Chapter 32
Optical Images

Conceptual Problems

1 •

Determine the Concept Yes. Note that a virtual image is ″seen″ because the eye focuses the diverging rays to form a real image on the retina. Similarly, the camera lens can focus the diverging rays onto the film.

2 •

Determine the Concept Yes; the mirror image is a left-handed coordinate system.

3 ••

(*a*) False. The virtual image formed by a concave mirror when the object is between the focal point and the vertex of the mirror depends on the distance of the object from the vertex.

(*b*) False. When the object is outside the focal point, the image is real.

(*c*) True.

(*d*) False. When the object is between the center of curvature and the focal point, the image is enlarged and real.

***4** ••

Determine the Concept Let s be the object distance and f the focal length of the mirror.

(*a*) If $s < f$, the image is virtual, upright, and larger than the object.

(*b*) If $s < f$, the image is virtual, upright, and larger than the object.

(*c*) If $s > 2f$, the image is real, inverted, and smaller than the object.

(*d*) If $f < s < 2f$, the image is real, inverted, and larger than the object.

5 ••

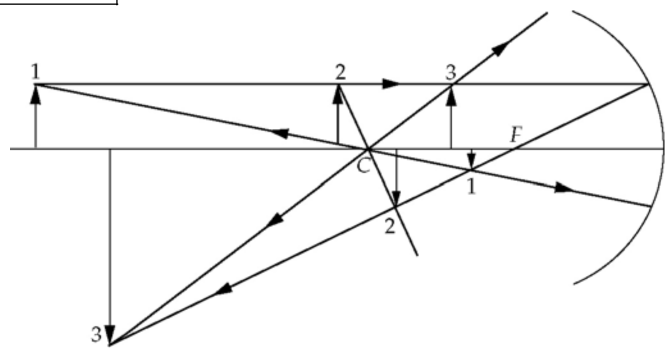
Determine the Concept A convex mirror always produces a virtual, upright image that is s than the object. It never produces an enlarged image.

6 ••

Determine the Concept They appear more distant because the images are smaller than they would be in a flat mirror.

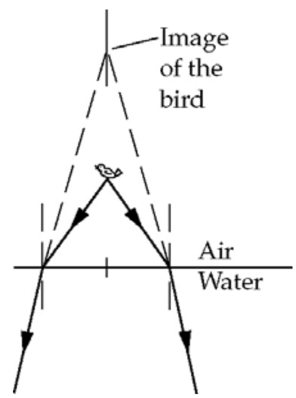
7 ••

Picture the Problem The ray diagram shows three object positions 1, 2, and 3 as the object is moved from a great distance toward the focal point F of a concave mirror. The real images corresponding to each of these object positions are labeled with the same numeral. $\boxed{(b) \text{ is correct.}}$



8 •

Picture the Problem The diagram shows two rays (from the bundle of rays) of light refracted at the air-water interface. Because the index of refraction of water is greater than that of air, the rays are bent toward the normal. The diver will, therefore, think that the rays are diverging from a point above the bird and so the bird appears to be farther from the surface than it actually is.



***9** •

Determine the Concept

(a) The lens will be positive if its index of refraction is greater than that of the surrounding medium and the lens is thicker in the middle than at the edges. Conversely, if the index of refraction of the lens is less than that of the surrounding medium, the lens will be positive if it is thinner at its center than at the edges.

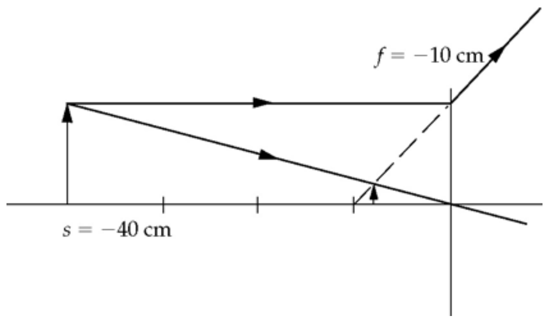
(b) The lens will be negative if its index of refraction is greater than that of the surrounding medium and the lens is thinner at the center than at the edges. Conversely, if the index of refraction of the lens is less than that of the surrounding medium, the lens will be negative if it is thicker at the center than at the edges.

10 •

Determine the Concept The focal length depends on the index of refraction, and *n* is a function of wavelength.

11 ••

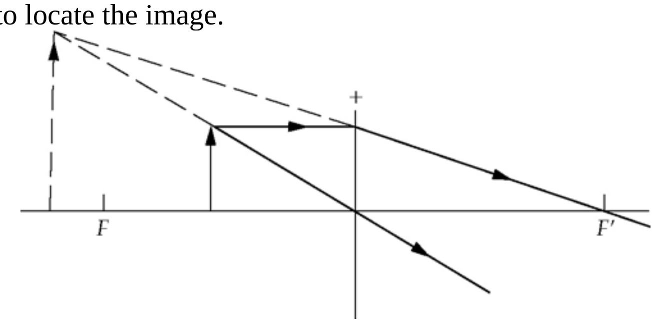
Picture the Problem We can use a ray diagram to determine the general features of the image. In the diagram shown, the parallel ray and central ray have been used to locate the image.



From the diagram, we see that the image is virtual (only one of the rays from the head of the object actually pass through the head of the image), upright, and diminished.
$\boxed{(d)\ \text{is correct.}}$

12 ••

Picture the Problem We can use a ray diagram to determine the general features of the image. In the diagram shown, the ray parallel to the principle axis and the central ray have been used to locate the image.



From the diagram, we see that the image is virtual (neither ray from the head of the object passes through the head of the image), upright, and enlarged. $\boxed{(c)\ \text{is correct.}}$

13 •

Determine the Concept The muscles in the eye change the thickness of the lens and thereby change the focal length of the lens to accommodate objects at different distances. A camera, on the other hand, has a fixed focal length so that focusing is accomplished by varying the distance between the lens and the film.

***14 •**

Determine the Concept The eye muscles of a farsighted person lack the ability to shorten the focal length of the lens in the eye sufficiently to form an image on the retina of the eye. A convex lens (a lens that is thicker in the middle than at the circumference) will bring the image forward onto the retina. $\boxed{(a)\text{ is correct.}}$

15 ••

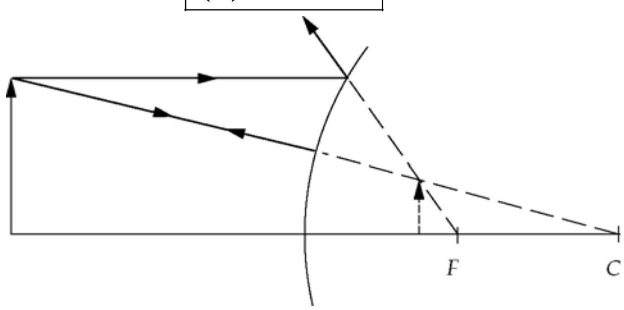
Determine the Concept Refraction of light at the water-cornea interface is less than at the air-cornea interface and so an image that would normally (that is, without a corrective lens) be in front of the retina, is formed on the retina. $\boxed{(b)\text{ is correct.}}$

16 ••

Determine the Concept A nearsighted person's lenses form sharp images (unless the person is also astigmatic) of nearby object's on the retinas of her eyes. The corrective lenses (convex) give a reduced image of the object and, therefore, should be removed. $\boxed{(b)\text{ is correct.}}$

***17 •**

Determine the Concept Referring to the ray diagram show below we note that the image is always virtual and diminished. $\boxed{(d)\text{ is correct.}}$



18 •

Determine the Concept Converging lenses can form real or virtual images that can be enlarged or reduced. $\boxed{(c)\text{ is correct.}}$

19 •

Picture the Problem We can apply the lens maker's equation to the air-glass lens and to the water-glass lens to find the ratio of their focal lengths.

Apply the lens maker's equation to the air-glass interface:

$$\frac{1}{f_{\text{air}}} = (1.6-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Apply the lens maker's equation to the water-glass interface:

$$\frac{1}{f_{\text{water}}}=(1.6-1.33)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

Divide the first of these equations by the second to obtain:

$$\frac{f_{\text{water}}}{f_{\text{air}}}=\frac{(1.6-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}{(1.6-1.33)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}=2.22$$

or $f_{\text{water}}=2.22\,f_{\text{air}}$ and $\boxed{(a)\text{ is correct.}}$

20 ••

(*a*) True.

(*b*) True.

(*c*) False. Where the rays intersect the axis of a spherical mirror depends on how far from the axis they are reflected from the mirror.

(*d*) True.

(*e*) False. The image distance for a virtual image is negative.

***21** •

Determine the Concept Microscopes ordinarily produce images (either the intermediate one produced by the objective or the one viewed through the eyepiece) that are larger than the object being viewed. A telescope, on the other hand, ordinarily produces images that are much reduced compared to the object. The object is normally viewed from a great distance and the telescope magnifies the angle subtended by the object.

Estimation and Approximation

22 ••

Picture the Problem We can use the lens-maker's equation to obtain a relationship between the two radii of curvature of the lenses we are to design.

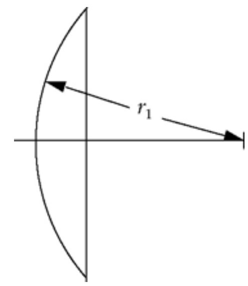
For a thin lens of focal length 27 cm and index of refraction of 1.6:

$$\frac{1}{27\,\text{cm}}=(1.6-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

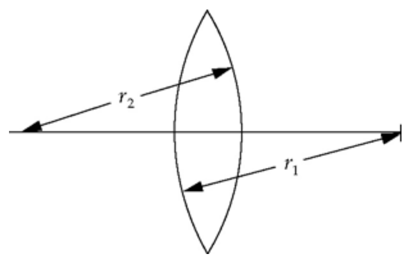
or

$$\frac{1}{r_1}-\frac{1}{r_2}=\frac{1}{16.2\,\text{cm}}$$

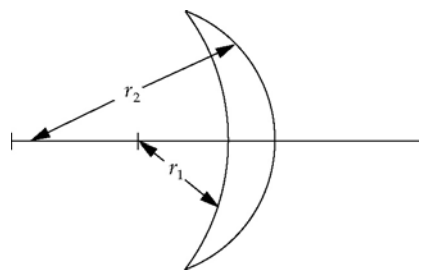
One solution is a plano-convex lens (one with a flat surface and a convex surface). Let $r_2 = \infty$. Then $r_1 = \boxed{16.2\,\text{cm}}$ and $r_2 = \boxed{\infty}$.



Another design is a double convex lens (one with both surfaces convex and radii of curvature that are equal in magnitude) obtained by letting $r_2 = -r_1$. Then $r_1 = \boxed{32.4\,\text{cm}}$ and $r_2 = \boxed{-32.4\,\text{cm}}$.



A third possibility is a double convex lens with unequal curvature, e.g., let $r_2 = 12$ cm. Then $r_1 = \boxed{6.89\,\text{cm}}$ and $r_2 = \boxed{12.0\,\text{cm}}$.



23 ••

Picture the Problem We can use the lens-maker's equation to obtain a relationship between the two radii of curvature of the lenses we are to design.

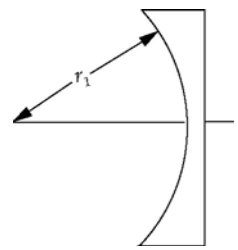
For a thin lens of focal length –27 cm and index of refraction of 1.6:

$$\frac{1}{-27\,\text{cm}} = (1.6-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

or

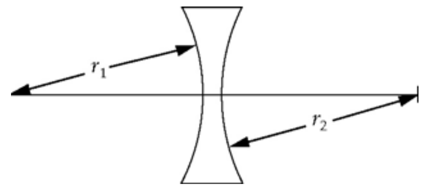
$$\frac{1}{r_1} - \frac{1}{r_2} = \frac{1}{-16.2\,\text{cm}}$$

One solution is a plano-concave lens (one with a flat surface and a concave surface), Let $r_2 = \infty$. Then $r_1 = \boxed{-16.2\,\text{cm}}$ and $r_2 = \boxed{\infty}$.

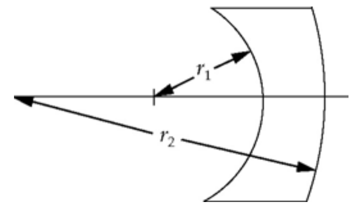


Another design is a biconcave lens

(one with both surfaces concave) by letting $r_2 = -r_1$. Then $r_1 = \boxed{-32.4\,\text{cm}}$ and $r_2 = \boxed{32.4\,\text{cm}}$.



A third possibility is a lens with r_2 = 8.1 cm. Then $r_1 = \boxed{5.40\,\text{cm}}$ and $r_2 = \boxed{16.2\,\text{cm}}$.



***24 ••**

Picture the Problem Because the focal length of a spherical lens depends on its radii of curvature and the magnification depends on the focal length, there is a practical upper limit to the magnification.

Use equation 32-20 to relate the magnification M of a simple magnifier to its focal length f:

$$M = \frac{x_{\text{np}}}{f}$$

Use the lens-maker's equation to relate the focal length of a lens to its radii of curvature and the index of refraction of the material from which it is constructed:

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

For a plano-convex lens, $r_2 = \infty$. Hence:

$$\frac{1}{f} = \frac{n-1}{r_1} \Rightarrow f = \frac{r_1}{n-1}$$

Substitute in the expression for M and simplify to obtain:

$$M = \frac{(n-1)x_{\text{np}}}{r_1}$$

Note that the smallest reasonable value for r_1 will maximize M.

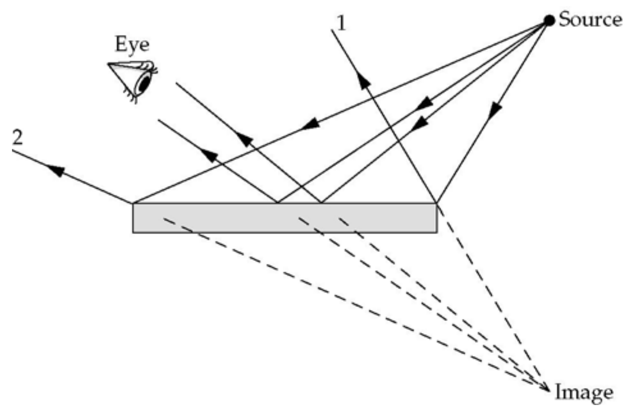
A reasonable smallest value for the radius of a magnifier is 1 cm. Use this value and n = 1.5 to estimate M_{max}:

$$M_{\text{max}} = \frac{(1.5-1)(25\,\text{cm})}{1\,\text{cm}} = \boxed{12.5}$$

Plane Mirrors

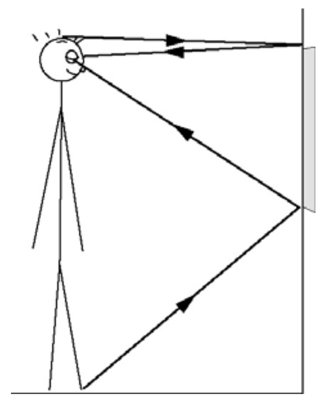
25 •

Determine the Concept Rays from the source and reflected by the mirror are shown. The reflected rays appear to diverge from the image. The eye can see the image if it is in the region between rays 1 and 2.



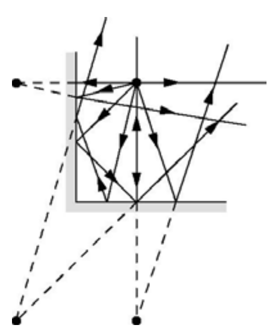
26 •

Determine the Concept The mirror must be half the height of the person, i.e., 81 cm. The top of the mirror must be 7.5 cm below the top of the head, or 154.5 cm above the floor. The bottom of the mirror must be 73.5 cm above the floor. A ray diagram showing rays from the person's feet and the top of her head reaching her eyes is shown to the right.



***27** •

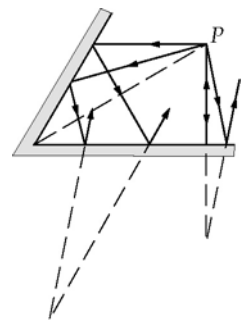
Determine the Concept Draw rays of light from the object that satisfy the law of reflection at the two mirror surfaces. Three virtual images are formed, as shown in the adjacent figure. The eye should be to the right and above the mirrors in order to see these images.



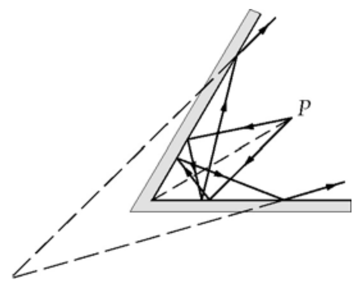
28 •

Determine the Concept Draw rays of light from the object that satisfy the law of reflection at the two mirror surfaces. The images are located at the intersection of the dashed lines (extensions of the reflected rays).

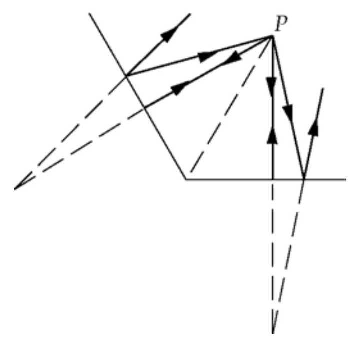
(*a*) The diagram to the right shows selected rays emanating from a point object (*P*) that form the two virtual images directly below the horizontal mirror:



The diagram to the right shows selected rays emanating from the point object (*P*) that form the image that lies on the bisector of the angle. There are two additional virtual images to the left of the mirror that is at 60° with the horizontal. Hence, the total number of images formed when a point object is on the bisector of the 60° angle is five.



(*b*) The diagram to the right shows selected rays emanating from a point object (*P*) that form the two virtual images at the intersection of the dashed lines (extensions of the reflected rays):



29 ••

Determine the Concept

(*a*) The first image in the mirror on the left is 10 cm behind the mirror. The mirror on the right forms an image 20 cm behind that mirror or 50 cm from the left mirror. This image will result in a second image 50 cm behind the left mirror. The first image in the left mirror is 40 cm from the right mirror and forms an image 40 cm behind the right mirror or 70 cm from the left mirror. That image gives an image 70 cm behind the left mirror. The fourth image behind the left mirror is 110 cm behind that mirror.

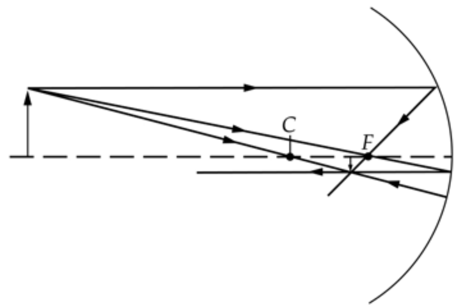
(*b*) Proceeding as in Part (*a*) for the mirror on the right, one finds the location of the images to be 20 cm, 40 cm, 80 cm, and 100 cm behind the right-hand mirror.

Spherical Mirrors

***30 ••**

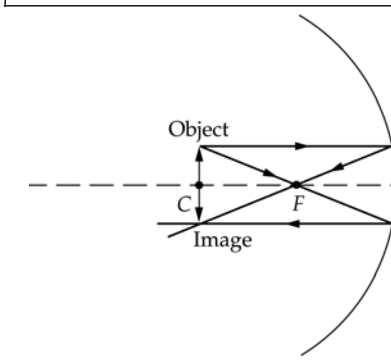
Picture the Problem The easiest rays to use in locating the image are 1) the ray parallel to the principal axis and passes through the focal point of the mirror, the ray that passes through the center of curvature of the spherical mirror and is reflected back on itself, and 2) the ray that passes through the focal point of the spherical mirror and is reflected parallel to the principal axis. We can use any two of these rays emanating from the top of the object to locate the image of the object.

(*a*) The ray diagram is shown to the right. The image is real, inverted, and reduced.



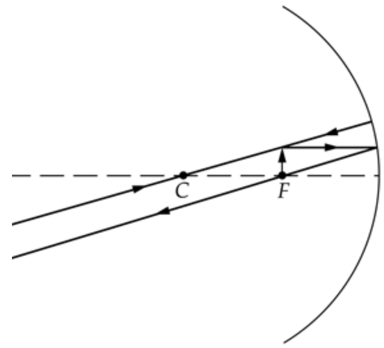
The image is real, inverted, and reduced.

(*b*) The ray diagram is shown to the right.



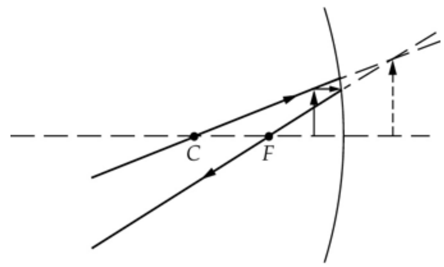
The image is real, inverted, and the same size as the object.

(c) The ray diagram is shown to the right. The object is at the focal point of the mirror.



The emerging rays are parallel and do not form an image.

(d) The ray diagram is shown to the right.



The image is virtual, erect, and enlarged.

31 •

Picture the Problem In describing the images, we must indicate where they are located, how large they are in relationship to the object, whether they are real or virtual, and whether they are upright or inverted. The object distance s, the image distance s', and the focal length of a mirror are related according to $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$, where $f=\frac{1}{2}r$ and r is the radius of curvature of the mirror. In this problem, f = 20 cm because r is positive for a concave mirror.

Solve the mirror equation for s':

$$s'=\frac{fs}{s-f}$$

(a) When s = 50 cm:

$$s'=\frac{(12\,\text{cm})(50\,\text{cm})}{50\,\text{cm}-12\,\text{cm}}=\boxed{15.8\,\text{cm}}$$

The lateral magnification of the image is:

$$m=-\frac{s'}{s}=-\frac{15.8\,\text{cm}}{50\,\text{cm}}=-0.316$$

Because the image distance is positive and the lateral magnification is less than one and negative, we can conclude that the image real, inverted, and reduced.

(*b*) When *s* = 24 cm:

$$s' = \frac{(12\,\text{cm})(24\,\text{cm})}{24\,\text{cm} - 12\,\text{cm}} = \boxed{24.0\,\text{cm}}$$

The lateral magnification of the image is:

$$m = -\frac{s'}{s} = -\frac{24\,\text{cm}}{24\,\text{cm}} = -1$$

Because the image distance is positive and the lateral magnification is one and negative, we can conclude that the image real, inverted, and the same size as the object.

(*c*) When *s* = 12 cm:

$$s' = \frac{(12\,\text{cm})(12\,\text{cm})}{12\,\text{cm} - 12\,\text{cm}} = \boxed{\infty}$$

and there is no image.

(*d*) When *s* = 8 cm:

$$s' = \frac{(12\,\text{cm})(8\,\text{cm})}{8\,\text{cm} - 12\,\text{cm}} = \boxed{-24.0\,\text{cm}}$$

The lateral magnification of the image is:

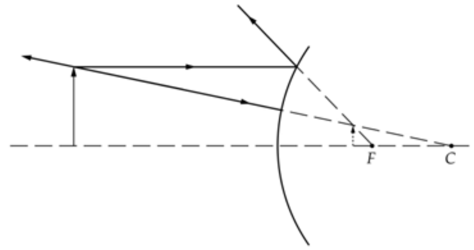
$$m = -\frac{s'}{s} = -\frac{-24\,\text{cm}}{8\,\text{cm}} = 3$$

Because the image distance is negative and the lateral magnification is three and positive, we can conclude that the image virtual, erect, and three times the size of the object.

32 ••

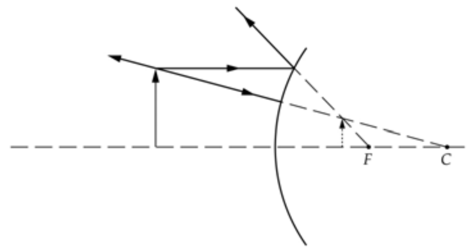
Picture the Problem The easiest rays to use in locating the image are 1) the ray parallel to the principal axis and passes through the focal point of the mirror, the ray that passes through the center of curvature of the spherical mirror and is reflected back on itself, and 2) the ray that passes through the focal point of the spherical mirror and is reflected parallel to the principal axis. We can use any two of these rays emanating from the top of the object to locate the image of the object.

(*a*) The ray diagram is shown to the right.



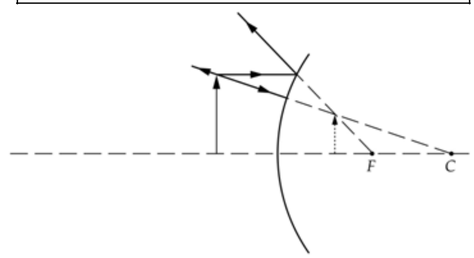
The image is virtual, upright, and reduced.

(*b*) The ray diagram is shown to the right.



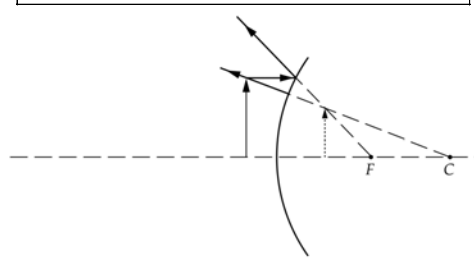
The image is virtual, upright, and reduced.

(*c*) The ray diagram is shown to the right.



The image is virtual, upright, and reduced.

(*d*) The ray diagram is shown to the right.



The image is virtual, upright, and reduced.

33 •

Picture the Problem In describing the images, we must indicate where they are located, how large they are in relationship to the object, whether they are real or virtual, and whether they are upright or inverted. The object distance *s*, the image distance *s*′, and the focal length of a mirror are related according to $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$, where $f=\frac{1}{2}r$ and *r* is the radius of curvature of the mirror. In this problem, $f = -20$ cm because *r* is negative for a convex mirror.

Solve the mirror equation for s':

$$s' = \frac{fs}{s - f}$$

(*a*) When s = 55 cm:

$$s' = \frac{(-12\,\text{cm})(55\,\text{cm})}{55\,\text{cm} - (-12\,\text{cm})} = \boxed{-9.85\,\text{cm}}$$

The lateral magnification of the image is:

$$m = -\frac{s'}{s} = -\frac{-9.85\,\text{cm}}{55\,\text{cm}} = 0.179$$

Because the image distance is negative and the lateral magnification is less than one in magnitude and positive, we can conclude that the image is virtual, upright, and reduced.

(*b*) When s = 24 cm:

$$s' = \frac{(-12\,\text{cm})(24\,\text{cm})}{24\,\text{cm} - (-12\,\text{cm})} = \boxed{-8.00\,\text{cm}}$$

The lateral magnification of the image is:

$$m = -\frac{s'}{s} = -\frac{-8\,\text{cm}}{24\,\text{cm}} = 0.333$$

Because the image distance is negative and the lateral magnification is less than one in magnitude and positive, we can conclude that the image is virtual, upright, and reduced.

(*c*) When s = 12 cm:

$$s' = \frac{(-12\,\text{cm})(12\,\text{cm})}{12\,\text{cm} - (-12\,\text{cm})} = \boxed{-6.00\,\text{cm}}$$

The lateral magnification of the image is:

$$m = -\frac{s'}{s} = -\frac{-6\,\text{cm}}{12\,\text{cm}} = \frac{1}{2}$$

Because the image distance is negative and the lateral magnification is one – half in magnitude and positive, we can conclude that the image virtual, upright, and half the size of the object.

(*d*) When s = 8 cm:

$$s' = \frac{(-12\,\text{cm})(8\,\text{cm})}{8\,\text{cm} - (-12\,\text{cm})} = \boxed{-4.80\,\text{cm}}$$

The lateral magnification of the image is:

$$m = -\frac{s'}{s} = -\frac{-4.80\,\text{cm}}{8\,\text{cm}} = 0.600$$

Because the image distance is negative and the lateral magnification is less than one and positive, we can conclude that the image is virtual, upright, and reduced.

34 •

Picture the Problem We can solve the mirror equation for 1/*s*′ and then examine the implications of $f < 0$ and $s > 0$.

Solve the mirror equation for 1/*s*′:

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{s - f}{sf}$$

For a convex mirror:

$$f < 0$$

With $s > 0$, the numerator is positive and the denominator negative. Consequently:

$$\frac{1}{s'} < 0 \Rightarrow \boxed{s' < 0}$$

***35** •

Picture the Problem We can use the mirror equation and the definition of the lateral magnification to find the radius of curvature of the mirror.

(*a*) Express the mirror equation:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} = \frac{2}{r}$$

Solve for *r*:

$$r = \frac{2ss'}{s' + s} \qquad (1)$$

The lateral magnification of the mirror is given by:

$$m = -\frac{s'}{s}$$

Solve for *s*′:

$$s' = -ms$$

Substitute for *s*′ in equation (1) to obtain:

$$r = \frac{-2ms}{1 - m}$$

Substitute numerical values and evaluate *r*:

$$r = \frac{-2(5.5)(2.1\,\text{cm})}{1 - 5.5} = \boxed{5.13\,\text{cm}}$$

(*b*) The mirror must be concave. A convex mirror always produces a diminished virtual image.

36 ••

Picture the Problem We can use the mirror equation and the relationship between the focal length of a mirror and its radius of curvature to find the location of the image. We can then use the definition of the lateral magnification of the mirror to find the height of the image formed in the mirror.

(*a*) and (*b*) Solve the mirror equation for for s':

$$s' = \frac{fs}{s - f}$$

Relate the focal length of the mirror to its radius of curvature:

$$f = \tfrac{1}{2}r$$

Substitute to obtain:

$$s' = \frac{\tfrac{1}{2}rs}{s - \tfrac{1}{2}r} = \frac{rs}{2s - r}$$

Substitute numerical values and evaluate s':

$$s' = \frac{(-1.2\,\text{m})(10\,\text{m})}{2(10\,\text{m}) - (-1.2\,\text{m})} = \boxed{-0.566\,\text{m}}$$

and

the image is 56.6 cm behind the mirror.

(*c*) Express the lateral magnification of the mirror:

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

Solve for y':

$$y' = -\frac{s'}{s}y$$

Substitute numerical values and evaluate y':

$$y' = -\frac{-0.566\,\text{m}}{10\,\text{m}}(2\,\text{m}) = \boxed{11.3\,\text{cm}}$$

37 ••

Picture the Problem We can use the mirror equation to locate the image formed in this mirror and the expression for the lateral magnification of the mirror to find the diameter of the image.

Solve the mirror equation for the location of the image of the moon:

$$s' = \frac{fs}{f - s}$$

Because $f = \frac{1}{2}r$:

$$s' = \frac{\frac{1}{2}rs}{\frac{1}{2}r - s} = \frac{rs}{r - 2s}$$

Substitute numerical values and evaluate s':

$$s' = \frac{(8\,\text{m})(3.8\times10^{8}\,\text{m})}{8\,\text{m} - 2(3.8\times10^{8}\,\text{m})} = \boxed{-4.00\,\text{m}}$$

Express the lateral magnification of the mirror:

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

Solve for y':

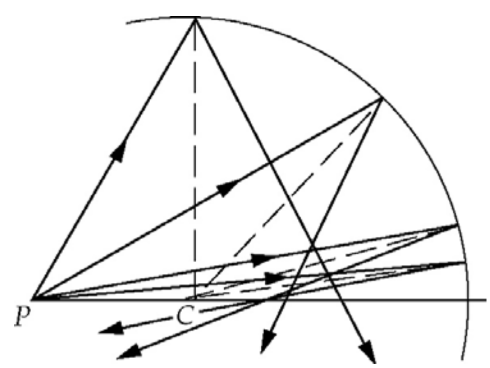
$$y' = -\frac{s'}{s}y$$

Substitute numerical values and evaluate y':

$$y' = -\frac{-4\,\text{m}}{3.8\times10^{8}\,\text{m}}(3.5\times10^{6}\,\text{m}) = \boxed{3.68\,\text{cm}}$$

38 ••

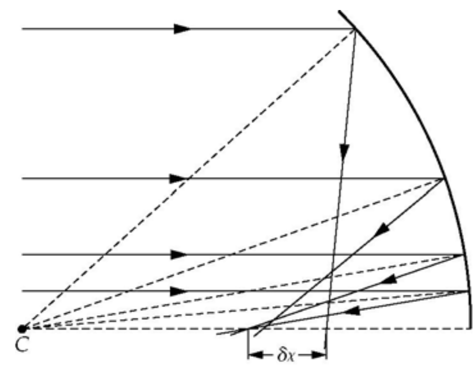
Picture the Problem The rays from the point object are shown in the diagram to the right. Note that the rays that reflect from the mirror far from the axis do not converge at the same point as those that reflect from the mirror close to the mirror axis. For the small-angle rays, the point of convergence is 4.5 cm from the mirror. The 60° ray crosses the axis at 3 cm from the mirror. Consequently, the image extends from 4.5 cm to 3.0 cm, or about 1.5 cm along the axis.



***39 ••**

Picture the Problem

(*a*) The figure to the right shows the mirror and the four rays drawn to scale. Using a calibrated ruler, the spread of the crossing points is $\delta x \approx 1.0$ cm. Note that the triangles formed by the center of curvature, the point of reflection on the mirror, and the point of intersection of the reflected ray and the mirror axis are isosceles triangles.



Express the equal angles of the isosceles triangles:

$$\theta_r = \sin^{-1}\left(\frac{y}{R}\right)$$

where y is the distance of the incoming ray from the mirror axis and R is the radius of curvature of the mirror.

Using the law of cosines, the distance between the point of intersection and the mirror is given by:

$$d = R\left\{1 - \left[2\cos\left(\sin^{-1}\left(\frac{y}{R}\right)\right)\right]^{-1}\right\}$$

Evaluate d for $y/R = 2/3$:

$$d = (6\,\text{cm})\left\{1 - \left[2\cos\left(\sin^{-1}\left(\frac{2}{3}\right)\right)\right]^{-1}\right\} = 1.975\,\text{cm}$$

Evaluate d for $y/R = 1/12$:

$$d = (6\,\text{cm})\left\{1 - \left[2\cos\left(\sin^{-1}\left(\frac{1}{12}\right)\right)\right]^{-1}\right\} = 2.990\,\text{cm}$$

Express the spread δx:

$$\delta x = 2.990\,\text{cm} - 1.975\,\text{cm} = \boxed{1.01\,\text{cm}}$$

in good agreement with the result obtained above.

(*b*) Evaluate *d* for *y*/*R* = 1/3:

$$d = (6\,\text{cm})\left\{1 - \left[2\cos\left(\sin^{-1}\left(\frac{1}{3}\right)\right)\right]^{-1}\right\}$$
$$= 2.818\,\text{cm}$$

Express the new spread $\delta x'$:

$$\delta x' = 2.990\,\text{cm} - 2.818\,\text{cm} = 0.172\,\text{cm}$$

Express the ratio of $\delta x'$ to δx:

$$\frac{\delta x'}{\delta x} = \frac{0.172\,\text{cm}}{1.01\,\text{cm}} = 17.0\%$$

By blocking off the edges of the mirror so that only paraxial rays within 2 cm of the mirror axis are reflected, the spread is reduced by 83.0%.

40 ••

Picture the Problem We can use the mirror equation to find the focal length of the mirror and then apply it a second time to find the object position after the mirror has been moved.

Solve the mirror equation for *f*:

$$f = \frac{ss'}{s' + s}$$

Substitute numerical values and evaluate *f*:

$$f = \frac{(100\,\text{cm})(75\,\text{cm})}{75\,\text{cm} + 100\,\text{cm}} = 42.86\,\text{cm}$$

Solve the mirror equation for *s*:

$$s = \frac{fs'}{s' - f}$$

Find *s* for *f* = –42.86 cm and *s*′ = – 35 cm:

$$s = \frac{(-42.86\,\text{cm})(-35\,\text{cm})}{-35\,\text{cm} - (-42.86\,\text{cm})} = 190.9\,\text{cm}$$

The distance *d* the mirror moved is:

$$d = 190.9\,\text{cm} - 100\,\text{cm} = \boxed{90.9\,\text{cm}}$$

41 ••

Picture the Problem We can use the mirror equation, with $s = \infty$, to find the image distance in the large mirror. Because this image serves as a virtual object for the small mirror, we can use the mirror equation a second time to find the focal length and, hence, the radius of curvature of the small mirror.

(*a*) Express the mirror equation:

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{r}$$

Because $s = \infty$: $\frac{1}{s'} = \frac{2}{r}$ and $s' = \frac{1}{2}r$

Substitute numerical values and evaluate s': $s' = \frac{1}{2}(5\,\text{m}) = 2.5\,\text{m}$

This image serves as a virtual object for the small mirror at $s = -0.5$ m. Solve the mirror equation for the focal length of the small mirror: $f_{\text{small}} = \frac{ss'}{s'+s}$

Substitute numerical values and evaluate f_{small}: $f_{\text{small}} = \frac{(-0.5\,\text{m})(2\,\text{m})}{2\,\text{m} + (-0.5\,\text{m})} = -0.667\,\text{m}$

The radius of curvature is twice the focal length:

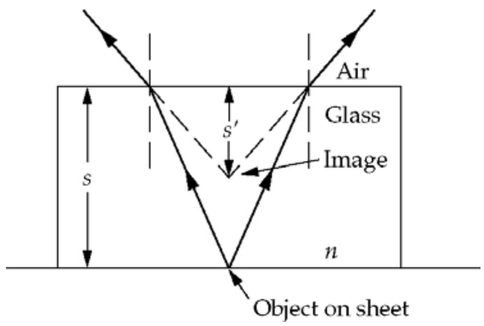
$$r_{\text{small}} = 2f_{\text{small}} = 2(-0.667\,\text{m}) = \boxed{-1.33\,\text{m}}$$

(*b*) $\boxed{\text{Because } f_{\text{small}} < 0, \text{ the small mirror is convex.}}$

Images Formed by Refraction

42 •

Picture the Problem The diagram shows two rays (from the bundle of rays) of light refracted at the glass-air interface. Because the index of refraction of air is less than that of water, the rays are bent away from the normal. The writing on the paper will, therefore, appear to be closer than it actually is. We can use the equation for refraction at a single surface to find the distance s'.



Use the equation for refraction at a single surface to relate the image and object distances: $\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{r}$

Here we have $n_1 = n$, $n_2 = 1$, and $r = \infty$. Therefore: $\frac{n}{s} + \frac{1}{s'} = 0$

Solve for s':

$$s' = -\frac{s}{n}$$

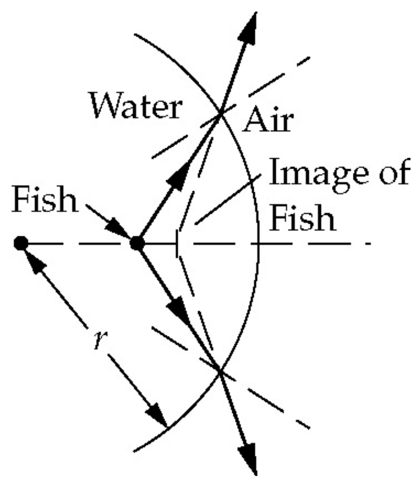
Substitute numerical values and evaluate s':

$$s' = -\frac{2\,\text{cm}}{1.5} = \boxed{-1.33\,\text{cm}}$$

where the minus sign tells us that the image is 1.33 cm below the glass surface.

43 •

Picture the Problem The diagram shows two rays (from the bundle of rays) of light refracted at the water-air interface. Because the index of refraction of air is less than that of water, the rays are bent away from the normal. The fish will, therefore, appear to be closer than it actually is. We can use the equation for refraction at a single surface to find the distance s'. We'll assume that the glass bowl is thin enough that we can ignore the refraction of the light passing through it.



(*a*) Use the equation for refraction at a single surface to relate the image and object distances:

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{r}$$

Here we have $n_1 = n$ and $n_2 = 1$. Therefore:

$$\frac{n}{s} + \frac{1}{s'} = \frac{1-n}{r}$$

Solve for s':

$$s' = \frac{rs}{s(1-n) - nr}$$

Substitute numerical values and evaluate s':

$$s' = \frac{(-20\,\text{cm})(10\,\text{cm})}{(10\,\text{cm})(1-1.33)-(1.33)(-20\,\text{cm})} = \boxed{-8.54\,\text{cm}}$$

where the minus sign tells us that the image is 8.54 cm from the front surface of the bowl.

(*b*) Repeat (*a*) with s = 30 cm:

$$s' = \frac{(-20\,\text{cm})(30\,\text{cm})}{(30\,\text{cm})(1-1.33)-(1.33)(-20\,\text{cm})} = \boxed{-35.9\,\text{cm}}$$

where the minus sign tells us that the image is 35.9 cm from the front surface of the bowl.

***44 ••**

Picture the Problem We can use the equation for refraction at a single surface to find the images corresponding to these three object positions. The signs of the image distances will tell us whether the images are real or virtual and the ray diagrams will confirm the correctness of our analytical solutions.

Use the equation for refraction at a single surface to relate the image and object distances:

$$\frac{n_1}{s}+\frac{n_2}{s'}=\frac{n_2-n_1}{r} \qquad (1)$$

Here we have n_1 = 1 and $n_2 = n$ = 1.5. Therefore:

$$\frac{1}{s}+\frac{n}{s'}=\frac{n-1}{r}$$

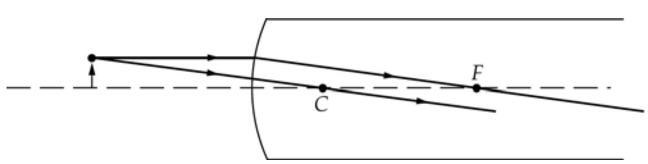
Solve for s':

$$s' = \frac{nrs}{s(n-1)-r}$$

(*a*) Substitute numerical values (s = 35 cm and r = 7.2 cm) and evaluate s':

$$s' = \frac{(1.5)(7.2\,\text{cm})(35\,\text{cm})}{(35\,\text{cm})(1.5-1)-(7.2\,\text{cm})} = \boxed{36.7\,\text{cm}}$$

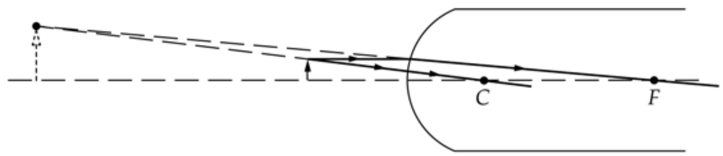
where the positive distance tells us that the image is 36.7 cm in back of the surface and is $\boxed{\text{real.}}$



(*b*) Substitute numerical values (s = 6.5 cm and r = 7.2 cm) and evaluate s':

$$s' = \frac{(1.5)(7.2\,\text{cm})(6.5\,\text{cm})}{(6.5\,\text{cm})(1.5-1)-(7.2\,\text{cm})} = \boxed{-17.8\,\text{cm}}$$

where the minus sign tells us that the image is 17.8 cm in front of the surface and is virtual.



(*c*) When $s = \infty$, equation (1) becomes:

$$\frac{n}{s'} = \frac{n-1}{r}$$

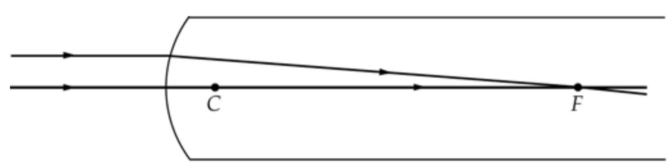
Solve for s':

$$s' = \frac{nr}{n-1}$$

Substitute numerical values and evaluate s':

$$s' = \frac{(1.5)(7.2\,\text{cm})}{1.5-1} = \boxed{21.6\,\text{cm}}$$

i.e., the image is at the focal point, is real, and of zero size.



45 ••

Picture the Problem We can use the equation for refraction at a single surface to find the image distance that corresponds to parallel light rays in the rod.

Use the equation for refraction at a single surface to relate the image and object distances:

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{r} \qquad (1)$$

Parallel rays imply that $s' = \infty$. Therefore:

$$\frac{1}{s} = \frac{n-1}{r}$$

Solve for s:

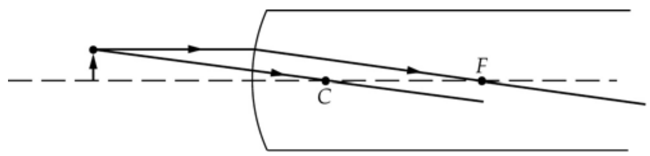
$$s = \frac{r}{n-1}$$

Substitute numerical values and

$$s = \frac{7.2\,\text{cm}}{1.5-1} = \boxed{14.4\,\text{cm}}$$

evaluate *s*:

The ray diagram is shown below:



46 ••

Picture the Problem We can use the equation for refraction at a single surface to find the images corresponding to these three object positions. The signs of the image distances will tell us whether the images are real or virtual and the ray diagrams will confirm the correctness of our analytical solutions.

Use the equation for refraction at a single surface to relate the image and object distances:

$$\frac{n_1}{s}+\frac{n_2}{s'}=\frac{n_2-n_1}{r} \qquad (1)$$

Here we have $n_1 = 1$ and $n_2 = n = 1.5$. Therefore:

$$\frac{1}{s}+\frac{n}{s'}=\frac{n-1}{r}$$

Solve for s':

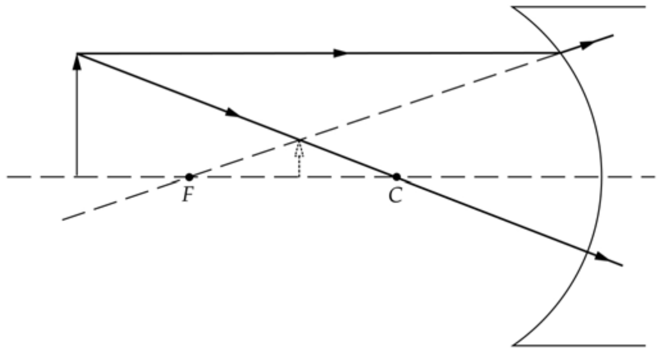
$$s'=\frac{nrs}{s(n-1)-r}$$

(*a*) Substitute numerical values ($s = 35$ cm and $r = -7.2$ cm) and evaluate s':

$$s'=\frac{(1.5)(-7.2\,\text{cm})(35\,\text{cm})}{(35\,\text{cm})(1.5-1)-(-7.2\,\text{cm})}$$

$$=\boxed{-15.3\,\text{cm}}$$

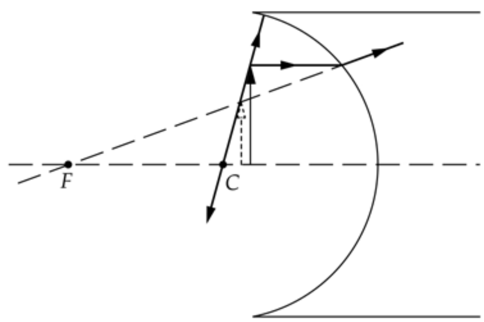
where the minus sign tells us that the image is 15.3 cm in front of the surface of the rod and is $\boxed{\text{virtual.}}$



(*b*) Substitute numerical values (s = 6.5 cm and r = –7.2 cm) and evaluate s':

$$s' = \frac{(1.5)(-7.2\,\text{cm})(6.5\,\text{cm})}{(6.5\,\text{cm})(1.5-1)-(-7.2\,\text{cm})} = \boxed{-6.72\,\text{cm}}$$

where the minus sign tells us that the image is 6.72 cm in front of the surface of the rod (located at the object) and is $\boxed{\text{virtual.}}$



(*c*) When $s = \infty$, equation (1) becomes:

$$\frac{n}{s'} = \frac{n-1}{r}$$

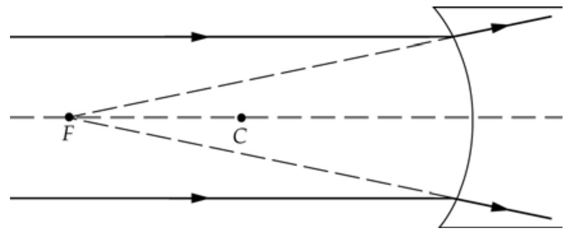
Solve for s':

$$s' = \frac{nr}{n-1}$$

Substitute numerical values and evaluate s':

$$s' = \frac{(1.5)\,(-7.2\,\text{cm})}{1.5-1} = \boxed{-21.6\,\text{cm}}$$

where the minus sign tells us that the image is 21.6 cm in front of the surface of the rod and is $\boxed{\text{virtual.}}$



47 ••

Picture the Problem We can use the equation for refraction at a single surface to find the images corresponding to these three object positions. The signs of the image distances will tell us whether the images are real or virtual and the ray diagrams will confirm the correctness of our analytical solutions.

Use the equation for refraction at a single surface to relate the image and object distances:

$$\frac{n_1}{s}+\frac{n_2}{s'}=\frac{n_2-n_1}{r} \quad (1)$$

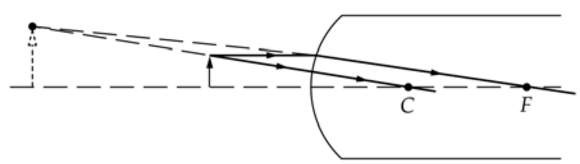
Solve for s':

$$s'=\frac{n_2 rs}{s(n_2-n_1)-n_1 r}$$

(*a*) Substitute numerical values (s = 35 cm, n_1 = 1.33, n_2 = 1.5, and r = 7.2 cm) and evaluate s':

$$s'=\frac{(1.5)(7.2\,\text{cm})(35\,\text{cm})}{(35\,\text{cm})(1.5-1.33)-(1.33)(7.2\,\text{cm})}=\boxed{-104\,\text{cm}}$$

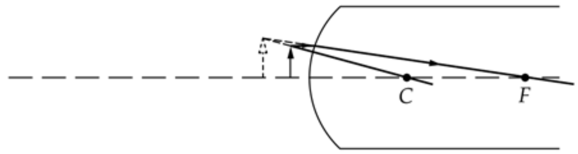
where the negative distance tells us that the image is 104 cm in front of the surface and is $\boxed{\text{virtual.}}$



(*b*) Substitute numerical values (s = 6.5 cm) and evaluate s':

$$s'=\frac{(1.5)(7.2\,\text{cm})(6.5\,\text{cm})}{(6.5\,\text{cm})(1.5-1.33)-(1.33)(7.2\,\text{cm})}=\boxed{-8.29\,\text{cm}}$$

where the minus sign tells us that the image is 8.29 cm in front of the surface and is $\boxed{\text{virtual.}}$



(*c*) When $s = \infty$, equation (1) becomes:

$$\frac{n_2}{s'} = \frac{n_2 - n_1}{r}$$

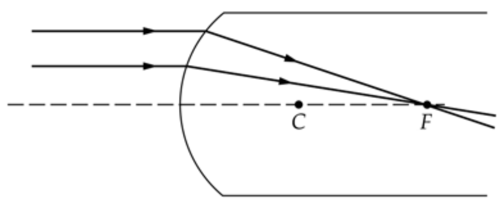
Solve for s':

$$s' = -\frac{n_2}{n_2 - n_1} r$$

Substitute numerical values and evaluate s':

$$s' = \frac{1.5}{1.5 - 1.33}(7.2\,\text{cm}) = \boxed{63.5\,\text{cm}}$$

i.e., the image is 63.5 cm to the right of the surface (at the focal point) and is $\boxed{\text{real.}}$



48 ••

Picture the Problem We can use the equation for refraction at a single surface to find the images corresponding to these three object positions. The signs of the image distances will tell us whether the images are real or virtual and the ray diagrams will confirm the correctness of our analytical solutions.

Use the equation for refraction at a single surface to relate the image and object distances:

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{r} \qquad (1)$$

Solve for s':

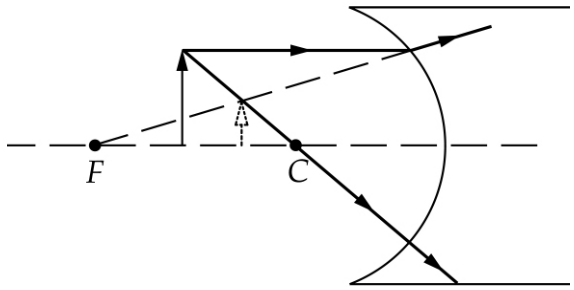
$$s' = \frac{n_2 r s}{s(n_2 - n_1) - n_1 r}$$

(*a*) Substitute numerical values (s = 35 cm) and evaluate s':

$$s' = \frac{(1.5)(-7.5\,\text{cm})(35\,\text{cm})}{(35\,\text{cm})(1.5 - 1.33) - (1.33)(-7.5\,\text{cm})}$$

$$= \boxed{-24.7\,\text{cm}}$$

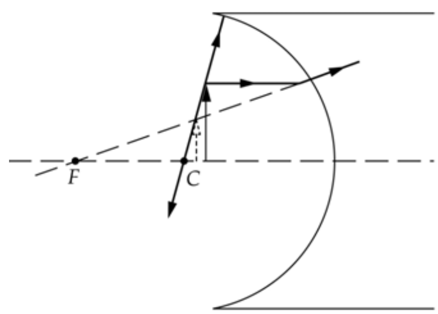
where the minus sign tells us that the image is 24.7 cm in front of the surface and is $\boxed{\text{virtual.}}$



(*b*) Substitute numerical values (s = 6.5 cm) and evaluate s':

$$s' = \frac{(1.5)(-7.5\,\text{cm})(6.5\,\text{cm})}{(6.5\,\text{cm})(1.5-1.33)-(1.33)(-7.5\,\text{cm})} = \boxed{-6.60\,\text{cm}}$$

where the minus sign tells us that the image is 6.60 cm in front of the surface and is $\boxed{\text{virtual.}}$



(*c*) When $s = \infty$, equation (1) becomes:

$$\frac{n_2}{s'} = \frac{n_2 - n_1}{r}$$

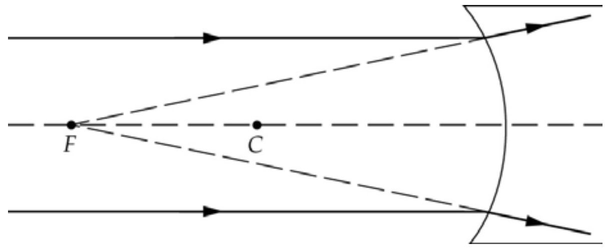
Solve for s':

$$s' = \frac{n_2 r}{n_2 - n_1}$$

Substitute numerical values and evaluate s':

$$s' = \frac{(1.5)(-7.5\,\text{cm})}{1.5-1.33} = \boxed{-66.2\,\text{cm}}$$

i.e., the image is at the focal point, is $\boxed{\text{virtual,}}$ and of zero size.



***49** ••

Picture the Problem We can use the equation for refraction at a single surface to find the images due to refraction at the ends of the glass rod. The image formed by the refraction at the first surface will serve as the object for the second surface. The sign of the final image distance will tell us whether the image is real or virtual.

(*a*) Use the equation for refraction at a single surface to relate the image and object distances at the first surface:

$$\frac{n_1}{s}+\frac{n_2}{s'}=\frac{n_2-n_1}{r} \qquad (1)$$

Solve for s':

$$s'=\frac{n_2 rs}{s(n_2-n_1)-n_1 r}$$

Substitute numerical values and evaluate s':

$$s'=\frac{(1.6)(8\,\text{cm})(20\,\text{cm})}{(20\,\text{cm})(1.6-1)-(8\,\text{cm})}=\boxed{64.0\,\text{cm}}$$

(*b*) The object for the second surface is 96 cm – 64 cm = 32 cm from the surface whose radius is 16 cm. Substitute numerical values and evaluate s':

$$s'=\frac{(1)(-16\,\text{cm})(32\,\text{cm})}{(32\,\text{cm})(1-1.6)-(1.6)(-16\,\text{cm})}=\boxed{-80.0\,\text{cm}}$$

(*c*) $\boxed{\text{The final image is } 96\text{ cm} - 80\text{ cm} = 16\text{ cm from the surface whose radius is 8 cm and is virtual.}}$

50 ••

Picture the Problem We can use the equation for refraction at a single surface to find the images due to refraction at the ends of the glass rod. The image formed by the refraction at the first surface will serve as the object for the second surface. The sign of the final image distance will tell us whether the image is real or virtual.

(*a*) Use the equation for refraction at a single surface to relate the image and object distances at the first surface:

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{r} \qquad (1)$$

Solve for *s*′:

$$s' = \frac{n_2 r s}{s(n_2 - n_1) - n_1 r}$$

Substitute numerical values and evaluate *s*′:

$$s' = \frac{(1.6)(16\,\text{cm})(20\,\text{cm})}{(20\,\text{cm})(1.6 - 1) - (16\,\text{cm})} = \boxed{-128\,\text{cm}}$$

(*b*) The object for the second surface is 96 cm + 128 cm = 224 cm from the surface whose radius is 8 cm. Substitute numerical values and evaluate *s*′:

$$s' = \frac{(1)(-8\,\text{cm})(224\,\text{cm})}{(224\,\text{cm})(1 - 1.6) - (1.6)(-8\,\text{cm})} = \boxed{14.7\,\text{cm}}$$

(*c*) $\boxed{\text{The final image is 14.7 cm from the far end of the rod and is real.}}$

Thin Lenses

51 •

Picture the Problem We can use the lens-maker's equation to find the focal length of each of the lenses.

The lens-maker's equation is:

$$\frac{1}{f} = (n - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

where the numerals 1 and 2 denote the first and second surfaces, respectively.

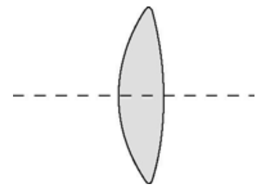
(*a*) For r_1 = 15 cm and r_2 = –26 cm:

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{15\,\text{cm}} - \frac{1}{-26\,\text{cm}}\right)$$

and

$$f = \boxed{19.0\,\text{cm}}$$

A double convex lens is shown to the right:



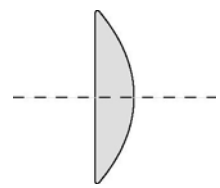
(*b*) For $r_1 = \infty$ and $r_2 = -15$ cm:

$$\frac{1}{f} = (1.5-1)\left(\frac{1}{\infty} - \frac{1}{-15\,\text{cm}}\right)$$

and

$$f = \boxed{30.0\,\text{cm}}$$

A plano-convex lens is shown to the right:



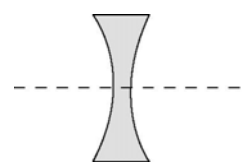
(*c*) For $r_1 = -15$ cm and $r_2 = +15$ cm:

$$\frac{1}{f} = (1.5-1)\left(\frac{1}{-15\,\text{cm}} - \frac{1}{15\,\text{cm}}\right)$$

and

$$f = \boxed{-15.0\,\text{cm}}$$

A double concave lens is shown to the right:



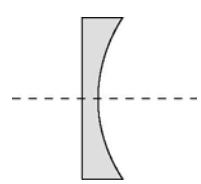
(*d*) For $r_1 = \infty$ and $r_2 = +26$ cm:

$$\frac{1}{f} = (1.5-1)\left(\frac{1}{\infty} - \frac{1}{26\,\text{cm}}\right)$$

and

$$f = \boxed{-52.0\,\text{cm}}$$

A plano-concave lens is shown to the right:



52 •

Picture the Problem We can use the lens-maker's equation to find the focal length of the lens.

The lens-maker's equation is:

$$\frac{1}{f}=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

where the numerals 1 and 2 denote the first and second surfaces, respectively.

Substitute numerical values to obtain:

$$\frac{1}{f}=(1.62-1)\left(\frac{1}{-100\,\text{cm}}-\frac{1}{-40\,\text{cm}}\right)$$

Solve for *f*:

$$f=\boxed{108\,\text{cm}}$$

***53** •

Picture the Problem We can use the lens-maker's equation to find the focal length of the lens and the thin-lens equation to locate the image. We can use $m=-\frac{s'}{s}$ to find the lateral magnification of the image.

(*a*) The lens-maker's equation is:

$$\frac{1}{f}=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

where the numerals 1 and 2 denote the first and second surfaces, respectively.

Substitute numerical values to obtain:

$$\frac{1}{f}=(1.45-1)\left(\frac{1}{-30\,\text{cm}}-\frac{1}{25\,\text{cm}}\right)$$

Solve for *f*:

$$f=\boxed{-30.3\,\text{cm}}$$

(*b*) Use the thin-lens equation to relate the image and object distances:

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

Solve for *s*′:

$$s'=\frac{fs}{s-f}$$

Substitute numerical values and evaluate *s*′:

$$s'=\frac{(-30.3\,\text{cm})(80\,\text{cm})}{80\,\text{cm}-(-30.3\,\text{cm})}=\boxed{-22.0\,\text{cm}}$$

(c) The lateral magnification of the image is given by:

$$m = -\frac{s'}{s}$$

Substitute numerical values and evaluate m:

$$m = -\frac{-22\,\text{cm}}{80\,\text{cm}} = \boxed{0.275}$$

(d) Because $s' < 0$ and $m > 0$, the image is $\boxed{\text{virtual and upright.}}$

54 •

Picture the Problem We can use the lens-maker's equation to find the focal length of each of the lenses described in the problem statement.

The lens-maker's equation is:

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

where the numerals 1 and 2 denote the first and second surfaces, respectively.

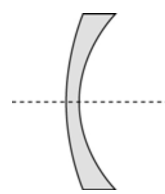
(a) For r_1 = 20 cm, r_2 = 10 cm:

$$\frac{1}{f} = (1.6-1)\left(\frac{1}{20\,\text{cm}} - \frac{1}{10\,\text{cm}}\right)$$

and

$$f = \boxed{-33.3\,\text{cm}}$$

A sketch of the lens is shown to the right:



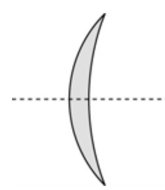
(b) For r_1 = 10 cm, r_2 = 20 cm:

$$\frac{1}{f} = (1.6-1)\left(\frac{1}{10\,\text{cm}} - \frac{1}{20\,\text{cm}}\right)$$

and

$$f = \boxed{33.3\,\text{cm}}$$

A sketch of the lens is shown to the right:



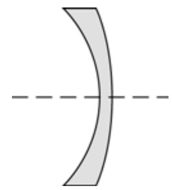
(c) For r_1 = –10 cm, r_2 = –20 cm:

$$\frac{1}{f} = (1.6-1)\left(\frac{1}{-10\,\text{cm}} - \frac{1}{-20\,\text{cm}}\right)$$

and

$$f = \boxed{-33.3\,\text{cm}}$$

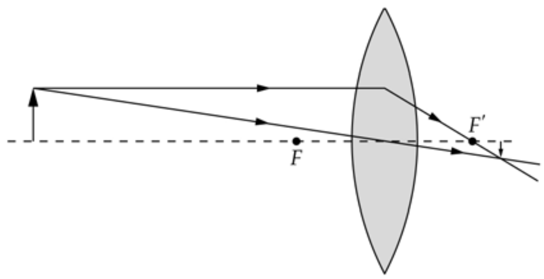
A sketch of the lens is shown to the right:



Remarks: Note that the lenses that are thicker on their axis than on their circumferences are positive (converging) lenses and those that are thinner on their axis are negative (diverging) lenses.

***55** •

Picture the Problem The parallel and central rays were used to locate the image in the diagram shown below. The power P of the lens, in diopters, can be found from $P = 1/f$ and the size of the image from $m = \frac{y'}{y} = -\frac{s'}{s}$.



The image is real, inverted, and diminished.

The thin-lens equation is:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

Solve for s':

$$s' = \frac{fs}{s-f}$$

Use the definition of the power of the lens to find its focal length:

$$f = \frac{1}{P} = \frac{1}{10\,\text{m}^{-1}} = 0.1\,\text{m} = 10\,\text{cm}$$

Substitute numerical values and evaluate s':

$$s' = \frac{(10\,\text{cm})(25\,\text{cm})}{25\,\text{cm} - 10\,\text{cm}} = \boxed{16.7\,\text{cm}}$$

Use the lateral magnification equation to relate the height of the image y' to the height y of the object and the image and object distances:

$$m=\frac{y'}{y}=-\frac{s'}{s}$$

Solve for y':

$$y'=-\frac{s'}{s}y$$

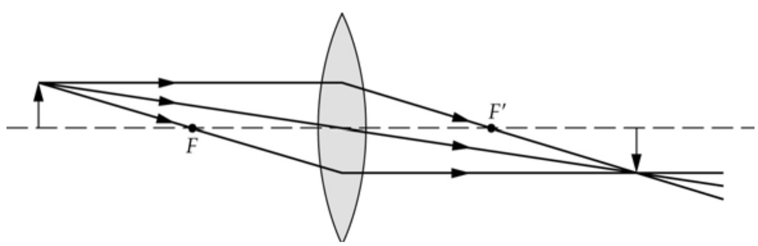
Substitute numerical values and evaluate y':

$$y'=-\frac{16.7\,\text{cm}}{25\,\text{cm}}(3\,\text{cm})=\boxed{-2.00\,\text{cm}}$$

Because $s'>0$ and $y'=-2.00$ cm, the image is real, inverted, and diminished in agreement with the ray diagram.

56 •

Picture the Problem The parallel and central rays were used to locate the image in the diagram shown below. The power P of the lens, in diopters, can be found from $P=1/f$ and the size of the image from $m=\frac{y'}{y}=-\frac{s'}{s}$.



The image is real and inverted and appears to be the same size as the object.

The thin-lens equation is:

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

Solve for s':

$$s'=\frac{fs}{s-f}$$

Use the definition of the power of the lens to find its focal length:

$$f=\frac{1}{P}=\frac{1}{10\,\text{m}^{-1}}=0.1\,\text{m}=10\,\text{cm}$$

Substitute numerical values and evaluate s':

$$s'=\frac{(10\,\text{cm})(20\,\text{cm})}{20\,\text{cm}-10\,\text{cm}}=\boxed{20.0\,\text{cm}}$$

Use the lateral magnification equation to relate the height of the image y' to the height y of the object and the image and object distances:

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

Solve for y':

$$y' = -\frac{s'}{s}y$$

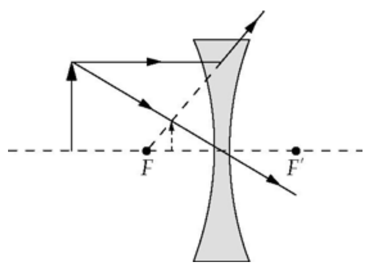
Substitute numerical values and evaluate y':

$$y' = -\frac{20\,\text{cm}}{20\,\text{cm}}(1\,\text{cm}) = \boxed{-1.00\,\text{cm}}$$

Because $s' > 0$ and $y' = -1\,\text{cm}$, the image is real, inverted, and the same size as the object in agreement with the ray diagram.

57 •

Picture the Problem The parallel and central rays were used to locate the image in the diagram shown below. The power P of the lens, in diopters, can be found from $P = 1/f$ and the size of the image from $m = \dfrac{y'}{y} = -\dfrac{s'}{s}$.



The image is virtual, upright, and diminished.

The thin-lens equation is:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

Solve for s':

$$s' = \frac{fs}{s-f}$$

Use the definition of the power of the lens to find its focal length:

$$f = \frac{1}{P} = \frac{1}{-10\,\text{m}^{-1}} = -0.1\,\text{m} = -10\,\text{cm}$$

Substitute numerical values and evaluate s':

$$s' = \frac{(-10\,\text{cm})(20\,\text{cm})}{20\,\text{cm} - (-10\,\text{cm})} = \boxed{-6.67\,\text{cm}}$$

Use the lateral magnification equation to relate the height of the image y' to the height y of the object and the image and object distances:

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

Solve for y':

$$y' = -\frac{s'}{s}y$$

Substitute numerical values and evaluate y':

$$y' = -\frac{-6.67\,\text{cm}}{20\,\text{cm}}(1.5\,\text{cm}) = \boxed{0.500\,\text{cm}}$$

$$\boxed{\begin{array}{l}\text{Because } s' < 0 \text{ and } y' = 0.500\text{ cm, the image is virtual, erect, and}\\ \text{about one-third the size of the object in agreement with the ray diagram.}\end{array}}$$

58 ••

Picture the Problem The parallel and central rays were used to locate the image in the diagram shown below. The power P of the lens, in diopters, can be found from $P = 1/f$ and the size of the image from $m = -\frac{s'}{s}$.

(a) A negative object distance implies that the object is a virtual object, i.e., that light rays converge on the object rather than diverge from the object. A virtual object can occur in a two-lens system when the first lens forms an image that is at a distance $-|s|$ from the second lens.

(b) The thin-lens equation is:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

Solve for s':

$$s' = \frac{fs}{s-f}$$

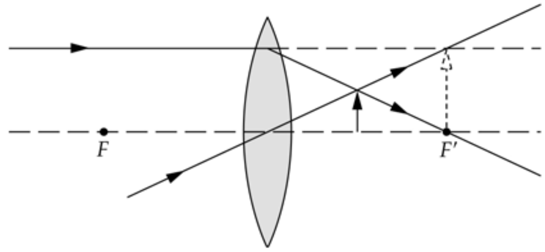
Substitute numerical values and evaluate s':

$$s' = \frac{(20\,\text{cm})(-20\,\text{cm})}{-20\,\text{cm} - (20\,\text{cm})} = \boxed{10.0\,\text{cm}}$$

The lateral magnification is:

$$m = -\frac{s'}{s} = -\frac{10\,\text{cm}}{-20\,\text{cm}} = \boxed{0.500}$$

The parallel and central rays were used to locate the image in the ray diagram shown below:



Because $s' > 0$ and $m > 0$, the image is real, erect, and one-half the size of the virtual object.

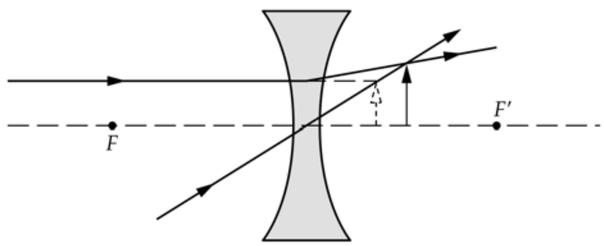
(*c*) Proceed as in (*b*) with $s = -10$ cm and $f = -30$ cm:

$$s' = \frac{(-30\,\text{cm})(-10\,\text{cm})}{-10\,\text{cm} - (-30\,\text{cm})} = \boxed{15.0\,\text{cm}}$$

and

$$m = -\frac{s'}{s} = -\frac{15\,\text{cm}}{-10\,\text{cm}} = \boxed{1.500}$$

The parallel and central rays were used to locate the image in the ray diagram shown below:

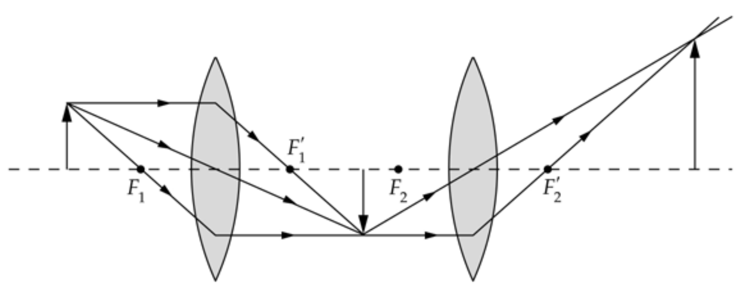


Because $s' > 0$ and $m = 1.5$, the image is real, erect, and one and one-half times the size of the virtual object.

***59 ••**

Picture the Problem We can apply the thin-lens equation to find the image formed in the first lens and then use this image as the object for the second lens.

(*a*) The parallel, central, and focal rays were used to locate the image formed by the first lens and the parallel and central rays to locate the image formed by the second lens.



Apply the thin-lens equation to express the location of the image formed by the first lens:

$$s_1' = \frac{f_1 s_1}{s_1 - f_1} \qquad (1)$$

Substitute numerical values and evaluate s_1':

$$s_1' = \frac{(10\,\text{cm})(20\,\text{cm})}{20\,\text{cm} - 10\,\text{cm}} = 20\,\text{cm}$$

Find the lateral magnification of the first image:

$$m_1 = -\frac{s_1'}{s} = -\frac{20\,\text{cm}}{20\,\text{cm}} = -1$$

Because the lenses are separated by 35 cm, the object distance for the second lens is 35 cm – 20 cm = 15 cm. Equation (1) applied to the second lens is:

$$s_2' = \frac{f_2 s_2}{s_2 - f_2}$$

Substitute numerical values and evaluate s_2':

$$s_2' = \frac{(10\,\text{cm})(15\,\text{cm})}{15\,\text{cm} - 10\,\text{cm}} = 30\,\text{cm}$$

and the final image is $\boxed{85.0\,\text{cm}}$ from the object.

Find the lateral magnification of the second image:

$$m_2 = -\frac{s_2'}{s} = -\frac{30\,\text{cm}}{15\,\text{cm}} = -2$$

$\boxed{\text{Because } s'_2 > 0 \text{ and } m = m_1 m_2 = 2\text{, the image is real, erect, and twice the size of the object.}}$

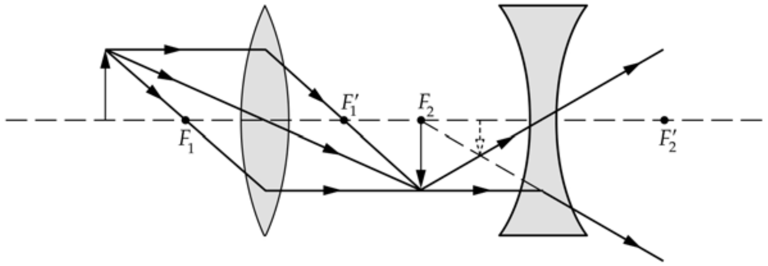
The overall lateral magnification of the image is the product of the magnifications of each image:

$$m = m_1 m_2 = (-1)(-2) = \boxed{2.00}$$

60 ••

Picture the Problem We can apply the thin-lens equation to find the image formed in the first lens and then use this image as the object for the second lens.

(*a*) The parallel, central, and focal rays were used to locate the image formed by the first lens and the parallel and central rays to locate the image formed by the second lens.



Apply the thin-lens equation to express the location of the image formed by the first lens:

$$s_1' = \frac{f_1 s_1}{s_1 - f_1} \qquad (1)$$

Substitute numerical values and evaluate s_1':

$$s_1' = \frac{(10\,\text{cm})(20\,\text{cm})}{20\,\text{cm} - 10\,\text{cm}} = 20\,\text{cm}$$

Find the lateral magnification of the first image:

$$m_1 = -\frac{s'_1}{s} = -\frac{20\,\text{cm}}{20\,\text{cm}} = -1$$

Because the lenses are separated by 35 cm, the object distance for the second lens is
35 cm – 20 cm = 15 cm. Equation (1) applied to the second lens is:

$$s_2' = \frac{f_2 s_2}{s_2 - f_2}$$

Substitute numerical values and evaluate s_2':

$$s_2' = \frac{(-15\,\text{cm})(15\,\text{cm})}{15\,\text{cm} - (-15\,\text{cm})} = -7.5\,\text{cm}$$

and the final image is $\boxed{47.5\,\text{cm}}$ from the object.

Find the lateral magnification of the second image:

$$m_2 = -\frac{s_2'}{s} = -\frac{-7.5\,\text{cm}}{15\,\text{cm}} = 0.5$$

Because $s'_2 < 0$ and $m = m_1 m_2 = -0.5$, the image is virtual, inverted, and half as large as the object.

The overall lateral magnification of the image is the product of the magnifications of each image:

$$m = m_1 m_2 = (-1)(0.5) = \boxed{-0.500}$$

61 ••

Picture the Problem We can use the thin-lens equation and the definition of the lateral magnification to show that $s = (m - 1)f/m$.

(*a*) Express the thin-lens equation:

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

Express the lateral magnification of the image and solve for *s*′:

$$m=-\frac{s'}{s}\Rightarrow s'=-ms$$

Substitute to obtain:

$$\frac{1}{s}+\frac{1}{-ms}=\frac{1}{f}$$

Solve for *s*:

$$s=\boxed{\frac{(m-1)f}{m}}$$

(*b*) The magnification *m* is:

$$m=-\frac{y'}{y}=-\frac{24\,\text{mm}}{1.75\,\text{m}}=-0.0137$$

Substitute numerical values and evaluate *s*:

$$s=\frac{(-0.0137-1)(50\,\text{mm})}{-0.0137}=\boxed{3.70\,\text{m}}$$

62 ••

Picture the Problem We can plot the first graph by solving the thin-lens equation for the image distance *s*′ and the second graph by using the definition of the magnification of the image.

(*a*) and (*b*) Solve the thin-lens equation for *s*′ to obtain:

$$s'=\frac{fs}{s-f}$$

The magnification of the image is given by:

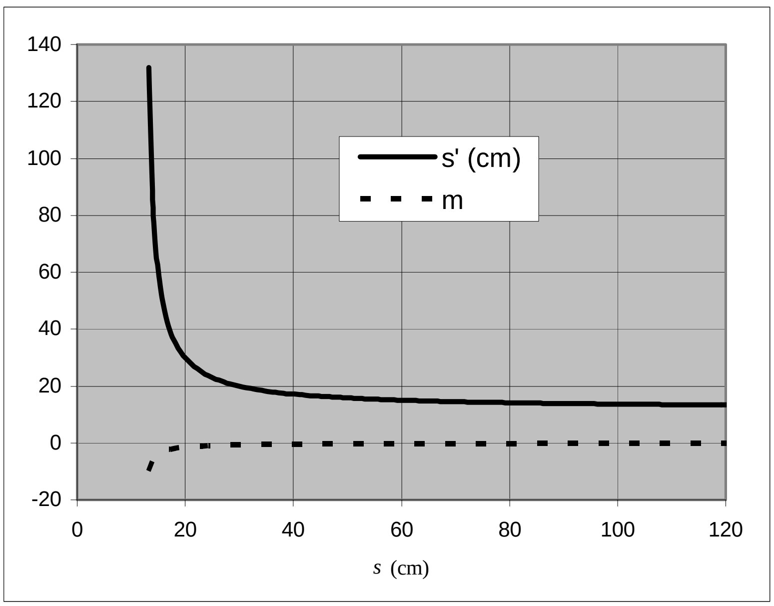
$$m=-\frac{s'}{s}$$

A spreadsheet program to calculate *s*′as a function of *s* is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Content/Formula	Algebraic Form
B1	12	f
A4	13.2	s
A5	A4 + 1	$s+\Delta s$
B4	B1*A4/(A4 – B1)	$\frac{fs}{s-f}$
C5	–B4/A4	$-\frac{s'}{s}$

	A	B	C
1	f=	12	cm
2			
3	s	s'	m
4	13.2	132.00	–10.00
5	14.2	77.45	–5.45
6	15.2	57.00	–3.75
7	16.2	46.29	–2.86
8	17.2	39.69	–2.31
9	18.2	35.23	–1.94
108	117.2	13.37	–0.11
109	118.2	13.36	–0.11
110	119.2	13.34	–0.11
111	120.2	13.33	–0.11

A graph of *s*′ as a function of *s* follows.



(*c*) The images are real and inverted for this range of object distances.

(*d*) The asymptotes of the graph of *s*′ versus *s* correspond to the focal length of the lens. The horizontal asymptote of the graph of *m* versus *s* indicates the fact that, as the object moves away from the lens, the image formed by the lens approaches the far focal point and its size approaches zero.

63 ••

Picture the Problem We can plot the first graph by solving the thin-lens equation for the image distance s' and the second graph by using the definition of the magnification of the image.

(*a*) and (*b*) Solve the thin-lens equation for s' to obtain:

$$s' = \frac{fs}{s - f}$$

The magnification of the image is given by:

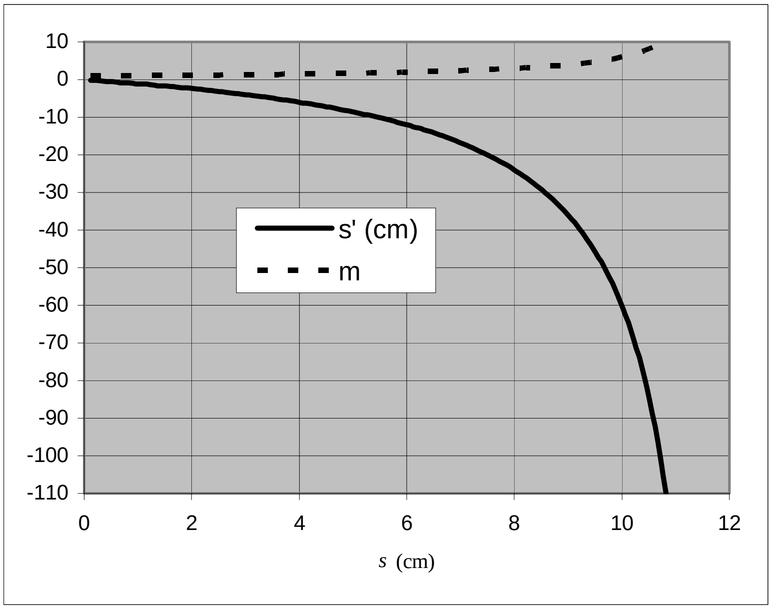
$$m = -\frac{s'}{s}$$

A spreadsheet program to calculate s' as a function of s is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Content/Formula	Algebraic Form
B1	12	f
A4	0.12	s
A5	A4 + 0.1	$s + \Delta s$
B4	B1*A4/(A4 – B1)	$\frac{fs}{s-f}$
C5	–B4/A4	$-\frac{s'}{s}$

	A	B	C
1	f=	12	cm
2			
3	s	s'	m
4	0.12	–0.12	1.01
5	0.22	–0.22	1.02
6	0.32	–0.33	1.03
7	0.42	–0.44	1.04
8	0.52	–0.54	1.05
9	0.62	–0.65	1.05
108	10.52	–85.30	8.11
109	10.62	–92.35	8.70
110	10.72	–100.50	9.37
111	10.82	–110.03	10.17

A graph of s' as a function of s follows.



(*c*) The images are virtual and erect for this range of object distances.

(*d*) The asymptote of the graph of s' versus s corresponds to the image approaching infinity as the object distance approaches the focal length of the lens. The horizontal asymptote of the graph of m versus s indicates that, as the object moves toward the lens, the height of the image formed by the lens approaches the height of the object.

***64** ••

Picture the Problem We can apply the thin-lens equation to find the image formed in the first lens and then use this image as the object for the second lens.

Apply the thin-lens equation to express the location of the image formed by the first lens:

$$s_1' = \frac{f_1 s_1}{s_1 - f_1} \qquad (1)$$

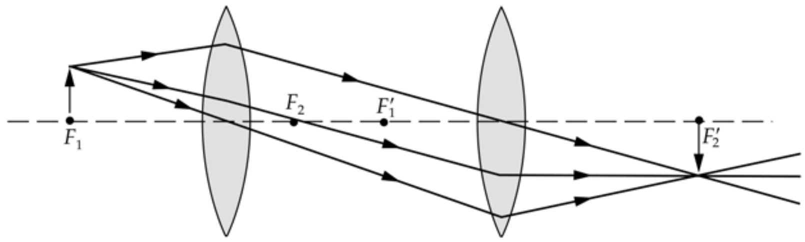
Substitute numerical values and evaluate s_1':

$$s_1' = \frac{(15\,\text{cm})(15\,\text{cm})}{15\,\text{cm} - 15\,\text{cm}} = \infty$$

With $s_1' = \infty$, the thin-lens equation applied to the second lens becomes:

$$\frac{1}{s_2'} = \frac{1}{f_2} \Rightarrow s_2' = f_2 = \boxed{15.0\,\text{cm}}$$

A ray diagram is shown below:



The final image is 50 cm from the object, real, inverted, and the same size as the object.

65 ••

Picture the Problem We can apply the thin-lens equation to find the image formed in the first lens and then use this image as the object for the second lens.

Apply the thin-lens equation to express the location of the image formed by the first lens:

$$s_1' = \frac{f_1 s_1}{s_1 - f_1} \qquad (1)$$

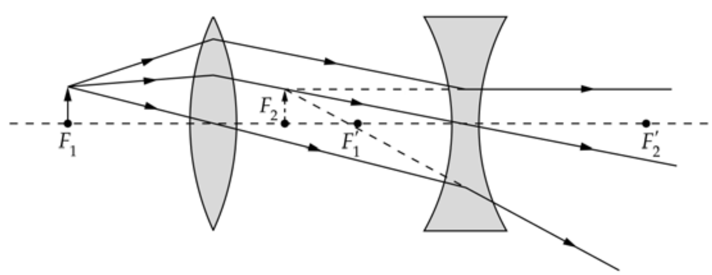
Substitute numerical values and evaluate s_1':

$$s_1' = \frac{(15\,\text{cm})(15\,\text{cm})}{15\,\text{cm} - 15\,\text{cm}} = \infty$$

With $s'_1 = \infty$, the thin-lens equation applied to the second lens becomes:

$$\frac{1}{s_2'} = \frac{1}{f_2} \Rightarrow s_2' = f_2 = \boxed{15.0\,\text{cm}}$$

A ray diagram is shown below:



The final image is 50 cm from the object, real, inverted, and the same size as the object.

66 •••

Picture the Problem We can substitute $x = s - f$ and $x' = s' - f$ in the thin-lens equation and the equation for the lateral magnification of an image to obtain Newton's equations.

Express the thin-lens equation:

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

If $x = s - f$ and $x' = s' - f$:

$$\frac{1}{x+f}+\frac{1}{x'+f}=\frac{1}{f}$$

Expand this expression to obtain:

$$f(x'+x+2f)=(x+f)(x'+f)$$
$$=xx'+xf+x'f+f^2$$

or, simplifying, $\boxed{xx'=f^2}$ (1)

The lateral magnification is:

$$m=-\frac{s'}{s}$$

or, because $x = s - f$ and $x' = s' - f$,

$$m=-\frac{x'+f}{x+f}$$

Solve equation (1) for x:

$$x=\frac{f^2}{x'}$$

Substitute for x and simplify to obtain:

$$m=-\frac{x'+f}{\frac{f^2}{x'}+f}=-\frac{x'+f}{\frac{f(f+x')}{x'}}$$
$$=\boxed{-\frac{x'}{f}}$$

The lateral magnification is also given by:

$$m=-\frac{x'+f}{x+f}$$

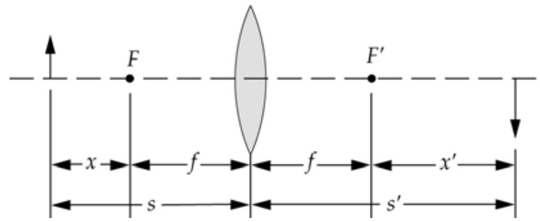
From equation (1) we have:

$$x'=\frac{f^2}{x}$$

Substitute to obtain:

$$m=-\frac{\frac{f^2}{x}+f}{x+f}=-\frac{f\left(\frac{f}{x}+1\right)}{x\left(1+\frac{f}{x}\right)}=\boxed{-\frac{f}{x}}$$

The variables x, f, s,and s' are shown in the sketch below:



67 •••

Picture the Problem The ray diagram shows the two lens positions and the corresponding image and object distances (denoted by the numerals 1 and 2). We can use the thin-lens equation relate the two sets of image and object distances to the focal length of the lens and then use the hint to express the relationships between these distances and the distances D and L to eliminate s_1, s_1', s_2, and s_2' and obtain an expression relating f, D, and L.

Relate the image and object distances for the two lens positions to the focal length of the lens:

$$\frac{1}{s_1}+\frac{1}{s_1'}=\frac{1}{f} \text{ and } \frac{1}{s_2}+\frac{1}{s_2'}=\frac{1}{f}$$

Solve for f to obtain:

$$f=\frac{s_1s_1'}{s_1+s_1'}=\frac{s_2s_2'}{s_2+s_2'} \qquad (1)$$

The distances D and L can be expressed in terms of the image and object distances:

$$D=s_1+s_1'=s_2+s_2'$$

and

$$L=s_2-s_1=s_1'-s_2'$$

Substitute for the sums of the image and object distances in equation (1) to obtain:

$$f=\frac{s_1s_1'}{D}=\frac{s_2s_2'}{D}$$

From the hint:

$$s_1=s_2' \text{ and } s_1'=s_2$$

Hence $D = s_1 + s_2$ and:

$$D-L=2s_1 \text{ and } D+L=2s_2$$

Take the product of $D - L$ and $D + L$ to obtain:

$$(D-L)(D+L)=D^2-L^2$$
$$=4s_1s_2=4s_1s_1'$$

From the thin-lens equation:

$$4s_1s_2=rs_1s_1'=4fD$$

Substitute to obtain:

$$4fD=D^2-L^2$$

Solve for f:

$$\boxed{f = \frac{D^2 - L^2}{4D}}$$

68 ••

Picture the Problem We can use results obtained in Problem 67 to find the focal length of the lens and the two locations of the lens with respect to the object.

(a) From Problem 77 we have:

$$f = \frac{D^2 - L^2}{4D}$$

Substitute numerical values and evaluate f:

$$f = \frac{(1.7\,\text{m})^2 - (0.72\,\text{m})^2}{4(1.7\,\text{m})} = \boxed{34.9\,\text{cm}}$$

(b) Solve the thin-lens equation for the image distance to obtain:

$$s' = \frac{fs}{f - s} \qquad (1)$$

In Problem 77 it was established that:

$$D - L = 2s_1 \text{ and } D + L = 2s_2$$

Solve for s_1 and s_2:

$$s_1 = \frac{D - L}{2} \text{ and } s_2 = \frac{D + L}{2}$$

Substitute numerical values and evaluate s_1 and s_2:

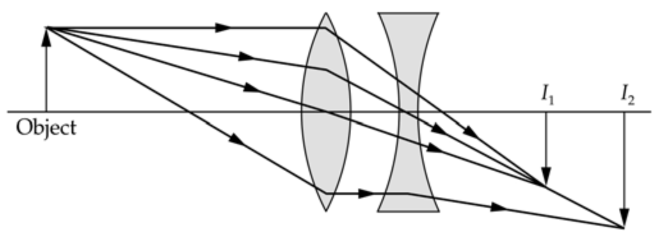
$$s_1 = \frac{170\,\text{cm} - 72\,\text{cm}}{2} = \boxed{49.0\,\text{cm}}$$

and

$$s_2 = \frac{170\,\text{cm} + 72\,\text{cm}}{2} = \boxed{121\,\text{cm}}$$

69 •••

Picture the Problem The ray diagram shows four rays from the head of the object that locate images I_1 and I_2. We can use the thin-lens equation to find the location of the image formed in the positive lens and then, knowing the separation of the two lenses, determine the object distance for the second lens and apply the thin lens a second time to find the location of the final image.



(*a*) Express the object-to-image distance *d*:

$$d = s_1 + 5\,\text{cm} + s_2' \qquad (1)$$

Apply the thin-lens equation to the positive lens:

$$\frac{1}{s_1} + \frac{1}{s_1'} = \frac{1}{f_1}$$

Solve for s_1':

$$s_1' = \frac{f_1 s_1}{s_1 - f_1}$$

Substitute numerical values and evaluate s_1':

$$s_1' = \frac{(8.5\,\text{cm})(17.5\,\text{cm})}{17.5\,\text{cm} - 8.5\,\text{cm}} = 16.53\,\text{cm}$$

Find the object distance for the negative lens:

$$s_2 = 5\,\text{cm} - s_1' = 5\,\text{cm} - 16.53\,\text{cm}$$
$$= -11.53\,\text{cm}$$

The image distance s_2' is given by:

$$s_2' = \frac{f_2 s_2}{s_2 - f_2}$$

Substitute numerical values and evaluate s_2':

$$s_2' = \frac{(-30\,\text{cm})(-11.53\,\text{cm})}{-11.53\,\text{cm} - (-30\,\text{cm})} = 18.7\,\text{cm}$$

Substitute numerical values in equation (1) and evaluate *d*:

$$d = 17.5\,\text{cm} + 5\,\text{cm} + 18.7\,\text{cm}$$
$$= \boxed{41.2\,\text{cm}}$$

(*b*) The overall lateral magnification is given by:

$$m = m_1 m_2$$

Express m_1 and m_2:

$$m_1 = -\frac{s_1'}{s_1} \text{ and } m_2 = -\frac{s_2'}{s_2}$$

Substitute to obtain:

$$m = \left(-\frac{s_1'}{s_1}\right)\left(-\frac{s_2'}{s_2}\right) = \frac{s_1' s_2'}{s_1 s_2}$$

Substitute numerical values and evaluate *m*:

$$m = \frac{(16.53\,\text{cm})(18.7\,\text{cm})}{(17.5\,\text{cm})(-11.53\,\text{cm})} = \boxed{-1.53}$$

$$\boxed{\text{Because } m < 0 \text{, the image is inverted. Because } s_2' > 0 \text{, the image is real.}}$$

Aberrations

***70** •

Determine the Concept Chromatic aberrations are a consequence of the differential refraction of light of differing wavelengths by lenses. $\boxed{(a)\text{ is correct.}}$

71 •

(*a*) False. Aberrations are a consequence of imperfections in lenses.

(*b*) True.

72 •

Picture the Problem We can use the lens-maker's equation to find the focal length the this lens for the two colors of light.

The lens-maker's equation relates the radii of curvature and the index of refraction to the focal length of the lens:

$$\frac{1}{f}=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

(*a*) For red light:

$$\frac{1}{f_{\text{red}}}=(1.47-1)\left(\frac{1}{10\,\text{cm}}-\frac{1}{-10\,\text{cm}}\right)$$

and

$$f_{\text{red}}=\boxed{10.6\,\text{cm}}$$

(*b*) For blue light:

$$\frac{1}{f_{\text{blue}}}=(1.53-1)\left(\frac{1}{10\,\text{cm}}-\frac{1}{-10\,\text{cm}}\right)$$

and

$$f_{\text{blue}}=\boxed{9.43\,\text{cm}}$$

The Eye

***73** ••

Picture the Problem The thin-lens equation relates the image and object distances to the power of a lens.

(*a*) Use the thin-lens equation to relate the image and object distances to the power of the lens:

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}=P$$

Because $s' = d$ and, for a distance object, $s = \infty$:

$$P_{\min} = \frac{1}{s'} = \boxed{\frac{1}{d}}$$

(b) If x_{np} is the closest distance an object could be and still remain in clear focus on the screen, equation (1) becomes:

$$P_{\max} = \boxed{\frac{1}{x_{np}} + \frac{1}{d}}$$

(c) Use our result in (a) to obtain:

$$P_{\min} = \frac{1}{2.5\,\text{cm}} = \boxed{40.0\,\text{D}}$$

Use the results of (a) and (b) to express the accommodation of the model eye:

$$A = P_{\max} - P_{\min} = \frac{1}{x_{np}} + \frac{1}{d} - \frac{1}{d} = \frac{1}{x_{np}}$$

Substitute numerical values and evaluate A:

$$A = \frac{1}{25\,\text{cm}} = \boxed{4.00\,\text{D}}$$

74 ••

Picture the Problem The thin-lens equation relates the image and object distances to the power of a lens.

(a) Use the thin-lens equation to relate the image and object distances to the power of the lens:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} = P$$

Because $s' = d$ and $s = x_{fp}$:

$$P_{\min} = \boxed{\frac{1}{x_{fp}} + \frac{1}{d}}$$

(b) To correct for the nearsightedness of this eye, we need a lens that will form an image 25 cm in front of the eye of an object at the eye's far point:

$$P_{\min} = \frac{1}{50\,\text{cm}} + \frac{1}{-25\,\text{cm}} = \boxed{-2.00\,\text{D}}$$

75 ••

Picture the Problem The thin-lens equation relates the image and object distances to the power of a lens.

(a) Use the thin-lens equation to relate the image and object distances to the power of the lens:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} = P$$

Because $s' = d$ and $s = x'_{np}$:

$$P'_{max} = \boxed{\frac{1}{x'_{np}} + \frac{1}{d}} \qquad (1)$$

(*b*) For a normal eye:

$$P_{max} = \frac{1}{x_{np}} + \frac{1}{d} \qquad (2)$$

The amount by which the power of the lens is too small is the difference between equations (2) and (1):

$$P_{max} - P'_{max} = \frac{1}{x_{np}} + \frac{1}{d} - \left(\frac{1}{x'_{np}} + \frac{1}{d}\right) = \boxed{\frac{1}{x_{np}} - \frac{1}{x'_{np}}}$$

(*c*) For x_{np} = 15 cm and x'_{np} = 150 cm:

$$P_{max} - P'_{max} = \frac{1}{15\,\text{cm}} - \frac{1}{150\,\text{cm}} = \boxed{6.00\,\text{D}}$$

76 •

Picture the Problem We can use the thin-lens equation to find the distance from the lens to the image and then take their difference to find the distance the lens would have to be moved.

Express the distance *d* that the lens would have to move:

$$d = s' - f$$

Solve the thin-lens equation for s':

$$s' = \frac{fs}{s - f}$$

Substitute to obtain:

$$d = \frac{fs}{s - f} - f$$

Substitute numerical values and evaluate *d*:

$$d = \frac{(2.5\,\text{cm})(25\,\text{cm})}{25\,\text{cm} - 2.5\,\text{cm}} - 2.5\,\text{cm} = \boxed{0.278\,\text{cm}}$$

That is, the lens would have to move 0.278 cm toward the object.

77 •

Picture the Problem We can apply the thin-lens equation for the two values of *s* to find Δf.

Express the change Δf in the focal length:

$$\Delta f = f_{s=3\,\text{m}} - f_{s=0.3\,\text{m}}$$

Solve the thin-lens equation for *s*:

$$f = \frac{ss'}{s'+s}$$

Substitute to obtain:

$$\Delta f = \frac{s_{3\text{m}}s'_{3\text{m}}}{s'_{3\text{m}} + s_{3\text{m}}} - \frac{s_{0.3\text{m}}s'_{0.3\text{m}}}{s'_{0.3\text{m}} + s_{0.3\text{m}}}$$

or, because $s'_{3\text{ m}} = s'_{0.3\text{ m}}$,

$$\Delta f = s'_{3\text{m}}\left[\frac{s_{3\text{m}}}{s'_{3\text{m}} + s_{3\text{m}}} - \frac{s_{0.3\text{m}}}{s'_{0.3\text{m}} + s_{0.3\text{m}}}\right]$$

Substitute numerical values and evaluate Δf:

$$\Delta f = (2.5\,\text{cm})\left[\frac{300\,\text{cm}}{2.5\,\text{cm}+300\,\text{cm}} - \frac{30\,\text{cm}}{2.5\,\text{cm}+30\,\text{cm}}\right] = 0.172\,\text{cm} = \boxed{1.72\,\text{mm}}$$

78 •

Picture the Problem We can use the thin-lens equation and the definition of the power of a lens to express the near point distance as a function of *P* and *s*.

From the thin-lens equation we have:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} = P$$

Solve for s':

$$s' = \frac{s}{Ps - 1}$$

Substitute numerical values and evaluate s':

$$s' = \frac{25\,\text{cm}}{(1.75\,\text{m}^{-1})(0.25\,\text{m}) - 1} = -44.4\,\text{cm}$$

$$\boxed{\text{The person's near point with lenses is 44.4 cm.}}$$

***79** •

Picture the Problem We can use the relationship between a distance measured along the arc of a circle and the angle subtended at its center to approximate the smallest angle the two points can subtend and the separation of the two points
20 m from the eye.

(*a*) Relate θ_{min} to the diameter of the eye and the distance between the activated cones:

$$d_{\text{eye}}\theta_{\text{min}} \approx 2\,\mu\text{m}$$

Solve for $\theta_{\min}$:

$$\theta_{\min} = \frac{2\ \mu\text{m}}{d_{\text{eye}}}$$

Substitute numerical values and evaluate $\theta_{\min}$:

$$\theta_{\min} = \frac{2\ \mu\text{m}}{2.5\,\text{cm}} = \boxed{80.0\ \mu\text{rad}}$$

(*b*) Let *D* represent the separation of the points *R* = 20 m from the eye to obtain:

$$D = R\theta_{\min} = (20\,\text{m})(80\ \mu\text{rad}) = \boxed{1.60\,\text{mm}}$$

80 ••

Picture the Problem We can use the thin-lens equation to find *f* and the definition of the power of a lens to find *P*.

(*a*) Solve the thin-lens equation for *f*:

$$f = \frac{ss'}{s' + s}$$

Noting that $s' < 0$, substitute numerical values and evaluate *f*:

$$f = \frac{(45\,\text{cm})(-80\,\text{cm})}{-80\,\text{cm} + 45\,\text{cm}} = \boxed{103\,\text{cm}}$$

(*b*) Use the definition of the power of a lens to obtain:

$$P = \frac{1}{f} = \frac{1}{1.03\,\text{m}} = \boxed{0.971\,\text{diopters}}$$

81 ••

Picture the Problem We can use the thin-lens equation to find *f* and the definition of the power of a lens to find *P*.

Express the required power of the lens:

$$P = \frac{1}{f}$$

The thin-lens equation is:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

For $s = \infty$:

$$\frac{1}{s'} = \frac{1}{f} \Rightarrow f = s'$$

Substitute for *f* to obtain:

$$P = \frac{1}{s'}$$

Substitute for s' and evaluate *P*:

$$P = \frac{1}{2.25\,\text{m}} = \boxed{0.444\,\text{diopters}}$$

82 ••

Picture the Problem We can use the lens-maker's equation with $s = \infty$ to find the radius of the cornea modeled as a homogeneous sphere with an index of refraction of 1.4.

Use the lens-maker's equation to relate the radius of the cornea to its index of refraction and that of air:

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{r}$$

Because $n_2 = n$, $n_1 = 1$, and $s = \infty$:

$$\frac{n}{s'} = \frac{n-1}{r}$$

Solve for r:

$$r = \frac{s'(n-1)}{n} = \left(1 - \frac{1}{n}\right)s'$$

Substitute numerical values and evaluate r:

$$r = \left(1 - \frac{1}{1.4}\right)(2.5\,\text{cm}) = \boxed{0.714\,\text{cm}}$$

The eye is not a homogeneous sphere. It is filled with a transparent liquid (vitreous humor) which has an index of refraction that is not known. If that index of refraction differs from 1.4, there is refraction at the inner surface of the cornea which will result in the formation of the image nearer the cornea's surface if $n > 1.4$ and farther if $n < 1.4$, where n is the index of refraction of the vitreous humor. If $n < 1.4$, then r as calculated above is too small.

83 ••

Picture the Problem We can use the definition of the power of a lens and the thin-lens equation to find the power of the lens that should be used in the glasses.

Express the power of the lens that should be used in the glasses:

$$P = P_{\text{eye}} + P_{\text{lens}} = \frac{1}{f_{\text{eye}}} + \frac{1}{f_{\text{glasses}}} \qquad (1)$$

Because the glasses are 2 cm from the eye:

$$s' = -80\,\text{cm} + 2\,\text{cm} = -78\,\text{cm}$$

and

$$s = 25\,\text{cm} - 2\,\text{cm} = 23\,\text{cm}$$

Apply the thin-lens equation to the eye with $s' = \infty$:

$$\frac{1}{s} = \frac{1}{f_{\text{eye}}} \Rightarrow f_{\text{eye}} = s$$

Apply the thin-lens equation to the glasses with $s = \infty$:

$$\frac{1}{s'} = \frac{1}{f_{\text{glasses}}} \Rightarrow f_{\text{glasses}} = s'$$

Substitute for f_{eye} and f_{glasses} in equation (1) to obtain:

$$P = \frac{1}{s} + \frac{1}{s'}$$

Substitute numerical values and evaluate P:

$$P = \frac{1}{0.23\,\text{m}} + \frac{1}{-0.78\,\text{m}} = \boxed{3.07\,\text{D}}$$

84 •••

Picture the Problem We can use the thin-lens equation and the distance from her eyes to her glasses to derive an expression for the location of her near point.

(a) Express her near point, x_{np}, at age 45 in terms of the location of her glasses:

$$x_{\text{np}} = |s'| + 2.2\,\text{cm} \qquad (1)$$

Because the glasses are 2.2 cm from her eye:

$$s = 25\,\text{cm} - 2.2\,\text{cm} = 22.8\,\text{cm}$$

Apply the thin-lens equation to the glasses:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f_{\text{glasses}}} = P$$

Solve for s':

$$s' = \frac{s}{Ps - 1} = \frac{1}{P - \frac{1}{s}}$$

Substitute in equation (1) to obtain:

$$x_{\text{np}} = \left|\frac{1}{P - \frac{1}{s}}\right| + 2.2\,\text{cm} \qquad (2)$$

Substitute numerical values and evaluate x_{np}:

$$x_{\text{np}} = \left|\frac{1}{2.1\,\text{m}^{-1} - \frac{1}{0.228\,\text{m}}}\right| + 2.2\,\text{cm} = \boxed{45.9\,\text{cm}}$$

(b) At age 55:

$$s = 40\,\text{cm} - 2.2\,\text{cm} = 37.8\,\text{cm}$$

Substitute numerical values in equation (2) and evaluate *s*′:

$$x_{np} = \left| \frac{1}{2.1\,\text{m}^{-1} - \dfrac{1}{0.378\,\text{m}}} \right| + 2.2\,\text{cm}$$

$$= \boxed{185\,\text{cm}}$$

(*c*) Solve the thin-lens equation for *f*:

$$f = \frac{ss'}{s' + s}$$

From the definition of *P*:

$$P = \frac{1}{f} = \frac{s' + s}{s's}$$

For $s = 22.8$ cm and $s' = 183.3$ cm:

$$P = \frac{183.3\,\text{cm} + 22.8\,\text{cm}}{(183.3\,\text{cm})(22.8\,\text{cm})} = \boxed{4.93\,\text{D}}$$

The Simple Magnifier

***85** •

Picture the Problem We can use the definitions of the magnifying power of a lens ($M = x_{np}/f$) and of the power of a lens ($P = 1/f$) to find the magnifying power of the given lens.

The magnifying power of the lens is given by:

$$M = \frac{x_{np}}{f} = Px_{np}$$

where *P* is the power of the lens.

Substitute numerical values and evaluate *M*:

$$M = (20\,\text{m}^{-1})(0.3\,\text{m}) = \boxed{6.00}$$

86 •

Picture the Problem We can use the definition of the magnifying power of a lens to find the required focal length so that this person’s lens will have magnification power of 5.

The magnifying power of the lens is given by:

$$M = \frac{x_{np}}{f}$$

Solve for *f*:

$$f = \frac{x_{np}}{M}$$

Substitute numerical values and evaluate *f*:

$$f = \frac{25\,\text{cm}}{5} = \boxed{5.00\,\text{cm}}$$

87 •

Picture the Problem We can use the definition of the magnifying power of a lens to find the magnifying power of this lens.

The magnifying power of the lens is given by:

$$M = \frac{x_{np}}{f}$$

Substitute numerical values and evaluate *M*:

$$M = \frac{35\,\text{cm}}{7\,\text{cm}} = \boxed{5.00}$$

88 ••

Picture the Problem Let the numerals 1 and 2 denote the 1st and 2nd persons, respectively. We can use the definition of magnifying power to find the effective magnifying power of the lens for each person. The relative height of the images on the retinas of the two persons is given by the ratio of the effective magnifying powers.

The magnifying power of the lens is given by:

$$M = \frac{x_{np}}{f}$$

Substitute numerical values and evaluate M_1 and M_2:

$$M_1 = \frac{25\,\text{cm}}{6\,\text{cm}} = \boxed{4.17}$$

and

$$M_2 = \frac{40\,\text{cm}}{6\,\text{cm}} = \boxed{6.67}$$

From the definition of magnifying power we have:

$$\frac{M_1}{M_2} = \frac{\frac{y_1}{f}}{\frac{y_2}{f}} = \frac{y_1}{y_2}$$

Substitute for M_1 and M_2 and evaluate the ratio of y_1 to y_2:

$$\frac{y_1}{y_2} = \frac{4.17}{6.67} = \boxed{0.625}$$

89 ••

Picture the Problem We can use the definition of angular magnification to find the expected angular magnification if the final image is at infinity and the thin-lens equation and the expression for the magnification of a thin lens to find the angular magnification when the final image is at 25 cm.

(*a*) Express the angular magnification when the final image is at infinity:

$$M = \frac{x_{np}}{f} = x_{np}P$$

where P is the power of the lens.

Substitute numerical values and evaluate M:

$$M = (25\,\text{cm})(12\,\text{m}^{-1}) = \boxed{3.00}$$

(*b*) Express the magnification of the lens when the final image is at 25 cm:

$$m = -\frac{s'}{s}$$

Solve the thin-lens equation for s:

$$s = \frac{fs'}{s' - f}$$

Substitute to obtain:

$$m = -\frac{s'}{\frac{fs'}{s' - f}} = -\frac{s' - f}{f} = -\frac{s'}{f} + 1$$
$$= 1 - s'P$$

Substitute numerical values and evaluate m:

$$m = 1 - (-0.25\,\text{m})(12\,\text{m}^{-1}) = \boxed{4}$$

***90 ••**

Picture the Problem We can use the definition of the angular magnification of a lens and the thin-lens equation to show that $M = \frac{x_{np}}{f} + 1$.

(*a*) Express the angular magnification of the simple magnifier in terms of the angles subtended by the object and the image:

$$M = \frac{\theta}{\theta_0} \qquad (1)$$

Solve the thin-lens equation for s:

$$s = \frac{fs'}{s' - f}$$

Because the image is virtual:

$$s' = -x_{np}$$

Substitute to obtain:

$$s = \frac{f(-x_{np})}{-x_{np} - f} = \frac{fx_{np}}{x_{np} + f}$$

Express the angle subtended by the object:

$$\theta_0 = \frac{y}{x_{np}}$$

where y is the height of the object.

Express the angle subtended by the image:

$$\theta = \frac{y}{s}$$

Substitute for s to obtain:

$$\theta = \frac{y}{\frac{fx_{np}}{x_{np} + f}} = \frac{y(x_{np} + f)}{fx_{np}}$$

Substitute in equation (1) and simplify:

$$M = \frac{\frac{y(x_{np} + f)}{fx_{np}}}{\frac{y}{x_{np}}} = \frac{x_{np} + f}{f} = \boxed{\frac{x_{np}}{f} + 1}$$

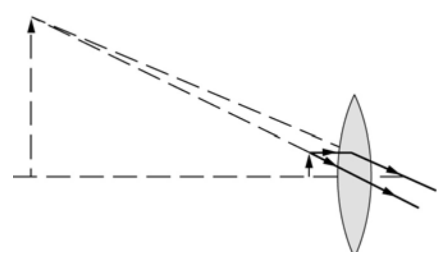
(*b*) In terms of the power of the magnifying lens:

$$M = x_{np}P + 1$$

The magnification of a 20-D lens for a person with a near point of 30 cm and the final image at the near point is:

$$M = (0.3\,\text{m})(20\,\text{m}^{-1}) + 1 = \boxed{7.00}$$

A ray diagram for this situation is shown to the right:



91 ••

Picture the Problem We can use the definitions of lateral and angular magnification and the result given in Problem 82 to show that, when the image of a simple magnifier is viewed at the near point, the lateral and angular magnifications are equal.

Express the lateral magnification of the lens:

$$M = \frac{x_{np}}{f}$$

Because the image is viewed at the near point, $f = s$ and:

$$M = \frac{x_{\text{np}}}{s}$$

From Problem 32-82:

$$M = \frac{x_{\text{np}}}{f} + 1$$

and

$$\frac{x_{\text{np}}}{s} = \frac{x_{\text{np}}}{f} + 1 \text{ or } \boxed{M_{\text{lateral}} = M_{\text{angular}}}$$

The Microscope

92 ••

Picture the Problem We can use the thin-lens equation to find the location of the object and the expression for the magnifying power of a microscope to find the magnifying power of the given microscope for a person whose near point is at 25 cm.

(*a*) Using the thin-lens equation, relate the object distance *s* to the focal length of the objective lens f_0:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f_0}$$

Solve for *s* to obtain:

$$s = \frac{f_0 s'}{s' - f_0}$$

From Figure 32-48, the image distance for the image formed by the objective lens is:

$$s' = f_0 + L = 1.7\,\text{cm} + 16\,\text{cm} = 17.7\,\text{cm}$$

Substitute numerical values and evaluate *s*:

$$s = \frac{(1.7\,\text{cm})(17.7\,\text{cm})}{17.7\,\text{cm} - 1.7\,\text{cm}} = \boxed{1.88\,\text{cm}}$$

(*b*) Express the magnifying power of a microscope:

$$M = -\frac{L}{f_0}\frac{x_{\text{np}}}{f_{\text{e}}}$$

Substitute numerical values and evaluate *M*:

$$M = -\frac{16\,\text{cm}}{1.7\,\text{cm}}\frac{25\,\text{cm}}{5.1\,\text{cm}} = \boxed{-46.1}$$

***93** ••

Picture the Problem The lateral magnification of the objective is $m_{\text{o}} = -L/f_{\text{o}}$ and the magnifying power of the microscope is $M = m_{\text{o}} M_{\text{e}}$.

(*a*) The lateral magnification of the objective is given by:

$$m_o = -\frac{L}{f_o}$$

Substitute numerical values and evaluate m_o:

$$m_o = -\frac{16\,\text{cm}}{8.5\,\text{mm}} = \boxed{-1.88}$$

(*b*) The magnifying power of the microscope is given by:

$$M = m_o M_e$$

where M_e is the angular magnification of the lens.

Substitute numerical values and evaluate M:

$$M = (-1.88)(10) = \boxed{-18.8}$$

94 ••

Picture the Problem We can find the tube length from the length of the tube to which the lenses are fastened and the focal lengths of the objective and eyepiece. We can use their definitions to find the lateral magnification of the objective and the magnifying power of the microscope. The distance of the object from the objective can be found using the thin-lens equation.

(*a*) The tube length L is given by:

$$L = D - f_o - f_e$$
$$= 0.30\,\text{m} - \frac{2}{20\,\text{m}^{-1}} = \boxed{20.0\,\text{cm}}$$

(*b*) The lateral magnification of the objective m_o is given by:

$$m_o = -\frac{L}{f_o} = -\frac{20\,\text{cm}}{5\,\text{cm}} = \boxed{-4.00}$$

(*c*) The magnifying power of the microscope is given by:

$$M = m_o M_e = m_o \frac{x_{np}}{f_e}$$

Substitute numerical values and evaluate M:

$$M = (-4)\frac{25\,\text{cm}}{5\,\text{cm}} = \boxed{-20.0}$$

(*d*) From the thin-lens equation we have:

$$\frac{1}{s_o} + \frac{1}{s_o'} = \frac{1}{f_o}$$

where $s_o' = f_o + L$

Substitute to obtain:

$$\frac{1}{s_o} + \frac{1}{f_o + L} = \frac{1}{f_o}$$

Solve for s_o:

$$s_o = \frac{f_o(f_o + L)}{L}$$

Substitute numerical values and evaluate s_o:

$$s_o = \frac{(5\,\text{cm})(5\,\text{cm} + 20\,\text{cm})}{20\,\text{cm}} = \boxed{6.25\,\text{cm}}$$

***95 ••**

Picture the Problem The magnifying power of a compound microscope is the product of the magnifying powers of the objective and the eyepiece.

Express the magnifying power of the microscope in terms of the magnifying powers of the objective and eyepiece:

$$M = m_o m_e \qquad (1)$$

From Problem 82, the magnification of the eyepiece is given by:

$$m_e = \frac{x_{np}}{f_e} + 1 = P_e x_{np} + 1$$

The magnification of the objective is given by:

$$m_o = -\frac{L}{f_o}$$

where $L = D - f_o - f_e$

Substitute to obtain:

$$m_o = -\frac{D - f_o - f_e}{f_o}$$

Substitute for m_e and m_o in equation (1) to obtain:

$$M = (P_e x_{np} + 1)\left(-\frac{D - f_o - f_e}{f_o}\right)$$

Substitute numerical values and evaluate M:

$$M = [(80\,\text{D})(0.25\,\text{m}) + 1]\left(-\frac{28\,\text{cm} - 2.22\,\text{cm} - 1.25\,\text{cm}}{2.22\,\text{cm}}\right) = \boxed{-232}$$

96 •••

Picture the Problem We can find the focal length of the eyepiece from its angular magnification and the near point of a normal eye. The location of the object such that it is in focus for a normal relaxed eye can be found from the lateral magnification of the eyepiece and the magnifying power of the microscope. Finally, we can use the thin-lens equation to find the focal length of the objective lens.

(*a*) Relate the focal length of the eyepiece to its angular magnifying power:

$$M_e = \frac{x_{np}}{f_e} \Rightarrow f_e = \frac{x_{np}}{M_e}$$

Substitute numerical values and evaluate f_e:

$$f_e = \frac{25\,\text{cm}}{15} = \boxed{1.67\,\text{cm}}$$

(*b*) Relate *s* to *s*′ through the lateral magnification of the objective:

$$m_o = -\frac{s'}{s} \Rightarrow s = -\frac{s'}{m_o}$$

Relate the magnifying power of the microscope *M* to the lateral magnification of its objective m_0 and the angular magnification of its eyepiece M_e:

$$M = m_o M_e$$

Solve for m_o:

$$m_o = \frac{M}{M_e}$$

Substitute to obtain:

$$s = -\frac{s'M_e}{M}$$

Evaluate *s*′:

$$s' = 22\,\text{cm} - f_e$$
$$= 22\,\text{cm} - 1.67\,\text{cm} = 20.33\,\text{cm}$$

Substitute numerical values and evaluate *s*:

$$s = -\frac{(20.33\,\text{cm})(15)}{-600} = \boxed{0.508\,\text{cm}}$$

(*c*) Solve the thin-lens equation for f_o:

$$f_o = \frac{ss'}{s' + s}$$

Substitute numerical values and evaluate f_o:

$$f_o = \frac{(0.508\,\text{cm})(20.33\,\text{cm})}{20.33\,\text{cm} + 0.508\,\text{cm}}$$
$$= \boxed{0.496\,\text{cm}}$$

The Telescope

97 •

Picture the Problem Because of the great distance to the moon, its image formed by the objective lens is at the focal point of the objective lens and we can use $D = f_o\theta$ to find the diameter D of the image of the moon. Because angle subtended by the final image at infinity is given by $\theta_e = M\theta_o = M\theta$, we can solve (*b*) and (*c*) together by first using $M = -f_o/f_e$ to find the magnifying power of the telescope.

(*a*) Relate the diameter D of the image of the moon to the image distance and the angle subtended by the moon:

$$D = s_o'\theta$$

Because the image of the moon is at the focal point of the objective lens:

$$s_o' = f_o$$

and

$$D = f_o\theta$$

Substitute numerical values and evaluate D:

$$D = (100\,\text{cm})(0.009\,\text{rad}) = \boxed{9.00\,\text{mm}}$$

(*b*) and (*c*) Relate the angle subtended by the final image at infinity to the magnification of the telescope and the angle subtended at the objective:

$$\theta_e = M\theta_o = M\theta$$

Express the magnifying power of the telescope:

$$M = -\frac{f_o}{f_e}$$

Substitute numerical values and evaluate M and θ_e:

$$M = -\frac{100\,\text{cm}}{5\,\text{cm}} = \boxed{-20.0}$$

and

$$\theta_e = (-20)(0.009\,\text{rad}) = \boxed{-0.180\,\text{rad}}$$

98 •

Picture the Problem Because of the great distance to the moon, its image formed by the objective lens is at the focal point of the objective lens and we can use $D = f_o\theta$ to find the diameter D of the image of the moon.

Relate the diameter D of the image of the moon to the image distance and the angle subtended by the moon:

$$D = s_o'\theta$$

Because the image of the moon is at

$$s_o' = f_o$$

the focal point of the objective lens: and

$$D = f_o\theta$$

Substitute numerical values and evaluate D:

$$D = (19.5\,\text{m})(0.009\,\text{rad}) = \boxed{17.6\,\text{cm}}$$

***99 ••**

Picture the Problem Because the light-gathering power of a mirror is proportional to its area, we can compare the light-gathering powers of these mirrors by finding the ratio of their areas. We can use the ratio of the focal lengths of the objective and eyepiece lenses to find the magnifying power of the Palomar telescope.

(a) Express the ratio of the light-gathering powers of the Palomar and Yerkes mirrors:

$$\frac{P_{\text{Palomar}}}{P_{\text{Yerkes}}} = \frac{A_{\text{Palomar mirror}}}{A_{\text{Yerkes mirror}}} = \frac{\frac{\pi}{4}d^2_{\text{Palomar mirror}}}{\frac{\pi}{4}d^2_{\text{Yerkes mirror}}} = \frac{d^2_{\text{Palomar mirror}}}{d^2_{\text{Yerkes mirror}}}$$

Substitute numerical values and evaluate $P_{\text{Palomar}}/P_{\text{Yerkes}}$:

$$\frac{P_{\text{Palomar}}}{P_{\text{Yerkes}}} = \frac{(200\,\text{in})^2}{(40\,\text{in})^2} = 25.0$$

or

$$P_{\text{Palomar}} = \boxed{(25.0)P_{\text{Yerkes}}}$$

(b) Express the magnifying power of the Palomar telescope:

$$M = -\frac{f_o}{f_e}$$

Substitute numerical values and evaluate M:

$$M = -\frac{1.68\,\text{m}}{1.25\,\text{cm}} = \boxed{-134}$$

100 ••

Picture the Problem We can use the expression for the magnifying power of a telescope and the fact that the length of a telescope is the sum of focal lengths of its objective and eyepiece lenses to obtain simultaneous equations in f_o and f_e.

The magnifying power of the telescope is given by:

$$M = -\frac{f_o}{f_e} = 7$$

The length of the telescope is the sum of the focal lengths of the objective and eyepiece lenses:

$$L = f_o + f_e = 32\,\text{cm}$$

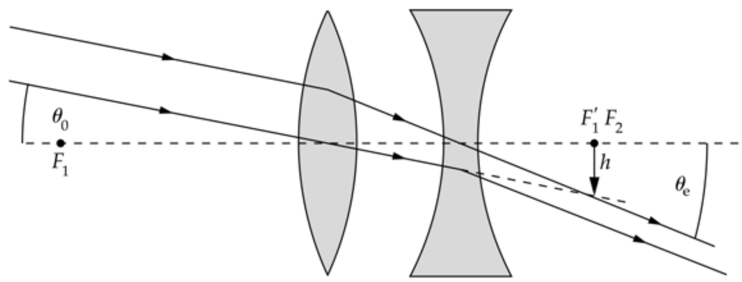
Solve these equations simultaneously to obtain:

$$f_o = \boxed{28.0\,\text{cm}} \text{ and } f_e = \boxed{4.00\,\text{cm}}$$

101 ••

Picture the Problem The magnification of a telescope is the ratio of the angle subtended at the eyepiece lens to the angle subtended at the objective lens. We can use the geometry of the ray diagram to express both θ_e and θ_o.

(*b*) The ray diagram is shown below:



(*a*) Express the magnifying power *M* of the telescope:

$$M = \frac{\theta_e}{\theta_o}$$

Because the image formed by the objective lens is at the focal point, F'_1:

$$\theta_o = \frac{h}{f_o}$$

where we have assumed that $\theta_o << 1$ so that $\tan\theta_o \approx \theta_o$.

Express the angle subtended by the eyepiece:

$$\theta_e = \frac{h}{f_e}$$ where f_e is negative.

Substitute to obtain:

$$M = \frac{\frac{h}{f_e}}{\frac{h}{f_o}} = \frac{f_o}{f_e}$$

and $M = \boxed{-\frac{f_o}{f_e}}$ is positive.

Remarks: Because the object for the eyepiece is at its focal point, the image is at infinity. As is also evident from the ray diagram, the image is virtual and upright.

102 ••

Picture the Problem We can use the thin-lens equation to find the image distance for the objective lens and the object distance for the eyepiece lens. The separation of the lenses is the sum of these distances. We can use the definition of the angular magnification and the angles subtended at the objective and eyepiece lenses to find the height of the final image.

(*a*) Solve the thin-lens equation for s_o':

$$s_o' = \frac{f_o s_o}{s_o - f_o}$$

Substitute numerical values and evaluate s_o':

$$s_o' = \frac{(1\,\text{m})(30\,\text{m})}{30\,\text{m} - 1\,\text{m}} = \boxed{103.45\,\text{cm}}$$

where we have kept more than three significant figures in the answer for use in (*c*) and (*d*).

(*b*) Solve the thin-lens equation for s_e:

$$s_e = \frac{f_e s_e'}{s_e' - f_e}$$

Noting that $s_e' = -25$ cm, substitute numerical values and evaluate s_e:

$$s_e = \frac{(-5\,\text{cm})(-25\,\text{cm})}{-25\,\text{cm} - (-5\,\text{cm})_e} = \boxed{-6.25\,\text{cm}}$$

where the minus sign tells us that the object of the eyepiece is virtual.

(*c*) Express the separation *D* of the lenses:

$$D = s_o' + s_e$$

Substitute numerical values and evaluate *D*:

$$D = 103.45\,\text{cm} - 6.25\,\text{cm} = \boxed{97.2\,\text{cm}}$$

(*d*) Express the height h' of the final image in terms of the magnification *M* of the telescope:

$$h' = Mh$$

The magnification of the telescope is the product of the magnifications of the objective and eyepiece lenses:

$$M = m_o m_e = \frac{s_o'}{s_o}\frac{s_e'}{s_e}$$

Substitute to obtain:

$$h' = \frac{s_o'}{s_o}\frac{s_e'}{s_e}h$$

Substitute numerical values and evaluate h':

$$h' = \left(\frac{103.45\,\text{cm}}{3000\,\text{cm}}\right)\left(\frac{-25\,\text{cm}}{-6.25\,\text{cm}}\right)(1.5\,\text{m}) = \boxed{20.7\,\text{cm}}$$

Express the angular magnification of the telescope:

$$M = \frac{\theta_e}{\theta_o}$$

The angle subtended by the object is:

$$\theta_o = \frac{h}{s_o}$$

The angle subtended by the image is:

$$\theta_e = \tan^{-1}\left(\frac{h'}{s_e}\right)$$

Substitute to obtain:

$$M = \frac{\tan^{-1}\left(\frac{h'}{s_e}\right)}{\frac{h}{s_o}} = \frac{s_o}{h}\tan^{-1}\left(\frac{h'}{s_e}\right)$$

Substitute numerical values and evaluate M:

$$M = \frac{30\,\text{m}}{1.5\,\text{m}}\tan^{-1}\left(\frac{20.7\,\text{cm}}{6.25\,\text{cm}}\right) = \boxed{25.6}$$

103 •••

Picture the Problem The roles of the objective and eyepiece lenses are reversed.

Express the magnifying power of the ″wrong end″ telescope:

$$M = -\frac{f_e}{f_o}$$

Substitute numerical values and evaluate M:

$$M = -\frac{1.5\,\text{cm}}{2.25\,\text{m}} = -6.67\times10^{-3} = \boxed{-1/150}$$

General Problems

104 •

Picture the Problem We can solve the thin-lens equation for s' and then argue that the signs of the numerator and denominator are such that their quotient is always negative.

Solve the thin-lens equation for s':

$$s' = \frac{fs}{s - f}$$

For a diverging lens:

$f < 0$ and $s > 0$ for a real object.

Consequently, the denominator is positive and the numerator is negative, so s' must always be negative.

***105** •

Picture the Problem We can express the distance Δs that the lens must move as the difference between the image distances when the object is at 30 m and when it is at infinity and then express these image distances using the thin-lens equation.

Express the distance Δs that the lens must move to change from focusing on an object at infinity to one at a distance of 30 m:

$$\Delta s = s'_{30} - s'_{\infty}$$

Solve the thin-lens equation for s':

$$s' = \frac{fs}{s - f}$$

Substitute and simplify to obtain:

$$\Delta s = \frac{fs_{30}}{s_{30} - f} - \frac{fs_{\infty}}{s_{\infty} - f} = \frac{fs_{30}}{s_{30} - f} - \frac{f}{1 - f/s_{\infty}} = f\left[\frac{s_{30}}{s_{30} - f} - 1\right]$$

Substitute numerical values and evaluate Δs:

$$\Delta s = (200\,\text{mm})\left[\frac{30\,\text{m}}{30\,\text{m} - 0.2\,\text{m}} - 1\right] = \boxed{1.34\,\text{mm}}$$

106 •

Picture the Problem We can express the distance Δs that the lens must move as the difference between the image distances when the object is at 30 m and when it is at infinity and then express these image distances using the thin-lens equation.

Express the distance Δs that the lens must move to change from focusing on an object at infinity to one at a

$$\Delta s = s'_{5} - s'_{\infty}$$

distance of 5 m:

Solve the thin-lens equation for s':

$$s' = \frac{fs}{s - f}$$

Substitute and simplify to obtain:

$$\Delta s = \frac{fs_5}{s_5 - f} - \frac{fs_\infty}{s_\infty - f} = \frac{fs_5}{s_5 - f} - \frac{f}{1 - f/s_\infty} = f\left[\frac{s_5}{s_5 - f} - 1\right]$$

Substitute numerical values and evaluate Δs:

$$\Delta s = (28\,\text{mm})\left[\frac{5\,\text{m}}{5\,\text{m} - 0.028\,\text{m}} - 1\right] = \boxed{0.158\,\text{mm}}$$

107 •

Picture the Problem We can use the thin-lens and magnification equations to obtain simultaneous equations that we can solve to find the image and object distances for the two situations described in the problem statement.

(*a*) Use the thin-lens equation to relate the image and object distances to the focal length of the lens:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

Because the image is twice as large as the object:

$$m = -\frac{s'}{s} \Rightarrow s' = -2s$$

Substitute to obtain:

$$\frac{1}{s} + \frac{1}{-2s} = \frac{1}{f}$$

Solve for s:

$$s = \tfrac{1}{2} f$$

Substitute numerical values and evaluate s and s':

$$s = \tfrac{1}{2}(10\,\text{cm}) = \boxed{5.00\,\text{cm}}$$

and

$$s' = -2(5\,\text{cm}) = \boxed{-10.0\,\text{cm}}$$

(*b*) If the image is inverted, then: $s' = 2s$ and $\frac{1}{s} + \frac{1}{2s} = \frac{1}{f}$

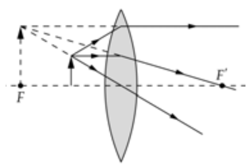
Solve for *s*: $s = \frac{3}{2} f$

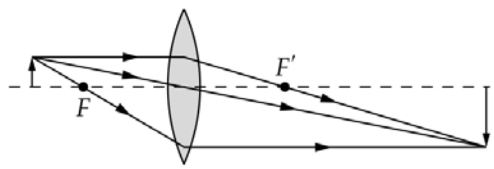
Substitute numerical values and evaluate *s* and *s*′: $s = \frac{1}{2}3(10\,\text{cm}) = \boxed{15.0\,\text{cm}}$

and

$s' = 2(15\,\text{cm}) = \boxed{30.0\,\text{cm}}$

The ray diagrams for (*a*) (left) and (*b*) (right) are shown below:

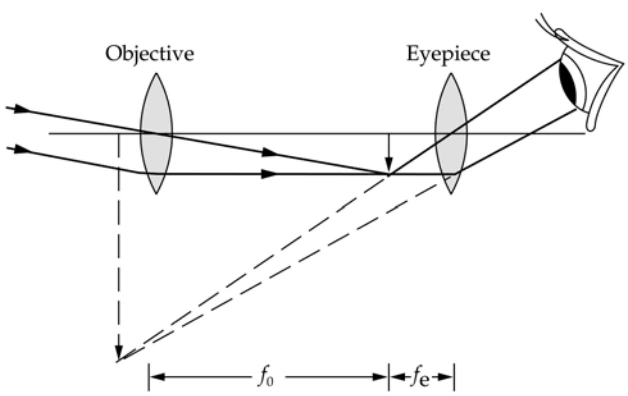




108 ••

(*a*) In an astronomical telescope the eyepiece (short focal length) and objective (long focal length) lenses are separated by the sum of their focal lengths. Given these two lenses, we'll use the 25 mm lens as the eyepiece lens and the 75 mm lens as the objective lens and mount them 100 mm apart. The angular magnification is then $M = \frac{f_0}{f_e} = \frac{75\,\text{mm}}{25\,\text{mm}} = \boxed{3}$.

(*b*) A ray diagram showing how rays from a distant object are magnified by an astronomical telescope follows. A real and inverted image of the distant object is formed by the objective lens near its second focal point. The eyepiece lens forms an enlarged and inverted image of the image formed by the objective lens.



109 ••

Determine the Concept

(*a*) Because the focal lengths appear in the magnification formula as a product, it would appear that it does not matter in which order we use them. The usual arrangement would be to use the shorter focal length lens as the objective but we get the same magnification in the reverse order. What difference does it make then? None in this problem. However, it is generally true that the smaller the focal length of a lens, the smaller its diameter. This condition makes it harder to use the shorter focal length lens, with its smaller diameter, as the eyepiece lens. If we separate the objective and eyepiece lenses by $L + f_e + f_o$ = 16 cm + 7.5 cm + 2.5 cm = $\boxed{26.0\,\text{cm},}$ the overall magnification will be

$$M = m_0 M_e = -\frac{L}{f_0}\frac{x_{np}}{f_e} = -\frac{16\,\text{cm}}{7.5\,\text{cm}}\frac{25\,\text{cm}}{2.5\,\text{cm}} = \boxed{-21.3}\,.$$

In a compound microscope, the lenses are separated by:

$$\delta = L + f_e + f_0$$

Substitute numerical values and evaluate δ:

$$\delta = 16\,\text{cm} + 7.5\,\text{cm} + 2.5\,\text{cm} = \boxed{26.0\,\text{cm}}$$

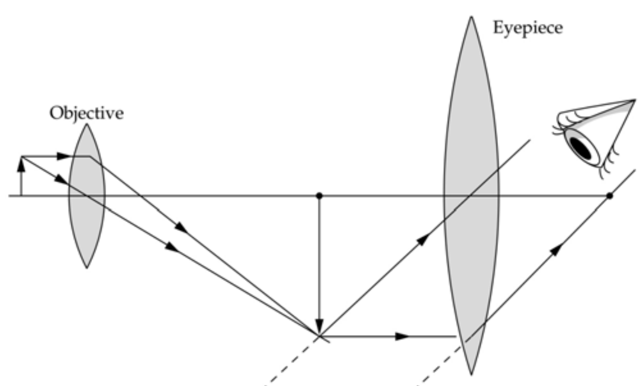
The overall magnification of a compound microscope is given by:

$$M = m_0 M_e = -\frac{L}{f_0}\frac{x_{np}}{f_e}$$

Substitute numerical values and evaluate M:

$$M = -\frac{16\,\text{cm}}{7.5\,\text{cm}}\frac{25\,\text{cm}}{2.5\,\text{cm}} = \boxed{-21.3}$$

(*b*) A ray diagram showing how rays from a near-by object are magnified by a compound microscope follows. A real and inverted image of the near-by object is formed by the objective lens at the first focal point of the eyepiece lens. The eyepiece lens forms an inverted and virtual image of this image at infinity.



***110 ••**

Picture the Problem We can use the equation for refraction at a single surface to locate the image of the fish and the expression for the magnification due to refraction at a spherical surface to find the magnification of the image.

(*a*) Use the equation describing refraction at a single surface to relate the image and object distances:

$$\frac{n_1}{s}+\frac{n_2}{s'}=\frac{n_2-n_1}{r}$$

Solve for s':

$$s'=\frac{n_2 rs}{(n_2-n_1)s-n_1 r}$$

Substitute numerical values and evaluate s':

$$s'=\frac{(1)(0.5\,\text{m})(2.5\,\text{m})}{(1-1.33)(2.5\,\text{m})-(1.33)(0.5\,\text{m})}=\boxed{-0.839\,\text{m}}$$

Note that the fish appears to be much closer to the diver than it actually is.

(*b*) Express the magnification due to refraction at a spherical surface:

$$m=-\frac{n_1 s'}{n_2 s}$$

Substitute numerical values and evaluate m:

$$m=-\frac{(1.33)(-0.839\,\text{m})}{(1)(2.5\,\text{m})}=\boxed{0.446}$$

Note that the fish appears to be smaller than it actually is.

111 ••

Picture the Problem We can use the thin-lens equation and the definition of the magnification of an image to determine where the person should stand.

Use the thin-lens equation to relate s and s':

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

The magnification of the image is given by:

$$m=-\frac{s'}{s}=-\frac{2.4\,\text{cm}}{175\,\text{cm}}=-1.37\times10^{-2}$$

and

$$s'=-ms$$

Substitute to obtain:

$$\frac{1}{s}-\frac{1}{ms}=\frac{1}{f}$$

Solve for *s*:

$$s=\left(1-\frac{1}{m}\right)f$$

Substitute numerical values and evaluate *s*:

$$s=\left(1-\frac{1}{-1.37\times10^{-2}}\right)(50\,\text{mm})$$
$$=\boxed{3.70\,\text{m}}$$

112 ••

Picture the Problem We can use the thin-lens equation and the definition of the magnification of an image to determine the ideal focal length of the lens.

Use the thin-lens equation to relate *s* and *s*′:

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

The magnification of the image is given by:

$$m=-\frac{s'}{s}=-\frac{3.6\,\text{cm}}{200\,\text{cm}}=-1.80\times10^{-2}$$

and

$$s'=-ms$$

Substitute to obtain:

$$\frac{1}{s}-\frac{1}{ms}=\frac{1}{f}$$

Solve for *f*:

$$f=\frac{s}{1-\frac{1}{m}}$$

Substitute numerical values and evaluate *f*:

$$f=\frac{30\,\text{m}}{1-\frac{1}{-1.80\times10^{-2}}}=\boxed{0.530\,\text{m}}$$

113 ••

Picture the Problem Let the numeral 1 refer to the first lens and the numeral 2 to the second lens. We apply the thin-lens equation twice; once to locate the image formed by the first lens and a second time to find the image formed by the second lens. The magnification of the image is the product of the magnifications produced by the two lenses.

(*a*) Solve the thin-lens equation for the location of the image formed by the first lens:

$$s_1' = \frac{f_1 s_1}{s_1 - f_1}$$

Substitute numerical values and evaluate s_1':

$$s_1' = \frac{(10\,\text{cm})(12\,\text{cm})}{12\,\text{cm} - 10\,\text{cm}} = 60.0\,\text{cm}$$

Because the second lens is 20 cm to the right of the first lens:

$$s_2 = 20\,\text{cm} - 60\,\text{cm} = -40\,\text{cm}$$

Solve the thin-lens equation for the location of the image formed by the second lens:

$$s_2' = \frac{f_2 s_2}{s_2 - f_2}$$

Substitute numerical values and evaluate s_2':

$$s_2' = \frac{(12.5\,\text{cm})(-40\,\text{cm})}{-40\,\text{cm} - 12.5\,\text{cm}} = \boxed{9.52\,\text{cm}}$$

i.e., the final image is 9.52 cm to the right of the second lens.

(*b*) Express the magnification of the final image:

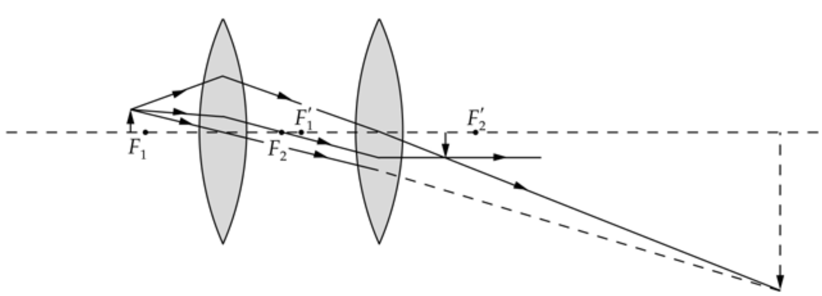
$$m = m_1 m_2 = \left(-\frac{s_1'}{s_1}\right)\left(-\frac{s_2'}{s_2}\right) = \frac{s_1' s_2'}{s_1 s_2}$$

Substitute numerical values and evaluate *m*:

$$m = \frac{(60\,\text{cm})(9.52\,\text{cm})}{(12\,\text{cm})(-40\,\text{cm})} = \boxed{-1.19}$$

i.e., the final image is about 20% larger than the object and is inverted.

(*c*) The ray diagram is shown in the figure. The enlarged, inverted image formed by the first lens serves as a virtual object for the second lens. The image formed from this virtual object is the real, inverted image shown in the ray diagram.



114 ••

Picture the Problem We can apply the equation for refraction at a surface to both surfaces of the lens and add the resulting equations to obtain an equation relating the image and object distances to the indices of refraction. We can then use the lens maker's equation to complete the derivation of the given relationship between *f* ′ and *f*.

(*a*) Relate *s* and *s*′ at the water-lens interface:

$$\frac{n_w}{s}+\frac{n}{s_1'}=\frac{n-n_w}{r_1}$$

Relate *s* and *s*′ at the lens-water interface:

$$\frac{n}{-s_1'}+\frac{n}{s'}=\frac{n_w-n}{r_2}$$

Add these equations to obtain:

$$n_w\left(\frac{1}{s}+\frac{1}{s'}\right)=(n-n_w)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

Let $\frac{1}{f'}=\frac{1}{s}+\frac{1}{s'}$ to obtain:

$$\frac{n_w}{f'}=(n-n_w)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

The lens-maker's equation is:

$$\frac{1}{f}=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

and

$$\frac{1}{r_1}-\frac{1}{r_2}=\frac{1}{(n-1)f}$$

Substitute to obtain:

$$\frac{n_w}{f'}=(n-n_w)\left(\frac{1}{(n-1)f}\right)$$

Solve for *f* ′:

$$f'=\boxed{\frac{n_w(n-1)}{n-n_w}f}$$

(*b*) Use the lens-maker's equation to find the focal length of the lens in air:

$$\frac{1}{f}=(1.5-1)\left(\frac{1}{-30\,\text{cm}}-\frac{1}{35\,\text{cm}}\right)$$

and

$$f=\boxed{-32.3\,\text{cm}}$$

Use the result derived in (*a*) to find *f* ′:

$$f'=\frac{(1.33)(1.5-1)}{1.5-1.33}(-32.3\,\text{cm})$$

$$=\boxed{-126\,\text{cm}}$$

***115 ••**

Picture the Problem Here we must consider refraction at each surface separately. To find the focal length we imagine the object at $s = \infty$, and find the image from the first refracting surface at s'_1. That image serves as the object for the second refracting surface. We'll find that this is a virtual image for the second refracting surface, i.e., s_2 is negative. Using the equation for refraction at a single surface a second time, we can locate the image formed by the second refracting surface by the virtual object at s_2. The location of that image is then the focal point of the thick lens. We'll let the numeral 1 denote the first surface and the numeral 2 the second surface. In part (*b*) we can proceed as in part (*a*) (except that now $n_1 = 1.33$ for the first refraction and $n_2 = 1.33$ for the second refraction) to determine the focal length in water, which we denote by f_w.

(*a*) Use the equation for refraction at a single surface to relate s_1 and s_1':

$$\frac{n_1}{s_1} + \frac{n_2}{s_1'} = \frac{n_2 - n_1}{r_1}$$

For $s_1 = \infty$:

$$\frac{n_2}{s_1'} = \frac{n_2 - n_1}{r_1}$$

Solve for s_1':

$$s_1' = \frac{n_2 r_1}{n_2 - n_1} \qquad (1)$$

Substitute numerical values and evaluate s_1':

$$s_1' = \frac{(1.5)(20\,\text{cm})}{1.5 - 1} = 60.0\,\text{cm}$$

The object distance s_2 for the second lens is:

$$s_2 = -(s_1' - 4\,\text{cm}) = -(60\,\text{cm} - 4\,\text{cm}) = -56\,\text{cm}$$

Solve the equation for refraction at a single surface for s_2':

$$s_2' = \frac{n_2 r_2 s_2}{(n_2 - n_1)s_2 - n_1 r_2} \qquad (2)$$

Substitute numerical values and evaluate s_2':

$$s_2' = \frac{(1)(-20\,\text{cm})(-56\,\text{cm})}{(1 - 1.5)(-56\,\text{cm}) - (1.5)(-20\,\text{cm})} = 19.3\,\text{cm}$$

Because f is measured from the center of the lens:

$$f = s_2' + 2\,\text{cm} = 19.3\,\text{cm} + 2\,\text{cm} = \boxed{21.3\,\text{cm}}$$

(*b*) Substitute numerical values in equation (1) and evaluate s_1':

$$s_1' = \frac{(1.5)(20\,\text{cm})}{1.5 - 1.33} = 176\,\text{cm}$$

The object distance s_2 for the second lens is:

$$s_2 = -(s_1' - 4\,\text{cm}) = -(176\,\text{cm} - 4\,\text{cm}) = -172\,\text{cm}$$

Substitute numerical values in equation (2) and evaluate s'_2:

$$s_2' = \frac{(1.33)(-20\,\text{cm})(-172\,\text{cm})}{(1.33 - 1.5)(-172\,\text{cm}) - (1.5)(-20\,\text{cm})} = 77.2\,\text{cm}$$

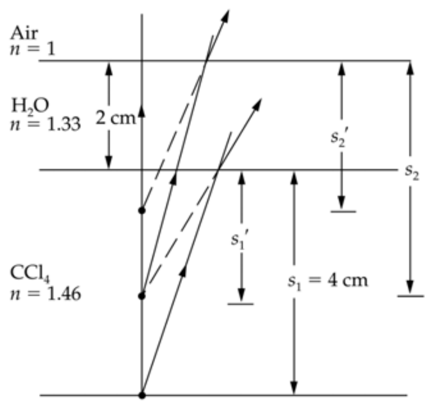
Because f_w is measured from the center of the lens:

$$f_w = s_2' + 2\,\text{cm} = 77.2\,\text{cm} + 2\,\text{cm} = \boxed{79.2\,\text{cm}}$$

Remarks: Note that if we use the expression given in Problem 114 we obtain f_w = 83.3 cm, in only moderate agreement with the exact result given above.

116 ••

Picture the Problem Let the numeral 1 denote the CCl_4-H_2O interface and the numeral 2 the H_2O-air interface. We can locate the final image by applying the equation for refraction at a single surface to both interfaces. The ray diagram shown below shows a spot at the bottom of the tank and the rays of light emanating from it that form the intermediate and final images.



Use the equation for refraction at a single surface to relate s and s' at the CCl_4-H_2O interface:

$$\frac{n_{CCl_4}}{s_1} + \frac{n_{H_2O}}{s_1'} = \frac{n_{H_2O} - n_{CCl_4}}{r}$$

or, because $r = \infty$,

$$\frac{n_{CCl_4}}{s_1} + \frac{n_{H_2O}}{s_1'} = 0$$

Solve for s_1':

$$s_1' = -\frac{n_{H_2O}s_1}{n_{CCl_4}} \qquad (1)$$

Substitute numerical values and evaluate s_1':

$$s_1' = -\frac{(1.33)(4\,\text{cm})}{1.46} = -3.64\,\text{cm}$$

The depth of this image, as viewed from the H_2O-air interface is:

$$s_2 = 2\,\text{cm} - s_1' = 2\,\text{cm} - (-3.64\,\text{cm}) = 5.64\,\text{cm}$$

At the H_2O-air interface equation (1) becomes:

$$s_2' = -\frac{n_{\text{air}} s_2}{n_{H_2O}}$$

Substitute numerical values and evaluate s_2':

$$s_2' = -\frac{(1)(5.64\,\text{cm})}{1.33} = -4.24\,\text{cm}$$

$$\boxed{\text{The apparent depth is } 4.24\,\text{cm.}}$$

117 ••

Picture the Problem The speed of the jogger as seen in the mirror is $v' = ds'/dt$. We can use the mirror equation to derive an expression for v' in terms of f and ds/dt.

Solve the mirror equation for s':

$$s' = \left(\frac{1}{f} - \frac{1}{s}\right)^{-1} \qquad (1)$$

Differentiate s' with respect to time to obtain:

$$v' = \frac{ds'}{dt} = \frac{d}{dt}\left(\frac{1}{f} - \frac{1}{s}\right)^{-1} = -\left(\frac{1}{f} - \frac{1}{s}\right)^{-2}\left(\frac{1}{s^2}\right)\frac{ds}{dt}$$

Simplify this result to obtain:

$$v' = -\left(\frac{s'}{s}\right)^2 v \qquad (2)$$

Rewrite equation (1) in terms of r:

$$s' = \left(\frac{2}{r} - \frac{1}{s}\right)^{-1}$$

Find s' when $s = 5$ m:

$$s' = \left(\frac{2}{-2\,\text{m}} - \frac{1}{5\,\text{m}}\right)^{-1} = -0.833\,\text{m}$$

Use equation (2) to find $|v'|$ when $|v| = 3.5$ m/s:

$$|v'| = \left|-\left(\frac{-0.833\,\text{m}}{5\,\text{m}}\right)^2 (3.5\,\text{m/s})\right| = \boxed{0.0971\,\text{m/s}}$$

118 ••

Picture the Problem Let the numerals 1 and 2 denote to the first and second refracting surfaces of the spherical lens, respectively, and follow the steps given in the hint.

Use the equation for refraction at a single surface to relate s_1 and s_1':

$$\frac{n_1}{s_1}+\frac{n_2}{s_1'}=\frac{n_2-n_1}{r_1}$$

When $s_1 = \infty$:

$$\frac{n_2}{s_1'}=\frac{n_2-n_1}{r_1}$$

Solve for s_1':

$$s_1'=\frac{n_2 r_1}{n_2-n_1}$$

Substitute numerical values and evaluate s_1':

$$s_1'=\frac{(1.5)(2\text{ mm})}{1.5-1}=6.00\text{ mm}$$

Because the thickness of the glass sphere is 4 mm:

$$s_2=4\text{ mm}-s_1'=4\text{ mm}-6\text{ mm}=-2\text{ mm}$$

Use the equation for refraction at a single surface to relate s_2 and s_2':

$$\frac{n_2}{s_2}+\frac{n_1}{s_2'}=\frac{n_1-n_2}{r_2}$$

Solve for s_2':

$$s_2'=\frac{n_1 r_2 s_2}{(n_1-n_2)s_2-n_2 r_2}$$

Substitute numerical values and evaluate s_2':

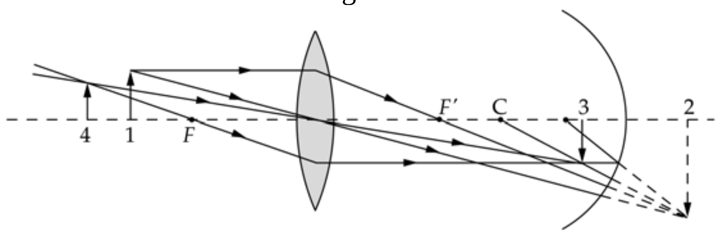
$$s_2'=\frac{(1)(-2\text{ mm})(-2\text{ mm})}{(1-1.5)(-2\text{ mm})-[1.5](-2\text{ mm})}=1.00\text{ mm}$$

Because $s_2'=1.00\text{ mm}=r/2,\ f=1.00\text{ mm}.$

119 •••

Picture the Problem We can use the thin-lens equation to locate the first image formed by the lens, the mirror equation to locate the image formed in the mirror, and the thin-lens equation a second time to locate the final image formed by the lens as the rays pass back through it.

(*b*) and (*c*) The ray diagram is shown below. The numeral 1 represents the object. The parallel and central rays from 1 are shown; one passes through the center of the lens, the other is paraxial and then passes through the focal point F'. The two rays intersect behind the mirror, and the image formed there, identified by the numeral 2, serves as a virtual object for the mirror. Two rays are shown emanating from this virtual image, one through the center of the mirror, the other passing through its focal point (halfway between C and the mirror surface) and then continuing as a paraxial ray. These two rays intersect in front of the mirror, forming a real image, identified by the numeral 3. Finally, the image 3 serves as a real object for the lens; again we show two rays, a paraxial ray that then passes through the focal point *F* and a ray through the center of the lens. These two rays intersect to form the final real, upright, and diminished image, identified as 4. To see this image the eye must be to the left of the image 4.



(*a*) Solve the thin-lens equation for s_1':

$$s_1' = \frac{fs_1}{s_1 - f}$$

Substitute numerical values and evaluate s_1':

$$s_1' = \frac{(10\,\text{cm})(15\,\text{cm})}{15\,\text{cm} - 10\,\text{cm}} = 30\,\text{cm}$$

Because the image formed by the lens is behind the mirror:

$$s_2 = 25\,\text{cm} - 30\,\text{cm} = -5\,\text{cm}$$

Solve the mirror equation for s_2':

$$s_2' = \frac{fs_2}{s_2 - f}$$

Substitute numerical values and evaluate s_2':

$$s_2' = \frac{(5\,\text{cm})(-5\,\text{cm})}{-5\,\text{cm} - 5\,\text{cm}} = 2.50\,\text{cm}$$ and the image is 22.5 cm from the lens; i.e., $s_3 = 22.5$ cm.

Solve the thin-lens equation for s_3':

$$s_3' = \frac{fs_3}{s_3 - f}$$

Substitute numerical values and evaluate s_3':

$$s_3' = \frac{(10\,\text{cm})(22.5\,\text{cm})}{22.5\,\text{cm} - 10\,\text{cm}} = \boxed{18.0\,\text{cm}}$$

***120** •••

Picture the Problem The mirror surfaces must be concave to create inverted images on reflection. Therefore, the lens is a diverging lens. Let the numeral 1 denote the lens in its initial orientation and the numeral 2 the lens in its second orientation. We can use the mirror equation to find the magnitudes of the radii of the lens' surfaces, the thin-lens equation to find its focal length, and the lens maker's equation to find its index of refraction.

Solve the mirror equation for $|r_1|$:

$$|r_1| = \frac{2s_1s_1'}{s_1' + s_1}$$

Substitute numerical values and evaluate $|r_1|$:

$$|r_1| = \frac{2(30\,\text{cm})(6\,\text{cm})}{6\,\text{cm} + 30\,\text{cm}} = 10.0\,\text{cm}$$

Solve the mirror equation for $|r_2|$:

$$|r_2| = \frac{2s_2s_2'}{s_2' + s_2}$$

Substitute numerical values and evaluate $|r_2|$:

$$|r_2| = \frac{2(30\,\text{cm})(10\,\text{cm})}{10\,\text{cm} + 30\,\text{cm}} = 15.0\,\text{cm}$$

Solve the thin-lens equation for f:

$$f = \frac{ss'}{s' + s}$$

Substitute numerical values and evaluate f:

$$f = \frac{(30\,\text{cm})(-7.5\,\text{cm})}{-7.5\,\text{cm} + 30\,\text{cm}} = -10.0\,\text{cm}$$

Solve the lens-maker's equation for n to obtain:

$$n = \frac{1}{f\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} + 1$$

Because the lens is a diverging lens, r_1 = –10 cm and r_2 = 15 cm. Substitute numerical values and evaluate n:

$$n = \frac{1}{(-10\,\text{cm})\left(\frac{1}{-10\,\text{cm}} - \frac{1}{15\,\text{cm}}\right)} + 1$$

$$= \boxed{1.60}$$

121 •••

Picture the Problem Assume that the object is very small compared to r so that all incident and reflected rays traverse 1 cm of water. The problem involves two refractions at the air–water interface and one reflection at the mirror. Let the numeral 1 refer to the first refraction at the air–water interface, the numeral 2 to the reflection in the mirror surface, and the numeral 3 to the second refraction at the water–air interface.

Use the equation for refraction at a single surface to relate s_1 and s_1':

$$\frac{n_1}{s_1} + \frac{n_2}{s_1'} = \frac{n_2 - n_1}{r}$$

or, because $r = \infty$,

$$\frac{n_1}{s_1} + \frac{n_2}{s_1'} = 0$$

Solve for s_1':

$$s_1' = -\frac{n_2 s_1}{n_1}$$

Let $n_2 = n$. Because $n_1 = 1$:

$$s_1' = -ns_1$$

Find the object distance for the mirror:

$$s_2 = 1 - s_1' = 1 + ns_1$$

where 1 has units of cm.

Solve the mirror equation for s_2':

$$s_2' = \left(\frac{2}{r} - \frac{1}{s_2}\right)^{-1}$$

Substitute for s_2:

$$s_2' = \left(\frac{2}{r} - \frac{1}{1 + ns_1}\right)^{-1}$$

Find the object distance s_3 for the water–air interface:

$$s_3 = 1 - s_2' = 1 - \left(\frac{2}{r} - \frac{1}{1 + ns_1}\right)^{-1}$$

Use the equation for refraction at a single surface to relate s_3 and s_3':

$$\frac{n_1}{s_3} + \frac{n_2}{s_3'} = \frac{n_2 - n_1}{r}$$

or, because $r = \infty$,

$$\frac{n_1}{s_3} + \frac{n_2}{s_3'} = 0$$

Solve for s_3':

$$s_3' = -\frac{n_2 s_3}{n_1}$$

Because $n_2 = 1$ and $n_1 = n$:

$$s_3' = -\frac{s_3}{n} = -\frac{1 - \left(\frac{2}{r} - \frac{1}{1 + ns_1}\right)^{-1}}{n}$$

Equate s_3' and s_1:

$$s_1 = -\frac{1 - \left(\frac{2}{r} - \frac{1}{1 + ns_1}\right)^{-1}}{n}$$

Simplify to obtain:

$$s_1^2 + \frac{2 - r}{n} s_1 + \frac{1 - r}{n^2} = 0$$

Substitute numerical values and simplify:

$$s_1^2 + \frac{2\,\text{cm} - 50\,\text{cm}}{1.33}s_1 + \frac{1\,\text{cm} - 50\,\text{cm}}{(1.33)^2} = 0$$

or

$$s_1^2 - 36.09s_1 - 27.70 = 0$$

where s_1 is in cm.

Solve for the positive value of s_1:

$$s_1 = \boxed{36.8\,\text{cm}}$$

122 •••

Picture the Problem We can use the lens maker's equation, in conjunction with the result given in Problem 114, to find the index of refraction of the liquid.

Solve the lens-maker's equation for n:

$$n = \frac{1}{f\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} + 1$$

Substitute numerical values and evaluate n:

$$n = \frac{1}{(27.5\,\text{cm})\left(\frac{1}{-17\,\text{cm}} - \frac{1}{-8\,\text{cm}}\right)} + 1$$

$$= 1.55$$

From Problem 114, the focal length of the lens in the liquid, f_L, is related to the focal length of the lens in air, f, according to:

$$f_L = \frac{n_L(n-1)}{n - n_L}f$$

Solve for n_L:

$$n_L = \frac{nf_L}{(n-1)f + f_L}$$

Substitute numerical values and evaluate n_L:

$$n_L = \frac{(1.55)(109\,\text{cm})}{(1.55-1)(27.5\,\text{cm}) + 109\,\text{cm}}$$

$$= \boxed{1.36}$$

123 •••

Picture the Problem The problem involves two refractions and one reflection. We can use the refraction at spherical surface equation and the mirror equation to find the images formed in the two refractions and one reflection. Let the numeral 1 refer to the first refraction at the air-glass interface, the numeral 2 to the reflection from the silvered surface, and the numeral 3 refer to the refraction at the glass-air interface.

(*a*) The image and object distances for the first refraction are related according to:

$$\frac{n_1}{s_1}+\frac{n_2}{s_1'}=\frac{n_2-n_1}{r}$$

Solve for s_1' to obtain:

$$s_1'=\frac{n_2 r s_1}{(n_2-n_1)s_1-n_1 r}$$

Substitute numerical values and evaluate s_1':

$$s_1'=\frac{(1.5)(10\,\text{cm})(30\,\text{cm})}{(1.5-1)(30\,\text{cm})-(1)(10\,\text{cm})}=90.0\,\text{cm}$$

The object for the mirror surface is behind the mirror and its distance from the surface of the mirror is:

$$s_2=20\,\text{cm}-90\,\text{cm}=-70\,\text{cm}$$

Use the mirror equation to relate s_2 and s_2':

$$\frac{1}{s_2}+\frac{1}{s_2'}=\frac{2}{r}$$

Solve for s_2':

$$s_2'=\frac{r s_2}{2s_2-r}$$

Substitute numerical values and evaluate s_2':

$$s_2'=\frac{(10\,\text{cm})(-70\,\text{cm})}{2(-70\,\text{cm})-10\,\text{cm}}=4.67\,\text{cm}$$

The object for the second refraction at the glass-air interface is in front of the mirrored surface and its distance from the glass-air interface is:

$$s_3=20\,\text{cm}-4.67\,\text{cm}=15.3\,\text{cm}$$

The image and object distances for the second refraction are related according to:

$$\frac{n_1}{s_3}+\frac{n_2}{s_3'}=\frac{n_2-n_1}{r}$$

Solve for s_3' to obtain:

$$s_3'=\frac{n_2 r s_3}{(n_2-n_1)s_3-n_1 r}$$

Noting that $r = -10$ cm, substitute numerical values and evaluate s_3':

$$s_3'=\frac{(1)(-10\,\text{cm})(15.3\,\text{cm})}{(1-1.5)(15.3\,\text{cm})-(1.5)(-10\,\text{cm})}=-20.8\,\text{cm}$$

The final image is $-20.8\text{ cm} + 20\text{ cm} = 0.8\text{ cm}$ behind the mirror surface.

(*b*) Proceed as in (*a*) with $s_1 = 20\text{ cm}$ to obtain $s_3 = -20\text{ cm}$ and the final image to be at the mirror surface.

124 •••

Picture the Problem We can solve the lens maker's equation for *f* and then differentiate with respect to *n* and simplify to obtain $df/f = -dn/(n-1)$.

(*a*) The lens maker's equation is:

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{1}{C}(n-1)$$

where $\frac{1}{C} = \frac{1}{r_1} - \frac{1}{r_2}$

Solve for *f*:

$$f = C(n-1)^{-1}$$

Differentiate *f* with respect to *n* and simplify:

$$\frac{df}{dn} = \frac{d}{dn}\left[C(n-1)^{-1}\right] = -C(n-1)^{-2} = -\frac{f(n-1)^{-2}}{(n-1)^{-1}} = -\frac{f}{n-1}$$

Solve for *df*/*f* :

$$\frac{df}{f} = \boxed{-\frac{dn}{n-1}}$$

(*b*) Express the focal length for blue light in terms of the focal length for red light:

$$f_{\text{blue}} = f_{\text{red}} + \Delta f \qquad (1)$$

Approximate *df*/*f* by $\Delta f/f$ and *dn* by Δn to obtain:

$$\frac{\Delta f}{f} \approx -\frac{\Delta n}{n-1}$$

Solve for Δf :

$$\Delta f = -\frac{f\,\Delta n}{n-1}$$

Substitute for Δf in equation (1) to obtain:

$$f_{\text{blue}} = f_{\text{red}} - \frac{f_{\text{red}}\,\Delta n}{n-1} = f_{\text{red}}\left(1 - \frac{\Delta n}{n_{\text{red}} - 1}\right)$$

Substitute numerical values and evaluate f_{blue}:

$$f_{\text{blue}} = (20\,\text{cm})\left(1 - \frac{1.53 - 1.47}{1.47 - 1}\right) = \boxed{17.4\,\text{cm}}$$

***125 •••**

Picture the Problem We examine the amount by which the image distance s' changes due to a change in s.

Solve the thin-lens equation for s':

$$s' = \left(\frac{1}{f} - \frac{1}{s}\right)^{-1}$$

Differentiate s' with respect to s:

$$\frac{ds'}{ds} = \frac{d}{ds}\left[\left(\frac{1}{f} - \frac{1}{s}\right)^{-1}\right] = -\frac{1}{\left(\frac{1}{f} - \frac{1}{s}\right)^2}\frac{1}{s^2} = -\frac{s'^2}{s^2} = -m^2$$

$$\boxed{\text{The image of an object of length } \Delta s \text{ will have a length } -m^2\Delta s.}$$