gamma)$ was plotted using the data in columns C and E.



The following graph of T as a function of $\ln(\gamma)$ was plotted using the data in columns C and F.


4
3
2
1
0
T (ps)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
ln(gamma)