Chapter 38
Solids and the Theory of Conduction

Conceptual Problems

1 •

Determine the Concept The energy lost by the electrons in collision with the ions of the crystal lattice appears as Joule heat (I^2R).

***2** •

Determine the Concept The resistivity of brass at 4 K is almost entirely due to the ″residual resistance, ″ the resistance due to impurities and other imperfections of the crystal lattice. In brass, the zinc ions act as impurities in copper. In pure copper, the resistivity at 4 K is due to its residual resistance, which is very low if the copper is very pure.

3 •

Picture the Problem The contact potential is given by $V_{\text{contact}} = \dfrac{\phi_1 - \phi_2}{e}$, where ϕ_1 and ϕ_2 are the work functions of the two different metals in contact with each other.

(*a*) Express the contact potential in terms of the work functions of the metals:

$$V_{\text{contact}} = \frac{\phi_1 - \phi_2}{e}$$

$$\boxed{\text{Examining Table 38-2, we see that the greatest difference between the work functions will occur when potassium and nickel are joined.}}$$

(*b*) Substitute numerical values and evaluate V_{contact}:

$$V_{\text{contact}} = \frac{(5.2\,\text{eV} - 2.1\,\text{eV})(1.60\times10^{-19}\ \text{J/eV})}{1.60\times10^{-19}\ \text{C}} = \boxed{3.10\,\text{V}}$$

4 •

Picture the Problem The contact potential is given by $V_{\text{contact}} = \dfrac{\phi_1 - \phi_2}{e}$, where ϕ_1 and ϕ_2 are the work functions of the two different metals in contact with each other.

(*a*) Express the contact potential in terms of the work functions of the metals:

$$V_{\text{contact}} = \frac{\phi_1 - \phi_2}{e}$$

$\boxed{\text{Examining Table 38 - 2, we see that the least difference between the work functions will occur when silver and gold are joined.}}$

(*b*) Substitute numerical values and evaluate V_{contact}:

$$V_{\text{contact}} = \frac{(4.8\,\text{eV} - 4.7\,\text{eV})(1.60\times10^{-19}\ \text{J/eV})}{1.60\times10^{-19}\ \text{C}} = \boxed{0.100\,\text{V}}$$

5 •

Determine the Concept If the valence band is only partially full, there are many available empty energy states in the band, and the electrons in the band can easily be raised to a higher energy state by an electric field. $\boxed{(c)\ \text{is correct.}}$

6 •

Determine the Concept Insulators are poor conductors of electricity because there is a large energy gap between the full valence band and the next higher band where electrons can exist. $\boxed{(b)\ \text{is correct.}}$

7 •

(*a*) True

(*b*) False. The classical free-electron theory predicts heat capacities for metals that are not observed experimentally.

(*c*) True

(*d*) False. The Fermi energy is the energy of the last filled (or half-filled) level at $T = 0$.

(*e*) True

(*f*) True

(*g*) False. Because semiconductors conduct current by electrons and holes, their conduction is in both directions.

***8** •

Determine the Concept The resistivity of copper increases with increasing temperature; the resistivity of (pure) silicon decreases with increasing temperature because the number of charge carriers increases.

9 •

Determine the Concept Because a gallium atom can accept electrons from the valence band of germanium to complete its four covalent bonds, $\boxed{(b)\ \text{is correct.}}$

10 •

Determine the Concept Because phosphorus has 3 electrons that it can donate to the conduction band of germanium without leaving holes in the valence band, $\boxed{(d)\text{ is correct.}}$

11 •

Determine the Concept The excited electron is in the conduction band and can conduct electricity. A hole is left in the valence band allowing the positive hole to move through the band also contributing to the current.

12 •

Determine the Concept

(a) Phosphorus and antimony will make n-type semiconductors since each has one more valence electron than silicon.

(b) Boron and thallium will make p-type semiconductors since each has one less valence electron than silicon.

13 •

Determine the Concept Because a *pn* junction solar cell has donor impurities on one side and acceptor impurities on the other, both electrons and holes are created. $\boxed{(c)\text{ is correct.}}$

Estimation and Approximation

14 •

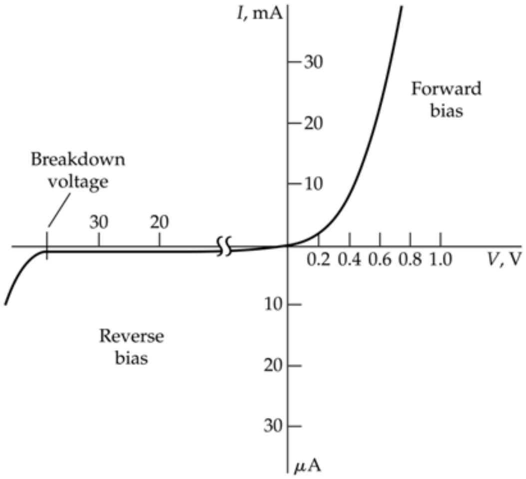
Picture the Problem We can use the list of tables on the inside back covers of volumes 1 and 2 to find tables of material properties. A representative sample is included in the following table in which all the units are SI:

Table	Material property	Largest value	Smallest value	Ratio (order of magnitude)
13-1	Mass density	22.5×10^{3} (Osmium)	0.08994 (Hydrogen)	10^{5}
20-3	Thermal conductivity	429 (Ag)	0.026 (air)	10^{4}
20-1	Thermal expansion	51×10^{-6} (ice)	10^{-6} (invar)	10^{2}
12-8	Tensile strength	520 (steel)	2 (concrete)	10^{2}

12-8	Young’s modulus	200 (steel)	9 (bone)	10
18-1	Heat capacity	4.18 (water)	0.900 (Al)	1

15 •

Picture the Problem Figure 38-21 is reproduced below. We can draw tangent lines at each of the voltages and estimate the slope. The differential resistance is the reciprocal of the slope.



V (V)	1/slope (Ω)
–20	$\boxed{\infty}$
+0.2	$\boxed{40}$
+0.4	$\boxed{20}$
+0.6	$\boxed{10}$
+0.8	$\boxed{5}$

Remarks: Note that, because of the difficulty in determining the slopes, these results are only approximations.

The Structure of Solids

16 •

Picture the Problem We can use the geometry of the ion to relate the volume per mole to the length of its side r_0 and the definition of density to express the volume per mole in terms of its molar mass and density.

Because the cube length/ion is r_0, the volume/mole is given by:

$$V_{\text{mol}} = 2N_{\text{A}}r_0^3$$

Solve for r_0:

$$r_0 = \sqrt[3]{\frac{V_{\text{mol}}}{2N_{\text{A}}}}$$

The volume/mole is given by:

$$V_{\text{mol}} = \frac{M}{\rho}$$

where M is the molar mass of KCl.

Substitute for V_{mol} in the expression for r_0:

$$r_0 = \sqrt[3]{\frac{M}{2\rho N_{\text{A}}}}$$

Substitute numerical values and evaluate r_0:

$$r_0 = \sqrt[3]{\frac{74.55\,\text{g/mol}}{2\left(1.984\,\text{g/cm}^3\right)\left(6.02\times10^{23}\text{ particles/mol}\right)}} = \boxed{0.315\,\text{nm}}$$

17 •

Picture the Problem We can use the definition of density and the geometry of the ions to compute the density of LiCl.

The density of LiCl is given by:

$$\rho = \frac{M_{\text{unit cell}}}{V_{\text{unit cell}}}$$

Express the volume of the unit cell:

$$V_{\text{unit cell}} = \left(2r_0\right)^3$$

Because the unit cell has four molecules, its mass is given by:

$$M_{\text{unit cell}} = \frac{4M}{N_{\text{A}}}$$

Substitute for $V_{\text{unit cell}}$ and $M_{\text{unit cell}}$ to obtain:

$$\rho = \frac{\dfrac{4M}{N_{\text{A}}}}{\left(2r_0\right)^3} = \frac{4M}{N_{\text{A}}\left(2r_0\right)^3}$$

Substitute numerical values and evaluate ρ:

$$\rho = \frac{4(42.4\,\text{g/mol})}{(6.02\times10^{23}\text{ particles/mol})\left[2(0.257\times10^{-9}\text{ m})\right]^3} = 2.07\times10^6\text{ g/m}^3\times\left(\frac{1\text{m}}{10^2\text{ cm}}\right)^3$$

$$= \boxed{2.07\,\text{g/cm}^3}$$

***18** •

Picture the Problem We can solve Equation 38-6 for *n*.

Equation 38-6 is:

$$U(r_0) = -\alpha\frac{ke^2}{r_0}\left(1-\frac{1}{n}\right)$$

and

$$\left|U(r_0)\right| = \alpha\frac{ke^2}{r_0}\left(1-\frac{1}{n}\right)$$

Solve for *n* to obtain:

$$n = \frac{1}{1-\dfrac{\left|U(r_0)\right|r_0}{\alpha ke^2}}$$

Substitute numerical values and evaluate *n*:

$$n = \frac{1}{1-\dfrac{(741\,\text{kJ/mol})\left(\dfrac{1\text{eV/ion pair}}{96.47\,\text{kJ/mol}}\right)(0.257\,\text{nm})}{(1.7476)(1.44\,\text{eV}\cdot\text{nm})}} = \boxed{4.64}$$

19 ••

Picture the Problem We can substitute numerical values in Equation 38-6 to evaluate $U(r_o)$ for $n = 8$ and $n = 10$.

(*a*) Equation 38-6 is:

$$U(r_0) = -\alpha\frac{ke^2}{r_0}\left(1-\frac{1}{n}\right)$$

Substitute numerical values and evaluate $U(r_0)$:

$$U(r_o) = -\frac{(1.7476)(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\,\text{J})^2}{0.208\times10^{-9}\,\text{m}}\left(1-\frac{1}{8}\right)\left(\frac{1\text{eV}}{1.60\times10^{-19}\,\text{J}}\right)$$

$$= \boxed{-10.6\,\text{eV}}$$

(*b*) The fractional change is given by:

$$\frac{\Delta U(r_0)}{U(r_0)} = \frac{U_{n=10}-U_{n=8}}{U_{n=8}} = \frac{U_{n=10}}{U_{n=8}} - 1$$

Substitute numerical values and evaluate $U(r_0)$ for $n = 10$:

$$U_{n=10}(r_\mathrm{o}) = -\frac{(1.7476)\left(8.99\times10^{9}\,\mathrm{N\cdot m^2/C^2}\right)\left(1.60\times10^{-19}\,\mathrm{J}\right)^2}{0.208\times10^{-9}\,\mathrm{m}}\left(1-\frac{1}{10}\right)\left(\frac{1\,\mathrm{eV}}{1.60\times10^{-19}\,\mathrm{J}}\right)$$
$$= -10.9\,\mathrm{eV}$$

Substitute numerical values and evaluate the fractional change in $U(r_0)$:

$$\frac{\Delta U(r_0)}{U(r_0)} = \frac{-10.9\,\mathrm{eV}}{-10.6\,\mathrm{eV}} - 1 = \boxed{-2.83\%}$$

A Microscopic Picture of Conduction

20 •

Picture the Problem We can use the expression for the volume occupied by one electron to show that $r_\mathrm{s} = \left(\frac{3}{4\pi n}\right)^{1/3}$.

(*a*) The volume occupied by one electron is:

$$\frac{1}{n} = \frac{4}{3}\pi r_\mathrm{s}^3$$

Solve for r_s:

$$r_\mathrm{s} = \boxed{\left(\frac{3}{4\pi n}\right)^{1/3}}$$

(*b*) From Table 38-1:

$$n_\mathrm{Cu} = 8.47\times10^{28}\,\mathrm{m^{-3}}$$

Substitute numerical values and evaluate r_s for copper:

$$r_\mathrm{s} = \sqrt[3]{\frac{3}{4\pi\left(8.47\times10^{28}\,\mathrm{m^{-3}}\right)}} = \boxed{0.141\,\mathrm{nm}}$$

21 •

Picture the Problem We can use the expression for the resistivity of the copper in terms of v_av and λ to find the classical value for the resistivity ρ of copper. In (*b*) we can use $v_\mathrm{av} = \sqrt{\frac{3kT}{m_\mathrm{e}}}$ to relate the average speed to the temperature.

(*a*) In terms of the mean free path and the mean speed, the resistivity is:

$$\rho = \frac{m_\mathrm{e} v_\mathrm{av}}{n_\mathrm{e} e^2 \lambda}$$

Substitute numerical values and evaluate ρ (see Table 38-1 for the free-electron number density of copper):

$$\rho = \frac{\left(9.11\times10^{-31}\text{ kg}\right)\left(1.17\times10^{5}\text{ m/s}\right)}{\left(8.47\times10^{28}\text{ m}^{-3}\right)\left(1.60\times10^{-19}\text{ C}\right)^{2}\left(0.4\,\text{nm}\right)} = \boxed{0.123\,\mu\Omega\cdot\text{m}}$$

(*b*) Relate the average speed of the electrons to the temperature:

$$v_{\text{av}} = \sqrt{\frac{3kT}{m_{\text{e}}}}$$

Substitute for v_{av} in the expression for ρ to obtain:

$$\rho = \frac{m_{\text{e}}}{n_{\text{e}}e^{2}\lambda}\sqrt{\frac{3kT}{m_{\text{e}}}} = \frac{1}{n_{\text{e}}e^{2}\lambda}\sqrt{3m_{\text{e}}kT}$$

Substitute numerical values and evaluate ρ:

$$\rho = \frac{\sqrt{3\left(9.11\times10^{-31}\text{ kg}\right)\left(1.38\times10^{-23}\text{ J/K}\right)\left(100\,\text{K}\right)}}{\left(8.47\times10^{28}\text{ m}^{-3}\right)\left(1.60\times10^{-19}\text{ C}\right)^{2}\left(0.4\,\text{nm}\right)} = \boxed{0.0708\,\mu\Omega\cdot\text{m}}$$

***22** ••

Picture the Problem We can use Equation 38-14 to estimate the resistivity of silicon.

(*a*) From Equation 38-14:

$$\rho = \frac{m_{\text{e}}v_{\text{av}}}{n_{\text{e}}e^{2}\lambda} \qquad (1)$$

The speed of the electrons is given by:

$$v_{\text{av}} = v_{\text{F}} = \sqrt{\frac{2E_{\text{F}}}{m_{\text{e}}}}$$

Substitute numerical values and evaluate v_{av}:

$$v_{\text{av}} = \sqrt{\frac{2\left(4.88\,\text{eV}\right)}{\left(9.11\times10^{-31}\text{kg}\right)}\,\frac{1.60\times10^{-19}\text{J}}{1\,\text{eV}}}$$
$$= 1.31\times10^{6}\,\text{m/s}$$

The electron density of Si is given by:

$$n_{\text{e}} = MN_{\text{A}}N_{\text{atom}}$$

where N_{atom} is the number of electrons per atom.

Substitute numerical values and evaluate n_{e}:

$$n_{\text{e}} = \left(2.41\times10^{3}\,\frac{\text{kg}}{\text{m}^{3}}\right)\left(\frac{6.02\times10^{23}\text{atoms}}{0.02809\,\text{kg}}\right)\left(\frac{2\text{ e}}{\text{atom}}\right) = 1.03\times10^{29}\,\text{e/m}^{3}$$

Substitute numerical values in equation (1) and evaluate ρ:

$$\rho=\frac{(9.11\times10^{-31}\,\text{kg})(1.31\times10^{6}\,\text{m/s})}{(1.60\times10^{-19}\,\text{C})^{2}(1.03\times10^{29}\,\text{e/m}^{3})(27.0\times10^{-9}\,\text{m})}=\boxed{1.66\times10^{-8}\,\Omega\cdot\text{m}}$$

(*b*) The accepted resistivity of 640 Ω·m is much greater than the calculated value. We assume that valence electrons will produce conduction in the material. Silicon is a semiconductor and a gap between the valence band and conduction band exists. Only electrons with sufficient energies will be found in the conduction band.

The Fermi Electron Gas

23 •

Picture the Problem The number density of free electrons is given by $n=\rho N_{\text{A}}/M$, where N_{A} is Avogadro's number, ρ is the density of the element, and M is its molar mass.

Relate the number density of free electrons to the density ρ and molar mass M of the element:

$$\frac{n}{N_{\text{A}}}=\frac{\rho}{M}\Rightarrow n=\frac{\rho N_{\text{A}}}{M}$$

(*a*) For Ag:

$$n_{\text{Ag}}=\frac{(10.5\,\text{g/cm}^{3})(6.02\times10^{23}\,\text{electrons/mol})}{107.87\,\text{g/mol}}=\boxed{5.86\times10^{22}\,\text{electrons/cm}^{3}}$$

(*b*) For Au:

$$n_{\text{Ag}}=\frac{(19.3\,\text{g/cm}^{3})(6.02\times10^{23}\,\text{electrons/mol})}{196.97\,\text{g/mol}}=\boxed{5.90\times10^{22}\,\text{electrons/cm}^{3}}$$

$$\boxed{\text{Both these results agree with the values in Table 38-1.}}$$

24 •

Picture the Problem The number of free electrons per atom n_{e} is given by $n_{\text{e}}=nM/\rho N_{\text{A}}$, where N_{A} is Avogadro's number, ρ is the density of the element, M is its molar mass, and n is the free electron number density for the element.

The number of free electrons per atom is given by:

$$n_{\text{e}}=\frac{nM}{\rho N_{\text{A}}}$$

Substitute numerical values and evaluate n_{e}:

$$n_e = \frac{\left(18.1\times10^{22}\ \text{electrons/cm}^3\right)\left(26.98\ \text{g/mol}\right)}{\left(2.7\ \text{g/cm}^3\right)\left(6.02\times10^{23}\ \text{electrons/mol}\right)} = \boxed{3.00}$$

25 •

Picture the Problem The number of free electrons per atom n_e is given by $n_e = nM/\rho N_A$,where N_A is Avogadro's number, ρ is the density of the element, M is its molar mass, and n is the free electron number density for the element.

The number of free electrons per atom is given by:

$$n_e = \frac{nM}{\rho N_A}$$

Substitute numerical values and evaluate n_e:

$$n_e = \frac{\left(14.8\times10^{22}\ \text{electrons/cm}^3\right)\left(118.69\ \text{g/mol}\right)}{\left(7.3\ \text{g/cm}^3\right)\left(6.022\times10^{23}\ \text{electrons/mol}\right)} = \boxed{4.00}$$

***26** •

Picture the Problem The Fermi temperature T_F is defined by $kT_F = E_F$, where E_F is the Fermi energy.

The Fermi temperature is given by:

$$T_F = \frac{E_F}{k}$$

(*a*) For Al:

$$T_F = \frac{11.7\ \text{eV}}{8.62\times10^{-5}\ \text{eV/K}} = \boxed{1.36\times10^5\ \text{K}}$$

(*b*) For K:

$$T_F = \frac{2.11\text{eV}}{8.62\times10^{-5}\ \text{eV/K}} = \boxed{2.45\times10^4\ \text{K}}$$

(*c*) For Sn:

$$T_F = \frac{10.2\ \text{eV}}{8.62\times10^{-5}\ \text{eV/K}} = \boxed{1.18\times10^5\ \text{K}}$$

27 •

Picture the Problem We can solve the expression for the Fermi energy for the speed of a conduction electron.

Express the Fermi energy in terms of the Fermi speed a conduction electron:

$$E_F = \tfrac{1}{2} m_e u_F^2$$

Solve for u_F:

$$u_F = \sqrt{\frac{2E_F}{m_e}}$$

(*a*) Substitute numerical values (see Table 38-1 for E_F) and evaluate u_F for Na:

$$u_F = \sqrt{\frac{2(3.24\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}{9.11\times10^{-31}\,\text{kg}}}$$

$$= \boxed{1.07\times10^{6}\,\text{m/s}}$$

(*b*) Substitute numerical values and evaluate u_F for Au:

$$u_F = \sqrt{\frac{2(5.53\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}{9.11\times10^{-31}\,\text{kg}}}$$

$$= \boxed{1.39\times10^{6}\,\text{m/s}}$$

(*c*) Substitute numerical values and evaluate u_F for Sn:

$$u_F = \sqrt{\frac{2(10.2\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}{9.11\times10^{-31}\,\text{kg}}}$$

$$= \boxed{1.89\times10^{6}\,\text{m/s}}$$

28 •

Picture the Problem The Fermi energy at $T = 0$ depends on the number of electrons per unit volume (the number density) according to $E_F = (0.365\,\text{eV}\cdot\text{nm}^2)(N/V)^{2/3}$.

The Fermi energy at $T = 0$ is given by:

$$E_F = (0.365\,\text{eV}\cdot\text{nm}^2)\left(\frac{N}{V}\right)^{2/3}$$

(*a*) For Al, $N/V = 18.1\times10^{22}$ electrons/cm^3 (see Table 38-1) and:

$$E_F = (0.365\,\text{eV}\cdot\text{nm}^2)(18.1\times10^{22}\,\text{electrons/cm}^3)^{2/3} = \boxed{11.7\,\text{eV}}$$

(*b*) For K, $N/V = 1.4\times10^{22}$ electrons/cm^3 and:

$$E_F = (0.365\,\text{eV}\cdot\text{nm}^2)(1.4\times10^{22}\,\text{electrons/cm}^3)^{2/3} = \boxed{2.12\,\text{eV}}$$

(*c*) For Sn, $N/V = 14.8\times10^{22}$ electrons/cm^3 and:

$$E_F = (0.365\,\text{eV}\cdot\text{nm}^2)(14.8\times10^{22}\,\text{electrons/cm}^3)^{2/3} = \boxed{10.2\,\text{eV}}$$

29 •

Picture the Problem The average energy of electrons in a Fermi gas at $T = 0$ is three-fifths of the Fermi energy.

The average energy at $T = 0$ is given by:

$$E_{\text{av}} = \tfrac{3}{5}E_{\text{F}}$$

(*a*) For copper, $E_{\text{F}} = 7.04$ eV (see Table 38-1) and:

$$E_{\text{av}} = (0.6)(7.04\,\text{eV}) = \boxed{4.22\,\text{eV}}$$

(*b*) For lithium, $E_{\text{F}} = 4.75$ eV and:

$$E_{\text{av}} = (0.6)(4.75\,\text{eV}) = \boxed{2.85\,\text{eV}}$$

30 •

Picture the Problem The Fermi energy at $T = 0$ is given by $E_{\text{F}} = (0.365\,\text{eV}\cdot\text{nm}^2)(N/V)^{3/2}$, where N/V is the free-electron number density and the Fermi temperature is related to the Fermi energy according to $kT_{\text{F}} = E_{\text{F}}$.

(*a*) The Fermi temperature for iron is given by:

$$T_{\text{F}} = \frac{E_{\text{F}}}{k}$$

Substitute numerical values (see Table 38-1) and evaluate T_{F}:

$$T_{\text{F}} = \frac{11.2\,\text{eV}}{8.62\times10^{-5}\,\text{eV/K}} = \boxed{1.30\times10^{5}\,\text{K}}$$

(*b*) The Fermi energy at $T = 0$ is given by:

$$E_{\text{F}} = (0.365\,\text{eV}\cdot\text{nm}^2)\left(\frac{N}{V}\right)^{3/2}$$

Substitute numerical values (see Table 38-1) and evaluate E_{F}:

$$E_{\text{F}} = (0.365\,\text{eV}\cdot\text{nm}^2)(17.0\times10^{22}\,\text{electrons/cm}^3)^{2/3} = \boxed{11.2\,\text{eV}}$$

***31** ••

Picture the Problem We can use $n_{\text{e}} = \rho V = \dfrac{\rho N_{\text{A}} N_{\text{atom}}}{m}$, where N_{atom} is the number of electrons per atom, to calculate the electron density of gold. The Fermi energy is given by $E_{\text{F}} = \tfrac{1}{2}m_{\text{e}}v_{\text{F}}^2$.

(*a*) The electron density of gold is given by:

$$n_{\text{e}} = \rho V = \frac{\rho N_{\text{A}} N_{\text{atom}}}{m}$$

Substitute numerical values and evaluate n_{e}:

$$n_e = \frac{\left(19.3\times10^3\,\frac{\text{kg}}{\text{m}^3}\right)\left(6.02\times10^{23}\,\text{atoms}\right)\left(\frac{1\,\text{e}}{1\,\text{atom}}\right)}{0.197\,\text{kg}} = \boxed{5.90\times10^{28}\ \text{e/m}^3}$$

(*b*) The Fermi energy is given by:

$$E_\text{F} = \tfrac{1}{2} m_e v_\text{F}^2$$

Substitute numerical values and evaluate E_F:

$$E_\text{F} = \frac{1}{2}\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.39\times10^6\,\text{m/s}\right)^2\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right) = \boxed{5.50\,\text{eV}}$$

(*c*) The factor by which the Fermi energy is higher than the *kT* energy at room temperature is:

$$f = \frac{E_\text{F}}{kT}$$

At room temperature kT = 0.026 eV. Substitute numerical values and evaluate *f*:

$$f = \frac{5.50\,\text{eV}}{0.026\,\text{eV}} = \boxed{212}$$

(*d*) E_F is 212 times *kT* at room temperature. There are so many free electrons present that most of them are crowded, as described by the Pauli exclusion principle, up to energies far higher than they would be according to the classical model.

***32** ••

Picture the Problem We can solve $PV = \tfrac{2}{3} NE_\text{av}$ for *P* and substitute for E_av in order to express *P* in terms of *N*/*V* and E_F.

Solve $PV = \tfrac{2}{3} NE_\text{av}$ for *P*:

$$P = \frac{2}{3}\left(\frac{N}{V}\right)E_\text{av}$$

Because $E_\text{av} = \tfrac{3}{5} E_\text{F}$:

$$P = \frac{2}{3}\left(\frac{N}{V}\right)\left(\frac{3}{5}E_\text{F}\right) = \frac{2}{5}\left(\frac{N}{V}\right)E_\text{F}$$

Substitute numerical values (see Table 38-1) and evaluate *P*:

$$\begin{aligned} P &= \frac{2}{5}\left(8.47\times10^{22}\ \text{electrons/cm}^3\right)\left(7.04\,\text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right) \\ &= \boxed{3.82\times10^{10}\ \text{N/m}^2} = 3.82\times10^{10}\ \text{N/m}^2 \times \frac{1\,\text{atm}}{101.325\times10^3\ \text{N/m}^2} \\ &= \boxed{3.77\times10^5\,\text{atm}} \end{aligned}$$

33 ••

Picture the Problem We can follow the procedure given in the problem statement to show that $P = \frac{2NE_\mathrm{F}}{5V} = CV^{-5/3}$ and $B = \frac{5}{3}P = \frac{2NE_\mathrm{F}}{3V}$.

(*a*) From Problem 32 we have:

$$PV = \tfrac{2}{3}NE_\mathrm{av}$$

Because $E_\mathrm{av} = \tfrac{3}{5}E_\mathrm{F}$:

$$P = \frac{2}{3}\left(\frac{N}{V}\right)\left(\frac{3}{5}E_\mathrm{F}\right) = \frac{2}{5}\left(\frac{N}{V}\right)E_\mathrm{F} \qquad (1)$$

The Fermi energy is given by:

$$E_\mathrm{F} = \frac{h^2}{8m_\mathrm{e}}\left(\frac{3N}{\pi V}\right)^{2/3} = \frac{h^2}{8m_\mathrm{e}}\left(\frac{3N}{\pi}\right)^{2/3}V^{-2/3}$$

Substitute to obtain:

$$P = \frac{2}{5}\left(\frac{N}{V}\right)\frac{h^2}{8m_\mathrm{e}}\left(\frac{3N}{\pi}\right)^{2/3}V^{-2/3}$$

$$= \frac{N^{5/3}h^2}{20m_\mathrm{e}}\left(\frac{3}{\pi}\right)^{2/3}V^{-5/3} = \boxed{CV^{-5/3}}$$

where $C = \frac{N^{5/3}h^2}{20m_\mathrm{e}}\left(\frac{3}{\pi}\right)^{2/3}$ is a constant.

(*b*) The bulk modulus is given by:

$$B = -V\frac{dP}{dV} = -V\frac{d}{dV}\left[CV^{-5/3}\right]$$

$$= -CV\left(-\frac{5}{3}V^{-8/3}\right) = \frac{5}{3}CV^{-5/3}$$

$$= \frac{5}{3}P$$

Substitute for *P* from equation (1) to obtain:

$$B = \frac{5}{3}\left[\frac{2}{5}\left(\frac{N}{V}\right)E_\mathrm{F}\right] = \boxed{\frac{2}{3}\left(\frac{N}{V}\right)E_\mathrm{F}}$$

(*c*) Substitute numerical values and evaluate *B* for copper:

$$B = \frac{2}{3}\left(8.47\times10^{22}\ \text{electrons/cm}^3\right)\left(7.04\,\text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right) = \boxed{63.6\times10^{9}\ \text{N/m}^2}$$

From Table 13-2:

$$B_\mathrm{Cu} = 140\ \text{GN/m}^2$$

Divide the calculated value for B by the value from Table 13-2 to obtain:

$$\frac{B}{B_{\text{Cu}}}=\frac{63.6\,\text{GN/m}^2}{140\,\text{GN/m}^2}=0.455$$

or

$$B=\boxed{0.454B_{\text{Cu}}}$$

34 •

Picture the Problem The contact potential is given by $V_{\text{contact}}=\dfrac{\phi_1-\phi_2}{e}$, where ϕ_1 and ϕ_2 are the work functions of the two different metals in contact with each other.

The contact potential is given by:

$$V_{\text{contact}}=\frac{\phi_1-\phi_2}{e}$$

(a) For Ag and Cu (see Table 38-1):

$$V_{\text{contact}}=\frac{(4.7\,\text{eV}-4.1\text{eV})(1.60\times10^{-19}\text{ J/eV})}{1.60\times10^{-19}\text{ C}}=\boxed{0.6\,\text{V}}$$

(b) For Ag and Ni:

$$V_{\text{contact}}=\frac{(5.2\,\text{eV}-4.7\,\text{eV})(1.60\times10^{-19}\text{ J/eV})}{1.60\times10^{-19}\text{ C}}=\boxed{0.5\,\text{V}}$$

(c) For Ca and Cu (see Table 38-1):

$$V_{\text{contact}}=\frac{(4.1\text{eV}-3.2\,\text{eV})(1.60\times10^{-19}\text{ J/eV})}{1.60\times10^{-19}\text{ C}}=\boxed{0.9\,\text{V}}$$

Heat Capacity Due to Electrons in a Metal

***35** ••

Picture the Problem We can use Equation 38-29 to find the molar specific heat of gold at constant volume and room temperature.

The molar specific heat is given by Equation 38-29:

$$c'_{\text{V}}=\frac{\pi^2RT}{2T_{\text{F}}}$$

The Fermi energy is given by:

$$E_{\text{F}}=kT_{\text{F}}\Rightarrow T_{\text{F}}=\frac{E_{\text{F}}}{k}$$

Substitute for T_F to obtain:

$$c'_V = \frac{\pi^2 RkT}{2E_F}$$

Substitute numerical values and evaluate c'_V :

$$c'_V = \frac{\pi^2 (8.31\,\text{J/mol K})(1.38\times10^{-23}\,\text{J/mol})\left(\dfrac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right)(300\,\text{K})}{2(5.53\,\text{eV})}$$

$$= \boxed{0.192\,\text{J/mol}\cdot\text{K}}$$

Remarks: The value 0.192 J/mol K is for a mole of gold atoms. Since each gold atom contributes one electron to the metal, a mole of gold corresponds to a mole of electrons.

Quantum Theory of Electrical Conduction

36 •

Picture the Problem We can solve the equation giving the resistivity of a conductor in terms of the mean free path and the mean speed for the mean free path and use the Fermi speeds from Problem 27 as the mean speeds.

In terms of the mean free path and the mean speed, the resistivity is:

$$\rho = \frac{m_e v_{av}}{ne^2\lambda}$$

Solve for λ to obtain:

$$\lambda = \frac{m_e v_{av}}{ne^2\rho}$$

From Problem 27 we have:

$$\mu_{F,Na} = 1.07\times10^6\ \text{m/s}$$

$$\mu_{F,Au} = 1.39\times10^6\ \text{m/s}$$

and

$$\mu_{F,Sn} = 1.89\times10^6\ \text{m/s}$$

Using the Fermi speeds as the average speeds, substitute numerical values and evaluate the mean free path of Na:

$$\lambda_{Na} = \frac{(9.11\times10^{-31}\,\text{kg})(1.07\times10^6\,\text{m/s})}{(2.65\times10^{22}\,\text{electrons/cm}^3)(1.60\times10^{-19}\,\text{C})^2(4.2\,\mu\Omega\cdot\text{cm})} = \boxed{34.2\,\text{nm}}$$

Proceed as above for Au:

$$\lambda_{\text{Au}} = \frac{\left(9.11\times10^{-31}\text{ kg}\right)\left(1.39\times10^{6}\text{ m/s}\right)}{\left(5.90\times10^{22}\text{ electrons/cm}^3\right)\left(1.60\times10^{-19}\text{ C}\right)^2\left(2.04\,\mu\Omega\cdot\text{cm}\right)} = \boxed{41.1\text{nm}}$$

Proceed as above for Sn:

$$\lambda_{\text{Sn}} = \frac{\left(9.11\times10^{-31}\text{ kg}\right)\left(1.89\times10^{6}\text{ m/s}\right)}{\left(14.8\times10^{22}\text{ electrons/cm}^3\right)\left(1.60\times10^{-19}\text{ C}\right)^2\left(10.6\,\mu\Omega\cdot\text{cm}\right)} = \boxed{4.29\text{ nm}}$$

***37** ••

Picture the Problem We can solve the resistivity equation for the mean free path and then substitute the Fermi speed for the average speed to express the mean free path as a function of the Fermi energy.

(*a*) In terms of the mean free path and the mean speed, the resistivity is:

$$\rho_{\text{i}} = \frac{m_{\text{e}}v_{\text{av}}}{ne^2\lambda_{\text{i}}} = \frac{m_{\text{e}}u_{\text{F}}}{ne^2\lambda_{\text{i}}}$$

Solve for λ to obtain:

$$\lambda_{\text{i}} = \frac{m_{\text{e}}u_{\text{F}}}{ne^2\rho_{\text{i}}}$$

Express the Fermi speed u_{F} in terms of the Fermi energy E_{F}:

$$u_{\text{F}} = \sqrt{\frac{2E_{\text{F}}}{m_{\text{e}}}}$$

Substitute to obtain:

$$\lambda_{\text{i}} = \frac{\sqrt{2m_{\text{e}}E_{\text{F}}}}{ne^2\rho_{\text{i}}}$$

Substitute numerical values (see Table 38-1) and evaluate λ_{i}:

$$\lambda_{\text{i}} = \frac{\sqrt{2\left(9.11\times10^{-31}\text{ kg}\right)\left(7.04\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)}}{\left(8.47\times10^{28}\text{ electrons/m}^3\right)\left(1.60\times10^{-19}\text{ C}\right)^2\left(10^{-8}\ \Omega\cdot\text{m}\right)} = \boxed{66.1\text{nm}}$$

(*b*) From Equation 38-16 we have:

$$\lambda = \frac{1}{n\pi r^2}$$

Solve for πr^2:

$$\pi r^2 = \frac{1}{n\lambda}$$

Substitute numerical values and evaluate πr^2:

$$\pi r^2 = \frac{1}{\left(8.47\times10^{28}\ \text{m}^{-3}\right)(66.1\,\text{nm})}$$

$$= 1.79\times10^{-22}\ \text{m}^2 = \boxed{1.79\times10^{-4}\ \text{nm}^2}$$

Band Theory of Solids

38 •

Picture the Problem We can use $E_g = hc/\lambda$ to calculate the energy gap for this semiconductor.

The lowest photon energy to increase conductivity is given by:

$$E_g = \frac{hc}{\lambda}$$

Substitute numerical values and evaluate E_g:

$$E_g = \frac{\left(6.63\times10^{-34}\,\text{J}\cdot\text{s}\right)\left(3\times10^{8}\,\text{m/s}\right)}{\left(380\times10^{-9}\,\text{m}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}$$

$$= \boxed{3.27\,\text{eV}}$$

***39** •

Picture the Problem We can relate the maximum photon wavelength to the energy gag using $\Delta E = hf = hc/\lambda$.

Express the energy gap as a function of the wavelength of the photon:

$$\Delta E = hf = \frac{hc}{\lambda}$$

Solve for λ:

$$\lambda = \frac{hc}{\Delta E}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{1.14\,\text{eV}} = \boxed{1.09\,\mu\text{m}}$$

40 •

Picture the Problem We can relate the maximum photon wavelength to the energy gag using $\Delta E = hf = hc/\lambda$.

Express the energy gap as a function of the wavelength of the photon:

$$\Delta E = hf = \frac{hc}{\lambda}$$

Solve for λ:

$$\lambda = \frac{hc}{\Delta E}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{0.74\,\text{eV}} = \boxed{1.68\,\mu\text{m}}$$

41 •

Picture the Problem We can relate the maximum photon wavelength to the energy gag using $\Delta E = hf = hc/\lambda$.

Express the energy gap as a function of the wavelength of the photon:

$$\Delta E = hf = \frac{hc}{\lambda}$$

Solve for λ:

$$\lambda = \frac{hc}{\Delta E}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{7.0\,\text{eV}} = \boxed{177\,\text{nm}}$$

42 ••

Picture the Problem We can use $\Delta E = hf = hc/\lambda$ to find the energy gap between these bands and $T = E_g/k$ to find the temperature for which kT equals this energy gap.

(*a*) Express the energy gap as a function of the wavelength of the photon:

$$\Delta E = \frac{hc}{\lambda}$$

Substitute numerical values and evaluate ΔE:

$$\Delta E = \frac{1240\,\text{eV}\cdot\text{nm}}{3.35\,\mu\text{m}} = \boxed{0.370\,\text{eV}}$$

(*b*) The temperature is related to the energy gap E_g according to:

$$T = \frac{E_g}{k}$$

Substitute numerical values and evaluate T:

$$T = \frac{0.370\,\text{eV}}{8.617\times10^{-5}\,\text{eV/K}} = \boxed{4.29\times10^{3}\,\text{K}}$$

Semiconductors

43 •

Picture the Problem We can use $\Delta E = kT$ to find the temperature for which $kT = 0.01$ eV

Express the temperature T in terms of the energy gap ΔE:

$$T = \frac{\Delta E}{k}$$

Substitute numerical values and evaluate T:

$$T=\frac{0.01\,\text{eV}}{1.38\times10^{-23}\ \text{J/K}\times\dfrac{1\,\text{eV}}{1.60\times10^{-19}\ \text{J}}}=\boxed{116\,\text{K}}$$

***44 ••**

Picture the Problem We can use $E=hf$ to find the energy gap of this semiconductor.

The energy gap of the semiconductor is given by:

$$E_g=hf=\frac{hc}{\lambda}$$

where

$hc = 1240$ eV·nm

Substitute numerical values and evaluate E_g:

$$E_g=\frac{1240\,\text{eV}\cdot\text{nm}}{1.85\,\mu\text{m}}=\boxed{0.670\,\text{eV}}$$

45 ••

Picture the Problem We can make the indicated substitutions in the expression for a_0 (= 0.0529 nm) to obtain an expression the Bohr radii for the outer electron as it orbits the impurity arsenic atom in silicon and germanium.

Make the indicated substitutions in the expression for a_0 to obtain:

$$a_B=\frac{\kappa\in_0 h^2}{\pi m_{\text{eff}}e^2}=\frac{\kappa\in_0 m_e h^2}{\pi m_e m_{\text{eff}}e^2}=\frac{\kappa m_e}{m_{\text{eff}}}\frac{\in_0 h^2}{\pi m_e e^2}=\frac{\kappa m_e}{m_{\text{eff}}}a_0$$

For silicon:

$$a_B=\frac{12m_e}{0.2m_e}(0.0529\,\text{nm})=\boxed{3.17\,\text{nm}}$$

For germanium:

$$a_B=\frac{16m_e}{0.1m_e}(0.0529\,\text{nm})=\boxed{8.46\,\text{nm}}$$

***46 ••**

Picture the Problem We can make the same substitutions we made in Problem 45 in the expression for E_1 (= 13.6 eV) to obtain an expression that we can use to estimate the binding energy of the extra electron of an impurity arsenic atom in silicon and germanium.

Make the indicated substitutions in the expression for E_1 to obtain:

$$E_1 = -\frac{e^2 m_{\text{eff}}}{8(\kappa \epsilon_0)^2 h^2} = -\frac{e^2 m_{\text{e}} m_{\text{eff}}}{8 m_{\text{e}} \kappa^2 \epsilon_0^2 h^2}$$

$$= -\frac{m_{\text{eff}}}{m_{\text{e}} \kappa^2 \epsilon_0^2} \frac{e^2 m_{\text{e}}}{8 \epsilon_0^2 h^2}$$

$$= -\frac{m_{\text{eff}}}{m_{\text{e}} \kappa^2 \epsilon_0^2} E_1$$

(*a*) For silicon:

$$E_1 = -\frac{0.2 m_{\text{e}}}{m_{\text{e}} (12)^2} (13.6\,\text{eV}) = \boxed{18.9\,\text{meV}}$$

(*b*) For germanium:

$$E_1 = -\frac{0.1 m_{\text{e}}}{m_{\text{e}} (16)^2} (13.6\,\text{eV}) = \boxed{5.31\,\text{meV}}$$

47 ••

Picture the Problem We can use the expression for the resistivity ρ of the sample as a function of the mean free path λ of the conduction electrons in conjunction with the expression for the average speed v_{av} of the electrons to derive an expression that we can use to calculate the mean free path of the electrons.

Express the resistivity ρ of the sample as a function of the mean free path λ of the conduction electrons:

$$\rho = \frac{m_{\text{e}} v_{\text{av}}}{n_{\text{e}} e^2 \lambda}$$

Solve for λ to obtain:

$$\lambda = \frac{m_{\text{e}} v_{\text{av}}}{n_{\text{e}} e^2 \rho} \qquad (1)$$

The average speed v_{av} of the electrons is given by:

$$v_{\text{av}} \approx v_{\text{rms}} = \sqrt{\frac{3kT}{m_{\text{e}}}}$$

Substitute for v_{av} in the expression for λ to obtain:

$$\lambda = \frac{m_{\text{e}}}{n_{\text{e}} e^2 \rho} \sqrt{\frac{3kT}{m_{\text{e}}}} = \frac{\sqrt{3k m_{\text{e}} T}}{n_{\text{e}} e^2 \rho}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{\sqrt{3(1.38\times10^{-23}\,\text{J/K})(0.2)(9.11\times10^{-31}\,\text{kg})(300\,\text{K})}}{(10^{16}\,\text{cm}^{-3})(1.6\times10^{-19}\,\text{C})^2(5\times10^{-3}\,\Omega\cdot\text{m})} = \boxed{37.2\,\text{nm}}$$

The number density of electrons n_{e} is related to the mass density ρ_{m}, Avogadro's number N_{A}, and the molar mass M.:

$$n_{\text{e}} = \frac{\rho_{\text{m}} N_{\text{A}}}{M}$$

Substitute numerical values (For copper, ρ = 8.93 g/cm^3 and M = 63.5 g/mol.) and evaluate n_e:

$$n_e = \frac{\left(8.93\,\text{g/cm}^3\right)\left(6.02\times10^{23}\,\text{electrons/mol}\right)}{63.5\,\text{g/mol}} = 8.47\times10^{28}\,\text{electrons/m}^3$$

Using equation (1), evaluate λ_{Cu} (see Table 25-1 for the resistivity of copper and Example 38-4 for u_F):

$$\lambda_{\text{Cu}} = \frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.57\times10^{6}\,\text{m/s}\right)}{\left(8.47\times10^{28}\,\text{electrons/m}^3\right)\left(1.6\times10^{-19}\,\text{C}\right)^2\left(1.7\times10^{-8}\,\Omega\cdot\text{m}\right)} = \boxed{38.8\,\text{nm}}$$

$$\boxed{\text{The mean free paths agree to within 4.02\%.}}$$

48 ••

Picture the Problem We can use the expression for the Hall coefficient to determine the type of impurity and the concentration of these impurities.

(*a*) and (*b*) The Hall coefficient is given by:

$$R = \frac{1}{nq} \qquad (1)$$

$$\boxed{\begin{array}{l}\text{Because } R > 0,\ q > 0 \text{ and conduction is by holes and the sample contains}\\ \text{acceptor impurities.}\end{array}}$$

Solve equation (1) for *n*:

$$n = \frac{1}{Rq}$$

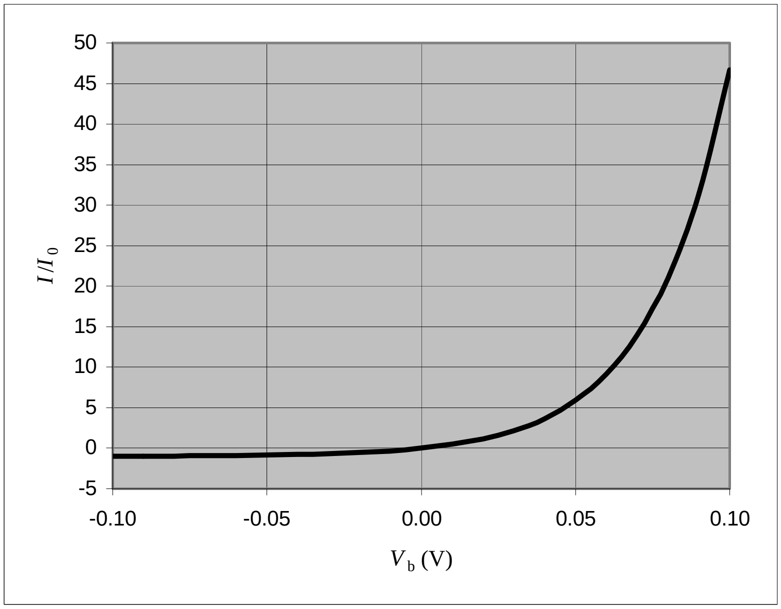
Substitute numerical values and evaluate *n*:

$$n = \frac{1}{\left(0.04\,\text{V}\cdot\text{m/A}\cdot\text{T}\right)\left(1.6\times10^{-19}\,\text{C}\right)} = \boxed{1.56\times10^{20}\,\text{m}^{-3}}$$

Semiconductor Junctions and Devices

49 ••

Picture the Problem The following graph of I/I_0 versus V_b was plotted using a spreadsheet program.



50 •

Picture the Problem The base current is the difference between the emitter current and the plate current.

The base current I_B is given by:

$$I_{\mathrm{B}} = I - I_{\mathrm{C}}$$

We're given that:

$$I_{\mathrm{C}} = 0.88I = 25.0\ \mathrm{mA}$$

Solve for I to obtain:

$$I = \frac{25.0\ \mathrm{mA}}{0.88} = 28.4\ \mathrm{mA}$$

Substitute numerical values for I and I_C and evaluate I_B:

$$I_{\mathrm{B}} = 28.4\ \mathrm{mA} - 25.0\ \mathrm{mA} = \boxed{3.4\ \mathrm{mA}}$$

***51** ••

Picture the Problem We can use its definition to compute the voltage gain of the amplifier.

The voltage gain of the amplifier is given by:

$$\text{Voltage gain} = \frac{I_{\mathrm{c}} R_L}{I_{\mathrm{b}} R_{\mathrm{b}}}$$

Substitute numerical values and evaluate the voltage gain:

$$\text{Voltage gain} = \frac{(0.5\ \mathrm{mA})(10\ \mathrm{k\Omega})}{(10\ \mu\mathrm{A})(2\ \mathrm{k\Omega})} = \boxed{250}$$

52 ••

Picture the Problem The number of electron-hole pairs N is related to the energy E of the incident beam and the energy gap E_g.

(*a*) The number of electron-hole pairs *N* is given by:

$$N=\frac{E}{E_{\mathrm{g}}}$$

Substitute numerical values and evaluate *N*:

$$N=\frac{660\,\mathrm{keV}}{0.72\,\mathrm{eV}}=\boxed{9.17\times10^{5}}$$

(*b*) The energy resolution of the detector is given by:

$$\frac{\Delta E}{E}=\frac{\Delta N}{N}$$

For $\Delta N = 1$ and $N=\sqrt{9.17\times10^{5}}$:

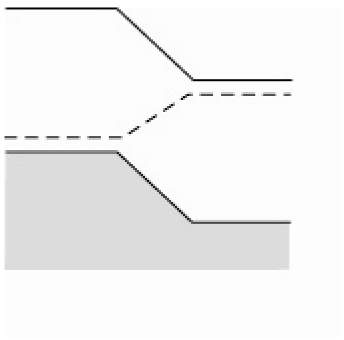
$$\frac{\Delta E}{E}=\frac{1}{\sqrt{9.17\times10^{5}}}=1.04\times10^{-3}$$

$$=\boxed{0.104\%}$$

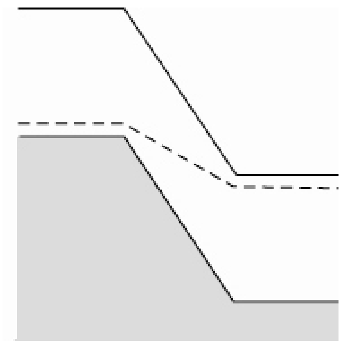
53 ••

Picture the Problem The nearly full valence band is shown shaded. The Fermi level is shown by the dashed line.

(*a*)



(*b*)



***54 ••**

Picture the Problem We can use Ohm's law and the expression for the current from Problem 49 to find the resistance for small reverse-and-forward bias voltages.

(*a*) Use Ohm's law to express the resistance:

$$R=\frac{V_{\mathrm{b}}}{I} \qquad (1)$$

From Problem 47, the current across a *pn* junction is given by:

$$I=I_0\left(e^{eV_{\mathrm{b}}/kT}-1\right) \qquad (2)$$

For $eV_{\mathrm{b}} << kT$:

$$e^{eV_{\mathrm{b}}/kT}-1\approx1+\frac{eV_{\mathrm{b}}}{kT}-1=\frac{eV_{\mathrm{b}}}{kT}$$

Substitute to obtain:

$$I=I_0\frac{eV_{\mathrm{b}}}{kT}$$

Substitute for I in equation (1) and simplify:

$$R=\frac{V_\text{b}}{I_0\dfrac{eV_\text{b}}{kT}}=\frac{kT}{eI_0}$$

Substitute numerical values and evaluate R:

$$R=\frac{(0.025\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}{(1.60\times10^{-19}\,\text{C})(10^{-9}\,\text{A})}$$
$$=\boxed{25.0\,\text{M}\Omega}$$

(*b*) Substitute equation (2) in equation (1) to obtain:

$$R=\frac{V_\text{b}}{I_0\left(e^{eV_\text{b}/kT}-1\right)}\qquad(3)$$

Evaluate $\dfrac{eV_\text{b}}{kT}$ for $V_\text{b}=-0.5$ V:

$$\frac{eV_\text{b}}{kT}=\frac{(1.60\times10^{-19}\,\text{C})(-0.5\,\text{V})}{(1.38\times10^{-23}\,\text{J/K})(293\,\text{K})}=-19.8$$

Evaluate equation (3) for $V_\text{b}=-0.5$ V:

$$R=\frac{-0.5\,\text{V}}{(10^{-9}\,\text{A})(e^{-19.8}-1)}=\boxed{500\,\text{M}\Omega}$$

(*c*) Evaluate $\dfrac{eV_\text{b}}{kT}$ for $V_\text{b}=+0.5$ V:

$$\frac{eV_\text{b}}{kT}=\frac{(1.60\times10^{-19}\,\text{C})(0.5\,\text{V})}{(1.38\times10^{-23}\,\text{J/K})(293\,\text{K})}=19.8$$

Evaluate equation (3) for $V_\text{b}=+0.5$ V:

$$R=\frac{0.5\,\text{V}}{(10^{-9}\,\text{A})(e^{19.8}-1)}=\boxed{1.26\,\Omega}$$

(*d*) Evaluate $R_\text{ac}=dV/dI$ to obtain:

$$R_\text{ac}=\frac{dV}{dI}=\left(\frac{dI}{dV}\right)^{-1}$$
$$=\left\{\frac{d}{dV}\left[I_0\left(e^{eV_\text{b}/kT}-1\right)\right]\right\}^{-1}$$
$$=\left\{\frac{eI_0}{kT}e^{eV_\text{b}/kT}\right\}^{-1}=\frac{kT}{eI_0}e^{-eV_\text{b}/kT}$$

Substitute numerical values and evaluate R_ac:

$$R_\text{ac}=(25\,\text{M}\Omega)e^{-19.8}=\boxed{0.0629\,\Omega}$$

55 ••

Picture the Problem We can use the Hall-effect equation to find the concentration of charge carriers in the slab of silicon. We can determine the semiconductor type by determining the directions of the magnetic and electric fields.

Use the expression for the Hall-effect voltage to relate the concentration of charge carriers n to

$$V_\text{H}=\frac{IB}{nte}$$

the Hall voltage V_H:

Solve for n to obtain:

$$n = \frac{IB}{teV_H}$$

Substitute numerical values and evaluate n:

$$n = \frac{(0.2\,\text{A})(0.4\,\text{T})}{(1.0\,\text{mm})(1.60\times10^{-19}\,\text{C})(5\,\text{mV})} = \boxed{1.00\times10^{23}\,\text{m}^{-3}}$$

Referring to Figure 26-28, note that $\vec{B}$ points out of the page. $\vec{E}$ is in the y direction. Therefore, the charge carriers are holes and the semiconductor is p-type.

The BCS Theory

56 •

Picture the Problem We can calculate E_g using $E_g = 3.5kT_c$ and find the wavelength of a photon having sufficient energy to break up Cooper pairs in tin at $T = 0$ using $\lambda = hc/E_g$.

(a) From Equation 38-24 we have:

$$E_g = 3.5kT_c$$

Substitute numerical values and evaluate E_g:

$$E_g = 3.5(8.617\times10^{-5}\,\text{eV/K})(3.72\,\text{K}) = \boxed{1.12\,\text{meV}}$$

Express the ratio of E_g to $E_{g,\text{measured}}$:

$$\frac{E_g}{E_{g,\text{measured}}} = \frac{1.12\,\text{meV}}{6\times10^{-4}\,\text{eV}} = 1.87$$

or

$$E_g \approx \boxed{2E_{g,\text{measured}}}$$

(b) The wavelength of a photon having sufficient energy to break up Cooper pairs in tin at $T = 0$ is given by:

$$\lambda = \frac{hc}{E_g}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{6\times10^{-4}\,\text{eV}} = 2.07\times10^{6}\,\text{nm} = \boxed{2.07\,\text{mm}}$$

***57 •**

Picture the Problem We can calculate E_g using $E_g = 3.5kT_c$ and find the wavelength of a photon having sufficient energy to break up Cooper pairs in tin at $T = 0$ using $\lambda = hc/E_g$.

(*a*) From Equation 38-24 we have:

$$E_g = 3.5kT_c$$

Substitute numerical values and evaluate E_g:

$$E_g = 3.5\left(8.62\times10^{-5}\ \text{eV/K}\right)\left(7.19\ \text{K}\right) = \boxed{2.17\ \text{meV}}$$

Express the ratio of E_g to $E_{g,\text{measured}}$:

$$\frac{E_g}{E_{g,\text{measured}}} = \frac{2.17\ \text{meV}}{2.73\times10^{-3}\ \text{eV}} = 0.795$$

or

$$E_g \approx \boxed{0.8E_{g,\text{measured}}}$$

(*b*) The wavelength of a photon having sufficient energy to break up Cooper pairs in tin at $T = 0$ is given by:

$$\lambda = \frac{hc}{E_g}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{1240\ \text{eV}\cdot\text{nm}}{2.73\times10^{-3}\ \text{eV}} = 4.54\times10^{5}\ \text{nm} = \boxed{0.454\ \text{mm}}$$

The Fermi-Dirac Distribution

58 ••

Picture the Problem We can evaluate the Fermi factor at the bottom of the conduction band for T near room temperature to show that this factor is given by $\exp(-E_g/2kT)$.

(*a*) At the bottom of the conduction band:

$e^{(E-E_F)/kT} = e^{E_g/2kT} >>> 1$ for T near room temperature.

We can neglect the 1 in the denominator of the Fermi function to obtain:

$$f\left(\tfrac{1}{2}E_g\right) = \frac{1}{e^{E_g/2kT}} = \boxed{e^{-E_g/2kT}}$$

Substitute numerical values and evaluate $f\left(\tfrac{1}{2}E_g\right)$ for T = 300 K:

$$f\left(\tfrac{1}{2}E_g\right)=\exp\left[\frac{-1\,\text{eV}}{2\left(8.62\times10^{-5}\ \text{eV/K}\right)\left(300\,\text{K}\right)}\right]=\boxed{4.01\times10^{-9}}$$

Given that low a probability of finding an electron in a state near the bottom of the conduction band, the exclusion principle has no significant impact on the distribution function. With 10^{22} valence electrons per cubic centimeter, the number of electrons in the conduction band will be about 4×10^{13} per cm^3.

(*b*) Evaluate $f\left(\tfrac{1}{2}E_g\right)$ for T = 300 K and E_g = 6 eV:

$$f\left(\tfrac{1}{2}E_g\right)=\exp\left[\frac{-6\,\text{eV}}{2\left(8.62\times10^{-5}\ \text{eV/K}\right)\left(300\,\text{K}\right)}\right]=\boxed{4.15\times10^{-51}}$$

The probability of finding even one electron in the conduction band is negligibly small (approximately 4×10^{-51}).

59 ••

Picture the Problem The number of energy states per unit volume per unit energy interval N is given by $N\approx g(E)\Delta E$, where N is only approximate, because ΔE is not infinitesimal and $g(E)=\dfrac{8\sqrt{2}\pi m_e^{3/2}V}{h^3}E^{1/2}$ is the density of states.

The number of states N is the product of the density of states and the energy interval:

$$N\approx g(E)\Delta E \qquad (1)$$

The density of states is given by:

$$g(E)=\frac{8\sqrt{2}\pi m_e^{3/2}V}{h^3}E^{1/2}$$

Substitute numerical values and evaluate $g(E)$:

$$\begin{aligned}g(E)&=\frac{8\sqrt{2}\pi\left(9.11\times10^{-31}\ \text{kg}\right)^{3/2}\left(1\times10^{-3}\ \text{m}\right)^3}{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)^3}\left(2.1\,\text{eV}\times\frac{1.60\times10^{-19}\ \text{J}}{\text{eV}}\right)^{1/2}\\&=6.15\times10^{37}\ \text{J}^{-1}\end{aligned}$$

Substitute numerical values in equation (1) and evaluate *N*:

$$N \approx \left(6.15\times10^{37}\ \text{J}^{-1}\right)\left(2.20\,\text{eV}-2.00\,\text{eV}\right)\times\frac{1.60\times10^{-19}\ \text{J}}{\text{eV}}=\boxed{1.97\times10^{18}}$$

***60 ••**

Picture the Problem Equation 38-22*a* expresses the dependence of the Fermi energy E_F on the number density of free electrons. Once we've determined the Fermi energy for silver, we can find the average electron energy from the Fermi energy for silver and then use the average electron energy to find the Fermi speed for silver.

(*a*) From Equation 38-22*a* we have:

$$E_F=\frac{h^2}{8m_e}\left(\frac{3N}{\pi V}\right)^{3/2}$$

Use Table 27-1 to find the free-electron number density *N*/*V* for silver:

$$\frac{N}{V}=5.86\text{x}10^{22}\ \frac{\text{electrons}}{\text{cm}^3}$$
$$=5.86\text{x}10^{28}\ \frac{\text{electrons}}{\text{m}^3}$$

Substitute numerical values and evaluate E_F:

$$E_F=\frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8\left(9.11\times10^{-31}\ \text{kg}\right)}\left[\frac{3\left(5.86\times10^{28}\ \text{electrons/m}^3\right)}{\pi}\right]^{2/3}\left(\frac{1\text{eV}}{1.60\times10^{-19}\ \text{J}}\right)$$
$$=\boxed{5.51\text{eV}}$$

(*b*) The average electron energy is given by:

$$E_{\text{av}}=\frac{3}{5}E_F$$

Substitute numerical values and evaluate *E*av:

$$E_{\text{av}}=\frac{3}{5}\left(5.51\,\text{eV}\right)=\boxed{3.31\text{eV}}$$

(*c*) Express the Fermi energy in terms of the Fermi speed of the electrons:

$$E_F=\tfrac{1}{2}m_e v_F^2$$

Solve for v_F:

$$v_F=\sqrt{\frac{2E_F}{m_e}}$$

Substitute numerical values and evaluate v_F:

$$v_F=\sqrt{\frac{2\left(3.31\text{eV}\right)}{9.11\times10^{-31}\ \text{kg}}\left(\frac{1.60\times10^{-19}\ \text{J}}{1\text{eV}}\right)}$$
$$=\boxed{1.08\times10^{6}\ \text{m/s}}$$

61 ••

Picture the Problem We can evaluate the $f(E_F)$ at $E = E_F$ to show that $F = 0.5$.

The Fermi factor is:

$$f(E)=\frac{1}{e^{(E-E_F)/kT}+1}$$

Evaluate $f(E_F)$:

$$f(E_F)=\frac{1}{e^{(E_F-E_F)/kT}+1}=\frac{1}{1+1}=\boxed{0.5}$$

62 ••

Picture the Problem We can find the difference between the energies at which the Fermi factor has the given values by solving the expression for Fermi factor for E and then deriving an expression for ΔE.

(*a*) The Fermi factor is:

$$f(E)=\frac{1}{e^{(E-E_F)/kT}+1}$$

Solve for E:

$$E = E_F + kT\ln\left(\frac{1}{f(E)}-1\right)$$

The difference between the energies is given by:

$$\begin{aligned}\Delta E &= E(0.1)-E(0.9)\\ &= E_F+\frac{(1.38\times10^{-23}\ \text{J/K})(300\ \text{K})}{1.60\times10^{-19}\ \text{J/eV}}\ln\left(\frac{1}{0.1}-1\right)\\ &\quad-\left\{E_F+\frac{(1.38\times10^{-23}\ \text{J/K})(300\ \text{K})}{1.60\times10^{-19}\ \text{J/eV}}\ln\left(\frac{1}{0.9}-1\right)\right\}\\ &=\frac{(1.38\times10^{-23}\ \text{J/K})(300\ \text{K})}{1.60\times10^{-19}\ \text{J/eV}}\left[\ln\left(\frac{1}{0.1}-1\right)-\ln\left(\frac{1}{0.9}-1\right)\right]\\ &=\boxed{0.114\ \text{eV}}\end{aligned}$$

(*b*) and (*c*) $\boxed{\text{Because } \Delta E \text{ is independent of } E_F,\ \Delta E \text{ is the same as in } (a).}$

***63** ••

Picture the Problem The probability that a conduction electron will have a given kinetic energy is given by the Fermi factor.

The Fermi factor is:

$$f(E)=\frac{1}{e^{(E-E_{\mathrm{F}})/kT}+1}$$

Because $E_{\mathrm{F}} - 4.9\text{ eV} >> 300k$:

$$f(4.9\,\mathrm{eV})=\frac{1}{0+1}=\boxed{1}$$

64 ••

Picture the Problem We can solve Equation 38-22a for V and substitute in Equation 38-41 to show that $g(E)=(3N/2)E_{\mathrm{F}}^{-3/2}E^{1/2}$.

From Equation 38-22a we have:

$$E_{\mathrm{F}}=\frac{h^2}{8m_{\mathrm{e}}}\left(\frac{3N}{\pi V}\right)^{2/3}$$

Solve for V to obtain:

$$V=\frac{3N}{\pi}\left(\frac{h^2}{8m_{\mathrm{e}}E_{\mathrm{F}}}\right)^{3/2}$$

The density $g(E)$ is given by Equation 38-41:

$$g(E)=\frac{8\pi\sqrt{2}m_{\mathrm{e}}^{3/2}V}{h^3}E^{1/2}$$

Substitute for V and simplify to obtain:

$$g(E)=\frac{8\pi\sqrt{2}m_{\mathrm{e}}^{3/2}}{h^3}\left[\frac{3N}{\pi}\left(\frac{h^2}{8m_{\mathrm{e}}E_{\mathrm{F}}}\right)^{3/2}\right]E^{1/2}$$

$$=\boxed{\frac{3N}{2}E_{\mathrm{F}}^{-3/2}E^{1/2}}$$

65 ••

Picture the Problem We can use the expression for $g(E)$ from Problem 64 to show that the average energy at $T = 0$ is $\frac{3}{5}E_{\mathrm{F}}$.

From Problem 64 we have:

$$g(E)=\frac{3N}{2}E_{\mathrm{F}}^{-3/2}E^{1/2}$$

Substitute in the expression for E_{av} and simplify to obtain:

$$E_{\text{av}} = \frac{1}{N}\int_0^{E_{\text{F}}} Eg(E)dE = \frac{1}{N}\int_0^{E_{\text{F}}} E\left(\frac{3N}{2}E_{\text{F}}^{-3/2}E^{1/2}\right)dE = \frac{3}{2}E_{\text{F}}^{-3/2}\int_0^{E_{\text{F}}} E^{3/2}dE$$

Integrate the expression for E_{av}:

$$E_{\text{av}} = \frac{3}{2}E_{\text{F}}^{-3/2}\int_0^{E_{\text{F}}} E^{3/2}dE = \frac{3}{2}E_{\text{F}}^{-3/2}\,\frac{2}{5}E_{\text{F}}^{5/2} = \boxed{\frac{3}{5}E_{\text{F}}}$$

66 ••

Picture the Problem We can integrate $g(E)$ from 0 to E_{F} to show that the total number of states is $\frac{2}{3}AE_{\text{F}}^{3/2}$.

(*a*) Integrate $g(E)$ from 0 to E_{F}:

$$N = \int_0^{E_{\text{F}}} AE^{1/2}dE = \boxed{\tfrac{2}{3}AE_{\text{F}}^{3/2}}$$

(*b*) Express the fraction of N within kT of E_{F}:

$$\frac{kTg(E_{\text{F}})}{N} = \frac{kTAE_{\text{F}}^{1/2}}{\frac{2}{3}AE_{\text{F}}^{3/2}} = \boxed{\frac{3kT}{2E_{\text{F}}}}$$

(*c*) Substitute numerical values and evaluate the expression obtained in (*b*) for copper:

$$\frac{3kT}{2E_{\text{F}}} = \frac{3(8.62\times10^{-5}\text{ eV/K})(300\text{ K})}{2(7.04\text{ eV})} = \boxed{5.51\times10^{-3}}$$

67 ••

Picture the Problem The probability that a conduction electron in metal is the Fermi factor.

Express the Fermi factor:

$$f(E) = \frac{1}{e^{(E-E_{\text{F}})/kT}+1}$$

Calculate the dimensionless exponent in the Fermi factor:

$$\frac{E-E_{\text{F}}}{kT} = \frac{5.49\text{ eV}-5.50\text{ eV}}{(8.62\times10^{-5}\text{ eV/K})(300\text{ K})} = -0.387$$

Use this result to calculate the Fermi factor:

$$f(5.49\,\mathrm{eV}) = \frac{1}{e^{-0.387}+1} = \boxed{0.596}$$

68 ••

Picture the Problem We can integrate the density-of-states function, Equation 38-41, to find the number of occupied states N. The fraction of these states that are within kT of E_F can then be found from the ratio of $kTg(E_\mathrm{F})$ to N.

The density of states function is:

$$g(E) = AE^{1/2}$$

where

$$A = \frac{8\pi\sqrt{2}m_\mathrm{e}^{3/2}V}{h^3}$$

Integrate $g(E)$ from 0 to E_F to find the total number of occupied states:

$$N = \int_0^{E_\mathrm{F}} AE^{1/2}dE = \tfrac{2}{3}AE_{E_\mathrm{F}}^{3/2}$$

Express the fraction of N within kT of E_F:

$$\frac{kTg(E_\mathrm{F})}{N} = \frac{kTAE_\mathrm{F}^{1/2}}{\tfrac{2}{3}AE_\mathrm{F}^{3/2}} = \boxed{\frac{3kT}{2E_\mathrm{F}}}$$

(*a*) Substitute numerical values and evaluate the expression obtained above for copper at $T = 77$ K:

$$\frac{3kT}{2E_\mathrm{F}} = \frac{3(8.62\times10^{-5}\,\mathrm{eV/K})(77\,\mathrm{K})}{2(7.04\,\mathrm{eV})} = \boxed{1.41\times10^{-3}}$$

(*b*) At $T = 300$ K:

$$\frac{3kT}{2E_\mathrm{F}} = \frac{3(8.62\times10^{-5}\,\mathrm{eV/K})(300\,\mathrm{K})}{2(7.04\,\mathrm{eV})} = \boxed{5.51\times10^{-3}}$$

69 ••

Picture the Problem The distribution function of electrons in the conduction band is given by $n(E) = g(E)f(E)$ where $f(E)$ is the Fermi factor and $g(E)$ is the density of states in terms of E_F.

Express the number of electrons n with energy E:

$$n(E) = g(E)f(E) \qquad (1)$$

where

$$g(E) = \frac{3N}{2}E_{E_\mathrm{F}}^{-3/2}E^{1/2}$$

and

$$f(E)=\frac{1}{e^{(E-E_\mathrm{F})/kT}+1}$$

The dimensionless exponent in the Fermi factor is:

$$\frac{E-E_\mathrm{F}}{kT}=\frac{E-\frac{1}{2}E_\mathrm{g}}{kT}>>1$$

and

$$\exp\left(\frac{E-\frac{1}{2}E_\mathrm{g}}{kT}\right)>>1$$

Hence:

$$f(E)=\frac{1}{e^{\left(E-\frac{1}{2}E_\mathrm{g}\right)/kT}}\approx e^{E_\mathrm{g}/2kT}e^{-E/kT}$$

Substitute in equation (1) and simplify to obtain:

$$n(E)=\boxed{\left(\tfrac{3}{2}NE_F^{-3/2}e^{E_\mathrm{g}/2kT}\right)E^{1/2}e^{-E/kT}}$$

There is an additional temperature dependence that arises from the fact that E_F depends on T. At room temperature, $\exp[(E-E_\mathrm{g}/2)/kT]\geq\exp(0.35\text{ eV}/0.0259\text{ eV})=7.4\times10^5$, so the approximation leading to the Boltzmann distribution is justified.

***70** •••

Picture the Problem We can follow the step-by-step procedure outlined in the problem statement to obtain the indicated results.

(*a*) The Fermi factor is:

$$f(E)=\frac{1}{e^{(E-E_\mathrm{F})/kT}+1}=\frac{1}{e^{-E_\mathrm{F}/kT}e^{E/kT}+1}$$

$$=\boxed{\frac{1}{Ce^{E/kT}+1}}$$

provided $C=e^{-E_\mathrm{F}/kT}$

(*b*) If $C>>e^{-E/kT}$:

$$f(E)=\frac{1}{Ce^{E/kT}+1}\approx\frac{1}{Ce^{E/kT}}=\boxed{Ae^{-E/kT}}$$

where $A=1/C$

(*c*) The energy distribution function is:

$$n(E)dE=g(E)dE\,f(E)$$

where

$$g(E)=\frac{8\pi\sqrt{2}m_\mathrm{e}^{3/2}V}{h^3}E^{1/2}$$

Substitute for $g(E)dE$ *and* $f(E)$ in the expression for N to obtain:

$$N=A\frac{8\pi\sqrt{2}m_\mathrm{e}^{3/2}V}{h^3}\int_0^\infty E^{1/2}e^{-E/kT}dE$$

The definite integral has the value:

$$\int_0^\infty E^{1/2}e^{-E/kT}dE=\frac{(kT)^{3/2}}{2}\sqrt{\pi}$$

Substitute to obtain:

$$N=A\frac{8\pi\sqrt{2}m_{\text{e}}^{3/2}V}{h^3}\frac{(kT)^{3/2}}{2}\sqrt{\pi}$$

Solve for A:

$$A=\boxed{\frac{\sqrt{2}h^3}{8\pi^{3/2}m_{\text{e}}^{3/2}}\left(\frac{N}{V}\right)\frac{1}{(kT)^{3/2}}}$$

(*d*) Evaluate A at T = 300 K:

$$A=\frac{\sqrt{2}\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)^3 n}{8\pi^{3/2}\left(9.11\times10^{-31}\ \text{kg}\right)^{3/2}\left[\left(1.38\times10^{-23}\ \text{J/K}\right)(300\,\text{K})\right]^{3/2}}\approx4\times10^{-26}n$$

where the units are SI.

$$\boxed{\begin{array}{l}\text{The valence electron concentration is typically about } 10^{39}\ \text{m}^{-3}\text{. To satisfy the}\\ \text{condition that } A<<1 \text{ at room temperature, } n \text{ should be less than } 10^{23}\ \text{m}^{-3}\text{, or}\\ \text{about one millionth of the valence electron concentration. Because } A \text{ depends}\\ \text{on } T^{-3/2}\text{, the electron concentration may be greater the higher the temperature.}\end{array}}$$

(*e*) $$\boxed{\begin{array}{l}10^{17}\ \text{cm}^{-3}=10^{23}\ \text{m}^{-3}\text{. So, according to the criterion in } (d)\text{, the classical}\\ \text{approximation is applicable.}\end{array}}$$

71 •••

Picture the Problem We can approximate the separation of electrons in the gas by $(V/N)^{1/3}$ and use the for A from Problem 70 and de Broglie's equation to express the separation d of electrons in terms of the de Broglie wavelength λ and the constant A.

The separation d of electrons is approximately:

$$d=\left(\frac{V}{N}\right)^{1/3}$$

From Problem 70:

$$\left(\frac{V}{N}\right)^{1/3}=\frac{2^{1/6}h}{8^{1/3}\pi^{1/2}m_{\text{e}}^{1/2}}\frac{1}{(kT)^{1/2}A^{1/3}}$$

Substitute to obtain:

$$\begin{aligned}d&=\frac{2^{1/6}h}{8^{1/3}\pi^{1/2}m_{\text{e}}^{1/2}}\frac{1}{(kT)^{1/2}A^{1/3}}\\&=\frac{2^{1/6}h}{\pi^{1/2}}\frac{1}{\sqrt{2m_{\text{e}}kT}A^{1/3}}\end{aligned}$$

Express the momentum of an electron in the gas in terms of its de Broglie wavelength λ:

$$p=\frac{h}{\lambda}=\sqrt{2mK}=\sqrt{2m_{\text{e}}kT}$$

Substitute for $\sqrt{2m_{\text{e}}kT}$ in the expression for d to obtain:

$$d=\frac{2^{1/6}h}{\pi^{1/2}}\frac{\lambda}{hA^{1/3}}=\frac{2^{1/6}}{\pi^{1/2}}\frac{\lambda}{A^{1/3}}$$

$$=0.633\frac{\lambda}{A^{1/3}}$$

$$\boxed{\text{Thus, if } A<<1, d>>\lambda}$$

72 •••

Picture the Problem We can follow the procedure outlined in the problem statement to determine the rms energy of a Fermi distribution.

Express the E_{rms} in terms of $g(E)$:

$$E_{\text{rms}}=\left(\frac{1}{N}\int_0^{E_{\text{F}}}g(E)E^2dE\right)^{1/2}$$

The density of states $g(E)$ is given by:

$$g(E)=\frac{3N}{2}E_{\text{F}}^{-3/2}E^{1/2}$$

Substitute to obtain:

$$E_{\text{rms}}=\left(\frac{1}{2NE_{\text{F}}^{3/2}}\int_0^{E_{\text{F}}}E^{3/2}dE\right)^{1/2}$$

Evaluate the integral and simplify:

$$E_{\text{rms}}=\sqrt{\frac{3}{7}}E_{\text{F}}=\boxed{0.655E_{\text{F}}}$$

$$\boxed{\begin{array}{l}E_{\text{rms}}>E_{\text{av}} \text{ because the process of averaging the square of the energy weighs}\\ \text{larger energies more heavily.}\end{array}}$$

General Problems

73 •

Picture the Problem The number of free electrons per atom n_{e} is given by $n_{\text{e}}=nM/\rho N_{\text{A}}$ where N_{A} is Avogadro's number, ρ is the density of the element, M is its molar mass, and n is the free electron number density for the element.

The number of electrons per atom is given by:

$$n_{\text{e}}=\frac{nM}{\rho N_{\text{A}}}$$

Substitute numerical values and evaluate n_e:

$$n_\mathrm{e} = \frac{\left(1.4\times10^{22}\ \text{electrons/cm}^3\right)\left(39.098\,\text{g/mol}\right)}{\left(0.851\,\text{g/cm}^3\right)\left(6.02\times10^{23}\ \text{electrons/mol}\right)} = \boxed{1.07}$$

74 •

Picture the Problem The number of free electrons per atom n_e is related to the number density of free electrons n by $n_\mathrm{e} = nM/\rho N_\mathrm{A}$, where N_A is Avogadro's number, ρ is the density of the element, and M is its molar mass.

The number of electrons per atom is given by:

$$n_\mathrm{e} = \frac{nM}{\rho N_\mathrm{A}}$$

Solve for n to obtain:

$$n = \frac{n_\mathrm{e}\rho N_\mathrm{A}}{M}$$

(*a*) Substitute numerical values and evaluate n for Mg:

$$n = \frac{(2)\left(1.74\,\text{g/cm}^3\right)\left(6.02\times10^{23}\ \text{electrons/mol}\right)}{24.31\,\text{g/mol}} = \boxed{8.62\times10^{22}\ \text{electrons/cm}^3}$$

(*b*) Substitute numerical values and evaluate n for Zn:

$$n = \frac{(2)\left(7.1\,\text{g/cm}^3\right)\left(6.02\times10^{23}\ \text{electrons/mol}\right)}{65.38\,\text{g/mol}} = \boxed{13.1\times10^{22}\ \text{electrons/cm}^3}$$

$$\boxed{\text{Both results agree with the values in Table 38 - 1 to within 1\%.}}$$

75 ••

Picture the Problem We can integrate $g(E)$ from 0 to E_F to show that the total number of states is $\frac{2}{3}AE_\mathrm{F}^{3/2}$ and then use this result to find the fraction of the free electrons that are above the Fermi energy at the given temperatures.

Integrate $g(E)$ from 0 to E_F:

$$N = \int_0^{E_\mathrm{F}} AE^{1/2}dE = \tfrac{2}{3}AE^{3/2}_{E_\mathrm{F}}$$

Express the fraction of N within kT of E_F:

$$\frac{kTg\left(E_\mathrm{F}\right)}{N} = \frac{kTAE_\mathrm{F}^{1/2}}{\frac{2}{3}AE_\mathrm{F}^{3/2}} = \frac{3kT}{2E_\mathrm{F}}$$

(*a*) Substitute numerical values and evaluate this fraction for copper at 300 K:

$$\frac{3kT}{2E_\mathrm{F}}=\frac{3\left(8.62\times10^{-5}\ \mathrm{eV/K}\right)\left(300\ \mathrm{K}\right)}{2\left(7.04\ \mathrm{eV}\right)}=\boxed{5.51\times10^{-3}}$$

(*b*) Evaluate the same fraction at 1000 K:

$$\frac{3kT}{2E_\mathrm{F}}=\frac{3\left(8.62\times10^{-5}\ \mathrm{eV/K}\right)\left(1000\ \mathrm{K}\right)}{2\left(7.04\ \mathrm{eV}\right)}=\boxed{1.84\times10^{-2}}$$

***76** ••

Picture the Problem The Fermi factor gives the probability of an energy state being occupied as a function of the energy of the state *E*, the Fermi energy E_F for the particular material, and the temperature *T*.

The Fermi factor is:

$$f(E)=\frac{1}{e^{(E-E_\mathrm{F})/kT}+1}$$

For 10 percent probability:

$$0.1=\frac{1}{e^{(E-E_\mathrm{F})/kT}+1}$$

or

$$e^{(E-E_\mathrm{F})/kT}=9$$

Take the natural logarithm of both sides of the equation to obtain:

$$\frac{E-E_\mathrm{F}}{kT}=\ln 9$$

Solve for *E* to obtain:

$$E=E_\mathrm{F}+kT\ln 9$$

From Table 37-1, E_F(Mn) = 11.0 eV. Substitute numerical values and evaluate *E*:

$$E=11.0\ \mathrm{eV}+\left(1.38\times10^{-23}\ \mathrm{J/K}\right)\left(1300\ \mathrm{K}\right)\left(\frac{1\ \mathrm{eV}}{1.60\times10^{-19}\ \mathrm{J}}\right)\ln 9=\boxed{11.2\ \mathrm{eV}}$$

77 ••

Picture the Problem The energy gap for the semiconductor is related to the wavelength of the emitted light according to $E_\mathrm{g}=hc/\lambda$.

Express the energy gap *E*g in terms of the wavelength λ of the emitted light:

$$E_\mathrm{g}=\frac{hc}{\lambda}$$

Solve for λ:

$$\lambda=\frac{hc}{E_\mathrm{g}}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{1.8\,\text{eV}} = \boxed{689\,\text{nm}}$$

Remarks: This wavelength is in the red portion of the visible spectrum.

***78** •••

Picture the Problem The rate of production of electron-hole pairs is the ratio of the incident energy to the energy required to produce an electron-hole pair.

(*a*)The number of electron-hole pairs *N* produced in one second is:

$$N = \frac{IA}{\frac{hc}{\lambda}} = \frac{IA\lambda}{hc}$$

Substitute numerical values and evaluate *N*:

$$N = \frac{(4.0\,\text{W/m}^2)(2\times10^{-4}\,\text{m}^2)(775\,\text{nm})}{(1240\,\text{eV}\cdot\text{nm})(1.60\times10^{-19}\,\text{J/eV})} = \boxed{3.12\times10^{15}\,\text{s}^{-1}}$$

(*b*) In the steady state, the rate of recombination equals the rate of generation. Therefore:

$$N = \boxed{3.12\times10^{15}\,\text{s}^{-1}}$$

(*c*) The power radiated equals the power absorbed:

$$P_{\text{rad}} = IA$$

Substitute numerical values and evaluate P_{rad}:

$$P_{\text{rad}} = (4.0\,\text{W/m}^2)(2\times10^{-4}\,\text{m}^2) = \boxed{0.800\,\text{mJ/s}}$$