Chapter 3

Vectors in Physics

Solutions to Even-numbered Conceptual Questions

2. Vectors $\vec{\mathbf{A}}$, $\vec{\mathbf{G}}$, and $\vec{\mathbf{J}}$ are all equal to one another. Also, vector $\vec{\mathbf{I}}$ is the same as vector $\vec{\mathbf{L}}$.

4. No. The component and the magnitude can be equal if the vector has only a single component. If the vector has more than one nonzero component, however, its magnitude will be greater than either of its components.

6. No. If a vector has a nonzero component, the smallest magnitude it can have is the magnitude of the component.

8. **(a)** The magnitude of $1.4\vec{\mathbf{A}}$ is equal to the magnitude of $2.2\vec{\mathbf{B}}$. These vectors point in different directions, however. **(b)** The x component of $1.4\vec{\mathbf{A}}$ is less than the y component of $2.2\vec{\mathbf{B}}$ because the x component is negative. The two components have the same magnitude, however.

10. The vector $\vec{\mathbf{A}}$ can point in the following directions: 45˚, 135˚, 225˚, and 315˚. In each of these directions $|A_x| = |A_y|$.

12. The vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ must point in the same direction.

14. The direction angle for this vector must be greater 180˚ and less than 270˚.

16. There are two possible direction angles for this vector, 135˚ and 315˚.

18. When sailing upwind, your speed relative to the wind is greater than the speed of the wind itself. If you sail downwind, however, you move with the wind, and its speed relative to you is decreased.

20. **(a)** Relative to the ground, the aircraft being refueled has a velocity of 125 m/s due east, the same as the KC-10A. **(b)** Relative to the KC-10A, the aircraft being refueled has zero velocity.

Solutions to Problems

1. y = height of the press box
x = distance to second base

$$\frac{y}{x} = \tan\theta$$

$$x = \frac{y}{\tan\theta} = \frac{38.0\text{ ft}}{\tan 15.0°} = \boxed{142\text{ ft}}$$

2. y = elevation
x = distance driven

(a) $\theta = \sin^{-1}\frac{y}{x} = \sin^{-1}\frac{550\text{ ft}}{(1.7\text{ mi})\left(\frac{5280\text{ ft}}{\text{mi}}\right)}$
$= \boxed{3.5°}$

(b) $x = \frac{y}{\sin\theta} = \left(\frac{150\text{ ft}}{\sin 3.51°}\right)\left(\frac{1\text{ mi}}{5280\text{ ft}}\right)$
$= \boxed{0.46\text{ mi}}$

3. $\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{6\text{ ft}}{100\text{ ft}} = \boxed{3.43°}$

4. (a) $r_x = r\cos\theta = (75\text{ m})\cos 35.0°$
$= \boxed{61\text{ m}}$
$r_y = r\sin\theta = (75\text{ m})\sin 35.0°$
$= \boxed{43\text{ m}}$

(b) $r_x = (75\text{ m})\cos 85.0° = \boxed{6.5\text{ m}}$
$r_y = (75\text{ m})\sin 85.0° = \boxed{75\text{ m}}$

5. (a) $(90\text{ ft})\hat{\mathbf{x}} + (90\text{ ft})\hat{\mathbf{y}}$

(b) $(90\text{ ft})\hat{\mathbf{y}}$

(c) $(0\text{ ft})\hat{\mathbf{x}} + (0\text{ ft})\hat{\mathbf{y}}$

6. h = height of the lighthouse
r = distance from the base to the edge of the rocky cliff
s = height of the sailor
d = distance of the ship from the rocks
$y = h - s$
$x = d + r$

$$x = \frac{y}{\tan\theta} = d + r = \frac{h-s}{\tan\theta}$$

$$d = \frac{h-s}{\tan\theta} - r = \frac{49\text{ ft} - 14\text{ ft}}{\tan 30.0°} - 19\text{ ft} = \boxed{42\text{ ft}}$$

7. The lengths between the atoms form an isosceles triangle. By drawing a line segment between the center of the oxygen atom and the midpoint of the hydrogen atoms, two right triangles are formed. The opposite angle to the segment between a hydrogen atom and the midpoint is 104.5°/2 = 52.25°. Half the distance between hydrogen atoms is given by

$$y = r\sin\theta = (0.96\text{ Å})\sin 52.25° = 0.759\text{ Å}$$

and the total distance is

$2y = 2(0.759\text{ Å}) = \boxed{1.5\text{ Å}}$.

8. (a) $\theta = \tan^{-1}\dfrac{r_y}{r_x} = \tan^{-1}\left(\dfrac{-9.5\text{ m}}{14\text{ m}}\right) = \boxed{-34°}$

(b) $r = \sqrt{r_x^2 + r_y^2} = \sqrt{(14\text{ m})^2 + (-9.5\text{ m})^2} = \boxed{17\text{ m}}$

(c) $\boxed{\text{The direction does not change. The magnitude is doubled.}}$

9. (a) Draw a diagonal from one corner of the square to its opposite corner. This segment is the diameter of the circle. The radius is half its length, which is $\boxed{\text{less}}$ than the length of a side of the square.

(b) $r = \dfrac{d}{2} = \dfrac{1}{2}\left(\dfrac{2.5\text{ m}}{\cos 45°}\right) = \dfrac{1}{2}(3.54\text{ m}) = \boxed{1.8\text{ m}}$

10. (a) The distance covered is

$510\text{ ft} + 250\text{ ft} = \boxed{760\text{ ft}}$.

The displacement is $\sqrt{(510\text{ ft})^2 + (250\text{ ft})^2} = 570\text{ ft}$.

(b) $\boxed{\text{about } 25°}$

(c) $\theta = \tan^{-1}\dfrac{y}{x} = \tan^{-1}\dfrac{250\text{ ft}}{510\text{ ft}} = \tan^{-1} 0.49 = \boxed{26°}$

11. $A_x = 50$

$A_y = 0$

$B_x = 120\cos(-70°) = 41.0$

$B_y = 120\sin(-70°) = -113$

(a) $|A_x| > |B_x|$, $\boxed{\vec{\mathbf{A}}}$

(b) $|B_y| > |A_y|$, $\boxed{\vec{\mathbf{B}}}$

12. i. 10.0 m north, 15.0 m west, 5.00 m south

$$r = \sqrt{(10.0\text{ m}-5.00\text{ m})^2 + (15.0\text{ m})^2} = \boxed{15.8\text{ m}}$$

Assume north is along the x-axis.

$$\theta = \tan^{-1}\left(\frac{15.0\text{ m}}{5.00\text{ m}}\right) = \boxed{71.6^\circ\text{ west of north}}$$

ii. 10.0 m north, 15.0 m west, 5.00 m north

$$r = \sqrt{(10.0\text{ m}+5.00\text{ m})^2 + (15.0\text{ m})^2} = \boxed{21.2\text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{15.0\text{ m}}{15.0\text{ m}}\right) = \boxed{45.0^\circ\text{ west of north}}$$

iii. 10.0 m north, 15.0 m east, 5.00 m south

$$r = \sqrt{(10.0\text{ m}-5.00\text{ m})^2 + (-15.0\text{ m})^2} = \boxed{15.8\text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{-15.0\text{ m}}{5.00\text{ m}}\right) = -71.6^\circ$$

$$= \boxed{71.6^\circ\text{ east of north}}$$

iv. 10.0 m north, 15.0 m east, 5.00 m north

$$r = \sqrt{(10.0\text{ m}+5.00\text{ m})^2 + (-15.0\text{ m})^2} = \boxed{21.2\text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{-15.0\text{ m}}{15.0\text{ m}}\right) = -45^\circ$$

$$= \boxed{45.0^\circ\text{ east of north}}$$

13. y = depth of whale
x = distance traveled horizontally

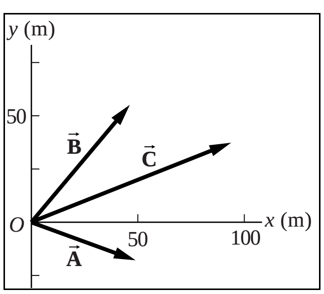
(a) $y = r\sin\theta = (150\text{ m})\sin(20.0^\circ)$

$$= \boxed{51\text{ m deep}}$$

(b) $x = r\cos\theta = (150\text{ m})\cos(20.0^\circ)$

$$= \boxed{140\text{ m}}$$

14. (a)



(b) $\vec{\mathbf{A}} = (50.0\text{ m})\cos(-20.0°)\hat{\mathbf{x}} + (50.0\text{ m})\sin(-20.0°)\hat{\mathbf{y}}$
$= (47.0\text{ m})\hat{\mathbf{x}} - (17.1\text{ m})\hat{\mathbf{y}}$

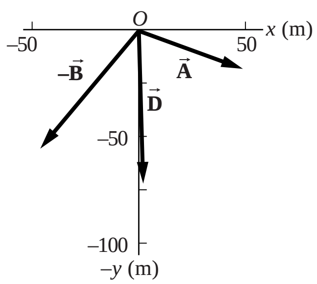
$\vec{\mathbf{B}} = (70.0\text{ m})\cos(50.0°)\hat{\mathbf{x}} + (70.0\text{ m})\sin(50.0°)\hat{\mathbf{y}}$
$= (45.0\text{ m})\hat{\mathbf{x}} + (53.6\text{ m})\hat{\mathbf{y}}$

$\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{C}}$
$= (47.0\text{ m} + 45.0\text{ m})\hat{\mathbf{x}} + (-17.1\text{ m} + 53.6\text{ m})\hat{\mathbf{y}}$
$= (92.0\text{ m})\hat{\mathbf{x}} + (36.5\text{ m})\hat{\mathbf{y}}$

$C = \sqrt{(92.0\text{ m})^2 + (36.5\text{ m})^2} = \boxed{99.0\text{ m}}$

$\theta = \tan^{-1}\left(\frac{36.5\text{ m}}{92.0\text{ m}}\right) = \boxed{21.6°}$

15. (a)



(b) $\vec{\mathbf{A}} = (50.0\text{ m})\cos(-20.0°)\hat{\mathbf{x}} + (50.0\text{ m})\sin(-20.0°)\hat{\mathbf{y}}$
$= (47.0\text{ m})\hat{\mathbf{x}} - (17.1\text{ m})\hat{\mathbf{y}}$

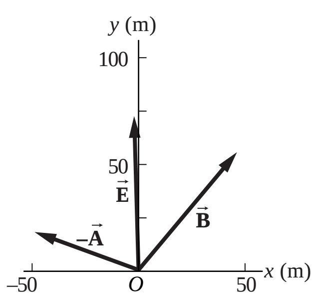
$\vec{\mathbf{B}} = (70.0\text{ m})\cos(50.0°)\hat{\mathbf{x}} + (70.0\text{ m})\sin(50.0°)\hat{\mathbf{y}}$
$= (45.0\text{ m})\hat{\mathbf{x}} + (53.6\text{ m})\hat{\mathbf{y}}$

$\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$
$= (47.0\text{ m} - 45.0\text{ m})\hat{\mathbf{x}} + (-17.1\text{ m} - 53.6\text{ m})\hat{\mathbf{y}}$
$= (2.0\text{ m})\hat{\mathbf{x}} - (70.7\text{ m})\hat{\mathbf{y}}$

$D = \sqrt{(2.0\text{ m})^2 + (-70.7\text{ m})^2} = \boxed{70.7\text{ m}}$

$\theta = \tan^{-1}\left(\frac{-70.7\text{ m}}{2.0\text{ m}}\right) = \boxed{-88°}$

16. (a)



(b) $\vec{\mathbf{A}} = (50.0\text{ m})\cos(-20.0°)\hat{\mathbf{x}} + (50.0\text{ m})\sin(-20.0°)\hat{\mathbf{y}}$
$= (47.0\text{ m})\hat{\mathbf{x}} - (17.1\text{ m})\hat{\mathbf{y}}$

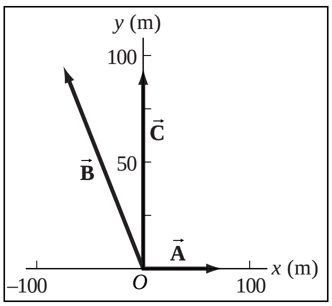
$\vec{\mathbf{B}} = (70.0\text{ m})\cos(50.0°)\hat{\mathbf{x}} + (70.0\text{ m})\sin(50.0°)\hat{\mathbf{y}}$
$= (45.0\text{ m})\hat{\mathbf{x}} + (53.6\text{ m})\hat{\mathbf{y}}$

$\vec{\mathbf{E}} = \vec{\mathbf{B}} - \vec{\mathbf{A}}$
$= (45.0\text{ m} - 47.0\text{ m})\hat{\mathbf{x}} + [53.6\text{ m} - (-17.1\text{ m})]\hat{\mathbf{y}}$
$= (-2.0\text{ m})\hat{\mathbf{x}} + (70.7\text{ m})\hat{\mathbf{y}}$

$$E = \sqrt{(-2.0\text{ m})^2 + (70.7\text{ m})^2} = \boxed{70.7\text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{70.7\text{ m}}{-2.0\text{ m}}\right) = -88.4° + 180° = \boxed{92°}$$

17. (a)



(b) $\vec{\mathbf{A}} = (75\text{ m})\hat{\mathbf{x}}$

$\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = (95\text{ m})\hat{\mathbf{y}}$

$\vec{\mathbf{B}} = \vec{\mathbf{C}} - \vec{\mathbf{A}} = (95\text{ m})\hat{\mathbf{y}} - (75\text{ m})\hat{\mathbf{x}}$

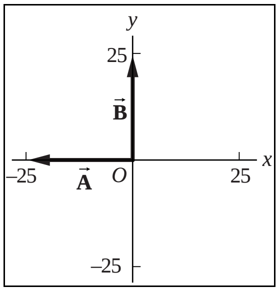
The magnitude is $\boxed{\text{about 120 m}}$. The direction is $\boxed{\text{about 130°}}$.

(c) $$B = \sqrt{(-75\text{ m})^2 + (95\text{ m})^2} = \boxed{121\text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{95\text{ m}}{-75\text{ m}}\right) = -52° + 180° = \boxed{128°}$$

18. (a) $$35 = \sqrt{A^2 + B^2}$$
$$35^2 = A^2 + B^2$$
$$B = \sqrt{35^2 - A^2} = \sqrt{35^2 - (-25)^2} = \sqrt{1225 - 625} = \sqrt{600} = \boxed{24.5}$$

(b)



19. **(a)** $\vec{\mathbf{A}}+\vec{\mathbf{B}}=(0+10)\hat{\mathbf{x}}+(-5+0)\hat{\mathbf{y}}$
$=10\hat{\mathbf{x}}-5\hat{\mathbf{y}}$

$\sqrt{10^2+(-5)^2}=\sqrt{125}=\boxed{5\sqrt{5}}$

$\theta=\tan^{-1}\left(\frac{-5}{10}\right)=\boxed{-26.6°}$

(b) $\vec{\mathbf{A}}-\vec{\mathbf{B}}=(0-10)\hat{\mathbf{x}}+(-5-0)\hat{\mathbf{y}}$
$=-10\hat{\mathbf{x}}-5\hat{\mathbf{y}}$

$\sqrt{(-10)^2+(-5)^2}=\boxed{5\sqrt{5}}$

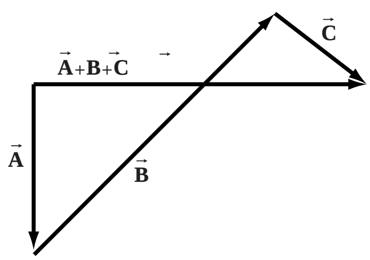
$\theta=\tan^{-1}\left(\frac{-5}{-10}\right)=\boxed{-153°}$

(c) $\vec{\mathbf{B}}-\vec{\mathbf{A}}=-(\vec{\mathbf{A}}-\vec{\mathbf{B}})=10\hat{\mathbf{x}}+5\hat{\mathbf{y}}$

$\sqrt{10^2+5^2}=\boxed{5\sqrt{5}}$

$\theta=\tan^{-1}\left(\frac{5}{10}\right)=\boxed{26.6°}$

20. **(a)**



$\left|\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}}\right|\approx\boxed{20\text{ m}}$

$\theta\approx\boxed{1.5°}$

(b) $\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}}=[0+(20.0\text{ m})\cos 45°+(7.0\text{ m})\cos(-30°)]\hat{\mathbf{x}}+[(-10.0\text{ m})+(20.0\text{ m})\sin 45°+(7.0\text{ m})\sin(-30°)]\hat{\mathbf{y}}$
$=(20\text{ m})\hat{\mathbf{x}}+(0.60\text{ m})\hat{\mathbf{y}}$

$$\left|\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}}\right|=\sqrt{(20\text{ m})^2+(0.60\text{ m})^2}=\boxed{20\text{ m}}$$

$$\theta=\tan^{-1}\frac{0.60\text{ m}}{20\text{ m}}=\boxed{1.7°}$$

21. $r_x=(65\text{ m})\cos(-40.0°)=50\text{ m}$
$r_y=(65\text{ m})\sin(-40.0°)=-42\text{ m}$
$\boxed{\Delta\vec{\mathbf{r}}=(50\text{ m})\hat{\mathbf{x}}-(42\text{ m})\hat{\mathbf{y}}}$

22. $A_x=(2.5\text{ m})\cos(140°)=\boxed{-1.9\text{ m}}$
$A_y=(2.5\text{ m})\sin(140°)=\boxed{1.6\text{ m}}$

23. $\vec{\mathbf{A}}=(6.1\text{ m})(-\hat{\mathbf{x}})=-(6.1\text{ m})\,\hat{\mathbf{x}}$ so $-3.7\vec{\mathbf{A}}=(-3.7)(-6.1\text{ m})\,\hat{\mathbf{x}}=(23\text{ m})\,\hat{\mathbf{x}}$

(a) $A_x=\boxed{23\text{ m}}$

(b) $A=\boxed{23\text{ m}}$

24. $-5.2\vec{\mathbf{A}}=(44\text{ m})\,\hat{\mathbf{x}}$
$\vec{\mathbf{A}}=(-8.5\text{ m})\,\hat{\mathbf{x}}$

(a) $A_x=\boxed{-8.5\text{ m}}$

(b) $A=\boxed{8.5\text{ m}}$

25. (a) $\theta=\tan^{-1}\left(\frac{-2.0\text{ m}}{5.0\text{ m}}\right)=\boxed{-22°}$

$$A=\sqrt{(5.0\text{ m})^2+(-2.0\text{ m})^2}=\sqrt{25\text{ m}^2+4.0\text{ m}^2}=\sqrt{29}\text{ m}=\boxed{5.4\text{ m}}$$

(b) $\theta=\tan^{-1}\left(\frac{5.0\text{ m}}{-2.0\text{ m}}\right)=\boxed{110°}$

$$B=\sqrt{(-2.0\text{ m})^2+(5.0\text{ m})^2}=\sqrt{4.0\text{ m}^2+25\text{ m}^2}=\sqrt{29}\text{ m}=\boxed{5.4\text{ m}}$$

(c) $\vec{\mathbf{A}}+\vec{\mathbf{B}} = (5.0\text{ m}-2.0\text{ m})\hat{\mathbf{x}}+(-2.0\text{ m}+5.0\text{ m})\hat{\mathbf{y}}$
$= (3.0\text{ m})\hat{\mathbf{x}}+(3.0\text{ m})\hat{\mathbf{y}}$

$$\theta = \tan^{-1}\left(\frac{3.0\text{ m}}{3.0\text{ m}}\right) = \tan^{-1}(1.0) = \boxed{45°}$$

$$\sqrt{(3.0\text{ m})^2+(3.0\text{ m})^2} = \sqrt{2(3.0\text{ m})^2} = 3\sqrt{2}\text{ m} = \boxed{4.2\text{ m}}$$

26. (a) $\theta = \tan^{-1}\left(\frac{-12\text{ m}}{25\text{ m}}\right) = \boxed{-26°}$

$A = \sqrt{(25\text{ m})^2+(-12\text{ m})^2} = \sqrt{769\text{ m}^2} = \boxed{28\text{ m}}$

(b) $\theta = \tan^{-1}\left(\frac{15\text{ m}}{2.0\text{ m}}\right) = \boxed{82°}$

$B = \sqrt{(2.0\text{ m})^2+(15\text{ m})^2} = \sqrt{229\text{ m}^2} = \boxed{15\text{ m}}$

(c) $\vec{\mathbf{A}}+\vec{\mathbf{B}} = (25\text{ m}+2.0\text{ m})\hat{\mathbf{x}}+(-12\text{ m}+15\text{ m})\hat{\mathbf{y}}$
$= (27\text{ m})\hat{\mathbf{x}}+(3.0\text{ m})\hat{\mathbf{y}}$

$\theta = \tan^{-1}\left(\frac{3.0\text{ m}}{27\text{ m}}\right) = \boxed{6.3°}$

$\sqrt{(27\text{ m})^2+(3.0\text{ m})^2} = \sqrt{738\text{ m}^2} = \boxed{27\text{ m}}$

27. (a) $\vec{\mathbf{A}}-\vec{\mathbf{B}} = (25\text{ m}-2\text{ m})\hat{\mathbf{x}}+(-12\text{ m}-15\text{ m})\hat{\mathbf{y}}$
$= \boxed{(23\text{ m})\hat{\mathbf{x}}-(27\text{ m})\hat{\mathbf{y}}}$

(b) $\vec{\mathbf{B}}-\vec{\mathbf{A}} = -(\vec{\mathbf{A}}-\vec{\mathbf{B}}) = \boxed{-(23\text{ m})\hat{\mathbf{x}}+(27\text{ m})\hat{\mathbf{y}}}$

28. $\vec{\mathbf{A}} = (1.5)(\cos 40°)\hat{\mathbf{x}}+(1.5)(\sin 40°)\hat{\mathbf{y}}$
$= \boxed{(1.1\text{ m})\hat{\mathbf{x}}+(0.96\text{ m})\hat{\mathbf{y}}}$
$\vec{\mathbf{B}} = (2.0)\cos(-19°)\hat{\mathbf{x}}+(2.0)\sin(-19°)\hat{\mathbf{y}}$
$= \boxed{(1.9\text{ m})\hat{\mathbf{x}}-(0.65\text{ m})\hat{\mathbf{y}}}$
$\vec{\mathbf{C}} = (1.0)\cos(180°-25°)\hat{\mathbf{x}}+(1.0)\sin(180°-25°)\hat{\mathbf{y}}$
$= \boxed{(-0.91\text{ m})\hat{\mathbf{x}}+(0.42\text{ m})\hat{\mathbf{y}}}$
$\vec{\mathbf{D}} = \boxed{(1.5\text{ m})\hat{\mathbf{y}}}$

29. $\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}}$
$= [(1.5\text{ m})\cos 40°+(1.0\text{ m})\cos 155°+(2.0\text{ m})\cos(-19°)]\hat{\mathbf{x}}+[(1.5\text{ m})\sin 40°+(1.0\text{ m})\sin 155°+(2.0\text{ m})\sin(-19°)]\hat{\mathbf{y}}$
$= \boxed{(2.1\text{ m})\hat{\mathbf{x}}+(0.74\text{ m})\hat{\mathbf{y}}}$

30. (a) The displacement magnitudes are equal. The *x* and *y* components are just switched.

(b) Move 1: $d = \sqrt{(-7.0)^2 + (3.5)^2} = \boxed{7.8\text{ cm}}$

$\theta = \tan^{-1}\dfrac{3.5}{-7.0} = -27° + 180° = \boxed{153°}$

Move 2: $d = \sqrt{(3.5)^2 + (7.0)^2} = \boxed{7.8\text{ cm}}$

$\theta = \tan^{-1}\left(\dfrac{7.0}{3.5}\right) = \boxed{63°}$

31. (a) $\vec{\mathbf{A}} = -(72\text{ m})\hat{\mathbf{x}} + (120\text{ m})\hat{\mathbf{y}}$

$\vec{\mathbf{B}} = -\vec{\mathbf{A}} = (72\text{ m})\hat{\mathbf{x}} - (120\text{ m})\hat{\mathbf{y}}$

$B = \sqrt{(72\text{ m})^2 + (-120\text{ m})^2} = \boxed{140\text{ m}}$

$\theta = \tan^{-1}\frac{B_y}{B_x} = \tan^{-1}\left(\frac{-120\text{ m}}{72\text{ m}}\right) = \boxed{-59°} = \boxed{59° \text{ south of east}}$

(b) The vector displacement is independent of the order in which its component displacements are taken. The initial displacement is the same, and there is $\boxed{\text{no change}}$ in the displacement for the homeward part of the trip.

32. $\vec{\mathbf{v}}_{\text{av}} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t} = \left(\frac{-72\text{ m}}{62\text{ min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\hat{\mathbf{x}} + \left(\frac{120\text{ m}}{62\text{ m}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\hat{\mathbf{y}}$

$= \left(-0.019\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{x}} + \left(0.032\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{y}}$

$v_{\text{av}} = \sqrt{\left(-0.019\ \frac{\text{m}}{\text{s}}\right)^2 + \left(0.032\ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{0.038\text{ m/s}}$

$\theta = \tan^{-1}\left(\frac{0.032\ \frac{\text{m}}{\text{s}}}{-0.022\ \frac{\text{m}}{\text{s}}}\right)$

$= -59° + 180°$

$= \boxed{31° \text{ west of north}}$

33. $r_{\text{W}} = \left(20.0\frac{\text{m}}{\text{s}}\right)(120\text{ s}) = 2400\text{ m}$

$r_{\text{S}} = \left(15\frac{\text{m}}{\text{s}}\right)(60.0\text{ s}) = 900\text{ m}$

Let north be along the *y*-axis.

$\theta = \tan^{-1}\left(\frac{-900\text{ m}}{-2400\text{ m}}\right)$

$= 20.6° + 180° = 201°$

$= \boxed{21° \text{ south of west}}$

$r = \sqrt{(2400\text{ m})^2 + (900\text{ m})^2} = \boxed{2.6\text{ km}}$

34. Let north be along the *y*-axis.

$\theta = \tan^{-1}\left(\frac{2500\text{ ft}}{1500\text{ ft}}\right) = 59° \text{ or } \boxed{31° \text{ east of north}}$

$r = \sqrt{(1500\text{ ft})^2 + (2500\text{ ft})^2} = 2915\text{ ft}$

$v_{\text{av}} = \frac{2915\text{ ft}}{3.0\text{ min}} = \boxed{970\text{ ft/min}}$

35. **(a)** $v_x = \left(3.25\ \frac{\text{m}}{\text{s}}\right)\cos(30.0°) = \boxed{2.81\ \text{m/s}}$

$v_y = \left(3.25\ \frac{\text{m}}{\text{s}}\right)\sin(30.0°) = \boxed{1.63\ \text{m/s}}$

(b) The components will be $\boxed{\text{halved}}$.

36. The only acceleration of the ball, after it left your hand, was due to the force of gravity, $\boxed{9.8\ \text{m/s}^2\ \text{downward}}$.
Check: (choose up as positive)
Up:

$v = v_0 - gt_1$

$t_1 = \frac{v_0 - v}{g} = \frac{4.5\ \frac{\text{m}}{\text{s}} - 0\ \frac{\text{m}}{\text{s}}}{9.8\ \frac{\text{m}}{\text{s}^2}} = 0.46\ \text{s}$

Down: $t_2 = \frac{v_0 - v}{g} = \frac{0\ \frac{\text{m}}{\text{s}} - \left(-4.5\ \frac{\text{m}}{\text{s}}\right)}{9.8\ \frac{\text{m}}{\text{s}^2}} = 0.46\ \text{s}$

$\text{total} = t_1 + t_2 = 2(0.46\ \text{s}) = 0.92\ \text{s}$

37. $a_{\text{av}} = \frac{\Delta v}{\Delta t} = \frac{10.0\ \frac{\text{m}}{\text{s}} - 0\ \frac{\text{m}}{\text{s}}}{3.00\ \text{s} – 0\ \text{s}} = 3.33\ \frac{\text{m}}{\text{s}^2}$

$g\sin\theta = \left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\sin 20.0° = 3.36\ \frac{\text{m}}{\text{s}^2}$

So, $a_{\text{av}} = g\sin\theta$ (with rounding errors).

38. $v = at\ \ (v_0 = 0)$

$$\begin{aligned} &= (g\sin\theta)t \\ &= \left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\sin(15.0°)(3.00\ \text{s}) \\ &= \boxed{7.62\ \text{m/s}} \end{aligned}$$

39. **(a)** $\vec{\mathbf{v}}_{\text{av}} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$

$\Delta t = t_{7.38} - t_0 = 7.38\ \text{d}\left(\frac{24.0\ \text{h}}{1\ \text{d}}\right)\left(\frac{3600\ \text{s}}{1\ \text{h}}\right) = 6.376\times10^5\ \text{s}$

$$\begin{aligned} \Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_{7.38} - \vec{\mathbf{r}}_0 &= 3.84\times10^8\ \text{m}\left\{\cos\left[\left(2.46\times10^{-6}\ \frac{\text{rad}}{\text{s}}\right)(6.376\times10^5\ \text{s})\right]\hat{\mathbf{x}}\right. \\ &\quad \left. + \sin\left[\left(2.46\times10^{-6}\ \frac{\text{rad}}{\text{s}}\right)(6.376\times10^5\ \text{s})\right]\hat{\mathbf{y}}\right\} - 3.48\times10^8\ \text{m}\{(\cos 0)\hat{\mathbf{x}} + (\sin 0)\hat{\mathbf{y}}\} \\ &= 3.84\times10^8\ \text{m}(0.0027\hat{\mathbf{x}} + 1.00\hat{\mathbf{y}}) - 3.48\times10^8\ \text{m}(1.00\hat{\mathbf{x}} - 0\hat{\mathbf{y}}) \\ &= 3.84\times10^8\ \text{m}(-1.00\hat{\mathbf{x}} + 1.00\hat{\mathbf{y}}) \end{aligned}$$

$\vec{\mathbf{v}}_{\text{av}} = \frac{3.84\times10^8\ \text{m}(-1.00\hat{\mathbf{x}} + 1.00\hat{\mathbf{y}})}{6.376\times10^5\ \text{s}} = 602.3\ \frac{\text{m}}{\text{s}}(-1.00\hat{\mathbf{x}} + 1.00\hat{\mathbf{y}})$

$$v_{\text{av}} = \sqrt{\left(-602.3\ \frac{\text{m}}{\text{s}}\right)^2 + \left(602.3\ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{852\ \frac{\text{m}}{\text{s}}}$$

$$\theta = \tan^{-1}\frac{v_{\text{av}\,y}}{v_{\text{av}\,x}} = \tan^{-1}\left(\frac{602.3\ \frac{\text{m}}{\text{s}}}{-602.3\ \frac{\text{m}}{\text{s}}}\right) = -45.0^\circ \text{ (in 2nd quadrant)}$$

$$\theta = 180^\circ - 45^\circ = \boxed{135^\circ}$$

(b) Since the distance traveled is one quarter of the moon's orbit, which is larger than the displacement during the same time interval, the instantaneous speed is greater.

40. (a) $\vec{\mathbf{a}}_{\text{av}} = \dfrac{\Delta\vec{\mathbf{v}}}{\Delta t}$

$$\Delta t = t_{0.100} - t_0 = (0.100\text{ d})\left(\frac{24.0\text{ h}}{1\text{ d}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right) - 0 = 8.64\times10^3\text{ s}$$

$$\Delta\vec{\mathbf{v}} = \vec{\mathbf{v}}_{0.100} - \vec{\mathbf{v}}_0$$

$$= \left(945\ \frac{\text{m}}{\text{s}}\right)\left\{-\sin\left[\left(2.46\times10^{-6}\ \frac{\text{rad}}{\text{s}}\right)(8.64\times10^3\text{ s})\right]\hat{\mathbf{x}} + \cos\left[\left(2.46\times10^{-6}\ \frac{\text{rad}}{\text{s}}\right)(8.64\times10^3\text{ s})\right]\hat{\mathbf{y}}\right\} - \left(945\ \frac{\text{m}}{\text{s}}\right)\{-(\sin 0)\hat{\mathbf{x}} + (\cos 0)\hat{\mathbf{y}}\}$$

$$= \left(945\ \frac{\text{m}}{\text{s}}\right)\{-(0.0213)\hat{\mathbf{x}} + 0.99977\hat{\mathbf{y}}\} - \left(945\ \frac{\text{m}}{\text{s}}\right)\{-0\hat{\mathbf{x}} + 1\hat{\mathbf{y}}\}$$

$$= \left(945\ \frac{\text{m}}{\text{s}}\right)[-(0.0213)\hat{\mathbf{x}} - (0.00023)\hat{\mathbf{y}}] = -\left(20.1\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{x}} - \left(0.217\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{y}}$$

$$\vec{\mathbf{a}}_{\text{av}} = \frac{-\left(20.1\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{x}} - \left(0.217\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{y}}}{8.64\times10^3\text{ s}} = -\left(2.33\times10^{-3}\ \frac{\text{m}}{\text{s}^2}\right)\hat{\mathbf{x}} - \left(2.5\times10^{-5}\ \frac{\text{m}}{\text{s}^2}\right)\hat{\mathbf{y}}$$

$$a_{\text{av}} = \boxed{2.33\times10^{-3}\ \frac{\text{m}}{\text{s}^2}}$$

$$\theta = 180^\circ + \tan^{-1}\left(\frac{2.5\times10^{-5}\ \frac{\text{m}}{\text{s}^2}}{2.33\times10^{-3}\ \frac{\text{m}}{\text{s}^2}}\right) = \boxed{181^\circ}$$

(b) $\Delta t = t_{0.01} - t_0 = 8.64\times10^2\text{ s}$

$$\Delta\vec{\mathbf{v}} = \vec{\mathbf{v}}_{0.01} - \vec{\mathbf{v}}_0$$

$$= \left(945\ \frac{\text{m}}{\text{s}}\right)\left\{-\sin\left[2.46\times10^{-6}\ \frac{\text{rad}}{\text{s}}\right](864\text{ s})\hat{\mathbf{x}} + \cos\left[\left(2.46\times10^{-6}\ \frac{\text{rad}}{\text{s}}\right)(864\text{ s})\right]\hat{\mathbf{y}}\right\} - \left(945\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{y}}$$

$$= \left(945\ \frac{\text{m}}{\text{s}}\right)\{-(0.00213)\hat{\mathbf{x}}] + 1.00000\hat{\mathbf{y}}\} - \left(945\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{y}}$$

$$= -\left(2.01\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{x}}$$

$$\vec{\mathbf{a}}_{\text{av}} = \frac{-\left(2.01\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{x}}}{864\text{ s}} = -\left(2.33\times10^{-3}\ \frac{\text{m}}{\text{s}^2}\right)\hat{\mathbf{x}}$$

$$a_{av} = \boxed{2.33\times 10^{-3}\ \frac{\text{m}}{\text{s}^2}}$$

$$\theta = \boxed{180°}$$

41. $v_{pg} =$ plane's speed with respect to the ground

$v_{ap} =$ attendant's speed with respect to the plane

$v_{ag} =$ attendant's speed with respect to the ground

The plane is traveling in the positive direction.

$$\begin{aligned} v_{ag} &= v_{ap} + v_{pg} \\ &= -1.22\ \frac{\text{m}}{\text{s}} + 16.5\ \frac{\text{m}}{\text{s}} \\ &= \boxed{15.3\ \text{m/s}} \end{aligned}$$

42. The x-component of the velocity was chosen perpendicular to the motion of the river. Therefore, the motion of the river will not affect the time it takes to travel across it.

$$v_x = \left(6.1\ \frac{\text{m}}{\text{s}}\right)\cos(25°) = 5.53\ \frac{\text{m}}{\text{s}}$$

$$t = \frac{25\ \text{m}}{5.53\ \frac{\text{m}}{\text{s}}} = \boxed{4.5\ \text{s}}$$

43. $v_{wg} =$ walkway's speed relative to the ground

$v_{yg} =$ your speed relative to the ground while on the walkway

$v_{yw} =$ your speed relative to the walkway

$$\begin{aligned} v_{yg} = v_{yw} + v_{wg} &= \frac{85\ \text{m}}{68\ \text{s}} + 2.2\ \frac{\text{m}}{\text{s}} \\ &= 1.25\ \frac{\text{m}}{\text{s}} + 2.2\ \frac{\text{m}}{\text{s}} \\ &= 3.45\ \frac{\text{m}}{\text{s}} \end{aligned}$$

$t =$ time to travel 85 m while on the walkway

$$= \frac{85\ \text{m}}{3.45\ \frac{\text{m}}{\text{s}}}$$

$$= \boxed{25\ \text{s}}$$

44. $v_{wg} =$ walkway's speed relative to the ground

$v_{yg} =$ your speed relative to the ground while on the walkway

$v_{yw} =$ your speed relative to the walkway

The walkway is moving in the positive direction.

$$v_{yg} = v_{yw} + v_{wg} = -1.3\ \frac{\text{m}}{\text{s}} + 2.2\ \frac{\text{m}}{\text{s}}$$
$$= 0.9\ \frac{\text{m}}{\text{s}}$$

t = time to travel the length of the walkway

$$= \frac{85\ \text{m}}{0.9\ \frac{\text{m}}{\text{s}}}$$
$$= \boxed{94\ \text{s}}$$

45. (a) $\vec{\mathbf{v}}_{pg}$ = velocity of the plane relative to the ground

$\vec{\mathbf{v}}_{pa}$ = velocity of the plane relative to the air

$\vec{\mathbf{v}}_{ag}$ = velocity of the air relative to the ground

$\vec{\mathbf{v}}_{pg} = \vec{\mathbf{v}}_{pa} + \vec{\mathbf{v}}_{ag}$

The plane needs to travel due north. Let north be along the positive y-axis, then

$$\vec{\mathbf{v}}_{pg} = \left(310\ \frac{\text{km}}{\text{h}}\right)(\cos\theta)\hat{\mathbf{x}} + \left(310\ \frac{\text{km}}{\text{h}}\right)(\sin\theta)\hat{\mathbf{y}} + \left(75\ \frac{\text{km}}{\text{h}}\right)\hat{\mathbf{x}}$$

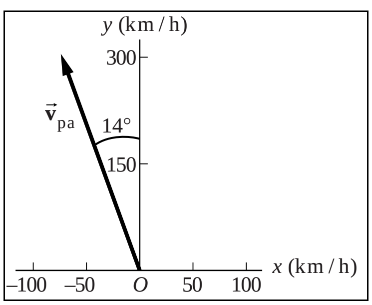
For the plane to travel due north, the net velocity in the x-direction relative to the ground must equal zero.

$$v_{pgx} = 0 = \left(310\ \frac{\text{km}}{\text{h}}\right)\cos\theta + \left(75\ \frac{\text{km}}{\text{h}}\right)$$

$$\cos\theta = \frac{-75\ \frac{\text{km}}{\text{h}}}{310\ \frac{\text{km}}{\text{h}}} = \frac{-15}{62}$$

$$\theta = \cos^{-1}\left(\frac{-15}{62}\right) = 104° = \boxed{14°\text{ west of north}}$$

(b)



$$v_{pax} = \left(310\ \frac{\text{km}}{\text{h}}\right)\cos 104° = -75\ \frac{\text{km}}{\text{h}}$$

$$v_{pay} = \left(310\ \frac{\text{km}}{\text{h}}\right)\sin 104° = 301\ \frac{\text{km}}{\text{h}}$$

(c) If the plane's speed is decreased, the angle should be $\boxed{\text{increased}}$.

46. $\vec{\mathbf{v}}_{pf}$ = passenger's velocity relative to the ferry

$\vec{\mathbf{v}}_{pw}$ = passenger's velocity relative to the water

$\vec{\mathbf{v}}_{fw}$ = ferry's velocity relative to the water

$$= \vec{\mathbf{v}}_{fp} + \vec{\mathbf{v}}_{pw}$$
$$= -\vec{\mathbf{v}}_{pf} + \vec{\mathbf{v}}_{pw}$$

Let north be along the positive x-axis, then

$$\vec{\mathbf{v}}_{\text{fw}} = -\left(1.50\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{x}} + \left(4.50\ \frac{\text{m}}{\text{s}}\right)\cos(30.0°)\mathbf{x} + \left(4.50\ \frac{\text{m}}{\text{s}}\right)\sin(30.0°)\hat{\mathbf{y}}$$
$$= \left(-1.50\ \frac{\text{m}}{\text{s}} + 3.90\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{x}} + \left(2.25\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{y}}$$
$$= \left(2.40\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{x}} + \left(2.25\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{y}}$$

$$\theta = \tan^{-1}\left(\frac{2.25\ \frac{\text{m}}{\text{s}}}{2.40\ \frac{\text{m}}{\text{s}}}\right) = \boxed{43.2° \text{ west of north}}$$

$$v_{\text{fw}} = \sqrt{\left(2.40\ \frac{\text{m}}{\text{s}}\right)^2 + \left(2.25\ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{3.29\ \text{m/s}}$$

47. Place the x-axis perpendicular to the flow of the river, such that the river is flowing in the negative y-direction.

$\vec{\mathbf{v}}_{\text{sw}}$ = jet ski's velocity relative to the water

$\vec{\mathbf{v}}_{\text{sg}}$ = jet ski's velocity relative to the ground

$\vec{\mathbf{v}}_{\text{wg}}$ = water's velocity relative to the ground

$$\vec{\mathbf{v}}_{\text{sw}} = \vec{\mathbf{v}}_{\text{sg}} + \vec{\mathbf{v}}_{\text{gw}} = \vec{\mathbf{v}}_{\text{sg}} - \vec{\mathbf{v}}_{\text{wg}}$$
$$= \left(9.5\ \frac{\text{m}}{\text{s}}\right)\cos(20.0°)\hat{\mathbf{x}} + \left(9.5\ \frac{\text{m}}{\text{s}}\right)\sin(20.0°)\hat{\mathbf{y}} - \left(-2.8\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{y}}$$
$$= \left(8.9\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{x}} + \left(3.25\ \frac{\text{m}}{\text{s}} + 2.8\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{y}}$$
$$= \left(8.9\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{x}} + \left(6.05\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{y}}$$

$$v_{\text{sw}} = \sqrt{\left(8.9\ \frac{\text{m}}{\text{s}}\right)^2 + \left(6.05\ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{11\ \text{m/s}}$$

48. $\vec{\mathbf{v}}_{\text{sg}}$ = jet ski's velocity relative to the ground

$\vec{\mathbf{v}}_{\text{sw}}$ = jet ski's velocity relative to the water

$\vec{\mathbf{v}}_{\text{wg}}$ = water's velocity relative to the ground

Place the x-axis perpendicular to the river, such that the river is flowing in the negative y-direction.

(a) For the jet ski's velocity relative to the ground to be perpendicular to the shore of the river, the jet ski's velocity in the y-direction relative to the ground must equal zero.

$$\vec{\mathbf{v}}_{\text{sg}} = \vec{\mathbf{v}}_{\text{sw}} + \vec{\mathbf{v}}_{\text{wg}}$$
$$= \left(12\ \frac{\text{m}}{\text{s}}\right)(\cos\theta)\hat{\mathbf{x}} + \left(12\ \frac{\text{m}}{\text{s}}\right)(\sin\theta)\hat{\mathbf{y}} - \left(2.8\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{y}}$$

$$v_{\text{sg}y} = 0 = \left(12\ \frac{\text{m}}{\text{s}}\right)\sin\theta - 2.8\ \frac{\text{m}}{\text{s}}$$

$$\sin\theta = \frac{2.8\ \frac{\text{m}}{\text{s}}}{12\ \frac{\text{m}}{\text{s}}} = 0.233$$

$$\theta = \sin^{-1}(0.233) = \boxed{13°}$$

(b) At higher speeds relative to the water, the angle necessary to compensate for the river's motion is smaller. The angle $\boxed{\text{decreases}}$.

49. (a) $\boxed{\text{Jet ski A}}$ reaches the opposite shore in the least amount of time. Both jet skis have the same speed relative to the water, but jet ski B's speed is shared between the *x*- and *y*-directions, whereas jet ski A's is all in the *x*-direction.

(b) $\vec{\mathbf{v}}_{\text{Aw}}$ = the velocity of jet ski A with respect to the water
$\vec{\mathbf{v}}_{\text{Bw}}$ = the velocity of jet ski B with respect to the water

Let $v = v_{\text{Aw}} = v_{\text{Bw}}$ ($v_{\text{Aw}} = v_{\text{Bw}}$ given) and place the *x*-axis perpendicular to the flow of the river, such that the river is flowing in the negative *y*-direction.

Then, $v_{\text{Aw}x} = v = \frac{w}{t_{\text{A}}}$ and $v_{\text{Bw}x} = v\cos 35° = \frac{w}{t_{\text{B}}}$ where *w* is the width of the river.

Combine and solve these two equations for $\frac{t_{\text{A}}}{t_{\text{B}}}$.

$$w = vt_{\text{A}} = vt_{\text{B}}\cos 35°$$

$$\boxed{\frac{t_{\text{A}}}{t_{\text{B}}} = \cos 35° = 0.82}$$

50. $\frac{y}{r} = \frac{\text{height}}{\text{length}} = \sin\theta$

$$\theta = \sin^{-1}\left(\frac{3.00\text{ ft}}{10.0\text{ ft}}\right) = \boxed{17.5°}$$

51. $2\vec{\mathbf{A}} + \vec{\mathbf{B}} = 2(12.1\text{ m})\hat{\mathbf{x}} + (-32.2\text{ m})\hat{\mathbf{y}}$

$$= (24.2\text{ m})\hat{\mathbf{x}} - (32.2\text{ m})\hat{\mathbf{y}}$$

$$\theta = \tan^{-1}\left(\frac{-32.2\text{ m}}{24.2\text{ m}}\right) = \boxed{-53.1°}$$

$$\text{magnitude} = \sqrt{(24.2\text{ m})^2 + (-32.2\text{ m})^2} = \sqrt{1622\text{ m}^2} = \boxed{40.3\text{ m}}$$

52.

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} = (13.8\text{ m})\hat{\mathbf{x}}$$

$$\vec{\mathbf{A}} - \vec{\mathbf{B}} = (-51.4\text{ m})\hat{\mathbf{x}}$$

$$-\vec{\mathbf{C}} = -(62.2\text{ m})\hat{\mathbf{x}}$$

Add: $2\vec{\mathbf{A}} = (-99.8\text{ m})\hat{\mathbf{x}}$

$$\boxed{\vec{\mathbf{A}} = (-49.9\text{ m})\hat{\mathbf{x}}}$$

$$(-49.9\text{ m})\hat{\mathbf{x}} - \vec{\mathbf{B}} = (-51.4\text{ m})\hat{\mathbf{x}}$$

$$\boxed{\vec{\mathbf{B}} = (1.5\text{ m})\hat{\mathbf{x}}}$$

53. $A_z = (65\text{ m})\cos 55° = \boxed{37\text{ m}}$

The projection of *A* into the *xy*-plane $(\vec{\mathbf{A}} - A_z\hat{\mathbf{z}})$
has magnitude (65 m)sin 55°, and so

$$A_x = (65\text{ m})(\sin 55°)\cos 35° = \boxed{44\text{ m}}$$

$$A_y = (65\text{ m})(\sin 55°)\sin 35° = \boxed{31\text{ m}}$$

54. At time $t = 0$ s:

$$v_{0x} = \left(5.00\ \frac{\text{m}}{\text{s}}\right)\cos 35.0° = 4.10\ \frac{\text{m}}{\text{s}}$$

$$v_{0y} = \left(5.00\ \frac{\text{m}}{\text{s}}\right)\sin 35.0° = 2.87\ \frac{\text{m}}{\text{s}}$$

At time $t = 2$ s:

$$v_{2x} = \left(6.00\ \frac{\text{m}}{\text{s}}\right)\cos(-50.0°) = 3.86\ \frac{\text{m}}{\text{s}}$$

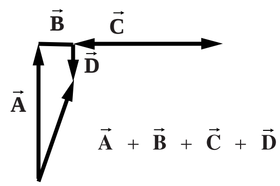
$$v_{2y} = \left(6.00\ \frac{\text{m}}{\text{s}}\right)\sin(-50.0°) = -4.60\ \frac{\text{m}}{\text{s}}$$

$$\vec{\mathbf{a}}_{\text{av}} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{\Delta v_x}{\Delta t}\hat{\mathbf{x}} + \frac{\Delta v_y}{\Delta t}\hat{\mathbf{y}}$$

$$= \left(\frac{3.86\ \frac{\text{m}}{\text{s}} - 4.10\ \frac{\text{m}}{\text{s}}}{2.00\ \text{s}}\right)\hat{\mathbf{x}} + \left(\frac{-4.60\ \frac{\text{m}}{\text{s}} - 2.87\ \frac{\text{m}}{\text{s}}}{2.00\ \text{s}}\right)\hat{\mathbf{y}}$$

$$= \boxed{(-0.12\ \text{m/s}^2)\hat{\mathbf{x}} + (-3.74\ \text{m/s}^2)\hat{\mathbf{y}}}$$

55. (a)



$$\left|\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} + \vec{\mathbf{D}}\right| \boxed{\approx 38\ \text{ft}}$$

(b) $\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} + \vec{\mathbf{D}} = (0 + 45\ \text{ft} - 35\ \text{ft} + 0)\hat{\mathbf{x}} + (51\,\text{ft} + 0 + 0 - 13\ \text{ft})\hat{\mathbf{y}}$

$$= (10\ \text{ft})\hat{\mathbf{x}} + (38\ \text{ft})\hat{\mathbf{y}}$$

$$\left|\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} + \vec{\mathbf{D}}\right| = \sqrt{(10\ \text{ft})^2 + (38\ \text{ft})^2}$$

$$= \boxed{39\ \text{ft}}$$

56. (a) $\vec{\mathbf{v}} = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}t = \left(16.6\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{x}} + 1.75\ \text{s}\left(-9.81\ \frac{\text{m}}{\text{s}^2}\right)\hat{\mathbf{y}} = \boxed{\left(16.6\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{x}} - \left(17.2\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{y}}}$

(b) $v = \sqrt{\left(16.6\ \frac{\text{m}}{\text{s}}\right)^2 + \left(-17.2\ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{23.9\ \text{m/s}}$

$$\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\left(\frac{-17.2\ \frac{\text{m}}{\text{s}}}{16.6\ \frac{\text{m}}{\text{s}}}\right) = -46.0°\ \text{or}\ \boxed{46.0°\ \text{below horizontal}}$$

57. $\vec{\mathbf{a}}_{\text{av}} = \dfrac{\Delta\vec{\mathbf{v}}}{\Delta t} = \dfrac{\vec{\mathbf{v}}_{t+\Delta t} - \vec{\mathbf{v}}_t}{\Delta t}$

$$= \frac{\left\{\left(16.6\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{x}} - \left[\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(t+\Delta t)\right]\hat{\mathbf{y}}\right\} - \left\{\left(16.6\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{x}} - \left[\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)t\right]\hat{\mathbf{y}}\right\}}{\Delta t}$$

$$= \frac{\left[\left(-9.81\ \frac{\text{m}}{\text{s}^2}\right)\Delta t\right]\hat{\mathbf{y}}}{\Delta t}$$

$$= \left(-9.81\ \frac{\text{m}}{\text{s}^2}\right)\hat{\mathbf{y}}$$

58. $\vec{\mathbf{v}}_{12}$ = velocity of plane 1 relative to plane 2

$\vec{\mathbf{v}}_{21}$ = velocity of plane 2 relative to plane 1

$\vec{\mathbf{v}}_{1\text{g}}$ = velocity of plane 1 relative to the ground

$\vec{\mathbf{v}}_{2\text{g}}$ = velocity of plane 2 relative to the ground

Let north be along the positive *x*-axis.

(a) $\vec{\mathbf{v}}_{12} = \vec{\mathbf{v}}_{1\text{g}} + \vec{\mathbf{v}}_{\text{g}2} = \vec{\mathbf{v}}_{1\text{g}} - \vec{\mathbf{v}}_{2\text{g}}$

$$= \left(12\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{x}} - \left[\left(7.5\ \frac{\text{m}}{\text{s}}\right)\cos(70.0°)\hat{\mathbf{x}} + \left(7.5\ \frac{\text{m}}{\text{s}}\right)\sin(70.0°)\hat{\mathbf{y}}\right]$$

$$= \left(12\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{x}} - \left(2.57\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{x}} - \left(7.05\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{y}}$$

$$= \left(9.43\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{x}} - \left(7.05\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{y}}$$

$$\theta = \tan^{-1}\left(\frac{-7.05\ \frac{\text{m}}{\text{s}}}{9.43\ \frac{\text{m}}{\text{s}}}\right) = -36.8° = \boxed{37° \text{ east of north}}$$

$$v_{12} = \sqrt{\left(9.43\ \frac{\text{m}}{\text{s}}\right)^2 + \left(-7.05\ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{12\ \text{m/s}}$$

(b) $\vec{\mathbf{v}}_{21} = -\vec{\mathbf{v}}_{12} = -\left(9.43\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{x}} + \left(7.05\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{y}}$

The direction is 180° from the direction of plane 1.
–37° + 180° = 143° = $\boxed{53° \text{ south of west}}$
The magnitude is the same: $\boxed{12\ \text{m/s}}$.

59. (a) In time *t*, relative to the ground, the bus moves a distance *y* and the rain falls a distance *x*, such that $y = v_{\text{bg}y}t$ and $x = v_{\text{rg}x}t$ where $v_{\text{bg}y} / v_{\text{rg}x}$ is the *y*/*x*-component of the velocity of the bus/rain relative to the ground. These distances, *x* and *y*, form the legs of a right triangle with an angle 15° adjacent to *x*, the vertical. Solve for *t*.

$$t = \frac{y}{v_{\text{bg}y}} = \frac{x}{v_{\text{rg}x}}$$

$$\frac{y}{x}=\frac{v_{\text{bg}y}}{v_{\text{rg}x}}=\tan 15°=0.27$$

$$\frac{v_{\text{rg}x}}{v_{\text{bg}y}}=\frac{1}{0.27}=\boxed{3.7}$$

(b) $v_{\text{rg}x}=3.7v_{\text{bg}y}=(3.7)\left(18\ \frac{\text{m}}{\text{s}}\right)=\boxed{67\ \text{m/s}}$

60. Draw a line segment from the center of the circle to the midpoint of one of the sides of the triangle. This segment, the radius, and half of the side of the triangle form a right triangle. Each corner of the triangle measures 60°, so the angle adjacent to the half of the triangle's side is 30°. So,

l = length of one half of the side of the triangle

$= r\cos\theta$

$$2l=2r\cos\theta=2(3.0\ \text{m})\cos 30°=\boxed{5.2\ \text{m}}$$

61. (a) $\vec{\mathbf{v}}_{\text{gs}}$ = velocity of surfer relative to shore

$v_{\text{gs}}=7.2\ \text{m/s}$

$v_{\text{gs}y}=1.3\ \text{m/s}=v_{\text{gs}}\sin\theta$

$$\theta=\sin^{-1}\left(\frac{v_{\text{gs}y}}{v_{\text{gs}}}\right)=\sin^{-1}\left(\frac{1.3\ \frac{\text{m}}{\text{s}}}{7.2\ \frac{\text{m}}{\text{s}}}\right)=\boxed{10°}$$

(b) $\vec{\mathbf{v}}_{\text{gs}}=\vec{\mathbf{v}}_{\text{gw}}+\vec{\mathbf{v}}_{\text{ws}}$

$$\begin{aligned}\vec{\mathbf{v}}_{\text{gw}}&=\vec{\mathbf{v}}_{\text{gs}}-\vec{\mathbf{v}}_{\text{ws}}\\&=\left[(v_{\text{gs}}\cos 10°)\hat{\mathbf{x}}+\left(1.3\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{y}}\right]-\left(1.3\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{y}}\\&=\left(7.2\ \frac{\text{m}}{\text{s}}\cos 10°\right)\hat{\mathbf{x}}\\&=\boxed{\left(7.1\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{x}}}\end{aligned}$$

(c) If the y-component stays the same, but the vector increases in length, the angle it makes with the x-axis must $\boxed{\text{decrease}}$.

62. (a) In order for the boat to move directly across the river the y-component of its velocity relative to the water must be upstream and must equal the speed of the water relative to the shore.

$$v_{\text{bw}y}=1.4\ \frac{\text{m}}{\text{s}}=v_{\text{bw}}\sin\theta$$

$$\theta=\sin^{-1}\left(\frac{v_{\text{bw}y}}{v_{\text{bw}}}\right)=\sin^{-1}\left(\frac{1.4\ \frac{\text{m}}{\text{s}}}{6.1\ \frac{\text{m}}{\text{s}}}\right)=\boxed{13°}$$

(b) The boat's speed across the river is $v_{\text{bw}x}=v_{\text{bw}}\cos 13°$.

$$t=\frac{d}{v_{\text{bw}x}}=\frac{25.0\ \text{m}}{\left(6.1\ \frac{\text{m}}{\text{s}}\right)(\cos 13°)}=\boxed{4.2\ \text{s}}$$

(c) To maintain the same y-component with a larger speed requires a smaller $\sin\theta$; hence, a smaller angle. The heading should be still upstream, but at a smaller angle than before, thus $\boxed{\text{more downstream}}$.

63.
$$A_x = -A \qquad A_y = 0$$
$$B_x = B\cos 30.0° = \frac{\sqrt{3}}{2}B \qquad B_y = B\sin 30.0° = \frac{1}{2}B$$
$$C_x = (15\text{ m})\cos(-40.0°) = 11.5\text{ m} \qquad C_y = (15\text{ m})\sin(-40.0°) = -9.64\text{ m}$$

$\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}} = 0$, so $-A+\frac{\sqrt{3}}{2}B+11.5\text{ m}=0$ and $0+\frac{1}{2}B-9.64\text{ m}=0.$

$\boxed{B = 2(9.64\text{ m}) = 19\text{ m}}$

$$A = \frac{\sqrt{3}}{2}B+11.5\text{ m} = \frac{\sqrt{3}}{2}(19.3\text{ m})+11.5\text{ m}$$

$\boxed{A = 28\text{ m}}$

64. $\vec{\mathbf{v}}_{12}$ = velocity of boat 1 relative to boat 2
$\vec{\mathbf{v}}_{21}$ = velocity of boat 2 relative to boat 1
$\vec{\mathbf{v}}_{1\text{m}}$ = velocity of boat 1 relative to the marina
$\vec{\mathbf{v}}_{2\text{m}}$ = velocity of boat 2 relative to the marina
Let north be along the y-axis.

$$\vec{\mathbf{v}}_{12} = \left(2.30\ \frac{\text{m}}{\text{s}}\right)\cos(50.0°)\hat{\mathbf{x}}+\left(2.30\ \frac{\text{m}}{\text{s}}\right)\sin(50.0°)\hat{\mathbf{y}} = \left(1.48\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{x}}+\left(1.76\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{y}} = -\vec{\mathbf{v}}_{21}$$

$$\vec{\mathbf{v}}_{1\text{m}} = \left(0.75\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{y}}$$

$$\vec{\mathbf{v}}_{2\text{m}} = \vec{\mathbf{v}}_{21}+\vec{\mathbf{v}}_{1\text{m}} = -\left(1.48\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{x}}-\left(1.76\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{y}}+\left(0.75\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{y}} = -\left(1.48\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{x}}-\left(1.01\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{y}}$$

$$v_{2\text{m}} = \sqrt{\left(-1.48\ \frac{\text{m}}{\text{s}}\right)^2+\left(-1.01\ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{1.80\text{ m/s}}$$

$$\theta = \tan^{-1}\left(\frac{-1.01\ \frac{\text{m}}{\text{s}}}{-1.48\ \frac{\text{m}}{\text{s}}}\right) = \boxed{34.3°\text{ south of west}}$$