Chapter 40
Nuclear Physics

Conceptual Problems

1 •

Determine the Concept Two or more nuclides with the same atomic number Z but different N and A numbers are called isotopes.

(*a*) Two other isotopes of ^{14}N are: ^{15}N, ^{16}N

(*b*) Two other isotopes of ^{56}Fe are: ^{54}Fe, ^{55}Fe

(*c*) Two other isotopes of ^{118}Sn are: ^{54}Fe, ^{55}Fe

2 •

Determine the Concept The parent of that series, ^{237}Np, has a half-life of 2×10^{6} y that is much shorter than the age of the earth. There is no naturally occurring Np remaining on earth.

3 •

Determine the Concept Generally, β-decay leaves the daughter nucleus neutron rich, i.e., above the line of stability. The daughter nucleus therefore tends to decay via β^- emission which converts a nuclear neutron to a proton.

***4** •

Determine the Concept ^{14}C is found on earth because it is constantly being formed by cosmic rays in the upper atmosphere in the reaction $^{14}\text{N} + \text{n} \rightarrow {}^{14}\text{C} + {}^{1}\text{H}$.

5 •

Determine the Concept It would make the dating unreliable because the current concentration of ^{14}C is not equal to that at some earlier time.

6 •

Determine the Concept An element with such a high Z value would either fission spontaneously or decay almost immediately by α emission (see Figure 40-3).

7 •

Determine the Concept The probability for neutron capture by the fissionable nucleus is large only for slow (thermal) neutrons. The neutrons emitted in the fission process are fast (high energy) neutrons and must be slowed to thermal neutrons before they are likely to be captured by another fissionable nucleus.

8 •

Determine the Concept The process of ″slowing down″ involves the sharing of energy of a fast neutron and another nucleus in an elastic collision. The fast particle will lose maximum energy in such a collision if the target particle is of the same mass as the incident particle. Hence, neutron-proton collisions are most effective in slowing down neutrons. However, ordinary water cannot be used as a moderator because protons will capture the slow neutrons and form deuterons.

9 •

Determine the Concept Beta decay occurs in nuclei that have too many or too few neutrons for stability. In β decay, A remains the same while Z either increases by 1 (β^- decay) or decreases by 1 (β^+ decay).

(*a*) The reaction is: $${}^{22}_{11}\text{Na} \rightarrow {}^{22}_{10}\text{Ne} + {}^{0}_{+1}\beta \Rightarrow \boxed{\beta^+}\ \text{decay}$$

(*b*) The reaction is: $${}^{24}_{11}\text{Na} \rightarrow {}^{24}_{12}\text{Mg} + {}^{0}_{-1}\beta \Rightarrow \boxed{\beta^-}\ \text{decay}$$

10 •

Advantages	Disadvantages
The reactor uses ^{238}U, which, by neutron capture and subsequent decays, produces ^{239}Pu. Thus plutonium isotope fissions by fast neutron capture. Thus, the breeder reactor uses the plentiful uranium isotope and does not need a moderator to slow the neutrons needed for fission.	The fraction of delayed neutrons emitted in the fission of ^{239}Pu is very small. Consequently, control of the fission reaction is very difficult, and the safety hazards are more severe than for the ordinary reactor that uses ^{235}U as fuel.

11 •

(*a*) False. The nucleus does not contain electrons.

(*b*) True.

(*c*) False. After two half-lives, three-fourths of the radioactive nuclei in a given sample have decayed.

(*d*) True (given an unlimited supply of ^{238}U).

12 •

Determine the Concept Pressure and temperature changes have no effect on the internal structure of the nucleus. They do have an effect on the electronic configuration; consequently, they can influence K-capture processes.

***13** •

Determine the Concept Knowing the parent nucleus and one of the decay products, we can use the conservation of charge, the conservation of energy, and the conservation of the number of nucleons to identify the participants in the decay.

(*a*) beta decay of ^{16}N $\boxed{{}^{16}_{7}\text{N}\rightarrow{}^{16}_{8}\text{O}+{}^{0}_{-1}\beta+{}^{0}_{0}\overline{\nu}+Q}$

(*b*) alpha decay of ^{248}Fm $\boxed{{}^{248}_{100}\text{Fm}\rightarrow{}^{244}_{98}\text{Cf}+{}^{4}_{2}\text{He}+Q}$

(*c*) positron decay of ^{12}N $\boxed{{}^{12}_{7}\text{N}\rightarrow{}^{12}_{6}\text{C}+{}^{0}_{+1}\beta+{}^{0}_{0}\nu+Q}$

(*d*) beta decay of ^{81}Se $\boxed{{}^{81}_{34}\text{Se}\rightarrow{}^{81}_{35}\text{Br}+{}^{0}_{-1}\beta+{}^{0}_{0}\overline{\nu}+Q}$

(*e*) positron decay of ^{61}Cu $\boxed{{}^{61}_{29}\text{Cu}\rightarrow{}^{61}_{28}\text{Ni}+{}^{0}_{+1}\beta+{}^{0}_{0}\nu+Q}$

(*f*) alpha decay of $^{228\text{Th}}$ $\boxed{{}^{228}_{90}\text{Th}\rightarrow{}^{224}_{88}\text{Ra}+{}^{4}_{2}\text{He}+Q}$

***14** •

Determine the Concept We can use the information regarding the daughter nuclei to write and balance equations for each of the reactions.

(*a*) $\boxed{{}^{240}_{94}\text{Pu}\rightarrow 3{}^{1}_{0}\text{n}+{}^{90}_{38}\text{Sr}+{}^{147}_{56}\text{Ba}}$

(*b*) $\boxed{{}^{72}_{32}\text{Ge}+{}^{4}_{2}\text{He}\rightarrow{}^{1}_{0}\text{n}+{}^{75}_{34}\text{Se}}$

(*c*) $\boxed{{}^{127}_{53}\text{I}+{}^{2}_{1}\text{H}\rightarrow{}^{1}_{0}\text{n}+{}^{128}_{54}\text{Xe}}$

(*d*) $\boxed{{}^{235}_{92}\text{U}+{}^{1}_{0}\text{n}\rightarrow 2{}^{1}_{0}\text{n}+{}^{113}_{47}\text{Ag}+{}^{121}_{45}\text{Rh}}$

(*e*) $\boxed{{}^{55}_{25}\text{Mn}+{}^{7}_{3}\text{Li}\rightarrow{}^{3}_{1}\text{H}+{}^{59}_{27}\text{Co}}$

(*f*) ${}^{238}_{92}\text{U}+{}^{1}_{0}\text{n}\rightarrow{}^{239}_{92}\text{U};\ {}^{239}_{92}\text{U}\rightarrow{}^{0}_{-1}\beta+{}^{0}_{0}\nu+{}^{239}_{93}\text{Np};\ {}^{239}_{93}\text{Np}\rightarrow{}^{0}_{-1}\beta+{}^{0}_{0}\nu+\boxed{{}^{239}_{94}\text{Pu}}$

Estimation and Approximation

15 •

Picture the Problem There is no table of half lives in the text although the information is mentioned in the alpha particle discussion for alpha decay (about 15 orders of magnitude). Mass density in an atom ranges roughly as the cube of the radius of an atom to that of the nucleus, also about 15 orders of magnitude. Nuclear masses only range 2 orders of magnitude.

Material property	Ratio (order of magnitude)
Mass density	10^{15}
Half life	10^{15}
Nuclear masses	2

16 ••

Picture the Problem The mass of ^{235}U required is given by $m_{235} = \frac{N}{N_A} M_{235}$, where M_{235} is the molecular mass of ^{235}U and N is the number of fissions required to produce 10^{20} J. The mass of deuterium and tritium required can be found similarly.

(*a*) Relate the mass of ^{235}U required to the number of fissions N required:

$$m_{235} = \frac{N}{N_A} M_{235} \qquad (1)$$

where M_{235} is the molecular mass of ^{235}U.

Determine N:

$$N = \frac{E_{\text{annual}}}{E_{\text{per fission}}}$$

Substitute numerical values and evaluate N:

$$N = \frac{10^{20}\ \text{J}}{200\,\text{MeV} \times \frac{1.60\times10^{-19}\ \text{J}}{\text{eV}}} = 3.13\times10^{30}$$

Substitute numerical values in equation (1) and evaluate m_{235}:

$$m_{235} = \frac{3.13\times10^{30}}{6.02\times10^{23}\ \text{nuclei/mol}}(235\,\text{g/mol}) = \boxed{5.20\times10^{6}\ \text{kg}}$$

(*b*) Relate the mass of ^{2}H and ^{3}H required to the number of fusions N required:

$$m_{^2\text{H}+^3\text{H}} = \frac{N}{N_\text{A}} M_{^2\text{H}+^3\text{H}} \qquad (2)$$

where $M_{^2\text{H}+^3\text{H}}$ is the molecular mass of ^{2}H + ^{3}H.

Determine N:

$$N = \frac{E_\text{annual}}{E_\text{per fission}}$$

Substitute numerical values and evaluate N:

$$N = \frac{10^{20}\ \text{J}}{18\,\text{MeV} \times \dfrac{1.60\times10^{-19}\ \text{J}}{\text{eV}}} = 3.47\times10^{31}$$

Substitute numerical values in equation (2) and evaluate $m_{^2\text{H}+^3\text{H}}$:

$$m_{235} = \frac{3.47\times10^{31}}{6.02\times10^{23}\ \text{nuclei/mol}}(5\,\text{g/mol}) = \boxed{2.88\times10^{6}\ \text{kg}}$$

Properties of Nuclei

***17 •**

Picture the Problem To find the binding energy of a nucleus we add the mass of its neutrons to the mass of its protons and then subtract the mass of the nucleus and multiply by c^2. To convert to MeV we multiply this result by 931.5 MeV/u. The binding energy per nucleon is the ratio of the binding energy to the mass number of the nucleus.

(*a*) For ^{12}C, Z = 6 and N = 6. Add the mass of the neutrons to that of the protons:

$$6m_\text{p} + 6m_\text{n} = 6\times1.007825\,\text{u} + 6\times1.008665\,\text{u} = 12.098940\,\text{u}$$

Subtract the mass of ^{12}C from this result:

$$(6m_\text{p} + 6m_\text{n}) - m_{^{12}\text{C}} = 12.098940\,\text{u} - 12\,\text{u} = 0.098940\,\text{u}$$

Multiply the mass difference by c^2 and convert to MeV:

$$E_\text{b} = (\Delta m)c^2 = (0.098940\,\text{u})c^2 \times \frac{931.5\,\text{MeV}/c^2}{1\text{u}} = \boxed{92.2\,\text{MeV}}$$

and the binding energy per nucleon is $\dfrac{E_b}{A} = \dfrac{92.2\,\text{MeV}}{12} = \boxed{7.68\,\text{MeV}}$

(b) For ^{56}Fe, Z = 26 and N = 30. Add the mass of the neutrons to that of the protons:

$$26m_p + 30m_n = 26\times 1.007825\,\text{u} + 30\times 1.008665\,\text{u} = 56.463400\,\text{u}$$

Subtract the mass of ^{56}Fe from this result:

$$(26m_p + 30m_n) - m_{^{12}\text{C}} = 56.463400\,\text{u} - 55.934942\,\text{u} = 0.528458\,\text{u}$$

Multiply the mass difference by c^2 and convert to MeV:

$$E_b = (\Delta m)c^2 = (0.528458\,\text{u})c^2 \times \frac{931.5\,\text{MeV}/c^2}{1\,\text{u}} = \boxed{492\,\text{MeV}}$$

and the binding energy per nucleon is $\dfrac{E_b}{A} = \dfrac{492\,\text{MeV}}{56} = \boxed{8.79\,\text{MeV}}$

(c) For ^{238}U, Z = 92 and N = 146. Add the mass of the neutrons to that of the protons:

$$92m_p + 146m_n = 92\times 1.007825\,\text{u} + 146\times 1.008665\,\text{u} = 239.984990\,\text{u}$$

Subtract the mass of ^{238}U from this result:

$$(92m_p + 146m_n) - m_{^{238}\text{U}} = 239.984990\,\text{u} - 238.050783\,\text{u} = 1.934207\,\text{u}$$

Multiply the mass difference by c^2 and convert to MeV:

$$E_b = (\Delta m)c^2 = (1.934207\,\text{u})c^2 \times \frac{931.5\,\text{MeV}/c^2}{1\,\text{u}} = \boxed{1802\,\text{MeV}}$$

and the binding energy per nucleon is $\dfrac{E_b}{A} = \dfrac{1802\,\text{MeV}}{238} = \boxed{7.57\,\text{MeV}}$

18 •

Picture the Problem To find the binding energy of a nucleus we add the mass of its neutrons to the mass of its protons and then subtract the mass of the nucleus and multiply by c^2. To convert to MeV we multiply this result by 931.5 MeV/u. The binding energy per nucleon is the ratio of the binding energy to the mass number of the nucleus.

(*a*) For ^{6}Li, Z = 3 and *N* = 3. Add the mass of the neutrons to that of the protons:

$$3m_\mathrm{p} + 3m_\mathrm{n} = 3\times 1.007825\,\mathrm{u} + 3\times 1.008665\,\mathrm{u} = 6.049470\,\mathrm{u}$$

Subtract the mass of ^{6}Li from this result:

$$(3m_\mathrm{p} + 3m_\mathrm{n}) - m_{^6\mathrm{L}i} = 6.049470\,\mathrm{u} - 6.015122\,\mathrm{u} = 0.034348\,\mathrm{u}$$

Multiply the mass difference by c^2 and convert to MeV:

$$E_\mathrm{b} = (\Delta m)c^2 = (0.034348\,\mathrm{u})c^2 \times \frac{931.5\,\mathrm{MeV}/c^2}{1\mathrm{u}} = \boxed{32.0\,\mathrm{MeV}}$$

and the binding energy per nucleon is $\dfrac{E_\mathrm{b}}{A} = \dfrac{32.0\,\mathrm{MeV}}{6} = \boxed{5.33\,\mathrm{MeV}}$

(*b*) For ^{39}K, Z = 19 and *N* = 20. Add the mass of the neutrons to that of the protons:

$$19m_\mathrm{p} + 20m_\mathrm{n} = 19\times 1.007825\,\mathrm{u} + 20\times 1.008665\,\mathrm{u} = 39.321975\,\mathrm{u}$$

Subtract the mass of ^{39}K from this result:

$$(19m_\mathrm{p} + 20m_\mathrm{n}) - m_{^{39}\mathrm{K}} = 39.321975\,\mathrm{u} - 38.963707\,\mathrm{u} = 0.358268\,\mathrm{u}$$

Multiply the mass difference by c^2 and convert to MeV:

$$E_\mathrm{b} = (\Delta m)c^2 = (0.358268\,\mathrm{u})c^2 \times \frac{931.5\,\mathrm{MeV}/c^2}{1\mathrm{u}} = \boxed{334\,\mathrm{MeV}}$$

and the binding energy per nucleon is $\dfrac{E_\mathrm{b}}{A} = \dfrac{334\,\mathrm{MeV}}{39} = \boxed{8.56\,\mathrm{MeV}}$

(*c*) For ^{208}Pb, Z = 82 and *N* = 126. Add the mass of the neutrons to that of the protons:

$$82m_\text{p} + 126m_\text{n} = 82\times1.007825\,\text{u} + 126\times1.008665\,\text{u} = 209.733440\,\text{u}$$

Subtract the mass of ^{208}Pb from this result:

$$(82m_\text{p} + 126m_\text{n}) - m_{^{208}\text{Pb}} = 209.733440\,\text{u} - 207.976636\,\text{u} = 1.756804\,\text{u}$$

Multiply the mass difference by c^2 and convert to MeV:

$$E_\text{b} = (\Delta m)c^2 = (1.756804\,\text{u})c^2 \times \frac{931.5\,\text{MeV}/c^2}{1\,\text{u}} = \boxed{1636\,\text{MeV}}$$

and the binding energy per nucleon is $\dfrac{E_\text{b}}{A} = \dfrac{1636\,\text{MeV}}{208} = \boxed{7.87\,\text{MeV}}$

19 •

Picture the Problem The nuclear radius is given by $R = R_0 A^{1/3}$ where R_0 = 1.2 fm.

(a) The radius of ^{16}O is: $R_{^{16}\text{O}} = (1.2\,\text{fm})(16)^{1/3} = \boxed{3.02\,\text{fm}}$

(b) The radius of ^{56}Fe is: $R_{^{56}\text{Fe}} = (1.2\,\text{fm})(56)^{1/3} = \boxed{4.59\,\text{fm}}$

(c) The radius of ^{197}Au is: $R_{^{197}\text{Au}} = (1.2\,\text{fm})(197)^{1/3} = \boxed{6.98\,\text{fm}}$

20 •

Picture the Problem The nuclear radius is given by $R = R_0 A^{1/3}$ where R_0 = 1.2 fm.

The radii of the daughter nuclei are given by:

$$R = R_0 A^{1/3} \qquad (1)$$

Because the ratio of the mass numbers of the daughter nuclei is 3 to 1:

$$A_1 = \frac{3}{4}(239) \text{ and } A_2 = \frac{1}{4}(239)$$

Substitute in equation (1) to obtain:

$$R_1 = (1.2\,\text{fm})\left(\frac{3\times239}{4}\right)^{1/3} = \boxed{6.77\,\text{fm}}$$

and

$$R_2 = (1.2\,\text{fm})\left(\frac{239}{4}\right)^{1/3} = \boxed{4.69\,\text{fm}}$$

***21** ••

Picture the Problem The speed of the neutrons can be found from their thermal energy. The time taken to reduce the intensity of the beam by one-half, from I to $I/2$, is the half-life of the neutron. Because the beam is monoenergetic, the neutrons all travel at the same speed.

(*a*) The thermal energy of the neutron is:

$$\begin{aligned} E_{\text{thermal}} &= kT \\ &= \left(1.38\times10^{-23}\ \text{J/K}\right)(25+273)\text{K} \\ &= \boxed{4.11\times10^{-21}\ \text{J}} \\ &= 4.11\times10^{-21}\ \text{J}\times\frac{1\text{eV}}{1.60\times10^{-19}\ \text{J}} \\ &= \boxed{25.7\,\text{meV}} \end{aligned}$$

(*b*) Equate E_{thermal} and the kinetic energy of the neutron to obtain:

$$E_{\text{thermal}} = \tfrac{1}{2}m_{\text{n}}v^2$$

Solve for v to obtain:

$$v = \sqrt{\frac{2E_{\text{thermal}}}{m_{\text{n}}}}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{\frac{2\left(4.11\times10^{-21}\ \text{J}\right)}{1.67\times10^{-27}\ \text{kg}}} = \boxed{2.22\,\text{km/s}}$$

(*c*) Relate the half-life, $t_{1/2}$, to the speed of the neutrons in the beam:

$$t_{1/2} = \frac{x}{v}$$

Substitute numerical values and evaluate $t_{1/2}$:

$$\begin{aligned} t_{1/2} &= \frac{1350\ \text{km}}{2.22\,\text{km/s}} = 608\,\text{s}\times\frac{1\text{min}}{60\,\text{s}} \\ &= \boxed{10.1\text{min}} \end{aligned}$$

22 •

Picture the Problem We can use the definition of density, the equation for the volume of a sphere, and the given approximation to calculate the density of nuclear matter in grams per cubic centimeter.

Express the density of a spherical nucleus:

$$\rho = \frac{m}{V}$$

The approximate mass is:

$$m = \left(1.66\times10^{-27}\ \text{kg}\right)A$$

Express the volume of the nucleus:

$$V = \tfrac{4}{3}\pi\left(R_0A^{1/3}\right)^3 = \tfrac{4}{3}\pi R_0^3A$$

Substitute for m and V to obtain:

$$\rho = \frac{\left(1.66\times10^{-27}\ \text{kg}\right)A}{\frac{4}{3}\pi R_0^3 A} = \frac{3\left(1.66\times10^{-27}\ \text{kg}\right)}{4\pi R_0^3}$$

Substitute numerical values and evaluate ρ:

$$\rho = \frac{3\left(1.66\times10^{-27}\ \text{kg}\right)}{4\pi(1.2\ \text{fm})^3} = 2.29\times10^{17}\ \text{kg/m}^3 = \boxed{2.29\times10^{14}\ \text{g/cm}^3}$$

23 ••

Picture the Problem The separation of the nuclei when they are just touching is the sum of their radii, which is given by $R = R_0A^{1/3}$.

The electrostatic potential energy of the system is given by:

$$U = k\frac{q_1q_2}{R} = k\frac{(Z_{\text{Mo}}e)(Z_{\text{La}}e)}{R_{\text{Mo}} + R_{\text{La}}}$$

where R is the distance from the center of the ^{95}Mo nucleus to the center of the ^{139}La nucleus.

The radii of the nuclei are:

$$R_{\text{Mo}} = R_{\text{o}}A_{\text{Mo}}^{1/3} \text{ and } R_{\text{La}} = R_{\text{o}}A_{\text{La}}^{1/3}$$

Substitute for R_{Mo} and R_{La} and simplify to obtain:

$$U = ke^2\frac{(Z_{\text{Mo}})(Z_{\text{La}})}{R_{\text{o}}A_{\text{Mo}}^{1/3} + R_{\text{o}}A_{\text{La}}^{1/3}} = \frac{ke^2}{R_{\text{o}}}\frac{(Z_{\text{Mo}})(Z_{\text{La}})}{\left(A_{\text{Mo}}^{1/3} + A_{\text{La}}^{1/3}\right)}$$

Substitute numerical values and evaluate U:

$$U = \frac{\left(8.99\times10^9\ \text{Nm}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(1.2\times10^{-15}\ \text{m}\right)}\frac{(42)(57)}{\left(95^{1/3} + 139^{1/3}\right)}$$
$$= 4.71\times10^{-11}\ \text{J}\times\frac{1\text{eV}}{1.60\times10^{-19}\ \text{J}} = \boxed{295\ \text{MeV}}$$

***24** ••

Picture the Problem The Heisenberg uncertainty principle relates the uncertainty in position, Δx, to the uncertainty in momentum, Δp, by $\Delta x\Delta p \geq \frac{1}{2}\hbar$.

Solve the Heisenberg equation for Δp:

$$\Delta p \approx \frac{\hbar}{2\Delta x}$$

Substitute numerical values and evaluate Δp:

$$\Delta p \approx \frac{1.05\times10^{-34}\,\text{J}\cdot\text{s}}{2\left(10\times10^{-15}\,\text{m}\right)} = 5.25\times10^{-21}\,\text{kg}\cdot\text{m/s}$$

The kinetic energy of the electron is given by:

$$K = pc$$

Substitute numerical values and evaluate K:

$$K = \left(5.25\times10^{-21}\,\text{kg}\cdot\text{m/s}\right)\left(3\times10^{8}\,\text{m/s}\right) = 1.58\times10^{-12}\,\text{J}\times\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}} = 9.88\,\text{MeV}$$

This result contradicts experimental observations that show that the energy of electrons in unstable atoms is of the order of 1 to 1000 eV.

Radioactivity

25 •

Picture the Problem The counting rate, as a function of the number of half-lives *n*, is given by $R = \left(\frac{1}{2}\right)^n R_0$.

(*a*) The counting rate after *n* half-lives is:

$$R = \left(\tfrac{1}{2}\right)^n R_0$$

Solve for *n* to obtain:

$$n = \frac{\ln\left(R/R_0\right)}{\ln\left(\frac{1}{2}\right)}$$

Substitute numerical values and evaluate *n*:

$$n = \frac{\ln\left(\dfrac{1000\,\text{counts/s}}{4000\,\text{counts/s}}\right)}{\ln\left(\frac{1}{2}\right)} = 2$$

Because there are two half-lives in 10 min:

$$t_{1/2} = \boxed{5\,\text{min}}$$

(*b*) At the end of 4 half-lives:

$$R = \left(\tfrac{1}{2}\right)^4\left(4000\,\text{counts/s}\right) = \boxed{250\,\text{Bq}}$$

26 •

Picture the Problem The counting rate, as a function of the number of half-lives *n*, is given by $R = \left(\frac{1}{2}\right)^n R_0$.

(*a*) When t = 4 min, two half-lives will have passed and n = 2:

$$R = \left(\tfrac{1}{2}\right)^2\left(2000\,\text{counts/s}\right) = \boxed{500\,\text{Bq}}$$

(*b*) When t = 6 min, three half-lives will have passed and n = 3:

$$R = \left(\tfrac{1}{2}\right)^3 (2000\,\text{counts/s}) = \boxed{250\,\text{Bq}}$$

(*c*) When t = 8 min, four half-lives will have passed and n = 4:

$$R = \left(\tfrac{1}{2}\right)^4 (2000\,\text{counts/s}) = \boxed{125\,\text{Bq}}$$

27 •

Picture the Problem The counting rate, as a function of the number of half-lives n, is given by $R = \left(\tfrac{1}{2}\right)^n R_0$ and the decay constant λ is related to the half-life by $t_{1/2} = \ln 2/\lambda$.

(*a*) Relate the counting rate at time t = 10 min to the counting rate at t = 0:

$$R_{10\,\text{min}} = \left(\tfrac{1}{2}\right)^n R_0$$

Solve for n:

$$n = \frac{\ln\left(R_{10\,\text{min}}/R_0\right)}{\ln\left(\tfrac{1}{2}\right)}$$

Substitute numerical values and evaluate n:

$$n = \frac{\ln\left(\dfrac{1000\,\text{counts/s}}{8000\,\text{counts/s}}\right)}{\ln\left(\tfrac{1}{2}\right)} = 3$$

Therefore, 3 half-lives have passed in 10 min:

$$3t_{1/2} = 10\,\text{min} \Rightarrow t_{1/2} = \boxed{200\,\text{s}}$$

(*b*) The decay constant λ is related to the half-life by:

$$t_{1/2} = \frac{\ln 2}{\lambda} \Rightarrow \lambda = \frac{\ln 2}{t_{1/2}}$$

Substitute for $t_{1/2}$ and evaluate λ:

$$\lambda = \frac{\ln 2}{200\,\text{s}} = \boxed{3.47\times10^{-3}\,\text{s}^{-1}}$$

(*c*) Six half-lives will have passed in 20 min:

$$R_{20\,\text{min}} = \left(\tfrac{1}{2}\right)^6 (8000\,\text{Bq}) = \boxed{125\,\text{Bq}}$$

28 •

Picture the Problem We can use $R = \lambda N_0 e^{-\lambda t}$ to show that the disintegration rate is approximately 1 Ci.

The decay rate is given by:

$$R = \lambda N_0 e^{-\lambda t}$$

where N_0 is the number of nuclei at t = 0.

The decay constant λ is related to the half-life by:

$$t_{1/2} = \frac{\ln 2}{\lambda} \Rightarrow \lambda = \frac{\ln 2}{t_{1/2}}$$

Substitute for $t_{1/2}$ and evaluate λ:

$$\lambda = \frac{\ln 2}{1620\,\text{y}} = 4.28\times10^{-4}\,\text{y}^{-1}\times\frac{\text{y}}{31.56\,\text{Ms}}$$
$$= 1.356\times10^{-11}\,\text{s}^{-1}$$

The number of nuclei at $t = 0$ is given by:

$N_0 = N_A/M$
where M is the atomic mass of radium and N_A is Avogadro's number.

Substitute numerical values and evaluate N_0:

$$N_0 = \frac{6.02\times10^{23}}{226} = 2.664\times10^{21}$$

Substitute numerical values for λ and N_0 and evaluate R:

$$R = \left(1.356\times10^{-11}\,\text{s}^{-1}\right)\left(2.664\times10^{21}\right)e^{-\left(1.356\times10^{-11}\,\text{s}^{-1}\right)(1200\,\text{s})} = 3.61\times10^{10}\,\text{s}^{-1}$$
$$\approx 3.7\times10^{10}\,\text{s}^{-1} = \boxed{1\,\text{Ci}}$$

29 •

Picture the Problem We can use $R = \left(\frac{1}{2}\right)^n R_0$ to relate the counting rate R to the number of half-lives n that have passed since $t = 0$. The detection efficiency depends on the probability that a radioactive decay particle will enter the detector and the probability that upon entering the detector it will produce a count. If the efficiency is 20 percent, the decay rate must be 5 times the counting rate.

(*a*) When $t = 2.4$ min, $n = 1$ and:

$$R_{2.4\,\text{min}} = \left(\tfrac{1}{2}\right)^1\left(1000\,\text{counts/s}\right) = \boxed{500\,\text{Bq}}$$

When $t = 4.8$ min, $n = 2$ and:

$$R_{4.8\,\text{min}} = \left(\tfrac{1}{2}\right)^2\left(1000\,\text{counts/s}\right) = \boxed{250\,\text{Bq}}$$

(*b*) The number of radioactive nuclei is related to the decay rate R, and the decay constant λ:

$$R = \lambda N \Rightarrow N = \frac{R}{\lambda} \qquad (1)$$

The decay constant is related to the half-life:

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{2.4\,\text{min}} = \frac{0.693}{144\,\text{s}}$$
$$= 4.813\times10^{-3}\,\text{s}^{-1}$$

Calculate the decay rate at $t = 0$

$$R_0 = 5\times1000\,\text{counts/s} = 5000\,\text{s}^{-1}$$

from the counting rate:

Substitute in equation (1) and evaluate N_0 at $t = 0$:

$$N_0 = \frac{R_0}{\lambda} = \frac{5000\,\text{s}^{-1}}{4.813\times10^{-3}\,\text{s}^{-1}} = \boxed{1.04\times10^{6}}$$

Calculate the decay rate at $t = 2.4$ min from the counting rate:

$$R_{2.4\,\text{min}} = 5\times500\,\text{counts/s} = 2500\,\text{s}^{-1}$$

Substitute in equation (1) and evaluate $N_{2.4\text{ min}}$ at $t = 0$:

$$N_{2.4\,\text{min}} = \frac{R_{2.4\,\text{min}}}{\lambda} = \frac{2500\,\text{s}^{-1}}{4.813\times10^{-3}\,\text{s}^{-1}} = \boxed{5.19\times10^{5}}$$

(*c*) The time at which the counting rate will be about 30 counts/s is the product of the number of half-lives that will have passed and the half-life:

$$t = nt_{1/2} \qquad (2)$$

The counting rate R after n half-lives is related to the counting rate at $t = 0$ by:

$$R = \left(\tfrac{1}{2}\right)^n R_0$$

Solve for n:

$$n = \frac{\ln(R/R_0)}{\ln\left(\frac{1}{2}\right)}$$

Substitute numerical values and evaluate n:

$$n = \frac{\ln(30\,\text{counts/s}/1000\,\text{counts/s})}{\ln\left(\frac{1}{2}\right)} = 5.059$$

Substitute numerical values for n and $t_{1/2}$ in equation (2) and evaluate t:

$$t = (5.059)(2.4\,\text{min}) = \boxed{12.1\,\text{min}}$$

30 •

Picture the Problem Knowing each of these reactions, we can use Table 40-1 to find the differences in the masses of the nuclei and then convert this difference into the energy released in each reaction.

(*a*) Write the reaction:

$${}^{226}\text{Ra}\rightarrow{}^{222}\text{Rn}+{}^{4}\text{He}$$

Use Table 40-1 to find ΔE:

$$\Delta E = \frac{931.5\,\text{MeV}/c^2}{1\,\text{u}}\left(226.025403\,\text{u} - 222.017571\,\text{u} - 4.002603\,\text{u}\right)c^2 = \boxed{4.87\,\text{MeV}}$$

(*b*) Write the reaction:

$$^{242}\text{Pu} \rightarrow {}^{238}\text{U} + {}^{4}\text{He}$$

Use Table 40-1 to find ΔE:

$$\Delta E = \frac{931.5\,\text{MeV}/c^2}{1\,\text{u}}\left(242.058737\,\text{u} - 238.050783\,\text{u} - 4.002603\,\text{u}\right)c^2 = \boxed{4.98\,\text{MeV}}$$

***31 ••**

Picture the Problem Each ^{239}Pu nucleus emits an alpha particle whose activity, *A*, depends on the decay constant of ^{239}Pu and on the number *N* of nuclei present in the ingested ^{239}Pu. We can find the decay constant from the half-life and the number of nuclei present from the mass ingested and the atomic mass of ^{239}Pu. Finally, we can use the dependence of the activity on time to find the time at which the activity be 1000 alpha particles per second.

(*a*) The activity of the nuclei present in the ingested ^{239}Pu is given by:

$$A = \lambda N \qquad (1)$$

Find the constant for the decay of ^{239}Pu:

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{0.693}{(24360\,\text{y})(31.56\,\text{Ms/y})} = 9.02\times10^{-13}\,\text{s}^{-1}$$

Express the number of nuclei present in the quantity of ^{239}Pu ingested:

$$N = m_{\text{Pu}}\frac{N_{\text{A}}}{M_{\text{Pu}}}$$

where M_{Pu} is the atomic mass of ^{239}Pu.

Substitute numerical values and evaluate *N*:

$$N = (2.0\,\mu\text{g})\left(\frac{6.02\times10^{23}\,\text{nuclei/mol}}{239\,\text{g/mol}}\right) = 5.04\times10^{15}\,\text{nuclei}$$

Substitute numerical values in equation (1) and evaluate *A*:

$$A = \left(9.02\times10^{-13}\,\text{s}^{-1}\right)\left(5.04\times10^{15}\,\alpha\right) = \boxed{4.55\times10^{3}\,\alpha/\text{s}}$$

(*b*) The activity varies with time according to:

$$A = A_o e^{-\lambda t}$$

Solve for t to obtain:

$$t = \frac{\ln\left(\frac{A}{A_o}\right)}{-\lambda}$$

Substitute numerical values and evaluate t:

$$t = \frac{\ln\left(\frac{1\times10^{3}\,\alpha/\mathrm{s}}{4.55\times10^{3}\,\alpha/\mathrm{s}}\right)}{-\left(9.02\times10^{-13}\,\mathrm{s}^{-1}\right)\left(\frac{31.56\,\mathrm{Ms}}{1\,\mathrm{y}}\right)} = \boxed{5.32\times10^{4}\ \mathrm{y}}$$

32 ••

Picture the Problem We can use conservation of energy and conservation of linear momentum to relate the momenta and kinetic energies of the nuclei to the decay's Q value

(a) Express the kinetic energies of the alpha particle and daughter nucleus:

$$K_\alpha = \tfrac{1}{2}m_\alpha v_\alpha^2 = \frac{p_\alpha^2}{2m_\alpha} \qquad (1)$$

and

$$K_\mathrm{Y} = \tfrac{1}{2}m_\mathrm{Y} v_\mathrm{Y}^2 = \frac{p_\mathrm{Y}^2}{2m_\mathrm{Y}} \qquad (2)$$

Solve equations (1) and (2) for p_α^2 and p_D^2:

$$p_\alpha^2 = 2m_\alpha K_\alpha$$

and

$$p_\mathrm{Y}^2 = 2m_\mathrm{Y} K_\mathrm{Y}$$

From the conservation of linear momentum we have:

$$\vec{p}_\mathrm{i} = \vec{p}_\mathrm{f}$$

or, because the parent is initially at rest,

$0 = p_\alpha - p_\mathrm{Y}$ and $p_\alpha = p_\mathrm{Y}$

Because the momenta are equal:

$$2m_\mathrm{Y}K_\mathrm{Y} = 2m_\alpha K_\alpha$$

Solve for K_Y:

$$K_\mathrm{Y} = \frac{m_\alpha}{m_\mathrm{Y}}K_\alpha = \frac{4}{A-4}K_\alpha$$

Because the daughter nucleus and the alpha particle share the Q-value:

$$Q = K_\mathrm{Y} + K_\alpha = \frac{4}{A-4}K_\alpha + K_\alpha = \left(\frac{4}{A-4}+1\right)K_\alpha = \left(\frac{A}{A-4}\right)K_\alpha$$

Solve for K_α:

$$K_\alpha = \boxed{\left(\frac{A-4}{A}\right)Q}$$

(*b*) Substitute for K_α in the expression for Q to obtain:

$$Q = K_Y + \left(\frac{A-4}{A}\right)Q$$

Solve for K_Y:

$$K_Y = Q - \left(\frac{A-4}{A}\right)Q = \boxed{\frac{4Q}{A}}$$

***33 ••**

Picture the Problem We can write the equation of the decay process by using the fact that the post-decay sum of the Z and A numbers must equal the pre-decay values of the parent nucleus. The Q value in the equations from Problem 32 is given by $Q = -(\Delta m)c^2$.

^{239}Pu undergoes alpha decay according to:

$$\boxed{{}^{239}_{94}\text{Pu} \rightarrow {}^{235}_{92}\text{U} + {}^{4}_{2}\alpha + Q}$$

The Q value for the decay is given by:

$$Q = \left[(m_{\text{Pu}}) - (m_{\text{U}} + m_\alpha)\right]\left(\frac{931.5\,\text{MeV}}{1\text{u}}\right)$$

Substitute numerical values and evaluate Q:

$$Q = \left[(239.052156\,\text{u}) - (235.043923\,\text{u} + 4.002603\,\text{u})\right]\left(\frac{931.5\text{MeV}}{1\text{u}}\right) = \boxed{5.24\,\text{MeV}}$$

From Problem 32, the kinetic energy of the alpha particle is given by:

$$K_\alpha = \left(\frac{A-4}{A}\right)Q$$

Substitute numerical values and evaluate K_α:

$$K_\alpha = \left(\frac{239-4}{239}\right)(5.24\,\text{MeV}) = \boxed{5.15\,\text{MeV}}$$

From Problem 32, the kinetic energy of the ^{239}U is given by:

$$K_{\text{U}} = \frac{4Q}{A}$$

Substitute numerical values and evaluate K_{U}:

$$K_{\text{U}} = \frac{4(5.24\,\text{MeV})}{239} = \boxed{87.7\,\text{keV}}$$

34 •

Picture the Problem We can find the age of the sample using $R_n = \left(\tfrac{1}{2}\right)^n R_0$ to find n and then applying $t = nt_{1/2}$.

Express the age of the bone in terms of the half-life of ^{14}C and the

$$t = nt_{1/2} \qquad (1)$$

number n of half-lives that have elapsed:

The decay rate R_n after n half-lives is related to the counting rate R_0 at $t = 0$ by:

$$R_n = \left(\tfrac{1}{2}\right)^n R_0$$

Solve for n:

$$n = \frac{\ln\left(R/R_0\right)}{\ln\left(\tfrac{1}{2}\right)}$$

Because there are 15.0 decays per minute per gram of carbon in a living organism:

$$R_0 = 15.0\,\frac{\text{decays}}{\text{min}\cdot\text{g}} \times \frac{1\,\text{min}}{60\,\text{s}} \times 175\,\text{g}$$
$$= 43.75\,\text{Bq}$$

Substitute numerical values for R and R_0 and evaluate n:

$$n = \frac{\ln\left(\dfrac{8.1\,\text{Bq}}{43.75\,\text{Bq}}\right)}{\ln\left(\tfrac{1}{2}\right)} = 2.433$$

Substitute numerical values in equation (1) and evaluate t:

$$t = (2.433)(5730\,\text{y}) = \boxed{13{,}940\,\text{y}}$$

35 •

Picture the Problem We can solve $R = R_0 e^{-\lambda t}$ for λ to find the decay constant of the sample and use $t_{1/2} = \dfrac{\ln 2}{\lambda}$ to find its half-life. The number of radioactive nuclei in the sample initially can be found from $R_0 = \lambda N_0$.

(*a*) The decay rate is given by:

$$R = R_0 e^{-\lambda t}$$

Solve for λ to obtain:

$$\lambda = \frac{\ln\left(\dfrac{R}{R_0}\right)}{-t}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{\ln\left(\dfrac{85.2\,\text{Bq}}{115\,\text{Bq}}\right)}{-2.25\,\text{h}} = \boxed{0.133\,\text{h}^{-1}}$$

The half-life is related to the decay constant:

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{0.133\,\text{h}^{-1}} = \boxed{5.20\,\text{h}}$$

(*b*) The number N_0 of radioactive nuclei in the sample initially is related to the decay constant λ and the initial decay rate R_0:

$$R_0 = \lambda N_0 \Rightarrow N_0 = \frac{R_0}{\lambda}$$

Substitute numerical values and evaluate N_0:

$$N_0 = \frac{115\,\text{Bq}}{0.133\,\text{h}^{-1} \times \dfrac{1\,\text{h}}{3600\,\text{s}}} = \boxed{3.11\times10^{6}}$$

***36** ••

Picture the Problem We can use $R_0 = \lambda N$ to find the initial activity of the sample and $R = R_0 e^{-\lambda t}$ to find the activity of the sample after 1.75 y.

(*a*) The initial activity of the sample is the product of the decay constant λ for ^{60}Co and the number of atoms N of ^{60}Co initially present in the sample:

$$R_0 = \lambda N \qquad (1)$$

Express N in terms of the mass m of the sample, the molar mass M of ^{60}Co, and Avogadro's number N_A:

$$N = \frac{m}{M} N_A$$

Substitute numerical values and evaluate N:

$$N = \left(\frac{1.00\times10^{-6}\,\text{g}}{60\,\text{g/mol}}\right)\left(6.02\times10^{23}\,\text{nuclei/mol}\right) = 1.00\times10^{16}\ \text{nuclei}$$

The decay constant is given by:

$$\lambda = \frac{\ln 2}{t_{1/2}}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{0.693}{(5.27\ \text{y})(31.56\,\text{Ms/y})} = 4.17\times10^{-9}\,\text{s}^{-1}$$

Substitute numerical values in equation (1) and evaluate A_0:

$$R_0 = \left(4.17\times10^{-9}\,\text{s}^{-1}\right)\left(1.00\times10^{16}\,\text{nuclei}\right) = 4.17\times10^{7}\ \text{s}^{-1} \times \frac{1\,\text{Ci}}{3.7\times10^{10}\ \text{s}^{-1}} = \boxed{1.13\,\text{mCi}}$$

(*b*) The activity varies with time according to:

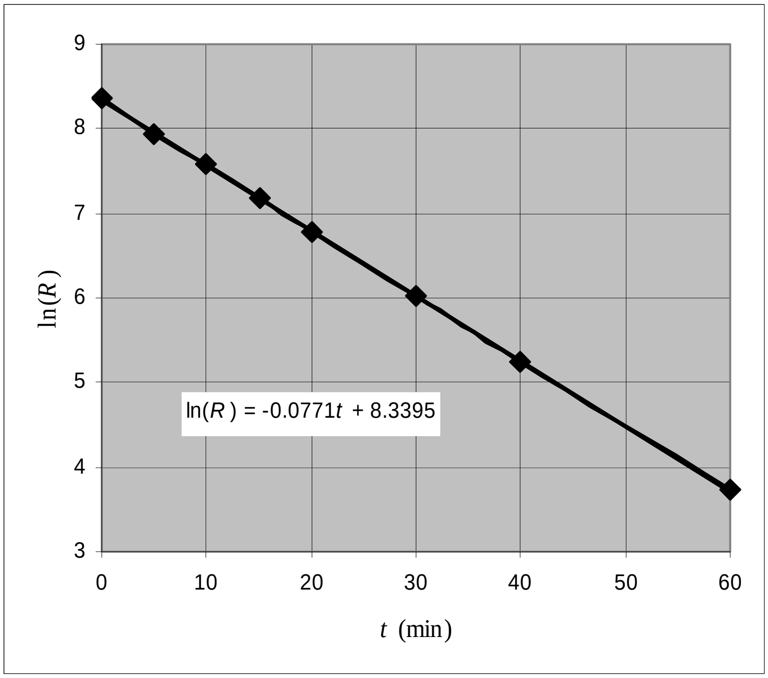
$$R = R_o e^{-\lambda t} = R_o e^{-\left(\frac{0.693t}{5.27\,\text{y}}\right)}$$

Evaluate R at t = 1.75 y:

$$R = (1.13\,\text{mCi})e^{-\left(\frac{0.693\times 1.75\,\text{y}}{5.27\,\text{y}}\right)} = \boxed{0.898\,\text{mCi}}$$

37 ••

Picture the Problem The following graph was plotted using a spreadsheet program. Excel's ″Add Trendline″ feature was used to determine the equation of the line.



The linearity and negative slope of this graph tell us that it represents an exponential decay.

The decay rate equation is:

$$R = R_0 e^{-\lambda t}$$

Take the natural logarithm of both sides of the equation to obtain:

$$\ln R = \ln e^{-\lambda t} + \ln R_0$$
$$= -\lambda t + \ln R_0$$

This equation is of the form:

$$y = mx + b$$

where $y = \ln R$, $x = t$, $m = -\lambda$, and $b = \ln R_0$.

The decay constant is the negative of the slope of the graph:

$$\lambda = \boxed{0.0771\,\text{min}^{-1}}$$

The half-life of the radioisotope is:

$$t_{1/2}=\frac{\ln 2}{\lambda}=\frac{\ln 2}{0.0771\,\text{min}^{-1}}=\boxed{8.99\,\text{min}}$$

38 ••

Picture the Problem We can solve Equation 40-7 for λ to show that $\lambda = t_1^{-1}\ln(R_0/R_1)$.

(*a*) Express the half-life as a function of the decay constant λ:

$$t_{1/2}=\frac{\ln 2}{\lambda} \qquad (1)$$

From Equation 40-7 it follows that:

$$\frac{R_0}{R_1}=e^{\lambda t}$$

Solve for λ:

$$\lambda=\frac{\ln\left(\frac{R_0}{R_1}\right)}{t}=\boxed{t^{-1}\ln\left(\frac{R_0}{R_1}\right)}$$

(*b*) Substitute numerical values for t, R_1, and R_0 and evaluate λ:

$$\lambda=\frac{1}{60\,\text{s}}\ln\left(\frac{1200\,\text{Bq}}{800\,\text{Bq}}\right)=\boxed{0.00676\,\text{s}^{-1}}$$

Use the decay constant to find the half-life:

$$t_{1/2}=\frac{\ln 2}{\lambda}=\frac{\ln 2}{0.00676\,\text{s}^{-1}}=\boxed{103\,\text{s}}$$

39 ••

Picture the Problem The required mass is given by M = (5 counts/min)/R, where R is the current counting rate per gram of carbon. We can use the assumed age of the casket to find the number of half-lives that have elapsed and $R=\left(\frac{1}{2}\right)^n R_0$ to find the current counting rate per gram of ^{14}C.

The mass of carbon required is:

$$M=\frac{5\,\text{counts/min}}{R} \qquad (1)$$

Because there were about 15.0 decays per minute per gram of the living wood, the counting rate per gram is:

$$R=\left(\tfrac{1}{2}\right)^n R_0=\left(\tfrac{1}{2}\right)^n(15\,\text{counts/min}\cdot\text{g})$$

We can find n from the assumed age of the casket and the half-life of ^{14}C:

$$n=\frac{18{,}000\,\text{y}}{5730\,\text{y}}=3.141$$

Substitute for n and evaluate R:

$$R = \left(\tfrac{1}{2}\right)^{3.141}\left(15\,\text{counts/min}\cdot\text{g}\right) = 1.70\,\text{counts/min}\cdot\text{g}$$

Substitute for R in equation (1) and evaluate M:

$$M = \frac{5\,\text{counts/min}}{1.70\,\text{counts/min}\cdot\text{g}} = \boxed{2.94\,\text{g}}$$

40 ••

Picture the Problem The decay constant λ can be found from the decay rate R and the number of radioactive nuclei N at the moment of interest and the half-life, in turn, can be found from the decay constant.

The decay rate R is related to the decay constant λ and the number of radioactive nuclei N at the moment of interest:

$$R = \lambda N \Rightarrow \lambda = \frac{R}{N} \qquad (1)$$

The number of radioactive nuclei N at the moment of interest can be found from Avogadro's number, the mass m of the sample, and the molar mass M of the sample:

$$N = N_{\text{A}}\frac{m}{M}$$

Substitute numerical values and evaluate N:

$$N = \left(6.02\times10^{23}\,\text{nuclei/mol}\right)\frac{10^{-3}\,\text{g}}{59.934\,\text{g/mol}} = 1.004\times10^{19}$$

Substitute numerical values in equation (1) and evaluate λ:

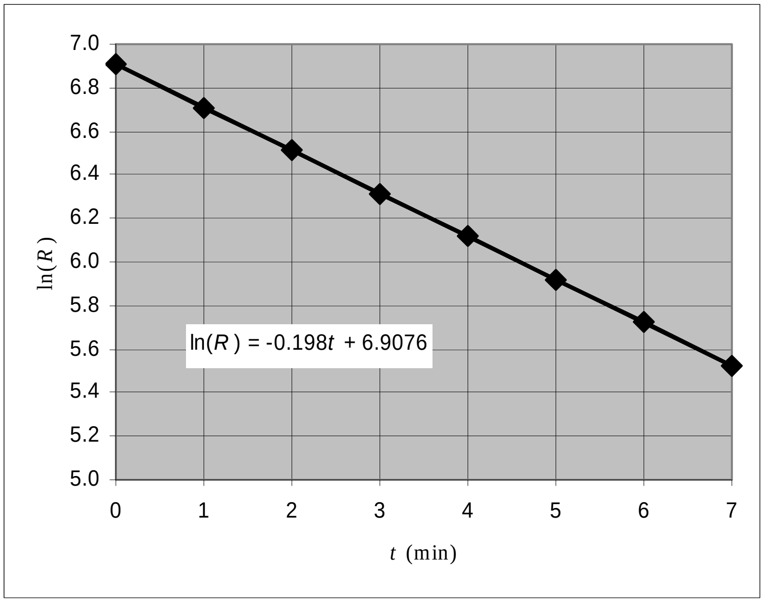
$$\lambda = \frac{1.131\,\text{Ci}\times\dfrac{3.7\times10^{10}\,\text{Bq}}{\text{Ci}}}{1.004\times10^{19}} = \boxed{4.17\times10^{-9}\,\text{s}^{-1}}$$

Find the half-life from the decay constant:

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{4.17\times10^{-9}\,\text{s}^{-1}} = 1.67\times10^{8}\,\text{s}\times\frac{1\,\text{y}}{31.56\,\text{Ms}} = \boxed{5.27\,\text{y}}$$

***41 ••**

Picture the Problem The following graph was plotted using a spreadsheet program. Excel's ″Add Trendline″ feature was used to determine the equation of the line.



The linearity and negative slope of this graph tells us that it represents an exponential decay.

The decay rate equation is: $R = R_0 e^{-\lambda t}$

Take the natural logarithm of both sides of the equation to obtain:

$$\ln R = \ln e^{-\lambda t} + \ln R_0$$
$$= -\lambda t + \ln R_0$$

This equation is of the form: $y = mx + b$

where $y = \ln R$, $x = t$, $m = -\lambda$, and $b = \ln R_0$.

The half-life of the radioisotope is:

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{0.198\,\text{min}^{-1}} = \boxed{3.50\,\text{min}}$$

42 ••

Picture the Problem We can use the decay rate equation $R = R_0 e^{-\lambda t}$ and the expression relating the half-life of a source to its decay constant to find the half-life of the sample. Solving the decay-rate equation for t will yield the time at which the activity level drops to any given value.

(*a*) The half-life of the material is given by:

$$t_{1/2}=\frac{\ln 2}{\lambda} \qquad (1)$$

The decay rate is given by:

$$R=R_0e^{-\lambda t} \qquad (2)$$

Solve for λ:

$$\lambda=\frac{\ln\left(\frac{R_0}{R}\right)}{t}$$

Substitute for λ in equation (1) to obtain:

$$t_{1/2}=\frac{\ln 2}{\frac{\ln\left(\frac{R_0}{R}\right)}{t}}=\frac{\ln 2}{\ln\left(\frac{R_0}{R}\right)}t$$

Substitute numerical values and evaluate $t_{1/2}$:

$$t_{1/2}=\frac{\ln 2}{\ln\left(\frac{115\,\text{decays/min}}{73.5\,\text{decays/min}}\right)}(101\,\text{h})=\boxed{156\,\text{h}}$$

(*b*) Solve equation (2) for t:

$$t=\frac{\ln\left(\frac{R}{R_0}\right)}{-\lambda}$$

Express λ in terms of $t_{1/2}$:

$$\lambda=\frac{\ln 2}{t_{1/2}}$$

Substitute for λ in the expression for t to obtain:

$$t=\frac{\ln\left(\frac{R}{R_0}\right)}{-\ln 2}t_{1/2}$$

Substitute numerical values and evaluate t:

$$t=\frac{\ln\left(\frac{10\,\text{decays/min}}{115\,\text{decays/min}}\right)}{-\ln 2}(156\,\text{h})=550\,\text{h}\times\frac{1\,\text{d}}{24\,\text{h}}=\boxed{22.9\,\text{d}}$$

43 ••

Picture the Problem We can use the decay rate equation $R=R_0e^{-\lambda t}$ and the expression relating the half-life of a source to its decay constant to find the age of the fossils.

The decay rate is given by:

$$R = R_0 e^{-\lambda t}$$

Solve for t to obtain:

$$t = \frac{\ln\left(\frac{R}{R_0}\right)}{-\lambda}$$

Express λ in terms of $t_{1/2}$:

$$\lambda = \frac{\ln 2}{t_{1/2}}$$

Substitute for λ in the expression for t to obtain:

$$t = \frac{\ln\left(\frac{R}{R_0}\right)}{-\ln 2} t_{1/2}$$

or, because the activity at any time is proportional to the number of radioactive nuclei present,

$$t = \frac{\ln\left(\frac{N_{\text{Rb}}}{N_{0,\text{Rb}}}\right)}{-\ln 2} t_{1/2} \qquad (1)$$

The number of ^{87}Sr nuclei present in the rocks is given by:

$$N_{\text{Sr}} = N_{0,\text{Rb}} - N_{\text{Rb}} \Rightarrow N_{0,\text{Rb}} = N_{\text{Sr}} + N_{\text{Rb}}$$

We're given that:

$$N_{\text{Sr}} = 0.01 N_{\text{Rb}} \Rightarrow \frac{N_{\text{Sr}}}{N_{\text{Rb}}} = 0.01$$

Express the ratio of $N_{0,\text{Rb}}$ to N_{Rb}:

$$\frac{N_{0,\text{Rb}}}{N_{\text{Rb}}} = \frac{N_{\text{Sr}} + N_{\text{Rb}}}{N_{\text{Rb}}} = \frac{N_{\text{Sr}}}{N_{\text{Rb}}} + 1$$
$$= 0.01 + 1 = 1.01$$

Substitute numerical values in equation (1) and evaluate the age of the fossils:

$$t = \frac{\ln\left(\frac{1}{1.01}\right)}{-\ln 2}\left(4.9\times10^{10}\text{ y}\right)$$
$$= \boxed{7.03\times10^{8}\text{ y}}$$

44 •••

Picture the Problem We can evaluate this integral by changing variables to obtain a form that we can find in a table of integrals.

Change variables by letting:

$$x = \lambda t$$

Then:

$$dx = \lambda dt\ ,\ dt = \frac{dx}{\lambda}\text{, and } t = \frac{x}{\lambda}$$

Substitute to obtain:

$$\tau = \int_0^\infty t\lambda e^{-\lambda t}dt = \int_0^\infty \frac{x}{\lambda}\lambda e^{-x}\frac{dx}{\lambda} = \frac{1}{\lambda}\int_0^\infty xe^{-x}dx$$

From integral tables:

$$\int_0^\infty xe^{-x}dx = 1$$

Substitute in the expression for τ to obtain:

$$\tau = \boxed{\frac{1}{\lambda}}$$

Nuclear Reactions

45 •

Picture the Problem We can use $Q = -(\Delta m)c^2$ to find the Q values for these reactions.

(a) Find the mass of each atom from Table 40-1:

$$m_{^1\mathrm{H}} = 1.007825\ \mathrm{u}$$
$$m_{^3\mathrm{H}} = 3.016049\ \mathrm{u}$$
$$m_{^3\mathrm{He}} = 3.016029\ \mathrm{u}$$
$$m_{\mathrm{n}} = 1.008665\,\mathrm{u}$$

Calculate the initial mass m_i of the incoming particles:

$$m_{\mathrm{i}} = 1.007825\ \mathrm{u} + 3.016049\ \mathrm{u} = 4.023874\ \mathrm{u}$$

Calculate the final mass m_f:

$$m_{\mathrm{f}} = 3.016029\ \mathrm{u} + 1.008665\ \mathrm{u} = 4.024694\ \mathrm{u}$$

Calculate the increase in mass:

$$\Delta m = m_{\mathrm{f}} - m_{\mathrm{i}} = 4.024694\ \mathrm{u} - 4.023874\ \mathrm{u} = 0.000820\ \mathrm{u}$$

Calculate the Q value:

$$Q = -(\Delta m)c^2 = -(0.000820\,\mathrm{u})c^2\left(931.5\frac{\mathrm{MeV}/c^2}{u}\right) = \boxed{-0.764\,\mathrm{MeV}}$$

(*b*) Proceed as in (*a*) to obtain:

$$Q = (0.003510\,\text{u})\left(931.5\,\frac{\text{MeV}/c^2}{\text{u}}\right)c^2$$
$$= \boxed{3.27\,\text{MeV}}$$

Remarks: Because *Q* < 0 for the first reaction, it is endothermic. Because *Q* > 0 for the second reaction, it is exothermic.

46 •

Picture the Problem We can use $Q = -(\Delta m)c^2$ to find the *Q* values for these reactions.

(*a*) Find the mass of each atom from Table 40-1:

$$m_{^2\text{H}} = 2.014102\ \text{u}$$
$$m_{^3\text{H}} = 3.016049\ \text{u}$$
$$m_{^1\text{H}} = 1.007825\ \text{u}$$

Calculate the initial mass m_i of the incoming particles:

$$m_i = 2(2.014102\ \text{u})$$
$$= 4.028204\ \text{u}$$

Calculate the final mass m_f:

$$m_f = 3.016049\ \text{u} + 1.007825\ \text{u}$$
$$= 4.023874\ \text{u}$$

Calculate the increase in mass:

$$\Delta m = m_f - m_i$$
$$= 4.023874\ \text{u} - 4.028204\ \text{u}$$
$$= -0.004330\ \text{u}$$

Calculate the *Q* value:

$$Q = -(\Delta m)c^2$$
$$= -(-0.004330\,\text{u})c^2\left(931.5\,\frac{\text{MeV}/c^2}{u}\right)$$
$$= \boxed{4.03\,\text{MeV}}$$

(*b*) Proceed as in (*a*) to obtain:

$$Q = -(\Delta m)c^2$$
$$= -(-0.019703\,\text{u})\left(931.5\,\frac{\text{MeV}/c^2}{\text{u}}\right)c^2$$
$$= \boxed{18.4\,\text{MeV}}$$

(*c*) Proceed as in (*a*) to obtain:

$$Q = -(\Delta m)c^2$$
$$= -(-0.005135\,\text{u})\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2$$
$$= \boxed{4.78\,\text{MeV}}$$

***47 ••**

Picture the Problem We can use $Q = -(\Delta m)c^2$ to find the Q values for this reaction.

(*a*) The masses of the atoms are:

$$m_{^{14}\text{C}} = 14.003242\,\text{u}$$
$$m_{^{14}\text{N}} = 14.003074\,\text{u}$$

Calculate the increase in mass:

$$\Delta m = m_\text{f} - m_\text{i}$$
$$= 14.003074\,\text{u} - 14.003242\,\text{u}$$
$$= -0.000168\,\text{u}$$

Calculate the Q value:

$$Q = -(\Delta m)c^2$$
$$= -(-0.000168\,\text{u})c^2\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)$$
$$= \boxed{0.156\,\text{MeV}}$$

(*b*)

The masses given are for atoms, not nuclei, so for nuclear masses the masses are too large by the atomic number times the mass of an electron. For the given nuclear reaction, the mass of the carbon atom is too large by $6m_\text{e}$ and the mass of the nitrogen atom is too large by $7m_\text{e}$. Subtracting $6m_\text{e}$ from both sides of the reaction equation leaves an extra electron mass on the right. Not including the mass of the beta particle (electron) is mathematically equivalent to explicitly subtracting $1m_\text{e}$ from the right side of the equation.

48 ••

Picture the Problem We can use $Q = -(\Delta m)c^2$ to find the Q values for this reaction.

(*a*) The masses of the atoms are:

$$m_{^{13}\text{N}} = 13.005738\,\text{u}$$
$$m_{^{13}\text{C}} = 13.003354\,\text{u}$$

For β^+ decay:

$$Q = (m_i - m_f - 2m_e)c^2 = (m_i - m_f)c^2 - 2m_e c^2$$

Calculate $m_i - m_f$:

$$m_i - m_f = 13.005738\,\text{u} - 13.003354\,\text{u} = 0.002384\,\text{u}$$

Calculate the Q value:

$$Q = (0.002384\,\text{u})c^2\left(931.5\frac{\text{MeV}/c^2}{u}\right) - 2(0.511\,\text{MeV}) = \boxed{1.20\,\text{MeV}}$$

(*b*) The atomic masses include the masses of the electrons of the neutral atoms. In this reaction the initial atom has 7 electrons and the final atom only has 6 electrons. Moreover, in addition to one electron not included in the atomic masses, a positron of mass equal to that of an electron is created. Consequently, one must add the rest energies of two electrons to the rest energy of the daughter atomic mass when calculating Q.

Fission and Fusion

***49** •

Picture the Problem The power output of the reactor is the product of the number of fissions per second and energy liberated per fission.

Express the required number N of fissions per second in terms of the power output P and the energy released per fission $E_{\text{per fission}}$:

$$N = \frac{P}{E_{\text{per fission}}}$$

Substitute numerical values and evaluate N:

$$N = \frac{500\,\text{MW}}{200\,\text{MeV}} = \frac{5\times10^{8}\,\frac{\text{J}}{\text{s}} \times \frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}}{200\,\text{MeV}} = \boxed{1.56\times10^{19}\,\text{s}^{-1}}$$

50 •

Picture the Problem If k = 1.1, the reaction rate after N generations is 1.1^N. We can find the number of generations by setting 1.1^N equal, in turn, to 2, 10, and 100 and solving for N. The time to increase by a given factor is the number of generations N needed to increase by that factor times the generation time.

(*a*) Set 1.1^N equal to 2 and solve for N:

$$(1.1)^N = 2$$
$$N \ln 1.1 = \ln 2$$
$$N = \frac{\ln 2}{\ln 1.1} = \boxed{7.27}$$

(*b*) Set 1.1^N equal to 10 and solve for N:

$$(1.1)^N = 10$$
$$N \ln 1.1 = \ln 10$$
$$N = \frac{\ln 10}{\ln 1.1} = \boxed{24.2}$$

(*c*) Set 1.1^N equal to 100 and solve for N:

$$(1.1)^N = 100$$
$$N \ln 1.1 = \ln 100$$
$$N = \frac{\ln 100}{\ln 1.1} = \boxed{48.3}$$

(*d*) Multiply the number of generations by the generation time:

$$t_2 = Nt_1 = (7.27)(1\,\text{ms}) = \boxed{7.27\,\text{ms}}$$
$$t_{10} = Nt_1 = (24.2)(1\,\text{ms}) = \boxed{24.2\,\text{ms}}$$
$$t_{100} = Nt_1 = (48.3)(1\,\text{ms}) = \boxed{48.3\,\text{ms}}$$

(*e*) Multiply the number of generations by the generation time:

$$t_2 = Nt_1 = (7.27)(100\,\text{ms}) = \boxed{0.727\,\text{s}}$$
$$t_{10} = Nt_1 = (24.2)(100\,\text{ms}) = \boxed{2.42\,\text{s}}$$
$$t_{100} = Nt_1 = (48.3)(100\,\text{ms}) = \boxed{4.83\,\text{s}}$$

***51** ••

Picture the Problem We can use $Q = -(\Delta m)c^2$, where $\Delta m = m_f - m_i$, to calculate the Q value.

The Q value is given by:

$$Q = -(\Delta m)c^2 \times \frac{931.5\,\text{MeV/c}^2}{1\text{u}}$$

Calculate the change in mass Δm:

$$\Delta m = m_f - m_i$$
$$= 94.905842\text{ u} + 138.906348\text{ u} + 2(1.008665\text{ u}) - (235.043923\text{ u} + 1.008665\text{ u})$$
$$= -0.223068\text{ u}$$

Substitute for Δm and evaluate Q:

$$Q = -(-0.223068\text{ u}) \times \frac{931.5\,\text{MeV}}{1\text{u}}$$
$$= \boxed{208\text{ MeV}}$$

The ratio of Q to U found in Problem 23 is:

$$\frac{Q}{U} = \frac{208\text{ MeV}}{236\,\text{MeV}} = \boxed{88.1\%}$$

52 ••

Picture the Problem We can find the number of neutrons per second in the generation of 4 W of power from the number of reactions per second.

The number of neutrons emitted per second is:

$$N_n = \tfrac{1}{2}N$$

where N is the number of reactions per second.

The number of reactions per second is:

$$N = 2\left(\frac{4\frac{\text{J}}{\text{s}} \times \frac{1\text{eV}}{1.60\times10^{-19}\text{ J}}}{3.27\,\text{MeV} + 4.03\,\text{MeV}}\right)$$
$$= 6.85\times10^{12}\,\text{s}^{-1}$$

Substitute for N and evaluate N_n:

$$N_n = \tfrac{1}{2}\left(6.85\times10^{12}\,\text{s}^{-1}\right)$$
$$= \boxed{3.43\times10^{12}\text{ neutrons/s}}$$

53 ••

Picture the Problem We can use the energy released in the reactions of Problem 50, together with the 17.6 MeV released in the reaction described in this problem, to find the energy released using 5 ^{2}H nuclei. Finding the number of D atoms in 4 L of H_2O, we can then find the energy produced if all of the ^{2}H nuclei undergo fusion.

Find the energy released using 5 ^{2}H nuclei:

$$Q = 3.27\,\text{MeV} + 4.03\,\text{MeV} + 17.6\,\text{MeV}$$
$$= 24.9\,\text{MeV}$$

The number of H atoms in 4 L of H_2O is:

$$N_H = 2\left(\frac{m_{H_2O}}{18\,\text{g/mol}}\right)N_A$$

Substitute numerical values and evaluate N_H:

$$N_H = 2\left(\frac{4\,\text{kg}}{18\,\text{g/mol}}\right)\left(6.02\times10^{23}\text{ atoms/mol}\right) = 2.676\times10^{26}$$

The number of D atoms in 4 L of H_2O is:

$$\begin{aligned}N_D &= \left(1.5\times10^{-4}\right)N_H\\ &= \left(1.5\times10^{-4}\right)\left(2.676\times10^{26}\right)\\ &= 4.01\times10^{22}\end{aligned}$$

The energy produced is given by:

$$E = \frac{N_D}{5}Q$$

Substitute numerical values and evaluate E:

$$\begin{aligned}E &= \frac{4.01\times10^{22}}{5}\left(24.9\,\text{MeV}\right)\\ &= 1.997\times10^{23}\text{ MeV}\times\frac{1.60\times10^{-19}\text{ J}}{\text{eV}}\\ &= \boxed{3.20\times10^{10}\text{ J}}\end{aligned}$$

***54 •••**

Picture the Problem We can use the conservation of momentum and the given *Q* value to find the final energies of both the ^{4}He nucleus and the neutron, assuming that the initial momentum of the system is zero.

Apply conservation of energy to obtain:

$$\begin{aligned}18.6\,\text{MeV} &= \tfrac{1}{2}m_{He}v_{He}^2 + \tfrac{1}{2}m_n v_n^2\\ &= K_{He} + K_n\end{aligned} \qquad (1)$$

Apply conservation of momentum to obtain:

$$m_{He}v_{He} + m_n v_n = 0 \qquad (2)$$

Solve equation (2) for v_{He}:

$$v_{He} = -\frac{m_n v_n}{m_{He}} \Rightarrow v_{He}^2 = \left(\frac{m_n}{m_{He}}\right)^2 v_n^2$$

Substitute for v_{He}^2 in equation (1):

$$18.6\,\text{MeV} = \tfrac{1}{2}m_{He}\left(\frac{m_n}{m_{He}}\right)^2 v_n^2 + \tfrac{1}{2}m_n v_n^2$$

or

$$18.6\,\text{MeV} = \tfrac{1}{2}m_\text{n}v_\text{n}^2\left(1+\frac{m_\text{n}}{m_\text{He}}\right)$$
$$= K_\text{n}\left(1+\frac{m_\text{n}}{m_\text{He}}\right)$$

Solve for K_n:

$$K_\text{n} = \frac{18.6\,\text{MeV}}{1+\dfrac{m_\text{n}}{m_\text{He}}}$$

Substitute numerical values for m_n and m_He and evaluate K_n:

$$K_\text{n} = \frac{18.6\,\text{MeV}}{1+\dfrac{1.008665\,\text{u}}{4.002603\,\text{u}}} = \boxed{14.86\,\text{MeV}}$$

Use equation (1) to find K_He:

$$K_\text{He} = 18.6\,\text{MeV} - K_\text{n}$$
$$= 18.6\,\text{MeV} - 14.86\,\text{MeV}$$
$$= \boxed{3.74\,\text{MeV}}$$

55 •••

Picture the Problem Adding the three reactions will yield their net effect. We can use $(\Delta m)c^2$ to find the rest energy released in the cycle and find the rate of proton consumption from the ratio of the sun's power output to the released per proton in fusion.

(a) Add the three reactions to obtain:

$${}^1\text{H} + {}^1\text{H} + {}^1\text{H} + {}^2\text{H} + {}^1\text{H} + {}^3\text{He} \rightarrow {}^2\text{H} + \beta^+ + \nu_\text{e} + {}^3\text{He} + \gamma + {}^4\text{He} + \beta^+ + \nu_\text{e}$$

Simplify to obtain:

$$\boxed{4\,{}^1\text{H} \rightarrow {}^4\text{He} + 2\beta^+ + 2\nu_\text{e} + \gamma}$$

(b) Express the rest energy released in this cycle:

$$(\Delta m)c^2 = (4m_\text{p} - m_\alpha - 4m_\text{e})c^2$$

Use Table 40-1 to find the masses of the participants in the reaction and evaluate $(\Delta m)c^2$:

$$(\Delta m)c^2 = [4(1.007825\text{ u}) - 4.002603\text{ u}]c^2 \times \frac{931.5\,\text{MeV}/c^2}{\text{u}} - 4(0.511\,\text{MeV})$$
$$= \boxed{24.7\,\text{MeV}}$$

(c) Express the rate R of proton consumption:

$$R = \frac{P}{E} \qquad (1)$$

where E is the energy released per proton in fusion.

Find N, the number of protons in the sun:

$$N = \frac{\frac{1}{2}m_{\text{sun}}}{m_{\text{p}}} = \frac{\frac{1}{2}\left(1.99\times10^{30}\ \text{kg}\right)}{1.67\times10^{-27}\ \text{kg}} = 5.96\times10^{56}$$

where we have assumed that protons constitute about half of the total mass of the sun.

The energy released per proton in fusion is:

$$E = \tfrac{1}{4}\left(26.7\,\text{MeV}\right) = 6.675\,\text{MeV} = 6.675\,\text{MeV}\times\frac{1.60\times10^{-19}\ \text{J}}{\text{eV}} = 1.07\times10^{-12}\ \text{J}$$

Substitute numerical values in equation (1) and evaluate R:

$$R = \frac{4\times10^{26}\ \text{W}}{1.07\times10^{-12}\ \text{J}} = \boxed{3.74\times10^{38}\ \text{s}^{-1}}$$

The time T for the consumption of all protons is:

$$T = \frac{N}{R} = \frac{5.96\times10^{56}}{3.74\times10^{38}\ \text{s}^{-1}} = 1.59\times10^{18}\ \text{s}\times\frac{1\,\text{y}}{31.56\,\text{Ms}} = \boxed{5.04\times10^{10}\ \text{y}}$$

General Problems

56 •

Picture the Problem We can use the values of k, e, h, and c and the appropriate conversion factors to show that ke^2 = 1.44 MeV·fm and hc = 1240 MeV·fm

(a) Evaluate ke^2 to obtain:

$$ke^2 = \left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2 = 2.307\times10^{-28}\ \text{N}\cdot\text{m}^2$$
$$= 2.307\times10^{-28}\ \text{J}\cdot\text{m}\times\frac{1\,\text{eV}}{1.60\times10^{-19}\ \text{J}} = 1.44\times10^{-9}\ \text{eV}\cdot\text{m}$$
$$= 1.44\times10^{-9}\ \text{eV}\cdot\text{m}\times\frac{1\,\text{fm}}{10^{-15}\ \text{m}}\times\frac{1\,\text{MeV}}{10^{6}\ \text{eV}} = \boxed{1.44\,\text{MeV}\cdot\text{fm}}$$

(*b*) Evaluate *hc* to obtain:

$$hc = (6.63\times10^{-34}\ \text{J}\cdot\text{s})(3\times10^{8}\ \text{m/s}) = 1.99\times10^{-25}\ \text{J}\cdot\text{m}$$
$$= 1.99\times10^{-25}\ \text{J}\cdot\text{m}\times\frac{1\text{eV}}{1.60\times10^{-19}\ \text{J}} = 1240\times10^{-9}\ \text{eV}\cdot\text{m}$$
$$= 1240\times10^{-9}\ \text{eV}\cdot\text{m}\times\frac{1\,\text{fm}}{10^{-15}\ \text{m}}\times\frac{1\,\text{MeV}}{10^{6}\ \text{eV}} = \boxed{1240\,\text{MeV}\cdot\text{fm}}$$

***57** •

Picture the Problem We can use the given information regarding the half-life of the source to find its decay constant. We can then plot a graph of the counting rate as a function of time.

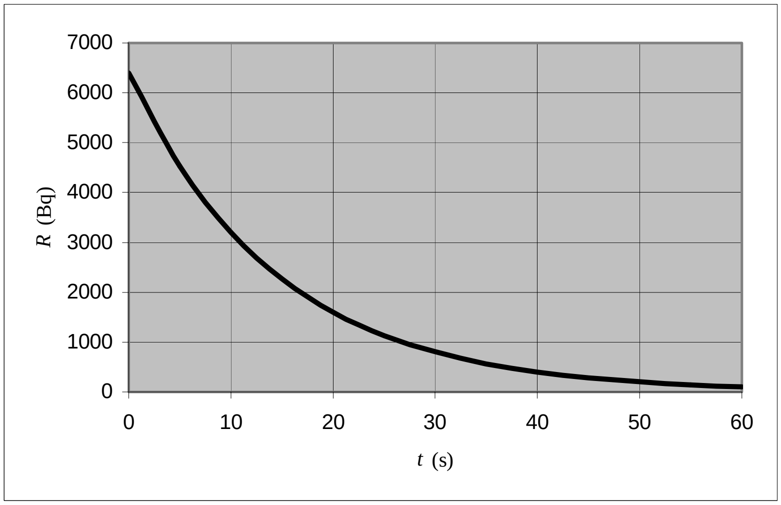
The decay constant is related to the half-life of the source:

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{10\,\text{s}} = \boxed{0.0693\text{s}^{-1}}$$

The activity of the source is given by:

$$R = R_0 e^{-\lambda t} = (6400\,\text{Bq})e^{-(0.0693\,\text{s}^{-1})t}$$

The following graph of $R = (6400\,\text{Bq})e^{-(0.0693\,\text{s}^{-1})t}$ was plotted using a spreadsheet program.



58 •

Picture the Problem The energy needed to remove a neutron is given by $Q = (\Delta m)c^2$ where Δm is the difference between the sum of the masses of the reaction products and the mass of the target nucleus.

(*a*) The reason is:

$$^{4}\text{He} + \text{Q} = {}^{3}\text{He} + \text{n}$$

The masses are (see Table 40-1):

$$m_{^{4}\text{He}} = 4.002603\ \text{u}$$
$$m_{^{3}\text{He}} = 3.016029\ \text{u}$$
$$m_{\text{n}} = 1.008665\,\text{u}$$

Calculate the final mass:

$$m_{\text{f}} = 3.016029\ \text{u} + 1.008665\ \text{u}$$
$$= 4.024694\ \text{u}$$

Calculate the increase in mass:

$$\Delta m = m_{\text{f}} - m_{\text{i}}$$
$$= 4.024694\ \text{u} - 4.002603\ \text{u}$$
$$= 0.022091\ \text{u}$$

Calculate the energy Q needed to remove a neutron from ^{4}He:

$$Q = (\Delta m)c^{2}$$
$$= (0.022091\text{u})c^{2}\left(931.5\frac{\text{MeV}/c^{2}}{\text{u}}\right)$$
$$= \boxed{20.6\,\text{MeV}}$$

(*b*) The reaction is:

$$^{7}\text{Li} + \text{Q} = {}^{6}\text{Li} + \text{n}$$

The masses are (see Table 40-1):

$$m_{^{7}\text{Li}} = 7.016004\ \text{u}$$
$$m_{^{6}\text{Li}} = 6.015122\ \text{u}$$
$$m_{\text{n}} = 1.008665\,\text{u}$$

Calculate the final mass:

$$m_{\text{f}} = 6.015122\ \text{u} + 1.008665\ \text{u}$$
$$= 7.023787\ \text{u}$$

Calculate the increase in mass:

$$\Delta m = m_{\text{f}} - m_{\text{i}}$$
$$= 7.023787\ \text{u} - 7.016004\ \text{u}$$
$$= 0.007783\ \text{u}$$

Calculate the energy Q needed to remove a neutron from ^{4}He:

$$Q = (\Delta m)c^{2}$$
$$= (0.007783\text{u})c^{2}\left(\frac{931.5\,\text{MeV}/c^{2}}{1\text{u}}\right)$$
$$= \boxed{7.25\,\text{MeV}}$$

59 •

Picture the Problem The maximum kinetic energy of the electron is given by $K_{max} = Q = \left(m_{^{14}\text{C}} - m_{^{14}\text{N}}\right)c^2$.

The maximum kinetic energy of the electron is the Q value for the reaction:

$$K_{max} = Q = \left(m_{^{14}\text{C}} - m_{^{14}\text{N}}\right)c^2$$

Find the mass of each atom from Table 40-1:

$$m_{^{14}\text{C}} = 14.003242\ \text{u}$$
$$m_{^{14}\text{N}} = 14.003074\ \text{u}$$

Calculate $\Delta m = m_{^{14}\text{C}} - m_{^{14}\text{N}}$:

$$\Delta m = 14.003242\ \text{u} - 14.003074\ \text{u} = 0.000168\,\text{u}$$

Calculate the maximum kinetic energy of the electron:

$$Q = (\Delta m)c^2 = (0.000168\,\text{u})c^2\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right) = \boxed{156\ \text{keV}}$$

60 •

Picture the Problem We can use the definition density to find the radius of the neutron star.

Relate the mass of the neutron star to the mass of the sun M, the volume V of the star and the nuclear density ρ:

$$M = \rho V = \tfrac{4}{3}\pi\rho R^3$$

where R is the radius of the star.

Solve for R:

$$R = \sqrt[3]{\frac{3M}{4\pi\rho}}$$

In Problem 20 it was established that:

$$\rho = 1.174\times10^{17}\ \text{kg/m}^3$$

Substitute numerical values and evaluate R:

$$R = \sqrt[3]{\frac{3\left(1.99\times10^{30}\ \text{kg}\right)}{4\pi\left(1.174\times10^{17}\ \text{kg/m}^3\right)}} = \boxed{15.9\,\text{km}}$$

***61 ••**

Picture the Problem We can show that ^{109}Ag is stable against alpha decay by demonstrating that its Q value is negative.

The Q value for this reaction is:

$$Q = -\left[\left(m_{\text{Rh}} + m_\alpha\right) - m_{\text{Ag}}\right]c^2\left(931.5\,\frac{\text{MeV}/c^2}{\text{u}}\right)$$

Substitute numerical values and evaluate Q:

$$\begin{aligned} Q &= -\left[\left(4.002603\text{ u} + 104.905250\text{ u}\right) - 108.904756\text{ u}\right]\left(931.5\text{MeV/u}\right) \\ &= \boxed{-2.88\text{ MeV}} \end{aligned}$$

Remarks: Alpha decay occurs spontaneously and the Q value will equal the sum of the kinetic energies of the alpha particle and the recoiling daughter nucleus, $Q = K_\alpha + K_{\text{D}}$. Kinetic energy cannot be negative; hence, alpha decay cannot occur unless the mass of the parent nucleus is greater than the sum of the masses of the alpha particle and daughter nucleus, $m_{\text{P}} > m_\alpha + m_{\text{D}}$. Alpha decay cannot take place unless the total rest mass decreases.

62 ••

Picture the Problem We can use $E_{\text{threshold}} = hf_{\text{threshold}} = hc/\lambda_{\text{threshold}}$, where $E_{\text{threshold}}$ is the binding energy of the deuteron, to find the threshold wavelength for the given nuclear reaction.

Express the threshold energy of the photon:

$$E_{\text{threshold}} = hf_{\text{threshold}} = \frac{hc}{\lambda_{\text{threshold}}}$$

Solve for the threshold wavelength:

$$\lambda_{\text{threshold}} = \frac{hc}{E_{\text{threshold}}} \qquad (1)$$

The threshold energy equals the binding energy of the deuteron:

$$E_{\text{threshold}} = E_{\text{B}} = \left[m_{\text{D}} - \left(m_{\text{p}} + m_{\text{n}}\right)\right]c^2 \times \frac{931.5\,\text{MeV}/c^2}{\text{u}}$$

Substitute numerical values and evaluate E_{th}, the energy that must be added to the deuteron that will cause it to fission:

$$\begin{aligned} E_{\text{threshold}} &= \left[2.014102\text{ u} - \left(1.007825\text{ u} + 1.008665\text{ u}\right)\right]\left(931.5\,\text{MeV/u}\right) \\ &= -2.22\text{ MeV} \times \frac{1.60\times10^{-19}\text{ J}}{\text{eV}} = -3.55\times10^{-13}\text{ J} \end{aligned}$$

Substitute numerical values in equation (1) and evaluate $\lambda_{\text{threshold}}$:

$$\lambda_{\text{threshold}} = \frac{(6.63\times10^{-34}\,\text{J}\cdot\text{s})(3.00\times10^{8}\,\text{m/s})}{3.55\times10^{-13}\,\text{J}} = 5.60\times10^{-13}\,\text{m} = \boxed{0.560\,\text{pm}}$$

63 •

Picture the Problem The activity of a radioactive source is the product of the number of radioactive nuclei present and their decay constant.

The activity of the isotope ^{40}K in the student is:

$$R = N_{40}\lambda = \frac{N_{40}\ln 2}{t_{1/2}} \qquad (1)$$

Find N, the number of K nuclei in the student:

$$N = 0.0036\frac{mN_{\text{A}}}{M}$$

where m is the mass of the student and M is the atomic mass of K.

Substitute numerical values and evaluate N:

$$N = 0.0036\frac{(60\,\text{kg})(6.02\times10^{23}\,\text{nuclei/mol})}{39.098\ \text{g/mol}} = 3.326\times10^{24}$$

The number N_{40} of ^{40}K nuclei in the student is the product of the relative abundance and the number of K nuclei in the student:

$$\begin{aligned} N_{40} &= \text{Relative abundance}\times N \\ &= (1.2\times10^{-4})(3.326\times10^{24}) \\ &= 3.991\times10^{20} \end{aligned}$$

Substitute numerical values in equation (1) and evaluate R:

$$R = \frac{(3.991\times10^{20})\ln 2}{1.3\times10^{9}\ \text{y}\times\dfrac{31.56\,\text{Ms}}{\text{y}}} = \boxed{6.74\times10^{3}\,\text{Bq}}$$

64 ••

Picture the Problem We can find the energy released in the reaction $\beta^{+}+\beta^{-}\to Q$ by recognizing that a total of 2 electron masses are converted into energy in this annihilation.

The energy released when a positron-electron pair annihilate is given by:

$$Q = E = 2m_{\text{e}}c^{2}$$

Substitute numerical values and evaluate Q:

$$Q = 2\left(9.11\times10^{-31}\,\text{kg}\right)\left(3\times10^{8}\,\text{m/s}\right)^2 = 1.64\times10^{-13}\,\text{J}\times\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}} = \boxed{1.02\,\text{MeV}}$$

65 ••

Picture the Problem We can use the fact that, after n half-lives, the decay rate of the ^{24}Na isotope is $R = \left(\frac{1}{2}\right)^n R_0$, where R_0 is its decay rate at $t = 0$.

The counting rate after n half-lives is related to the initial counting rate:

$$R = \left(\tfrac{1}{2}\right)^n R_0$$

Divide both sides of the equation by the volume V of blood in the patient:

$$\frac{R}{V} = \left(\tfrac{1}{2}\right)^n \frac{R_0}{V}$$

We're given that $n = 2/3$, $R_0 = 600$ kBq, and, after n half-lives, the decay rate per unit volume is 60 Bq/mL:

$$60\,\text{Bq/mL} = \left(\tfrac{1}{2}\right)^{2/3} \frac{600\,\text{kBq}}{V}$$

Solve for and evaluate V:

$$V = \left(\tfrac{1}{2}\right)^{2/3} \frac{600\,\text{kBq}}{60\,\text{Bq/mL}} = 6.30\times10^{3}\,\text{mL}$$
$$= \boxed{6.30\,\text{L}}$$

***66** ••

Picture the Problem We can solve this problem in the center of mass reference frame for the general case of an α particle in a head-on collision with a nucleus of atomic mass M u and then substitute data for a nucleus of ^{197}Au and a nucleus of ^{10}B.

In the CM frame, the kinetic energy is:

$$K_{\text{CM}} = \frac{K_{\text{lab}}}{1+\dfrac{m_\alpha}{M}} = \frac{K_{\text{lab}}}{1+\dfrac{4\,\text{u}}{M}}$$

At the point of closest approach:

$$K_{\text{CM}} = \frac{kq_1q_2}{R_{\min}} = \frac{k(2e)(Ze)}{R_{\min}} = \frac{ke^2\,2Z}{R_{\min}}$$

or, because $ke^2 = 1.44$ MeV·fm,

$$K_{\text{CM}} = \frac{(1.44\,\text{MeV}\cdot\text{fm})(2Z)}{R_{\min}}$$

Solve for $R_{\min}$ to obtain:

$$R_{\min} = \frac{(1.44\,\text{MeV}\cdot\text{fm})(2Z)}{K_{\text{CM}}} \qquad (1)$$

(*a*) Neglecting the recoil of the target nucleus is equivalent to replacing K_{CM} by K_{lab}. Evaluate equation (1) for ^{197}Au:

$$R_{min} = \frac{(1.44\,\text{MeV}\cdot\text{fm})(2\times 79)}{8\,\text{MeV}} = \boxed{28.4\,\text{fm}}$$

Evaluate equation (1) for ^{10}B:

$$R_{min} = \frac{(1.44\,\text{MeV}\cdot\text{fm})(2\times 5)}{8\,\text{MeV}} = \boxed{1.80\,\text{fm}}$$

(*b*) Find K_{CM} for the ^{197}Au nucleus:

$$K_{CM} = \frac{8\,\text{MeV}}{1+\frac{4\,\text{u}}{197\,\text{u}}} = 7.841\,\text{MeV}$$

Substitute numerical values in equation (1) and evaluate R_{min}:

$$R_{min} = \frac{(1.44\,\text{MeV}\cdot\text{fm})(2\times 79)}{7.841\,\text{MeV}} = \boxed{29.0\,\text{fm}}$$

Note that this result is about 2% greater that R_{min} calculated ignoring recoil.

Find K_{CM} for the ^{10}B nucleus:

$$K_{CM} = \frac{8\,\text{MeV}}{1+\frac{4\,\text{u}}{10\,\text{u}}} = 5.714\,\text{MeV}$$

Substitute numerical values in equation (1) and evaluate R_{min}:

$$R_{min} = \frac{(1.44\,\text{MeV}\cdot\text{fm})(2\times 5)}{5.714\,\text{MeV}} = \boxed{2.52\,\text{fm}}$$

Note that this result is about 40% greater that R_{min} calculated ignoring recoil.

67 ••

Picture the Problem The allowed energy levels in a one-dimensional infinite square well are given by Equation 35-13: $E_n = n^2\left(\frac{h^2}{8mL^2}\right)$.

(*a*) The lowest energy of a nucleon of mass 1 u in the well corresponds to $n = 1$:

$$E_1 = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8(1\,\text{u})\left(1.66\times10^{-27}\ \text{kg/u}\right)(3\,\text{fm})^2}$$
$$= 3.678\times10^{-12}\ \text{J}\times\frac{1\,\text{eV}}{1.60\times10^{-19}\ \text{J}}$$
$$= \boxed{23.0\,\text{MeV}}$$

(*b*) Because neutrons are fermions, there can be only two per state:

$$E = 2(E_1+E_2+E_3+E_4+E_5+E_6) = 2\left(E_1+2^2E_1+3^2E_1+4^2E_1+5^2E_1+6^2E_1\right)$$
$$= 182E_1 = 182(23.0\,\text{MeV}) = \boxed{4.19\,\text{GeV}}$$

(c) Find E for 4 protons and 4 neutrons:

$$E = 4(E_1+E_2+E_3) = 4\left(E_1+2^2E_1+3^2E_1\right)$$
$$= 56E_1 = 56(23.0\,\text{MeV}) = \boxed{1.29\,\text{GeV}}$$

68 ••

Picture the Problem We can apply BE = $(\Delta m)c^2$ to the model to find the binding energies and the binding energies/bond.

(*a*) Find the binding energy BE for this model:

$$\text{BE} = \left(4m_\alpha - m_{^{16}\text{O}}\right)c^2$$
$$= \left[4(4.002603\ \text{u}) - 15.994915\ \text{u}\right]c^2$$
$$= (0.015497\ \text{u})c^2$$

There are 6 bonds for the regular tetrahedron:

$$\frac{\text{BE}}{\text{bond}} = \tfrac{1}{6}\text{BE} = \tfrac{1}{6}(0.015497\,\text{u})c^2$$
$$= \tfrac{1}{6}(0.015497\,\text{u})c^2\times\frac{931.5\,\text{MeV/u}}{c^2}$$
$$= \boxed{2.406\,\text{MeV}}$$

(*b*) ^{12}C has 3 pairwise α particle bonds. Find the total BE for ^{12}C with this model:

$$\text{BE}\left(^{12}\text{C}\right) = 3\times\text{BE}\left(^{4}\text{He}\right) + 3(2.406\,\text{MeV})$$

Calculate BE(^{4}He):

$$\text{BE}\left({}^{4}\text{He}\right)=\left[2\left(m_{\text{p}}+m_{\text{n}}\right)-m_{{}^{4}\text{He}}\right]c^{2}$$

$$=\left[2\left(1.007825\,\text{u}+1.008665\,\text{u}\right)-4.002603\,\text{u}\right]c^{2}\times\frac{931.5\,\text{MeV}/c^{2}}{\text{u}}$$

$$=28.30\,\text{MeV}$$

Substitute numerical values and evaluate $\text{BE}\left({}^{12}\text{C}\right)$:

$$\text{BE}\left({}^{12}\text{C}\right)=3\left(28.30\,\text{MeV}\right)+3\left(2.406\,\text{MeV}\right)=\boxed{92.1\,\text{MeV}}$$

Use Table 40-1 to find $\text{BE}\left({}^{12}\text{C}\right)$:

$$\text{BE}\left({}^{12}\text{C}\right)=\left[6\left(m_{\text{p}}+m_{\text{n}}\right)-m_{{}^{12}\text{C}}\right]c^{2}$$

$$=\left[6\left(1.007825\,\text{u}+1.008665\,\text{u}\right)-12.000000\,\text{u}\right]c^{2}\times\frac{931.5\,\text{MeV}/c^{2}}{\text{u}}$$

$$=92.2\,\text{MeV}$$

Note that this result is good agreement with the model.

69 ••

Picture the Problem We can separate the variables in the differential equation $dN/dt = R_\text{p} - \lambda N$ and integrate to express N as a function of t. When $dN/dt \approx 0$, $R_\text{p} - \lambda N_\infty = 0$, a condition we can use to find N_∞.

(*a*) Separate the variables in the differential equation to obtain:

$$\frac{dN}{R_\text{p}-\lambda N}=dt$$

Integrate the left side of the equation from 0 to N and the right side from 0 to t to obtain:

$$\int_0^N\frac{dN'}{R_\text{p}-\lambda N'}=\int_0^t dt'$$

Let $u = R_\text{p} - \lambda N'$. Then:

$$du=-\lambda dN'$$

and

$$\int_0^N\frac{dN'}{R_\text{p}-\lambda N'}=-\frac{1}{\lambda}\int_{\ell_1}^{\ell_2}\frac{du}{u}=-\frac{1}{\lambda}\ln u\Big|_{\ell_1}^{\ell_2}$$

$$=-\frac{1}{\lambda}\ln\left(R_\text{p}-\lambda N'\right)\Big|_0^N$$

$$=-\frac{1}{\lambda}\ln\left(R_\text{p}-\lambda N\right)+\frac{1}{\lambda}\ln\left(R_\text{p}\right)$$

$$=\frac{1}{\lambda}\ln\left(\frac{R_\text{p}}{R_\text{p}-\lambda N}\right)$$

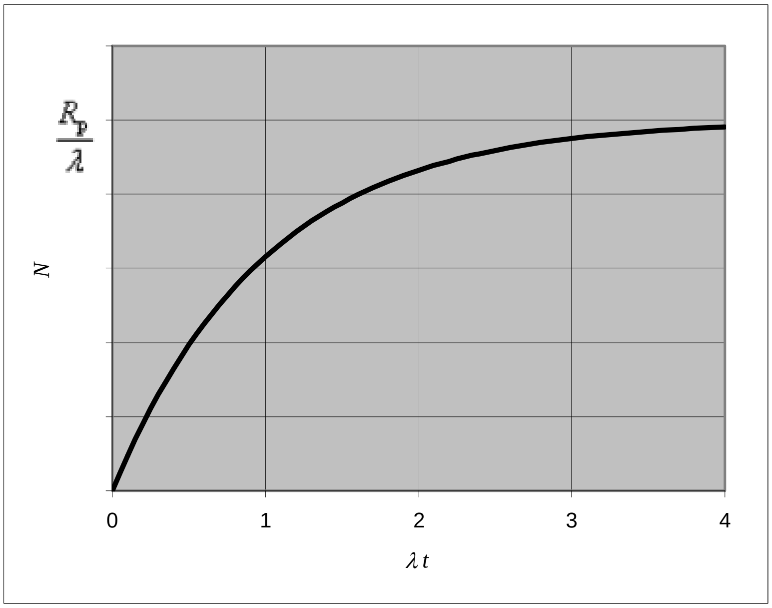
Because $\int_0^t dt' = t$:

$$\frac{1}{\lambda}\ln\left(\frac{R_p}{R_p - \lambda N}\right) = t$$

Solve for N to obtain:

$$N = \frac{R_p}{\lambda}\left(1 - e^{-\lambda t}\right)$$

The following graph of $N(t) = (R_p/\lambda)(1 - e^{-\lambda t})$ was plotted using a spreadsheet program. Note that $N(t)$ approaches R_p/λ in the same manner that the charge on a capacitor approaches the value CV.



(b) When $dN/dt = 0$:

$$R_p - \lambda N_\infty = 0 \Rightarrow N_\infty = \frac{R_p}{\lambda}$$

The decay constant is:

$$\lambda = \frac{\ln 2}{t_{1/2}}$$

Substitute for λ to obtain:

$$N_\infty = \frac{R_p}{\ln 2} t_{1/2}$$

Substitute numerical values and evaluate N_∞:

$$N_\infty = \frac{100\,\mathrm{s}^{-1}}{\ln 2}\left(10\,\mathrm{min} \times \frac{60\,\mathrm{s}}{\mathrm{min}}\right) = \boxed{8.66 \times 10^4}$$

***70 ••**

Picture the Problem The mass of ^{235}U required is given by $m_{235} = \frac{N}{N_A} M_{235}$, where M_{235} is the molecular mass of ^{235}U and N is the number of fissions required to produce 7.0×10^{19} J.

Relate the mass of ^{235}U required to the number of fissions N required:

$$m_{235} = \frac{N}{N_A} M_{235} \qquad (1)$$

where M_{235} is the molecular mass of ^{235}U.

Determine N:

$$N = \frac{E_{\text{annual}}}{E_{\text{per fission}}}$$

Substitute numerical values and evaluate N:

$$N = \frac{7.0\times10^{19}\ \text{J}}{200\,\text{MeV}\times\frac{1.60\times10^{-19}\ \text{J}}{\text{eV}}} = 2.18\times10^{30}$$

Substitute numerical values in equation (1) and evaluate m_{235}:

$$m_{235} = \frac{2.18\times10^{30}}{6.02\times10^{23}\ \text{nuclei/mol}}(235\,\text{g/mol}) = \boxed{8.51\times10^{5}\ \text{kg}}$$

71 ••

Picture the Problem In the ground state of a one-dimensional infinite square well of length L the wavelength of a particle is $2L$. We can use de Broglie's equation to find p for the particle and the relationship $E^2 = E_0^2 + p^2c^2$ with $E_0 << pc$ to show that $E \approx pc$.

(*a*) In the ground state of a one-dimensional infinite square well of length L:

$$\lambda = 2L = 2(2\,\text{fm}) = \boxed{4.00\,\text{fm}}$$

(*b*) Use de Broglie's relation to obtain:

$$p = \frac{h}{\lambda} = \frac{hc}{\lambda c}$$

Substitute numerical values and evaluate p:

$$p = \frac{1240\,\text{eV}\cdot\text{nm}}{(4\,\text{fm})c} = \boxed{310\,\text{MeV}/c}$$

(*c*) Relate the total energy of the electron to its rest energy and

$$E^2 = E_0^2 + p^2c^2 = p^2c^2\left(1+\frac{E_0^2}{p^2c^2}\right)$$

momentum:

Because $E_0 << pc$: $E^2 \approx p^2c^2 \Rightarrow E \approx \boxed{pc}$

(*d*) The kinetic energy of an electron in the ground state of this well is given by:

$$K = E - E_0 \approx E = pc$$
$$= (310\,\text{MeV}/c)c = \boxed{310\,\text{MeV}}$$

72 ••

Picture the Problem When a single proton is removed from a ^{12}C nucleus, a ^{11}B nucleus remains and we can use $Q = \Delta mc^2$ to determine the minimum energy required to remove a proton.

The nuclear reaction is: $^{12}_{6}\text{C} + Q \rightarrow {}^{11}_{5}\text{B} + {}^{1}_{1}\text{H}$

The minimum energy Q required is: $Q = \left(m_{^{11}\text{B}} + m_{^{1}\text{H}} - m_{^{12}\text{C}}\right)c^2$

Substitute numerical values and evaluate Q:

$$Q = [(11.009306\,\text{u} + 1.007825\,\text{u}) - 12.000000\,\text{u}]\left(\frac{931.5\,\text{MeV}}{\text{u}}\right) = \boxed{16.0\,\text{MeV}}$$

***73** •••

Picture the Problem The momentum of the electron is related to its total energy through $E^2 = p^2c^2 + E_0^2$ and its total relativistic energy E is the sum of its kinetic and rest energies.

(*a*) Relate the total energy of the electron to its momentum and rest energy:

$$E^2 = p^2c^2 + E_0^2 \qquad (1)$$

The total relativistic energy E of the electron is the sum of its kinetic energy and its rest energy:

$$E = K + E_0$$

Substitute for E in equation (1) to obtain:

$$(K + E_0)^2 = p^2c^2 + E_0^2$$

Solve for p:

$$p = \frac{\sqrt{K(K + 2E_0)}}{c}$$

Substitute numerical values and evaluate p:

$$p = \frac{\sqrt{(0.782\,\text{MeV})(0.782\,\text{MeV} + 2\times 0.511\,\text{MeV})}}{c} = \boxed{1.188\,\text{MeV}/c}$$

(*b*) Because $p_p = -p_e$:

$$K_p = \frac{p_p^2}{2m_p}$$

Substitute numerical values (see Table 7-1 for the rest energy of a proton) and evaluate K_p:

$$K_p = \frac{(1.188\,\text{MeV}/c)^2}{2(938.28\,\text{MeV}/c^2)} = \boxed{752\,\text{eV}}$$

(*c*) The percent correction is:

$$\frac{K_p}{K} = \frac{752\,\text{eV}}{0.782\,\text{MeV}} = \boxed{0.0962\%}$$

74 •••

Picture the Problem Conservation of momentum and conservation of energy allow us to find the final velocities. Because the initial kinetic energy of the nucleus is zero, its final kinetic energy equals the energy lost by the neutron.

(*a*) Apply conservation of momentum to the collision to obtain:

$$(m+M)V = mv_L$$

Solve for *V*:

$$V = \boxed{\frac{mv_L}{m+M}}$$

(*b*) In the CM frame, $V_{Mi} = V$ and so:

$$V_{Mi} = \boxed{V}$$

In the CM frame, $V_f = -V_i$ and so:

$$V_{Mf} = \boxed{-V}$$

(*c*) Use conservation of momentum to obtain one relation for the final velocities:

$$mv_L = mv_f + MV_{Mf} \qquad (1)$$

The equality of the initial and final kinetic energies provides a second equation relating the two final velocities. This is implemented by equating the speeds of recession and approach:

$$V_{Mf} - v_f = -(V_{Mi} - v_L) = 0 + v_L$$

and so

$$v_f = V_{Mf} - v_L$$

To eliminate v_f, substitute in equation (1) :

$$mv_L = m(V_{Mf} - v_L) + MV_{Mf}$$

Solve for V_{Mf}:

$$V_{Mf} = \boxed{\frac{2m}{M+m}v_L}$$

(*d*) The kinetic energy of the nucleus after the collision in the laboratory frame is:

$$K_M = \tfrac{1}{2}MV_{Mf}^2$$

Substitute for V_{Mf} and simplify to obtain:

$$K_{Mf} = \tfrac{1}{2}M\left(\frac{2m}{M+m}v_L\right)^2 = \boxed{\frac{4mM}{(M+m)^2}\left(\tfrac{1}{2}mv_L^2\right)}$$

(*e*) The fraction of the energy lost by the neutron in the elastic collision is given by:

$$\frac{\Delta E}{E} = \frac{-K_{Mf}}{\frac{1}{2}mv_L^2} = \frac{\frac{4mM}{(M+m)^2}\left(\frac{1}{2}mv_L^2\right)}{\frac{1}{2}mv_L^2} = \frac{4mM}{(M+m)^2} = \frac{4mM}{M^2\left(1+\frac{m}{M}\right)^2} = \boxed{\frac{\frac{4m}{M}}{\left(1+\frac{m}{M}\right)^2}}$$

75 •••

Picture the Problem We can use the result of Problem 74, part (*e*), to find the fraction $f = E_f/E_0$ of its initial energy lost per collision and then use this result to show that, after N collisions, $E = (0.714)^N E_0$.

(*a*) Determine $f = E_f/E_0$ per collision:

$$f = \frac{E_0 - \Delta E}{E_0} = 1 - \frac{\Delta E}{E_0}$$

From Problem 74, part (*e*):

$$\frac{\Delta E}{E_0} = \frac{4m}{M\left(1+\frac{m}{M}\right)^2}$$

Substitute for $\Delta E/E_0$ in the expression for f to obtain:

$$f = 1 - \frac{4m}{M\left(1+\frac{m}{M}\right)^2}$$

Substitute numerical values and evaluate *f*:

$$f = 1 - \frac{4(1.008665\,\text{u})}{12.000000\,\text{u}\left(1 + \frac{1.008665\,\text{u}}{12.000000\,\text{u}}\right)^2} = 0.714$$

After *N* collisions:

$$E_{fN} = f^N E_0 = \boxed{(0.714)^N E_0} \quad (1)$$

(*b*) Solve equation (1) for *N*:

$$N = \frac{\ln\left(\frac{E_{fN}}{E_0}\right)}{\ln(0.714)}$$

Substitute numerical values and evaluate *N*:

$$N = \frac{\ln\left(\frac{0.02\,\text{eV}}{2\,\text{MeV}}\right)}{\ln(0.714)} = 54.7$$

$$\boxed{\begin{array}{l}\text{55 head-on collisions are required to reduce the energy of the neutron}\\ \text{from 2 MeV to 0.02 eV.}\end{array}}$$

76 •••

Picture the Problem We can use the result of Problem 74, part (*e*), to find the fraction $f = E_\text{f}/E_0$ of its initial energy lost per collision. Note the difference between the energy loss per collision specified here and that of the preceding problem. In the preceding problem it was assumed that all collisions are head-on collisions.

(*a*) Determine $f = E_\text{f}/E_0$ per collision:

$$f = \frac{E_0 - \Delta E}{E_0} = 1 - \frac{\Delta E}{E_0}$$

In a collision with a hydrogen atom:

$$\frac{\Delta E}{E_0} = 0.63$$

and so

$$f = 0.37$$

After *N* collisions:

$$E_{fN} = f^N E_0 = (0.37)^N E_0 \quad (1)$$

Solve equation (1) for *N*:

$$N = \frac{\ln\left(\frac{E_{fN}}{E_0}\right)}{\ln(0.37)} \quad (2)$$

Substitute numerical values and evaluate N:

$$N=\frac{\ln\left(\frac{0.02\,\text{eV}}{2\,\text{MeV}}\right)}{\ln(0.37)}=18.5$$

$$\boxed{\text{19 head-on collisions with an atom of hydrogen are required to reduce the energy of the neutron from 2 MeV to 0.02 eV.}}$$

(b) In a collision with a carbon atom:

$$\frac{\Delta E}{E_0}=0.11$$

and so

$$f=0.89$$

Equation (2) becomes:

$$N=\frac{\ln\left(\frac{E_{fN}}{E_0}\right)}{\ln(0.89)}$$

Substitute numerical values and evaluate N:

$$N=\frac{\ln\left(\frac{0.02\,\text{eV}}{2\,\text{MeV}}\right)}{\ln(0.89)}=158$$

$$\boxed{\text{158 head-on collisions with an atom of carbon are required to reduce the energy of the neutron from 2 MeV to 0.02 eV.}}$$

***77** •••

Picture the Problem We can differentiate $N_B(t)=\frac{N_{A0}\lambda_A}{\lambda_B-\lambda_A}\left(e^{-\lambda_A t}-e^{-\lambda_B t}\right)$ with respect to t to show that it is the solution to the differential equation $dN_B/dt=\lambda_A N_A-\lambda_B N_B$.

(a) The rate of change of N_B is the rate of generation of B nuclei minus the rate of decay of B nuclei. The generation rate is equal to the decay rate of A nuclei, which equals $\lambda_A N_A$. The decay rate of B nuclei is $\lambda_B N_B$.

(b) We're given that:

$$\frac{dN_B}{dt}=\lambda_A N_A-\lambda_B N_B \quad (1)$$

$$N_B(t)=\frac{N_{A0}\lambda_A}{\lambda_B-\lambda_A}\left(e^{-\lambda_A t}-e^{-\lambda_B t}\right) \quad (2)$$

$$N_A=N_{A0}e^{-\lambda_A t} \quad (3)$$

Differentiate equation (2) with respect to t to obtain:

$$\frac{d}{dt}\left[N_{\mathrm{B}}(t)\right]=\frac{N_{\mathrm{A0}}\lambda_{\mathrm{A}}}{\lambda_{\mathrm{B}}-\lambda_{\mathrm{A}}}\frac{d}{dt}\left[\left(e^{-\lambda_{\mathrm{A}}t}-e^{-\lambda_{\mathrm{B}}t}\right)\right]=\frac{N_{\mathrm{A0}}\lambda_{\mathrm{A}}}{\lambda_{\mathrm{B}}-\lambda_{\mathrm{A}}}\left[-\lambda_{\mathrm{A}}e^{-\lambda_{\mathrm{A}}t}+\lambda_{\mathrm{B}}e^{-\lambda_{\mathrm{B}}t}\right]$$

Substitute this derivative in equation (1) to get:

$$\frac{N_{\mathrm{A0}}\lambda_{\mathrm{A}}}{\lambda_{\mathrm{B}}-\lambda_{\mathrm{A}}}\left[-\lambda_{\mathrm{A}}e^{-\lambda_{\mathrm{A}}t}+\lambda_{\mathrm{B}}e^{-\lambda_{\mathrm{B}}t}\right]=\lambda_{\mathrm{A}}N_{\mathrm{A0}}e^{-\lambda_{\mathrm{A}}t}-\lambda_{\mathrm{B}}\left[\frac{N_{\mathrm{A0}}\lambda_{\mathrm{A}}}{\lambda_{\mathrm{B}}-\lambda_{\mathrm{A}}}\left(e^{-\lambda_{\mathrm{A}}t}-e^{-\lambda_{\mathrm{B}}t}\right)\right]$$

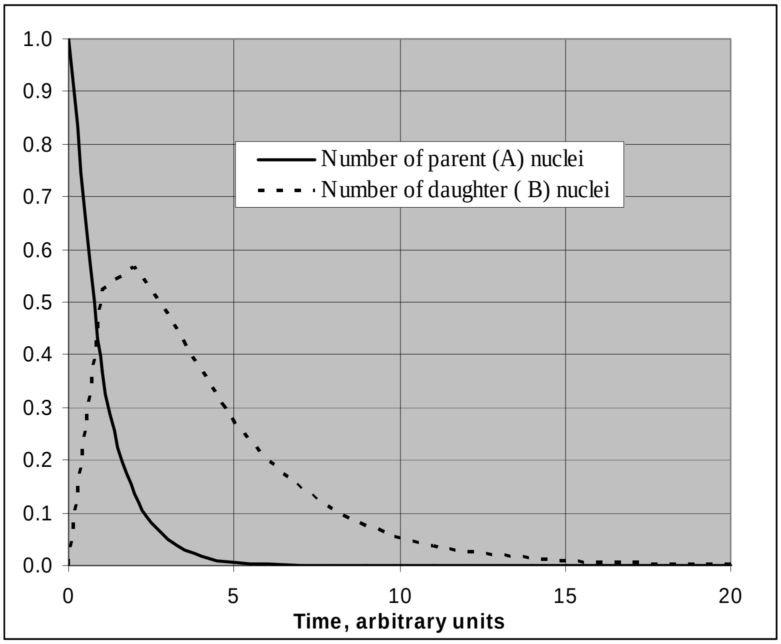
Multiply both sides by $\dfrac{\lambda_{\mathrm{B}}-\lambda_{\mathrm{A}}}{\lambda_{\mathrm{B}}\lambda_{\mathrm{A}}}$ and simplify to obtain:

$$\begin{aligned}\frac{N_{\mathrm{A0}}}{\lambda_{\mathrm{B}}}\left[-\lambda_{\mathrm{A}}e^{-\lambda_{\mathrm{A}}t}+\lambda_{\mathrm{B}}e^{-\lambda_{\mathrm{B}}t}\right]&=\frac{\lambda_{\mathrm{B}}-\lambda_{\mathrm{A}}}{\lambda_{\mathrm{B}}}N_{\mathrm{A0}}e^{-\lambda_{\mathrm{A}}t}-N_{\mathrm{A0}}\left(e^{-\lambda_{\mathrm{A}}t}-e^{-\lambda_{\mathrm{B}}t}\right)\\&=N_{\mathrm{A0}}e^{-\lambda_{\mathrm{A}}t}-\frac{N_{\mathrm{A0}}\lambda_{\mathrm{A}}}{\lambda_{\mathrm{B}}}e^{-\lambda_{\mathrm{A}}t}-N_{\mathrm{A0}}e^{-\lambda_{\mathrm{A}}t}+N_{\mathrm{A0}}e^{-\lambda_{\mathrm{B}}t}\\&=-\frac{N_{\mathrm{A0}}\lambda_{\mathrm{A}}}{\lambda_{\mathrm{B}}}e^{-\lambda_{\mathrm{A}}t}+N_{\mathrm{A0}}e^{-\lambda_{\mathrm{B}}t}\\&=\frac{N_{\mathrm{A0}}}{\lambda_{\mathrm{B}}}\left[-\lambda_{\mathrm{A}}e^{-\lambda_{\mathrm{A}}t}+\lambda_{\mathrm{B}}e^{-\lambda_{\mathrm{B}}t}\right]\end{aligned}$$

which is an identity and confirms that equation (2) is the solution to equation (1).

(*c*) If $\lambda_{\mathrm{A}}>\lambda_{\mathrm{B}}$ the denominator and the expression in the parentheses are both negative for $t>0$. If $\lambda_{\mathrm{A}}<\lambda_{\mathrm{B}}$ the denominator and the expression in the parentheses are both positive for $t>0$.

(*d*) The following graph was plotted using a spreadsheet program.



78 •••

Picture the Problem We can express the time at which the number of isotope B nuclei will be a maximum by setting dN_B/dt equal to zero and solving for t.

From Problem 77 we have:

$$\frac{dN_B}{dt} = \lambda_A N_A - \lambda_B N_B = 0 \text{ for extrema}$$

Replace $\lambda_A N_A$ by $\lambda_A N_{A0} e^{-\lambda_A t}$ and N_B by $\frac{N_{A0}\lambda_A}{\lambda_B - \lambda_A}\left(e^{-\lambda_A t} - e^{-\lambda_B t}\right)$:

$$\lambda_A N_{A0} e^{-\lambda_A t} - \lambda_B \frac{N_{A0}\lambda_A}{\lambda_B - \lambda_A}\left(e^{-\lambda_A t} - e^{-\lambda_B t}\right) = 0$$

Simplify to obtain:

$$e^{-\lambda_A t} - \frac{\lambda_B}{\lambda_B - \lambda_A}\left(e^{-\lambda_A t} - e^{-\lambda_B t}\right) = 0$$

$$(\lambda_B - \lambda_A)e^{-\lambda_A t} - \lambda_B\left(e^{-\lambda_A t} - e^{-\lambda_B t}\right) = 0$$

Remove the parentheses and combine like terms to obtain:

$$\lambda_A e^{-\lambda_A t} = \lambda_B e^{-\lambda_B t}$$

Solve for t:

$$t = \boxed{\frac{\ln(\lambda_B/\lambda_A)}{\lambda_B - \lambda_A}}$$

Remarks: Note that all we've shown is that an *extreme value* exists at $\boldsymbol{t = \dfrac{\ln(\lambda_B/\lambda_A)}{\lambda_B - \lambda_A}}$**. To show that this value for *t* maximizes N_B, we need to either 1) examine the second derivative at this value for *t*, or 2) plot a graph of N_B as a function of time (see Problem 77) .**

79 •••

Picture the Problem We can show that, provided $\tau_A >> \tau_B$, $e^{-\lambda_A t} - e^{-\lambda_B t} \approx 1$ and $\dfrac{\lambda_A}{\lambda_B - \lambda_A} \approx \dfrac{\lambda_A}{\lambda_B}$ and, hence, that $N_B = (\lambda_A/\lambda_B)N_A$.

We have, from Problem 77 (*b*):

$$N_B(t) = \frac{N_A \lambda_A}{\lambda_B - \lambda_A}\left(e^{-\lambda_A t} - e^{-\lambda_B t}\right) \quad (1)$$

Because $\tau_A >> \tau_B$:

$$\lambda_A << \lambda_B$$

When several years have passed, because $\lambda_A t << 1$:

$$e^{-\lambda_A t} - e^{-\lambda_B t} \approx 1 \quad (2)$$

Also, when $\lambda_A << \lambda_B$:

$$\frac{\lambda_A}{\lambda_B - \lambda_A} \approx \frac{\lambda_A}{\lambda_B} \quad (3)$$

Substitute (2) and (3) in (1) to obtain:

$$N_B(t) = \boxed{\frac{\lambda_A}{\lambda_B} N_A}$$