Chapter 29
Alternating-Current Circuits

Conceptual Problems

***1** •

Determine the Concept Because the rms current through the resistor is given by $I_{rms} = \mathcal{E}_{rms}/R$ and both $\mathcal{E}_{rms}$ and R are independent of frequency, $\boxed{(b) \text{ is correct.}}$

2 •

Picture the Problem We can use the relationship between V and V_{peak} to decide the effect of doubling the rms voltage on the peak voltage.

Express the initial rms voltage in terms of the peak voltage:

$$V_{rms} = \frac{V_{max}}{\sqrt{2}}$$

Express the doubled rms voltage in terms of the new peak voltage V'_{max}:

$$2V_{rms} = \frac{V'_{max}}{\sqrt{2}}$$

Divide the second of these equations by the first and simplify to obtain:

$$\frac{2V_{rms}}{V_{rms}} = \frac{\frac{V'_{max}}{\sqrt{2}}}{\frac{V_{max}}{\sqrt{2}}} \text{ or } 2 = \frac{V'_{max}}{V_{max}}$$

Solve for V'_{max}:

$$V'_{max} = 2V_{max} \text{ and } \boxed{(a) \text{ is correct.}}$$

3 •

Determine the Concept The inductance of an inductor is determined by the details of its construction and is independent of the frequency of the circuit. The inductive reactance, on the other hand, is frequency dependent. $\boxed{(b) \text{ is correct.}}$

4 •

Determine the Concept The inductive reactance of an inductor varies with the frequency according to $X_L = \omega L.$ Hence, doubling ω will double X_L. $\boxed{(a) \text{ is correct.}}$

***5 •**

Determine the Concept The capacitive reactance of an capacitor varies with the frequency according to $X_C = 1/\omega C$. Hence, doubling ω will halve X_C. $\boxed{(c) \text{ is correct.}}$

6 •

Determine the Concept Yes to both questions. While the current in the inductor is increasing, the inductor absorbs power from the generator. When the current in the inductor reverses direction, the inductor supplies power to the generator.

7 •

Determine the Concept Yes to both questions. While charge is accumulating on the capacitor, the capacitor absorbs power from the generator. When the capacitor is discharging, it supplies power to the generator.

8 •

Picture the Problem We can use the definitions of the capacitive reactance and inductive reactance to find the SI units of *LC*.

Use its definition to express the inductive reactance:

$$X_L = 2\pi f L$$

Solve for *L*:

$$L = \frac{X_L}{2\pi f}$$

Use its definition to express the capacitive reactance:

$$X_C = \frac{1}{2\pi f C}$$

Solve for *C*:

$$C = \frac{1}{2\pi f X_C}$$

Express the product of *L* and *C*:

$$LC = \frac{X_L}{2\pi f}\frac{1}{2\pi f X_C} = \frac{X_L}{4\pi^2 f^2 X_C}$$

Because the units of X_L and X_C cancel, the units of *LC* are those of $1/f^2$ or s^2.
$\boxed{(a) \text{ is correct.}}$

***9 ••**

Determine the Concept To make an *LC* circuit with a small resonance frequency requires a large inductance and large capacitance. Neither is easy to construct.

10 •

(a) True. The Q factor and the width of the resonance curve at half power are related according to $Q = \omega_0/\Delta\omega$; i.e., they are inversely proportional to each other.

(b) True. The impedance of an *RLC* circuit is given by $Z = \sqrt{R^2 + (X_L - X_C)^2}$. At resonance $X_L = X_C$ and so $Z = R$.

(c) True. The phase angle δ is related to X_L and X_C according to $\delta = \tan^{-1}\frac{X_L - X_C}{R}$. At resonance $X_L = X_C$ and so $\delta = 0$.

11 •

Determine the Concept Yes. The power factor is defined to be $\cos\delta = R/Z$ and, because Z is frequency dependent, so is $\cos\delta$.

***12** •

Determine the Concept Yes; the bandwidth must be wide enough to accommodate the modulation frequency.

13 •

Determine the Concept Because the power factor is defined to be $\cos\delta = R/Z$, if R = 0, then the power factor is zero.

14 •

Determine the Concept A transformer is a device used to raise or lower the voltage in a circuit without an appreciable loss of power. $\boxed{(c)\text{ is correct.}}$

15 •

True. If energy is to be conserved, the product of the current and voltage must be constant.

16 ••

Picture the Problem Let the subscript 1 denote the primary and the subscript 2 the secondary. Assuming no loss of power in the transformer, we can equate the power in the primary circuit to the power in the secondary circuit and solve for the current in the primary windings.

Assuming no loss of power in the transformer: $P_1 = P_2$

Substitute for P_1 and P_2 to obtain: $I_1V_1 = I_2V_2$

Solve for I_1: $I_1 = I_2\frac{V_2}{V_1} = \frac{I_2V_2}{V_1} = \frac{P_2}{V_1}$

and $\boxed{(b)\text{ is correct}}$

17 •

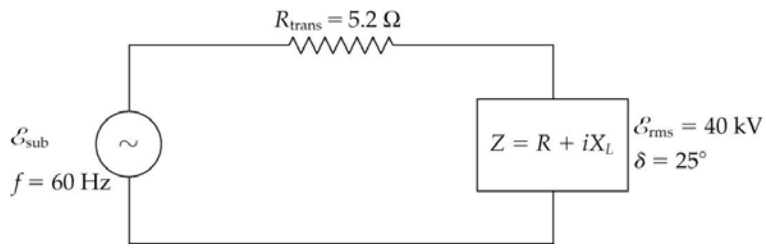
(*a*) False. The effective (rms) value of the current is not zero.

(*b*) True. The reactance of a capacitor goes to zero as *f* approaches very high frequencies.

Estimation and Approximation

***18** ••

Picture the Problem We can find the resistance and inductive reactance of the plant's total load from the impedance of the load and the phase constant. The current in the power lines can be found from the total impedance of the load the potential difference across it and the rms voltage at the substation by applying Kirchhoff's loop rule to the substation-transmission wires-load circuit. The power lost in transmission can be found from $P_{\text{trans}} = I_{\text{rms}}^2R_{\text{trans}}$. We can find the cost savings by finding the difference in the power lost in transmission when the phase angle is reduced to 18°. Finally, we can find the capacitance that is required to reduce the phase angle to 18° by first finding the capacitive reactance using the definition of $\tan\delta$ and then applying the definition of capacitive reactance to find *C*.



(*a*) Relate the resistance and inductive reactance of the plant's total load to *Z* and δ: $R = Z\cos\delta$ and $X_L = Z\sin\delta$

Express *Z* in terms of the current *I* in the power lines and voltage $\mathcal{E}_{\text{rms}}$ at the plant: $Z = \frac{\mathcal{E}_{\text{rms}}}{I}$

Express the power delivered to the plant in terms of $\mathcal{E}_{\text{rms}}$, I_{rms}, and δ and: $P_{\text{av}} = \mathcal{E}_{\text{rms}}I_{\text{rms}}\cos\delta$ and solve for I_{rms}:

$$I_{rms} = \frac{P_{av}}{\mathcal{E}_{rms}\cos\delta} \qquad (1)$$

Substitute to obtain:

$$Z = \frac{\varepsilon_{rms}^2\cos\delta}{P_{av}}$$

Substitute numerical values and evaluate Z:

$$Z = \frac{(40\,\text{kV})^2\cos 25°}{2.3\,\text{MW}} = 630\,\Omega$$

Substitute numerical values and evaluate R and X_L:

$$R = (630\,\Omega)\cos 25° = \boxed{571\,\Omega}$$

and

$$X_L = (630\,\Omega)\sin 25° = \boxed{266\,\Omega}$$

(b) Use equation (1) to find the current in the power lines:

$$I_{rms} = \frac{2.3\,\text{MW}}{(40\,\text{kV})\cos 25°} = \boxed{63.4\,\text{A}}$$

Apply Kirchhoff's loop rule to the circuit:

$$\mathcal{E}_{sub} - I_{rms}R_{trans} - IZ_{tot} = 0$$

Solve for ε_{sub}:

$$\mathcal{E}_{sub} = I_{rms}(R_{trans} + Z_{tot})$$

Evaluate Z_{tot}:

$$Z_{tot} = \sqrt{R^2 + X_L^2} = \sqrt{(571\Omega)^2 + (266\,\Omega)^2} = 630\,\Omega$$

Substitute numerical values and evaluate ε_{sub}:

$$\mathcal{E}_{sub} = (63.4\,\text{A})(5.2\,\Omega + 630\,\Omega) = \boxed{40.3\,\text{kV}}$$

(c) The power lost in transmission is:

$$P_{trans} = I_{rms}^2 R_{trans} = (63.4\,\text{A})^2(5.2\,\Omega) = \boxed{20.9\,\text{kW}}$$

(d) Express the cost savings ΔC in terms of the difference in energy consumption ($P_{25°} - P_{18°}$)Δt and the per-unit cost u of the energy:

$$\Delta C = (P_{25°} - P_{18°})\Delta t u$$

Express the power list in

$$P_{18°} = I_{18°}^2 R_{trans}$$

transmission when $\delta = 18°$:

Find the current in the transmission lines when $\delta = 18°$:

$$I_{18°} = \frac{2.3\,\text{MW}}{(40\,\text{kV})\cos 18°} = 60.5\,\text{A}$$

Evaluate $P_{18°}$:

$$P_{18°} = (60.5\,\text{A})^2(5.2\,\Omega) = 19.0\,\text{kW}$$

Substitute numerical values and evaluate ΔC:

$$\Delta C = (20.9\,\text{kW} - 19.0\,\text{kW})(16\,\text{h/d})(30\,\text{d/month})(\$0.07/\text{kW}\cdot\text{h}) = \boxed{\$63.84}$$

Relate the new phase angle δ to the inductive reactance X_L, the reactance due to the added capacitance X_C, and the resistance of the load R:

$$\tan\delta = \frac{X_L - X_C}{R}$$

Solve for and evaluate X_C:

$$X_C = X_L - R\tan\delta = 266\,\Omega - (571\,\Omega)\tan 18° = 80.5\,\Omega$$

Substitute numerical values and evaluate C:

$$C = \frac{1}{2\pi(60\,\text{s}^{-1})(80.5\,\Omega)} = \boxed{33.0\,\mu\text{F}}$$

Alternating Current Generators

19 •

Picture the Problem We can use the relationship $\mathcal{E}_{max} = 2\pi NBAf$ between the maximum emf induced in the coil and its frequency to find f when ε_{max} is given and ε_{max} when f is given .

(*a*) Relate the induced emf to the angular frequency of the coil:

$$\mathcal{E} = \mathcal{E}_{max}\cos\omega t$$

where

$$\mathcal{E}_{max} = NBA\omega = 2\pi NBAf$$

Solve for f:

$$f = \frac{\mathcal{E}_{max}}{2\pi NBA}$$

Substitute numerical values and evaluate f:

$$f = \frac{10\,\text{V}}{2\pi(200)(0.5\,\text{T})(4\times10^{-4}\,\text{m}^2)} = \boxed{39.8\,\text{Hz}}$$

(*b*) From (*a*) we have: $\mathcal{E}_{max} = NBA\omega = 2\pi NBAf$

Substitute numerical values and evaluate $\mathcal{E}_{max}$:

$$\mathcal{E}_{max} = 2\pi(200)(0.5\,\text{T})(4\times10^{-4}\,\text{m}^2)(60\,\text{s}^{-1}) = \boxed{15.1\,\text{V}}$$

20 •

Picture the Problem We can use the relationship $\mathcal{E}_{max} = 2\pi NBAf$ between the maximum emf induced in the coil and the magnetic field in which it is rotating to find *B* required to generate a given emf at a given frequency.

Relate the induced emf to the magnetic field in which the coil is rotating: $\mathcal{E}_{max} = NBA\omega = 2\pi NBAf$

Solve for *B*:

$$B = \frac{\mathcal{E}_{max}}{2\pi NfA}$$

Substitute numerical values and evaluate *B*:

$$B = \frac{10\,\text{V}}{2\pi(200)(60\,\text{s}^{-1})(4\times10^{-4}\,\text{m}^2)} = \boxed{0.332\,\text{T}}$$

***21** •

Picture the Problem We can use the relationship $\mathcal{E}_{max} = 2\pi NBAf$ to relate the maximum emf generated to the area of the coil, the number of turns of the coil, the magnetic field in which the coil is rotating, and the frequency at which it rotates.

(*a*) Relate the induced emf to the magnetic field in which the coil is rotating:

$$\mathcal{E}_{max} = NBA\omega = 2\pi NBAf \quad (1)$$

Substitute numerical values and evaluate $\mathcal{E}_{max}$:

$$\mathcal{E}_{max} = 2\pi(300)(0.4\,\text{T})(2\times10^{-2}\,\text{m})(1.5\times10^{-2}\,\text{m})(60\,\text{s}^{-1}) = \boxed{13.6\,\text{V}}$$

(*b*) Solve equation (1) for *f*:

$$f = \frac{\mathcal{E}_{max}}{2\pi NBA}$$

Substitute numerical values and evaluate *f*:

$$f=\frac{110\,\text{V}}{2\pi(300)(0.4\,\text{T})(2\times10^{-2}\,\text{m})(1.5\times10^{-2}\,\text{m})}=\boxed{486\,\text{Hz}}$$

22 •

Picture the Problem We can use the relationship $\mathcal{E}_{max}=2\pi NBAf$ to relate the maximum emf generated to the area of the coil, the number of turns of the coil, the magnetic field in which the coil is rotating, and the frequency at which it rotates.

Relate the induced emf to the magnetic field in which the coil is rotating:

$$\mathcal{E}_{max}=NBA\omega=2\pi NBAf$$

Solve for B:

$$B=\frac{\mathcal{E}_{max}}{2\pi NfA}$$

Substitute numerical values and evaluate B:

$$B=\frac{24\,\text{V}}{2\pi(300)(60\,\text{s}^{-1})(2\times10^{-2}\,\text{m})(1.5\times10^{-2}\,\text{m})}=\boxed{0.707\,\text{T}}$$

Alternating Current in a Resistor

***23** •

Picture the Problem We can use $P_{av}=\mathcal{E}_{rms}I_{rms}$ to find I_{rms}, $I_{max}=\sqrt{2}I_{rms}$ to find I_{max}, and $P_{max}=I_{max}\mathcal{E}_{max}$ to find P_{max}.

(*a*) Relate the average power delivered by the source to the rms voltage across the bulb and the rms current through it:

$$P_{av}=\mathcal{E}_{rms}I_{rms}$$

Solve for and evaluate I_{rms}:

$$I_{rms}=\frac{P_{av}}{\mathcal{E}_{rms}}=\frac{100\,\text{W}}{120\,\text{V}}=\boxed{0.833\,\text{A}}$$

(*b*) Express I_{max} in terms of I_{rms}:

$$I_{max}=\sqrt{2}I_{rms}$$

Substitute for I_{rms} and evaluate I_{max}:

$$I_{max}=\sqrt{2}(0.833\,\text{A})=\boxed{1.18\,\text{A}}$$

(*c*) Express the maximum power in terms of the maximum voltage and maximum current:

$$P_{max}=I_{max}\mathcal{E}_{max}$$

Substitute numerical values and evaluate P_{max}:

$$P_{max} = (1.18\,A)\sqrt{2}(120\,V) = \boxed{200\,W}$$

24 •

Picture the Problem We can $I_{max} = \sqrt{2}I_{rms}$ to find the largest current the breaker can carry and $P_{av} = I_{rms}V_{rms}$ to find the average power supplied by this circuit.

(*a*) Express I_{max} in terms of I_{rms}:

$$I_{max} = \sqrt{2}I_{rms} = \sqrt{2}(15\,A) = \boxed{21.2\,A}$$

(*b*) Relate the average power to the rms current and voltage:

$$P_{av} = I_{rms}V_{rms} = (15\,A)(120\,V) = \boxed{1.80\,kW}$$

Alternating Current in Inductors and Capacitors

25 •

Picture the Problem We can use $X_L = \omega L$ to find the reactance of the inductor at any frequency.

Express the inductive reactance as a function of *f*:

$$X_L = \omega L = 2\pi f L$$

(*a*) At $f = 60$ Hz:

$$X_L = 2\pi(60\,s^{-1})(1\,mH) = \boxed{0.377\,\Omega}$$

(*b*) At $f = 600$ Hz:

$$X_L = 2\pi(600\,s^{-1})(1\,mH) = \boxed{3.77\,\Omega}$$

(*c*) At $f = 6$ kHz:

$$X_L = 2\pi(6000\,s^{-1})(1\,mH) = \boxed{37.7\,\Omega}$$

26 •

(*a*) Relate the reactance of the inductor to its inductance:

$$X_L = \omega L = 2\pi f L$$

Solve for and evaluate *L*:

$$L = \frac{X_L}{2\pi f} = \frac{100\,\Omega}{2\pi(80\,s^{-1})} = \boxed{0.199\,H}$$

(*b*) At 160 Hz:

$$X_L = 2\pi(160\,s^{-1})(0.199\,H) = \boxed{200\,\Omega}$$

27 •

Picture the Problem We can equate the reactances of the capacitor and the inductor and then solve for the frequency.

Express the reactance of the inductor:

$$X_L = \omega L = 2\pi f L$$

Express the reactance of the capacitor:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

Equate these reactances to obtain:

$$2\pi f L = \frac{1}{2\pi f C}$$

Solve for *f* to obtain:

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

Substitute numerical values and evaluate *f*:

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{(10\,\mu\text{F})(1\,\text{mH})}} = \boxed{1.59\,\text{kHz}}$$

28 •

Picture the Problem We can use $X_C = 1/\omega C$ to find the reactance of the capacitor at any frequency.

Express the capacitive reactance as a function of *f*:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

(*a*) At *f* = 60 Hz:

$$X_C = \frac{1}{2\pi(60\,\text{s}^{-1})(1\,\text{nF})} = \boxed{2.65\,\text{M}\Omega}$$

(*b*) At *f* = 6 kHz:

$$X_C = \frac{1}{2\pi(6000\,\text{s}^{-1})(1\,\text{nF})} = \boxed{26.5\,\text{k}\Omega}$$

(*c*) At *f* = 6 MHz:

$$X_C = \frac{1}{2\pi(6\times10^6\,\text{s}^{-1})(1\,\text{nF})} = \boxed{26.5\,\Omega}$$

***29** •

Picture the Problem We can use $I_{max} = \varepsilon_{max}/X_C$ and $X_C = 1/\omega C$ to express *I*max as a function of ε_{max}, *f*, and *C*. Once we've evaluate I_{max}, we can use $I_{rms} = I_{max}/\sqrt{2}$ to find I_{rms}.

Express I_{max} in terms of ε_{max} and X_C:

$$I_{max} = \frac{\mathcal{E}_{max}}{X_C}$$

Express the capacitive reactance:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

Substitute to obtain:

$$I_{max} = 2\pi f C \mathcal{E}_{max}$$

(a) Substitute numerical values and evaluate I_{max}:

$$I_{max} = 2\pi\left(20\,\text{s}^{-1}\right)\left(20\,\mu\text{F}\right)\left(10\,\text{V}\right) = \boxed{25.1\,\text{mA}}$$

(b) Express I_{rms} in terms of I_{max}:

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{25.1\,\text{mA}}{\sqrt{2}} = \boxed{17.8\,\text{mA}}$$

30 •

Picture the Problem We can use $X_C = 1/\omega C = 1/2\pi f C$ to relate the reactance of the capacitor to the frequency.

Using its definition, express the reactance of a capacitor:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

Solve for f to obtain:

$$f = \frac{1}{2\pi C X_C}$$

(a) Find f when X_C = 1 Ω:

$$f = \frac{1}{2\pi(10\,\mu\text{F})(1\,\Omega)} = \boxed{15.9\,\text{kHz}}$$

(b) Find f when X_C = 100 Ω:

$$f = \frac{1}{2\pi(10\,\mu\text{F})(100\,\Omega)} = \boxed{159\,\text{Hz}}$$

(c) Find f when X_C = 0.01 Ω:

$$f = \frac{1}{2\pi(10\,\mu\text{F})(0.01\,\Omega)} = \boxed{1.59\,\text{MHz}}$$

31 ••

Picture the Problem We can use the trigonometric identity $\cos\theta + \cos\phi = 2\cos\frac{1}{2}(\theta+\phi)\cos\frac{1}{2}(\theta-\phi)$ to find the sum of the phasors V_1 and V_2 and then use this sum to express I as a function of time. In (b) we'll use a phasor diagram to obtain the same result and in (c) we'll use the phasor diagram appropriate to the given voltages to express the current as a function of time.

(*a*) Express the current in the resistor:

$$I = \frac{V}{R} = \frac{V_1 + V_2}{R}$$

Use the trigonometric identity $\cos\theta + \cos\phi = 2\cos\frac{1}{2}(\theta+\phi)\cos\frac{1}{2}(\theta-\phi)$ to find $V_1 + V_2$:

$$V_1 + V_2 = (5.0\,\text{V})[\cos(\omega t - \alpha) + \cos(\omega t + \alpha)] = (5\,\text{V})[2\cos\tfrac{1}{2}(2\omega t)\cos\tfrac{1}{2}(-2\alpha)]$$
$$= (10\,\text{V})\cos\frac{\pi}{6}\cos\omega t = (8.66\,\text{V})\cos\omega t$$

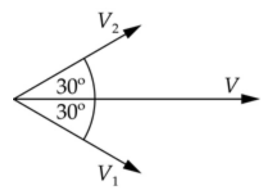
Substitute to obtain:

$$I = \frac{(8.66\,\text{V})\cos\omega t}{25\,\Omega} = \boxed{(0.346\,\text{A})\cos\omega t}$$

(*b*) Express the magnitude of the current in *R*:

$$|I| = \frac{|V|}{R}$$

The phasor diagram for the voltages is shown to the right.



Use vector addition to find $|V|$:

$$|V| = 2|V_1|\cos 30° = 2(5\,\text{V})\cos 30°$$
$$= 8.66\,\text{V}$$

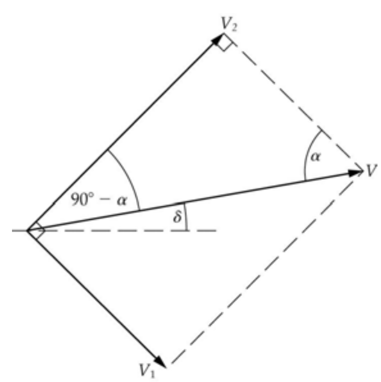
Substitute to obtain:

$$|I| = \frac{8.66\,\text{V}}{25\,\Omega} = 0.346\,\text{A}$$

and

$$I = \boxed{(0.346\,\text{A})\cos\omega t}$$

(*c*) The phasor diagram is shown to the right. Note that the phase angle between V_1 and V_2 is now 90°.



Use the Pythagorean theorem to find $|V|$:

$$|V| = \sqrt{|V_1|^2 + |V_2|^2} = \sqrt{(5\,\text{V})^2 + (7\,\text{V})^2}$$
$$= 8.60\,\text{V}$$

Express I as a function of t:

$$I = \frac{|V|}{R}\cos(\omega t + \delta)$$

where

$$\delta = 45° - (90° - \alpha) = \alpha - 45°$$
$$= \tan^{-1}\left(\frac{7\,\text{V}}{5\,\text{V}}\right) - 45° = 9.46° = 0.165\,\text{rad}$$

Substitute numerical values and evaluate I:

$$I = \frac{8.60\,\text{V}}{25\,\Omega}\cos(\omega t + 0.165\,\text{rad})$$
$$= \boxed{(0.344\,\text{A})\cos(\omega t + 0.165\,\text{rad})}$$

LC and *RLC* Circuits without a Generator

***32** •

Picture the Problem We can use $X_L = \omega L$ and $X_C = 1/\omega C$ to show the $1/\sqrt{LC}$ has the unit s^{-1}. Alternatively, we can use the dimensions of C and L to establish this result.

Substitute the units for L and C in the expression $1/\sqrt{LC}$ to obtain:

$$\frac{1}{\sqrt{\text{H}\cdot\text{F}}} = \frac{1}{\sqrt{(\Omega\cdot\text{s})\left(\frac{\text{s}}{\Omega}\right)}} = \frac{1}{\sqrt{\text{s}^2}} = \boxed{\text{s}^{-1}}$$

Alternatively, use the defining equation ($C = Q/V$) for capacitance to obtain the dimension of C:

$$[C] = \frac{[Q]}{[V]}$$

Solve the defining equation ($V = L\,dI/dt$) for inductance to obtain the dimension of L:

$$[L] = \frac{[V]}{\left[\frac{dI}{dt}\right]} = \frac{[V]}{\frac{[Q]}{[T]^2}} = \frac{[V][T]^2}{[Q]}$$

Express the dimension of $1/\sqrt{LC}$:

$$\left[\frac{1}{\sqrt{LC}}\right] = \left[\frac{1}{\sqrt{[L][C]}}\right] = \left[\frac{1}{\sqrt{\frac{[V][T]^2}{[Q]}\frac{[Q]}{[V]}}}\right]$$
$$= \left[\frac{1}{\sqrt{[T]^2}}\right] = \frac{1}{[T]}$$

Because the SI unit of time is the second, we've shown that $1/\sqrt{LC}$ has units of $\boxed{\text{s}^{-1}.}$

33 •

Picture the Problem We can use $T = 2\pi/\omega$ and $\omega = 1/\sqrt{LC}$ to relate T (and hence f) to L and C.

(*a*) Express the period of oscillation of the *LC* circuit:

$$T = \frac{2\pi}{\omega}$$

For an *LC* circuit:

$$\omega = \frac{1}{\sqrt{LC}}$$

Substitute to obtain:

$$T = 2\pi\sqrt{LC} \qquad (1)$$

Substitute numerical values and evaluate T:

$$T = 2\pi\sqrt{(2\,\text{mH})(20\,\mu\text{F})} = \boxed{1.26\,\text{ms}}$$

(*b*) Solve equation (1) for L to obtain:

$$L = \frac{T^2}{4\pi^2 C} = \frac{1}{4\pi^2 f^2 C}$$

Substitute numerical values and evaluate L:

$$L = \frac{1}{4\pi^2(60\,\text{s}^{-1})^2(80\,\mu\text{F})} = \boxed{88.0\,\text{mH}}$$

34 ••

Picture the Problem We can use the expression $f_0 = 1/2\pi\sqrt{LC}$ for the resonance frequency of an *LC* circuit to show that each circuit oscillates with the same frequency. In (*b*) we can use $I_{max} = \omega Q_0$, where Q_0 is the charge of the capacitor at time zero, and the definition of capacitance $Q_0 = CV$ to express I_{max} in terms of ω, C and V.

Express the resonance frequency for an *LC* circuit:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

(*a*) Express the product of L and C for each circuit:

Circuit 1: L_1C_1,

Circuit 2: $L_2C_2 = (2L_1)(\tfrac{1}{2}C_1) = L_1C_1$,

and

Circuit 3: $L_3C_3 = (\tfrac{1}{2}L_1)(2C_1) = L_1C_1$

Because $L_1C_1 = L_2C_2 = L_3C_3$, the resonance frequencies of the three circuits are the same.

(*b*) Express I_{max} in terms of the

$$I_{max} = \omega Q_0$$

charge stored in the capacitor:

Express Q_0 in terms of the capacitance of the capacitor and the potential difference across the capacitor:

$$Q_0 = CV$$

Substitute to obtain:

$$I_{\max} = \omega CV$$

or, for ω and V constant,

$$I_{\max} \propto C$$

$$\boxed{\text{The circuit with } C = C_3 \text{ has the greatest } I_{\max}.}$$

35 ••

Picture the Problem We can use $U = \frac{1}{2}CV^2$ to find the energy stored in the electric field of the capacitor, $\omega_0 = 2\pi f_0 = 1/\sqrt{LC}$ to find f_0, and $I_{\max} = \omega Q_0$ and $Q_0 = CV$ to find $I_{\max}$.

(*a*) Express the energy stored in the system as a function of C and V:

$$U = \tfrac{1}{2}CV^2$$

Substitute numerical values and evaluate U:

$$U = \tfrac{1}{2}(5\,\mu\text{F})(30\,\text{V})^2 = \boxed{2.25\,\text{mJ}}$$

(*b*) Express the resonance frequency of the circuit in terms of L and C:

$$\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}}$$

Solve for f_0:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Substitute numerical values and evaluate f_0:

$$f_0 = \frac{1}{2\pi\sqrt{(10\,\text{mH})(5\,\mu\text{F})}} = \boxed{712\,\text{Hz}}$$

(*c*) Express $I_{\max}$ in terms of the charge stored in the capacitor:

$$I_{\max} = \omega Q_0$$

Express Q_0 in terms of the capacitance of the capacitor and the potential difference across the capacitor:

$$Q_0 = CV$$

Substitute to obtain: $I_{max} = \omega CV$

Substitute numerical values and evaluate I_{max}:

$$I_{max} = 2\pi\left(712\,\text{s}^{-1}\right)(5\,\mu\text{F})(30\,\text{V}) = \boxed{0.671\,\text{A}}$$

36 •

Picture the Problem We can use its definition to find the power factor of the circuit and $I_{rms} = \varepsilon/Z$ to find the rms current in the circuit. In (*c*) we can use $P_{av} = I_{rms}^2 R$ to find the average power supplied to the circuit.

(*a*) Express the power factor of the circuit:

$$\cos\delta = \frac{R}{Z}$$

Express Z for the circuit:

$$Z = \sqrt{R^2 + X_L^2}$$

Substitute to obtain:

$$\cos\delta = \frac{R}{\sqrt{R^2 + X_L^2}} = \frac{R}{\sqrt{R^2 + (2\pi fL)^2}}$$

Substitute numerical values and evaluate $\cos\delta$:

$$\cos\delta = \frac{100\,\Omega}{\sqrt{(100\,\Omega)^2 + \left[2\pi\left(60\,\text{s}^{-1}\right)(0.4\,\text{H})\right]^2}} = \boxed{0.553}$$

(*b*) Express the rms current in terms of the rms voltage and the impedance of the circuit:

$$I_{rms} = \frac{\varepsilon}{Z} = \frac{\varepsilon}{\sqrt{R^2 + (2\pi fL)^2}}$$

Substitute numerical values and evaluate I_{rms}:

$$I_{rms} = \frac{120\,\text{V}}{\sqrt{(100\,\Omega)^2 + \left[2\pi\left(60\,\text{s}^{-1}\right)(0.4\,\text{H})\right]^2}} = \boxed{0.663\,\text{A}}$$

(*c*) Express the average power supplied to the circuit in terms of the rms current and the resistance of the inductor:

$$P_{av} = I_{rms}^2 R$$

Substitute numerical values and evaluate P_{av}:

$$P_{av} = (0.663\,\text{A})^2(100\,\Omega) = \boxed{44.0\,\text{W}}$$

***37 ••**

Picture the Problem Let Q represent the instantaneous charge on the capacitor and apply Kirchhoff's loop rule to obtain the differential equation for the circuit. We can then solve this equation to obtain an expression for the charge on the capacitor as a function of time and, by differentiating this expression with respect to time, an expression for the current as a function of time. We'll use a spreadsheet program to plot the graphs.

Apply Kirchhoff's loop rule to a clockwise loop just after the switch is closed:

$$\frac{Q}{C}+L\frac{dI}{dt}=0$$

Because $I = dQ/dt$:

$$L\frac{d^2Q}{dt^2}+\frac{Q}{C}=0 \text{ or } \frac{d^2Q}{dt^2}+\frac{1}{LC}Q=0$$

The solution to this equation is:

$$Q(t)=Q_0\cos(\omega t-\delta)$$

where $\omega=\sqrt{\dfrac{1}{LC}}$

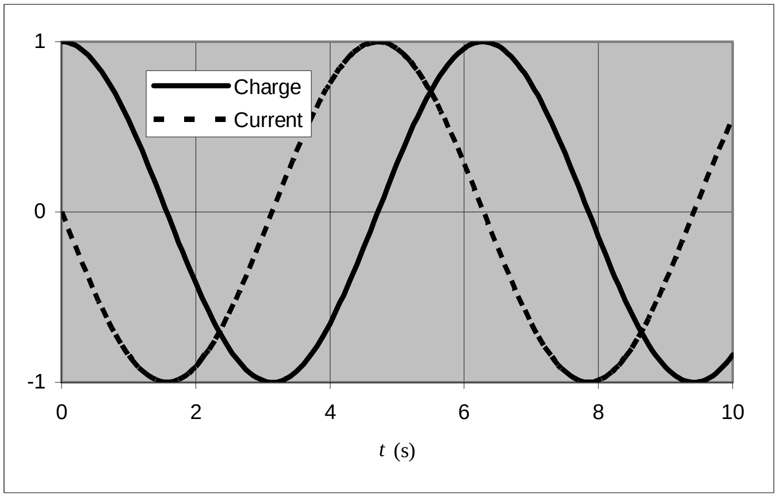
Because $Q(0) = Q_0$, $\delta = 0$ and:

$$Q(t)=Q_0\cos\omega t$$

The current in the circuit is the derivative of Q with respect to t:

$$I=\frac{dQ}{dt}=\frac{d}{dt}[Q_0\cos\omega t]=-\omega Q_0\sin\omega t$$

(a) A spreadsheet program was used to plot the following graph showing both the charge on the capacitor and the current in the circuit as functions of time. L, C, and Q_0 were all arbitrarily set equal to one to obtain these graphs. Note that the current leads the charge by one-fourth of a cycle or 90°.



(b) The equation for the current is:

$$I=-\omega Q_0\sin\omega t \quad (1)$$

The sine and cosine functions are related through the identity:

$$-\sin\theta=\cos\left(\theta+\frac{\pi}{2}\right)$$

Use this identity to rewrite equation (1):

$$I = -\omega Q_0 \sin \omega t = \boxed{\omega Q_0 \cos\left(\omega t + \frac{\pi}{2}\right)}$$

showing that the current leads the charge by 90°.

RL Circuits with a Generator

38 ••

Picture the Problem We can express the ratio of V_R to V_L and solve this expression for the resistance R of the circuit. In (*b*) we can use the fact that, in an *LR* circuit, V_L leads V_R by 90° to find the ac input voltage.

(*a*) Express the potential differences across R and L in terms of the common current through these components:

$$V_L = IX_L = I\omega L$$

and

$$V_R = IR$$

Divide the second of these equations by the first to obtain:

$$\frac{V_R}{V_L} = \frac{IR}{I\omega L} = \frac{R}{\omega L}$$

Solve for R:

$$R = \left(\frac{V_R}{V_L}\right)\omega L$$

Substitute numerical values and evaluate R:

$$R = \left(\frac{30\,\text{V}}{40\,\text{V}}\right)2\pi\left(60\,\text{s}^{-1}\right)(1.4\,\text{H}) = \boxed{396\,\Omega}$$

(*b*) Because V_R leads V_L by 90° in an *LR* circuit:

$$V = \sqrt{V_R^2 + V_L^2}$$

Substitute numerical values and evaluate V:

$$V = \sqrt{(30\,\text{V})^2 + (40\,\text{V})^2} = \boxed{50.0\,\text{V}}$$

39 ••

Picture the Problem We can solve the expression for the impedance in an *LR* circuit for the inductive reactance and then use the definition of X_L to find L.

Express the impedance of the coil in terms of its resistance and inductive reactance:

$$Z = \sqrt{R^2 + X_L^2}$$

Solve for X_L to obtain:

$$X_L = \sqrt{Z^2 - R^2}$$

Express X_L in terms of L:

$$X_L = 2\pi fL$$

Equate these two expressions to obtain:

$$2\pi fL = \sqrt{Z^2 - R^2}$$

Solve for L:

$$L = \frac{\sqrt{Z^2 - R^2}}{2\pi f}$$

Substitute numerical values and evaluate L:

$$L = \frac{\sqrt{(200\,\Omega)^2 - (80\,\Omega)^2}}{2\pi(1\,\text{kHz})} = \boxed{29.2\,\text{mH}}$$

40 ••

Picture the Problem We can express the two output voltage signals as the product of the current from each source and R = 1 kΩ. We can find the currents due to each source using the given voltage signals and the definition of the impedance for each of them.

(*a*) Express the voltage signals observed at the output side of the transmission line in terms of the potential difference across the resistor:

$$V_{1,\,\text{out}} = I_1R$$

and

$$V_{2,\,\text{out}} = I_2R$$

Express I_1 and I_2:

$$I_1 = \frac{V_1}{Z_1} = \frac{(10\,\text{V})\cos 100t}{\sqrt{(10^3\,\Omega)^2 + [(100\,\text{s}^{-1})(1\,\text{H})]^2}} = (9.95\,\text{mA})\cos 100t$$

and

$$I_2 = \frac{V_2}{Z_2} = \frac{(10\,\text{V})\cos 10^4 t}{\sqrt{(10^3\,\Omega)^2 + [(10^4\,\text{s}^{-1})(1\,\text{H})]^2}} = (0.995\,\text{mA})\cos 10^4 t$$

Substitute for I_1 and I_2 to obtain:

$$V_{1,\,\text{out}} = (10^3\,\Omega)(9.95\,\text{mA})\cos 100t = \boxed{(9.95\,\text{V})\cos 100t}$$

and

$$V_{2,\,\text{out}} = (10^3\,\Omega)(0.995\,\text{mA})\cos 10^4 t = \boxed{(0.995\,\text{V})\cos 10^4 t}$$

(*b*) Express the ratio of $V_{1,\text{out}}$ to $V_{2,\text{out}}$:

$$\frac{V_{1,\text{out}}}{V_{2,\text{out}}} = \frac{9.95\,\text{V}}{0.995\,\text{V}} = \boxed{10.0}$$

41 ••

Picture the Problem The average power supplied to coil is related to the power factor by $P_{\text{av}} = \mathcal{E}_{\text{rms}} I_{\text{rms}} \cos\delta$. In (*b*) we can use $P_{\text{av}} = I_{\text{rms}}^2 R$ to find *R*. Because the inductance *L* is related to the resistance *R* and the phase angle δ according to $X_L = \omega L = R\tan\delta$, we can use this relationship to find the resistance of the coil. Finally, we can decide whether the current leads or lags the voltage by noting whether X_L is less than or greater than *R*.

(*a*) Express the average power supplied to the coil in terms of the power factor of the circuit:

$$P_{\text{av}} = \mathcal{E}_{\text{rms}} I_{\text{rms}} \cos\delta$$

Solve for the power factor:

$$\cos\delta = \frac{P_{\text{av}}}{\mathcal{E}_{\text{rms}} I_{\text{rms}}}$$

Substitute numerical values and evaluate $\cos\delta$:

$$\cos\delta = \frac{60\,\text{W}}{(120\,\text{V})(1.5\,\text{A})} = \boxed{0.333}$$

(*b*) Express the power supplied by the source in terms of the resistance of the coil:

$$P_{\text{av}} = I_{\text{rms}}^2 R$$

Solve for and evaluate *R*:

$$R = \frac{P_{\text{av}}}{I_{\text{rms}}^2} = \frac{60\,\text{W}}{(1.5\,\text{A})^2} = \boxed{26.7\,\Omega}$$

(*c*) Relate the inductive reactance to the resistance and phase angle:

$$X_L = \omega L = R\tan\delta$$

Solve for *L*:

$$L = \frac{R\tan\delta}{\omega} = \frac{R\tan\left(\cos^{-1}0.333\right)}{2\pi f}$$

Substitute numerical values and evaluate *L*:

$$L = \frac{(26.7\,\Omega)\tan 70.5^\circ}{2\pi\left(60\,\text{s}^{-1}\right)} = \boxed{0.200\,\text{H}}$$

(*d*) Evaluate X_L:

$$X_L = (26.7\,\Omega)\tan 70.5^\circ = 75.4\,\Omega$$

Because $X_L > R$, the circuit is inductive and:

$$\boxed{I \text{ lags } \mathcal{E} \text{ by } 70.5^\circ}\,.$$

42 ••

Picture the Problem We can use $I_{max} = \mathcal{E}_{max} \Big/ \sqrt{R^2 + (\omega L)^2}$ and $V_{L,max} = I_{max} X_L = \omega L I_{max}$ to find the maximum current in the circuit and the maximum voltage across the inductor. Once we've found $V_{L,max}$ we can find $V_{L,rms}$ using $V_{L,rms} = V_{L,max} \Big/ \sqrt{2}$. We can use $P_{av} = \frac{1}{2} I_{max}^2 R$ to find the average power dissipation, and $U_{L,max} = \frac{1}{2} L I_{max}^2$ to find the maximum energy stored in the magnetic field of the inductor. The average energy stored in the magnetic field of the inductor can be found from $U_{L,av} = \int P_{av} dt$.

Express the maximum current in the circuit:

$$I_{max} = \frac{\mathcal{E}_{max}}{Z} = \frac{\mathcal{E}_{max}}{\sqrt{R^2 + (\omega L)^2}}$$

Substitute numerical values and evaluate I_{max}:

$$I_{max} = \frac{345\,\text{V}}{\sqrt{(40\,\Omega)^2 + \left[(150\pi\,\text{s}^{-1})(36\,\text{mH})\right]^2}} = \boxed{7.94\,\text{A}}$$

Relate the maximum voltage across the inductor to the current flowing through it:

$$V_{L,max} = I_{max} X_L = \omega L I_{max}$$

Substitute numerical values and evaluate $V_{L,max}$:

$$V_{L,max} = (150\pi\,\text{s}^{-1})(36\,\text{mH})(7.94\,\text{A}) = \boxed{135\,\text{V}}$$

$V_{L,rms}$ is related to $V_{L,max}$ according to:

$$V_{L,rms} = \frac{V_{L,max}}{\sqrt{2}} = \frac{135\,\text{V}}{\sqrt{2}} = \boxed{95.5\,\text{V}}$$

Relate the average power dissipation to I_{max} and R:

$$P_{av} = \tfrac{1}{2} I_{max}^2 R$$

Substitute numerical values and evaluate P_{av}:

$$P_{av} = \tfrac{1}{2}(7.94\,\text{A})^2(40\,\Omega) = \boxed{1.26\,\text{kW}}$$

The maximum energy stored in the magnetic field of the inductor is:

$$U_{L,max} = \tfrac{1}{2} L I_{max}^2 = \tfrac{1}{2}(36\,\text{mH})(7.94\,\text{A})^2 = \boxed{1.13\,\text{J}}$$

The definition of $U_{L,av}$ is:

$$U_{L,av} = \frac{1}{T}\int_0^T U(t)\,dt$$

$U(t)$ is given by:

$$U(t)=\frac{1}{2}L[I(t)]^2$$

Substitute for $U(t)$ to obtain:

$$U_{L,\text{av}}=\frac{L}{2T}\int_0^T[I(t)]^2dt$$

Evaluating the integral yields:

$$U_{L,\text{av}}=\frac{L}{2T}\left[\frac{1}{2}I_{\max}^2\right]T=\frac{1}{4}LI_{\max}^2$$

Substitute numerical values and evaluate $U_{L,\text{av}}$:

$$U_{L,\text{av}}=\frac{1}{4}(36\,\text{mH})(7.94\,\text{A})^2=\boxed{0.567\,\text{J}}$$

43 ••

Picture the Problem We can use the definition of the power factor to find the relationship between X_L and R when f = 60 Hz and then use the definition of X_L to relate the inductive reactance at 240 Hz to the inductive reactance at 60 Hz. We can then use the definition of the power factor to determine its value at 240 Hz.

Using the definition of the power factor, relate R and X_L:

$$\cos\delta=\frac{R}{Z}=\frac{R}{\sqrt{R^2+X_L^2}} \qquad (1)$$

Square both sides of the equation to obtain:

$$\cos^2\delta=\frac{R^2}{R^2+X_L^2}$$

Solve for $X_L^2(60\,\text{Hz})$:

$$X_L^2(60\,\text{Hz})=R^2\left(\frac{1}{\cos^2\delta}-1\right)$$

Substitute for $\cos\delta$ and simplify to obtain:

$$X_L^2(60\,\text{Hz})=R^2\left(\frac{1}{(0.866)^2}-1\right)=\tfrac{1}{3}R^2$$

Use the definition of X_L to obtain:

$$X_L^2(f)=4\pi f^2L^2$$

and

$$X_L^2(f')=4\pi f'^2L^2$$

Divide the second of these equations by the first to obtain:

$$\frac{X_L^2(f')}{X_L^2(f)}=\frac{4\pi f'^2L^2}{4\pi f^2L^2}=\frac{f'^2}{f^2}$$

$$\frac{X_L^2(f')}{X_L^2(f)}=\frac{4\pi f'^2L^2}{4\pi f^2L^2}=\frac{f'^2}{f^2}$$

or

$$X_L^2(f') = \left(\frac{f'}{f}\right)^2 X_L^2(f)$$

Substitute numerical values to obtain:

$$X_L^2(240\,\text{Hz}) = \left(\frac{240\,\text{s}^{-1}}{60\,\text{s}^{-1}}\right)^2 X_L^2(60\,\text{Hz})$$
$$= 16\left(\frac{1}{3}R^2\right) = \frac{16}{3}R^2$$

Substitute in equation (1) to obtain:

$$(\cos\delta)_{240\,\text{Hz}} = \frac{R}{\sqrt{R^2 + \frac{16}{3}R^2}}$$
$$= \sqrt{\frac{3}{19}} = \boxed{0.397}$$

***44** ••

Picture the Problem We can apply Kirchhoff's loop rule to obtain expressions for I_R and I_L and then use trigonometric identities to show that $I = I_R + I_L = I_{\max} \cos(\omega t - \delta)$, where $\tan\delta = R/X_L$ and $I_{\max} = \varepsilon_{\max}/Z$ with $Z^{-2} = R^{-2} + X_L^{-2}$.

(*a*) Apply Kirchhoff's loop rule to a clockwise loop that includes the source and the resistor:

$$\mathcal{E}_{\max}\cos\omega t - I_R R = 0$$

Solve for I_R:

$$I_R = \boxed{\frac{\mathcal{E}_{\max}}{R}\cos\omega t}$$

(*b*) Apply Kirchhoff's loop rule to a clockwise loop that includes the source and the inductor:

$$\mathcal{E}_{\max}\cos(\omega t - 90°) - I_L X_L = 0$$

because the current lags the potential difference across the inductor by 90°.

Solve for I_L:

$$I_L = \boxed{\frac{\mathcal{E}_{\max}}{X_L}\cos(\omega t - 90°)}$$

(*c*) Express the current drawn from the source in terms of $I_{\max}$ and the phase constant δ:

$$I = I_R + I_L = I_{\max}\cos(\omega t - \delta)$$

Use a trigonometric identity to expand $\cos(\omega t - \delta)$:

$$I = I_{\max}(\cos\omega t\cos\delta + \sin\omega t\sin\delta)$$
$$= I_{\max}\cos\omega t\cos\delta + I_{\max}\sin\omega t\sin\delta$$

From our results in (*a*):

$$I = I_R + I_L = \frac{\mathcal{E}_{\max}}{R}\cos\omega t + \frac{\mathcal{E}_{\max}}{X_L}\cos(\omega t - 90°) = \frac{\mathcal{E}_{\max}}{R}\cos\omega t + \frac{\mathcal{E}_{\max}}{X_L}\sin\omega t$$

A useful trigonometric identity is:

$$A\cos\omega t + B\sin\omega t = \sqrt{A^2 + B^2}\cos(\omega t - \delta)$$

where

$$\delta = \tan^{-1}\frac{B}{A}$$

Apply this identity to obtain:

$$I = \sqrt{\left(\frac{\mathcal{E}_{\max}}{R}\right)^2 + \left(\frac{\mathcal{E}_{\max}}{X_L}\right)^2}\cos(\omega t - \delta) \quad (1)$$

and

$$\delta = \tan^{-1}\left(\frac{\dfrac{\mathcal{E}_{\max}}{X_L}}{\dfrac{\mathcal{E}_{\max}}{R}}\right) = \tan^{-1}\left(\frac{R}{X_L}\right) \quad (2)$$

Simplify equation (1) and rewrite equation (2) to obtain:

$$I = \sqrt{\left(\frac{\mathcal{E}_{\max}}{R}\right)^2 + \left(\frac{\mathcal{E}_{\max}}{X_L}\right)^2}\cos(\omega t - \delta) = \mathcal{E}_{\max}\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L}\right)^2}\cos(\omega t - \delta) = \mathcal{E}_{\max}\sqrt{\left(\frac{1}{Z}\right)^2}\cos(\omega t - \delta) = \boxed{\frac{\mathcal{E}_{\max}}{Z}\cos(\omega t - \delta)}$$

where

$$\tan\delta = \boxed{\frac{R}{X_L}} \text{ and } \frac{1}{Z^2} = \frac{1}{R^2} + \frac{1}{X_L^2}$$

45 ••

Picture the Problem We can use the complex numbers method to find the impedances of the parallel portion of the circuit and the total impedance of the circuit. We can then use

Kirchhoff's loop rule to obtain an expression for the current drawn form the source. Knowing the current drawn from the source, we can find the potential difference across the parallel portion of the circuit and then use this information to find the currents drawn by the load and the inductor.

(*a*) Express the rms currents in R, C, and R_L:

$$I_{R,\,\text{rms}} = \frac{\mathcal{E}_{\text{rms}}}{Z},\ I_{R_L,\,\text{rms}} = \frac{V_{\text{p,rms}}}{R_L}\text{ ,and}$$

$$I_{L,\,\text{rms}} = \frac{V_{\text{p,rms}}}{X_L}$$

Express the total impedance of the circuit:

$$Z = R + Z_{\text{p}}$$

where Z_{p} is the impedance of the parallel branch of the circuit.

Use complex numbers to relate Z_{p} to R_L and X_C:

$$\frac{1}{Z_{\text{p}}} = \frac{1}{R_L} + \frac{1}{iX_L} = \frac{R_L + iX_L}{iR_L X_L}$$

or

$$Z_{\text{p}} = \frac{iR_L X_L}{R_L + iX_L}$$

Multiple the numerator and denominator of this fraction by the complex conjugate of $R_L + iX_L$ and simplify to obtain:

$$Z_{\text{p}} = \frac{iR_L X_L}{R_L + iX_L}\frac{R_L - iX_L}{R_L - iX_L} = \frac{R_L X_L^2}{R_L^2 + X_L^2} + i\frac{R_L^2 X_L}{R_L^2 + X_L^2}$$

Substitute numerical values and evaluate X_L:

$$X_L = \omega L = 2\pi f L = 2\pi\left(500\,\text{s}^{-1}\right)\left(3.2\,\text{mH}\right) = 10.1\,\Omega$$

Substitute numerical values and evaluate Z_{p}:

$$Z_{\text{p}} = \frac{(20\,\Omega)(10.1\,\Omega)^2}{(20\,\Omega)^2 + (10.1\,\Omega)^2} + i\frac{(20\,\Omega)^2(10.1\,\Omega)}{(20\,\Omega)^2 + (10.1\,\Omega)^2} = 4.06\,\Omega + i(8.05\,\Omega)$$

and

$$\left|Z_{\text{p}}\right| = \sqrt{(4.06\,\Omega)^2 + (8.05\,\Omega)^2} = 9.02\,\Omega$$

Substitute to evaluate Z:

$$Z = 4\,\Omega + 4.03\,\Omega + i(8.05\,\Omega) = 8.03\,\Omega + i(8.05\,\Omega)$$

and

$$|Z| = \sqrt{(8.03\,\Omega)^2 + (8.05\,\Omega)^2} = 11.4\,\Omega$$

Express and evaluate the power factor:

$$\cos\delta = \frac{R}{Z} = \frac{8.05\,\Omega}{11.4\,\Omega} = 0.706$$

Apply Kirchhoff's loop rule to obtain:

$$\mathcal{E}_{\text{rms}} - I_{R,\text{rms}}|Z| = 0$$

Solve for and evaluate $I_{R,\text{ rms}}$:

$$I_{R,\text{ rms}} = \frac{\mathcal{E}_{\text{rms}}}{|Z|} = \frac{100\,\text{V}/\sqrt{2}}{11.4\,\Omega} = \boxed{6.20\,\text{A}}$$

Express and evaluate $V_{\text{p, rms}}$:

$$V_{\text{p, rms}} = I_{R_L,\text{rms}}|Z_{\text{p}}| = (6.20\,\text{A})(9\,\Omega) = 55.8\,\text{V}$$

Substitute numerical values and evaluate $I_{R,\text{ rms}}$:

$$I_{R_L,\text{ rms}} = \frac{55.8\,\text{V}}{20\,\Omega} = \boxed{2.79\,\text{A}}$$

Substitute numerical values and evaluate $I_{L,\text{ rms}}$:

$$I_{L,\text{ rms}} = \frac{55.8\,\text{V}}{10.1\,\Omega} = \boxed{5.52\,\text{A}}$$

(*b*) Proceed as in (*a*) with f = 2000 Hz. Substitute numerical values and evaluate X_L:

$$X_L = \omega L = 2\pi f L = 2\pi(2000\,\text{s}^{-1})(3.2\,\text{mH}) = 40.2\,\Omega$$

Substitute numerical values and evaluate Z_{p}:

$$Z_{\text{p}} = \frac{(20\,\Omega)(40.2\,\Omega)^2}{(20\,\Omega)^2 + (40.2\,\Omega)^2} + i\frac{(20\,\Omega)^2(40.2\,\Omega)}{(20\,\Omega)^2 + (40.2\,\Omega)^2} = 16.0\,\Omega + i(7.98\,\Omega)$$

and

$$|Z_{\text{p}}| = \sqrt{(16.0\,\Omega)^2 + (7.98\,\Omega)^2} = 17.9\,\Omega$$

Substitute to evaluate Z:

$$Z = 4\,\Omega + 16.0\,\Omega + i(7.97\,\Omega) = 20.0\,\Omega + i(7.98\,\Omega)$$

and

$$|Z| = \sqrt{(20.0\,\Omega)^2 + (7.98\,\Omega)^2} = 21.5\,\Omega$$

Find the power factor:

$$\cos\delta = \frac{R}{Z} = \frac{20.0\,\Omega}{21.5\,\Omega} = 0.930$$

Apply Kirchhoff's loop rule to obtain:

$$\mathcal{E}_{\text{rms}} - I_{R,\text{rms}}|Z| = 0$$

Solve for and evaluate $I_{R,\text{ rms}}$:

$$I_{R,\text{ rms}} = \frac{\mathcal{E}_{\text{rms}}}{|Z|} = \frac{100\,\text{V}/\sqrt{2}}{21.5\,\Omega} = \boxed{3.29\,\text{A}}$$

Express and evaluate $V_{\text{p, rms}}$:

$$V_{\text{p, rms}} = I_{R_L,\text{rms}}|Z_{\text{p}}| = (3.29\,\text{A})(17.9\,\Omega) = 58.9\,\text{V}$$

Substitute numerical values and evaluate $I_{R,\text{ rms}}$:

$$I_{R_L,\text{ rms}} = \frac{58.9\,\text{V}}{20\,\Omega} = \boxed{2.95\,\text{A}}$$

Substitute numerical values and evaluate $I_{L,\text{ rms}}$:

$$I_{L,\text{ rms}} = \frac{58.9\,\text{V}}{40.2\,\Omega} = \boxed{1.47\,\text{A}}$$

(*c*) Express the fraction of the power dissipated in the resistor:

$$\frac{P_{L,\text{rms}}}{P_{\text{tot}}} = \frac{I^2_{R_L,\text{ rm}}R_L}{\mathcal{E}_{\text{rms}}I_{R,\text{ rms}}\cos\delta}$$

Evaluate this fraction for f = 500 Hz:

$$\left.\frac{P_{L,\text{rms}}}{P_{\text{tot}}}\right|_{f=500\text{ Hz}} = \frac{(2.79\text{A})^2(20\,\Omega)}{\left(\frac{100\,\text{V}}{\sqrt{2}}\right)(6.20\,\text{A})(0.706)} = 0.503 = \boxed{50.3\%}$$

When f = 2000 Hz:

$$\left.\frac{P_{L,\text{rms}}}{P_{\text{tot}}}\right|_{f=2000\text{ Hz}} = \frac{(2.95\,\text{A})^2(20\,\Omega)}{\left(\frac{100\,\text{V}}{\sqrt{2}}\right)(3.29\,\text{A})(0.930)} = 0.804 = \boxed{80.4\%}$$

46 ••

Picture the Problem We can treat the ac and dc components separately. For the dc component, L acts like a short circuit. For convenience we let ε_1 denote the maximum value of the ac emf. We can use $P = \mathcal{E}_1^2/R_{1,2}$ to find the power dissipated in the resistors due to the dc source. We'll apply Kirchhoff's loop rule the loop including L, R_1, and R_2 to derive an expression for the power dissipated in the resistors due to the ac source. Note that only the power dissipated in the resistor R_2 due to the ac source is frequency dependent.

(*a*) Express the total power dissipated in R_1 and R_2:

$$P = P_{dc} + P_{ac} \qquad (1)$$

Express and evaluate the dc power dissipated in R_1 and R_2:

$$P_{1,dc} = \frac{\mathcal{E}_2^2}{R_1} = \frac{(16\,\text{V})^2}{10\,\Omega} = 25.6\,\text{W}$$

and

$$P_{2,dc} = \frac{\mathcal{E}_2^2}{R_2} = \frac{(16\,\text{V})^2}{8\,\Omega} = 32.0\,\text{W}$$

Express and evaluate the average ac power dissipated in R_1:

$$P_{1,ac} = \frac{1}{2}\frac{\mathcal{E}_1^2}{R_1} = \frac{1}{2}\frac{(20\,\text{V})^2}{10\,\Omega} = 20.0\,\text{W}$$

Apply Kirchhoff's loop rule to a clockwise loop that includes R_1, L, and R_2:

$$R_1 I_1 - Z_2 I_2 = 0$$

Solve for I_2:

$$I_2 = \frac{R_1}{Z_2} I_1 = \frac{R_1}{Z_2}\frac{\mathcal{E}_1}{R_1} = \frac{\mathcal{E}_1}{Z_2}$$

Express the average ac power dissipated in R_2:

$$P_{2,\,ac} = \tfrac{1}{2} I_2^2 R_2 = \tfrac{1}{2}\left(\frac{\mathcal{E}_1}{Z_2}\right)^2 R_2 = \frac{1}{2}\frac{\mathcal{E}_1^2 R_2}{Z_2^2}$$

Substitute numerical values and evaluate $P_{2,\,ac}$:

$$P_{2,\,ac} = \frac{1}{2}\left[\frac{(20\,\text{V})^2(8\,\Omega)}{(8\,\Omega)^2 + \left(2\pi\{100\,\text{s}^{-1}\}\{6\,\text{mH}\}\right)^2}\right] = 20.5\,\text{W}$$

Substitute in equation (1) to obtain:

$$P_1 = 25.6\,\text{W} + 20.0\,\text{W} = \boxed{45.6\,\text{W}}$$

$$P_2 = 32.0\,\text{W} + 20.5\,\text{W} = \boxed{52.5\,\text{W}}$$

and

$$P = P_1 + P_2 = \boxed{98.1\text{W}}$$

(*b*) Proceed as in (*a*) to evaluate $P_{2,\,ac}$ with $f = 200$ Hz:

$$P_{2,\,ac} = \frac{1}{2}\left[\frac{(20\,\text{V})^2(8\,\Omega)}{(8\,\Omega)^2 + \left(2\pi\{200\,\text{s}^{-1}\}\{6\,\text{mH}\}\right)^2}\right] = 13.2\,\text{W}$$

Substitute in equation (1) to obtain:

$$P_1 = 25.6\,\text{W} + 20.0\,\text{W} = \boxed{45.6\,\text{W}}$$

$$P_2 = 32.0\,\text{W} + 13.2\,\text{W} = \boxed{45.2\,\text{W}}$$

and

$$P = P_1 + P_2 = \boxed{90.8\,\text{W}}$$

(*c*) Proceed as in (*a*) to evaluate $P_{2,\,\text{ac}}$ with f = 800 Hz:

$$P_{2,\,\text{ac}} = \frac{1}{2}\frac{(20\,\text{V})^2(8\,\Omega)}{\left[(8\,\Omega)^2 + \left(2\pi\left(800\,\text{s}^{-1}\right)(6\,\text{mH})\right)^2\right]} = 1.64\,\text{W}$$

Substitute in equation (1) to obtain:

$$P_1 = 25.6\,\text{W} + 20.0\,\text{W} = \boxed{45.6\,\text{W}}$$

$$P_2 = 32.0\,\text{W} + 1.64\,\text{W} = \boxed{33.6\,\text{W}}$$

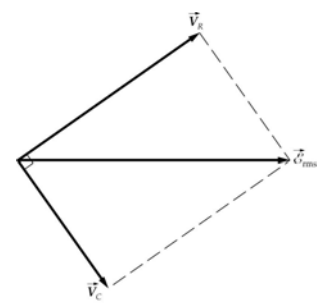
and

$$P = P_1 + P_2 = \boxed{79.2\,\text{W}}$$

47 ••

Picture the Problem We can use the phasor diagram for an *RC* circuit to find the voltage across the resistor.

Sketch the phasor diagram for the voltages in the circuit:



Use the Pythagorean theorem to express V_R:

$$V_R = \sqrt{\mathcal{E}_{\text{rms}}^2 - V_C^2}$$

Substitute numerical values and evaluate V_R:

$$V_R = \sqrt{(100\,\text{V})^2 - (80\,\text{V})^2} = \boxed{60.0\,\text{V}}$$

Filters and Rectifiers

***48** ••

Picture the Problem We can use Kirchhoff's loop rule to obtain a differential equation relating the input, capacitor, and resistor voltages. We'll then assume a solution to this equation that is a linear combination of sine and cosine terms with coefficients that we can find by substitution in the differential equation. Repeating this process for the output side of the filter will yield the desired equation.

Apply Kirchhoff's loop rule to the input side of the filter to obtain:

$$V_{in} - V - IR = 0$$

where V is the potential difference across the capacitor.

Substitute for V_{in} and I to obtain:

$$V_{peak}\cos\omega t - V - R\frac{dQ}{dt} = 0$$

Because $Q = CV$:

$$\frac{dQ}{dt} = \frac{d}{dt}[CV] = C\frac{dV}{dt}$$

Substitute for dQ/dt to obtain:

$$V_{peak}\cos\omega t - V - RC\frac{dV}{dt} = 0$$

the differential equation describing the potential difference across the capacitor.

Assume a solution of the form:

$$V = V_c\cos\omega t + V_s\sin\omega t$$

Substitution of this assumed solution and its first derivative in the differential equations, followed by equating the coefficients of the sine and cosine terms, yields two coupled linear equations:

$$V_c + \omega RCV_s = V_{peak}$$

and

$$V_s - \omega RCV_c = 0$$

Solve these equations simultaneously to obtain:

$$V_c = \frac{1}{1 + (\omega RC)^2}V_{peak}$$

and

$$V_s = \frac{\omega RC}{1 + (\omega RC)^2}V_{peak}$$

Note that the output voltage is the voltage across the resistor and that it is phase shifted relative to the input voltage:

$$V_{out} = V_H\cos(\omega t - \delta)$$

where V_H is the amplitude of the signal.

Assume that V_H is of the form:

$$V_H(t) = v_c\cos\omega t + v_s\sin\omega t$$

The input, output, and capacitor voltages are related according to:

$$V_H(t) = V_{in}(t) - V(t)$$

Substitute for $V_H(t)$, $V_{peak}(t)$, and $V(t)$ and use the previously established values for V_c and V_s to obtain:

$$v_c = V_{peak} - V_c$$

and

$$v_s = -V_s$$

Substitute for V_c and V_s to obtain:

$$v_c = \frac{(\omega RC)^2}{1 + (\omega RC)^2}V_{peak}$$

and

$$v_s = -\frac{\omega RC}{1+(\omega RC)^2}V_{peak}$$

V_H, v_c, and v_s are related according to the Pythagorean relationship:

$$V_H = \sqrt{v_c^2 + v_s^2}$$

Substitute for v_c and v_s to obtain:

$$V_H = \frac{\omega RC}{\sqrt{1+(\omega RC)^2}}V_{peak} = \boxed{\frac{V_{peak}}{\sqrt{1+\left(\frac{1}{\omega RC}\right)^2}}}$$

49 ••

Picture the Problem We can use some of the intermediate results from Problem 48 to express the tangent of the phase constant.

(*a*) Because, as was shown in Problem 48, $V_H = \sqrt{v_c^2 + v_s^2}$:

$$\tan\delta = \frac{v_s}{v_c}$$

Also from Problem 48:

$$v_c = \frac{(\omega RC)^2}{1+(\omega RC)^2}V_{peak}$$

and

$$v_s = -\frac{\omega RC}{1+(\omega RC)^2}V_{peak}$$

Substitute to obtain:

$$\tan\delta = \frac{-\frac{\omega RC}{1+(\omega RC)^2}V_{peak}}{\frac{(\omega RC)^2}{1+(\omega RC)^2}V_{peak}} = \boxed{-\frac{1}{\omega RC}}$$

(*b*) Solve for δ:

$$\delta = \tan^{-1}\left[-\frac{1}{\omega RC}\right]$$

As $\omega \to 0$:

$$\delta \to \boxed{-90°}$$

(*c*) As $\omega \to \infty$:

$$\delta \to \boxed{0}$$

50 ••

Picture the Problem We can use the results obtained in Problems 48 and 49 to find $f_{3\,dB}$ and to plot graphs of log(V_{out}) versus log(f) and δ versus log(f).

(*a*) Express the ratio V_{out}/V_{in}:

$$\frac{V_{out}}{V_{in}} = \frac{\dfrac{V_{peak}}{\sqrt{1+\left(\dfrac{1}{\omega RC}\right)^2}}}{V_{peak}} = \frac{1}{\sqrt{1+\left(\dfrac{1}{\omega RC}\right)^2}}$$

When $V_{out} = V_{in}/\sqrt{2}$:

$$\frac{1}{\sqrt{1+\left(\dfrac{1}{\omega RC}\right)^2}} = \frac{1}{\sqrt{2}}$$

Square both sides of the equation and solve for ωRC to obtain:

$$\omega RC = 1 \Rightarrow \omega = \frac{1}{RC} \Rightarrow f_{3\,\text{dB}} = \frac{1}{2\pi RC}$$

Substitute numerical values and evaluate $f_{3\text{ dB}}$:

$$f_{3\,\text{dB}} = \frac{1}{2\pi(20\,\text{k}\Omega)(15\,\text{nF})} = \boxed{531\,\text{Hz}}$$

(*b*) From Problem 48 we have:

$$V_{out} = \frac{V_{peak}}{\sqrt{1+\left(\dfrac{1}{\omega RC}\right)^2}}$$

From Problem 49 we have:

$$\delta = \tan^{-1}\left[-\frac{1}{\omega RC}\right]$$

Rewrite these expressions in terms of $f_{3\text{ dB}}$ to obtain:

$$V_{out} = \frac{V_{peak}}{\sqrt{1+\left(\dfrac{1}{2\pi fRC}\right)^2}} = \frac{V_{peak}}{\sqrt{1+\left(\dfrac{f_{3\,\text{dB}}}{f}\right)^2}}$$

and

$$\delta = \tan^{-1}\left[-\frac{1}{2\pi fRC}\right] = \tan^{-1}\left[-\frac{f_{3\,\text{dB}}}{f}\right]$$

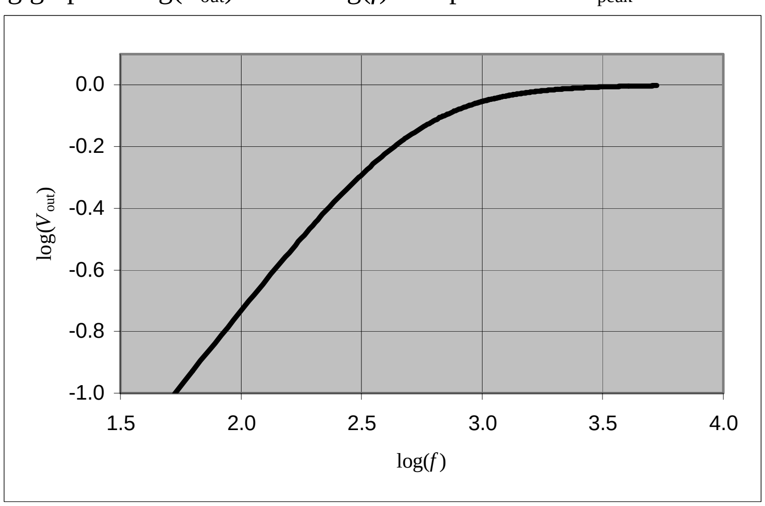
A spreadsheet program to generate the data for a graph of V_{out} versus f and δ versus f is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
B1	2.00E+03	R
B2	1.50E–08	C
B3	1	V_{peak}
B4	531	$f_{3\text{ dB}}$
A8	53	$0.1f_{3\text{ dB}}$

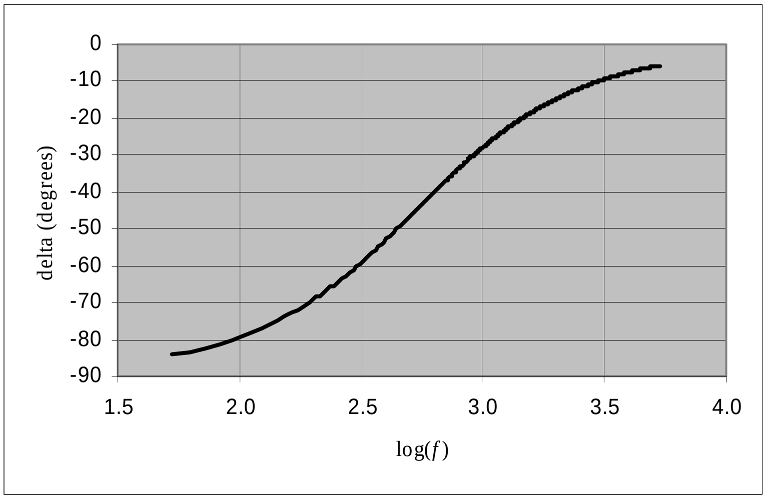
C8	$B$3/SQRT(1+(1($B$4/A8))^2)	$\dfrac{V_{\text{peak}}}{\sqrt{1+\left(\dfrac{f_{3\,\text{dB}}}{f}\right)^2}}$
D8	LOG(C8)	$\log(V_{\text{out}})$
E8	ATAN(–$B$4/A8)	$\tan^{-1}\left[-\dfrac{f_{3\,\text{dB}}}{f}\right]$
F8	E8*180/PI()	δ in degrees

	A	B	C	D	E	F
1	R=	2.00E+04	ohms			
2	C=	1.50E–08	F			
3	V_peak=	1	V			
4	f_3 dB=	531	Hz			
5						
6						
7	f	log(f)	V_out	log(V_out)	delta(rad)	delta(deg)
8	53	1.72	0.099	–1.003	–1.471	–84.3
9	63	1.80	0.118	–0.928	–1.453	–83.2
10	73	1.86	0.136	–0.865	–1.434	–82.2
11	83	1.92	0.155	–0.811	–1.416	–81.1
55	523	2.72	0.702	–0.154	–0.793	–45.4
56	533	2.73	0.709	–0.150	–0.783	–44.9
57	543	2.73	0.715	–0.146	–0.774	–44.3
531	5283	3.72	0.995	–0.002	–0.100	–5.7
532	5293	3.72	0.995	–0.002	–0.100	–5.7
533	5303	3.72	0.995	–0.002	–0.100	–5.7
534	5313	3.73	0.995	–0.002	–0.100	–5.7

The following graph of $\log(V_{\text{out}})$ versus $\log(f)$ was plotted for $V_{\text{peak}} = 1$ V.



A graph of δ in degrees as a function of $\log(f)$ follows.



Referring to the spreadsheet program, we see that when $f = f_{3\,\text{dB}}$, $\delta \approx \boxed{-44.9°.}$ This result is in good agreement with its calculated value of –45.0°.

51 •••

Picture the Problem We can use Kirchhoff's loop rule to obtain a differential equation relating the input, capacitor, and resistor voltages. Because the voltage drop across the resistor is small compared to the voltage drop across the capacitor, we can express the voltage drop across the capacitor in terms of the input voltage.

Apply Kirchhoff's loop rule to the input side of the filter to obtain:

$$V_{\text{in}} - V_C - IR = 0$$

where V_C is the potential difference across the capacitor.

Substitute for V_{in} and I to obtain:

$$V_{\text{peak}} \cos \omega t - V_{\text{c}} - R\frac{dQ}{dt} = 0$$

Because $Q = CV_C$:

$$\frac{dQ}{dt} = \frac{d}{dt}\left[CV_C\right] = C\frac{dV_C}{dt}$$

Substitute for dQ/dt to obtain:

$$V_{\text{peak}} \cos \omega t - V_C - RC\frac{dV_C}{dt} = 0$$

the differential equation describing the potential difference across the capacitor.

Because the voltage drop across the resistor is very small compared to the voltage drop across the capacitor:

$$V_{\text{peak}} \cos \omega t - V_C \approx 0$$

and

$$V_C \approx V_{\text{peak}} \cos \omega t$$

Consequently, the potential difference across the resistor is given by:

$$V_R = RC\frac{dV_C}{dt} \approx \boxed{RC\frac{d}{dt}\left[V_{\text{peak}} \cos \omega t\right]}$$

52 ••

Picture the Problem We can use the expression for V_H from Problem 48 and the definition of β given in the problem to show that every time the frequency is halved, the output drops by 6 dB.

From Problem 48:

$$V_H = \frac{V_{peak}}{\sqrt{1+\left(\frac{1}{\omega RC}\right)^2}}$$

or

$$\frac{V_H}{V_{peak}} = \frac{1}{\sqrt{1+\left(\frac{1}{\omega RC}\right)^2}}$$

Express this ratio in terms of f and $f_{3\,dB}$:

$$\frac{V_H}{V_{peak}} = \frac{1}{\sqrt{1+\left(\frac{f_{3\,dB}}{f}\right)^2}} = \frac{f}{\sqrt{f_{3\,dB}^2\left(1+\frac{f^2}{f_{3\,dB}^2}\right)}}$$

For $f << f_{3dB}$:

$$\frac{V_H}{V_{peak}} \approx \frac{f}{\sqrt{f_{3\,dB}^2\left(1+\frac{f^2}{f_{3\,dB}^2}\right)}} = \frac{f}{f_{3\,dB}}$$

From the definition of β we have:

$$\beta = 20\log_{10}\frac{V_H}{V_{peak}}$$

Substitute for V_H/V_{peak} to obtain:

$$\beta = 20\log_{10}\frac{f}{f_{3\,dB}}$$

Doubling the frequency yields:

$$\beta' = 20\log_{10}\frac{2f}{f_{3\,dB}}$$

The change in decibel level is:

$$\begin{aligned}\Delta\beta &= \beta' - \beta \\ &= 20\log_{10}\frac{2f}{f_{3\,dB}} - 20\log_{10}\frac{f}{f_{3\,dB}} \\ &= 20\log_{10}2 = \boxed{6.02\,\text{dB}}\end{aligned}$$

***53** ••

Picture the Problem We can express the instantaneous power dissipated in the resistor and then use the fact that the average value of the square of the cosine function over one cycle is ½ to establish the given result.

The instantaneous power $P(t)$ dissipated in the resistor is:

$$P(t)=\frac{V_{\text{out}}^2}{R}$$

The output voltage V_{out} is:

$$V_{\text{out}}=V_{\text{H}}\cos(\omega t-\delta)$$

From Problem 48:

$$V_{\text{H}}=\frac{V_{\text{peak}}}{\sqrt{1+\left(\frac{1}{\omega RC}\right)^2}}$$

Substitute in the expression for $P(t)$ to obtain:

$$P(t)=\frac{V_{\text{H}}^2}{R}\cos^2(\omega t-\delta)$$

$$=\frac{V_{\text{peak}}^2}{R\left[1+\left(\frac{1}{\omega RC}\right)^2\right]}\cos^2(\omega t-\delta)$$

Because the average value of the square of the cosine function over one cycle is ½:

$$P_{\text{ave}}=\frac{V_{\text{peak}}^2}{2R\left[1+\left(\frac{1}{\omega RC}\right)^2\right]}$$

Simplify this expression to obtain:

$$\boxed{P_{\text{ave}}=\frac{V_{\text{peak}}^2}{2R}\left(\frac{(\omega RC)^2}{1+(\omega RC)^2}\right)}$$

54 ••

Picture the Problem We can solve the expression for V_{H} from Problem 48 for the required capacitance of the capacitor.

From Problem 48:

$$V_{\text{H}}=\frac{V_{\text{peak}}}{\sqrt{1+\left(\frac{1}{\omega RC}\right)^2}}$$

We require that:

$$\frac{V_{\text{H}}}{V_{\text{peak}}}=\frac{1}{\sqrt{1+\left(\frac{1}{\omega RC}\right)^2}}=\frac{1}{10}$$

or

$$\sqrt{1+\left(\frac{1}{\omega RC}\right)^2}=10$$

Solve for C to obtain:

$$C=\frac{1}{\sqrt{99}\,\omega R}=\frac{1}{2\pi\sqrt{99}\,Rf}$$

Substitute numerical values and evaluate C:

$$C = \frac{1}{2\pi\sqrt{99}(20\,\text{k}\Omega)(60\,\text{Hz})} = \boxed{13.3\,\text{nF}}$$

55 ••

Picture the Problem We can use Kirchhoff's loop rule to obtain a differential equation relating the input, capacitor, and resistor voltages. We'll then assume a solution to this equation that is a linear combination of sine and cosine terms with coefficients that we can find by substitution in the differential equation. The solution to these simultaneous equations will yield the amplitude of the output voltage.

Apply Kirchhoff's loop rule to the input side of the filter to obtain:

$$V_{\text{in}} - IR - V = 0$$

where V is the potential difference across the capacitor.

Substitute for V_{in} and I to obtain:

$$V_{\text{peak}}\cos\omega t - R\frac{dQ}{dt} - V = 0$$

Because $Q = CV$:

$$\frac{dQ}{dt} = \frac{d}{dt}[CV] = C\frac{dV}{dt}$$

Substitute for dQ/dt to obtain:

$$V_{\text{peak}}\cos\omega t - RC\frac{dV}{dt} - V = 0$$

the differential equation describing the potential difference across the capacitor.

Assume a solution of the form:

$$V = V_{\text{c}}\cos\omega t + V_{\text{s}}\sin\omega t$$

Substitution of this assumed solution and its first derivative in the differential equation, followed by equating the coefficients of the sine and cosine terms, yields two coupled linear equations:

$$V_{\text{c}} + \omega RCV_{\text{s}} = V_{\text{peak}}$$

and

$$V_{\text{s}} - \omega RCV_{\text{c}} = 0$$

Solve these equations simultaneously to obtain:

$$V_{\text{c}} = \frac{1}{1+(\omega RC)^2}V_{\text{peak}}$$

and

$$V_{\text{s}} = \frac{\omega RC}{1+(\omega RC)^2}V_{\text{peak}}$$

Note that the output voltage is the voltage across the capacitor and that it is phase shifted relative to the input voltage:

$$V_{\text{out}} = V_L\cos(\omega t - \delta)$$

where V_L is the amplitude of the signal.

V_L, V_{c}, and V_{s} are related according to the Pythagorean relationship:

$$V_L = \sqrt{V_{\text{c}}^2 + V_{\text{s}}^2}$$

Substitute for V_c and V_s to obtain:

$$V_L = \sqrt{\left[\frac{1}{1+(\omega RC)^2}V_{\text{peak}}\right]^2 + \left[\frac{\omega RC}{1+(\omega RC)^2}V_{\text{peak}}\right]^2}$$

Simplify algebraically to obtain:

$$V_L = \boxed{\frac{V_{\text{peak}}}{\sqrt{1+(\omega RC)^2}}}$$

$$\boxed{\text{As } f \to 0, V_L \to V_{\text{peak}}. \text{ As } f \to \infty, V_L \to 0.}$$

56 ••

Picture the Problem We can use some of the intermediate results from Problem 55 to express the tangent of the phase constant.

From Problem 55:

$$V_L = \sqrt{V_c^2 + V_s^2}$$

where $\tan\delta = \dfrac{V_s}{V_c}$

Also from Problem 55:

$$V_c = \frac{1}{1+(\omega RC)^2}V_{\text{peak}}$$

and

$$V_s = \frac{\omega RC}{1+(\omega RC)^2}V_{\text{peak}}$$

Substitute to obtain:

$$\tan\delta = \frac{\dfrac{\omega RC}{1+(\omega RC)^2}V_{\text{peak}}}{\dfrac{1}{1+(\omega RC)^2}V_{\text{peak}}} = \boxed{\omega RC}$$

Solve for δ:

$$\delta = \tan^{-1}(\omega RC)$$

As $\omega \to 0$:

$$\delta \to \boxed{0}$$

(c) As $\omega \to \infty$:

$$\delta \to \boxed{90°}$$

Remarks: See the spreadsheet solution in the following problem for additional evidence that our answer for Part (*c*) is correct.

***57** ••

Picture the Problem We can use the expressions for V_L and δ derived in Problem 56 to plot the graphs of V_L versus f and δ versus f for the low-pass filter of Problem 55. We'll simplify the spreadsheet program by expressing both V_L and δ as functions of $f_{3\text{ dB}}$.

From Problem 56 we have:

$$V_L = \frac{V_{\text{peak}}}{\sqrt{1+(\omega RC)^2}}$$

and

$$\delta = \tan^{-1}(\omega RC)$$

Rewrite each of these expressions in terms of $f_{3\text{ dB}}$ to obtain:

$$V_L = \frac{V_{\text{peak}}}{\sqrt{1+(2\pi fRC)^2}} = \frac{V_{\text{peak}}}{\sqrt{1+\left(\frac{f}{f_{3\text{dB}}}\right)^2}}$$

and

$$\delta = \tan^{-1}(2\pi fRC) = \tan^{-1}\left(\frac{f}{f_{3\text{dB}}}\right)$$

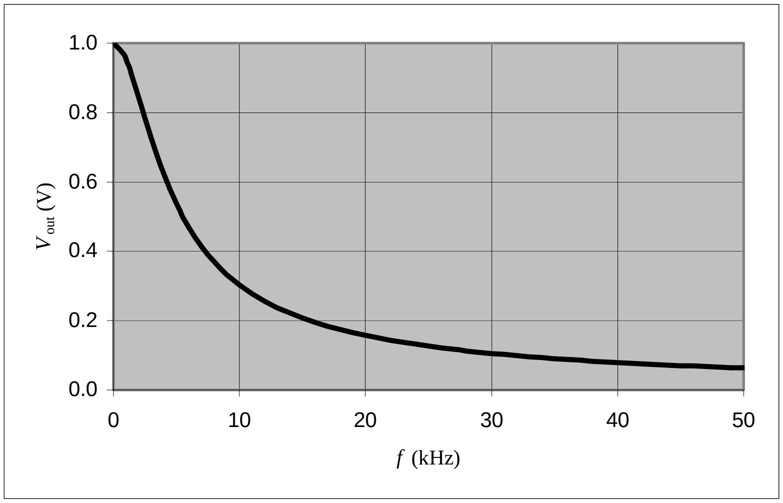
A spreadsheet program to generate the data for graphs of V_L versus f and δ versus f for the low-pass filter is shown below. Note that V_{peak} has been arbitrarily set equal to 1 V. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
B1	2.00E+03	R
B2	5.00E–09	C
B3	1	V_{peak}
B4	(2*PI()*$B$1*$B$2)^–1	$f_{3\text{ dB}}$
B8	$B$3/SQRT(1+((A8/$B$4)^2))	$\frac{V_{\text{peak}}}{\sqrt{1+\left(\frac{f}{f_{3\text{dB}}}\right)^2}}$
C8	ATAN(A8/$B$4)	$\tan^{-1}\left(\frac{f}{f_{3\text{dB}}}\right)$
D8	C8*180/PI()	δ in degrees

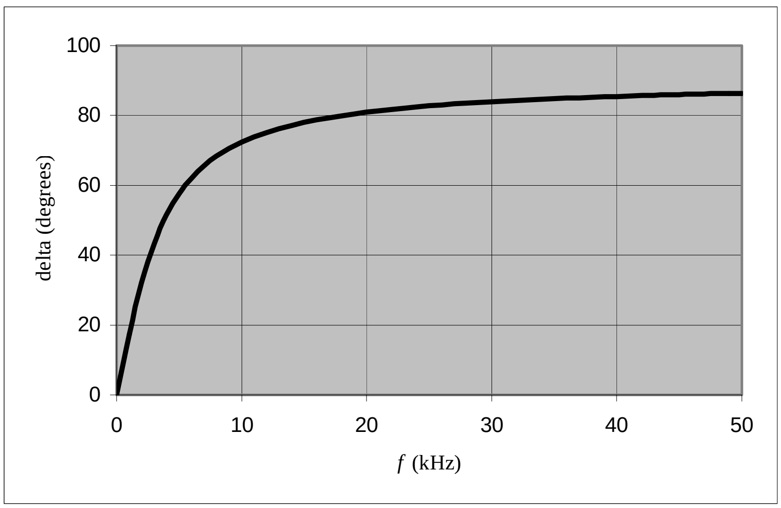
	A	B	C	D
1	R=	1.00E+04	ohms	
2	C=	5.00E–09	F	
3	V_peak=	1	V	
4	f_3 dB=	3.183	kHz	
5				
6	f(kHz)	V_out	delta(rad)	delta(deg)
7	0	1.000	0.000	0.0
8	1	0.954	0.304	17.4
9	2	0.847	0.561	32.1
10	3	0.728	0.756	43.3
54	47	0.068	1.503	86.1
55	48	0.066	1.505	86.2
56	49	0.065	1.506	86.3

57	50	0.064	1.507	86.4

A graph of V_{out} as a function of f follows:



A graph of δ as a function of f follows:



58 •••

Picture the Problem We can use Kirchhoff's loop rule to obtain a differential equation relating the input, capacitor, and resistor voltages. We'll then assume a solution to this equation that is a linear combination of sine and cosine terms with coefficients that we can find by substitution in the differential equation. The solution to these simultaneous equations will yield the amplitude of the output voltage.

Apply Kirchhoff's loop rule to the input side of the filter to obtain:

$$V_{in} - IR - V_C = 0$$

where V_C is the potential difference across the capacitor.

Substitute for V_{in} and I to obtain:

$$V_{peak} \cos \omega t - R\frac{dQ}{dt} - V_C = 0$$

Because $Q = CV_C$:

$$\frac{dQ}{dt} = \frac{d}{dt}[CV_C] = C\frac{dV_C}{dt}$$

Substitute for dQ/dt to obtain:

$$V_{\text{peak}} \cos\omega t - RC\frac{dV_C}{dt} - V_C = 0$$

the differential equation describing the potential difference across the capacitor.

The output voltage is the voltage across the capacitor. Because this voltage is small :

$$V_{\text{peak}} \cos\omega t - RC\frac{dV_C}{dt} \approx 0$$

Separate the variables in this differential equation and solve for V_C:

$$V_C = \boxed{\frac{1}{RC}\int V_{\text{peak}} \cos\omega t dt}$$

***59** •••

Picture the Problem We can apply Kirchhoff's loop rule to both the input side and output side of the trap filter to obtain an expression for the impedance of the trap. Requiring that the impedance of the trap be zero will yield the frequency at which the circuit rejects signals. Defining the bandwidth as $\Delta\omega = \left|\omega - \omega_{trap}\right|$ and requiring that $\left|Z_{trap}\right| = R$ will yield an expression for the bandwidth and reveal its dependence on R.

Apply Kirchhoff's loop rule to the output of the trap circuit to obtain:

$$V_{\text{out}} - IX_L - IX_C = 0$$

Solve for V_{out}:

$$V_{\text{out}} = I\left(X_L + X_C\right) = IZ_{trap} \qquad (1)$$

where $Z_{trap} = X_L + X_C$

Apply Kirchhoff's loop rule to the input of the trap circuit to obtain:

$$V_{\text{in}} - IR - IX_L - IX_C = 0$$

Solve for I:

$$I = \frac{V_{\text{in}}}{R + X_L + X_C} = \frac{V_{\text{in}}}{R + Z_{trap}}$$

Substitute for I in equation (1) to obtain:

$$V_{\text{out}} = V_{\text{in}}\frac{Z_{trap}}{R + Z_{trap}}$$

Because $X_L = i\omega L$ and $X_C = \frac{-i}{\omega C}$:

$$Z_{trap} = i\left(\omega L - \frac{1}{\omega C}\right)$$

Note that $Z_{trap} = 0$ and $V_{\text{out}} = 0$ provided:

$$\omega = \boxed{\frac{1}{\sqrt{LC}}}$$

Let the bandwidth $\Delta\omega$ be:

$$\Delta\omega = \left|\omega - \omega_{trap}\right| \qquad (2)$$

Let the frequency bandwidth to be defined by the frequency at which $\left|Z_{trap}\right| = R$. Then:

$$\omega L - \frac{1}{\omega C} = R$$

or

$$\omega^2 LC - 1 = \omega RC$$

Because $\omega_{trap} = \dfrac{1}{\sqrt{LC}}$:

$$\left(\frac{\omega}{\omega_{trap}}\right)^2 - 1 = \omega RC$$

For $\omega \approx \omega_{trap}$:

$$\left(\frac{\omega^2 - \omega_{trap}^2}{\omega_{trap}}\right) \approx \omega_{trap} RC$$

Solve for $\omega^2 - \omega_{trap}^2$:

$$\omega^2 - \omega_{trap}^2 = \left(\omega - \omega_{trap}\right)\left(\omega + \omega_{trap}\right)$$

Because $\omega \approx \omega_{trap}$, $\omega - \omega_{trap} \approx 2\omega_{trap}$:

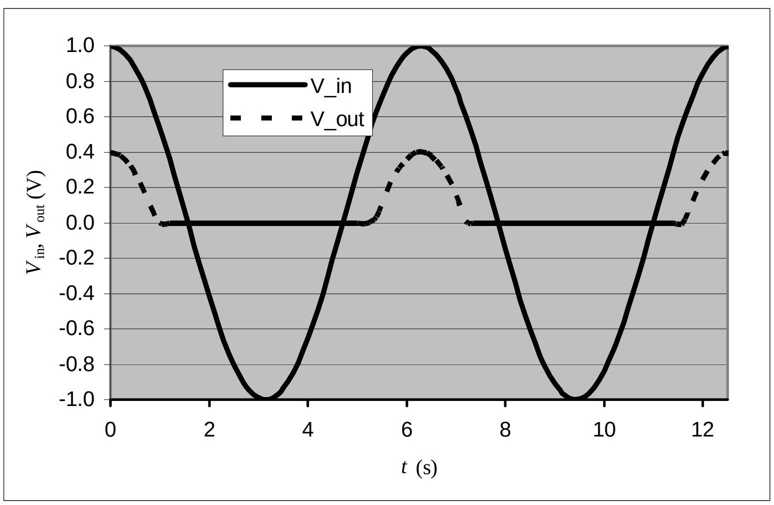
$$\omega^2 - \omega_{trap}^2 \approx 2\omega_{trap}\left(\omega - \omega_{trap}\right)$$

Substitute in equation (2) to obtain:

$$\Delta\omega = \left|\omega - \omega_{trap}\right| = \frac{RC\omega_{trap}^2}{2} = \boxed{\frac{R}{2L}}$$

60 ••

Picture the Problem For voltages greater than 0.6 V, the output voltage will mirror the input voltage minus a 0.6 V drop. But when the voltage swings below 0.6 V, the output voltage will be 0. A spreadsheet program was used to plot the following graph. The angular frequency and the peak voltage were both arbitrarily set equal to one.



61 ••

Picture the Problem We can use the decay of the potential difference across the capacitor to relate the time constant for the *RC* circuit to the frequency of the input signal. Expanding the exponential factor in the expression for V_C will allow us to find the approximate value for *C* that will limit the variation in the output voltage by less than 50 percent (or any other percentage).

The voltage across the capacitor is given by:

$$V_C = V_{\text{in}} e^{-t/RC}$$

Expand the exponential factor to obtain:

$$e^{-t/RC} \approx 1 - \frac{1}{RC}t$$

For a decay of less than 50 percent:

$$1 - \frac{1}{RC}t \le 0.5$$

Solve for *C* to obtain:

$$C \le \frac{2}{R}t$$

Because the voltage goes positive every cycle, $t = 1/60$ s and:

$$C \le \frac{2}{1\,\text{k}\Omega}\left(\frac{1}{60}\text{s}\right) = \boxed{33.3\,\mu\text{F}}$$

LC Circuits with a Generator

62 ••

Picture the Problem We know that the current leads the voltage across and capacitor and lags the voltage across an inductor. We can use $I_{L,\max} = \mathcal{E}_{\max}/X_L$ and $I_{C,\max} = \mathcal{E}_{\max}/X_C$ to find the amplitudes of these currents. The current in the generator will vanish under resonance conditions, i.e., when $|I_L| = |I_C|$. To find the currents in the inductor and capacitor at resonance, we can use the common potential difference across them and their reactances … together with our knowledge of the phase relationships mentioned above.

(*a*) Express the amplitudes of the currents through the inductor and the capacitor:

$$I_{L,\max} = \frac{\mathcal{E}_{\max}}{X_L}$$

and

$$I_{C,\max} = \frac{\mathcal{E}_{\max}}{X_C}$$

Express X_L and X_C:

$$X_L = \omega L \text{ and } X_C = \frac{1}{\omega C}$$

Substitute to obtain:

$$I_{L,\max} = \frac{100\,\text{V}}{(4\,\text{H})\omega}$$

$$= \boxed{\frac{25\,\text{V/H}}{\omega}\text{, lagging } \mathcal{E} \text{ by } 90°}$$

and

$$I_{C,\max} = \frac{100\,\text{V}}{\frac{1}{(25\,\mu\text{F})\omega}}$$

$$= \boxed{\begin{array}{l}(2.5\times10^{-3}\,\text{V}\cdot\text{F})\omega,\\ \text{leading } \mathcal{E} \text{ by } 90°\end{array}}$$

(*b*) Express the condition that $I = 0$:

$$|I_L| = |I_C|$$

or

$$\frac{\mathcal{E}}{\omega L} = \frac{\mathcal{E}}{\frac{1}{\omega C}} = \omega C\mathcal{E}$$

Solve for ω to obtain:

$$\omega = \frac{1}{\sqrt{LC}}$$

Substitute numerical values and evaluate ω:

$$\omega = \frac{1}{\sqrt{(4\,\text{H})(25\,\mu\text{F})}} = \boxed{100\,\text{rad/s}}$$

(*c*) Express the current in the inductor at $\omega = \omega_0$:

$$I_L = \left(\frac{25\,\text{V/H}}{100\,\text{s}^{-1}}\right)\cos[(100\,\text{rad/s})t - 90°]$$

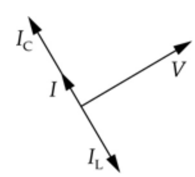
$$= \boxed{(0.250\,\text{A})\sin[(100\,\text{s}^{-1})t]}$$

Express the current in the capacitor at $\omega = \omega_0$:

$$I_C = (2.5\times10^{-3}\,\text{V}\cdot\text{F})(100\,\text{s}^{-1})\times\cos[(100\,\text{rad/s})t + 90°]$$

$$= \boxed{-(0.25\,\text{A})\sin[(100\,\text{s}^{-1})t]}$$

(*d*) The phasor diagram is shown to the right.



63 ••

Picture the Problem We can differentiate Q with respect to time to find I as a function of time. In (*b*) we can find C by using $\omega = 1/\sqrt{LC}$. The energy stored in the magnetic field of the inductor is given by $U_m = \frac{1}{2}LI^2$ and the energy stored in the electric field of the capacitor by $U_e = \frac{1}{2}\frac{Q^2}{C}$.

(*a*) Use the definition of current to obtain:

$$I = \frac{dQ}{dt} = \frac{d}{dt}\left[(15\,\mu\text{C})\cos\left(1250\,t + \frac{\pi}{4}\right)\right]$$
$$= -(15\,\mu\text{C})(1250\,\text{s}^{-1})\sin\left(1250\,t + \frac{\pi}{4}\right)$$
$$= \boxed{-(18.75\,\text{mA})\sin\left(1250\,t + \frac{\pi}{4}\right)}$$

(*b*) Relate C to L and ω:

$$\omega = \frac{1}{\sqrt{LC}}$$

Solve for C to obtain:

$$C = \frac{1}{\omega^2 L}$$

Substitute numerical values and evaluate C:

$$C = \frac{1}{(1250\,\text{s}^{-1})^2(28\,\text{mH})} = \boxed{22.86\,\mu\text{F}}$$

(*c*) Express and evaluate the magnetic energy U_m:

$$U_m = \tfrac{1}{2}LI^2 = \tfrac{1}{2}(28\,\text{mH})(18.75\,\text{mA})^2\sin^2\left(1250t + \frac{\pi}{4}\right)$$
$$= \boxed{(4.92\,\mu\text{J})\sin^2\left(1250t + \frac{\pi}{4}\right)}$$

Express and evaluate the electrical energy U_e:

$$U_e = \tfrac{1}{2}\frac{Q^2}{C}$$
$$= \tfrac{1}{2}\frac{(15\,\mu\text{F})^2}{22.86\,\mu\text{F}}\cos^2\left(1250\,t + \frac{\pi}{4}\right)$$
$$= \boxed{(4.92\,\mu\text{J})\cos^2\left(1250\,t + \frac{\pi}{4}\right)}$$

The total energy stored in the electric and magnetic fields is:

$$U = U_{\mathrm{m}} + U_{\mathrm{e}} = (4.92\,\mu\mathrm{J})\sin^2\left(1250t + \frac{\pi}{4}\right) + (4.92\,\mu\mathrm{J})\cos^2\left(1250t + \frac{\pi}{4}\right) = \boxed{4.92\,\mu\mathrm{J}}$$

***64 •••**

Picture the Problem We can use the definition of the capacitance of a dielectric-filled capacitor and the expression for the resonance frequency of an *LC* circuit to derive an expression for the fractional change in the thickness of the dielectric in terms of the resonance frequency and the frequency of the circuit when the dielectric is under compression. We can then use this expression for $\Delta t/t$ to calculate the value of Young's modulus for the dielectric material.

Use its definition to express Young's modulus of the dielectric material:

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{\Delta P}{\Delta t/t} \qquad (1)$$

Letting *t* be the initial thickness of the dielectric, express the initial capacitance of the capacitor:

$$C_0 = \frac{\kappa \in_0 A}{t}$$

Express the capacitance of the capacitor when it is under compression:

$$C_{\mathrm{c}} = \frac{\kappa \in_0 A}{t - \Delta t}$$

Express the resonance frequency of the capacitor before the dielectric is compressed:

$$\omega_0 = \frac{1}{\sqrt{C_0 L}} = \frac{1}{\sqrt{\dfrac{\kappa \in_0 AL}{t}}}$$

When the dielectric is compressed:

$$\omega_{\mathrm{c}} = \frac{1}{\sqrt{C_{\mathrm{c}} L}} = \frac{1}{\sqrt{\dfrac{\kappa \in_0 AL}{t - \Delta t}}}$$

Express the ratio of ω_{c} to ω_0 and simplify to obtain:

$$\frac{\omega_{\mathrm{c}}}{\omega_0} = \frac{\sqrt{\dfrac{\kappa \in_0 AL}{t}}}{\sqrt{\dfrac{\kappa \in_0 AL}{t - \Delta t}}} = \sqrt{1 - \frac{\Delta t}{t}}$$

Expand the radical binomially to obtain:

$$\frac{\omega_{\mathrm{c}}}{\omega_0} = \left(1 - \frac{\Delta t}{t}\right)^{1/2} \approx 1 - \frac{\Delta t}{2t}$$

provided $\Delta t \ll t$.

Solve for $\Delta t/t$:

$$\frac{\Delta t}{t}=2\left(1-\frac{\omega_c}{\omega_0}\right)$$

Substitute in equation (1) to obtain:

$$Y=\frac{\Delta P}{2\left(1-\frac{\omega_c}{\omega_0}\right)}$$

Substitute numerical values and evaluate Y:

$$Y=\frac{(800\,\text{atm})(101.325\,\text{kPa/atm})}{2\left(1-\frac{116\,\text{MHz}}{120\,\text{MHz}}\right)}=\boxed{1.22\times10^{9}\ \text{N/m}^2}$$

65 •••

Picture the Problem We can model this capacitor as the equivalent of two capacitors connected in parallel. Let C_1 be the capacitance of the dielectric-filled capacitor and C_2 be the capacitance of the air-filled capacitor. We'll derive expressions for the capacitances of the parallel capacitors and add these expressions to obtain $C(x)$. We can then use the given resonance frequency when $x = w/2$ and the given value for L to evaluate C_0. In Part (*b*) we can use our result for $C(x)$ and the relationship between f, L, and $C(x)$ at resonance to express $f(x)$.

(*a*) Express the equivalent capacitance of the two capacitors in parallel:

$$C(x)=C_1+C_2=\frac{\kappa\in_0 A_1}{d}+\frac{\in_0 A_2}{d}\quad(1)$$

Express A_2 in terms of the total area of a capacitor plate A, w, and the distance x:

$$\frac{A_2}{A}=\frac{x}{w}\Rightarrow A_2=A\frac{x}{w}$$

Express A_1 in terms of A and A_2:

$$A_1=A-A_2=A\left(1-\frac{x}{w}\right)$$

Substitute in equation (1) and simplify to obtain:

$$C(x)=\frac{\kappa\in_0 A}{d}\left(1-\frac{x}{w}\right)+\frac{\in_0 A}{d}\frac{x}{w}=\frac{\in_0 A}{d}\left[\kappa\left(1-\frac{x}{w}\right)+\frac{x}{w}\right]=\kappa C_0\left[1-\frac{\kappa-1}{\kappa w}x\right]$$

where $C_0 = \frac{\epsilon_0 A}{d}$

Find $C(w/2)$:

$$C\left(\frac{w}{2}\right) = \kappa C_0\left[1 - \frac{\kappa-1}{\kappa w}\frac{w}{2}\right] = \kappa C_0\left[1 - \frac{\kappa-1}{2\kappa}\right] = C_0\frac{\kappa+1}{2}$$

Express the resonance frequency of the circuit in terms of L and $C(x)$:

$$f(x) = \frac{1}{2\pi\sqrt{LC(x)}} \qquad (2)$$

Evaluate $f(w/2)$:

$$f\left(\frac{w}{2}\right) = \frac{1}{2\pi\sqrt{LC_0\frac{\kappa+1}{2}}} = \frac{1}{2\pi}\sqrt{\frac{2}{(\kappa+1)LC_0}}$$

Solve for C_0 to obtain:

$$C_0 = \frac{1}{2\pi^2 f^2\left(\frac{w}{2}\right)L(\kappa+1)}$$

Substitute numerical values and evaluate C_0:

$$C_0 = \frac{1}{2\pi^2(90\,\text{MHz})^2(2\,\text{mH})(4.8+1)} = \boxed{5.39\times10^{-16}\ \text{F}}$$

(*b*) Substitute for $C(x)$ in equation (2) to obtain:

$$f(x) = \frac{1}{2\pi\sqrt{L\kappa C_0\left[1 - \frac{\kappa-1}{\kappa w}x\right]}}$$

Substitute numerical values and evaluate $f(x)$:

$$f(x) = \frac{1}{2\pi\sqrt{(2\,\text{mH})(4.8)(5.39\times10^{-16}\ \text{F})\left[1 - \frac{4.8-1}{4.8(0.2\,\text{m})}x\right]}} = \boxed{\frac{70.0\,\text{MHz}}{\sqrt{1-(3.96\,\text{m}^{-1})x}}}$$

RLC Circuits with a Generator

66 •

Picture the Problem We can use the expression for the resonance frequency of a series *RLC* circuit to obtain an expression for C as a function of f.

Express the resonance frequency as a function of L and C:

$$\omega = 2\pi f = \frac{1}{\sqrt{LC}}$$

Solve for C to obtain:

$$C = \frac{1}{4\pi^2 f^2 L}$$

Substitute numerical values and evaluate the smallest value for C:

$$C_{\text{smallest}} = \frac{1}{4\pi^2 (1600\,\text{kHz})^2 (1\,\mu\text{H})} = 9.89\,\text{nF}$$

Substitute numerical values and evaluate the largest value for C:

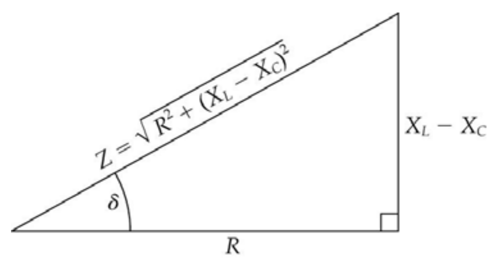
$$C_{\text{largest}} = \frac{1}{4\pi^2 (500\,\text{kHz})^2 (1\,\mu\text{H})} = 101\,\text{nF}$$

Therefore:

$$\boxed{9.89\,\text{nF} \le C \le 101\,\text{nF}}$$

67 •

Picture the Problem The diagram shows the relationship between δ, X_L, X_C, and R. We can use this reference triangle to express the power factor for the circuit in Example 29-5. In (*b*) we can use the reference triangle to relate ω to $\tan\delta$.



(*a*) Express the power factor for the circuit:

$$\cos\delta = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Evaluate X_L and X_C:

$$X_L = \omega L = (400\,\text{s}^{-1})(2\,\text{H}) = 800\,\Omega$$

and

$$X_C = \frac{1}{\omega C} = \frac{1}{(400\,\text{s}^{-1})(2\,\mu\text{F})} = 1250\,\Omega$$

Substitute numerical values and evaluate $\cos\delta$:

$$\cos\delta = \frac{20\,\Omega}{\sqrt{(20\,\Omega)^2 + (800\,\Omega - 1250\,\Omega)^2}} = \boxed{0.0444}$$

(*b*) Express $\tan\delta$:

$$\tan\delta = \frac{X_L - X_C}{R} = \frac{\omega L - \dfrac{1}{\omega C}}{R}$$

Rewrite this equation explicitly as a quadratic equation in ω:

$$LC\omega^2 - CR\tan\delta\omega - 1 = 0$$

Substitute numerical values to obtain:

$$[(2\,\text{H})(2\,\mu\text{F})]\omega^2 - [(2\,\mu\text{F})(20\,\Omega)\tan(\cos^{-1}0.5)]\omega - 1 = 0$$

or

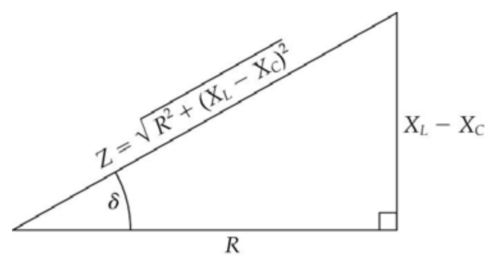
$$(4\times10^{-6}\,\text{s}^2)\omega^2 \pm (69.3\times10^{-6}\,\text{s})\omega - 1 = 0$$

Solve for ω to obtain:

$$\omega = \boxed{491\,\text{rad/s}} \text{ or } \omega = \boxed{509\,\text{rad/s}}$$

68 •

Picture the Problem The diagram shows the relationship between δ, X_L, X_C, and R. We can use this reference triangle to express the power factor for the given circuit. In (*b*) we can find the rms current from the rms potential difference and the impedance of the circuit. We can find the average power delivered by the source from the rms current and the resistance of the resistor.



(*a*) The power factor is defined to be:

$$\cos\delta = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

With no inductance in the circuit:

$$X_L = 0$$

and

$$\cos\delta = \frac{R}{\sqrt{R^2 + X_C^2}} = \frac{R}{\sqrt{R^2 + \dfrac{1}{\omega^2 C^2}}}$$

Substitute numerical values and evaluate $\cos\delta$:

$$\cos\delta = \frac{80\,\Omega}{\sqrt{(80\,\Omega)^2 + \frac{1}{(400\,\text{s}^{-1})^2(20\,\mu\text{F})^2}}} = \boxed{0.539}$$

(*b*) Express the rms current in the circuit:

$$I_{\text{rms}} = \frac{\mathcal{E}_{\text{rms}}}{Z} = \frac{\frac{\mathcal{E}_{\max}}{\sqrt{2}}}{\sqrt{R^2 + X_C^2}} = \frac{\mathcal{E}_{\max}}{\sqrt{2}\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

Insert numerical values and evaluate I_{rms}:

$$I_{\text{rms}} = \frac{20\,\text{V}}{\sqrt{2}\sqrt{(80\,\Omega)^2 + \frac{1}{(400\,\text{s}^{-1})^2(20\,\mu\text{F})^2}}} = \boxed{95.3\,\text{mA}}$$

(*c*) Express and evaluate the average power delivered by the generator:

$$P_{\text{av}} = I_{\text{rms}}^2 R = (95.3\,\text{mA})^2(80\,\Omega) = \boxed{0.727\,\text{W}}$$

***69 ••**

Picture the Problem The impedance of an ac circuit is given by $Z = \sqrt{R^2 + (X_L - X_C)^2}$. We can evaluate the given expression for P_{av} first for $X_L = X_C = 0$ and then for $R = 0$.

(*a*) For $X = 0$, $Z = R$ and:

$$P_{\text{av}} = \frac{R\mathcal{E}_{\text{rms}}^2}{Z^2} = \frac{R\mathcal{E}_{\text{rms}}^2}{R^2} = \boxed{\frac{\mathcal{E}_{\text{rms}}^2}{R}}$$

(*b*), (*c*) If $R = 0$, then:

$$P_{\text{av}} = \frac{R\mathcal{E}_{\text{rms}}^2}{Z^2} = \frac{(0)\mathcal{E}_{\text{rms}}^2}{(X_L - X_C)^2} = \boxed{0}$$

Remarks: Recall that there is no energy dissipation in an ideal inductor or capacitor.

70 ••

Picture the Problem We can use $\omega_0 = 1/\sqrt{LC}$ to find the resonant frequency of the circuit, $I_{\text{rms}} = \mathcal{E}_{\text{rms}}/R$ to find the rms current at resonance, the definitions of X_C and X_L to find these reactances at ω = 8000 rad/s, the definitions of Z and I_{rms} to find the impedance and rms current at ω = 8000 rad/s, and the definition of the phase angle to find δ.

(*a*) Express the resonant frequency ω_0 in terms of L and C:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Substitute numerical values and evaluate ω_0:

$$\omega_0 = \frac{1}{\sqrt{(10\,\text{mH})(2\,\mu\text{F})}} = \boxed{7.07\times10^3\ \text{rad/s}}$$

(*b*) Relate the rms current at resonance to ε_{rms} and the impedance of the circuit at resonance:

$$I_{rms} = \frac{\varepsilon_{rms}}{R} = \frac{\varepsilon_{max}}{\sqrt{2}R} = \frac{100\,\text{V}}{\sqrt{2}(5\,\Omega)} = \boxed{14.1\,\text{A}}$$

(*c*) Express and evaluate X_C and X_L at ω = 8000 rad/s:

$$X_C = \frac{1}{\omega C} = \frac{1}{(8000\,\text{s}^{-1})(2\,\mu\text{F})} = \boxed{62.5\,\Omega}$$

and

$$X_L = \omega L = (8000\,\text{s}^{-1})(10\,\text{mH}) = \boxed{80.0\,\Omega}$$

(*d*) Express the impedance in terms of the reactances, substitute the results from (*c*), and evaluate Z:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(5\,\Omega)^2 + (80\,\Omega - 62.5\,\Omega)^2} = \boxed{18.2\,\Omega}$$

Relate the rms current at ω = 8000 rad/s to ε_{rms} and the impedance of the circuit at this frequency:

$$I_{rms} = \frac{\varepsilon_{rms}}{Z} = \frac{\varepsilon_{max}}{\sqrt{2}Z} = \frac{100\,\text{V}}{\sqrt{2}(18.2\,\Omega)} = \boxed{3.89\,\text{A}}$$

(*e*) Using its definition, express and evaluate δ:

$$\delta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{80\,\Omega - 62.5\,\Omega}{5\,\Omega}\right) = \boxed{74.1^\circ}$$

71 ••

Picture the Problem We can use $f_0 = 1/2\pi\sqrt{LC}$ to find the resonant frequency of the circuit, the definitions of X_C and X_L to find these reactances at f = 1000 Hz, the definitions of Z and I_{rms} to find the impedance and rms current at f= 1000 Hz, and the definition of the phase angle to find δ.

(*a*) Express the resonant frequency f_0 in terms of L and C:

$$f_0=\frac{1}{2\pi\sqrt{LC}}$$

Substitute numerical values and evaluate f_0:

$$f_0=\frac{1}{2\pi\sqrt{(10\,\text{mH})(2\,\mu\text{F})}}=\boxed{1.13\,\text{kHz}}$$

(*b*) Express and evaluate X_C and X_L at f = 1000 Hz:

$$X_C=\frac{1}{2\pi fC}=\frac{1}{2\pi(1000\,\text{s}^{-1})(2\,\mu\text{F})}$$
$$=\boxed{79.6\,\Omega}$$

and

$$X_L=2\pi fL=2\pi(1000\,\text{s}^{-1})(10\,\text{mH})$$
$$=\boxed{62.8\,\Omega}$$

(*c*) Express the impedance in terms of the reactances, substitute the results from (*b*), and evaluate Z:

$$Z=\sqrt{R^2+(X_L-X_C)^2}$$
$$=\sqrt{(5\,\Omega)^2+(62.8\,\Omega-79.6\,\Omega)^2}$$
$$=\boxed{17.5\,\Omega}$$

Relate the rms current at f = 1000 Hz to ε_{rms} and the impedance of the circuit at this frequency:

$$I_{\text{rms}}=\frac{\varepsilon_{\text{rms}}}{Z}=\frac{\varepsilon_{\text{max}}}{\sqrt{2}Z}=\frac{100\,\text{V}}{\sqrt{2}(17.5\,\Omega)}$$
$$=\boxed{4.04\,\text{A}}$$

(*d*) Using its definition, express and evaluate δ:

$$\delta=\tan^{-1}\left(\frac{X_L-X_C}{R}\right)$$
$$=\tan^{-1}\left(\frac{62.8\,\Omega-79.6\,\Omega}{5\,\Omega}\right)=\boxed{-73.4^\circ}$$

72 ••

Picture the Problem Note that the reactances and, hence, the impedance of an ac circuit are frequency dependent. We can use the definitions of X_L, X_C, and Z, δ, and $\cos\delta$ to find the phase angle and the power factor of the circuit at the given frequencies.

Express the phase angle δ and the power factor $\cos\delta$ for the circuit:

$$\delta=\tan^{-1}\left(\frac{X_L-X_C}{R}\right) \qquad (1)$$

and

$$\cos\delta = \frac{R}{Z} \qquad (2)$$

(*a*) Evaluate X_L, X_C, and Z at $f = 900$ Hz:

$$X_L = 2\pi f L = 2\pi\left(900\,\text{s}^{-1}\right)\left(10\,\text{mH}\right) = 56.5\,\Omega,$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi\left(900\,\text{s}^{-1}\right)\left(2\,\mu\text{F}\right)} = 88.4\,\Omega,$$

and

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{(5\,\Omega)^2 + (56.5\,\Omega - 88.4\,\Omega)^2} = 32.3\,\Omega$$

Substitute in equations (1) and (2) to obtain:

$$\delta = \tan^{-1}\left(\frac{56.5\,\Omega - 88.4\,\Omega}{5\,\Omega}\right) = \boxed{-81.1^\circ}$$

and

$$\cos\delta = \frac{5\,\Omega}{32.3\,\Omega} = \boxed{0.155}$$

(*b*) Evaluate X_L, X_C, and Z at $f = 1.1$ kHz:

$$X_L = 2\pi f L = 2\pi\left(1100\,\text{s}^{-1}\right)\left(10\,\text{mH}\right) = 69.1\,\Omega,$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi\left(1100\,\text{s}^{-1}\right)\left(2\,\mu\text{F}\right)} = 72.3\,\Omega,$$

and

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{(5\,\Omega)^2 + (69.1\,\Omega - 72.3\,\Omega)^2} = 5.94\,\Omega$$

Substitute in equations (1) and (2) to obtain:

$$\delta = \tan^{-1}\left(\frac{69.1\,\Omega - 72.3\,\Omega}{5\,\Omega}\right) = \boxed{-32.6^\circ}$$

and

$$\cos\delta = \frac{5\,\Omega}{5.94\,\Omega} = \boxed{0.842}$$

(c) Evaluate X_L, X_C, and Z at f = 1.3 kHz:

$$X_L = 2\pi fL = 2\pi\left(1300\,\text{s}^{-1}\right)(10\,\text{mH}) = 81.7\,\Omega,$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi\left(1300\,\text{s}^{-1}\right)(2\,\mu\text{F})} = 61.2\,\Omega,$$

and

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(5\,\Omega)^2 + (81.7\,\Omega - 61.2\,\Omega)^2} = 21.1\,\Omega$$

Substitute in equations (1) and (2) to obtain:

$$\delta = \tan^{-1}\left(\frac{81.7\,\Omega - 61.2\,\Omega}{5\,\Omega}\right) = \boxed{76.3°}$$

and

$$\cos\delta = \frac{5\,\Omega}{21.1\,\Omega} = \boxed{0.237}$$

73 ••

Picture the Problem The Q factor of the circuit is given by $Q = \omega_0 L/R$, the resonance width by $\Delta f = f_0/Q = \omega_0/2\pi Q$, and the power factor by $\cos\delta = R/Z$. Because Z is frequency dependent, we'll need to find X_C and X_L at ω = 8000 rad/s in order to evaluate $\cos\delta$.

Using their definitions, express the Q factor and the resonance width of the circuit:

$$Q = \frac{\omega_0 L}{R} \qquad (1)$$

and

$$\Delta f = \frac{f_0}{Q} = \frac{\omega_0}{2\pi Q} \qquad (2)$$

(a) Express and evaluate the resonance frequency for the circuit:

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(10\,\text{mH})(2\,\mu\text{F})}} = 7.07\times10^3\ \text{rad/s}$$

Substitute numerical values in equation (1) and evaluate Q:

$$Q = \frac{\left(7.07\times10^3\ \text{rad/s}\right)(10\,\text{mH})}{5\,\Omega} = \boxed{14.1}$$

(b) Substitute numerical values in equation (2) and evaluate Δf:

$$\Delta f = \frac{7.07\times10^3\ \text{rad/s}}{2\pi(14.1)} = \boxed{79.8\,\text{Hz}}$$

(*c*) Express the power factor of the circuit:

$$\cos\delta = \frac{R}{Z}$$

Evaluate X_L, X_C, and Z at $\omega = 8000$ rad/s:

$$X_L = \omega L = \left(8000\,\text{s}^{-1}\right)\left(10\,\text{mH}\right) = 80.0\,\Omega,$$

$$X_C = \frac{1}{\omega C} = \frac{1}{\left(8000\,\text{s}^{-1}\right)\left(2\,\mu\text{F}\right)} = 62.5\,\Omega,$$

and

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{\left(5\,\Omega\right)^2 + \left(80\,\Omega - 62.5\,\Omega\right)^2} = 18.2\,\Omega$$

Substitute numerical values and evaluate $\cos\delta$:

$$\cos\delta = \frac{5\,\Omega}{18.2\,\Omega} = \boxed{0.275}$$

***74 ••**

Picture the Problem We can use its definition, $Q = f_0/\Delta f$ to find the Q factor for the circuit.

Express the Q factor for the circuit:

$$Q = \frac{f_0}{\Delta f}$$

Substitute numerical values and evaluate Q:

$$Q = \frac{100.1\,\text{MHz}}{0.05\,\text{MHz}} = \boxed{2002}$$

75 ••

Picture the Problem We can use $I = \mathcal{E}_{\text{max}}/Z$ to find the current in the coil and the definition of the phase angle to evaluate δ. We can equate *XL* and *XC* to find the capacitance required so that the current and the voltage are in phase. Finally, we can find the voltage measured across the capacitor by using $V_C = IX_C$.

(*a*) Express the current in the coil in terms of the potential difference across it and its impedance:

$$I = \frac{\mathcal{E}_{\text{max}}}{Z}$$

Substitute numerical values and evaluate I:

$$I = \frac{100\,\text{V}}{10\,\Omega} = \boxed{10.0\,\text{A}}$$

(*b*) Express and evaluate the phase angle δ:

$$\delta = \cos^{-1}\frac{R}{Z} = \sin^{-1}\frac{X}{Z}$$
$$= \sin^{-1}\left(\frac{8\,\Omega}{10\,\Omega}\right) = \boxed{53.1°}$$

(*c*) Express the condition on the reactances that must be satisfied if the current and voltage are to be in phase:

$$X_L = X_C \text{ or } X_L = \frac{1}{\omega C}$$

Solve for C to obtain:

$$C = \frac{1}{\omega X_L} = \frac{1}{2\pi f X_L}$$

Substitute numerical values and evaluate C:

$$C = \frac{1}{2\pi\left(60\,\text{s}^{-1}\right)\left(8\,\Omega\right)} = \boxed{332\,\mu\text{F}}$$

(*d*) Express the potential difference across the capacitor:

$$V_C = IX_C$$

Relate the current I in the circuit to the impedance of the circuit when $X_L = X_C$:

$$I = \frac{V}{R}$$

Substitute to obtain:

$$V_C = \frac{VX_C}{R} = \frac{V}{2\pi fCR}$$

Relate the impedance of the circuit to the resistance of the coil:

$$Z = \sqrt{R^2 + X^2}$$

Solve for and evaluate the resistance of the coil:

$$R = \sqrt{Z^2 - X^2} = \sqrt{\left(10\,\Omega\right)^2 - \left(8\,\Omega\right)^2}$$
$$= 6\,\Omega$$

Substitute numerical values and evaluate V_C:

$$V_C = \frac{100\,\text{V}}{2\pi\left(60\,\text{s}^{-1}\right)\left(332\,\mu\text{F}\right)\left(6\,\Omega\right)} = \boxed{133\,\text{V}}$$

76 ••

Picture the Problem We can find C using $V_C = I_{\text{rms}}X_C$ and I_{rms} from the potential difference across the inductor. In the absence of resistance in the circuit, the measured rms voltage across both the capacitor and inductor is $V = \left|V_L - V_C\right|$.

(*a*) Relate the capacitance *C* to the potential difference across the capacitor:

$$V_C = I_{\text{rms}} X_C = \frac{I_{\text{rms}}}{2\pi f C}$$

Solve for *C* to obtain:

$$C = \frac{I_{\text{rms}}}{2\pi f V_C}$$

Use the potential difference across the inductor to express the rms current in the circuit:

$$I_{\text{rms}} = \frac{V_L}{X_L} = \frac{V_L}{2\pi f L}$$

Substitute to obtain:

$$C = \frac{V_L}{(2\pi f)^2 L V_C}$$

Substitute numerical values and evaluate *C*:

$$C = \frac{50\text{ V}}{\left[2\pi\left(60\text{ s}^{-1}\right)\right]^2 (0.25\text{ H})(75\text{ V})} = \boxed{18.8\ \mu\text{F}}$$

(*b*) Express the measured rms voltage V across both the capacitor and the inductor when $R = 0$:

$$V = \left|V_L - V_C\right|$$

Substitute numerical values and evaluate *V*:

$$V = \left|50\text{ V} - 75\text{ V}\right| = \boxed{25.0\text{ V}}$$

77 ••

Picture the Problem We can rewrite Equation 29-51 in terms of ω, *L*, and *C* and factor *L* from the resulting expression to obtain the given equation. In (*b*) and (*c*) we can use the expansions for $\cot^{-1}x$ and $\tan^{-1}x$ to approximate δ at very low and very high frequencies.

(*a*) From Equation 29-51:

$$\tan\delta = \frac{\omega L - 1/\omega C}{R} = \frac{\omega^2 L - 1/C}{\omega R} = \frac{L\left(\omega^2 - 1/LC\right)}{\omega R} = \boxed{\frac{L\left(\omega^2 - \omega_0^2\right)}{\omega R}}$$

(*b*) Rewrite $\tan\delta$ as:

$$\tan\delta = \frac{\omega L}{R} - \frac{1}{\omega RC} \qquad (1)$$

For $\omega \ll 1$:

$$\tan\delta \approx -\frac{1}{\omega RC}$$

and

$\cot\delta = -\omega RC$ or $\delta = \cot^{-1}(-\omega RC)$

Use the expansion for $\cot^{-1}x$ to obtain:

$$\cot^{-1}x = \pm\frac{\pi}{2} - x$$

Recall that, for negative values of the argument, the angle approaches $-\pi/2$*, to obtain:

$$-\frac{\pi}{2} - \delta = -\omega RC$$

or

$$\delta = \boxed{-\frac{\pi}{2} + \omega RC}$$

(*c*) For $\omega >> 1$, equation (1) becomes:

$$\tan\delta \approx \frac{\omega L}{R} \text{ or } \delta \approx \tan^{-1}\frac{\omega L}{R}$$

Use the expansion for $\tan^{-1}x$ to obtain:

$$\tan^{-1}x = \frac{\pi}{2} - \frac{1}{x} \text{ or } \delta = \boxed{\frac{\pi}{2} - \frac{R}{\omega L}}$$

***You can easily confirm this using your graphing calculator.**

78 ••

Picture the Problem We can use the definition of the power factor to express $\cos\delta$ in the absence of an inductor and simplify the resulting equation to obtain the equation given above.

(*a*) Express the power factor for a series *RLC* circuit:

$$\cos\delta = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

With no inductance in the circuit,
$X_L = 0$ and:

$$\cos\delta = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (-X_C)^2}}$$

Substitute for X_C and simplify to obtain:

$$\cos\delta = \frac{R}{\sqrt{R^2 + \frac{1}{(\omega C)^2}}} = \frac{R}{R\sqrt{1 + \frac{1}{(\omega RC)^2}}} = \boxed{\frac{\omega RC}{\sqrt{1 + (\omega RC)^2}}}$$

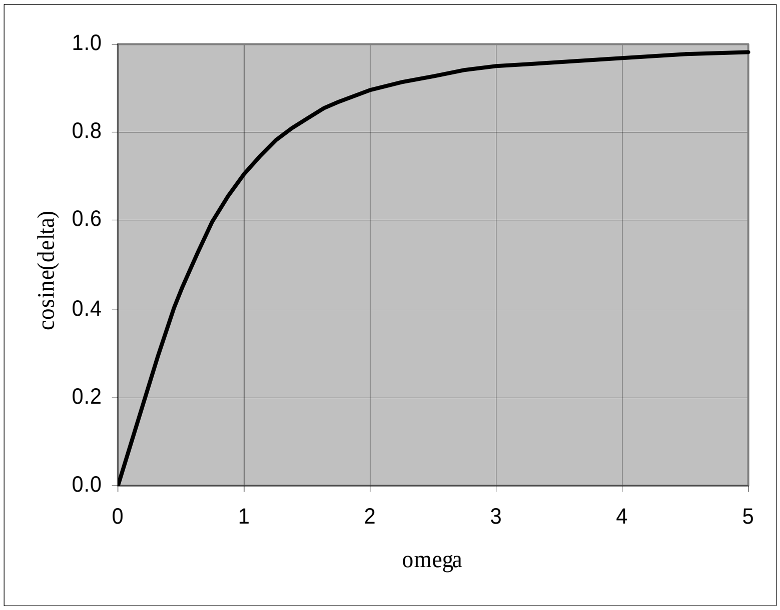
(*b*)A spreadsheet program to generate the data for a graph of $\cos\delta$ versus ωRC is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
A2	0.0	ωRC
A3	A2 + 0.5	$\omega RC + 0.5$
B2	A2/(1 +A2^2)^(0.5)	$\dfrac{\omega RC}{\sqrt{1+(\omega RC)^2}}$

	A	B	C
1	R=	1	ohm
2	C=	1	F

3			
4	omega	cos(delta)	
5	0.0	0.000	
6	0.5	0.447	
7	1.0	0.707	
13	4.0	0.970	
14	4.5	0.976	
15	5.0	0.981	

The following graph of cos δ as a function of ω was plotted using the data in the above table. Note that both R and C were set equal to 1.



***79** ••

Picture the Problem We can find the rms current in the circuit and then use it to find the potential differences across each of the circuit elements. We can use phasor diagrams and our knowledge the phase shifts between the voltages across the three circuit elements to find the voltage differences across their combinations.

(a) Express the potential difference between points A and B in terms of I_{rms} and X_L:

$$V_{AB} = I_{\text{rms}} X_L \qquad (1)$$

Express I_{rms} in terms of ε and Z:

$$I_{\text{rms}} = \frac{\mathcal{E}}{Z} = \frac{\mathcal{E}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Evaluate X_L and X_C to obtain:

$$X_L = 2\pi fL = 2\pi(60\,\text{s}^{-1})(137\,\text{mH}) = 51.6\,\Omega$$

and

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(60\,\text{s}^{-1})(25\,\mu\text{F})} = 106.1\,\Omega$$

Substitute numerical values and evaluate I_{rms}:

$$I_{\text{rms}} = \frac{115\,\text{V}}{\sqrt{(50\,\Omega)^2 + (51.6\,\Omega - 106.1\,\Omega)^2}} = 1.55\,\text{A}$$

Substitute numerical values in equation (1) and evaluate V_{AB}:

$$V_{AB} = (1.55\,\text{A})(51.6\,\Omega) = \boxed{80.0\,\text{V}}$$

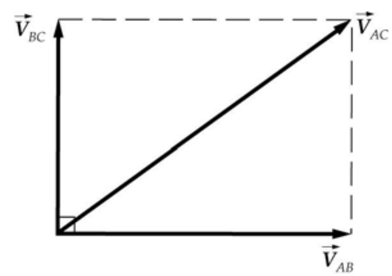
(*b*) Express the potential difference between points *B* and *C* in terms of I_{rms} and *R*:

$$V_{BC} = I_{\text{rms}}R = (1.55\,\text{A})(50\,\Omega) = \boxed{77.5\,\text{V}}$$

(*c*) Express the potential difference between points *C* and *D* in terms of I_{rms} and X_C:

$$V_{CD} = I_{\text{rms}}X_C = (1.55\,\text{A})(106.1\,\Omega) = \boxed{164\,\text{V}}$$

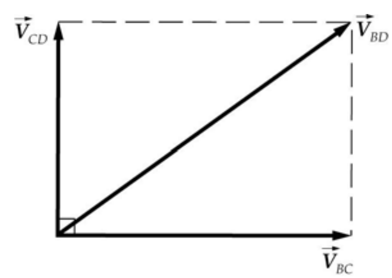
(*d*) The voltage across the inductor lags the voltage across the resistor as shown in the phasor diagram to the right:



Use the Pythagorean theorem to find V_{AC}:

$$V_{AC} = \sqrt{V_{AB}^2 + V_{BC}^2} = \sqrt{(80.0\,\text{V})^2 + (77.5\,\text{V})^2} = \boxed{111\,\text{V}}$$

(*e*) The voltage across the inductor lags the voltage across the resistor as shown in the phasor diagram to the right:



Use the Pythagorean theorem to find V_{BD}:

$$V_{BD} = \sqrt{V_{CD}^2 + V_{BC}^2} = \sqrt{(164\,\text{V})^2 + (77.5\,\text{V})^2} = \boxed{181\,\text{V}}$$

80 ••

Picture the Problem We can use $P_{av} = \mathcal{E}_{rms} I_{rms} \cos\delta$ to find the power supplied to the circuit and $P_{av} = I_{rms}^2 R$ to find the resistance. In (*c*) we can relate the capacitive reactance to the impedance, inductive reactance, and resistance of the circuit and solve for the capacitance *C*. We can use the condition on X_L and X_C at resonance to find the capacitance or inductance you would need to add to the circuit to make the power factor equal to 1.

(*a*) Express the power supplied to the circuit in terms of ε_{rms}, I_{rms}, and the power factor $\cos\delta$:

$$P_{av} = \mathcal{E}_{rms} I_{rms} \cos\delta$$

Substitute numerical values and evaluate P_{av}:

$$P_{av} = (120\,\text{V})(11\,\text{A})\cos 45° = \boxed{933\,\text{W}}$$

(*b*) Relate the power dissipated in the circuit to the resistance of the resistor:

$$P_{av} = I_{rms}^2 R \text{ or } R = \frac{P_{av}}{I_{rms}^2}$$

Substitute numerical values and evaluate *R*:

$$R = \frac{933\,\text{W}}{(11\,\text{A})^2} = \boxed{7.71\,\Omega}$$

(*c*) Express the capacitance of the capacitor in terms of its reactance:

$$C = \frac{1}{\omega X_C} = \frac{1}{2\pi f X_C} \qquad (1)$$

Relate the capacitive reactance to the impedance, inductive reactance, and resistance of the circuit:

$$Z^2 = R^2 + (X_L - X_C)^2$$

Express the impedance of the circuit in terms of the rms emf ε and the rms current I_{rms}:

$$Z^2 = \frac{\mathcal{E}^2}{I_{rms}^2}$$

Substitute to obtain:

$$\frac{\mathcal{E}^2}{I_{rms}^2} = R^2 + (X_L - X_C)^2$$

Solve for $\left|X_L - X_C\right|$:

$$\left|X_L - X_C\right| = \sqrt{\frac{\mathcal{E}^2}{I_{\text{rms}}^2} - R^2}$$

Note that because I leads ε, the circuit is capacitive and $X_C > X_L$. Hence:

$$\left|X_L - X_C\right| = -\left(X_L - X_C\right)$$

and

$$X_C = X_L + \sqrt{\frac{\mathcal{E}^2}{I_{\text{rms}}^2} - R^2} = 2\pi fL + \sqrt{\frac{\mathcal{E}^2}{I_{\text{rms}}^2} - R^2}$$

Substitute numerical values and evaluate X_C:

$$X_C = 2\pi\left(60\,\text{s}^{-1}\right)\left(0.05\,\text{H}\right) + \sqrt{\frac{\left(120\,\text{V}\right)^2}{\left(11\,\text{A}\right)^2} - \left(7.71\,\Omega\right)^2} = 26.6\,\Omega$$

Substitute in equation (1) and evaluate C:

$$C = \frac{1}{2\pi\left(60\,\text{s}^{-1}\right)\left(26.6\,\Omega\right)} = \boxed{99.7\,\mu\text{F}}$$

(d) Express the relationship between X_L and X_C when $\cos\delta = 1$:

$$X_L = X_C$$

Because $X_L = 18.1\ \Omega$, we could make $X_L = X_C$ by adding 7.75 Ω of inductive reactance to the circuit. Find the *series* inductance equivalent to 7.75 Ω of inductive reactance:

$$L = \frac{X_L}{2\pi f} = \frac{7.75\,\Omega}{2\pi\left(60\,\text{s}^{-1}\right)} = \boxed{20.6\,\text{mH}}$$

Alternatively, we could make $X_L = X_C$ by reducing the capacitive reactance by 7.75 Ω. Find the capacitive reactance that you have to added in *parallel* to the existing capacitive reactance to reduce the equivalent capacitive reactance by 7.75 Ω:

$$\frac{1}{18.1\,\Omega} = \frac{1}{26.6\,\Omega} + \frac{1}{X_C}$$

and

$$X_C = 56.6\,\Omega$$

Find the capacitance corresponding to a capacitive reactance of 56.6 Ω:

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi\left(60\,\text{s}^{-1}\right)\left(56.6\,\Omega\right)} = \boxed{46.9\,\mu\text{F}}$$

81 ••

Picture the Problem We can find X_C using the equation relating X_C, X_L, R, and $\tan\delta$ and then solve the defining equation for X_C for C.

Express the capacitance of the circuit in terms of its capacitive reactance:

$$C = \frac{1}{\omega X_C} = \frac{1}{2\pi f X_C}$$

Express the phase angle δ in terms of X_L, X_C, and R:

$$\tan\delta = \frac{X_L - X_C}{R}$$

Solve for X_C to obtain:

$$X_C = 2\pi f L - R\tan\delta$$

Substitute numerical values and evaluate X_C:

$$X_C = 2\pi\left(500\,\text{s}^{-1}\right)\left(0.15\,\text{H}\right) - \left(35\,\Omega\right)\tan 75° = 341\,\Omega$$

Substitute numerical values and evaluate C:

$$C = \frac{1}{2\pi\left(500\,\text{s}^{-1}\right)\left(341\,\Omega\right)} = \boxed{0.933\,\mu\text{F}}$$

82 ••

Picture the Problem We can use the condition on X_L and X_C at resonance to find f_0. By expressing the phase angle δ in terms of X_L, X_C, and R we can obtain a quadratic equation in ω that we can solve for the frequencies corresponding to the given phase angles. We can then use these frequencies to express the ratios of f to f_0 for the given phase angles.

(*a*) Relate X_C and X_L at resonance:

$$X_L = X_C$$

or

$$2\pi f_0 L = \frac{1}{2\pi f_0 C}$$

Solve for f_0:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

Substitute numerical values and evaluate f_0:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{\left(0.35\,\text{H}\right)\left(5\,\mu\text{F}\right)}} = \boxed{120\,\text{Hz}}$$

(*b*) Express the phase angle δ in terms of X_L, X_C, and R:

$$\tan\delta = \frac{X_L - X_C}{R}$$

or

$$R\tan\delta = \omega L - \frac{1}{\omega C}$$

Rewrite this equation explicitly as a quadratic equation to obtain:

$$LC\omega^2 - (RC\tan\delta)\omega - 1 = 0$$

Substitute numerical values and simplify to obtain:

$$(1.75\times10^{-6}\,\mathrm{F\cdot H})\omega^2 - \left[(2\times10^{-3}\,\Omega\cdot\mathrm{H})\tan\delta\right]\omega - 1 = 0$$

or

$$(1\,\mathrm{F\cdot H})\omega^2 - \left[(1.14\times10^{3}\,\Omega\cdot\mathrm{H})\tan\delta\right]\omega - 5.71\times10^{5} = 0$$

For $\delta = 60°$:

$$(1\,\mathrm{F\cdot H})\omega^2 - (1.97\times10^{3}\,\Omega\cdot\mathrm{H})\omega - 5.71\times10^{5} = 0$$

Solve for the positive value of ω:

$$\omega = 2.23\times10^{3}\,\mathrm{s^{-1}}$$

and

$$f = \frac{\omega}{2\pi} = \frac{2.23\times10^{3}\,\mathrm{s^{-1}}}{2\pi} = 355\,\mathrm{Hz}$$

Calculate the ratio f/f_0:

$$\frac{f}{f_0} = \frac{355\,\mathrm{Hz}}{120\,\mathrm{Hz}} = \boxed{2.96}$$

For $\delta = -60°$:

$$(1\,\mathrm{F\cdot H})\omega^2 + (1.97\times10^{3}\,\Omega\cdot\mathrm{H})\omega - 5.71\times10^{5} = 0$$

Solve for the positive value of ω and then for f:

$$\omega = 256\,\mathrm{s^{-1}}$$

and

$$f = \frac{\omega}{2\pi} = \frac{256\,\mathrm{s^{-1}}}{2\pi} = 40.7\,\mathrm{Hz}$$

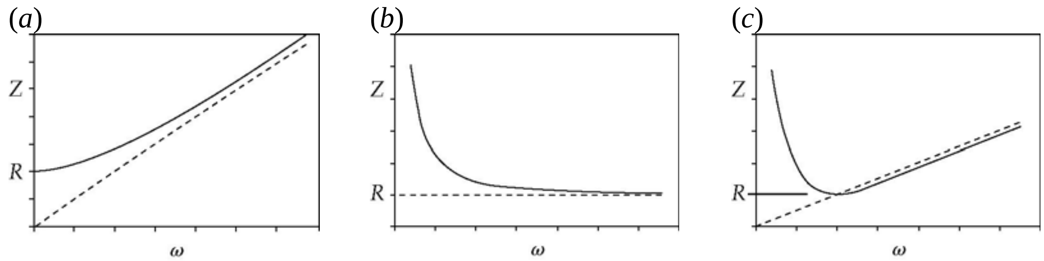
Calculate the ratio f/f_0:

$$\frac{f}{f_0} = \frac{40.7\,\mathrm{Hz}}{120\,\mathrm{Hz}} = \boxed{0.339}$$

Remarks: Note that these ratios are reciprocals of each other.

83 ••

Picture the Problem The impedance for the three circuits as functions of the angular frequency is shown in the three figures below. Also shown in each figure (dashed line) is the asymptotic approach for large angular frequencies.



***84 ••**

Picture the Problem We can substitute for X_L and X_C in Equation 29-48 and simplify the resulting equation to obtain the given equation for I_{max}.

Equation 29-48 is:

$$I_{max} = \frac{\mathcal{E}_{max}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Substitute for X_L and X_C to obtain:

$$I_{max} = \frac{\mathcal{E}_{max}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

Simplify algebraically to obtain:

$$I_{max} = \frac{\mathcal{E}_{max}}{\sqrt{R^2 + \omega^2 L^2\left(1 - \frac{1}{\omega^2 LC}\right)^2}} = \frac{\mathcal{E}_{max}}{\sqrt{R^2 + \omega^2 L^2\left(1 - \frac{\omega_0^2}{\omega^2}\right)^2}} = \frac{\mathcal{E}_{max}}{\sqrt{R^2 + \frac{L^2}{\omega^2}\left(\omega^2 - \omega_0^2\right)^2}}$$

$$= \frac{\mathcal{E}_{max}}{\frac{1}{\omega}\sqrt{\omega^2 R^2 + L^2\left(\omega^2 - \omega_0^2\right)^2}} = \boxed{\frac{\omega \mathcal{E}_{max}}{\sqrt{\omega^2 R^2 + L^2\left(\omega^2 - \omega_0^2\right)^2}}}$$

85 ••

Picture the Problem We can use the constraints on L and C at resonance and the given values for X_L and X_C to obtain simultaneous equations that we can solve for L and C. In (*b*) we can find Q from its definition and in (*c*) we can calculate I_{max} from ε_{max} and Z.

(*a*) Relate X_L and X_C at resonance:

$$X_L = X_C \text{ or } \omega_0 L = \frac{1}{\omega_0 C}$$

Solve for the product of L and C:

$$LC = \frac{1}{\omega_0^2} = \frac{1}{\left(10^4\ \text{rad/s}\right)^2} = 10^{-8}\ \text{s}^2 \qquad (1)$$

Express X_C and X_L:

$$X_C = \frac{1}{\omega C} = 16\,\Omega$$

and

$$X_L = \omega L = 4\,\Omega$$

Eliminate ω between these equations to obtain:

$$\frac{L}{C} = 64\,\Omega^2 \qquad (2)$$

Solve equations (1) and (2) simultaneously to obtain:

$$L = \boxed{0.800\,\text{mH}} \text{ and } C = \boxed{12.5\,\mu\text{F}}$$

(b) Express Q in terms of R, L, and ω_0:

$$Q = \frac{\omega_0 L}{R}$$

Substitute numerical values and evaluate Q:

$$Q = \frac{\left(10^4\ \text{rad/s}\right)\left(0.800\,\text{mH}\right)}{5\,\Omega} = \boxed{1.60}$$

(c) Relate the maximum current in the circuit to ε_{max} and Z:

$$I_{\text{max}} = \frac{\varepsilon_{\text{max}}}{Z} = \frac{\varepsilon_{\text{max}}}{\sqrt{R^2 + \left(X_L - X_C\right)^2}}$$

Substitute numerical values and evaluate I_{max}:

$$I_{\text{max}} = \frac{26\,\text{V}}{\sqrt{(5\,\Omega)^2 + (4\,\Omega - 16\,\Omega)^2}} = \boxed{2.00\,\text{A}}$$

86 ••

Picture the Problem We can find the maximum current in the circuit from the maximum voltage across the capacitor and the reactance of the capacitor. To find the range of inductance that is safe to use we can express Z^2 for the circuit in terms of $\varepsilon_{\text{max}}^2$ and I_{max}^2 and solve the resulting quadratic equation for L.

(a) Express the maximum current in terms of the maximum potential difference across the capacitor and its reactance:

$$I_{\text{max}} = \frac{V_{C,\text{max}}}{X_C} = \omega C V_{C,\text{max}}$$

Substitute numerical values and evaluate I_{max}:

$$I_{max} = (2500\,\text{rad/s})(8\,\mu\text{F})(150\,\text{V}) = \boxed{3.00\,\text{A}}$$

(*b*) Relate the maximum current in the circuit to the emf of the source and the impedance of the circuit:

$$I_{max} = \frac{\mathcal{E}_{max}}{Z} \text{ or } Z^2 = \frac{\mathcal{E}_{max}^2}{I_{max}^2}$$

Express Z^2 in terms of R, X_L, and X_C:

$$Z^2 = R^2 + (X_L - X_C)^2$$

Substitute to obtain:

$$\frac{\mathcal{E}_{max}^2}{I_{max}^2} = R^2 + (X_L - X_C)^2$$

Evaluate X_C:

$$X_C = \frac{1}{\omega C} = \frac{1}{(2500\,\text{rad/s})(8\,\mu\text{F})} = 50\,\Omega$$

Substitute numerical values to obtain:

$$\frac{(200\,\text{V})^2}{(3\,\text{A})^2} = (60\,\Omega)^2 + ((2500\,\text{rad/s})L - 50\,\Omega)^2$$

or

$$844\,\Omega^2 = \left[(2500\,\text{s}^{-1})L - 50\,\Omega\right]^2$$

Solve for L to obtain:

$$L = \frac{50\,\Omega \pm \sqrt{844\,\Omega^2}}{2500\,\text{s}^{-1}}$$

Denoting the solutions as L_+ and L_-, find the values for the inductance:

$$L_+ = 31.6\,\text{mH and } L_- = 8.38\,\text{mH}$$

Express the ranges for L:

$$\boxed{8.00\,\text{mH} < L < 8.38\,\text{mH}}$$

and

$$\boxed{31.6\,\text{mH} < L < 40.0\,\text{mH}}$$

87 ••

Picture the Problem We can find the impedance of the circuit from the applied emf and the current drawn by the device. In (*b*) we can use $P_{av} = I_{rms}^2 R$ to find R and the definition of the impedance of a series *RLC* circuit to find $X = X_L - X_C$.

(*a*) Express the impedance of the device in terms of the current it

$$Z = \frac{\mathcal{E}}{I}$$

draws and the emf provided by the power line:

Substitute numerical values to obtain:

$$Z = \frac{120\,\text{V}}{10\,\text{A}} = \boxed{12.0\,\Omega}$$

(*b*) Use the relationship between the average power supplied to the device and the rms current it draws to find *R*:

$$P_{av} = I_{rms}^2 R$$

and

$$R = \frac{P_{av}}{I_{rms}^2} = \frac{720\,\text{W}}{(10\,\text{A})^2} = \boxed{7.20\,\Omega}$$

Express the impedance of a series *RLC* circuit:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

or

$$Z^2 = R^2 + (X_L - X_C)^2$$

Solve for $X_L - X_C$:

$$X = X_L - X_C = \sqrt{Z^2 - R^2}$$

Substitute numerical values and evaluate *X*:

$$X = \sqrt{(12\,\Omega)^2 - (7.2\,\Omega)^2} = \boxed{9.60\,\Omega}$$

(*c*) $\boxed{\text{If the current leads the emf, the reactance is capacitive.}}$

***88 ••**

Picture the Problem We can use the fact that when the current is a maximum, $X_L = X_C$, to find the inductance of the circuit. In (*b*), we can find *I*max from ε_{max} and the impedance of the circuit at resonance.

(*a*) Relate X_L and X_C at resonance:

$$X_L = X_C \text{ or } \omega_0 L = \frac{1}{\omega_0 C}$$

Solve for *L* to obtain:

$$L = \frac{1}{\omega_0^2 C}$$

Substitute numerical values and evaluate *L*:

$$L = \frac{1}{(5000\,\text{s}^{-1})^2 (10\,\mu\text{F})} = \boxed{4.00\,\text{mH}}$$

(*b*) Noting that, at resonance, *X* = 0, express I_{max} in terms of the applied emf and the impedance of the circuit at resonance:

$$I_{max} = \frac{\mathcal{E}_{max}}{Z} = \frac{10\,\text{V}}{100\,\Omega} = \boxed{0.100\,\text{A}}$$

89 ••

Picture the Problem We can use Ohm's law to express the current through the resistor as a function of time. Because the resistor and capacitor are in parallel they have the same potential difference across them … the emf of the source. We can relate the charge on the capacitor as a function of time to its capacitance and the potential difference across it and differentiate this expression with respect to time to express $I_C(t)$. We can then apply Kirchhoff's junction rule to express the total current drawn from the source. Using the results of (*a*) and (*b*) we can show that $I = I_R + I_C = I_{max}\cos(\omega t + \delta)$, where $\tan\delta = R/X_C$ and $I_{max} = \varepsilon_{max}/Z$ with $Z^{-2} = R^{-2} + X_C^{-2}$.

(*a*) Apply Ohm's law to obtain:

$$I_R(t) = \frac{V(t)}{R} = \frac{\mathcal{E}_{max}\cos\omega t}{R} = \boxed{\frac{\mathcal{E}_{max}}{R}\cos\omega t}$$

(*b*) Express the potential difference across the capacitor in terms of the instantaneous charge on the capacitor:

$$V_C(t) = \frac{q(t)}{C} \text{ or } q(t) = CV_C(t)$$

Differentiate $q(t)$ to express the current to the capacitor:

$$I_C(t) = \frac{dq(t)}{dt} = C\frac{d}{dt}(V_C(t)) = C\frac{d}{dt}(\mathcal{E}_{max}\cos\omega t) = -\omega C\mathcal{E}_{max}\sin\omega t$$

Use the definition of X_C and the trigonometric identity $\cos(\alpha + 90°) = -\sin\alpha$ to obtain:

$$I_C(t) = \boxed{\frac{\mathcal{E}_{max}}{X_C}\cos(\omega t + 90°)}$$

(*c*) Apply Kirchhoff's junction rule to obtain:

$$I = I_R + I_C = \frac{\mathcal{E}_{max}}{R}\cos\omega t + \frac{\mathcal{E}_{max}}{X_C}\cos(\omega t + 90°) = \frac{\mathcal{E}_{max}}{R}\cos\omega t - \frac{\mathcal{E}_{max}}{X_C}\sin\omega t$$

We know that the current is also expressible in the form:

$$I = I_{max}\cos(\omega t + \delta)$$

Expand this expression, using the formula for the cosine of the sum of two angles, to obtain:

$$I = I_{\max}\cos\omega t\cos\delta - I_{\max}\sin\omega t\sin\delta$$

Equate these expressions and rewrite the resulting equation to obtain:

$$\left(\frac{\mathcal{E}_{\max}}{R} - I_{\max}\cos\delta\right)\cos\omega t - \left(\frac{\mathcal{E}_{\max}}{X_C} - I_{\max}\sin\delta\right) = 0$$

Express the conditions that must be satisfied if this equation is to be true for all values of t:

$$\frac{\mathcal{E}_{\max}}{R} - I_{\max}\cos\delta = 0$$

and

$$\frac{\mathcal{E}_{\max}}{X_C} - I_{\max}\sin\delta = 0$$

Rewrite these equations as:

$$I_{\max}\sin\delta = \frac{\mathcal{E}_{\max}}{X_C} \qquad (1)$$

and

$$I_{\max}\cos\delta = \frac{\mathcal{E}_{\max}}{R} \qquad (2)$$

Divide equation (1) by equation (2) and simplify to obtain:

$$\tan\delta = \boxed{\frac{R}{X_C}}$$

Square equations (1) and (2) and add to obtain:

$$I_{\max}^2\sin^2\delta + I_{\max}^2\cos^2\delta = I_{\max}^2\left(\sin^2\delta + \cos^2\delta\right) = I_{\max}^2 = \left(\frac{\mathcal{E}_{\max}}{X_C}\right)^2 + \left(\frac{\mathcal{E}_{\max}}{R}\right)^2 = \mathcal{E}_{\max}^2\left(\frac{1}{X_C^2} + \frac{1}{R^2}\right) = \frac{\mathcal{E}_{\max}^2}{Z^2}$$

or

$$I_{\max} = \boxed{\frac{\mathcal{E}_{\max}}{Z}}$$

where $Z^{-2} = \boxed{X_C^{-2} + R^{-2}}$

***90 ••**

Picture the Problem Because we'll need to use it repeatedly in solving this problem, we'll begin by using complex numbers to derive an expression for the impedance Z_p of the parallel combination of C with L and R_L in series. The total impedance of the circuit is then $Z = R + Z_p$. We can apply Kirchhoff's loop rule to obtain expressions for the voltages across the load resistor with S either open or closed.

Use complex numbers to relate Z_p to R_L, X_L, and X_C:

$$\frac{1}{Z_p} = \frac{1}{-iX_C} + \frac{1}{R_L + iX_L} = \frac{R_L + i(X_L - X_C)}{X_C X_L - iR_L X_C}$$

or

$$Z_p = \frac{X_C X_L - iR_L X_C}{R_L + i(X_L - X_C)}$$

Multiple the numerator and denominator of this fraction by the complex conjugate of $R_L + i(X_L - X_C)$:

$$Z_p = \frac{X_C X_L - iR_L X_C}{R_L + i(X_L - X_C)} \frac{R_L - i(X_L - X_C)}{R_L - i(X_L - X_C)}$$

Simplify to obtain:

$$Z_p = \frac{R_L X_C^2}{R_L^2 + (X_L - X_C)^2} - i\frac{X_C\left[R_L^2 + X_L(X_L - X_C)\right]}{R_L^2 + (X_L - X_C)^2} \quad (1)$$

(*a*) **S is closed**. Because L is shorted:

$$X_L = 0$$

Evaluate X_C:

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(10\,\text{s}^{-1})(8\,\mu\text{F})} = 1.99\,\text{k}\Omega$$

Substitute numerical values in equation (1) and evaluate Z_p, Z, $|Z|$, and δ:

$$Z_p = 30\,\Omega - i(0.452\,\Omega),$$

$$Z = 40\,\Omega - i(0.452\,\Omega),$$

and

$$|Z| = \sqrt{(40\,\Omega)^2 + (0.452\,\Omega)^2} = \boxed{40.0\,\Omega}$$

In Problem 29-77 we showed that for a parallel combination of a resistor and capacitor, the phase angle δ is given by:

$$\delta = \tan^{-1}\left(\frac{R}{X_C}\right)$$

Substitute numerical values and evaluate δ:

$$\delta = \tan^{-1}\left(\frac{40\,\Omega}{-0.452\,\Omega}\right) = \boxed{-89.4°}$$

No phasor diagram is shown because it is impossible to represent it to scale.

(*b*) **<u>S is open; i.e., the inductor is in the circuit</u>**. Find X_L:

$$X_L = \omega L = 2\pi f L = 2\pi\left(10\,\text{s}^{-1}\right)(0.15\,\text{H}) = 9.42\,\Omega$$

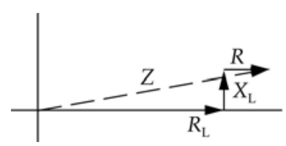
Substitute numerical values in equation (1) and evaluate Z_p, Z, $|Z|$, and δ:

$$Z_\text{p} = 30.3\,\Omega + i(9.01\,\Omega),$$
$$Z = 40.3\,\Omega + i(9.01\,\Omega),$$
$$|Z| = \sqrt{(40.3\,\Omega)^2 + (9.01\,\Omega)^2} = \boxed{41.3\,\Omega}$$

and

$$\delta = \tan^{-1}\left(\frac{X}{R}\right) = \tan^{-1}\left(\frac{9.01\Omega}{40.3\Omega}\right) = \boxed{12.6°}$$

The phasor diagram for this case is shown to the right.



(*c*) **<u>S is closed.</u>** Apply Kirchhoff's loop rule to a loop including the source, R, and R_L:

$$\mathcal{E} - IR - V_{R_L} = 0$$

Solve for V_{R_L}:

$$V_{R_L} = \mathcal{E} - IR$$

Express the current I in the circuit:

$$I = \frac{\mathcal{E}}{Z}$$

Substitute and simplify to obtain:

$$V_{R_L} = \mathcal{E} - \frac{\mathcal{E}R}{Z} = \left(1 - \frac{R}{|Z|}\right)\mathcal{E}_\text{max}\cos(\omega t - \delta)$$

From (*a*) we have:

$$Z_\text{p} = 30\,\Omega - i(0.452\,\Omega),$$
$$Z = 40\,\Omega - i(0.452\,\Omega),$$
$$|Z| = 40.0\,\Omega\text{, and}$$

$$\delta = \tan^{-1}\left(\frac{-0.452}{40}\right) = -0.647° \approx 0°$$

Substitute numerical values to obtain:

$$V_{R_L} = \left(1 - \frac{10\,\Omega}{40\,\Omega}\right)(100\,\text{V})\cos\left[\left(20\,\text{s}^{-1}\right)\pi t\right] = \boxed{(75\,\text{V})\cos\left[\left(20\,\text{s}^{-1}\right)\pi t\right]}$$

<u>S is open</u>. Apply Kirchhoff's loop rule to a loop including the source, R, L, and R_L when S is open:

$$\mathcal{E} - IR - IX_L - V_{R_L} = 0$$

Solve for V_{R_L}:

$$V_{R_L} = \mathcal{E} - IR - IX_L = \mathcal{E} - I(R + X_L)$$

Express the current I in the circuit:

$$I = \frac{\mathcal{E}}{Z}$$

Substitute to obtain:

$$V_{R_L} = \left(1 - \frac{R + X_L}{|Z|}\right)\mathcal{E}_{\max}\cos(\omega t - \delta)$$

Substitute numerical values and evaluate Z_p and Z:

$$Z_p = 30.3\,\Omega + i(9.01\,\Omega),$$

$$Z = 40.3\,\Omega + i(9.01\,\Omega),$$

$$|Z| = 41.3\,\Omega,$$

and

$$\delta = \tan^{-1}\left(\frac{X_L}{R + R_L}\right) = \tan^{-1}\left(\frac{9.42\,\Omega}{40.3\,\Omega}\right) = 13.2°$$

Substitute numerical values and evaluate V_{R_L}:

$$V_{R_L} = \left(1 - \frac{10\,\Omega + 9.42\,\Omega}{41.3\,\Omega}\right) \times (100\,\text{V})\cos\left[\left(20\,\text{s}^{-1}\right)\pi t - 13.2°\right] = \boxed{(53.0\,\text{V})\cos\left[\left(20\,\text{s}^{-1}\right)\pi t - 13.2°\right]}$$

(*d*) Find X_L and X_C when $f = 1000$ Hz:

$$X_L = 2\pi\left(1000\,\text{s}^{-1}\right)(0.15\,\text{H}) = 942\,\Omega$$

and

$$X_C = \frac{1}{2\pi\left(1000\,\text{s}^{-1}\right)(8\,\mu\text{F})} = 19.9\,\Omega$$

S is closed. $X_L = 0$, and Z_p simplifies to:

$$Z_p = \frac{R_L X_C^2}{R_L^2 + X_C^2} - i\frac{R_L^2 X_C}{R_L^2 + X_C^2}$$

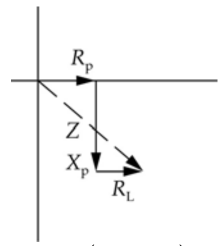
Substitute numerical values in equation (1) and evaluate Z_p, Z, $|Z|$, and δ:

$$Z_p = 9.17\,\Omega - i(13.8\,\Omega),$$
$$Z = 19.17\,\Omega - i(13.8\,\Omega),$$
$$|Z| = \sqrt{(19.17\,\Omega)^2 + (13.8\,\Omega)^2} = \boxed{23.6\,\Omega}$$

and

$$\delta = \tan^{-1}\left(\frac{-13.8\,\Omega}{19.17\,\Omega}\right) = \boxed{-35.7°}$$

A phasor diagram for this circuit is shown to the right,



S is open. Substitute numerical values in equation (1) and evaluate Z_p, Z, $|Z|$, and δ:

$$Z_p = 0.0140\,\Omega - i(20.3\,\Omega),$$
$$Z = 10.0\,\Omega - i(20.3\,\Omega),$$
$$|Z| = \sqrt{(10.0\,\Omega)^2 + (20.3\,\Omega)^2} = \boxed{22.6\,\Omega}$$

and

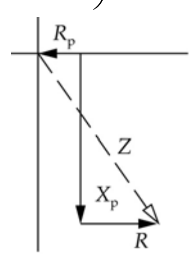
Find the total impedance, its magnitude, and phase angle for the circuit:

$$Z = 10.0\,\Omega - i(20.4\,\Omega),$$
$$Z = \sqrt{(10.0\,\Omega)^2 + (20.4\,\Omega)^2} = \boxed{22.7\,\Omega}$$

and

$$\delta = \tan^{-1}\left(\frac{-20.4\,\Omega}{10\,\Omega}\right) = \boxed{-63.9°}$$

The phasor diagram is shown to the right.



(*e*)

> The load voltage at the higher frequency is much more attenuated with S open, while opening S does not reduce the low frequency load voltage significantly. Therefore, S open is the better arrangement for a low-pass filter.

91 ••

Picture the Problem We can find the resonant frequency of any parallel ac circuit by setting the imaginary part of the reciprocal of the impedance equal to zero. In (*b*) we can use complex numbers to find the impedance of each branch of the circuit and then relate the common potential difference across each branch to its impedance and the current in the resistors.

(*a*) Express the reciprocal of the impedance of the circuit:

$$\frac{1}{Z}=\frac{1}{R_1-iX_C}+\frac{1}{R_2+iX_L}$$

Rewrite this expression with a common denominator and simplify to obtain:

$$\frac{1}{Z}=\frac{(R_1+R_2)+i(X_L-X_C)}{(R_1R_2+X_CX_L)+i(R_1X_L-R_2X_C)}$$

Multiply this expression by 1 in the form of the complex conjugate of the denominator divided by itself and simplify (separate the real part of the expression from the imaginary part) to obtain:

$$\frac{1}{Z}=\frac{(R_1+R_2)(R_1R_2+X_CX_L)+(X_L-X_C)(R_1X_L-R_2X_C)}{(R_1R_2+X_CX_L)^2+(R_1X_L-R_2X_C)^2}+i\frac{(X_L-X_C)(R_1R_2+X_CX_L)-(R_1+R_2)(R_1X_L-R_2X_C)}{(R_1R_2+X_CX_L)^2+(R_1X_L-R_2X_C)^2}$$

Set the imaginary part of $1/Z$ equal to zero to obtain:

$$(X_L-X_C)(R_1R_2+X_CX_L)-(R_1+R_2)(R_1X_L-R_2X_C)=0$$

Substitute numerical values for R_1 and R_2 (suppress the units to save space and make the resulting equation more readable) to obtain:

$$\left(\omega_0L-\frac{1}{\omega_0C}\right)\left(8+\frac{L}{C}\right)-6\left(2\omega_0L-\frac{4}{\omega_0C}\right)=0$$

Simplify this equation by clearing the fractions and combining like terms to obtain:

$$\left(8LC^2+L^2C-12LC^2\right)\omega_0^2=L-16C$$

Solve for ω_0:

$$\omega_0=\sqrt{\frac{L-16C}{8LC^2+L^2C-12LC^2}}$$

Substitute numerical values for L and C and evaluate ω_0:

$$\omega_0=\sqrt{\frac{1.15\times10^{-2}}{4.28\times10^{-9}}}=\boxed{1.64\times10^{3}\ \text{rad/s}}$$

(*b*) Express the currents in each branch at resonance:

$$I_C = \frac{\mathcal{E}}{|Z_C|} \text{ and } I_L = \frac{\mathcal{E}}{|Z_L|}$$

Evaluate $Z_{C,\text{res}}$ and $|Z_{C,\text{res}}|$:

$$Z_{C,\text{res}} = 2\,\Omega - i\frac{1}{(1.64\times10^3\text{ s}^{-1})(30\times10^{-6}\text{ F})} = 2\,\Omega - i(20.3\,\Omega),$$

$$|Z_{C,\text{res}}| = \sqrt{(2\,\Omega)^2 + (20.3\,\Omega)^2} = 20.4\,\Omega,$$

Substitute to obtain:

$$I_{C,\text{rms}} = \frac{40\text{ V}}{\sqrt{2}(20.4\,\Omega)} = \boxed{1.39\text{ A}}$$

and

$$\delta_C = \tan^{-1}\left(\frac{-20.3\,\Omega}{2\,\Omega}\right) = \boxed{-84.4^\circ}$$

Evaluate $Z_{L,\text{res}}$ and $|Z_{L,\text{res}}|$:

$$Z_{L,\text{res}} = 4\,\Omega + i(1.64\times10^3\text{ s}^{-1})(12\times10^{-3}\text{ H}) = 4\,\Omega + i(19.7\,\Omega),$$

$$|Z_{L,\text{res}}| = \sqrt{(4\,\Omega)^2 + (19.7\,\Omega)^2} = 20.1\,\Omega,$$

Substitute to obtain:

$$I_{L,\text{rms}} = \frac{40\text{ V}}{\sqrt{2}(20.1\,\Omega)} = \boxed{1.41\text{ A}}$$

and

$$\delta_L = \tan^{-1}\left(\frac{19.7\,\Omega}{4\,\Omega}\right) = \boxed{78.5^\circ}$$

Express and evaluate the rms current supplied by the source:

$$\begin{aligned} I_{\text{rms}} &= I_{L,\text{rms}}\cos\delta_L + I_{C,\text{rms}}\cos\delta_C \\ &= (1.41\text{ A})\cos 78.5^\circ \\ &\quad + (1.39\text{ A})\cos(-84.4^\circ) \\ &= \boxed{0.417\text{ A}} \end{aligned}$$

92 ••

Picture the Problem We can use its definition to express Q in terms of ω_0 and $\Delta\omega$. By expressing the current drawn from the source we can obtain an expression for the energy stored in the system each cycle and then use this result to establish the relationship between ω, R, L, and C when the energy stored per cycle is at half-maximum. Finally, we can solve the resulting equation for the values of ω that will allow us to determine $\Delta\omega$.

The definition of Q is:

$$Q = \frac{\omega_0}{\Delta\omega}$$

where $\Delta\omega$ is the width of the resonance at half maximum.

Express the resonance frequency of the circuit:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Substitute to obtain:

$$Q = \frac{1}{\sqrt{LC}\,\Delta\omega} \qquad (1)$$

Express the current to the capacitor:

$$I_C = \frac{V}{X_C} = \omega C V$$

with I_C leading V by 90°.

Express the current in the inductor:

$$I_L = \frac{V}{X_L} = \frac{V}{\omega L}$$

with I_L lagging V by 90°.

Express the current in the resistor:

$$I_R = \frac{V}{R}$$

with I_R in phase with V.

Express the total current drawn from the source:

$$I = \frac{V}{Z} = V\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{\omega L} - \omega C\right)^2}$$

$$= \frac{V}{R}\sqrt{1 + R^2\left(\frac{1}{\omega L} - \omega C\right)^2}$$

At resonance, the reactive term is zero and the total current is the current in the resistor:

$$I_0 = \frac{V}{R}$$

Substitute to obtain:

$$I = I_0\sqrt{1 + R^2\left(\frac{1}{\omega L} - \omega C\right)^2}$$

Express the total energy stored in the circuit per cycle:

$$U_{\text{tot}} = \frac{Q_0^2}{2C}$$

where Q_0 is the maximum charge on the capacitor.

Relate the maximum value of the current to the maximum value of the charge:

$$I_{max} = \omega Q_0$$

Substitute to obtain:

$$U_{tot} = \frac{I_{max}^2}{2\omega^2 C} = \frac{1}{2\omega^2 C}\frac{V^2}{R^2} = \frac{1}{2\omega^2 C}\frac{I_0^2}{1 + R^2\left(\frac{1}{\omega L} - \omega C\right)^2}$$

At resonance we have:

$$U_{tot,res} = \frac{I_0^2}{2\omega^2 C}$$

At half $U_{tot,res}$:

$$\frac{1}{2}U_{tot,res} = \frac{I_0^2}{4\omega^2 C} = \frac{1}{2\omega^2 C}\frac{I_0^2}{1 + R^2\left(\frac{1}{\omega L} - \omega C\right)^2}$$

or

$$\frac{1}{2} = \frac{1}{1 + R^2\left(\frac{1}{\omega L} - \omega C\right)^2}$$

Solve for $R\left(\omega C - \frac{1}{\omega L}\right)$ to obtain:

$$R\left(\omega C - \frac{1}{\omega L}\right) = \pm 1 \qquad (2)$$

Rewrite equation (2) explicitly as a quadratic equation:

$$RLC\omega^2 \pm L\omega - R = 0$$

Letting + denote the roots with a positive coefficient of ω and – the roots with a negative coefficient, solve this equation for ω_+ and ω_-:

$$\omega_+ = \frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

and

$$\omega_- = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

Express $\Delta\omega$:

$$\Delta\omega = \omega_+ - \omega_- = \frac{1}{RC}$$

Substitute in equation (1) to obtain:

$$Q = \frac{RC}{\sqrt{LC}} = \boxed{R\sqrt{\frac{C}{L}}}$$

93 ••

Picture the Problem We can use the expression for the resonance frequency derived by equating the capacitive and inductive reactances at resonance to express ω_0 in terms of L and C. In (b) we can use the result derived in Problem 92 to find R from Q, L, and C.

(a) Express the resonance frequency ω_0 in terms of L and C:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Solve for C to obtain:

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{4\pi^2 f^2 L}$$

Substitute numerical values and evaluate C:

$$C = \frac{1}{4\pi^2\left(4\times10^3\ \text{s}^{-1}\right)^2\left(4\,\text{mH}\right)} = \boxed{0.396\,\mu\text{F}}$$

(b) From Problem 92 we have:

$$Q = R\sqrt{\frac{C}{L}}$$

Solve for R to obtain:

$$R = Q\sqrt{\frac{L}{C}}$$

Substitute numerical values and evaluate R:

$$R = 8\sqrt{\frac{4\,\text{mH}}{0.396\,\mu\text{F}}} = \boxed{804\,\Omega}$$

94 ••

Picture the Problem We can use the expression for the resonance frequency derived by equating the capacitive and inductive reactances at resonance to express ω_0 in terms of L and C. We can use the result derived in Problem 92 to find the Q-value resulting from halving the capacitance and to find the resistance necessary to give $Q = 8$.

Express the resonance frequency ω_0 in terms of L and C:

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ or } f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Substitute numerical values and evaluate f_0:

$$f_0 = \frac{1}{2\pi\sqrt{\frac{1}{2}(4\,\text{mH})(0.396\,\mu\text{F})}} = \boxed{5.66\,\text{kHz}}$$

From Problem 92 we have:

$$Q = R\sqrt{\frac{C}{L}} \qquad (1)$$

Letting C' represent the halved capacitance, express Q':

$$Q' = R\sqrt{\frac{C'}{L}}$$

Divide Q' by Q and simplify to obtain:

$$\frac{Q'}{Q} = \frac{R\sqrt{\frac{C'}{L}}}{R\sqrt{\frac{C}{L}}} = \sqrt{\frac{C'}{C}}$$

Because $C' = \frac{1}{2}C$:

$$Q' = \frac{Q}{\sqrt{2}} = \frac{8}{\sqrt{2}} = \boxed{5.66}$$

Solve equation (1) for R to obtain:

$$R = Q\sqrt{\frac{L}{C}}$$

Substitute numerical values and evaluate R:

$$R = 8\sqrt{\frac{4\,\text{mH}}{\frac{1}{2}(0.396\,\mu\text{F})}} = \boxed{1.14\,\text{k}\Omega}$$

95 ••

Picture the Problem We can use its definition to find the resonance frequency of this series *RLC* circuit and the fact that, at resonance, $Z = R$, to find the resonance current. Because, at resonance $V_L = V_C$, we can find the voltage across either element from the product of the current and its reactance. In (*c*) we can use the definition of the Q factor to find the angular frequency corresponding to $f = f_0 + \frac{1}{2}\Delta f$ and then use this result to find X_L, X_C, and Z at this frequency. Finally, we can use these values for X_L, X_C, and Z to find the rms current and the rms voltages across the inductor and capacitor.

(*a*) Express the resonance frequency f_0 in terms of L and C:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Substitute numerical values and evaluate f_0:

$$f_0 = \frac{1}{2\pi\sqrt{(36\,\text{mH})(4\,\text{nF})}} = \boxed{13.26\,\text{kHz}}$$

(*b*) At resonance, $Z = R$ and:

$$I = \frac{\mathcal{E}}{R} = \frac{20\text{ V}}{100\,\Omega} = \boxed{200\,\text{mA}}$$

Express and evaluate the equal (at resonance) rms voltages across the capacitor and the inductor:

$$\begin{aligned} V_C = V_L = IX_L = \omega_0 IL = 2\pi f_0 IL \\ = 2\pi(13.26\,\text{kHz})(0.2\,\text{A})(36\,\text{mH}) \\ = \boxed{600\,\text{V}} \end{aligned}$$

(*c*) Express the rms current in the circuit and the rms voltages across the inductor and capacitor:

$$I = \frac{\mathcal{E}}{Z},\ V_L = IX_L,\text{ and } V_C = IX_C$$

Express the Q factor for an RLC circuit:

$$Q = \frac{\omega_0 L}{R} \approx \frac{\omega_0}{\Delta\omega} = \frac{f_0}{\Delta f}$$

Solve for Δf:

$$\Delta f = \frac{R}{\omega_0 L} f_0$$

Express f:

$$f = f_0 + \frac{R}{2\omega_0 L} f_0 = f_0\left(1 + \frac{R}{2\omega_0 L}\right)$$

Substitute numerical values and evaluate f and ω:

$$\begin{aligned} f &= (13.26\,\text{kHz}) \\ &\quad\times\left(1 + \frac{100\,\Omega}{4\pi(13.26\,\text{kHz})(36\,\text{mH})}\right) \\ &= 13.48\,\text{kHz} \end{aligned}$$

and

$$\omega = 2\pi f = 2\pi(13.48\,\text{kHz}) = 84.7\,\text{krad/s}$$

Calculate X_L and X_C at 84.7 krad/s:

$$\begin{aligned} X_L = \omega L = (84.7\,\text{krad/s})(36\,\text{mH}) \\ = 3.05\,\text{k}\Omega \end{aligned}$$

and

$$X_C = \frac{1}{\omega C} = \frac{1}{(84.7\,\text{krad/s})(4\,\text{nF})} = 2.95\,\text{k}\Omega$$

Now we can find Z:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(100\,\Omega)^2 + (3.05\,\text{k}\Omega - 2.95\,\text{k}\Omega)^2} = 141\,\Omega$$

Substitute numerical values and evaluate I, V_L, and V_C:

$$I = \frac{20\,\text{V}}{141\,\Omega} = \boxed{142\,\text{mA}},$$

$$V_L = (142\,\text{mA})(3.05\,\text{k}\Omega) = \boxed{433\,\text{V}},$$

and

$$V_C = (142\,\text{mA})(2.95\,\text{k}\Omega) = \boxed{419\,\text{V}}$$

96 •••

Picture the Problem We can use complex numbers to find the impedance in the branches of the given circuit. We can then use Kirchhoff's loop rule to find the currents in the branches and a current phasor diagram to find the total current and its phase relative to the applied voltage.

(*a*) Use complex numbers to find Z_L, $|Z_L|$, and δ_L:

$$Z_L = R_2 + iX_L = 40\,\Omega + i(30\,\Omega),$$

$$|Z_L| = \sqrt{R_2^2 + X_L^2} = \sqrt{(40\,\Omega)^2 + (30\,\Omega)^2} = \boxed{50.0\,\Omega}$$

and

$$\delta_L = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{30\,\Omega}{40\,\Omega}\right) = \boxed{36.9^\circ}$$

Use complex numbers to find Z_C, $|Z_C|$, and δ_C:

$$Z_C = R_1 + iX_C = 10\,\Omega - i(10\,\Omega),$$

$$|Z_C| = \sqrt{R_1^2 + X_C^2} = \sqrt{(10\,\Omega)^2 + (10\,\Omega)^2} = \boxed{14.1\,\Omega}$$

and

$$\delta_C = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{-10\,\Omega}{10\,\Omega}\right) = \boxed{-45.0^\circ}$$

(*b*) Apply Kirchhoff's loop rule to the source and the inductive branch to obtain:

$$V - I_L X_L = 0$$

or

$$I_L = \frac{V}{Z_L} = \frac{110\text{ V}}{50\,\Omega}$$
$$= \boxed{2.20\text{ A lagging the voltage by }36.9°}$$

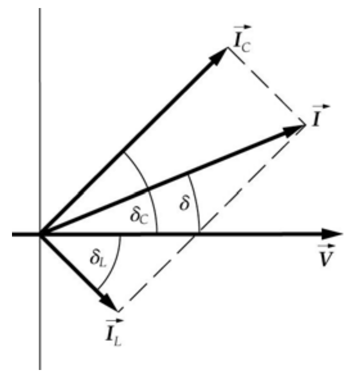
Apply Kirchhoff's loop rule to the source and the capacitive branch to obtain:

$$V - I_C X_C = 0$$

or

$$I_C = \frac{V}{Z_C} = \frac{110\text{ V}}{14.1\Omega}$$
$$= \boxed{7.80\text{ A leading the voltage by }45.0°}$$

(*c*) The current phasor diagram is shown to the right.



Express the total current in terms of its horizontal and vertical components:

$$I = \sqrt{I_{\text{hor}}^2 + I_{\text{vert}}^2}$$

and

$$\delta = \tan^{-1}\left(\frac{I_{\text{vert}}}{I_{\text{hor}}}\right)$$

Find the horizontal component I_{hor} of the total current:

$$I_{\text{hor}} = I_C\cos\delta_C + I_L\cos\delta_L$$
$$= (7.80\text{ A})\cos 45° + (2.20\text{ A})\cos 36.9°$$
$$= 7.27\text{ A}$$

Find the vertical component I_{vert} of the total current:

$$I_{\text{vert}} = I_C\sin\delta_C - I_L\sin\delta_L$$
$$= (7.80\text{ A})\sin 45° - (2.20\text{ A})\sin 36.9°$$
$$= 4.19\text{ A}$$

Substitute numerical values and evaluate I and δ:

$$I = \sqrt{(4.19\text{ A})^2 + (7.27\text{ A})^2} = \boxed{8.39\text{ A}}$$

and

$$\delta = \tan^{-1}\left(\frac{4.19\text{ A}}{7.27\text{ A}}\right) = \boxed{30.0°}$$

Remarks: The total current leads the applied voltage by 30.0°.

***97 •••**

Picture the Problem We can manipulate Equation 29-47 into a form that has the ratio of *L* to *R* in it and then use the definition of *Q* to eliminate *L* and *R*. In (*b*) we can approximate $\omega^2 - \omega_0^2$, near resonance, as $2\omega_0\Delta\omega$ and substitute in the result from (*a*) to obtain the desired result.

(*a*) From Equation 29-47:

$$\tan\delta = \frac{\omega L - 1/\omega C}{R} = \frac{\omega^2 L - 1/C}{\omega R} = \frac{L\left(\omega^2 - 1/LC\right)}{\omega R} = \frac{L\left(\omega^2 - \omega_0^2\right)}{\omega R}$$

Express *Q* in terms of ω_0, *L* and *R*:

$$Q = \frac{\omega_0 L}{R}$$

Solve for *L*/*R* to obtain:

$$\frac{L}{R} = \frac{Q}{\omega_0}$$

Substitute to obtain:

$$\tan\delta = \boxed{\frac{Q\left(\omega^2 - \omega_0^2\right)}{\omega\omega_0}} \qquad (1)$$

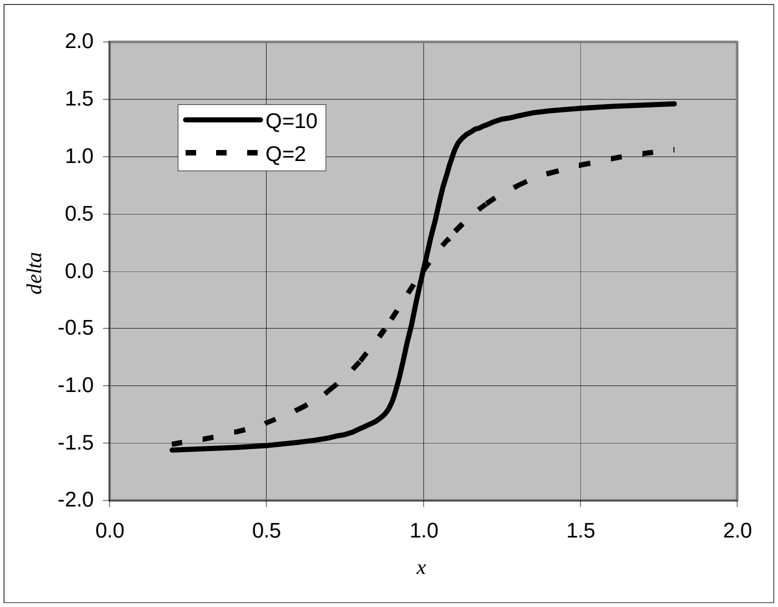
(*b*) Near resonance:

$$\omega^2 - \omega_0^2 = \left(\omega + \omega_0\right)\left(\omega - \omega_0\right) \approx 2\omega_0\Delta\omega$$

Substitute in equation (1) to obtain:

$$\tan\delta = \frac{Q\left(2\omega_0\Delta\omega\right)}{\omega\omega_0} = \boxed{\frac{2Q\left(\omega - \omega_0\right)}{\omega}}$$

(*c*) A following graph of δ as a function of $x = \omega/\omega_0$ was plotted using a spreadsheet program. The solid curve is for a high-*Q* circuit and the dashed curve is for a low-*Q* circuit.



98 •••

Picture the Problem We can rewrite Equation 29-45 in terms of the current and then differentiate Equation 29-46. Substituting for I, dI/dt, X_L, and X_C will allow us to use the trigonometric identities for the sine and cosine of the sum of two angles to rewrite the equation in such a form that we can equate the coefficients of $\sin\omega t$ and $\cos\omega t$ to obtain Equation 29-47 and an equation that is satisfied provided Z is given by Equation 29-49.

Rewrite Equation 29-45 in terms of the current:

$$L\frac{dI}{dt}+RI+\frac{1}{C}\int Idt=\mathcal{E}_{\max}\cos\omega t$$

Equation 29-46 is:

$$I=I_{\max}\cos(\omega t-\delta)$$

Differentiate Equation 29-50 with respect to time to obtain:

$$\frac{dI}{dt}=-\omega I_{\max}\sin(\omega t-\delta)$$

Evaluate $\int Idt$:

$$\int Idt=I_{\max}\int\cos(\omega t-\delta)dt$$
$$=\frac{I_{\max}}{\omega}\sin(\omega t-\delta)$$

Substitute to obtain:

$$L(-\omega I_{\max}\sin(\omega t-\delta))$$
$$+RI_{\max}\cos(\omega t-\delta)$$
$$+\frac{1}{C}\left(\frac{I_{\max}}{\omega}\sin(\omega t-\delta)\right)=\mathcal{E}_{\max}\cos\omega t$$

Divide through by I_{max} to obtain:

$$L(-\omega\sin(\omega t-\delta)) + R\cos(\omega t-\delta) + \frac{1}{C}\left(\frac{1}{\omega}\sin(\omega t-\delta)\right) = \frac{\mathcal{E}_{max}}{I_{max}}\cos\omega t$$

Use the definitions of X_L, X_C, and Z to obtain:

$$-X_L\sin(\omega t-\delta)+R\cos(\omega t-\delta) + X_C\sin(\omega t-\delta) = Z\cos\omega t$$

Use the trigonometric identities for $\sin(\alpha+\beta)$ and $\cos(\alpha+\beta)$ to obtain:

$$-X_L(\sin\omega t\cos\delta-\cos\omega t\sin\delta)+R(\cos\omega t\cos\delta+\sin\omega t\sin\delta) + X_C(\sin\omega t\cos\delta-\cos\omega t\sin\delta) = Z\cos\omega t$$

Collect the terms in $\sin\omega t$ and $\cos\omega t$:

$$(-X_L\cos\delta+R\sin\delta+X_C\cos\delta)\sin\omega t + (R\cos\delta-X_C\sin\delta+X_L\sin\delta)\cos\omega t = Z\cos\omega t$$

Equate the coefficients of $\sin\omega t$ and $\cos\omega t$ to obtain:

$$-X_L\cos\delta+R\sin\delta+X_C\cos\delta=0$$

and

$$R\cos\delta-X_C\sin\delta+X_L\sin\delta=Z$$

Solve the first of these equations for $\tan\delta$:

$$\boxed{\tan\delta=\frac{X_L-X_C}{R}}$$ Equation 29-47

Rewrite the second equation as:

$$R-X_C\tan\delta+X_L\tan\delta=\frac{Z}{\cos\delta}$$

or

$$(X_L-X_C)\tan\delta+R=\frac{Z}{\cos\delta}$$

Simplify this equation to obtain Equation 29-49:

$$Z=\boxed{\sqrt{R^2+(X_L-X_C)^2}}$$

99 •••

Picture the Problem In (*a*) we can apply Kirchhoff's loop rule to obtain the 2nd order differential equation relating the charge on the capacitor to the time. In (*b*) we'll assume a solution of the form $Q=Q_{max}\cos\omega t$, differentiate it twice, and substitute for d^2Q/dt^2 and Q to show that the assumed solution satisfies the DE provided

$Q_{max} = -\frac{\varepsilon_{max}}{L(\omega^2 - \omega_0^2)}$. In (*c*) we'll use our results from (*a*) and (*b*) to establish the for I_{max} given in the problem statement.

(*a*) Apply Kirchhoff's loop rule to obtain:

$$\varepsilon - \frac{Q}{C} - L\frac{dI}{dt} = 0$$

Substitute for ε and rearrange the differential equation to obtain:

$$L\frac{dI}{dt} + \frac{Q}{C} = \varepsilon_{max}\cos\omega t$$

Because $I = dQ/dt$:

$$\boxed{L\frac{d^2Q}{dt^2} + \frac{Q}{C} = \varepsilon_{max}\cos\omega t}$$

(*b*) Assume that the solution is:

$$Q = Q_{max}\cos\omega t$$

Differentiate the assumed solution twice to obtain:

$$\frac{dQ}{dt} = -\omega Q_{max}\sin\omega t$$

and

$$\frac{d^2Q}{dt^2} = -\omega^2 Q_{max}\cos\omega t$$

Substitute in the differential equation to obtain:

$$-\omega^2 LQ_{max}\cos\omega t + \frac{Q_{max}}{C}\cos\omega t = \varepsilon_{max}\cos\omega t$$

Factor $\cos\omega t$ from the left-hand side of the equation:

$$\left(-\omega^2 LQ_{max} + \frac{Q_{max}}{C}\right)\cos\omega t = \varepsilon_{max}\cos\omega t$$

If this equation is to hold for all values of t it must be true that:

$$-\omega^2 LQ_{max} + \frac{Q_{max}}{C} = \varepsilon_{max}$$

or

$$Q_{max} = \frac{\varepsilon_{max}}{-\omega^2 L + \frac{1}{C}}$$

Factor L from the denominator and substitute for $1/LC$ to obtain:

$$Q_{max} = \frac{\varepsilon_{max}}{L\left(-\omega^2 + \frac{1}{LC}\right)} = \boxed{\frac{-\varepsilon_{max}}{L(\omega^2 - \omega_0^2)}}$$

(*c*) From (*a*) and (*b*) we have:

$$I=\frac{dQ}{dt}=-\omega Q_{\max}\sin\omega t$$
$$=\frac{\omega\mathcal{E}_{\max}}{L\left(\omega^2-\omega_0^2\right)}\sin\omega t$$
$$=I_{\max}\sin\omega t$$

where

$$I_{\max}=\frac{\omega\mathcal{E}_{\max}}{L\left|\omega^2-\omega_0^2\right|}=\frac{\mathcal{E}_{\max}}{\frac{L}{\omega}\left|\omega^2-\omega_0^2\right|}$$
$$=\frac{\mathcal{E}_{\max}}{\left|\omega L-\frac{1}{\omega C}\right|}=\frac{\mathcal{E}_{\max}}{\left|X_L-X_C\right|}$$

If $\omega > \omega_0$, $X_L > X_C$ and the current lags the voltage by 90°. Therefore:

$$I=I_{\max}\sin\omega t=\boxed{I_{\max}\cos\left(\omega t-90°\right)}$$

If $\omega < \omega_0$, $X_L < X_C$ and the current leads the voltage by 90°. Therefore:

$$I=-I_{\max}\sin\omega t=\boxed{I_{\max}\cos\left(\omega t+90°\right)}$$

100 •••

Picture the Problem We can use the condition determining the half-power points to obtain a quadratic equation that we can solve for the frequencies corresponding to the half-power points. Expanding these solutions binomially will lead us to the result that $\Delta\omega = \omega_2 - \omega_1 \approx R/L$. We can then use the definition of Q to complete the proof that $Q \approx \omega_0/\Delta\omega$.

Equation 29-58 is:

$$P_{\text{av}}=\frac{\mathcal{E}_{\text{rms}}^2R\omega^2}{L^2\left(\omega^2-\omega_0^2\right)^2+\omega^2R^2}$$

The half-power points will occur when the denominator is twice the value near resonance, that is, when:

$$L^2(\omega^2-\omega_0^{\,2})^2=\omega^2R^2\approx\omega_0^{\,2}R^2$$

or

$$\left(\frac{L}{R}\right)^2\left(\omega^2-\omega_0^2\right)^2=\omega_0^2$$

Let ω_1 and ω_2 be the solutions of this equation. Then:

$$\left(\frac{L}{R}\right)^2\left(\omega_1^2-\omega_0^2\right)^2=\omega_0^2$$

and

$$\left(\frac{L}{R}\right)^2\left(\omega_2^2-\omega_0^2\right)^2=\omega_0^2$$

Solve these equations for ω_1 and ω_2 to obtain:

$$\omega_1 = \omega_0\left(1 - \frac{R}{\omega_0 L}\right)^{1/2}$$

and

$$\omega_2 = \omega_0\left(1 + \frac{R}{\omega_0 L}\right)^{1/2}$$

Expand these solutions binomially to obtain:

$$\omega_1 = \omega_0\left(1 - \frac{R}{2\omega_0 L} + \text{higher order terms}\right)$$

and

$$\omega_2 = \omega_0\left(1 + \frac{R}{2\omega_0 L} + \text{higher order terms}\right)$$

For $R << X_L$ (a condition that holds for a sharply peaked resonance):

$$\omega_1 \approx \omega_0\left(1 - \frac{R}{2\omega_0 L}\right),$$

$$\omega_2 \approx \omega_0\left(1 + \frac{R}{2\omega_0 L}\right),$$

and

$$\Delta\omega = \omega_2 - \omega_1 \approx \frac{R}{L}.$$

From the definition of Q:

$$\frac{R}{L} = \frac{\omega_0}{Q}$$

Substitute to obtain:

$$\Delta\omega \approx \frac{\omega_0}{Q}$$

Solve for Q:

$$Q \approx \boxed{\frac{\omega_0}{\Delta\omega}}$$

101 •••

Picture the Problem We'll differentiate $Q = Q_0 e^{-Rt/2L}\cos\omega' t$ twice and substitute this function and both its derivatives in the differential equation of the circuit. Rewriting the resulting equation in the form A$\cos\omega' t$ + B$\sin\omega' t$ = 0 will reveal that B vanishes. Requiring that A$\cos\omega' t$ = 0 hold for all values of t will lead to $\omega' = \sqrt{(1/LC) - (R/2L)^2}$.

Equation 29-47*b* is:

$$L\frac{d^2Q}{dt^2} + \frac{Q}{C} + R\frac{dQ}{dt} = 0$$

Assume a solution of the form: $Q = Q_0 e^{-Rt/2L} \cos\omega' t$

Differentiate $Q(t)$ twice to obtain:

$$\frac{dQ}{dt} = -Q_0 e^{-Rt/2L}\left[\omega' \sin\omega' t + \frac{R}{2L}\cos\omega' t\right]$$

and

$$\frac{d^2Q}{dt^2} = Q_0 e^{-Rt/2L}\left[\left(\frac{R^2}{4L^2} - \omega'^2\right)\cos\omega' t + \frac{R\omega'}{L}\sin\omega' t\right]$$

Substitute these derivatives in the differential equation and simplify to obtain:

$$LQ_0 e^{-Rt/2L}\left[\left(\frac{R^2}{4L^2} - \omega'^2\right)\cos\omega' t + \frac{R\omega'}{L}\sin\omega' t\right] + \frac{Q_0}{C} e^{-Rt/2L}\cos\omega' t - RQ_0 e^{-Rt/2L}\left[\omega' \sin\omega' t + \frac{R}{2L}\cos\omega' t\right] = 0$$

or

$$L\left[\left(\frac{R^2}{4L^2} - \omega'^2\right)\cos\omega' t + \frac{R\omega'}{L}\sin\omega' t\right] + \frac{1}{C}\cos\omega' t - R\left[\omega' \sin\omega' t + \frac{R}{2L}\cos\omega' t\right] = 0$$

Rewrite this equation in the form $A\cos\omega' t + B\sin\omega' t = 0$:

$$(R\omega' - R\omega')\sin\omega' t + \left[L\left(\frac{R^2}{4L^2} - \omega'^2\right) + \frac{1}{C} - \frac{R^2}{2L}\right]\cos\omega' t = 0$$

or

$$\left[\left(\frac{R^2}{4L} - L\omega'^2 + \frac{1}{C} - \frac{R^2}{2L}\right)\right]\cos\omega' t = 0$$

If this equation is to hold for all values of t, its coefficient must vanish:

$$\frac{R^2}{4L} - L\omega'^2 + \frac{1}{C} - \frac{R^2}{2L} = 0$$

Solve for ω':

$$\omega' = \boxed{\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}},$$

the condition that must be satisfied if $Q = Q_0 e^{-Rt/2L}\cos\omega' t$ is the solution to Equation 29-47*b*.

***102 •••**

Picture the Problem We can use $L = \mu_0 n^2 A\ell$ to determine the inductance of the empty solenoid and the resonance condition to find the capacitance of the sample-free circuit when the resonance frequency of the circuit is 6.0000 MHz. By expressing L as a function of f_0 and then evaluating df_0/dL and approximating the derivative with $\Delta f_0/\Delta L$, we can evaluate χ from its definition.

(*a*) Express the inductance of an air-core solenoid:

$$L = \mu_0 n^2 A\ell$$

Substitute numerical values and evaluate L:

$$L = \left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(\frac{400}{0.04\,\text{m}}\right)^2\frac{\pi}{4}(0.003\,\text{m})^2(0.04\,\text{m}) = \boxed{35.5\,\mu\text{H}}$$

(*b*) Express the condition for resonance in the LC circuit:

$$X_L = X_C$$

or

$$2\pi f_0 L = \frac{1}{2\pi f_0 C} \qquad (1)$$

Solve for C to obtain:

$$C = \frac{1}{4\pi^2 f_0^2 L}$$

Substitute numerical values and evaluate C:

$$C = \frac{1}{4\pi^2(6\,\text{MHz})(35.5\,\mu\text{H})} = \boxed{119\,\mu\text{F}}$$

(*c*) Express the sample's susceptibility in terms of L and ΔL:

$$\chi = \frac{\Delta L}{L} \qquad (2)$$

Solve equation (1) for f_0:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Differentiate f_0 with respect to L:

$$\frac{df_0}{dL} = \frac{1}{2\pi\sqrt{C}}\frac{d}{dL}L^{-1/2} = -\frac{1}{4\pi\sqrt{C}}L^{-3/2}$$

$$= -\frac{1}{4\pi L\sqrt{LC}} = -\frac{f_0}{2L}$$

Approximate df_0/dL by $\Delta f_0/\Delta L$:

$$\frac{\Delta f_0}{\Delta L} = -\frac{f_0}{2L} \text{ or } \frac{\Delta f_0}{f_0} = -\frac{\Delta L}{2L}$$

Substitute in equation (2) to obtain:

$$\chi = -2\frac{\Delta f_0}{f_0}$$

Substitute numerical values and evaluate χ:

$$\chi = -2\left(\frac{5.9989\,\text{MHz} - 6.0000\,\text{MHz}}{6.0000\,\text{MHz}_0}\right) = \boxed{3.67\times10^{-4}}$$

103 •••

Picture the Problem We can find the angular frequency ω for the circuit in Problem 91 such that the magnitudes of the reactances of the two parallel branches are equal by equating the reactances in the two branches. We can use $P = \frac{1}{2}I^2R = (\mathcal{E}/Z)^2(R/2)$, where Z is, in turn, Z_L and Z_C, to find the power dissipated in each resistor.

(*a*) When the reactances of the parallel branches are equal:

$$X_L = X_C \text{ and } \omega = \frac{1}{\sqrt{LC}}$$

Substitute numerical values to obtain:

$$\omega = \frac{1}{\sqrt{(12\,\text{mH})(30\,\mu\text{F})}} = \boxed{1.67\,\text{krad/s}}$$

(*b*) Express the power dissipation in a resistor in an ac circuit:

$$P = \frac{1}{2}I^2R = \frac{1}{2}\left(\frac{\mathcal{E}}{Z}\right)^2 R$$

Find Z_L and $|Z_L|$ at 1.67 krad/s:

$$Z_L = R_2 + i\omega L = 4\,\Omega + i(1.67\,\text{krad/s})(12\,\text{mH}) = 4\,\Omega + i(20.0\,\Omega)$$

and

$$|Z_L| = \sqrt{(4\,\Omega)^2 + (20\,\Omega)^2} = 20.4\,\Omega$$

Find Z_C and $|Z_C|$ at 1.67 krad/s:

$$Z_C = R_1 - i\frac{1}{\omega C} = 2\,\Omega - i\frac{1}{(1.67\,\text{krad/s})(30\,\mu\text{F})} = 2\,\Omega - i(20.0\,\Omega)$$

and

$$|Z_C| = \sqrt{(2\,\Omega)^2 + (20.0\,\Omega)^2} = 20.1\,\Omega$$

Evaluate the power dissipated in R_1 and R_2:

$$P_1 = \frac{1}{2}\left(\frac{\mathcal{E}}{Z_C}\right)^2 R_1 = \frac{1}{2}\left(\frac{40\,\text{V}}{20.1\,\Omega}\right)^2 (2\,\Omega)$$
$$= \boxed{3.96\,\text{W}}$$

and

$$P_2 = \frac{1}{2}\left(\frac{\mathcal{E}}{Z_L}\right)^2 R_2 = \frac{1}{2}\left(\frac{40\,\text{V}}{20.4\,\Omega}\right)^2 (4\,\Omega)$$
$$= \boxed{7.69\,\text{W}}$$

104 •••

Picture the Problem We can equate the power dissipated in the two resistors to obtain a relationship between the currents in and the resistances of the two branches. Expressing the currents in terms of the impedances of the two branches and the common potential difference across them will lead us to an equation that is quadratic in ω^2 that we can solve for ω. In (*b*) we can use complex numbers to find the reactances of each of the two parallel branches and then use these results to draw the phasor diagram of (*c*). We can use the results of (*b*) to find the impedance of the circuit in (*d*).

(*a*) Express the condition under which the power dissipation in the two resistors is the same:

$$I_1^2 R_1 = I_2^2 R_2 \text{ or } \frac{I_1^2}{I_2^2} = \frac{R_2}{R_1}$$

Express the ratio of the squares of the currents in the two resistors:

$$\frac{I_1^2}{I_1^2} = \left(\frac{\dfrac{\mathcal{E}}{Z_C}}{\dfrac{\mathcal{E}}{Z_L}}\right)^2 = \frac{Z_L^2}{Z_C^2}$$

Equate these expressions to obtain:

$$\frac{R_2}{R_1} = \frac{Z_L^2}{Z_C^2} = \frac{R_2^2 + X_L^2}{R_1^2 + X_C^2}$$

or

$$\frac{R_2}{R_1}\left(R_1^2 + X_C^2\right) = R_2^2 + X_L^2$$

Substitute for X_L, X_C, and the ratio of R_2 to R_1 to obtain:

$$2\left(4\,\Omega^2 + \frac{1}{\omega^2 C^2}\right) = 16\,\Omega^2 + \omega^2 L^2$$

or

$$8\,\Omega^2 + \frac{2}{\omega^2 C^2} = 16\,\Omega^2 + \omega^2 L^2$$

Combine like terms and clear fractions to obtain:

$$L^2C^2\omega^4 + \left(8\,\Omega^2\right)C^2\omega^2 - 2 = 0$$

Substitute numerical values to obtain:

$$\left(1.30\times10^{-13}\,\text{s}^4\right)\omega^4 + \left(7.20\times10^{-9}\,\text{s}^2\right)\omega^2 - 2 = 0$$

Use the ″solver″ capability of your calculator to solve for ω^2:

$$\omega^2 = 3.89\times10^6\,(\text{rad/s})^2$$

and

$$\omega = \boxed{1.97\times10^3\ \text{rad/s}}$$

(*b*) Express and evaluate Z_C, $|Z_C|$, and δ_C:

$$Z_C = R_1 - i\frac{1}{\omega C}$$

$$= 2\,\Omega - i\frac{1}{\left(1.97\times10^3\ \text{rad/s}\right)\left(30\,\mu\text{F}\right)}$$

$$= 2\,\Omega - i\left(16.9\,\Omega\right)$$

$$|Z_C| = \sqrt{(2\,\Omega)^2 + (16.9\,\Omega)^2} = \boxed{17.0\,\Omega}$$

and

$$\delta_C = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{-16.9\,\Omega}{2\,\Omega}\right)$$

$$= \boxed{-83.3^\circ}$$

Express and evaluate Z_L, $|Z_L|$, and δ_L:

$$Z_L = R_2 + i\omega L$$

$$= 4\,\Omega + i\left(1.97\times10^3\ \text{rad/s}\right)\left(12\,\text{mH}\right)$$

$$= 4\,\Omega + i\left(23.6\,\Omega\right)$$

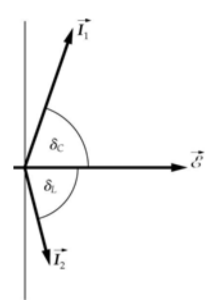
$$|Z_L| = \sqrt{(4\,\Omega)^2 + (23.6\,\Omega)^2} = \boxed{23.9\,\Omega}$$

and

$$\delta_L = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{23.9\,\Omega}{4\,\Omega}\right)$$

$$= \boxed{80.5^\circ}$$

(*c*) The applied voltage and the currents in the two branches are shown on the phasor diagram to the right.



(*d*) Express the impedance of the circuit and simplify to obtain:

$$Z = \frac{Z_L Z_C}{Z_L + Z_C} = \frac{[4\,\Omega + i(23.6\,\Omega)][2\,\Omega - i(16.9\,\Omega)]}{4\,\Omega + i(23.6\,\Omega) + 2\,\Omega - i(16.9\,\Omega)} = \frac{407\,\Omega^2 - i(20.4\,\Omega^2)}{6\,\Omega + i(6.70\,\Omega)}$$

Multiply *Z* by 1 in the form of the complex conjugate of $6\ \Omega + i(6.70\Omega)$ divided by itself and simplify to obtain:

$$Z = \left(\frac{407\,\Omega^2 - i(20.4\,\Omega^2)}{6\,\Omega + i(6.70\,\Omega)}\right) \times \left(\frac{6\,\Omega - i(6.70\,\Omega)}{6\,\Omega - i(6.70\,\Omega)}\right) = \frac{2.58\times10^3\,\Omega^3 - i(2.85\times10^3\,\Omega^3)}{80.9\,\Omega^2} = 31.9\,\Omega - i(35.2\,\Omega)$$

Find the magnitude of the circuit's impedance and the phase angle for the circuit:

$$|Z| = \sqrt{(31.9\,\Omega)^2 + (35.2\,\Omega)^2} = \boxed{47.5\,\Omega}$$

and

$$\delta = \tan^{-1}\frac{X_C}{R} = \tan^{-1}\left(\frac{-35.2\,\Omega}{31.9\,\Omega}\right) = \boxed{-47.8^\circ}$$

The Transformer

***105** •

Picture the Problem Let the subscript 1 denote the primary and the subscript 2 the secondary. We can use $V_2N_1 = V_1N_2$ and $N_1I_1 = N_2I_2$ to find the turn ratio and the primary current when the transformer connections are reversed.

(*a*) Relate the number of primary and secondary turns to the primary and secondary voltages:

$$V_2N_1 = V_1N_2 \qquad (1)$$

Solve for and evaluate the ratio N_2/N_1:

$$\frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{24\,\text{V}}{120\,\text{V}} = \boxed{\frac{1}{5}}$$

(*b*) Relate the current in the primary to the current in the secondary and

$$I_1 = \frac{N_2}{N_1} I_2$$

to the turns ratio:

Express the current in the primary winding in terms of the voltage across it and its impedance:

$$I_2 = \frac{V_2}{Z_2}$$

Substitute to obtain:

$$I_1 = \frac{N_2}{N_1}\frac{V_2}{Z_2}$$

Substitute numerical values and evaluate I_1:

$$I_1 = \left(\frac{5}{1}\right)\left(\frac{120\,\text{V}}{12\,\Omega}\right) = \boxed{50.0\,\text{A}}$$

106 •

Picture the Problem Let the subscript 1 denote the primary and the subscript 2 the secondary. We can decide whether the transformer is a step-up or step-down transformer by examining the ratio of the number of turns in the secondary to the number of terms in the primary. We can relate the open-circuit voltage in the secondary to the primary voltage and the turns ratio.

(*a*) $\boxed{\text{Because there are fewer turns in the secondary than in the primary it is a step-down transformer.}}$

(*b*) Relate the open-circuit voltage V_2 in the secondary to the voltage V_1 in the primary:

$$V_2 = \frac{N_2}{N_1}V_1$$

Substitute numerical values and evaluate V_2:

$$V_2 = \frac{8}{400}(120\,\text{V}) = \boxed{2.40\,\text{V}}$$

(*c*) Because there are no losses:

$$V_1 I_1 = V_2 I_2$$

Solve for and evaluate I_2:

$$I_2 = \frac{V_1}{V_2}I_1 = \frac{120\,\text{V}}{2.40\,\text{V}}(0.1\text{A}) = \boxed{5.00\,\text{A}}$$

107 •

Picture the Problem Let the subscript 1 denote the primary and the subscript 2 the secondary. We can use $I_1V_1 = I_2V_2$ to find the current in the primary and $V_2N_1 = V_1N_2$ to find the number of turns in the secondary.

(*a*) Because we have 100 percent efficiency:

$$I_1V_1 = I_2V_2$$

Solve for and evaluate I_1:

$$I_1 = I_2 \frac{V_2}{V_1} = (20\,\text{A}) \frac{9\,\text{V}}{120\,\text{V}} = \boxed{1.50\,\text{A}}$$

(*b*) Relate the number of primary and secondary turns to the primary and secondary voltages:

$$V_2 N_1 = V_1 N_2$$

Solve for the ratio N_2/N_1:

$$N_2 = \frac{V_2}{V_1} N_1$$

Substitute numerical values and evaluate N_2/N_1:

$$N_2 = \frac{9\,\text{V}}{120\,\text{V}}(250) = 18.8 \approx \boxed{19}$$

108 •

Picture the Problem We can relate the input and output voltages to the number of turns in the primary and secondary using $V_2 N_1 = V_1 N_2$.

Relate the output voltages V_2 to the input voltage V_1 and the number of turns in the primary N_1 and secondary N_2:

$$V_2 = \frac{N_2}{N_1} V_1$$

Solve for N_2:

$$N_2 = N_1 \frac{V_2}{V_1}$$

Evaluate N_2 for V_2 = 2.5 V:

$$N_2 = (500)\left(\frac{2.5\,\text{V}}{120\,\text{V}}\right) = \boxed{10.4}$$

Evaluate N_2 for V_2 = 7.5 V:

$$N_2 = (500)\left(\frac{7.5\,\text{V}}{120\,\text{V}}\right) = \boxed{31.3}$$

Evaluate N_2 for V_2 = 9 V:

$$N_2 = (500)\left(\frac{9\,\text{V}}{120\,\text{V}}\right) = \boxed{37.5}$$

109 •

Picture the Problem We can relate the input and output voltages to the number of turns in the primary and secondary using $V_2 N_1 = V_1 N_2$.

Relate the output voltages V_2 to the input voltage V_1 and the number of

$$V_2 = \frac{N_2}{N_1} V_1$$

turns in the primary N_1 and secondary N_2:

Solve for N_1:

$$N_1 = N_2 \frac{V_1}{V_2}$$

Substitute numerical values and evaluate N_1:

$$N_1 = (400)\left(\frac{2000\,\text{V}}{240\,\text{V}}\right) = \boxed{3333}$$

***110** ••

Picture the Problem Note: In a simple circuit maximum power transfer from source to load requires that the load resistance equals the internal resistance of the source. We can use Ohm's law and the relationship between the primary and secondary currents and the primary and secondary voltages and the turns ratio of the transformer to derive an expression for the turns ratio as a function of the effective resistance of the circuit and the resistance of the speaker(s).

Express the effective loudspeaker resistance at the primary of the transformer:

$$R_{\text{eff}} = \frac{V_1}{I_1}$$

Relate V_1 to V_2, N_1, and N_2:

$$V_1 = V_2 \frac{N_1}{N_2}$$

Express I_1 in terms of I_2, N_1, and N_2:

$$I_1 = I_2 \frac{N_2}{N_1}$$

Substitute to obtain:

$$R_{\text{eff}} = \frac{V_2 \dfrac{N_1}{N_2}}{I_2 \dfrac{N_2}{N_1}} = \left(\frac{V_2}{I_2}\right)\left(\frac{N_1}{N_2}\right)^2$$

Solve for N_1/N_2:

$$\frac{N_1}{N_2} = \sqrt{\frac{I_2 R_{\text{eff}}}{V_2}} = \sqrt{\frac{R_{\text{eff}}}{R_2}} \qquad (1)$$

Evaluate N_1/N_2 for $R_{\text{eff}} = R_{\text{int}}$:

$$\frac{N_1}{N_2} = \sqrt{\frac{2000\,\Omega}{8\,\Omega}} = \boxed{15.8}$$

Express the power delivered to the two speakers connected in parallel:

$$P_{\text{sp}} = I_1^2 R_{\text{eff}} \qquad (2)$$

Find the equivalent resistance R_{sp} of the two 8-Ω speakers in parallel:

$$\frac{1}{R_{sp}} = \frac{1}{8\,\Omega} + \frac{1}{8\,\Omega} = \frac{2}{8\,\Omega} = \frac{1}{4\,\Omega}$$

and

$$R_{sp} = 4\,\Omega$$

Solve equation (1) for R_{eff} to obtain:

$$R_{eff} = R_2\left(\frac{N_1}{N_2}\right)^2$$

Substitute numerical values and evaluate R_{eff}:

$$R_{eff} = (4\,\Omega)(15.8)^2 = 999\,\Omega$$

Find the current drawn from the source:

$$I_1 = \frac{V}{R_{tot}} = \frac{12\,\text{V}}{2000\,\Omega + 999\,\Omega} = 4.00\,\text{mA}$$

Substitute numerical values in equation (2) and evaluate the power delivered to the parallel speakers:

$$P_{sp} = (4\,\text{mA})^2(999\,\Omega) = \boxed{16.0\,\text{mW}}$$

111 ••

Picture the Problem We can substitute $I_2 = V_2/Z$ in Equation 29-62 to show that $I_1 = \varepsilon/[(N_1/N_2)^2 Z]$ and then use this result in $Z_{eff} = \varepsilon/I_1$ to show that $Z_{eff} = (N_1/N_2)^2 Z$.

From Equation 29-62 we have:

$$I_1 = \frac{N_2}{N_1} I_2$$

or, because $I_2 = V_2/Z$,

$$I_1 = \frac{N_2}{N_1}\frac{V_2}{Z}$$

From Equation 29-61 we have:

$$V_2 = \frac{N_2}{N_1} V_1 = \frac{N_2}{N_1}\varepsilon$$

Substitute to obtain:

$$I_1 = \frac{N_2}{N_1}\frac{\frac{N_2}{N_1}\varepsilon}{Z} = \left(\frac{N_2}{N_1}\right)^2\frac{\varepsilon}{Z}$$

$$= \boxed{\frac{\varepsilon}{(N_1/N_2)^2 Z}}$$

Express the effective impedance Z_{eff} of the speaker in terms of ε and I_1:

$$Z_{eff} = \frac{\varepsilon}{I_1}$$

Substitute for I_1 to obtain:

$$Z_{\text{eff}} = \frac{\mathcal{E}}{\dfrac{\mathcal{E}}{(N_1/N_2)^2 Z}} = \boxed{(N_1/N_2)^2 Z}$$

General Problems

112 •

Picture the Problem We can use $P_{\text{av}} = \mathcal{E}_{\text{rms}} I_{\text{rms}}$ to find the rms current and $I_{\max} = \sqrt{2} I_{\text{rms}}$ to find the maximum current drawn by the dryer.

(*a*) Express the average power delivered by the source in terms of $\mathcal{E}_{\text{rms}}$ and I_{rms}:

$$P_{\text{av}} = \mathcal{E}_{\text{rms}} I_{\text{rms}}$$

Solve for and evaluate I_{rms}:

$$I_{\text{rms}} = \frac{P_{\text{av}}}{\mathcal{E}_{\text{rms}}} = \frac{5\,\text{kW}}{240\,\text{V}} = \boxed{20.8\,\text{A}}$$

(*b*) Relate the maximum current $I_{\max}$ to the rms current I_{rms}:

$$I_{\max} = \sqrt{2} I_{\text{rms}} = \sqrt{2}(20.8\,\text{A}) = \boxed{29.5\,\text{A}}$$

(*c*) Proceed as in (*a*) and (*b*) to obtain:

$$I_{\text{rms}} = \boxed{41.6\,\text{A}} \text{ and } I_{\max} = \boxed{59.0\,\text{A}}$$

113 •

Picture the Problem We can use its definition to find the reactance of the capacitor at the given frequencies.

Express the reactance of a capacitor:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

(*a*) Evaluate X_C at $f = 60$ Hz:

$$X_C = \frac{1}{2\pi(60\,\text{Hz})(10\,\mu\text{F})} = \boxed{265\,\Omega}$$

(*b*) Evaluate X_C at $f = 6$ kHz:

$$X_C = \frac{1}{2\pi(6\,\text{kHz})(10\,\mu\text{F})} = \boxed{2.65\,\Omega}$$

(*a*) Evaluate X_C at $f = 6$ MHz:

$$X_C = \frac{1}{2\pi(6\,\text{MHz})(10\,\mu\text{F})} = \boxed{2.65\,\text{m}\Omega}$$

114 ••

Picture the Problem We can use its definition, $I_{\text{rms}} = \sqrt{\left(I^2\right)_{\text{av}}}$ to relate the rms current to the current carried by the resistor and find $\left(I^2\right)_{\text{av}}$ by integrating I^2.

(*a*) Express the rms current in terms of the $\left(I^2\right)_{\text{av}}$:

$$I_{\text{rms}} = \sqrt{\left(I^2\right)_{\text{av}}}$$

Evaluate I^2:

$$\begin{aligned} I^2 &= \left[(5\,\text{A})\sin 120\pi t + (7\,\text{A})\sin 240\pi t\right]^2 \\ &= \left(25\,\text{A}^2\right)\sin^2 120\pi t + \left(70\,\text{A}^2\right)\sin 120\pi t \sin 240\pi t + \left(49\,\text{A}^2\right)\sin^2 240\pi t \end{aligned}$$

Find $\left(I^2\right)_{\text{av}}$ by integrating I^2 from $t = 0$ to $t = T = 2\pi/\omega$ and dividing by T:

$$\begin{aligned} \left(I^2\right)_{\text{av}} = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} &\left\{\left(25\,\text{A}^2\right)\sin^2 120\pi t + \left(70\,\text{A}^2\right)\sin 120\pi t \sin 240\pi t \right. \\ &\left. + \left(49\,\text{A}^2\right)\sin^2 240\pi t\right\}dt \end{aligned}$$

Use the trigonometric identity $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ to simplify and evaluate the 1st and 3rd integrals and recognize that the middle term is of the form sin*x*sin2*x* to obtain:

$$\left(I^2\right)_{\text{av}} = 12.5\,\text{A}^2 + 0 + 24.5\,\text{A}^2 = 37.0\,\text{A}^2$$

Substitute to obtain:

$$I_{\text{rms}} = \sqrt{37.0\,\text{A}^2} = \boxed{6.08\,\text{A}}$$

(*b*) Relate the power dissipated in the resistor to its resistance and the rms current in it:

$$P = I_{\text{rms}}^2 R$$

Substitute numerical values and evaluate P:

$$P = (6.08\,\text{A})^2(12\,\Omega) = \boxed{444\,\text{W}}$$

(*c*) Express the rms voltage across the resistor in terms of R and I_{rms}:

$$V_{\text{rms}} = I_{\text{rms}} R$$

Substitute numerical values and evaluate V_{rms}:

$$V_{\text{rms}} = (6.08\,\text{A})(12\,\Omega) = \boxed{73.0\,\text{V}}$$

***115** ••

Picture the Problem The average of any quantity over a time interval ΔT is the integral of the quantity over the interval divided by ΔT. We can use this definition to find both the average of the voltage squared, $\left(V^2\right)_{\text{av}}$ and then use the definition of the rms voltage.

(*a*) From the definition of V_{rms} we have:

$$V_{\text{rms}} = \sqrt{\left(V_0^2\right)_{\text{av}}}$$

Noting that $-V_0^2 = V_0^2$, evaluate V_{rms}:

$$V_{\text{rms}} = \sqrt{V_0^2} = V_0 = \boxed{12.0\,\text{V}}$$

(*b*) Noting that the voltage during the second half of each cycle is now zero, express the voltage during the first half cycle of the time interval $\frac{1}{2}\Delta T$:

$$V = V_0$$

Express the square of the voltage during this half cycle:

$$V^2 = V_0^2$$

Calculate $\left(V^2\right)_{\text{av}}$ by integrating V^2 from $t = 0$ to $t = \frac{1}{2}\Delta T$ and dividing by ΔT:

$$\left(V^2\right)_{\text{av}} = \frac{V_0^2}{\Delta T}\int_0^{\frac{1}{2}\Delta T} dt = \frac{V_0^2}{\Delta T}\left[t\right]_0^{\frac{1}{2}\Delta T} = \tfrac{1}{2}V_0^2$$

Substitute to obtain:

$$V_{\text{rms}} = \sqrt{\tfrac{1}{2}V_0^2} = \frac{V_0}{\sqrt{2}} = \frac{12\,\text{V}}{\sqrt{2}} = \boxed{8.49\,\text{V}}$$

116 ••

Picture the Problem We can use the definitions of I_{rms} and V_{rms} to find the rms value of the waveform and the average power delivered by the pulse generator.

(*a*) From the definition of I_{rms} we have:

$$I_{\text{rms}} = \sqrt{\left(I^2\right)_{\text{av}}}$$

Evaluate $\left(I^2\right)_{\text{av}}$ over 1 s:

$$\left(I^2\right)_{\text{av}} = \frac{(0.1\,\text{s})I^2 + (0.9\,\text{s})(0)}{1\,\text{s}} = \frac{(0.1\,\text{s})I^2}{1\,\text{s}} = (0.1)(15\,\text{A})^2 = 22.5\,\text{A}^2$$

Substitute to obtain:

$$I_{\text{rms}} = \sqrt{22.5\,\text{A}^2} = \boxed{4.74\,\text{A}}$$

(*b*) Express the average power delivered by the pulse generator in terms of I_{rms} and V_{rms}:

$$P_{\text{av}} = I_{\text{rms}} V_{\text{rms}}$$

From the definition of V_{rms} we have:

$$V_{\text{rms}} = \sqrt{\left(V^2\right)_{\text{av}}}$$

Evaluate $\left(V^2\right)_{\text{av}}$ over 1 s:

$$\left(V^2\right)_{\text{av}} = \frac{(0.1\,\text{s})V^2 + (0.9\,\text{s})(0)}{1\,\text{s}} = \frac{(0.1\,\text{s})V^2}{1\,\text{s}} = (0.1)(100\,\text{V})^2 = 1000\,\text{V}^2$$

Evaluate V_{rms}:

$$V_{\text{rms}} = \sqrt{1000\,\text{V}^2} = 31.6\,\text{V}$$

Substitute numerical values and evaluate P_{av}:

$$P_{\text{av}} = (4.74\,\text{A})(31.6\,\text{V}) = \boxed{150\,\text{W}}$$

117 ••

Picture the Problem We can use the definition of capacitance to find the charge on each capacitor and the definition of current to find the steady-state current in the circuit. We can find the maximum and minimum energy stored in the capacitors using $U = \frac{1}{2}C_{\text{eq}}V^2$, where V is either the maximum or the minimum potential difference across the capacitors.

(*a*) Use the definition of capacitance to express the charge on each capacitor:

$Q_1 = C_1V_1$ and $Q_2 = C_2V_2$

or, because the capacitors are in parallel,

$Q_1 = C_1V$ and $Q_2 = C_2V$

where $V = \mathcal{E} + 24\,\text{V}$

Substitute to obtain:

$$Q_1 = C_1(\mathcal{E} + 24\,\text{V}) = (3\,\mu\text{F})[(20\,\text{V})\cos(120\pi t) + 24\,\text{V}] = \boxed{(60\,\mu\text{F})\cos(120\pi t) + 72\,\mu\text{F}}$$

and

$$Q_2 = C_2(\mathcal{E} + 24\,\text{V}) = (1.5\,\mu\text{F})[(20\,\text{V})\cos(120\pi t) + 24\,\text{V}] = \boxed{(30\,\mu\text{F})\cos(120\pi t) + 36\,\mu\text{F}}$$

(*b*) Express the steady-state current as the rate at which charge is being delivered to the capacitors:

$$I = \frac{dQ}{dt} = \frac{d}{dt}\left(Q_1 + Q_2\right)$$

Substitute for Q_1 and Q_2 and evaluate I:

$$\begin{aligned} I &= \frac{d}{dt}\big[(60\ \mu\text{F})\cos(120\pi t) + 72\ \mu\text{F} \\ &\quad + (30\ \mu\text{F})\cos(120\pi t) + 36\ \mu\text{F}\big] \\ &= -120\pi(60\ \mu\text{F})\sin(120\pi t) \\ &\quad -120\pi(30\ \mu\text{F})\sin(120\pi t) \\ &= \boxed{-(33.9\ \text{mA})\sin(120\pi t)} \end{aligned}$$

(*c*) Express U_{max} in terms of the maximum potential difference across the capacitors:

$$U_{max} = \tfrac{1}{2} C_{eq} V_{max}^2$$

Because V_{max} = 44 V and $C_{eq} = C_1 + C_2 = 3\ \mu\text{F} + 1.5\ \mu\text{F} = 4.5\ \mu\text{F}$:

$$U_{max} = \tfrac{1}{2}(4.5\ \mu\text{F})(44\ \text{V})^2 = \boxed{4.36\ \text{mJ}}$$

(*d*) Express U_{min} in terms of the minimum potential difference across the capacitors:

$$U_{min} = \tfrac{1}{2} C_{eq} V_{min}^2$$

The minimum energy stored in the capacitors occurs when $V_{min} = 24\ \text{V} - \varepsilon_{max} = 4\ \text{V}$:

$$U_{min} = \tfrac{1}{2}(4.5\ \mu\text{F})(4\ \text{V})^2 = \boxed{36.0\ \mu\text{J}}$$

118 ••

Picture the Problem The average of any quantity over a time interval ΔT is the integral of the quantity over the interval divided by ΔT. We can use this definition to find both the average current I_{av}, and the average of the current squared, $\left(I^2\right)_{av}$

From the definition of I_{av} and I_{rms} we have:

$$I_{av} = \frac{1}{\Delta T}\int_0^{\Delta T} I dt \ \text{ and } \ I_{rms} = \sqrt{\left(I^2\right)_{av}}$$

(*a*) Express the current during the first half cycle of time interval ΔT:

$$I = \frac{4}{\Delta T} t$$

where I is in A when t and T are in seconds.

Evaluate I_{av}:

$$I_{av} = \frac{1}{\Delta T}\int_0^{\Delta T}\frac{4}{\Delta T}t\,dt = \frac{4}{(\Delta T)^2}\int_0^{\Delta T}t\,dt$$
$$= \frac{4}{(\Delta T)^2}\left[\frac{t^2}{2}\right]_0^{\Delta T} = \boxed{2.00\text{ A}}$$

Express the square of the current during this half cycle:

$$I^2 = \frac{16}{(\Delta T)^2}t^2$$

Noting that the average value of the squared current is the same for each time interval ΔT, calculate $\left(I^2\right)_{av}$ by integrating I^2 from $t = 0$ to $t = \Delta T$ and dividing by ΔT:

$$\left(I^2\right)_{av} = \frac{1}{\Delta T}\int_0^{\Delta T}\frac{16}{(\Delta T)^2}t^2dt$$
$$= \frac{16}{(\Delta T)^3}\left[\frac{t^3}{3}\right]_0^{\Delta T} = \frac{16}{3}$$

Substitute in the expression for I_{rms} to obtain:

$$I_{rms} = \sqrt{\frac{16}{3}\text{A}^2} = \boxed{2.31\text{A}}$$

(*b*) Noting that the current during the second half of each cycle is zero, express the current during the first half cycle of the time interval $\frac{1}{2}\Delta T$:

$$I = 4\text{ A}$$

Evaluate I_{av}:

$$I_{av} = \frac{4\text{ A}}{\Delta T}\int_0^{\frac{1}{2}\Delta T}dt = \frac{4\text{ A}}{\Delta T}[t]_0^{\frac{1}{2}\Delta T} = \boxed{2.00\text{ A}}$$

Express the square of the current during this half cycle:

$$I^2 = 16\text{ A}^2$$

Calculate $\left(I^2\right)_{av}$ by integrating I^2 from $t = 0$ to $t = \frac{1}{2}\Delta T$ and dividing by ΔT:

$$\left(I^2\right)_{av} = \frac{16\text{ A}^2}{\Delta T}\int_0^{\frac{1}{2}\Delta T}dt$$
$$= \frac{16\text{ A}^2}{\Delta T}[t]_0^{\frac{1}{2}\Delta T} = 8\text{ A}^2$$

Substitute in the expression for I_{rms} to obtain:

$$I_{rms} = \sqrt{8\text{ A}^2} = \boxed{2.83\text{ A}}$$

119 ••

Picture the Problem We can apply Kirchhoff's loop rule to express the current in the circuit in terms of the emfs of the sources and the resistance of the resistor. We can then find I_{max} and I_{min} by considering the conditions under which the time-dependent factor in I will be a maximum or a minimum. The average of any quantity over a time interval ΔT is the integral of the quantity over the interval divided by ΔT. We can use this definition to find average of the current squared, $\left(I^2\right)_{av}$ and then I_{rms}.

Apply Kirchhoff's loop rule to obtain:

$$\mathcal{E}_1 + \mathcal{E}_2 - IR = 0$$

Solve for I:

$$I = \frac{\mathcal{E}_1 + \mathcal{E}_2}{R}$$

Substitute numerical values to obtain:

$$I = \frac{(20\,\text{V})\cos\left(2\pi\left(180\,\text{s}^{-1}\right)t\right) + 18\,\text{V}}{36\,\Omega} = 0.5\,\text{A} + (0.556\,\text{A})\cos\left(1131\,\text{s}^{-1}\right)t$$

Express the condition that must be satisfied if the current is to be a maximum:

$$\cos\left(1131\,\text{s}^{-1}\right)t = 1$$

Evaluate I_{max}:

$$I_{max} = 0.5\,\text{A} + 0.556\,\text{A} = \boxed{1.06\,\text{A}}$$

Express the condition that must be satisfied if the current is to be a minimum:

$$\cos\left(1131\,\text{s}^{-1}\right)t = -1$$

Evaluate I_{min}:

$$I_{min} = 0.5\,\text{A} - 0.556\,\text{A} = \boxed{-0.0560\,\text{A}}$$

Because the average value of $\cos\omega t = 0$:

$$I_{av} = \boxed{0.500\,\text{A}}$$

Express and evaluate the average current delivered by the source whose emf is $\mathcal{E}_2$:

$$I_2 = \frac{\mathcal{E}_2}{R} = \frac{18\,\text{V}}{36\,\Omega} = 0.5\,\text{A}$$

Because $I_1 = (0.556\,\text{A})\cos\left(1131\,\text{s}^{-1}\right)t$:

$$\left(I_1^2\right)_{av} = \frac{1}{5.56\,\text{ms}}\int_0^{5.56\,\text{ms}} (0.556\,\text{A})^2 \cos^2\left(1131\,\text{s}^{-1}\right)t\,dt$$

Use the trigonometric identity $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$ to obtain:

$$\left(I_1^2\right)_{\rm av} = \frac{0.309\,{\rm A}^2}{2(5.56\,{\rm ms})}\int_0^{5.56\,{\rm ms}}\left(1+\cos 2\left(1131\,{\rm s}^{-1}\right)t\right)dt$$
$$= \left(27.8\,{\rm A}^2/{\rm s}\right)\left[t + \frac{1}{2262\,{\rm s}^{-1}}\sin\left(2262\,{\rm s}^{-1}\right)t\right]_0^{5.56\,{\rm ms}}$$

Evaluate $\left(I_1^2\right)_{\rm av}$:

$$\left(I_1^2\right)_{\rm av} = \left(27.8\,{\rm A}^2/{\rm s}\right)\left[5.56\,{\rm ms} + \frac{1}{2262\,{\rm s}^{-1}}\sin\left(2262\,{\rm s}^{-1}\right)(5.56\,{\rm ms})\right] = 0.1543\,{\rm A}^2$$

Express $\left(I^2\right)_{\rm av}$:

$$\left(I^2\right)_{\rm av} = \left(I_1^2\right)_{\rm av} + \left(I_2^2\right)_{\rm av} = 0.1543\,{\rm A}^2 + (0.5\,{\rm A})^2 = 0.4043\,{\rm A}^2$$

Evaluate $I_{\rm rms}$:

$$I_{\rm rms} = \sqrt{\left(I^2\right)_{\rm av}} = \sqrt{0.4043\,{\rm A}^2} = \boxed{0.636\,{\rm A}}$$

***120** ••

Picture the Problem We can apply Kirchhoff's loop rule to obtain an expression for charge on the capacitor as a function of time. Differentiating this expression with respect to time will give us the current in the circuit. We can then find $I_{\rm max}$ and $I_{\rm min}$ by considering the conditions under which the time-dependent factor in I will be a maximum or a minimum. We can use the maximum value of the current to find $I_{\rm rms}$.

Apply Kirchhoff's loop rule to obtain:

$$\mathcal{E}_1 + \mathcal{E}_2 - \frac{q(t)}{C} = 0$$

Substitute numerical values and solve for $q(t)$:

$$q(t) = (2\,\mu{\rm F})(20\,{\rm V})\cos\left(1131\,{\rm s}^{-1}\right)t + (2\,\mu{\rm F})(18\,{\rm V}) = (40\,\mu{\rm C})\cos\left(1131\,{\rm s}^{-1}\right)t + 36\,\mu{\rm C}$$

Differentiate this expression with respect to t to obtain the current as a function of time:

$$I = \frac{dq}{dt} = \frac{d}{dt}\left[(40\,\mu{\rm C})\cos\left(1131\,{\rm s}^{-1}\right)t + 36\,\mu{\rm C}\right] = \boxed{-(45.2\,{\rm mA})\sin\left(1131\,{\rm s}^{-1}\right)t}$$

Express the condition that must be

$$\sin\left(1131\,{\rm s}^{-1}\right)t = 1$$

satisfied if the current is to be a minimum:

and

$$I_{min} = \boxed{-45.2\,\text{mA}}$$

Express the condition that must be satisfied if the current is to be a maximum:

$$\sin\left(1131\,\text{s}^{-1}\right)t = -1$$

and

$$I_{max} = \boxed{45.2\,\text{mA}}$$

Because the dc source sees the capacitor as an open circuit and the average value of the sine function over a period is zero:

$$I_{av} = \boxed{0}$$

Because the peak current is 45.2 mA:

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{45.2\,\text{mA}}{\sqrt{2}} = \boxed{32.0\,\text{mA}}$$

21 ••

Picture the Problem The inductance acts as a short circuit to the constant voltage source. The current is infinite at all times. Consequently, $I_{max} = I_{rms} = \infty$; there is no minimum current.