Chapter 34
Wave-Particle Duality and Quantum Physics

Conceptual Problems

***1 •**

Determine the Concept The Young double-slit experiment, the diffraction of light by a small aperture, and the J.J. Thomson cathode-ray experiment all demonstrated the wave nature of electromagnetic radiation. Only the photoelectric effect requires an explanation based on the quantization of electromagnetic radiation. $\boxed{(c)\text{ is correct.}}$

2 ••

Determine the Concept Since the power radiated by a source is the energy radiated per unit area and per unit time, it is directly proportional to the energy. The energy radiated varies inversely with the wavelength ($E = hc/\lambda$); i.e., the longer the wavelength, the less energy is associated with the electromagnetic radiation. $\boxed{(b)\text{ is correct.}}$

3 •

(*a*) True

(*b*) False. The work function of a metal is a property of the metal and is independent of the frequency of the incident light.

(*c*) True

(*d*) True

4 •

Determine the Concept In the photoelectric effect, the number of electrons emitted per second is a function of the light intensity, proportional to the light intensity, independent of the work function of the emitting surface and independent of the frequency of the light. $\boxed{(b)\text{ is correct.}}$

***5 •**

Determine the Concept The threshold wavelength for emission of photoelectrons is related to the work function of a metal through $\phi = hc/\lambda_t$. Hence $\lambda_t = hc/\phi$ and $\boxed{(a)\text{ is correct.}}$

6 ••

Determine the Concept In order for electrons to be emitted hc/λ must be greater than ϕ. Evidently, $hc/\lambda_1 < \phi$, but $hc/\lambda_2 > \phi$.

7 •

(*a*) True

(*b*) True

(*c*) True

(*d*) False. Electrons are too small to be resolved by an electron microscope.

8 •

Determine the Concept If the de Broglie wavelengths of an electron and a proton are equal, their momenta must be equal. Since $m_p > m_e$, $v_p < v_e$. Response (*c*) is correct.

9 •

Picture the Problem The kinetic energy of a particle can be expressed, in terms of its momentum, as $K = \frac{p^2}{2m}$. We can use the equality of the kinetic energies and the fact that $m_e < m_p$ to determine the relative sizes of their de Broglie wavelengths.

Express the equality of the kinetic energies of the proton and electron in terms of their momenta and masses:

$$\frac{p_p^2}{2m_p} = \frac{p_e^2}{2m_e}$$

Use the de Broglie relation for the wavelength of matter waves to obtain:

$$\frac{h^2}{2m_p\lambda_p^2} = \frac{h^2}{2m_e\lambda_e^2}$$

or

$$m_p\lambda_p^2 = m_e\lambda_e^2$$

Since $m_e < m_p$:

$$\lambda_p^2 < \lambda_e^2 \text{ and } \lambda_e > \lambda_p$$

and $\boxed{(c)\text{ is correct.}}$

10 •

Determine the Concept Yes. <*x*> can equal a value for which $P(x)$ is zero. An example is the asymmetric well for all even numbered states.

***11 •**

Determine the Concept In the photoelectric effect, an electron absorbs the energy of a single photon. Therefore, $K_{max} = hf - \phi$, independently of the number of photons incident on the surface. However, the number of photons incident on the surface determines the number of electrons that are emitted.

12 ••

Picture the Problem The probability of a particular event occurring is the number of ways that event can occur divided by the number of possible outcomes. The expectation value, on the other hand, is the average value of the experiment.

(*a*) Find the probability of a 1 coming up when the die is thrown:

$$P(1) = \frac{3}{6} = \boxed{\frac{1}{2}}$$

(*b*) Find the average value of a large number of throws of the die:

$$\langle n \rangle = \frac{3\times 1 + 3\times 2}{6} = \boxed{1.5}$$

13 ••

Determine the Concept According to quantum theory, the average value of many measurements of the same quantity will yield the expectation value of that quantity. However, any single measurement may differ from the expectation value.

Estimation and Approximation

14 ••

Picture the Problem From Einstein's photoelectric equation we have $K_{max} = hf - \phi$, which is of the form $y = mx + b$, where the slope is h and the K_{max}-intercept is the work function. Hence we should plot a graph of K_{max} versus f in order to obtain a straight line whose slope will be an experimental value for Planck's constant.

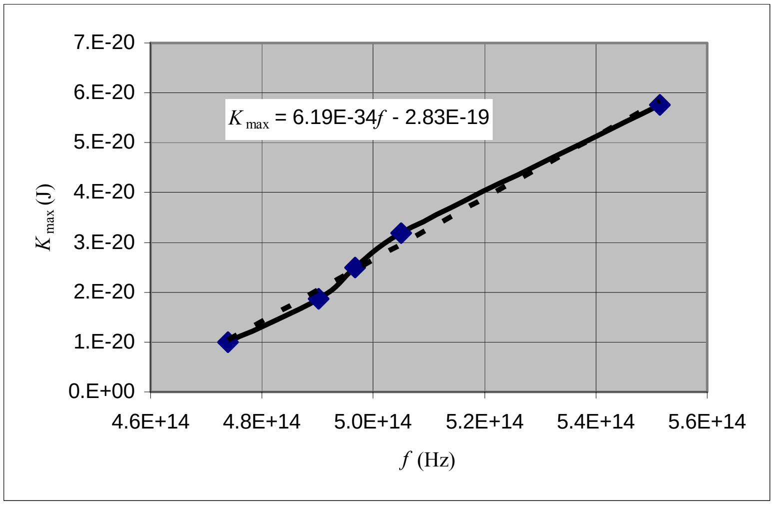
(*a*) The spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
A3	544	λ (nm)
B3	0.36	K_{max} (eV)
C3	A3*10^−19	λ (m)
D3	3*10^8/C3	c/λ
E3	B3*1.6*10^−19	K_{max} (J)

lambda	Kmax	lambda	f=c/lambda	Kmax
(nm)	(eV)	(m)	(Hz)	(J)

544	0.36	5.44E–07	5.51E+14	5.76E–20
594	0.199	5.94E–07	5.05E+14	3.18E–20
604	0.156	6.04E–07	4.97E+14	2.50E–20
612	0.117	6.12E–07	4.90E+14	1.87E–20
633	0.062	6.33E–07	4.74E+14	9.92E–21

The following graph was plotted from the data shown in the above table. Excel's ″Add Trendline″ was used to fit a linear function to the data.



(*b*) From the regression line we note that the experimental value for Planck's constant is:

$$h_{\exp} = \boxed{6.19\times10^{-34}\ \text{J}\cdot\text{s}}$$

(*c*) Express the percent difference between $h_{\exp}$ and h:

$$\%\,\text{diff} = \frac{h-h_{\exp}}{h} = 1-\frac{h_{\exp}}{h}$$

$$= 1-\frac{6.19\times10^{-34}\ \text{J}\cdot\text{s}}{6.63\times10^{-34}\ \text{J}\cdot\text{s}} = \boxed{6.64\%}$$

15 ••

Picture the Problem From Einstein's photoelectric equation we have $K_{\max} = hf - \phi$, which is of the form $y = mx + b$, where the slope is h and the $K_{\max}$-intercept is the work function. Hence we should plot a graph of $K_{\max}$ versus f in order to obtain a straight line whose intercept will be an experimental value for the work function.

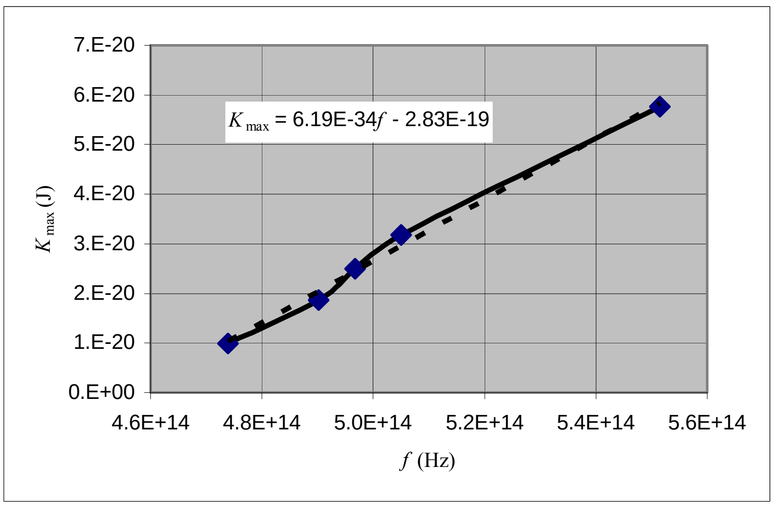
(*a*) The spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
A3	544	λ (nm)

B3	0.36	K_{max} (eV)
C3	A3*10^–19	λ (m)
D3	3*10^8/C3	c/λ
E3	B3*1.6*10^–19	K_{max} (J)

lambda (nm)	Kmax (eV)	lambda (m)	f=c/lambda (Hz)	Kmax (J)
544	0.36	5.44E–07	5.51E+14	5.76E–20
594	0.199	5.94E–07	5.05E+14	3.18E–20
604	0.156	6.04E–07	4.97E+14	2.50E–20
612	0.117	6.12E–07	4.90E+14	1.87E–20
633	0.062	6.33E–07	4.74E+14	9.92E–21

The following graph was plotted from the data shown in the above table. Excel's "Add Trendline" was used to fit a linear function to the data.



(*b*) From the regression line we note that the experimental value for the work function ϕ is:

$$\phi_{exp} = 2.83\times10^{-19}\ \text{J}\times\frac{1\,\text{eV}}{1.6\times10^{-19}\ \text{J}} = \boxed{1.77\,\text{eV}}$$

(*c*) $\boxed{\text{The value of } \phi_{exp} = 1.77\text{ eV is closest to the work function for cesium.}}$

***16 ••**

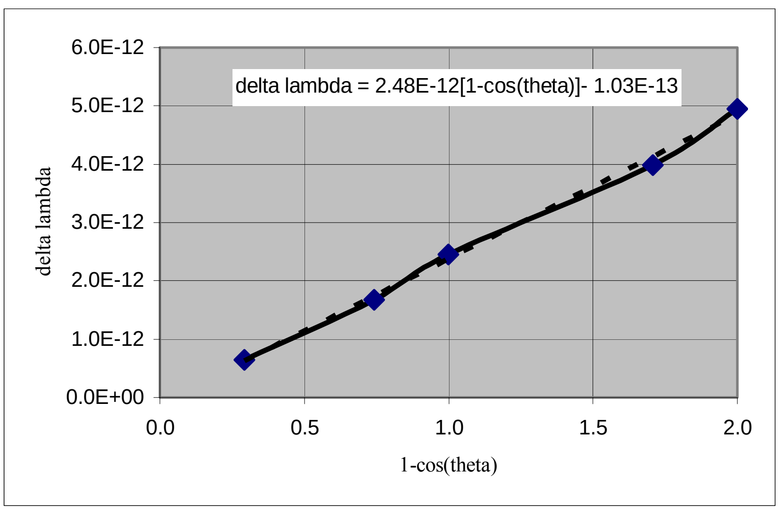
Picture the Problem From the Compton-scattering equation we have $\lambda_2 - \lambda_1 = \lambda_C(1-\cos\theta)$, where $\lambda_C = h/m_e c$ is the Compton wavelength. Note that this equation is of the form $y = mx + b$ provided we let $y = \lambda_2 - \lambda_1$ and $x = 1 - \cos\theta$. Thus, we can linearize the Compton equation by plotting $\Delta\lambda = \lambda_2 - \lambda_1$ as a function of $1-\cos\theta$. The slope of the resulting graph will yield an experimental value for the Compton wavelength.

(*a*) The spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
A3	45	θ (deg)
B3	1 – cos(A3*PI()/180)	$1-\cos\theta$
C3	6.47E^–13	$\Delta\lambda = \lambda_2 - \lambda_1$

θ	$1-\cos\theta$	$\lambda_2-\lambda_1$
(deg)		
45	0.293	6.47E–13
75	0.741	1.67E–12
90	1.000	2.45E–12
135	1.707	3.98E–12
180	2.000	4.95E–12

The following graph was plotted from the data shown in the above table. Excel's ″Add Trendline″ was used to fit a linear function to the data. The regression line is
$\Delta\lambda = 2.48\times10^{-12}(1-\cos\theta) - 1.03\times10^{-13}$



From the regression line we note that the experimental value for the Compton wavelength $\lambda_{C,\exp}$ is:

$$\lambda_{C,\exp} = \boxed{2.48\times10^{-12}\ \text{m}}$$

The Compton wavelength is given by:

$$\lambda_C = \frac{h}{m_e c} = \frac{hc}{m_e c^2}$$

Substitute numerical values and evaluate λ_C:

$$\lambda_C = \frac{1240\ \text{eV}\cdot\text{nm}}{5.11\times10^5\ \text{eV}} = 2.43\times10^{-12}\ \text{m}$$

Express the percent difference between λ_C and $\lambda_{C,exp}$:

$$\%\,\text{diff} = \frac{\lambda_{C,exp} - \lambda_{exp}}{\lambda_{exp}} = \frac{\lambda_{C,exp}}{\lambda_{exp}} - 1$$

$$= \frac{2.48\times10^{-12}\ \text{m}}{2.43\times10^{-12}\ \text{m}} - 1 = \boxed{2.06\%}$$

***17 ••**

Picture the Problem The de Broglie wavelength of an object is given by $\lambda = h/p$, where p is the momentum of the object.

The de Broglie wavelength of an object, in terms of its mass m and speed v, is:

$$\lambda = \frac{h}{mv}$$

The values in the following table were obtained using the internet:

Type of ball	m	v_{max}
	(g)	(m/s)
Baseball	142	44
Tennis	57	54
Golf	57	42
Soccer	250	31

The de Broglie wavelength of a baseball, moving with its maximum speed, is:

$$\lambda = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(0.142\ \text{kg})(44\ \text{m/s})} = 1.06\times10^{-34}\ \text{m}$$

Proceed as above to obtain the values shown in the table:

Type of ball	m	v_{max}	λ
	(g)	(m/s)	(m)
Baseball	142	44	1.06×10^{-34}
Tennis	57	54	2.15×10^{-34}
Golf	57	42	2.77×10^{-34}
Soccer	250	31	0.855×10^{-34}

$\boxed{\text{Examination of the table indicates that the soccer ball has the shortest de Broglie wavelength.}}$

The Particle Nature of Light: Photons

18 •

Picture the Problem We can find the photon energy for an electromagnetic wave of a given frequency f from $E = hf$ where h is Planck's constant.

(*a*) For f = 100 MHz:

$$\begin{aligned} E &= hf \\ &= \left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)(100\,\text{MHz}) \\ &= \boxed{6.63\times10^{-26}\ \text{J}} \\ &= 6.63\times10^{-26}\ \text{J}\times\frac{1\,\text{eV}}{1.60\times10^{-19}\ \text{J}} \\ &= \boxed{4.14\times10^{-7}\ \text{eV}} \end{aligned}$$

(*b*) For f = 900 kHz:

$$\begin{aligned} E &= hf \\ &= \left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)(900\,\text{kHz}) \\ &= \boxed{5.96\times10^{-28}\ \text{J}} \\ &= 5.96\times10^{-28}\ \text{J}\times\frac{1\,\text{eV}}{1.60\times10^{-19}\ \text{J}} \\ &= \boxed{3.73\times10^{-9}\ \text{eV}} \end{aligned}$$

19 •

Picture the Problem The energy of a photon, in terms of its frequency, is given by $E=hf$.

(*a*) Express the frequency of a photon in terms of its energy and evaluate f for E = 1 eV:

$$\begin{aligned} f &= \frac{E}{h} = \frac{1\,\text{eV}}{4.14\times10^{-15}\ \text{eV}\cdot\text{s}} \\ &= \boxed{2.42\times10^{14}\ \text{Hz}} \end{aligned}$$

(*b*) For E = 1 keV:

$$\begin{aligned} f &= \frac{1\,\text{keV}}{4.14\times10^{-15}\ \text{eV}\cdot\text{s}} \\ &= \boxed{2.42\times10^{17}\ \text{Hz}} \end{aligned}$$

(*c*) For E = 1 MeV:

$$\begin{aligned} f &= \frac{1\,\text{MeV}}{4.14\times10^{-15}\ \text{eV}\cdot\text{s}} \\ &= \boxed{2.42\times10^{20}\ \text{Hz}} \end{aligned}$$

***20** •

Picture the Problem We can use $E = hc/\lambda$ to find the photon energy when we are given the wavelength of the radiation.

(*a*) Express the photon energy as a function of wavelength and evaluate E for λ = 450 nm:

$$E = \frac{hc}{\lambda} = \frac{1240\,\text{eV}\cdot\text{nm}}{450\,\text{nm}} = \boxed{2.76\,\text{eV}}$$

(*b*) For λ = 550 nm:

$$E = \frac{1240\,\text{eV}\cdot\text{nm}}{550\,\text{nm}} = \boxed{2.25\,\text{eV}}$$

(*c*) For λ = 650 nm:

$$E = \frac{1240\,\text{eV}\cdot\text{nm}}{650\,\text{nm}} = \boxed{1.91\,\text{eV}}$$

21 •

Picture the Problem We can use $E = hc/\lambda$ to find the photon energy when we are given the wavelength of the radiation.

(*a*) Express the photon energy as a function of wavelength and evaluate E for λ = 0.1 nm:

$$E = \frac{hc}{\lambda} = \frac{1240\,\text{eV}\cdot\text{nm}}{0.1\,\text{nm}} = \boxed{12.4\,\text{keV}}$$

(*b*) For λ =1 fm:

$$E = \frac{1240\,\text{eV}\cdot\text{nm}}{10^{-6}\,\text{nm}} = \boxed{1.24\,\text{GeV}}$$

22 ••

Picture the Problem We can express the density of photons in the beam as the number of photons per unit volume. The number of photons per unit volume is, in turn, the ratio of the power of the laser to the energy of the photons and the volume occupied by the photons emitted in one second is the product of the cross-sectional area of the beam and the speed at which the photons travel, i.e., the speed of light.

Express the density of photons in the beam as a function of the number of photons emitted per second and the volume occupied by those photons:

$$\rho = \frac{N}{V}$$

Relate the number of photons emitted per second to the power of the laser and the energy of the photons:

$$N = \frac{P}{E} = \frac{P\lambda}{hc}$$

Express the volume containing the photons emitted in one second as a function of the cross sectional area of the beam:

$$V = Ac$$

Substitute to obtain:

$$\rho = \frac{P\lambda}{hc^2 A}$$

Substitute numerical values and evaluate ρ:

$$\rho = \frac{(3\,\text{mW})(632\,\text{nm})}{(6.63\times10^{-34}\,\text{J}\cdot\text{s})(3\times10^{8}\,\text{m/s})^2\left(\frac{\pi}{4}(1\,\text{mm})^2\right)} = \boxed{4.05\times10^{13}\,\text{m}^{-3}}$$

***23 •**

Picture the Problem The number of photons per unit volume is, in turn, the ratio of the power of the laser to the energy of the photons and the volume occupied by the photons emitted in one second is the product of the cross-sectional area of the beam and the speed at which the photons travel; i.e., the speed of light.

Relate the number of photons emitted per second to the power of the laser and the energy of the photons:

$$N = \frac{P}{E} = \frac{P\lambda}{hc}$$

Substitute numerical values and evaluate N:

$$N = \frac{(2.5\,\text{mW})(1.55\,\mu\text{m})}{(6.63\times10^{-34}\,\text{J}\cdot\text{s})(3\times10^{8}\,\text{m/s})} = \boxed{1.95\times10^{16}\,\text{s}^{-1}}$$

The Photoelectric Effect

24 •

Picture the Problem The threshold wavelength and frequency for emission of photoelectrons is related to the work function of a metal through $\phi = hf_t = hc/\lambda_t$. We can use Einstein's photoelectric equation $K_{max} = \frac{hc}{\lambda} - \phi$ to find the maximum kinetic energy of the electrons for the given wavelengths of the incident light.

(*a*) Express the threshold frequency in terms of the work function for tungsten and evaluate f_t:

$$f_t = \frac{\phi}{h} = \frac{4.58\,\text{eV}}{4.14\times10^{-15}\,\text{eV}\cdot\text{s}} = \boxed{1.11\times10^{15}\,\text{Hz}}$$

Using $v = f\lambda$, express the threshold wavelength in terms of the threshold

$$\lambda_t = \frac{v}{f_t} = \frac{3\times10^{8}\,\text{m/s}}{1.11\times10^{15}\,\text{Hz}} = \boxed{270\,\text{nm}}$$

frequency and evaluate λ_t:

(*b*) Using Einstein's photoelectric equation, relate the maximum kinetic energy of the electrons to their wavelengths and evaluate K_{max}:

$$K_{max} = E - \phi = hf - \phi = \frac{hc}{\lambda} - \phi$$
$$= \frac{1240\,\text{eV}\cdot\text{nm}}{200\,\text{nm}} - 4.58\,\text{eV}$$
$$= \boxed{1.62\,\text{eV}}$$

(*c*) Evaluate K_{max} for $\lambda = 250$ nm:

$$K_{max} = \frac{1240\,\text{eV}\cdot\text{nm}}{250\,\text{nm}} - 4.58\,\text{eV}$$
$$= \boxed{0.380\,\text{eV}}$$

25 •

Picture the Problem We can use the Einstein equation for photon energy to find the energy of an incident photon and his photoelectric equation to relate the work function for potassium to the maximum energy of the photoelectrons. The threshold wavelength can be found from $\lambda_t = hc/\phi$.

(*a*) Use the Einstein equation for photon energy to relate the energy of the incident photon to its wavelength:

$$E = \frac{hc}{\lambda} = \frac{1240\,\text{eV}\cdot\text{nm}}{300\,\text{nm}} = \boxed{4.13\,\text{eV}}$$

(*b*) Using Einstein's photoelectric equation, relate the work function for potassium to the maximum kinetic energy of the photoelectrons:

$$K_{max} = E - \phi$$

Solve for and evaluate ϕ:

$$\phi = E - K_{max} = 4.13\,\text{eV} - 2.03\,\text{eV}$$
$$= \boxed{2.10\,\text{eV}}$$

(*c*) Proceed as in (*b*) with $E = hc/\lambda$:

$$K_{max} = \frac{hc}{\lambda} - \phi$$
$$= \frac{1240\,\text{eV}\cdot\text{nm}}{430\,\text{nm}} - 2.10\,\text{eV}$$
$$= \boxed{0.784\,\text{eV}}$$

(*d*) Express the threshold wavelength as a function of potassium's work function and evaluate λ_t:

$$\lambda_t = \frac{hc}{\phi} = \frac{1240\,\text{eV}\cdot\text{nm}}{2.10\,\text{eV}} = \boxed{590\,\text{nm}}$$

26 •

Picture the Problem We can find the work function for silver using $\phi = hc/\lambda_t$ and the maximum kinetic energy of the electrons using Einstein's photoelectric equation.

(*a*) Express the work function for silver as a function of the threshold wavelength:

$$\phi = \frac{hc}{\lambda_t} = \frac{1240\,\text{eV}\cdot\text{nm}}{262\,\text{nm}} = \boxed{4.73\,\text{eV}}$$

(*b*) Using Einstein's photoelectric equation, relate the work function for silver to the maximum kinetic energy of the photoelectrons:

$$K_{\text{max}} = E - \phi = \frac{hc}{\lambda} - \phi$$
$$= \frac{1240\,\text{eV}\cdot\text{nm}}{175\,\text{nm}} - 4.73\,\text{eV}$$
$$= \boxed{2.36\,\text{eV}}$$

27 •

Picture the Problem We can find the threshold frequency and wavelength for cesium using $\phi = hf_t = hc/\lambda_t$ and the maximum kinetic energy of the electrons using Einstein's photoelectric equation.

(*a*) Use the Einstein equation for photon energy to express and evaluate the threshold wavelength for cesium:

$$\lambda_t = \frac{hc}{\phi} = \frac{1240\,\text{eV}\cdot\text{nm}}{1.9\,\text{eV}} = \boxed{653\,\text{nm}}$$

Use $v = f\lambda$ to find the threshold frequency:

$$f_t = \frac{v}{\lambda_t} = \frac{3\times10^8\ \text{m/s}}{653\,\text{nm}}$$
$$= \boxed{4.59\times10^{14}\ \text{Hz}}$$

(*b*) Using Einstein's photoelectric equation, relate the maximum kinetic energy of the photoelectrons to the wavelength of the incident light and evaluate K_{max} for $\lambda = 250$ nm:

$$K_{\text{max}} = \frac{hc}{\lambda} - \phi$$
$$= \frac{1240\,\text{eV}\cdot\text{nm}}{250\,\text{nm}} - 1.90\,\text{eV}$$
$$= \boxed{3.06\,\text{eV}}$$

(*c*) Proceed as above with $\lambda = 350$ nm:

$$K_{\text{max}} = \frac{1240\,\text{eV}\cdot\text{nm}}{350\,\text{nm}} - 1.90\,\text{eV}$$
$$= \boxed{1.64\,\text{eV}}$$

***28 ••**

Picture the Problem We can use Einstein's photoelectric equation to find the work function of this surface and then apply it a second time to find the maximum kinetic energy of the photoelectrons when the surface is illuminated with light of wavelength 365 nm.

Use Einstein's photoelectric equation to relate the maximum kinetic energy of the emitted electrons to their total energy and the work function of the surface:

$$K_{\text{max}} = \frac{hc}{\lambda} - \phi$$

Using Einstein's photoelectric equation, find the work function of the surface:

$$\phi = E - K_{\text{max}} = \frac{hc}{\lambda} - K_{\text{max}}$$
$$= \frac{1240\,\text{eV}\cdot\text{nm}}{780\,\text{nm}} - 0.37\,\text{eV}$$
$$= 1.22\,\text{eV}$$

Substitute for ϕ and λ and evaluate K_{max}:

$$K_{\text{max}} = \frac{1240\,\text{eV}\cdot\text{nm}}{410\,\text{nm}} - 1.22\,\text{eV}$$
$$= \boxed{1.80\,\text{eV}}$$

Compton Scattering

29 •

Picture the Problem We can calculate the shift in wavelength using the Compton relationship $\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta)$.

The shift in wavelength is given by:

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta)$$

Substitute numerical values and evaluate $\Delta\lambda$:

$$\Delta\lambda = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(3\times10^{8}\ \text{m/s}\right)}\left(1-\cos 60°\right) = \boxed{1.21\,\text{pm}}$$

30 •

Picture the Problem We can calculate the scattering angle using the Compton relationship $\Delta\lambda = \frac{h}{m_e c}\left(1-\cos\theta\right)$.

Using the Compton scattering equation, relate the shift in wavelength to the scattering angle:

$$\Delta\lambda = \frac{h}{m_e c}\left(1-\cos\theta\right)$$

Solve for θ:

$$\theta = \cos^{-1}\left(1-\frac{m_e c}{h}\Delta\lambda\right)$$

Substitute numerical values and evaluate θ:

$$\theta = \cos^{-1}\left(1-\frac{\left(9.11\times10^{-31}\ \text{kg}\right)\left(3\times10^{8}\ \text{m/s}\right)}{6.63\times10^{-34}\ \text{J}\cdot\text{s}}\left(0.33\,\text{pm}\right)\right) = \boxed{30.2°}$$

31 •

Picture the Problem We can calculate the shift in wavelength using the Compton relationship $\Delta\lambda = \frac{h}{m_e c}\left(1-\cos\theta\right)$.

Express the wavelength of the incident photons in terms of the fractional change in wavelength:

$$\frac{\Delta\lambda}{\lambda} = 2.3\% \Rightarrow \lambda = \frac{\Delta\lambda}{0.023}$$

Using the Compton scattering equation, relate the shift in wavelength to the scattering angle:

$$\Delta\lambda = \frac{h}{m_e c}\left(1-\cos\theta\right)$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{0.023\left(9.11\times10^{-31}\ \text{kg}\right)\left(3\times10^{8}\ \text{m/s}\right)}\left(1-\cos 135°\right) = \boxed{180\,\text{pm}}$$

***32** •

Picture the Problem We can use the Einstein equation for photon energy to find the energy of both the incident and scattered photon and the Compton scattering equation to find the wavelength of the scattered photon.

(*a*) Use the Einstein equation for photon energy to obtain:

$$E = \frac{hc}{\lambda_1} = \frac{1240\,\text{eV}\cdot\text{nm}}{0.0711\,\text{nm}} = \boxed{17.4\,\text{keV}}$$

(*b*) Express the wavelength of the scattered photon in terms of its pre-scattering wavelength and the shift in its wavelength during scattering:

$$\lambda_2 = \lambda_1 + \Delta\lambda = \lambda_1 + \frac{h}{m_e c}(1-\cos\theta)$$

Substitute numerical values and evaluate λ_2:

$$\lambda_2 = 0.0711\,\text{nm} + \frac{6.63\times10^{-34}\,\text{J}\cdot\text{s}}{(9.11\times10^{-31}\,\text{kg})(3\times10^{8}\,\text{m/s})}(1-\cos 180°) = \boxed{0.0760\,\text{nm}}$$

(*c*) Use the Einstein equation for photon energy to obtain:

$$E = \frac{hc}{\lambda_2} = \frac{1240\,\text{eV}\cdot\text{nm}}{0.0760\,\text{nm}} = \boxed{16.3\,\text{keV}}$$

33 •

Picture the Problem Compton used X rays of wavelength 71.1 pm. Let the direction the incident photon (and the recoiling electron) is moving be the positive direction. We can use $p = h/\lambda$ to find the momentum of the incident photon and the conservation of momentum to find its momentum after colliding with the electron.

Use the expression for the momentum of a photon to find the momentum of Compton's photons:

$$p_1 = \frac{h}{\lambda_1} = \frac{6.63\times10^{-34}\,\text{J}\cdot\text{s}}{71.1\,\text{pm}} = \boxed{9.32\times10^{-24}\,\text{kg}\cdot\text{m/s}}$$

Using the Compton scattering equation, relate the shift in wavelength to the scattering angle:

$$\lambda_2 = \lambda_1 + \lambda_C(1-\cos\theta)$$

Substitute numerical values and evaluate λ_2:

$$\lambda_2 = 71.1\,\text{pm} + (2.43\times10^{-12}\,\text{m})(1-\cos 180°) = 76.0\,\text{pm}$$

Apply conservation of momentum to obtain:

$$p_1 = p_e - p_2 \Rightarrow p_e = p_1 - p_2$$

Substitute for p_1 and p_2 and evaluate p_e:

$$p_e = 9.32\times10^{-24}\ \text{kg}\cdot\text{m/s} - \left(-\frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{76.0\ \text{pm}}\right) = \boxed{1.80\times10^{-23}\ \text{kg}\cdot\text{m/s}}$$

34 ••

Picture the Problem We can calculate the shift in wavelength using the Compton relationship $\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta) = \lambda_C(1-\cos\theta)$ and use conservation of energy to find the kinetic energy of the scattered electron.

(*a*) Use the Compton scattering equation to find the change in wavelength of the photon:

$$\Delta\lambda = \lambda_C(1-\cos\theta) = (2.43\times10^{-12}\ \text{m})(1-\cos90^\circ) = \boxed{2.43\ \text{pm}}$$

(*b*) Use conservation of energy to relate the change in the kinetic energy of the electron to the energies of the incident and scattered photon:

$$\Delta E_e = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2}$$

Find the wavelength of the scattered photon:

$$\lambda_2 = \lambda_1 + \Delta\lambda = 6\ \text{pm} + 2.43\ \text{pm} = 8.43\ \text{pm}$$

Substitute and evaluate the kinetic energy of the electron (equal to the change in its energy since it was stationary prior to the collision with the photon):

$$\Delta E_e = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2} = 1240\ \text{eV}\cdot\text{nm}\left(\frac{1}{6\ \text{pm}} - \frac{1}{8.43\ \text{pm}}\right) = \boxed{59.6\ \text{keV}}$$

35 ••

Picture the Problem We can find the number of head-on collisions required to double the wavelength of the incident photon by dividing the required change in wavelength by the change in wavelength per collision. The change in wavelength per collision can be found using the Compton scattering equation.

Express the number of collisions required in terms of the change in wavelength per collision:

$$N = \frac{\Delta\lambda}{\Delta\lambda/\text{collision}}$$

Using the Compton scattering equation, express the wavelength shift per collision:

$$\Delta\lambda = \lambda_C(1-\cos\theta)$$

Substitute numerical values and evaluate $\Delta\lambda$:

$$\Delta\lambda = (2.43\times10^{-12}\text{ m})(1-\cos180°) = 4.86\text{ pm}$$

Substitute and evaluate N:

$$N = \frac{200\text{ pm}}{4.86\text{ pm}} = \boxed{42}$$

Electrons and Matter Waves

36 •

Picture the Problem From Equation 34-16 we have $\lambda = \frac{1.226}{\sqrt{K}}\text{nm}$ provided K is in electron volts.

(*a*) For K = 2.5 eV:

$$\lambda = \frac{1.226}{\sqrt{2.5}}\text{nm} = \boxed{0.775\text{ nm}}$$

(*b*) For K = 250 eV:

$$\lambda = \frac{1.226}{\sqrt{250}}\text{nm} = \boxed{0.0775\text{ nm}}$$

(*c*) For K = 2.5 keV:

$$\lambda = \frac{1.226}{\sqrt{2500}}\text{nm} = \boxed{0.0245\text{ nm}}$$

(*d*) For K = 25 keV:

$$\lambda = \frac{1.226}{\sqrt{25000}}\text{nm} = \boxed{7.75\text{ pm}}$$

37 •

Picture the Problem We can use its definition to find the de Broglie wavelength of this electron.

Use its definition to express the de Broglie wavelength of the electron in terms of its momentum:

$$\lambda = \frac{h}{p} = \frac{h}{m_e v}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{6.63\times10^{-34}\ \mathrm{J\cdot s}}{(9.11\times10^{-31}\ \mathrm{kg})(2.5\times10^{5}\ \mathrm{m/s})} = \boxed{2.91\,\mathrm{nm}}$$

38 •

Picture the Problem We can find the momentum of the electron from the de Broglie equation and its kinetic energy from $\lambda = \frac{1.226}{\sqrt{K}}\,\mathrm{nm}$, where K is in eV.

(*a*) Use the de Broglie relation to express the momentum of the electron:

$$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}\ \mathrm{J\cdot s}}{200\,\mathrm{nm}} = \boxed{3.31\times10^{-27}\ \mathrm{kg\cdot m/s}}$$

(*b*) Use the electron wavelength equation to relate the electron's wavelength to its kinetic energy:

$$\lambda = \frac{1.226}{\sqrt{K}}\,\mathrm{nm}$$

Solve for and evaluate K:

$$K = \left(\frac{1.226\,\mathrm{eV^{1/2}nm}}{200\,\mathrm{nm}}\right)^2 = \boxed{3.76\times10^{-5}\ \mathrm{eV}}$$

***39** ••

Picture the Problem The momenta of these particles can be found from their kinetic energies and speeds. Their de Broglie wavelengths are given by $\lambda = h/p$.

(*a*) The momentum of a particle p, in terms of its kinetic energy K, is given by:

$$p = \sqrt{2mK}$$

Substitute numerical values and evaluate p_e:

$$p_e = \sqrt{2m_e K} = \sqrt{2\left(9.11\times10^{-31}\ \text{kg}\right)\left(150\,\text{keV}\times\frac{1.6\times10^{-19}\ \text{C}}{\text{eV}}\right)}$$
$$= \boxed{2.09\times10^{-22}\ \text{N}\cdot\text{s}}$$

Substitute numerical values and evaluate p_p:

$$p_p = \sqrt{2m_p K} = \sqrt{2\left(1.67\times10^{-27}\ \text{kg}\right)\left(150\,\text{keV}\times\frac{1.6\times10^{-19}\ \text{C}}{\text{eV}}\right)}$$
$$= \boxed{8.95\times10^{-21}\ \text{N}\cdot\text{s}}$$

Substitute numerical values and evaluate p_α:

$$p_\alpha = \sqrt{2m_\alpha K} = \sqrt{2\left(4\,\text{u}\times\frac{1.66\times10^{-27}\ \text{kg}}{\text{u}}\right)\left(150\,\text{keV}\times\frac{1.6\times10^{-19}\ \text{C}}{\text{eV}}\right)}$$
$$= \boxed{1.79\times10^{-20}\ \text{N}\cdot\text{s}}$$

(*b*) The de Broglie wavelengths of the particles are given by:

$$\lambda = \frac{h}{p}$$

Substitute numerical values and evaluate λ_p:

$$\lambda_p = \frac{h}{p_p} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{8.95\times10^{-21}\ \text{N}\cdot\text{s}} = \boxed{7.41\times10^{-14}\ \text{m}}$$

Substitute numerical values and evaluate λ_e:

$$\lambda_e = \frac{h}{p_e} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{2.09\times10^{-22}\ \text{N}\cdot\text{s}} = \boxed{3.17\times10^{-12}\ \text{m}}$$

Substitute numerical values and evaluate λ_α:

$$\lambda_\alpha = \frac{h}{p_\alpha} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{1.79\times10^{-20}\ \text{N}\cdot\text{s}} = \boxed{3.70\times10^{-14}\ \text{m}}$$

40 •

Picture the Problem The wavelength associated with a particle of mass m and kinetic energy K is given by Equation 34-15 as $\lambda = \dfrac{1240\,\text{eV}\cdot\text{nm}}{\sqrt{2mc^2K}}$.

Substitute numerical data in Equation 34-15 to obtain:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{\sqrt{2(940\,\text{MeV})(0.02\,\text{eV})}} = \boxed{0.202\,\text{nm}}$$

41 •

Picture the Problem The wavelength associated with a particle of mass m and kinetic energy K is given by Equation 34-15 as $\lambda = \dfrac{1240\,\text{eV}\cdot\text{nm}}{\sqrt{2mc^2K}}$.

Substitute numerical data in Equation 34-15 to obtain:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{\sqrt{2(938\,\text{MeV})(2\,\text{MeV})}} = 2.02\times10^{-5}\ \text{nm} = \boxed{20.2\,\text{fm}}$$

***42** •

Picture the Problem We can use its definition to calculate the de Broglie wavelength of this proton.

Use its definition to express the de Broglie wavelength of the proton:

$$\lambda_\text{p} = \frac{h}{p_\text{p}} = \frac{h}{m_\text{p}v_\text{p}}$$

Substitute numerical values and evaluate λ_p:

$$\lambda = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(1.67\times10^{-27}\ \text{kg})[0.003(3\times10^{8}\ \text{m/s})]} = \boxed{0.441\,\text{pm}}$$

43 •

Picture the Problem We can solve Equation 34-15 ($\lambda = \dfrac{1240\,\text{eV}\cdot\text{nm}}{\sqrt{2mc^2K}}$) for the kinetic energy of the proton and use the rest energy of a proton $mc^2 = 938$ MeV to simplify our computation.

Solve Equation 34-15 for the kinetic energy of the proton:

$$K = \frac{(1240\,\text{eV}\cdot\text{nm})^2}{2mc^2\lambda^2}$$

(*a*) Substitute numerical values and evaluate K for $\lambda = 1$ nm:

$$K = \frac{(1240\,\text{eV}\cdot\text{nm})^2}{2(938\,\text{MeV})(1\,\text{nm})^2} = \boxed{0.820\,\text{meV}}$$

(*b*) Evaluate K for $\lambda = 1$ nm:

$$K = \frac{(1240\,\text{eV}\cdot\text{nm})^2}{2(938\,\text{MeV})(10^{-6}\,\text{nm})^2} = \boxed{820\,\text{MeV}}$$

44 •

Picture the Problem We'll need to convert oz and mph into SI units. Then we can use its definition to calculate the de Broglie wavelength of the baseball.

Use its definition to express the de Broglie wavelength of the baseball:

$$\lambda_{\text{baseball}} = \frac{h}{p_{\text{baseball}}} = \frac{h}{m_{\text{baseball}}v_{\text{baseball}}}$$

Substitute numerical values and evaluate $\lambda_{\text{baseball}}$:

$$\lambda_{\text{baseball}} = \frac{6.63\times10^{-34}\,\text{J}\cdot\text{s}}{\left(5\,\text{oz}\times\frac{1\,\text{lb}}{16\,\text{oz}}\times\frac{1\,\text{kg}}{2.20\,\text{lb}}\right)\left(95\,\frac{\text{mi}}{\text{h}}\times\frac{0.447\,\frac{\text{m}}{\text{s}}}{\frac{\text{mi}}{\text{h}}}\right)} = 1.10\times10^{-34}\,\text{m}$$

For the tennis ball:

$$\lambda_{\text{tennis ball}} = \frac{h}{p_{\text{tennis ball}}} = \frac{h}{m_{\text{tennis ball}}v_{\text{tennis balll}}}$$

Substitute numerical values and evaluate $\lambda_{\text{tennis ball}}$:

$$\lambda_{\text{tennis balll}} = \frac{6.63\times10^{-34}\,\text{J}\cdot\text{s}}{\left(2\,\text{oz}\times\frac{1\,\text{lb}}{16\,\text{oz}}\times\frac{1\,\text{kg}}{2.20\,\text{lb}}\right)\left(130\,\frac{\text{mi}}{\text{h}}\times\frac{0.447\,\frac{\text{m}}{\text{s}}}{\frac{\text{mi}}{\text{h}}}\right)} = 2.01\times10^{-34}\,\text{m}$$

$$\boxed{\text{The tennis ball has the longer de Broglie wavelength.}}$$

Remarks: Because $\lambda = h/p$, we could have solved the problem by determining which ball has the smaller momentum.

45 •

Picture the Problem If K is in electron volts, the wavelength of a particle is given by $\lambda = \frac{1.226}{\sqrt{K}}\text{ nm}$ provided K is in eV.

Evaluate λ for $K = 54$ eV:

$$\lambda = \frac{1.226}{\sqrt{K}}\text{ nm} = \frac{1.226}{\sqrt{54}}\text{ nm} = \boxed{0.167\text{ nm}}$$

46 •

Picture the Problem We can use $\lambda = \frac{1.226}{\sqrt{K}}\text{ nm}$, where K is in eV, to find the energy of electrons whose wavelength is λ.

Relate the wavelength of the electrons to their kinetic energy:

$$\lambda = \frac{1.226}{\sqrt{K}}\text{ nm}$$

Solve for K:

$$K = \left(\frac{1.226\text{ nm}\cdot\text{eV}^{1/2}}{\lambda}\right)^2$$

Substitute numerical values and evaluate K:

$$K = \left(\frac{1.226\text{ nm}\cdot\text{eV}^{1/2}}{0.257\text{ nm}}\right)^2 = \boxed{22.8\text{ eV}}$$

***47** •

Picture the Problem We can use $\lambda = \frac{1.226}{\sqrt{K}}\text{ nm}$, where K is in eV, to find the wavelength of 70-keV electrons.

Relate the wavelength of the electrons to their kinetic energy:

$$\lambda = \frac{1.23}{\sqrt{K}}\text{ nm}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{1.226}{\sqrt{70\times10^3\text{ eV}}}\text{ nm} = \boxed{4.63\text{ pm}}$$

48 •

Picture the Problem We can use its definition to calculate the de Broglie wavelength of a neutron with speed 10^6 m/s.

Use its definition to express the de Broglie wavelength of the neutron:

$$\lambda_n = \frac{h}{p_n} = \frac{h}{m_n v_n}$$

Substitute numerical values and evaluate λ_n:

$$\lambda_n = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(1.67\times10^{-27}\ \text{kg})(10^6\ \text{m/s})} = \boxed{0.397\ \text{pm}}$$

Wave-Particle Duality

49 •

Picture the Problem In order for diffraction to occur, the diameter of the aperture *d* must be approximately equal to the de Broglie wavelength of the spherical object. We can use the de Broglie relationship to find the size of the aperture necessary for this object to show diffraction.

Express the de Broglie wavelength of the spherical object:

$$\lambda_{\text{object}} = \frac{h}{p_{\text{object}}} = \frac{h}{m_{\text{object}} v_{\text{object}}}$$

Substitute numerical values and evaluate λ_{object}:

$$\lambda_{\text{object}} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(4\times10^{-3}\ \text{kg})(100\ \text{m/s})} = \boxed{1.66\times10^{-33}\ \text{m}}$$

$$\boxed{\begin{array}{l}\text{This is many orders of magnitude smaller than even the diameter of a proton}\\ \text{and so no common objects would be able to squeeze through such an aperture.}\end{array}}$$

50 •

Picture the Problem In order for diffraction to occur, the size of the object must be approximately λ. The wavelength of the neutron is given by $\lambda = \frac{1240\ \text{eV}\cdot\text{nm}}{\sqrt{2mc^2K}}$. The rest energy of the neutron is mc^2 = 940 MeV.

Substitute numerical values and evaluate λ:

$$\lambda = \frac{1240\ \text{eV}\cdot\text{nm}}{\sqrt{2(940\ \text{MeV})(10\ \text{MeV})}} = 9.04\times10^{-6}\ \text{nm} \approx \boxed{10\ \text{fm}}$$

$$\boxed{\begin{array}{l}\text{This wavelength is of the same order - of - magnitude as a nuclear diameter}\\ \text{and so nuclei would be suitable targets to demonstrate the wave nature of}\\ \text{neutrons with this energy.}\end{array}}$$

51 •

Picture the Problem We can use $\lambda = \frac{1.226}{\sqrt{K}}\,\text{nm}$, where K is in eV, to find the wavelength of 200-eV electrons. In order for diffraction to occur, the size of the target must be approximately λ.

Relate the wavelength of the electrons to their kinetic energy:

$$\lambda = \frac{1.226}{\sqrt{K}}\,\text{nm}$$

Substitute numerical values and evaluate λ:

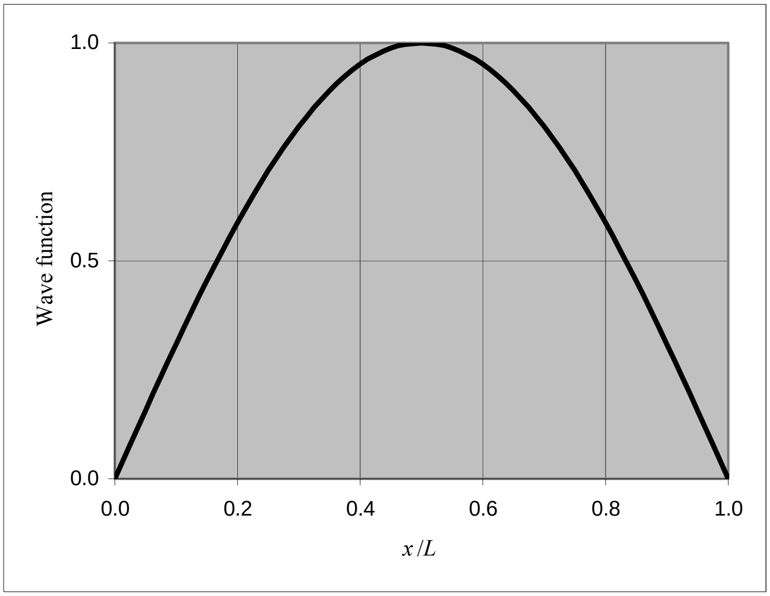
$$\lambda = \frac{1.226}{\sqrt{200\,\text{eV}}}\,\text{nm} = \boxed{0.0867\,\text{nm}}$$

This distance is of the order of the size of an atom and so atoms would be suitable targets to demonstrate the wave nature of electrons with this energy.

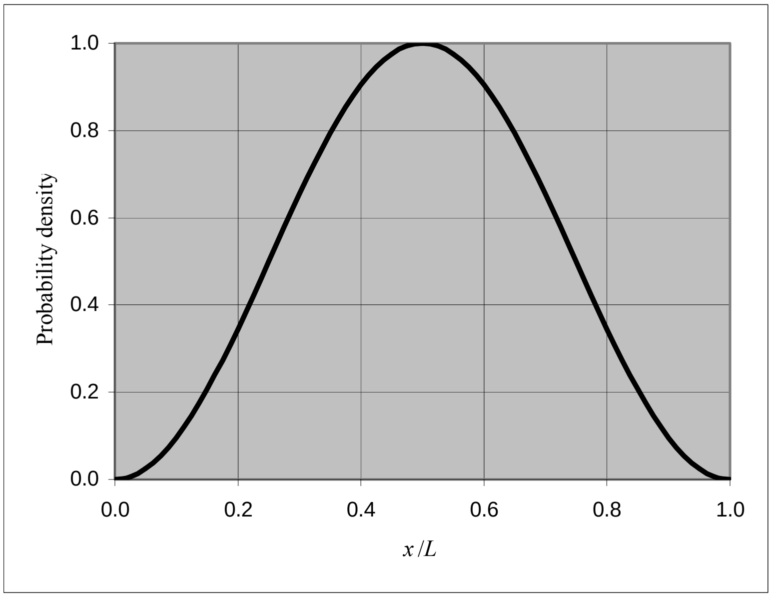
A Particle in a Box

***52** ••

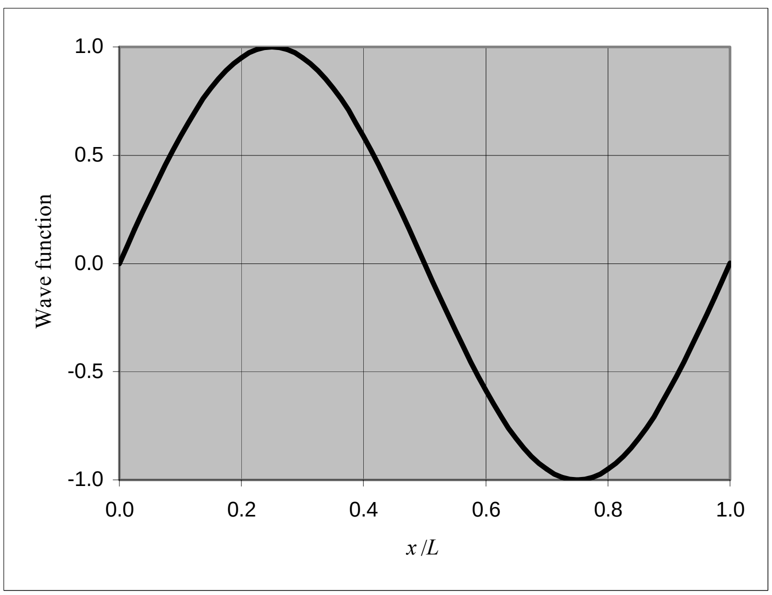
Picture the Problem The wave function for state n is $\psi_n(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$. The following graphs were plotted using a spreadsheet program. The graph of $\psi(x)$ for $n = 1$ is shown below:



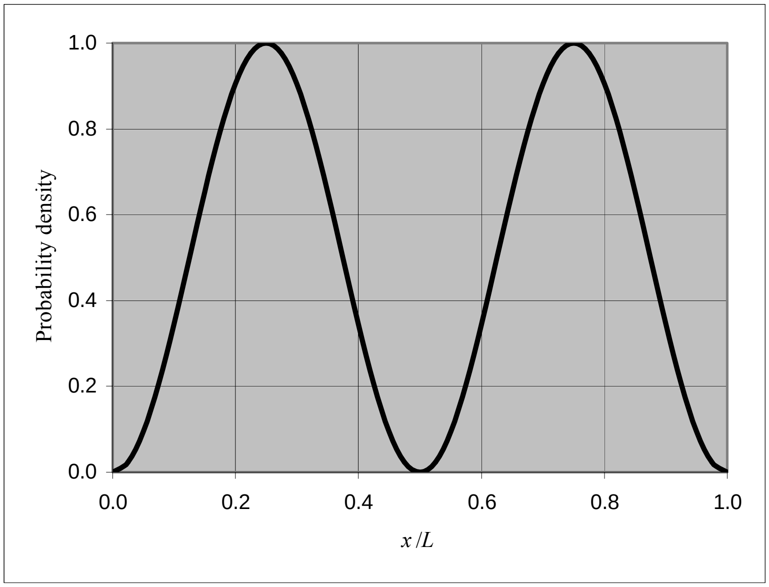
The graph of $\psi^2(x)$ for $n = 1$ is shown below:



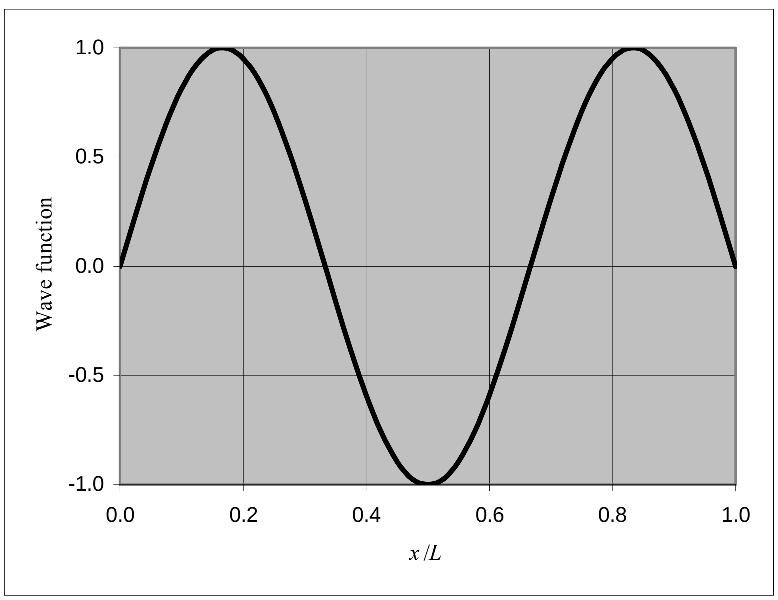
The graph of $\psi(x)$ for $n = 2$ is shown below:



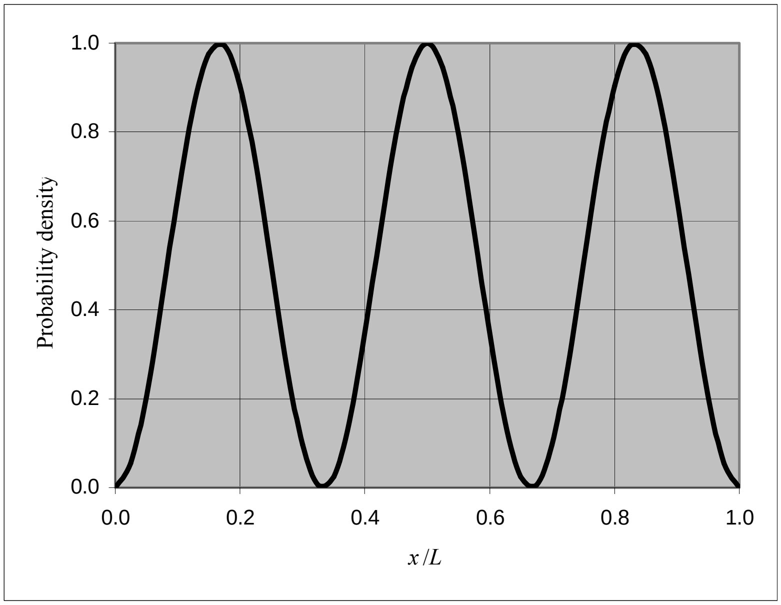
The graph of $\psi^2(x)$ for $n = 2$ is shown below:



The graph of $\psi(x)$ for $n = 3$ is shown below:



The graph of $\psi^2(x)$ for $n = 3$ is shown below:



53 ••

Picture the Problem We can find the ground-state energy using $E_1 = \dfrac{h^2}{8mL^2}$ and the energies of the excited states using $E_n = n^2 E_1$. The wavelength of the electromagnetic radiation emitted when the proton transitions from one state to another is given by the Einstein equation for photon energy ($E = \dfrac{hc}{\lambda}$).

(*a*) Express the ground-state energy:

$$E_1 = \frac{h^2}{8m_p L^2}$$

Substitute numerical values and evaluate E_1:

$$E_1 = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8\left(1.67\times10^{-27}\ \text{kg}\right)\left(10^{-15}\ \text{m}\right)^2}\frac{1\,\text{eV}}{1.6\times10^{-19}\ \text{J}} = \boxed{206\,\text{MeV}}$$

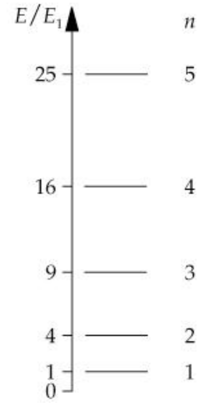
Find the energies of the first two excited states:

$$E_2 = 2^2 E_1 = 4(206\,\text{MeV}) = \boxed{824\,\text{MeV}}$$

and

$$E_3 = 3^2 E_1 = 9(206\,\text{MeV}) = \boxed{1.85\,\text{GeV}}$$

The energy-level diagram for this system is shown to the right:



(b) Relate the wavelength of the electromagnetic radiation emitted during a proton transition to the energy released in the transition:

$$\lambda = \frac{hc}{\Delta E} = \frac{1240\,\text{eV}\cdot\text{nm}}{\Delta E}$$

For the $n = 2$ to $n = 1$ transition:

$$\Delta E = E_2 - E_1 = 4E_1 - E_1 = 3E_1$$

Substitute numerical values and evaluate $\lambda_{2\to1}$:

$$\lambda_{2\to1} = \frac{1240\,\text{eV}\cdot\text{nm}}{3E_1} = \frac{1240\,\text{eV}\cdot\text{nm}}{3(206\,\text{MeV})} = \boxed{2.01\,\text{fm}}$$

(c) For the $n = 3$ to $n = 2$ transition:

$$\Delta E = E_3 - E_2 = 9E_1 - 4E_1 = 5E_1$$

Substitute numerical values and evaluate $\lambda_{3\to2}$:

$$\lambda_{3\to2} = \frac{1240\,\text{eV}\cdot\text{nm}}{5E_1} = \frac{1240\,\text{eV}\cdot\text{nm}}{5(206\,\text{MeV})} = \boxed{1.20\,\text{fm}}$$

(d) For the $n = 3$ to $n = 1$ transition:

$$\Delta E = E_3 - E_1 = 9E_1 - E_1 = 8E_1$$

Substitute numerical values and evaluate $\lambda_{3\to1}$:

$$\lambda_{3\to1} = \frac{1240\,\text{eV}\cdot\text{nm}}{8E_1} = \frac{1240\,\text{eV}\cdot\text{nm}}{8(206\,\text{MeV})} = \boxed{0.752\,\text{fm}}$$

54 ••

Picture the Problem We can find the ground-state energy using $E_1 = \dfrac{h^2}{8mL^2}$ and the energies of the excited states using $E_n = n^2 E_1$. The wavelength of the electromagnetic radiation emitted when the proton transitions from one state to another is given by the

Einstein equation for photon energy ($E = \dfrac{hc}{\lambda}$).

(a) Express the ground-state energy:

$$E_1 = \frac{h^2}{8m_\mathrm{p}L^2}$$

Substitute numerical values and evaluate E_1:

$$E_1 = \frac{\left(6.63\times10^{-34}\ \mathrm{J\cdot s}\right)^2}{8\left(1.67\times10^{-27}\ \mathrm{kg}\right)\left(0.2\ \mathrm{nm}\right)^2}\frac{1\,\mathrm{eV}}{1.60\times10^{-19}\ \mathrm{J}} = \boxed{5.14\,\mathrm{meV}}$$

Find the energies of the first two excited states:

$$E_2 = 2^2 E_1 = 4(5.14\,\mathrm{meV}) = \boxed{20.6\,\mathrm{meV}}$$

and

$$E_3 = 3^2 E_1 = 9(5.14\,\mathrm{meV}) = \boxed{46.3\,\mathrm{meV}}$$

(b) Relate the wavelength of the electromagnetic radiation emitted during a proton transition to the energy released in the transition:

$$\lambda = \frac{hc}{\Delta E} = \frac{1240\,\mathrm{eV\cdot nm}}{\Delta E}$$

For the $n = 2$ to $n = 1$ transition:

$$\Delta E = E_2 - E_1 = 4E_1 - E_1 = 3E_1$$

Substitute numerical values and evaluate $\lambda_{2\to1}$:

$$\lambda_{2\to1} = \frac{1240\,\mathrm{eV\cdot nm}}{3E_1} = \frac{1240\,\mathrm{eV\cdot nm}}{3(5.14\,\mathrm{meV})} = \boxed{80.4\,\mu\mathrm{m}}$$

(c) For the $n = 3$ to $n = 2$ transition:

$$\Delta E = E_3 - E_2 = 9E_1 - 4E_1 = 5E_1$$

Substitute numerical values and evaluate $\lambda_{3\to2}$:

$$\lambda_{3\to2} = \frac{1240\,\mathrm{eV\cdot nm}}{5E_1} = \frac{1240\,\mathrm{eV\cdot nm}}{5(5.14\,\mathrm{meV})} = \boxed{48.2\,\mu\mathrm{m}}$$

(d) For the $n = 3$ to $n = 1$ transition:

$$\Delta E = E_3 - E_1 = 9E_1 - E_1 = 8E_1$$

Substitute numerical values and evaluate $\lambda_{3\to1}$:

$$\lambda_{3\to1} = \frac{1240\,\mathrm{eV\cdot nm}}{8E_1} = \frac{1240\,\mathrm{eV\cdot nm}}{8(5.14\,\mathrm{meV})} = \boxed{30.2\,\mu\mathrm{m}}$$

Calculating Probabilities and Expectation Values

55 ••

Picture the Problem The probability of finding the particle in some range Δx is $\psi^2 dx$. The interval $\Delta x = 0.002L$ is so small that we can neglect the variation in $\psi(x)$ and just compute $\psi^2\Delta x$.

Express the probability of finding the particle in the interval Δx:

$$P = P(x)\Delta x = \psi^2(x)\Delta x$$

Express the wave function for a particle in the ground state:

$$\psi_1(x) = \sqrt{\frac{2}{L}}\sin\frac{\pi x}{L}$$

Substitute to obtain:

$$P = \frac{2}{L}\sin^2\frac{\pi x}{L}\Delta x = \frac{2}{L}\left(\sin^2\frac{\pi x}{L}\right)(0.002L) = 0.004\sin^2\frac{\pi x}{L}$$

(*a*) Evaluate P at $x = L/2$:

$$P = 0.004\sin^2\frac{\pi L}{2L} = 0.004\sin^2\frac{\pi}{2} = \boxed{0.004}$$

(*b*) Evaluate P at $x = 2L/3$:

$$P = 0.004\sin^2\frac{2\pi L}{3L} = 0.004\sin^2\frac{2\pi}{3} = \boxed{0.003}$$

(*c*) Evaluate P at $x = L$:

$$P = 0.004\sin^2\frac{\pi L}{L} = 0.004\sin^2\pi = \boxed{0}$$

***56** ••

Picture the Problem The probability of finding the particle in some range Δx is $\psi^2 dx$. The interval $\Delta x = 0.002L$ is so small that we can neglect the variation in $\psi(x)$ and just compute $\psi^2\Delta x$.

Express the probability of finding the particle in the interval Δx:

$$P = P(x)\Delta x = \psi^2(x)\Delta x$$

Express the wave function for a particle in its first excited state:

$$\psi_2(x)=\sqrt{\frac{2}{L}}\sin\frac{2\pi x}{L}$$

Substitute to obtain:

$$P=\frac{2}{L}\sin^2\frac{2\pi x}{L}\Delta x$$
$$=\frac{2}{L}\left(\sin^2\frac{2\pi x}{L}\right)(0.002L)$$
$$=0.004\sin^2\frac{2\pi x}{L}$$

(a) Evaluate P at $x=L/2$:

$$P=0.004\sin^2\frac{2\pi L}{2L}=0.004\sin^2\pi$$
$$=\boxed{0}$$

(b) Evaluate P at $x=2L/3$:

$$P=0.004\sin^2\frac{4\pi L}{3L}=0.004\sin^2\frac{4\pi}{3}$$
$$=\boxed{0.003}$$

(c) Evaluate P at $x=L$:

$$P=0.004\sin^2\frac{2\pi L}{L}=0.004\sin^2 2\pi$$
$$=\boxed{0}$$

57 ••

Picture the Problem We'll use $\langle f(x)\rangle=\int f(x)\psi^2(x)dx$ with $\psi_n(x)=\sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$.

(a) Express $\psi(x)$ for the $n = 2$ state:

$$\psi_2(x)=\sqrt{\frac{2}{L}}\sin\frac{2\pi x}{L}$$

Express $\langle x\rangle$ using the $n = 2$ wave function:

$$\langle x\rangle=\int_0^L\frac{2x}{L}\sin^2\frac{2\pi x}{L}dx$$

Change variables by letting $\theta=\frac{2\pi x}{L}$. Then:

$$x=\frac{L}{2\pi}\theta,$$
$$d\theta=\frac{2\pi}{L}dx,\text{ and}$$
$$dx=\frac{L}{2\pi}d\theta$$

and the limits on θ are 0 and 2π.

Substitute to obtain:

$$\langle x\rangle = \frac{2}{L}\int_0^L \left(\frac{L}{2\pi}\theta\right)\sin^2\theta\left(\frac{L}{2\pi}d\theta\right) = \frac{L}{2\pi^2}\int_0^{2\pi}\theta\sin^2\theta\,d\theta$$

Using a table of integrals, evaluate the integral:

$$\langle x\rangle = \frac{L}{2\pi^2}\left[\frac{\theta^2}{4} - \frac{\theta\sin 2\theta}{4} - \frac{\cos 2\theta}{8}\right]_0^{2\pi} = \frac{L}{2\pi^2}\left[\pi^2 - \frac{1}{8} + \frac{1}{8}\right] = \boxed{\frac{L}{2}}$$

(*b*) Express $\langle x^2\rangle$ using the $n = 2$ wave function:

$$\langle x^2\rangle = \int_0^L \frac{2x^2}{L}\sin^2\frac{2\pi x}{L}dx$$

Change variables as in (*a*) and substitute to obtain:

$$\langle x^2\rangle = \frac{2}{L}\int_0^L \left(\frac{L}{2\pi}\theta\right)^2\sin^2\theta\left(\frac{L}{2\pi}d\theta\right) = \frac{L^2}{4\pi^3}\int_0^{2\pi}\theta^2\sin^2\theta\,d\theta$$

Using a table of integrals, evaluate the integral:

$$\langle x^2\rangle = \frac{L^2}{4\pi^3}\left[\frac{\theta^3}{6} - \left(\frac{\theta^2}{4} - \frac{1}{8}\right)\sin 2\theta - \frac{\theta\cos 2\theta}{4}\right]_0^{2\pi} = \frac{L^2}{4\pi^3}\left[\frac{4\pi^3}{3} - \frac{\pi}{2}\right]$$
$$= L^2\left(\frac{1}{3} - \frac{1}{8\pi^2}\right) = \boxed{0.321L^2}$$

58 ••

Picture the Problem We'll use $\langle f(x)\rangle = \int f(x)\psi^2(x)dx$ with $\psi_n(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$. In Part (*c*) we'll use $P(x) = \psi_2^2(x)$ to determine the probability of finding the particle in some small region dx centered at $x = ½L$.

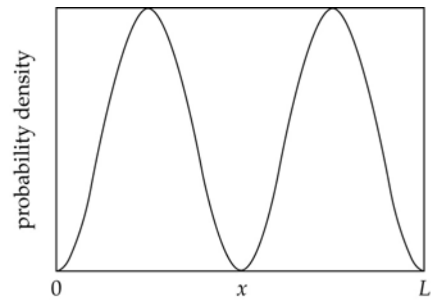
(*a*) Express the wave function for a particle in its first excited state:

$$\psi_2(x) = \sqrt{\frac{2}{L}}\sin\frac{2\pi x}{L}$$

Square both sides of the equation to obtain:

$$\psi_2^2(x) = \frac{2}{L}\sin^2\frac{2\pi x}{L}$$

The graph of $\psi^2(x)$ as a function of x is shown to the right:



(*b*) Express $\langle x\rangle$ using the $n = 2$ wave function:

$$\langle x\rangle = \int_0^L \frac{2x}{L}\sin^2\frac{2\pi x}{L}\,dx$$

Change variables by letting $\theta = \dfrac{2\pi x}{L}$. Then:

$$x = \frac{L}{3\pi}\theta,$$

$$d\theta = \frac{3\pi}{L}dx, \text{ and}$$

$$dx = \frac{L}{3\pi}d\theta$$

and the limits on θ are 0 and 2π.

Substitute to obtain:

$$\langle x\rangle = \frac{2}{L}\int_0^L\left(\frac{L}{2\pi}\theta\right)\sin^2\theta\left(\frac{L}{2\pi}d\theta\right)$$

$$= \frac{L}{2\pi^2}\int_0^{2\pi}\theta\sin^2\theta\,d\theta$$

Using a table of integrals, evaluate the integral:

$$\langle x\rangle = \frac{L}{2\pi^2}\left[\frac{\theta^2}{4} - \frac{\theta\sin 2\theta}{4} - \frac{\cos 2\theta}{8}\right]_0^{2\pi}$$

$$= \frac{L}{2\pi^2}\left[\pi^2 - \frac{1}{8} + \frac{1}{8}\right] = \boxed{\frac{L}{2}}$$

(*c*) Express $P(x)$:

$$P(x) = \psi_2^2(x)$$

$$= \frac{2}{L}\sin^2\frac{2\pi x}{L}dx$$

Evaluate $P(L/2)$:

$$P\left(\frac{L}{2}\right) = \frac{2}{L}\sin^2\frac{2\pi}{L}\cdot\frac{L}{2}$$

$$= \frac{2}{L}\sin^2\pi = 0$$

Because $P(L/2) = 0$:

$$P\left(\frac{L}{2}\right)dx = \boxed{0}$$

(*d*) $\boxed{\text{The answers to Parts } (b) \text{ and } (c) \text{ are not contradictory. } (b) \text{ states that the average value of measurements of the position of the particle will yield } L/2\text{, even though the probability that any one measurement of position will yield } L/2 \text{ is zero.}}$

59 ••

Picture the Problem We can find the constant A by applying the normalization condition $\int_{-\infty}^{\infty}\psi^2(x)dx = 1$ and finding the value for A that satisfies this condition. As soon as we have found the normalization constant, we can calculate the probability of the finding the particle in the region $-a \leq x \leq a$ using $P = \int_{-a}^{a}\psi^2(x)dx$.

(*a*) Express the normalization condition:

$$\int_{-\infty}^{\infty}\psi^2(x)dx = 1$$

Substitute $\psi(x) = Ae^{-|x|/a}$:

$$\int_{-\infty}^{\infty}\left(Ae^{-|x|/a}\right)^2 dx = 2A^2\int_0^{\infty}e^{-2x/a}dx$$

From integral tables:

$$\int_0^{\infty}e^{-\alpha x}dx = \frac{1}{\alpha}$$

Therefore:

$$2A^2\int_0^{\infty}e^{-2x/a}dx = 2A^2\left(\frac{a}{2}\right) = aA^2 = 1$$

Solve for A:

$$A = \boxed{\frac{1}{\sqrt{a}}}$$

(*b*) Express the normalized wave function:

$$\psi(x) = \frac{1}{\sqrt{a}}e^{-|x|/a}$$

The probability of finding the particle in the region $-a \leq x \leq a$ is:

$$P = \int_{-a}^{a}\psi^2(x)dx = 2\int_0^{a}\frac{1}{a}e^{-2x/a}dx$$

$$= \frac{2}{a}\int_0^{a}e^{-2x/a}dx = 1 - e^{-2} = \boxed{0.865}$$

60 ••

Picture the Problem The probability density for the particle in its ground state is given by $P(x)=\frac{2}{L}\sin^2\frac{\pi}{L}x$. We'll evaluate the integral of $P(x)$ between the limits specified in (*a*), (*b*), and (*c*).

Express $P(x)$ for $0 < x < d$:

$$P(x)=\frac{2}{L}\int_0^d \sin^2\frac{\pi}{L}x\,dx$$

Change variables by letting $\theta=\frac{\pi}{L}x$. Then:

$$x=\frac{L}{\pi}\theta,$$

$$d\theta=\frac{\pi}{L}dx, \text{ and}$$

$$dx=\frac{L}{\pi}d\theta$$

Substitute to obtain:

$$P(x)=\frac{2}{L}\int_0^{\theta'}\sin^2\theta\left(\frac{L}{\pi}d\theta\right)$$

$$=\frac{2}{\pi}\int_0^{\theta'}\sin^2\theta d\theta$$

Using a table of integrals, evaluate $\int_0^{\theta'}\sin^2\theta\,d\theta$:

$$P(x)=\frac{2}{\pi}\left[\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right]_0^{\theta'} \qquad (1)$$

(*a*) Noting that the limits on θ are 0 and $\pi/2$, evaluate equation (1) over the interval $0 < x < \frac{1}{2}L$:

$$P(x)=\frac{2}{\pi}\left[\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right]_0^{\pi/2}=\frac{2}{\pi}\left[\frac{\pi}{4}\right]$$

$$=\boxed{0.500}$$

(*b*) Noting that the limits on θ are 0 and $\pi/3$, Evaluate equation (1) over the interval $0 < x < L/3$:

$$P(x)=\frac{2}{\pi}\left[\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right]_0^{\pi/3}$$

$$=\frac{2}{\pi}\left[\frac{\pi}{6}-\frac{\sin 2\pi/3}{4}\right]$$

$$=\frac{1}{3}-\frac{\sqrt{3}}{4\pi}=\boxed{0.196}$$

(*c*) Noting that the limits on θ are 0 and $3\pi/4$, Evaluate equation (1) over the interval $0 < x < 3L/4$:

$$P(x) = \frac{2}{\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{3\pi/4}$$
$$= \frac{2}{\pi}\left[\frac{3\pi}{8} - \frac{\sin 6\pi/4}{4}\right]$$
$$= \frac{3}{4} + \frac{1}{2\pi} = \boxed{0.909}$$

61 ••

Picture the Problem The probability density for the particle in its first excited state is given by $P(x) = \frac{2}{L}\sin^2\frac{2\pi}{L}x$. We'll evaluate the integral of $P(x)$ between the limits specified in (*a*), (*b*), and (*c*).

Express $P(x)$ for $0 < x < d$:

$$P(x) = \frac{2}{L}\int_0^d \sin^2\frac{2\pi}{L}x\,dx$$

Change variables by letting $\theta = \frac{2\pi}{L}x$. Then:

$$x = \frac{L}{2\pi}\theta,$$
$$d\theta = \frac{2\pi}{L}dx, \text{ and}$$
$$dx = \frac{L}{2\pi}d\theta$$

Substitute to obtain:

$$P(x) = \frac{2}{L}\int_0^{\theta'} \sin^2\theta\left(\frac{L}{2\pi}d\theta\right)$$
$$= \frac{1}{\pi}\int_0^{\theta'} \sin^2\theta d\theta$$

Using a table of integrals, evaluate $\int_0^{\theta'} \sin^2\theta\,d\theta$:

$$P(x) = \frac{1}{\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\theta'} \qquad (1)$$

(*a*) Noting that the limits on θ are 0 and π, evaluate equation (1) over the interval $0 < x < \frac{1}{2}L$:

$$P(x) = \frac{1}{\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi} = \frac{1}{\pi}\left[\frac{\pi}{2}\right]$$
$$= \boxed{0.500}$$

(*b*) Noting that the limits on θ are 0 and $2\pi/3$, evaluate equation (1) over the interval $0 < x < L/3$:

$$P(x) = \frac{1}{\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{2\pi/3} = \frac{1}{\pi}\left[\frac{2\pi}{6} - \frac{\sin 4\pi/3}{4}\right] = \frac{1}{3} + \frac{\sqrt{3}/2}{4\pi} = \boxed{0.402}$$

(*c*) Noting that the limits on θ are 0 and $3\pi/2$, evaluate equation (1) over the interval $0 < x < 3L/4$:

$$P(x) = \frac{1}{\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{3\pi/2} = \frac{1}{\pi}\left[\frac{3\pi}{4}\right] = \boxed{0.750}$$

62 ••

Picture the Problem Classically, $\langle x\rangle = \int xP(x)dx$ and $\langle x^2\rangle = \int x^2P(x)dx$.

Evaluate $\langle x\rangle$ with $P(x) = 1/L$:

$$\langle x\rangle = \int_0^L \frac{x}{L}dx = \left[\frac{x^2}{2L}\right]_0^L = \boxed{\frac{L}{2}}$$

Evaluate $\langle x^2\rangle$ with $P(x) = 1/L$:

$$\langle x^2\rangle = \int_0^L \frac{x^2}{L}dx = \left[\frac{x^3}{3L}\right]_0^L = \boxed{\frac{L^2}{3}}$$

63 ••

Picture the Problem We'll use $\langle f(x)\rangle = \int f(x)\psi^2(x)dx$ with $\psi_n(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$ to show that $\langle x\rangle = \frac{L}{2}$ and $\langle x^2\rangle = \frac{L^2}{3} = \frac{L^2}{2n^2\pi^2}$.

(*a*) Express $\langle x\rangle$ for a particle in the *n*th state:

$$\langle x\rangle = \int_0^L \frac{2x}{L}\sin^2\frac{n\pi x}{L}dx$$

Change variables by letting $\theta = \frac{n\pi x}{L}$. Then:

$$x = \frac{L}{n\pi}\theta,$$

$$d\theta = \frac{n\pi}{L}dx, \text{ and}$$

$$dx = \frac{L}{n\pi}d\theta$$

and the limits on θ are 0 and $n\pi$.

Substitute to obtain:

$$\langle x\rangle = \frac{2}{L}\int_0^{n\pi}\left(\frac{L}{n\pi}\theta\right)\sin^2\theta\left(\frac{L}{n\pi}d\theta\right) = \frac{2L}{n^2\pi^2}\int_0^{n\pi}\theta\sin^2\theta\,d\theta$$

Using a table of integrals, evaluate the integral:

$$\langle x\rangle = \frac{2L}{n^2\pi^2}\left[\frac{\theta^2}{4}-\frac{\theta\sin 2\theta}{4}-\frac{\cos 2\theta}{8}\right]_0^{n\pi}$$
$$= \frac{2L}{n^2\pi^2}\left[\frac{n^2\pi^2}{4}-\frac{n\pi\sin 2n\pi}{4}-\frac{\cos 2n\pi}{8}+\frac{1}{8}\right] = \frac{2L}{n^2\pi^2}\left[\frac{n^2\pi^2}{4}-\frac{1}{8}+\frac{1}{8}\right]$$
$$= \boxed{\frac{L}{2}}$$

Express $\langle x^2\rangle$ for a particle in the nth state:

$$\langle x^2\rangle = \int_0^L \frac{2x^2}{L}\sin^2\frac{n\pi x}{L}dx$$

Change variables by letting $\theta = \frac{n\pi x}{L}$. Then:

$$x = \frac{L}{n\pi}\theta,$$
$$d\theta = \frac{n\pi}{L}dx, \text{ and}$$
$$dx = \frac{L}{n\pi}d\theta$$

and the limits on θ are 0 and $n\pi$.

Substitute to obtain:

$$\langle x^2\rangle = \frac{2}{L}\int_0^{n\pi}\left(\frac{L}{n\pi}\theta\right)^2\sin^2\theta\left(\frac{L}{n\pi}d\theta\right) = \frac{2L^2}{n^3\pi^3}\int_0^{n\pi}\theta^2\sin^2\theta\,d\theta$$

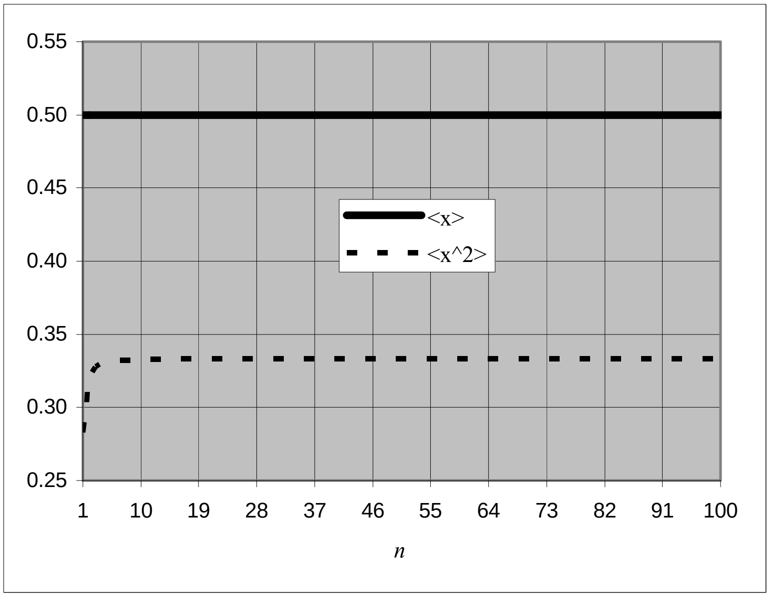
Using a table of integrals, evaluate the integral:

$$\langle x^2\rangle = \frac{2L^2}{n^3\pi^3}\left[\frac{\theta^3}{6}-\left(\frac{\theta^2}{4}-\frac{1}{8}\right)\sin 2\theta-\frac{\theta\cos 2\theta}{4}\right]_0^{n\pi} = \frac{2L^2}{n^3\pi^3}\left[\frac{n^3\pi^3}{6}-\frac{n\pi}{4}\right]$$
$$= \boxed{\frac{L^2}{3}-\frac{L^2}{2n^2\pi^2}}$$

(*b*) $\boxed{\text{For large values of } n\text{, the result agrees with the classical value of } L^2/3 \text{ given in Problem 62.}}$

***64 ••**

Picture the Problem From Problem 63 we have $\langle x\rangle = \frac{L}{2}$ and $\langle x^2\rangle = \frac{L^2}{3} - \frac{L^2}{2n^2\pi^2}$. A spreadsheet program was used to plot the following graphs of <*x*> and <*x*^{2}> as a function of *n*.



$$\boxed{\text{As } n \to \infty, \langle x^2\rangle \to \frac{L^2}{3}}$$

65 ••

Picture the Problem For the ground state, *n* = 1 and so we'll evaluate $\langle f(x)\rangle = \int f(x)\psi^2(x)dx$ using $\psi_1(x) = \sqrt{\frac{2}{L}}\cos\frac{\pi x}{L}$.

Because $\psi_1^2(x)$ is an even function of *x*, $x\psi_1^2(x)$ is an odd function of *x*. It follows that the integral of $x\psi_1^2(x)$ between –*L*/2 and *L*/2 is zero. Thus:

$\langle x\rangle = \boxed{0}$ for all values of *n*.

Express $\langle x^2\rangle$:

$$\langle x^2\rangle = \frac{2}{L}\int_{-L/2}^{L/2} x^2\cos^2\frac{\pi}{L}x\,dx$$

Change variables by letting $\theta = \frac{\pi x}{L}$.

Then:

$$x = \frac{L}{\pi}\theta,$$

$$d\theta = \frac{\pi}{L}dx,\text{ and}$$

$$dx = \frac{L}{\pi}d\theta$$

and the limits on θ are $-\pi/2$ and $\pi/2$.

Substitute to obtain:

$$\langle x^2\rangle = \frac{2}{L}\int_{-\pi/2}^{\pi/2}\left(\frac{L}{\pi}\theta\right)^2\cos^2\theta\left(\frac{L}{\pi}d\theta\right)$$

$$= \frac{2L^2}{\pi^3}\int_{-\pi/2}^{\pi/2}\theta^2\cos^2\theta\,d\theta$$

Use a trigonometric identity to rewrite the integrand:

$$\langle x^2\rangle = \frac{2L^2}{\pi^3}\int_{-\pi/2}^{\pi/2}\theta^2\left(1-\sin^2\theta\right)d\theta$$

$$= \frac{2L^2}{\pi^3}\left[\int_{-\pi/2}^{\pi/2}\theta^2 d\theta - \int_{-\pi/2}^{\pi/2}\theta^2\sin^2\theta\,d\theta\right]$$

Evaluate the second integral by looking it up in the tables:

$$\langle x^2\rangle = \frac{2L^2}{\pi^3}\left[\frac{\theta^3}{3} - \left\{\frac{\theta^3}{6} - \left(\frac{\theta^2}{4}-\frac{1}{8}\right)\sin 2\theta - \frac{\theta\cos 2\theta}{4}\right\}\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{2L^2}{\pi^3}\left[\frac{\theta^3}{6} + \left(\frac{\theta^2}{4}-\frac{1}{8}\right)\sin 2\theta + \frac{\theta\cos 2\theta}{4}\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{2L^2}{\pi^3}\left[\frac{\pi^3}{48} + \left(\frac{\pi^2}{16}-\frac{1}{8}\right)\sin\pi + \frac{\pi\cos\pi}{8}\right.$$

$$\left. + \frac{\pi^3}{48} + \left(\frac{\pi^2}{16}-\frac{1}{8}\right)\sin\pi + \frac{\pi\cos\pi}{8}\right]$$

$$= \frac{2L^2}{\pi^3}\left[\frac{\pi^3}{24} - \frac{\pi}{4}\right] = \boxed{L^2\left[\frac{1}{12} - \frac{1}{2\pi^2}\right]}$$

Remarks: The result differs from that of Example 34-8. Since we have shifted the origin by $\Delta x = L/2$, we could have arrived at the above result, without performing the integration, by subtracting $(\Delta x)^2 = L^2/4$ from $\langle x^2 \rangle$ as given in Example 34-8.

66 ••

Picture the Problem For the first excited state, $n = 2$, and so we'll evaluate $\langle f(x)\rangle = \int f(x)\psi^2(x)dx$ using $\psi_2(x) = \sqrt{\frac{2}{L}}\sin\frac{2\pi x}{L}$.

Since $\psi_2^2(x)$ is an even function of x, $x\psi_2^2(x)$ is an odd function of x. It follows that the integral of $x\psi_2^2(x)$ between $-L/2$ and $L/2$ is zero. Thus:

$$\langle x \rangle = \boxed{0}$$

Express $\langle x^2 \rangle$:

$$\langle x^2 \rangle = \frac{2}{L}\int_{-L/2}^{L/2} x^2 \sin^2\frac{2\pi}{L}x dx$$

Change variables by letting $\theta = \frac{2\pi x}{L}$. Then:

$$x = \frac{L}{2\pi}\theta,$$

$$d\theta = \frac{2\pi}{L}dx, \text{ and}$$

$$dx = \frac{L}{2\pi}d\theta$$

and the limits on θ are $-\pi$ and π.

Substitute to obtain:

$$\langle x^2 \rangle = \frac{2}{L}\int_{-\pi}^{\pi}\left(\frac{L}{2\pi}\theta\right)^2 \sin^2\theta\left(\frac{L}{2\pi}d\theta\right)$$

$$= \frac{L^2}{4\pi^3}\int_{-\pi}^{\pi}\theta^2 \sin^2\theta\, d\theta$$

Evaluate the integral by looking it up in the tables:

$$\begin{aligned}
\left\langle x^2\right\rangle &= \frac{L^2}{4\pi^3}\left[\frac{\theta^3}{6}-\left(\frac{\theta^2}{4}-\frac{1}{8}\right)\sin 2\theta-\frac{\theta\cos 2\theta}{4}\right]_{-\pi}^{\pi}\\
&= \frac{L^2}{4\pi^3}\left[\frac{\pi^3}{6}-\left(\frac{\pi^2}{4}-\frac{1}{8}\right)\sin 2\pi-\frac{\pi\cos 2\pi}{4}\right.\\
&\qquad\left.+\frac{\pi^3}{6}-\left(\frac{\pi^2}{4}-\frac{1}{8}\right)\sin 2\pi-\frac{\pi\cos 2\pi}{4}\right]\\
&= \frac{L^2}{4\pi^3}\left[\frac{\pi^3}{6}-\frac{\pi}{4}+\frac{\pi^3}{6}-\frac{\pi}{4}\right]\\
&= \frac{L^2}{4\pi^3}\left[\frac{\pi^3}{3}-\frac{\pi}{2}\right]=\boxed{L^2\left[\frac{1}{12}-\frac{1}{8\pi^2}\right]}
\end{aligned}$$

Remarks: The result differs from that of Example 34-8. Since we have shifted the origin by $\Delta x = L/2$, we could have arrived at the above result, without performing the integration, by subtracting $(\Delta x)^2 = L^2/4$ from $\left\langle x^2\right\rangle$ as given in Example 34-8.

General Problems

***67 •**

Picture the Problem We can use the Einstein equation for photon energy to find the energy of each photon in the beam. The intensity of the energy incident on the surface is the ratio of the power delivered by the beam to its delivery time. Hence, we can express the energy incident on the surface in terms of the intensity of the beam.

(*a*) Use the Einstein equation for photon energy to express the energy of each photon in the beam:

$$E_{\text{photon}} = hf = \frac{hc}{\lambda}$$

Substitute numerical values and evaluate E_{photon}:

$$E_{\text{photon}} = \frac{1240\,\text{eV}\cdot\text{nm}}{400\,\text{nm}} = \boxed{3.10\,\text{eV}}$$

(*b*) Relate the energy incident on a surface of area A to the intensity of the beam:

$$E = IA\Delta t$$

Substitute numerical values and evaluate E:

$$E = (100\ \text{W/m}^2)(10^{-4}\ \text{m}^2)(1\,\text{s})$$
$$= 0.01\,\text{J} \times \frac{1\,\text{eV}}{1.60\times10^{-19}\ \text{J}}$$
$$= \boxed{6.25\times10^{16}\ \text{eV}}$$

(*c*) Express the number of photons striking this area in 1 s as the ratio of the total energy incident on the surface to the energy delivered by each photon:

$$N = \frac{E}{E_{\text{photon}}} = \frac{6.25\times10^{16}\ \text{ev}}{3.10\,\text{eV}}$$
$$= \boxed{2.02\times10^{16}}$$

68 •

Picture the Problem The particle's *n*th-state energy is $E_n = n^2\dfrac{h^2}{8mL^2}$. We can find n by solving this equation for n and substituting the particle's kinetic energy for E_n.

Express the energy of the particle when it is in its *n*th state:

$$E_n = n^2\frac{h^2}{8mL^2}$$

Solve for n:

$$n = \frac{L}{h}\sqrt{8mE_n}$$

Express the energy (kinetic) of the particle:

$$E_n = \tfrac{1}{2}mv^2$$

Substitute to obtain:

$$n = \frac{2mvL}{h}$$

Substitute numerical values and evaluate n:

$$n = \frac{2(10^{-9}\ \text{kg})(10^{-3}\ \text{m/s})(10^{-2}\ \text{m})}{6.63\times10^{-34}\ \text{J}\cdot\text{s}}$$
$$= 3.02\times10^{19} \approx \boxed{3\times10^{19}}$$

69 •

Picture the Problem We can use the fact that the uncertainties are given by $\Delta x/L$ = 0.01 percent and $\Delta p/p$ = 0.01 percent to find Δx and Δp.

(*a*) Assuming that $\Delta x/L$ = 0.01 percent, find Δx:

$$\Delta x = 10^{-4}(L) = 10^{-4}(10^{-2}\ \text{m}) = \boxed{1.00\ \mu\text{m}}$$

Assuming that $\Delta p/p = 0.01$ percent, find Δp:

$$\Delta p = 10^{-4}mv = 10^{-4}\left(10^{-9}\text{ kg}\right)\left(10^{-3}\text{ m/s}\right) = \boxed{10^{-16}\text{ kg}\cdot\text{m/s}}$$

(*b*) Evaluate $(\Delta x\Delta p)/\hbar$:

$$\frac{\Delta x\Delta p}{\hbar} = \frac{(1\ \mu\text{m})\left(10^{-16}\text{ kg}\cdot\text{m/s}\right)}{1.054\times10^{-34}} = \boxed{0.949\times10^{12}}$$

70 •

Picture the Problem We can estimate the number of emitted photons from the ratio of the total energy in the flash to the energy of a single photon.

Letting N be the number of emitted photons, express the ratio of the total energy in the flash to the energy of a single photon:

$$N = \frac{E}{E_{\text{photon}}}$$

Relate the energy in the flash to the power produced:

$$E = P\Delta t$$

Express the energy of a single photon as a function of its wavelength:

$$E_{\text{photon}} = \frac{hc}{\lambda}$$

Substitute to obtain:

$$N = \frac{P\Delta t\lambda}{hc}$$

Substitute numerical values and evaluate N:

$$N = \frac{\left(5\times10^{15}\text{ W}\right)\left(10^{-12}\text{ s}\right)\left(400\times10^{-9}\text{ m}\right)}{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3\times10^{8}\text{ m/s}\right)} = \boxed{1.01\times10^{22}}$$

71 •

Picture the Problem We can use the electron wavelength equation $\lambda = \frac{1.23}{\sqrt{K}}\text{nm},$ where K is in eV to find the minimum energy required to see an atom.

Relate the energy of the electron to the size of an atom (the wavelength of the electron):

$$\lambda = \frac{1.23}{\sqrt{K}}\text{nm}$$

provided K is in eV.

Solve for K:

$$K = \frac{\left(1.23\,\text{eV}^{1/2}\cdot\text{nm}\right)^2}{\lambda^2}$$

Substitute numerical values and evaluate K:

$$K = \frac{\left(1.23\,\text{eV}^{1/2}\cdot\text{nm}\right)^2}{(0.1\,\text{nm})^2} = \boxed{151\,\text{eV}}$$

72 •

Picture the Problem The flea's de Broglie wavelength is $\lambda = h/p$, where p is the flea's momentum immediately after takeoff. We can use a constant acceleration equation to find the flea's speed and, hence, momentum immediately after takeoff.

Express the de Broglie wavelength of the flea immediately after takeoff:

$$\lambda = \frac{h}{p} = \frac{h}{mv_0}$$

Using a constant acceleration equation, express the height the flea can jump as a function of its takeoff speed:

$$v^2 = v_0^2 + 2a\Delta y$$

or, since $v = 0$ and $a = -g$,

$$v_0 = \sqrt{2g\Delta y}$$

Substitute to obtain:

$$\lambda = \frac{h}{m\sqrt{2g\Delta y}}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{6.63\times10^{-34}\,\text{J}\cdot\text{s}}{\left(8\times10^{-6}\,\text{kg}\right)\sqrt{2\left(9.81\,\text{m/s}^2\right)(0.2\,\text{m})}} = \boxed{4.18\times10^{-29}\,\text{m}}$$

***73 ••**

Picture the Problem We can relate the fraction of the photons entering the eye to ratio of the area of the pupil to the area of a sphere of radius R. We can find the number of photons emitted by the source from the rate at which it emits and the energy of each photon which we can find using the Einstein equation.

Letting r be the radius of the pupil, $N_{\text{entering eye}}$ the number of photons per second entering the eye, and N_{emitted} the number of photons emitted by the source per second, express the fraction of the light energy entering the eye at a distance R from the

$$\frac{N_{\text{entering eye}}}{N_{\text{emitted}}} = \frac{A_{\text{eye}}}{4\pi R^2} = \frac{\pi r^2}{4\pi R^2} = \frac{r^2}{4R^2}$$

source:

Solve for R to obtain:

$$R=\frac{r}{2}\sqrt{\frac{N_{\text{emitted}}}{N_{\text{entering eye}}}} \qquad (1)$$

Find the number of photons emitted by the source per second:

$$N_{\text{emitted}}=\frac{P}{E_{\text{photon}}}$$

Using the Einstein equation, express the energy of the photons:

$$E_{\text{photon}}=\frac{hc}{\lambda}$$

Substitute numerical values and evaluate E_{photon}:

$$E_{\text{photon}}=\frac{1240\,\text{eV}\cdot\text{nm}}{600\,\text{nm}}=2.07\,\text{eV}$$

Substitute and evaluate N_{emitted}:

$$N_{\text{emitted}}=\frac{100\,\text{W}}{(2.07\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}=3.02\times10^{20}\,\text{s}^{-1}$$

Substitute for N_{emitted} in equation (1) and evaluate R:

$$R=\frac{3.5\,\text{mm}}{2}\sqrt{\frac{3.02\times10^{20}\,\text{s}^{-1}}{20\,\text{s}^{-1}}}=\boxed{6.80\times10^{3}\,\text{km}}$$

74 ••

Picture the Problem The intensity of the light such that one photon per second passes through the pupil is the ratio of the energy of one photon to the product of the area of the pupil and time interval during which the photon passes through the pupil. We'll use the Einstein equation to express the energy of the photon.

Use its definition to relate the intensity of the light to the energy of a 600-nm photon:

$$I_{1\,\text{photon}}=\frac{P}{A}=\frac{E_{1\,\text{photon}}}{A\Delta t}$$

Using the Einstein equation, express the energy of a 600-nm photon:

$$E_{1\,\text{photon}}=\frac{hc}{\lambda}$$

Substitute for $E_{1\text{ photon}}$ to obtain:

$$I_{1\,\text{photon}}=\frac{hc}{\lambda A\Delta t}$$

Substitute numerical values and evaluate $I_{1\text{ photon}}$:

$$I_{1\text{photon}} = \frac{(1240\,\text{eV}\cdot\text{nm})(1.602\times10^{-19}\ \text{J/eV})}{(600\,\text{nm})\left[\frac{\pi}{4}(5\times10^{-3}\ \text{m})^2\right](1\,\text{s})} = \boxed{1.69\times10^{-14}\ \text{W/m}^2}$$

75 ••

Picture the Problem We can find the intensity at a distance of 1.5 m directly from its definition. The number of photons striking the surface each second can be found from the ratio of the energy incident on the surface to the energy of a 650-nm photon.

(*a*) Use its definition to express the intensity of the light as a function of distance from the light bulb:

$$I = \frac{P}{A} = \frac{P}{4\pi R^2}$$

Substitute numerical data to obtain:

$$I = \frac{90\ \text{W}}{4\pi(1.5\,\text{m})^2} = \boxed{3.18\ \text{W/m}^2}$$

(*b*) Express the number of photons per second that strike the surface as the ratio of the energy incident on the surface to the energy of a 650-nm photon:

$$N = \frac{IA}{E_{\text{photon}}}$$

where A is the area of the surface.

Use the Einstein equation to express the energy of the 650-nm photons:

$$E = \frac{hc}{\lambda}$$

Substitute to obtain:

$$N = \frac{IA\lambda}{hc}$$

Substitute numerical values and evaluate N:

$$N = \frac{(3.18\ \text{W/m}^2)(10^{-4}\ \text{m}^2)(650\,\text{nm})}{(1240\,\text{eV}\cdot\text{nm})(1.60\times10^{-19}\ \text{J/eV})} = \boxed{1.04\times10^{15}}$$

76 ••

Picture the Problem The maximum kinetic energy of the photoelectrons is related to the frequency of the incident photons and the work function of the cathode material through the Einstein equation. We can apply this equation to the two sets of data and solve the resulting equations simultaneously for the work function.

Using the Einstein equation, relate the maximum kinetic energy of the emitted electrons to the frequency of the incident photons and the work function of the cathode material:

$$K_{\text{max}} = hf - \phi = \frac{hc}{\lambda} - \phi$$

Substitute numerical data for the light of wavelength λ_1:

$$1.8\,\text{eV} = \frac{hc}{\lambda_1} - \phi$$

Substitute numerical data for the light of wavelength $\lambda_1/2$:

$$5.5\,\text{eV} = \frac{hc}{\lambda_1/2} - \phi = \frac{2hc}{\lambda_1} - \phi$$

Solve these equations simultaneously for ϕ to obtain:

$$\phi = \boxed{1.90\,\text{eV}}$$

77 ••

Picture the Problem We can use the Einstein equation to express the energy of the scattered photon in terms of its wavelength and the Compton scattering equation to relate this wavelength to the scattering angle and the pre-scattering wavelength.

Express the energy of the scattered photon E' as a function of their wavelength λ':

$$E' = \frac{hc}{\lambda'}$$

Express the wavelength of the scattered photon as a function of the scattering angle θ:

$$\lambda' = \frac{h}{m_e c}(1 - \cos\theta) + \lambda$$

where λ is the wavelength of the incident photon.

Substitute and simplify to obtain:

$$E' = \frac{hc}{\dfrac{h}{m_e c}(1 - \cos\theta) + \lambda} = \frac{\dfrac{hc}{\lambda}}{\dfrac{hc}{m_e c^2 \lambda}(1 - \cos\theta) + 1} = \boxed{\frac{E}{\dfrac{E}{m_e c^2}(1 - \cos\theta) + 1}}$$

78 ••

Picture the Problem While we can work with either of the transitions described in the problem statement, we'll use the first transition in which radiation of wavelength 114.8 nm is emitted. We can express the energy released in the transition in terms of the difference between the energies in the two states and solve the resulting equation for n.

Express the energy of the emitted radiation as the particle goes from the nth to $n-1$ state:

$$\Delta E = E_n - E_{n-1}$$

Express the energy of the particle in nth state:

$$E_n = n^2 E_1$$

Express the energy of the particle in the $n-1$ state:

$$E_{n-1} = (n-1)^2 E_1$$

Substitute and simplify to obtain:

$$\Delta E = n^2 E_1 - (n-1)^2 E_1 = (2n-1)E_1 = \frac{hc}{\lambda}$$

Solve for n:

$$n = \frac{hc}{2\lambda E_1} + \frac{1}{2}$$

Substitute numerical values and evaluate n:

$$n = \frac{1240\,\text{eV}\cdot\text{nm}}{2(114.8\,\text{nm})(1.2\,\text{eV})} + \frac{1}{2} = \boxed{5}$$

***79** ••

Picture the Problem We can use the expression for the energy of a particle in a well to find the energy of the most energetic electron in the uranium atom.

Relate the energy of an electron in the uranium atom to its quantum number n:

$$E_n = n^2\left(\frac{h^2}{8mL^2}\right)$$

Substitute numerical values and evaluate E_{92}:

$$E_{92} = (92)^2\left[\frac{(6.63\times10^{-34}\,\text{J}\cdot\text{s})^2}{8(9.11\times10^{-31}\,\text{kg})(0.05\,\text{nm})^2} \times \frac{1\,\text{eV}}{1.6\times10^{-19}\,\text{J}}\right] = \boxed{1.28\,\text{MeV}}$$

The rest energy of an electron is:

$$m_e c^2 = (9.11\times10^{-31}\ \text{kg})(3\times10^8\ \text{m/s})^2\left(\frac{1\,\text{eV}}{1.6\times10^{-19}\ \text{J}}\right) = 0.512\,\text{MeV}$$

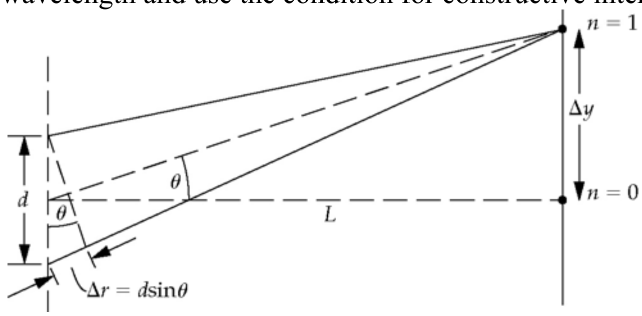
Express the ratio of E_{92} to $m_e c^2$:

$$\frac{E_{92}}{m_e c^2} = \frac{1.28\,\text{MeV}}{0.512\,\text{MeV}} = 2.50$$

$$\boxed{\text{The energy of the most energetic electron is approximately 2.5 times the rest-mass energy of an electron.}}$$

80 ••

Picture the Problem We can express the kinetic energy of an electron in the beam in terms of its momentum. We can use the de Broglie relationship to relate the electron's momentum to its wavelength and use the condition for constructive interference to find λ.



Express the kinetic energy of an electron in terms of its momentum:

$$K = \frac{p^2}{2m} \qquad (1)$$

Using the de Broglie relationship, relate the momentum of an electron to its momentum:

$$p = \frac{h}{\lambda}$$

Substitute for p in equation (1) to obtain:

$$K = \frac{h^2}{2m\lambda^2} \qquad (2)$$

The condition for constructive interference is:

$$d\sin\theta = n\lambda$$

where d is the slit separation and $n = 0, 1, 2, \ldots$

Solve for λ:

$$\lambda = \frac{d\sin\theta}{n}$$

For $\theta << 1$, $\sin\theta$ is also given by:

$$\sin\theta \approx \frac{\Delta y}{L}$$

Substitute for $\sin\theta$ to obtain:

$$\lambda = \frac{d\Delta y}{nL}$$

Substitute for λ in equation (2) to obtain:

$$K = \frac{n^2 L^2 h^2}{2md^2(\Delta y)^2}$$

Substitute numerical values (n = 1) and evaluate K:

$$K = \frac{(1)^2(1.5\,\text{m})^2(6.63\times10^{-34}\,\text{J}\cdot\text{s})^2}{2(9.11\times10^{-31}\,\text{kg})(54\,\text{nm})^2(0.68\,\text{mm})^2}\frac{1\,\text{eV}}{1.6\times10^{-19}\,\text{J}} = \boxed{2.52\,\text{keV}}$$

81 ••

Picture the Problem The maximum kinetic energy of the photoelectrons is related to the frequency of the incident photons and the work function of the illuminated surface through the Einstein equation. We can apply this equation to either set of data and solve the resulting equations simultaneously for the work function of the surface and the wavelength of the incident photons.

Using the Einstein equation, relate the maximum kinetic energy of the emitted electrons to the frequency of the incident photons and the work function of the cathode material:

$$K_{\text{max}} = hf - \phi = \frac{hc}{\lambda} - \phi$$

Substitute numerical data for the light of wavelength λ:

$$1.2\,\text{eV} = \frac{hc}{\lambda} - \phi$$

Substitute numerical values for the light of wavelength λ':

$$1.76\,\text{eV} = \frac{hc}{\lambda'} - \phi = \frac{hc}{0.8\lambda} - \phi$$

Solve these equations simultaneously for ϕ to obtain:

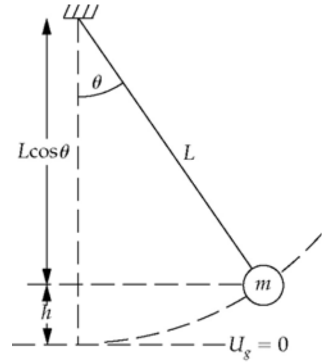
$$\phi = \boxed{1.04\,\text{eV}}$$

Substitute in either of the equations and solve for λ:

$$\lambda = \boxed{554\,\text{nm}}$$

82 ••

Picture the Problem The diagram shows the pendulum with an angular displacement θ. The energy of the oscillator is equal to its initial potential energy $mgh = mgL(1 - \cos\theta)$. We can find n by equating this initial energy to $E_n = \left(n + \frac{1}{2}\right)hf_0$ and solving for n. In part (b) we'll express the ratio of ΔE_n to E_n and solve for Δn.



(a) Express the nth-state energy as a function of the frequency of the pendulum:

$$E_n = \left(n + \tfrac{1}{2}\right)hf_0 = \left(n + \tfrac{1}{2}\right)\frac{h}{2\pi}\sqrt{\frac{g}{L}}$$

Express the energy of the pendulum:

$$E_n = mgL(1 - \cos\theta)$$

Substitute to obtain:

$$mgL(1 - \cos\theta) = \left(n + \tfrac{1}{2}\right)\frac{h}{2\pi}\sqrt{\frac{g}{L}}$$

Solve for n:

$$n = \frac{2\pi m\sqrt{g}\,L^{3/2}(1 - \cos\theta)}{h} - \frac{1}{2}$$

Substitute numerical values and evaluate n:

$$n = \frac{2\pi(0.3\,\text{kg})\sqrt{9.81\,\text{m/s}^2}\,(1\,\text{m})^{3/2}(1 - \cos 10°)}{6.63\times10^{-34}\,\text{J}\cdot\text{s}} - \frac{1}{2} = \boxed{1.35\times10^{32}}$$

(b) Express the ratio of ΔE_n to E_n:

$$\frac{\Delta E_n}{E_n} = \frac{\left(n + \Delta n + \frac{1}{2}\right)hf_0 - \left(n + \frac{1}{2}\right)hf_0}{\left(n + \frac{1}{2}\right)hf_0}$$

$$= \frac{\Delta n}{n + \frac{1}{2}} = 10^{-4}$$

Solve for and evaluate Δn:

$$\Delta n = 10^{-4}\left(n + \tfrac{1}{2}\right) \approx 10^{-4}n = \boxed{1.35\times10^{28}}$$

***83** ••

Picture the Problem We can use the fact that the energy of the nth state is related to the energy of the ground state according to $E_n = n^2E_1$ to express the fractional change in energy in terms of n and then examine this ratio as n grows without bound.

(*a*) Express the ratio $(E_{n+1} - E_n)/E_n$:

$$\frac{E_{n+1} - E_n}{E_n} = \frac{(n+1)^2 - n^2}{n^2} = \frac{2n+1}{n^2}$$

$$= \frac{2}{n} + \frac{1}{n^2} \approx \boxed{\frac{2}{n}}$$

for $n >> 1$.

(*b*) Evaluate $\dfrac{E_{1001} - E_{1000}}{E_{1000}}$:

$$\frac{E_{1001} - E_{1000}}{E_{1000}} \approx \frac{2}{1000} = \boxed{0.2\%}$$

(*c*) $\boxed{\text{Classically, the energy is continuous. For very large values of } n\text{, the energy difference between adjacent levels is infinitesimal.}}$

84 ••

Picture the Problem We can apply the definition of power in conjunction with the de Broglie equation for the energy of a photon to derive an expression for the average power produced by the laser.

The average power produced by the laser is:

$$P = \frac{\Delta E}{\Delta t}$$

Use the de Broglie equation to express the energy of the emitted photons:

$$\Delta E = Nhf = \frac{Nhc}{\lambda}$$

where N is number of photons in each pulse.

Substitute for ΔE to obtain:

$$P = \frac{Nhc}{\lambda \Delta t}$$

Substitute numerical values and evaluate P:

$$P = \frac{(5\times10^{9})(6.63\times10^{-34}\text{ J}\cdot\text{s})(3\times10^{8}\text{ m/s})}{(850\text{ nm})(10^{-8}\text{ s})} = \boxed{117\text{ mW}}$$

Remarks: Note that the pulse length has no bearing on the solution.

85 ••

Picture the Problem We can find the rate at which energy is delivered to the atom using the definitions of power and intensity. We can also use the definition of power to determine how much time is required for an amount of energy equal to the work function to fall on one atom.

(*a*) Relate the energy per second (power) falling on an atom to the intensity of the incident radiation:

$$P = \frac{\Delta E}{\Delta t} = IA$$

Substitute numerical values and evaluate *P*:

$$P = (0.01\,\text{W/m}^2)(0.01\times10^{-18}\,\text{m}^2)$$
$$= 10^{-22}\,\frac{\text{J}}{\text{s}}\times\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}$$
$$= \boxed{6.25\times10^{-4}\,\text{eV/s}}$$

(*b*) Classically:

$$\Delta t = \frac{\Delta E}{P} = \frac{\phi}{P}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{2\,\text{eV}}{6.25\times10^{-4}\,\text{eV/s}} = 3200\,\text{s}$$
$$= \boxed{53.3\,\text{min}}$$