Chapter 27
Sources of the Magnetic Field

Conceptual Problems

***1 •**

Picture the Problem The electric forces are described by Coulomb's law and the laws of attraction and repulsion of charges and are independent of the fact the charges are moving. The magnetic interaction is, on the other hand, dependent on the motion of the charges. Each moving charge constitutes a current that creates a magnet field at the location of the other charge.

(*a*) The electric forces are repulsive; the magnetic forces are attractive (the two charges moving in the same direction act like two currents in the same direction).

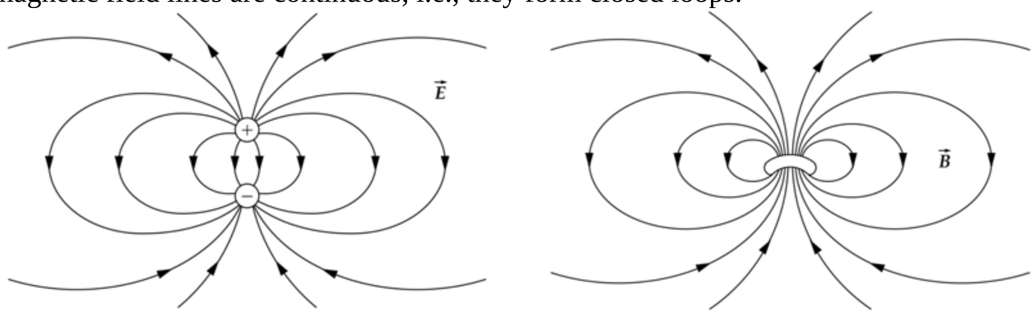
(*b*) The electric forces are again repulsive; the magnetic forces are also repulsive.

2 •

No. The magnitude of the field depends on the location within the loop.

3 •

Picture the Problem The field lines for the electric dipole are shown in the sketch to the left and the field lines for the magnetic dipole are shown in the sketch to the right. Note that, while the far fields (the fields far from the dipoles) are the same, the near fields (the fields between the two charges and inside the current loop/magnetic dipole) are not, and that, in the region between the two charges, the electric field is in the opposite direction to that of the magnetic field at the center of the magnetic dipole. It is especially important to note that while the electric field lines begin and terminate on electric charges, the magnetic field lines are continuous, i.e., they form closed loops.



4 •

Determine the Concept Applying the right-hand rule to the wire to the left we see that the magnetic field due to its current is out of the page at the midpoint. Applying the right-hand rule to the wire to the right we see that the magnetic field due to its current is out of

the page at the midpoint. Hence, the sum of the magnetic fields is out of the page as well.
$\boxed{(c) \text{ is correct.}}$

5 •

Determine the Concept While we could express the force wire 1 exerts on wire 2 and compare it to the force wire 2 exerts on wire 1 to show that they are the same, it is simpler to recognize that these are action and reaction forces. $\boxed{(a) \text{ is correct.}}$

***6** •

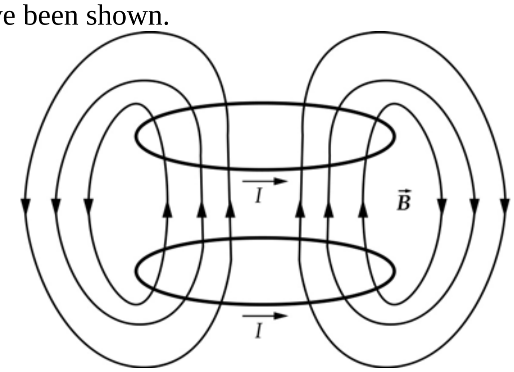
Determine the Concept Applying the right-hand rule to the wire to the left we see that the magnetic field due to the current points to west at all points north of the wire.
$\boxed{(c) \text{ is correct.}}$

7 •

Determine the Concept At points to the west of the vertical wire, the magnetic field due to its current exerts a downward force on the horizontal wire and at points to the east it exerts an upward force on the horizontal wire. Hence, the net magnetic force is zero and
$\boxed{(e) \text{ is correct.}}$

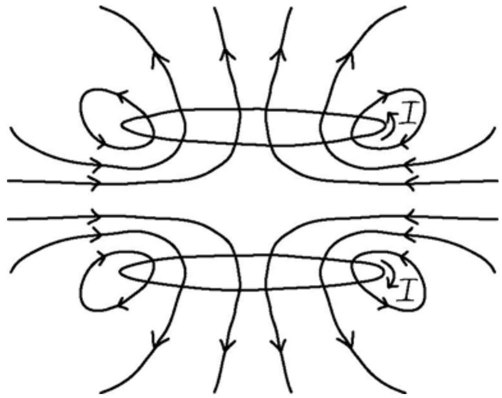
8 •

Picture the Problem The field-line sketch follows. An assumed direction for the current in the coils is shown in the diagram. Note that the field is stronger in the region between the coaxial coils and that the field lines have neither beginning nor ending points as do electric-field lines. Because there are an uncountable infinity of lines, only a representative few have been shown.



***9** •

Picture the Problem The field-line sketch is shown below. An assumed direction for the current in the coils is shown in the diagram. Note that the field lines never begin or end and that they do not touch or cross each other. Because there are an uncountable infinity of lines, only a representative few have been shown.



10 •

Determine the Concept Because all of these statements regarding Ampère's law are true, $\boxed{(e) \text{ is correct.}}$

11 •

(*a*) True

(*b*) True

***12 •**

Determine the Concept The magnetic susceptibility χ_m is defined by the equation $\vec{M} = \chi_m \dfrac{\vec{B}_{app}}{\mu_0}$, where $\vec{M}$ is the magnetization vector and $\vec{B}_{app}$ is the applied magnetic field. For paramagnetic materials, χ_m is a small positive number that depends on temperature, whereas for diamagnetic materials, it is a small negative constant independent of temperature. $\boxed{(a) \text{ is correct.}}$

13 •

(*a*) False. The magnetic field due to a current element is perpendicular to the current element.

(*b*) True

(*c*) False. The magnetic field due to a long wire varies inversely with the distance from the wire.

(*d*) False. Ampère's law is easier to apply if there is a high degree of symmetry, but is valid in all situations.

(*e*) True

14 •

Determine the Concept Yes. The classical relation between magnetic moment and angular momentum is $\vec{\boldsymbol{\mu}} = \frac{q}{2m}\vec{\boldsymbol{L}}$. Thus, if its charge density is zero, a particle with angular momentum will not have a magnetic moment.

15 •

Determine the Concept No. The classical relation between magnetic moment and angular momentum is $\vec{\boldsymbol{\mu}} = \frac{q}{2m}\vec{\boldsymbol{L}}$. Thus, if the angular momentum of the particle is zero, its magnetic moment will also be zero.

16 •

Determine the Concept Yes, there is angular momentum associated with the magnetic moment. The magnitude of $\vec{\boldsymbol{L}}$ is extremely small, but very sensitive experiments have demonstrated its presence (Einstein-de Haas effect).

17 •

Determine the Concept From Ampère’s law, the current enclosed by a closed path within the tube is zero, and from the cylindrical symmetry it follows that $B = 0$ everywhere within the tube.

***18** •

Determine the Concept The force per unit length experienced by each segment of the wire, due to the currents in the other segments of the wire, will be equal. These equal forces will result in the wire tending to form a circle.

19 •

Determine the Concept H_2, CO_2, and N_2 are diamagnetic ($\chi_m < 0$); O_2 is paramagnetic ($\chi_m > 0$).

Estimation and Approximation

20 ••

Picture the Problem We can use the definition of the magnetization of the earth’s core to find its volume and radius.

(*a*) Express the magnetization of the earth’s core in terms of the magnetic moment of the earth and the volume of the core:

$$M = \frac{\mu}{V}$$

Solve for and evaluate V:

$$V = \frac{\mu}{M} = \frac{9\times10^{22}\ \text{A}\cdot\text{m}^2}{1.5\times10^9\ \text{A/m}} = \boxed{6.00\times10^{13}\ \text{m}^3}$$

(b)Assuming a spherical core centered with the earth:

$$V = \tfrac{4}{3}\pi r^3$$

Solve for r:

$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

Substitute numerical values and evaluate r:

$$r = \sqrt[3]{\frac{3\left(6\times10^{13}\ \text{m}^3\right)}{4\pi}} = \boxed{2.43\times10^4\ \text{m}}$$

***21 ••**

Picture the Problem We can model the lightning bolt as a current in a long wire and use the expression for the magnetic field due to such a current to estimate the transient magnetic field 100 m from the lightning bolt.

The magnetic field due to the current in a long, straight wire is:

$$B = \frac{\mu_0}{4\pi}\frac{2I}{r}$$

where r is the distance from the wire.

Assuming that the height of the cloud is 1 km, the charge transfer will take place in roughly 10^{-3} s and the current associated with this discharge is:

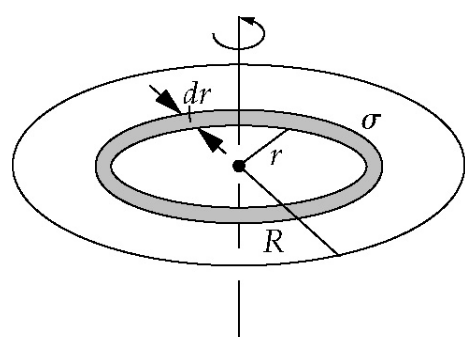
$$I = \frac{\Delta Q}{\Delta t} = \frac{30\ \text{C}}{10^{-3}\ \text{s}} = 3\times10^4\ \text{A}$$

Substitute numerical values and evaluate B:

$$B = \frac{4\pi\times10^{-7}\ \text{N/A}^2}{4\pi}\frac{2\left(3\times10^4\ \text{A}\right)}{100\ \text{m}} = \boxed{60.0\ \mu\text{T}}$$

***22 ••**

Picture the Problem A rotating disk with total charge Q and surface charge density σ is shown in the diagram. We can find Q by deriving an expression for the magnetic field B at the center of the disk due to its rotation. We'll use Ampere's law to express the field dB at the center of the disk due to the element of current dI and then integrate over r to find B.



Applying Ampere's law to a circular current loop of radius r we obtain:

$$B = \frac{\mu_0 I}{2r}$$

The B field at the center of an annular ring on a rotating disk of radius r and thickness dr is:

$$dB = \frac{\mu_0}{2r} dI \qquad (1)$$

If σ represents the surface charge density, then the current in the annular ring is given by:

$$dI = \frac{\sigma(2\pi r)}{T} dr, \text{ where } \sigma = \frac{Q}{\pi R^2}$$

Because $T = \frac{2\pi}{\omega}$:

$$dI = \sigma\omega r dr$$

Substitute for dI in equation (1) to obtain:

$$dB = \frac{\mu_0}{2r} \sigma\omega r dr = \frac{\mu_0 \sigma\omega}{2} dr$$

Integrate from $r = 0$ to R to obtain:

$$B = \frac{\mu_0 \sigma\omega}{2} \int_0^R dr = \frac{\mu_0 \sigma\omega R}{2}$$

Substitution for σ yields:

$$B = \frac{\mu_0 \left(\frac{Q}{\pi R^2}\right) \omega R}{2} = \frac{\mu_0 Q\omega}{2\pi R}$$

Solve for Q to obtain:

$$Q = \frac{2\pi RB}{\mu_0 \omega}$$

Substitute numerical values and evaluate Q:

$$Q = \frac{2\pi(10^7 \text{ m})(0.1\text{T})}{(4\pi \times 10^{-7} \text{ N/A}^2)(10^{-2} \text{ rad/s})} = \boxed{5.00 \times 10^{14} \text{ C}}$$

The electric field above the sunspot is given by:

$$E = \frac{\sigma}{2\epsilon_0} = \frac{Q}{2\pi \epsilon_0 R^2}$$

Substitute numerical values and evaluate E:

$$E = \frac{5.00 \times 10^{14} \text{ C}}{2\pi(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(10^7 \text{ m})^2} = \boxed{90.0 \text{ GN/C}}$$

The Magnetic Field of Moving Point Charges

23 •

Picture the Problem We can substitute for $\vec{v}$ and q in the equation describing the magnetic field of the moving charged particle ($\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$), evaluate r and $\hat{r}$ for each of the given points of interest, and substitute to find $\vec{B}$.

Express the magnetic field of the moving charged particle:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} = \left(10^{-7}\ \text{N/A}^2\right)\left(12\ \mu\text{C}\right)\frac{(30\ \text{m/s})\hat{i}\times\hat{r}}{r^2} = \left(36.0\ \text{pT}\cdot\text{m}^2\right)\frac{\hat{i}\times\hat{r}}{r^2}$$

(*a*) Find *r* and $\hat{r}$ for the particle at (0, 2 m) and the point of interest at the origin:

$$\vec{r} = -(2\,\text{m})\hat{j},\ r = 2\,\text{m},\ \text{and}\ \hat{r} = -\hat{j}$$

Substitute and evaluate $\vec{B}(0,0)$:

$$\vec{B}(0,0) = \left(36.0\,\text{pT}\cdot\text{m}^2\right)\frac{\hat{i}\times\left(-\hat{j}\right)}{(2\,\text{m})^2} = \boxed{-(9.00\,\text{pT})\hat{k}}$$

(*b*) Find *r* and $\hat{r}$ for the particle at (0, 2 m) and the point of interest at (0, 1 m):

$$\vec{r} = -(1\,\text{m})\hat{j},\ r = 1\,\text{m},\ \text{and}\ \hat{r} = -\hat{j}$$

Substitute and evaluate $\vec{B}(0,1\text{m})$:

$$\vec{B}(0,1\text{m}) = \left(36.0\,\text{pT}\cdot\text{m}^2\right)\frac{\hat{i}\times\left(-\hat{j}\right)}{(1\text{m})^2} = \boxed{-(36.0\,\text{pT})\hat{k}}$$

(*c*) Find *r* and $\hat{r}$ for the particle at (0, 2 m) and the point of interest at (0, 3 m):

$$\vec{r} = (1\text{m})\hat{j},\ r = 1\text{m},\ \text{and}\ \hat{r} = \hat{j}$$

Substitute and evaluate $\vec{B}(0,3\text{m})$:

$$\vec{B}(0,3\text{m}) = \left(36.0\,\text{pT}\cdot\text{m}^2\right)\frac{\hat{i}\times\hat{j}}{(1\text{m})^2} = \boxed{(36.0\,\text{pT})\hat{k}}$$

(*d*) Find *r* and $\hat{\boldsymbol{r}}$ for the particle at (0, 2 m) and the point of interest at (0, 4 m):

$$\vec{\boldsymbol{r}} = (2\,\text{m})\hat{\boldsymbol{j}},\ r = 2\,\text{m},\ \text{and}\ \hat{\boldsymbol{r}} = \hat{\boldsymbol{j}}$$

Substitute and evaluate $\vec{\boldsymbol{B}}(0,4\,\text{m})$:

$$\vec{\boldsymbol{B}}(0,4\,\text{m}) = \left(36.0\,\text{pT}\cdot\text{m}^2\right)\frac{\hat{\boldsymbol{i}}\times\hat{\boldsymbol{j}}}{(2\,\text{m})^2} = \boxed{(9.00\,\text{pT})\hat{\boldsymbol{k}}}$$

24 •

Picture the Problem We can substitute for $\vec{\boldsymbol{v}}$ and *q* in the equation describing the magnetic field of the moving charged particle ($\vec{\boldsymbol{B}} = \frac{\mu_0}{4\pi}\frac{q\vec{\boldsymbol{v}}\times\hat{\boldsymbol{r}}}{r^2}$), evaluate *r* and $\hat{\boldsymbol{r}}$ for each of the given points of interest, and substitute to find $\vec{\boldsymbol{B}}$.

The magnetic field of the moving charged particle is given by:

$$\vec{\boldsymbol{B}} = \frac{\mu_0}{4\pi}\frac{q\vec{\boldsymbol{v}}\times\hat{\boldsymbol{r}}}{r^2} = \left(10^{-7}\ \text{N/A}^2\right)(12\,\mu\text{C})\frac{(30\,\text{m/s})\hat{\boldsymbol{i}}\times\hat{\boldsymbol{r}}}{r^2} = \left(36.0\,\text{pT}\cdot\text{m}^2\right)\frac{\hat{\boldsymbol{i}}\times\hat{\boldsymbol{r}}}{r^2}$$

(*a*) Find *r* and $\hat{\boldsymbol{r}}$ for the particle at (0, 2 m) and the point of interest at (1 m, 3 m):

$$\vec{\boldsymbol{r}} = (1\,\text{m})\hat{\boldsymbol{i}} + (1\,\text{m})\hat{\boldsymbol{j}},\ r = \sqrt{2}\,\text{m},\ \text{and}\ \hat{\boldsymbol{r}} = \frac{1}{\sqrt{2}}\hat{\boldsymbol{i}} + \frac{1}{\sqrt{2}}\hat{\boldsymbol{j}}$$

Substitute for $\hat{\boldsymbol{r}}$ and evaluate $\vec{\boldsymbol{B}}(1\text{m},3\text{m})$:

$$\vec{\boldsymbol{B}}(1\text{m},3\text{m}) = \left(36.0\,\text{pT}\cdot\text{m}^2\right)\times\frac{\hat{\boldsymbol{i}}\times\left(\frac{1}{\sqrt{2}}\hat{\boldsymbol{i}} + \frac{1}{\sqrt{2}}\hat{\boldsymbol{j}}\right)}{\left(\sqrt{2}\,\text{m}\right)^2} = \frac{\left(36.0\,\text{pT}\cdot\text{m}^2\right)}{\sqrt{2}}\frac{\hat{\boldsymbol{k}}}{\left(\sqrt{2}\,\text{m}\right)^2} = \boxed{(12.7\,\text{pT})\hat{\boldsymbol{k}}}$$

(*b*) Find *r* and $\hat{\boldsymbol{r}}$ for the particle at (0, 2 m) and the point of interest at (2 m, 2 m):

$$\vec{\boldsymbol{r}} = (2\,\text{m})\hat{\boldsymbol{i}},\ r = 2\,\text{m},\ \text{and}\ \hat{\boldsymbol{r}} = \hat{\boldsymbol{i}}$$

Substitute for $\hat{\boldsymbol{r}}$ and evaluate $\vec{\boldsymbol{B}}(2\,\text{m},2\,\text{m})$:

$$\vec{\boldsymbol{B}}(2\,\text{m},2\,\text{m}) = \left(36.0\,\text{pT}\cdot\text{m}^2\right)\frac{\hat{\boldsymbol{i}}\times\hat{\boldsymbol{i}}}{(2\,\text{m})^2} = \boxed{0}$$

(*c*) Find *r* and $\hat{\boldsymbol{r}}$ for the particle at (0, 2 m) and the point of interest at (2 m, 3 m):

$$\vec{\boldsymbol{r}} = (2\,\text{m})\hat{\boldsymbol{i}} + (1\,\text{m})\hat{\boldsymbol{j}},\ r = \sqrt{5}\,\text{m}, \text{ and}$$
$$\hat{\boldsymbol{r}} = \frac{2}{\sqrt{5}}\hat{\boldsymbol{i}} + \frac{1}{\sqrt{5}}\hat{\boldsymbol{j}}$$

Substitute for $\hat{\boldsymbol{r}}$ and evaluate $\vec{\boldsymbol{B}}(2\,\text{m},3\,\text{m})$:

$$\vec{\boldsymbol{B}}(2\,\text{m},3\,\text{m}) = \left(36.0\,\text{pT}\cdot\text{m}^2\right)\frac{\hat{\boldsymbol{i}}\times\left(\frac{2}{\sqrt{5}}\hat{\boldsymbol{i}} + \frac{1}{\sqrt{5}}\hat{\boldsymbol{j}}\right)}{\left(\sqrt{5}\,\text{m}\right)^2} = \boxed{(3.22\,\text{pT})\hat{\boldsymbol{k}}}$$

25 •

Picture the Problem We can substitute for $\vec{\boldsymbol{v}}$ and *q* in the equation describing the magnetic field of the moving proton ($\vec{\boldsymbol{B}} = \frac{\mu_0}{4\pi}\frac{q\vec{\boldsymbol{v}}\times\hat{\boldsymbol{r}}}{r^2}$), evaluate *r* and $\hat{\boldsymbol{r}}$ for each of the given points of interest, and substitute to find $\vec{\boldsymbol{B}}$.

The magnetic field of the moving proton is given by:

$$\vec{\boldsymbol{B}} = \frac{\mu_0}{4\pi}\frac{q\vec{\boldsymbol{v}}\times\hat{\boldsymbol{r}}}{r^2} = \left(10^{-7}\ \text{N/A}^2\right)\left(1.60\times10^{-19}\,\text{C}\right)\frac{\left[\left(10^4\ \text{m/s}\right)\hat{\boldsymbol{i}} + \left(2\times10^4\ \text{m/s}\right)\right]\hat{\boldsymbol{j}}\times\hat{\boldsymbol{r}}}{r^2}$$
$$= \left(1.60\times10^{-22}\ \text{T}\cdot\text{m}^2\right)\frac{\left(\hat{\boldsymbol{i}} + 2\hat{\boldsymbol{j}}\right)\times\hat{\boldsymbol{r}}}{r^2}$$

(*a*) Find *r* and $\hat{\boldsymbol{r}}$ for the proton at (3 m, 4 m) and the point of interest at (2 m, 2 m):

$$\vec{\boldsymbol{r}} = -(1\,\text{m})\hat{\boldsymbol{i}} - (2\,\text{m})\hat{\boldsymbol{j}},\ r = \sqrt{5}\,\text{m}, \text{ and}$$
$$\hat{\boldsymbol{r}} = -\frac{1}{\sqrt{5}}\hat{\boldsymbol{i}} - \frac{2}{\sqrt{5}}\hat{\boldsymbol{j}}$$

Substitute for $\hat{\boldsymbol{r}}$ and evaluate $\vec{\boldsymbol{B}}(1\,\text{m},3\,\text{m})$:

$$\vec{\boldsymbol{B}}(1\,\text{m},3\,\text{m}) = \left(1.60\times10^{-22}\ \text{T}\cdot\text{m}^2\right)\frac{\left(\hat{\boldsymbol{i}} + 2\hat{\boldsymbol{j}}\right)\times\left(-\frac{1}{\sqrt{5}}\hat{\boldsymbol{i}} - \frac{2}{\sqrt{5}}\hat{\boldsymbol{j}}\right)}{r^2}$$
$$= \frac{\left(1.60\times10^{-22}\ \text{T}\cdot\text{m}^2\right)}{\sqrt{5}}\left[\frac{-2\hat{\boldsymbol{k}} + 2\hat{\boldsymbol{k}}}{\left(\sqrt{5}\,\text{m}\right)^2}\right] = \boxed{0}$$

(*b*) Find *r* and $\hat{\boldsymbol{r}}$ for the proton at (3 m, 2 m) and the point of interest at (6 m, 4 m):

$$\vec{r} = (3\,\text{m})\hat{\boldsymbol{i}}\,,\ r = 3\,\text{m}\,,\ \text{and}\ \hat{\boldsymbol{r}} = \hat{\boldsymbol{i}}$$

Substitute for $\hat{\boldsymbol{r}}$ and evaluate $\vec{\boldsymbol{B}}(6\,\text{m},4\,\text{m})$:

$$\vec{\boldsymbol{B}}(6\,\text{m},4\,\text{m}) = \left(1.60\times10^{-22}\ \text{T}\cdot\text{m}^2\right)\frac{\left(\hat{\boldsymbol{i}}+2\hat{\boldsymbol{j}}\right)\times\hat{\boldsymbol{i}}}{(3\,\text{m})^2} = \left(1.60\times10^{-22}\ \text{T}\cdot\text{m}^2\right)\left(\frac{-2\hat{\boldsymbol{k}}}{9\,\text{m}^2}\right)$$

$$= \boxed{-\left(3.56\times10^{-23}\ \text{T}\right)\hat{\boldsymbol{k}}}$$

(*c*) Find *r* and $\hat{\boldsymbol{r}}$ for the proton at (3 m, 4 m) and the point of interest at the (3 m, 6 m):

$$\vec{r} = (2\,\text{m})\hat{\boldsymbol{j}}\,,\ r = 2\,\text{m}\,,\ \text{and}\ \hat{\boldsymbol{r}} = \hat{\boldsymbol{j}}$$

Substitute for $\hat{\boldsymbol{r}}$ and evaluate $\vec{\boldsymbol{B}}(3\,\text{m},6\,\text{m})$:

$$\vec{\boldsymbol{B}}(3\,\text{m},6\,\text{m}) = \left(1.60\times10^{-22}\ \text{T}\cdot\text{m}^2\right)\frac{\left(\hat{\boldsymbol{i}}+2\hat{\boldsymbol{j}}\right)\times\hat{\boldsymbol{j}}}{(2\,\text{m})^2} = \left(1.60\times10^{-22}\ \text{T}\cdot\text{m}^2\right)\left(\frac{\hat{\boldsymbol{k}}}{4\,\text{m}^2}\right)$$

$$= \boxed{\left(4.00\times10^{-23}\ \text{T}\right)\hat{\boldsymbol{k}}}$$

26 •

Picture the Problem The centripetal force acting on the orbiting electron is the Coulomb force between the electron and the proton. We can apply Newton's 2nd law to the electron to find its orbital speed and then use the expression for the magnetic field of a moving charge to find *B*.

Express the magnetic field due to the motion of the electron:

$$B = \frac{\mu_0}{4\pi}\frac{ev}{r^2}$$

Apply $\sum F_{\text{radial}} = ma_{\text{c}}$ to the electron:

$$\frac{ke^2}{r^2} = m\frac{v^2}{r}$$

Solve for *v* to obtain:

$$v = \sqrt{\frac{ke^2}{mr}}$$

Substitute and simplify to obtain:

$$B = \frac{\mu_0}{4\pi}\frac{e}{r^2}\sqrt{\frac{ke^2}{mr}} = \frac{\mu_0 e^2}{4\pi r^2}\sqrt{\frac{k}{mr}}$$

Substitute numerical values and evaluate B:

$$B = \frac{\left(10^{-7}\text{ N/A}^2\right)\left(1.6\times10^{-19}\text{ C}\right)^2}{\left(5.29\times10^{-11}\text{ m}\right)^2}\sqrt{\frac{8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2}{\left(9.11\times10^{-31}\text{ kg}\right)\left(5.29\times10^{-11}\text{ m}\right)}} = \boxed{12.5\,\text{T}}$$

***27** ••

Picture the Problem We can find the ratio of the magnitudes of the magnetic and electrostatic forces by using the expression for the magnetic field of a moving charge and Coulomb's law. Note that v and $\vec{r}$, where $\vec{r}$ is the vector from one charge to the other, are at right angles. The field $\vec{B}$ due to the charge at the origin at the location (0, b, 0) is perpendicular to v and $\vec{r}$.

Express the magnitude of the magnetic force on the moving charge at (0, b, 0):

$$F_B = qvB = \frac{\mu_0}{4\pi}\frac{q^2v^2}{b^2}$$

and, applying the right hand rule, we find that the direction of the force is toward the charge at the origin; i.e., the magnetic force between the two moving charges is attractive.

Express the magnitude of the repulsive electrostatic interaction between the two charges:

$$F_E = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{b^2}$$

Express the ratio of F_B to F_E and simplify to obtain:

$$\frac{F_B}{F_E} = \frac{\dfrac{\mu_0}{4\pi}\dfrac{q^2v^2}{b^2}}{\dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{b^2}} = \varepsilon_0\mu_0 v^2 = \boxed{\frac{v^2}{c^2}}$$

where c is the speed of light in a vacuum.

The Magnetic Field of Currents: The Biot-Savart Law

28 •

Picture the Problem We can substitute for $\vec{v}$ and q in the Biot-Savart relationship ($d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{Id\vec{\ell}\times\hat{r}}{r^2}$), evaluate r and $\hat{r}$ for each of the points of interest, and substitute to find $d\vec{B}$.

Express the Biot-Savart law for the given current element:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell}\times\hat{r}}{r^2}$$
$$= \left(10^{-7}\ \text{N/A}^2\right)\frac{(2\ \text{A})(2\ \text{mm})\hat{k}\times\hat{r}}{r^2}$$
$$= \left(0.400\ \text{nT}\cdot\text{m}^2\right)\frac{\hat{k}\times\hat{r}}{r^2}$$

(*a*) Find r and $\hat{r}$ for the point whose coordinates are (3 m, 0, 0):

$\vec{r} = (3\ \text{m})\hat{i}$, $r = 3\ \text{m}$, and $\hat{r} = \hat{i}$

Evaluate $d\vec{B}$ at (3 m, 0, 0):

$$d\vec{B}(3\ \text{m},0,0) = \left(0.400\ \text{nT}\cdot\text{m}^2\right)\frac{\hat{k}\times\hat{i}}{(3\ \text{m})^2}$$
$$= \boxed{(44.4\ \text{pT})\hat{j}}$$

(*b*) Find r and $\hat{r}$ for the point whose coordinates are (–6 m, 0, 0):

$\vec{r} = -(6\ \text{m})\hat{i}$, $r = 6\ \text{m}$, and $\hat{r} = -\hat{i}$

Evaluate $d\vec{B}$ at (–6 m, 0, 0):

$$d\vec{B}(-6\ \text{m},0,0) = \left(0.400\ \text{nT}\cdot\text{m}^2\right)\frac{\hat{k}\times\left(-\hat{i}\right)}{(6\ \text{m})^2}$$
$$= \boxed{-(11.1\ \text{pT})\hat{j}}$$

(*c*) Find r and $\hat{r}$ for the point whose coordinates are (0, 0, 3 m):

$\vec{r} = (3\ \text{m})\hat{k}$, $r = 3\ \text{m}$, and $\hat{r} = \hat{k}$

Evaluate $d\vec{B}$ at (0, 0, 3 m):

$$d\vec{B}(0,0,3\ \text{m}) = \left(0.400\ \text{nT}\cdot\text{m}^2\right)\frac{\hat{k}\times\hat{k}}{(3\ \text{m})^2}$$
$$= \boxed{0}$$

(*d*) Find r and $\hat{r}$ for the point whose coordinates are (0, 3 m, 0):

$\vec{r} = (3\ \text{m})\hat{j}$, $r = 3\ \text{m}$, and $\hat{r} = \hat{j}$

Evaluate $d\vec{B}$ at (0, 3 m, 0):

$$d\vec{B}(0,3\,\text{m},0) = \left(0.400\,\text{nT}\cdot\text{m}^2\right)\frac{\hat{k}\times\hat{j}}{(3\,\text{m})^2}$$

$$= \boxed{-(44.4\,\text{pT})\hat{i}}$$

29 •

Picture the Problem We can substitute for $\vec{v}$ and q in the Biot-Savart relationship ($d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell}\times\hat{r}}{r^2}$), evaluate r and $\hat{r}$ for (0, 3 m, 4 m), and substitute to find $d\vec{B}$.

Express the Biot-Savart law for the given current element:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell}\times\hat{r}}{r^2}$$

$$= \left(10^{-7}\ \text{N/A}^2\right)\frac{(2\,\text{A})(2\,\text{mm})\hat{k}\times\hat{r}}{r^2}$$

$$= \left(0.400\,\text{nT}\cdot\text{m}^2\right)\frac{\hat{k}\times\hat{r}}{r^2}$$

Find r and $\hat{r}$ for the point whose coordinates are (0, 3 m, 4 m):

$$\vec{r} = (3\,\text{m})\hat{j} + (4\,\text{m})\hat{k}\ ,$$

$$r = 5\,\text{m}\ ,$$

and

$$\hat{r} = \frac{3}{5}\hat{j} + \frac{4}{5}\hat{k}$$

Evaluate $d\vec{B}$ at (3 m, 0, 0):

$$d\vec{B}(3\,\text{m},0,0) = \left(0.400\,\text{nT}\cdot\text{m}^2\right)\frac{\hat{k}\times\left(\frac{3}{5}\hat{j}+\frac{4}{5}\hat{k}\right)}{(5\,\text{m})^2} = \boxed{-(9.60\,\text{pT})\hat{i}}$$

***30** •

Picture the Problem We can substitute for $\vec{v}$ and q in the Biot-Savart relationship ($d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell}\times\hat{r}}{r^2}$), evaluate r and $\hat{r}$ for the given points, and substitute to find $d\vec{B}$.

Express the Biot-Savart law for the given current element:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell}\times\hat{r}}{r^2}$$

$$= \left(10^{-7}\ \text{N/A}^2\right)\frac{(2\,\text{A})(2\,\text{mm})\hat{k}\times\hat{r}}{r^2}$$

$$= \left(0.400\,\text{nT}\cdot\text{m}^2\right)\frac{\hat{k}\times\hat{r}}{r^2}$$

(*a*) Find r and $\hat{\boldsymbol{r}}$ for the point whose coordinates are (2 m, 4 m, 0):

$\vec{r} = (2\,\text{m})\hat{\boldsymbol{i}} + (4\,\text{m})\hat{\boldsymbol{j}}$,

$r = 2\sqrt{5}\,\text{m}$,

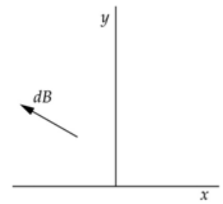
and

$$\hat{\boldsymbol{r}} = \frac{2}{2\sqrt{5}}\hat{\boldsymbol{i}} + \frac{4}{2\sqrt{5}}\hat{\boldsymbol{j}} = \frac{1}{\sqrt{5}}\hat{\boldsymbol{i}} + \frac{2}{\sqrt{5}}\hat{\boldsymbol{j}}$$

Evaluate $d\vec{B}$ at (2 m, 4 m, 0):

$$d\vec{B}(2\,\text{m},4\,\text{m},0) = (0.400\,\text{nT}\cdot\text{m}^2)\frac{\hat{\boldsymbol{k}}\times\left(\frac{1}{\sqrt{5}}\hat{\boldsymbol{i}} + \frac{2}{\sqrt{5}}\hat{\boldsymbol{j}}\right)}{(2\sqrt{5}\,\text{m})^2} = \boxed{-(17.9\,\text{pT})\hat{\boldsymbol{i}} + (8.94\,\text{pT})\hat{\boldsymbol{j}}}$$

The diagram is shown to the right:



(*b*) Find r and $\hat{\boldsymbol{r}}$ for the point whose coordinates are (2 m, 0, 4 m):

$\vec{r} = (2\,\text{m})\hat{\boldsymbol{i}} + (4\,\text{m})\hat{\boldsymbol{k}}$,

$r = 2\sqrt{5}\,\text{m}$,

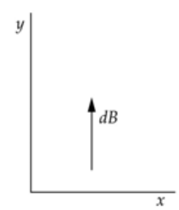
and

$$\hat{\boldsymbol{r}} = \frac{2}{2\sqrt{5}}\hat{\boldsymbol{i}} + \frac{4}{2\sqrt{5}}\hat{\boldsymbol{k}} = \frac{1}{\sqrt{5}}\hat{\boldsymbol{i}} + \frac{2}{\sqrt{5}}\hat{\boldsymbol{k}}$$

Evaluate $d\vec{B}$ at (2 m, 0, 4 m):

$$d\vec{B}(2\,\text{m},0,4\,\text{m}) = (0.400\,\text{nT}\cdot\text{m}^2)\frac{\hat{\boldsymbol{k}}\times\left(\frac{1}{\sqrt{5}}\hat{\boldsymbol{i}} + \frac{2}{\sqrt{5}}\hat{\boldsymbol{k}}\right)}{(2\sqrt{5}\,\text{m})^2} = \boxed{(8.94\,\text{pT})\hat{\boldsymbol{j}}}$$

The diagram is shown to the right:



$\vec{B}$ Due to a Current Loop

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Picture the Problem We can use $B_x = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{(x^2 + R^2)^{3/2}}$ to find B on the axis of the current loop.

Express B on the axis of a current loop:

$$B_x = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{\left(x^2 + R^2\right)^{3/2}}$$

Substitute numerical values to obtain:

$$B_x = \left(10^{-7}\ \text{N/A}^2\right)\frac{2\pi(0.03\,\text{m})^2(2.6\,\text{A})}{\left(x^2 + (0.03\,\text{m})^2\right)^{3/2}} = \frac{1.47\times10^{-9}\ \text{T}\cdot\text{m}^3}{\left(x^2 + (0.03\,\text{m})^2\right)^{3/2}}$$

(*a*) Evaluate B at the center of the loop:

$$B(0) = \frac{1.47\times10^{-9}\ \text{T}\cdot\text{m}^3}{\left(0 + (0.03\,\text{m})^2\right)^{3/2}} = \boxed{54.5\,\mu\text{T}}$$

(*b*) Evaluate B at x = 1 cm:

$$B(0.01\text{m}) = \frac{1.47\times10^{-9}\ \text{T}\cdot\text{m}^3}{\left((0.01\,\text{m})^2 + (0.03\,\text{m})^2\right)^{3/2}} = \boxed{46.5\,\mu\text{T}}$$

(*c*) Evaluate B at x = 2 cm:

$$B(0.02\,\text{m}) = \frac{1.47\times10^{-9}\ \text{T}\cdot\text{m}^3}{\left((0.02\,\text{m})^2 + (0.03\,\text{m})^2\right)^{3/2}} = \boxed{31.4\,\mu\text{T}}$$

(*d*) Evaluate B at x = 35 cm:

$$B(0.35\,\text{m}) = \frac{1.47\times10^{-9}\ \text{T}\cdot\text{m}^3}{\left((0.35\,\text{m})^2 + (0.03\,\text{m})^2\right)^{3/2}} = \boxed{33.9\,\text{nT}}$$

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Picture the Problem We can solve $B_x = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{\left(x^2 + R^2\right)^{3/2}}$ for I with $x = 0$ and substitute the earth's magnetic field at the equator to find the current in the loop that would produce a magnetic field equal to that of the earth.

Express B on the axis of the current loop:

$$B_x = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{\left(x^2 + R^2\right)^{3/2}}$$

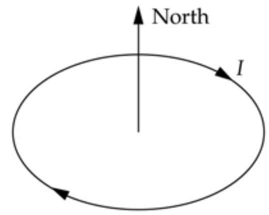
Solve for I with x = 0:

$$I = \frac{4\pi}{\mu_0}\frac{R}{2\pi}B_x$$

Substitute numerical values and evaluate I:

$$I = \frac{1}{\left(10^{-7}\ \text{N/A}^2\right)}\frac{(0.1\,\text{m})^3}{2\pi(0.1\,\text{m})^2}(0.7\,\text{G})\left(\frac{1\,\text{T}}{10^4\,\text{G}}\right) = \boxed{11.1\,\text{A}}$$

The orientation of the loop and current is shown in the sketch:



33 ••

Picture the Problem We can solve $B_x = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{\left(x^2+R^2\right)^{3/2}}$ for B_0, express the ratio of B_x to B_0, and solve the resulting equation for x.

Express B on the axis of the current loop:

$$B_x = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{\left(x^2+R^2\right)^{3/2}}$$

Evaluate B_x for $x = 0$:

$$B_0 = \frac{\mu_0}{4\pi}\frac{2\pi I}{R}$$

Express the ratio of B_x to B_0:

$$\frac{B_x}{B_0} = \frac{\frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{\left(x^2+R^2\right)^{3/2}}}{\frac{\mu_0}{4\pi}\frac{2\pi I}{R}} = \frac{R^3}{\left(x^2+R^2\right)^{3/2}}$$

Solve for x to obtain:

$$x = R\sqrt{\left(\frac{B_0}{B_x}\right)^{2/3} - 1} \qquad (1)$$

(a) Evaluate equation (1) for $B_x = 0.\,1B_0$:

$$x = 10\,\text{cm}\sqrt{\left(\frac{B_0}{0.1B_0}\right)^{2/3} - 1} = \boxed{19.1\,\text{cm}}$$

(b) Evaluate equation (1) for $B_x = 0.\,01B_0$:

$$x = 10\,\text{cm}\sqrt{\left(\frac{B_0}{0.01B_0}\right)^{2/3} - 1} = \boxed{45.3\,\text{cm}}$$

(*a*) Evaluate equation (1) for $B_x = 0.001B_0$:

$$x = 10\,\text{cm}\sqrt{\left(\frac{B_0}{0.001B_0}\right)^{2/3} - 1} = \boxed{99.5\,\text{cm}}$$

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Picture the Problem We can solve $B_x = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{\left(x^2+R^2\right)^{3/2}}$ for I with $x = 0$ and substitute the earth's magnetic field at the equator to find the current in the loop that would produce a magnetic field equal to that of the earth.

Express B on the axis of the current loop:

$$B_x = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{\left(x^2+R^2\right)^{3/2}}$$

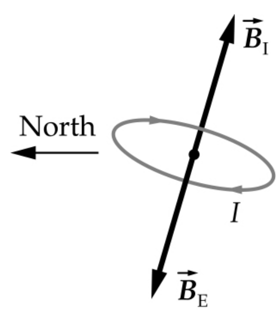
Solve for I with $x = 0$ and $B_x = B_E$:

$$I = \frac{4\pi}{\mu_0}\frac{R}{2\pi}B_E$$

Substitute numerical values and evaluate I:

$$I = \frac{1}{\left(10^{-7}\,\text{N/A}^2\right)}\frac{0.085\,\text{m}}{2\pi}(0.7\,\text{G})\left(\frac{1\,\text{T}}{10^4\,\text{G}}\right) = \boxed{9.47\,\text{A}}$$

The normal to the plane of the loop must be in the direction of the earth's field, and the current must be counterclockwise as seen from above. Here $\vec{B}_E$ denotes the earth's field and $\vec{B}_I$ the field due to the current in the coil.



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Picture the Problem We can use the expression for the magnetic field on the axis of a current loop and the expression for the electric field on the axis of ring of charge Q to plot graphs of B_x/B_0 and $E(x)/(kQ/R^2)$ as functions of x/R.

(*a*) Express B_x on the axis of a current loop:

$$B_x = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{\left(x^2+R^2\right)^{3/2}}$$

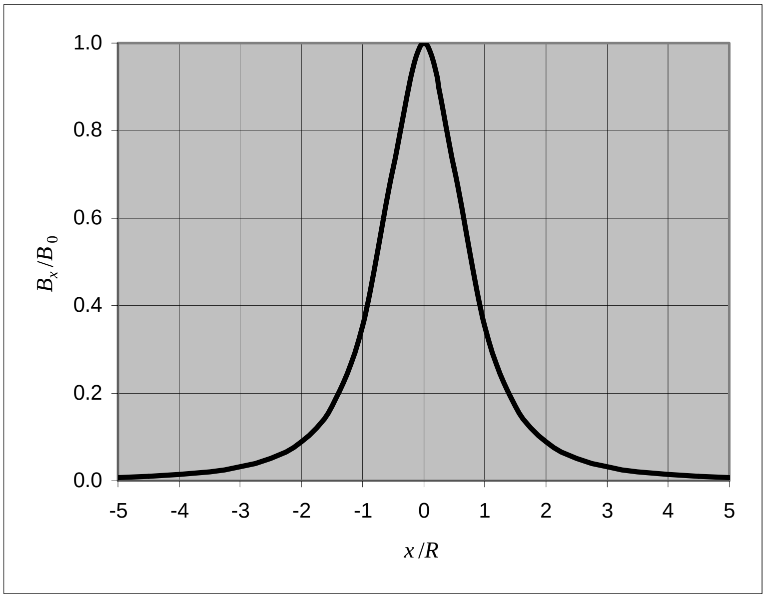
Express B_0 at the center of the loop:

$$B_0 = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{\left(R^2\right)^{3/2}} = \frac{\mu_0 I}{2R}$$

Express the ratio of B_x to B_0 and simplify to obtain:

$$\frac{B_x}{B_0} = \frac{1}{\left(1 + \frac{x^2}{R^2}\right)^{3/2}}$$

The graph of B_x/B_0 as a function of x/R shown below was plotted using a spreadsheet program:



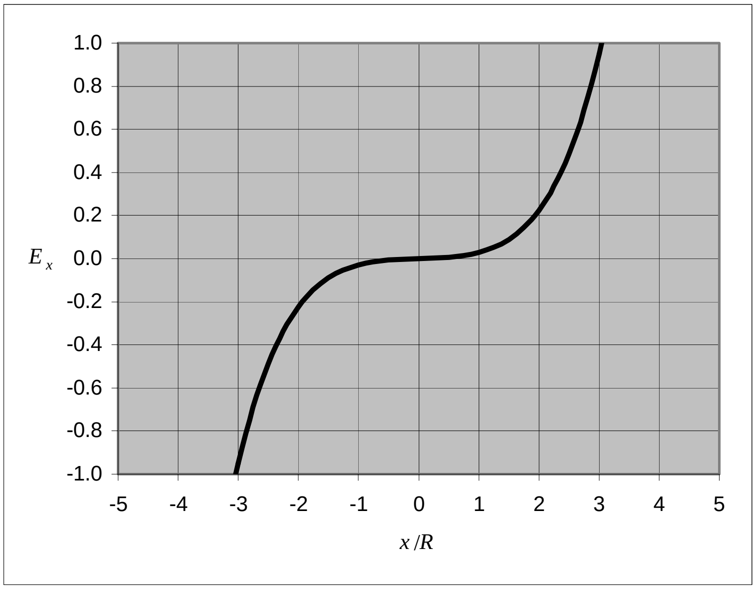
Express E_x on the axis due to a ring of radius R carrying a total charge Q:

$$E(x) = \frac{kQx}{\left(R^2 + x^2\right)^{3/2}} = \frac{kQ}{R^2} \frac{\frac{x}{R}}{\left(1 + \frac{x^2}{R^2}\right)^{3/2}}$$

Divide both sides of this equation by kQ/R^2 to obtain:

$$\frac{E(x)}{\frac{kQ}{R^2}} = \frac{\frac{x}{R}}{\left(1 + \frac{x^2}{R^2}\right)^{3/2}}$$

The graph of E_x as a function of x/R shown below was plotted using a spreadsheet program. Here $E(x)$ is normalized, i.e., we've set $kQ/R^2 = 100$.



(*b*) Express the magnetic field on the x axis due to the loop centered at $x = 0$:

$$B_1(x) = \frac{\mu_0 R^2 I}{2\left(x^2 + R^2\right)^{3/2}} = \frac{\mu_0 I}{2R\left(1 + \frac{x^2}{R^2}\right)^{3/2}}$$

where N is the number of turns.

Because $B_0 = \frac{\mu_0 I}{2R}$:

$$B_1(x) = \frac{B_0}{\left(1 + \frac{x^2}{R^2}\right)^{3/2}}$$

or

$$\frac{B_1(x)}{B_0} = \left[1 + \left(\frac{x}{R}\right)^2\right]^{-3/2}$$

Express the magnetic field on the x axis due to the loop centered at $x = R$:

$$B_2(x) = \frac{\mu_0 R^2 I}{2\left[(R - x)^2 + R^2\right]^{3/2}}$$

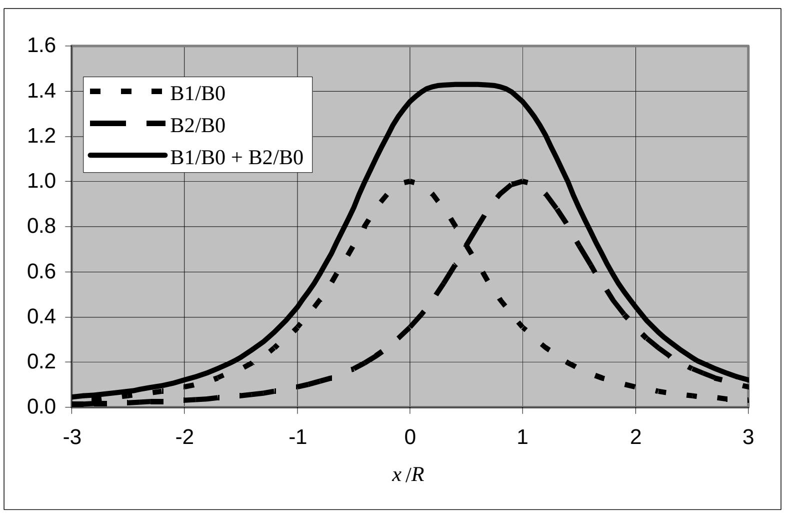
Simplify this expression to obtain:

$$B_2(x)=\frac{\mu_0 R^2 I}{2\left[(R-x)^2+R^2\right]^{3/2}}=\frac{\mu_0 I}{2R\left[\left(1-\frac{x}{R}\right)^2+1\right]^{3/2}}=\frac{B_0}{\left[\left(1-\frac{x}{R}\right)^2+1\right]^{3/2}}$$

or

$$\frac{B_2(x)}{B_0}=\left[\left(1-\frac{x}{R}\right)^2+1\right]^{-3/2}$$

The graphs of B_1/B_0, B_2/B_0, and $B_1/B_0 + B_2/B_0$ as functions of x/R with the second loop displaced by $d = R$ from the center of the first loop along the x axis shown below were plotted using a spreadsheet program.



Note that, midway between the two loops, $dB(x)/dx = 0$. Also, when $d = R$, $B(x)$ is nearly flat at the midpoint which shows that in the region midway between the two coils $B(x)$ is nearly constant.

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Picture the Problem Let the origin be midway between the coils so that one of them is centered at $x = -r/2$ and the other is centered at $x = r/2$. Let the numeral 1 denote the coil centered at $x = -r/2$ and the numeral 2 the coil centered at $x = r/2$. We can express the magnetic field in the region between the coils as the sum of the magnetic fields B_1 and B_2 due to the two coils.

Express the magnetic field on the x axis due to the coil centered at $x = -r/2$:

$$B_1(x) = \frac{\mu_0 N r^2 I}{2\left[\left(\frac{r}{2}+x\right)^2 + r^2\right]^{3/2}}$$

where N is the number of turns.

Express the magnetic field on the x axis due to the coil centered at $x = r/2$:

$$B_2(x) = \frac{\mu_0 N r^2 I}{2\left[\left(\frac{r}{2}-x\right)^2 + r^2\right]^{3/2}}$$

Add these equations to express the total magnetic field along the x axis:

$$B_x(x) = B_1(x) + B_2(x) = \frac{\mu_0 N r^2 I}{2\left[\left(\frac{r}{2}+x\right)^2 + r^2\right]^{3/2}} + \frac{\mu_0 N r^2 I}{2\left[\left(\frac{r}{2}-x\right)^2 + r^2\right]^{3/2}}$$

$$= \frac{\mu_0 N r^2 I}{2}\left(\left[\left(\frac{r}{2}+x\right)^2 + r^2\right]^{-3/2} + \left[\left(\frac{r}{2}-x\right)^2 + r^2\right]^{-3/2}\right)$$

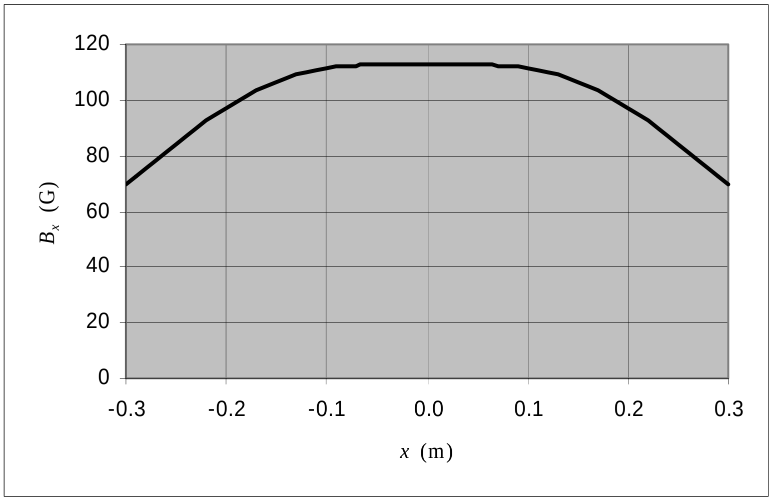
The spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
B1	1.13×10^{-7}	μ_0
B2	0.30	r
B3	250	N
B3	15	I
B5	0.5*$B$1*$B$3*($B$2^2)*$B$4	$\text{Coeff} = \frac{\mu_0 N r^2 I}{2}$
A8	–0.30	$-r$
B8	$B$5*(($B$2/2+A8)^2+$B$2^2)^(–3/2)	$\frac{\mu_0 N r^2 I}{2}\left[\left(\frac{r}{2}+x\right)^2 + r^2\right]^{-3/2}$
C8	$B$5* (($B$2/2–A8)^2+$B$2^2)^(–3/2)	$\frac{\mu_0 N r^2 I}{2}\left[\left(\frac{r}{2}-x\right)^2 + r^2\right]^{-3/2}$
D8	10^4(B8+C8)	$B_x = 10^4(B_1 + B_2)$

	A	B	C	D
1	mu_0=	1.26E-06	N/A^2	
2	r=	0.3	m	
3	N=	250	turns	

4	I=	15	A	
5	Coeff=	2.13E–04		
6				
7	x	B_1	B_2	B(x)
8	–0.30	5.63E–03	1.34E–03	70
9	–0.29	5.86E–03	1.41E–03	73
10	–0.28	6.08E–03	1.48E–03	76
11	–0.27	6.30E–03	1.55E–03	78
12	–0.26	6.52E–03	1.62E–03	81
13	–0.25	6.72E–03	1.70E–03	84
14	–0.24	6.92E–03	1.78E–03	87
15	–0.23	7.10E–03	1.87E–03	90
61	0.23	1.87E–03	7.10E–03	90
62	0.24	1.78E–03	6.92E–03	87
63	0.25	1.70E–03	6.72E–03	84
64	0.26	1.62E–03	6.52E–03	81
65	0.27	1.55E–03	6.30E–03	78
66	0.28	1.48E–03	6.08E–03	76
67	0.29	1.41E–03	5.86E–03	73
68	0.30	1.34E–03	5.63E–03	70

The following graph of B_x as a function of x was plotted using the data in the above table.



The maximum value of B_x is 113 G. Twenty percent of this maximum value is 23 G. Referring to the table of values we see that the field is within 20 percent of 113 G in the interval $\boxed{-0.23\,\text{m} < x < 0.23\,\text{m}.}$

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Picture the Problem Let the numeral 1 denote the coil centered at the origin and the numeral 2 the coil centered at $x = R$. We can express the magnetic field in the region between the coils as the sum of the magnetic fields due to the two coils and then evaluate the derivatives at $x = R/2$.

Express the magnetic field on the x axis due to the coil centered at $x = 0$:

$$B_1(x) = \frac{\mu_0 NR^2 I}{2\left(x^2 + R^2\right)^{3/2}}$$

where N is the number of turns.

Express the magnetic field on the x axis due to the coil centered at $x = R$:

$$B_2(x) = \frac{\mu_0 NR^2 I}{2\left[(x-R)^2 + R^2\right]^{3/2}}$$

Add these equations to express the total magnetic field along the x axis:

$$B_x(x) = B_1(x) + B_2(x) = \frac{\mu_0 NR^2 I}{2\left(x^2 + R^2\right)^{3/2}} + \frac{\mu_0 NR^2 I}{2\left[(x-R)^2 + R^2\right]^{3/2}}$$
$$= \frac{\mu_0 NR^2 I}{2}\left(\frac{1}{\left(x^2 + R^2\right)^{3/2}} + \frac{1}{\left[(x-R)^2 + R^2\right]^{3/2}}\right)$$

Evaluate x_1 and x_2 at $x = R/2$:

$$x_1\left(\tfrac{1}{2}R\right) = \sqrt{\tfrac{1}{4}R^2 + R^2} = \left(\tfrac{5}{4}R^2\right)^{1/2}$$

and

$$x_2\left(\tfrac{1}{2}R\right) = \sqrt{\left(\tfrac{1}{2}R - R\right)^2 + R^2} = \left(\tfrac{5}{4}R^2\right)^{1/2}$$

Differentiate B_x with respect to x to obtain:

$$\frac{dB_x}{dx} = \frac{\mu_0 NR^2 I}{2}\frac{d}{dx}\left(\frac{1}{x_1^3} + \frac{1}{x_2^3}\right)$$
$$= \frac{\mu_0 NR^2 I}{2}\left(\frac{x}{x_1^5} + \frac{x-R}{x_2^5}\right)$$

Evaluate dB_x/dx at $x = R/2$ to obtain:

$$\left.\frac{dB_x}{dx}\right|_{x=\frac{1}{2}R} = \frac{\mu_0 NR^2 I}{2}\left(\frac{\frac{1}{2}R}{\left(\frac{5}{4}R^2\right)^{5/2}} + \frac{-\frac{1}{2}R}{\left(\frac{5}{4}R^2\right)^{5/2}}\right) = \boxed{0}$$

Differentiate dB_x/dx with respect to x to obtain:

$$\frac{d^2 B_x}{dx^2} = \frac{\mu_0 NR^2 I}{2}\frac{d}{dx}\left(\frac{x}{x_1^5} + \frac{x-R}{x_2^5}\right) = \frac{\mu_0 NR^2 I}{2}\left(\frac{1}{x_1^5} + \frac{1}{x_2^5} - \frac{5x^2}{x_1^7} - \frac{5(x-R)^2}{x_2^7}\right)$$

Evaluate d^2B_x/dx^2 at $x = R/2$ to obtain:

$$\left.\frac{d^2B_x}{dx^2}\right|_{x=\frac{1}{2}R} = \frac{\mu_0 NR^2 I}{2}\left(\frac{1}{\left(\frac{5}{4}R^2\right)^{5/2}} + \frac{1}{\left(\frac{5}{4}R^2\right)^{5/2}} - \frac{\frac{5}{4}R^2}{\left(\frac{5}{4}R^2\right)^{7/2}} - \frac{\frac{5}{4}R^2}{\left(\frac{5}{4}R^2\right)^{7/2}}\right) = \boxed{0}$$

Differentiate d^2B_x/dx^2 with respect to x to obtain:

$$\frac{d^3B_x}{dx^3} = \frac{\mu_0 NR^2 I}{2}\frac{d}{dx}\left(\frac{1}{x_1^5} + \frac{1}{x_2^5} - \frac{5x^2}{x_1^7} - \frac{5(x-R)^2}{x_2^7}\right)$$

$$= \frac{\mu_0 NR^2 I}{2}\left(\frac{35x^3}{x_1^9} - \frac{15x}{x_1^7} - \frac{15(x-R)}{x_2^7} - \frac{35(x-R)^3}{x_2^9}\right)$$

Evaluate d^3B_x/dx^3 at $x = R/2$ to obtain:

$$\left.\frac{d^3B_x}{dx^3}\right|_{x=\frac{1}{2}R} = \frac{\mu_0 NR^2 I}{2}\left(\frac{\frac{35}{8}R^3}{\left(\frac{5}{4}R^2\right)^{9/2}} - \frac{\frac{15}{2}R}{\left(\frac{5}{4}R^2\right)^{7/2}} - \frac{-\frac{15}{2}R}{\left(\frac{5}{4}R^2\right)^{7/2}} - \frac{-\frac{35}{8}R^3}{\left(\frac{5}{4}R^2\right)^{9/2}}\right) = \boxed{0}$$

***38** •••

Picture the Problem Let the origin be midway between the coils so that one of them is centered at $x = -r\sqrt{3}/2$ and the other is centered at $x = r\sqrt{3}/2$. Let the numeral 1 denote the coil centered at $x = -r\sqrt{3}/2$ and the numeral 2 the coil centered at $x = r\sqrt{3}/2$. We can express the magnetic field in the region between the coils as the difference of the magnetic fields B_1 and B_2 due to the two coils.

Express the magnetic field on the x axis due to the coil centered at $x = -r\sqrt{3}/2$:

$$B_1(x) = \frac{\mu_0 N r^2 I}{2\left[\left(\frac{r\sqrt{3}}{2} + x\right)^2 + r^2\right]^{3/2}}$$

where N is the number of turns.

Express the magnetic field on the x axis due to the coil centered at $x = r\sqrt{3}/2$:

$$B_2(x) = \frac{\mu_0 N r^2 I}{2\left[\left(\frac{r\sqrt{3}}{2} - x\right)^2 + r^2\right]^{3/2}}$$

Subtract these equations to express the total magnetic field along the x axis:

$$B_x(x) = B_1(x) - B_2(x) = \frac{\mu_0 N r^2 I}{2\left[\left(\frac{r\sqrt{3}}{2}+x\right)^2 + r^2\right]^{3/2}} - \frac{\mu_0 N r^2 I}{2\left[\left(\frac{r\sqrt{3}}{2}-x\right)^2 + r^2\right]^{3/2}}$$

$$= \frac{\mu_0 N r^2 I}{2}\left(\left[\left(\frac{r\sqrt{3}}{2}+x\right)^2 + r^2\right]^{-3/2} - \left[\left(\frac{r\sqrt{3}}{2}-x\right)^2 + r^2\right]^{-3/2}\right)$$

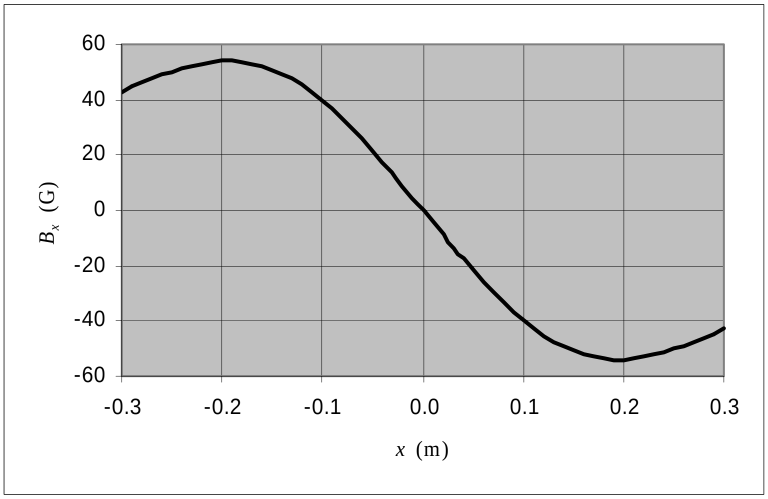
The spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
B1	1.26×10^{-6}	μ_0
B2	0.30	r
B3	250	N
B3	15	I
B5	0.5*$B$1*$B$3*($B$2^2)*$B$4	$\text{Coeff} = \frac{\mu_0 N r^2 I}{2}$
A8	–0.30	$-r$
B8	$B$5*(($B$2*SQRT(3)/2+A8)^2 +$B$2^2)^(–3/2)	$\frac{\mu_0 N r^2 I}{2}\left[\left(\frac{r\sqrt{3}}{2}+x\right)^2 + r^2\right]^{-3/2}$
C8	$B$5* (($B$2*SQRT(3)/2–A8)^2 +$B$2^2)^(–3/2)	$\frac{\mu_0 N r^2 I}{2}\left[\left(\frac{r\sqrt{3}}{2}-x\right)^2 + r^2\right]^{-3/2}$
D8	10^4*(B8–C8)	$B_x = B_1 - B_2$

	A	B	C	D
1	mu_0=	1.26E–06	N/A^2	
2	r=	0.3	m	
3	N=	250	turns	
4	I=	15	A	
5	Coeff=	2.13E–04		
6				
7	x	B_1	B_2	B(x)
8	–0.30	5.63E–03	1.34E–03	68.4
9	–0.29	5.86E–03	1.41E–03	68.9
10	–0.28	6.08E–03	1.48E–03	69.2
11	–0.27	6.30E–03	1.55E–03	69.2
12	–0.26	6.52E–03	1.62E–03	68.9
13	–0.25	6.72E–03	1.70E–03	68.4
14	–0.24	6.92E–03	1.78E–03	67.5
15	–0.23	7.10E–03	1.87E–03	66.4

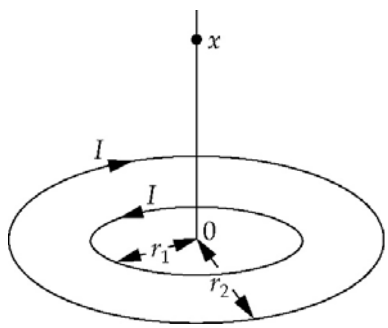
61	0.23	1.87E–03	7.10E–03	–66.4
62	0.24	1.78E–03	6.92E–03	–67.5
63	0.25	1.70E–03	6.72E–03	–68.4
64	0.26	1.62E–03	6.52E–03	–68.9
65	0.27	1.55E–03	6.30E–03	–69.2
66	0.28	1.48E–03	6.08E–03	–69.2
67	0.29	1.41E–03	5.86E–03	–68.9
68	0.30	1.34E–03	5.63E–03	–68.4

The following graph of B_x as a function of x was plotted using the data in the above table.



39 ••

Picture the Problem The diagram shows the two coils of radii r_1 and r_2 with the currents flowing in the directions given. We can use the expression for B on the axis of a current loop to express the difference of the fields due to the two loops at a distance x from their common center. We'll denote each field by the subscript identifying the radius of the current loop.



The magnitude of the field on the x axis due to the current in the inner loop is:

$$B_1 = \frac{\mu_0}{4\pi}\frac{2\pi r_1^2 I}{\left(x^2 + r_1^2\right)^{3/2}} = \frac{\mu_0 r_1^2 I}{2\left(x^2 + r_1^2\right)^{3/2}}$$

The magnitude of the field on the x axis due to the current in the outer loop is:

$$B_2 = \frac{\mu_0}{4\pi}\frac{2\pi r_2^2 I}{\left(x^2 + r_2^2\right)^{3/2}} = \frac{\mu_0 r_2^2 I}{2\left(x^2 + r_2^2\right)^{3/2}}$$

The resultant field at x is the difference between B_1 and B_2:

$$B_x(x) = B_1(x) - B_2(x) = \frac{\mu_0 r_1^2 I}{2\left(x^2 + r_1^2\right)^{3/2}} - \frac{\mu_0 r_2^2 I}{2\left(x^2 + r_2^2\right)^{3/2}}$$

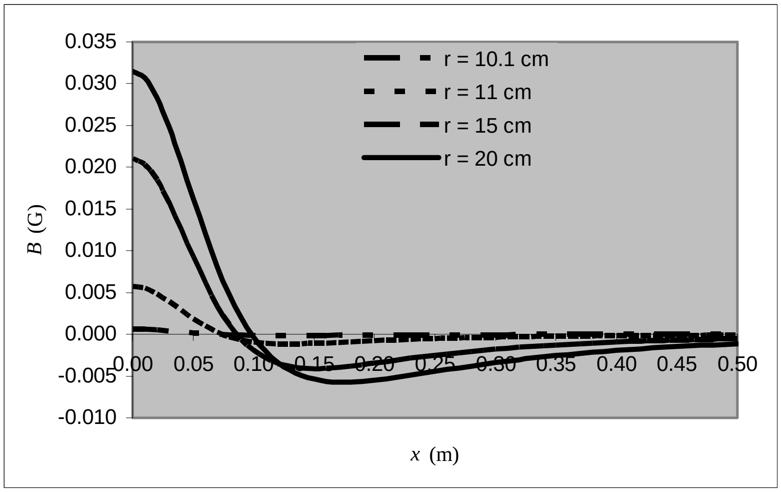
(*a*) The spreadsheet program to calculate B_x as a function of x, for r_2 = 10.1 cm, is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
B1	1.26×10^{-6}	μ_0
B2	0.1	r_1
B3	1	I
B4	0.101	r_2
A7	0	x
B7	0.5*$B$1*$B$2^2*$B$3/(A7^2+$B$2^2)^(3/2)	$\frac{\mu_0 r_1^2 I}{2\left(x^2 + r_1^2\right)^{3/2}}$
C7	0.5*$B$1*$B$4^2*$B$3/(A7^2+$B$4^2)^(3/2)	$\frac{\mu_0 r_2^2 I}{2\left(x^2 + r_2^2\right)^{3/2}}$
D7	10^4*(B7–C7)	$B_x(x) = B_1(x) - B_2(x)$

The spreadsheet for B_x when r = 10.1 cm follows. The other three tables are similar.

	A	B	C	D
1	mu_0=	1.26E–06	N/A^2	
2	r_1=	0.1	m	
3	I=	1	A	
4	r_2=	0.101	m	
5	r_2=	0.11	m	
6	r_2=	0.15	m	
7	r_2=	0.2	m	
8				
9	x	B_1	B_2	B_x
10	0.00	6.30E–06	6.24E–06	6.24E–04
11	0.01	6.21E–06	6.15E–06	5.97E–04
12	0.02	5.94E–06	5.89E–06	5.21E–04
13	0.03	5.54E–06	5.49E–06	4.14E–04
14	0.04	5.04E–06	5.01E–06	2.95E–04
15	0.05	4.51E–06	4.49E–06	1.81E–04
56	0.46	6.04E–08	6.15E–08	–1.13E–05
57	0.47	5.68E–08	5.78E–08	–1.07E–05
58	0.48	5.34E–08	5.45E–08	–1.01E–05
59	0.49	5.04E–08	5.13E–08	–9.51E–06
60	0.50	4.75E–08	4.84E–08	–8.99E–06

The following graph shows $B(x)$ for r = 10.1 cm, 11 cm, 15 cm, and 20 cm.



40 •••

Picture the Problem We can approximate $B(x)$ by using the result from Problem 39 for the field due to a single coil of radius r and evaluating $B(x) \approx \frac{\partial B}{\partial r}\Delta r$ at $r = r_1$.

The magnetic field at a distance x on the axis of a coil of radius r is given by:

$$B(x) = \frac{\mu_0 I}{4\pi}\frac{2\pi r^2}{\left(x^2 + r^2\right)^{3/2}}$$

Express $B(x)$ in terms of the rate of change of B with respect to r:

$$B(x) \approx \frac{\partial B}{\partial r}\Delta r \qquad (1)$$

Evaluate the partial derivative of B with respect to r:

$$\frac{\partial}{\partial r}\left[\frac{\mu_0 I}{4\pi}\left(\frac{2\pi r^2}{\left(x^2+r^2\right)^{3/2}}\right)\right]=\frac{\mu_0 I}{4\pi}\frac{\partial}{\partial r}\left[\frac{2\pi r^2}{\left(x^2+r^2\right)^{3/2}}\right]$$

$$=\frac{\mu_0 I}{4\pi}\left[\frac{\left(x^2+r^2\right)^{3/2}\frac{\partial}{\partial r}\left(2\pi r^2\right)-2\pi r^2\frac{\partial}{\partial r}\left[\left(x^2+r^2\right)^{3/2}\right]}{\left(x^2+r_1^2\right)^3}\right]$$

$$=\frac{\mu_0 I}{4\pi}\left[\frac{\left(x^2+r^2\right)^{3/2}\left(4\pi r\right)-2\pi r^2\left[\frac{3}{2}\left(x^2+r^2\right)^{1/2}\left(2r\right)\right]}{\left(x^2+r_1^2\right)^3}\right]$$

$$=\mu_0 Ir\left[\frac{\left(x^2+r^2\right)^{3/2}-\frac{3}{2}r^2\left(x^2+r^2\right)^{1/2}}{\left(x^2+r^2\right)^3}\right]$$

$$=\mu_0 Ir\left[\frac{2\left(x^2+r^2\right)^{3/2}-3r^2\left(x^2+r^2\right)^{1/2}}{2\left(x^2+r^2\right)^3}\right]$$

$$=\mu_0 Ir\left(x^2+r^2\right)^{1/2}\left[\frac{2\left(x^2+r^2\right)-3r^2}{2\left(x^2+r^2\right)^3}\right]$$

$$=\frac{\mu_0 I}{2}\left[\frac{2x^2r-r^3}{\left(x^2+r^2\right)^{5/2}}\right]$$

Evaluate $\partial B/\partial x$ at $r = r_1$ to obtain:

$$\left.\frac{\partial B}{\partial r}\right|_{r=r_1}=\frac{\mu_0 I}{2}\left[\frac{2x^2r_1-r_1^3}{\left(x^2+r_1^2\right)^{5/2}}\right]$$

Substitute in equation (1) to obtain:

$$\boxed{B(x)\approx\left(\frac{\mu_0 I\Delta r}{2}\right)\left[\frac{2x^2r_1-r_1^3}{\left(x^2+r_1^2\right)^{5/2}}\right]}$$

Remarks: This solution shows that the field due to two coils separated by Δ*r* can be approximated by the given expression.

41 •••

Picture the Problem We can factor x from the denominator of the equation from Problem 40 to show that $B(x)\approx\left(\frac{\mu_0 I\Delta r}{2}\right)\left(\frac{2r_1}{x^3}\right)$.

From Problem 40:

$$B(x)\approx\left(\frac{\mu_0 I\Delta r}{2}\right)\left[\frac{2x^2r_1-r_1^3}{\left(x^2+r_1^2\right)^{5/2}}\right]$$

Factor x^2 from the denominator of the expression to obtain:

$$B(x) \approx \left(\frac{\mu_0 I \Delta r}{2}\right)\left[\frac{2r_1x^2 - r_1^3}{x^5\left(1+\frac{r_1^2}{x^2}\right)^{5/2}}\right]$$

For $x >> r_1$:

$$x^5\left(1+\frac{r_1^2}{x^2}\right)^{5/2} \approx x^5$$

Substitute and simplify to obtain:

$$B(x) \approx \left(\frac{\mu_0 I \Delta r}{2}\right)\left(\frac{2r_1x^2 - r_1^3}{x^5}\right)$$

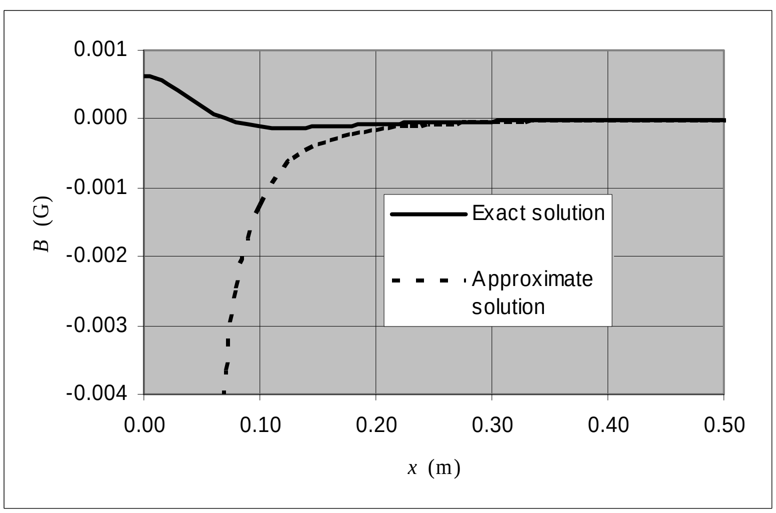
$$= \left(\frac{\mu_0 I \Delta r}{2}\right)\left(\frac{2r_1x^2}{x^5} - \frac{r_1^3}{x^5}\right)$$

$$= \left(\frac{\mu_0 I \Delta r}{2}\right)\left(\frac{2r_1}{x^3} - \frac{r_1^3}{x^5}\right)$$

For $x >> r$:

$$\boxed{B(x) \approx \left(\frac{\mu_0 I \Delta r}{2}\right)\left(\frac{2r_1}{x^3}\right)}$$

The spreadsheet-generated graph that follows provides a comparison of the exact and approximate fields. Note that the two solutions agree for large values of *x*.



Straight-Line Current Segments

42 •

Picture the Problem The magnetic field due to the current in a long straight wire is given by $B=\frac{\mu_0}{4\pi}\frac{2I}{R}$ where I is the current in the wire and R is the distance from the wire.

Express the magnetic field due to a long straight wire:

$$B=\frac{\mu_0}{4\pi}\frac{2I}{R}$$

Substitute numerical values to obtain:

$$B=\left(10^{-7}\ \text{T}\cdot\text{m/A}\right)\frac{2(10\ \text{A})}{R}=\frac{2.00\times10^{-6}\ \text{T}\cdot\text{m}}{R}$$

(*a*) Evaluate B at R = 10 cm:

$$B(10\ \text{cm})=\frac{2.00\times10^{-6}\ \text{T}\cdot\text{m}}{0.1\ \text{m}}=\boxed{20.0\ \mu\text{T}}$$

(*b*) Evaluate B at R = 50 cm:

$$B(50\ \text{cm})=\frac{2.00\times10^{-6}\ \text{T}\cdot\text{m}}{0.5\ \text{m}}=\boxed{4.00\ \mu\text{T}}$$

(*c*) Evaluate B at R = 2 m:

$$B(2\ \text{m})=\frac{2.00\times10^{-6}\ \text{T}\cdot\text{m}}{2\ \text{m}}=\boxed{1.00\ \mu\text{T}}$$

Problems 43 to 48 refer to Figure 27-45, which shows two long straight wires in the xy plane and parallel to the x axis. One wire is at y = –6 cm and the other is at y = +6 cm. The current in each wire is 20 A.

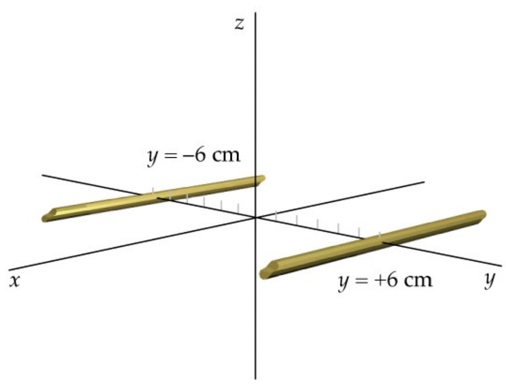


Figure 27-45 Problems 43-48

***43** •

Picture the Problem Let + denote the wire (and current) at y = +6 cm and – the wire (and current) at y = –6 cm. We can use $B=\frac{\mu_0}{4\pi}\frac{2I}{R}$ to find the magnetic field due to each of the current carrying wires and superimpose the magnetic fields due to the currents in the wires to find B at the given points on the y axis. We can apply the right-hand rule to find the direction of each of the fields and, hence, of $\vec{B}$.

(*a*) Express the resultant magnetic field at $y = -3$ cm:

$$\vec{B}(-3\,\text{cm}) = \vec{B}_+(-3\,\text{cm}) + \vec{B}_-(-3\,\text{cm})$$

Find the magnitudes of the magnetic fields at $y = -3$ cm due to each wire:

$$B_+(-3\,\text{cm}) = (10^{-7}\,\text{T}\cdot\text{m/A})\frac{2(20\,\text{A})}{0.09\,\text{m}} = 44.4\,\mu\text{T}$$

and

$$B_-(-3\,\text{cm}) = (10^{-7}\,\text{T}\cdot\text{m/A})\frac{2(20\,\text{A})}{0.03\,\text{m}} = 133\,\mu\text{T}$$

Apply the right-hand rule to find the directions of $\vec{B}_+$ and $\vec{B}_-$:

$$\vec{B}_+(-3\,\text{cm}) = (44.4\,\mu\text{T})\hat{k}$$

and

$$\vec{B}_-(-3\,\text{cm}) = -(133\,\mu\text{T})\hat{k}$$

Substitute to obtain:

$$\vec{B}(-3\,\text{cm}) = (44.4\,\mu\text{T})\hat{k} - (133\,\mu\text{T})\hat{k} = \boxed{-(88.6\,\mu\text{T})\hat{k}}$$

(*b*) Express the resultant magnetic field at $y = 0$:

$$\vec{B}(0) = \vec{B}_+(0) + \vec{B}_-(0)$$

Because $\vec{B}_+(0) = -\vec{B}_-(0)$:

$$\vec{B}(0) = \boxed{0}$$

(*c*) Proceed as in (*a*) to obtain:

$$\vec{B}_+(3\,\text{cm}) = (133\,\mu\text{T})\hat{k},$$

$$\vec{B}_-(3\,\text{cm}) = -(44.4\,\mu\text{T})\hat{k},$$

and

$$\vec{B}(3\,\text{cm}) = (133\,\mu\text{T})\hat{k} - (44.4\,\mu\text{T})\hat{k} = \boxed{-(88.6\,\mu\text{T})\hat{k}}$$

(*d*) Proceed as in (*a*) with $y = 9$ cm to obtain:

$$\vec{B}_+(9\,\text{cm}) = -(133\,\mu\text{T})\hat{k},$$

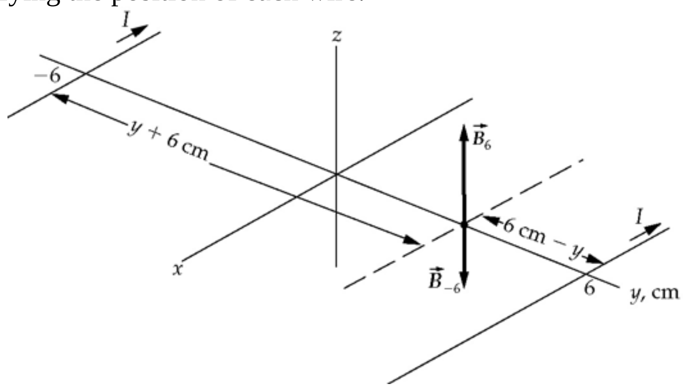
$$\vec{B}_-(9\,\text{cm}) = -(26.7\,\mu\text{T})\hat{k},$$

and

$$\vec{B}(9\,\text{cm}) = -(133\,\mu\text{T})\hat{k} - (26.7\,\mu\text{T})\hat{k}$$
$$= \boxed{-(160\,\mu\text{T})\hat{k}}$$

44 ••

Picture the Problem The diagram shows the two wires with the currents flowing in the negative *x* direction. We can use the expression for *B* due to a long, straight wire to express the difference of the fields due to the two currents. We'll denote each field by the subscript identifying the position of each wire.



The field due to the current in the wire located at $y = 6$ cm is:

$$B_6 = \frac{\mu_0}{4\pi}\frac{2I}{0.06\,\text{m} - y}$$

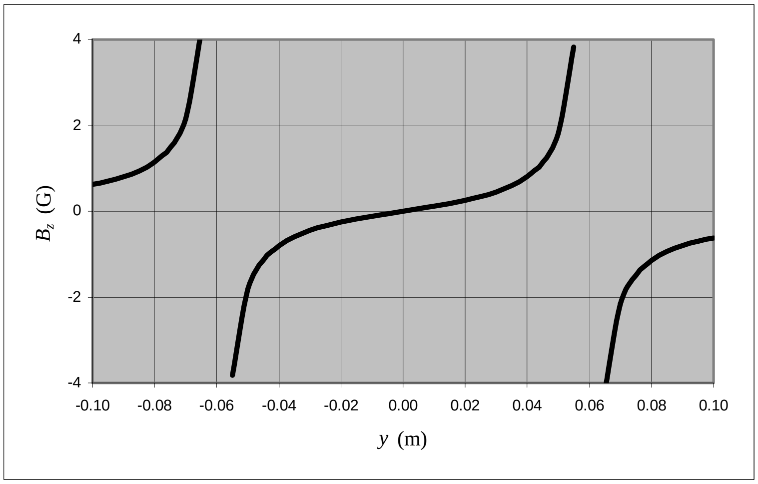
The field due to the current in the wire located at $y = -6$ cm is:

$$B_{-6} = \frac{\mu_0}{4\pi}\frac{2I}{0.06\,\text{m} + y}$$

The resultant field B_z is the difference between B_6 and B_{-6}:

$$B_z = B_6 - B_{-6} = \frac{\mu_0}{4\pi}\frac{I}{0.06\,\text{m} - y} - \frac{\mu_0}{4\pi}\frac{I}{0.06\,\text{m} - y} = \frac{\mu_0 I}{4\pi}\left(\frac{1}{0.06\,\text{m} - y} - \frac{1}{0.06\,\text{m} + y}\right)$$

The following graph of B_z as a function of y was plotted using a spreadsheet program:



45 •

Picture the Problem Let + denote the wire (and current) at y = +6 cm and – the wire (and current) at y = –6 cm. We can use $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$ to find the magnetic field due to each of the current carrying wires and superimpose the magnetic fields due to the currents in the wires to find B at the given points on the y axis. We can apply the right-hand rule to find the direction of each of the fields and, hence, of $\vec{B}$.

(*a*) Express the resultant magnetic field at y = –3 cm:

$$\vec{B}(-3\,\text{cm}) = \vec{B}_+(-3\,\text{cm}) + \vec{B}_-(-3\,\text{cm})$$

Find the magnitudes of the magnetic fields at y = –3 cm due to each wire:

$$B_+(-3\,\text{cm}) = \left(10^{-7}\,\text{T}\cdot\text{m/A}\right)\frac{2(20\,\text{A})}{0.09\,\text{m}} = 44.4\,\mu\text{T}$$

and

$$B_-(-3\,\text{cm}) = \left(10^{-7}\,\text{T}\cdot\text{m/A}\right)\frac{2(20\,\text{A})}{0.03\,\text{m}} = 133\,\mu\text{T}$$

Apply the right-hand rule to find the directions of $\vec{B}_+$ and $\vec{B}_-$:

$$\vec{B}_+(-3\,\text{cm}) = -(44.4\,\mu\text{T})\hat{k}$$

and

$$\vec{B}_-(-3\,\text{cm}) = -(133\,\mu\text{T})\hat{k}$$

Substitute to obtain:

$$\vec{B}(-3\,\text{cm}) = -(44.4\,\mu\text{T})\hat{k} - (133\,\mu\text{T})\hat{k} = \boxed{-(177\,\mu\text{T})\hat{k}}$$

(*b*) Express the resultant magnetic field at $y = 0$:

$$\vec{B}(0) = \vec{B}_+(0) + \vec{B}_-(0)$$

Find the magnitudes of the magnetic fields at $y = 0$ cm due to each wire:

$$B_+(0) = \left(10^{-7}\ \text{T}\cdot\text{m/A}\right)\frac{2(20\,\text{A})}{0.06\,\text{m}} = 66.7\ \mu\text{T}$$

and

$$B_-(0) = \left(10^{-7}\ \text{T}\cdot\text{m/A}\right)\frac{2(20\,\text{A})}{0.06\,\text{m}} = 66.7\ \mu\text{T}$$

Apply the right-hand rule to find the directions of $\vec{B}_+$ and $\vec{B}_-$:

$$\vec{B}_+(0) = -(66.7\ \mu\text{T})\hat{k}$$

and

$$\vec{B}_-(0) = -(66.7\ \mu\text{T})\hat{k}$$

Substitute to obtain:

$$\vec{B}(0) = -(66.7\ \mu\text{T})\hat{k} - (66.7\ \mu\text{T})\hat{k} = \boxed{-(133\ \mu\text{T})\hat{k}}$$

(*c*) Proceed as in (*a*) with $y = +3$ cm to obtain:

$$\vec{B}_+(3\,\text{cm}) = -(133\ \mu\text{T})\hat{k},$$
$$\vec{B}_-(3\,\text{cm}) = -(44.4\ \mu\text{T})\hat{k},$$

and

$$\vec{B}(3\,\text{cm}) = -(133\ \mu\text{T})\hat{k} - (44.4\ \mu\text{T})\hat{k} = \boxed{-(177\ \mu\text{T})\hat{k}}$$

(*d*) Proceed as in (*a*) with $y = +9$ cm to obtain:

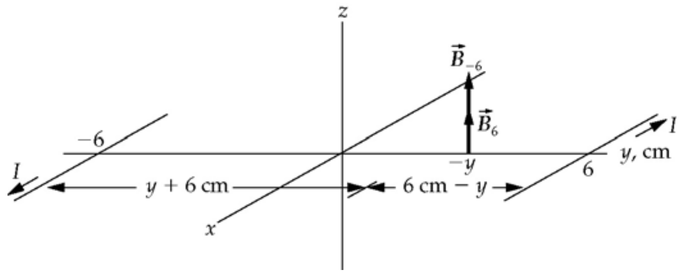
$$\vec{B}_+(9\,\text{cm}) = (133\ \mu\text{T})\hat{k},$$
$$\vec{B}_-(9\,\text{cm}) = -(26.7\ \mu\text{T})\hat{k},$$

and

$$\vec{B}(9\,\text{cm}) = (133\ \mu\text{T})\hat{k} - (26.7\ \mu\text{T})\hat{k} = \boxed{(106\ \mu\text{T})\hat{k}}$$

46 ••

Picture the Problem The diagram shows the two wires with the currents flowing in the negative *x* direction. We can use the expression for *B* due to a long, straight wire to express the difference of the fields due to the two currents. We'll denote each field by the subscript identifying the position of each wire.



The field due to the current in the wire located at y = 6 cm is:

$$B_6 = \frac{\mu_0}{4\pi}\frac{2I}{0.06\,\text{m} - y}$$

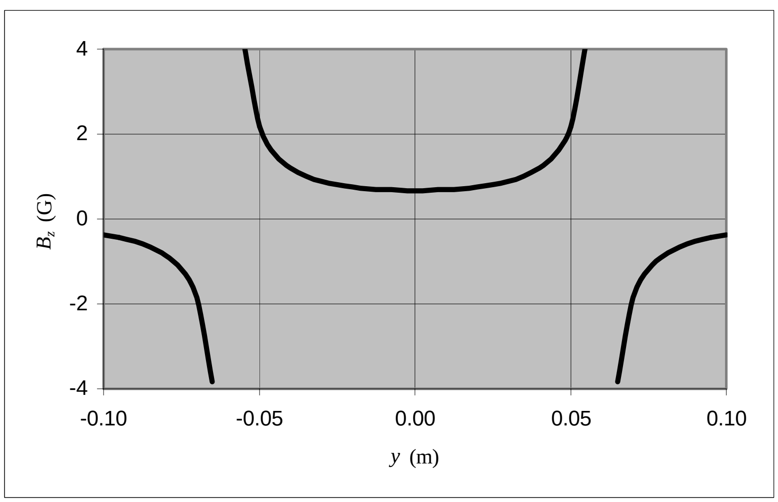
The field due to the current in the wire located at y = –6 cm is:

$$B_{-6} = \frac{\mu_0}{4\pi}\frac{2I}{0.06\,\text{m} + y}$$

The resultant field B_z is the sum of B_6 and B_{-6}:

$$B_z = B_6 - B_{-6} = \frac{\mu_0}{4\pi}\frac{I}{0.06\,\text{m} - y} + \frac{\mu_0}{4\pi}\frac{I}{0.06\,\text{m} - y} = \frac{\mu_0 I}{4\pi}\left(\frac{1}{0.06\,\text{m} - y} + \frac{1}{0.06\,\text{m} + y}\right)$$

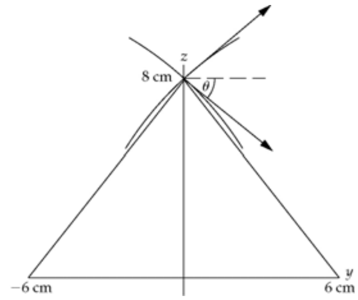
The following graph of B_z as a function of y was plotted using a spreadsheet program:



47 •

Picture the Problem Let + denote the wire (and current) at y = +6 cm and – the wire (and current) at y = –6 cm. We can use $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$ to find the magnetic field due to each of the current carrying wires and superimpose the magnetic fields due to the currents in the wires to find B at the given points on the z axis.

(*a*) Apply the right-hand rule to show that, for the currents parallel and in the negative *x* direction, the directions of the fields are as shown to the right:



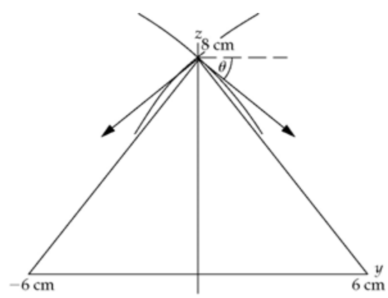
Express the magnitudes of the magnetic fields at $z = +8$ cm due to the current-carrying wires at $y = -6$ cm and $y = +6$ cm:

$$B_{z-} = B_{z+} = \left(10^{-7}\ \text{T}\cdot\text{m/A}\right) \times \frac{2(20\,\text{A})}{\sqrt{(0.06\,\text{m})^2 + (0.08\,\text{m})^2}} = 40.0\ \mu\text{T}$$

Noting that the *z* components add to zero, express the resultant magnetic field at $z = +8$ cm:

$$\vec{B}(z = 8\,\text{cm}) = 2(40.0\ \mu\text{T})\sin\theta\,\hat{j} = 2(40.0\ \mu\text{T})(0.8)\,\hat{j} = \boxed{(64.0\ \mu\text{T})\,\hat{j}}$$

(*b*) Apply the right-hand rule to show that, for the currents antiparallel with the current in the wire at $y = -6$ cm in the negative *x* direction, the directions of the fields are as shown to the right:



Noting that the *y* components add to zero, express the resultant magnetic field at $z = +8$ cm:

$$\vec{B}(z = 8\,\text{cm}) = -2(40.0\ \mu\text{T})\cos\theta\,\hat{k} = -2(40.0\ \mu\text{T})(0.6)\,\hat{k} = \boxed{-(48.0\ \mu\text{T})\,\hat{k}}$$

48 •

Picture the Problem Let + denote the wire (and current) at $y = +6$ cm and – the wire (and current) at $y = -6$ cm. The forces per unit length the wires exert on each other are action and reaction forces and hence are equal in magnitude. We can use $F = I\ell B$ to express the force on either wire and $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$ to express the magnetic field at the location of either wire due to the current in the other.

Express the force exerted on either wire:

$$F = I\ell B$$

Express the magnetic field at either location due to the current in the wire at the other location:

$$B = \frac{\mu_0}{4\pi}\frac{2I}{R}$$

Substitute to obtain:

$$F = I\ell\left(\frac{\mu_0}{4\pi}\frac{2I}{R}\right) = \frac{2\ell\mu_0}{4\pi}\frac{I^2}{R} = \frac{\ell\mu_0}{2\pi}\frac{I^2}{R}$$

Divide both sides of the equation by ℓ to obtain:

$$\frac{F}{\ell} = \frac{2\mu_0}{4\pi}\frac{I^2}{R}$$

Substitute numerical values and evaluate F/ℓ:

$$\frac{F}{\ell} = \frac{2\left(10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(20\ \text{A}\right)^2}{0.12\ \text{m}} = \boxed{667\ \mu\text{N/m}}$$

49 •

Picture the Problem We can use $\frac{F}{\ell} = \frac{2\mu_0}{4\pi}\frac{I^2}{R}$ to relate the force per unit length each current-carrying wire exerts on the other to their common current.

(*a*) Because the currents repel, they are antiparallel.

(*b*)Express the force per unit length experienced by each wire:

$$\frac{F}{\ell} = \frac{2\mu_0}{4\pi}\frac{I^2}{R}$$

Solve for I:

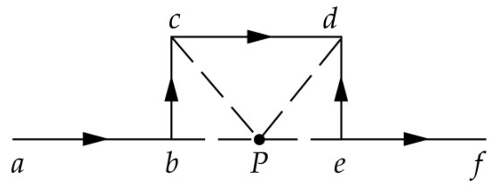
$$I = \sqrt{\frac{4\pi R}{2\mu_0}\frac{F}{\ell}}$$

Substitute numerical values and evaluate I:

$$I = \sqrt{\frac{\left(8.6\ \text{cm}\right)}{2\left(10^{-7}\ \text{T}\cdot\text{m/A}\right)}\left(3.6\ \text{nN/m}\right)} = \boxed{39.3\ \text{mA}}$$

50 ••

Picture the Problem Note that the current segments *a*-*b* and *e*-*f* do not contribute to the magnetic field at point *P*. The current in the segments *b*-*c*, *c*-*d*, and *d*-*e* result in a magnetic field at *P* that points into the plane of the paper. Note that the angles *bPc* and *ePd* are 45° and use the expression for *B* due to a straight wire segment to find the contributions to the field at P of segments *bc*, *cd*, and *de*.



Express the resultant magnetic field at P:

$$B = B_{bc} + B_{cd} + B_{de}$$

Express the magnetic field due to a straight line segment:

$$B = \frac{\mu_0}{4\pi}\frac{I}{R}\left(\sin\theta_1 + \sin\theta_2\right) \qquad (1)$$

Use equation (1) to express B_{bc} and B_{de}:

$$B_{bc} = \frac{\mu_0}{4\pi}\frac{I}{R}\left(\sin 45° + \sin 0°\right) = \frac{\mu_0}{4\pi}\frac{I}{R}\sin 45°$$

Use equation (1) to express B_{cd}:

$$B_{cd} = \frac{\mu_0}{4\pi}\frac{I}{R}\left(\sin 45° + \sin 45°\right) = 2\frac{\mu_0}{4\pi}\frac{I}{R}\sin 45°$$

Substitute to obtain:

$$B = \frac{\mu_0}{4\pi}\frac{I}{R}\sin 45° + 2\frac{\mu_0}{4\pi}\frac{I}{R}\sin 45° + \frac{\mu_0}{4\pi}\frac{I}{R}\sin 45° = 4\frac{\mu_0}{4\pi}\frac{I}{R}\sin 45°$$

Substitute numerical values and evaluate B:

$$B = 4\left(10^{-7}\ \text{T}\cdot\text{m/A}\right)\frac{8\,\text{A}}{0.01\,\text{m}}\sin 45° = \boxed{226\,\mu\text{T}}$$

51 ••

Picture the Problem The forces acting on the wire are the upward magnetic force F_B and the downward gravitational force mg, where m is the mass of the wire. We can use a condition for translational equilibrium and the expression for the force per unit length between parallel current-carrying wires to relate the required current to the mass of the wire, its length, and the separation of the two wires.

Apply $\sum F_y = 0$ to the floating

$$F_B - mg = 0$$

wire to obtain:

Express the repulsive force acting on the upper wire:

$$F_B = 2\frac{\mu_0}{4\pi}\frac{I^2\ell}{R}$$

Substitute to obtain:

$$2\frac{\mu_0}{4\pi}\frac{I^2\ell}{R} - mg = 0$$

Solve for *I*:

$$I = \sqrt{\frac{4\pi mgR}{2\mu_0\ell}}$$

Substitute numerical values and evaluate *I*:

$$I = \sqrt{\frac{(14\times10^{-3}\ \text{kg})(9.81\,\text{m/s}^2)(1.5\times10^{-3}\ \text{m})}{2(10^{-7}\ \text{T}\cdot\text{m/A})(0.16\,\text{m})}} = \boxed{80.2\,\text{A}}$$

***52 ••**

Picture the Problem Note that the forces on the upper wire are away from and directed along the lines to the lower wire and that their horizontal components cancel. We can use $\frac{F}{\ell} = 2\frac{\mu_0}{4\pi}\frac{I^2}{R}$ to find the resultant force in the upward direction (the *y* direction) acting on the top wire. In part (*b*) we can use the right-hand rule to determine the directions of the magnetic fields at the upper wire due to the currents in the two lower wires and use $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$ to find the magnitude of the resultant field due to these currents.

(*a*) Express the force per unit length each of the lower wires exerts on the upper wire:

$$\frac{F}{\ell} = 2\frac{\mu_0}{4\pi}\frac{I^2}{R}$$

Noting that the horizontal components add up to zero, express the net upward force per unit length on the upper wire:

$$\sum\frac{F_y}{\ell} = 2\frac{\mu_0}{4\pi}\frac{I^2}{R}\cos 30° + 2\frac{\mu_0}{4\pi}\frac{I^2}{R}\cos 30° = 4\frac{\mu_0}{4\pi}\frac{I^2}{R}\cos 30°$$

Substitute numerical values and evaluate $\sum \frac{F_y}{\ell}$:

$$\sum \frac{F_y}{\ell} = 4\left(10^{-7}\ \text{T}\cdot\text{m/A}\right)\frac{(15\,\text{A})^2}{0.1\text{m}}\cos 30° = \boxed{7.79\times 10^{-4}\ \text{N/m}}$$

(*b*) Noting, from the geometry of the wires, the magnetic field vectors both are at an angle of 30° with the horizontal and that their *y* components cancel, express the resultant magnetic field:

$$\vec{\boldsymbol{B}} = 2\frac{\mu_0}{4\pi}\frac{2I}{R}\cos 30°\hat{\boldsymbol{i}}$$

Substitute numerical values and evaluate *B*:

$$B = 2\left(10^{-7}\ \text{T}\cdot\text{m/A}\right)\frac{2(15\,\text{A})}{0.1\text{m}}\cos 30° = \boxed{52.0\,\mu\text{T}}$$

53 ••

Picture the Problem Note that the forces on the upper wire are away from the lower left hand wire and toward the lower right hand wire and that, due to symmetry, their vertical components cancel. We can use $\frac{F}{\ell} = 2\frac{\mu_0}{4\pi}\frac{I^2}{R}$ to find the resultant force in the *x* direction (to the right) acting on the top wire. In part (*b*) we can use the right-hand rule to determine the directions of the magnetic fields at the upper wire due to the currents in the two lower wires and use $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$ to find the magnitude of the resultant field due to these currents.

(*a*) Express the force per unit length each of the lower wires exerts on the upper wire:

$$\frac{F}{\ell} = 2\frac{\mu_0}{4\pi}\frac{I^2}{R}$$

Noting that the vertical components add up to zero, express the net force per unit length acting to the right on the upper wire:

$$\sum \frac{F_x}{\ell} = 2\frac{\mu_0}{4\pi}\frac{I^2}{R}\cos 60° + 2\frac{\mu_0}{4\pi}\frac{I^2}{R}\cos 60° = 4\frac{\mu_0}{4\pi}\frac{I^2}{R}\cos 60°$$

Substitute numerical values and evaluate $\sum \frac{F_x}{\ell}$:

$$\sum \frac{F_x}{\ell} = 4\left(10^{-7}\ \mathrm{T\cdot m/A}\right)\frac{(15\ \mathrm{A})^2}{0.1\ \mathrm{m}}\cos 60°$$
$$= \boxed{4.50\times10^{-4}\ \mathrm{N/m}}$$

(*b*) Noting, from the geometry of the wires, that the magnetic field vectors both are at an angle of 30° with the horizontal and that their *x* components cancel, express the resultant magnetic field:

$$\vec{B} = -2\frac{\mu_0}{4\pi}\frac{2I}{R}\sin 30°\,\hat{\boldsymbol{j}}$$

Substitute numerical values and evaluate *B*:

$$B = 2\left(10^{-7}\ \mathrm{T\cdot m/A}\right)\frac{2(15\ \mathrm{A})}{0.1\ \mathrm{m}}\sin 30°$$
$$= \boxed{30.0\ \mu\mathrm{T}}$$

54 ••

Picture the Problem Let the numeral 1 denote the current flowing in the positive *x* direction and the magnetic field resulting from it and the numeral 2 denote the current flowing in the positive *y* direction and the magnetic field resulting from it. We can express the magnetic field anywhere in the *xy* plane using $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$ and the right-hand rule and then impose the condition that $\vec{B} = 0$ to determine the set of points that satisfy this condition.

Express the resultant magnetic field due to the two current-carrying wires:

$$\vec{B} = \vec{B}_1 + \vec{B}_2$$

Express the magnetic field due to the current flowing in the positive *x* direction:

$$\vec{B}_1 = \frac{\mu_0}{4\pi}\frac{2I_1}{y}\hat{\boldsymbol{k}}$$

Express the magnetic field due to the current flowing in the positive *y* direction:

$$\vec{B}_2 = -\frac{\mu_0}{4\pi}\frac{2I_2}{x}\hat{\boldsymbol{k}}$$

Substitute to obtain:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2I}{y}\hat{\boldsymbol{k}} - \frac{\mu_0}{4\pi}\frac{2I}{x}\hat{\boldsymbol{k}}$$
$$= \left(\frac{\mu_0}{4\pi}\frac{2I}{y} - \frac{\mu_0}{4\pi}\frac{2I}{x}\right)\hat{\boldsymbol{k}}$$

because $I = I_1 = I_2$.

For $\vec{B} = 0$:

$$\frac{\mu_0}{4\pi}\frac{2I}{y} - \frac{\mu_0}{4\pi}\frac{2I}{x} = 0 \Rightarrow x = y.$$

Hence, $\vec{B} = 0$ along a line that makes an angle of $45°$ with the x axis.

55 ••

Picture the Problem Let the numeral 1 denote the current flowing along the positive z axis and the magnetic field resulting from it and the numeral 2 denote the current flowing in the wire located at x = 10 cm and the magnetic field resulting from it. We can express the magnetic field anywhere in the xy plane using $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$ and the right-hand rule and then impose the condition that $\vec{B} = 0$ to determine the current that satisfies this condition.

(*a*) Express the resultant magnetic field due to the two current-carrying wires:

$$\vec{B} = \vec{B}_1 + \vec{B}_2$$

Express the magnetic field at x = 2 cm due to the current flowing in the positive z direction:

$$\vec{B}_1(x = 2\,\text{cm}) = \frac{\mu_0}{4\pi}\frac{2I_1}{2\,\text{cm}}\hat{j}$$

Express the magnetic field at x = 2 cm due to the current flowing in the wire at x = 10 cm:

$$\vec{B}_2(x = 2\,\text{cm}) = -\frac{\mu_0}{4\pi}\frac{2I_2}{8\,\text{cm}}\hat{j}$$

Substitute to obtain:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2I_1}{2\,\text{cm}}\hat{j} - \frac{\mu_0}{4\pi}\frac{2I_2}{8\,\text{cm}}\hat{j}$$

$$= \left(\frac{\mu_0}{4\pi}\frac{2I_1}{2\,\text{cm}} - \frac{\mu_0}{4\pi}\frac{2I_2}{8\,\text{cm}}\right)\hat{j}$$

For $\vec{B} = 0$:

$$\frac{\mu_0}{4\pi}\frac{2I_1}{2\,\text{cm}} - \frac{\mu_0}{4\pi}\frac{2I_2}{8\,\text{cm}} = 0$$

or

$$\frac{I_1}{2\,\text{cm}} - \frac{I_2}{8\,\text{cm}} = 0$$

Solve for and evaluate I_2:

$$I_2 = 4I_1 = 4(20\,\text{A}) = \boxed{80.0\,\text{A}}$$

(*b*) Express the magnetic field at x = 5 cm:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2I_1}{5\,\text{cm}}\hat{j} - \frac{\mu_0}{4\pi}\frac{2I_2}{5\,\text{cm}}\hat{j} = \frac{2\mu_0}{4\pi(5\,\text{cm})}(I_1 - I_2)\hat{j}$$

Substitute numerical values and evaluate $\vec{B}(x = 5\,\text{cm})$:

$$\vec{B} = \frac{2(10^{-7}\,\text{T}\cdot\text{m/A})}{5\,\text{cm}}(20\,\text{A} - 80\,\text{A})\hat{j} = \boxed{-(0.240\,\text{mT})\hat{j}}$$

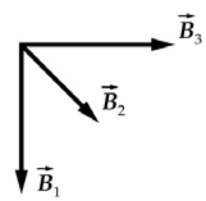
56 ••

Picture the Problem Choose a coordinate system with its origin at the lower left-hand corner of the square, the positive x axis to the right and the positive y axis upward. We can use $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$ and the right-hand rule to find the magnitude and direction of the magnetic field at the unoccupied corner due to each of the currents, and superimpose these fields to find the resultant field.

(*a*) Express the resultant magnetic field at the unoccupied corner:

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 \qquad (1)$$

When all the currents are into the paper their magnetic fields at the unoccupied corner are as shown to the right:



Express the magnetic field at the unoccupied corner due to the current I_1:

$$\vec{B}_1 = -\frac{\mu_0}{4\pi}\frac{2I}{L}\hat{j}$$

Express the magnetic field at the unoccupied corner due to the current I_2:

$$\vec{B}_2 = \frac{\mu_0}{4\pi}\frac{2I}{L\sqrt{2}}\cos 45°(\hat{i} - \hat{j}) = \frac{\mu_0}{4\pi}\frac{2I}{2L}(\hat{i} - \hat{j})$$

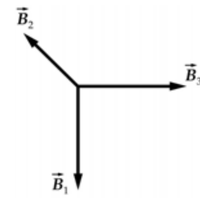
Express the magnetic field at the unoccupied corner due to the current I_3:

$$\vec{B}_3 = \frac{\mu_0}{4\pi}\frac{2I}{L}\hat{i}$$

Substitute in equation (1) and simplify to obtain:

$$\vec{B} = -\frac{\mu_0}{4\pi}\frac{2I}{L}\hat{j} + \frac{\mu_0}{4\pi}\frac{2I}{2L}\left(\hat{i}-\hat{j}\right) + \frac{\mu_0}{4\pi}\frac{2I}{L}\hat{i} = \frac{\mu_0}{4\pi}\frac{2I}{L}\left(-\hat{j} + \frac{1}{2}\left(\hat{i}-\hat{j}\right) + \hat{i}\right)$$
$$= \frac{\mu_0}{4\pi}\frac{2I}{L}\left[\left(1+\frac{1}{2}\right)\hat{i} + \left(-1-\frac{1}{2}\right)\hat{j}\right] = \boxed{\frac{3\mu_0 I}{4\pi L}\left[\hat{i}-\hat{j}\right]}$$

(*b*) When I_2 is out of the paper the magnetic fields at the unoccupied corner are as shown to the right:



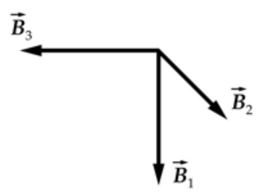
Express the magnetic field at the unoccupied corner due to the current I_2:

$$\vec{B}_2 = \frac{\mu_0}{4\pi}\frac{2I}{L\sqrt{2}}\cos 45°\left(-\hat{i}+\hat{j}\right) = \frac{\mu_0}{4\pi}\frac{2I}{2L}\left(-\hat{i}+\hat{j}\right)$$

Substitute in equation (1) and simplify to obtain:

$$\vec{B} = -\frac{\mu_0}{4\pi}\frac{2I}{L}\hat{j} + \frac{\mu_0}{4\pi}\frac{2I}{2L}\left(-\hat{i}+\hat{j}\right) + \frac{\mu_0}{4\pi}\frac{2I}{L}\hat{i} = \frac{\mu_0}{4\pi}\frac{2I}{L}\left(-\hat{j} + \frac{1}{2}\left(-\hat{i}+\hat{j}\right) + \hat{i}\right)$$
$$= \frac{\mu_0}{4\pi}\frac{2I}{L}\left[\left(1-\frac{1}{2}\right)\hat{i} + \left(-1+\frac{1}{2}\right)\hat{j}\right] = \frac{\mu_0}{4\pi}\frac{2I}{L}\left[\frac{1}{2}\hat{i} - \frac{1}{2}\hat{j}\right] = \boxed{\frac{\mu_0 I}{4\pi L}\left[\hat{i}-\hat{j}\right]}$$

(*c*) When I_1 and I_2 are in and I_3 is out of the paper the magnetic fields at the unoccupied corner are as shown to the right:



From (*a*) or (*b*) we have:

$$\vec{B}_1 = -\frac{\mu_0}{4\pi}\frac{2I}{L}\hat{j}$$

From (*a*) we have:

$$\vec{B}_2 = \frac{\mu_0}{4\pi}\frac{2I}{L\sqrt{2}}\cos 45°\left(\hat{i}-\hat{j}\right) = \frac{\mu_0}{4\pi}\frac{2I}{2L}\left(\hat{i}-\hat{j}\right)$$

Express the magnetic field at the unoccupied corner due to the current I_3:

$$\vec{B}_3 = -\frac{\mu_0}{4\pi}\frac{2I}{L}\hat{i}$$

Substitute in equation (1) and simplify to obtain:

$$\vec{B} = -\frac{\mu_0}{4\pi}\frac{2I}{L}\hat{\boldsymbol{j}} + \frac{\mu_0}{4\pi}\frac{2I}{2L}\left(\hat{\boldsymbol{i}} - \hat{\boldsymbol{j}}\right) - \frac{\mu_0}{4\pi}\frac{2I}{L}\hat{\boldsymbol{i}} = \frac{\mu_0}{4\pi}\frac{2I}{L}\left(-\hat{\boldsymbol{j}} + \frac{1}{2}\left(\hat{\boldsymbol{i}} - \hat{\boldsymbol{j}}\right) - \hat{\boldsymbol{i}}\right)$$

$$= \frac{\mu_0}{4\pi}\frac{2I}{L}\left[\left(-1 + \frac{1}{2}\right)\hat{\boldsymbol{i}} + \left(-1 - \frac{1}{2}\right)\hat{\boldsymbol{j}}\right] = \boxed{\frac{\mu_0 I}{4\pi L}\left[-\hat{\boldsymbol{i}} - 3\hat{\boldsymbol{j}}\right]}$$

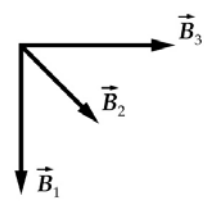
***57** ••

Picture the Problem Choose a coordinate system with its origin at the lower left-hand corner of the square, the positive *x* axis to the right and the positive *y* axis upward. Let the numeral 1 denote the wire and current in the upper left-hand corner of the square, the numeral 2 the wire and current in the lower left-hand corner (at the origin) of the square, and the numeral 3 the wire and current in the lower right-hand corner of the square. We can use $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$ and the right-hand rule to find the magnitude and direction of the magnetic field at, say, the upper right-hand corner due to each of the currents, superimpose these fields to find the resultant field, and then use $F = I\ell B$ to find the force per unit length on the wire.

(*a*) Express the resultant magnetic field at the upper right-hand corner:

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 \qquad (1)$$

When all the currents are into the paper their magnetic fields at the upper right-hand corner are as shown to the right:



Express the magnetic field due to the current I_1:

$$\vec{B}_1 = -\frac{\mu_0}{4\pi}\frac{2I}{a}\hat{\boldsymbol{j}}$$

Express the magnetic field due to the current I_2:

$$\vec{B}_2 = \frac{\mu_0}{4\pi}\frac{2I}{a\sqrt{2}}\cos 45°\left(\hat{\boldsymbol{i}} - \hat{\boldsymbol{j}}\right)$$

$$= \frac{\mu_0}{4\pi}\frac{2I}{2a}\left(\hat{\boldsymbol{i}} - \hat{\boldsymbol{j}}\right)$$

Express the magnetic field due to the current I_3:

$$\vec{B}_3 = \frac{\mu_0}{4\pi}\frac{2I}{a}\hat{\boldsymbol{i}}$$

Substitute in equation (1) and simplify to obtain:

$$\vec{B} = -\frac{\mu_0}{4\pi}\frac{2I}{a}\hat{j} + \frac{\mu_0}{4\pi}\frac{2I}{2a}\left(\hat{i} - \hat{j}\right) + \frac{\mu_0}{4\pi}\frac{2I}{a}\hat{i} = \frac{\mu_0}{4\pi}\frac{2I}{a}\left(-\hat{j} + \frac{1}{2}\left(\hat{i} - \hat{j}\right) + \hat{i}\right)$$
$$= \frac{\mu_0}{4\pi}\frac{2I}{a}\left[\left(1 + \frac{1}{2}\right)\hat{i} + \left(-1 - \frac{1}{2}\right)\hat{j}\right] = \frac{3\mu_0 I}{4\pi a}\left[\hat{i} - \hat{j}\right]$$

Using the expression for the magnetic force on a current-carrying wire, express the force per unit length on the wire at the upper right-hand corner:

$$\frac{F}{\ell} = BI \qquad (2)$$

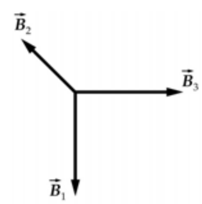
Substitute to obtain:

$$\frac{\vec{F}}{\ell} = \frac{3\mu_0 I^2}{4\pi a}\left[\hat{i} - \hat{j}\right]$$

and

$$\frac{F}{\ell} = \sqrt{\left(\frac{3\mu_0 I^2}{4\pi a}\right)^2 + \left(\frac{3\mu_0 I^2}{4\pi a}\right)^2}$$
$$= \boxed{\frac{3\sqrt{2}\mu_0 I^2}{4\pi a}}$$

(*b*) When the current in the upper right-hand corner of the square is out of the page, and the currents in the wires at adjacent corners are oppositely directed, the magnetic fields at the upper right-hand are as shown to the right:



Express the magnetic field at the upper right-hand corner due to the current I_2:

$$\vec{B}_2 = \frac{\mu_0}{4\pi}\frac{2I}{a\sqrt{2}}\cos 45°\left(-\hat{i} + \hat{j}\right)$$
$$= \frac{\mu_0}{4\pi}\frac{2I}{2a}\left(-\hat{i} + \hat{j}\right)$$

Using $\vec{B}_1$ and $\vec{B}_3$ from (*a*), substitute in equation (1) and simplify to obtain:

$$\vec{B} = -\frac{\mu_0}{4\pi}\frac{2I}{a}\hat{j} + \frac{\mu_0}{4\pi}\frac{2I}{2a}\left(-\hat{i} + \hat{j}\right) + \frac{\mu_0}{4\pi}\frac{2I}{a}\hat{i} = \frac{\mu_0}{4\pi}\frac{2I}{a}\left(-\hat{j} + \frac{1}{2}\left(-\hat{i} + \hat{j}\right) + \hat{i}\right)$$
$$= \frac{\mu_0}{4\pi}\frac{2I}{a}\left[\left(1 - \frac{1}{2}\right)\hat{i} + \left(-1 + \frac{1}{2}\right)\hat{j}\right] = \frac{\mu_0}{4\pi}\frac{2I}{a}\left[\frac{1}{2}\hat{i} - \frac{1}{2}\hat{j}\right] = \frac{\mu_0 I}{4\pi a}\left[\hat{i} - \hat{j}\right]$$

Substitute in equation (2) to obtain:

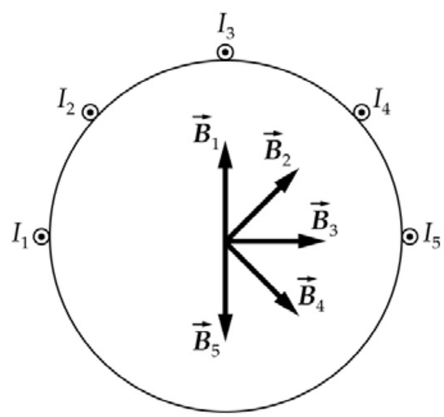
$$\frac{\vec{F}}{\ell} = \frac{\mu_0 I^2}{4\pi a}\left[\hat{i} - \hat{j}\right]$$

and

$$\frac{F}{\ell} = \sqrt{\left(\frac{\mu_0 I^2}{4\pi a}\right)^2 + \left(\frac{\mu_0 I^2}{4\pi a}\right)^2} = \boxed{\frac{\sqrt{2}\mu_0 I^2}{4\pi a}}$$

58 ••

Picture the Problem The configuration is shown in the adjacent figure. Here the *z* axis points out of the plane of the paper, the *x* axis points to the right, the *y* axis points up. We can use $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$ and the right-hand rule to find the magnetic field due to the current in each wire and add these magnetic fields vectorially to find the resultant field.



Express the resultant magnetic field on the *z* axis:

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 + \vec{B}_5$$

$\vec{B}_1$ is given by:

$$\vec{B}_1 = B\hat{j}$$

$\vec{B}_2$ is given by:

$$\vec{B}_2 = (B\cos 45°)\hat{i} + (B\sin 45°)\hat{j}$$

$\vec{B}_3$ is given by:

$$\vec{B}_3 = B\hat{i}$$

$\vec{B}_4$ is given by:

$$\vec{B}_4 = (B\cos 45°)\hat{i} - (B\sin 45°)\hat{j}$$

$\vec{B}_5$ is given by:

$$\vec{B}_5 = -B\hat{j}$$

Substitute for $\vec{B}_1$, $\vec{B}_2$, $\vec{B}_3$, $\vec{B}_4$, and $\vec{B}_5$ and simplify to obtain:

$$\begin{aligned}\vec{B} &= B\hat{j} + (B\cos 45°)\hat{i} + (B\sin 45°)\hat{j} + B\hat{i} + (B\cos 45°)\hat{i} - (B\sin 45°)\hat{j} - B\hat{j} \\ &= (B\cos 45°)\hat{i} + B\hat{i} + (B\cos 45°)\hat{i} = (B + 2B\cos 45°)\hat{i} = \left(1 + \sqrt{2}\right)B\hat{i}\end{aligned}$$

Express B due to each current at $z = 0$:

$$B = \frac{\mu_0}{4\pi}\frac{2I}{R}$$

Substitute to obtain:

$$\boxed{\vec{B} = \left(1+\sqrt{2}\right)\frac{\mu_0 I}{2\pi R}\,\hat{\boldsymbol{i}}}$$

$\vec{B}$ Due to a Current in a Solenoid

59 •

Picture the Problem We can use $B_x = \frac{1}{2}\mu_0 nI\left(\frac{b}{\sqrt{b^2+R^2}}+\frac{a}{\sqrt{a^2+R^2}}\right)$ to find B at any point on the axis of the solenoid. Note that the number of turns per unit length for this solenoid is 300 turns/0.3 m = 1000 turns/m.

Express the magnetic field at any point on the axis of the solenoid:

$$B_x = \frac{1}{2}\mu_0 nI\left(\frac{b}{\sqrt{b^2+R^2}}+\frac{a}{\sqrt{a^2+R^2}}\right)$$

Substitute numerical values to obtain:

$$B_x = \frac{1}{2}\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)(1000)(2.6\,\text{A})\left(\frac{b}{\sqrt{b^2+(0.012\,\text{m})^2}}+\frac{a}{\sqrt{a^2+(0.012\,\text{m})^2}}\right)$$
$$= (1.63\,\text{mT})\left(\frac{b}{\sqrt{b^2+(0.012\,\text{m})^2}}+\frac{a}{\sqrt{a^2+(0.012\,\text{m})^2}}\right)$$

(*a*) Evaluate B_x for $a = b = 0.15$ m:

$$B_x = (1.63\,\text{mT})\left(\frac{0.15\,\text{m}}{\sqrt{(0.15\,\text{m})^2+(0.012\,\text{m})^2}}+\frac{0.15\,\text{m}}{\sqrt{(0.15\,\text{m})^2+(0.012\,\text{m})^2}}\right) = \boxed{3.25\,\text{mT}}$$

(*b*) Evaluate B_x for $a = 0.1$ m and $b = 0.2$ m:

$$B_x(0.2\,\text{m}) = (1.63\,\text{mT})\left(\frac{0.2\,\text{m}}{\sqrt{(0.2\,\text{m})^2+(0.012\,\text{m})^2}}+\frac{0.1\,\text{m}}{\sqrt{(0.1\,\text{m})^2+(0.012\,\text{m})^2}}\right)$$
$$= \boxed{3.25\,\text{mT}}$$

(*c*) Evaluate B_x (= B_{end}) for $a = 0$ and $b = 0.3$ m:

$$B_x = (1.63\,\text{mT})\left(\frac{0.3\,\text{m}}{\sqrt{(0.3\,\text{m})^2 + (0.012\,\text{m})^2}}\right) = \boxed{1.63\,\text{mT}}$$

Note that $B_{\text{end}} = \frac{1}{2} B_{\text{center}}$.

***60** •

Picture the Problem We can use $B_x = \mu_0 nI$ to find the approximate magnetic field on the axis and inside the solenoid.

Express B_x as a function of n and I:

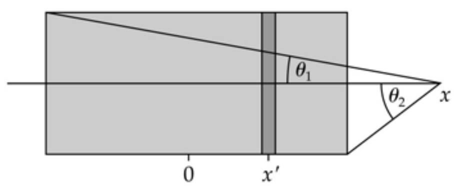
$$B_x = \mu_0 nI$$

Substitute numerical values and evaluate B_x:

$$B_x = \left(4\pi \times 10^{-7}\ \text{N/A}^2\right)\left(\frac{600}{2.7\,\text{m}}\right)(2.5\,\text{A}) = \boxed{0.698\,\text{mT}}$$

61 •••

Picture the Problem The solenoid, extending from $x = -\ell/2$ to $x = \ell/2$, with the origin at its center, is shown in the diagram. To find the field at the point whose coordinate is x outside the solenoid we can determine the field at x due to an infinitesimal segment of the solenoid of width dx' at x', and then integrate from $x = -\ell/2$ to $x = \ell/2$. The segment may be considered as a coil ndx' carrying a current I.



Express the field dB at the axial point whose coordinate is x:

$$dB_x = \frac{\mu_0}{4\pi} \frac{2\pi R^2 I}{\left[(x - x')^2 + R^2\right]^{3/2}} dx'$$

Integrate dB_x from $x = -\ell/2$ to $x = \ell/2$ to obtain:

$$B_x = \frac{\mu_0 nIR^2}{2} \int_{-\ell/2}^{\ell/2} \frac{dx'}{\left[(x - x')^2 + R^2\right]^{3/2}} = \frac{\mu_0 nI}{2}\left(\frac{x + \ell/2}{\sqrt{(x + \ell/2)^2 + R^2}} - \frac{x - \ell/2}{\sqrt{(x - \ell/2)^2 + R^2}}\right)$$

Refer to the diagram to express $\cos\theta_1$ and $\cos\theta_2$:

$$\cos\theta_1 = \frac{x + \frac{1}{2}\ell}{\left[R^2 + \left(x + \frac{1}{2}\ell\right)^2\right]^{1/2}}$$

and

$$\cos\theta_2 = \frac{x - \frac{1}{2}\ell}{\left[R^2 + \left(x - \frac{1}{2}\ell\right)^2\right]^{1/2}}$$

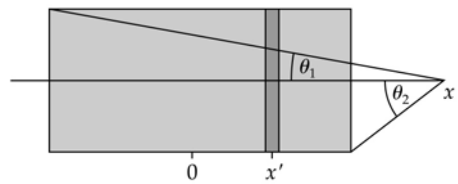
Substitute to obtain:

$$B = \boxed{\tfrac{1}{2}\mu_0 nI\left(\cos\theta_1 - \cos\theta_2\right)}$$

62 •••

Picture the Problem We can use Equation 27-35, together with the small angle approximation for the cosine and tangent functions, to show that θ_1 and θ_2 are as given and that B is given by Equation 27-37.

(*a*) The angles θ_1 and θ_2 are shown in the diagram. Note that $\tan\theta_1 = R/(x + \ell/2)$ and $\tan\theta_2 = R/(x - \ell/2)$.



Apply the small angle approximation $\tan\theta \approx \theta$ to obtain:

$$\theta_1 \approx \boxed{\frac{R}{x + \frac{1}{2}\ell}}$$

and

$$\theta_2 \approx \boxed{\frac{R}{x - \frac{1}{2}\ell}}$$

(*b*) Express the magnetic field outside the solenoid:

$$B = \tfrac{1}{2}\mu_0 nI\left(\cos\theta_1 - \cos\theta_2\right)$$

Apply the small angle approximation for the cosine function to obtain:

$$\cos\theta_1 = 1 - \tfrac{1}{2}\left(\frac{R}{x + \frac{1}{2}\ell}\right)^2$$

and

$$\cos\theta_2 = 1 - \tfrac{1}{2}\left(\frac{R}{x - \frac{1}{2}\ell}\right)^2$$

Substitute and simplify to obtain:

$$B = \tfrac{1}{2}\mu_0 nI\left[1 - \tfrac{1}{2}\left(\frac{R}{x + \frac{1}{2}\ell}\right)^2 - 1 + \tfrac{1}{2}\left(\frac{R}{x - \frac{1}{2}\ell}\right)^2\right] = \tfrac{1}{4}\mu_0 nIR^2\left[\frac{1}{\left(x - \frac{1}{2}\ell\right)^2} - \frac{1}{\left(x + \frac{1}{2}\ell\right)^2}\right]$$

Let $r_1 = x - \frac{1}{2}\ell$ be the distance to the near end of the solenoid, $r_1 = x + \frac{1}{2}\ell$ the distance to the far

$$B = \boxed{\frac{\mu_0}{4\pi}\left(\frac{q_m}{r_1^2} - \frac{q_m}{r_2^2}\right)}$$

end, and $q_m = nI\pi R^2 = \mu/\ell$, where $\mu = nI\pi R^2$ is the magnetic moment of the solenoid to obtain:

Ampère's Law

***63** •

Picture the Problem We can apply Ampère's law to a circle centered on the axis of the cylinder and evaluate this expression for $r < R$ and $r > R$ to find B inside and outside the cylinder.

Apply Ampère's law to a circle centered on the axis of the cylinder:

$$\oint_C \vec{B}\cdot d\vec{\ell} = \mu_0 I_C$$

Note that, by symmetry, the field is the same everywhere on this circle.

Evaluate this expression for $r < R$:

$$\oint_C \vec{B}_{\text{inside}}\cdot d\vec{\ell} = \mu_0(0) = 0$$

Solve for B_{inside} to obtain:

$$B_{\text{inside}} = \boxed{0}$$

Evaluate this expression for $r > R$:

$$\oint_C \vec{B}_{\text{outside}}\cdot d\vec{\ell} = B(2\pi R) = \mu_0 I$$

Solve for B_{outside} to obtain:

$$B_{\text{outside}} = \boxed{\frac{\mu_0 I}{2\pi R}}$$

64 •

Picture the Problem We can use Ampère's law, $\oint_C \vec{B}\cdot d\vec{\ell} = \mu_0 I_C$, to find the line integral $\oint_C \vec{B}\cdot d\vec{\ell}$ for each of the three paths.

(*a*) Evaluate $\oint_C \vec{B}\cdot d\vec{\ell}$ for C_1:

$$\oint_{C_1} \vec{B}\cdot d\vec{\ell} = \boxed{\mu_0(8\,\text{A})}$$

Evaluate $\oint_C \vec{B}\cdot d\vec{\ell}$ for C_2:

$$\oint_{C_2} \vec{B}\cdot d\vec{\ell} = \mu_0(8\,\text{A} - 8\,\text{A}) = \boxed{0}$$

Evaluate $\oint_C \vec{B}\cdot d\vec{\ell}$ for C_3:

$$\oint_{C_1} \vec{B}\cdot d\vec{\ell} = \boxed{-\mu_0(8\,\text{A})}$$ because the field is opposite the direction of integration.

(*b*) None of the paths can be used to find B at a general point because there the current configuration does not have cylindrical symmetry.

65 •

Picture the Problem Let the current in the wire and outer shell be I. We can apply Ampère's law to a circle, concentric with the inner wire, of radius r to find B at points between the wire and the shell far from the ends ($r < R$), and outside the cable ($r > R$).

(*a*) Apply Ampère's law for $r < R$:

$$\oint_C \vec{B}_{r<R}\cdot d\vec{\ell} = B_{r<R}(2\pi r) = \mu_0 I$$

Solve for $B_{r<R}$ to obtain:

$$B_{r<R} = \boxed{\frac{\mu_0 I}{2\pi r}}$$

(*b*) Apply Ampère's law for $r > R$:

$$\oint_C \vec{B}_{r>R}\cdot d\vec{\ell} = \mu_0(0)$$

Solve for $B_{r>R}$ to obtain:

$$B_{r>R} = \boxed{0}$$

66 ••

Picture the Problem. Let the radius of the wire be a. We can apply Ampère's law to a circle, concentric with the center of the wire, of radius r to find B at various distances from the center of the wire.

Express Ampère's law:

$$\oint_C \vec{B}\cdot d\vec{\ell} = \mu_0 I_C$$

Using the fact that the current is uniformly distributed over the cross-sectional area of the wire, relate the current enclosed by a circle of radius r to the total current I carried by the wire:

$$\frac{I_C}{\pi r^2} = \frac{I}{\pi a^2}$$

or

$$I_C = I\frac{r^2}{a^2}$$

Substitute and evaluate the integral to obtain:

$$B_r(2\pi r) = \frac{\mu_0 r^2}{a^2} I$$

Solve for $B_{r<a}$:

$$B_{r<a} = \frac{\mu_0 r}{2\pi a^2} I \qquad (1)$$

For $r \geq a$:

$$\oint_C \vec{B}_{r\geq a}\cdot d\vec{\ell} = B_{r\geq a}(2\pi r) = \mu_0 I$$

Solve for $B_{r\geq a}$:

$$B_{r\geq a} = \frac{\mu_0 I}{2\pi r} \qquad (2)$$

(*a*) Use equation (1) to evaluate *B*(0.1 cm):

$$B(0.1\,\text{cm}) = \frac{(4\pi\times10^{-7}\ \text{N/A}^2)(0.001\,\text{m})}{2\pi(0.005\,\text{m})^2}(100\,\text{A}) = \boxed{8.00\times10^{-4}\ \text{T}}$$

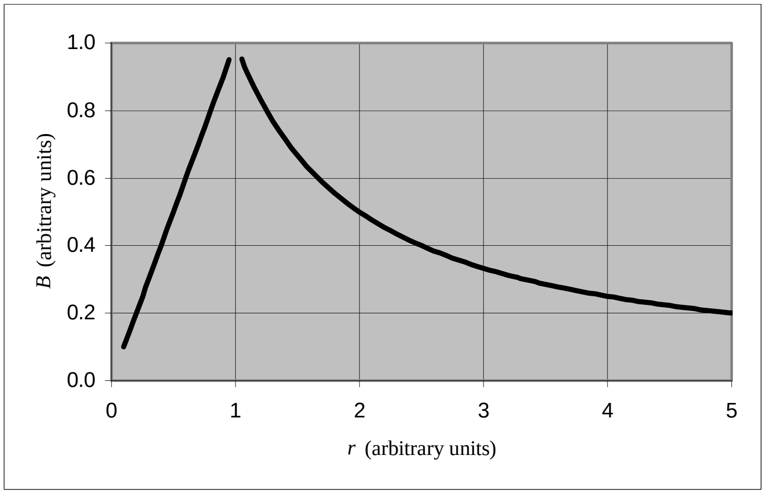
(*b*) Use either equation to evaluate *B* at the surface of the wire:

$$B(0.005\,\text{cm}) = \frac{(4\pi\times10^{-7}\ \text{N/A}^2)(0.005\,\text{m})}{2\pi(0.005\,\text{m})^2}(100\,\text{A}) = \boxed{4.00\times10^{-3}\ \text{T}}$$

(*c*) Use equation (2) to evaluate *B*(0.7 cm):

$$B(0.007\,\text{m}) = \frac{(4\pi\times10^{-7}\ \text{N/A}^2)(100\,\text{A})}{2\pi(0.007\,\text{m})} = \boxed{2.86\times10^{-3}\ \text{T}}$$

(*d*) A graph of *B* as a function of *r* follows:



***67 ••**

Determine the Concept The contour integral consists of four portions, two horizontal portions for which $\oint_C \vec{B}\cdot d\vec{\ell} = 0$, and two vertical portions. The portion within the magnetic field gives a nonvanishing contribution, whereas the portion outside the field gives no contribution to the contour integral. Hence, the contour integral has a finite value. However, it encloses no current; thus, it appears that Ampère's law is violated. What this demonstrates is that there must be a fringing field so that the contour integral does vanish.

68 ••

Picture the Problem Let r_1 = 1 mm, r_2 = 2 mm, and r_3 = 3 mm and apply Ampère's law in each of the three regions to obtain expressions for B in each part of the coaxial cable and outside the coaxial cable.

Apply Ampère's law to a circular path of radius $r < r_1$ to obtain:

$$B_{r<r_1}(2\pi r) = \mu_0 I_C$$

Because the current is uniformly distributed over the cross section of the inner wire:

$$\frac{I_C}{\pi r^2} = \frac{I}{\pi r_1^2} \Rightarrow I_C = \frac{r^2}{r_1^2} I$$

Substitute for I_C to obtain:

$$B_{r<r_1}(2\pi r) = \mu_0 \frac{r^2}{r_1^2} I$$

Solve for $B_{r<r_1}$:

$$B_{r<r_1} = \frac{2\mu_0 I}{4\pi} \frac{r}{r_1^2} \qquad (1)$$

Apply Ampère's law to a circular path of radius $r_1 < r < r_2$ to obtain:

$$B_{r_1<r<r_2}(2\pi r) = \mu_0 I$$

Solve for $B_{r_1<r<r_2}$:

$$B_{r_1<r<r_2} = \frac{2\mu_0 I}{4\pi} \frac{1}{r} \qquad (2)$$

Apply Ampère's law to a circular path of radius $r_2 < r < r_3$ to obtain:

$$B_{r_2<r<r_3}(2\pi r) = \mu_0 I_C = \mu_0 (I - I')$$

where I' is the current in the outer conductor at a distance less than r from the center of the inner conductor.

Because the current is uniformly distributed over the cross section of the outer conductor:

$$\frac{I'}{\pi r^2 - \pi r_2^2} = \frac{I}{\pi r_3^2 - \pi r_2^2}$$

Solve for I':

$$I' = \frac{r^2 - r_2^2}{r_3^2 - r_2^2} I$$

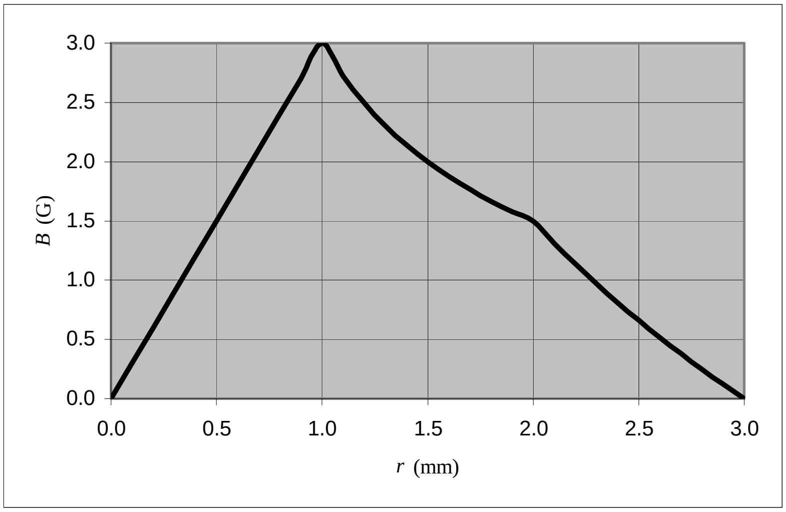
Substitute for I' to obtain:

$$B_{r_2<r<r_3}(2\pi r) = \mu_0 \left(I - \frac{r^2 - r_2^2}{r_3^2 - r_2^2} I \right)$$

Solve for $B_{r_1<r<r_2}$:

$$B_{r_2<r<r_3} = \frac{2\mu_0 I}{4\pi} \left(1 - \frac{r^2 - r_2^2}{r_3^2 - r_2^2} \right) \qquad (3)$$

A spreadsheet program was used to plot the following graph of equations (1), (2), and (3).



Apply Ampère's law to a circular path of radius $r > r_3$ to obtain:

$$B_{r>r_3}(2\pi r) = \mu_0 I_C = \mu_0(I - I) = 0$$

and $B_{r>r_3} = \boxed{0}$

69 ••

Picture the Problem We can use Ampère's law to calculate *B* because of the high degree of symmetry. The current through *C* depends on whether *r* is less than or the inner radius *a*, greater than the inner radius *a* but less than the outer radius *b*, or greater than the outer radius *b*.

(*a*) Apply Ampère's law to a circular path of radius $r < a$ to obtain:

$$\oint_C \vec{B}_{r<a} \cdot d\vec{\ell} = \mu_0 I_C = \mu_0(0) = 0$$

and

$$B_{r<a} = \boxed{0}$$

(*b*) Use the uniformity of the current over the cross-section of the conductor to express the current I' enclosed by a circular path whose radius satisfies the condition $a < r < b$:

$$\frac{I'}{\pi(r^2 - a^2)} = \frac{I}{\pi(b^2 - a^2)}$$

Solve for $I_C = I'$:

$$I_C = I' = I\,\frac{r^2 - a^2}{b^2 - a^2}$$

Substitute in Ampère's law to obtain:

$$\oint_C \vec{B}_{a<r<b} \cdot d\vec{\ell} = B_{a<r<b}(2\pi r) = \mu_0 I' = \mu_0 I \frac{r^2 - a^2}{b^2 - a^2}$$

Solve for $B_{a<r<b}$:

$$B_{a<r<b} = \boxed{\frac{\mu_0 I}{2\pi r} \frac{r^2 - a^2}{b^2 - a^2}}$$

(*c*) Express I_C for $r > b$:

$$I_C = I$$

Substitute in Ampère's law to obtain:

$$\oint_C \vec{B}_{r>b} \cdot d\vec{\ell} = B_{r>b}(2\pi r) = \mu_0 I$$

Solve for $B_{r>b}$:

$$B_{r>b} = \boxed{\frac{\mu_0 I}{2\pi r}}$$

70 ••

Picture the Problem The number of turns enclosed within the rectangular area is *na*. Denote the corners of the rectangle, starting in the lower left-hand corner and proceeding counterclockwise, as 1, 2, 3, and 4. We can apply Ampère's law to each side of this rectangle in order to evaluate $\oint_C \vec{B} \cdot d\vec{\ell}$.

Express the integral around the closed path C as the sum of the integrals along the sides of the rectangle:

$$\oint_C \vec{B} \cdot d\vec{\ell} = \int_{1\to 2} \vec{B} \cdot d\vec{\ell} + \int_{2\to 3} \vec{B} \cdot d\vec{\ell} + \int_{3\to 4} \vec{B} \cdot d\vec{\ell} + \int_{4\to 1} \vec{B} \cdot d\vec{\ell}$$

Evaluate $\int_{1\to 2} \vec{B} \cdot d\vec{\ell}$:

$$\int_{1\to 2} \vec{B} \cdot d\vec{\ell} = aB$$

For the paths $2 \to 3$ and $4 \to 1$, $\vec{B}$ is either zero (outside the solenoid) or is perpendicular to $d\vec{\ell}$ and so:

$$\int_{2\to 3} \vec{B} \cdot d\vec{\ell} = \int_{4\to 1} \vec{B} \cdot d\vec{\ell} = 0$$

For the path $3 \to 4$, $\vec{B}$ =0 and:

$$\int_{3\to 4} \vec{B} \cdot d\vec{\ell} = 0$$

Substitute in Ampère's law to obtain:

$$\oint_C \vec{B} \cdot d\vec{\ell} = aB + 0 + 0 + 0 = aB = \mu_0 I_C = \mu_0 naI$$

Solve for B to obtain: $B = \boxed{\mu_0 nI}$

71 ••

Picture the Problem The magnetic field inside a tightly wound toroid is given by $B = \mu_0 NI/(2\pi r)$, where $a < r < b$ and a and b are the inner and outer radii of the toroid.

Express the magnetic field of a toroid:

$$B = \frac{\mu_0 NI}{2\pi r}$$

(a) Substitute numerical values and evaluate B(1.1 cm):

$$B(1.1\,\text{cm}) = \frac{(4\pi \times 10^{-7}\ \text{N/A}^2)(1000)(1.5\,\text{A})}{2\pi(1.1\,\text{cm})} = \boxed{27.3\,\text{mT}}$$

(b) Substitute numerical values and evaluate B(1.5 cm):

$$B(1.5\,\text{cm}) = \frac{(4\pi \times 10^{-7}\ \text{N/A}^2)(1000)(1.5\,\text{A})}{2\pi(1.5\,\text{cm})} = \boxed{20.0\,\text{mT}}$$

***72** ••

Picture the Problem In parts (a), (b), and (c) we can use a right-hand rule to determine the direction of the magnetic field at points above and below the infinite sheet of current. In part (d) we can evaluate $\oint_C \vec{B} \cdot d\vec{\ell}$ around the specified path and equate it to $\mu_0 I_C$ and solve for B.

(a) $\boxed{\text{At } P \text{ the magnetic field points to the right (i.e., in the } -\hat{i} \text{ direction) since its vertical components cancel.}}$

(b) $\boxed{\text{Because the sheet is infinite, the same argument used in (a) applies; B is in the } -\hat{i} \text{ direction.}}$

(c) $\boxed{\text{Below the sheet the magnetic field points to the left, i.e., in the } \hat{i} \text{ direction. The vertical components cancel.}}$

(d) Express $\oint_C \vec{B} \cdot d\vec{\ell}$, in the counterclockwise direction, for the given path:

$$\oint_C \vec{B} \cdot d\vec{\ell} = 2 \int_{\text{parallel}} \vec{B} \cdot d\vec{\ell} + 2 \int_{\perp} \vec{B} \cdot d\vec{\ell}$$

For the paths perpendicular to the sheet, $\vec{B}$ and $d\vec{\ell}$ are perpendicular to each other and:

$$\int_{\perp} \vec{B}\cdot d\vec{\ell} = 0$$

For the paths parallel to the sheet, $\vec{B}$ and $d\vec{\ell}$ are in the same direction and:

$$\int_{\text{parallel}} \vec{B}\cdot d\vec{\ell} = Bw$$

Substitute to obtain:

$$\oint_C \vec{B}\cdot d\vec{\ell} = 2\int_{\text{parallel}} \vec{B}\cdot d\vec{\ell} = 2Bw = \mu_0 I_C = \mu_0(\lambda w)$$

Solve for B:

$$B = \tfrac{1}{2}\mu_0\lambda \text{ and } \vec{B}_{\text{above}} = \boxed{-\tfrac{1}{2}\mu_0\lambda\hat{\boldsymbol{i}}}$$

Magnetization and Magnetic Susceptibility

73 •

Picture the Problem We can use $B = B_{\text{app}} = \mu_0 nI$ to find B and B_{app} at the center when there is no core in the solenoid and $B = B_{\text{app}} + \mu_0 M$ when there is an iron core with a magnetization $M = 1.2\times10^6$ A/m.

(*a*) Express the magnetic field, in the absence of a core, in the solenoid :

$$B = B_{\text{app}} = \mu_0 nI$$

Substitute numerical values and evaluate B and B_{app}:

$$B = B_{\text{app}} = \left(4\pi\times10^{-7}\text{ N/A}^2\right)\left(\frac{400}{0.2\,\text{m}}\right)(4\,\text{A}) = \boxed{10.1\,\text{mT}}$$

(*b*) With an iron core with a magnetization $M = 1.2\times10^6$ A/m present:

$$B_{\text{app}} = \boxed{10.1\,\text{mT}}$$

and

$$B = B_{\text{app}} + \mu_0 M = 10.1\,\text{mT} + \left(4\pi\times10^{-7}\text{ N/A}^2\right)\left(1.2\times10^6\text{ A/m}\right) = \boxed{1.52\,\text{T}}$$

74 •

Picture the Problem We can use $B = B_{\text{app}} = \mu_0 nI$ to find B and B_{app} at the center when there is no core in the solenoid and $B = B_{\text{app}} + \mu_0 M$ when there is an aluminum core. We can use $M = \chi_m \frac{B_{app}}{\mu_0}$ to find the magnetization of the core with the aluminum present.

Express the magnetic field, in the absence of a core, in the solenoid :

$$B = B_{app} = \mu_0 nI$$

Substitute numerical values and evaluate B and B_{app}:

$$B = B_{app} = \left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(\frac{400}{0.2\,\text{m}}\right)(4\,\text{A})$$
$$= \boxed{10.1\,\text{mT}}$$

Express the magnetization in the core with the aluminum present:

$$M = \chi_m \frac{B_{app}}{\mu_0}$$

Use Table 27-1 to find the value of χ_m for aluminum:

$$\chi_{m,\,Al} = 2.3\times10^{-5}$$

Substitute numerical values and evaluate M:

$$M = 2.3\times10^{-5}\,\frac{10.1\,\text{mT}}{4\pi\times10^{-7}\ \text{N/A}^2}$$
$$= \boxed{0.185\,\text{A/m}}$$

75 •

Picture the Problem We can use $B_{app} = \mu_0 nI$ to find B_{app} at the center of the tungsten core in the solenoid. The magnetization is related to B_{app} and χ_m according to $M = \chi_m B_{app}/\mu_0 = \chi_m nI$ and we can use $B = B_{app}(1+\chi_m)$ to find B.

Express the magnetic field, for a tungsten core, in the solenoid :

$$B_{app} = \mu_0 nI$$

Substitute numerical values and evaluate B_{app}:

$$B_{app} = \left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(\frac{400}{0.2\,\text{m}}\right)(4\,\text{A})$$
$$= \boxed{10.053\,\text{mT}}$$

Express the magnetization in the core with the aluminum present:

$$M = \chi_m \frac{B_{app}}{\mu_0} = \chi_m nI$$

Use Table 27-1 to find the value of χ_m for tungsten:

$$\chi_{m,\,tungstenl} = 6.8\times10^{-5}$$

Substitute numerical values and evaluate M:

$$M = \left(6.8\times10^{-5}\right)\left(\frac{400}{0.2\,\text{m}}\right)(4\,\text{A}) = \boxed{0.544\,\text{A/m}}$$

Express B in terms of B_{app} and χ_{m}:

$$B = B_{\text{app}}\left(1+\chi_{\text{m}}\right)$$

Substitute numerical values and evaluate B:

$$B = (10.053\,\text{mT})\left(1+6.8\times10^{-5}\right) = \boxed{10.054\,\text{mT}}$$

76 •

Picture the Problem We can use $B = B_{\text{app}}\left(1+\chi_{\text{m}}\right)$ to relate B and B_{app} to the magnetic susceptibility of tungsten. Dividing both sides of this equation by B_{app} and examining the value of $\chi_{\text{m, tungsten}}$ will allow us to decide whether the field inside the solenoid decreases or increases when the core is removed.

Express the magnetic field inside the solenoid with the tungsten core present B in terms of B_{app} and χ_{m}:

$$B = B_{\text{app}}\left(1+\chi_{\text{m}}\right)$$

where B_{app} is the magnetic field in the absence of the tungsten core.

Express the ratio of B to B_{app}:

$$\frac{B}{B_{\text{app}}} = 1+\chi_{\text{m}} \qquad (1)$$

(*a*) Because $\chi_{\text{m, tungsten}} > 0$:

$$B > B_{\text{app}}$$

and

$\boxed{B \text{ will decrease when the tungsten core is removed.}}$

(*b*) From equation (1) the fractional change is:

$$\chi_{\text{m}} = 6.8\times10^{-5} = \boxed{6.8\times10^{-3}\%}$$

77 •

Picture the Problem We can use $B = B_{\text{app}}\left(1+\chi_{\text{m}}\right)$ to relate B and B_{app} to the magnetic susceptibility of liquid sample.

Express the magnetic field inside the solenoid with the liquid sample present B in terms of B_{app} and χ_{m},

$$B = B_{\text{app}}\left(1+\chi_{\text{m, sample}}\right)$$

where B_{app} is the magnetic field in the absence of the liquid sample.

sample:

The fractional change in the magnetic field in the core is:

$$\frac{\Delta B}{B_{\text{app}}} = \chi_{\text{m, sample}}$$

Substitute numerical values and evaluate $\chi_{\text{m, sample}}$:

$$\chi_{\text{m, sample}} = \frac{\Delta B}{B_{\text{app}}} = -0.004\%$$

$$= \boxed{-4.00\times10^{-5}}$$

78 •

Picture the Problem We can use $B = B_{\text{app}} = \mu_0 nI$ to find B and B_{app} at the center when there is no core in the solenoid and $B = B_{\text{app}}(1+\chi_{\text{m}})$ when there is an aluminum or silver core.

(*a*) Express the magnetic field, in the absence of a core, in the solenoid:

$$B = B_{\text{app}} = \mu_0 nI$$

Substitute numerical values and evaluate B and B_{app}:

$$B = B_{\text{app}} = \left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(\frac{50}{0.01\,\text{m}}\right)(10\,\text{A}) = \boxed{62.8\,\text{mT}}$$

(*b*) With an aluminum core:

$$B = B_{\text{app}}(1+\chi_{\text{m}})$$

Use Table 27-1 to find the value of χ_{m} for aluminum:

$$\chi_{\text{m, Al}} = 2.3\times10^{-5}$$

and

$$1+\chi_{\text{m, Al}} = 1+2.3\times10^{-5} \approx 1$$

Substitute numerical values and evaluate B and B_{app}:

$$B = B_{\text{app}} = \left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(\frac{50}{0.01\,\text{m}}\right)(10\,\text{A}) = \boxed{62.8\,\text{mT}}$$

(*c*) With a silver core:

$$B = B_{\text{app}}(1+\chi_{\text{m}})$$

Use Table 27-1 to find the value of χ_{m} for silver:

$$\chi_{\text{m, Ag}} = -2.6\times10^{-5}$$

and

$$1+\chi_{\text{m, Ag}} = 1-2.6\times10^{-5} \approx 1$$

Substitute numerical values and evaluate B and B_{app}:

$$B = B_{app} = \left(4\pi \times 10^{-7}\ \text{N/A}^2\right)\left(\frac{50}{0.01\,\text{m}}\right)(10\,\text{A}) = \boxed{62.8\,\text{mT}}$$

***79 ••**

Picture the Problem We can use the data in the table and $B_{app} = \mu_0 nI$ to plot B versus B_{app}. We can find K_m using $B = K_m B_{app}$.

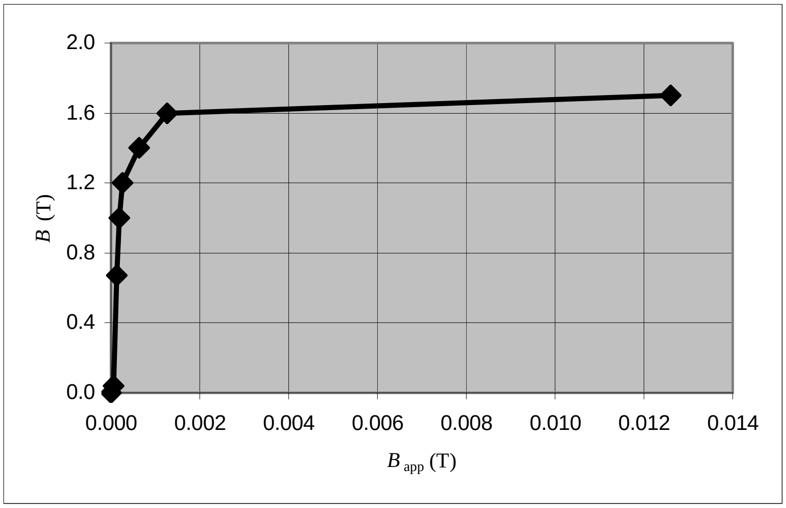
We can find the applied field B_{app} for a long solenoid using:

$$B_{app} = \mu_0 nI$$

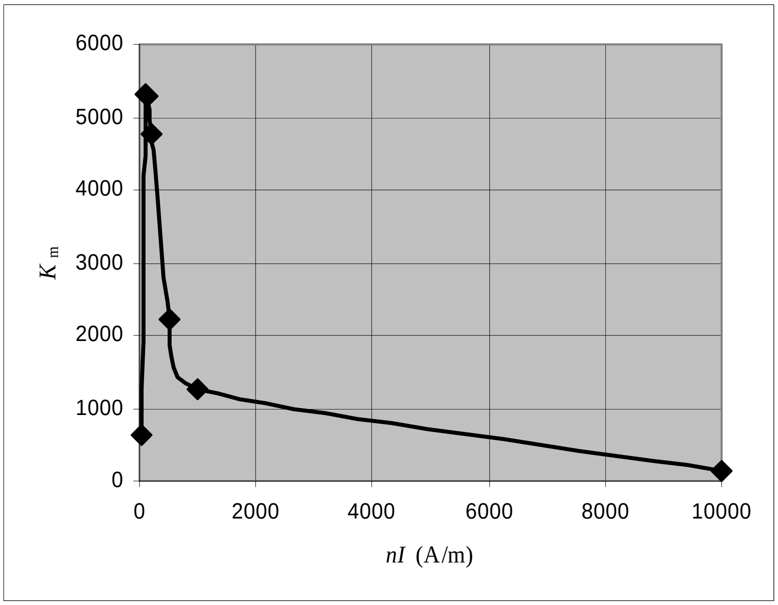
K_m can be found from B_{app} and B using:

$$K_m = \frac{B}{B_{app}}$$

The following graph was plotted using a spreadsheet program. The abscissa values for the graph were obtained by multiplying nI by μ_0. B initially rises rapidly, and then becomes nearly flat. This is characteristic of a ferromagnetic material.



The graph of K_m versus nI shown below was also plotted using a spreadsheet program. Note that K_m becomes quite large for small values of nI but then diminishes. A more revealing graph would be to plot $B/(nI)$, which would be quite large for small values of nI and then drop to nearly zero at nI = 10,000 A/m, corresponding to saturation of the magnetization.



80 ••

Picture the Problem We can use the definition of the magnetization of a sample to find M and the relationship between the Bohr magneton and the magnetic moment of the sample to find the number of electrons aligned in the sample. In part (*c*) we can express the magnetic moment of the disk in terms of the amperian surface current and solve for the latter.

(*a*) Express the magnetization of the sample in terms of its magnetic moment and volume:

$$M = \frac{\mu}{V} = \frac{\mu}{\pi r^2 d}$$

Substitute numerical values and evaluate M:

$$M = \frac{1.5\times10^{-2}\ \text{A}\cdot\text{m}^2}{\pi(1.4\ \text{cm})^2(0.3\ \text{cm})} = \boxed{8.12\times10^{3}\ \text{A/m}}$$

(*b*) Relate the magnetic moment of the sample to the Bohr magneton:

$$\mu = N\mu_\text{B}$$

Solve for and evaluate N:

$$N = \frac{\mu}{\mu_\text{B}} = \frac{1.5\times10^{-2}\ \text{A}\cdot\text{m}^2}{9.27\times10^{-24}\ \text{A}\cdot\text{m}^2} = \boxed{1.62\times10^{21}}$$

(*c*) Express the magnetic moment of the disk in terms of the amperian

$$\mu = AI$$

surface current:

Solve for I and substitute for μ to obtain:

$$I = \frac{\mu}{A} = \frac{MV}{A} = \frac{MAt}{A} = Mt$$

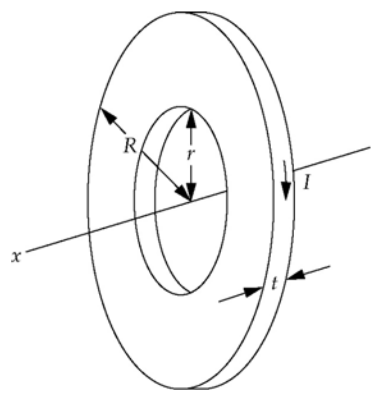
where t is the thickness of the disk.

Substitute numerical values and evaluate I:

$$I = \left(8.12\times10^3\ \text{A/m}\right)\left(0.3\,\text{cm}\right) = \boxed{24.4\,\text{A}}$$

81 ••

Picture the Problem We can imagine the cylinder with the hole cut out as the superposition of two uniform cylinders with radii r and R, respectively, and magnetization $-M$ and M, respectively. We can use the expression for B on the axis of a current loop to express the difference of the fields due to the two cylinders at a distance x from their common center. We'll denote each field by the subscript identifying the radius of the current loop.



From Problem 39 we have:

$$B_r = \frac{\mu_0}{4\pi}\frac{2\pi r^2 I}{\left(x^2+r^2\right)^{3/2}} = \frac{\mu_0 r^2 I}{2\left(x^2+r^2\right)^{3/2}}$$

and

$$B_R = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{\left(x^2+R^2\right)^{3/2}} = \frac{\mu_0 R^2 I}{2\left(x^2+R^2\right)^{3/2}}$$

The resultant field at x is the difference between B_R and B_r:

$$B_x(x) = B_R(x) - B_r(x) = \frac{\mu_0 R^2 I}{2\left(x^2+R^2\right)^{3/2}} - \frac{\mu_0 r^2 I}{2\left(x^2+r^2\right)^{3/2}} = \frac{\mu_0 I}{2}\left[\frac{R^2}{\left(x^2+R^2\right)^{3/2}} - \frac{r^2}{\left(x^2+r^2\right)^{3/2}}\right]$$

The resultant magnetization of the disks is $M = B/\mu_0$:

$$M(x) = \frac{I}{2}\left[\frac{R^2}{\left(x^2+R^2\right)^{3/2}} - \frac{r^2}{\left(x^2+r^2\right)^{3/2}}\right]$$

The magnetization current is the product of M and the thickness of the disks:

The magnetization is related to the amperian current:

$$M=\frac{dI_{\text{amperian}}}{d\ell}\Rightarrow I_{\text{amperian}}=\int_0^t Md\ell$$

Substitute for M to obtain:

$$I_{\text{amperian}}=\int_0^t \frac{I}{2}\left[\frac{R^2}{\left(x^2+R^2\right)^{3/2}}-\frac{r^2}{\left(x^2+r^2\right)^{3/2}}\right]d\ell=\boxed{\frac{It}{2}\left[\frac{R^2}{\left(x^2+R^2\right)^{3/2}}-\frac{r^2}{\left(x^2+r^2\right)^{3/2}}\right]}$$

Atomic Magnetic Moments

***82 ••**

Picture the Problem We can find the magnetic moment of a nickel atom μ from its relationship the saturation magnetization M_S using $M_S=n\mu$ where n is the number of molecules. n, in turn, can be found from Avogadro's number, the density of nickel, and its molar mass using $n=\frac{N_A\rho}{\text{M}}$.

Express the saturation magnetic field in terms of the number of molecules per unit volume and the magnetic moment of each molecule:

$$M_S=n\mu$$

or

$$\mu=\frac{M_S}{n}$$

Express the number of molecules per unit volume in terms of Avogadro's number N_A, the molecular mass M, and the density ρ:

$$n=\frac{N_A\rho}{\text{M}}$$

Substitute and simplify to obtain:

$$\mu=\frac{M_S}{\frac{N_A\rho}{\text{M}}}=\frac{\mu_0 M_S}{\frac{\mu_0 N_A\rho}{\text{M}}}=\frac{\mu_0 M_S M}{\mu_0 N_A\rho}$$

Substitute numerical values and evaluate μ:

$$\mu=\frac{(0.61\,\text{T})\left(58.7\times10^{-3}\ \text{kg/mol}\right)}{\left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(6.02\times10^{23}\ \text{atoms/mol}\right)\left(8.7\,\text{g/cm}^3\right)}=5.44\times10^{-24}\ \text{A}\cdot\text{m}^2$$

Express the value of 1 Bohr magneton:

$$\mu_B=9.27\times10^{-24}\ \text{A}\cdot\text{m}^2$$

Divide μ by μ_B to obtain:

$$\frac{\mu}{\mu_B}=\frac{5.44\times10^{-24}\ \text{A}\cdot\text{m}^2}{9.27\times10^{-24}\ \text{A}\cdot\text{m}^2}=0.587$$

or

$$\mu=\boxed{0.587\mu_B}$$

83 ••

Picture the Problem We can find the magnetic moment of a cobalt atom μ from its relationship to the saturation magnetization M_S using $M_S=n\mu$, where n is the number of molecules. n, in turn, can be found from Avogadro's number, the density of cobalt, and its molar mass using $n=\frac{N_A\rho}{\text{M}}$.

Express the saturation magnetic field in terms of the number of molecules per unit volume and the magnetic moment of each molecule:

$$M_S=n\mu$$

or

$$\mu=\frac{M_S}{n}$$

Express the number of molecules per unit volume in terms of Avogadro's number N_A, the molecular mass M, and the density ρ:

$$n=\frac{N_A\rho}{\text{M}}$$

Substitute and simplify to obtain:

$$\mu=\frac{M_S}{\frac{N_A\rho}{\text{M}}}=\frac{\mu_0M_S}{\frac{\mu_0N_A\rho}{\text{M}}}=\frac{\mu_0M_SM}{\mu_0N_A\rho}$$

Substitute numerical values and evaluate μ:

$$\mu=\frac{(1.79\ \text{T})(58.9\times10^{-3}\ \text{kg/mol})}{(4\pi\times10^{-7}\ \text{N/A}^2)(6.02\times10^{23}\ \text{atoms/mol})(8.9\ \text{g/cm}^3)}=1.57\times10^{-23}\ \text{A}\cdot\text{m}^2$$

Express the value of 1 Bohr magneton:

$$\mu_B=9.27\times10^{-24}\ \text{A}\cdot\text{m}^2$$

Divide μ by μ_B to obtain:

$$\frac{\mu}{\mu_B}=\frac{1.57\times10^{-23}\ \text{A}\cdot\text{m}^2}{9.27\times10^{-24}\ \text{A}\cdot\text{m}^2}=1.69$$

or

$$\mu=\boxed{1.69\mu_B}$$

Paramagnetism

84 •

Picture the Problem We can show that $\chi_m = \mu\mu_0 M_s/3kT$ by equating Curie's law and the equation that defines χ_m ($M = \chi_m \frac{B_{app}}{\mu_0}$) and solving for χ_m.

Express Curie's law:

$$M = \frac{1}{3}\frac{\mu B_{app}}{kT}M_S$$

where M_S is the saturation value.

Express the magnetization of the substance in terms of its magnetic susceptibility χ_m:

$$M = \chi_m \frac{B_{app}}{\mu_0}$$

Equate these expressions to obtain:

$$\chi_m \frac{B_{app}}{\mu_0} = \frac{1}{3}\frac{\mu B_{app}}{kT}M_S$$

or

$$\frac{\chi_m}{\mu_0} = \frac{1}{3}\frac{\mu}{kT}M_S$$

Solve for χ_m to obtain:

$$\chi_m = \boxed{\frac{\mu_0 \mu M_S}{3kT}}$$

85 ••

Picture the Problem We can use the assumption that $M = fM_S$ and Curie's law to solve these equations simultaneously for the fraction f of the molecules have their magnetic moments aligned with the external magnetic field.

(*a*) Assume that some fraction f of the molecules have their magnetic moments aligned with the external magnetic field and that the rest of the molecules are randomly oriented and so do not contribute to the magnetic field:

$$M = fM_S$$

From Curie's law we have:

$$M = \frac{1}{3}\frac{\mu B_{app}}{kT}M_S$$

Equate these expressions and solve for f to obtain:

$$fM_S = \frac{1}{3}\frac{\mu B_{app}}{kT}M_S$$

and

$$f = \boxed{\frac{\mu B}{3kT}}$$

because B given in the problem statement is the external magnetic field B_{app}.

(*b*) Substitute numerical values and evaluate f:

$$f = \frac{\left(9.27\times10^{-24}\text{ A}\cdot\text{m}^2\right)(1\text{T})}{3\left(1.381\times10^{-23}\text{ J/K}\right)(300\text{ K})} = \boxed{7.46\times10^{-4}}$$

***86 ••**

Picture the Problem In (*a*) we can express the saturation magnetic field in terms of the number of molecules per unit volume and the magnetic moment of each molecule and use $n = N_A\rho/\text{M}$ to express the number of molecules per unit volume in terms of Avogadro's number N_A, the molecular mass M, and the density ρ. We can use $\chi_m = \mu_0\mu M_S/3kT$ from Problem 84 to calculate χ_m.

(*a*) Express the saturation magnetic field in terms of the number of molecules per unit volume and the magnetic moment of each molecule:

$$M_S = n\mu_B$$

Express the number of molecules per unit volume in terms of Avogadro's number N_A, the molecular mass M, and the density ρ:

$$n = \frac{N_A\rho}{\text{M}}$$

Substitute to obtain:

$$M_S = \frac{N_A\rho}{\text{M}}\mu_B$$

Substitute numerical values and evaluate M_S:

$$M_S = \frac{\left(6.02\times10^{23}\text{ atoms/mol}\right)\left(2.7\times10^3\text{ kg/m}^3\right)\left(9.27\times10^{-24}\text{ A}\cdot\text{m}^2\right)}{27\text{ g/mol}}$$

$$= \boxed{5.58\times10^5\text{ A/m}}$$

and

$$B_S = \mu_0 M_S = \left(4\pi\times10^{-7}\text{ N/A}^2\right)\left(5.58\times10^5\text{ A/m}\right) = \boxed{0.701\text{T}}$$

(*b*) From Problem 84 we have:

$$\chi_m = \frac{\mu_0 \mu M_S}{3kT}$$

Substitute numerical values and evaluate χ_m:

$$\chi_m = \frac{(4\pi \times 10^{-7}\ \text{N/A}^2)(9.27\times 10^{-24}\ \text{A}\cdot\text{m}^2)(5.58\times 10^5\ \text{A/m})}{3(1.381\times 10^{-23}\ \text{J/K})(300\ \text{K})} = \boxed{5.23\times 10^{-4}}$$

(*c*) $\boxed{\text{In calculating } \chi_m \text{ in } (b) \text{ we neglected any diamagnetic effects.}}$

87 ••

Picture the Problem We can use Equation 27-17 to express B_{app} and Equation 27-21 to express B in terms of B_{app} and M.

Express B_{app} inside a tightly wound toroid:

$$B_{app} = \boxed{\frac{\mu_0 NI}{2\pi a}} \text{ for } R - r < a < R + r$$

The resultant field B in the ring is the sum of B_{app} and $\mu_0 M$:

$$B = B_{app} + \mu_0 M = \boxed{\frac{\mu_0 NI}{2\pi a} + \mu_0 M}$$

88 ••

Picture the Problem We can find the magnetization using $M = \chi_m B_{app}/\mu_0$ and the magnetic field using $B = B_{app}(1 + \chi_m)$.

(*a*) Using Equation 27-22, express the magnetization M in terms of χ_m and B_{app}:

$$M = \chi_m \frac{B_{app}}{\mu_0}$$

Express B_{app} inside a tightly wound toroid:

$$B_{app} = \frac{\mu_0 NI}{2\pi r_{mean}}$$

Substitute to obtain:

$$M = \chi_m \frac{\dfrac{\mu_0 NI}{2\pi r_{mean}}}{\mu_0} = \chi_m \frac{NI}{2\pi r_{mean}}$$

Substitute numerical values and evaluate M:

$$M = \frac{(4\times 10^{-3})(2000)(15\ \text{A})}{2\pi(0.2\ \text{m})} = \boxed{95.5\ \text{A/m}}$$

(*b*) Express B in terms of B_{app} and χ_m:

$$B = B_{app}(1 + \chi_m)$$

Substitute for B_{app} to obtain:

$$B = \frac{\mu_0 NI}{2\pi r_{mean}}(1 + \chi_m)$$

Substitute numerical values and evaluate B:

$$B = \frac{(4\pi \times 10^{-7}\ \text{N/A}^2)(2000)(15\,\text{A})}{2\pi(0.2\,\text{m})}(1 + 4\times 10^{-3}) = \boxed{30.1\,\text{mT}}$$

(*c*) Express the fractional increase in B produced by the liquid oxygen:

$$\frac{\Delta B}{B} = \frac{B - B_{app}}{B} = \frac{B_{app}(1+\chi_m) - B_{app}}{B} = \frac{\chi_m B_{app}}{B} = \frac{\chi_m}{1+\chi_m} = \frac{1}{\frac{1}{\chi_m} + 1}$$

Substitute numerical values and evaluate $\Delta B/B$:

$$\frac{\Delta B}{B} = \frac{1}{\frac{1}{4\times 10^{-3}} + 1} = 3.98\times 10^{-3} = \boxed{0.398\%}$$

89 ••

Picture the Problem We can use $B = B_{app}(1 + \chi_m)$ and $B_{app} = \frac{\mu_0 NI}{2\pi r_{mean}} = \mu_0 nI$ to find B within the substance and $M = \chi_m \frac{B_{app}}{\mu_0}$ to find the magnitude of the magnetization.

(*a*) Express the magnetic field B within the substance in terms of B_{app} and χ_m:

$$B = B_{app}(1 + \chi_m)$$

Express B_{app} inside the toroid:

$$B_{app} = \frac{\mu_0 NI}{2\pi r_{mean}} = \mu_0 nI$$

Substitute to obtain:

$$B = \mu_0 nI(1 + \chi_m)$$

Substitute numerical values and evaluate B:

$$B=\left(4\pi\times10^{-7}\text{ N/A}^2\right)\left(60\times10^2\text{ m}^{-1}\right)(4\text{ A})\left(1+2.9\times10^{-4}\right)=\boxed{30.2\text{ mT}}$$

(*b*) Express the magnetization *M* in terms of χ_m and B_{app}:

$$M=\chi_m\frac{B_{app}}{\mu_0}$$

Substitute for B_{app} to obtain:

$$M=\chi_m\frac{\mu_0 nI}{\mu_0}=\chi_m nI$$

Substitute numerical values and evaluate *M*:

$$M=\left(2.9\times10^{-4}\right)\left(6000\text{ m}^{-1}\right)(4\text{ A})=\boxed{6.96\text{ A/m}}$$

(*c*) If there were no paramagnetic core present:

$$B=B_{app}=\boxed{30.2\text{ mT}}$$

Ferromagnetism

***90 •**

Picture the Problem We can use $B=K_m B_{app}$ to find *B* and $M=(K_m-1)B_{app}/\mu_0$ to find *M*.

Express *B* in terms of *M* and K_m:

$$B=K_m B_{app}$$

Substitute numerical values and evaluate *B*:

$$B=(5500)\left(1.57\times10^{-4}\text{ T}\right)=\boxed{0.864\text{ T}}$$

Relate *M* to K_m and B_{app}:

$$M=(K_m-1)\frac{B_{app}}{\mu_0}\approx\frac{K_m B_{app}}{\mu_0}$$

Substitute numerical values and evaluate *M*:

$$M=\frac{(5500)\left(1.57\times10^{-4}\text{ T}\right)}{4\pi\times10^{-7}\text{ N/A}^2}=\boxed{6.87\times10^5\text{ A/m}}$$

91 ••

Picture the Problem We can relate the permeability μ of annealed iron to χ_m using $\mu=(1+\chi_m)\mu_0$, find χ_m using Equation 27-22 ($M=\chi_m\frac{B_{app}}{\mu_0}$), and use its definition ($K_m=1+\chi_m$) to evaluate K_m.

Express the permeability μ of annealed iron in terms of its magnetic susceptibility χ_m:

$$\mu = (1+\chi_m)\mu_0 \qquad (1)$$

Using Equation 27-22, express the magnetization M in terms of χ_m and B_{app}:

$$M = \chi_m \frac{B_{app}}{\mu_0}$$

Solve for and evaluate χ_m (see Table 27-2 for the product of μ_0 and M):

$$\chi_m = \frac{\mu_0 M}{B_{app}} = \frac{2.16\,\text{T}}{0.201\,\text{T}} = 10.75$$

Use its definition to express and evaluate the relative permeability K_m:

$$K_m = 1+\chi_m = 1+10.75 = \boxed{11.75}$$

Substitute numerical values in equation (1) and evaluate μ:

$$\mu = (1+10.75)\left(4\pi\times10^{-7}\ \text{N/A}^2\right) = \boxed{1.48\times10^{-5}\,\text{N/A}^2}$$

92 ••

Picture the Problem We can use the relationship between the magnetic field on the axis of a solenoid and the current in the solenoid to find the minimum current is needed in the solenoid to demagnetize the magnet.

Relate the magnetic field on the axis of a solenoid to the current in the solenoid:

$$B_x = \mu_0 n I$$

Solve for I to obtain:

$$I = \frac{B_x}{\mu_0 n}$$

Let $B_{app} = B_x$ to obtain:

$$I = \frac{B_{app}}{\mu_0 n}$$

Substitute numerical values and evaluate I:

$$I = \frac{5.53\times10^{-2}\ \text{T}}{\left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(\dfrac{600}{0.15\,\text{m}}\right)} = \boxed{11.0\,\text{A}}$$

93 ••

Picture the Problem We can use the equation describing the magnetic field on the axis of a solenoid, as a function of the current in the solenoid, to find B_{app}. We can then use $B = B_{app} + \mu_0 M$ to find M and $B = K_m B_{app}$ to evaluate K_m.

(*a*) Relate the magnetic field on the axis of a solenoid to the current in the solenoid:

$$B_x = \mu_0 nI$$

Substitute numerical values to obtain:

$$B_{app} = \left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(50\ \text{cm}^{-1}\right)\left(2\ \text{A}\right) = \boxed{12.6\ \text{mT}}$$

(*b*) Relate M to B and B_{app}:

$$B = B_{app} + \mu_0 M$$

Solve for and evaluate M:

$$M = \frac{B - B_{app}}{\mu_0} = \frac{1.72\ \text{T} - 12.6\ \text{mT}}{4\pi\times10^{-7}\ \text{N/A}^2} = \boxed{1.36\times10^6\ \text{A/m}}$$

(*c*) Express B in terms of K_m and B_{app}:

$$B = K_m B_{app}$$

Solve for and evaluate K_m:

$$K_m = \frac{B}{B_{app}} = \frac{1.72\ \text{T}}{12.6\ \text{mT}} = \boxed{137}$$

94 ••

Picture the Problem We can use the equation describing the magnetic field on the axis of a solenoid, as a function of the current in the solenoid, to find B_{app}. We can then use $B = B_{app} + \mu_0 M$ to find M and $B = K_m B_{app}$ to evaluate K_m.

(*a*) Relate the magnetic field on the axis of the solenoid to the current in the solenoid:

$$B_x = \mu_0 nI$$

Substitute numerical values and evaluate B_{app}:

$$B_{app} = \left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(50\ \text{cm}^{-1}\right)\left(0.2\ \text{A}\right) = \boxed{1.26\ \text{mT}}$$

(*b*) Relate M to B and B_{app}:

$$B = B_{app} + \mu_0 M$$

Solve for M:

$$M = \frac{B - B_{\text{app}}}{\mu_0}$$

Substitute numerical values and evaluate M:

$$M = \frac{1.58\,\text{T} - 1.26\,\text{mT}}{4\pi\times10^{-7}\ \text{N/A}^2} = \boxed{1.26\times10^{6}\ \text{A/m}}$$

(c) Express B in terms of K_{m} and B_{app}:

$$B = K_{\text{m}} B_{\text{app}}$$

Solve for and evaluate K_{m}:

$$K_{\text{m}} = \frac{B}{B_{\text{app}}} = \frac{1.58\,\text{T}}{1.26\,\text{mT}} = \boxed{1.25\times10^{3}}$$

95 ••

Picture the Problem The magnetic field in the core of a hollow solenoid is related to the current in its coils according to $B_x = B_{\text{app}} = \mu_0 nI$. The presence of the iron increases the magnetic field by a factor of K_{m}. In part (b), requiring that the magnetic field be unchanged when the iron core is removed will allow us to find the current that will produce the same field within the solenoid.

(a) Relate the magnetic field on the axis of the solenoid to the current in the solenoid:

$$B_x = B_{\text{app}} = \mu_0 nI$$

Express B in terms of B_{app}:

$$B = K_{\text{m}} B_{\text{app}}$$

Substitute to obtain:

$$B = K_{\text{m}} \mu_0 nI$$

Substitute numerical values and evaluate B:

$$B = 1200\left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(2000\,\text{m}^{-1}\right)\left(20\,\text{mA}\right) = \boxed{60.3\,\text{mT}}$$

(b) We require, that with the iron core removed, the magnetic field is unchanged:

$$B = K_{\text{m}} \mu_0 nI = \mu_0 nI_0$$

Solve for and evaluate I_0:

$$I_0 = K_{\text{m}} I = 1200\left(20\,\text{mA}\right) = \boxed{24.0\,\text{A}}$$

***96 ••**

Picture the Problem Because the wires carry equal currents in opposite directions, the magnetic field midway between them will be twice that due to either current alone and will be greater, by a factor of K_m, than it would be in the absence of the insulator. We can use Ampère's law to find the field, due to either current, at the midpoint of the plane of the wires and $d\vec{F} = Id\vec{\ell} \times \vec{B}$ to find the force per unit length on either wire.

(*a*) Relate the magnetic field in the insulator to the magnetic field in its absence:

$$B = K_m B_{app}$$

Apply Ampère's law to a closed circular path a distance *r* from a current-carrying wire to obtain:

$$\oint_C \vec{B} \cdot d\vec{\ell} = B_{app}(2\pi r) = \mu_0 I_C = \mu_0 I$$

Solve for B_{app} to obtain:

$$B_{app} = \frac{\mu_0 I}{2\pi r}$$

Because there are two current carrying wires, with their currents in opposite directions, the fields are additive and:

$$B = 2K_m \frac{\mu_0 I}{2\pi r} = \frac{K_m \mu_0 I}{\pi r}$$

Substitute numerical values and evaluate *B*:

$$B = \frac{120(4\pi \times 10^{-7}\ \text{N/A}^2)(40\ \text{A})}{\pi(0.02\ \text{m})} = \boxed{96.0\ \text{mT}}$$

(*b*) Express the force per unit length experienced by either wire due to the current in the other:

$$\frac{F}{\ell} = BI$$

Apply Ampère's law to obtain:

$$\oint_C \vec{B} \cdot d\vec{\ell} = B(2\pi r) = \mu_0 I_C = \mu_0 I$$

where *r* is the separation of the wires.

Solve for *B*:

$$B = \frac{\mu_0 I}{2\pi r} \text{ and } B_{app} = \frac{K_m \mu_0 I}{2\pi r}$$

Substitute to obtain:

$$\frac{F}{\ell} = \frac{K_m \mu_0 I^2}{2\pi r}$$

Substitute numerical values and evaluate $\frac{F}{\ell}$:

$$\frac{F}{\ell} = \frac{120\left(4\pi \times 10^{-7}\ \text{N/A}^2\right)(40\ \text{A})^2}{2\pi(0.04\ \text{m})} = \boxed{0.960\ \text{N/m}}$$

97 ••

Picture the Problem We can use $B = B_{\text{app}} + \mu_0 M$ and the expression for the magnetic field inside a tightly wound toroid to find the magnetization *M*. We can find K_m from its definition, $\mu = K_m \mu_0$ to find μ, and $K_m = 1 + \chi_m$ to find χ_m for the iron sample.

(*a*) Relate the magnetization to *B* and B_{app}:

$$B = B_{\text{app}} + \mu_0 M$$

Solve for *M*:

$$M = \frac{B - B_{\text{app}}}{\mu_0}$$

Express the magnetic field inside a tightly wound toroid:

$$B_{\text{app}} = \frac{\mu_0 NI}{2\pi r}$$

Substitute and simplify to obtain:

$$M = \frac{B - \dfrac{\mu_0 NI}{2\pi r}}{\mu_0} = \frac{B}{\mu_0} - \frac{NI}{2\pi r}$$

Substitute numerical values and evaluate *M*:

$$M = \frac{1.8\ \text{T}}{4\pi \times 10^{-7}\ \text{N/A}^2} - \frac{2000(10\ \text{A})}{2\pi(0.2\ \text{m})} = \boxed{1.42 \times 10^6\ \text{A/m}}$$

(*b*) Use its definition to express K_m:

$$K_m = \frac{B}{B_{\text{app}}} = \frac{B}{\dfrac{\mu_0 NI}{2\pi r}} = \frac{2\pi r B}{\mu_0 NI}$$

Substitute numerical values and evaluate K_m:

$$K_m = \frac{2\pi(0.2\ \text{m})(1.8\ \text{T})}{\left(4\pi \times 10^{-7}\ \text{N/A}^2\right)(2000)(10\ \text{A})} = \boxed{90.0}$$

Now that we know K_m we can find μ using:

$$\mu = K_m \mu_0 = 90\left(4\pi \times 10^{-7}\ \text{N/A}^2\right) = \boxed{1.13 \times 10^{-4}\ \text{T} \cdot \text{m/A}}$$

Relate χ_m to K_m:

$$K_m = 1 + \chi_m$$

Solve for and evaluate χ_m: $\chi_m = K_m - 1 = \boxed{89.0}$

98 ••

Picture the Problem We can substitute the expression for applied magnetic field ($B_{app} = \dfrac{\mu_0 NI}{2\pi r}$) in the defining equation for K_m ($B = K_m B_{app}$) to obtain an expression for the magnetic field B in the toroid.

Relate the magnetic field in the toroid to the relative permeability of its core:

$$B = K_m B_{app}$$

Express the applied magnetic field in the toroid in terms of the current in its winding:

$$B_{app} = \frac{\mu_0 NI}{2\pi r}$$

Substitute to obtain:

$$B = \frac{K_m \mu_0 NI}{2\pi r}$$

Express the number of turns N of wire in terms of the number of turns per unit length n:

$$N = 2\pi rn$$

Substitute to obtain:

$$B = K_m \mu_0 nI$$

Substitute numerical values and evaluate B:

$$B = 500\left(4\pi \times 10^{-7}\ \text{N/A}^2\right)\left(60\,\text{cm}^{-1}\right)\left(0.2\,\text{A}\right) = \boxed{0.754\,\text{T}}$$

99 ••

Picture the Problem We can use Ampère's law to obtain expressions for the magnetic field inside the wire, inside the ferromagnetic material, and in the region outside the insulating ferromagnetic material.

(*a*) Apply Ampère's law to a circle of radius r < 1 mm and concentric with the center of the wire:

$$\oint_C \vec{B} \cdot d\vec{\ell} = B(2\pi r) = \mu_0 I_C$$

Assuming that the current is distributed uniformly over the cross-sectional area of the wire (uniform current density), express I_C in terms

$$\frac{I_C}{\pi r^2} = \frac{I}{\pi R^2}$$

or

of the total current I: $I_C = \frac{r^2}{R^2} I$

Substitute to obtain: $B(2\pi r) = \frac{\mu_0 I r^2}{R^2}$

Solve for B: $B = \frac{\mu_0 I}{2\pi R^2} r$

Substitute numerical values and evaluate B:

$$B = \frac{(4\pi \times 10^{-7}\ \text{N/A}^2)(40\ \text{A})}{2\pi (1\ \text{mm})^2} r = \boxed{(8.00\ \text{T/m})r}$$

(b) Relate the magnetic field inside the ferromagnetic material to the magnetic field due to the current in the wire: $B = K_\text{m} B_\text{app}$

Apply Ampère's law to a circle of radius 1 mm < r < 4 mm and concentric with the center of the wire:

$$\oint_C \vec{B} \cdot d\vec{\ell} = B_\text{app}(2\pi r) = \mu_0 I_C = \mu_0 I$$

Solve for B_app: $B_\text{app} = \frac{\mu_0 I}{2\pi r}$

Substitute to obtain: $B = \frac{K_\text{m} \mu_0 I}{2\pi r}$

Substitute numerical values and evaluate B:

$$B = \frac{400(4\pi \times 10^{-7}\ \text{N/A}^2)(40\ \text{A})}{2\pi r} = \boxed{(3.20 \times 10^{-3}\ \text{T} \cdot \text{m})\frac{1}{r}}$$

(c) Apply Ampère's law to a circle of radius r > 4 mm and concentric with the center of the wire:

$$\oint_C \vec{B} \cdot d\vec{\ell} = B(2\pi r) = \mu_0 I_C = \mu_0 I$$

Solve for B: $B = \frac{\mu_0 I}{2\pi r}$

Substitute numerical values and evaluate B:

$$B = \frac{\left(4\pi \times 10^{-7}\ \text{N/A}^2\right)\left(40\ \text{A}\right)}{2\pi r} = \boxed{\left(8.00 \times 10^{-6}\ \text{T}\cdot\text{m}\right)\frac{1}{r}}$$

(*d*) Note that the field in the ferromagnetic region is that which would be produced in a nonmagnetic region by a current of $400I$ = 1600 A. The ampèrian current on the inside of the surface of the ferromagnetic material must therefore be (1600 – 40) A = 1560 A in the direction of I. On the outside surface there must then be an ampèrian current of 1560 A in the opposite direction.

General Problems

100 •

Picture the Problem Because point P is on the line connecting the straight segments of the conductor, these segments do not contribute to the magnetic field at P. Hence, we can use the expression for the magnetic field at the center of a current loop to find B_P.

Express the magnetic field at the center of a current loop:

$$B = \frac{\mu_0 I}{2R}$$

where R is the radius of the loop.

Express the magnetic field at the center of half a current loop:

$$B = \frac{1}{2}\frac{\mu_0 I}{2R} = \frac{\mu_0 I}{4R}$$

Substitute numerical values and evaluate B:

$$B = \frac{\left(4\pi \times 10^{-7}\ \text{N/A}^2\right)\left(15\ \text{A}\right)}{4\left(0.2\ \text{m}\right)} = \boxed{2.36 \times 10^{-5}\ \text{T}}$$

***101** •

Picture the Problem Let out of the page be the positive x direction. Because point P is on the line connecting the straight segments of the conductor, these segments do not contribute to the magnetic field at P. Hence, the resultant magnetic field at P will be the sum of the magnetic fields due to the current in the two semicircles, and we can use the expression for the magnetic field at the center of a current loop to find $\vec{B}_P$.

Express the resultant magnetic field at P:

$$\vec{B}_P = \vec{B}_1 + \vec{B}_2$$

Express the magnetic field at the center of a current loop:

$$B=\frac{\mu_0 I}{2R}$$

where R is the radius of the loop.

Express the magnetic field at the center of half a current loop:

$$B=\frac{1}{2}\frac{\mu_0 I}{2R}=\frac{\mu_0 I}{4R}$$

Express $\vec{B}_1$ and $\vec{B}_2$:

$$\vec{B}_1=\frac{\mu_0 I}{4R_1}\hat{i}$$

and

$$\vec{B}_2=-\frac{\mu_0 I}{4R_2}\hat{i}$$

Substitute to obtain:

$$\vec{B}_P=\frac{\mu_0 I}{4R_1}\hat{i}-\frac{\mu_0 I}{4R_2}\hat{i}=\boxed{\frac{\mu_0 I}{4}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\hat{i}}$$

102 ••

Picture the Problem We can express B as a function of N, I, and R using $B=\dfrac{\mu_0 NI}{2R}$ and eliminate R by relating ℓ to R through $\ell=2\pi RN$.

Express the magnetic field at the center of a coil of N turns and radius R:

$$B=\frac{\mu_0 NI}{2R}$$

Relate ℓ to the number of turns N:

$$\ell=2\pi RN$$

Solve for R to obtain:

$$R=\frac{\ell}{2\pi N}$$

Substitute to obtain:

$$B=\frac{\mu_0 NI}{2\dfrac{\ell}{2\pi N}}=\boxed{\frac{\mu_0 \pi N^2 I}{\ell}}$$

103 ••

Picture the Problem The magnetic field at P (which is out of the page) is the sum of the magnetic fields due to the three parts of the wire. Let the numerals 1, 2, and 3 denote the left-hand, center (short), and right-hand wires. We can then use the expression for B due to a straight wire segment to find each of these fields and their sum.

Express the resultant magnetic field at point P:

$$B_P = B_1 + B_2 + B_3$$

Because $B_1 = B_3$:

$$B_P = 2B_1 + B_2$$

Express the magnetic field due to a straight wire segment:

$$B = \frac{\mu_0}{4\pi}\frac{I}{R}\left(\sin\theta_1 + \sin\theta_2\right)$$

For wires 1 and 3 (the long wires), $\theta_1 = 90°$ and $\theta_2 = 45°$:

$$B_1 = \frac{\mu_0}{4\pi}\frac{I}{a}\left(\sin 90° + \sin 45°\right) = \frac{\mu_0}{4\pi}\frac{I}{a}\left(1 + \frac{1}{\sqrt{2}}\right)$$

For wire 2, $\theta_1 = \theta_2 = 45°$:

$$B_2 = \frac{\mu_0}{4\pi}\frac{I}{a}\left(\sin 45° + \sin 45°\right) = \frac{\mu_0}{4\pi}\frac{I}{a}\left(\frac{2}{\sqrt{2}}\right)$$

Substitute and simplify to obtain:

$$B_P = 2\left[\frac{\mu_0}{4\pi}\frac{I}{a}\left(1 + \frac{1}{\sqrt{2}}\right)\right] + \frac{\mu_0}{4\pi}\frac{I}{a}\left(\frac{2}{\sqrt{2}}\right) = \frac{\mu_0}{2\pi}\frac{I}{a}\left(1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{\mu_0}{2\pi}\frac{I}{a}\left(1 + \frac{2}{\sqrt{2}}\right) = \boxed{\frac{\mu_0}{2\pi}\frac{I}{a}\left(1 + \sqrt{2}\right)}$$

***104 ••**

Picture the Problem Depending on the direction of the wire, the magnetic field due to its current (provided this field is a large enough fraction of the earth's magnetic field) will either add to or subtract from the earth's field and moving the compass over the ground in the vicinity of the wire will indicate the direction of the current.

Apply Ampère's law to a circle of radius r and concentric with the center of the wire:

$$\oint_C \vec{\boldsymbol{B}} \cdot d\vec{\ell} = B_{\text{wire}}\left(2\pi r\right) = \mu_0 I_C = \mu_0 I$$

Solve for B to obtain:

$$B_{\text{wire}} = \frac{\mu_0 I}{2\pi r}$$

Substitute numerical values and evaluate B_{wire}:

$$B_{wire} = \frac{(4\pi \times 10^{-7}\ \text{N/A}^2)(50\ \text{A})}{2\pi(2\ \text{m})} = 0.0500\ \text{G}$$

Express the ratio of B_{wire} to B_{earth}:

$$\frac{B_{wire}}{B_{earth}} = \frac{0.05\ \text{G}}{0.7\ \text{G}} \approx 7\%$$

Thus, the field of the current-carrying wire should be detectable with a good compass.

If the cable runs east-west, its magnetic field is in the north-south direction and thus either adds to or subtracts from the earth's field, depending on the current direction and location of the compass. Moving the compass over the region one should be able to detect the change.

If the cable runs north-south, its magnetic field is perpendicular to that of the earth, and moving the compass about one should observe a change in the direction of the compass needle.

105 ••

Picture the Problem Let I_1 and I_2 represent the currents of 20 A and 5 A, $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$, and $\vec{F}_4$ the forces that act on the horizontal wire at the top of the loop, and the other wires following the current in a counterclockwise direction, and $\vec{B}_1$, $\vec{B}_2$, $\vec{B}_3$, and $\vec{B}_4$ the magnetic fields at these wires due to I_1. Let the positive x direction be to the right and the positive y direction be upward. Note that only the components into or out of the paper of $\vec{B}_1$, $\vec{B}_2$, $\vec{B}_3$, and $\vec{B}_4$ contribute to the forces $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$, and $\vec{F}_4$, respectively.

(*a*) Express the forces $\vec{F}_2$ and $\vec{F}_4$ in terms of I_2 and $\vec{B}_2$ and $\vec{B}_4$:

$$\vec{F}_2 = I_2\vec{\ell}_2 \times \vec{B}_2$$

and

$$\vec{F}_4 = I_2\vec{\ell}_4 \times \vec{B}_4$$

Express $\vec{B}_2$ and $\vec{B}_4$:

$$\vec{B}_2 = -\frac{\mu_0}{4\pi}\frac{2I_1}{R_1}\hat{k}$$

and

$$\vec{B}_4 = -\frac{\mu_0}{4\pi}\frac{2I_1}{R_4}\hat{k}$$

Substitute to obtain:

$$\vec{F}_2 = -I_2\ell_2\hat{j}\times\left(-\frac{\mu_0}{4\pi}\frac{2I_1}{R_1}\hat{k}\right) = \frac{\mu_0\ell_2 I_1 I_2}{2\pi R_2}\hat{i}$$

and

$$\vec{F}_4 = I_2\ell_4\hat{j}\times\left(-\frac{\mu_0}{4\pi}\frac{2I_1}{R_4}\hat{k}\right) = -\frac{\mu_0\ell_4 I_1 I_2}{2\pi R_4}\hat{i}$$

Substitute numerical values and evaluate $\vec{F}_2$ and $\vec{F}_4$:

$$\vec{F}_2 = \frac{(4\pi\times10^{-7}\ \text{N/A}^2)(0.1\,\text{m})(20\,\text{A})(5\,\text{A})}{2\pi(0.02\,\text{m})}\hat{i} = \boxed{(1.00\times10^{-4}\ \text{N})\hat{i}}$$

and

$$\vec{F}_4 = -\frac{(4\pi\times10^{-7}\ \text{N/A}^2)(0.1\,\text{m})(20\,\text{A})(5\,\text{A})}{2\pi(0.07\,\text{m})}\hat{i} = \boxed{(-0.286\times10^{-4}\ \text{N})\hat{i}}$$

(*b*) Express the net force acting on the coil:

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 \qquad (1)$$

Because the lengths of segments 1 and 3 are the same and the currents in these segments are in opposite directions:

$$\vec{F}_1 + \vec{F}_3 = 0$$

and

$$\vec{F}_{\text{net}} = \vec{F}_2 + \vec{F}_4$$

Substitute for $\vec{F}_2$ and $\vec{F}_4$ in equation (1) and simplify to obtain:

$$\vec{F}_{\text{net}} = (-0.250\times10^{-4}\ \text{N})\hat{j} + (1.00\times10^{-4}\ \text{N})\hat{i} + (0.250\times10^{-4}\ \text{N})\hat{j} + (-0.286\times10^{-4}\ \text{N})\hat{i} = \boxed{(0.714\times10^{-4}\ \text{N})\hat{i}}$$

106 ••

Picture the Problem Let out of the page be the positive *x* direction and the numerals 40 and 60 refer to the circular arcs whose radii are 40 cm and 60 cm. Because point *P* is on the line connecting the straight segments of the conductor, these segments do not contribute to the magnetic field at *P*. Hence the resultant magnetic field at *P* will be the sum of the magnetic fields due to the current in the two circular arcs and we can use the expression for the magnetic field at the center of a current loop to find $\vec{B}_P$.

Express the resultant magnetic field at P:

$$\vec{B}_P = \vec{B}_{40} + \vec{B}_{60}$$

Express the magnetic field at the center of a current loop:

$$B = \frac{\mu_0 I}{2R}$$

where R is the radius of the loop.

Express the magnetic field at the center of one-sixth of a current loop:

$$B = \frac{1}{6}\frac{\mu_0 I}{2R} = \frac{\mu_0 I}{12R}$$

Express $\vec{B}_{40}$ and $\vec{B}_{60}$:

$$\vec{B}_{40} = -\frac{\mu_0 I}{12R_{40}}\hat{\boldsymbol{i}}$$

and

$$\vec{B}_{60} = \frac{\mu_0 I}{12R_{60}}\hat{\boldsymbol{i}}$$

Substitute to obtain:

$$\vec{B}_P = -\frac{\mu_0 I}{12R_{40}}\hat{\boldsymbol{i}} + \frac{\mu_0 I}{12R_{60}}\hat{\boldsymbol{i}} = \frac{\mu_0 I}{12}\left(\frac{1}{R_{60}} - \frac{1}{R_{40}}\right)\hat{\boldsymbol{i}}$$

Substitute numerical values and evaluate $\vec{B}_P$:

$$\vec{B}_P = \frac{(4\pi\times10^{-7}\ \text{N/A}^2)(8\,\text{A})}{12}\left(\frac{1}{0.6\,\text{m}} - \frac{1}{0.4\,\text{m}}\right)\hat{\boldsymbol{i}} = \boxed{(-6.98\times10^{-7}\ \text{T})\hat{\boldsymbol{i}}}$$

107 ••

Picture the Problem Let the positive x direction be into the page and the numerals 20 and 40 refer to the circular arcs whose radii are 20 cm and 40 cm. Because point P is on the line connecting the straight segments of the conductor, these segments do not contribute to the magnetic field at P and the resultant field at P is the sum of the fields due to the two semicircular current loops.

Express the resultant magnetic field at P:

$$\vec{B}_P = \vec{B}_{20} + \vec{B}_{40}$$

Express the magnetic field at the center of a circular current loop:

$$B = \frac{\mu_0 I}{2R}$$

where R is the radius of the loop.

Express the magnetic field at the center of half a circular current loop:

$$B = \frac{1}{2}\frac{\mu_0 I}{2R} = \frac{\mu_0 I}{4R}$$

Express $\vec{B}_{20}$ and $\vec{B}_{40}$:

$$\vec{B}_{20} = \frac{\mu_0 I}{4R_{20}}\hat{i} \text{ and } \vec{B}_{40} = \frac{\mu_0 I}{4R_{40}}\hat{i}$$

Substitute to obtain:

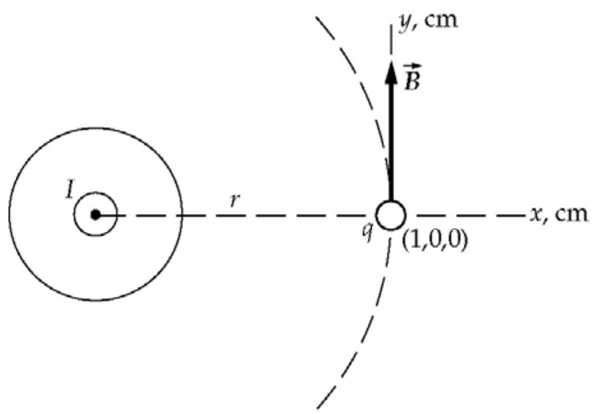
$$\vec{B}_P = \frac{\mu_0 I}{4R_{20}}\hat{i} + \frac{\mu_0 I}{4R_{40}}\hat{i} = \frac{\mu_0 I}{4}\left(\frac{1}{R_{20}} + \frac{1}{R_{40}}\right)\hat{i}$$

Substitute numerical values and evaluate B_P:

$$\vec{B}_P = \frac{(4\pi \times 10^{-7}\ \text{N/A}^2)(3\,\text{A})}{4}\left(\frac{1}{0.2\,\text{m}} + \frac{1}{0.4\,\text{m}}\right)\hat{i} = \boxed{(7.07\ \mu\text{T})\hat{i}}$$

***108 ••**

Picture the Problem Chose the coordinate system shown to the right. Then the current is in the positive z direction. Assume that the electron is at (1 cm, 0, 0). We can use $\vec{F} = q\vec{v} \times \vec{B}$ to relate the magnetic force on the electron to $\vec{v}$ and $\vec{B}$ and $\vec{B} = \frac{\mu_0}{4\pi}\frac{2I}{r}\hat{j}$ to express the magnetic field at the location of the electron. We'll need to express $\vec{v}$ for each of the three situations described in the problem in order to evaluate $\vec{F} = q\vec{v} \times \vec{B}$.



Express the magnetic force acting on the electron:

$$\vec{F} = q\vec{v} \times \vec{B}$$

Express the magnetic field due to the current in the wire as a function of distance from the wire:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2I}{r}\hat{j}$$

Substitute to obtain:

$$\vec{F} = q\vec{v} \times \frac{\mu_0}{4\pi}\frac{2I}{r}\hat{j} = \frac{2q\mu_0 I}{4\pi r}\left(\vec{v} \times \hat{j}\right) \quad (1)$$

(*a*) Express the velocity of the electron when it moves directly away from the wire:

$$\vec{v} = v\hat{i}$$

Substitute to obtain:

$$\vec{F} = \frac{2q\mu_0 I}{4\pi r}\left(v\hat{i} \times \hat{j}\right) = \frac{2q\mu_0 Iv}{4\pi r}\hat{k}$$

Substitute numerical values and evaluate $\vec{F}$:

$$\vec{F} = \frac{2\left(4\pi \times 10^{-7}\ \text{N/A}^2\right)\left(-1.6\times 10^{-19}\ \text{C}\right)\left(5\times 10^{6}\ \text{m/s}\right)\left(20\ \text{A}\right)\hat{k}}{4\pi\left(0.01\,\text{m}\right)}$$

$$= \boxed{\left(-3.20\times 10^{-16}\ \text{N}\right)\hat{k}}$$

(*b*) Express $\vec{v}$ when the electron is traveling parallel to the wire in the direction of the current:

$$\vec{v} = v\hat{k}$$

Substitute in equation (1) to obtain:

$$\vec{F} = \frac{2q\mu_0 I}{4\pi r}\left(v\hat{k} \times \hat{j}\right) = -\frac{2q\mu_0 Iv}{4\pi r}\hat{i}$$

Substitute numerical values and evaluate $\vec{F}$:

$$\vec{F} = -\frac{2\left(4\pi \times 10^{-7}\ \text{N/A}^2\right)\left(-1.6\times 10^{-19}\ \text{C}\right)\left(5\times 10^{6}\ \text{m/s}\right)\left(20\ \text{A}\right)\hat{i}}{4\pi\left(0.01\,\text{m}\right)} = \boxed{\left(3.20\times 10^{-16}\ \text{N}\right)\hat{i}}$$

(*c*) Express $\vec{v}$ when the electron is traveling perpendicular to the wire and tangent to a circle around the wire:

$$\vec{v} = v\hat{j}$$

Substitute in equation (1) to obtain:

$$\vec{F} = \frac{2q\mu_0 I}{4\pi r}\left(v\hat{j} \times \hat{j}\right) = \boxed{0}$$

109 ••

Picture the Problem We can apply Ampère's law to derive expressions for the magnetic field as a function of the distance from the center of the wire.

Apply Ampère's law to a closed circular path of radius $r < r_0$ to obtain:

$$B_{r<r_0}(2\pi r) = \mu_0 I_C$$

Because the current is uniformly distributed over the cross section of the wire:

$$\frac{I_C}{\pi r^2} = \frac{I}{\pi r_0^2} \Rightarrow I_C = \frac{r^2}{r_0^2} I$$

Substitute to obtain:

$$B_{r<r_0}(2\pi r) = \frac{\mu_0 r^2 I}{r_0^2}$$

Solve for $B_{r<r_0}$:

$$B_{r<r_0} = \frac{\mu_0 r I}{2\pi r_0^2} = \frac{\mu_0}{4\pi}\frac{2I}{r_0^2} r \qquad (1)$$

Apply Ampère's law to a closed circular path of radius $r > r_0$ to obtain:

$$B_{r>r_0}(2\pi r) = \mu_0 I_C = \mu_0 I$$

Solve for $B_{r>r_0}$:

$$B_{r>r_0} = \frac{\mu_0}{4\pi}\frac{2I}{r} \qquad (2)$$

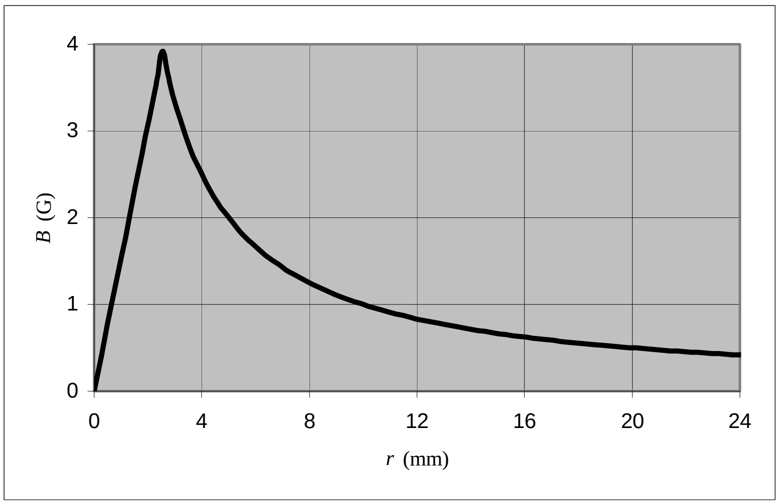
The spreadsheet program to calculate B as a function of r in the interval $0 \le r \le 10r_0$ is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
B1	1.00E–07	$\frac{\mu_0}{4\pi}$
B2	5	I
B3	1	I
A6	2.55E–03	r (m)
B6	0.00E+00	r (mm)
C6	10^4*$B$1*2*$B$2*A6/$B$3^2	$\frac{\mu_0}{4\pi}\frac{2I}{r_0^2} r$
C17	10^4*$B$1*2*$B$2*A6/A17	$\frac{\mu_0}{4\pi}\frac{2I}{r}$

	A	B	C
1	mu/4pi=	1.00E–07	N/A^2

2	I=	5	A
3	r_0=	2.55E–03	m
4			
5	r (m)	r (mm)	B (T)
6	0.00E+00	0.00E+00	0.00E+00
7	2.55E–04	2.55E–01	3.92E–01
8	5.10E–04	5.10E–01	7.84E–01
9	7.65E–04	7.65E–01	1.18E+00
10	1.02E–03	1.02E+00	1.57E+00
102	2.45E–02	2.45E+01	4.08E–01
103	2.47E–02	2.47E+01	4.04E–01
104	2.50E–02	2.50E+01	4.00E–01
105	2.52E–02	2.52E+01	3.96E–01
106	2.55E–02	2.55E+01	3.92E–01

A graph of B as a function of r follows.



110 ••

Picture the Problem We can use $\vec{\tau} = \vec{\mu} \times \vec{B}$ to find the torque exerted on the small coil (magnetic moment = $\vec{\mu}$) by the magnetic field $\vec{B}$ due to the current in the large coil.

Relate the torque exerted by the large coil on the small coil to the magnetic moment $\vec{\mu}$ of the small coil and the magnetic field $\vec{B}$ due to the current in the large coil:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

or, because the planes of the two coils are perpendicular,

$$\tau = \mu B$$

Express the magnetic moment of the small coil:

$$\mu = NIA$$

where I is the current in the coil, N is the number of turns in the coil, and A is the cross-sectional area of the coil.

Express the magnetic field at the center of the large coil:

$$B = \frac{N'\mu_0 I'}{2R}$$

where I' is the current in the large coil, N' is the number of turns in the coil, and R is its radius.

Substitute to obtain:

$$\tau = \frac{NN'II'A\mu_0}{2R}$$

Substitute numerical values and evaluate τ:

$$\tau = \frac{(50)(20)(4\,\text{A})(1\,\text{A})\pi(0.5\,\text{cm})^2(4\pi\times10^{-7}\ \text{N/A}^2)}{2(10\,\text{cm})} = \boxed{1.97\ \mu\text{N}\cdot\text{m}}$$

***111 ••**

Picture the Problem We can apply Newton's 2nd law for rotational motion to obtain the differential equation of motion of the bar magnet. While this equation is not linear, we can use a small-angle approximation to render it linear and obtain an expression for the square of the angular frequency that we can solve for κ when there is an external field and for the period T in the absence of an external field.

Apply $\sum\tau = I\alpha$ to the bar magnet when $B \neq 0$ to obtain the differential equation of motion for the magnet:

$$-\kappa\theta - \mu B\sin\theta = I\frac{d^2\theta}{dt^2}$$

where I is the moment of inertia of the magnet about an axis through its point of suspension.

For small displacements from equilibrium (θ << 1):

$$-\kappa\theta - \mu B\theta \approx I\frac{d^2\theta}{dt^2}$$

Rewrite the differential equation as:

$$I\frac{d^2\theta}{dt^2} + (\kappa + \mu B)\theta = 0$$

or

$$\frac{d^2\theta}{dt^2} + \left(\frac{\kappa + \mu B}{I}\right)\theta = 0$$

Because the coefficient of the linear term is the square of the angular frequency, we have:

$$\omega^2 = \frac{\kappa + \mu B}{I} \qquad (1)$$

Express the moment of inertia (see Table 9-1) of the bar magnet about an axis through its center:

$$I = \tfrac{1}{12}mL^2$$

Substitute to obtain:

$$\omega^2 = \frac{\kappa + \mu B}{\tfrac{1}{12}mL^2}$$

Solve for κ to obtain:

$$\kappa = \tfrac{1}{12}mL^2\omega^2 - \mu B = \tfrac{1}{12}mL^2\left(\frac{4\pi^2}{T^2}\right) - \mu B$$

$$= \frac{\pi^2 mL^2}{3T^2} - \mu B$$

Substitute numerical values and evaluate κ:

$$\kappa = \frac{\pi^2(0.8\,\text{kg})(0.16\,\text{m})^2}{3(0.5\,\text{s})^2} - (0.12\,\text{A}\cdot\text{m}^2)(0.2\,\text{T}) = \boxed{0.246\,\text{N}\cdot\text{m/rad}}$$

Substitute $B = 0$ and $\omega = 2\pi/T$ in equation (1) to obtain:

$$\frac{4\pi^2}{T^2} = \frac{\kappa}{I}$$

Solve for T:

$$T = 2\pi\sqrt{\frac{I}{\kappa}} = 2\pi\sqrt{\frac{mL^2}{12\kappa}} = \pi L\sqrt{\frac{m}{3\kappa}}$$

Substitute numerical values and evaluate T:

$$T = \pi(0.16\,\text{m})\sqrt{\frac{0.8\,\text{kg}}{3(0.246\,\text{N}\cdot\text{m/rad})}}$$

$$= \boxed{0.523\,\text{s}}$$

112 ••

Picture the Problem We can apply Newton's 2nd law for rotational motion to obtain the differential equation of motion of the bar magnet. While this equation is not linear, we can use a small-angle approximation to render it linear and obtain an expression for the square of the angular frequency that we can solve for the frequency f of the motion.

Apply $\sum\tau = I\alpha$ to the bar magnet to obtain the differential equation of motion for the magnet:

$$-\mu B\sin\theta = I\frac{d^2\theta}{dt^2}$$

where I is the moment of inertia of the magnet about an axis through its point of suspension.

For small displacements from equilibrium ($\theta << 1$):

$$-\mu B\theta \approx I\frac{d^2\theta}{dt^2}$$

Rewrite the differential equation as:

$$I\frac{d^2\theta}{dt^2} + \mu B\theta = 0$$

or

$$\frac{d^2\theta}{dt^2} + \frac{\mu B}{I}\theta = 0$$

Because the coefficient of the linear term is the square of the angular frequency, we have:

$$\omega^2 = \frac{\mu B}{I}$$

Solve for ω to obtain:

$$\omega = \boxed{\sqrt{\frac{\mu B}{I}}}$$

113 ••

Picture the Problem We can use the potential energy of the displaced bar magnet to find the force acting on it to return it to its equilibrium position. While this restoring force is not, in general, linear, we can use a binomial expansion to show that for displacements that are small compared to the radius of the coil, the restoring force is linear and, hence, the motion of the bar magnet is simple harmonic motion. We can then apply Newton's 2nd law to obtain the differential equation of motion of the bar magnet and use the coefficient of the linear term to express the period of the motion.

Express the potential energy of the displaced bar magnet:

$$U = -\mu B$$

Express the magnetic field on the axis of the current loop:

$$B = \frac{\mu_0}{4\pi}\frac{2\pi NR^2 I}{\left(x^2 + R^2\right)^{3/2}}$$

where I is the current in the loop and R is its radius.

Substitute to obtain:

$$U = -\frac{\mu_0}{4\pi}\frac{2\pi\mu NR^2 I}{\left(x^2 + R^2\right)^{3/2}}$$

Differentiate U with respect to x to find the restoring force acting on the bar magnet:

$$F_x = -\frac{dU}{dx} = \tfrac{1}{2}\mu_0\mu NR^2I\frac{d}{dx}\left[\left(x^2+R^2\right)^{-3/2}\right] = -\frac{3\mu_0\mu NR^2I}{2}\left[\frac{1}{\left(x^2+R^2\right)^{5/2}}\right]x$$

Factor R^2 from the radical to obtain:

$$F_x = -\frac{3\mu_0\mu NR^2I}{2R^5}\left[\frac{1}{\left(1+\frac{x^2}{R^2}\right)^{5/2}}\right]x = -\frac{3\mu_0\mu NI}{2R^3}\left(1+\frac{x^2}{R^2}\right)^{-5/2}x$$

Expand the radical factor to obtain:

$$\left(1+\frac{x^2}{R^2}\right)^{-5/2} = 1-\frac{5}{2}\frac{x^2}{R^2} + \text{higher order terms}$$

For $x << R$:

$$\left(1+\frac{x^2}{R^2}\right)^{-5/2} \approx 1$$

Substitute in F_x to obtain:

$$F_x = -\frac{3\mu_0\mu NI}{2R^3}x$$

Thus, we've shown that the bar magnet experiences a linear restoring force and, hence, its motion will be simple harmonic motion.

Apply $\sum\vec{F} = m\vec{a}$ to the bar magnet to obtain:

$$-\frac{3\mu_0\mu NI}{2R^3}x = m\frac{d^2x}{dt^2}$$

or

$$\frac{d^2x}{dt^2} + \frac{3\mu_0\mu NI}{2mR^3}x = 0$$

Because the coefficient of the linear term is the square of the angular frequency we have:

$$\omega^2 = \frac{4\pi^2}{T^2} = \frac{3\mu_0\mu NI}{2mR^3}$$

Solve for T to obtain:

$$T = 2\pi\sqrt{\frac{2mR^3}{3\mu_0\mu NI}}$$

Substitute numerical values and evaluate T:

$$T = 2\pi\sqrt{\frac{2(0.1\,\text{kg})(0.1\,\text{m})^3}{3(4\pi\times10^{-7}\ \text{N/A}^2)(0.04\,\text{A}\cdot\text{m}^2)(100)(5\,\text{A})}} = \boxed{10.2\,\text{s}}$$

114 ••

Picture the Problem We can apply Newton's 2nd law for rotational motion to obtain the differential equation of motion of the bar magnet. While this equation is not linear, we can use a small-angle approximation to render it linear and obtain an expression for the square of the angular frequency that we can solve for the frequency f of the motion.

Apply $\sum\tau = I\alpha$ to the bar magnet to obtain the differential equation of motion for the magnet:

$$-\mu B\sin\theta = I\frac{d^2\theta}{dt^2}$$

where I is the moment of inertia of the magnet about an axis through its point of suspension.

For small displacements from equilibrium ($\theta << 1$):

$$-\mu B\theta \approx I\frac{d^2\theta}{dt^2}$$

Rewrite the differential equation as:

$$I\frac{d^2\theta}{dt^2} + \mu B\theta = 0$$

or

$$\frac{d^2\theta}{dt^2} + \frac{\mu B}{I}\theta = 0$$

Because the coefficient of the linear term is the square of the angular frequency, we have:

$$\omega^2 = 4\pi^2 f^2 = \frac{\mu B}{I}$$

where f is the frequency of oscillation.

Solve for f to obtain:

$$f = \frac{1}{2\pi}\sqrt{\frac{\mu B}{I}}$$

or, because $\mu = 2.2N\mu_B$ where N is the number of iron atoms in the bar magnet,

$$f = \frac{1}{2\pi}\sqrt{\frac{2.2N\mu_B B}{I}}$$

From Table 9-1 we have:

$$I = \tfrac{1}{12}mL^2 = \tfrac{1}{12}\rho VL^2$$

Express the number of iron atoms in terms of Avogadro's number and the atomic weight of iron M:

$$\frac{N}{N_A} = \frac{m}{M} = \frac{\rho V}{M}$$

and

$$N = \frac{N_A \rho V}{M}$$

Substitute for I and N and simplify to obtain:

$$f = \frac{1}{2\pi}\sqrt{\frac{2.2N_A\rho V\mu_B B}{\frac{1}{12}\rho VL^2 M}} = \frac{1}{\pi L}\sqrt{\frac{6.6N_A\mu_B B}{M}}$$

Substitute numerical values and evaluate f:

$$f = \frac{1}{\pi(0.08\,\text{m})}\sqrt{\frac{6.6(6.02\times10^{23}\,/\,\text{mol})(9.27\times10^{-24}\,\text{A}\cdot\text{m}^2)(0.5\times10^{-4}\,\text{T})}{55.85\,\text{g/mol}}} = \boxed{0.723\,\text{Hz}}$$

115 ••

Picture the Problem We can solve the equation for the frequency f of the compass needle given in Problem 112 for magnetic dipole moment of the needle. In Parts (*b*) and (*c*) we can use their definitions to find the magnetization M and the amperian current I_{amperian}.

(*a*) In Problem 112 it is established that the frequency of the compass needle is:

$$f = \frac{1}{2\pi}\sqrt{\frac{\mu B}{I}}$$

where I is the moment of inertia of the needle.

Solve for μ to obtain:

$$\mu = \frac{4\pi^2 f^2 I}{B}$$

Express the moment of inertia of the needle:

$$I = \tfrac{1}{12}mL^2 = \tfrac{1}{12}\rho VL^2 = \tfrac{1}{12}\rho\pi r^2 L^3$$

Substitute to obtain:

$$\mu = \frac{\pi^3 f^2 \rho r^2 L^3}{3B}$$

Substitute numerical values and evaluate μ:

$$\mu = \frac{\pi^3\left(1.4\,\text{s}^{-1}\right)^2\left(7.96\times10^3\ \text{kg/m}^3\right)\left(0.85\times10^{-3}\ \text{m}\right)^2\left(0.03\,\text{m}\right)^3}{3\left(0.6\times10^{-4}\ \text{T}\right)} = \boxed{5.24\times10^{-2}\ \text{A}\cdot\text{m}^2}$$

(*b*) Use its definition to express the magnetization *M*:

$$M = \frac{\mu}{V}$$

Substitute to obtain:

$$M = \frac{\mu}{V} = \frac{\pi^3 f^2 \rho r^2 L^3}{3BV} = \frac{\pi^2 f^2 \rho L^2}{3B}$$

Substitute numerical values and evaluate *M*:

$$M = \frac{\pi^2\left(1.4\,\text{s}^{-1}\right)^2\left(7.96\times10^3\ \text{kg/m}^3\right)\left(0.03\,\text{m}\right)^2}{3\left(0.6\times10^{-4}\ \text{T}\right)} = \boxed{7.70\times10^5\ \text{A/m}}$$

(*c*) Express and evaluate the amperian current on the surface of the needle:

$$I_{\text{amperian}} = ML = \left(7.70\times10^5\ \text{A/m}\right)\left(0.03\,\text{m}\right) = \boxed{2.31\times10^4\ \text{A}}$$

***116 ••**

Picture the Problem We can use the definition of angular momentum and Equation 27-27, together with the definition of the magnetization *M* of the iron bar, to derive an expression for the rotational angular velocity of the bar just after it has been demagnetized.

Assuming its angular momentum to be conserved, use the definition of *L* to express the angular momentum of the iron bar just after it has been demagnetized:

$$L = I\omega$$

Solve for the angular velocity ω:

$$\omega = \frac{L}{I}$$

Assuming that Equation 27-27 holds yields:

$$L = \frac{2m}{q}\mu = \frac{2m_e}{e}MV = \frac{2m_e}{e}M\pi r^2\ell$$

where *r* is the radius of the bar and ℓ its length.

Modeling the bar as a cylinder, express its moment of inertia with respect to its axis:

$$I = \tfrac{1}{2}mr^2 = \tfrac{1}{2}\rho V r^2 = \tfrac{1}{2}\rho\pi r^4\ell$$

Substitute to obtain:

$$\omega = \frac{\frac{2m_e}{e}M\pi r^2\ell}{\frac{1}{2}\rho\pi r^4\ell} = \frac{4m_e M}{e\rho r^2}$$

Substitute numerical values (see Table 13-1 for the density of iron) and evaluate ω:

$$\omega = \frac{4\left(9.11\times10^{-31}\text{ kg}\right)\left(1.72\times10^{6}\text{ A/m}\right)}{\left(1.6\times10^{-19}\text{ C}\right)\left(7.96\times10^{3}\text{ kg/m}^3\right)\left(0.01\text{ m}\right)^2} = \boxed{4.92\times10^{-5}\text{ rad/s}}$$

117 ••

Picture the Problem The dipole moment of the bar is given by $\mu = 2.219N\mu_B$, where N is the number of atoms in the bar. We can express N in terms of Avogadro's number, the density of iron, the volume of the bar, and the atomic weight of iron. We can use the definition of torque to find the torque that must be supplied to hold the iron bar perpendicular to the given magnetic field.

(*a*) Express the magnetic dipole moment of the magnetized iron bar:

$$\mu = 2.219N\mu_B$$

where N is the number of iron atoms in the bar.

Express the number of iron atoms in terms of Avogadro's number and the atomic weight of iron M:

$$\frac{N}{N_A} = \frac{m}{\text{M}} = \frac{\rho V}{\text{M}}$$

and

$$N = \frac{N_A\rho V}{\text{M}}$$

Substitute to obtain:

$$\mu = \frac{2.219N_A\rho V\mu_B}{\text{M}} = \frac{2.219N_A\rho\ell A\mu_B}{\text{M}}$$

Substitute numerical values and evaluate μ:

$$\mu = \frac{2.219\left(6.02\times10^{23}\text{ mol}^{-1}\right)\left(7.96\times10^{3}\text{ kg/m}^3\right)\left(0.2\text{ m}\right)}{55.85\times10^{-3}\text{ kg/mol}} \times\left(2\times10^{-4}\text{ m}^2\right)\left(9.27\times10^{-24}\text{ A}\cdot\text{m}^2\right) = \boxed{70.6\text{ A}\cdot\text{m}^2}$$

(*b*) Express the torque required to hold the iron bar perpendicular to the magnetic field:

$$\tau = \mu B \sin\theta = \mu B \sin 90° = \mu B$$

Substitute numerical values and evaluate τ:

$$\tau = \left(70.6\,\mathrm{A\cdot m^2}\right)\left(0.25\,\mathrm{T}\right) = \boxed{17.7\,\mathrm{N\cdot m}}$$

***118 ••**

Picture the Problem Note that B_e and B_c are perpendicular to each other and that the resultant magnetic field is at an angle θ with north. We can use trigonometry to relate B_c and B_e and express B_c in terms of the geometry of the coil and the current flowing in it.

Express B_c in terms of B_e:

$$B_c = B_e \tan\theta$$

where θ is the angle of the resultant field from north.

Express the field B_c due to the current in the coil:

$$B_c = \frac{N\mu_0 I}{2R}$$

where N is the number of turns.

Substitute to obtain:

$$\frac{N\mu_0 I}{2R} = B_e \tan\theta$$

Solve for I:

$$I = \boxed{\frac{2RB_e}{\mu_0 N}\tan\theta}$$

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Picture the Problem Let the positive x direction be out of the page. We can use the expressions for the magnetic fields due to an infinite straight line and a circular loop to express the net magnetic field at the center of the circular loop. We can set this net field to zero and solve for r.

Express the net magnetic field at the center of circular loop:

$$\vec{\boldsymbol{B}} = \vec{\boldsymbol{B}}_{\text{loop}} + \vec{\boldsymbol{B}}_{\text{line}}$$

Letting R represent the radius of the loop, express $\vec{\boldsymbol{B}}_{\text{loop}}$:

$$\vec{\boldsymbol{B}}_{\text{loop}} = -\frac{\mu_0 I}{2R}\hat{\boldsymbol{i}}$$

Express the magnetic field due to the current in the infinite straight line:

$$\vec{\boldsymbol{B}}_{\text{line}} = \frac{\mu_0 I}{2\pi r}\hat{\boldsymbol{i}}$$

Substitute to obtain:

$$\vec{B} = -\frac{\mu_0 I}{2R}\hat{i} + \frac{\mu_0 I}{2\pi r}\hat{i} = \left(-\frac{\mu_0 I}{2R} + \frac{\mu_0 I}{2\pi r}\right)\hat{i}$$

If $\vec{B} = 0$, then:

$$-\frac{\mu_0 I}{2R} + \frac{\mu_0 I}{2\pi r} = 0$$

or

$$-\frac{1}{R} + \frac{1}{\pi r} = 0$$

Solve for r:

$$r = \frac{R}{\pi}$$

Substitute numerical values and evaluate r:

$$r = \frac{10\,\text{cm}}{\pi} = \boxed{3.18\,\text{cm}}$$

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Picture the Problem Note that only the current in the section of wire of length 2*a* contributes to the field at *P*. Hence, we can use the expression for *B* due to a straight wire segment to find the magnetic field at *P*. In Part (*b*) we can use our result from (*a*), together with the value for θ when the polygon has *N* sides, to obtain an expression for *B* at the center of a polygon of *N* sides.

Express the magnetic field at *P* due to a straight wire segment:

$$B_P = \frac{\mu_0}{4\pi}\frac{I}{R}\left(\sin\theta_1 + \sin\theta_2\right)$$

Because $\theta_1 = \theta_2 = \theta$:

$$B_P = \frac{\mu_0}{4\pi}\frac{I}{R}\left(2\sin\theta\right) = \left(\frac{\mu_0}{2\pi}\frac{I}{R}\right)\sin\theta$$

Refer to the figure to obtain:

$$\sin\theta = \frac{a}{\sqrt{a^2 + R^2}}$$

Substitute to obtain:

$$B_P = \boxed{\frac{\mu_0 a I}{2\pi R\sqrt{a^2 + R^2}}}$$

(*b*) Express θ for an *N*-sided polygon:

$$\theta = \frac{\pi}{N}$$

Because each side of the polygon contributes to *B* an amount equal to that obtained in (*a*):

$$B = \boxed{\left(\frac{N\mu_0 I}{2\pi R}\right)\sin\left(\frac{\pi}{N}\right)}$$

As $N \to \infty$:

$$\sin\left(\frac{\pi}{N}\right) \to \frac{\pi}{N}$$

and

$$B \to \left(\frac{N\mu_0 I}{2\pi R}\right)\left(\frac{\pi}{N}\right) = \boxed{\frac{\mu_0 I}{2R}}$$, the expression for the magnetic field at the center of a current-carrying circular loop.

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Picture the Problem We can use Ampère's law to derive expressions for $B(r)$ for $r < R$, $r = R$, and $r > R$ that we can evaluate for the given distances from the center of the cylindrical conductor.

Apply Ampère's law to a closed circular path a distance $r < R$ from the center of the cylindrical conductor to obtain:

$$\oint_C \vec{B} \cdot d\vec{\ell} = B(r)(2\pi r) = \mu_0 I_C = \mu_0 I(r)$$

Solve for $B(r)$ to obtain:

$$B(r) = \frac{\mu_0 I(r)}{2\pi r}$$

Substitute for $I(r)$:

$$B(r) = \frac{\mu_0 (50\ \text{A/m}) r}{2\pi\, r} = \frac{\mu_0 (50\ \text{A/m})}{2\pi}$$

(*a*) and (*b*) Noting that B is independent of r, substitute numerical values and evaluate $B(5\ \text{cm})$ and $B(10\ \text{cm})$:

$$B(5\,\text{cm}) = B(10\,\text{cm}) = \frac{(4\pi \times 10^{-7}\ \text{N/A}^2)(50\ \text{A/m})}{2\pi} = \boxed{10.0\ \mu\text{T}}$$

(*c*) Apply Ampère's law to a closed circular path a distance $r > R$ from the center of the cylindrical conductor to obtain:

$$\oint_C \vec{B} \cdot d\vec{\ell} = B(r)(2\pi r) = \mu_0 I_C = \mu_0 I(R)$$

Solve for $B(r)$:

$$B(r) = \frac{\mu_0 I(R)}{2\pi r}$$

Substitute numerical values and evaluate $B(20\ \text{cm})$:

$$B(20\,\text{cm}) = \frac{\left(4\pi\times10^{-7}\ \text{N/A}^2\right)(50\ \text{A/m})(0.1\,\text{m})}{2\pi(0.2\,\text{m})} = \boxed{5.00\ \mu\text{T}}$$

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Picture the Problem The field $\vec{B}$ due to the 10-A current is in the *yz* plane. The net force on the wires of the square along the *y* direction cancel and do not contribute to a net torque or force. We can use $\vec{\tau} = \vec{l}\times\vec{F}$, $\vec{F} = I\vec{\ell}\times\vec{B}$, and the expression for the magnetic field due to an infinite straight wire to express the torque acting on each of the wires and hence, the net torque acting on the loop.

(*a*) Express the torque on the loop:

$$\vec{\tau} = \vec{l}\times\vec{F}$$

where $\vec{l}$ is the lever arm.

Express the magnetic force on a current element:

$$\vec{F} = I\vec{\ell}\times\vec{B}$$

Express the magnetic field at the wire at y = 10 cm:

$$\vec{B}_{y=10} = \frac{\mu_0}{4\pi}\frac{2I}{R}\frac{1}{\sqrt{2}}\left(-\hat{j}-\hat{k}\right)$$

where

$$R = \sqrt{(0.1\,\text{m})^2 + (0.1\,\text{m})^2} = 0.141\,\text{m}\,.$$

Substitute numerical values and evaluate $\vec{B}_{y=10}$:

$$\vec{B}_{y=10} = \frac{4\pi\times10^{-7}\ \text{N/A}^2}{4\pi\sqrt{2}}\frac{2(10\ \text{A})}{0.141\,\text{m}}\left(-\hat{j}-\hat{k}\right) = \left(1.00\times10^{-5}\ \text{T}\right)\left(-\hat{j}-\hat{k}\right)$$

Proceed similarly to obtain:

$$\vec{B}_{y=-10} = \left(1.42\times10^{-5}\ \text{T}\right)\left(-\hat{j}+\hat{k}\right)$$

Evaluate $\vec{F}_{y=10}$:

$$\begin{aligned}\vec{F}_{y=10} &= I\vec{\ell}\times\vec{B}_{y=10} = (5\ \text{A})(0.2\,\text{m})\hat{i}\times\left(1.00\times10^{-5}\ \text{T}\right)\left(-\hat{j}-\hat{k}\right)\\ &= \left(1.00\times10^{-5}\ \text{N}\right)\left[\hat{i}\times\left(-\hat{j}-\hat{k}\right)\right] = \left(1.00\times10^{-5}\ \text{N}\right)\left(-\hat{k}+\hat{j}\right)\end{aligned}$$

Evaluate $\vec{F}_{y=-10}$:

$$\begin{aligned}\vec{F}_{y=-10} &= (5\ \text{A})(-0.2\,\text{m})\hat{i}\times\left(1.00\times10^{-5}\ \text{T}\right)\left(-\hat{j}+\hat{k}\right)\\ &= \left(-1.00\times10^{-5}\ \text{N}\right)\left[\hat{i}\times\left(-\hat{j}+\hat{k}\right)\right] = \left(1.00\times10^{-5}\ \text{N}\right)\left(\hat{k}+\hat{j}\right)\end{aligned}$$

Express and evaluate the net force acting on the loop:

$$\vec{F} = \vec{F}_{y=10} + \vec{F}_{y=-10} = \left(1.00\times10^{-5}\ \text{N}\right)\left(-\hat{k}+\hat{j}\right) + \left(1.00\times10^{-5}\ \text{N}\right)\left(\hat{k}+\hat{j}\right)$$
$$= \left(2.00\times10^{-5}\ \text{N}\right)\hat{j}$$

Express and evaluate the torque about the x axis acting on the loop:

$$\tau = (0.1\,\text{m})\left(2.00\times10^{-5}\ \text{N}\right) = \boxed{2.00\times10^{-6}\ \text{N}\cdot\text{m}}$$

(*b*) The net force acting on the loop is the sum of the forces acting on the four sides (see the next to last step in (*a*)):

$$\vec{F} = \vec{F}_{y=10} + \vec{F}_{y=-10} = \boxed{\left(2.00\times10^{-5}\ \text{N}\right)\hat{j}}$$

123 ••

Picture the Problem The force acting on the lower wire is given by $F_{\text{lower wire}} = I\ell B$, where I is the current in the lower wire, ℓ is the length of the wire on the balance, and B is the magnetic field at the location of the lower wire due to the current in the upper wire. We can apply Ampère's law to find B at the location of the wire on the pan of the balance.

The force experienced by the lower wire is given by:

$$F_{\text{lower wire}} = I\ell B$$

Apply Ampère's law to a closed circular path of radius r centered on the upper wire to obtain:

$$B(2\pi r) = \mu_0 I_C = \mu_0 I$$

Solve for B to obtain:

$$B = \frac{\mu_0 I}{2\pi r}$$

Substitute for B in the expression for the force on the lower wire:

$$F_{\text{lower wire}} = I\ell\left(\frac{\mu_0 I}{2\pi r}\right) = \frac{\mu_0 \ell I^2}{2\pi r}$$

Solve for I to obtain:

$$I = \sqrt{\frac{2\pi r F_{\text{lower wire}}}{\mu_0 \ell}}$$

Noting that the force on the lower wire is the increase in the reading of the balance, substitute numerical values and evaluate I:

$$I = \sqrt{\frac{2\pi(2\,\text{cm})\left(5\times10^{-6}\ \text{kg}\right)}{\left(4\pi\times10^{-7}\ \text{N/A}^2\right)(10\,\text{cm})}} = \boxed{2.24\,\text{A}}$$

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Picture the Problem We can use a proportion to relate minimum current detectible using this balance to its sensitivity and to the current and change in balance reading from Problem 123.

The minimum current $I_{\min}$ detectible is to the sensitivity of the balance as the current in Problem 123 is to the change in the balance reading in Problem 123:

$$\frac{I_{\min}}{0.1\,\text{mg}} = \frac{2.24\,\text{A}}{5.0\,\text{mg}}$$

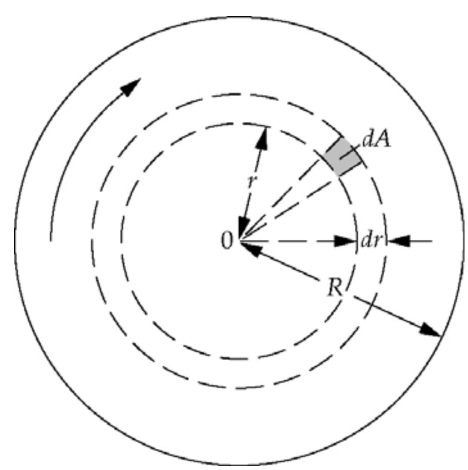
Solve for and evaluate $I_{\min}$:

$$I_{\min} = (0.1\,\text{mg})\left(\frac{2.24\,\text{A}}{5.0\,\text{mg}}\right) = \boxed{44.8\,\text{mA}}$$

The "standard" current balance can be made very sensitive by increasing the length (i.e., moment arm) of the wire balance, which one cannot do with this kind; however, this is compensated somewhat by the high sensitivity of the electronic balance.

***125** •••

Picture the Problem The diagram shows the rotating disk and the circular strip of radius *r* and width *dr* with charge *dq*. We can use the definition of surface charge density to express *dq* in terms of *r* and *dr* and the definition of current to show that $dI = \omega\sigma r\,dr$. We can then use this current and expression for the magnetic field on the axis of a current loop to obtain the results called for in (*b*) and (*c*).



(*a*) Express the total charge *dq* that passes a given point on the circular strip once each period:

$$dq = \sigma dA = 2\pi\sigma r dr$$

Using its definition, express the current in the element of width *dr*:

$$dI = \frac{dq}{dt} = \frac{2\pi\sigma r dr}{\frac{2\pi}{\omega}} = \boxed{\omega\sigma r dr}$$

(*c*) Express the magnetic field dB_x at a distance x along the axis of the disk due to the current loop of radius r and width dr:

$$dB_x = \frac{\mu_0}{4\pi}\frac{2\pi r^2 dI}{\left(x^2+r^2\right)^{3/2}} = \frac{\mu_0 \omega\sigma r^3}{2\left(x^2+r^2\right)^{3/2}}dr$$

Integrate from $r = 0$ to $r = R$ to obtain:

$$B_x = \frac{\mu_0\omega\sigma}{2}\int_0^R \frac{r^3}{\left(x^2+r^2\right)^{3/2}}dr = \boxed{\frac{\mu_0\omega\sigma}{2}\left(\frac{R^2+2x^2}{\sqrt{R^2+x^2}}-2x\right)}$$

(*b*) Evaluate B_x for $x = 0$:

$$B_x(0) = \frac{\mu_0\omega\sigma}{2}\left(\frac{R^2}{\sqrt{R^2}}\right) = \boxed{\tfrac{1}{2}\mu_0\sigma\omega R}$$

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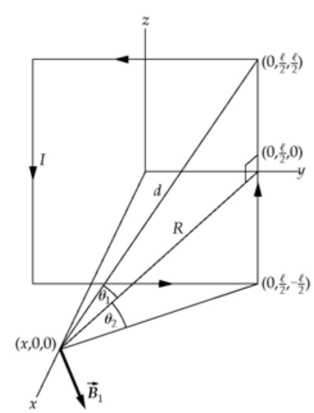
Picture the Problem From the symmetry of the system it is evident that the fields due to each segment of length ℓ are the same in magnitude. We can express the magnetic field at $(x,0,0)$ due to one side (segment) of the square, find its component in the x direction, and then multiply by four to find the resultant field.



Express B due to a straight wire segment:

$$B = \frac{\mu_0}{4\pi}\frac{I}{R}\left(\sin\theta_1+\sin\theta_2\right)$$

where R is the perpendicular distance from the wire segment to the field point.

Use $\theta_1 = \theta_2$ and $R = \sqrt{x^2+\ell^2/4}$ to express B due to one side at $(x,0,0)$:

$$B_1(x,0,0) = \frac{\mu_0}{4\pi}\frac{I}{\sqrt{x^2+\frac{\ell^2}{4}}}\left(2\sin\theta_1\right) = \frac{\mu_0}{2\pi}\frac{I}{\sqrt{x^2+\frac{\ell^2}{4}}}\left(\sin\theta_1\right)$$

Referring to the diagram, express $\sin\theta_1$:

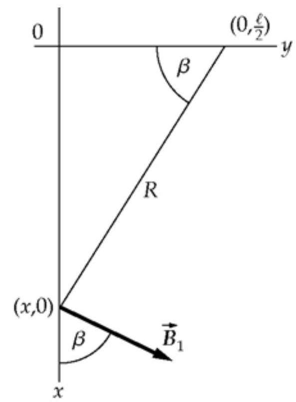
$$\sin\theta_1 = \frac{\frac{\ell}{2}}{d} = \frac{\frac{\ell}{2}}{\sqrt{x^2 + \frac{\ell^2}{2}}}$$

Substitute to obtain:

$$B_1(x,0,0) = \frac{\mu_0}{2\pi}\frac{I}{\sqrt{x^2+\frac{\ell^2}{4}}}\frac{\frac{\ell}{2}}{\sqrt{x^2+\frac{\ell^2}{2}}}$$

$$= \frac{\mu_0 I}{4\pi\sqrt{x^2+\frac{\ell^2}{4}}}\frac{\ell}{\sqrt{x^2+\frac{\ell^2}{2}}}$$

By symmetry, the sum of the *y* and *z* components of the fields due to the four segments must vanish, whereas the *x* components will add. The diagram to the right is a view of the *xy* plane showing the relationship between $\vec{B}_1$ and the angle β it makes with the *x* axis.



Express B_{1x}:

$$B_{1x} = B_1\cos\beta$$

Substitute and simplify to obtain:

$$B_{1x} = \frac{\mu_0 I}{4\pi\sqrt{x^2+\frac{\ell^2}{4}}}\frac{\ell}{\sqrt{x^2+\frac{\ell^2}{2}}}\frac{\frac{\ell}{2}}{\sqrt{x^2+\frac{\ell^2}{4}}}$$

$$= \frac{\mu_0 I\ell^2}{8\pi\left(x^2+\frac{\ell^2}{4}\right)\sqrt{x^2+\frac{\ell^2}{2}}}$$

The resultant magnetic field is the sum of the fields due to the 4 wire segments (sides of the square):

$$\vec{B} = 4B_{1x}\hat{i}$$

$$= \boxed{\frac{\mu_0 I\ell^2}{2\pi\left(x^2+\frac{\ell^2}{4}\right)\sqrt{x^2+\frac{\ell^2}{2}}}\hat{i}}$$

Factor x^2 from the two factors in the denominator to obtain:

$$\vec{B} = \frac{\mu_0 I\ell^2}{2\pi x^2\left(1+\frac{\ell^2}{4x^2}\right)\sqrt{x^2\left(1+\frac{\ell^2}{2x^2}\right)}}\hat{i}$$

$$= \frac{\mu_0 I\ell^2}{2\pi x^3\left(1+\frac{\ell^2}{4x^2}\right)\sqrt{\left(1+\frac{\ell^2}{2x^2}\right)}}\hat{i}$$

For $x >> \ell$:

$$\vec{B} \approx \frac{\mu_0 I\ell^2}{2\pi x^3}\hat{i} = \boxed{\frac{\mu_0 \mu}{2\pi x^3}\hat{i}}$$

where $\mu = I\ell^2$.