Chapter 30
Maxwell's Equations and Electromagnetic Waves

Conceptual Problems

***1 •**

(*a*) False. Maxwell's equations apply to both time-independent and time-dependent fields.

(*b*) True

(*c*) True

(*d*) True

(*e*) False. The magnitudes of the electric and magnetic field vectors are related according to $E = cB$.

(*f*) True

2 ••

Determine the Concept Two changes would be required. Gauss's law for magnetism would become $\oint_S B_n dA = \mu_0 q_m$ and Faraday's law would become $\oint_C \vec{E}\cdot d\vec{\ell} = -\frac{d}{dt}\int_S B_n dA - \frac{I_m}{\epsilon_0}$, where I_m is the current associated with the motion of the magnetic poles.

3 •

Determine the Concept X rays have greater frequencies whereas light waves have longer wavelengths (see Table 30-1).

***4 •**

Determine the Concept The frequencies of ultraviolet radiation are greater than those of infrared radiation (see Table 30-1).

5 •

Determine the Concept Consulting Table 30-1 we see that FM radio and televisions waves have wavelengths of the order of a few meters.

6 •

Determine the Concept The dipole antenna detects the *electric* field, the loop antenna detects the *magnetic* field of the wave.

7 •

Determine the Concept The dipole antenna should be in the horizontal plane and normal to the line from the transmitter to the receiver.

***8** •

Determine the Concept A red plastic filter absorbs all the light incident on it except for the red light and a green plastic filter absorbs all the light incident on it except for the green light. If the red beam is incident on a red filter it will pass through, whereas, if it is incident on the green filter it will be absorbed. Because the green filter absorbs more energy than does the red filter, the laser beam will exert a greater force on the green filter.

Estimation and Approximation

9 ••

Picture the Problem We'll assume that the plastic bead has the same density as water. Applying a condition for translational equilibrium to the bead will allow us to relate the gravitational force acting on it to the force exerted by the laser beam. Because the force exerted by the laser beam is related to the radiation pressure and the radiation pressure to the intensity of the beam, we'll be able to find the beam's intensity. Knowing the beam's intensity, we find the total power needed to lift the bead.

Apply $\sum F_y = 0$ to the bead:

$$F_{\text{by laser beam}} - mg = 0$$

Relate the force exerted by the laser beam to the radiation pressure exerted by the beam:

$$F_{\text{by laser beam}} = P_r A = \frac{1}{4}\pi d^2 P_r$$

Substitute to obtain:

$$\frac{1}{4}\pi d^2 P_r - mg = 0$$

The radiation pressure P_r is the quotient of the intensity I and the speed of light c:

$$P_r = \frac{I}{c}$$

Substitute for P_r to obtain:

$$\frac{1}{4}\frac{\pi d^2 I}{c} - mg = 0 \qquad (1)$$

Express the mass of the bead:

$$m = \rho V = \frac{1}{6}\pi\rho d^3$$

Substitute for m in equation (1) to obtain:

$$\frac{1}{4}\frac{\pi d^2 I}{c} - \frac{1}{6}\pi\rho d^3 g = 0$$

Solve for I:

$$I = \frac{2}{3}c\rho\, dg$$

Substitute numerical values and evaluate I:

$$I = \frac{2}{3}\left(3\times10^8\text{ m/s}\right)\left(10^3\text{ kg/m}^3\right)\left(15\,\mu m\right)\left(9.81\text{m/s}^2\right) = \boxed{2.94\times10^7\text{ W/m}^2}$$

The power needed is the product of the beam intensity and the cross-sectional area of the bead:

$$P = IA_{\text{bead}} = \frac{1}{4}\pi d^2 I$$

Substitute numerical values and evaluate P:

$$P = \frac{1}{4}\pi\left(15\,\mu\text{m}\right)^2\left(2.94\times10^7\text{ W/m}^2\right) = \boxed{5.20\,\text{mW}}$$

10 •••

Picture the Problem The net force acting on the spacecraft is the difference between the repulsive force due to radiation pressure and the attractive gravitational force. We can apply Newton's 2nd law to the spacecraft and solve the resulting equation for the acceleration of the spacecraft. Because the acceleration turns out to be a function of r, we'll need to integrate a to find v^2. We'll assume that the sail absorbs all of the radiation incident on it.

Apply Newton's 2nd law to the spacecraft (including sail) to obtain:

$$F_r - F_g = ma$$

Solve for a:

$$a = \frac{F_r - F_g}{m}$$

Assuming that the sail absorbs all of the incident solar radiation:

$$F_r = P_r A = \frac{IA}{c}$$

where A is the area of the sail.

Because $I = \frac{P_s}{4\pi r^2}$:

$$F_r = \frac{P_s A}{4\pi r^2 c}$$

Substitute for F_r and F_g to obtain:

$$a = \frac{\frac{P_s A}{4\pi r^2 c} - \frac{GM_s m}{r^2}}{m} = \frac{P_s A}{4\pi r^2 mc} - \frac{GM_s}{r^2}$$

$$= \frac{\frac{P_s A}{4\pi c} - GM_s m}{mr^2}$$

Neglecting the gravitational term:

$$a = \boxed{\frac{P_s A}{4\pi r^2 mc}}$$

(b) Because a is a function of r, the velocity must be found by integration. Note that:

$$a = \frac{dv}{dt} = \frac{dv}{dr}\frac{dr}{dt} = v\frac{dv}{dr} \Rightarrow vdv = adr$$

Substitute for a and integrate v' from v_0 to v and r' from r_0 to r:

$$\int_{v_0}^{v} v'dv' = \tfrac{1}{2}\left(v^2 - v_0^2\right) = \left(\frac{\frac{P_s A}{4\pi c} - GM_s m}{m}\right)\int_{r_0}^{r}\frac{dr'}{r'^2} = \left(\frac{\frac{P_s A}{4\pi c} - GM_s m}{m}\right)\left(\frac{1}{r_0} - \frac{1}{r}\right)$$

Solve for v^2 to obtain:

$$v^2 = v_0^2 + 2\left(\frac{\frac{P_s A}{4\pi c} - GM_s m}{m}\right)\left(\frac{1}{r_0} - \frac{1}{r}\right)$$

Ignore the gravitational term to obtain:

$$v^2 = \boxed{v_0^2 + \left(\frac{P_s A}{2\pi mc}\right)\left(\frac{1}{r_0} - \frac{1}{r}\right)}$$

(c) This scheme is not likely to work effectively. For any reasonable mass, the surface mass density of the sail would have to be extremely small and the sail would have to be huge. Additionally, unless struts are built into the sail, it would collapse during the acceleration of the spacecraft.

11 ••

Picture the Problem We can use $I = E_{rms}B_{rms}/\mu_0$ and $B_{rms} = E_{rms}/c$ to express E_{rms} in terms of I. We can then use $B_{rms} = E_{rms}/c$ to find B_{rms}. The average power output of the sun is given by $P_{av} = 4\pi R^2 I$ where R is the earth-sun distance. The intensity and the radiation pressure at the surface of the sun can be found from the definitions of these physical quantities.

(*a*) Express the intensity I of the radiation as a function of its average power and the distance r from the station:

$$I = \frac{E_{rms}B_{rms}}{\mu_0} = \frac{E_{rms}^2}{c\mu_0}$$

Solve for E_{rms}:

$$E_{rms} = \sqrt{c\mu_0 I}$$

Substitute numerical values and evaluate E_{rms}:

$$E_{rms} = \sqrt{(3\times10^8\text{ m/s})(4\pi\times10^{-7}\text{ N/A}^2)(1.37\text{ kW/m}^2)} = \boxed{719\text{ V/m}}$$

Use $B_{rms} = E_{rms}/c$ to evaluate B_{rms}:

$$B_{rms} = \frac{719\text{ V/m}}{3\times10^8\text{ m/s}} = \boxed{2.40\ \mu\text{T}}$$

(*b*) Express the average power output of the sun in terms of the solar constant:

$$P_{av} = 4\pi R^2 I$$

where R is the earth-sun distance.

Substitute numerical values and evaluate P_{av}:

$$P_{av} = 4\pi(1.5\times10^{11}\text{ m})^2(1.37\text{ kW/m}^2) = \boxed{3.87\times10^{26}\text{ W}}$$

(*c*) Express the intensity at the surface of the sun in terms of the sun's average power output and radius r:

$$I = \frac{P_{av}}{4\pi r^2}$$

Substitute numerical values and evaluate I at the surface of the sun:

$$I = \frac{3.87\times10^{26}\text{ W}}{4\pi(6.96\times10^8\text{ m})^2} = \boxed{6.36\times10^7\text{ W/m}^2}$$

Express the radiation pressure in terms of the intensity:

$$P_r = \frac{I}{c}$$

Substitute numerical values and evaluate P_r:

$$P_r = \frac{6.36\times10^7\text{ W/m}^2}{3\times10^8\text{ m/s}} = \boxed{0.212\text{ Pa}}$$

***12 ••**

Picture the Problem We can find the radiation pressure force from the definition of pressure and the relationship between the radiation pressure and the intensity of the radiation from the sun. We can use Newton's law of gravitation to find the gravitational force the sun exerts on the earth.

The radiation pressure exerted on the earth is given by:

$$P_r = \frac{F_r}{A} \Rightarrow F_r = P_r A$$

where A is the cross-sectional area of the earth.

Express the radiation pressure in terms of the intensity of the radiation I from the sun:

$$P_r = \frac{I}{c}$$

Substituting for P_r and A yields:

$$F_r = \frac{I\pi R^2}{c}$$

Substitute numerical values and evaluate F_r:

$$F_r = \frac{\pi\left(1370\text{ W/m}^2\right)\left(6370\text{ km}\right)^2}{3\times10^8\text{ m/s}} = \boxed{5.82\times10^8\text{ N}}$$

The gravitational force exerted on the earth by the sun is given by:

$$F = \frac{Gm_{\text{sun}}m_{\text{earth}}}{r^2}$$

where r is the radius of the earth's orbit.

Substitute numerical values and evaluate F:

$$F = \frac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\text{ kg}\right)\left(5.98\times10^{24}\text{ kg}\right)}{\left(1.5\times10^{11}\text{ m}\right)^2} = 3.53\times10^{22}\text{ N}$$

Express the ratio of the force due radiation pressure F_r to the gravitational force F:

$$\frac{F_r}{F} = \frac{5.82\times10^8\text{ N}}{3.53\times10^{22}\text{ N}} = 1.65\times10^{-14}$$

$$\boxed{\text{The gravitational force is greater by a factor of approximately } 10^{14}.}$$

***13 ••**

Picture the Problem We can find the radiation pressure force from the definition of pressure and the relationship between the radiation pressure and the intensity of the radiation from the sun. We can use Newton's law of gravitation to find the gravitational force the sun exerts on Mars.

The radiation pressure exerted on Mars is given by:

$$P_r = \frac{F_r}{A} \Rightarrow F_r = P_r A$$

where A is the cross-sectional area of Mars.

Express the radiation pressure on Mars in terms of the intensity of the radiation I_{Mars} from the sun:

$$P_r = \frac{I_{Mars}}{c}$$

Substituting for P_r and A yields:

$$F_r = \frac{I_{Mars}\pi R_{Mars}^2}{c}$$

Express the ratio of the solar constant at the earth I_{earth} to the solar constant I_{Mars} at Mars:

$$\frac{I_{Mars}}{I_{earth}} = \left(\frac{r_{earth}}{r_{Mars}}\right)^2 \Rightarrow I_{Mars} = I_{earth}\left(\frac{r_{earth}}{r_{Mars}}\right)^2$$

Substitute for I_{Mars} to obtain:

$$F_r = \frac{I_{earth}\pi R_{Mars}^2}{c}\left(\frac{r_{earth}}{r_{Mars}}\right)^2$$

Substitute numerical values and evaluate F_r:

$$F_r = \frac{\pi\left(1370\text{ W/m}^2\right)\left(3395\text{ km}\right)^2}{3\times10^8\text{ m/s}}\left(\frac{1.50\times10^{11}\text{ m}}{2.29\times10^{11}\text{ m}}\right)^2 = \boxed{7.09\times10^7\text{ N}}$$

The gravitational force exerted on Mars by the sun is given by:

$$F = \frac{Gm_{sun}m_{Mars}}{r^2} = \frac{Gm_{sun}\left(0.11m_{earth}\right)}{r^2}$$

where r is the radius of Mars' orbit.

Substitute numerical values and evaluate F:

$$F = \frac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\text{ kg}\right)\left(0.11\right)\left(5.98\times10^{24}\text{ kg}\right)}{\left(2.29\times10^{11}\text{ m}\right)^2} = 1.66\times10^{21}\text{ N}$$

Express the ratio of the force due radiation pressure F_r to the gravitational force F:

$$\frac{F_r}{F} = \frac{7.09\times10^7\text{ N}}{1.66\times10^{21}\text{ N}} = 4.27\times10^{-14}$$

Because the ratio of these forces is 1.65×10^{-14} for the earth and 4.27×10^{-14} for Mars, Mars has the larger ratio. The reason that the ratio is higher for Mars is that the dependence of the radiation pressure on the distance from the Sun is the same for both forces (r^{-2}), whereas the dependence on the radii of the planets is different. Radiation pressure varies as R^2, whereas the gravitational force varies as R^3 (assuming that the two planets have the same density, an assumption that is nearly true). Consequently, the ratio of the forces goes as $R^2/R^3 = R^{-1}$. Because Mars is smaller than earth, the ratio is larger.

***14 ••**

Picture the Problem We can use Newton's 2nd law to express the acceleration of an atom in terms of the net force acting on the atom and the relationship between radiation pressure and the intensity of the beam to find the net force. Once we know the acceleration of an atom, we can use the definition of acceleration to find the stopping time for a rubidium atom at room temperature.

(*a*) Apply $\sum F = ma$ to the atom to obtain:

$$F = ma$$

where F is the force exerted by the laser beam.

The radiation pressure P_r and intensity of the beam I are related according to:

$$P_r = \frac{F}{A} = \frac{I}{c}$$

Solve for F to obtain:

$$F = \frac{IA}{c} = \frac{I\lambda^2}{c}$$

Substitute for F in the expression of Newton's 2nd law to obtain:

$$\frac{I\lambda^2}{c} = ma$$

Solve for a:

$$a = \frac{I\lambda^2}{mc}$$

Substitute numerical values and evaluate a:

$$a = \frac{\left(10\,\text{W/m}^2\right)\left(780\,\text{nm}\right)^2}{\left(85\frac{\text{g}}{\text{mol}} \times \frac{1\,\text{mol}}{6.02\times10^{23}\,\text{particles}}\right)\left(3\times10^8\,\text{m/s}\right)} = \boxed{1.44\times10^5\,\text{m/s}^2}$$

(*b*) Using the definition of acceleration, express the stopping time Δt of the atom:

$$\Delta t = \frac{v_{\text{final}} - v_{\text{initial}}}{a}$$

Because $v_{\text{final}} \approx 0$:

$$\Delta t \approx \frac{-v_{\text{initial}}}{a}$$

Using the rms speed as the initial speed of an atom, relate v_{initial} to the temperature of the gas:

$$v_{\text{initial}} = v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

Substitute in the expression for the stopping time to obtain:

$$\Delta t = -\frac{1}{a}\sqrt{\frac{3kT}{m}}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = -\frac{1}{-1.44\times10^{5}\ \text{m/s}^2}\sqrt{\frac{3\left(1.38\times10^{-23}\ \text{J/K}\right)\left(300\,\text{K}\right)}{\left(85\frac{\text{g}}{\text{mol}}\times\frac{1\,\text{mol}}{6.02\times10^{23}\ \text{particles}}\right)}} = \boxed{2.06\,\text{ms}}$$

Maxwell's Displacement Current

15 •

Picture the Problem We can differentiate the expression for the electric field between the plates of a parallel-plate capacitor to find the rate of change of the electric field and the definitions of the conduction current and electric flux to compute I_d.

(*a*) Express the electric field between the plates of the parallel-plate capacitor:

$$E = \frac{Q}{\epsilon_0 A}$$

Differentiate this expression with respect to time to obtain an expression for the rate of change of the electric field:

$$\frac{dE}{dt} = \frac{d}{dt}\left[\frac{Q}{\epsilon_0 A}\right] = \frac{1}{\epsilon_0 A}\frac{dQ}{dt} = \frac{I}{\epsilon_0 A}$$

Substitute numerical values and evaluate dE/dt:

$$\frac{dE}{dt} = \frac{5\,\text{A}}{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\pi\left(0.023\,\text{m}\right)^2} = \boxed{3.40\times10^{14}\ \text{V/m}\cdot\text{s}}$$

(*b*) Express the displacement current I_d:

$$I_d = \epsilon_0 \frac{d\phi_e}{dt}$$

Substitute for the electric flux to obtain:

$$I_d = \epsilon_0 \frac{d}{dt}\left[EA\right] = \epsilon_0 A\frac{dE}{dt}$$

Substitute numerical values and evaluate I_d:

$$I_d = \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\pi\left(0.023\,\text{m}\right)^2\left(3.40\times10^{14}\ \text{V/m}\cdot\text{s}\right) = \boxed{5.00\,\text{A}}$$

16 •

Picture the Problem We can express the displacement current in terms of the electric flux and differentiate the resulting expression to obtain I_d in terms dE/dt.

Express the displacement current I_d:

$$I_d = \epsilon_0 \frac{d\phi_e}{dt}$$

Substitute for the electric flux to obtain:

$$I_d = \epsilon_0 \frac{d}{dt}[EA] = \epsilon_0 A\frac{dE}{dt}$$

Because $E = (0.05\,\text{N/C})\sin 2000t$:

$$I_d = \epsilon_0 A\frac{d}{dt}[(0.05\,\text{N/C})\sin 2000t] = (2000\,\text{s}^{-1})\epsilon_0 A(0.05\,\text{N/C})\cos 2000t$$

I_d will have its maximum value when cos 2000t = 1. Hence:

$$I_{d,max} = (2000\,\text{s}^{-1})\epsilon_0 A(0.05\,\text{N/C})$$

Substitute numerical values and evaluate $I_{d,max}$:

$$I_d = (2000\,\text{s}^{-1})(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(1\,\text{m}^2)(0.05\,\text{N/C}) = \boxed{8.85\times10^{-10}\,\text{A}}$$

17 ••

Picture the Problem We can use Ampere's law to a circular path of radius r between the plates and parallel to their surfaces to obtain an expression relating B to the current enclosed by the amperian loop. Assuming that the displacement current is uniformly distributed between the plates, we can relate the displacement current enclosed by the circular loop to the conduction current I.

Apply Ampere's law to a circular path of radius r between the plates and parallel to their surfaces to obtain:

$$\oint_C \vec{B}\cdot d\vec{\ell} = 2\pi rB = \mu_0 I_{\text{enclosed}} = \mu_0 I$$

Assuming that the displacement current is uniformly distributed:

$$\frac{I}{\pi r^2} = \frac{I_d}{\pi R^2} \Rightarrow I = \frac{r^2}{R^2}I_d$$

where R is the radius of the circular plates.

Substitute to obtain:

$$2\pi rB = \frac{\mu_0 r^2}{R^2}I_d$$

Solve for B:

$$B = \frac{\mu_0 r}{2\pi R^2} I_{\text{d}}$$

Substitute numerical values and evaluate B:

$$B(r) = \frac{(4\pi \times 10^{-7}\ \text{N/A}^2)(5\,\text{A})}{2\pi (0.023\,\text{m})^2} r = \boxed{(1.89\times 10^{-3}\ \text{T/m})r}$$

18 ••

Picture the Problem We can use the definitions of the displacement current and electric flux, together with the expression for the capacitance of an air-core-parallel-plate capacitor to show that $I_{\text{d}} = C\,dV/dt$.

(*a*) Use its definition to express the displacement current I_{d}:

$$I_{\text{d}} = \epsilon_0 \frac{d\phi_{\text{e}}}{dt}$$

Substitute for the electric flux to obtain:

$$I_{\text{d}} = \epsilon_0 \frac{d}{dt}[EA] = \epsilon_0 A \frac{dE}{dt}$$

Because $E = V/d$:

$$I_{\text{d}} = \epsilon_0 A \frac{d}{dt}\left[\frac{V}{d}\right] = \frac{\epsilon_0 A}{d}\frac{dV}{dt}$$

The capacitance of an air-core-parallel-plate capacitor whose plates have area A and that are separated by a distance d is given by:

$$C = \frac{\epsilon_0 A}{d}$$

Substitute to obtain:

$$I_{\text{d}} = \boxed{C\frac{dV}{dt}}$$

(*b*) Substitute in the expression derived in (*a*) to obtain:

$$I_{\text{d}} = (5\,\text{nF})\frac{d}{dt}[(3\,\text{V})\cos 500\pi t] = -(5\,\text{nF})(3\,\text{V})(500\pi\,\text{s}^{-1})\sin 500\pi t = \boxed{-(23.6\,\mu\text{A})\sin 500\pi t}$$

***19** ••

Picture the Problem We can use the conservation of charge to find I_{d}, the definitions of the displacement current and electric flux to find dE/dt, and Ampere's law to evaluate $\vec{B}\cdot d\vec{\ell}$ around the given path.

(*a*) From conservation of charge we know that:

$$I_d = I = \boxed{10.0\text{ A}}$$

(*b*) Express the displacement current I_d:

$$I_d = \epsilon_0 \frac{d\phi_e}{dt} = \epsilon_0 \frac{d}{dt}[EA] = \epsilon_0 A \frac{dE}{dt}$$

Substitute for *dE*/*dt*:

$$\frac{dE}{dt} = \frac{I_d}{\epsilon_0 A}$$

Substitute numerical values and evaluate *dE*/*dt*:

$$\frac{dE}{dt} = \frac{10\text{ A}}{(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(0.5\text{ m}^2)} = \boxed{2.26\times10^{12}\ \frac{\text{V}}{\text{m}\cdot\text{s}}}$$

(*c*) Apply Ampere's law to a circular path of radius *r* between the plates and parallel to their surfaces to obtain:

$$\oint_C \vec{B}\cdot d\vec{\ell} = \mu_0 I_{\text{enclosed}}$$

Assuming that the displacement current is uniformly distributed:

$$\frac{I_{\text{enclosed}}}{\pi r^2} = \frac{I_d}{A} \Rightarrow I_{\text{enclosed}} = \frac{\pi r^2}{A} I_d$$

where *R* is the radius of the circular plates.

Substitute for I_{enclosed} to obtain:

$$\oint_C \vec{B}\cdot d\vec{\ell} = \frac{\mu_0 \pi r^2}{A} I_d$$

Substitute numerical values and evaluate $\oint_C \vec{B}\cdot d\vec{\ell}$:

$$\oint_C \vec{B}\cdot d\vec{\ell} = \frac{(4\pi\times10^{-7}\text{ N/A}^2)\pi(0.1\text{ m})^2(10\text{ A})}{0.5\text{ m}^2} = \boxed{7.90\times10^{-7}\text{ T}\cdot\text{m}}$$

20 •••

Picture the Problem If $Q = Q_0 e^{-t/\tau}$ is the charge on the capacitor plates, then the conduction current *I* = *dQ*/*dt*. We can use $I_d = \epsilon_0 \frac{d\phi_e}{dt}$ to find the displacement current and $I_b = \frac{dQ_b}{dt}$ to find the current due to the rate of change of the bound charges. The total current is the sum of *I*, I_d, and I_b.

(*a*) The conduction current is given by:

$$I = \frac{dQ}{dt}$$

The charge on the capacitor varies with time according to:

$$Q = Q_0 e^{-t/\tau}, \text{ where } \tau = RC$$

Substitute for Q to obtain:

$$I = \frac{d}{dt}\left[Q_0 e^{-t/\tau}\right] = \boxed{\frac{Q_0}{\tau} e^{-t/\tau}}$$

This current is in the direction of the electric field, which is from the positive plate to the negative plate. By choosing the positive sign for this current we define this to be the positive direction.

(*b*) The displacement current is given by:

$$I_\text{d} = \in_0 \frac{d\phi_\text{e}}{dt} = \in_0 \frac{d}{dt}[EA] = \in_0 A\frac{dE}{dt}$$

Relate the electric field E to the potential difference V between the plates and the separation of the plates d:

$$E = \frac{V}{d}$$

Substitute to obtain:

$$I_\text{d} = \in_0 A\frac{d}{dt}\left[\frac{V}{d}\right] = \frac{\in_0 A}{d}\frac{dV}{dt}$$

or, because $C = \frac{\kappa \in_0 A}{d}$,

$$I_\text{d} = \frac{C}{\kappa}\frac{dV}{dt}$$

V varies with time according to:

$$V = V_0 e^{-t/\tau} = \frac{Q_0}{C} e^{-t/\tau}$$

Substituting in the expression for I_d yields:

$$I_\text{d} = \frac{C}{\kappa}\frac{d}{dt}\left[\frac{Q_0}{C} e^{-t/\tau}\right] = -\frac{Q_0}{\kappa\tau} e^{-t/\tau}$$

$$= \boxed{-\frac{1}{\kappa} I}$$

(*c*) As the voltage across the dielectric decreases the magnitude of the bound charges also decreases. The current I_b due to the flow of these bound charges though a

$I_\text{b} = \frac{dQ_\text{b}}{dt}$ where Q_b is the bound charge on the surface of the dielectric next to the plate with charge Q.

stationary surface is given by:

It follows that Q and Q_b are opposite in sign and are related by Equation 24-27:

$$Q_b = -\left(1 - \frac{1}{\kappa}\right)Q$$

Substitute in the expression for I_b and carry out the differentiation to obtain:

$$I_b = \frac{d}{dt}\left[-\left(1 - \frac{1}{\kappa}\right)Q\right] = -\left(1 - \frac{1}{\kappa}\right)\frac{dQ}{dt}$$
$$= \boxed{-\left(1 - \frac{1}{\kappa}\right)I}$$

(*d*) Add the currents found in (*a*), (*b*), and (*c*) to obtain:

$$I_{total} = I + I_d + I_b$$
$$= I - \frac{1}{\kappa}I - \left(1 - \frac{1}{\kappa}\right)I$$
$$= \boxed{0}$$

Remarks: In more sophisticated treatments of electrodynamics it is conventional to refer to the sum $I_d + I_b$ as the displacement current.

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Picture the Problem We can find the conduction current as a function of time using $I = V(t)/R$ and substituting for $V(t)$. We can use $I_d = \epsilon_0 \phi_e$ to obtain an expression for the displacement current I_d as a function of time. Finally, equating the conduction and displacement currents will yield an expression for the time at which they are equal.

(*a*) Express the conduction current in terms of the potential difference between the plates of the capacitor:

$$I = \frac{V(t)}{R} = \frac{AV(t)}{\rho d}$$

Substitute for $V(t)$ to obtain:

$$I = \boxed{\frac{(0.01\,\text{V/s})A}{\rho d}t}$$

(*b*) The displacement current is given by:

$$I_d = \epsilon_0 \frac{d}{dt}(EA) = \epsilon_0 \frac{d}{dt}\left(\frac{V}{d}A\right)$$
$$= \frac{\epsilon_0 A}{d}\frac{dV}{dt}$$

Substitute for V and simplify to obtain:

$$I_d = \frac{\epsilon_0 A}{d}\frac{d}{dt}\left[(0.01\,\text{V/s})t\right] = \boxed{\frac{(0.01\,\text{V/s})\epsilon_0 A}{d}}$$

(*c*) Set $I_d = I$ to obtain:

$$\frac{(0.01\,\text{V/s})\epsilon_0 A}{d} = \frac{A(0.01\,\text{V/s})}{\rho d}t$$

Solve for t:

$$t = \boxed{\epsilon_0 \rho}$$

22 ••

Picture the Problem We can use $I_d = \epsilon_0 \frac{d\phi_e}{dt}$ and the relationship between the voltage across the plates and the electric field between them to find the displacement current. The conduction current between the plates is given by $I = \frac{V}{R} = \frac{AV}{\rho d}$ where A is the area of the plates and d is their separation.

(*a*) The displacement current is given by:

$$I_d = \epsilon_0 \frac{d\phi_e}{dt} = \epsilon_0 \frac{d}{dt}[EA] = \epsilon_0 A\frac{dE}{dt}$$

Relate the electric field E to the potential difference V between the plates and the separation of the plates d:

$$E = \frac{V}{d}$$

Substitute to obtain:

$$I_d = \epsilon_0 A\frac{d}{dt}\left[\frac{V}{d}\right] = \frac{\epsilon_0 A}{d}\frac{dV}{dt}$$

V varies with time according to:

$$V = V_0 \cos\omega t$$

Substituting in the expression for I_d yields:

$$I_d = \frac{\epsilon_0 A}{d}\frac{d}{dt}\left[V_0 \cos\omega t\right] = -\frac{\epsilon_0 \pi r^2 V_0}{\omega d}\sin\omega t$$

Substitute numerical values and evaluate I_d:

$$I_d = -\frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\pi\left(20\ \text{cm}\right)^2\left(40\ \text{V}\right)}{\left(120\pi\ \text{rad/s}\right)\left(1\ \text{mm}\right)}\sin\left(120\pi\ \text{rad/s}\right)t$$
$$= \boxed{-\left(1.18\times10^{-10}\ \text{A}\right)\sin\left(120\pi\ \text{rad/s}\right)t}$$

(*b*) The conduction current between the plates is given by:

$$I = \frac{V}{R} = \frac{AV}{\rho d} = \frac{AV_0}{\rho d}\cos\omega t$$

Substitute numerical values and simplify to obtain:

$$I = \frac{\pi\left(0.2\ \text{m}\right)^2\left(40\ \text{V}\right)}{\left(10^4\ \Omega\cdot\text{m}\right)\left(10^{-3}\ \text{m}\right)}\cos\left(120\pi\ \text{rad/s}\right)t$$
$$= \boxed{\left(0.503\ \text{A}\right)\cos\left(120\pi\ \text{rad/s}\right)t}$$

***23 •••**

Picture the Problem We can follow the step-by-step instructions in the problem statement to show that Equation 30-4 gives the same result for *B* as that given in Part (*a*).

(*a*) Express the magnetic field at *P* using the expression for *B* due to a straight wire segment:

$$B_P = \frac{\mu_0}{4\pi}\frac{I}{R}\left(\sin\theta_1 + \sin\theta_2\right)$$

where

$$\sin\theta_1 = \sin\theta_2 = \frac{a}{\sqrt{R^2 + a^2}}$$

Substitute for $\sin\theta_1$ and $\sin\theta_2$ to obtain:

$$B_P = \frac{\mu_0}{4\pi}\frac{I}{R}\frac{2a}{\sqrt{R^2 + a^2}}$$
$$= \boxed{\frac{\mu_0 Ia}{2\pi R}\frac{1}{\sqrt{R^2 + a^2}}}$$

(*b*) Express the electric flux through the circular strip of radius *r* and width *dr* in the *yz* plane:

$$d\phi_e = E_x dA = E_x\left(2\pi r dr\right)$$

The electric field due to the dipole is:

$$E_x = \frac{2kQ}{r^2 + a^2}\cos\theta_1 = \frac{2kQa}{\left(r^2 + a^2\right)^{3/2}}$$

Substitute for E_x to obtain:

$$d\phi_{\text{e}} = E_x dA = \frac{2kQa}{\left(r^2+a^2\right)^{3/2}}\left(2\pi\, rdr\right) = \frac{2Qa}{4\pi\in_0\left(r^2+a^2\right)^{3/2}}\left(2\pi\, rdr\right) = \boxed{\frac{Qa}{\in_0\left(r^2+a^2\right)^{3/2}}rdr}$$

(c) Multiply both sides of the expression for ϕ_{e} by $\in_0$:

$$\in_0\, d\phi_{\text{e}} = \frac{Qa}{\left(r^2+a^2\right)^{3/2}}rdr$$

Integrate r from 0 to R to obtain:

$$\in_0\,\phi_{\text{e}} = Qa\int_0^R \frac{rdr}{\left(r^2+a^2\right)^{3/2}} = Qa\left(\frac{-1}{\sqrt{R^2+a^2}}+\frac{1}{a}\right) = \boxed{Q\left(1-\frac{a}{\sqrt{R^2+a^2}}\right)}$$

(d) The displacement current is defined to be:

$$I_{\text{d}} = \in_0\frac{d\phi_{\text{e}}}{dt} = \frac{d}{dt}\left[Q\left(1-\frac{a}{\sqrt{R^2+a^2}}\right)\right] = \left(1-\frac{a}{\sqrt{R^2+a^2}}\right)\frac{dQ}{dt} = -I\left(1-\frac{a}{\sqrt{R^2+a^2}}\right)$$

The total current is the sum of I and I_{d}:

$$I + I_{\text{d}} = I - I\left(1-\frac{a}{\sqrt{R^2+a^2}}\right) = \boxed{I\frac{a}{\sqrt{R^2+a^2}}}$$

(e) Apply Equation 30-4 (the generalized form of Ampere's law) to obtain:

$$\oint_C \vec{\boldsymbol{B}}\cdot d\vec{\ell} = 2\pi RB = \mu_0\left(I + I_{\text{d}}\right)$$

Solve for B:

$$B = \frac{\mu_0}{2\pi R}\left(I + I_{\text{d}}\right)$$

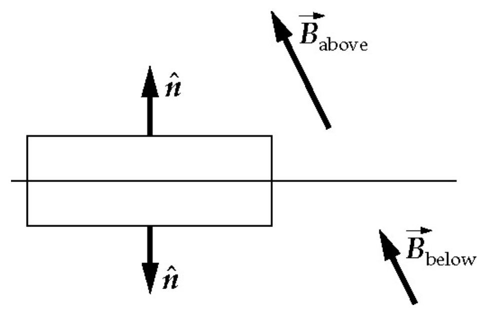
Substitute for $I + I_d$ from (*d*) to obtain:

$$B = \frac{\mu_0}{2\pi R}\left(I\frac{a}{\sqrt{R^2+a^2}}\right) = \boxed{\frac{\mu_0 Ia}{2\pi R}\frac{1}{\sqrt{R^2+a^2}}}$$

Maxwell's Equations and the Electromagnetic Spectrum

24 ••

Picture the Problem The figure shows the end view of a pillbox surrounding a small area *dA* of the surface. The normal components of the magnetic field, $\vec{B}_{\text{n, top}}$ and $\vec{B}_{\text{n, bottom}}$, are shown with different magnitudes. When performing the surface integral the normal to the surface is outward, as shown in the figure.



Apply Gauss's law for magnetism to the pillbox to obtain:

$$\oint_S \vec{B}\cdot\hat{n}dA = \int_{\text{bottom surface}}\vec{B}\cdot\hat{n}dA + \int_{\text{lateral surface}}\vec{B}\cdot\hat{n}dA + \int_{\text{top surface}}\vec{B}\cdot\hat{n}dA = 0$$

Because the horizontal component of $\vec{B}$ is zero, $\int_{\text{lateral surface}}\vec{B}\cdot\hat{n}dA = 0$, and:

$$\oint_S \vec{B}\cdot\hat{n}dA = \int_{\text{bottom surface}}\vec{B}\cdot\hat{n}dA + \int_{\text{top surface}}\vec{B}\cdot\hat{n}dA = 0 \qquad (1)$$

Because $\vec{B}$ and $\hat{n}$ are oppositely directed at the bottom surface:

$$\int_{\text{bottom surface}}\vec{B}_{\text{below}}\cdot\hat{n}dA = -B_{\text{n,below}}A$$

Because $\vec{B}$ and $\hat{n}$ are parallel at the top surface:

$$\int_{\text{top surface}}\vec{B}_{\text{below}}\cdot\hat{n}dA = B_{\text{n,above}}A$$

Substitute in equation (1) to obtain:

$$-B_{\text{n, below}}A + B_{\text{n, above}}A = 0$$

Solve for $B_{\text{n,top}}$:

$\boxed{B_{\text{n, above}} = B_{\text{n, below}}}$; i.e., the normal component of $\vec{B}$ is continuous across the surface.

***25 •**

Picture the Problem We can use $c = f\lambda$ to find the wavelengths corresponding to the given frequencies.

Solve $c = f\lambda$ for λ:

$$\lambda = \frac{c}{f}$$

(*a*) For $f = 1000$ kHz:

$$\lambda = \frac{3\times10^8\ \text{m/s}}{1000\times10^3\ \text{s}^{-1}} = \boxed{300\ \text{m}}$$

(*b*) For $f = 100$ MHz:

$$\lambda = \frac{3\times10^8\ \text{m/s}}{100\times10^6\ \text{s}^{-1}} = \boxed{3.00\ \text{m}}$$

***26 •**

Picture the Problem We can use $c = f\lambda$ to find the frequency corresponding to the given wavelength.

Solve $c = f\lambda$ for f:

$$f = \frac{c}{\lambda}$$

Substitute numerical values and evaluate f:

$$f = \frac{3\times10^8\ \text{m/s}}{3\times10^{-2}\ \text{m}} = 10^{10}\ \text{Hz} = \boxed{10.0\ \text{GHz}}$$

27 •

Picture the Problem We can use $c = f\lambda$ to find the frequency corresponding to the given wavelength.

Solve $c = f\lambda$ for f:

$$f = \frac{c}{\lambda}$$

Substitute numerical values and evaluate f:

$$f = \frac{3\times10^8\ \text{m/s}}{0.1\times10^{-9}\ \text{m}} = \boxed{3.00\times10^{18}\ \text{Hz}}$$

Electric Dipole Radiation

28 ••

Picture the Problem We can use the intensity I_1 at a distance $r = 10$ m and at an angle $\theta = 90°$ to find the proportionality constant in the expression for the intensity of radiation from an electric dipole and then use the resulting equation to find the intensity at the given distances and angles.

Express the intensity of radiation as a function of r and θ:

$$I(\theta,r)=\frac{C}{r^2}\sin^2\theta \qquad (1)$$

where C is a constant of proportionality.

Express I(90°,10 m):

$$I(90°,10\,\text{m})=I_1=\frac{C}{(10\,\text{m})^2}\sin^2 90°$$
$$=\frac{C}{100\,\text{m}^2}$$

Solve for C:

$$C=\left(100\,\text{m}^2\right)I_1$$

Substitute in equation (1) to obtain:

$$I(\theta,r)=\frac{\left(100\,\text{m}^2\right)I_1}{r^2}\sin^2\theta \qquad (2)$$

(*a*) Evaluate equation (2) for r = 30 m and θ = 90°:

$$I(90°,30\,\text{m})=\frac{\left(100\,\text{m}^2\right)I_1}{(30\,\text{m})^2}\sin^2 90°$$
$$=\boxed{\tfrac{1}{9}I_1}$$

(*b*) Evaluate equation (2) for r = 10 m and θ = 45°:

$$I(45°,10\,\text{m})=\frac{\left(100\,\text{m}^2\right)I_1}{(10\,\text{m})^2}\sin^2 45°$$
$$=\boxed{\tfrac{1}{2}I_1}$$

(*c*) Evaluate equation (2) for r = 20 m and θ = 30°:

$$I(30°,20\,\text{m})=\frac{\left(100\,\text{m}^2\right)I_1}{(20\,\text{m})^2}\sin^2 30°$$
$$=\boxed{\tfrac{1}{16}I_1}$$

29 ••

Picture the Problem We can use the intensity I_1 at a distance r = 10 m and at an angle θ = 90° to find the proportionality constant in the expression for the intensity of radiation from an electric dipole and then use the resulting equation to find the angle for a given intensity and distance and the distance corresponding to a given intensity and angle.

Express the intensity of radiation as a function of r and θ:

$$I(\theta,r)=\frac{C}{r^2}\sin^2\theta \qquad (1)$$

where C is a constant of proportionality.

Express $I(90°,10\text{ m})$:

$$I(90°,10\,\text{m}) = I_1 = \frac{C}{(10\,\text{m})^2}\sin^2 90° = \frac{C}{100\,\text{m}^2}$$

Solve for C:

$$C = (100\,\text{m}^2)I_1$$

Substitute in equation (1) to obtain:

$$I(\theta,r) = \frac{(100\,\text{m}^2)I_1}{r^2}\sin^2\theta \qquad (2)$$

(*a*) For $r = 5$ m and $I(\theta,r) = I_1$:

$$I_1 = \frac{(100\,\text{m}^2)I_1}{(5\,\text{m})^2}\sin^2\theta$$

or

$$\sin^2\theta = \tfrac{1}{4}$$

Solve for θ to obtain:

$$\theta = \sin^{-1}\tfrac{1}{2} = \boxed{30.0°}$$

(*b*) For $\theta = 45°$ and $I(\theta,r) = I_1$:

$$I_1 = \frac{(100\,\text{m}^2)I_1}{r^2}\sin^2 45°$$

or

$$r^2 = \tfrac{1}{2}(100\,\text{m}^2)$$

Solve for r to obtain:

$$r = \sqrt{\tfrac{1}{2}(100\,\text{m}^2)} = \boxed{7.07\,\text{m}}$$

30 ••

Picture the Problem We can use the intensity I at a distance $r = 4000$ m and at an angle $\theta = 90°$ to find the proportionality constant in the expression for the intensity of radiation from an electric dipole and then use the resulting equation to find the intensity at sea level and 1.5 km from the transmitter.

Express the intensity of radiation as a function of r and θ:

$$I(\theta,r) = \frac{C}{r^2}\sin^2\theta \qquad (1)$$

where C is a constant of proportionality.

Use the given data to obtain:

$$4\times10^{-12}\,\text{W/m}^2 = \frac{C}{(4\,\text{km})^2}\sin^2 90° = \frac{C}{(4\,\text{km})^2}$$

Solve for C:

$$C = (4\,\text{km})^2\left(4\times10^{-12}\ \text{W/m}^2\right) = 6.40\times10^{-5}\ \text{W}$$

Substitute in equation (1) to obtain:

$$I(\theta, r) = \frac{6.40\times10^{-5}\ \text{W}}{r^2}\sin^2\theta \qquad (2)$$

For a point at sea level and 1.5 km from the transmitter:

$$\theta = \tan^{-1}\frac{2\,\text{km}}{1.5\,\text{km}} = 53.1°$$

Evaluate I(53.1°,1.5 km):

$$I(53.1°, 1.5\,\text{km}) = \frac{6.40\times10^{-5}\ \text{W}}{(1.5\,\text{km})^2}\sin^2 53.1° = \boxed{18.2\,\text{pW/m}^2}$$

31 •••

Picture the Problem The intensity of radiation from an electric dipole is equal to $I_0(\sin^2\theta)/r^2$, where θ is the angle between the electric dipole moment and the position vector $\vec{\boldsymbol{r}}$. We can integrate the intensity to express the total power radiated by the antenna and use this result to evaluate I_0. Knowing I_0 we can find the intensity at a horizontal distance of 120 km directly in front of the station.

Express the intensity of the signal as a function of r and θ:

$$I(r, \theta) = I_0\frac{\sin^2\theta}{r^2}$$

At a horizontal distance of 120 km from the station and directly in front of it:

$$I(120\,\text{km}, 90°) = I_0\frac{\sin^2 90°}{(120\,\text{km})^2} = \frac{I_0}{(120\,\text{km})^2} \qquad (1)$$

From the definition of intensity we have:

$$dP = I\,dA$$

and

$$P_{\text{tot}} = \iint I(r,\theta)\,dA$$

where, in polar coordinates,

$$dA = r^2\sin\theta\,d\theta\,d\phi$$

Substitute for dA to obtain:

$$P_{\text{tot}} = \int_0^{2\pi}\int_0^{\pi} I(r,\theta)\,r^2\sin\theta\,d\theta\,d\phi$$

Substitute for $I(r,\theta)$:

$$P_{\text{tot}} = I_0 \int_0^{2\pi}\int_0^{\pi} \sin^3\theta\, d\theta\, d\phi$$

From integral tables we find that:

$$\int_0^{\pi} \sin^3\theta d\theta = -\tfrac{1}{3}\cos\theta\left(\sin^2\theta + 2\right)\Big]_0^{\pi} = \frac{4}{3}$$

Substitute and integrate with respect to ϕ to obtain:

$$P_{\text{tot}} = \frac{4}{3} I_0 \int_0^{2\pi} d\phi = \frac{4}{3} I_0 [\phi]_0^{2\pi} = \frac{8\pi}{3} I_0$$

Solve for I_0:

$$I_0 = \frac{3}{8\pi} P_{\text{tot}}$$

Substitute for P_{tot} and evaluate I_0:

$$I_0 = \frac{3}{8\pi}(500\,\text{kW}) = 59.7\,\text{kW}$$

Substitute for I_0 in equation (1) and evaluate $I(120\text{ km},90°)$:

$$I(120\,\text{km},90°) = \frac{59.7\,\text{kW}}{(120\,\text{km})^2} = \boxed{4.15\,\mu\text{W/m}^2}$$

Express the number of photons incident on an area A in time Δt:

$$\frac{N}{A\Delta t} = \frac{N}{(P/I)\Delta t} = \frac{NI}{P\Delta t} = \frac{NI}{E} = \frac{I}{E/N} = \frac{I}{hf}$$

Substitute numerical values and evaluate I/hf:

$$\frac{I}{hf} = \frac{4.15\,\mu\text{W/m}^2}{(6.63\times10^{-34}\,\text{J}\cdot\text{s})(1.20\,\text{MHz})} = 5.21\times10^{21}\,\frac{\text{photons}}{\text{m}^2\cdot\text{s}} = \boxed{5.22\times10^{17}\,\frac{\text{photons}}{\text{cm}^2\cdot\text{s}}}$$

***32 •••**

Picture the Problem The intensity of radiation from an electric dipole is given by $I_0(\sin^2\theta)/r^2$, where θ is the angle between the electric dipole moment and the position vector $\vec{r}$. We can integrate the intensity to express the total power radiated by the antenna and use this result to evaluate I_0. Knowing I_0 we can find the total power radiated by the station.

From the definition of intensity we

$$dP = IdA$$

have:

and

$$P_{\text{tot}} = \iint I(r,\theta)\,dA$$

where, in polar coordinates,

$$dA = r^2 \sin\theta\, d\theta\, d\phi$$

Substitute for dA to obtain:

$$P_{\text{tot}} = \int_0^{2\pi}\int_0^{\pi} I(r,\theta) r^2 \sin\theta\, d\theta\, d\phi$$

Express the intensity of the signal as a function of r and θ:

$$I(r,\theta) = I_0 \frac{\sin^2\theta}{r^2} \qquad (1)$$

Substitute for $I(r,\theta)$:

$$P_{\text{tot}} = I_0 \int_0^{2\pi}\int_0^{\pi} \sin^3\theta\, d\theta\, d\phi$$

From integral tables we find that:

$$\int_0^{\pi} \sin^3\theta d\theta = -\tfrac{1}{3}\cos\theta\left(\sin^2\theta + 2\right)\Big]_0^{\pi} = \frac{4}{3}$$

Substitute and integrate with respect to ϕ to obtain:

$$P_{\text{tot}} = \frac{4}{3} I_0 \int_0^{2\pi} d\phi = \frac{4}{3} I_0 [\phi]_0^{2\pi} = \frac{8\pi}{3} I_0$$

From equation (1) we have:

$$I_0 = \frac{I(r,\theta) r^2}{\sin^2\theta}$$

Substitute to obtain:

$$P_{\text{tot}} = \frac{8\pi}{3} \frac{I(r,\theta) r^2}{\sin^2\theta}$$

or, because $\theta = 90°$,

$$P_{\text{tot}} = \frac{8\pi}{3} I(r) r^2$$

Substitute numerical values and evaluate P_{tot}:

$$P_{\text{tot}} = \frac{8\pi}{3}\left(2\times10^{-13}\ \text{W/m}^2\right)(30\,\text{km})^2 = \boxed{1.51\,\text{mW}}$$

33 •••

Picture the Problem The intensity of radiation from the airport's vertical dipole antenna is given by $I_0(\sin^2\theta)/r^2$, where θ is the angle between the electric dipole moment and the position vector $\vec{r}$. We can integrate the intensity to express the total power radiated by the antenna and use this result to evaluate I_0. Knowing I_0 we can find the intensity of the signal at the plane's elevation and distance from the airport.

Express the intensity of the signal as a function of r and θ:

$$I(r,\theta) = I_0 \frac{\sin^2\theta}{r^2} \qquad (1)$$

From the definition of intensity we have:

$$dP = I\,dA$$

and

$$P_{\text{tot}} = \iint I(r,\theta)\,dA$$

where, in polar coordinates,

$$dA = r^2 \sin\theta\, d\theta\, d\phi$$

Substitute for dA to obtain:

$$P_{\text{tot}} = \int_0^{2\pi}\int_0^{\pi} I(r,\theta) r^2 \sin\theta\, d\theta\, d\phi$$

Substitute for $I(r,\theta)$:

$$P_{\text{tot}} = I_0 \int_0^{2\pi}\int_0^{\pi} \sin^3\theta\, d\theta\, d\phi$$

From integral tables we find that:

$$\int_0^{\pi} \sin^3\theta\, d\theta = -\tfrac{1}{3}\cos\theta\left(\sin^2\theta + 2\right)\Big]_0^{\pi} = \frac{4}{3}$$

Substitute and integrate with respect to ϕ to obtain:

$$P_{\text{tot}} = \frac{4}{3} I_0 \int_0^{2\pi} d\phi = \frac{4}{3} I_0 [\phi]_0^{2\pi} = \frac{8\pi}{3} I_0$$

Solve for I_0:

$$I_0 = \frac{3}{8\pi} P_{\text{tot}}$$

Substitute for I_0 in equation (1):

$$I(r,\theta) = \frac{3P_{\text{tot}}}{8\pi} \frac{\sin^2\theta}{r^2}$$

At the elevation of the plane:

$$\theta = \tan^{-1}\left(\frac{2500\,\text{m}}{4000\,\text{m}}\right) = 32.0°$$

and

$$r = \sqrt{(2500\,\text{m})^2 + (4000\,\text{m})^2} = 4717\,\text{m}$$

Substitute numerical values and evaluate $I(4717\text{ m}, 32°)$:

$$I(4717\,\text{m}, 32°) = \frac{3(100\,\text{W})}{8\pi} \frac{\sin^2 32°}{(4717\,\text{m})^2} = \boxed{0.151\,\mu\text{W/m}^2}$$

Energy and Momentum in an Electromagnetic Wave

34 •

Picture the Problem We can use *P*r = *I*/*c* to find the radiation pressure. The intensity of the electromagnetic wave is related to the rms values of its electric and magnetic fields according to $I = E_{rms}B_{rms}/\mu_0$, where $B_{rms} = E_{rms}/c$.

(*a*) Express the radiation pressure in terms of the intensity of the wave:

$$P_r = \frac{I}{c}$$

Substitute numerical values and evaluate P_r:

$$P_r = \frac{100\,\text{W/m}^2}{3\times10^8\ \text{m/s}} = \boxed{0.333\,\mu\text{Pa}}$$

(*b*) Relate the intensity of the electromagnetic wave to E_{rms} and B_{rms}:

$$I = \frac{E_{rms}B_{rms}}{\mu_0}$$

or, because $B_{rms} = E_{rms}/c$,

$$I = \frac{E_{rms}\,E_{rms}/c}{\mu_0} = \frac{E_{rms}^2}{\mu_0 c}$$

Solve for E_{rms}:

$$E_{rms} = \sqrt{\mu_0 cI}$$

Substitute numerical values and evaluate E_{rms}:

$$E_{rms} = \sqrt{\left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(3\times10^8\ \text{m/s}\right)\left(100\,\text{W/m}^2\right)} = \boxed{194\,\text{V/m}}$$

(*c*) Express B_{rms} in terms of E_{rms}:

$$B_{rms} = \frac{E_{rms}}{c}$$

Substitute numerical values and evaluate B_{rms}:

$$B_{rms} = \frac{194\,\text{V/m}}{3\times10^8\ \text{m/s}} = \boxed{0.647\,\mu\text{T}}$$

35 •

Picture the Problem The rms values of the electric and magnetic fields are found from their amplitudes by dividing by the square root of two. The rms values of the electric and magnetic fields are related according to $B_{rms} = E_{rms}/c$. We can find the intensity of the radiation using $I = E_{rms}B_{rms}/\mu_0$ and the radiation pressure using $P_r = I/c$.

(*a*) Relate E_{rms} to E_0:

$$E_{rms} = \frac{E_0}{\sqrt{2}} = \frac{400\ \text{V/m}}{\sqrt{2}} = \boxed{283\,\text{V/m}}$$

(*b*) Find B_{rms} from E_{rms}:

$$B_{rms} = \frac{E_{rms}}{c} = \frac{283\,\text{V/m}}{3\times10^{8}\ \text{m/s}} = \boxed{0.943\,\mu\text{T}}$$

(*c*) The intensity of an electromagnetic wave is given by:

$$I = \frac{E_{rms}B_{rms}}{\mu_0}$$

Substitute numerical values and evaluate I:

$$I = \frac{(283\,\text{V/m})(0.943\,\mu\text{T})}{4\pi\times10^{-7}\ \text{N/A}^2} = \boxed{212\,\text{W/m}^2}$$

(*d*) Express the radiation pressure in terms of the intensity of the wave:

$$P_r = \frac{I}{c}$$

Substitute numerical values and evaluate P_r:

$$P_r = \frac{212\,\text{W/m}^2}{3\times10^{8}\ \text{m/s}} = \boxed{0.707\,\mu\text{Pa}}$$

36 •

Picture the Problem Given E_{rms}, we can find B_{rms} using $B_{rms} = E_{rms}/c$. The average energy density of the wave is given by $u_{av} = E_{rms}B_{rms}/\mu_0 c$ and the intensity of the wave by $I = u_{av}c$.

(*a*) Express B_{rms} in terms of E_{rms}:

$$B_{rms} = \frac{E_{rms}}{c}$$

Substitute numerical values and evaluate B_{rms}:

$$B_{rms} = \frac{400\,\text{V/m}}{3\times10^{8}\ \text{m/s}} = \boxed{1.33\,\mu\text{T}}$$

(*b*) The average energy density u_{av} is given by:

$$u_{av} = \frac{E_{rms}B_{rms}}{\mu_0 c}$$

Substitute numerical values and evaluate u_{av}:

$$u_{av} = \frac{(400\,\text{V/m})(1.33\,\mu\text{T})}{(4\pi\times10^{-7}\ \text{N/A}^2)(3\times10^{8}\ \text{m/s})} = \boxed{1.41\,\mu\text{J/m}^3}$$

(*c*) Express the intensity as the product of the average energy density and the speed of light in a vacuum:

$$I = u_{av}c$$

Substitute numerical values and evaluate I:

$$I = \left(1.41\,\mu\text{J/m}^3\right)\left(3\times10^8\text{ m/s}\right) = \boxed{423\,\text{W/m}^2}$$

37 •

Picture the Problem We can simplify the units of cB to show that this product has the same units as E.

Express the units of cB and simplify:

$$\frac{\text{m}}{\text{s}}\times\text{T} = \frac{\text{m}}{\text{s}}\times\frac{\text{N}}{\text{A}\cdot\text{m}} = \frac{\text{m}}{\text{s}}\times\frac{\text{N}}{\frac{\text{C}}{\text{s}}\cdot\text{m}} = \frac{\text{N}}{\text{C}} = \frac{\text{N}}{\text{C}}\times\frac{\text{m}}{\text{m}} = \frac{\text{J}}{\text{C}\cdot\text{m}} = \boxed{\frac{\text{V}}{\text{m}}}$$

***38** •

Picture the Problem Given B_{rms}, we can find E_{rms} using $E_{\text{rms}} = cB_{\text{rms}}$. The average energy density of the wave is given by $u_{\text{av}} = E_{\text{rms}}B_{\text{rms}}/\mu_0 c$ and the intensity of the wave by $I = u_{\text{av}}c$.

(*a*) Express E_{rms} in terms of B_{rms}:

$$E_{\text{rms}} = cB_{\text{rms}}$$

Substitute numerical values and evaluate E_{rms}:

$$E_{\text{rms}} = \left(3\times10^8\text{ m/s}\right)\left(0.245\,\mu\text{T}\right) = \boxed{73.5\,\text{V/m}}$$

(*b*) The average energy density u_{av} is given by:

$$u_{\text{av}} = \frac{E_{\text{rms}}B_{\text{rms}}}{\mu_0 c}$$

Substitute numerical values and evaluate u_{av}:

$$u_{\text{av}} = \frac{\left(73.5\,\text{V/m}\right)\left(0.245\,\mu\text{T}\right)}{\left(4\pi\times10^{-7}\text{ N/A}^2\right)\left(3\times10^8\text{ m/s}\right)} = \boxed{47.8\,\text{nJ/m}^3}$$

(*c*) Express the intensity as the product of the average energy density and the speed of light in a vacuum:

$$I = u_{\text{av}}c$$

Substitute numerical values and evaluate I:

$$I = \left(47.8\,\text{nJ/m}^3\right)\left(3\times10^8\text{ m/s}\right) = \boxed{14.3\,\text{W/m}^2}$$

39 ••

Picture the Problem We can find the force exerted on the card using the definition of pressure and the relationship between radiation pressure and the intensity of the electromagnetic wave. Note that, when the card reflects all the radiation incident on it, conservation of momentum requires that the force is doubled.

(*a*) Using the definition of pressure, express the force exerted on the card by the radiation:

$$F = P_r A$$

Relate the radiation pressure to the intensity of the wave:

$$P_r = \frac{I}{c}$$

Substitute to obtain:

$$F = \frac{IA}{c}$$

Substitute numerical values and evaluate *F*:

$$F = \frac{\left(200\,\text{W/m}^2\right)\left(0.2\,\text{m}\right)\left(0.3\,\text{m}\right)}{3\times10^8\ \text{m/s}} = \boxed{40.0\,\text{nN}}$$

(*b*) If the card reflects all of the radiation incident on it, the force exerted on the card is doubled:

$$F = \boxed{80.0\,\text{nN}}$$

40 ••

Picture the Problem Only the normal component of the radiation pressure exerts a force on the card.

(*a*) Using the definition of pressure, express the force exerted on the card by the radiation:

$$F = 2P_r A\cos\theta$$

where the factor of 2 is a consequence of the fact that the card reflects the radiation incident on it.

Relate the radiation pressure to the intensity of the wave:

$$P_r = \frac{I}{c}$$

Substitute to obtain:

$$F = \frac{2IA\cos\theta}{c}$$

Substitute numerical values and evaluate F:

$$F = \frac{2(200\ \text{W/m}^2)(0.2\ \text{m})(0.3\ \text{m})\cos 30°}{3\times10^8\ \text{m/s}} = \boxed{69.3\ \text{nN}}$$

***41 ••**

Picture the Problem We can use $I = P_{av}/4\pi r^2$ and $I = E_{rms}B_{rms}/\mu_0$ to express E_{rms} in terms of P_{av} and the distance r from the station.

Express the intensity I of the radiation as a function of its average power and the distance r from the station:

$$I = \frac{P_{av}}{4\pi r^2}$$

The intensity is also given by:

$$I = \frac{E_{rms}B_{rms}}{\mu_0} = \frac{E_{rms}^2}{c\mu_0} = \frac{E_{max}^2}{2c\mu_0}$$

Equate these expressions to obtain:

$$\frac{P_{av}}{4\pi r^2} = \frac{E_{max}^2}{2c\mu_0}$$

Solve for E_{max}:

$$E_{max} = \sqrt{\frac{c\mu_0 P_{av}}{2\pi}}\left(\frac{1}{r}\right)$$

(a) Substitute numerical values and evaluate E_{max} for $r = 500$ m:

$$E_{max}(500\ \text{m}) = \sqrt{\frac{(3\times10^8\ \text{m/s})(4\pi\times10^{-7}\ \text{N/A}^2)(50\ \text{kW})}{2\pi}}\left(\frac{1}{500\ \text{m}}\right) = \boxed{3.46\ \text{V/m}}$$

Use $B_{max} = E_{max}/c$ to evaluate B_{max}:

$$B_{max} = \frac{3.46\ \text{V/m}}{3\times10^8\ \text{m/s}} = \boxed{11.5\ \text{nT}}$$

(b) Substitute numerical values and evaluate E_{max} for $r = 5$ km:

$$E_{max}(5\ \text{km}) = \sqrt{\frac{(3\times10^8\ \text{m/s})(4\pi\times10^{-7}\ \text{N/A}^2)(50\ \text{kW})}{2\pi}}\left(\frac{1}{5\ \text{km}}\right) = \boxed{0.346\ \text{V/m}}$$

Use $B_{max} = E_{max}/c$ to evaluate B_{max}:

$$B_{max} = \frac{0.346\ \text{V/m}}{3\times10^8\ \text{m/s}} = \boxed{1.15\ \text{nT}}$$

(c) Substitute numerical values and evaluate E_{max} for $r = 50$ km:

$$E_{max}(500\,\text{m}) = \sqrt{\frac{(3\times10^8\ \text{m/s})(4\pi\times10^{-7}\ \text{N/A}^2)(50\,\text{kW})}{2\pi}}\left(\frac{1}{50\,\text{km}}\right) = \boxed{0.0346\,\text{V/m}}$$

Use $B_{max} = E_{max}/c$ to evaluate B_{max}:

$$B_{max} = \frac{0.0346\,\text{V/m}}{3\times10^8\ \text{m/s}} = \boxed{0.115\,\text{nT}}$$

42 ••

Picture the Problem We can use $I = P_{av}/A$ to express E_{rms} in terms of I. We can then use $B_{rms} = E_{rms}/c$ to find B_{rms}. The average power output of the sun is given by $P_{av} = 4\pi R^2 I$, where R is the earth-sun distance. The intensity and the radiation pressure at the surface of the sun can be found from the definitions of these physical quantities.

(*a*) From the definition of intensity we have:

$$I = \frac{P_{av}}{A} = \frac{4P_{av}}{\pi d^2}$$

Substitute numerical values and evaluate I:

$$I = \frac{4(1.5\,\text{mW})}{\pi(10^{-3}\ \text{m})^2} = \boxed{1.91\text{kW/m}^2}$$

(*b*) Express the intensity I of the radiation as a function of its average power and the distance r from the station:

$$I = \frac{E_{rms}B_{rms}}{\mu_0} = \frac{E_{rms}^2}{c\mu_0}$$

Solve for E_{rms}:

$$E_{rms} = \sqrt{c\mu_0 I}$$

Substitute numerical values and evaluate E_{rms}:

$$E_{rms} = \sqrt{(3\times10^8\ \text{m/s})(4\pi\times10^{-7}\ \text{N/A}^2)(1.91\text{kW/m}^2)} = \boxed{849\,\text{V/m}}$$

Use $B_{rms} = E_{rms}/c$ to evaluate B_{rms}:

$$B_{rms} = \frac{849\,\text{V/m}}{3\times10^8\ \text{m/s}} = \boxed{2.83\,\mu\text{T}}$$

(*d*) Express the radiation pressure in terms of the intensity:

$$P_r = \frac{I}{c}$$

Substitute numerical values and evaluate P_r:

$$P_r = \frac{1.91\times10^3\ \text{W/m}^2}{3\times10^8\ \text{m/s}} = \boxed{6.37\,\mu\text{Pa}}$$

***43 ••**

Picture the Problem We can use $I = E_{rms}B_{rms}/\mu_0$ and $B_{rms} = E_{rms}/c$ to express E_{rms} in terms of I. We can then use $B_{rms} = E_{rms}/c$ to find B_{rms}.

Express the intensity I of the radiation as a function of its average power and the distance r from the station:

$$I = \frac{E_{rms}B_{rms}}{\mu_0} = \frac{E_{rms}^2}{c\mu_0}$$

Solve for E_{rms}:

$$E_{rms} = \sqrt{c\mu_0 I}$$

Use the definition of intensity to relate the intensity of the electromagnetic wave to the power in the beam:

$$I = \frac{P}{A} = \frac{I_{\text{trans. line}}V}{A}$$

Substitute for I to obtain:

$$E_{rms} = \sqrt{\frac{c\mu_0 I_{\text{trans. line}}V}{A}}$$

Substitute numerical values and evaluate E_{rms}:

$$E_{rms} = \sqrt{\frac{(3\times10^8\ \text{m/s})(4\pi\times10^{-7}\ \text{N/A}^2)(10^3\ \text{A})(750\ \text{kV})}{50\ \text{m}^2}} = \boxed{75.2\ \text{kV/m}}$$

Use $B_{rms} = E_{rms}/c$ to evaluate B_{rms}:

$$B_{rms} = \frac{75.2\ \text{kV/m}}{3\times10^8\ \text{m/s}} = \boxed{0.251\ \text{mT}}$$

44 ••

Picture the Problem The spatial length L of the pulse is the product of its speed c and duration Δt. We can find the energy density within the pulse using its definition ($u = U/V$). The electric amplitude of the pulse is related to the energy density in the beam according to $u = \epsilon_0 E^2$ and we can find B from E using $B = E/c$.

(*a*) The spatial length L of the pulse is the product of its speed c and duration Δt:

$$L = c\Delta t$$

Substitute numerical values and evaluate L:

$$L = (3\times10^8\ \text{m/s})(10\ \text{ns}) = \boxed{3.00\ \text{m}}$$

(*b*) The energy density within the pulse is the energy of the beam per unit volume:

$$u = \frac{U}{V} = \frac{U}{\pi r^2 L}$$

Substitute numerical values and evaluate *u*:

$$u = \frac{20\,\text{J}}{\pi(2\,\text{mm})^2(3.00\,\text{m})} = \boxed{531\,\text{kJ/m}^3}$$

(*c*) *E* is related to *u* according to:

$$u = \epsilon_0 E_{\text{rms}}^2 = \tfrac{1}{2}\epsilon_0 E_0^2$$

Solve for E_0 to obtain:

$$E_0 = \sqrt{\frac{2u}{\epsilon_0}}$$

Substitute numerical values and evaluate E_0:

$$E_0 = \sqrt{\frac{2(531\text{ kJ/m}^3)}{8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2}} = \boxed{346\,\text{MV/m}}$$

Use $B_0 = E_0/c$ to find B_0:

$$B_0 = \frac{346\text{ MV/m}}{3\times10^8\text{ m/s}} = \boxed{1.15\,\text{T}}$$

***45 ••**

Picture the Problem We can determine the direction of propagation of the wave, its wavelength, and its frequency by examining the argument of the cosine function. We can find *E* from $|\vec{S}| = E^2/\mu_0 c$ and *B* from $B = E/c$. Finally, we can use the definition of the Poynting vector and the given expression for $\vec{S}$ to find $\vec{E}$ and $\vec{B}$.

(*a*) $\boxed{\text{Because the argument of the cosine function is of the form } kx-\omega t, \text{ the wave propagates in the positive } x \text{ direction.}}$

(*b*) Examining the argument of the cosine function, we note that the wave number *k* of the wave is:

$$k = \frac{2\pi}{\lambda} = 10\,\text{m}^{-1}$$

Solve for and evaluate λ:

$$\lambda = \frac{2\pi}{10\,\text{m}^{-1}} = \boxed{0.628\,\text{m}}$$

Examining the argument of the cosine function, we note that the angular frequency ω of the wave is:

$$\omega = 2\pi f = 3\times10^9\,\text{s}^{-1}$$

Solve for and evaluate f to obtain:

$$f = \frac{3\times10^9\ \text{s}^{-1}}{2\pi} = \boxed{477\,\text{MHz}}$$

(c) Express the magnitude of $\vec{S}$ in terms of E:

$$\left|\vec{S}\right| = \frac{E^2}{\mu_0 c}$$

Solve for E:

$$E = \sqrt{\mu_0 c\left|\vec{S}\right|}$$

Substitute numerical values and evaluate E:

$$E = \sqrt{\left(3\times10^8\ \text{m/s}\right)\left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(100\ \text{W/m}^2\right)} = 194\ \text{V/m}$$

Because $\vec{S}(x,t) = \left(100\ \text{W/m}^2\right)\cos^2\left[10x - \left(3\times10^9\right)t\right]\hat{\boldsymbol{i}}$ and $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$:

$$\boxed{\vec{E}(x,t) = (194\ \text{V/m})\cos\left[10x - \left(3\times10^9\right)t\right]\hat{\boldsymbol{j}}}$$

Use $B = E/c$ to evaluate B:

$$B = \frac{194\ \text{V/m}}{3\times10^8\ \text{m/s}} = 0.647\ \mu\text{T}$$

Because $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$, the direction of $\vec{B}$ must be such that the cross product of $\vec{E}$ with $\vec{B}$ is in the positive x direction:

$$\boxed{\vec{B}(x,t) = (0.647\ \mu\text{T})\cos\left[10x - \left(3\times10^9\right)t\right]\hat{\boldsymbol{k}}}$$

46 ••

Picture the Problem We can use the definition of the electric field between the plates of the parallel-plate capacitor and the definition of the displacement current to show that the displacement current in the capacitor is equal to the conduction current in the capacitor leads. In (b) we can use the definition of the Poynting vector and the directions of the electric and magnetic fields to determine the direction of the Poynting vector between the capacitor plates. In (c), we'll demonstrate that the flux of $\vec{S}$ into the region between the plates is equal to the rate of change of the energy stored in the capacitor by evaluating these quantities separately and showing that they are equal.

(a) The electric field between the plates of the capacitor is given by:

$$E = \frac{V(t)}{d} = \frac{V}{d}\left(1 - e^{-t/RC}\right)$$

The displacement current is proportional to the rate at which the

$$I_D(t) = \epsilon_0\frac{d\phi_e}{dt} = \epsilon_0\frac{d}{dt}(AE) = \epsilon_0 A\frac{dE}{dt}$$

flux is changing between the plates:

Substitute for E and carry out the details of the differentiation to obtain:

$$I_D(t) = \epsilon_0 A \frac{d}{dt}\left[\frac{V}{d}\left(1-e^{-t/RC}\right)\right] = \frac{\epsilon_0 AV}{d}\frac{d}{dt}\left[\left(1-e^{-t/RC}\right)\right] = \frac{\epsilon_0 AV}{d}\frac{d}{dt}\left[-e^{-t/RC}\right] = \frac{\epsilon_0 AV}{dRC}e^{-t/RC}$$

Because the capacitance of an air-filled-parallel-plate capacitor is given by $C = \frac{\epsilon_0 A}{d}$:

$$I_D(t) = \frac{CV}{RC}e^{-t/RC} = \boxed{I(t)}$$

(b) Apply Ampere's law to a closed circular path of radius r (the radius of the capacitor plates) to obtain:

$$B(2\pi r) = \mu_0 I_C = \mu_0 I_D$$

Substitute for I_D from (a):

$$B(2\pi r) = \mu_0 \epsilon_0 \frac{\pi r^2 V}{d(RC)}e^{-t/RC}$$

Solve for B to obtain:

$$B = \mu_0 \epsilon_0 \frac{rV}{2d(RC)}e^{-t/RC}$$

> Because $\vec{E}$ is perpendicular to the plates of the capacitor and $\vec{B}$ is tangent to circles that are concentric and whose center is through the middle of the capacitor plates, $\vec{S}$ points radially inward toward the center of the capacitor.

(c) The magnitude of the Poynting vector is:

$$\left|\vec{S}\right| = I = \frac{BE}{\mu_0}$$

Substitute for B and E and simplify to obtain:

$$I = \boxed{\frac{\epsilon_0}{2}\frac{V^2 r}{d^2 RC}e^{-t/RC}\left(1-e^{-t/RC}\right)}$$

The total power is:

$$P = \frac{dE}{dt} = 2\pi r d I$$

Substitute for I to obtain:

$$\frac{dE}{dt}=\in_0\frac{V^2\pi r^2}{dRC}e^{-t/RC}\left(1-e^{-t/RC}\right)$$

Because the capacitance of an air-filled-parallel-plate capacitor is given by $C=\dfrac{\in_0\,\pi\,r^2}{d}$:

$$\frac{dE}{dt}=\frac{V^2}{R}e^{-t/RC}\left(1-e^{-t/RC}\right) \qquad (1)$$

The energy in the capacitor at any time is:

$$E=\frac{1}{2}C[V(t)]^2$$

Differentiate E with respect to time to obtain:

$$\frac{dE}{dt}=\frac{d}{dt}\left[\frac{1}{2}C(V(t))^2\right]=CV(t)\frac{dV(t)}{dt}$$

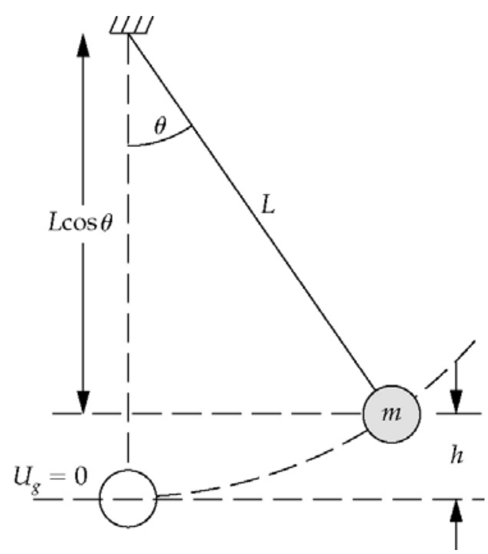
Substitute for $V(t)$ and complete the differentiation to obtain:

$$\frac{dE}{dt}=\frac{V^2}{R}e^{-t/RC}\left(1-e^{-t/RC}\right) \qquad (2)$$

The equivalence of equations (1) and (2) proves that the flux of $\vec{S}$ into this region is equal to the rate of change of the energy stored in the capacitor.

47 ••

Picture the Problem The diagram shows the displacement of the pendulum bob, through an angle θ, as a consequence of the complete absorption of the radiation incident on it. We can use conservation of energy (mechanical energy is conserved *after* the collision) to relate the maximum angle of deflection of the pendulum to the initial momentum of the pendulum bob. Because the displacement of the bob during the absorption of the pulse is negligible, we can use conservation of momentum (conserved *during* the collision) to equate the momentum of the electromagnetic pulse to the initial momentum of the bob.



Apply conservation of energy to obtain:

$$K_f-K_i+U_f-U_i=0$$

or, since $U_i = K_f = 0$ and $K_i = p_i^2/2m$,

$$-\frac{p_i^2}{2m}+U_f=0$$

U_f is given by:

$$U_f=mgh=mgL(1-\cos\theta)$$

Substitute for U_f:

$$-\frac{p_i^2}{2m}+mgL(1-\cos\theta)=0$$

Solve for θ to obtain:

$$\theta=\cos^{-1}\left(1-\frac{p_i^2}{2m^2gL}\right)$$

Use conservation of momentum to relate the momentum of the electromagnetic pulse to the initial momentum p_i of the pendulum bob:

$$p_{\text{em wave}}=\frac{U}{c}=\frac{P\Delta t}{c}=p_i$$

where Δt is the duration of the pulse.

Substitute for p_i:

$$\theta=\cos^{-1}\left(1-\frac{P^2(\Delta t)^2}{2m^2c^2gL}\right)$$

Substitute numerical values and evaluate θ:

$$\theta=\cos^{-1}\left(1-\frac{(1000\,\text{MW})^2(200\,\text{ns})^2}{2(10\,\text{mg})^2(3\times10^8\ \text{m/s})^2(9.81\,\text{m/s}^2)(0.04\,\text{m})}\right)=\boxed{6.10\times10^{-3}\ \text{degrees}}$$

Remarks: The solution presented here is valid only if the displacement of the bob during the absorption of the pulse is negligible. (Otherwise, the horizontal component of the momentum of the pulse-bob system is not conserved during the collision.) We can show that the displacement during the pulse-bob collision is small by solving for the speed of the bob after absorbing the pulse. Applying conservation of momentum ($mv = P(\Delta t)/c$) and solving for v gives $v = 6.67\times10^{-7}$ m/s. This speed is so slow compared to c, we can conclude that the duration of the collision is extremely close to 200 ns (the time for the pulse to travel its own length). Traveling at 6.67×10^{-7} m/s for 200 ns, the bob would travel 1.33×10^{-13} m—a distance 1000 times smaller that the diameter of a hydrogen atom. (Since 6.67×10^{-7} m/s is the maximum speed of the bob during the collision, the bob would actually travel less than 1.33×10^{-13} m during the collision.)

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Picture the Problem We can use the definitions of pressure and the relationship between radiation pressure and the intensity of the radiation to find the force due to radiation pressure on one of the mirrors.

(*a*) Because only about 0.01 percent of the energy inside the laser "leaks out", the average power of the radiation incident on one of the mirrors is:

$$P = \frac{15\,\text{W}}{10^{-4}} = \boxed{1.50\times10^{5}\ \text{W}}$$

(*b*) Use the definition of radiation pressure to obtain:

$$P_r = \frac{F}{A}$$

where *F* is the force due to radiation pressure and *A* is the area of the mirror on which the radiation is incident.

The radiation pressure is also related to the intensity of the radiation:

$$P_r = \frac{2I}{c} = \frac{2P}{Ac}$$

where *P* is the power of the laser and the factor of 2 is due to the fact that the mirror is essentially totally reflecting.

Equate the two expression for the radiation pressure and solve for *F*:

$$\frac{F}{A} = \frac{2P}{Ac} \Rightarrow F = \frac{2P}{c}$$

Substitute numerical values and evaluate *F*:

$$F = \frac{2\left(1.50\times10^{5}\ \text{W}\right)}{3\times10^{8}\ \text{m/s}} = \boxed{1.00\,\text{mN}}$$

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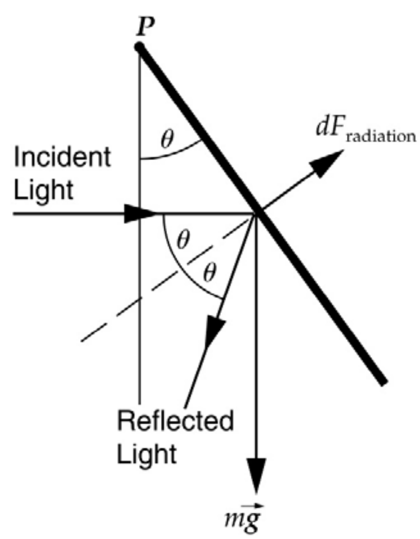
Picture the Problem The card, pivoted at point *P*, is shown in the diagram. Note that the force exerted by the radiation acts along the dashed line. Let the length of the card be ℓ, the width of the card be *w*, and the force acting on an area $dA = w\,dx$ be $dF_{\text{radiation}}$. We can find the total torque exerted on the card due to radiation pressure by integrating $d\tau_{\text{radiation}}$ over the length ℓ of the card and then relate the intensity of the light to the angle θ by applying the condition for rotational equilibrium to the card.



Express the torque, due to *F*, acting at a distance *x* from *P*:

$$d\tau_{\text{radiation}} = x\,dF_{\text{radiation}}$$

Relate $dF_{\text{radiation}}$ to the intensity of the light:

$$dF_{\text{radiation}} = \frac{2I}{c}\cos\theta\, dA$$

where the factor of 2 arises from the total reflection of the radiation incident on the mirror.

Substitute to obtain:

$$d\tau_{\text{radiation}} = \frac{2I}{c}\cos\theta\, xdA = \frac{2I}{c}\cos\theta\, xwdx$$

Integrate x from 0 to ℓ:

$$\tau_{\text{radiation}} = \frac{2Iw}{c}\cos\theta\int_0^{\ell} xdx = \frac{2Iw}{c}\cos\theta\left(\frac{\ell^2}{2}\right) = \frac{IA\ell}{c}\cos\theta$$

Apply $\sum\tau_P = 0$ to the card:

$$\frac{IA\ell}{c}\cos\theta - \left(\tfrac{1}{2}\ell\sin\theta\right)mg = 0$$

Solve for I to obtain:

$$I = \frac{mgc}{2A}\tan\theta$$

Substitute numerical values and evaluate I:

$$I = \frac{(2\,\text{g})(9.81\,\text{m/s}^2)(3\times10^8\ \text{m/s})}{2(0.1\,\text{m})(0.15\,\text{m})}\tan 1^\circ = \boxed{3.42\,\text{MW/m}^2}$$

The Wave Equation for Electromagnetic Waves

50 •

Picture the Problem We can show that Equation 30-17*a* is satisfied by the wave function E_y by showing that the ratio of $\partial^2E_y/\partial x^2$ to $\partial^2E_y/\partial t^2$ is $1/c^2$ where $c = \omega/k$.

Differentiate $E_y = E_0\sin(kx - \omega t)$ with respect to x:

$$\frac{\partial E_y}{\partial x} = \frac{\partial}{\partial x}\left[E_0\sin(kx-\omega t)\right] = kE_0\cos(kx-\omega t)$$

Evaluate the second partial derivative of E_y with respect to x:

$$\frac{\partial^2 E_y}{\partial x^2} = \frac{\partial}{\partial x}\left[kE_0\cos(kx-\omega t)\right] = -k^2E_0\sin(kx-\omega t) \qquad (1)$$

Differentiate $E_y = E_0 \sin(kx - \omega t)$ with respect to t:

$$\frac{\partial E_y}{\partial t} = \frac{\partial}{\partial t}\left[E_0 \sin(kx - \omega t)\right] = -\omega E_0 \cos(kx - \omega t)$$

Evaluate the second partial derivative of E_y with respect to t:

$$\frac{\partial^2 E_y}{\partial t^2} = \frac{\partial}{\partial t}\left[-\omega E_0 \cos(kx - \omega t)\right] = -\omega^2 E_0 \sin(kx - \omega t) \quad (2)$$

Divide equation (1) by equation (2) to obtain:

$$\frac{\dfrac{\partial^2 E_y}{\partial x^2}}{\dfrac{\partial^2 E_y}{\partial t^2}} = \frac{-k^2 E_0 \sin(kx - \omega t)}{-\omega^2 E_0 \sin(kx - \omega t)} = \frac{k^2}{\omega^2}$$

or

$$\frac{\partial^2 E_y}{\partial x^2} = \frac{k^2}{\omega^2}\frac{\partial^2 E_y}{\partial t^2} = \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2}$$

provided $c = \omega/k$.

51 •

Picture the Problem Substitute numerical values and evaluate c:

$$c = \frac{1}{\sqrt{\left(4\pi \times 10^{-7}\ \text{N/A}^2\right)\left(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)}} = \boxed{3.00 \times 10^8\ \text{m/s}}$$

***52** •••

Picture the Problem We can use Figures 30-10 and 30-11and a derivation similar to that in the text to obtain the given results.

In Figure 30-11, replace B_z by E_z. For Δx small:

$$E_z(x_2) = E_z(x_1) + \frac{\partial E_z}{\partial x}\Delta x$$

Evaluate the line integral of $\vec{E}$ around the rectangular area $\Delta x \Delta z$:

$$\oint \vec{E} \cdot d\vec{\ell} \approx -\frac{\partial E_z}{\partial x}\Delta x \Delta z \quad (1)$$

Express the magnetic flux through the same area:

$$\int_S B_n dA = B_y \Delta x \Delta z$$

Apply Faraday's law to obtain:

$$\oint \vec{E} \cdot d\vec{\ell} \approx -\frac{\partial}{\partial t}\int_S B_n dA = -\frac{\partial}{\partial t}\left(B_y \Delta x \Delta z\right) = -\frac{\partial B_y}{\partial t}\Delta x \Delta z$$

Substitute in equation (1) to obtain:

$$-\frac{\partial E_z}{\partial x}\Delta x\Delta z = -\frac{\partial B_y}{\partial t}\Delta x\Delta z$$

or

$$\boxed{\frac{\partial E_z}{\partial x} = \frac{\partial B_y}{\partial t}}$$

In Figure 30-10, replace E_y by B_y and evaluate the line integral of $\vec{\boldsymbol{B}}$ around the rectangular area $\Delta x\Delta z$:

$$\oint \vec{\boldsymbol{B}}\cdot d\vec{\ell} = \mu_0 \in_0 \int_S E_n dA$$

provided there are no conduction currents.

Evaluate these integrals to obtain:

$$\boxed{\frac{\partial B_y}{\partial x} = \mu_0 \in_0 \frac{\partial E_z}{\partial t}}$$

(*b*) Using the first result obtained in (*a*), find the second partial derivative of E_z with respect to x:

$$\frac{\partial}{\partial x}\left(\frac{\partial E_z}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial B_y}{\partial t}\right)$$

or

$$\frac{\partial^2 E_z}{\partial x^2} = \frac{\partial}{\partial t}\left(\frac{\partial B_y}{\partial x}\right)$$

Use the second result obtained in (*a*) to obtain:

$$\frac{\partial^2 E_z}{\partial x^2} = \frac{\partial}{\partial t}\left(\mu_0\varepsilon_0\frac{\partial E_z}{\partial t}\right) = \mu_0 \in_0 \frac{\partial^2 E_z}{\partial t^2}$$

or, because $\mu_0 \in_0 = 1/c^2$,

$$\boxed{\frac{\partial^2 E_z}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E_z}{\partial t^2}}\ .$$

Using the second result obtained in (*a*), find the second partial derivative of B_y with respect to x:

$$\frac{\partial}{\partial x}\left(\frac{\partial B_y}{\partial x}\right) = \mu_0 \in_0 \frac{\partial}{\partial x}\left(\frac{\partial E_z}{\partial t}\right)$$

or

$$\frac{\partial^2 B_y}{\partial x^2} = \mu_0 \in_0 \frac{\partial}{\partial t}\left(\frac{\partial E_z}{\partial x}\right)$$

Use the second result obtained in (*a*) to obtain:

$$\frac{\partial^2 B_y}{\partial x^2} = \mu_0 \in_0 \frac{\partial}{\partial t}\left(\frac{\partial B_y}{\partial t}\right) = \mu_0 \in_0 \frac{\partial^2 B_y}{\partial t^2}$$

or, because $\mu_0 \in_0 = 1/c^2$,

$$\boxed{\frac{\partial^2 B_y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 B_y}{\partial t^2}}\ .$$

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Picture the Problem We can show that these functions satisfy the wave equations by differentiating them twice (using the chain rule) with respect to *x* and *t* and equating the expressions for the second partial of *f* with respect to *u*.

Let $u = x - vt$. Then:

$$\frac{\partial f}{\partial x} = \frac{\partial u}{\partial x}\frac{\partial f}{\partial u} = \frac{\partial f}{\partial u}$$

and

$$\frac{\partial f}{\partial t} = \frac{\partial u}{\partial t}\frac{\partial f}{\partial u} = -v\frac{\partial f}{\partial u}$$

Express the second derivatives of *f* with respect to *x* and *t* to obtain:

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial u^2}$$

and

$$\frac{\partial^2 f}{\partial t^2} = v^2\frac{\partial^2 f}{\partial u^2}$$

Thus, for any $f(u)$:

$$\boxed{\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}}$$

Let $u = x + vt$. Then:

$$\frac{\partial f}{\partial x} = \frac{\partial u}{\partial x}\frac{\partial f}{\partial u} = \frac{\partial f}{\partial u}$$

and

$$\frac{\partial f}{\partial t} = \frac{\partial u}{\partial t}\frac{\partial f}{\partial u} = v\frac{\partial f}{\partial u}$$

Express the second derivatives of *f* with respect to *x* and *t* to obtain:

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial u^2}$$

and

$$\frac{\partial^2 f}{\partial t^2} = v^2\frac{\partial^2 f}{\partial u^2}$$

Thus, for any $f(u)$:

$$\boxed{\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}}$$

General Problems

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Picture the Problem We can substitute the appropriate units and simplify to show that the units of the Poynting vector are watts per square meter and that those of radiation pressure are newtons per square meter.

(*a*) Express the units of $\vec{S}$ and simplify:

$$\frac{\frac{\text{V}}{\text{m}}\times\text{T}}{\frac{\text{N}}{\text{A}^2}}=\frac{\frac{\text{J}}{\text{C}\cdot\text{m}}\times\frac{\text{N}}{\text{C}\cdot\frac{\text{m}}{\text{s}}}}{\frac{\text{N}}{\text{A}^2}}$$

$$=\frac{\frac{\text{J}}{\text{C}}}{\frac{\text{s}}{\text{C}}}=\frac{\text{J}}{\text{s}}=\boxed{\text{W}}$$

(*b*) Express the units of P_r and simply:

$$\frac{\frac{\text{W}}{\text{m}^2}}{\frac{\text{m}}{\text{s}}}=\frac{\frac{\text{J}}{\text{s}\cdot\text{m}^2}}{\frac{\text{m}}{\text{s}}}=\frac{\frac{\text{N}\cdot\text{m}}{\text{m}^2}}{\text{m}}=\boxed{\frac{\text{N}}{\text{m}^2}}$$

55 ••

Determine the Concept The current induced in a loop antenna is proportional to the time-varying magnetic field. For maximum signal, the antenna's plane should make an angle θ = 0° with the line from the antenna to the transmitter. For any other angle, the induced current is proportional to cos θ. The intensity of the signal is therefore proportional to cos θ.

56 ••

Picture the Problem We can use $c = f\lambda$ to find the wavelength. Examination of the argument of the cosine function will reveal the direction of propagation of the wave. We can find the magnitude, wave number, and angular frequency of the electric vector from the given information and the result of (*a*) and use these results to obtain $\vec{E}(z, t)$. Finally, we can use its definition to find the Poynting vector.

(*a*) Relate the wavelength of the wave to its frequency and the speed of light:

$$\lambda=\frac{c}{f}$$

Substitute numerical values and evaluate λ:

$$\lambda=\frac{3\times10^8\ \text{m/s}}{100\text{MHz}}=\boxed{3.00\,\text{m}}$$

From the sign of the argument of the cosine function and the spatial dependence on z, we can conclude that the wave propagates in the z direction.

(b) Express the amplitude of $\vec{E}$:

$$E = cB = \left(3\times10^{8}\text{ m/s}\right)\left(10^{-8}\text{ T}\right) = 3.00\text{ V/m}$$

Find the angular frequency and wave number of the wave:

$$\omega = 2\pi f = 2\pi\left(100\text{ MHz}\right) = 6.28\times10^{8}\text{ s}^{-1}$$

and

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{3.00\text{ m}} = 2.09\text{ m}^{-1}$$

Because $\vec{S}$ is in the positive z direction, $\vec{E}$ must be in the negative y direction in order to satisfy the Poynting vector expression:

$$\boxed{\vec{E}(z,t) = -\left(3.00\text{ V/m}\right)\cos\left[\left(2.09\text{ m}^{-1}\right)z - \left(6.28\times10^{8}\text{ s}^{-1}\right)t\right]\hat{j}}$$

(c) Use its definition to express the Poynting vector:

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = \frac{-\left(3.00\text{ V/m}\right)\left(10^{-8}\text{ T}\right)}{4\pi\times10^{-7}\text{ N/A}^2}\cos^2\left[\left(2.09\text{ m}^{-1}\right)z - \left(6.28\times10^{8}\text{ s}^{-1}\right)t\right]\left(\hat{j}\times\hat{i}\right)$$

or

$$\boxed{\vec{S} = \left(23.9\text{ mW/m}^2\right)\cos^2\left[\left(2.09\text{ m}^{-1}\right)z - \left(6.28\times10^{8}\text{ s}^{-1}\right)t\right]\hat{k}}$$

The intensity of the wave is the average magnitude of the Poynting vector. The average value of the square of the cosine function is 1/2:

$$I = \left|\vec{S}\right| = \tfrac{1}{2}\left(23.9\text{ mW/m}^2\right) = \boxed{12.0\text{ mW/m}^2}$$

***57 ••**

Picture the Problem The maximum rms voltage induced in the loop is given by $\mathcal{E}_{\text{rms}} = A\omega B_0/\sqrt{2}$, where A is the area of the loop, B_0 is the amplitude of the magnetic field, and ω is the angular frequency of the wave. We can use the definition of density and the expression for the intensity of an electromagnetic wave to derive an expression for B_0.

The maximum induced rms emf occurs when the plane of the loop is perpendicular to $\vec{B}$:

$$\mathcal{E}_{\text{rms}} = \frac{A\omega B_0}{\sqrt{2}} = \frac{\pi R^2\omega B_0}{\sqrt{2}} \qquad (1)$$

where R is the radius of loop of wire.

From the definition of intensity we have:

$$I = \frac{P}{4\pi r^2}$$

where r is the distance from the transmitter.

The intensity is also given by:

$$I = \frac{E_0 B_0}{2\mu_0} = \frac{B_0^2 c}{2\mu_0}$$

Substitute to obtain:

$$\frac{B_0^2 c}{2\mu_0} = \frac{P}{4\pi r^2}$$

Solve for B_0:

$$B_0 = \frac{1}{r}\sqrt{\frac{\mu_0 P}{2\pi c}}$$

Substitute in equation (1) to obtain:

$$\mathcal{E}_{\text{rms}} = \frac{\pi R^2 (2\pi f)}{\sqrt{2}\, r}\sqrt{\frac{\mu_0 P}{2\pi c}} = \frac{R^2 f}{\sqrt{2}\, r}\sqrt{\frac{2\pi^3 \mu_0 P}{c}}$$

Substitute numerical values and evaluate ε_{rms}:

$$\mathcal{E}_{\text{rms}} = \frac{(0.3\,\text{m})^2 (100\,\text{MHz})}{\sqrt{2}(10^5\,\text{m})}\sqrt{\frac{2\pi^3 (4\pi\times 10^{-7}\,\text{N/A}^2)(50\,\text{kW})}{3\times 10^8\,\text{m/s}}} = \boxed{7.25\,\text{mV}}$$

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Picture the Problem The voltage induced in the piece of wire is the product of the electric field and the length of the wire. The maximum rms voltage induced in the loop is given by $\mathcal{E} = A\omega B_0$, where A is the area of the loop, B_0 is the amplitude of the magnetic field, and ω is the angular frequency of the wave.

(*a*) Because E is independent of x:

$$V = E\ell$$

where ℓ is the length of the wire.

Substitute numerical values and evaluate V:

$$V = \left[(10^{-4}\,\text{N/C})\cos 10^6 t\right](0.5\,\text{m}) = \boxed{(50.0\,\mu\text{V})\cos 10^6 t}$$

(*b*) The voltage induced in a loop is given by:

$$\mathcal{E} = \omega B_0 A$$

where A is the area of the loop and B_0 is the amplitude of the magnetic field.

Eliminate B_0 in favor of E_0 and substitute for A to obtain:

$$\mathcal{E} = \frac{\omega E_0 \pi R^2}{c}$$

Substitute numerical values and evaluate $\mathcal{E}$:

$$\mathcal{E} = \frac{(10^6\ \text{s}^{-1})(10^{-4}\ \text{N/C})\pi(0.2\ \text{m})^2}{3\times 10^8\ \text{m/s}} = \boxed{41.9\ \text{nV}}$$

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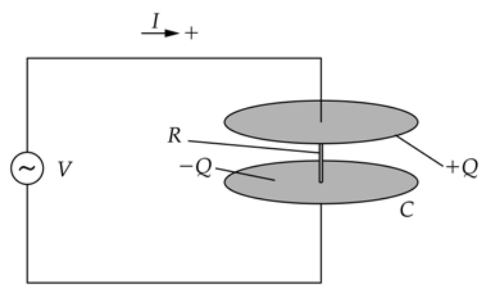
Picture the Problem Some of the charge entering the capacitor passes through the resistive wire while the rest of it accumulates on the upper plate. The total current is the rate at which the charge passes through the resistive wire plus the rate at which it accumulates on the upper plate. The magnetic field between the capacitor plates is due to both the current in the resistive wire and the displacement current though a surface bounded by a circle a distance r from the resistive wire. The phase difference between the supplied current and the applied voltage may be calculated using a phasor diagram.



(a) The current drawn by the capacitor is the sum of the conduction current through the resistance wire and dQ/dt, where Q is the charge on the upper plate of the capacitor:

$$I = I_c + \frac{dQ}{dt} \qquad (1)$$

Express the conduction current I_c in terms of the potential difference between the plates and the resistance of the wire:

$$I_c = \frac{V}{R} = \frac{V_0}{R}\sin\omega t$$

Express the displacement current between the capacitor plates. Let C be the capacitance of the capacitor:

$$Q = CV$$

so

$$\frac{dQ}{dt} = C\frac{dV}{dt} = \omega C V_0 \cos\omega t$$

Substitute in equation (1):

$$I = \frac{V_0}{R}\sin \omega t + \omega C V_0 \cos \omega t \qquad (2)$$

Using Equation 24-10 for the capacitance of a parallel-plate capacitor with plate area *A* and plate separation *d* we have:

$$C = \frac{\epsilon_0 A}{d} = \frac{\epsilon_0 \pi a^2}{d}$$

Substituting for *C* equation 2 gives:

$$\boxed{I = V_0\left(\frac{1}{R}\sin \omega t + \frac{\omega \epsilon_0 \pi a^2}{d}\cos \omega t\right)}$$

(*b*) Apply the generalized form of Ampere's law to a circular path of radius *r* centered within the plates of the capacitor, where I'_{d} is the displacement current through the flat surface *S* bounded by the path and I_{c} is the conduction current through the same surface:

$$\oint_C \vec{\boldsymbol{B}} \cdot d\vec{\ell} = \mu_0\left(I_{\mathrm{c}} + I'_{\mathrm{d}}\right)$$

By symmetry the line integral is *B* times the circumference of the circle of radius *r*:

$$B(2\pi r) = \mu_0\left(I_{\mathrm{c}} + I'_{\mathrm{d}}\right) \qquad (3)$$

In the region between the capacitor plates there is a uniform electric field due to the surface charges +*Q* and –*Q*. The associated displacement current through *S* is:

$$I'_{\mathrm{d}} = \epsilon_0 \frac{d\phi_{\mathrm{e}}}{dt} = \epsilon_0 \frac{d}{dt}\left(A'E\right)$$

$$= \epsilon_0 A' \frac{dE}{dt} = \epsilon_0 \pi r^2 \frac{dE}{dt}$$

provided $(r \le a)$

To evaluate the displacement current we first must evaluate *E* everywhere on *S*. Near the surface of a conductor $E = \sigma/\epsilon_0$ (Equation 22-25), where σ is the surface charge density:

$E = \sigma/\epsilon_0$, where $\sigma = Q/A = Q/\left(\pi a^2\right)$

so

$$E = \frac{Q}{\epsilon_0 \pi a^2}$$

Substituting for E in the equation for I'_{d} gives:

$$I'_{\mathrm{d}}=\epsilon_0\,\pi r^2\frac{dE}{dt}=\epsilon_0\,\pi r^2\frac{d}{dt}\left(\frac{Q}{\epsilon_0\,\pi a^2}\right)$$
$$=\frac{r^2}{d^2}\frac{dQ}{dt}=\frac{r^2}{d^2}\frac{d}{dt}\left(V_0\sin\omega t\right)$$
$$=\omega\frac{r^2}{d^2}V_0\cos\omega t$$

Substituting for I_{c} and I'_{d} in equation (3) and solving for B gives:

$$B(r)=\frac{\mu_0\left(I_{\mathrm{c}}+I'_{\mathrm{d}}\right)}{2\pi r}$$
$$=\frac{\mu_0}{2\pi r}\left(\frac{V_0}{R}\sin\omega t+\omega\frac{r^2}{a^2}V_0\cos\omega t\right)$$
$$=\boxed{\frac{\mu_0 V_0}{2\pi r}\left(\frac{1}{R}\sin\omega t+\omega\frac{r^2}{a^2}\cos\omega t\right)}$$

(*c*) Both the charge Q and the conduction current I_{c} are in phase with V. However, dQ/dt, which is equal to the displacement current I_{d} through S for $r \geq a$, lags V by 90°. (Mathematically, cos ωt lags behind sin ωt by 90°.) The voltage V leads the current $I = I_{\mathrm{c}} + I_{\mathrm{d}}$ by phase angle δ. The current relation is expressed in terms of the current amplitudes:

$$I=I_{\mathrm{c}}+I_{\mathrm{d}}$$

or

$$I_{\max}\sin(\omega t+\delta)=I_{\mathrm{c,max}}\sin\omega t+I_{\mathrm{d,max}}\cos\omega t$$

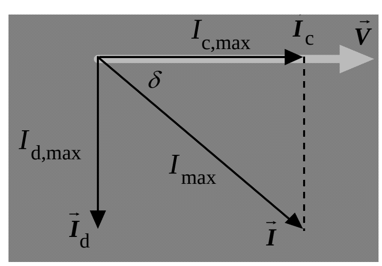
The values of the conduction and displacement current amplitudes are obtained by comparison with the answer to part (*a*):

$$I_{\mathrm{c,max}}=\frac{V_0}{R}$$

and

$$I_{\mathrm{d,max}}=\frac{\omega\,\epsilon_0\,\pi a^2 V_0}{d}$$

A phasor diagram for adding the currents I_{c} and I_{d} is shown to the right. The conduction current I_{c} is in phase with the voltage V across the resistor and I_{d} lags behind it by 90°:



From the phasor diagram we have:

$$\tan\delta = \frac{I_{\text{d,max}}}{I_{\text{c,max}}} = \frac{V_0 \dfrac{\omega \in_0 \pi a^2}{d}}{V_0/R}$$

$$= \frac{R\omega \in_0 \pi a^2}{d}$$

so

$$\delta = \boxed{\tan^{-1}\left(\frac{R\omega \in_0 \pi a^2}{d}\right)}$$

Remarks: The capacitor and the resistive wire are connected in parallel. The potential difference across each of them is the applied voltage V_0 sin ωt.

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Picture the Problem The total force on the surface is the sum of the force due to the reflected radiation and the force due to the absorbed radiation. From the conservation of momentum, the force due to the 10 kW that are reflected is twice the force due to the 10 kW that are absorbed.

Express the total force on the surface:

$$F_{\text{tot}} = F_{\text{r}} + F_a$$

Substitute for F_{r} and F_{a} to obtain:

$$F_{\text{tot}} = \frac{2\left(\frac{1}{2}P\right)}{c} + \frac{\frac{1}{2}P}{c} = \frac{3P}{2c}$$

Substitute numerical values and evaluate F_{tot}:

$$F_{\text{tot}} = \frac{3(20\,\text{kW})}{2(3\times10^8\ \text{m/s})} = \boxed{0.100\,\text{mN}}$$

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Picture the Problem We can use the definition of the Poynting vector and the relationship between $\vec{B}$ and $\vec{E}$ to find the instantaneous Poynting vectors for each of the resultant wave motions and the fact that the time average of the cross product term is zero for $\omega_1 \neq \omega_2$, and ½ for the square of cosine function to find the time-averaged Poynting vectors.

(*a*) Because $\vec{E}_1$ and $\vec{E}_2$ propagate in the *x* direction:

$$\vec{E}\times\vec{B} = \mu_0 S\hat{\boldsymbol{i}} \Rightarrow \vec{B} = B\hat{\boldsymbol{k}}$$

Express *B* in terms of E_1 and E_2:

$$B = \frac{1}{c}(E_1 + E_2)$$

Substitute for E_1 and E_2 to obtain:

$$\vec{B} = \frac{1}{c}\left[E_{1,0}\cos(k_1x-\omega_1t)+E_{2,0}\cos(k_2x-\omega_2t+\delta)\right]\hat{k}$$

Express the instantaneous Poynting vector for the resultant wave motion:

$$\begin{aligned}\vec{S} &= \frac{1}{\mu_0}\left(E_{1,0}\cos(k_1x-\omega_1t)+E_{2,0}\cos(k_2x-\omega_2t+\delta)\right)\hat{j}\\ &\quad\times\frac{1}{c}\left(E_{1,0}\cos(k_1x-\omega_1t)+E_{2,0}\cos(k_2x-\omega_2t+\delta)\right)\hat{k}\\ &= \frac{1}{\mu_0 c}\left(E_{1,0}\cos(k_1x-\omega_1t)+E_{2,0}\cos(k_2x-\omega_2t+\delta)\right)^2\left(\hat{j}\times\hat{k}\right)\\ &= \boxed{\begin{aligned}&\frac{1}{\mu_0 c}\left[E_{1,0}^2\cos^2(k_1x-\omega_1t)+2E_{1,0}E_{2,0}\cos(k_1x-\omega_1t)\right.\\ &\qquad\left.\times\cos(k_2x-\omega_2t+\delta)+E_{2,0}^2\cos^2(k_2x-\omega_2t+\delta)\right]\hat{i}\end{aligned}}\end{aligned}$$

(*b*) The time average of the cross product term is zero for $\omega_1 \neq \omega_2$, and the time average of the square of the cosine terms is ½:

$$\boxed{\vec{S}_{\text{av}} = \frac{1}{2\mu_0 c}\left[E_{1,0}^2 + E_{2,0}^2\right]\hat{i}}$$

(*c*) In this case $\vec{B}_2 = -B\hat{k}$ because the wave with $k = k_2$ propagates in the $-\hat{i}$ direction. The magnetic field is then:

$$\vec{B} = \frac{1}{c}\left[E_{1,0}\cos(k_1x-\omega_1t)-E_{2,0}\cos(k_2x+\omega_2t+\delta)\right]\hat{k}$$

Express the instantaneous Poynting vector for the resultant wave motion:

$$\begin{aligned}\vec{S} &= \frac{1}{\mu_0}\left(E_{1,0}\cos(k_1x-\omega_1t)+E_{2,0}\cos(k_2x-\omega_2t+\delta)\right)\hat{j}\\ &\quad\times\frac{1}{c}\left(E_{1,0}\cos(k_1x-\omega_1t)-E_{2,0}\cos(k_2x+\omega_2t+\delta)\right)\hat{k}\\ &= \boxed{\frac{1}{\mu_0 c}\left[E_{1,0}^2\cos^2(k_1x-\omega_1t)-E_{2,0}^2\cos^2(k_2x+\omega_2t+\delta)\right]\hat{i}}\end{aligned}$$

(*d*) The time average of the square of the cosine terms is ½:

$$\boxed{\vec{S}_{\text{av}} = \frac{1}{2\mu_0 c}\left[E_{1,0}^2 - E_{2,0}^2\right]\hat{i}}$$

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Picture the Problem We can use the definitions of power and intensity to express the area of the surface as a function of *P*, *I*, and the efficiency ε.

Use the definition of power to relate the required surface area to the intensity of the solar radiation:

$$P = \frac{E}{t}\varepsilon = IA\varepsilon$$

where ε is the efficiency of the system.

Solve for A to obtain:

$$A = \frac{P}{I\varepsilon}$$

Substitute numerical values and evaluate A:

$$A = \frac{25\,\text{kW}}{0.3\left(0.75\,\text{kW/m}^2\right)} = \boxed{111\,\text{m}^2}$$

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Picture the Problem We can use the relationship between the average value of the Poynting vector (the intensity), E_0, and B_0 to find B_0. The application of Faraday's law will allow us to find the emf induced in the antenna. The emf induced in a 2-m wire oriented in the direction of the electric field can be found using $\mathcal{E} = E\ell$ and the relationship between E and B.

(*a*) The intensity of the signal is related the amplitude of the magnetic field in the wave:

$$S_{\text{av}} = I = \frac{E_0 B_0}{2\mu_0} = \frac{cB_0^2}{2\mu_0}$$

Solve for B_0:

$$B_0 = \sqrt{\frac{2\mu_0 I}{c}}$$

Substitute numerical values and evaluate B_0:

$$B_0 = \sqrt{\frac{2\left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(10^{-14}\ \text{W/m}^2\right)}{3\times10^8\ \text{m/s}}} = \boxed{9.15\times10^{-15}\ \text{T}}$$

(*b*) Apply Faraday's law to the antenna coil to obtain:

$$\left|\mathcal{E}\right| = \frac{d}{dt}(BA) = A\frac{d}{dt}\left(NK_{\text{m}}B_0 \sin\omega t\right) = NK_{\text{m}}AB_0\omega\cos\omega t$$

Substitute numerical values and evaluate $\left|\mathcal{E}\right|$:

$$\left|\mathcal{E}\right| = 2000(200)\pi(0.01\,\text{m})^2\left(9.15\times10^{-15}\ \text{T}\right)\left[2\pi(140\,\text{kHz})\right]\cos\left[2\pi(140\,\text{kHz})\right]t = \boxed{(1.01\,\mu\text{V})\cos\left(8.80\times10^5\ \text{s}^{-1}\right)t}$$

(*c*) The voltage induced in the wire

$$\mathcal{E} = E\ell$$

is the product of its length ℓ and the amplitude of electric field E_0:

Relate E to B:

$$E = cB = cB_0 \sin \omega t$$

Substitute for E to obtain:

$$\mathcal{E} = c\ell B_0 \sin \omega t$$

Substitute numerical values and evaluate $\left|\mathcal{E}\right|$:

$$\begin{aligned}\mathcal{E} &= \left(3\times10^{8}\ \text{m/s}\right)\left(2\ \text{m}\right)\left(9.15\times10^{-15}\ \text{T}\right)\sin\left[2\pi\left(140\ \text{kHz}\right)\right]t \\ &= \boxed{\left(5.49\ \mu\text{V}\right)\sin\left(8.80\times10^{5}\ \text{s}^{-1}\right)t}\end{aligned}$$

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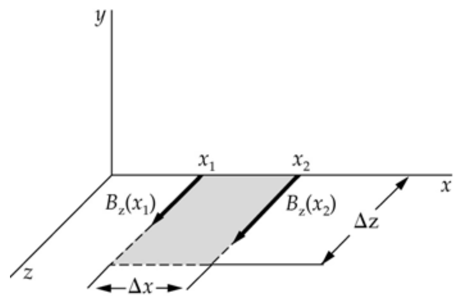
Picture the Problem We'll choose the curve with sides Δx and Δz in the xy plane shown in the diagram and apply Equation 30-6d to show that $\frac{\partial B_z}{\partial x} = -\mu_0 \in_0 \frac{\partial E_y}{\partial t}$.



Because Δx is very small, we can approximate the difference in B_z at the points x_1 and x_2 by:

$$B_z(x_2) - B_z(x_1) = \Delta B \approx \frac{\partial B_z}{\partial x}\Delta x$$

Then:

$$\oint_C \vec{\boldsymbol{B}}\cdot d\vec{\ell} \approx \mu_0 \in_0 \frac{\partial E_y}{\partial t}\Delta x\Delta z$$

The flux of the electric field through this curve is approximately:

$$\int_S E_\text{n}\, dA = E_y \Delta x \Delta y$$

Apply Faraday's law to obtain:

$$\frac{\partial B_z}{\partial x}\Delta x\Delta z = -\mu_0 \in_0 \frac{\partial E_y}{\partial t}\Delta x\Delta z$$

or

$$\boxed{\frac{\partial B_z}{\partial x} = -\mu_0 \in_0 \frac{\partial E_y}{\partial t}}$$

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Picture the Problem We can use Ohm's law to relate the electric field E in the conductor to I, ρ, and a and Ampere's law to find the magnetic field B just outside the conductor.

Knowing $\vec{E}$ and $\vec{B}$ we can find $\vec{S}$ and, using its normal component, show that the rate of energy flow into the conductor equals I^2R, where R is the resistance.

(*a*) Apply Ohm's law to the cylindrical conductor to obtain:

$$V = IR = \frac{I\rho L}{A} = \frac{I\rho L}{\pi a^2} = EL$$

Solve for E:

$$E = \boxed{\frac{I\rho}{\pi a^2}}$$

(*b*) Apply Ampere's law to a circular path of radius a at the surface of the cylindrical conductor:

$$\int_C \vec{B}\cdot d\vec{\ell} = B(2\pi a) = \mu_0 I_{\text{enclosed}} = \mu_0 I$$

Solve for B to obtain:

$$B = \boxed{\frac{\mu_0 I}{2\pi a}}$$

(*c*) The electric field at the surface of the conductor is in the direction of the current and the magnetic field at the surface is tangent to the surface. Use the results of (*a*) and (*b*) and the right-hand rule to evaluate $\vec{S}$:

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

$$= \frac{1}{\mu_0}\left(\frac{I\rho}{\pi a^2}\right)\hat{u}_{\text{parallel}} \times \left(\frac{\mu_0 I}{2\pi a}\right)\hat{u}_{\text{tangent}}$$

$$= \boxed{-\frac{I^2\rho}{2\pi^2 a^3}\hat{r}}$$

where $\hat{r}$ is a unit vector directed radially outward from the cylindrical conductor.

(*d*) The flux through the surface of the conductor into the conductor is:

$$\oint S_n dA = S(2\pi aL)$$

Substitute for S_n, the *inward* component of $\vec{S}$, and simplify to obtain:

$$\oint S_n dA = \frac{I^2\rho}{2\pi^2 a^3}(2\pi aL) = \frac{I^2\rho L}{\pi a^2}$$

Since $R = \frac{\rho L}{A} = \frac{\rho L}{\pi a^2}$:

$$\oint S_n dA = \boxed{I^2 R}$$

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Picture the Problem We can use Faraday's law to express the induced electric field at a distance $r < R$ from the solenoid axis in terms of the rate of change of magnetic flux and $B = n\mu_0 at$ to express B in terms of the current in the windings of the solenoid. We can use the results of (*a*) to find the magnitude and direction of the Poynting vector $\vec{S}$ at the cylindrical surface $r = R$ just inside the solenoid windings. In part (c) we'll use the definition of flux and the expression for the magnetic energy in a given region to show that the flux of $\vec{S}$ into the solenoid equals the rate of increase of the magnetic energy inside the solenoid.

(*a*) Apply Faraday's law to a circular path of radius $r < R$:

$$\oint_C E \cdot d\ell = E(2\pi r) = -\frac{d\phi_m}{dt}$$

Solve for E to obtain:

$$E = -\frac{1}{2\pi r}\frac{d\phi_m}{dt} \qquad (1)$$

Express the magnetic field inside a long solenoid:

$$B = n\mu_0 I = n\mu_0 at$$

The magnetic flux through a circle of radius r is:

$$\phi_m = BA = n\mu_0 at\pi r^2$$

Substitute in equation (1) to obtain:

$$E = -\frac{1}{2\pi r}\frac{d}{dt}\left[n\mu_0 at\pi r^2\right] = \boxed{-\frac{n\mu_0 a r}{2}}$$

(*b*) Express the magnitude of $\vec{S}$ at $r = R$:

$$S = \frac{EB}{\mu_0}$$

At the cylindrical surface just inside the windings:

$$B = n\mu_0 at$$

Substitute to obtain:

$$S = \frac{\left(\frac{n\mu_0 a R}{2}\right)(n\mu_0 at)}{\mu_0} = \frac{n^2\mu_0 a^2 Rt}{2}$$

Because the field $\vec{E}$ is tangential and directed so as to give an induced current that opposes the increase in $\vec{B}$, $\vec{E}\times\vec{B}$ is a vector that points toward the axis of the solenoid. Hence:

$$\vec{S} = \boxed{-\frac{n^2\mu_0 a^2 Rt}{2}\hat{r}}$$

where $\hat{r}$ is a unit vector that points radially outward.

(*c*) Consider a cylindrical surface of length L and radius R. Because $\vec{S}$ points inward, the energy flowing

$$\oint S_n dA = 2\pi RLS = 2\pi RL\left(\frac{n^2\mu_0 a^2 Rt}{2}\right)$$
$$= n^2\pi\mu_0 R^2 La^2 t$$

into the solenoid per unit time is:

Express the magnetic energy in the solenoid:

$$U_B = u_m V = \frac{B^2}{2\mu_0}\left(\pi R^2 L\right) = \frac{\left(\mu_0 nat\right)^2}{2\mu_0}\left(\pi R^2 L\right) = \frac{n^2 \pi \mu_0 R^2 L a^2 t^2}{2}$$

Evaluate dU_B/dt:

$$\frac{dU_B}{dt} = \frac{d}{dt}\left[\frac{n^2 \pi \mu_0 R^2 L a^2 t^2}{2}\right] = \boxed{n^2 \pi \mu_0 R^2 L a^2 t} = \oint S_n dA$$

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Picture the Problem We can use a condition for translational equilibrium to obtain an expression relating the forces due to gravity and radiation pressure that act on the particles. We can express the force due to radiation pressure in terms of the radiation pressure and the effective cross sectional area of the particles and the radiation pressure in terms of the intensity of the solar radiation. We can solve the resulting equation for r.

Apply the condition for translational equilibrium to the particle:

$$F_r - F_g = 0$$

or, since $F_r = P_r A$ and $F_g = mg$,

$$P_r A - \frac{GM_s m}{R^2} = 0 \qquad (1)$$

The radiation pressure P_r depends on the intensity of the radiation I:

$$P_r = \frac{I}{c}$$

The intensity of the solar radiation at a distance R is:

$$I = \frac{P}{4\pi R^2}$$

Substitute to obtain:

$$P_r = \frac{P}{4\pi R^2 c}$$

Substitute for P_r, A, and m in equation (1):

$$\frac{P}{4\pi R^2 c}\left(\pi r^2\right) - \frac{\frac{4}{3}\pi r^3 \rho G M_s}{R^2} = 0$$

Solve for R to obtain:

$$r = \frac{3P}{16\pi\rho c G M_s}$$

Substitute numerical values and evaluate r:

$$r = \frac{3\left(3.83\times10^{26}\ \text{W}\right)}{16\pi\left(1\,\text{g/cm}^3\right)\left(3\times10^8\ \text{m/s}\right)\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\ \text{kg}\right)} = \boxed{0.574\ \mu\text{m}}$$

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Picture the Problem

(a) At a perfectly conducting surface $\vec{E} = 0$. Therefore, the sum of the electric fields of the incident and reflected wave must add to zero, and so $\vec{E}_\text{i} = -\vec{E}_\text{r}$.

(b) Let the incident and reflected waves be described by:

$$E_\text{i} = E_{0y}\cos(\omega t - kx)$$

and

$$E_\text{r} = -E_{0y}\cos(\omega t + kx)$$

Use the trigonometric identity $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ to obtain:

$$\begin{aligned} E_\text{i} + E_\text{r} &= E_{0y}\cos(\omega t - kx) - E_{0y}\cos(\omega t + kx) = E_{0y}\left[\cos(\omega t - kx) - \cos(\omega t + kx)\right] \\ &= E_{0y}\left[\cos\omega t\cos(-kx) - \sin\omega t\sin(-kx) - \cos\omega t\cos kx + \sin\omega t\sin(kx)\right] \\ &= E_{0y}\left[\cos\omega t\cos kx + \sin\omega t\sin kx - \cos\omega t\cos kx + \sin\omega t\sin kx\right] \\ &= \boxed{2E_{0y}\sin\omega t\sin kx}, \text{ the equation of a standing wave.} \end{aligned}$$

(c) Because $\vec{E}\times\vec{B} = \mu_0\vec{S}$ and $\vec{S}$ is in the direction of propagation of the wave, we see that for the incident wave $B_\text{i} = B_z\cos(\omega t - kx)$. Since both $\vec{S}$ and E_y are reversed for the reflected wave, $B_r = B_z\cos(\omega t + kx)$. So the magnetic field vectors are in the direction at the reflecting surface and add at that surface. Hence $\vec{B} = 2\vec{B}_r$.

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Picture the Problem Let the point source be a distance a above the plane. Consider a ring of radius r and thickness dr in the plane and centered at the point directly below the light source. Express the force of force on this ring and integrate the resulting expression to obtain F.

The intensity anywhere along this infinitesimal ring is $P/4\pi(r^2 + a^2)$ and the element of force dF on this ring of area $2\pi rdr$ is given by:

$$dF = \frac{P\,rdr}{c\left(r^2 + a^2\right)} \frac{a}{\sqrt{r^2 + a^2}} = \frac{Pardr}{c\left(r^2 + a^2\right)^{3/2}}$$

where we have taken into account that only the normal component of the incident radiation contributes to the force on the plane, and that the plane is a perfectly reflecting plane.

Integrate dF from $r = 0$ to $r = \infty$:

$$F = \frac{Pa}{c}\int_0^\infty \frac{rdr}{\left(r^2 + a^2\right)^{3/2}}$$

From integral tables:

$$\int_0^\infty \frac{rdr}{\left(r^2 + a^2\right)^{3/2}} = \frac{-1}{\sqrt{r^2 + a^2}}\Bigg]_0^\infty = \frac{1}{a}$$

Substitute to obtain:

$$F = \frac{Pa}{c}\left(\frac{1}{a}\right) = \frac{P}{c}$$

Substitute numerical values and evaluate F:

$$F = \frac{1\,\text{MW}}{3\times10^8\ \text{m/s}} = \boxed{3.33\,\text{mN}}$$