Chapter 28
Magnetic Induction

Conceptual Problems

***1 •**

Determine the Concept We know that the magnetic flux (in this case the magnetic field because the area of the conducting loop is constant and its orientation is fixed) must be changing so the only issues are whether the field is increasing or decreasing and in which direction. Because the direction of the magnetic field associated with the clockwise current is into the page, the changing field that is responsible for it must be either increasing out of the page (not included in the list of possible answers) or a decreasing field directed into the page. $\boxed{(d)\text{ is correct.}}$

2 •

Determine the Concept Note that when R is constant, $\boldsymbol{B}$ in the loop to the right points out of the paper.

(a) If R increases, I decreases and so does B. By Lenz's law, the induced current is counterclockwise.

(b) If R decreases, the induced current is clockwise.

3 ••

Determine the Concept If the counterclockwise current in loop A increases, so does the magnetic flux through B. To oppose this increase in flux, the induced current in loop B will by clockwise. If the counterclockwise current in loop A decreases, so does the magnetic flux through B. To oppose this decrease in flux, the induced current in loop B will be counterclockwise. We can use $\vec{F} = I\vec{\ell} \times \vec{B}$ to determine the direction of the forces on each loop and, hence, whether they will attract or repel each other.

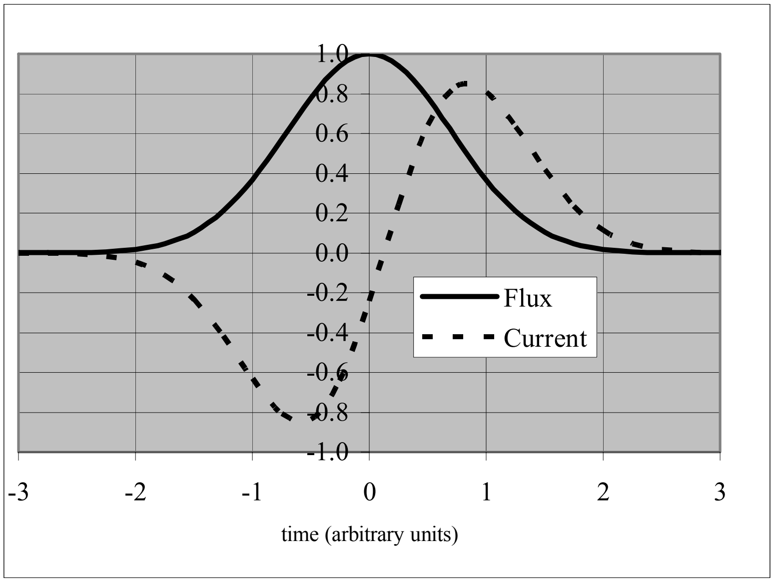
(a) If the current in B is clockwise the loops repel one another.

(b) If the current in B is counterclockwise the loops attract one another.

4 ••

Determine the Concept We know that, as the magnet moves to the right, the flux through the loop first increases until the magnet is half way through the loop and then decreases. Because the flux first increases and then decreases, the current will change directions, having its maximum values when the flux is changing most rapidly.

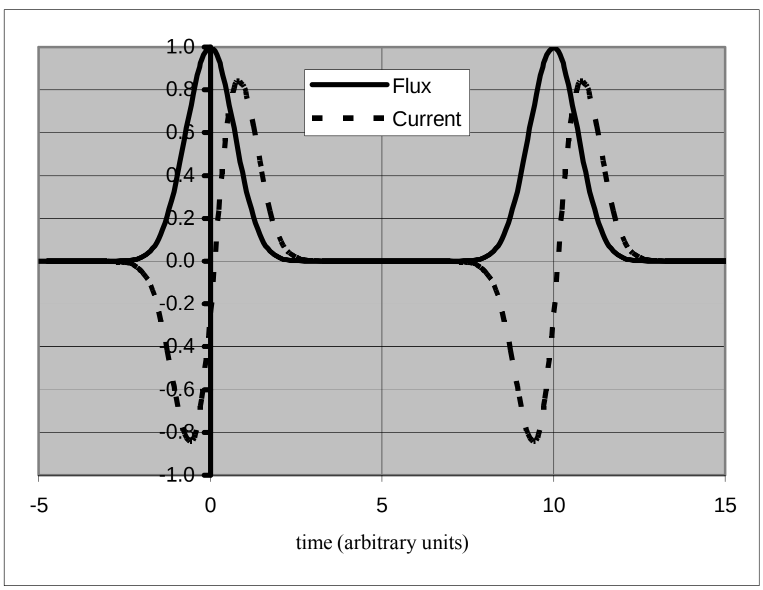
(a) and (b) The following graph shows the flux and the induced current as a function of time as the bar magnet passes through the coil. When the center of the magnet passes through the plane of the coil $d\phi_m/dt = 0$ and the current is zero.



5 ••

Determine the Concept Because the magnet moves with simple harmonic motion, the flux and the induced current will vary sinusoidally. The current will be a maximum wherever the flux is changing most rapidly and will be zero wherever the flux is momentarily constant.

(*a*), (*b*) The following graph shows the flux, ϕ_m, and the induced current (proportional to $-d\phi_m/dt$) in the loop as a function of time.



***6** •

Determine the Concept The magnetic energy stored in an inductor is given by $U_m = \frac{1}{2}LI^2$. Doubling I quadruples U_m. $\boxed{(c) \text{ is correct.}}$

7 •

Determine the Concept The protection is needed because if the current is suddenly interrupted, the resulting emf generated across the inductor due to the large flux change can blow out the inductor. The diode allows the current to flow (in a loop) even when the switch is opened.

8 •

Determine the Concept The inductance of a coil depends on the product $n^2\ell$, where n is the number of turns per unit length and ℓ is the length of the coil. If n increases by a factor of 3, ℓ will decrease by the same factor, because the inductors are made from the same length of wire. Hence, the inductance increases by a factor of $(3)^2(1/3) = \boxed{3}$.

9 •

(*a*) False. The induced emf in a circuit is proportional to *the rate of change of* the magnetic flux through the circuit.

(*b*) True.

(*c*) True.

(*d*) False. The inductance of a solenoid is determined by its length, cross-sectional area, number of turns per unit length, and the permeability of the matter in its core.

(*e*) True.

***10** •

Determine the Concept In the configuration shown in (*a*), energy is dissipated by eddy currents from the emf induced by the pendulum movement. In the configuration shown in (*b*), the slits inhibit the eddy currents and the braking effect is greatly reduced.

11 •

Determine the Concept The time varying magnetic field of the magnet sets up eddy currents in the metal tube. The eddy currents establish a magnetic field with a magnetic moment opposite to that of the moving magnet; thus the magnet is slowed down. If the tube is made of a nonconducting material, there are no eddy currents.

12 ••

Determine the Concept When the current is turned on, the increasing magnetic field in the coil induces a large emf in the ring. As described by Lenz's law, the direction of the current resulting from this induced emf is in such a direction that its magnetic field opposes the changing flux in the coil, i.e., the current induced in the ring will be in such a direction that the magnetic field in the coil will repel it. The demonstration will not work if a slot is cut in the ring, because the emf will not be able to induce a current in the ring.

Estimation and Approximation

***13 ••**

Picture the Problem We can use Faraday's law to relate the induced emf to the angular velocity with which the students turn the jump rope.

(*a*) It seems unlikely that the students could turn the "jump rope" wire faster than 5 revolutions per second. This corresponds to a maximum angular velocity of:

$$\omega = 5\frac{\text{rev}}{\text{s}}\times\frac{2\pi\,\text{rad}}{\text{rev}} = \boxed{31.4\,\text{rad/s.}}$$

(*b*) The magnetic flux ϕ_m through the rotating circular loop of wire varies sinusoidally with time according to:

$$\phi_m = BA\sin\omega t$$

and

$$\frac{d\phi_m}{dt} = BA\omega\cos\omega t$$

Because the average value of the cosine function, over one revolution, is ½, the average rate at which the flux changes through the circular loop is:

$$\left.\frac{d\phi_m}{dt}\right|_{\text{av}} = \tfrac{1}{2}BA\omega = \tfrac{1}{2}\pi r^2 B\omega$$

From Faraday's law, the magnitude of the induced emf in the loop is:

$$\mathcal{E} = \frac{d\phi_m}{dt} = \tfrac{1}{2}\pi r^2 B\omega$$

Substitute numerical values and evaluate $\mathcal{E}$:

$$\mathcal{E} = \tfrac{1}{2}\pi\left(\frac{1.5\,\text{m}}{2}\right)^2\left(0.7\,\text{G}\times\frac{1\,\text{T}}{10^4\,\text{G}}\right)(31.4\,\text{rad/s}) = \boxed{1.94\,\text{mV}}$$

(*c*) $\boxed{\text{No. To generate an emf of 1 V, the students would have to rotate the jump rope about 500 times faster.}}$

(*d*) $\boxed{\text{The use of multiple strands of lighter wire (so that the composite wire could be rotated at the same angular speed) looped several times around would increase the induced emf.}}$

14 •

Picture the Problem We can compare the energy density stored in the earth's electric field to that of the earth's magnetic field by finding their ratio.

The energy density in an electric field E is given by:

$$u_e = \frac{1}{2} \in_0 E^2$$

The energy density in a magnetic field B is given by:

$$u_m = \frac{B^2}{2\mu_0}$$

Express the ratio of u_m to u_e to obtain:

$$\frac{u_m}{u_e} = \frac{\frac{B^2}{2\mu_0}}{\frac{1}{2} \in_0 E^2} = \frac{B^2}{\mu_0 \in_0 E^2}$$

Substitute numerical values and evaluate u_m / u_e:

$$\frac{u_m}{u_e} == \frac{\left(5\times10^{-5}\text{ T}\right)^2}{\left(4\pi\times10^{-7}\text{ N/A}^2\right)\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)\left(100\text{ V/m}\right)^2} = 2.25\times10^4$$

or

$$u_m = \boxed{\left(2.25\times10^4\right)u_e}$$

15 ••

Picture the Problem We can apply Faraday's law to estimate the maximum emf induced by the lightning strike in the antenna.

Use Faraday's law to express the magnitude of the induced emf in antenna:

$$\mathcal{E} = \frac{d\phi_m}{dt} = \frac{d}{dt}[BA]$$

where A is the area of the antenna.

Because the lightning strike has such a short duration:

$$\mathcal{E} \approx \frac{BA}{\Delta t}$$

The magnetic field induced in the loop is given by:

$$B = \frac{\mu_0}{4\pi}\frac{2I}{r} = \frac{\mu_0 I}{2\pi r}$$

where r is the distance from the antenna to the lightning strike.

Substitute for B to obtain:

$$\mathcal{E} = \frac{\mu_0 IA}{2\pi r\Delta t}$$

Substitute numerical values and evaluate $\mathcal{E}$:

$$\mathcal{E} = \frac{\left(4\pi\times10^{-7}\text{ N/A}^2\right)\left(\frac{30\text{ C}}{1\,\mu\text{s}}\right)\left(0.1\text{ m}^2\right)}{2\pi(300\text{ m})(1\,\mu\text{s})}$$

$$= \boxed{2.00\text{ kV}}$$

Magnetic Flux

16 •

Picture the Problem Because the surface is a plane with area A and $\vec{B}$ is constant in magnitude and direction over the surface and makes an angle θ with the unit normal vector, we can use $\phi_m = BA\cos\theta$ to find the magnetic flux through the coil.

Substitute for B and A to obtain:

$$\phi_m = \left(2000\,\text{G}\cdot\frac{1\,\text{T}}{10^4\,\text{G}}\right)\left(5\times10^{-2}\,\text{m}\right)^2\cos\theta$$
$$= \left(5.00\times10^{-4}\,\text{Wb}\right)\cos\theta$$

(*a*) For $\theta = 0°$:

$$\phi_m = \left(5.00\times10^{-4}\,\text{Wb}\right)\cos 0° = 5.00\times10^{-4}\,\text{Wb} = \boxed{0.500\,\text{mWb}}$$

(*b*) For $\theta = 30°$:

$$\phi_m = \left(5.00\times10^{-4}\,\text{Wb}\right)\cos 30° = 4.33\times10^{-4}\,\text{Wb} = \boxed{0.433\,\text{mWb}}$$

(*c*) For $\theta = 60°$:

$$\phi_m = \left(5.00\times10^{-4}\,\text{Wb}\right)\cos 60° = 2.50\times10^{-4}\,\text{Wb} = \boxed{0.250\,\text{mWb}}$$

(*d*) For $\theta = 90°$:

$$\phi_m = \left(5.00\times10^{-4}\,\text{Wb}\right)\cos 90° = \boxed{0}$$

***17** •

Picture the Problem Because the coil defines a plane with area A and $\vec{B}$ is constant in magnitude and direction over the surface and makes an angle θ with the unit normal vector, we can use $\phi_m = NBA\cos\theta$ to find the magnetic flux through the coil.

Substitute for N, B, and A to obtain:

$$\phi_m = NB\pi r^2\cos\theta = 25\left(0.7\,\text{G}\cdot\frac{1\,\text{T}}{10^4\,\text{G}}\right)\pi\left(5\times10^{-2}\,\text{m}\right)^2\cos\theta$$
$$= \left(1.37\times10^{-5}\,\text{Wb}\right)\cos\theta$$

(*a*) When the plane of the coil is horizontal, $\theta = 90°$:

$$\phi_m = (1.37\times10^{-5}\ \text{Wb})\cos 90° = \boxed{0}$$

(*b*) When the plane of the coil is vertical with its axis pointing north, $\theta = 0°$:

$$\phi_m = (1.37\times10^{-5}\ \text{Wb})\cos 0° = \boxed{1.37\times10^{-5}\ \text{Wb}}$$

(*c*) When the plane of the coil is vertical with its axis pointing east, $\theta = 90°$:

$$\phi_m = (1.37\times10^{-5}\ \text{Wb})\cos 90° = \boxed{0}$$

(*d*) When the plane of the coil is vertical with its axis making an angle of 30° with north, $\theta = 30°$:

$$\phi_m = (1.37\times10^{-5}\ \text{Wb})\cos 30° = \boxed{1.19\times10^{-5}\ \text{Wb}}$$

18 •

Picture the Problem Because the square coil defines a plane with area A and $\vec{\boldsymbol{B}}$ is constant in magnitude and direction over the surface and makes an angle θ with the unit normal vector, we can use $\phi_m = NBA\cos\theta$ to find the magnetic flux through the coil.

Substitute for N, B, and A to obtain:

$$\begin{aligned}\phi_m &= NBA\cos\theta \\ &= 14(1.2\ \text{T})(5\times10^{-2}\ \text{m})^2\cos\theta \\ &= (42.0\ \text{mWb})\cos\theta\end{aligned}$$

(*a*) For $\theta = 0°$:

$$\phi_m = (42.0\ \text{mWb})\cos 0° = \boxed{42.0\ \text{mWb}}$$

(*b*) For $\theta = 60°$:

$$\phi_m = (42.0\ \text{mWb})\cos 60° = \boxed{21.0\ \text{mWb}}$$

19 •

Picture the Problem Noting that the flux through the base must also penetrate the spherical surface, we can apply its definition to express ϕ_m.

Apply the definition of magnetic flux to obtain:

$$\phi_m = AB = \boxed{\pi R^2 B}$$

20 ••

Picture the Problem We can use $\phi_m = NBA\cos\theta$ to express the magnetic flux through the solenoid and $B = \mu_0 nI$ to relate the magnetic field in the solenoid to the current in its coils.

Express the magnetic flux through a coil with N turns:

$$\phi_m = NBA\cos\theta$$

Express the magnetic field inside a long solenoid:

$$B = \mu_0 nI$$

where n is the number of turns per unit length.

Substitute to obtain:

$$\phi_m = N\mu_0 nIA\cos\theta$$

or, because $n = N/L$ and $\theta = 0°$,

$$\phi_m = \frac{N^2\mu_0 IA}{L} = \frac{N^2\mu_0 I\pi r^2}{L}$$

Substitute numerical values and evaluate ϕ_m:

$$\phi_m = \frac{(400)^2(4\pi\times10^{-7}\ \text{N/A}^2)(3\ \text{A})\pi(0.01\ \text{m})^2}{0.25\ \text{m}} = \boxed{7.58\times10^{-4}\ \text{Wb}}$$

21 ••

Picture the Problem We can use $\phi_m = NBA\cos\theta$ to express the magnetic flux through the solenoid and $B = \mu_0 nI$ to relate the magnetic field in the solenoid to the current in its coils.

Express the magnetic flux through a coil with N turns:

$$\phi_m = NBA\cos\theta$$

Express the magnetic field inside a long solenoid:

$$B = \mu_0 nI$$

where n is the number of turns per unit length.

Substitute to obtain:

$$\phi_m = N\mu_0 nIA\cos\theta$$

or, because $n = N/L$ and $\theta = 0°$,

$$\phi_m = \frac{N^2\mu_0 IA}{L} = \frac{N^2\mu_0 I\pi r^2}{L}$$

Substitute numerical values and evaluate ϕ_m:

$$\phi_m = \frac{(800)^2\left(4\pi\times10^{-7}\ \text{N/A}^2\right)(2\ \text{A})\pi(0.02\ \text{m})^2}{0.3\ \text{m}} = \boxed{6.74\times10^{-3}\ \text{Wb}}$$

22 ••

Picture the Problem We can apply the definitions of magnet flux and of the dot product to find the flux for the given unit vectors.

Apply the definition of magnetic flux to the coil to obtain:

$$\phi_m = N\int_S \vec{B}\cdot\hat{n}\,dA$$

Because $\vec{B}$ is constant:

$$\phi_m = N\vec{B}\cdot\hat{n}\int_S dA = N\left(\vec{B}\cdot\hat{n}\right)A = N\left(\vec{B}\cdot\hat{n}\right)\pi r^2$$

Evaluate $\vec{B}$:

$$\vec{B} = (0.4\,T)\hat{i}$$

Substitute numerical values and simplify to obtain:

$$\phi_m = (15)[(0.4\ \text{T})]\pi(0.04\ \text{m})^2 = \left(0.0302\ \text{T}\cdot\text{m}^2\right)\hat{i}\cdot\hat{n}$$

(*a*) Evaluate ϕ_m for $\hat{n} = \hat{i}$:

$$\phi_m = \left(0.0302\ \text{T}\cdot\text{m}^2\right)\hat{i}\cdot\hat{i} = \boxed{0.0302\ \text{Wb}}$$

(*b*) Evaluate ϕ_m for $\hat{n} = \hat{j}$:

$$\phi_m = \left(0.0302\ \text{T}\cdot\text{m}^2\right)\hat{i}\cdot\hat{j} = \boxed{0}$$

(*c*) Evaluate ϕ_m for $\hat{n} = \left(\hat{i}+\hat{j}\right)/\sqrt{2}$:

$$\phi_m = \left(0.0302\ \text{T}\cdot\text{m}^2\right)\hat{i}\cdot\frac{\left(\hat{i}+\hat{j}\right)}{\sqrt{2}} = \frac{0.0302\ \text{T}\cdot\text{m}^2}{\sqrt{2}} = \boxed{0.0213\ \text{Wb}}$$

(*d*) Evaluate ϕ_m for $\hat{n} = \hat{k}$:

$$\phi_m = \left(0.0302\ \text{T}\cdot\text{m}^2\right)\hat{i}\cdot\hat{k} = \boxed{0}$$

(*e*) Evaluate ϕ_m for $\hat{n} = 0.6\hat{i} + 0.8\hat{j}$:

$$\begin{aligned}\phi_m &= \left(0.0302\ \text{T}\cdot\text{m}^2\right)\hat{i}\cdot\left(0.6\hat{i}+0.8\hat{j}\right) = 0.6\left(0.0302\ \text{T}\cdot\text{m}^2\right)\hat{i}\cdot\hat{i} \\ &\quad + 0.8\left(0.0302\ \text{T}\cdot\text{m}^2\right)\hat{i}\cdot\hat{j} \\ &= 0.6\left(0.0302\ \text{T}\cdot\text{m}^2\right) = \boxed{0.0181\ \text{Wb}}\end{aligned}$$

23 ••

Picture the Problem The magnetic field outside the solenoid is, to a good approximation, zero. Hence, the flux through the loop is the flux in the core of the solenoid. The magnetic field inside the solenoid is uniform. Hence, the flux through this small loop is given by the same expression with R_3 replacing R_1:

(*a*) Express the flux through the large circular loop outside the solenoid:

$$\phi_{\mathrm{m}} = NBA = \boxed{\mu_0 nIN\pi R_1^2}$$

(*b*) Express the flux through the small loop inside the solenoid:

$$\phi_{\mathrm{m}} = NBA = \boxed{\mu_0 nIN\pi R_3^2}$$

***24 ••**

Picture the Problem We can use the hint to set up the element of area dA and express the flux $d\phi_{\mathrm{m}}$ through it and then carry out the details of the integration to express ϕ_{m}.

(*a*) Express the flux through the strip of area dA:

$$d\phi_{\mathrm{m}} = BdA$$

where $dA = bdx$.

Express B at a distance x from a long, straight wire:

$$B = \frac{\mu_0}{4\pi}\frac{2I}{x} = \frac{\mu_0}{2\pi}\frac{I}{x}$$

Substitute to obtain:

$$d\phi_{\mathrm{m}} = \frac{\mu_0}{2\pi}\frac{I}{x}bdx = \frac{\mu_0 Ib}{2\pi}\frac{dx}{x}$$

Integrate from $x = d$ to $x = d + a$:

$$\phi_{\mathrm{m}} = \frac{\mu_0 Ib}{2\pi}\int_d^{d+a}\frac{dx}{x} = \boxed{\frac{\mu_0 Ib}{2\pi}\ln\frac{d+a}{d}}$$

(*b*) Substitute numerical values and evaluate ϕ_{m}:

$$\phi_{\mathrm{m}} = \frac{\left(4\pi\times10^{-7}\ \mathrm{N/A^2}\right)(20\ \mathrm{A})(0.1\ \mathrm{m})}{2\pi}\ln\left(\frac{7\ \mathrm{cm}}{2\ \mathrm{cm}}\right) = \boxed{5.01\times10^{-7}\ \mathrm{Wb}}$$

25 •••

Picture the Problem Consider an element of area $dA = Ldr$ where $r \le R$. We can use its definition to express $d\phi_{\mathrm{m}}$ through this area in terms of B and Ampere's law to express B as a function of I. The fact that the current is uniformly distributed over the cross-sectional area of the conductor allows us to set up a proportion from which we can obtain I as a function of r. With these substitutions in place we can integrate $d\phi_{\mathrm{m}}$ to obtain ϕ_{m}/L.

Express the flux $d\phi_m$ through an area Ldr:

$$d\phi_m = BdA = BLdr \qquad (1)$$

Apply Ampere's law to the current contained inside a cylindrical region of radius $r < R$:

$$\oint_C \vec{B}\cdot d\vec{\ell} = 2\pi rB = \mu_0 I_C$$

and

$$B = \frac{\mu_0 I_C}{2\pi r}$$

Using the fact that the current I is uniformly distributed over the cross-sectional area of the conductor, express its variation with distance r from the center of the conductor:

$$\frac{I(r)}{I} = \frac{\pi r^2}{\pi R^2}$$

or

$$I(r) = I_C = I\frac{r^2}{R^2}$$

Substitute and simplify to obtain:

$$B = \frac{\mu_0 I}{2\pi r}\frac{r^2}{R^2} = \frac{\mu_0 I}{2\pi R^2}r$$

Substitute in equation (1):

$$d\phi_m = \frac{\mu_0 LI}{2\pi R^2}rdr$$

Integrate $d\phi_m$ from $r = 0$ to $r = R$ to obtain:

$$\phi_m = \frac{\mu_0 LI}{2\pi R^2}\int_0^R rdr = \frac{\mu_0 LI}{4\pi}$$

Divide both sides of this equation by L to express the magnetic flux per unit length:

$$\frac{\phi_m}{L} = \boxed{\frac{\mu_0 I}{4\pi}}$$

26 •••

Picture the Problem We can use its definition to express the flux through the rectangular region and Ampere's law to relate the magnetic field to the current in the wire and the position of the long straight wire.

(*a*) Note that for $0 \le x \le b$, B is symmetric about the wire, into the paper for the region below the wire and out of the paper for the region above the wire. Thus, for the area $2(b - x)a$:

$$\phi_{m,net} = 0$$

To find the flux through the

$$d\phi_m = BdA$$

remaining area of the rectangle, express the flux through a strip of area dA:

where $dA = adx$.

Using Ampere's law, express B at a distance x from a long, straight wire:

$$B = \frac{\mu_0}{4\pi}\frac{2I}{x} = \frac{\mu_0}{2\pi}\frac{I}{x}$$

Substitute to obtain:

$$d\phi_{\mathrm{m}} = \frac{\mu_0}{2\pi}\frac{I}{x}adx = \frac{\mu_0 Ia}{2\pi}\frac{dx}{x}$$

For $0 \le x \le b$, integrate from $x' = b - x$ to $x' = x$:

$$\phi_{\mathrm{m}}(0 \le x \le b) = \frac{\mu_0 Ia}{2\pi}\int_{b-x}^{x}\frac{dx'}{x'}$$

$$= \boxed{\frac{\mu_0 Ia}{2\pi}\ln\left(\frac{x}{b-x}\right)}$$

For $x \ge b$, integrate from $x' = x$ to $x' = x + b$:

$$\phi_{\mathrm{m}}(x \ge b) = \frac{\mu_0 Ia}{2\pi}\int_{x}^{x+b}\frac{dx'}{x'}$$

$$= \boxed{\frac{\mu_0 Ia}{2\pi}\ln\left(\frac{x+b}{x}\right)}$$

(*b*) From the expressions derived in (*a*) we see that $|\phi_{\mathrm{m}}| \to \infty$ as:

$$\boxed{x \to 0}$$

The flux is a minimum ($\phi_{\mathrm{m}} = 0$) for:

$\boxed{x = \frac{1}{2}b}$ as expected from symmetry.

Induced EMF and Faraday's Law

***27 •**

Picture the Problem We can find the induced emf by applying Faraday's law to the loop. The application of Ohm's law will yield the induced current in the loop and we can find the rate of joule heating using $P = I^2R$.

(*a*) Apply Faraday's law to express the induced emf in the loop in terms of the rate of change of the magnetic field:

$$|\mathcal{E}| = \frac{d\phi_{\mathrm{m}}}{dt} = \frac{d}{dt}(AB) = A\frac{dB}{dt} = \pi R^2\frac{dB}{dt}$$

Substitute numerical values and

$$|\mathcal{E}| = \pi(0.05\,\mathrm{m})^2(40\,\mathrm{mT/s}) = \boxed{0.314\,\mathrm{mV}}$$

evaluate $|\mathcal{E}|$:

(*b*) Using Ohm's law, relate the induced current to the induced voltage and the resistance of the loop and evaluate *I*:

$$I = \frac{\mathcal{E}}{R} = \frac{0.314\,\text{mV}}{0.4\,\Omega} = \boxed{0.785\,\text{mA}}$$

(*c*) Express the rate at which power is dissipated in a conductor in terms of the induced current and the resistance of the loop and evaluate *P*:

$$P = I^2R = (0.785\,\text{mA})^2(0.4\,\Omega)$$
$$= \boxed{0.247\,\mu\text{W}}$$

28 •

Picture the Problem Given ϕ_m as a function of time, we can use Faraday's law to express $\mathcal{E}$ as a function of time.

(*a*) Apply Faraday's law to express the induced emf in the loop in terms of the rate of change of the magnetic field:

$$\mathcal{E} = -\frac{d\phi_m}{dt} = -\frac{d}{dt}\left[\left(t^2 - 4t\right)\times 10^{-1}\,\text{Wb}\right]$$
$$= -(2t - 4)\times 10^{-1}\,\text{Wb/s}$$
$$= \boxed{-(0.2t - 0.4)\,\text{V}}$$

(*b*) Evaluate ϕ_m at $t = 0$:

$$\phi_m(0\,\text{s}) = \left[(0)^2 - 4(0)\right]\times 10^{-1}\,\text{Wb} = \boxed{0}$$

Evaluate $\mathcal{E}$ at $t = 0$:

$$\mathcal{E}(0\,\text{s}) = -[0.2(0) - 0.4]\text{V}$$
$$= \boxed{0.400\,\text{V}}$$

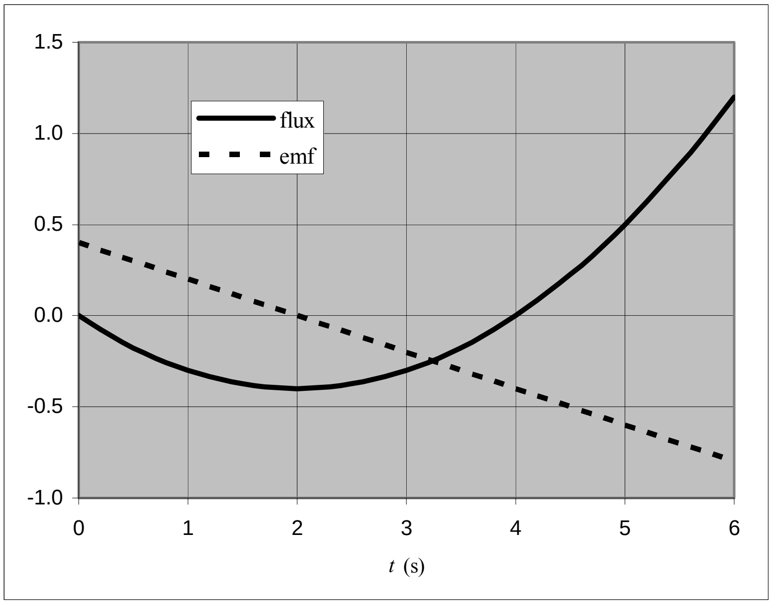
Proceed as above to complete the table to the right:

t	ϕ_m	$\mathcal{E}$
(s)	(Wb)	(V)
0	0	0
2	–0.400	0
4	0	–0.400
6	1.20	–0.800

29 •

Picture the Problem We can find the time at which the flux is a minimum by looking for the lowest point on the graph of $\mathcal{E}$ versus *t* and the emf at this time by determining the value of *V* at this time from the graph. We can interpret the graphs to find the times at which the flux is zero and the corresponding values of the emf.

(a) The flux, ϕ_m, and the induced emf, $\mathcal{E}$, are shown as functions of t in the following graph. The solid curve represents ϕ_m, the dashed curve represents $\mathcal{E}$.



(b) $\boxed{\text{Referring to the graph, we see that the flux is a minimum at } t = 2\text{ s and that } V = 0 \text{ at this instant.}}$

(c) $\boxed{\text{The flux is zero at } t = 0 \text{ and } t = 4\text{ s. At these times, } \mathcal{E} = 0.4\text{ V and } -0.4\text{ V, respectively.}}$

30 •

Picture the Problem We can use its definition to find the magnetic flux through the solenoid and Faraday's law to find the emf induced in the solenoid when the external field is reduced to zero in 1.4 s.

(a) Express the magnetic flux through the solenoid in terms of N, B, A, and θ:

$$\phi_m = NBA\cos\theta = NB\pi R^2\cos\theta$$

Substitute numerical values and evaluate ϕ_m:

$$\phi_m = (400)(0.06\text{ T})\pi(0.008\text{ m})^2\cos 50° = \boxed{3.10\text{ mWb}}$$

(*b*) Apply Faraday's law to obtain:

$$\mathcal{E} = -\frac{d\phi_m}{dt} = -\frac{0 - 3.10\,\text{mWb}}{1.4\,\text{s}} = \boxed{2.22\,\text{mV}}$$

***31 ••**

Picture the Problem We can use the definition of average current to express the total charge passing through the coil as a function of I_{av}. Because the induced current is proportional to the induced emf and the induced emf, in turn, is given by Faraday's law, we can express ΔQ as a function of the number of turns of the coil, the magnetic field, the resistance of the coil, and the area of the coil. Knowing the reversal time, we can find the average current from its definition and the average emf in the coil from Ohm's law.

(*a*) Express the total charge that passes through the coil in terms of the induced current:

$$\Delta Q = I_{av}\Delta t$$

Relate the induced current to the induced emf:

$$I = I_{av} = \frac{\mathcal{E}}{R}$$

Using Faraday's law, express the induced emf in terms of ϕ_m:

$$\mathcal{E} = -\frac{\Delta\phi_m}{\Delta t}$$

Substitute and simplify to obtain:

$$\Delta Q = \frac{\mathcal{E}}{R}\Delta t = \frac{-\frac{\Delta\phi_m}{\Delta t}}{R}\Delta t = -\frac{2\phi_m}{R} = -\frac{2NBA}{R} = -\frac{2NB\left(\frac{\pi}{4}d^2\right)}{R} = -\frac{NB\pi d^2}{2R}$$

where d is the diameter of the coil.

Substitute numerical values and evaluate ΔQ:

$$\Delta Q = -\frac{(100)(1\,\text{T})\pi(0.02\,\text{m})^2}{2(50\,\Omega)} = \boxed{-1.26\,\text{mC}}$$

(*b*) Apply the definition of average current to obtain:

$$I_{av} = \frac{\Delta Q}{\Delta t} = \frac{1.26\,\text{mC}}{0.1\,\text{s}} = \boxed{12.6\,\text{mA}}$$

(*c*) Using Ohm's law, relate the average emf in the coil to the average current:

$$\mathcal{E}_{av} = I_{av}R = (12.6\,\text{mA})(50\,\Omega)$$
$$= \boxed{630\,\text{mV}}$$

32 ••

Picture the Problem We can use the definition of average current to express the total charge passing through the coil as a function of I_{av}. Because the induced current is proportional to the induced emf and the induced emf, in turn, is given by Faraday's law, we can express ΔQ as a function of the number of turns of the coil, the magnetic field, the resistance of the coil, and the area of the coil.

Express the total charge that passes through the coil in terms of the induced current:

$$\Delta Q = I_{av}\Delta t$$

Relate the induced current to the induced emf:

$$I = I_{av} = \frac{\mathcal{E}}{R}$$

Using Faraday's law, express the induced emf in terms of ϕ_m:

$$\mathcal{E} = -\frac{\Delta\phi_m}{\Delta t}$$

Substitute to obtain:

$$\Delta Q = \frac{\mathcal{E}}{R}\Delta t = \frac{-\dfrac{\Delta\phi_m}{\Delta t}}{R}\Delta t$$
$$= -\frac{2\phi_m}{R} = -\frac{2NBA}{R}$$

Substitute numerical values and evaluate ΔQ:

$$\Delta Q = \left|-\frac{2(1000)(0.7\times10^{-4}\,\text{T})(300\times10^{-4}\,\text{m}^2)}{15\,\Omega}\right| = \boxed{0.280\,\text{mC}}$$

33 ••

Picture the Problem We can use Faraday's law to express the earth's magnetic field at this location in terms of the induced emf and Ohm's law to relate the induced emf to the charge that passes through the current integrator.

Using Faraday's law, express the induced emf in terms of the change in the magnetic flux as the coil is rotated through 90°:

$$\mathcal{E} = \left|-\frac{\Delta\phi_m}{\Delta t}\right| = \frac{NBA}{\Delta t} = \frac{NB\pi r^2}{\Delta t}$$

Solve for B:

$$B = \frac{\mathcal{E}\Delta t}{N\pi r^2}$$

Using Ohm's law, relate the induced emf to the induced current:

$$\mathcal{E} = IR = \frac{\Delta Q}{\Delta t}R$$

where ΔQ is the charge that passes through the current integrator.

Substitute to obtain:

$$B = \frac{\frac{\Delta Q}{\Delta t}R\Delta t}{N\pi r^2} = \frac{\Delta QR}{N\pi r^2}$$

Substitute numerical values and evaluate B:

$$B = \frac{(9.4\,\mu\text{C})(20\,\Omega)}{(300)\pi(0.05\,\text{m})^2} = \boxed{79.8\,\mu\text{T}}$$

34 ••

Picture the Problem We can use Faraday's law to express the induced emf in the coil in terms of the rate of change of the magnetic flux. We can use its definition to express the magnetic flux through the rectangular region and Ampere's law to relate the magnetic field to the current in the wire and the position of the long straight wire.

(*a*) Apply Faraday's law to relate the induced emf to the changing magnetic flux:

$$\mathcal{E} = -\frac{d\phi_{\text{m}}}{dt} \qquad (1)$$

Note that for $0 \le x \le b$, B is symmetric about the wire, into the paper for the region below the wire and out of the paper for the region above the wire. Thus, for the area $2(b - x)a$:

$$\phi_{\text{m,net}} = 0$$

To find the flux through the remaining area of the rectangle, express the flux through a strip of area dA:

$$d\phi_{\text{m}} = BdA$$

where $dA = adx$.

Using Ampere's law, express B at a distance x from a long, straight wire:

$$B = \frac{\mu_0}{4\pi}\frac{2I}{x} = \frac{\mu_0}{\pi}\frac{t}{x}$$

Substitute to obtain:

$$d\phi_m = \frac{\mu_0}{\pi}\frac{t}{x}a\,dx = \frac{\mu_0 ta}{\pi}\frac{dx}{x}$$

For $0 \le x \le b$, integrate from $x' = b - x$ to $x' = x$:

$$\phi_m(0 \le x \le b) = \frac{\mu_0 ta}{\pi}\int_{b-x}^{x}\frac{dx'}{x'} = \frac{\mu_0 ta}{\pi}\ln\left(\frac{x}{b-x}\right)$$

Differentiate this expression with respect to time to obtain:

$$\frac{d\phi_m}{dt} = \frac{d}{dt}\left[\frac{\mu_0 ta}{\pi}\ln\left(\frac{x}{b-x}\right)\right] = \frac{\mu_0 a}{\pi}\ln\left(\frac{x}{b-x}\right)$$

Substitute in equation (1) and evaluate $\mathcal{E}$ for $x = b/4$:

$$\mathcal{E} = -\frac{\mu_0 a}{\pi}\ln\left(\frac{b/4}{b-b/4}\right) = -\frac{\mu_0 a}{\pi}\ln\left(\frac{1}{3}\right) = \boxed{1.10\frac{\mu_0 a}{\pi}}$$

(*b*) Using Ohm's law, express and evaluate *R*:

$$R = \frac{\mathcal{E}}{I} = \frac{1.10\mu_0 a}{\pi I} = \frac{1.10(4\pi\times10^{-7}\ \text{N/A}^2)(1.5\,\text{m})}{\pi(0.1\,\text{A})} = \boxed{6.60\ \mu\Omega}$$

Because the magnetic flux due to *I* is increasing into the page, the induced current will be in such a direction that its magnetic field will oppose this increase; i.e, it will be out of the page. Thus the induced current is counterclockwise.

35 ••

Picture the Problem We can use Faraday's law to express the induced emf in the coil in terms of the rate of change of the magnetic flux. We can use its definition to express the magnetic flux through the rectangular region and Ampere's law to relate the magnetic field to the current in the wire and the position of the long straight wire.

(*a*) Apply Faraday's law to relate the induced emf to the changing magnetic flux:

$$\mathcal{E} = -\frac{d\phi_m}{dt} \qquad (1)$$

Note that for $0 \le x \le b$, B is symmetric about the wire, into the paper for the region below the wire and out of the paper for the region above the wire. Thus, for the area $2(b-x)a$:

$$\phi_{\text{m,net}} = 0$$

To find the flux through the remaining area of the rectangle, express the flux through a strip of area dA:

$$d\phi_{\text{m}} = BdA$$

where $dA = adx$.

Using Ampere's law, express B at a distance x from a long, straight wire:

$$B = \frac{\mu_0}{4\pi}\frac{2I}{x} = \frac{\mu_0}{\pi}\frac{t}{x}$$

Substitute to obtain:

$$d\phi_{\text{m}} = \frac{\mu_0}{\pi}\frac{t}{x}adx = \frac{\mu_0 ta}{\pi}\frac{dx}{x}$$

For $0 \le x \le b$, integrate from $x' = b - x$ to $x' = x$:

$$\phi_{\text{m}}(0 \le x \le b) = \frac{\mu_0 ta}{\pi}\int_{b-x}^{x}\frac{dx'}{x'} = \frac{\mu_0 ta}{\pi}\ln\left(\frac{x}{b-x}\right)$$

Differentiate this expression with respect to time to obtain:

$$\frac{d\phi_{\text{m}}}{dt} = \frac{d}{dt}\left[\frac{\mu_0 ta}{\pi}\ln\left(\frac{x}{b-x}\right)\right] = \frac{\mu_0 a}{\pi}\ln\left(\frac{x}{b-x}\right)$$

Substitute in equation (1) and evaluate $\mathcal{E}$ for $x = b/3$:

$$\mathcal{E} = -\frac{\mu_0 a}{\pi}\ln\left(\frac{b/3}{b-b/3}\right) = -\frac{\mu_0 a}{\pi}\ln\left(\frac{1}{2}\right) = \boxed{0.693\frac{\mu_0 a}{\pi}}$$

(*b*) Using Ohm's law, express and evaluate R:

$$R = \frac{\mathcal{E}}{I} = \frac{0.693\mu_0 a}{\pi I} = \frac{0.693(4\pi\times10^{-7}\ \text{N/A}^2)(1.5\,\text{m})}{\pi(0.1\,\text{A})} = \boxed{4.16\ \mu\Omega}$$

Because the magnetic flux due to I is increasing into the page, the induced current will be in such a direction that its magnetic field will oppose this increase, i.e, it will be out of the page. Thus, the induced current is counterclockwise.

Motional EMF

***36 •**

Picture the Problem We can apply the equation for the force on a charged particle moving in a magnetic field to find the magnetic force acting on an electron in the rod. We can use $\vec{E} = \vec{v} \times \vec{B}$ to find E and $V = E\ell$, where ℓ is the length of the rod, to find the potential difference between its ends.

(*a*) Relate the magnetic force on an electron in the rod to the speed of the rod, the electronic charge, and the magnetic field in which the rod is moving:

$$\vec{F} = q\vec{v} \times \vec{B}$$
and
$$F = qvB\sin\theta$$

Substitute numerical values and evaluate F:

$$F = \left(1.6\times10^{-19}\ \text{C}\right)\left(8\ \text{m/s}\right)\left(0.05\ \text{T}\right)\sin 90° = \boxed{6.40\times10^{-20}\ \text{N}}$$

(*b*) Express the electrostatic field $\vec{E}$ in the rod in terms of the magnetic field $\vec{B}$:

$$\vec{E} = \vec{v} \times \vec{B}$$
and
$$E = vB\sin\theta$$

Substitute numerical values and evaluate B:

$$E = \left(8\ \text{m/s}\right)\left(0.05\ \text{T}\right)\sin 90° = \boxed{0.400\ \text{V/m}}$$

(*c*) Relate the potential difference between the ends of the rod to its length ℓ and the electric field E:

$$V = E\ell$$

Substitute numerical values and evaluate V:

$$V = \left(0.4\ \text{V/m}\right)\left(0.3\ \text{m}\right) = \boxed{0.120\ \text{V}}$$

37 •

Picture the Problem We can use $\vec{E} = \vec{v} \times \vec{B}$ to relate the speed of the rod to the electric field in the rod and magnetic field in which it is moving and $V = E\ell$ to relate the electric field in the rod to the potential difference between its ends.

Express the electrostatic field $\vec{E}$ in the rod in terms of the magnetic field $\vec{B}$ and solve for v:

$$\vec{E} = \vec{v} \times \vec{B}$$

and

$$v = \frac{E}{B\sin\theta}$$

Relate the potential difference between the ends of the rod to its length ℓ and the electric field E and solve for E:

$$V = E\ell \Rightarrow E = \frac{V}{\ell}$$

Substitute for E to obtain:

$$v = \frac{V}{B\ell\sin\theta}$$

Substitute numerical values and evaluate v:

$$v = \frac{6\,\mathrm{V}}{(0.05\,\mathrm{T})(0.3\,\mathrm{m})} = \boxed{400\,\mathrm{m/s}}$$

38 •

Picture the Problem Because the speed of the rod is constant, an external force must act on the rod to counter the magnetic force acting on the induced current. We can use the motional-emf equation $\mathcal{E} = vB\ell$ to evaluate the induced emf, Ohm's law to find the current in the circuit, Newton's 2nd law to find the force needed to move the rod with constant velocity, and $P = Fv$ to find the power input by the force.

(*a*) Relate the induced emf in the circuit to the speed of the rod, the magnetic field, and the length of the rod:

$$\mathcal{E} = vB\ell = (10\,\mathrm{m/s})(0.8\,\mathrm{T})(0.2\,\mathrm{m}) = \boxed{1.60\,\mathrm{V}}$$

(*b*) Using Ohm's law, relate the current in the circuit to the induced emf and the resistance of the circuit:

$$I = \frac{\mathcal{E}}{R} = \frac{1.6\,\mathrm{V}}{2\,\Omega} = \boxed{0.800\,\mathrm{A}}$$

Note that, because the rod is moving to the right, the flux in the region defined by the rod, the rails, and the resistor is increasing. Hence, in accord with Lenz's law, the current must be counterclockwise if its magnetic field is to counter this increase in flux.

(*c*) Because the rod is moving with constant velocity, the net force acting on it must be zero. Apply

$$\sum F_x = F - F_\mathrm{m} = 0$$

and

Newton's 2nd law to relate F to the magnetic force F_m:

$$F = F_m = BI\ell$$
$$= (0.8\,\text{T})(0.8\,\text{A})(0.2\,\text{m}) = \boxed{0.128\,\text{N}}$$

(d) Express the power input by the force in terms of the force and the velocity of the rod:

$$P = Fv = (0.128\,\text{N})(10\,\text{m/s}) = \boxed{1.28\,\text{W}}$$

(e) The rate of Joule heat production is given by:

$$P = I^2R = (0.8\,\text{A})^2(2\,\Omega) = \boxed{1.28\,\text{W}}$$

39 ••

Picture the Problem We'll need to determine how long it takes for the loop to completely enter the region in which there is a magnetic field, how long it is in the region, and how long it takes to leave the region. Once we know these times, we can use its definition to express the magnetic flux as a function of time. We can use Faraday's law to find the induced emf as a function of time.

(a) Find the time required for the loop to enter the region where there is a uniform magnetic field:

$$t = \frac{\ell_{\text{side of loop}}}{v} = \frac{10\,\text{cm}}{2.4\,\text{cm/s}} = 4.17\,\text{s}$$

Letting w represent the width of the loop, express and evaluate ϕ_m for $0 < t < 4.17\,\text{s}$:

$$\phi_m = NBA = NBwvt$$
$$= (1.7\,\text{T})(0.05\,\text{m})(0.024\,\text{m/s})t$$
$$= (2.04\,\text{mWb/s})t$$

Find the time during which the loop is fully in the region where there is a uniform magnetic field:

$$t = \frac{\ell_{\text{side of loop}}}{v} = \frac{10\,\text{cm}}{2.4\,\text{cm/s}} = 4.17\,\text{s}$$

i.e., the loop will begin to exit the region when t = 8.33 s.

Express ϕ_m for $4.17\,\text{s} < t < 8.33\,\text{s}$:

$$\phi_m = NBA = NB\ell w$$
$$= (1.7\,\text{T})(0.1\,\text{m})(0.05\,\text{m})$$
$$= 8.50\,\text{mWb}$$

The left-end of the loop will exit the field when t = 12.5 s. Express ϕ_m for $8.33\,\text{s} < t < 12.5\,\text{s}$:

$$\phi_m = mt + b$$

where m is the slope of the line and b is the ϕ_m-intercept.

For t = 8.33 s and ϕ_m = 8.50 mWb:

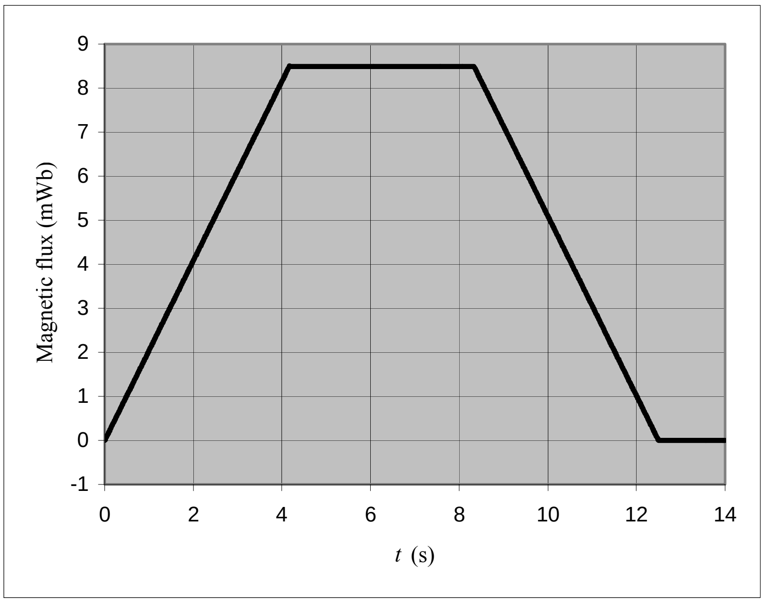
$$8.50\,\text{mWb} = m(8.33\,\text{s}) + b \qquad (1)$$

For t = 12.5 s and ϕ_m = 0: $$0 = m(12.5\,\mathrm{s}) + b \qquad (2)$$

Solve equations (1) and (2) simultaneously to obtain: $$\phi_m = -(2.04\,\mathrm{mWb/s})t + 25.5\,\mathrm{mWb}$$

The loop will be completely out of the magnetic field when t > 12.5 s and: $$\phi_m = 0$$

The following graph of $\phi_m(t)$ was plotted using a spreadsheet program.



(b) Using Faraday's law, relate the induced emf to the magnetic flux: $$\mathcal{E} = -\frac{d\phi_m}{dt}$$

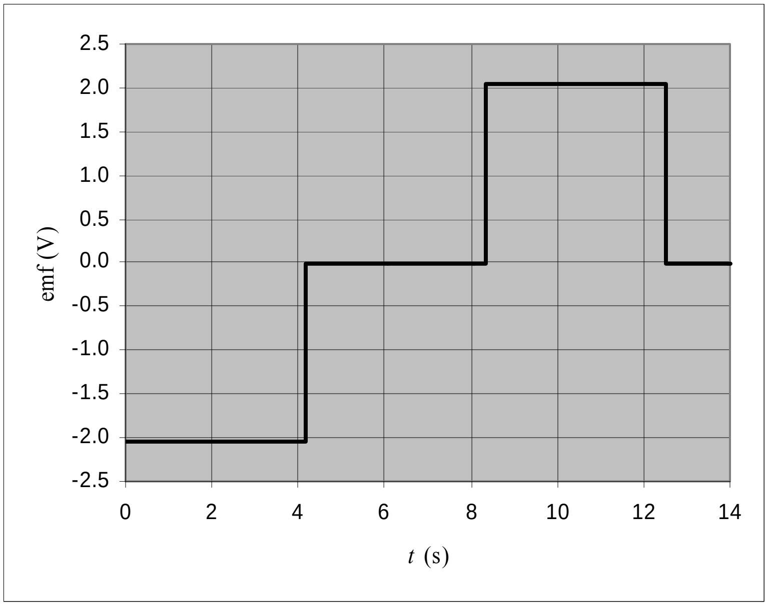
During the interval $0 < t < 4.17\,\mathrm{s}$: $$\mathcal{E} = -\frac{d}{dt}\left[(2.04\,\mathrm{mWb/s})t\right] = -2.04\,\mathrm{mV}$$

During the interval $4.17\,\mathrm{s} < t < 8.33\,\mathrm{s}$: $$\mathcal{E} = -\frac{d}{dt}\left[8.50\,\mathrm{mWb}\right] = 0$$

During the interval $8.33\,\mathrm{s} < t < 12.5\,\mathrm{s}$: $$\mathcal{E} = -\frac{d}{dt}\left[(-2.04\,\mathrm{mWb/s})t + 25.5\,\mathrm{mWb}\right] = 2.04\,\mathrm{mV}$$

For t > 12.5 s: $$\mathcal{E} = 0$$

The following graph of $\varepsilon(t)$ was plotted using a spreadsheet program.



40 ••

Picture the Problem The rod is executing simple harmonic motion in the *xy* plane, i.e., in a plane perpendicular to the magnetic field. The emf induced in the rod is a consequence of its motion in this magnetic field and is given by $|\mathcal{E}| = vB\ell$. Because we're given the position of the oscillator as a function of time, we can differentiate this expression to obtain *v*.

Express the motional emf in terms of *v*, *B*, and ℓ:

$$|\mathcal{E}| = vB\ell = B\ell\frac{dx}{dt}$$

Evaluate *dx*/*dt*:

$$\begin{aligned}\frac{dx}{dt} &= \frac{d}{dt}[(2\,\text{cm})\cos 120\pi t]\\ &= -(2\,\text{cm})(120\,\text{s}^{-1})\pi\sin 120\pi t\\ &= -(7.54\,\text{m/s})\sin 120\pi t\end{aligned}$$

Substitute numerical values and evaluate $|\mathcal{E}|$:

$$|\mathcal{E}| = -(1.2\,\text{T})(0.15\,\text{m})(7.54\,\text{m/s})\sin 120\pi t = \boxed{-(1.36\,\text{V})\sin 120\pi t}$$

41 ••

Picture the Problem Let *m* be the mass of the rod and *F* be the net force acting on it due to the current in it. We can obtain the equation of motion of the rod by applying

Newton's 2nd law to relate its acceleration to B, I, and ℓ. The net emf that drives I in this circuit is the emf of the battery minus the emf induced in the rod as a result of its motion.

(*a*) Letting the direction of motion of the rod be the positive x direction, apply $\sum F_x = ma_x$ to the rod:

$$BI\ell = m\frac{dv}{dt} \qquad (1)$$

where

$$I = \frac{\mathcal{E} - B\ell v}{R} \qquad (2)$$

Substitute to obtain:

$$\frac{dv}{dt} = \boxed{\frac{B\ell}{mR}(\mathcal{E} - B\ell v)}$$

(*b*) Express the condition on dv/dt when the rod has achieved its terminal speed:

$$\frac{B\ell}{mR}(\mathcal{E} - B\ell v_\text{t}) = 0$$

Solve for v_t to obtain:

$$v_\text{t} = \boxed{\frac{\mathcal{E}}{B\ell}}$$

(*c*) Substitute v_t for v in equation (2) to obtain:

$$I = \frac{\mathcal{E} - B\ell\dfrac{\mathcal{E}}{B\ell}}{R} = \boxed{0}$$

***42 ••**

Picture the Problem In Example 28-9 it is shown that the speed of the rod is given by $v = v_0 e^{-(B^2\ell^2/mR)t}$. We can use the definition of power and the expression for a motional emf to express the power dissipated in the resistance in terms of B, ℓ, v, and R. We can then separate the variables and integrate over all time to show that the total energy dissipated is equal to the initial kinetic energy of the rod.

Express the power dissipated in terms of ε and R:

$$P = \frac{\mathcal{E}^2}{R}$$

Express ε as a function of B, ℓ, and v:

$$\mathcal{E} = B\ell v$$

where

$$v = v_0 e^{-(B^2\ell^2/mR)t}$$

Substitute to obtain:

$$P = \frac{(B\ell v)^2}{R}$$

The total energy dissipated as the rod comes to rest is obtained by integrating $dE = P\,dt$:

$$E = \int_0^\infty \frac{(B\ell v)^2}{R}dt$$

$$= \int_0^\infty \frac{\left(B\ell v_0 e^{-\left(B^2\ell^2/mR\right)t}\right)^2}{R}dt$$

$$= \frac{B^2\ell^2 v_0^2}{R}\int_0^\infty e^{-2\left(B^2\ell^2/mR\right)t}dt$$

Evaluate the integral (by changing variables to $u = -\dfrac{2B^2\ell^2}{mR}$) to obtain:

$$E = \frac{B^2\ell^2 v_0^2}{R}\left(\frac{mR}{2B^2\ell^2}\right) = \boxed{\tfrac{1}{2}mv_0^2}$$

43 ••

Picture the Problem In Example 28-9 it is shown that the speed of the rod is given by $v = v_0 e^{-\left(B^2\ell^2/mR\right)t}$. We can write v as dx/dt, separate the variables and integrate to find the total distance traveled by the rod.

Apply the result from Example 28-9 to obtain:

$$\frac{dx}{dt} = v_0 e^{-Ct}$$

where

$$\mathrm{C} = \frac{B^2\ell^2}{mR}$$

Separate variables and integrate x' from 0 to x and t' from 0 to ∞:

$$\int_0^x dx' = v_0\int_0^\infty e^{-Ct'}dt$$

Evaluate the integrals to obtain:

$$x = \frac{v_0}{\mathrm{C}}$$

Substitute for C and simplify:

$$x = \boxed{\frac{mv_0R}{B^2\ell^2}}$$

44 ••

Picture the Problem Let m be the mass of the rod. The net force acting on the rod is due to the current in it. We can obtain the equation of motion for the rod by applying Newton's 2nd law to relate its acceleration to B, I, and ℓ. The net emf that drives I in this circuit is the emf of the capacitor minus the emf induced in the rod as a result of its motion.

(a) Letting the direction of motion of the rod be the positive x direction, apply $\sum F_x = ma_x$ to the rod:

$$BI\ell = m\frac{dv}{dt} \quad (1)$$

where

$$I = \frac{\frac{Q}{C} - B\ell v}{R} \quad (2)$$

Solve equation (1) for I:

$$I = \frac{m}{B\ell}\frac{dv}{dt}$$

or, because the capacitor is discharging,

$$-\frac{dQ}{dt} = \frac{m}{B\ell}\frac{dv}{dt}$$

Simplify to obtain:

$$dQ = -\frac{m}{B\ell}dv$$

Integrate Q' from Q_0 to Q and v' from 0 to v:

$$\int_{Q_0}^{Q} dQ' = -\frac{m}{B\ell}\int_0^v dv'$$

and

$$Q = Q_0 - \frac{m}{B\ell}v$$

Substitute in equation (2) to obtain:

$$I = \frac{\frac{Q_0 - \frac{m}{B\ell}v}{C} - B\ell v}{R} = \frac{Q_0 - \frac{m}{B\ell}v}{CR} - \frac{B\ell v}{R}$$

Substitute in equation (1) to obtain the equation of motion of the rod:

$$\frac{dv}{dt}=\frac{B\ell}{mR}\left(\frac{Q_0-\dfrac{m}{B\ell}v}{C}-B\ell v\right)$$

$$=\boxed{\frac{B\ell Q_0}{mRC}-\left(\frac{1}{RC}+\frac{B^2\ell^2}{mR}\right)v}$$

(*b*) When the rod has achieved its terminal speed:

$$BI\ell=m\frac{dv}{dt}=0$$

and

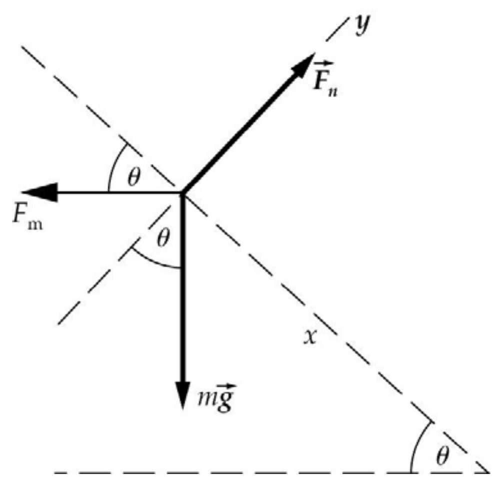
$$I=\frac{\dfrac{Q_f}{C}-B\ell v_t}{R}=0$$

Solve for v_t to obtain:

$$v_t=\boxed{\frac{Q_f}{CB\ell}}$$

***45 ••**

Picture the Problem The free-body diagram shows the forces acting on the rod as it slides down the inclined plane. The retarding force is the component of F_m acting up the incline, i.e., in the $-x$ direction. We can express F_m using the expression for the force acting on a conductor moving in a magnetic field. Recognizing that only the horizontal component of the rod's velocity $\vec{v}$ produces an induced emf, we can apply the expression for a motional emf in conjunction with Ohm's law to find the induced current in the rod. In part (*b*) we can apply Newton's 2nd law to obtain an expression for dv/dt and set this expression equal to zero to obtain v_t.



(*a*) Express the retarding force acting on the rod:

$$F=F_m\cos\theta \qquad (1)$$

where

$$F_m=I\ell B$$

and I is the current induced in the rod as a consequence of its motion in the magnetic field.

Express the induced emf due to the motion of the rod in the magnetic field:

$$\mathcal{E} = B\ell v\cos\theta$$

Using Ohm's law, relate the current I in the circuit to the induced emf:

$$I = \frac{\mathcal{E}}{R} = \frac{B\ell v\cos\theta}{R}$$

Substitute in equation (1) to obtain:

$$F = \left(\frac{B\ell v\cos\theta}{R}\right)\ell B\cos\theta = \boxed{\frac{B^2\ell^2 v}{R}\cos^2\theta}$$

(*b*) Apply $\sum F_x = ma_x$ to the rod:

$$mg\sin\theta - \frac{B^2\ell^2 v}{R}\cos^2\theta = m\frac{dv}{dt}$$

and

$$\frac{dv}{dt} = g\sin\theta - \frac{B^2\ell^2 v}{mR}\cos^2\theta$$

When the rod reaches its terminal velocity v_t, $dv/dt = 0$ and:

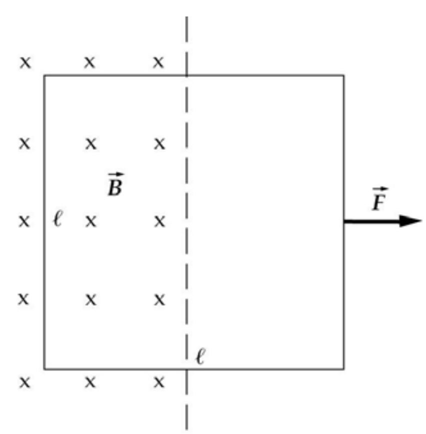
$$0 = g\sin\theta - \frac{B^2\ell^2 v_t}{mR}\cos^2\theta$$

Solve for v_t to obtain:

$$v_t = \boxed{\frac{mgR\sin\theta}{B^2\ell^2\cos^2\theta}}$$

46 ••

Picture the Problem The diagram shows the square loop being pulled from the magnetic field $\vec{B}$ by the constant force $\vec{F}$. The time required to pull the loop out of the magnetic field depends on the terminal speed of the loop. We can apply Newton's 2nd law and use the expressions for the magnetic force on a moving wire in a magnetic field to obtain the equation of motion for the loop and, from this equation, an expression for the terminal speed of the loop.



Apply $\sum\vec{F} = m\vec{a}$ to the square loop to obtain:

$$F - F_m = m\frac{dv}{dt} \qquad (1)$$

The magnetic force is given by:

$$F_{\mathrm{m}} = I\ell B = \frac{\mathcal{E}\,\ell\, B}{R}$$

where R is the resistance of the loop.

Substitute for Fm in equation (1) to obtain:

$$F - \frac{\mathcal{E}\,\ell\, B}{R} = m\frac{dv}{dt} \qquad (2)$$

The induced emf ε is related to the speed of the loop:

$$\mathcal{E} = vB\ell$$

Substitute for in equation (2) to obtain the equation of motion of the loop:

$$F - \frac{\ell^2 B^2}{R} v = m\frac{dv}{dt}$$

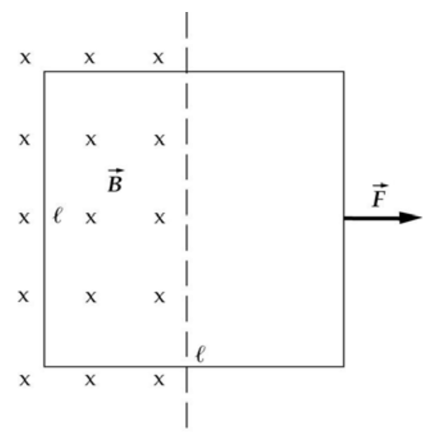
When the loop reaches its terminal speed, $dv/dt = 0$ and:

$$F - \frac{\ell^2 B^2}{R} v_{\mathrm{t}} = 0 \Rightarrow v_{\mathrm{t}} = \frac{R}{\ell^2 B^2} F$$

This result tells us that doubling F doubles the terminal speed v_{t}. Hence, doubling F will halve the time required to pull the loop from the magnetic field and $\boxed{(c)\text{ is correct.}}$

47 ••

Picture the Problem The diagram shows the square loop being pulled from the magnetic field $\vec{B}$ by the constant force $\vec{F}$. The time required to pull the loop out of the magnetic field depends on the terminal speed of the loop. We can apply Newton's 2nd law and use the expressions for the magnetic force on a moving wire in a magnetic field to obtain the equation of motion for the loop and, from this equation, an expression for the terminal speed of the loop.



Apply $\sum \vec{F} = m\vec{a}$ to the square loop to obtain:

$$F - F_{\mathrm{m}} = m\frac{dv}{dt} \qquad (1)$$

The magnetic force is given by:

$$F_{\mathrm{m}} = I\ell B = \frac{\mathcal{E}\,\ell\, B}{R}$$

where R is the resistance of the loop.

Substitute for F_{m} in equation (1) to obtain:

$$F - \frac{\mathcal{E}\,\ell\, B}{R} = m\frac{dv}{dt} \qquad (2)$$

The induced emf ε is related to the speed of the loop:

$$\mathcal{E} = vB\ell$$

Substitute for in equation (2) to obtain the equation of motion of the loop:

$$F - \frac{\ell^2 B^2}{R} v = m\frac{dv}{dt}$$

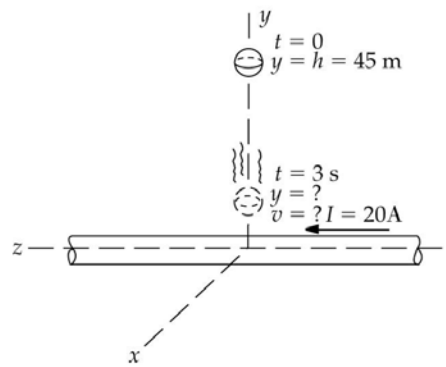
When the loop reaches its terminal speed, $dv/dt = 0$ and:

$$F - \frac{\ell^2 B^2}{R} v_t = 0 \Rightarrow v_t = \frac{R}{\ell^2 B^2} F$$

This result tells us that halving R halves the terminal speed v_t. Hence, halving R will double the time required to pull the loop from the magnetic field and $\boxed{(b)\text{ is correct.}}$

48 ••

Picture the Problem The diagram shows the initial position of the sphere and its position at $t = 3$ s. We can find the velocity of the sphere and the magnetic field when $t = 3$ s and use $\vec{E} = \vec{v} \times \vec{B}$ to find $\vec{E}$. We can find the voltage across the sphere at this time from the electric field at its center and its diameter.



(*a*) Relate the electric field at the center of the sphere to the magnetic field at that location:

$$\vec{E} = \vec{v} \times \vec{B}$$

Express the magnetic field as a function of the distance y from the current-carrying wire:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2I}{y}\left(-\hat{i}\right) = -\frac{\mu_0}{2\pi}\frac{I}{y}\hat{i}$$

Using a constant-acceleration equation, find the position of the sphere at $t = 3$ s:

$$y = y_0 + v_{0,y}\Delta t + \tfrac{1}{2}a(\Delta t)^2$$

or, because $y_0 = h$, $v_{0,y} = 0$, and $a = -g$,

$$y = h - \tfrac{1}{2}g(\Delta t)^2 = 45\,\text{m} - \tfrac{1}{2}\left(9.81\,\text{m/s}^2\right)(3\,\text{s})^2 = 0.855\,\text{m}$$

Substitute and evaluate $\vec{B}$:

$$\vec{B} = -\frac{\mu_0}{2\pi}\frac{I}{y}\hat{i} = -\frac{4\pi\times10^{-7}\,\text{N/A}^2}{2\pi}\frac{20\,\text{A}}{0.855\,\text{m}}\hat{i} = \left(-4.68\times10^{-6}\,\text{T}\right)\hat{i}$$

Using a constant-acceleration equation, find the velocity of the sphere at $t = 3$ s:

$$\vec{v} = \vec{v}_0 + \vec{a}\Delta t$$

or, because $\vec{v}_0 = 0$ and $\vec{a} = -g\,\hat{j}$

$$\vec{v} = -g\Delta t\,\hat{j} = -\left(9.81\,\text{m/s}^2\right)(3\,\text{s})\hat{j} = (-29.4\,\text{m/s})\hat{j}$$

Substitute and evaluate $\vec{E}$:

$$\vec{E} = (-29.4\,\text{m/s})\hat{j} \times \left(-4.68\times10^{-6}\,\text{T}\right)\hat{i} = \boxed{(-0.138\,\text{mV/m})\hat{k}}$$

(*b*) The potential difference across the sphere depends on the electric field at the center of the sphere and the diameter of the sphere:

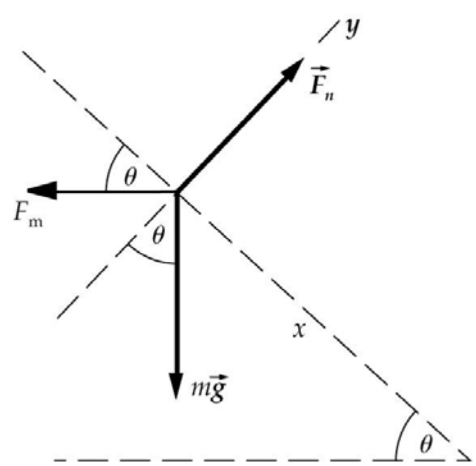
$$V = 2RE$$

Substitute numerical values and evaluate V:

$$V = 2(0.02\,\text{m})(0.138\,\text{mV/m}) = \boxed{5.52\,\mu\text{V}}$$

49 ••

Picture the Problem The free-body diagram shows the forces acting on the rod as it slides down the inclined plane. The retarding force is the component of F_m acting up the incline; i.e., in the $-x$ direction. We can express F_m using the expression for the force acting on a conductor moving in a magnetic field. We can apply the expression for a motional emf in conjunction with Ohm's law to find the induced current in the rod. In part (*b*) we can apply Newton's 2nd law to obtain an expression for dv/dt and set this expression equal to zero to obtain v_t.



(*a*) Noting that only the horizontal component of the rod's velocity $\vec{v}$ produces an induced emf, express $\mathcal{E}$ due to the motion of the rod in the magnetic field:

$$\mathcal{E} = B\ell v\cos\theta$$

Substitute numerical values and evaluate $\mathcal{E}$:

$$\mathcal{E} = (1.2\,\text{T})(15\,\text{m})v(\cos 30°) = \boxed{(15.6\,\text{T}\cdot\text{m})v}$$

(*b*) Apply Newton's 2nd law to the rod:

$$mg\sin\theta - F_{\text{m}}\cos\theta = m\frac{dv}{dt}$$

and

$$\frac{dv}{dt} = g\sin\theta - \frac{F_{\text{m}}}{m}\cos\theta \qquad (1)$$

where

$$F_{\text{m}} = I\ell B$$

and I is the current induced in the rod as a consequence of its motion in the magnetic field.

Using Ohm's law, relate the current I in the circuit to the induced emf:

$$I = \frac{\mathcal{E}}{R} = \frac{B\ell v\cos\theta}{R}$$

and

$$F_{\text{m}} = \frac{B^2\ell^2 v\cos\theta}{R}$$

Substitute in equation (1) to obtain the equation of motion of the rod:

$$\frac{dv}{dt} = g\sin\theta - \frac{B^2\ell^2 v}{mR}\cos^2\theta$$

When the rod reaches its terminal velocity v_{t}, $dv/dt = 0$:

$$0 = g\sin\theta - \frac{B^2\ell^2 v_{\text{t}}}{mR}\cos^2\theta$$

Solve for v_{t}:

$$v_{\text{t}} = \frac{mgR\sin\theta}{B^2\ell^2\cos^2\theta}$$

Substitute numerical values and evaluate v_{t}:

$$v_{\text{t}} = \frac{(0.4\,\text{kg})(9.81\,\text{m/s}^2)(2\,\Omega)\sin 30°}{(1.2\,\text{T})^2(15\,\text{m})^2\cos^2 30°} = \boxed{1.61\,\text{cm/s}}$$

50 •••

Picture the Problem Let F_{f} be the friction force between the rails and cylinder, F_{m} the magnetic force on the cylinder, and I_{m} the cylinder's moment of inertia. Because the current through the rod is uniformly distributed, we can treat the current as though it were concentrated at the center of the rod. We can find the magnitude of $\vec{\boldsymbol{B}}$ by applying Newton's 2nd law to the cylinder. The application of Ohm's law to the circuit will allow us to express the net force acting on the cylinder in terms of its speed. Setting this net force equal to zero will lead us to a value for the terminal velocity of the cylinder. We can use the definition of kinetic energy (both translational and rotational) to find the kinetic energy of the cylinder when it has reached its terminal velocity.

(*a*) Apply $\sum F_x = ma_x$ to the cylinder:

$$F_{\mathrm{m}} - F_{\mathrm{f}} = ma_x$$

or

$$BIa - F_{\mathrm{f}} = m\frac{dv}{dt} = mr\frac{d\omega}{dt}$$

Apply $\sum \tau = I\alpha$ to the cylinder:

$$F_{\mathrm{f}} r = I_{\mathrm{m}}\frac{d\omega}{dt}$$

Solve for F_{f} and substitute to obtain:

$$BIa - \frac{I_{\mathrm{m}}}{r}\frac{d\omega}{dt} = mr\frac{d\omega}{dt}$$

Solve for $r\dfrac{d\omega}{dt}$:

$$r\frac{d\omega}{dt} = \frac{BIa}{m + \dfrac{I_{\mathrm{m}}}{r^2}} = \frac{BIa}{m + \dfrac{\frac{1}{2}mr^2}{r^2}} = \frac{2BIa}{3m}$$

or

$$\frac{dv}{dt} = \frac{2BIa}{3m} \qquad (1)$$

Solve for B:

$$B = \frac{3m\dfrac{dv}{dt}}{2Ia}$$

Apply Ohm's law to the circuit to find I:

$$I = \frac{\mathcal{E}}{R} = \frac{12\,\mathrm{V}}{6\,\Omega} = 2\,\mathrm{A}$$

Substitute numerical values and evaluate B:

$$B = \frac{3(4\,\mathrm{kg})(0.1\,\mathrm{m/s^2})}{2(2\mathrm{A})(0.4\,\mathrm{m})} = \boxed{0.750\,\mathrm{T}}$$

Apply $\vec{F} = I\vec{\ell} \times \vec{B}$ to determine the direction of $\vec{B}$:

$$\boxed{\vec{B}\text{ is downward.}}$$

(*b*) Multiply both sides of equation (1) by m to express the net force acting on the cylinder:

$$F_{\mathrm{net}} = m\frac{dv}{dt} = \frac{2BIa}{3}$$

Use Ohm's law to express the current as a function of the emf of

$$I = \frac{\mathcal{E} - Bav}{R}$$

the battery and the induced emf in the cylinder:

Substitute to express the net force acting on the cylinder as a function of the velocity of the cylinder:

$$F_{\text{net}} = \frac{2B\left(\frac{\mathcal{E} - Bav}{R}\right)a}{3} = \boxed{\frac{2Ba\mathcal{E}}{3R} - \frac{2B^2a^2}{3R}v}$$

(*c*) Set $F_{\text{net}} = 0$ and solve for the terminal velocity of the cylinder:

$$\frac{2Ba\mathcal{E}}{3R} - \frac{2B^2a^2}{3R}v_{\text{t}} = 0$$

and

$$v_{\text{t}} = \frac{\mathcal{E}}{Ba} = \frac{12\,\text{V}}{(0.75\,\text{T})(0.4\,\text{m})} = \boxed{40.0\,\text{m/s}}$$

(*d*) Express the total kinetic energy of the cylinder when it has reached its terminal velocity:

$$K = \tfrac{1}{2}mv_{\text{t}}^2 + \tfrac{1}{2}I_{\text{m}}\omega_{\text{t}}^2 = \tfrac{1}{2}mv_{\text{t}}^2 + \tfrac{1}{2}\left(\tfrac{1}{2}mr^2\right)\frac{v_{\text{t}}^2}{r^2} = \tfrac{3}{4}mv_{\text{t}}^2$$

Substitute numerical values and evaluate *K*:

$$K = \tfrac{3}{4}(4\,\text{kg})(40\,\text{m/s})^2 = \boxed{4.80\,\text{kJ}}$$

***51 •••**

Picture the Problem We can use the expression for a motional emf and Ampere's law to express the net emf induced in the moving loop. We can also use express the magnetic flux through the loop and apply Faraday's law to obtain the same result.

(*a*) Express the motional emf induced in the segments parallel to the current-carrying wire:

$$\mathcal{E} = B(x)vb$$

Using Ampere's law, express $B(d + vt)$ and $B(d + a + vt)$:

$$B(d + vt) = \frac{\mu_0 I}{2\pi(d + vt)}$$

and

$$B(d + a + vt) = \frac{\mu_0 I}{2\pi(d + a + vt)}$$

Substitute to express $\mathcal{E}_1$ for the near wire and $\mathcal{E}_2$ for the far wire:

$$\mathcal{E}_1 = \frac{\mu_0 Ivb}{2\pi(d + vt)}$$

and

$$\mathcal{E}_1 = \frac{\mu_0 Ivb}{2\pi(d + a + vt)}$$

Noting that the emfs both point upward and hence oppose one another, express the net emf induced in the loop:

$$\begin{aligned}\mathcal{E} &= \mathcal{E}_1 - \mathcal{E}_2 \\ &= \frac{\mu_0 Ivb}{2\pi(d + vt)} - \frac{\mu_0 Ivb}{2\pi(d + a + vt)} \\ &= \boxed{\frac{\mu_0 Ivb}{2\pi}\left(\frac{1}{d + vt} - \frac{1}{d + a + vt}\right)}\end{aligned}$$

The motion of the segments perpendicular to the long wire does not change the flux through the rectangular loop. Consequently, these segments do not contribute to the the induced emf.

(*b*) From Faraday's law we have:

$$\mathcal{E} = -\frac{d\phi_{\mathrm{m}}}{dt}$$

Express the magnetic flux in an area of length b and width vdt:

$$d\phi_{\mathrm{m}} = B(x)dA = B(x)b\,dx$$

where, from Ampere's law,

$$B(x) = \frac{\mu_0 I}{2\pi x}$$

Substitute and integrate from $x = d + vt$ to $d + a + vt$:

$$\begin{aligned}\phi_{\mathrm{m}} &= \int_{d+vt}^{d+a+vt} B(x)dx = \frac{\mu_0 Ib}{2\pi}\int_{d+vt}^{d+a+vt}\frac{dx}{x} \\ &= \frac{\mu_0 Ib}{2\pi}\ln\left[\frac{d + a + vt}{d + vt}\right]\end{aligned}$$

Differentiate with respect to time and simplify to obtain:

$$\begin{aligned}\mathcal{E} &= -\frac{d}{dt}\left[\frac{\mu_0 Ib}{2\pi}\ln\frac{d + a + vt}{d + vt}\right] = -\frac{\mu_0 Ib}{2\pi}\frac{d}{dt}\left[\ln\frac{d + a + vt}{d + vt}\right] \\ &= -\frac{\mu_0 Ib}{2\pi}\left[\left(\frac{d + vt}{d + a + vt}\right)\left(\frac{(d + vt)v - (d + a + vt)v}{(d + vt)^2}\right)\right] \\ &= -\frac{\mu_0 Ibv}{2\pi}\left[\frac{(d + vt) - (d + a + vt)}{(d + vt)(d + a + vt)}\right] = -\frac{\mu_0 Ibv}{2\pi}\left[\frac{1}{d + a + vt} - \frac{1}{d + vt}\right] \\ &= \boxed{\frac{\mu_0 Ibv}{2\pi}\left[\frac{1}{d + vt} - \frac{1}{d + a + vt}\right]}\end{aligned}$$

52 •••

Picture the Problem We can use $\vec{F} = q\vec{v} \times \vec{B}$ to express the magnetic force acting on the moving charged body. Expressing the emf induced in a segment of the rod of length *dr* and integrating this expression over the length of the rod will lead us to an expression for the induced emf.

(*a*) Using the equation for the magnetic force on a moving charged body, express the force acting on the charged body a distance *r* from the pivot:

$$\vec{F} = q\vec{v} \times \vec{B}$$

and

$$F = qvB\sin\theta$$

Because $\vec{v} \perp \vec{B}$ and $v = r\omega$:

$$F = \boxed{qBr\omega}$$

(*b*) Use the motional emf equation to express the emf induced in a segment of the rod of length *dr* and at a distance *r* from the pivot:

$$d\mathcal{E} = Brdv = Br\omega dr$$

Integrate this expression from $r = 0$ to $r = \ell$ to obtain:

$$\int_0^{\mathcal{E}} d\mathcal{E}' = B\omega\int_0^{\ell} rdr$$

and

$$\mathcal{E} = \boxed{\tfrac{1}{2}B\omega\ell^2}$$

(*c*) Using Faraday's law, relate the induced emf to the rate at which the flux changes:

$$|\mathcal{E}| = \frac{d\phi_m}{dt}$$

Express the area *dA*, for any value of θ, between *r* and $r + dr$:

$$dA = r\theta dr$$

Integrate from $r = 0$ to $r = \ell$ to obtain:

$$A = \theta\int_0^{\ell} rdr = \tfrac{1}{2}\theta\ell^2$$

Using its definition, express the magnetic flux through this area:

$$\phi_m = BA = \boxed{\tfrac{1}{2}B\ell^2\theta}$$

Differentiate ϕ_m with respect to time to obtain:

$$|\mathcal{E}| = \frac{d}{dt}\left[\tfrac{1}{2}B\ell^2\theta\right] = \tfrac{1}{2}B\ell^2\frac{d\theta}{dt} = \boxed{\tfrac{1}{2}B\ell^2\omega}$$

Inductance

53 •

Picture the Problem We can use $\phi_m = LI$ and the dependence of I on t to find the magnetic flux through the coil. We can apply Faraday's law to find the induced emf in the coil.

(*a*) Use the definition of self-inductance to express ϕ_m:

$$\phi_m = LI$$

Express I as a function of time:

$$I = 3\,\text{A} + (200\,\text{A/s})t$$

Substitute to obtain:

$$\phi_m = L[3\,\text{A} + (200\,\text{A/s})t]$$

Substitute numerical values and express ϕ_m:

$$\phi_m = (8\,\text{H})[3\,\text{A} + (200\,\text{A/s})t] = \boxed{24\,\text{Wb} + (1600\,\text{H}\cdot\text{A/s})t}$$

(*b*) Use Faraday's law to relate ε, L, and dI/dt:

$$\mathcal{E} = -L\frac{dI}{dt}$$

Substitute numerical values and evaluate ε:

$$\mathcal{E} = -(8\,\text{H})(200\,\text{A/s}) = \boxed{-1.60\,\text{kV}}$$

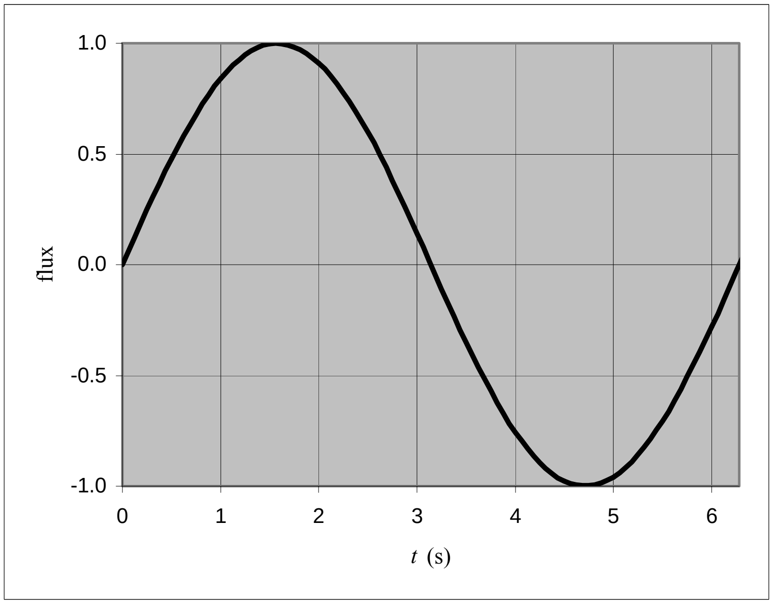
***54** •

Picture the Problem We can apply $\phi_m = LI$ to find ϕ_m and Faraday's law to find the self-induced emf as functions of time.

Use the definition of self-inductance to express ϕ_m:

$$\phi_m = LI = \boxed{LI_0 \sin 2\pi ft}$$

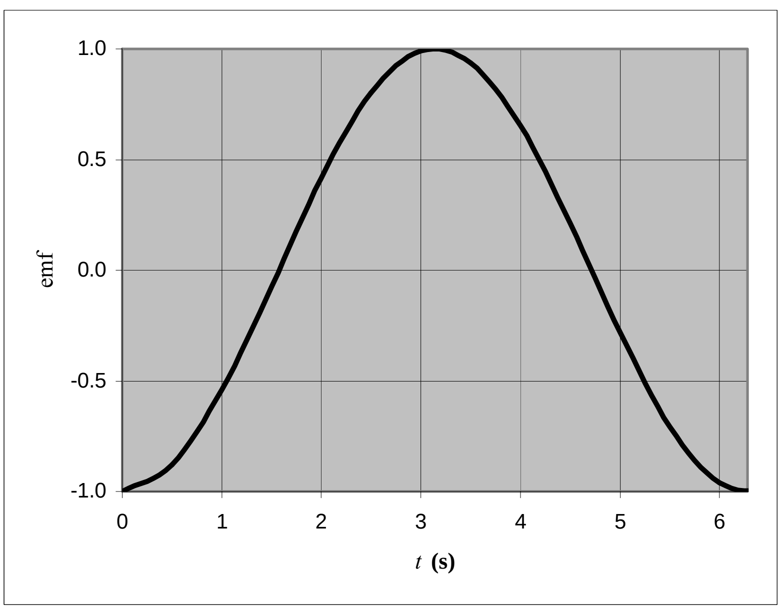
The graph of the flux ϕ_m as a function of time shown below was plotted using a spreadsheet program. The maximum value of the flux is LI_0 and we have chosen $2\pi f = 1$ rad/s.



Apply Faraday's law to relate ε, L, and dI/dt:

$$\varepsilon = -L\frac{dI}{dt} = -L\frac{d}{dt}\left[I_0 \sin 2\pi ft\right]$$
$$= \boxed{-2\pi fLI_0 \cos 2\pi ft}$$

The graph of the emf ε as a function of time shown below was plotted using a spreadsheet program. The maximum value of the induced emf is $2\pi fLI_0$ and we have chosen $2\pi f = 1$ rad/s.



55 ••

Picture the Problem We can use $B = \mu_0 nI$ to find the magnetic field on the axis at the center of the solenoid and the definition of magnetic flux to evaluate ϕ_m. We can use the definition of magnetic flux in terms of L and I to find the self-inductance of the solenoid. Finally, we can use Faraday's law to find the induced emf in the solenoid when the current changes at 150 A/s.

(*a*) Apply the expression for B inside a long solenoid to express and evaluate B:

$$B = \mu_0 nI = \left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(\frac{400}{0.25\ \text{m}}\right)(3\ \text{A}) = \boxed{6.03\ \text{mT}}$$

(*b*) Apply the definition of magnetic flux to obtain:

$$\phi_m = NBA = (400)(6.03\ \text{mT})\pi(0.01\ \text{m})^2 = \boxed{7.58\times10^{-4}\ \text{Wb}}$$

(*c*) Relate the self-inductance of the solenoid to the magnetic flux through it and its current:

$$L = \frac{\phi_m}{I} = \frac{7.58\times10^{-4}\ \text{Wb}}{3\ \text{A}} = \boxed{0.253\ \text{mH}}$$

(*d*) Apply Faraday's law to obtain:

$$\mathcal{E} = -L\frac{dI}{dt} = -(0.253\ \text{mH})(150\ \text{A/s}) = \boxed{-38.0\ \text{mV}}$$

56 ••

Picture the Problem We can find the mutual inductance of the two coaxial solenoids using $M_{2,1} = \dfrac{\phi_{m2}}{I_1} = \mu_0 n_2 n_1 \ell \pi r_1^2$.

Substitute numerical values and evaluate $M_{2,1}$:

$$M_{2,1} = \left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(\frac{300}{0.25\ \text{m}}\right)\left(\frac{1000}{0.25\ \text{m}}\right)(0.25\ \text{m})\pi(0.02\ \text{m})^2 = \boxed{1.89\ \text{mH}}$$

***57** ••

Picture the Problem Note that the current in the two parts of the wire is in opposite directions. Consequently, the total flux in the coil is zero. We can find the resistance of the wire-wound resistor from the length of wire used and the resistance per unit length.

Because the total flux in the coil is zero:

$$L = \boxed{0}$$

Express the total resistance of the wire:

$$R = \left(18\,\frac{\Omega}{\text{m}}\right)L = \left(18\,\frac{\Omega}{\text{m}}\right)(9\,\text{m}) = \boxed{162\,\Omega}$$

58 •••

Picture the Problem We can apply Kirchhoff's loop rule to the galvanometer circuit to relate the potential difference across L_2 to the potential difference across R_2. Integration of this equation over time will yield an equation that relates the mutual inductance between the two coils to the steady-state current in circuit 1 and the charge that flows through the galvanometer.

Apply Kirchhoff's loop rule to the galvanometer circuit:

$$M\frac{dI_1}{dt} + L_2\frac{dI_2}{dt} - R_2I_2 = 0$$

or

$$MdI_1 + L_2dI_2 - R_2I_2dt = 0$$

Integrate each term from $t = 0$ to $t = \infty$:

$$M\int_0^\infty dI_1 + L_2\int_0^\infty dI_2 - R_2\int_0^\infty I_2dt = 0$$

and

$$MI_{1\infty} + L_2I_{2\infty} - R_2Q = 0$$

Because $I_{2\infty} = 0$:

$$MI_{1\infty} - R_2Q = 0$$

Solve for M:

$$M = \frac{R_2Q}{I_{1\infty}}$$

Substitute numerical values and evaluate M:

$$M = \frac{(300\,\Omega)(2\times10^{-4}\,\text{C})}{5\,\text{A}} = \boxed{12.0\,\text{mH}}$$

59 •••

Picture the Problem We can use Ampere's law to express the magnetic field inside the rectangular toroid and the definition of magnetic flux to express ϕ_m through the toroid. We can then use the definition of self-inductance of a solenoid to express L.

Using the definition of the self-inductance of a solenoid, express L in terms of ϕ_m, N, and I:

$$L = \frac{N\phi_\text{m}}{I} \qquad (1)$$

Apply Ampere's law to a closed path or radius $a < r < b$:

$$\oint_C \vec{B}\cdot d\vec{\ell} = B2\pi r = \mu_0 I_C$$

or, because $I_C = NI$,

$$B2\pi r = \mu_0 NI$$

Solve for B to obtain:

$$B = \frac{\mu_0 NI}{2\pi r}$$

Express the flux in a strip of height H and width dr:

$$d\phi_m = BHdr$$

Substitute for B and integrate $d\phi_m$ from $r = a$ to $r = b$ to obtain:

$$\phi_m = \frac{\mu_0 NIH}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 NIH}{2\pi}\ln\left(\frac{b}{a}\right)$$

Substitute in equation (1) and simplify to obtain:

$$L = \boxed{\frac{\mu_0 N^2 H}{2\pi}\ln\left(\frac{b}{a}\right)}$$

Magnetic Energy

60 •

Picture the Problem The current in an LR circuit, as a function of time, is given by $I = I_f\left(1 - e^{-t/\tau}\right)$, where $I_f = \varepsilon_0/R$ and $\tau = L/R$. The energy stored in the inductor under steady-state conditions is stored in its magnetic field and is given by $U_m = \frac{1}{2}LI_f^2$.

(a) Express and evaluate I_f:

$$I_f = \frac{\varepsilon_0}{R} = \frac{24\,\text{V}}{12\,\Omega} = \boxed{2.00\,\text{A}}$$

(b) Express and evaluate the energy stored in an inductor:

$$U_m = \frac{1}{2}LI_f^2 = \frac{1}{2}(2\,\text{H})(2\,\text{A})^2 = \boxed{4.00\,\text{J}}$$

***61 ••**

Picture the Problem We can examine the ratio of u_m to u_E with $E = cB$ and $c = 1/\sqrt{\varepsilon_0\mu_0}$ to show that the electric and magnetic energy densities are equal.

Express the ratio of the energy density in the magnetic field to the energy density in the electric field:

$$\frac{u_m}{u_E} = \frac{\dfrac{B^2}{2\mu_0}}{\frac{1}{2}\varepsilon_0 E^2} = \frac{B^2}{\mu_0\varepsilon_0 E^2}$$

Substitute $E = cB$:

$$\frac{u_m}{u_E} = \frac{B^2}{\mu_0\varepsilon_0 c^2 B^2} = \frac{1}{\mu_0\varepsilon_0 c^2}$$

Substitute for c:

$$\frac{u_m}{u_E}=\frac{\mu_0\varepsilon_0}{\mu_0\varepsilon_0}=1 \Rightarrow \boxed{u_m=u_E}$$

62 ••

Picture the Problem We can use $L=\mu_0 n^2 A\ell$ to find the inductance of the solenoid and $B=\mu_0 nI$ to find the magnetic field inside it.

(*a*) Express the magnetic energy stored in the solenoid:

$$U_m=\tfrac{1}{2}LI^2$$

Relate the inductance of the solenoid to its dimensions and properties:

$$L=\mu_0 n^2 A\ell$$

Substitute to obtain:

$$U_m=\tfrac{1}{2}\mu_0 n^2 A\ell I^2$$

Substitute numerical values and evaluate U_m:

$$\begin{aligned}U_m&=\tfrac{1}{2}\left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(\frac{2000}{0.3\,\text{m}}\right)^2\\&\quad\times\left(4\times10^{-4}\ \text{m}^2\right)(0.3\,\text{m})(4\,\text{A})^2\\&=\boxed{53.6\,\text{mJ}}\end{aligned}$$

(*b*) The magnetic energy per unit volume in the solenoid is:

$$\begin{aligned}\frac{U_m}{V}&=\frac{U_m}{A\ell}=\frac{53.6\,\text{mJ}}{\left(4\times10^{-4}\ \text{m}^2\right)(0.3\,\text{m})}\\&=\boxed{447\ \text{J/m}^3}\end{aligned}$$

(*c*) Express the magnetic field in the solenoid in terms of n and I:

$$\begin{aligned}B&=\mu_0 nI=\mu_0\frac{N}{\ell}I\\&=\frac{\left(4\pi\times10^{-7}\ \text{N/A}^2\right)(2000)(4\,\text{A})}{0.3\,\text{m}}\\&=\boxed{33.5\,\text{mT}}\end{aligned}$$

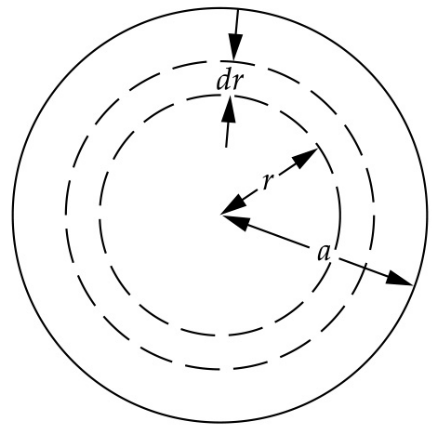
(*d*) The magnetic energy density is:

$$\begin{aligned}u_m&=\frac{B^2}{2\mu_0}=\frac{(33.5\,\text{mT})^2}{2\left(4\pi\times10^{-7}\ \text{N/A}^2\right)}\\&=\boxed{447\ \text{J/m}^3}\end{aligned}$$

in agreement with our result in Part (*b*).

63 ••

Picture the Problem Consider a cylindrical annulus of thickness dr at a radius $r < a$. We can use its definition to express the total magnetic energy dU_m inside the cylindrical annulus and divide both sides of this expression by the length of the wire to express the magnetic energy per unit length dU'_m. Integration of this expression will give us the magnetic energy per unit length within the wire.



Express the magnetic energy within the cylindrical annulus:

$$dU_m = \frac{B^2}{2\mu_0} V_{\text{annulus}} = \frac{B^2}{2\mu_0} 2\pi r \ell dr = \frac{B^2}{\mu_0} \pi r \ell dr$$

Divide both sides of the equation by ℓ to express the magnetic energy per unit length dU'_m:

$$dU'_m = \frac{B^2}{\mu_0} \pi r dr \qquad (1)$$

Use Ampere's law to express the magnetic field inside the wire at a distance $r < a$ from its center:

$$2\pi r B = \mu_0 I_C$$

and

$$B = \frac{\mu_0 I_C}{2\pi r}$$

where I_C is the current inside the cylinder of radius r.

Because the current is uniformly distributed over the cross-sectional area of the wire:

$$\frac{I_C}{I} = \frac{\pi r^2}{\pi a^2} \Rightarrow I_C = \frac{r^2}{a^2} I$$

Substitute to obtain:

$$B = \frac{\mu_0 r I}{2\pi a^2}$$

Substitute for B in equation (1) to obtain:

$$dU'_m = \frac{\left(\frac{\mu_0 r I}{2\pi a^2}\right)^2}{\mu_0} \pi r dr = \frac{\mu_0 I^2}{4\pi a^4} r^3 dr$$

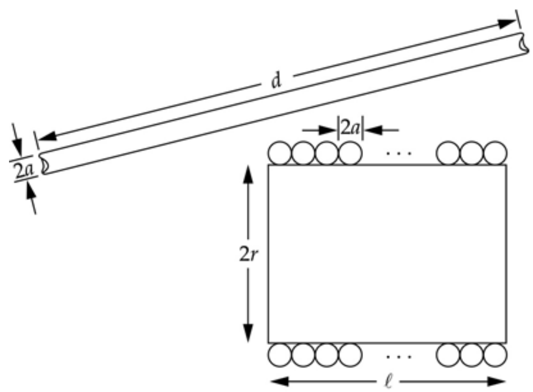
Integrate dU'_m from $r = 0$ to $r = a$:

$$U'_m = \frac{\mu_0 I^2}{4\pi a^4}\int_0^a r^3 dr = \frac{\mu_0 I^2}{4\pi a^4}\cdot\frac{a^4}{4} = \boxed{\frac{\mu_0 I^2}{16\pi}}$$

Remarks: Note that the magnetic energy per unit length is independent of the radius of the cylinder and depends only on the total current.

***64 ••**

Picture the Problem The wire of length d and radius a is shown in the diagram, as is the inductor constructed with this wire and whose inductance L is to be found. We can use the equation for the self-inductance of a cylindrical inductor to derive an expression for L.



The self-inductance of an inductor with length ℓ, cross-sectional area A, and number of turns per unit length n is:

$$L = \mu_0 n^2 A\ell \qquad (1)$$

The number of turns N is given by:

$$N = \frac{\ell}{2a}$$

The number of turns per unit length n is:

$$n = \frac{N}{\ell} = \frac{1}{2a}$$

Assuming that $a << r$, the length of the wire d is related to n and r:

$$d = N(2\pi r) = \left(\frac{\ell}{2a}\right)2\pi r = \frac{\pi r}{a}\ell$$

Solve for ℓ to obtain:

$$\ell = \frac{ad}{\pi r}$$

Substitute for ℓ, A, and n in equation (1) to obtain:

$$L = \mu_0\left(\frac{1}{2a}\right)^2(\pi r^2)\left(\frac{ad}{\pi r}\right) = \boxed{\mu_0\left(\frac{rd}{4a}\right)}$$

65 •

Picture the Problem We can substitute numerical values in the expression derived in Problem 64 to find the self-inductance of the inductor.

From Problem 64 we have:

$$L=\frac{\mu_0 rR}{4a}$$

Substitute numerical values and evaluate L:

$$L=\frac{\left(4\pi\times10^{-7}\ \text{N/A}^2\right)(0.25\,\text{cm})(10\,\text{cm})}{4(0.5\,\text{mm})}=\boxed{0.157\ \mu\text{H}}$$

66 ••

Picture the Problem We can find the number of turns on the coil from the length of the superconducting wire and the cross-sectional radius of the coil. We can use $B=(\mu_0 NI)/(2\pi r_{\text{mean}})$ to find the magnetic field at the mean radius. We can find the energy density in the magnetic field from $u_{\text{m}}=B^2/(2\mu_0)$ and the total energy stored in the toroid by multiplying u_{m} by the volume of the toroid.

(*a*) Express the number of turns in terms of the length of the wire L and length required per turn $2\pi r$:

$$N=\frac{L}{2\pi r}=\frac{1000\,\text{m}}{2\pi(0.02\,\text{m})}=\boxed{7958}$$

(*b*) Use the equation for B inside a tightly wound toroid to find the magnetic field at the mean radius:

$$B=\frac{\mu_0 NI}{2\pi r_{\text{mean}}}=\frac{\left(4\pi\times10^{-7}\ \text{N/A}^2\right)(7958)(400\,\text{A})}{2\pi(0.25\,\text{m})}=\boxed{2.55\,\text{T}}$$

(*c*) Express and evaluate the energy density in the magnetic field:

$$u_{\text{m}}=\frac{B^2}{2\mu_0}=\frac{(2.55\,\text{T})^2}{2\left(4\pi\times10^{-7}\ \text{N/A}^2\right)}=\boxed{2.59\times10^{6}\ \text{J/m}^3}$$

Relate the total energy stored in the toroid to the energy density in its magnetic field and the volume of the toroid:

$$U_{\text{m}}=u_{\text{m}}V_{\text{toroid}}$$

Think of the toroid as a cylinder of radius r and height $2\pi r_{\text{mean}}$ to obtain:

$$V_{\text{toroid}}=\pi r^2\left(2\pi r_{\text{mean}}\right)=2\pi^2 r^2 r_{\text{mean}}$$

Substitute for V_{toroid} to obtain:

$$U_{\text{m}}=2\pi^2 r^2 r_{\text{mean}}u_{\text{m}}$$

Substitute numerical values and evaluate U_m:

$$U_m = 2\pi^2(0.02\,\text{m})^2(0.25\,\text{m})(2.59\times10^6\ \text{J/m}^3) = \boxed{5.11\,\text{kJ}}$$

RL Circuits

67 •

Picture the Problem We can find the current using $I = I_f\left(1-e^{-t/\tau}\right)$ where $I_f = \varepsilon_0/R$ and $\tau = L/R$ and its rate of change by differentiating this expression with respect to time.

Express the dependence of the current on I_f and τ:

$$I = I_f\left(1-e^{-t/\tau}\right)$$

Evaluate I_f and τ:

$$I_f = \frac{\varepsilon_0}{R} = \frac{100\,\text{V}}{8\,\Omega} = 12.5\,\text{A}$$

and

$$\tau = \frac{L}{R} = \frac{4\,\text{H}}{8\,\Omega} = 0.5\,\text{s}$$

Substitute to obtain:

$$I = (12.5\,\text{A})\left(1-e^{-t/0.5\,\text{s}}\right) = (12.5\,\text{A})\left(1-e^{-2t\,\text{s}^{-1}}\right)$$

Express dI/dt:

$$\frac{dI}{dt} = (12.5\,\text{A})\left(-e^{-2t\,\text{s}^{-1}}\right)\left(-2\,\text{s}^{-1}\right) = (25\,\text{A/s})e^{-2t\,\text{s}^{-1}}$$

(*a*) When $t = 0$:

$$I = (12.5\,\text{A})\left(1-e^{0}\right) = \boxed{0}$$

and

$$\frac{dI}{dt} = (25\,\text{A/s})e^{0} = \boxed{25.0\,\text{A/s}}$$

(*b*) When $t = 0.1$ s:

$$I = (12.5\,\text{A})\left(1-e^{-0.2}\right) = \boxed{2.27\,\text{A}}$$

and

$$\frac{dI}{dt} = (25\,\text{A/s})e^{-0.2} = \boxed{20.5\,\text{A/s}}$$

(*c*) When $t = 0.5$ s:

$$I = (12.5\,\text{A})\left(1-e^{-1}\right) = \boxed{7.90\,\text{A}}$$

and

$$\frac{dI}{dt} = (25\,\text{A/s})e^{-1} = \boxed{9.20\,\text{A/s}}$$

(*d*) When $t = 1.0$ s:

$$I = (12.5\,\text{A})\left(1 - e^{-2}\right) = \boxed{10.8\,\text{A}}$$

and

$$\frac{dI}{dt} = (25\,\text{A/s})e^{-2} = \boxed{3.38\,\text{A/s}}$$

68 •

Picture the Problem We can find the current using $I = I_0 e^{-t/\tau}$, where I_0 is the current at time $t = 0$ and $\tau = L/R$.

Express the current as a function of time:

$$I = I_0 e^{-t/\tau} = (2\,\text{A})e^{-t/\tau}$$

Evaluate τ:

$$\tau = \frac{L}{R} = \frac{1\,\text{mH}}{10\,\Omega} = 10^{-4}\,\text{s}$$

Substitute to obtain:

$$I = (2\,\text{A})e^{-10^4 t\,\text{s}^{-1}}$$

(*a*) When $t = 0.5$ ms:

$$I = (2\,\text{A})e^{-10^4\left(0.5\times10^{-3}\,\text{s}\right)\text{s}^{-1}} = (2\,\text{A})e^{-5}$$
$$= \boxed{13.5\,\text{mA}}$$

(*b*) When $t = 10$ ms:

$$I = (2\,\text{A})e^{-10^4\left(10\times10^{-3}\,\text{s}\right)\text{s}^{-1}} = (2\,\text{A})e^{-100}$$
$$= 7.44\times10^{-44}\,\text{A} \approx \boxed{0}$$

***69** ••

Picture the Problem We can find the current using $I = I_\text{f}\left(1 - e^{-t/\tau}\right)$, where $I_\text{f} = \mathcal{E}_0/R$, and $\tau = L/R$, and its rate of change by differentiating this expression with respect to time.

Express the dependence of the current on I_f and τ:

$$I = I_\text{f}\left(1 - e^{-t/\tau}\right)$$

Evaluate I_f and τ:

$$I_\text{f} = \frac{\mathcal{E}_0}{R} = \frac{12\,\text{V}}{3\,\Omega} = 4\,\text{A}$$

and

$$\tau = \frac{L}{R} = \frac{0.6\,\text{H}}{3\,\Omega} = 0.2\,\text{s}$$

Substitute to obtain:

$$I = (4\,\text{A})\left(1 - e^{-t/0.2\,\text{s}}\right) = (4\,\text{A})\left(1 - e^{-5t\,\text{s}^{-1}}\right)$$

Express dI/dt:

$$\frac{dI}{dt} = (4\,\text{A})\left(-e^{-5t\,\text{s}^{-1}}\right)\left(-5\,\text{s}^{-1}\right) = (20\,\text{A/s})e^{-5t\,\text{s}^{-1}}$$

(*a*) Find the current at $t = 0.5$ s:

$$I(0.5\,\text{s}) = (4\,\text{A})\left(1 - e^{-5(0.5\,\text{s})\text{s}^{-1}}\right) = 3.67\,\text{A}$$

The rate at which the battery supplies power at $t = 0.5$ s is:

$$P(0.5\,\text{s}) = I(0.5\,\text{s})\mathcal{E} = (3.67\,\text{A})(12\,\text{V}) = \boxed{44.0\,\text{W}}$$

(*b*) The rate of joule heating is:

$$P_\text{J}(0.5\,\text{s}) = [I(0.5\,\text{s})]^2 R = (3.67\,\text{A})^2(3\,\Omega) = \boxed{40.4\,\text{W}}$$

(*c*) Using the expression for the magnetic energy stored in an inductor, express the rate at which energy is being stored:

$$\frac{dU_\text{L}}{dt} = \frac{d}{dt}\left[\tfrac{1}{2}LI^2\right] = LI\frac{dI}{dt}$$

Substitute for *L, I,* and *dI*/*dt* to obtain:

$$\frac{dU_\text{L}}{dt} = \frac{d}{dt}\left[\tfrac{1}{2}LI^2\right] = LI\frac{dI}{dt}$$

Substitute numerical values and evaluate $\frac{dU_\text{L}}{dt}$:

$$\frac{dU_\text{L}}{dt} = (0.6\,\text{H})(4\,\text{A})\left(1 - e^{-5t\,\text{s}^{-1}}\right)(20\,\text{A/s})e^{-5t\,\text{s}^{-1}} = (48\,\text{W})\left(1 - e^{-5t\,\text{s}^{-1}}\right)e^{-5t\,\text{s}^{-1}}$$

Evaluate this expression for $t = 0.5$ s:

$$\frac{dU_\text{L}}{dt} = (48\,\text{W})\left(1 - e^{-5(0.5\,\text{s})\text{s}^{-1}}\right)e^{-5(0.5\,\text{s})\text{s}^{-1}} = (48\,\text{W})\left(1 - e^{-2.5}\right)e^{-2.5} = \boxed{3.62\,\text{W}}$$

Remarks: Note that, to a good approximation, $dU_L/dt = P - P_J$.

70 ••

Picture the Problem We can find the current using $I = I_f\left(1 - e^{-t/\tau}\right)$, where $I_f = \varepsilon_0/R$ and $\tau = L/R$, and its rate of change by differentiating this expression with respect to time.

Express the dependence of the current on I_f and τ:

$$I = I_f\left(1 - e^{-t/\tau}\right)$$

Evaluate I_f and τ:

$$I_f = \frac{\mathcal{E}_0}{R} = \frac{12\,\text{V}}{3\,\Omega} = 4\,\text{A}$$

and

$$\tau = \frac{L}{R} = \frac{0.6\,\text{H}}{3\,\Omega} = 0.2\,\text{s}$$

Substitute to obtain:

$$I = (4\,\text{A})\left(1 - e^{-t/0.2\,\text{s}}\right) = (4\,\text{A})\left(1 - e^{-5t\,\text{s}^{-1}}\right)$$

Express dI/dt:

$$\frac{dI}{dt} = (4\,\text{A})\left(-e^{-5t\,\text{s}^{-1}}\right)\left(-5\,\text{s}^{-1}\right) = (20\,\text{A/s})e^{-5t\,\text{s}^{-1}}$$

(*a*) Find the current at $t = 1$ s:

$$I(1\,\text{s}) = (4\,\text{A})\left(1 - e^{-5(1\,\text{s})\text{s}^{-1}}\right) = 3.97\,\text{A}$$

The rate at which the battery supplies power at $t = 1$ s:

$$P(1\,\text{s}) = I(1\,\text{s})\mathcal{E} = (3.97\,\text{A})(12\,\text{V}) = \boxed{47.7\,\text{W}}$$

Find the current at $t = 100$ s:

$$I(100\,\text{s}) = (4\,\text{A})\left(1 - e^{-5(100\,\text{s})\text{s}^{-1}}\right) = 4.00\,\text{A}$$

The rate at which the battery supplies power at $t = 100$ s:

$$P(100\,\text{s}) = I(100\,\text{s})\mathcal{E} = (4\,\text{A})(12\,\text{V}) = \boxed{48.0\,\text{W}}$$

(*b*) The rate of joule heating at $t = 1$ s is:

$$P_J(1\,\text{s}) = [I(1\,\text{s})]^2 R = (3.97\,\text{A})^2(3\,\Omega) = \boxed{47.3\,\text{W}}$$

The rate of joule heating at $t = 100$ s is:

$$P_J(100\,\text{s}) = (4\,\text{A})^2(3\Omega) = \boxed{48.0\,\text{W}}$$

Using the expression for the magnetic energy stored in an inductor, express the rate at which energy is being stored:

$$\frac{dU_L}{dt} = \frac{d}{dt}\left[\tfrac{1}{2}LI^2\right] = LI\frac{dI}{dt}$$

Substitute for L, I and dI/dt to obtain:

$$\begin{aligned}\frac{dU_L}{dt} &= \frac{d}{dt}\left[\tfrac{1}{2}LI^2\right] = LI\frac{dI}{dt}\\ &= (0.6\,\text{H})(4\,\text{A})\left(1 - e^{-5t\,\text{s}^{-1}}\right)\\ &\quad\times(20\,\text{A/s})e^{-5t\,\text{s}^{-1}}\\ &= (48\,\text{W})\left(1 - e^{-5t\,\text{s}^{-1}}\right)e^{-5t\,\text{s}^{-1}}\end{aligned}$$

Evaluate dU_L/dt for $t = 1$ s:

$$\begin{aligned}\frac{dU_L}{dt} &= (48\,\text{W})\left(1 - e^{-5(1\,\text{s})\text{s}^{-1}}\right)e^{-5(1\,\text{s})\text{s}^{-1}}\\ &= (48\,\text{W})\left(1 - e^{-5}\right)e^{-5}\\ &= \boxed{0.321\,\text{W}}\end{aligned}$$

Evaluate dU_L/dt for $t = 100$ s:

$$\begin{aligned}\frac{dU_L}{dt} &= (48\,\text{W})\left(1 - e^{-5(100\,\text{s})\text{s}^{-1}}\right)e^{-5(100\,\text{s})\text{s}^{-1}}\\ &= (48\,\text{W})\left(1 - e^{-500}\right)e^{-500}\\ &= \boxed{0}\end{aligned}$$

Remarks: Note that, to a good approximation, $dU_L/dt = P - P_J$.

71 ••

Picture the Problem If the current is initially zero in an LR circuit, its value at some later time t is given by $I = I_f\left(1 - e^{-t/\tau}\right)$, where $I_f = \varepsilon_0/R$ and $\tau = L/R$ is the time constant for the circuit. We can find the time constant of the circuit from the given information and then use the definition of the time constant to find the self-inductance.

(a) Express the current in the circuit as a function of time:

$$I = I_f\left(1 - e^{-t/\tau}\right) \text{ where } \tau = \frac{L}{R} \qquad (1)$$

Express the current when $t = 4$ s:

$$0.5I_f = I_f\left(1 - e^{-4\,\text{s}/\tau}\right)$$

or

$$0.5 = 1 - e^{-4\,\text{s}/\tau} \Rightarrow 0.5 = e^{-4\,\text{s}/\tau}$$

Take logarithms of both sides of this equation to obtain:

$$\ln\tfrac{1}{2} = -\frac{4\,\text{s}}{\tau}$$

Solve for and evaluate τ:

$$\tau = \frac{4\,\text{s}}{\ln 2} = \boxed{5.77\,\text{s}}$$

(*b*) Solve equation (1) for and evaluate L:

$$L = R\tau = (5\,\Omega)(5.77\,\text{s}) = \boxed{28.9\,\text{H}}$$

72 ••

Picture the Problem If the current is initially zero in an *LR* circuit, its value at some later time *t* is given by $I = I_\text{f}\left(1 - e^{-t/\tau}\right)$, where $I_\text{f} = \varepsilon_0/R$ and $\tau = L/R$ is the time constant for the circuit. We can find the number of time constants that must elapse before the current reaches any given fraction of its final value by solving this equation for t/τ.

Express the fraction of its final value to which the current has risen as a function of time:

$$\frac{I}{I_\text{f}} = 1 - e^{-t/\tau}$$

Solve for t/τ:

$$\frac{t}{\tau} = -\ln\left(1 - \frac{I}{I_\text{f}}\right)$$

(*a*) Evaluate t/τ for $I/I_\text{f} = 0.9$:

$$\left.\frac{t}{\tau}\right|_{90\%} = -\ln(1 - 0.9) = \boxed{2.30}$$

(*b*) Evaluate t/τ for $I/I_\text{f} = 0.99$:

$$\left.\frac{t}{\tau}\right|_{99\%} = -\ln(1 - 0.99) = \boxed{4.61}$$

(*c*) Evaluate t/τ for $I/I_\text{f} = 0.999$:

$$\left.\frac{t}{\tau}\right|_{99.9\%} = -\ln(1 - 0.999) = \boxed{6.91}$$

73 ••

Picture the Problem If the current is initially zero in an *LR* circuit, its value at some later time *t* is given by $I = I_\text{f}\left(1 - e^{-t/\tau}\right)$, where $I_\text{f} = \varepsilon_0/R$ and $\tau = L/R$ is the time constant for the circuit. We can find the rate of increase of the current by differentiating *I* with respect to time and the time for the current to reach any given fraction of its initial value by solving for *t*.

(*a*) Express the current in the circuit as a function of time:

$$I = \frac{\mathcal{E}_0}{R}\left(1 - e^{-t/\tau}\right)$$

Express the initial rate of increase of the current by differentiating this expression with respect to time:

$$\frac{dI}{dt} = \frac{\mathcal{E}_0}{R}\frac{d}{dt}\left(1 - e^{-t/\tau}\right) = \frac{\mathcal{E}_0}{R}\left(-e^{-t/\tau}\right)\left(-\frac{1}{\tau}\right) = \frac{\mathcal{E}_0}{\tau R}e^{-\frac{R}{L}t} = \frac{\mathcal{E}_0}{L}e^{-\frac{R}{L}t}$$

Evaluate dI/dt at $t = 0$ to obtain:

$$\left.\frac{dI}{dt}\right|_{t=0} = \frac{\mathcal{E}_0}{L}e^0 = \frac{12\,\text{V}}{4\,\text{mH}} = \boxed{3.00\,\text{kA/s}}$$

(*b*) When $I = 0.5I_f$:

$$0.5 = 1 - e^{-t/\tau} \Rightarrow e^{-t/\tau} = 0.5$$

Evaluate dI/dt with $e^{-t/\tau} = 0.5$ to obtain:

$$\left.\frac{dI}{dt}\right|_{e^{-t/\tau}=0.5} = 0.5\frac{\mathcal{E}_0}{L} = 0.5\left(\frac{12\,\text{V}}{4\,\text{mH}}\right) = \boxed{1.50\,\text{kA/s}}$$

(*c*) Calculate I_f from ε and R:

$$I_f = \frac{\mathcal{E}_0}{R} = \frac{12\,\text{V}}{150\,\Omega} = \boxed{80.0\,\text{mA}}$$

(*d*) When $I = 0.99I_f$:

$$0.99 = 1 - e^{-t/\tau} \Rightarrow e^{-t/\tau} = 0.01$$

Solve for and evaluate t:

$$t = -\tau\ln(0.01) = -\frac{L}{R}\ln(0.01) = -\frac{4\,\text{mH}}{150\,\Omega}\ln(0.01) = \boxed{0.123\,\text{ms}}$$

74 ••

Picture the Problem If the current is initially zero in an LR circuit, its value at some later time t is given by $I = I_f\left(1 - e^{-t/\tau}\right)$, where $I_f = \varepsilon_0/R$ and $\tau = L/R$ is the time constant for the circuit. We can find the time for the current to reach any given value by solving this equation for t.

Evaluate I_f and τ:

$$I_f = \frac{\mathcal{E}_0}{R} = \frac{250\,\text{V}}{8\,\Omega} = 31.25\,\text{A}$$

and

$$\tau = \frac{L}{R} = \frac{50\,\text{H}}{8\,\Omega} = 6.25\,\text{s}$$

Solve $I = I_f\left(1 - e^{-t/\tau}\right)$ for t:

$$t = -\tau \ln\left(1 - \frac{I}{I_f}\right)$$
$$= -(6.25\,\text{s})\ln\left(1 - \frac{I}{31.25\,\text{A}}\right)$$

(*a*) Evaluate t for $I = 10$ A:

$$t\big|_{10\,\text{A}} = -(6.25\,\text{s})\ln\left(1 - \frac{10\,\text{A}}{31.25\,\text{A}}\right)$$
$$= \boxed{2.41\,\text{s}}$$

(*b*) Evaluate t for $I = 30$ A:

$$t\big|_{30\,\text{A}} = -(6.25\,\text{s})\ln\left(1 - \frac{30\,\text{A}}{31.25\,\text{A}}\right)$$
$$= \boxed{20.1\,\text{s}}$$

***75 •••**

Picture the Problem The self-induced emf in the inductor is proportional to the rate at which the current through it is changing. Under steady-state conditions, $dI/dt = 0$ and so the self-induced emf in the inductor is zero. We can use Kirchhoff's loop rule to obtain the current through and the voltage across the inductor as a function of time.

(*a*) Because, under steady-state conditions, the self-induced emf in the inductor is zero and because the inductor has negligible resistance, we can apply Kirchhoff's loop rule to the loop that includes the source, the 10-Ω resistor, and the inductor to find the current drawn from the battery and flowing through the inductor and the 10-Ω resistor:

$$10\,\text{V} - (10\,\Omega)I = 0$$

and

$$I = \frac{10\,\text{V}}{10\,\Omega} = \boxed{1.00\,\text{A}}$$

By applying Kirchhoff's junction rule at the junction between the resistors, we can conclude that:

$$I_{100\text{-}\Omega\text{ resistor}} = I_{\text{battery}} - I_{\text{inductor}} = \boxed{0}$$

(*b*) When the switch is closed, the current cannot immediately go to zero in the circuit because of the inductor. For a time, a current will circulate in the circuit loop between the inductor and the 100-Ω resistor. Because the current flowing through this circuit is initially 1 A, the voltage drop across the 100-Ω resistor is initially $\boxed{100\,\text{V.}}$ Conservation of energy (Kirchhoff's loop rule) requires that the voltage drop across the inductor is also $\boxed{100\,\text{V.}}$

(*c*) Apply Kirchhoff's loop rule to the *RL* circuit to obtain:

$$L\frac{dI}{dt}+IR=0$$

The solution to this differential equation is:

$$I(t)=I_0e^{-\frac{R}{L}t}=I_0e^{-\frac{t}{\tau}}$$

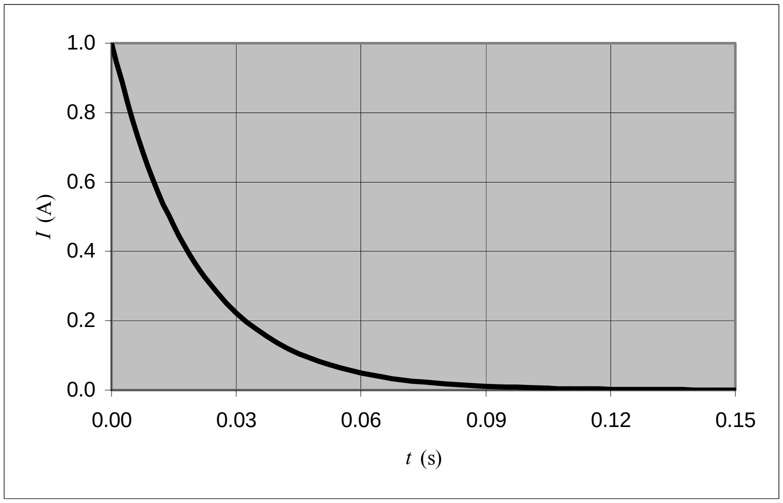
$$\text{where } \tau=\frac{L}{R}=\frac{2\,\text{H}}{100\,\Omega}=0.02\,\text{s}$$

A spreadsheet program to generate the data for graphs of the current and the voltage across the inductor as functions of time is shown below. The formulas used to calculate the quantities in the columns are as follows:

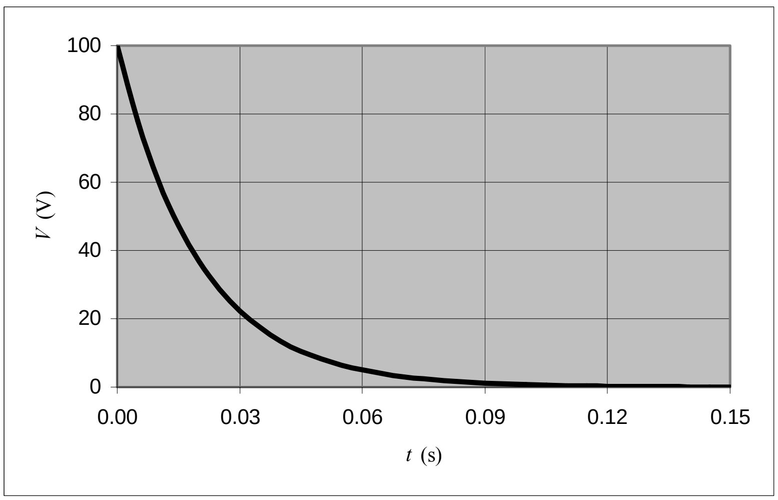
Cell	Formula/Content	Algebraic Form
B1	2	L
B2	100	R
B3	1	I_0
A6	0	t_0
B6	$B$3*EXP((−$B$2/$B$1)*A6)	$I_0e^{-\frac{R}{L}t}$

	A	B	C
1	L=	2	H
2	R=	100	ohms
3	I_0=	1	A
4			
5	t	I(t)	V(t)
6	0.000	1.00E+00	100.00
7	0.005	7.79E−01	77.88
8	0.010	6.07E−01	60.65
9	0.015	4.72E−01	47.24
10	0.020	3.68E−01	36.79
11	0.025	2.87E−01	28.65
12	0.030	2.23E−01	22.31
32	0.130	1.50E−03	0.15
33	0.135	1.17E−03	0.12
34	0.140	9.12E−04	0.09
35	0.145	7.10E−04	0.07
36	0.150	5.53E−04	0.06

The following graph of the current in the inductor as a function of time was plotted using the data in columns A and B of the spreadsheet program.



The following graph of the voltage across the inductor as a function of time was plotted using the data in columns A and C of the spreadsheet program.



76 ••

Picture the Problem We can evaluate the derivative of Equation 28-26 with respect to time at $t = 0$ to find the slope of the linear function of current as a function of time. Because the I-intercept of this equation is I_0, we can evaluate $I(t)$ at $t = \tau$ to show that the current is zero after one time constant.

Equation 28-26 describes the current in an LR circuit from which the source has been removed:

$$I = I_0 e^{-\frac{t}{\tau}}$$

Differentiate this expression with respect to t to obtain:

$$\frac{dI}{dt} = I_0 \frac{d}{dt} e^{-\frac{t}{\tau}} = I_0 e^{-\frac{t}{\tau}}\left(-\frac{1}{\tau}\right) = -\frac{I_0}{\tau} e^{-\frac{t}{\tau}}$$

Evaluate dI/dt at $t = 0$:

$$\left.\frac{dI}{dt}\right|_{t=0} = -\frac{I_0}{\tau}$$

Assuming that the current decreases steadily at this rate, express I as a linear function of t to obtain:

$$I(t) = -\frac{I_0}{\tau} t + I_0$$

Evaluate this function when $t = \tau$:

$$I(\tau) = -\frac{I_0}{\tau}\tau + I_0 = \boxed{0}$$

as was to have been shown.

77 ••

Picture the Problem The current in an initially energized but source-free RL circuit is given by $I = I_0 e^{-t/\tau}$. We can find τ from this equation and then use its definition to evaluate L.

(a) Express the current in the RL circuit as a function of time:

$$I = I_0 e^{-t/\tau}$$

Solve for and evaluate τ:

$$\tau = -\frac{t}{\ln\left(\frac{I}{I_0}\right)} = -\frac{45\,\text{ms}}{\ln\left(\frac{1.5\,\text{A}}{2.5\,\text{A}}\right)} = \boxed{88.1\,\text{ms}}$$

(b) Using the definition of the inductive time constant, relate L to R:

$$L = \tau R$$

Substitute numerical values and evaluate L:

$$L = (0.0881\,\text{s})(0.4\,\Omega) = \boxed{35.2\,\text{mH}}$$

78 •

Picture the Problem We can model this coil as a resistance-free inductor in series with an inductance-free resistor and express the potential difference across the coil as the sum of the potential differences across the inductor and the resistor. We can then use the given data to obtain two equations in the unknowns R and L and solve these equations simultaneously for the resistance and self-inductance of the coil.

Express the potential difference across the coil as the sum of the potential difference across a resistor and the potential difference across an inductor:

$$V = V_R + V_L = IR + L\frac{dI}{dt}$$

When $I = 5$ A and $dI/dt = 10$ A/s:

$$140\,\text{V} = (5\,\text{A})R + (10\,\text{A/s})L$$

When $I = 5$ A and $dI/dt = -10$ A/s:

$$60\,\text{V} = (5\,\text{A})R - (10\,\text{A/s})L$$

Add these equations to obtain:

$$200\,\text{V} = (10\,\text{A})R$$

and

$$R = \frac{200\,V}{10\,A} = \boxed{20.0\,\Omega}$$

Substitute in either of the equations to obtain:

$$L = \boxed{4.00\,\text{H}}$$

79 ••

Picture the Problem We can use the definition of inductance to express the rate at which the current changes through the inductors and the resistor and the result of Problem 88 to find the effective inductance in the circuit. We can find the final/steady-state current by applying Ohm's law.

(*a*) Express the rate of change of the current through the resistor:

$$\frac{dI_R}{dt} = \frac{\mathcal{E}}{L_{\text{eff}}}$$

Using the result given in Problem 88, find L_{eff}:

$$\frac{1}{L_{\text{eff}}} = \frac{1}{8\,\text{mH}} + \frac{1}{4\,\text{mH}}$$

and

$$L_{\text{eff}} = 2.67\,\text{mH}$$

Substitute numerical values and evaluate dI_R/dt at $t = 0$:

$$\left.\frac{dI_R}{dt}\right|_{t=0} = \frac{24\,\text{V}}{2.67\,\text{mH}} = \boxed{9.00\,\text{kA/s}}$$

Express the rate of change of the current through the 8-mH inductor:

$$\frac{dI_{8\,\text{mH}}}{dt}=\frac{\varepsilon}{L_{8\,\text{mH}}} \quad (1)$$

Express the rate of change of the current through the 4-mH inductor:

$$\frac{dI_{4\,\text{mH}}}{dt}=\frac{\varepsilon}{L_{4\,\text{mH}}} \quad (2)$$

Because $IR = 0$ when $t = 0$:

$$V_{L_{\text{eff}}}=V_{8\,\text{mH}}=V_{4\,\text{mH}}=24\,\text{V}$$

Substitute numerical values in equation (1) and evaluate $dI_{8\,\text{mH}}/dt$:

$$\frac{dI_{8\,\text{mH}}}{dt}=\frac{24\,\text{V}}{8\,\text{mH}}=\boxed{3.00\,\text{kA/s}}$$

Substitute numerical values in equation (2) and evaluate $dI_{4\,\text{mH}}/dt$:

$$\frac{dI_{4\,\text{mH}}}{dt}=\frac{24\,\text{V}}{4\,\text{mH}}=\boxed{6.00\,\text{kA/s}}$$

(*b*) After a long time has passed, the inductors will act as a short and the final current will be determined solely by the resistance in the circuit:

$$I_{\text{f}}=\frac{\varepsilon}{R}=\frac{24\,\text{V}}{15\,\Omega}=\boxed{1.60\,\text{A}}$$

***80 ••**

Picture the Problem If the current is initially zero in an *LR* circuit, its value at some later time *t* is given by $I=I_{\text{f}}\left(1-e^{-t/\tau}\right)$, where $I_{\text{f}}=\varepsilon_0/R$ and $\tau=L/R$ is the time constant for the circuit. We can find the time at which the power dissipation in the resistor equals the rate at which magnetic energy is stored in the inductor by equating expressions for these rates and using the expression for *I* and its rate of change.

Express the rate at which magnetic energy is stored in the inductor:

$$\frac{dU_L}{dt}=\frac{d}{dt}\left[\tfrac{1}{2}LI^2\right]=LI\frac{dI}{dt}$$

Express the rate at which power is dissipated in the resistor:

$$P=I^2R$$

Equate these expressions to obtain:

$$I^2R=LI\frac{dI}{dt}$$

Simplify to obtain:

$$I=\tau\frac{dI}{dt} \quad (1)$$

Express the current and its rate of change:

$$I = I_f\left(1 - e^{-t/\tau}\right)$$

and

$$\frac{dI}{dt} = I_f \frac{d}{dt}\left(1 - e^{-t/\tau}\right) = -I_f e^{-t/\tau}\left(-\frac{1}{\tau}\right)$$
$$= \frac{I_f}{\tau} e^{-t/\tau}$$

Substitute in equation (1) to obtain:

$$I_f\left(1 - e^{-t/\tau}\right) = \tau\left(\frac{I_f}{\tau} e^{-t/\tau}\right)$$

or

$$1 - e^{-t/\tau} = e^{-t/\tau} \Rightarrow 1 = 2e^{-t/\tau}$$

Solve for t:

$$t = -\tau \ln \tfrac{1}{2}$$

Using $\tau = 333\ \mu\text{s}$ from Example 28-11, evaluate t to obtain:

$$t = -(333\ \mu\text{s})\ln \tfrac{1}{2} = \boxed{231\ \mu\text{s}}$$

81 •••

Picture the Problem We can integrate $dE/dt = \mathcal{E}_0 I,$ where $I = I_f\left(1 - e^{-t/\tau}\right)$, to find the energy supplied by the battery, $dE_J/dt = I^2 R$ to find the energy dissipated in the resistor, and $U_L(\tau) = \frac{1}{2}L(I(\tau))^2$ to express the energy that has been stored in the inductor when $t = \tau$.

(*a*) Express the rate at which energy is supplied by the battery:

$$\frac{dE}{dt} = \mathcal{E}_0 I$$

Express the current in the circuit as a function of time:

$$I = \frac{\mathcal{E}_0}{R}\left(1 - e^{-t/\tau}\right)$$

Substitute to obtain:

$$\frac{dE}{dt} = \frac{\mathcal{E}_0^2}{R}\left(1 - e^{-t/\tau}\right)$$

Separate variables and integrate from $t = 0$ to $t = \tau$ to obtain:

$$E = \frac{\mathcal{E}_0^2}{R}\int_0^\tau \left(1 - e^{-t/\tau}\right)dt$$
$$= \frac{\mathcal{E}_0^2}{R}\left[\tau - \left(-\tau e^{-1} + \tau\right)\right]$$
$$= \frac{\mathcal{E}_0^2}{R}\frac{\tau}{e} = \frac{\mathcal{E}_0^2 L}{R^2 e}$$

Substitute numerical values and evaluate E:

$$E=\frac{(12\,\text{V})^2(0.6\,\text{H})}{(3\,\Omega)^2 e}=\boxed{3.53\,\text{J}}$$

(*b*) Express the rate at which energy is being dissipated in the resistor:

$$\frac{dE_{\text{J}}}{dt}=I^2R=\left[\frac{\mathcal{E}_0}{R}\left(1-e^{-t/\tau}\right)\right]^2 R$$
$$=\frac{\mathcal{E}_0^2}{R}\left(1-2e^{-t/\tau}+e^{-2t/\tau}\right)$$

Separate variables and integrate from $t = 0$ to $t = \tau$ to obtain:

$$E_{\text{J}}=\frac{\mathcal{E}_0^2}{R}\int_0^{\tau}\left(1-2e^{-t/\tau}+e^{-2t/\tau}\right)dt$$
$$=\frac{\mathcal{E}_0^2}{R}\left(\frac{2\tau}{e}-\frac{\tau}{2}-\frac{\tau}{2e^2}\right)$$
$$=\frac{\mathcal{E}_0^2 L}{R^2}\left(\frac{2}{e}-\frac{1}{2}-\frac{1}{2e^2}\right)$$

Substitute numerical values and evaluate E_{J}:

$$E_{\text{J}}=\frac{(12\,\text{V})^2(0.6\,\text{H})}{(3\,\Omega)^2}\left(\frac{2}{e}-\frac{1}{2}-\frac{1}{2e^2}\right)$$
$$=\boxed{1.61\,\text{J}}$$

(*c*) Express the energy stored in the inductor when $t = \tau$:

$$U_L(\tau)=\tfrac{1}{2}L\left(I(\tau)\right)^2$$
$$=\tfrac{1}{2}L\left(\frac{\mathcal{E}_0}{R}\left(1-e^{-1}\right)\right)^2$$
$$=\frac{L\mathcal{E}_0^2}{2R^2}\left(1-e^{-1}\right)^2$$

Substitute numerical values and evaluate E_{L}:

$$U_L(\tau)=\frac{(0.6\,\text{H})(12\,\text{V})^2}{2(3\,\Omega)^2}\left(1-e^{-1}\right)^2$$
$$=\boxed{1.92\,\text{J}}$$

Remarks: Note that, as we would expect from energy conservation, $E = E_{\text{J}} + E_{\text{L}}$.

General Problems

82 •

Picture the Problem We can apply the definition of magnetic flux to find the flux through the coil in its two orientations with respect to the magnetic field.

(*a*) Using its definition, express the magnetic flux through the coil:

$$\phi_m = NBA\cos\theta = NB\pi r^2\cos\theta$$
$$= (6)(0.5\,\mathrm{T})\pi(0.03\,\mathrm{m})^2\cos 0°$$
$$= \boxed{8.48\,\mathrm{mWb}}$$

(*b*) Proceed as in (*a*) with $\theta = 20°$:

$$\phi_m = NBA\cos\theta = NB\pi r^2\cos\theta$$
$$= (6)(0.5\,\mathrm{T})\pi(0.03\,\mathrm{m})^2\cos 20°$$
$$= \boxed{7.97\,\mathrm{mWb}}$$

83 •

Picture the Problem We can apply the definition of magnetic flux to find the flux through the coil in its two orientations with respect to the magnetic field and then use Faraday's law to find the emfs induced in the coil.

Using Faraday's law, express the emf induced in the coil:

$$\mathcal{E} = -\frac{\Delta\phi_m}{\Delta t} = -\frac{\phi_{m,f} - \phi_{m,i}}{\Delta t} = \frac{\phi_{m,i}}{\Delta t}$$

because $\phi_{m,f} = 0$

(*a*) Using its definition, express the magnetic flux through the coil:

$$\phi_m = NBA\cos\theta = NB\pi r^2\cos\theta$$

Substitute to obtain:

$$\mathcal{E} = \frac{NB\pi r^2\cos\theta}{\Delta t}$$

Substitute numerical values and evaluate ε:

$$\mathcal{E} = \frac{(6)(0.5\,\mathrm{T})\pi(0.03\,\mathrm{m})^2\cos 0°}{1.2\,\mathrm{s}}$$
$$= \boxed{7.07\,\mathrm{mV}}$$

(*b*) Proceed as in (*a*) with $\theta = 20°$:

$$\mathcal{E} = \frac{(6)(0.5\,\mathrm{T})\pi(0.03\,\mathrm{m})^2\cos 20°}{1.2\,\mathrm{s}}$$
$$= \boxed{6.64\,\mathrm{mV}}$$

84 •

Picture the Problem We can apply Faraday's and Ohm's laws to obtain expressions for the induced emf that we can equate and solve for the rate at which the perpendicular magnetic field must change to induce a current of 4.0 A in the coil.

Using Faraday's law, relate the induced emf in the coil to the

$$|\mathcal{E}| = \frac{d\phi_m}{dt} = NA\frac{dB}{dt}$$

changing magnetic flux:

Using Ohm's law, relate the induced emf to the resistance of the coil and the current in it:

$$|\mathcal{E}| = IR$$

Equate these expressions and solve for dB/dt:

$$NA\frac{dB}{dt} = IR$$

and

$$\frac{dB}{dt} = \frac{IR}{NA} = \frac{IR}{N\pi r^2}$$

Substitute numerical values and evaluate dB/dt:

$$\frac{dB}{dt} = \frac{(4\,\text{A})(25\,\Omega)}{(100)\pi(0.04\,\text{m})^2} = \boxed{199\,\text{T/s}}$$

***85 ••**

Picture the Problem We can apply Faraday's law and the definition of magnetic flux to derive an expression for the induced emf in the coil (potential difference between the slip rings). In part (b) we can solve this equation for ω under the given conditions.

(*a*) Use Faraday's law to express the induced emf:

$$\mathcal{E} = -\frac{d\phi_\text{m}}{dt}$$

Using the definition of magnetic flux, relate the magnetic flux through the loop to its angular velocity:

$$\phi_\text{m}(t) = NBA\cos\omega t$$

Substitute to obtain:

$$\mathcal{E} = -\frac{d}{dt}[NBA\cos\omega t]$$
$$= -NBab\omega(-\sin\omega t)$$
$$= \boxed{NBab\omega\sin\omega t}$$

(*b*) Express the condition under which $\varepsilon = \varepsilon_{\text{max}}$:

$$\sin\omega t = 1$$

Solve for and evaluate ω under this condition:

$$\omega = \frac{\mathcal{E}_{max}}{NBab}$$
$$= \frac{110\,\text{V}}{(1000)(2\,\text{T})(0.01\,\text{m})(0.02\,\text{m})}$$
$$= \boxed{275\,\text{rad/s}}$$

86 ••

Picture the Problem We can apply Faraday's law and the definition of magnetic flux to derive an expression for the induced emf in the rotating coil gaussmeter.

Use Faraday's law to express the induced emf:

$$\mathcal{E} = -\frac{d\phi_m}{dt}$$

Using the definition of magnetic flux, relate the magnetic flux through the loop to its angular velocity:

$$\phi_m(t) = NBA\cos\omega t$$

Substitute to obtain:

$$\mathcal{E} = -\frac{d}{dt}[NBA\cos\omega t]$$
$$= -NBA\omega(-\sin\omega t)$$
$$= NBA\omega\sin\omega t = \mathcal{E}_{max}\sin\omega t$$

where

$$\mathcal{E}_{max} = NBA\omega$$

Substitute numerical values and evaluate ε_{max}:

$$\mathcal{E}_{max} = (400)(0.45\,\text{T})(1.4\times10^{-4}\,\text{m}^2)\left(180\frac{\text{rev}}{\text{min}}\times\frac{2\pi\,\text{rad}}{\text{rev}}\times\frac{1\,\text{min}}{60\,\text{s}}\right) = \boxed{0.475\,\text{V}}$$

The maximum induced emf occurs at the moment the plane of the coil is parallel to the magnetic field $\vec{B}$. At this instant, ϕ_m is zero, but $\mathcal{E}$ is a maximum.

87 ••

Picture the Problem We can use the equality of the currents in the inductors connected in series and the additive nature of the total induced emf across the inductors to show that the inductances are additive.

Relate the total induced emf to the effective inductance L_{eff} and the rate at which the current is changing in the inductors:

$$\mathcal{E} = L_{\text{eff}} \frac{dI}{dt}$$

Because the inductors L_1 and L_2 are in series:

$$I_1 = I_2 = I$$

and

$$\frac{dI_1}{dt} = \frac{dI_2}{dt} = \frac{dI}{dt}$$

Express the total induced emf:

$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2 = L_1 \frac{dI}{dt} + L_2 \frac{dI}{dt}$$

$$= (L_1 + L_2)\frac{dI}{dt}$$

Substitute in equation (1) and simplify to obtain:

$$L_{\text{eff}} = \boxed{L_1 + L_2}$$

***88 ••**

Picture the Problem We can use the common potential difference across the parallel combination of inductors and the fact that the current into the parallel combination is the sum of the currents through each inductor to find an expression of the equivalent inductance.

Define L_{eff} by:

$$L_{\text{eff}} = \frac{\mathcal{E}}{dI/dt}$$

or

$$\frac{dI}{dt} = \mathcal{E}\frac{1}{L_{\text{eff}}} \qquad (1)$$

Relate the common potential difference across the inductors to their inductances and the rate at which the current is changing in each:

$$\mathcal{E}_1 = L_1 \frac{dI_1}{dt} \qquad (2)$$

and

$$\mathcal{E}_2 = L_2 \frac{dI_2}{dt} \qquad (3)$$

Because the current divides at the parallel junction:

$$I = I_1 + I_2$$

and

$$\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$$

Solve equations (2) and (3) for dI_1/dt and dI_2/dt and substitute to obtain:

$$\frac{dI}{dt} = \frac{\mathcal{E}_1}{L_1} + \frac{\mathcal{E}_2}{L_2}$$

Express the relationship between an emf $\mathcal{E}$ applied across the parallel combination of inductors and the emfs $\mathcal{E}_1$ and $\mathcal{E}_2$ across the individual inductors:

$$\mathcal{E} = \mathcal{E}_1 = \mathcal{E}_2$$

Substitute to obtain:

$$\frac{dI}{dt} = \frac{\mathcal{E}}{L_1} + \frac{\mathcal{E}}{L_2} = \mathcal{E}\left(\frac{1}{L_1} + \frac{1}{L_2}\right)$$

Substitute in equation (1) and solve for $1/L_{\text{eff}}$:

$$\frac{1}{L_{\text{eff}}} = \boxed{\frac{1}{L_1} + \frac{1}{L_2}}$$

***89 ••**

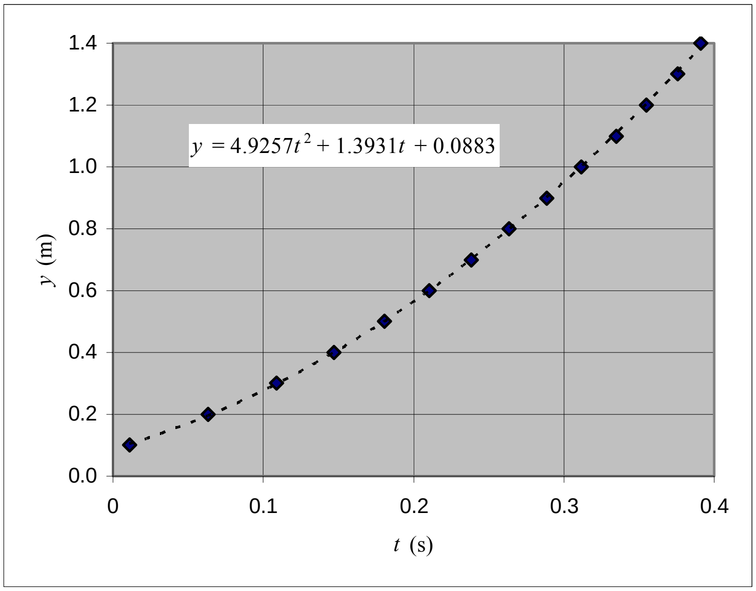
Picture the Problem

(*a*) $\boxed{\text{As the magnet passes through the coil, it induces an emf because of the changing flux through the coil. This allows the coil to "sense" when the magnet is passing through it.}}$

(*b*) $\boxed{\text{One cannot use a cylinder made of conductive material because eddy currents induced in it by a falling magnet would slow the magnet.}}$

(*c*) $\boxed{\text{As the magnet approaches the loop, the flux increases, resulting in the increasing voltage signal. When the magnet is passing the coil, the flux goes from increasing to decreasing, so the induced emf becomes zero and then negative. The time at which the induced emf is zero is the time at which the magnet is at the center of the coil.}}$

(*d*) Each time represents a point when the distance has increased by 10 cm. The following graph of distance versus time was plotted using a spreadsheet program. The regression curve, obtained using Excel's "Add Trendline" feature, is shown as a dashed line.



The coefficient of the second-degree term is $\frac{1}{2}g$. Consequently,

$$g = 2\left(4.9257\,\text{m/s}^2\right) = \boxed{9.85\,\text{m/s}^2}$$

90 ••

Picture the Problem The current equals the induced emf divided by the resistance. We can calculate the emf induced in the circuit as the coil moves by calculating the rate of change of the flux through the coil. The flux is proportional to the area of the coil in the magnetic field. We can find the direction of the current from Lenz's law.

(*a*) and (*c*) Express the magnitude of the induced current:

$$I = \frac{|\mathcal{E}|}{R} \tag{1}$$

Using Faraday's law, express the magnitude of the induced emf:

$$|\mathcal{E}| = \frac{d\phi_m}{dt}$$

When the coil is moving to the right (or to the left), the flux does not change (until the coil leaves the region of magnetic field). Thus:

$$|\mathcal{E}| = \frac{d\phi_m}{dt} = 0$$

and

$$I = \frac{|\mathcal{E}|}{R} = \boxed{0}$$

(*b*) and (*d*) Letting *x* represent the length of the side of the rectangular coil that is in the magnetic field, express the magnetic flux through the coil:

$$\phi_m = NBwx$$

Compute the rate of change of the flux when the coil is moving up or down:

$$\frac{d\phi_m}{dt} = NBw\frac{dx}{dt}$$
$$= (80)(1.4\,\mathrm{T})(0.25\,\mathrm{m})(2\,\mathrm{m/s})$$
$$= 56.0\,\mathrm{V}$$

Substitute in equation (1) to obtain:

$$I = \frac{56\,\mathrm{V}}{24\,\Omega} = \boxed{2.33\,\mathrm{A}}$$

(*b*) When the coil is moving upward, the outward flux increases and the induced current will be in the sense as to produce inward flux. *I* is clockwise.

(*d*) When the coil is moving downward, the outward flux decreases and the induced current will be in the sense as to produce outward flux. *I* is counterclockwise.

***91 ••**

Picture the Problem We can apply Faraday's law and the definition of magnetic flux to derive an expression for the induced emf in the coil. We can then apply Ohm's law to find the induced current as a function of time. Note that only half of the loop is in the magnetic field.

Apply Ohm's law to relate the induced current to the induced emf:

$$I(t) = \frac{\varepsilon(t)}{R} \qquad (1)$$

Use Faraday's law to express the induced emf:

$$\varepsilon(t) = -\frac{d\phi_m(t)}{dt}$$

Using the definition of magnetic flux, relate the magnetic flux through the loop to its angular velocity:

$$\phi_m(t) = NBA\cos\omega t$$

Substitute to obtain:

$$\varepsilon(t) = -\frac{d}{dt}\left[NBA\cos\omega t\right]$$
$$= -NBA\omega(-\sin\omega t)$$
$$= NBA\omega\sin\omega t$$

Substitute in equation (1) to obtain:

$$I(t) = \frac{NBA\omega}{R}\sin\omega t$$

Substitute numerical values and evaluate $I(t)$:

$$I(t)=\frac{(80)(1.4\,\mathrm{T})(0.25\,\mathrm{m})(0.15\,\mathrm{m})(2\,\mathrm{rad/s})}{24\Omega}\sin(2\,\mathrm{rad/s})t$$
$$=\boxed{(0.350\,\mathrm{A})\sin(2\,\mathrm{rad/s})t}$$

92 ••

Picture the Problem We can use the laws of Ohm and Faraday to express the charge dQ passing through the coil in time dt and integrate this expression to show that $Q = N(\phi_{m1} - \phi_{m2})/R$.

Use Ohm's law to express the induced current in terms of the induced emf:

$$\frac{dQ}{dt}=\frac{\mathcal{E}}{R}\Rightarrow dQ=\frac{\mathcal{E}}{R}dt$$

Apply Faraday's law to obtain:

$$dQ=-\frac{N}{R}\frac{d\phi_{\mathrm{m}}}{dt}dt=-\frac{N}{R}d\phi_{\mathrm{m}}$$

Integrate dQ from 0 to Q and $d\phi_m = \phi_{m1}$ to ϕ_{m2} to obtain:

$$\int_0^Q dQ'=-\frac{N}{R}\int_{\phi_{\mathrm{m1}}}^{\phi_{\mathrm{m2}}}d\phi_{\mathrm{m}}$$

and

$$Q=\boxed{\frac{N}{R}(\phi_{\mathrm{m1}}-\phi_{\mathrm{m2}})}$$

93 ••

Picture the Problem We can apply Faraday's law to relate the induced electric field E to the rates at which the magnetic flux is changing at distances $r < R$ and $r > R$ from the axis of the solenoid.

(*a*) Apply Faraday's law to relate the induced electric field to the magnetic flux in the solenoid within a cylindrical region of radius $r < R$:

$$\int_C \vec{E}\cdot d\vec{\ell}=-\frac{d\phi_{\mathrm{m}}}{dt}$$

or

$$E(2\pi r)=-\frac{d\phi_{\mathrm{m}}}{dt} \qquad (1)$$

Express the field within the solenoid:

$$B=\mu_0 nI$$

Express the magnetic flux through an area for which $r < R$:

$$\phi_{\mathrm{m}}=BA=\pi r^2\mu_0 nI$$

Substitute in equation (1) to obtain:

$$E(2\pi r) = -\frac{d}{dt}\left[\pi r^2 \mu_0 nI\right] = -\pi r^2 \mu_0 n \frac{dI}{dt}$$

Because $I = I_0 \sin \omega t$:

$$E = -\tfrac{1}{2} r\mu_0 n \frac{d}{dt}\left[I_0 \sin \omega t\right] = \boxed{-\tfrac{1}{2} r\mu_0 n I_0 \omega \cos \omega t}$$

(*b*) Proceed as in (*a*) with $r > R$ to obtain:

$$E(2\pi r) = -\frac{d}{dt}\left[\pi R^2 \mu_0 nI\right] = -\pi R^2 \mu_0 n \frac{dI}{dt} = -\pi R^2 \mu_0 n I_0 \omega \cos \omega t$$

Solve for E to obtain:

$$E = \boxed{-\frac{\mu_0 n R^2 I_0 \omega}{2r} \cos \omega t}$$

94 •••

Picture the Problem The system exhibits cylindrical symmetry, so one can use Ampère's law to determine B inside the inner cylinder, between the cylinders, and outside the outer cylinder. We can use $u_m = B^2/2\mu_0$ and the expression for B from part (a) to express the magnetic energy density in the region between the cylinders. We can integrate this expression for u_m over the volume between the cylinders to find the total magnetic energy in a volume of length ℓ. Finally, we can use our result in part (*c*) and $U_m = \tfrac{1}{2} LI^2$ to find the self-inductance of the cylinders per unit length.

(*a*) For $r < r_1$ and for $r > r_2$ the net enclosed current is zero; consequently, in these regions:

$$B = \boxed{0}$$

For $r_1 < r < r_2$:

$$2\pi r B = \mu_0 I_C \Rightarrow B = \boxed{\frac{\mu_0 I}{2\pi r}}$$

(*b*) Express the magnetic energy density in the region between the cylinders:

$$u_m = \frac{B^2}{2\mu_0}$$

Substitute for B and simplify to obtain:

$$u_m = \frac{\left(\frac{\mu_0 I}{2\pi r}\right)^2}{2\mu_0} = \boxed{\frac{\mu_0 I^2}{8\pi^2 r^2}}$$

(*c*) Express the magnetic energy dU_m in the cylindrical element of volume dV:

$$dU_m = u_m dV = \frac{\mu_0 I^2}{8\pi^2 r^2}(\ell 2\pi r dr)$$
$$= \frac{\mu_0 I^2 \ell}{4\pi} \cdot \frac{dr}{r}$$

Integrate this expression from $r = r_1$ to $r = r_2$ to obtain:

$$U_m = \frac{\mu_0 I^2 \ell}{4\pi}\int_{r_1}^{r_2}\frac{dr}{r} = \boxed{\frac{\mu_0}{4\pi} I^2 \ell \ln\frac{r_2}{r_1}}$$

(*d*) Express the energy in the magnetic field in terms of L and I:

$$U_m = \tfrac{1}{2} L I^2$$

Solve for L:

$$L = \frac{2U_m}{I^2}$$

From our result in (*c*):

$$\frac{U_m}{I^2} = \frac{\mu_0}{4\pi}\ell \ln\frac{r_2}{r_1}$$

Substitute to obtain:

$$L = 2\left(\frac{\mu_0}{4\pi}\ell \ln\frac{r_2}{r_1}\right) = \frac{\mu_0}{2\pi}\ell \ln\frac{r_2}{r_1}$$

Express the ratio L/ℓ:

$$\frac{L}{\ell} = \boxed{\frac{\mu_0}{2\pi}\ln\frac{r_2}{r_1}}$$

95 •••

Picture the Problem We can use its definition to express the magnetic flux through a rectangular element of area dA and then integrate from $r = r_1$ to $r = r_2$ to express the total flux through the region. Substituting in $L = \phi_m/I$ will yield the same result found in Part (*d*) of Problem 94.

Use the definition of self-inductance to relate the magnetic flux through the region of interest to the current I:

$$L = \frac{\phi_m}{I} \qquad (1)$$

Consider a strip of unit length ℓ and width dr at a distance r from the

$$d\phi_m = BdA = B\ell dr = Bdr$$

because $\ell = 1$.

axis. The flux through this area is given by:

Apply Ampere's law to express the magnetic field at a distance r from the axis:

$$2\pi rB = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

Substitute to obtain:

$$d\phi_{\text{m}} = \frac{\mu_0 I}{2\pi}\frac{dr}{r}$$

Integrate from $r = r_1$ to $r = r_2$ to obtain:

$$\phi_{\text{m}} = \frac{\mu_0 I}{2\pi}\int_{r_1}^{r_2}\frac{dr}{r}$$

and

$$\phi_{\text{m}} = \frac{\mu_0 I}{2\pi}\ln\frac{r_2}{r_1}$$

Substitute in equation (1) to obtain:

$$L = \boxed{\frac{\mu_0}{2\pi}\ln\frac{r_2}{r_1}}$$

***96 •••**

Picture the Problem We can use $I = \varepsilon/R$ and $\varepsilon = Bv\ell$ to find the current induced in the loop and Lenz's law to determine its direction. We can apply the equation for the force on a current-carrying wire to find the net magnetic force acting on the loop and then sum the forces to find the net force on the loop. Separating the variables in the differential equation and integrating will lead us to an expression for $v(t)$ and a second integration to an expression for $y(t)$. We can solve the latter equation for $y = 1.40$ m to find the time it takes the loop to exit the magnetic field and our expression for $v(t)$ to find its exit speed. Finally, we can use a constant-acceleration equation to find its exit speed in the absence of the magnetic field.

(*a*) Relate the magnitude of the induced current to the induced emf and the resistance of the loop:

$$I = \frac{\mathcal{E}}{R}$$

Relate the induced emf to the motion of the loop:

$$\mathcal{E} = Bv\ell$$

Substitute for ε to obtain:

$$I = \boxed{\frac{B\ell}{R}v}$$

As the loop falls, the flux into the page decreases. The direction of the induced current is such that its magnetic field opposes this decrease, i.e., clockwise.

(*b*) Express the velocity-dependent force that acts on the loop in terms of the current in the loop:

$$F_v = BI\ell$$

Substitute for I to obtain:

$$F_v = B\left(\frac{B\ell}{R}\right)v\ell = \boxed{\frac{B^2\ell^2}{R}v}$$

Apply $d\vec{\boldsymbol{F}} = Id\vec{\ell}\times\vec{\boldsymbol{B}}$ to the horizontal portion of the loop that is in the magnetic field to conclude that the net magnetic force is upward.

Note that the magnetic force on the left side of the loop is to the left and the magnetic force on the right side of the loop is to the right.

(*c*) The net force acting on the loop is the difference between the downward gravitational force and the upward magnetic force:

$$F_{\text{net}} = mg - F_v = \boxed{mg - \frac{B^2\ell^2}{R}v}$$

(*d*) Apply Newton's 2nd law of motion to the loop to obtain its equation of motion:

$$mg - \frac{B^2\ell^2}{R}v = m\frac{dv}{dt}$$

or

$$\boxed{\frac{dv}{dt} = g - \frac{B^2\ell^2}{mR}v}$$

Factor g to obtain an alternate form of the equation of motion:

$$\frac{dv}{dt} = g\left(1 - \frac{B^2\ell^2}{mgR}v\right) = \boxed{g\left(1 - \frac{v}{v_t}\right)}$$

where $v_t = \dfrac{mgR}{B^2\ell^2}$

(*e*) Separate the variables to obtain:

$$\frac{dv}{g - \frac{B^2\ell^2}{mR}v} = dt$$

or

$$\frac{dv}{a - bv} = dt$$

where $a = g$ and $b = \dfrac{B^2\ell^2}{mR}$

Integrate v' from 0 to v and t' from 0 to t:

$$\int_0^v \frac{dv'}{a - bv'} = \int_0^t dt' \Rightarrow -\frac{1}{b}\ln\left(\frac{a - bv}{a}\right) = t$$

Transform from logarithmic to exponential form and solve for v to obtain:

$$v(t) = \frac{a}{b}\left(1 - e^{-bt}\right)$$

Noting that $v_\text{t} = \dfrac{a}{b}$, we have:

$$v(t) = \boxed{v_\text{t}\left(1 - e^{-t/\tau}\right)}$$

where $\tau = \dfrac{v_\text{t}}{a} = \dfrac{v_\text{t}}{g}$.

(*f*) Write v as dy/dt and separate variables to obtain:

$$dy = v_\text{t}\left(1 - e^{-t/\tau}\right)dt$$

Integrate y' from 0 to y and t' from 0 to t:

$$\int_0^y dy' = v_\text{t}\int_0^t \left(1 - e^{-t'/\tau}\right)dt'$$

and

$$y(t) = \boxed{v_\text{t}\left[t - \tau\left(1 - e^{-t/\tau}\right)\right]}$$

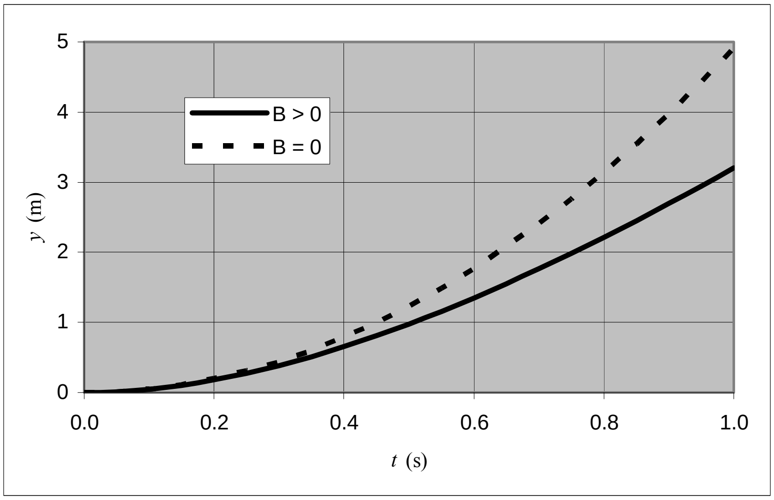
(*g*) A spreadsheet program to generate the data for graphs of position y as a function of time t is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
B1	0.05	m
B2	0.2	R
B3	0.4	B
B4	0.3	L
B5	$B$1*$B$7*$B$2/($B$3^2*$B$4^2)	v_t
B6	$B$5/$B$7	τ
B7	9.81	g
A10	0.00	t
B10	$B$5*(A10–$B$6*(1–EXP(–A10/$B$6)))	y
C10	0.5*$B$7*A10^2	$\frac{1}{2}gt^2$

	A	B	C
1	m=	0.05	kg
2	R=	0.2	ohms
3	B=	0.4	T
4	L=	0.3	m
5	vt=	6.813	m/s
6	tau=	0.694	s
7	g=	9.81	m/s^2
8			
9	t	y	y (no B)
10	0.00	0.000	0.000
11	0.05	0.012	0.012
12	0.10	0.047	0.049
13	0.15	0.103	0.110
14	0.20	0.179	0.196
15	0.25	0.273	0.307
16	0.30	0.384	0.441
17	0.35	0.511	0.601
18	0.40	0.654	0.785
19	0.45	0.809	0.993
20	0.50	0.978	1.226
21	0.55	1.159	1.484
22	0.60	1.351	1.766
23	0.65	1.553	2.072
24	0.70	1.764	2.403
25	0.75	1.985	2.759
26	0.80	2.214	3.139
27	0.85	2.451	3.544
28	0.90	2.695	3.973
29	0.95	2.946	4.427
30	1.00	3.202	4.905

Examining the table, we see that $y = 1.4$ m when $t \approx \boxed{0.60\,\text{s.}}$

The following graph shows y as a function of t for $B \neq 0$ (solid curve) and $B = 0$ (dashed curve).



97 •••

Picture the Problem We can use the expression for the period of this spring-and-mass oscillator to find the spring constant κ. We can express the induced current in the loop by relating it to the induced emf and relating the induced emf to the velocity of the loop. Knowing that the loop is executing SHM, we can express its velocity as a sinusoidal function of time. We can use the expression for the magnetic force on a current-carrying wire in a magnetic field to express the damping force acting on the loop.

(*a*) Express the period of the mass-spring system:

$$T = 2\pi\sqrt{\frac{m}{\kappa}}$$

Solve for κ to obtain:

$$\kappa = \frac{4\pi^2 m}{T^2}$$

Substitute numerical values and evaluate κ:

$$\kappa = \frac{4\pi^2(0.5\,\text{kg})}{(0.8\,\text{s})^2} = \boxed{30.8\,\text{N/m}}$$

(*b*) Express the current in the loop in terms of its resistance and the induced emf:

$$I = \frac{\mathcal{E}}{R}$$

Relate the induced emf in the wire to the motion of the wire:

$$\mathcal{E} = Bv\ell$$

or, because $\ell = w$ (where w is the width of the loop),

$$\mathcal{E} = Bvw$$

Express the position of the mass-spring system as a function of time:

$$y = y_0 \sin \omega t$$

Differentiate this expression with respect to time to express the velocity of the system:

$$v = \frac{dy}{dt} = y_0 \omega \cos \omega t$$

Substitute in our expression for I to obtain:

$$I = \boxed{\frac{B y_0 \omega w}{R} \cos \omega t}$$

(c) Express the damping force F_d acting on the loop:

$$F_d = BIw$$

Substitute for I and simplify to obtain:

$$F_d = -Bw \frac{B y_0 \omega w}{R} \cos \omega t = -\frac{B^2 w^2}{R} y_0 \omega \cos \omega t$$

Because $v = y_0 \omega \cos \omega t$:

$$F_d = -\frac{B^2 w^2}{R} v = \boxed{-\beta v}$$

where $\beta = \boxed{\frac{B^2 w^2}{R}}$.

(d) Choosing the static equilibrium position of the coil as the origin, apply $\sum \vec{F} = m\vec{a}$ to the coil when it is displaced slightly from this equilibrium position to obtain:

$$-F_d - F_r = m \frac{d^2 y}{dt^2}$$

where F_r is the restoring force exerted by the plastic spring.

Substituting for F_r and F_d yields the differential equation describing the motion of the coil:

$$-\beta \frac{dy}{dt} - \kappa y = m \frac{d^2 y}{dt^2}$$

or

$$\frac{d^2 y}{dt^2} + \frac{\beta}{m} \frac{dy}{dt} + \frac{\kappa}{m} y = 0$$

Note: compare this equation to Equation 14-35 on page 446 of Volume 1 of your textbook.

For weak damping, the solution to this differential equation is:

$$y(t) = (y_0 \cos \omega t) e^{-(\beta/2m)t}$$

Note: see Equation 14-36 on page 447 of your textbook.

Differentiate $y(t)$ with respect to time to obtain the velocity of the coil:

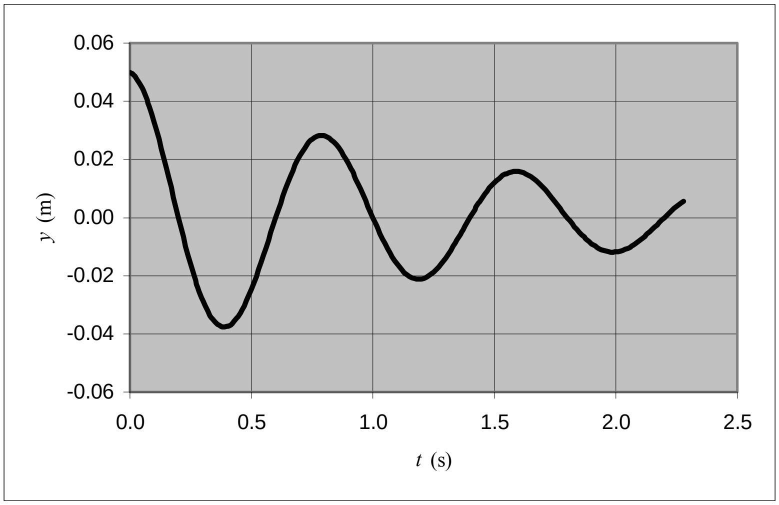
$$v(t) = -\left(\omega y_0 \sin \omega t + \frac{\beta y_0}{2m} \cos \omega t\right) e^{-\left(\frac{\beta}{2m}\right)t}$$

A spreadsheet program to generate the data for graphs of position y and velocity v as functions of time t is shown below. The formulas used to calculate the quantities in the columns are as follows:

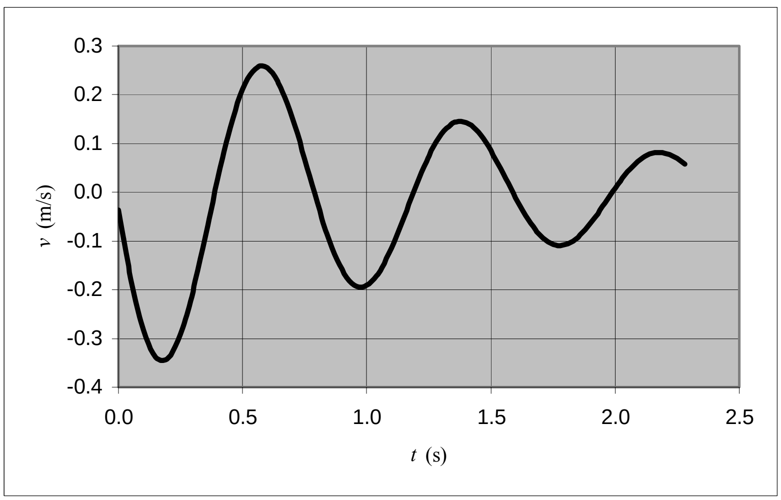
Cell	Formula/Content	Algebraic Form
B1	0.05	y_0
B2	0.8	T
B3	0.4	B
B4	0.2	R
B5	0.3	w
B6	0.05	m
B7	2*PI()/$B$2	ω
B8	$B$3^2*$B$5^2/$B$4	β
A11	0.00	t
B11	$B$1*COS($B$7*A11)* EXP((-$B$8/(2*$B$6))*A11)	$y(t)$
C11	–($B$1*$B$7*SIN($B$7*A11) +($B$8*$B$1/(2*$B$6))*COS($B$7*A11)) *EXP((–$B$8)/(2*$B$6))*A11)	$v(t)$

1	A	B	C
2	y_0=	0.05	m
3	T=	0.8	s
4	B=	0.4	T
5	R=	0.2	ohms
6	w=	0.3	m
7	m=	0.05	kg
8	omega=	7.85	s^-1
9	beta=	0.072	kg/s
10			
11	t	y	v
12	0.00	0.050	–0.036
13	0.01	0.049	–0.066
14	0.02	0.049	–0.096
15	0.03	0.048	–0.124
16	0.04	0.046	–0.151
17	0.05	0.045	–0.177
235	2.24	0.003	0.072
236	2.25	0.004	0.069
237	2.26	0.004	0.066
238	2.27	0.005	0.062
239	2.28	0.006	0.057

The graph of $y(t)$ follows:



The graph of $v(t)$ follows:



98 •••

Picture the Problem If the coil is twisted through an angle θ, a restoring torque equal to $\kappa\theta$ acts on it return it its equilibrium position. However, if it rotates back with angular speed $\omega = d\theta/dt,$ there will be an emf induced in the coil. The direction of the current resulting from this induced emf will be such that its magnetic field will oppose the change in flux resulting from the rotation of the coil. The net effect is that the motion of the coil is damped. We can apply Newton's 2nd law to relate the net restoring torque to the moment of inertia of the coil and its angular acceleration and use the laws of Faraday and Ohm to find the emf and current induced in the coil.

Apply $\sum\tau = I\alpha$ to the rotating coil to obtain:

$$\tau_{\text{restoring}} - \tau_{\text{retarding}} = I\frac{d^2\theta}{dt^2}$$

The magnitude of the retarding (damping) torque is given by:

$$\tau_{\text{retarding}} = NiBA\cos\theta$$

where i is the current induced in the coil whose cross-sectional area is A.

Substitute for $\tau_{\text{restoring}}$ and $\tau_{\text{retarding}}$ to obtain:

$$-\kappa\theta - NiBA\cos\theta = I\frac{d^2\theta}{dt^2} \qquad (1)$$

Apply Faraday's law to express the emf induced in the coil:

$$\mathcal{E} = -\frac{d}{dt}(NBA\sin\theta) = -(NBA\cos\theta)\frac{d\theta}{dt}$$

From Ohm's law, the magnitude of the induced current i in the coil is:

$$i = \frac{\mathcal{E}}{R} = \frac{NBA\cos\theta}{R}\frac{d\theta}{dt}$$

Substitute for the induced current i in equation (1) to obtain:

$$-\kappa\theta - \frac{N^2B^2A^2\cos^2\theta}{R}\frac{d\theta}{dt} = I\frac{d^2\theta}{dt^2}$$

For small displacements from equilibrium, $\cos\theta \approx 1$ and:

$$-\kappa\theta - \frac{N^2B^2A^2}{R}\frac{d\theta}{dt} \approx I\frac{d^2\theta}{dt^2}$$

Rearrange terms to obtain the differential equation of motion of the coil:

$$\frac{d^2\theta}{dt^2} + \frac{N^2B^2A^2}{RI}\frac{d\theta}{dt} + \frac{\kappa}{I}\theta \approx 0$$

Let $\beta = \frac{N^2B^2A^2}{RI}$ and $\omega = \sqrt{\frac{\kappa}{I}}$ to obtain:

$$\frac{d^2\theta}{dt^2} + \beta\frac{d\theta}{dt} + \omega^2\theta \approx 0$$

The solution to this second-order, homogeneous, linear differential equation with constant coefficients is:

$$\theta(t) = \boxed{\theta_0 e^{-(\beta/2)t}\cos\omega t}$$