Chapter 33
Interference and Diffraction

Conceptual Problems

***1 •**

Determine the Concept The energy is distributed nonuniformly in space; in some regions the energy is below average (destructive interference), in others it is higher than average (constructive interference).

2 •

Determine the Concept Coherent sources have a constant phase difference. The pairs of light sources that satisfy this criterion are (*b*), (*c*), and (*e*).

3 •

Determine the Concept The thickness of the air space between the flat glass and the lens is approximately proportional to the square of *d*, the diameter of the ring. Consequently, the separation between adjacent rings is proportional to 1/*d*.

4 ••

Determine the Concept The distance between adjacent fringes is so small that the fringes are not resolved by the eye.

5 ••

Determine the Concept If the film is thick, the various colors (i.e., different wavelengths) will give constructive and destructive interference at that thickness. Consequently, what one observes is the reflected intensity of white light.

***6 •**

(*a*) The phase change on reflection from the front surface of the film is 180°; the phase change on reflection from the back surface of the film is 0°. As the film thins toward the top, the phase change associated with the film's thickness becomes negligible and the two reflected waves interfere destructively.

(*b*) The first constructive interference will arise when $t = \lambda/4$. Therefore, the first band will be violet (shortest visible wavelength).

(*c*) When viewed in transmitted light, the top of the film is white, since no light is reflected. The colors of the bands are those complementary to the colors seen in reflected light; i.e., the top band will be red.

7 •

Determine the Concept The first zeroes in the intensity occur at angles given by $\sin\theta = \lambda/a$. Hence, decreasing *a* increases θ and the diffraction pattern becomes wider.

8 •

Determine the Concept Equation 33-2 expresses the condition for an intensity maximum i slit interference. Here *d* is the slit separation, λ the wavelength of the light, *m* an integer, a the angle at which the interference maximum appears.

Equation 33-11 expresses the condition for the first minimum in single-slit diffraction. Here *a* is the width of the slit, λ the wavelength of the light, and θ the angle at which the first minimum appears, assuming $m = 1$.

9 •

Picture the Problem We can solve $d\sin\theta = m\lambda$ for θ with $m = 1$ to express the location of the first-order maximum as a function of the wavelength of the light.

The interference maxima in a diffraction pattern are at angles θ given by:

$$d\sin\theta = m\lambda$$

where *d* is the separation of the slits and $m = 0, 1, 2, \ldots$

Solve for the angular location θ_1 of the first-order maximum :

$$\theta_1 = \sin^{-1}\left(\frac{\lambda}{d}\right)$$

Because $\lambda_{\text{green light}} < \lambda_{\text{red light}}$:

$\theta_{\text{green light}} < \theta_{\text{red light}}$ and $\boxed{(a) \text{ is correct.}}$

***10** •

Determine the Concept The distance on the screen to *m*th bright fringe is given by $y_m = m\dfrac{\lambda L}{d}$, where *L* is the distance from the slits to the screen and *d* is the separation of the slits. Because the index of refraction of air is slightly larger than the index of refraction of a vacuum, the introduction of air reduces λ to λ/n and decreases y_m. Because the separation of the fringes is $y_m - y_{m-1}$, the separation of the fringes decreases and $\boxed{(b) \text{ is correct.}}$

11 •

(*a*) False. When destructive interference of light waves occurs, the energy is no longer distributed evenly. For example, light from a two-slit device forms a pattern with very bright and very dark parts. There is practically no energy at the dark fringes and a great deal of energy at the bright fringe. The total energy over the entire pattern equals the energy from one slit plus the energy from the second slit. Interference re-distributes the energy.

(*b*) True

(*c*) True

(*d*) True

(*e*) True

Estimation and Approximation

***12** •

Picture the Problem We'll assume that the diameter of the pupil of the eye is 5 mm and that the wavelength of light is 600 nm. Then we can use the expression for the minimum angular separation of two objects than can be resolved by the eye and the relationship between this angle and the width of an object and the distance from which it is viewed to support the claim.

Relate the width w of an object that can be seen at a height h to the critical angular separation α_c:

$$\tan\alpha_c = \frac{w}{h}$$

Solve for w:

$$w = h\tan\alpha_c$$

The minimum angular separation α_c of two point objects that can just be resolved by an eye depends on the diameter D of the eye and the wavelength λ of light:

$$\alpha_c = 1.22\frac{\lambda}{D}$$

Substitute for α_c in the expression for w to obtain:

$$w = h\tan\left(1.22\frac{\lambda}{D}\right)$$

In low-earth orbit:

$$w = (400\,\text{km})\tan\left(1.22\frac{600\,\text{nm}}{5\,\text{mm}}\right) = 58.6\,\text{m}$$

Because the width of the Great Wall is about 5 m, a naked eye would not be able to see it from the moon.

At a distance equal to that of the distance of the moon from earth:

$$w=\left(3.84\times10^{8}\ \text{m}\right)\tan\left(1.22\frac{600\,\text{nm}}{5\,\text{mm}}\right)=56.2\,\text{km}$$

Because the width of the Great Wall is about 5 m, a naked eye would not be able to see it from the moon.

13 •

Picture the Problem We can use $\sin\theta=1.22\frac{\lambda}{D}$ to relate the diameter D of the opaque-disk water droplets to the angular diameter θ of a coronal ring and to the wavelength of light. We'll assume a wavelength of 500 nm.

The angle θ subtended by the first diffraction minimum is related to the wavelength λ of light and the diameter D of the opaque-disk water droplet:

$$\sin\theta=1.22\frac{\lambda}{D}$$

Because of the great distance to the cloud of water droplets, $\theta \ll 1$ and:

$$\theta\approx1.22\frac{\lambda}{D}$$

Solve for D to obtain:

$$D=\frac{1.22\lambda}{\theta}$$

Substitute numerical values and evaluate D:

$$D=\frac{1.22(500\,\text{nm})}{10^{\circ}\times\frac{\pi\,\text{rad}}{180^{\circ}}}=\boxed{3.50\ \mu\text{m}}$$

14 •

Picture the Problem We can use $\sin\theta=1.22\frac{\lambda_n}{D}$ to relate the diameter D of a microsphere to the angular diameter θ of a coronal ring and to the wavelength of light in water.

The angle θ subtended by the first diffraction minimum is related to the wavelength λ_n of light in water and the diameter D of the microspheres:

$$\sin\theta=1.22\frac{\lambda_n}{D}=1.22\frac{\lambda}{nD}$$

Because $\theta \ll 1$:

$$\theta\approx1.22\frac{\lambda}{nD}$$

Substitute numerical values and evaluate θ:

$$\theta \approx \frac{1.22(632.8\,\text{nm})}{1.33(5\,\mu\text{m})} = 0.116\,\text{rad} = \boxed{6.65°}$$

15 •

Picture the Problem We can use $\sin\theta = 1.22\dfrac{\lambda}{D}$ to relate the diameter *D* of a pollen grain to the angular diameter θ of a coronal ring and to the wavelength of light. We'll assume a wavelength of 450 nm for blue light and 650 nm for red light.

The angle θ subtended by the first diffraction minimum is related to the wavelength λ of light and to the diameter *D* of the microspheres:

$$\sin\theta = 1.22\frac{\lambda}{D}$$

Because $\theta << 1$:

$$\theta \approx 1.22\frac{\lambda}{nD}$$

Substitute numerical values and evaluate θ for red light:

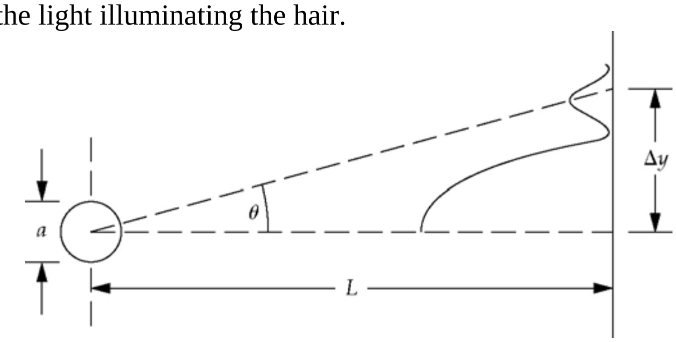
$$\theta_{\text{red}} \approx \frac{1.22(650\,\text{nm})}{25\,\mu\text{m}} = 3.17\times10^{-2}\,\text{rad} = \boxed{1.82°}$$

Substitute numerical values and evaluate θ for blue light:

$$\theta_{\text{blue}} \approx \frac{1.22(450\,\text{nm})}{25\,\mu\text{m}} = 2.20\times10^{-2}\,\text{rad} = \boxed{1.26°}$$

***16** ••

Picture the Problem The diagram shows the hair whose diameter *d* = *a*, the screen a distance *L* from the hair, and the separation Δy of the first diffraction peak from the center. We can use the geometry of the experiment to relate Δy to *L* and *a* and the condition for diffraction maxima to express θ in terms of the diameter of the hair and the wavelength of the light illuminating the hair.



Relate θ to Δy:

$$\tan\theta = \frac{\Delta y}{L}$$

Solve for Δy: $\Delta y = L\tan\theta$

Diffraction maxima occur where: $a\sin\theta = \left(m+\frac{1}{2}\right)\lambda$ where m = 1, 2, 3, …

Solve for θ to obtain: $\theta = \sin^{-1}\left[\frac{\left(m+\frac{1}{2}\right)\lambda}{a}\right]$

Substitute for θ in the expression for Δy to obtain: $\Delta y = L\tan\left\{\sin^{-1}\left[\frac{\left(m+\frac{1}{2}\right)\lambda}{a}\right]\right\}$

For the first peak, m = 1. Substitute numerical values and evaluate Δy:

$$\Delta y = (10\,\text{m})\tan\left\{\sin^{-1}\left[\frac{\left(1+\frac{1}{2}\right)(632.8\,\text{nm})}{70\,\mu\text{m}}\right]\right\} = \boxed{13.6\,\text{cm}}$$

Phase Difference and Coherence

17 •

Picture the Problem A path difference Δr contributes a difference δ given by $\delta = \frac{\Delta r}{\lambda}360°$.

(*a*) Relate a path difference Δr to a phase shift δ: $\delta = \frac{\Delta r}{\lambda}360°$ (1)

Solve for Δr: $\Delta r = \frac{\delta\lambda}{360°}$

Substitute numerical values and evaluate Δr: $\Delta r = \frac{(180°)(600\,\text{nm})}{360°} = \boxed{300\,\text{nm}}$

(*b*) Substitute numerical values in equation (1) and evaluate δ: $\delta = \frac{300\,\text{nm}}{800\,\text{nm}}360° = \boxed{135°}$

18 •

Picture the Problem The wavelength of light in a medium whose index of refraction is n is the ratio of the wavelength of the light in air divided by n. The number of wavelengths of light contained in a given distance is the ratio of the distance to the wavelength of light in the given medium. The difference in phase between the two waves is the sum of a π phase shift in the reflected wave and a phase shift due to the additional distance traveled by the wave reflected from the bottom of the water–air interface.

(*a*) Express the wavelength of light in water in terms of the wavelength of light in air:

$$\lambda_n = \frac{\lambda}{n} = \frac{500\,\text{nm}}{1.33} = \boxed{376\,\text{nm}}$$

(*b*) Relate the number of wavelengths *N* to the thickness *t* of the film and the wavelength of light in water:

$$N = \frac{2t}{\lambda_n} = \frac{2\times10^{-4}\,\text{cm}}{376\,\text{nm}} = \boxed{5.32}$$

(*c*) Express the phase difference as the sum of the phase shift due to reflection and the phase shift due to the additional distance traveled by the wave reflected from the bottom of the water–air interface:

$$\delta = \delta_{\text{reflection}} + \delta_{\text{additional distance traveled}}$$
$$= \pi + \frac{2t}{\lambda_n}2\pi = \pi + 2\pi N$$

Substitute for *N* and evaluate δ:

$$\delta = \pi\,\text{rad} + 2\pi(5.32\,\text{rad}) = \boxed{11.6\pi\,\text{rad}}$$

or, subtracting 11.6π rad from 12π rad,

$$\delta = \boxed{0.4\pi\,\text{rad}}$$

***19 ••**

Picture the Problem The difference in phase depends on the path difference according to $\delta = \frac{\Delta r}{\lambda}360°$. The path difference is the difference in the distances of (0, 15 cm) and (3 cm, 14 cm) from the origin.

Relate a path difference Δr to a phase shift δ:

$$\delta = \frac{\Delta r}{\lambda}360°$$

The path difference Δr is:

$$\Delta r = 15\,\text{cm} - \sqrt{(3\,\text{cm})^2 + (14\,\text{cm})^2}$$
$$= 0.682\,\text{cm}$$

Substitute numerical values and evaluate δ:

$$\delta = \frac{0.682\,\text{cm}}{1.5\,\text{cm}}360° = \boxed{164°}$$

Interference in Thin Films

20 •

Picture the Problem Because the *m*th fringe occurs when the path difference 2*t* equals *m* wavelengths, we can express the additional distance traveled by the light in air as an $m\lambda$. The thickness of the wedge, in turn, is related to the angle of the wedge and the distance from its vertex to the *m*th fringe.

(*a*) The first band is dark because the phase difference due to reflection by the back surface of the top plate and the top surface of the bottom plate is 180°

(*b*) The *m*th fringe occurs when the path difference 2*t* equals *m* wavelengths:

$$2t = m\lambda$$

Relate the thickness of the air wedge to the angle of the wedge:

$$\theta = \frac{t}{x} \Rightarrow t = x\theta$$

where we've used a small-angle approximation to replace an arc length by the length of a chord.

Substitute to obtain:

$$2x\theta = m\lambda$$

Solve for θ:

$$\theta = \frac{m\lambda}{2x} = \frac{1}{2}\frac{m}{x}\lambda$$

Substitute numerical values and evaluate θ:

$$\theta = \frac{1}{2}\left(\frac{5}{\text{cm}}\right)(700\,\text{nm}) = \boxed{1.75\times10^{-4}\ \text{rad}}$$

***21** ••

Picture the Problem The condition that one sees *m* fringes requires that the path difference between light reflected from the bottom surface of the top slide and the top surface of the bottom slide is an integer multiple of a wavelength of the light.

The *m*th fringe occurs when the path difference 2*d* equals *m* wavelengths:

$$2d = m\lambda \Rightarrow d = \frac{m\lambda}{2}$$

Because the nineteenth (but not the twentieth) bright fringe can be seen,

$$\left(m - \tfrac{1}{2}\right)\frac{\lambda}{2} < d < \left(m + \tfrac{1}{2}\right)\frac{\lambda}{2}$$

the limits on d must be: where $m = 19$

Substitute numerical values to obtain:

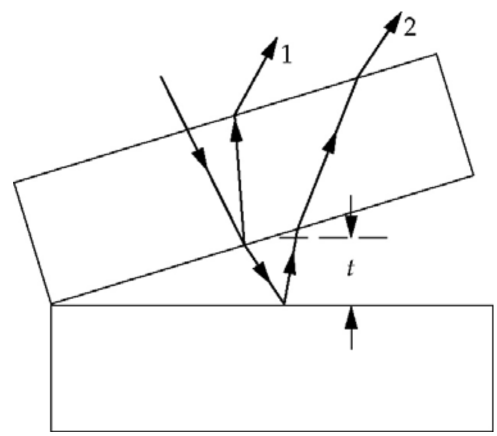
$$\left(19-\tfrac{1}{2}\right)\frac{590\,\text{nm}}{2} < d < \left(19+\tfrac{1}{2}\right)\frac{590\,\text{nm}}{2}$$

or

$$\boxed{5.46\,\mu\text{m} < d < 5.75\,\mu\text{m}}$$

22 ••

Picture the Problem The light reflected from the top surface of the bottom plate (wave 2 in the diagram) is phase shifted relative to the light reflected from the bottom surface of the top plate (wave 1 in the diagram). This phase difference is the sum of a phase shift of π (equivalent to a $\lambda/2$ path difference) resulting from reflection plus a phase shift due to the additional distance traveled.



Relate the extra distance traveled by wave 2 to the distance equivalent to the phase change due to reflection and to the condition for constructive interference:

$$2t + \tfrac{1}{2}\lambda = \lambda, 2\lambda, 3\lambda, \ldots$$

or

$$2t = \tfrac{1}{2}\lambda, \tfrac{3}{2}\lambda, \tfrac{5}{2}\lambda, \ldots$$

and

$2t = \left(m + \tfrac{1}{2}\right)\lambda$ where $m = 0, 1, 2, \ldots$ and λ is the wavelength of light in air.

Solve for m:

$$m = \frac{2t}{\lambda} - \frac{1}{2} = \frac{2(2r)}{\lambda} - \frac{1}{2} = \frac{4r}{\lambda} - \frac{1}{2}$$

where r is the radius of the wire.

Substitute numerical values and evaluate m:

$$m = \frac{4(0.025\,\text{mm})}{600\,\text{nm}} - \frac{1}{2} = \boxed{166}$$

23 ••

Picture the Problem We can use the condition for destructive interference in a thin film to find its thickness. Once we've found the thickness of the film, we can use the condition for constructive interference to find the wavelengths in the visible portion of the spectrum that will be brightest in the reflected interference pattern and the condition for destructive interference to find the wavelengths of light missing from the reflected light when the film is placed on glass with an index of refraction greater than that of the film.

(*a*) Express the condition for destructive interference in the thin film:

$$2t + \tfrac{1}{2}\lambda' = \tfrac{1}{2}\lambda', \tfrac{3}{2}\lambda', \tfrac{5}{2}\lambda', \ldots$$

or

$$2t = \lambda', 2\lambda', 3\lambda', \ldots$$

or

$$2t = m\lambda' = m\frac{\lambda}{n} \qquad (1)$$

where m = 1, 2, 3, … and λ' is the wavelength of the light in the film.

Solve for λ:

$$\lambda = \frac{2nt}{m}$$

Substitute for the missing wavelengths to obtain:

$$450\,\text{nm} = \frac{2nt}{m} \text{ and } 360\,\text{nm} = \frac{2nt}{m+1}$$

Divide the first of these equations by the second and simplify to obtain:

$$\frac{450\,\text{nm}}{360\,\text{nm}} = \frac{\frac{2nt}{m}}{\frac{2nt}{m+1}} = \frac{m+1}{m}$$

Solve for m:

$$m = 4 \text{ for } \lambda = 450\,\text{nm}$$

Solve equation (1) for t:

$$t = \frac{m\lambda}{2n}$$

Substitute numerical values and evaluate t:

$$t = \frac{4(450\,\text{nm})}{2(1.5)} = \boxed{600\,\text{nm}}$$

(*b*) Express the condition for constructive interference in the thin film:

$$2t + \tfrac{1}{2}\lambda' = \lambda', 2\lambda', 3\lambda', \ldots$$

or

$$2t = \tfrac{1}{2}\lambda', \tfrac{3}{2}\lambda', \tfrac{5}{2}\lambda', \ldots = \left(m + \tfrac{1}{2}\right)\lambda' \qquad (1)$$

where λ' is the wavelength of light in the oil and m = 0, 1, 2, …

Substitute for λ' to obtain:

$$2t = \left(m + \tfrac{1}{2}\right)\frac{\lambda}{n}$$

where n is the index of refraction of the film.

Solve for λ:

$$\lambda = \frac{2nt}{m+\frac{1}{2}}$$

Substitute numerical values and simplify to obtain:

$$\lambda = \frac{2(1.5)(600\,\text{nm})}{m+\frac{1}{2}} = \frac{1800\,\text{nm}}{m+\frac{1}{2}}$$

Substitute for *m* and evaluate λ to obtain the following table:

m	0	1	2	3	4	5
λ (nm)	3600	1200	720	514	400	327

From the table, we see that the only wavelengths in the visible spectrum are 720 nm, 514 nm, and 400 nm.

(*c*) Because the index of refraction of the glass is greater than that of the film, the light reflected from the film-glass interface will be shifted by $\frac{1}{2}\lambda$ (as is the wave reflected from the top surface) and the condition for destructive interference becomes:

$$2t = \tfrac{1}{2}\lambda', \tfrac{3}{2}\lambda', \tfrac{5}{2}\lambda', \ldots$$

or

$$2t = \left(m+\tfrac{1}{2}\right)\frac{\lambda}{n}$$

where *n* is the index of refraction of the film and *m* = 0, 1, 2, …

Solve for λ:

$$\lambda = \frac{2nt}{m+\frac{1}{2}}$$

Substitute numerical values and simplify to obtain:

$$\lambda = \frac{2(1.5)(600\,\text{nm})}{m+\frac{1}{2}} = \frac{1800\,\text{nm}}{m+\frac{1}{2}}$$

Substitute for *m* and evaluate λ to obtain the following table:

m	0	1	2	3	4	5
λ (nm)	3600	1200	720	514	400	327

From the table we see that the missing wavelengths in the visible spectrum are 720 nm, 514 nm, and 400 nm.

24 ••

Picture the Problem Because there is a $\frac{1}{2}\lambda$ phase change due to reflection at both the air-oil and oil-water interfaces, the condition for constructive interference is that twice the thickness of the oil film equal an integer multiple of the wavelength of light in the film.

Express the condition for constructive interference:

$$2t = \lambda', 2\lambda', 3\lambda', ...$$

or

$$2t = m\lambda' \quad (1)$$

where λ' is the wavelength of light in the oil = 1, 2, 3, …

Substitute for λ' to obtain:

$$2t = m\frac{\lambda}{n}$$

Solve for t:

$$t = \frac{m\lambda}{2n}$$

Substitute numerical values and evaluate t:

$$t = \frac{(2)(650\,\text{nm})}{2(1.22)} = \boxed{533\,\text{nm}}$$

25 ••

Picture the Problem Because there is a $\frac{1}{2}\lambda$ phase change due to reflection at both the air-oil and oil-glass interfaces, the condition for constructive interference is that twice the thickness of the oil film equal an integer multiple of the wavelength of light in the film.

Express the condition for constructive interference:

$$2t = \lambda', 2\lambda', 3\lambda', ... = m\lambda' \quad (1)$$

where λ' is the wavelength of light in the oil and m = 0, 1, 2, …

Substitute for λ' to obtain:

$$2t = m\frac{\lambda}{n}$$

where n is the index of refraction of the oil.

Solve for λ:

$$\lambda = \frac{2nt}{m}$$

Substitute for the predominant wavelengths to obtain:

$$690\,\text{nm} = \frac{2nt}{m} \text{ and } 460\,\text{nm} = \frac{2nt}{m+1}$$

Divide the first of these equations by the second and simplify to obtain:

$$\frac{690\,\text{nm}}{460\,\text{nm}} = \frac{\frac{2nt}{m}}{\frac{2nt}{m+1}} = \frac{m+1}{m}$$

Solve for *m*: $m = 2$ for $\lambda = 690\,\text{nm}$

Solve equation (1) for *t*:

$$t = \frac{m\lambda}{2n}$$

Substitute numerical values and evaluate *t*:

$$t = \frac{(2)(690\,\text{nm})}{2(1.45)} = \boxed{476\,\text{nm}}$$

***26 ••**

Picture the Problem Because the index of refraction of air is less than that of the oil, there is a phase shift of π rad ($\frac{1}{2}\lambda$) in the light reflected at the air-oil interface. Because the index of refraction of the oil is greater than that of the glass, there is no phase shift in the light reflected from the oil-glass interface. We can use the condition for constructive interference to determine *m* for λ = 700 nm and then use this value in our equation describing constructive interference to find the thickness *t* of the oil film.

Express the condition for constructive interference between the waves reflected from the air-oil interface and the oil-glass interface:

$$2t + \tfrac{1}{2}\lambda' = \lambda', 2\lambda', 3\lambda', \ldots$$

or

$$2t = \tfrac{1}{2}\lambda', \tfrac{3}{2}\lambda', \tfrac{5}{2}\lambda', \ldots = \left(m + \tfrac{1}{2}\right)\lambda' \qquad (1)$$

where λ' is the wavelength of light in the oil and m = 0, 1, 2, …

Substitute for λ' and solve for λ to obtain:

$$\lambda = \frac{2nt}{m + \frac{1}{2}}$$

Substitute the predominant wavelengths to obtain:

$$700\,\text{nm} = \frac{2nt}{m + \frac{1}{2}} \text{ and } 500\,\text{nm} = \frac{2nt}{m + \frac{3}{2}}$$

Divide the first of these equations by the second to obtain:

$$\frac{700\,\text{nm}}{500\,\text{nm}} = \frac{\dfrac{2nt}{m + \frac{1}{2}}}{\dfrac{2nt}{m + \frac{3}{2}}} = \frac{m + \frac{3}{2}}{m + \frac{1}{2}}$$

Solve for *m*: $m = 2$ for $\lambda = 700$ nm

Solve equation (1) for *t*:

$$t = \left(m + \tfrac{1}{2}\right)\frac{\lambda}{2n}$$

Substitute numerical values and evaluate *t*:

$$t = \left(2 + \tfrac{1}{2}\right)\frac{700\,\text{nm}}{2(1.45)} = \boxed{603\,\text{nm}}$$

Newton's Rings

***27 ••**

Picture the Problem This arrangement is essentially identical to a ″thin film″ configuration, except that the ″film″ is air. A phase change of 180° ($\frac{1}{2}\lambda$) occurs at the top of the flat glass plate. We can use the condition for constructive interference to derive the result given in (*a*) and use the geometry of the lens on the plate to obtain the result given in (*b*). We can then use these results in the remaining parts of the problem.

(*a*) The condition for constructive interference is:

$$2t+\tfrac{1}{2}\lambda=\lambda,2\lambda,3\lambda,\ldots$$

or

$$2t=\tfrac{1}{2}\lambda,\tfrac{3}{2}\lambda,\tfrac{5}{2}\lambda,\ldots=\left(m+\tfrac{1}{2}\right)\lambda$$

where λ is the wavelength of light in air and m = 0, 1, 2, …

Solve for t:

$$t=\boxed{\left(m+\tfrac{1}{2}\right)\frac{\lambda}{2},m=0,1,2,\ldots} \qquad (1)$$

(*b*) From Figure 33-39 we have:

$$r^2+(R-t)^2=R^2$$

or

$$R^2=r^2+R^2-2Rt+t^2$$

For $t << R$ we can neglect the last term to obtain:

$$R^2\approx r^2+R^2-2Rt$$

Solve for r:

$$r=\boxed{\sqrt{2Rt}} \qquad (2)$$

(*c*) $\boxed{\text{The transmitted pattern is complementary to the reflected pattern.}}$

(*d*) Square equation (2) and substitute for t from equation (1) to obtain:

$$r^2=\left(m+\tfrac{1}{2}\right)R\lambda$$

Solve for m:

$$m=\frac{r^2}{R\lambda}-\frac{1}{2}$$

Substitute numerical values and evaluate m:

$$m=\frac{(2\,\text{cm})^2}{(10\,\text{m})(590\,\text{nm})}-\frac{1}{2}=67$$

and so there will be $\boxed{68}$ bright fringes.

(*e*) The diameter of the m^{th} fringe is:

$$D = 2r = 2\sqrt{\left(m+\tfrac{1}{2}\right)R\lambda}$$

Noting that $m = 5$ for the sixth fringe, substitute numerical values and evaluate D:

$$D = 2\sqrt{\left(5+\tfrac{1}{2}\right)(10\,\text{m})(590\,\text{nm})} = \boxed{1.14\,\text{cm}}$$

(*f*) The wavelength of the light in the film becomes $\lambda_{air}/n = 444$ nm. The separation between fringes is reduced and the number of fringes that will be seen is increased by the factor $n = 1.33$.

28 ••

Picture the Problem This arrangement is essentially identical to a ″thin film″ configuration, except that the ″film″ is air. A phase change of 180° ($\tfrac{1}{2}\lambda$) occurs at the top of the flat glass plate. We can use the condition for constructive interference and the results of Problem 27(*b*) to determine the radii of the first and second bright fringes in the reflected light.

The condition for constructive interference is:

$$2t + \tfrac{1}{2}\lambda = \lambda, 2\lambda, 3\lambda, \ldots$$

or

$$2t = \tfrac{1}{2}\lambda, \tfrac{3}{2}\lambda, \tfrac{5}{2}\lambda, \ldots = \left(m+\tfrac{1}{2}\right)\lambda$$

where λ is the wavelength of light in air and $m = 0, 1, 2, \ldots$

Solve for t:

$$t = \left(m+\tfrac{1}{2}\right)\frac{\lambda}{2}, m = 0, 1, 2, \ldots$$

From Problem 27(*b*):

$$r = \sqrt{2tR}$$

Substitute for t to obtain:

$$r = \sqrt{\left(m+\tfrac{1}{2}\right)\lambda R}$$

The first fringe corresponds to $m = 0$:

$$r = \sqrt{\tfrac{1}{2}(520\,\text{nm})(2\,\text{m})} = \boxed{0.721\,\text{mm}}$$

The second fringe corresponds to $m = 1$:

$$r = \sqrt{\tfrac{3}{2}(520\,\text{nm})(2\,\text{m})} = \boxed{1.25\,\text{mm}}$$

29 ••

Picture the Problem This arrangement is essentially identical to a ″thin film″ configuration, except that the ″film″ is oil. A phase change of 180° ($\frac{1}{2}\lambda$) occurs at lens-oil interface. We can use the condition for constructive interference and the results from Problem 27(*b*) to determine the radii of the first and second bright fringes in the reflected light.

The condition for constructive interference is:

$$2t+\tfrac{1}{2}\lambda' = \lambda', 2\lambda', 3\lambda', ...$$

or

$$2t = \tfrac{1}{2}\lambda', \tfrac{3}{2}\lambda', \tfrac{5}{2}\lambda', ... = \left(m+\tfrac{1}{2}\right)\lambda'$$

where λ' is the wavelength of light in the oil and m = 0, 1, 2, …

Substitute for λ' and solve for t:

$$t = \left(m+\tfrac{1}{2}\right)\frac{\lambda}{2n}, m = 0, 1, 2, ...$$

where λ is the wavelength of light in air.

From Equation 33-29:

$$r = \sqrt{2tR}$$

Substitute for t to obtain:

$$r = \sqrt{\left(m+\tfrac{1}{2}\right)\frac{\lambda R}{n}}$$

The first fringe corresponds to m = 0:

$$r = \sqrt{\frac{1}{2}\frac{(520\,\text{nm})(2\,\text{m})}{1.82}} = \boxed{0.535\,\text{mm}}$$

The second fringe corresponds to m = 1:

$$r = \sqrt{\frac{3}{2}\frac{(520\,\text{nm})(2\,\text{m})}{1.82}} = \boxed{0.926\,\text{mm}}$$

Two-Slit Interference Pattern

***30** •

Picture the Problem The number of bright fringes per unit distance is the reciprocal of the separation of the fringes. We can use the expression for the distance on the screen to the mth fringe to find the separation of the fringes.

Express the number N of bright fringes per centimeter in terms of the separation of the fringes:

$$N = \frac{1}{\Delta y} \qquad (1)$$

Express the distance on the screen to the mth and (m + 1)st bright fringe:

$$y_m = m\frac{\lambda L}{d} \text{ and } y_{m+1} = (m+1)\frac{\lambda L}{d}$$

Subtract the second of these equations from the first to obtain:

$$\Delta y = \frac{\lambda L}{d}$$

Substitute in equation (1) to obtain:

$$N = \frac{d}{\lambda L}$$

Substitute numerical values and evaluate *N*:

$$N = \frac{1\,\text{mm}}{(600\,\text{nm})(2\,\text{m})} = \boxed{8.33\,\text{cm}^{-1}}$$

31 •

Picture the Problem We can use the expression for the distance on the screen to the *m*th and (*m* + 1)st bright fringes to obtain an expression for the separation Δ*y* of the fringes as a function of the separation of the slits *d*. Because the number of bright fringes per unit length *N* is the reciprocal of Δ*y*, we can find *d* from *N*, *λ*, and *L*.

Express the distance on the screen to the *m*th and (*m* + 1)st bright fringe:

$$y_m = m\frac{\lambda L}{d} \text{ and } y_{m+1} = (m+1)\frac{\lambda L}{d}$$

Subtract the second of these equations from the first to obtain:

$$\Delta y = \frac{\lambda L}{d}$$

Solve for *d*:

$$d = \frac{\lambda L}{\Delta y}$$

Because the number of fringes per unit length *N* is the reciprocal of Δ*y*:

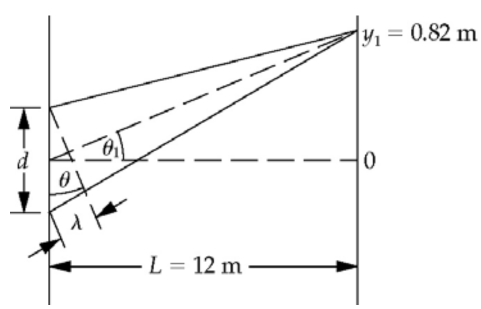
$$d = N\lambda L$$

Substitute numerical values and evaluate *d*:

$$d = (28\,\text{cm}^{-1})(589\,\text{nm})(3\,\text{m}) = \boxed{4.95\,\text{mm}}$$

32 •

Picture the Problem We can use the geometry of the setup, represented to the right, to find the separation of the slits. To find the number of interference maxima that can be observed we can apply the equation describing two-slit interference maxima and require that $\sin\theta \le 1$.



Because $d \ll L$, we can approximate $\sin\theta_1$ as:

$$\sin\theta_1 \approx \frac{\lambda}{d}$$

Solve for d to obtain:

$$d \approx \frac{\lambda}{\sin\theta_1} \qquad (1)$$

From the right triangle whose sides are L and y_1 we have:

$$\sin\theta_1 = \frac{0.82\,\text{m}}{\sqrt{(12\,\text{m})^2 + (0.82\,\text{m})^2}} = 0.06817$$

Substitute numerical values in equation (1) and evaluate d:

$$d \approx \frac{633\,\text{nm}}{0.06817} = \boxed{9.29\,\mu\text{m}}$$

(b) The equation describing two-slit interference maxima is:

$$d\sin\theta = m\lambda, m = 0, 1, 2, \ldots$$

Because $\sin\theta \le 1$ determines the maximum number of interference fringes that can be seen:

$$d = m_{\text{max}}\lambda$$

Solve for m_{max}:

$$m_{\text{max}} = \frac{d}{\lambda}$$

Substitute numerical values and evaluate m_{max}:

$$m_{\text{max}} = \frac{9.29\,\mu\text{m}}{633\,\text{nm}} = 14$$ because m must be an integer.

Because there are 14 fringes on either side of the central maximum:

$$N = 2m_{\text{max}} + 1 = 2(14) + 1 = \boxed{29}$$

33 ••

Picture the Problem We can use the equation for the distance on a screen to the *m*th bright fringe to derive an expression for the spacing of the maxima on the screen. In (c) we can use this same relationship to express the slit separation d.

(a) Express the distance on the screen to the mth and (m + 1)st bright fringe:

$$y_m = m\frac{\lambda L}{d} \text{ and } y_{m+1} = (m+1)\frac{\lambda L}{d}$$

Subtract the second of these equations from the first to obtain:

$$\Delta y = \frac{\lambda L}{d} \qquad (1)$$

Substitute numerical values and evaluate Δy:

$$\Delta y = \frac{(500\,\text{nm})(1\text{m})}{1\text{cm}} = \boxed{50.0\,\mu\text{m}}$$

(*b*) Not with the unaided eye. The separation is too small to be observed with the naked eye.

(*c*) Solve equation (1) for *d*:

$$d = \frac{\lambda L}{\Delta y}$$

Substitute numerical values and evaluate *d*:

$$d = \frac{(500\,\text{nm})(1\,\text{m})}{1\,\text{mm}} = \boxed{0.500\,\text{mm}}$$

34 ••

Picture the Problem Let the separation of the slits be *d*. We can find the total path difference when the light is incident at an angle ϕ and set this result equal to an integer multiple of the wavelength of the light to obtain the given equation.

Express the total path difference:

$$\Delta\ell = d\sin\phi + d\sin\theta_\text{m}$$

The condition for constructive interference is:

$$\Delta\ell = m\lambda$$

where *m* is an integer.

Substitute to obtain:

$$d\sin\phi + d\sin\theta_\text{m} = m\lambda$$

Divide both sides of the equation by *d* to obtain:

$$\boxed{\sin\phi + \sin\theta_\text{m} = \frac{m\lambda}{d}}$$

***35** ••

Picture the Problem Let the separation of the slits be *d*. We can find the total path difference when the light is incident at an angle ϕ and set this result equal to an integer multiple of the wavelength of the light to relate the angle of incidence on the slits to the direction of the transmitted light and its wavelength.

Express the total path difference:

$$\Delta\ell = d\sin\phi + d\sin\theta$$

The condition for constructive interference is:

$$\Delta\ell = m\lambda$$

where *m* is an integer.

Substitute to obtain:

$$d\sin\phi + d\sin\theta = m\lambda$$

Divide both sides of the equation by *d* to obtain:

$$\sin\phi + \sin\theta = \frac{m\lambda}{d}$$

Set $\theta = 0$ and solve for λ:

$$\lambda = \frac{d\sin\phi}{m}$$

Substitute numerical values and simplify to obtain:

$$\lambda = \frac{(2.5\,\mu\text{m})\sin 30°}{m} = \frac{1.25\,\mu\text{m}}{m}$$

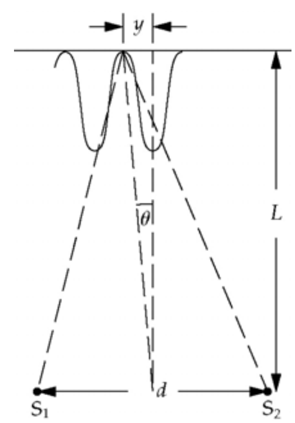
Evaluate λ for positive integral values of m:

m	λ (nm)
1	1250
2	625
3	417
4	313

From the table we can see that 625 nm and 417 nm are in the visible portion of the electromagnetic spectrum.

36 ••

Picture the Problem The diagram shows the two speakers, S_1 and S_2, the central-bright image and the first-order image to the left of the central-bright image. The distance y is measured from the center of the central-bright image. We can apply the conditions for constructive and destructive interference from two sources and use the geometry of the speakers and microphone to find the distance to the first interference minimum and the distance to the first interference maximum.



Relate the distance Δy to the first minimum from the center of the central maximum to θ and the distance L from the speakers to the plane of the microphone:

$$\tan\theta = \frac{y}{L}$$

Solve for y to obtain:

$$y = L\tan\theta \qquad (1)$$

Interference minima occur where:

$$d\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda$$

where $m = 0, 1, 2, 3, \ldots$

Solve for θ to obtain:

$$\theta = \sin^{-1}\left[\frac{\left(m + \frac{1}{2}\right)\lambda}{d}\right]$$

Relate the wavelength λ of the sound waves to the speed of sound v and the frequency f of the sound:

$$\lambda = \frac{v}{f}$$

Substitute for λ in the expression for θ to obtain:

$$\theta = \sin^{-1}\left[\frac{\left(m+\frac{1}{2}\right)v}{df}\right]$$

Substitute for θ in equation (1):

$$y = L\tan\left\{\sin^{-1}\left[\frac{\left(m+\frac{1}{2}\right)v}{df}\right]\right\} \qquad (2)$$

Noting that the first minimum corresponds to $m = 0$, substitute numerical values and evaluate Δy:

$$y_{\text{1st min}} = (1\,\text{m})\tan\left\{\sin^{-1}\left[\frac{\left(\frac{1}{2}\right)(343\,\text{m/s})}{(5\,\text{cm})(10\,\text{kHz})}\right]\right\} = \boxed{0.365\,\text{m}}$$

The maxima occur where:

$$d\sin\theta = m\lambda$$
where $m = 1, 2, 3, \ldots$

For diffraction maxima, equation (2) becomes:

$$\Delta y = L\tan\left\{\sin^{-1}\left[\frac{mv}{af}\right]\right\}$$

Noting that the first maximum corresponds to $m = 1$, substitute numerical values and evaluate Δy:

$$y_{\text{1st max}} = (1\,\text{m})\tan\left\{\sin^{-1}\left[\frac{(1)(343\,\text{m/s})}{(5\,\text{cm})(10\,\text{kHz})}\right]\right\} = \boxed{0.943\,\text{m}}$$

Diffraction Pattern of a Single Slit

37 •

Picture the Problem We can use the expression locating the first zeroes in the intensity to find the angles at which these zeroes occur as a function of the slit width a.

The first zeroes in the intensity occur at angles given by:

$$\sin\theta = \frac{\lambda}{a}$$

Solve for θ:

$$\theta = \sin^{-1}\left(\frac{\lambda}{a}\right)$$

(a) For $a = 1$ mm:

$$\theta = \sin^{-1}\left(\frac{600\,\text{nm}}{1\,\text{mm}}\right) = \boxed{0.600\,\text{mrad}}$$

(*b*) For *a* = 0.1 mm:

$$\theta = \sin^{-1}\left(\frac{600\,\text{nm}}{0.1\,\text{mm}}\right) = \boxed{6.00\,\text{mrad}}$$

(*c*) For *a* = 0.01 mm:

$$\theta = \sin^{-1}\left(\frac{600\,\text{nm}}{0.01\,\text{mm}}\right) = \boxed{60.0\,\text{mrad}}$$

38 •

Picture the Problem We can use the expression locating the first zeroes in the intensity to find the wavelength of the radiation as a function of the angle at which the first diffraction minimum is observed and the width of the plate.

The first zeroes in the intensity occur at angles given by:

$$\sin\theta = \frac{\lambda}{a}$$

Solve for λ:

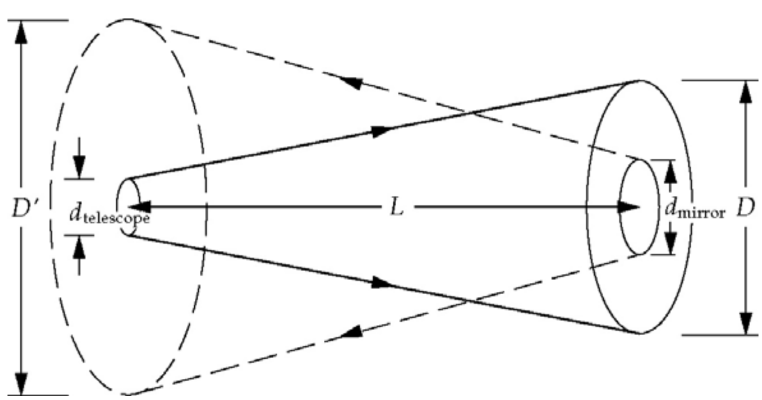
$$\lambda = a\sin\theta$$

Substitute numerical values and evaluate λ:

$$\lambda = (5\,\text{cm})\sin 37° = \boxed{3.01\,\text{cm}}$$

***39 ••**

Picture the Problem The diagram shows the beam expanding as it travels to the moon and that portion of it that is reflected from the mirror on the moon expanding as it returns to earth. We can express the diameter of the beam at the moon as the product of the beam divergence angle and the distance to the moon and use the equation describing diffraction at a circular aperture to find the beam divergence angle. We can follow this same procedure to find the diameter of the beam when it gets back to the earth. In Parts (*c*) and (*d*) we can use the dependence of the power in a beam on its cross-sectional area to find the fraction of the power of the beam that is reflected back to earth and the fraction of the original beam energy that is recaptured upon return to earth.



(*a*) Relate the diameter *D* of the beam at the moon to the distance to the moon *L* and the beam divergence angle θ:

$$D \approx \theta L$$

The angle θ subtended by the first diffraction minimum is related to the wavelength λ of the light and the diameter of the telescope opening $d_{\text{telescope}}$ by:

$$\sin\theta = 1.22\frac{\lambda}{d_{\text{telescope}}}$$

Because $\theta << 1$, $\sin\theta \approx \theta$ and:

$$\theta \approx 1.22\frac{\lambda}{d_{\text{telescope}}}$$

Substitute for θ in equation (1) to obtain:

$$D = \frac{1.22L\lambda}{d_{\text{telescope}}}$$

Substitute numerical values and evaluate D:

$$D = \left(3.82\times10^{8}\text{ m}\right)\left[\frac{1.22(500\text{ nm})}{6\text{ in}\times\frac{2.54\text{ cm}}{\text{in}}\times\frac{1\text{ m}}{10^{2}\text{ cm}}}\right] = \boxed{1.53\text{ km}}$$

(b) The portion of the beam reflected back to the earth will be that portion incident on the mirror, so the diffraction angle is:

$$\theta \approx 1.22\frac{\lambda}{d_{\text{mirror}}}$$

The beam will expand back to:

$$D' = L\left[1.22\frac{\lambda}{d_{\text{mirror}}}\right]$$

Substitute numerical values and evaluate D':

$$D' = \left(3.82\times10^{8}\text{ m}\right)\left[\frac{1.22(500\text{ nm})}{20\text{ in}\times\frac{2.54\text{ cm}}{\text{in}}\times\frac{1\text{ m}}{10^{2}\text{ cm}}}\right] = \boxed{459\text{ m}}$$

(c) Because the power of the beam is proportional to its cross-sectional area, the fraction of the power that is reflected back to the earth is the ratio of the area of the mirror to the area of the expanded beam at the moon:

$$\frac{P'}{P} = \frac{A_{\text{mirror}}}{A_{\text{beam}}} = \frac{\frac{\pi}{4}d_{\text{mirror}}^{2}}{\frac{\pi}{4}D^{2}} = \left(\frac{d_{\text{mirror}}}{D}\right)^{2}$$

Substitute for D to obtain:

$$\frac{P'}{P}=\left(\frac{\dfrac{d_{\text{mirror}}}{1.22L\lambda}}{d_{\text{telescope}}}\right)^2=\left(\frac{d_{\text{mirror}}d_{\text{telescope}}}{1.22L\lambda}\right)^2 \quad (1)$$

Substitute numerical values and evaluate P'/P:

$$\frac{P'}{P}=\left[\frac{(20\,\text{in})(6\,\text{in})\left(\dfrac{2.54\,\text{cm}}{\text{in}}\right)^2}{1.22(3.82\times10^8\ \text{m})(500\,\text{nm})}\right]^2$$

$$=\boxed{1.10\times10^{-7}}$$

(d) The angular spread of the beam from reflection from the 20-in mirror is given by:

$$\theta\approx1.22\frac{\lambda}{d_{\text{mirror}}}$$

The diameter D' of the beam on return to earth will be:

$$D'\approx1.22L\frac{\lambda}{d_{\text{mirror}}}$$

Letting P'' represent the power intercepted by the telescope, we have:

$$\frac{P''}{P'}=\frac{A_{\text{telescope}}}{A_{\text{beam}}}=\frac{\dfrac{\pi}{4}d_{\text{telescope}}^2}{\dfrac{\pi}{4}D'^2}$$

$$=\left(\frac{d_{\text{telescope}}}{D'}\right)^2$$

Substitute for D' and simplify:

$$\frac{P''}{P'}=\left(\frac{d_{\text{telescope}}d_{\text{mirror}}}{1.22L\lambda}\right)^2 \quad (2)$$

Multiply equation (2) by equation (1) and simplify to obtain:

$$\frac{P''}{P'}\frac{P'}{P}=\frac{P''}{P}=\left(\frac{d_{\text{telescope}}d_{\text{mirror}}}{1.22L\lambda}\right)^2\left(\frac{d_{\text{mirror}}d_{\text{telescope}}}{1.22L\lambda}\right)^2=\left(\frac{d_{\text{mirror}}d_{\text{telescope}}}{1.22L\lambda}\right)^4$$

Substitute numerical values and evaluate P''/P:

$$\frac{P''}{P}=\left[\frac{(20\,\text{in})(6\,\text{in})\left(\dfrac{2.54\,\text{cm}}{\text{in}}\right)^2}{1.22(3.82\times10^8\ \text{m})(500\,\text{nm})}\right]^4$$

$$=\boxed{1.21\times10^{-14}}$$

Interference-Diffraction Pattern of Two Slits

40 •

Picture the Problem We need to find the value of *m* for which the *m*th interference maximum coincides with the first diffraction minimum. Then there will be $N = 2m - 1$ fringes in the central maximum.

The number of fringes *N* in the central maximum is:

$$N = 2m - 1 \qquad (1)$$

Relate the angle θ_1 of the first diffraction minimum to the width *a* of the slits of the diffraction grating:

$$\sin\theta_1 = \frac{\lambda}{a}$$

Express the angle θ_m corresponding to the *m*th interference maxima in terms of the separation *d* of the slits:

$$\sin\theta_m = \frac{m\lambda}{d}$$

Because we require that $\theta_1 = \theta_m$, we can equate these expressions to obtain:

$$\frac{m\lambda}{d} = \frac{\lambda}{a}$$

Solve for and evaluate *m*:

$$m = \frac{d}{a} = \frac{5a}{a} = 5$$

Substitute in equation (1) to obtain:

$$N = 2(5) - 1 = \boxed{9}$$

If $d = na$:

$$m = \frac{d}{a} = \frac{na}{a} = n$$

and

$$N = \boxed{2n - 1}$$

41 ••

Picture the Problem We can equate the sine of the angle at which the first diffraction minimum occurs to the sine of the angle at which the fifth interference maximum occurs to find *a*. We can then find the number of bright interference fringes seen in the central diffraction maximum using $N = 2m - 1$.

(*a*) Relate the angle θ_1 of the first diffraction minimum to the width *a* of the slits of the diffraction grating:

$$\sin\theta_1 = \frac{\lambda}{a}$$

Express the angle θ_5 corresponding to the mth fifth interference maxima maximum in terms of the separation *d* of the slits:

$$\sin\theta_5 = \frac{5\lambda}{d}$$

Because we require that $\theta_1 = \theta_{m5}$, we can equate these expressions to obtain:

$$\frac{5\lambda}{d} = \frac{\lambda}{a}$$

Solve for and evaluate *ma*:

$$a = \frac{d}{5} = \frac{0.1\text{mm}}{5} = \boxed{20.0\,\mu\text{m}}$$

(*b*) Because *m* = 5:

$$N = 2m - 1 = 2(5) - 1 = \boxed{9}$$

42 ••

Picture the Problem We can equate the sine of the angle at which the first diffraction minimum occurs to the sine of the angle at which the *m*th interference maximum occurs to find *m*. We can then find the number of bright interference fringes seen in the central diffraction maximum using $N = 2m - 1$.

The number of fringes *N* in the central maximum is:

$$N = 2m - 1 \qquad (1)$$

Relate the angle θ_1 of the first diffraction minimum to the width *a* of the slits of the diffraction grating:

$$\sin\theta_1 = \frac{\lambda}{a}$$

Express the angle θ_m corresponding to the *m*th interference maxima in terms of the separation *d* of the slits:

$$\sin\theta_m = \frac{m\lambda}{d}$$

Because we require that $\theta_1 = \theta_m$, we can equate these expressions to obtain:

$$\frac{m\lambda}{d} = \frac{\lambda}{a}$$

Solve for *m*:

$$m = \frac{d}{a}$$

Substitute in equation (1) to obtain:

$$N = \frac{2d}{a} - 1$$

Substitute numerical values and evaluate N:

$$N = \frac{2(0.2\,\text{mm})}{0.01\,\text{mm}} - 1 = \boxed{39}$$

***43 ••**

Determine the ConceptPicture the Problem There are 8 interference fringes on each side of the central maximum. The secondary diffraction maximum is half as wide as the central one. It follows that it will contain 8 interference maxima.

44 ••

Picture the Problem We can equate the sine of the angle at which the first diffraction minimum occurs to the sine of the angle at which the *m*th interference maximum occurs to find *m*. We can then find the number of bright interference fringes seen in the central diffraction maximum using $N = 2m - 1.$ In (*b*) we can use the expression relating the intensity in a single-slit diffraction pattern to phase constant $\phi = \frac{2\pi}{\lambda} a \sin\theta$ to find the ratio of the intensity of the third interference maximum to the side of the centerline to the intensity of the center interference maximum.

(*a*) The number of fringes N in the central maximum is:

$$N = 2m - 1 \qquad (1)$$

Relate the angle θ_1 of the first diffraction minimum to the width a of the slits of the diffraction grating:

$$\sin\theta_1 = \frac{\lambda}{a}$$

Express the angle θ_m corresponding to the *m*th interference maxima in terms of the separation d of the slits:

$$\sin\theta_m = \frac{m\lambda}{d}$$

Because we require that $\theta_1 = \theta_m$, we can equate these expressions to obtain:

$$\frac{m\lambda}{d} = \frac{\lambda}{a}$$

Solve for m:

$$m = \frac{d}{a}$$

Substitute in equation (1) to obtain:

$$N = \frac{2d}{a} - 1$$

Substitute numerical values and evaluate N:

$$N = \frac{2(0.15\,\text{mm})}{0.03\,\text{mm}} - 1 = \boxed{9}$$

(*b*) Express the intensity for a single-slit diffraction pattern as a function of the phase difference ϕ:

$$I = I_0\left(\frac{\sin\frac{1}{2}\phi}{\frac{1}{2}\phi}\right)^2 \qquad (2)$$

where $\phi = \frac{2\pi}{\lambda}a\sin\theta$

For $m = 3$:

$$\sin\theta_3 = \frac{3\lambda}{d}$$

and

$$\phi = \frac{2\pi}{\lambda}a\sin\theta_3 = \frac{2\pi}{\lambda}a\left(\frac{3\lambda}{d}\right) = 6\pi\left(\frac{a}{d}\right)$$

Substitute numerical values and evaluate ϕ:

$$\phi = 6\pi\left(\frac{0.03\,\text{mm}}{0.15\,\text{mm}}\right) = \frac{6\pi}{5}$$

Solve equation (2) for the ratio of I_3 to I_0:

$$\frac{I}{I_0} = \left(\frac{\sin\frac{1}{2}\phi}{\frac{1}{2}\phi}\right)^2$$

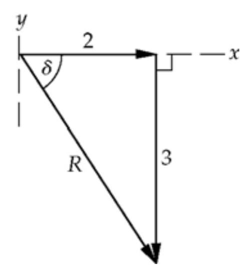
Substitute numerical values and evaluate I_3/I_0:

$$\frac{I_3}{I_0} = \left[\frac{\sin\frac{1}{2}\left(\frac{6\pi}{5}\right)}{\frac{1}{2}\left(\frac{6\pi}{5}\right)}\right]^2 = \boxed{0.255}$$

Using Phasors to Add Harmonic Waves

45 •

Picture the Problem Chose the coordinate system shown in the phasor diagram. We can use the standard methods of vector addition to find the resultant of the two waves.



The resultant of the two waves is of the form:

$$E = R\sin(\omega t + \delta)$$

Express $\vec{R}$ in vector form:

$$\vec{R} = 2\hat{i} - 3\hat{j}$$

Find the magnitude of $\vec{R}$:

$$R = \sqrt{(2)^2 + (-3)^2} = 3.61$$

Find the phase angle δ between $\vec{R}$ and $\vec{E}_1$:

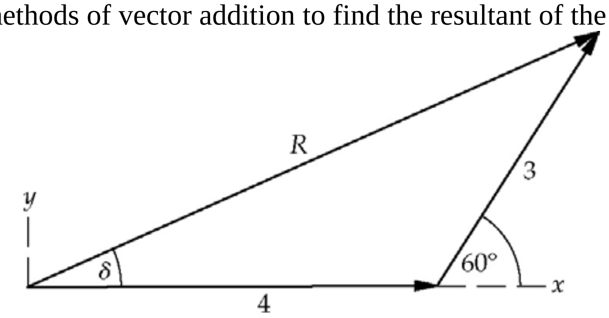
$$\delta = \tan^{-1}\left(\frac{-3}{2}\right) = -56.3°$$

Substitute to obtain:

$$E = \boxed{3.61\sin(\omega t - 56.3°)}$$

***46 •**

Picture the Problem Chose the coordinate system shown in the phasor diagram. We can use the standard methods of vector addition to find the resultant of the two waves.



The resultant of the two waves is of the form:

$$E = R\sin(\omega t + \delta)$$

Express the x component of $\vec{R}$:

$$R_x = 4 + 3\cos 60° = 5.50$$

Express the y component of $\vec{R}$:

$$R_y = 0 + 3\sin 60° = 2.60$$

Find the magnitude of $\vec{R}$:

$$R = \sqrt{(5.50)^2 + (2.60)^2} = 6.08$$

Find the phase angle δ between $\vec{R}$ and $\vec{E}_1$:

$$\delta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{2.60}{5.50}\right) = 25.3°$$

Substitute to obtain:

$$E = \boxed{6.08\sin(\omega t + 25.3°)}$$

Remarks: We could have used the law of cosines to find *R* and the law of sines to find δ.

47 ••

Picture the Problem We can evaluate the expression for the intensity for a single-slit diffraction pattern at the second secondary maximum to express I_2 in terms of I_0.

The intensity at the second secondary maximum is given by:

$$I_2 = I_0\left[\frac{\sin\frac{1}{2}\phi}{\frac{1}{2}\phi}\right]^2$$

where

$$\phi = \frac{2\pi}{\lambda} a \sin\theta$$

At this second secondary maximum:

$$a\sin\theta = \frac{5}{2}\lambda$$

and

$$\phi = \frac{2\pi}{\lambda}\left(\frac{5\lambda}{2}\right) = 5\pi$$

Substitute for ϕ and evaluate I_2:

$$I_2 = I_0\left[\frac{\sin\left(\frac{5\pi}{2}\right)}{\frac{5\pi}{2}}\right]^2 = \boxed{0.0162 I_0}$$

48 ••

Picture the Problem We can use phasor concepts to find the phase angle δ in terms of the number of phasors N (three in this problem) forming a closed polygon of N sides at the minima and then use this information to express the path difference Δr for each of these locations. Applying a small angle approximation, we can obtain an expression for y that we can evaluate for enough of the path differences to establish the pattern given in the problem statement.

Express the phase angle δ in terms of the number of phasors N forming a closed polygon of N sides at the first minimum:

$$\delta = \frac{2\pi}{N}$$

Express the path difference Δr in terms of $\sin\theta$ and the separation d of the slits:

$$\Delta r = d\sin\theta$$

or, provided the small angle approximation is valid,

$$\Delta r = \frac{yd}{L}$$

where L is the distance to the screen.

Solve for y:

$$y = \frac{L}{d}\Delta r \quad y = \frac{L}{2d}\delta$$

For three equally spaced sources, the phase angle corresponding to the first minimum is:

$$\delta = \frac{2\pi}{3} \text{ and } \Delta r = \frac{\lambda}{2\pi}\delta = \tfrac{1}{3}\lambda$$

Substitute to obtain:

$$y_1 = \frac{L}{d}\left(\frac{\lambda}{3}\right) = (1)\frac{\lambda L}{3d}$$

The phase angle corresponding to the second minimum is:

$$\delta = \frac{1}{2}\left(\frac{2\pi}{3}\right) \text{ and } \Delta r = \frac{\lambda}{2\pi}\delta = \tfrac{2}{3}\lambda$$

Substitute to obtain:

$$y_2 = \frac{L}{d}\left(\frac{2\lambda}{3}\right) = (2)\frac{\lambda L}{3d}$$

When the path difference is λ, we have an interference maximum.

The path difference corresponding to the fourth minimum is:

$$\Delta r = \tfrac{4}{3}\lambda$$

Substitute to obtain:

$$y_2 = \frac{L}{d}\left(\frac{4\lambda}{3}\right) = (4)\frac{\lambda L}{3d}$$

Continue in this manner to obtain:

$$\boxed{y_{\min} = \frac{n\lambda L}{3d}, n = 1, 2, 4, 5, 7, 8, ...}$$

(*b*) For L = 1 m, $\lambda = 5\times10^{-7}$ m, and d = 0.1 mm:

$$2y_{\min} = \frac{2(500\,\text{nm})(1\,\text{m})}{3(0.1\,\text{mm})} = \boxed{3.33\,\text{mm}}$$

49 ••

Picture the Problem We can use phasor concepts to find the phase angle δ in terms of the number of phasors N (four in this problem) forming a closed polygon of N sides at the minima and then use this information to express the path difference Δr for each of these locations. Applying a small angle approximation, we can obtain an expression for y that we can evaluate for enough of the path differences to establish the pattern given in the problem statement.

Express the phase angle δ in terms of the number of phasors N forming a closed polygon of N sides at the first minimum:

$$\delta = \frac{2\pi}{N}$$

Express the path difference Δr in terms of $\sin\theta$ and the separation d of the slits:

$$\Delta r = d\sin\theta$$

or, provided the small angle approximation is valid,

$$\Delta r = \frac{yd}{L}$$

where L is the distance to the screen.

Solve for y:

$$y = \frac{L}{d}\Delta r$$

For four equally spaced sources, the phase angle corresponding to the first minimum is:

$$\delta = \frac{\pi}{2} \text{ and } \Delta r = \frac{\lambda}{2\pi}\delta = \tfrac{1}{4}\lambda$$

Substitute to obtain:

$$y_1 = \frac{L}{d}\left(\frac{\lambda}{4}\right) = (1)\frac{\lambda L}{4d}$$

The phase angle corresponding to the second minimum is:

$$\delta = \pi \text{ and } \Delta r = \frac{\lambda}{2\pi}\delta = \tfrac{1}{2}\lambda$$

Substitute to obtain:

$$y_2 = \frac{L}{d}\left(\frac{\lambda}{2}\right) = (2)\frac{\lambda L}{4d}$$

The phase angle angle corresponding to the third minimum is:

$$\delta = \frac{3\pi}{2} \text{ and } \Delta r = \frac{\lambda}{2\pi}\left(\frac{3\pi}{2}\right) = \frac{3\lambda}{4}$$

Substitute to obtain:

$$y_3 = \frac{L}{d}\left(\frac{3\lambda}{4}\right) = (3)\frac{\lambda L}{4d}$$

Continue in this manner to obtain:

$$\boxed{y_{\text{min}} = \frac{n\lambda L}{4d}, n = 1, 2, 3, 5, 6, 7, 9, \ldots}$$

(*b*) For L = 2 m, $\lambda = 6\times10^{-7}$ m, d = 0.1 mm, and n = 1:

$$2y_{\text{min}} = \frac{2(600\,\text{nm})(2\,\text{m})}{4(0.1\text{mm})} = \boxed{6.00\,\text{mm}}$$

For two slits:

$$2y_{\text{min}} = \frac{2\left(m + \frac{1}{2}\right)\lambda L}{d}$$

For L = 2 m, $\lambda = 6\times10^{-7}$ m, d = 0.1 mm, and m = 0:

$$2y_{\text{min}} = \frac{(600\,\text{nm})(2\,\text{m})}{0.1\text{mm}} = 12.0\,\text{mm}$$

The width for four sources is half the width for two sources.

50 ••

Picture the Problem We can use $\sin\theta = \lambda/a$ to find the first zeros in the intensity pattern. The four-slit interference maxima occur at angles given by $d\sin\theta = m\lambda, m = 0,1,2,\ldots$. In (*c*) we can use the result of Problem 49 to find the angular spread between the central interference maximum and the first interference minimum on either side of it. In (*d*) we'll proceed as in Example 33-6, using a phasor diagram for a four-slit grating, to find the resultant amplitude at a given point in the intensity pattern as a function of the phase constant δ, that, in turn, is a function of the angle θ that determines the location of a point in the interference pattern.

(*a*) The first zeros in the intensity occur at angles given by:

$$\sin\theta = \frac{\lambda}{a}$$

Solve for θ:

$$\theta = \sin^{-1}\left(\frac{\lambda}{a}\right)$$

Substitute numerical values and evaluate θ:

$$\theta = \sin^{-1}\left(\frac{480\,\text{nm}}{2\,\mu\text{m}}\right) = \boxed{0.242\,\text{rad}}$$

(*b*) The four-slit interference maxima occur at angles given by:

$$d\sin\theta = m\lambda, m = 0,1,2,\ldots$$

Solve for θ_m:

$$\theta_m = \sin^{-1}\left[\frac{m\lambda}{d}\right]$$

Substitute numerical values to obtain:

$$\theta_m = \sin^{-1}\left[\frac{m(480\,\text{nm})}{6\,\mu\text{m}}\right] = \sin^{-1}(0.08m)$$

Evaluate θ_m for m = 0, 1, 2, and 3:

$$\theta_0 = \sin^{-1}[0(0.08)] = \boxed{0}$$

$$\theta_1 = \sin^{-1}[1(0.08)] = \boxed{80.1\,\text{mrad}}$$

$$\theta_2 = \sin^{-1}[2(0.08)] = \boxed{0.161\,\text{rad}}$$

$$\theta_3 = \sin^{-1}[3(0.08)] = 0.242\,\text{rad}$$

where θ_3 will not be seen as it coincides with the first minimum in the diffraction pattern.

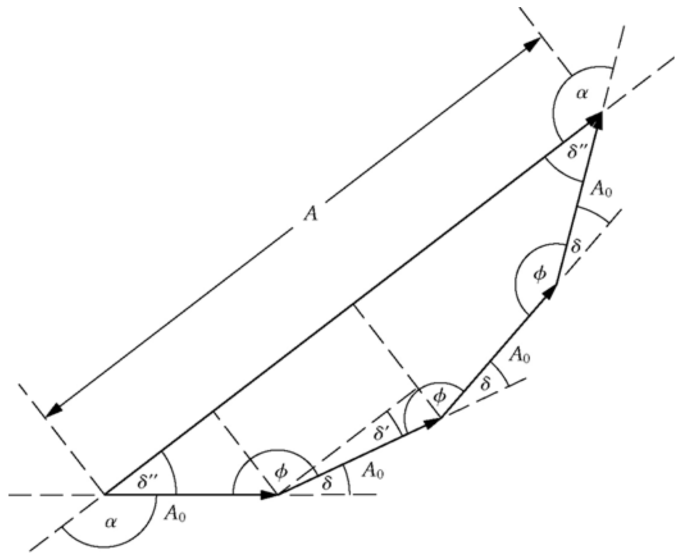
(*c*) From Problem 49 we have:

$$\theta_{\min} = \frac{n\lambda}{4d}$$

For $n = 1$:

$$\theta_{min} = \frac{480\,\text{nm}}{4(6\,\mu\text{m})} = \boxed{0.0200\,\text{rad}}$$

(*d*) Use the phasor method to show the superposition of four waves of the same amplitude A_0 and constant phase difference $\delta = \frac{2\pi}{\lambda} d \sin\theta$.



Express A in terms of δ' and δ'':

$$A = 2(A_0 \cos\delta'' + A_0 \cos\delta') \quad (1)$$

Because the sum of the external angles of a polygon equals 2π:

$$2\alpha + 3\delta = 2\pi$$

Examining the phasor diagram we see that:

$$\alpha + \delta'' = \pi$$

Eliminate α and solve for δ'' to obtain:

$$\delta'' = \tfrac{3}{2}\delta$$

Because the sum of the internal angles of a polygon of n sides is $(n - 2)\pi$:

$$3\phi + 2\delta'' = 3\pi$$

From the definition of a straight angle we have:

$$\phi - \delta' + \delta = \pi$$

Eliminate ϕ between these equations to obtain:

$$\delta' = \tfrac{1}{2}\delta$$

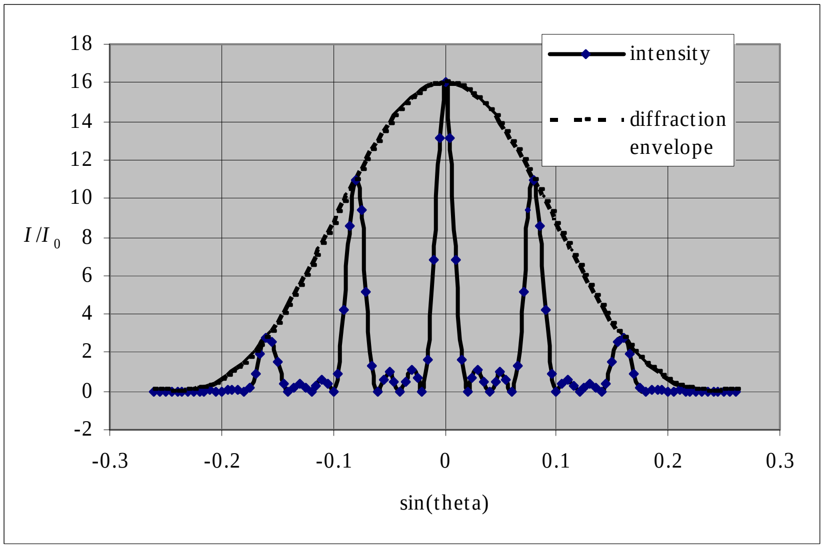
Substitute for δ'' and δ' in equation (1) to obtain:

$$A = 2A_0\left(\cos\tfrac{3}{2}\delta + \cos\tfrac{1}{2}\delta\right)$$

Because the intensity is proportional to the square of the amplitude of the resultant wave:

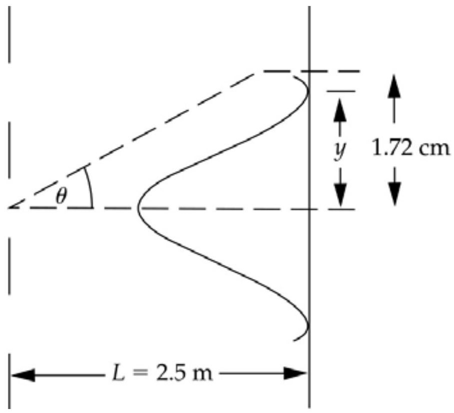
$$I = 4I_0\left(\cos\tfrac{3}{2}\delta + \cos\tfrac{1}{2}\delta\right)^2$$

The following graph of I/I_0 as a function of $\sin\theta$ was plotted using a spreadsheet program. The diffraction envelope was plotted using $\frac{I}{I_0} = 4^2\left(\frac{\sin\frac{1}{2}\phi}{\frac{1}{2}\phi}\right)^2$, where $\phi = \frac{2\pi}{\lambda}a\sin\theta$. Note the excellent agreement with the results calculated in (*a*), (*b*), and (*c*).



51 •••

Picture the Problem We can find the phase constant δ from the geometry of the diagram to the right. Using the value of δ found in this fashion we can express the intensity at the point 1.72 cm from the centerline in terms of the intensity on the centerline. On the centerline, the amplitude of the resultant wave is 3 times that of each individual wave and the intensity is 9 times that of each source acting separately.



(*a*) Express δ for the adjacent slits:

$$\delta = \frac{2\pi}{\lambda} d \sin\theta$$

For small angles, $\sin\theta \approx \tan\theta$:

$$\sin\theta \approx \tan\theta = \frac{y}{L}$$

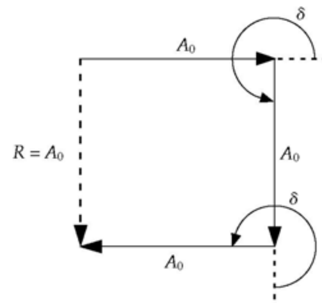
Substitute to obtain:

$$\delta = \frac{2\pi dy}{\lambda L}$$

Substitute numerical values and evaluate δ:

$$\delta = \frac{2\pi(0.06\,\text{mm})(1.72\,\text{cm})}{(550\,\text{nm})(2.5\,\text{m})} = \frac{3\pi}{2}\,\text{rad} = 270°$$

The three phasors, 270° apart, are shown in the diagram to the right. Note that they form three sides of a square. Consequently, their sum, shown as the resultant *R*, equals the magnitude of one of the phasors.



(*b*) Express the intensity at the point 1.72 cm from the centerline:

$$I \propto R^2$$

Because $I_0 \propto 9R^2$:

$$\frac{I}{I_0} = \frac{R^2}{9R^2} \Rightarrow I = \frac{I_0}{9}$$

Substitute for I_0 and evaluate *I*:

$$I = \frac{0.05\,\text{W/m}^2}{9} = \boxed{5.56\,\text{mW/m}^2}$$

***52 •••**

Picture the Problem We can use the phasor diagram shown in Figure 33-26 to determine the first three values of ϕ that produce subsidiary maxima. Setting the derivative of Equation 33-19 equal to zero will yield a transcendental equation whose roots are the values of ϕ corresponding to the maxima in the diffraction pattern.

(*a*) Referring to Figure 33-26 we see that the first subsidiary maximum occurs when:

$$\phi = 3\pi$$

A minimum occurs when:

$$\phi = 4\pi$$

Another maximum occurs when: $\phi = 5\pi$

Thus, subsidiary maxima occur when: $\phi = (2n+1)\pi,\ n = 1, 2, 3, \ldots$
and the first three subsidiary maxima are at $\phi = 3\pi$, 5π, and 7π.

(*b*) The intensity in the single-slit diffraction pattern is given by:

$$I = I_0\left(\frac{\sin\frac{1}{2}\phi}{\frac{1}{2}\phi}\right)^2$$

Set the derivative of this expression equal to zero for extrema:

$$\frac{dI}{d\phi} = 2I_0\left(\frac{\sin\frac{1}{2}\phi}{\frac{1}{2}\phi}\right)\left[\frac{\frac{1}{4}\phi\cos\frac{1}{2}\phi - \frac{1}{2}\sin\frac{1}{2}\phi}{\left(\frac{1}{2}\phi\right)^2}\right] = 0 \text{ for relative maxima and minima}$$

Simplify to obtain the transcendental equation: $\tan\frac{1}{2}\phi = \frac{1}{2}\phi$

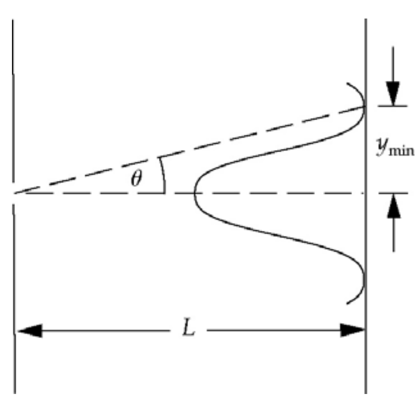
Solve this equation numerically (use the ″Solver″ function of your calculator) to obtain: $\phi = \boxed{2.86\pi, 4.92\pi, \text{and } 6.94\pi}$

Remarks: Note that our results in (*b*) are smaller than the approximate values found in (*a*) by 4.80%, 1.63%, and 0.865% and that the agreement improves as *n* increases.

Diffraction and Resolution

53 •

Picture the Problem We can use $\theta = 1.22\frac{\lambda}{D}$ to find the angle between the central maximum and the first diffraction minimum for a Fraunhofer diffraction pattern and the diagram to the right to find the distance between the central maximum and the first diffraction minimum on a screen 8 m away from the pinhole.



(*a*) The angle between the central maximum and the first diffraction $\theta = 1.22\frac{\lambda}{D}$

minimum for a Fraunhofer diffraction pattern is given by:

Substitute numerical values and evaluate θ:

$$\theta = 1.22\frac{700\text{ nm}}{0.1\text{ mm}} = \boxed{8.54\text{ mrad}}$$

(*b*) Referring to the diagram, we see that:

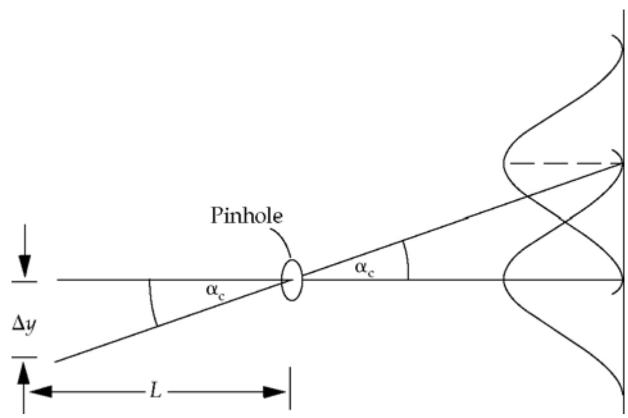
$$y = L\tan\theta$$

Substitute numerical values and evaluate *y*:

$$y = (8\text{ m})\tan(8.54\text{ mrad}) = \boxed{6.83\text{ cm}}$$

54 •

Picture the Problem We can apply Rayleigh's criterion to the overlapping diffraction patterns and to the diameter *D* of the pinhole to obtain an expression that we can solve for Δy.



Rayleigh's criterion is satisfied provided:

$$\alpha_c = 1.22\frac{\lambda}{D}$$

Relate α_c to the separation Δy of the light sources:

$$\alpha_c \approx \frac{\Delta y}{L} \text{ provided } \alpha_c << 1.$$

Equate these expressions to obtain:

$$\frac{\Delta y}{L} = 1.22\frac{\lambda}{D}$$

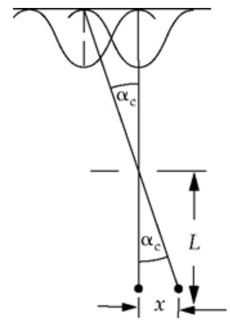
Solve for Δy:

$$\Delta y = 1.22\frac{\lambda L}{D}$$

Substitute numerical values and evaluate Δy:

$$\Delta y = 1.22\frac{(700\text{ nm})(10\text{ m})}{0.1\text{mm}} = \boxed{8.54\text{cm}}$$

***55** •



Picture the Problem We can use Rayleigh's criterion for slits and the geometry of the diagram to the right showing the overlapping diffraction patterns to express x in terms of λ, L, and the width a of the slit.

Referring to the diagram, relate α_c, L, and x:

$$\alpha_c \approx \frac{x}{L}$$

For slits, Rayleigh's criterion is:

$$\alpha_c = \frac{\lambda}{a}$$

Equate these two expressions to obtain:

$$\frac{x}{L} = \frac{\lambda}{a}$$

Solve for x:

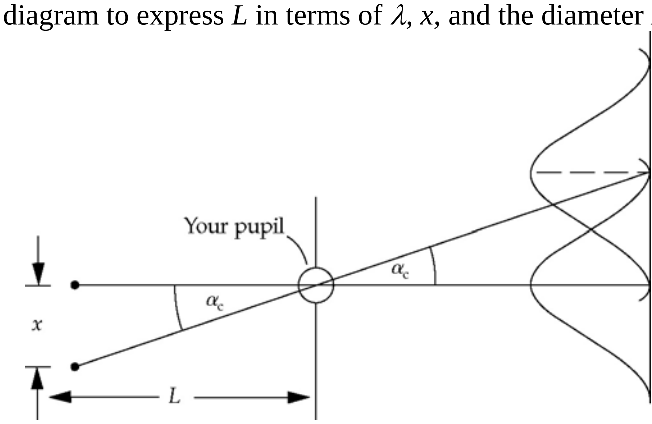
$$x = \frac{\lambda L}{a}$$

Substitute numerical values and evaluate x:

$$x = \frac{(700\text{ nm})(5\text{ m})}{0.5\text{ mm}} = \boxed{7.00\text{ mm}}$$

56 •

Picture the Problem We can use Rayleigh's criterion for circular apertures and the geometry of the diagram to express L in terms of λ, x, and the diameter D of your pupil.



Referring to the diagram, relate α_c, L, and x:

$$\alpha_c \approx \frac{x}{L}$$

For circular apertures, Rayleigh's criterion is:

$$\alpha_c = 1.22\frac{\lambda}{D}$$

Equate these two expressions to obtain:

$$\frac{x}{L} = 1.22\frac{\lambda}{D}$$

Solve for L:

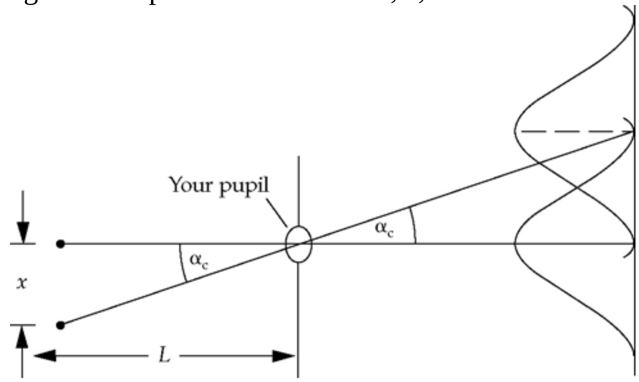
$$L = \frac{xD}{1.22\lambda}$$

Substitute numerical values and evaluate L:

$$L = \frac{(112\,\text{cm})(5\,\text{mm})}{1.22(550\,\text{nm})} = \boxed{8.35\,\text{km}}$$

57 •

Picture the Problem We can use Rayleigh's criterion for circular apertures and the geometry of the diagram to express L in terms of λ, x, and the diameter D of your pupil.



Referring to the diagram, relate α_c, L, and x:

$$\alpha_c \approx \frac{x}{L}$$

For circular apertures, Rayleigh's criterion is:

$$\alpha_c = 1.22\frac{\lambda}{D}$$

Equate these two expressions to obtain:

$$\frac{x}{L} = 1.22\frac{\lambda}{D}$$

Solve for L:

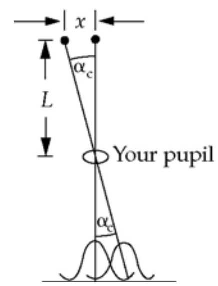
$$L = \frac{xD}{1.22\lambda}$$

Substitute numerical values and evaluate L:

$$L = \frac{(6.5\,\text{cm})(5\,\text{mm})}{1.22(550\,\text{nm})} = \boxed{484\,\text{m}}$$

58 ••

Picture the Problem We can use Rayleigh's criterion for circular apertures and the geometry of the diagram to the right showing the overlapping diffraction patterns to express L in terms of λ, x, and the diameter D of your pupil.



(*a*) Referring to the diagram, relate α_c, L, and x:

$$\alpha_c \approx \frac{x}{L} \text{ provided } \alpha << 1$$

For circular apertures, Rayleigh's criterion is:

$$\alpha_c = 1.22\frac{\lambda}{D}$$

Equate these two expressions to obtain:

$$\frac{x}{L} = 1.22\frac{\lambda}{D}$$

Solve for L:

$$L = \frac{xD}{1.22\lambda}$$

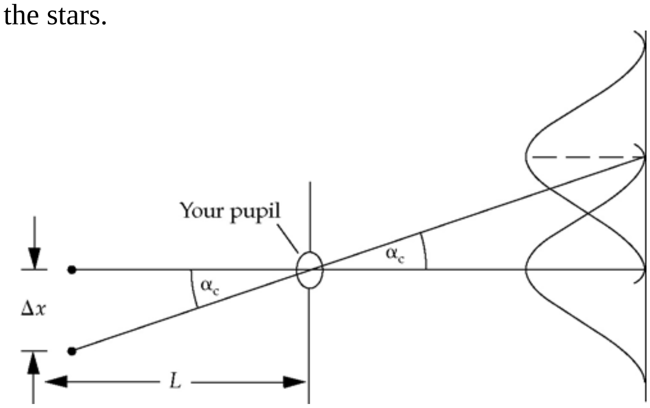
Substitute numerical values and evaluate L:

$$L = \frac{(6\,\text{mm})(5\,\text{mm})}{1.22(500\,\text{nm})} = \boxed{49.2\,\text{m}}$$

(*b*) Because L is inversely proportional to λ, the holes can be resolved better with violet light which has a shorter wavelength.

59 ••

Picture the Problem We can use Rayleigh's criterion for circular apertures and the geometry of the diagram to obtain an expression we can solve for the minimum separation Δx of the stars.



(*a*) Rayleigh's criterion is satisfied provided:

$$\alpha_c = 1.22\frac{\lambda}{D}$$

Relate α_c to the separation Δx of the light sources:

$$\alpha_c \approx \frac{\Delta x}{L} \text{ because } \alpha_c << 1$$

Equate these expressions to obtain:

$$\frac{\Delta x}{L} = 1.22\frac{\lambda}{D}$$

Solve for Δx:

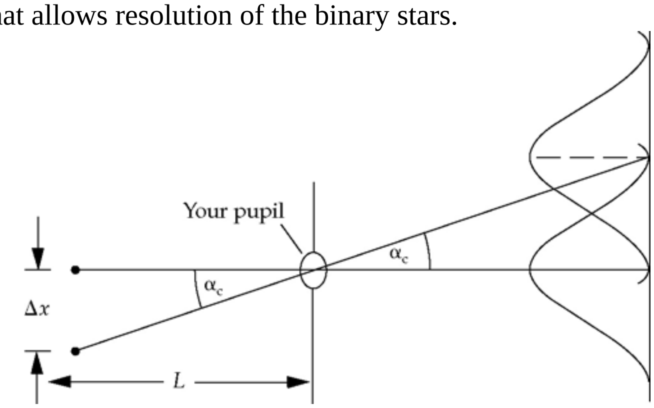
$$\Delta x = 1.22\frac{\lambda L}{D}$$

Substitute numerical values and evaluate Δx:

$$\Delta x = 1.22\frac{(550\,\text{nm})\left(4c\cdot\text{y}\times\frac{9.461\times10^{15}\,\text{m}}{1c\cdot\text{y}}\right)}{200\,\text{in}\times\frac{2.54\,\text{cm}}{1\,\text{in}}} = \boxed{5.00\times10^{9}\,\text{m}}$$

***60** ••

Picture the Problem We can use Rayleigh's criterion for circular apertures and the geometry of the diagram to obtain an expression we can solve for the minimum diameter *D* of the pupil that allows resolution of the binary stars.



(*a*) Rayleigh's criterion is satisfied provided:

$$\alpha_c = 1.22\frac{\lambda}{D}$$

Solve for *D*:

$$D = 1.22\frac{\lambda}{\alpha_c}$$

Substitute numerical values and evaluate D:

$$D = 1.22\frac{550\,\text{nm}}{14''\times\frac{1°}{3600''}\times\frac{\pi\,\text{rad}}{180°}} = \boxed{9.89\,\text{mm}} \approx 1\,\text{cm}$$

Diffraction Gratings

61 •

Picture the Problem We can solve $d\sin\theta = m\lambda$ for θ with $m = 1$ to express the location of the first-order maximum as a function of the wavelength of the light.

The interference maxima in a diffraction pattern are at angles θ given by:

$$d\sin\theta = m\lambda$$

where d is the separation of the slits and m = 0, 1, 2, …

Solve for the angular location θ_m of the maxima :

$$\theta_m = \sin^{-1}\left(\frac{m\lambda}{d}\right)$$

Relate the number of slits N per centimeter to the separation d of the slits:

$$N = \frac{1}{d}$$

Substitute to obtain:

$$\theta_m = \sin^{-1}(mN\lambda)$$

Evaluate θ_1 for λ = 434 nm:

$$\theta_1 = \sin^{-1}\left[\left(2000\,\text{cm}^{-1}\right)(434\,\text{nm})\right] = \boxed{86.9\,\text{mrad}}$$

Evaluate θ_1 for λ = 410 nm:

$$\theta_1 = \sin^{-1}\left[\left(2000\,\text{cm}^{-1}\right)(410\,\text{nm})\right] = \boxed{82.1\,\text{mrad}}$$

***62** •

Picture the Problem We can solve $d\sin\theta = m\lambda$ for λ with $m = 1$ to express the location of the first-order maximum as a function of the angles at which the first-order images are found.

The interference maxima in a diffraction pattern are at angles θ given by:

$$d\sin\theta = m\lambda$$

where d is the separation of the slits and m = 0, 1, 2, …

Solve for λ:

$$\lambda = \frac{d\sin\theta}{m}$$

Relate the number of slits N per centimeter to the separation d of the slits:

$$N = \frac{1}{d}$$

Let m =1 and substitute for d to obtain:

$$\lambda = \frac{d\sin\theta}{N}$$

Substitute numerical values and evaluate λ_1 for θ_1 = 9.72 $\times10^{-2}$ rad:

$$\lambda_1 = \frac{\sin\left(9.72\times10^{-2}\text{ rad}\right)}{2000\text{ cm}^{-1}} = \boxed{485\text{ nm}}$$

Substitute numerical values and evaluate λ_1 for θ_2 = 1.32 $\times10^{-1}$ rad:

$$\lambda_1 = \frac{\sin\left(1.32\times10^{-1}\text{ rad}\right)}{2000\text{ cm}^{-1}} = \boxed{658\text{ nm}}$$

63 •

Picture the Problem We can solve $d\sin\theta = m\lambda$ for θ with m = 1 to express the location of the first-order maximum as a function of the wavelength of the light.

The interference maxima in a diffraction pattern are at angles θ given by:

$$d\sin\theta = m\lambda$$

where d is the separation of the slits and m = 0, 1, 2, …

Solve for the angular location θ_m of the maxima :

$$\theta_m = \sin^{-1}\left(\frac{m\lambda}{d}\right)$$

Relate the number of slits N per centimeter to the separation d of the slits:

$$N = \frac{1}{d}$$

Substitute to obtain:

$$\theta_m = \sin^{-1}\left(mN\lambda\right)$$

Evaluate θ_1 for λ = 434 nm:

$$\theta_1 = \sin^{-1}\left[\left(15000\text{ cm}^{-1}\right)\left(434\text{ nm}\right)\right] = 0.7089\text{ rad} = \boxed{40.6°}$$

Evaluate θ_1 for λ = 410 nm:

$$\theta_1 = \sin^{-1}\left[\left(15000\text{ cm}^{-1}\right)\left(410\text{ nm}\right)\right] = 0.6624\text{ rad} = \boxed{38.0°}$$

64 •

Picture the Problem We can use the grating equation with $\sin\theta = 1$ and $m = 5$ to find the longest wavelength that can be observed in the fifth-order spectrum with the given grating spacing.

The interference maxima are at angles θ given by:

$$d\sin\theta = m\lambda,\quad m = 1, 2, 3, \ldots$$

Solve for λ:

$$\lambda = \frac{d\sin\theta}{m}$$

Evaluate λ for $\sin\theta = 1$ and $m = 5$:

$$\lambda = \frac{d}{5} = \frac{\dfrac{1}{4000\,\text{cm}^{-1}}}{5} = \boxed{500\,\text{nm}}$$

65 •

Picture the Problem We can use the grating equation to find the angle at which normally incident blue light will be diffracted by the *Morpho's* wings.

The grating equation is:

$$d\sin\theta = m\lambda$$

where $m = 1, 2, 3, \ldots$

Solve for θ to obtain:

$$\theta = \sin^{-1}\left[\frac{m\lambda}{d}\right]$$

Substitute numerical values and evaluate θ_1:

$$\theta = \sin^{-1}\left[\frac{(1)(440\,\text{nm})}{880\,\text{nm}}\right] = \boxed{30.0°}$$

66 ••

Picture the Problem We can use the grating equation to find the angular separation of the first-order spectrum of the two lines. In (b) we can apply the definition of the resolving power of the grating to find the width of the grating that must be illuminated for the lines to be resolved.

(*a*) Express the angular separation in the first-order spectrum of the two lines:

$$\Delta\theta = \theta_{579} - \theta_{577}$$

Solve the grating equation for θ:

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right)$$

Substitute to obtain:

$$\Delta\theta = \sin^{-1}\left[\frac{m(579\text{ nm})}{\frac{1}{2000\text{ cm}^{-1}}}\right] - \sin^{-1}\left[\frac{m(577\text{ nm})}{\frac{1}{2000\text{ cm}^{-1}}}\right]$$

For $m = 1$:

$$\Delta\theta = \sin^{-1}\left[\frac{(1)(579\text{ nm})}{\frac{1}{2000\text{ cm}^{-1}}}\right] - \sin^{-1}\left[\frac{(1)(577\text{ nm})}{\frac{1}{2000\text{ cm}^{-1}}}\right] = \boxed{0.0231°}$$

(*b*) Express the width of the beam necessary for these lines to be resolved:

$$w = Nd \qquad (1)$$

Relate the resolving power of the diffraction grating to the number of slits *N* that must be illuminated in order to resolve these wavelengths in the *m*th order:

$$\frac{\lambda}{\Delta\lambda} = mN$$

For $m = 1$:

$$N = \frac{\lambda}{\Delta\lambda}$$

Substitute in equation (1) to obtain:

$$w = \frac{\lambda d}{\Delta\lambda}$$

Letting λ be the average of the two wavelengths, substitute numerical values and evaluate *w*:

$$w = \frac{(578\text{ nm})\left(\frac{1}{2000\text{ cm}^{-1}}\right)}{2\text{ nm}} = \boxed{1.45\text{ mm}}$$

***67 ••**

Picture the Problem We can use the grating equation $d\sin\theta = m\lambda, m = 1, 2, 3, \ldots$ to express the order number in terms of the slit separation *d*, the wavelength of the light λ, and the angle θ.

The interference maxima in the diffraction pattern are at angles θ

$$d\sin\theta = m\lambda, m = 1, 2, 3, \ldots$$

given by:

Solve for m:

$$m = \frac{d \sin\theta}{\lambda}$$

If one is to see the complete spectrum:

$$\sin\theta \le 1 \text{ and } m \le \frac{d}{\lambda}$$

Evaluate m_{max}:

$$m_{max} \frac{\frac{1}{4800\,\text{cm}^{-1}}}{\lambda_{max}} = \frac{\frac{1}{4800\,\text{cm}^{-1}}}{700\,\text{nm}} = 2.98$$

Because $m_{max} = 2.98$, one can see the complete spectrum only for $m = 1$ and 2.

Express the condition for overlap:

$$m_1\lambda_1 \ge m_2\lambda_2$$

Because 700 nm < 2 × 400 nm, there is no overlap of the second-order spectrum into the first-order spectrum; however, there is overlap of long wavelengths in the second order with short wavelengths in the third-order spectrum.

68 ••

Picture the Problem We can use the grating equation and the resolving power of the grating to derive an expression for the angle at which you should look to see a wavelength of 510 nm in the fourth order.

The interference maxima in the diffraction pattern are at angles θ given by:

$$d \sin\theta = m\lambda, m = 1, 2, 3, \ldots \qquad (1)$$

The resolving power R is given by:

$$R = mN$$

where N is the number of slits and m is the order number.

Relate d to the width w of the grating:

$$d = \frac{w}{N}$$

Substitute for N to obtain:

$$d = \frac{mw}{R}$$

Substitute for d in equation (1) to obtain:

$$\frac{mw}{R}\sin\theta = m\lambda$$

Solve for θ:

$$\theta = \sin^{-1}\left(\frac{R\lambda}{w}\right)$$

Substitute numerical values and evaluate θ:

$$\theta = \sin^{-1}\left[\frac{(22{,}000)(510\,\text{nm})}{5\,\text{cm}}\right] = \boxed{13.0°}$$

69 ••

Picture the Problem The distance on the screen to the *m*th bright fringe can be found using $y_m = m\lambda L/d$,where d is the slit separation. We can use $\theta_{\min} = \lambda/Nd = \Delta y/2L$ to find the width of the central maximum and the $R = mN$, where N is the number of slits in the grating, to find the resolution in the first order.

(*a*) The distance on the screen to the *m*th bright fringe is given by:

$$y_m = m\frac{\lambda L}{d}$$

or, because $d = n^{-1}$,

$$y_m = mn\lambda L$$

Substitute numerical values to obtain:

$$y_m = m\left(4000\,\text{cm}^{-1}\right)(589\,\text{nm})(1.5\,\text{m}) = (0.353\,\text{m})m$$

Evaluate y_1 and y_2:

$$y_1 = (0.353\,\text{m})(1) = \boxed{0.353\,\text{m}}$$

and

$$y_2 = (0.353\,\text{m})(2) = \boxed{0.706\,\text{m}}$$

(*b*) The angle $\theta_{\min}$ that locates the first minima in the diffraction pattern is given by:

$$\theta_{\min} = \frac{\lambda}{Nd} = \frac{\Delta y}{2L}$$

where Δy is the width of the central maximum.

Solve for Δy:

$$\Delta y = \frac{2L\lambda}{Nd}$$

Substitute numerical values and evaluate Δy:

$$\Delta y = \frac{2(1.5\,\text{m})(589\,\text{nm})}{(8000\,\text{lines})\left(\frac{1}{4000\,\text{cm}^{-1}}\right)} = \boxed{88.4\,\mu\text{m}}$$

(*c*) The resolution *R* in the *m*th order is given by:

$$R = mN$$

Substitute numerical values and evaluate *R*:

$$R = (1)(8000) = \boxed{8000}$$

70 ••

Picture the Problem The width of the grating *w* is the product of its number of lines *N* and the separation of its slits *d*. Because the resolution of the grating is a function of the average wavelength, the difference in the wavelengths, and the order number, we can express *w* in terms of these quantities.

Express the width *w* of the grating as a function of the number of lines *N* and the slit separation *d*:

$$w = Nd$$

The resolving power *R* of the grating is given by:

$$R = \frac{\lambda}{\Delta\lambda} = mN$$

Solve for *N* to obtain:

$$N = \frac{\lambda}{m\Delta\lambda}$$

Substitute for *N* in the expression for *w* to obtain:

$$w = \frac{\lambda d}{m\Delta\lambda}$$

Letting λ be the average of the given wavelengths, substitute numerical values and evaluate *w*:

$$w = \frac{\frac{1}{2}(519.313\,\text{nm} + 519.322\,\text{nm})\left(\frac{1}{8400\,\text{cm}^{-1}}\right)}{2(519.322\,\text{nm} - 519.313\,\text{nm})} = \boxed{3.43\,\text{cm}}$$

***71** ••

Picture the Problem We can use the expression for the resolving power of a grating to find the resolving power of the grating capable of resolving these two isotopic lines in the third-order spectrum. Because the total number of the slits of the grating *N* is related to width *w* of the illuminated region and the number of lines per centimeter of the grating and the resolving power *R* of the grating, we can use this relationship to find the number of lines per centimeter of the grating

The resolving power of a diffraction grating is given by:

$$R=\frac{\lambda}{|\Delta\lambda|}=mN \qquad (1)$$

Substitute numerical values and evaluate R:

$$R=\frac{546.07532}{|546.07532-546.07355|}=\boxed{3.09\times10^{5}}$$

Express n, be the number of lines per centimeter of the grating, in terms of the total number of slits N of the grating and the width w of the grating:

$$n=\frac{N}{w}$$

From equation (1) we have:

$$N=\frac{R}{m}$$

Substitute to obtain:

$$n=\frac{R}{mw}$$

Substitute numerical values and evaluate n:

$$n=\frac{3.09\times10^{5}}{(3)(2\text{ cm})}=\boxed{5.15\times10^{4}\text{ cm}^{-1}}$$

72 ••

Picture the Problem We can differentiate the grating equation implicitly to obtain an expression for the number of lines per centimeter n as a function of $\cos\theta$ and $d\theta/d\lambda$. We can use the Pythagorean identity $\sin^2\theta+\cos^2\theta=1$ and the grating equation to write $\cos\theta$ in terms of n, m, and λ. Making this substitution and approximating $d\theta/d\lambda$ by $\Delta\theta/\Delta\lambda$ will yield an expression for n in terms of m, λ, $\Delta\lambda$, and $\Delta\theta$.

(*a*) The grating equation is:

$$d\sin\theta=m\lambda,\ m=0,1,2,\ldots \qquad (1)$$

Differentiate both sides of this equation with respect to λ:

$$\frac{d}{d\lambda}(d\sin\theta)=\frac{d}{d\lambda}(m\lambda)$$

or

$$d\cos\theta\frac{d\theta}{d\lambda}=m$$

Because $n = 1/d$:

$$\cos\theta\frac{d\theta}{d\lambda}=nm$$

Solve for n to obtain:

$$n = \frac{1}{m}\cos\theta \frac{d\theta}{d\lambda}$$

Approximate $d\theta/d\lambda$ by $\Delta\theta/\Delta\lambda$:

$$n = \frac{1}{m}\frac{\Delta\theta}{\Delta\lambda}\cos\theta$$

Substitute for $\cos\theta$:

$$n = \frac{1}{m}\frac{\Delta\theta}{\Delta\lambda}\sqrt{1-\sin^2\theta}$$

From equation (1):

$$\sin\theta = \frac{m\lambda}{d} = nm\lambda$$

Substitute to obtain:

$$n = \frac{1}{m}\frac{\Delta\theta}{\Delta\lambda}\sqrt{1-n^2m^2\lambda^2}$$

Solve for n:

$$n = \frac{1}{m\sqrt{\lambda^2 + \left(\frac{\Delta\lambda}{\Delta\theta}\right)^2}}$$

Substitute numerical values and evaluate n:

$$n = \frac{1}{3\sqrt{\left(\frac{480\text{ nm}+500\text{ nm}}{2}\right)^2 + \left(\frac{500\text{ nm}-480\text{ nm}}{12°\times\frac{\pi\text{ rad}}{180°}}\right)^2}} = 6.677\times10^5\text{ m}^{-1}$$

$$= \boxed{6677\text{ cm}^{-1}}$$

(b) Express m_{max} in terms of d and λ_{max}:

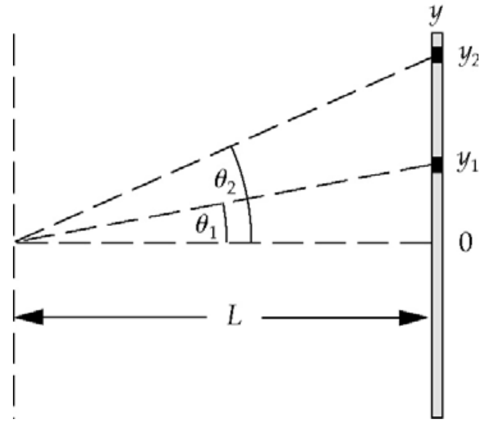
$$m_{\text{max}} = \frac{d}{\lambda_{\text{max}}} = \frac{1}{n\lambda_{\text{max}}}$$

Substitute numerical values and evaluate m_{max}:

$$m_{\text{max}} = \frac{1}{\left(6677\text{ cm}^{-1}\right)\left(500\text{ nm}\right)} = \boxed{3}$$

73 ••

Picture the Problem We can use the grating equation and the geometry of the diagram to derive an expression for the separation $\Delta y = y_2 - y_1$ of the spectral lines in terms of the distance L to the screen, the wavelengths of the resolved lines, and the number of grating slits per centimeter n. We will assume that the angle θ_2 is small and then verify that this is a justified assumption.



(*a*) The grating equation is:

$$d\sin\theta = m\lambda,\ m = 0, 1, 2, \ldots$$

Assuming that $\theta_2 \ll 1$ and $m = 2$:

$$\sin\theta_2 \approx \tan\theta_2 = \frac{y}{L}$$

Substitute to obtain:

$$d\frac{y}{L} = m\lambda$$

Solve for y:

$$y = \frac{mL\lambda}{d}$$

Letting the numerals 1 and 2 refer to the spectral lines, express $y_2 - y_1$:

$$\Delta y = y_2 - y_1 = \frac{mL}{d}(\lambda_2 - \lambda_1)$$

Solve for d to obtain:

$$d = \frac{mL}{y_2 - y_1}(\lambda_2 - \lambda_1)$$

The number of lines per centimeter n is the reciprocal of d:

$$n = \frac{y_2 - y_1}{mL(\lambda_2 - \lambda_1)}$$

Substitute numerical values and evaluate n:

$$n = \frac{8.4\,\text{cm}}{(2)(8\,\text{m})(590\,\text{nm} - 520\,\text{nm})}$$

$$= \boxed{750\,\text{cm}^{-1}}$$

To confirm our assumption that $\theta_2 \ll 1$, solve the grating equation for θ_2:

$$\theta_2 = \sin^{-1}\left(\frac{2\lambda}{d}\right) = \sin^{-1}(2\lambda n)$$

Substitute numerical values and evaluate θ_2:

$$\theta_2 = \sin^{-1}\left[2(590\,\text{nm})(750\,\text{cm}^{-1})\right] = 8.86\times10^{-2} << 1$$

Because $\theta_2 << 1$:

$\sin\theta_2 \approx \tan\theta_2 \approx \theta_2$, as was assumed above.

(*b*) The separation of the wavelengths is given by:

$$\Delta y = \frac{mL}{d}(\lambda_2 - \lambda_1) = mLn(\lambda_2 - \lambda_1)$$

For $m = 1$:

$$\Delta y = (1)(8\,\text{m})(750\,\text{cm}^{-1})(590\,\text{nm} - 520\,\text{nm}) = \boxed{4.20\,\text{cm}}$$

For $m = 3$:

$$\Delta y = (3)(8\,\text{m})(750\,\text{cm}^{-1})(590\,\text{nm} - 520\,\text{nm}) = \boxed{12.6\,\text{cm}}$$

74 •••

Picture the Problem We can differentiate the grating equation implicitly and approximate $d\theta/d\lambda$ by $\Delta\theta/\Delta\lambda$ to obtain an expression $\Delta\theta$ as a function of *m*, *n*, $\Delta\lambda$, and $\cos\theta$. We can use the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ and the grating equation to write $\cos\theta$ in terms of *n*, *m*, and λ. Making these substitutions will yield the given equation.

The grating equation is:

$$d\sin\theta = m\lambda,\ m = 0, 1, 2, \ldots \qquad (1)$$

Differentiate both sides of this equation with respect to λ:

$$\frac{d}{d\lambda}(d\sin\theta) = \frac{d}{d\lambda}(m\lambda)$$

or

$$d\cos\theta\frac{d\theta}{d\lambda} = m$$

Because $n = 1/d$:

$$\cos\theta\frac{d\theta}{d\lambda} = nm$$

Solve for *n* to obtain:

$$n = \frac{1}{m}\cos\theta\frac{d\theta}{d\lambda}$$

Approximate $d\theta/d\lambda$ by $\Delta\theta/\Delta\lambda$:

$$n = \frac{1}{m}\frac{\Delta\theta}{\Delta\lambda}\cos\theta$$

Solve for $\Delta\theta$:

$$\Delta\theta = \frac{nm\Delta\lambda}{\cos\theta}$$

Substitute for $\cos\theta$:

$$\Delta\theta = \frac{nm\Delta\lambda}{\sqrt{1-\sin^2\theta}}$$

From equation (1):

$$\sin\theta = \frac{m\lambda}{d} = nm\lambda$$

Substitute to obtain:

$$\Delta\theta = \frac{nm\Delta\lambda}{\sqrt{1-n^2m^2\lambda^2}}$$

Simplify by dividing the numerator and denominator by *nm*:

$$\Delta\theta = \frac{\Delta\lambda}{\frac{1}{nm}\sqrt{1-n^2m^2\lambda^2}} = \frac{\Delta\lambda}{\sqrt{\frac{1-n^2m^2\lambda^2}{n^2m^2}}} = \boxed{\frac{\Delta\lambda}{\sqrt{\frac{1}{n^2m^2}-\lambda^2}}}$$

75 •••

Picture the Problem We can use the grating equation and the geometry of the grating to derive an expression for ϕ_m in terms of the order number *m*, the wavelength of the light λ, and the groove separation *a*.

(*a*) The grating equation is:

$$d\sin\theta = m\lambda,\ m = 0, 1, 2, \ldots \qquad (1)$$

Because ϕ and θ_i have their left and right sides mutually perpendicular:

$$\theta_i = \phi_m$$

Substitute to obtain:

$$d\sin\phi_m = m\lambda$$

Solve for ϕ_m:

$$\phi_m = \boxed{\sin^{-1}\left(\frac{m\lambda}{d}\right)}$$

(*b*) For *m* = 2:

$$\phi_2 = \sin^{-1}\left(\frac{(2)(450\ \text{nm})}{\frac{1}{10{,}000\ \text{cm}^{-1}}}\right) = \boxed{64.2°}$$

76 •••

Picture the Problem We can follow the procedure outlined in the problem statement to obtain $R = \lambda/\Delta\lambda = mN$.

(*a*) Express the relationship between the phase difference ϕ and the path difference Δr:

$$\frac{\phi}{2\pi} = \frac{\Delta r}{\lambda} \Rightarrow \phi = \frac{2\pi\Delta r}{\lambda}$$

Because $\Delta r = d\sin\theta$:

$$\phi = \boxed{\frac{2\pi d}{\lambda}\sin\theta}$$

(*b*) Differentiate this expression with respect to θ to obtain:

$$\frac{d\phi}{d\theta} = \frac{d}{d\theta}\left[\frac{2\pi d}{\lambda}\sin\theta\right] = \frac{2\pi d}{\lambda}\cos\theta$$

Solve for $d\phi$:

$$d\phi = \boxed{\frac{2\pi d}{\lambda}\cos\theta\, d\theta}$$

(*c*) From (*b*):

$$d\theta = \frac{\lambda d\phi}{2\pi d\cos\theta}$$

Substitute $2\pi/N$ for $d\phi$ to obtain:

$$d\theta = \boxed{\frac{\lambda}{Nd\cos\theta}} \qquad \text{33-30}$$

(*d*) Equation 33-27 is:

$$d\sin\theta = m\lambda, m = 0, 1, 2, \ldots$$

Differentiate this expression implicitly with respect to λ to obtain:

$$\frac{d}{d\lambda}[d\sin\theta] = \frac{d}{d\lambda}[m\lambda]$$

or

$$d\cos\theta\frac{d\theta}{d\lambda} = m$$

Solve for $d\theta$ to obtain:

$$d\theta = \boxed{\frac{md\lambda}{d\cos\theta}} \qquad \text{33-31}$$

(*e*) Equate the two expressions for $d\theta$ obtained in (*c*) and (*d*):

$$\frac{\lambda}{Nd\cos\theta} = \frac{md\lambda}{d\cos\theta}$$

Solve for $R = \lambda/\Delta\lambda$:

$$R = \boxed{\frac{\lambda}{d\lambda} = mN}$$

General Problems

***77 •**

Picture the Problem We can apply the condition for constructive interference to find the angular position of the first maximum on the screen. Note that, due to reflection, the wave from the image is 180° out of phase with that from the source.

(*a*) Because $y_0 << L$, the distance from the mirror to the first maximum is given by:

$$y_0 = L\theta_0 \qquad (1)$$

Express the condition for constructive interference:

$$d\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda, m = 0, 1, 2, \ldots$$

Solve for θ:

$$\theta = \sin^{-1}\left[\left(m + \tfrac{1}{2}\right)\frac{\lambda}{d}\right]$$

For the first maximum, $m = 0$ and:

$$\theta_0 = \sin^{-1}\left[\left(\tfrac{1}{2}\right)\frac{\lambda}{d}\right]$$

Substitute in equation (1) to obtain:

$$y_0 = L\sin^{-1}\left[\left(\tfrac{1}{2}\right)\frac{\lambda}{d}\right]$$

Because the image of the slit is as far behind the mirror's surface as the slit is in front of it, $d = 2$ mm. Substitute numerical values and evaluate y_0:

$$y_0 = (1\,\text{m})\sin^{-1}\left[\left(\tfrac{1}{2}\right)\frac{600\,\text{nm}}{2\,\text{mm}}\right] = \boxed{0.150\,\text{mm}}$$

(*b*) The separation of the fringes on the screen is given by:

$$\Delta y = \frac{\lambda L}{d}$$

The number of dark bands per centimeter is the reciprocal of the fringe separation:

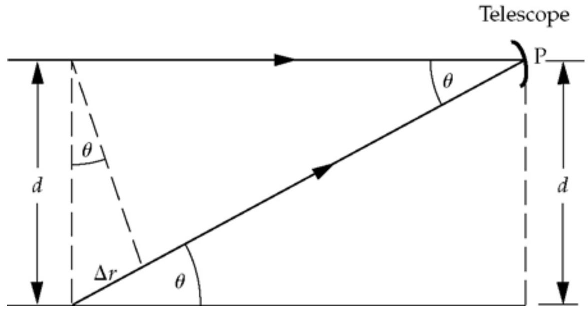
$$n = \frac{1}{\Delta y} = \frac{d}{\lambda L}$$

Substitute numerical values and evaluate n:

$$n = \frac{2\,\text{mm}}{(600\,\text{nm})(1\,\text{m})} = \boxed{3.33\times10^{3}\,\text{m}^{-1}}$$

78 ••

Picture the Problem The light from the radio galaxy reaches the radio telescope by two paths; one coming directly from the galaxy and the other reflected from the surface of the lake. The latter is phase shifted 180°, relative to the former, by reflection from the surface of the lake. We can use the condition for constructive interference of two waves to find the angle above the horizon at which the light from the galaxy will interfere constructively.



Because the reflected light is phase shifted by 180°, the condition for constructive interference at point P is:

$$\Delta r = \left(m + \tfrac{1}{2}\right)\lambda$$

where $m = 0, 1, 2, \ldots$

Referring to the figure, note that:

$$\sin\theta \approx \frac{\Delta r}{d} \Rightarrow \theta = \sin^{-1}\left[\frac{\Delta r}{d}\right]$$

Substitute for Δr to obtain:

$$\theta = \sin^{-1}\left[\frac{\left(m + \frac{1}{2}\right)\lambda}{d}\right]$$

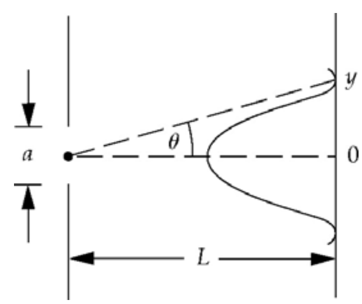
Noting that $m = 0$ for the first interference maximum, substitute numerical values and evaluate θ_0:

$$\theta_0 = \sin^{-1}\left[\frac{\frac{1}{2}(20\,\text{cm})}{20\,\text{m}}\right] = 5.00\times10^{-3}\,\text{rad}$$

$$= \boxed{0.286^\circ}$$

79 •

Picture the Problem We can use the condition determining the location of points of zero intensity in a diffraction pattern to express the location of the first zero in terms of *y* and *L*. The width of the central maximum can then be found from $\Delta y = 2y$.



Express the horizontal length of the principal diffraction maximum on the screen:

$$\Delta y = 2y \qquad (1)$$

Referring to the diagram, relate the angle θ to the distances *y* and *L*:

$$\tan\theta = \frac{y}{L}$$

or, because $\theta \ll 1$, $\tan\theta \approx \sin\theta$ and

$$\sin\theta = \frac{y}{L}$$

The points of zero intensity for a single-slit diffraction pattern are determined by the condition:

$$a\sin\theta = m\lambda, m = 1, 2, \ldots$$

Substitute for $\sin\theta$ to obtain:

$$\frac{ay}{L} = m\lambda$$

Solve for y:

$$y = m\frac{\lambda L}{a}$$

Substitute for y in equation (1):

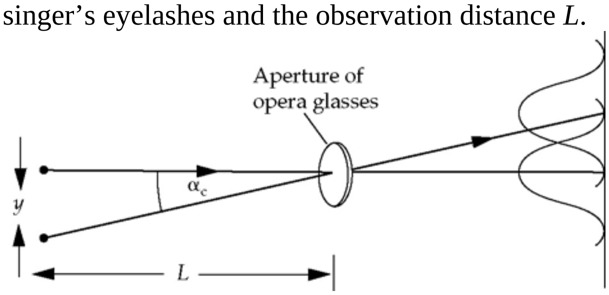
$$\Delta y = 2m\frac{\lambda L}{a}$$

At the first diffraction minimum, m = 1. Substitute numerical values and evaluate Δy:

$$\Delta y = 2(1)\frac{(700\,\text{nm})(6\,\text{m})}{0.5\,\text{mm}} = \boxed{1.68\,\text{cm}}$$

80 •

Picture the Problem We can use the Rayleigh criterion to express α_c in terms of λ and the diameter of the opera glasses lens D and the geometry of the problem to relate α_c to separation y of the singer's eyelashes and the observation distance L.



The critical angular separation, according to Rayleigh's criterion, is:

$$\alpha_c = 1.22\frac{\lambda}{D}$$

Given that $\alpha_c \ll 1$, it is also given by:

$$\alpha_c \approx \frac{y}{L}$$

Equating these two expressions yields:

$$\frac{y}{L} = 1.22\frac{\lambda}{D}$$

Solve for D to obtain:

$$D = 1.22\frac{\lambda L}{y}$$

Substitute numerical values and evaluate D:

$$D = 1.22\frac{(550\,\text{nm})(25\,\text{m})}{0.5\,\text{mm}} = \boxed{33.6\,\text{mm}}$$

81 •

Picture the Problem The resolving power of a telescope is the ability of the instrument to resolve two objects that are close together. Hence we can use Rayleigh's criterion as the resolving power of the Arecibo telescope.

Rayleigh's criterion for resolution is:

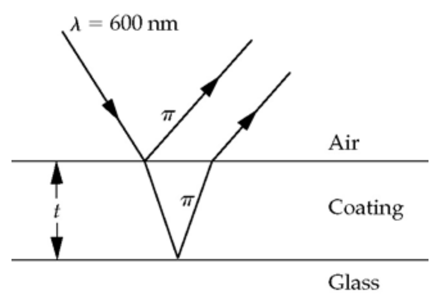
$$\alpha_c = 1.22\frac{\lambda}{D}$$

Substitute numerical values and evaluate α_c:

$$\alpha_c = 1.22\frac{3.2\,\text{cm}}{300\,\text{m}} = \boxed{0.130\,\text{mrad}}$$

***82** ••

Picture the Problem Note that reflection at both surfaces involves a phase shift of π rad. We can apply the condition for destructive interference to find the thickness t of the nonreflective coating.



The condition for destructive interference is:

$$2t = \left(m + \tfrac{1}{2}\right)\lambda_{\text{coating}} = \left(m + \tfrac{1}{2}\right)\frac{\lambda_{\text{air}}}{n_{\text{coating}}}$$

Solve for t:

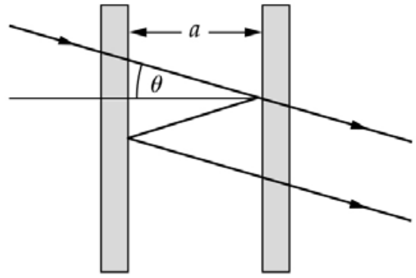
$$t = \left(m + \tfrac{1}{2}\right)\frac{\lambda_{\text{air}}}{2n_{\text{coating}}}$$

Evaluate t for $m = 0$:

$$t = \left(\tfrac{1}{2}\right)\frac{600\,\text{nm}}{2(1.30)} = \boxed{115\,\text{nm}}$$

83 ••

Picture the Problem The *Fabry-Perot interferometer* is shown in the figure. For constructive interference in the transmitted light the path difference must be an integral multiple of the wavelength of the light. This path difference can be found using the geometry of the interferometer.



Express the path difference between the two rays that emerge from the interferometer:

$$\Delta r = \frac{2a}{\cos\theta}$$

For constructive interference we Require that:

$$\Delta r = m\lambda, m = 0, 1, 2, \ldots$$

Equate these expressions to obtain:

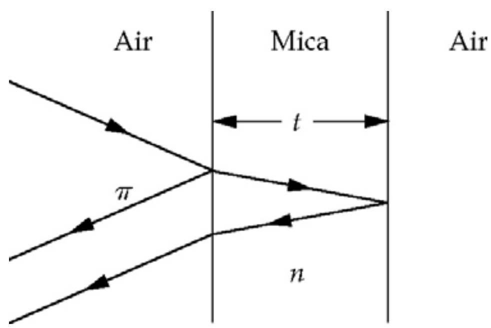
$$m\lambda = \frac{2a}{\cos\theta}$$

Solve for *a* to obtain:

$$a = \boxed{\frac{m\lambda}{2}\cos\theta}$$

84 ••

Picture the Problem The gaps in the spectrum of the visible light are the result of destructive interference between the incident light and the reflected light. Noting that there is a π rad phase shift at the first air-mica interface, we can use the condition for destructive interference to find the index of refraction *n* of the mica sheet.



Because there is a π rad phase shift at the first air-mica interface, the condition for destructive interference is:

$$2t = m\lambda_{\text{mica}} = m\frac{\lambda_{\text{air}}}{n}, m = 1, 2, 3, \ldots$$

Solve for *n*:

$$n = m\frac{\lambda_{\text{air}}}{2t} \qquad (1)$$

For λ = 474 nm:

$$2t = (474\,\text{nm})m$$

For λ = 421 nm:

$$2t = (421\,\text{nm})(m+1)$$

Equate these two expressions for $2t$ and solve for m to obtain:

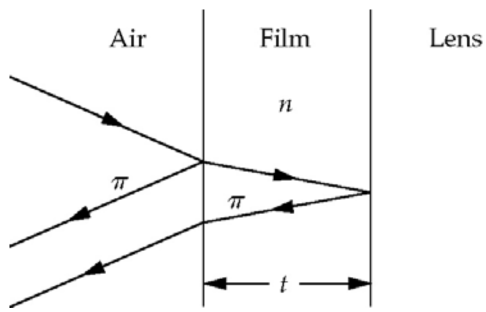
$m = 8$ for $\lambda = 474$ nm

Substitute numerical values in equation (1) and evaluate n:

$$n = 8\frac{474\,\text{nm}}{2(1.2\,\mu\text{m})} = \boxed{1.58}$$

85 ••

Picture the Problem Note that the light reflected at both the air-film and film-lens interfaces undergoes a π rad phase shift. We can use the condition for destructive interference between the light reflected from the air-film interface and the film-lens interface to find the thickness of the film. In (*c*) we can find the factor by which light of the given wavelengths is reduced by this film from $I \propto \cos^2 \frac{1}{2}\delta$.



(*a*) Express the condition for destructive interference between the light reflected from the air-film interface and the film-lens interface:

$$2t = \left(m+\tfrac{1}{2}\right)\lambda_{film} = \left(m+\tfrac{1}{2}\right)\frac{\lambda_{air}}{n} \qquad (1)$$

where m = 0, 1, 2, …

Solve for t:

$$t = \left(m+\tfrac{1}{2}\right)\frac{\lambda_{air}}{2n}$$

Evaluate t for $m = 0$:

$$t = \left(\frac{1}{2}\right)\frac{540\,\text{nm}}{2(1.38)} = \boxed{97.8\,\text{nm}}$$

(*b*) Solve equation (1) for λ_{air}:

$$\lambda_{air} = \frac{2tn}{m+\frac{1}{2}}$$

Evaluate λ_{air} for $m = 1$:

$$\lambda_{air} = \frac{2(97.8\,\text{nm})(1.38)}{1+\frac{1}{2}} = 180\,\text{nm}$$

$$\boxed{\text{No; because 180 nm is not in the visible portion of the spectrum.}}$$

(*c*) Express the reduction factor f as

$$f = \cos^2 \tfrac{1}{2}\delta \qquad (2)$$

a function of the phase difference δ between the two reflected waves:

Relate the phase difference to the path difference Δr:

$$\frac{\delta}{2\pi}=\frac{\Delta r}{\lambda_{\text{film}}}\Rightarrow\delta=2\pi\left(\frac{\Delta r}{\lambda_{\text{film}}}\right)$$

Because $\Delta r = 2t$:

$$\delta=2\pi\left(\frac{2t}{\lambda_{\text{film}}}\right)$$

Substitute in equation (2) to obtain:

$$f=\cos^2\left[\tfrac{1}{2}2\pi\left(\frac{2t}{\lambda_{\text{film}}}\right)\right]=\cos^2\left[\frac{2\pi t}{\lambda_{\text{film}}}\right]$$
$$=\cos^2\left[\frac{2\pi nt}{\lambda_{\text{air}}}\right]$$

Evaluate f for $\lambda = 400$ nm:

$$f_{400}=\cos^2\left[\frac{2\pi(1.38)(97.8\,\text{nm})}{400\,\text{nm}}\right]$$
$$=\boxed{0.273}$$

Evaluate f for $\lambda = 700$ nm:

$$f_{400}=\cos^2\left[\frac{2\pi(1.38)(97.8\,\text{nm})}{700\,\text{nm}}\right]$$
$$=\boxed{0.124}$$

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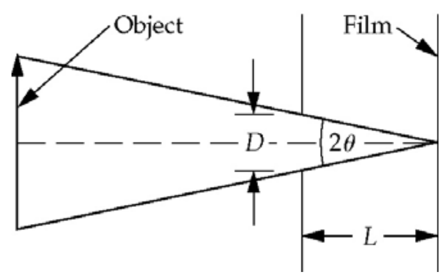
Picture the Problem As indicated in the problem statement, we can find the optimal size of the pinhole by equating the angular width of the object at the film and the angular width of the diffraction pattern.



Express the angular width of the a distant object at the film in terms of the diameter D of the pinhole and the distance L from the pinhole to the object:

$$2\theta=\frac{D}{L}\Rightarrow\theta=\frac{D}{2L}$$

Using Rayleigh's criterion, express the angular width of the diffraction

$$\theta_{\text{diffraction}}=1.22\frac{\lambda}{D}$$

pattern:

Equate these two expressions to obtain:

$$\frac{D}{2L}=1.22\frac{\lambda}{D}$$

Solving for D yields:

$$D=\sqrt{2.44\lambda L}$$

Substitute numerical values and evaluate D:

$$D=\sqrt{2.44(550\,\text{nm})(10\,\text{cm})}$$
$$=\boxed{0.366\,\text{mm}}$$

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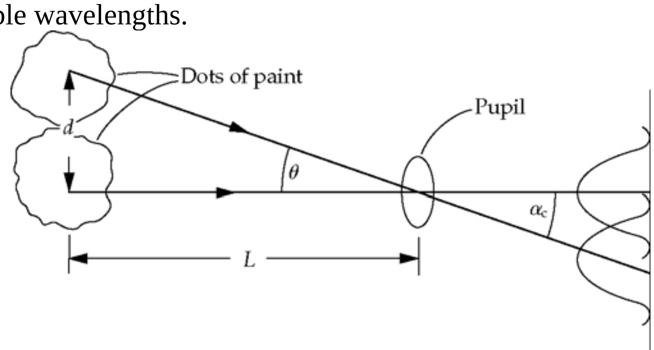
Picture the Problem We can use the geometry of the dots and the pupil of the eye and Rayleigh's criterion to find the greatest viewing distance that ensures that the effect will work for all visible wavelengths.



Referring to the diagram, express the angle subtended by the adjacent dots:

$$\theta\approx\frac{d}{L}$$

Letting the diameter of the pupil of the eye be D, apply Rayleigh's criterion to obtain:

$$\alpha_c=1.22\frac{\lambda}{D}$$

Set $\theta=\alpha_c$ to obtain:

$$\frac{d}{L}=1.22\frac{\lambda}{D}$$

Solve for L:

$$L=\frac{Dd}{1.22\lambda}$$

Evaluate L for the *shortest* wavelength light in the visible portion of the spectrum:

$$L=\frac{(3\,\text{mm})(2\,\text{mm})}{1.22(400\,\text{nm})}=\boxed{12.3\,\text{m}}$$

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Picture the Problem It is given that with one tube evacuated and one full of air at 1-atm pressure, there are 198 more wavelengths of light in the tube full of air than in the evacuated tube of the same length. We can use this condition to obtain an equation that expresses this difference in terms of L, λ_n, and λ_0. We can obtain a second equation relating λ_n, n, and λ_0 ($\lambda_n = \frac{\lambda_0}{n}$) and solve the two equations simultaneously to find n.

(*a*) The wavelengths are related by:

$$\lambda_n = \frac{\lambda_0}{n}$$

The number of wavelengths in length L is the length L divided by the wavelength. Thus:

$$\frac{L}{\lambda_n} - \frac{L}{\lambda_0} = 198$$

Substitute for λ_n:

$$\frac{nL}{\lambda_0} - \frac{L}{\lambda_0} = 198$$

Solve for λ_n to obtain:

$$n = 1 + \frac{198\lambda_0}{L}$$

Substitute numerical values and evaluate n:

$$n = 1 + 198\left(\frac{589\,\text{nm}}{0.4\,\text{m}}\right) = \boxed{1.0002916}$$

(*b*) Replace 198 with 198 ± 0.25 and assume that the uncertainties in L and λ_0 are negligible:

$$n = 1 + \frac{\lambda_0}{L}(198 \pm 0.25) = \boxed{1.0002916 \pm 0.0000004}$$

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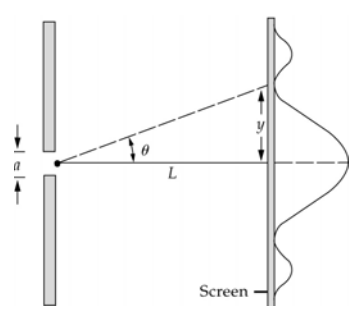
Picture the Problem We can use the condition that determines points of zero intensity for a single slit diffraction pattern and the geometry of the slit and screen shown in the diagram to derive the given width of the central maximum on the screen.



(*a*) The points of zero intensity for a single-slit diffraction pattern are given by:

$$a\sin\theta = m\lambda, m = 1, 2, 3, \ldots \qquad (1)$$

Relate the half-width y of the diffraction pattern to θ and L:

$$\tan\theta = \frac{y}{L}$$

Because θ is very small, $\tan\theta \approx \sin\theta$ and:

$$\sin\theta \approx \frac{y}{L}$$

Substitute for $\sin\theta$ in equation (1) to obtain:

$$a\frac{y}{L} \approx m\lambda$$

Solve for y:

$$y \approx m\frac{\lambda L}{a}$$

The width of the central maximum (m = 1) is:

$$2y \approx \boxed{\frac{2\lambda L}{a}}$$

(b) Set $a = \dfrac{2L\lambda}{a}$ and simplify to obtain:

$$2y \approx \frac{2\lambda L}{\frac{2L\lambda}{a}} = \boxed{a}$$