Chapter 4

Two-Dimensional Kinematics

Answers to Even-numbered Conceptual Questions

2. The y component of velocity is first positive and then negative in a symmetric fashion. As a result, the average y component of velocity is zero. The x component of velocity, on the other hand, is always $v_0 \cos\theta$. Therefore, the projectile's average velocity has a magnitude of $v_0 \cos\theta$ and points in the positive x direction.

4. Yes. A projectile at the top of its trajectory has a velocity that is horizontal, while at the same time its acceleration is vertical.

6. In the case of finite air resistance, the range of a projectile is greater when the launch angle is less than 45°, as we can see in Figure 4-9.

8. Just before it lands, this projectile is moving downward with the same speed it had when it was launched. In addition, if it was launched upward at an angle θ above the x axis, it is moving in a direction that is an angle θ below the x axis just before it lands. Therefore, its velocity just before landing is $\vec{\mathbf{v}} = (2\text{ m/s})\hat{\mathbf{x}} + (-4\text{ m/s})\hat{\mathbf{y}}$.

10. Maximum height depends on the initial speed squared. Therefore, to reach twice the height, projectile 1 must have an initial speed that is the square root of 2 times greater than the initial speed of projectile 2. It follows that the ratio of the speeds is the square root of 2.

12. Both divers are in the air for the same length of time, and both move with constant speed in the x direction. If diver 2 covers twice the horizontal distance, it follows that this diver had twice the initial speed of diver 1.

14. **(a)** In the order of increasing initial speed, we have projectiles (a), (b) and (c). To see this, note that projectile (c) has the smallest launch angle. Therefore, if this projectile is to reach the same height as projectiles (a) and (b) – that is, for it to have the same y component of velocity as the other projectiles – it must have a larger initial speed to make up for its smaller launch angle. **(b)** Because these projectiles all have the same initial y component of velocity, they all have the same time of flight.

16. The tomato lands on the road in front of you. This follows from the fact that its horizontal speed is the same as yours during the entire time of its fall.

18. A coin's time of flight is precisely the same in an elevator moving with constant speed as it is on the ground. On the ground, the coin's equation of motion is $y_c = y_0 - \frac{1}{2}gt^2$, and it lands when $y_c = 0$ (ground level). In the elevator, the coin's equation of motion is $y_c = y_0 + v_0 t - \frac{1}{2}gt^2$, where v_0 is the speed of the elevator; the equation of motion of the elevator floor is $y_e = v_0 t$. The coin lands when these two values of y are equal to one another. Setting $y_c = y_e$, we see that the term $v_0 t$ cancels; hence, the time of landing is the same as it was for the coin flip on the ground.

20. **(a)** The launch angle in this case is 50° above the positive x axis. **(b)** In this case, the launch angle is 150° counterclockwise from the positive x axis, or, equivalently, 30° above the negative x axis.

Solutions to Problems

1. Separate $\vec{\mathbf{v}}$ into *x*- and *y*-components. Let north be along the *x*-axis.

$$v_x = \left(2.8\ \frac{\text{m}}{\text{s}}\right)\cos 38° = 2.21\ \frac{\text{m}}{\text{s}}$$

$$v_y = \left(2.8\ \frac{\text{m}}{\text{s}}\right)\sin 38° = 1.72\ \frac{\text{m}}{\text{s}}$$

(a) $$y = v_y t = \left(1.72\ \frac{\text{m}}{\text{s}}\right)(35.0\text{ min})\left(\frac{60\text{ s}}{\text{min}}\right) = \boxed{3.6\text{ km}}$$

(b) $$x = v_x t = \left(2.21\ \frac{\text{m}}{\text{s}}\right)(35.0\text{ min})\left(\frac{60\text{ s}}{\text{min}}\right) = \boxed{4.6\text{ km}}$$

2. Let north be along the *y*-axis.

(a) $$v_x = \left(1.60\ \frac{\text{m}}{\text{s}}\right)\cos 15.0° = 1.55\ \frac{\text{m}}{\text{s}}$$

$$t = \frac{x}{v_x} = \frac{20.0\text{ m}}{1.55\ \frac{\text{m}}{\text{s}}} = \boxed{12.9\text{ s}}$$

(b) $$v_y = \left(1.60\ \frac{\text{m}}{\text{s}}\right)\sin 15.0° = 0.414\ \frac{\text{m}}{\text{s}}$$

$$t = \frac{y}{v_y} = \frac{30.0\text{ m}}{0.414\ \frac{\text{m}}{\text{s}}} = \boxed{72.5\text{ s}}$$

3. (a) $$a_x = \left(2.0\ \frac{\text{m}}{\text{s}^2}\right)\cos 5.5° = 1.99\ \frac{\text{m}}{\text{s}^2}$$

$$x = \frac{1}{2}a_x t^2 = \frac{1}{2}\left(1.99\ \frac{\text{m}}{\text{s}^2}\right)(12\text{ s})^2 = \boxed{140\text{ m}}$$

(b) $$a_y = \left(2.0\ \frac{\text{m}}{\text{s}^2}\right)\sin 5.5° = 0.192\ \frac{\text{m}}{\text{s}^2}$$

$$y = \frac{1}{2}a_y t^2 = \frac{1}{2}\left(0.192\ \frac{\text{m}}{\text{s}^2}\right)(12\text{ s})^2 = \boxed{14\text{ m}}$$

4. (a) $$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 = 0 + 0 + \frac{1}{2}\left(-4.4\ \frac{\text{m}}{\text{s}^2}\right)(5.0)^2 = \boxed{-55\text{ m}}$$

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 = 0 + \left(6.2\ \frac{\text{m}}{\text{s}}\right)(5.0\text{ s}) + 0 = \boxed{31\text{ m}}$$

(b) $$v_x = v_{0x} + a_x t = 0 + \left(-4.4\ \frac{\text{m}}{\text{s}^2}\right)(5.0\text{ s}) = \boxed{-22\text{ m/s}}$$

$$v_y = v_{0y} + a_y t = 6.2\ \frac{\text{m}}{\text{s}} + 0 = \boxed{6.2\text{ m/s}}$$

(c) The x-component of the velocity continually increases in the negative direction, so the speed $\boxed{\text{increases with time}}$.

5. (a) $x = x_0 + v_x t$

$$t = \frac{x - x_0}{v_x} = \frac{6.20\text{ cm} - 0\text{ cm}}{2.10 \times 10^9\ \frac{\text{cm}}{\text{s}}} = \boxed{2.95 \times 10^{-9}\text{ s}}$$

(b) $$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 = 0\text{ cm} + 0\text{ cm} + \frac{1}{2}\left(5.30 \times 10^{17}\ \frac{\text{cm}}{\text{s}^2}\right)(2.95 \times 10^{-9}\text{ s})^2 = \boxed{2.31\text{ cm}}$$

6. (a) Canoeist 1's 45° path determines an isosceles right triangle whose legs measure 1.0 km. So canoeist 2's path determines a right triangle whose legs measure 1.0 km and 0.5 km. Then for canoeist 2,

$$\theta = \tan^{-1}\frac{1.0\text{ km}}{0.5\text{ km}} = 63° \text{ north of west} = \boxed{27° \text{ west of north.}}$$

(b) $vt = d,$ so for $t_1 = t_2,$ we need

$\frac{d_1}{v_1} = \frac{d_2}{v_2}$ and then

$$v_2 = \frac{d_2}{d_1}v_1 = \frac{\sqrt{(0.5\text{ km})^2 + (1.0\text{ km})^2}}{\sqrt{(1.0\text{ km})^2 + (1.0\text{ km})^2}}\left(1.35\ \frac{\text{m}}{\text{s}}\right) = \boxed{1.1\text{ m/s}}$$

7. $$y = h - \left(\frac{g}{2v_{0x}{}^2}\right)x^2$$

$$\frac{h - y}{x^2} = \frac{g}{2v_{0x}{}^2}$$

$$v_{0x} = \sqrt{\left(\frac{g}{2}\right)\left(\frac{x^2}{h - y}\right)} = \sqrt{\left(\frac{9.81\ \frac{\text{m}}{\text{s}^2}}{2}\right)\frac{(15\text{ m})^2}{(0.52\text{ m})}} = \boxed{46\text{ m/s}}$$

8. There is no initial component of velocity in the y-direction.

$$v_y{}^2 = v_{0y}{}^2 - 2g\Delta y = 0 - 2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0\text{ m} - 108\text{ m}) = 2119\ \frac{\text{m}^2}{\text{s}^2}$$

$$v = \sqrt{v_x{}^2 + v_y{}^2} = \sqrt{\left(3.60\ \frac{\text{m}}{\text{s}}\right)^2 + 2119\ \frac{\text{m}^2}{\text{s}^2}} = \boxed{46.2\text{ m/s}}$$

9. There is no initial component of velocity in the y-direction.

$$v_y{}^2 = v_{0y}{}^2 - 2g\Delta y = 0 - 2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0\ \text{m} - 3.00\ \text{m}) = 58.9\ \frac{\text{m}^2}{\text{s}^2}$$

$$v = \sqrt{v_x{}^2 + v_y{}^2} = \sqrt{\left(1.75\ \frac{\text{m}}{\text{s}}\right)^2 + 58.9\ \frac{\text{m}^2}{\text{s}^2}} = \boxed{7.87\ \text{m/s}}$$

10. There is no initial component of velocity in the y-direction.
The time of travel is

$$t = \frac{x}{v_x} = \frac{8.75\ \text{m}}{6.95\ \frac{\text{m}}{\text{s}}} = 1.259\ \text{s}$$

$$\Delta y = -\frac{1}{2}g_Z t^2$$

$$g_Z = \frac{-2\Delta y}{t^2} = \frac{-2(-1.40\ \text{m})}{(1.259\ \text{s})^2} = \boxed{1.77\ \text{m/s}^2}$$

11. (a) The time the ball travels is $t = \frac{x}{v_x} = \frac{18\ \text{m}}{32\ \frac{\text{m}}{\text{s}}} = 0.5625\ \text{s}.$

$$h - y = \frac{1}{2}gt^2 = \frac{1}{2}\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.5625\ \text{s})^2 = \boxed{1.6\ \text{m}}$$

(b) The time the ball travels is less, therefore the drop distance $\boxed{\text{decreases}}$.

(c) The moon's gravity is less, therefore the drop distance $\boxed{\text{decreases}}$.

12. (a) $x = v_x t = \left(22\ \frac{\text{m}}{\text{s}}\right)(0.45\ \text{s}) = \boxed{9.9\ \text{m}}$

(b) $h - y = \frac{1}{2}gt^2 = \frac{1}{2}\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.45\ \text{s})^2 = \boxed{0.99\ \text{m}}$

13. (a) If air resistance is ignored, $v_{\text{fx}} = \boxed{2.70\ \text{m/s}}$.

(b) $v_y = -gt = -\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(2.10\ \text{s}) = \boxed{-20.6\ \text{m/s}}$

(c) [The speed of the clam in the x-direction would increase] with the speed of the crow, but [the speed in the y-direction would stay the same]. The speed of the crow determines v_x and gravity determines v_y.

14. (a) The – 45° direction of motion indicates that, just prior to landing, the climber is falling with a speed equal to his horizontal speed. So,

$$\Delta y = \frac{v_{0y}{}^2 - v_y{}^2}{2g} = \frac{0 - \left(-8.0\ \frac{\text{m}}{\text{s}}\right)^2}{2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)} = -3.3\ \text{m}$$

The height difference is $\boxed{3.3\ \text{m}}$.

(b) $v_y = -gt$

$$t = \frac{-v_y}{g} = \frac{-\left(-8.0 \ \frac{\text{m}}{\text{s}}\right)}{9.81 \ \frac{\text{m}}{\text{s}^2}} = 0.815 \text{ s}$$

$$x = v_x t = \left(8.0 \ \frac{\text{m}}{\text{s}}\right)(0.815 \text{ s}) = 6.5 \text{ m}$$

$\boxed{\text{The climber lands 3.5 m beyond the far edge of the 3.0-m crevasse.}}$

15. (a) The time the sparrow travels is $t = \frac{x}{v_x} = \frac{0.500 \text{ m}}{1.80 \ \frac{\text{m}}{\text{s}}} = 0.2778 \text{ s}.$

In this time, the sparrow has fallen

$$h - y = \frac{1}{2}gt^2 = \frac{1}{2}\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)(0.2778 \text{ s})^2 = \boxed{0.378 \text{ m}}$$

(b) The time interval decreases, so the distance of fall $\boxed{\text{decreases}}$.

16. The pumpkin will fall the 9.0 m in t seconds, given by

$$\Delta y = -\frac{1}{2}gt^2$$

$$t = \sqrt{\frac{-2\Delta y}{g}} = \sqrt{\frac{-2(-9.0 \text{ m})}{9.81 \ \frac{\text{m}}{\text{s}^2}}} = 1.355 \text{ s}$$

The initial horizontal speed needed is $v_x = \frac{x}{t} = \frac{3.5 \text{ m}}{1.355 \text{ s}} = \boxed{2.6 \text{ m/s}}$.

17. (a) $v_x = 3.3 \ \frac{\text{m}}{\text{s}}$

$$v_y = -gt = -\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)(0.75 \text{ s}) = -7.358 \ \frac{\text{m}}{\text{s}}$$

$$v = \sqrt{\left(3.3 \ \frac{\text{m}}{\text{s}}\right)^2 + \left(-7.358 \ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{8.1 \text{ m/s}}$$

$$\theta = \tan^{-1}\left(\frac{-7.358 \ \frac{\text{m}}{\text{s}}}{3.3 \ \frac{\text{m}}{\text{s}}}\right) = \boxed{-66°}$$

(b) $\Delta y = -\frac{1}{2}gt^2$

$$t = \sqrt{\frac{-2\Delta y}{g}} = \sqrt{\frac{-2(-9.0 \text{ m})}{\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)}} = 1.355 \text{ s}$$

$$v_y = -gt = -\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)(1.355 \text{ s}) = -13.3 \ \frac{\text{m}}{\text{s}}$$

$$v = \sqrt{\left(3.3 \ \frac{\text{m}}{\text{s}}\right)^2 + \left(-13.3 \ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{14 \text{ m/s}}$$

$$\theta = \tan^{-1}\left(\frac{-13.3 \ \frac{\text{m}}{\text{s}}}{3.3 \ \frac{\text{m}}{\text{s}}}\right) = \boxed{-76°}$$

18. (a) The circumference of the Ferris wheel is $C = 2\pi r = 2\pi(5.00 \text{ m}) = 31.416 \text{ m}$.

So, the average speed is $v = \frac{C}{\Delta t} = \frac{31.416 \text{ m}}{32.0 \text{ s}} = \boxed{0.982 \text{ m/s}}$.

(b) At the top of the wheel, $v_x = 0.982 \frac{\text{m}}{\text{s}}$ and $v_y = 0$.

$$\Delta y = -\frac{1}{2}gt^2$$

$$t = \sqrt{\frac{-2\Delta y}{g}} = \sqrt{\frac{-2(-11.75 \text{ m})}{9.81 \frac{\text{m}}{\text{s}^2}}} = 1.548 \text{ s}$$

$$d = v_x t = \left(0.982 \frac{\text{m}}{\text{s}}\right)(1.548 \text{ s}) = \boxed{1.52 \text{ m}}$$

19. (a) The swimmer traveled horizontally 1.96 m in $t = \frac{x}{v_x} = \frac{1.88 \text{ m}}{2.62 \frac{\text{m}}{\text{s}}} = 0.7176 \text{ s}$.

The height of the board above the water is

$$h - y = \frac{1}{2}gt^2 = \frac{1}{2}\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(0.7176 \text{ s})^2 = \boxed{2.53 \text{ m}}$$

(b) It takes the $\boxed{\text{same time}}$ to reach the water. Gravity and the vertical distance determine the time, not the horizontal speed.

20. (a) The time for the ball to drop 555 ft is given by

$$h - y = \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2(h-y)}{g}} = \sqrt{\frac{2(555 \text{ ft})\left(\frac{1 \text{ m}}{3.281 \text{ ft}}\right)}{9.81 \frac{\text{m}}{\text{s}^2}}} = 5.873 \text{ s}$$

The horizontal distance traveled in this time is $x = v_x t = \left(5.00 \frac{\text{m}}{\text{s}}\right)(5.873 \text{ s}) = \boxed{29.4 \text{ m}}$.

(b) $v_y = -gt = -\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(5.873 \text{ s}) = -57.6 \frac{\text{m}}{\text{s}}$

$$v = \sqrt{\left(-57.6 \frac{\text{m}}{\text{s}}\right)^2 + \left(5.00 \frac{\text{m}}{\text{s}}\right)^2} = \boxed{57.8 \text{ m/s}}$$

$$\theta = \tan^{-1}\left(\frac{-57.6 \frac{\text{m}}{\text{s}}}{5.00 \frac{\text{m}}{\text{s}}}\right) = \boxed{-85.0°}$$

21. The horizontal distance the ball travels is given by $x = v_0\sqrt{\frac{2h}{g}}$ and $\frac{h}{x} = \tan\theta$.

So,

$$\frac{h}{\tan\theta} = v_0\sqrt{\frac{2h}{g}}$$

$$\frac{h^2}{\tan^2\theta} = \frac{2v_0{}^2 h}{g}$$

$$h = \frac{2v_0{}^2}{g}\tan^2\theta$$

$$= \frac{2\left(4.20\ \frac{\text{m}}{\text{s}}\right)^2}{9.81\ \frac{\text{m}}{\text{s}^2}}\tan^2(-30.0°)$$

$$= \boxed{1.20\ \text{m}}$$

22. (a) The acceleration is due only to gravity.

$$\vec{\mathbf{a}} = \boxed{9.81\ \text{m/s}^2\ \text{downward}}$$

(b) $v_y{}^2 = -2g\Delta y$

$$v_y = \sqrt{-2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(-0.75\ \text{m})} = 3.836\ \frac{\text{m}}{\text{s}}$$

$$v_x = \sqrt{v^2 - v_y^2} = \sqrt{\left(4.0\ \frac{\text{m}}{\text{s}}\right)^2 - \left(3.836\ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{1.1\ \text{m/s}}$$

(c) $v_x = \sqrt{\left(5.0\ \frac{\text{m}}{\text{s}}\right)^2 - \left(3.836\ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{3.2\ \text{m/s}}$

23. (a) $v_x = v_0\cos\theta = \left(17.0\ \frac{\text{m}}{\text{s}}\right)\cos 35.0° = \boxed{13.9\ \text{m/s}}$

(b) $v_{0y} = v_0\sin\theta = \left(17.0\ \frac{\text{m}}{\text{s}}\right)\sin 35.0° = 9.75\ \frac{\text{m}}{\text{s}}$

At the top of the ball's trajectory, $v_y = 0$, so

$$v_y = v_{0y} - gt = 0$$

$$t = \frac{v_{0y}}{g} = \frac{9.75\ \frac{\text{m}}{\text{s}}}{9.81\ \frac{\text{m}}{\text{s}^2}} = 0.994\ \text{s}$$

The total time is just twice this.

$2t = 2(0.994\ \text{s}) = \boxed{1.99\ \text{s}}$

24. $v_{0y} = v_0 \sin\theta = \left(17.0\ \frac{\text{m}}{\text{s}}\right)\sin 35.0° = 9.75\ \frac{\text{m}}{\text{s}}$

The trajectory is symmetric.

$v_{\text{f}y} = -v_{0y} = \boxed{-9.75\ \text{m/s}}$

$\theta_{\text{f}} = -\theta_0 = \boxed{-35.0°}$

25. $v_x = \frac{x}{t} = \frac{1.30\ \text{m}}{1.25\ \text{s}} = 1.04\ \frac{\text{m}}{\text{s}} = v_{0x}$

$v_0 = \frac{v_{0x}}{\cos\theta} = \frac{1.04\ \frac{\text{m}}{\text{s}}}{\cos 35.0°} = \boxed{1.27\ \text{m/s}}$

26. At the peak of the ball's trajectory, $v_y = 0$, so $v_y = v_0 \sin\theta - gt = 0.$

$$t = \frac{v_0 \sin\theta}{g} = \frac{\left(9.50\ \frac{\text{m}}{\text{s}}\right)\sin 25.0°}{9.81\ \frac{\text{m}}{\text{s}^2}} = 0.40926\ \text{s}$$

The total time is twice this.

$2t = 2(0.40926\ \text{s}) = \boxed{0.819\ \text{s}}$

27. Determine the time.

$$y = (v_0 \sin\theta)t - \frac{1}{2}gt^2$$

$$-0.80\ \text{m} = \left(4.3\ \frac{\text{m}}{\text{s}}\right)\sin(-15°)t - \frac{1}{2}\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)t^2 = \left(-1.113\ \frac{\text{m}}{\text{s}}\right)t - \left(4.905\ \frac{\text{m}}{\text{s}^2}\right)t^2$$

$$0 = \left(4.905\ \frac{\text{m}}{\text{s}^2}\right)t^2 + \left(1.113\ \frac{\text{m}}{\text{s}}\right)t - 0.80\ \text{m}$$

$$t = \frac{-1.113\ \frac{\text{m}}{\text{s}} \pm \sqrt{\left(1.113\ \frac{\text{m}}{\text{s}}\right)^2 - 4\left(4.905\ \frac{\text{m}}{\text{s}^2}\right)(-0.80\ \text{m})}}{2\left(4.905\ \frac{\text{m}}{\text{s}^2}\right)}$$

$= -0.1135\ \text{s} \pm 0.4195$

$= 0.306\ \text{s}$ (using the positive solution)

The horizontal distance traveled is

$$x = v_x t = (v_0 \cos\theta)t = \left(4.3\ \frac{\text{m}}{\text{s}}\right)\cos(-15°)(0.306\ \text{s}) = \boxed{1.3\ \text{m}}$$

28. Determine the time.

$$y = (v_0 \sin\theta)t - \frac{1}{2}gt^2$$

$$-0.80\ \text{m} = \left(4.3\ \frac{\text{m}}{\text{s}}\right)\sin(15°)t - \frac{1}{2}\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)t^2 = \left(1.113\ \frac{\text{m}}{\text{s}}\right)t - \left(4.905\ \frac{\text{m}}{\text{s}^2}\right)t^2$$

$$0 = \left(4.905\ \frac{\text{m}}{\text{s}^2}\right)t^2 - \left(1.113\ \frac{\text{m}}{\text{s}}\right)t - 0.80\ \text{m}$$

$$t = \frac{1.113\ \frac{\text{m}}{\text{s}} \pm \sqrt{\left(-1.113\ \frac{\text{m}}{\text{s}}\right)^2 - 4\left(4.905\ \frac{\text{m}}{\text{s}^2}\right)(-0.80\ \text{m})}}{2\left(4.905\ \frac{\text{m}}{\text{s}^2}\right)}$$
$$= 0.1135\ \text{s} \pm 0.4195$$
$$= 0.533\ \text{s (using the positive solution)}$$

So, $x = v_x t = (v_0 \cos\theta)t = \left(4.3\dfrac{\text{m}}{\text{s}}\right)\cos(15°)(0.533\,\text{s}) = \boxed{2.2\,\text{m}}$.

29. (a) The speed of snowball A is the same as that of snowball B, because the total speed is independent of launch angle.

(b) Determine the time it takes for a snowball to reach the ground.

$$y = v_{0y}t - \frac{1}{2}gt^2$$
$$0 = \frac{1}{2}gt^2 - v_{0y}t + y$$

Solve for t using the quadratic equation.

$$t = \frac{v_{0y}}{g} \pm \frac{1}{g}\sqrt{{v_{0y}}^2 - 2gy}$$

For A:

$$t = \frac{-13\ \frac{\text{m}}{\text{s}}}{9.81\ \frac{\text{m}}{\text{s}^2}} \pm \frac{1}{9.81\ \frac{\text{m}}{\text{s}^2}}\sqrt{\left(-13\ \frac{\text{m}}{\text{s}}\right)^2 - 2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(-7.0\ \text{m})}$$
$$= -1.325\ \text{s} \pm 1.784\ \text{s}$$
$$= 0.459\ \text{s (using the positive solution)}$$

$$v = v_y = v_{0y} - gt = -13\ \frac{\text{m}}{\text{s}} - \left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.459\ \text{s}) = -18\ \frac{\text{m}}{\text{s}}$$

$|v| = \boxed{18\ \text{m/s}}$

For B:

$$t = \frac{\left(13\ \frac{\text{m}}{\text{s}}\right)\sin 25°}{9.81\ \frac{\text{m}}{\text{s}^2}} \pm \frac{1}{9.81\ \frac{\text{m}}{\text{s}^2}}\sqrt{\left(13\ \frac{\text{m}}{\text{s}}\right)^2 \sin^2 25° - 2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(-7.0\ \text{m})}$$
$$= 0.560\ \text{s} \pm 1.32\ \text{s}$$
$$= 1.88\ \text{s (using the positive solution)}$$

$$v_x = v_{0x} = \left(13\ \frac{\text{m}}{\text{s}}\right)\cos 25° = 11.78\ \frac{\text{m}}{\text{s}}$$

$$v_y = v_{0y} - gt = \left(13\ \frac{\text{m}}{\text{s}}\right)\sin 25° - \left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(1.88\ \text{s}) = -12.95\ \frac{\text{m}}{\text{s}}$$

$$v = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{\left(11.78\ \frac{\text{m}}{\text{s}}\right)^2 + \left(-12.95\ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{18\ \text{m/s}}$$

30. Snowball A is traveling in the $\boxed{\text{negative } y\text{-direction}}$.
For B:

$$v_x = v_{0x} = v\cos\theta = \left(13\ \frac{\text{m}}{\text{s}}\right)\cos(25°) = 11.78\ \frac{\text{m}}{\text{s}}$$

Determine the time it takes for the snowball to reach the ground.

$$y = v_{0y}t - \frac{1}{2}gt^2$$

$$0 = \frac{1}{2}gt^2 - v_{0y}t + y$$

$$t = \frac{v_{0y} \pm \sqrt{v_{0y}^{\ 2} - 2gy}}{g}$$

$$= \frac{\left(13\ \frac{\text{m}}{\text{s}}\right)\sin(25°) \pm \sqrt{\left[\left(13\ \frac{\text{m}}{\text{s}}\right)\sin(25)°\right]^2 - 2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(-7.0\text{ m})}}{9.81\ \frac{\text{m}}{\text{s}^2}}$$

$$= 0.560\text{ s} \pm 1.32\text{ s}$$

$$= 1.88\text{ s (using the positive solution)}$$

So,

$$v_y = v_{0y} - gt = \left(13\ \frac{\text{m}}{\text{s}}\right)\sin(25°) - \left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(1.88\text{ s}) = -12.9\ \frac{\text{m}}{\text{s}}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-12.9\ \frac{\text{m}}{\text{s}}}{11.78\ \frac{\text{m}}{\text{s}}}\right) = \boxed{48° \text{ below the horizontal}}$$

31. **(a)** The range of a projectile is given by $R = (v_0 / g)^2 \sin 2\theta$. This equation has its maximum, for some v_0, when sin 2*q* = 1. The sine of 90° equals one, therefore the optimal angle is 90°/2 = 45°.

So, $R = \dfrac{\left(30.0\ \frac{\text{m}}{\text{s}}\right)^2}{9.81\ \frac{\text{m}}{\text{s}^2}}\sin 90° = \boxed{91.7\text{ m}}$.

(b) The minimum speed of the ball occurs when $v_x = v_{0x}$ and $v_y = 0$.

$$v_{0x} = \left(30.0\ \frac{\text{m}}{\text{s}}\right)\cos 45° = \boxed{21.2\ \text{m/s}}$$

Since v_x is constant, any v_y will increase the speed.

32. $y_{\text{max}} = \dfrac{v_0^{\ 2}\sin^2\theta}{2g} = \dfrac{\left(30.0\ \frac{\text{m}}{\text{s}}\right)^2 \sin^2 45°}{2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)} = \boxed{22.9\text{ m}}$

33. The maximum height is achieved at time $t = \frac{1}{2}(4.50\text{ s}) = 2.25\text{ s},$ and at that time $v_y = 0.$

Since $v_y = v_{0y} - gt,$

$$v_{0y} = \left(9.81\frac{\text{m}}{\text{s}^2}\right)(2.25\,\text{s}) = 22.07\ \frac{\text{m}}{\text{s}}$$

So, $v_0 = \dfrac{22.07\ \frac{\text{m}}{\text{s}}}{\sin 63.0°} = \boxed{24.8\text{ m/s}}.$

34. **(a)** $v_x = v_0 \cos\theta = \left(13\ \frac{\text{m}}{\text{s}}\right)\cos 24° = 11.9\ \frac{\text{m}}{\text{s}}$

$$t = \frac{x}{v_x} = \frac{4.2\text{ m}}{11.9\ \frac{\text{m}}{\text{s}}} = \boxed{0.35\text{ s}}$$

(b) $y = (v_0 \sin\theta)t - \frac{1}{2}gt^2 = \left(13\ \frac{\text{m}}{\text{s}}\right)\sin(24°)(0.353\,\text{s}) - \frac{1}{2}\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.353\,\text{s})^2 = \boxed{1.3\,\text{m}}$

35. **(a)** $v_x = v_0 \cos\theta = \left(13\ \frac{\text{m}}{\text{s}}\right)\cos 24° = 11.9\ \frac{\text{m}}{\text{s}}$

$$t = \frac{x}{v_x} = \frac{4.2\,\text{m}}{11.88\frac{\text{m}}{\text{s}}} = 0.354\text{ s}$$

$$v_y = v_0 \sin\theta - gt = \left(13\ \frac{\text{m}}{\text{s}}\right)\sin 24° - \left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.3535\text{ s}) = 1.82\ \frac{\text{m}}{\text{s}}$$

$$v = \sqrt{\left(11.9\ \frac{\text{m}}{\text{s}}\right)^2 + \left(1.82\ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{12\ \text{m/s}}$$

$$\theta = \tan^{-1}\left(\frac{1.82\ \frac{\text{m}}{\text{s}}}{11.9\ \frac{\text{m}}{\text{s}}}\right) = \boxed{8.7°}$$

(b) $\boxed{\text{No}}$; the y-component of the velocity is still positive.

36. The average speed of a rider on the Ferris wheel is $v = \frac{x}{t} = \frac{C}{t} = \frac{2\pi r}{t} = \frac{2\pi(5.00\text{ m})}{32.0\text{ s}} = 0.982\ \frac{\text{m}}{\text{s}}.$

From your perspective, the wheel is rotating clockwise. Place the origin at the bottom of the wheel. Find the position of the keys when they are lost.

$x_0 = r\cos\theta_0 = (5.00\text{ m})\cos 135° = -3.54\text{ m (left of center)}$

$$\begin{aligned} y_0 = r\sin\theta_0 + r &= (5.00\text{ m})\sin 135° + 5.00\,\text{m} \\ &= 8.54\text{ m (above the base of the wheel)} \\ &= 8.54\text{ m} + 1.75\text{ m} \\ &= 10.29\text{ m (above the ground)} \end{aligned}$$

Find the initial components of velocity.

$$v_{0x} = \left(0.982\ \frac{\text{m}}{\text{s}}\right)\cos 45° = 0.6944\ \frac{\text{m}}{\text{s}}$$

$$v_{0y} = \left(0.982\ \frac{\text{m}}{\text{s}}\right)\sin 45° = 0.6944\ \frac{\text{m}}{\text{s}}$$

Now, $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$, so when the keys hit the ground,

$$\frac{1}{2}gt^2 - v_{0y}t - y_0 = 0$$

$$t = \frac{v_{0y} \pm \sqrt{v_{0y}{}^2 + 2gy_0}}{g} = \frac{0.6944\ \frac{\text{m}}{\text{s}} \pm \sqrt{\left(0.6944\ \frac{\text{m}}{\text{s}}\right)^2 + 2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(10.29\,\text{m})}}{9.81\frac{\text{m}}{\text{s}^2}} = -1.38\text{ s}, 1.52\text{ s}$$

The negative result is extraneous and so is ignored.

$$x = x_0 + v_{0x}t = -3.54\text{ m} + \left(0.6944\ \frac{\text{m}}{\text{s}}\right)(1.52\text{ s}) = -2.48\text{ m}$$

The keys land 2.48 m left of the base.

37. $y = (v_0 \sin\theta)t - \frac{1}{2}gt^2 = \left(2.25\ \frac{\text{m}}{\text{s}}\right)\sin(35.0°)(1.60\text{ s}) - \frac{1}{2}\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(1.60\text{ s})^2 = -10.5\text{ m}$

The girl was 10.5 m above the water.

38. At the projectile's highest point, $v_y = 0$.

So, $v_x = v_{0x} = \frac{v_0}{4}$.

$$v_{0x} = v_0 \cos\theta$$

$$\cos\theta = \frac{v_{0x}}{v_0} = \left(\frac{v_0}{4}\right)\left(\frac{1}{v_0}\right) = \frac{1}{4}$$

$$\theta = \cos^{-1}\left(\frac{1}{4}\right) = 75.5°$$

39. $R = \frac{v_0{}^2}{g}\sin 2\theta$

$$v_0{}^2 = \frac{gR}{\sin 2\theta}$$

$$v_0 = \sqrt{\frac{gR}{\sin 2\theta}} = \sqrt{\frac{\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(4086\text{ ft})\left(\frac{0.3048\text{ m}}{1\text{ ft}}\right)}{\sin 90.0°}} = 111\text{ m/s}$$

40. $v_y{}^2 = v_0{}^2 \sin^2\theta - 2g\Delta y$

$$\Delta y = \frac{v_0{}^2 \sin^2\theta - v_y{}^2}{2g}$$

$v_y = 0$ when the dolphin passes through the hoop, so

$$\Delta y = \frac{v_0{}^2 \sin^2\theta}{2g} = \frac{\left(12.0\ \frac{\text{m}}{\text{s}}\right)^2 \sin^2 40.0°}{2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)} = 3.03\text{ m}$$

41. $v_0 = 24.8\ \frac{\text{m}}{\text{s}}$ (see Problem 33)

$\theta = 63.0°$

$$R = \frac{v_0^2}{g}\sin 2\theta = \frac{\left(24.8\ \frac{\text{m}}{\text{s}}\right)^2}{9.81\ \frac{\text{m}}{\text{s}^2}}\sin 126° = \boxed{50.7\ \text{m}}$$

42. $v_{0x} = \frac{x}{t} = \frac{92.2\ \text{m}}{4.30\ \text{s}} = 21.44\ \frac{\text{m}}{\text{s}}$

$v_y = 0$ when the ball reaches its maximum height. This happens at $t = \frac{4.30\ \text{s}}{2}$.

$v_y = v_{0y} - gt = 0$

$$v_{0y} = gt = \left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(2.15\ \text{s}) = 21.09\ \frac{\text{m}}{\text{s}}$$

$$\theta = \tan^{-1}\frac{v_{0y}}{v_{0x}} = \tan^{-1}\left(\frac{21.09\ \frac{\text{m}}{\text{s}}}{21.44\ \frac{\text{m}}{\text{s}}}\right) = \boxed{44.5°}$$

$$v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} = \sqrt{\left(21.44\ \frac{\text{m}}{\text{s}}\right)^2 + \left(21.09\ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{30.1\ \text{m/s}}$$

43. (a) $R = \frac{v_0^2}{g}\sin 2\theta$

$$v_0^2 = \frac{gR}{\sin 2\theta}$$

$$v_0 = \sqrt{\frac{gR}{\sin 2\theta}} = \sqrt{\frac{\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(296\ \text{ft})\left(\frac{1\ \text{m}}{3.28\ \text{ft}}\right)}{\sin 90.0°}} = \boxed{29.8\ \text{m/s}}$$

(b) At the ball's maximum height, $v_y = 0$ and $t = \frac{1}{2}t_{\text{total}}$.

$v_y = v_0 \sin\theta - gt = 0$

$$t = \frac{v_0 \sin\theta}{g} = \frac{\left(29.75\ \frac{\text{m}}{\text{s}}\right)\sin 45°}{9.81\ \frac{\text{m}}{\text{s}^2}} = 2.144\ \text{s}$$

$$t_{\text{total}} = 2(2.144\ \text{s}) = \boxed{4.29\ \text{s}}$$

44. (a) The maximum height of the ball appears to be about 2.5 times the height of the cart. So $y = 2.5(11\ \text{cm}) = 28$ cm.

At the top $v_y = 0$.

$v_y^2 = v_{0y}^2 - 2g\Delta y = 0$

$$v_{0y} = \sqrt{2g\Delta y} = \sqrt{2(9.81\ \text{m/s}^2)(0.28\ \text{m})} = \boxed{2.3\ \text{m/s}}$$

(b) During 2 flashes the ball appears to fall about 10 cm.

$$\Delta y = v_{0y}t + \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2\Delta y}{g}} = \sqrt{\frac{2(0.10 \text{ m})}{9.81 \text{ m/s}^2}} = 0.14 \text{ s}$$

The time between flashes is then half this value, or $\boxed{0.07 \text{ s}}$.

45. (a)

$$5.00 \text{ m/s} = \sqrt{(1.12 \text{ m/s})^2 + v_y^2}$$

$$25.0 \text{ m}^2/\text{s}^2 = (1.12 \text{ m/s})^2 + v_y^2$$

$$v_y = \pm\sqrt{25.0 \text{ m}^2/\text{s}^2 - (1.12 \text{ m/s})^2} = \pm 4.873 \text{ m/s}$$

Use the positive value since the bale is rising when the speed first equals 5.00 m/s.

$$v_y = v_{0y} - gt$$

$$t = \frac{v_{0y} - v_y}{g} = \frac{8.85 \text{ m/s} - 4.87 \text{ m/s}}{9.81 \text{ m/s}^2} = \boxed{0.405 \text{ s}}$$

(b) If it is moving at 45° below the horizontal then

$$v_y = -v_{0x} = -1.12 \text{ m/s}$$

$$v_y = v_{0y} - gt$$

$$-1.12 \text{ m/s} = 8.85 \text{ m/s} - (9.81 \text{ m/s}^2)t$$

$$t = \frac{-9.97 \text{ m/s}}{-9.81 \text{ m/s}^2} = \boxed{1.02 \text{ s}}$$

(c) The initial vertical velocity component will be larger, so it will rise higher and stay longer in the air. Time in the air will $\boxed{\text{increase}}$.

46. (a)

$$v_x = v_{0x} = v_0 \cos\theta = \left(10.2 \ \frac{\text{m}}{\text{s}}\right)\cos 25.0° = 9.24 \ \frac{\text{m}}{\text{s}}$$

$$v_y = v_{0y} - gt = v_0 \sin\theta - gt = \left(10.2 \ \frac{\text{m}}{\text{s}}\right)\sin 25.0° - \left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)(0.250 \text{ s}) = 1.86 \ \frac{\text{m}}{\text{s}}$$

$$v = \sqrt{v_x{}^2 + v_y{}^2} = \sqrt{\left(9.24 \ \frac{\text{m}}{\text{s}}\right)^2 + \left(1.86 \ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{9.43 \text{ m/s}}$$

$$\theta = \tan^{-1}\left(\frac{1.86 \ \frac{\text{m}}{\text{s}}}{9.24 \ \frac{\text{m}}{\text{s}}}\right) = \boxed{11.4°}$$

(b)

$$v_y = \left(10.2 \ \frac{\text{m}}{\text{s}}\right)\sin 25.0° - \left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)(0.500 \text{ s}) = -0.594 \ \frac{\text{m}}{\text{s}}$$

$$v = \sqrt{\left(9.24 \ \frac{\text{m}}{\text{s}}\right)^2 + \left(-0.594 \ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{9.26 \text{ m/s}}$$

$$\theta = \tan^{-1}\left(\frac{-0.594 \ \frac{\text{m}}{\text{s}}}{9.24 \ \frac{\text{m}}{\text{s}}}\right) = \boxed{-3.68°}$$

(c) The ball had reached its greatest height $\boxed{\text{before}}$ $t = 0.500$ s because at $t = 0.500$ s, $v_y < 0$.

47. $v_y = 0$ when the ball has reached its maximum height, so

$$v_y = 0 = v_0 \sin\theta - gt$$

$$\sin\theta = \frac{gt}{v_0}$$

$$\theta = \sin^{-1}\frac{gt}{v_0} = \sin^{-1}\frac{\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.750\ \text{s})}{10.2\ \frac{\text{m}}{\text{s}}} = \boxed{46.2°}$$

48. **(a)** $R = \frac{{v_0}^2}{g}\sin 2\theta = \frac{\left(42.0\ \frac{\text{m}}{\text{s}}\right)^2}{\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}\sin 70.0° = \boxed{169\ \text{m}}$

(b) $\boxed{R \text{ is proportional to } \sin 2\theta.}$

$\sin 2(35°) = \sin 70° = \sin 110° = \sin 2(55.0°)$

$\boxed{\theta_0 = 55.0°}$

49. **(a)** At the lava's maximum height, $v_y = 0$.

$$v_y{}^2 = v_{0y}{}^2 - 2a\Delta y = 0$$

$$v_{0y} = \sqrt{2a\Delta y} = \sqrt{2\left(1.80\ \frac{\text{m}}{\text{s}^2}\right)(2.00\times10^5\ \text{m}-0)} = \boxed{849\ \text{m/s}}$$

(b) The maximum height reached by the lava would be $\boxed{\text{less}}$ on Earth due to Earth's greater gravity.

50. **(a)** To answer part (b), it must be assumed that the train has a component of velocity in the westward direction. Choose the x-axis to be along north.

$$x = v_x t = (v\cos\theta)t$$

$$\cos\theta = \frac{x}{vt} = \frac{150\ \text{m}}{\left(27\ \frac{\text{m}}{\text{s}}\right)(10.0\ \text{s})} = \frac{5}{9}$$

$$\theta = \cos^{-1}\frac{5}{9} = 56°$$

The train is traveling in a direction $\boxed{56° \text{ west of north}}$.

(b) $y = v_y t = v(\sin\theta)t = \left(27\ \frac{\text{m}}{\text{s}}\right)(\sin 56.3°)(10.0\ \text{s}) = \boxed{220\ \text{m}}$

51. $v_y = v_{0y} = \boxed{4.6\ \text{m/s}}$

52. $x = v_0\sqrt{\frac{2h}{g}}$

$$x^2 = v_0{}^2\left(\frac{2h}{g}\right)$$

$$h = \frac{gx^2}{2v_0{}^2} = \frac{\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(1.95\ \text{m})^2}{2\left(4.87\ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{0.786\ \text{m}}$$

53. (a) $\vec{\mathbf{v}}_{\text{cork,obs}} = \vec{\mathbf{v}}_{\text{bal,obs}} + \vec{\mathbf{v}}_{\text{cork,bal}} = \boxed{\left(2.0\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{y}} + \left(5.0\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{x}}}$

$$v_{\text{cork,obs}} = \sqrt{\left(5.0\ \frac{\text{m}}{\text{s}}\right)^2 + \left(2.0\ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{5.4\ \text{m/s}}$$

$$\theta = \tan^{-1}\left(\frac{2.0\ \frac{\text{m}}{\text{s}}}{5.0\ \frac{\text{m}}{\text{s}}}\right) = 22°$$

$\vec{\mathbf{v}}_{\text{cork,obs}} = 5.4$ m/s at $\boxed{22° \text{ above the horizontal}}$.

(b) $y_{\text{max}} = \frac{v_{0y}{}^2}{2g} + y_0 = \frac{\left(2.0\ \frac{\text{m}}{\text{s}}\right)^2}{2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)} + 6.0\ \text{m} = \boxed{6.2\ \text{m}}$

(c) $v_y = 0$ when the cork reaches its maximum height.

$$v_y = v_{0y} - gt_1 = 0$$

$$t_1 = \frac{v_{0y}}{g} = \frac{2.0\ \frac{\text{m}}{\text{s}}}{9.81\ \frac{\text{m}}{\text{s}^2}} = 0.204\ \text{s}$$

The time it takes for the cork to fall from its maximum height to the ground is given by

$h = \frac{1}{2}gt_2{}^2.$

$$t_2 = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(6.20\ \text{m})}{9.81\ \frac{\text{m}}{\text{s}^2}}} = 1.124\ \text{s}$$

$$t = t_1 + t_2 = 0.204\ \text{s} + 1.124\ \text{s} = \boxed{1.3\ \text{s}}$$

54. (a) $\vec{\mathbf{v}}_{\text{cork,obs}} = \vec{\mathbf{v}}_{\text{bal,obs}} + \vec{\mathbf{v}}_{\text{cork,bal}} = \boxed{\left(-2.0\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{y}} + \left(5.0\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{x}}}$

$$\vec{\mathbf{v}}_{\text{cork,obs}} = \sqrt{\left(5.0\ \frac{\text{m}}{\text{s}}\right)^2 + \left(-2.0\ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{5.4\ \text{m/s}}$$

$$\theta = \tan^{-1}\left(\frac{-2.0\ \frac{\text{m}}{\text{s}}}{5.0\ \frac{\text{m}}{\text{s}}}\right) = -22°$$

$\vec{\mathbf{v}}_{\text{cork,obs}} = 5.4$ m/s at $\boxed{22° \text{ below the horizontal}}$.

(b) The balloon is descending. The cork is ejected with a below-horizontal velocity, 6 m above the ground. The maximum height is $\boxed{6\text{ m}}$.

(c) $\Delta y = v_{0y}t - \frac{1}{2}gt^2$

$0 = \frac{1}{2}gt^2 - v_{0y}t + \Delta y$

Use the quadratic formula to solve for t.

$$t = \frac{v_{0y}}{g} \pm \frac{\sqrt{v_{0y}{}^2 - r\left(\frac{1}{2}g\right)\Delta y}}{g}$$

$$= \frac{-2.0\ \frac{\text{m}}{\text{s}}}{9.81\ \frac{\text{m}}{\text{s}^2}} \pm \frac{\sqrt{\left(-2.0\ \frac{\text{m}}{\text{s}}\right)^2 - 2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(-6.0\text{ m})}}{9.81\ \frac{\text{m}}{\text{s}^2}}$$

$$= -0.204\text{ s} \pm 1.125\text{ s}$$

$$= \boxed{0.92\text{ s}}$$

55. (a) Use the vector magnitude formula and $v_y = v_{0y} - gt$ to determine v_{0y}. Then use the relationship of v_0 and v_{0y} to determine θ.

(b)

$$\begin{aligned} & v_0{}^2 = v_{0y}{}^2 + v_{0x}{}^2 \\ - \quad & v^2 = v_y{}^2 + v_{0x}{}^2 \\ \hline & v_0{}^2 - v^2 = v_{0y}{}^2 - v_y{}^2 \end{aligned}$$

$$\left(12.0\ \frac{\text{m}}{\text{s}}\right)^2 - \left(11.3\ \frac{\text{m}}{\text{s}}\right)^2 = v_{0y}{}^2 - v_y{}^2$$

$$= v_{0y}{}^2 - (v_{0y} - gt)^2$$

$$= v_{0y}{}^2 - v_{0y}{}^2 + 2gv_{0y}t - g^2t^2$$

$$16.31\ \frac{\text{m}^2}{\text{s}^2} = 2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.250\text{ s})v_{0y} - \left(9.81\ \frac{\text{m}}{\text{s}^2}\right)^2(0.250\text{ s})^2$$

$$\left(4.905\ \frac{\text{m}}{\text{s}}\right)v_{0y} = 16.31\ \frac{\text{m}^2}{\text{s}^2} + 6.01\ \frac{\text{m}^2}{\text{s}^2}$$

$$v_{0y} = \frac{22.32\ \frac{\text{m}^2}{\text{s}^2}}{4.905\ \frac{\text{m}}{\text{s}}} = 4.55\ \frac{\text{m}}{\text{s}}$$

$$v_{0y} = v_0 \sin\theta$$

$$\theta = \sin^{-1}\frac{v_{0y}}{v_0} = \sin^{-1}\left(\frac{4.55\ \frac{\text{m}}{\text{s}}}{12.0\ \frac{\text{m}}{\text{s}}}\right) = \boxed{22.3^\circ}$$

56. (a) $v_x = 0$ just before the particle turns around.

$v_x = v_{0x} + a_x t = 0$

$$t = \frac{-v_{0x}}{ax} = \frac{-2.40\ \frac{\text{m}}{\text{s}}}{-1.90\ \frac{\text{m}}{\text{s}^2}} = 1.263\ \text{s}$$

$$x = v_{0x}t + \frac{1}{2}a_x t^2 = \left(2.40\ \frac{\text{m}}{\text{s}}\right)(1.263\ \text{s}) + \frac{1}{2}\left(-1.90\ \frac{\text{m}}{\text{s}^2}\right)(1.263\ \text{s})^2 = \boxed{1.52\ \text{m}}$$

(b) $v_y = a_y t = \left(3.20\ \frac{\text{m}}{\text{s}^2}\right)(1.263\ \text{s}) = 4.04\ \text{m/s}$

$$\boxed{\vec{\mathbf{v}} = (4.04\ \text{m/s})\hat{\mathbf{y}}}$$

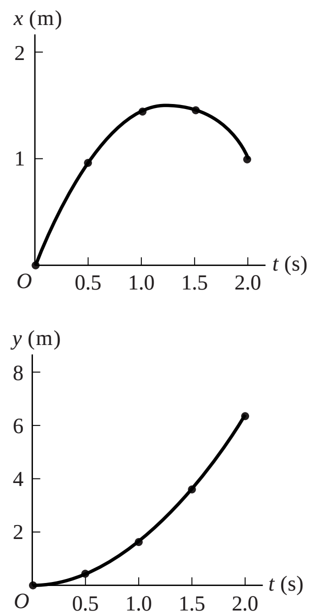
(c) $x = v_{0x}t + \frac{1}{2}a_x t^2$

$y = \frac{1}{2}a_y t^2$

for $t = 0.500$ s
$x = 0.963$ m
$y = 0.400$ m

for $t = 1.00$ s
$x = 1.46$ m
$y = 3.60$ m

for $t = 2.00$ s
$x = 1.00$ m
$y = 6.40$ m



57. (a) $y = y_0 + (v_0 \sin\theta)t - \frac{1}{2}gt^2 = 0$

$\frac{1}{2}gt^2 - (v_0 \sin\theta)t - y_0 = 0$

Use the quadratic formula to solve for t.

$$t = \frac{v_0 \sin\theta}{g} \pm \frac{\sqrt{v_0^2 \sin^2\theta - 4\left(\frac{1}{2}g\right)(-y_0)}}{g}$$

$$= \frac{\left(2.62\ \frac{\text{m}}{\text{s}}\right)\sin 60.5^\circ}{9.81\ \frac{\text{m}}{\text{s}^2}} \pm \frac{\sqrt{\left(2.62\ \frac{\text{m}}{\text{s}}\right)^2 \sin^2 60.5^\circ + 2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.455\ \text{m})}}{9.81\ \frac{\text{m}}{\text{s}^2}}$$

$$= \boxed{0.616\ \text{s}}$$

$t = -0.151$ is rejected, since $t \geq 0$.

(b) $x = (v_0 \cos\theta)t = \left(2.62\ \frac{\text{m}}{\text{s}}\right)(\cos 60.5^\circ)(0.6155\ \text{s}) = \boxed{0.794\ \text{m}}$

58. **(a)** $y = y_0 + (v_0 \sin\theta)t - \frac{1}{2}gt^2 = 0$

$\frac{1}{2}gt^2 - (v_0 \sin\theta)t - y_0 = 0$

Use the quadratic formula to solve for *t*.

$$t = \frac{v_0 \sin\theta}{g} \pm \frac{\sqrt{{v_0}^2 \sin^2\theta - 4\left(\frac{1}{2}g\right)(-y_0)}}{g}$$

$$= \frac{\left(2.62\ \frac{\text{m}}{\text{s}}\right)\sin(-30°)}{9.81\ \frac{\text{m}}{\text{s}^2}} \pm \frac{\sqrt{\left(2.62\ \frac{\text{m}}{\text{s}}\right)^2 \sin^2(-30°) + 2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.455\ \text{m})}}{9.81\ \frac{\text{m}}{\text{s}^2}}$$

$$= -0.134\ \text{s} \pm 0.333\ \text{s}$$

$$= \boxed{0.199\ \text{s}}$$

(b) $x = (v_0 \cos\theta)t = \left(2.62\ \frac{\text{m}}{\text{s}}\right)\cos(-30°)(0.199\ \text{s}) = \boxed{0.452\ \text{m}}$

59. Determine the time it takes for the shot-put to reach the ground.

$y = h + v_{0y}t - \frac{1}{2}gt^2 = 0$

$0 = \frac{1}{2}gt^2 - v_{0y}t - h$

Use the quadratic formula to solve for *t*.

$$t = \frac{v_{0y}}{g} + \frac{1}{g}\sqrt{{v_{0y}}^2 + 2gh}$$

$$= \frac{v_{0y}}{g}\left(1 + \sqrt{1 + \frac{2gh}{{v_{0y}}^2}}\right)$$

$$= \frac{\left(3.50\ \frac{\text{m}}{\text{s}}\right)\sin\theta}{9.81\ \frac{\text{m}}{\text{s}^2}}\left(1 + \sqrt{1 + \frac{2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(5.00\ \text{ft})\left(\frac{0.3048\ \text{m}}{1\ \text{ft}}\right)}{\left(3.50\ \frac{\text{m}}{\text{s}}\right)^2 \sin^2\theta}}\right)$$

(a) $t = 0.693$ s

$x = v_{0x}t = \left(3.50\ \frac{\text{m}}{\text{s}}\right)(\cos 20.0°)(0.693\ \text{s}) = \boxed{2.28\ \text{m}}$

(b) $t = 0.764$ s

$x = \left(3.50\ \frac{\text{m}}{\text{s}}\right)(\cos 30.0°)(0.764\ \text{s}) = \boxed{2.32\ \text{m}}$

(c) $t = 0.832$ s

$x = \left(3.50\ \frac{\text{m}}{\text{s}}\right)(\cos 40.0°)(0.832\ \text{s}) = \boxed{2.23\ \text{m}}$

60. $v_y = 0$ at the ball's maximum height.

$$v_y = v_0 \sin\theta - g\left(\frac{T}{2}\right) = 0$$

$$v_0 = \frac{gT}{2\sin\theta} = \frac{\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(2.50\ \text{s})}{2\sin 40.0^\circ} = \boxed{19.1\ \text{m/s}}$$

61. (a) $\boxed{12.1\ \text{m/s}}$ The minimum speed occurs at the $\boxed{\text{top}}$ of its motion where $v_y = 0$.

(b) $\vec{\mathbf{v}}_{\text{cg}} = \vec{\mathbf{v}}_{\text{ct}} + \vec{\mathbf{v}}_{\text{tg}} = (4.38\ \text{m/s})\hat{\mathbf{y}} + (12.1\ \text{m/s})\hat{\mathbf{x}}$

$$v_{\text{cg}} = \sqrt{(4.38\ \text{m/s})^2 + (12.1\ \text{m/s})^2} = \boxed{12.9\ \text{m/s}}$$

$$\theta = \tan^{-1}\left(\frac{4.38\ \text{m/s}}{12.1\ \text{m/s}}\right) = \boxed{19.9^\circ}$$

(c) $$y_{\max} = \frac{(v_0 \sin\theta)^2}{2g} = \frac{[(12.87\ \text{m/s})(\sin 19.9^\circ)]^2}{2(9.81\ \text{m/s}^2)} = \boxed{0.978\ \text{m}}$$

(d) $v_y^2 = v_{0y}^2 - 2g\Delta y$ At top, $v_y = 0$.

$$\Delta y = \frac{v_{0y}^2}{2g} = \frac{(4.38\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} = \boxed{0.978\ \text{m}}$$

62. (a) $v_y^2 = v_{0y}^2 - 2g\Delta y$ At top, $v_y = 0$.

$$v_{0y} = \sqrt{2g\Delta y} = \sqrt{2(9.81\ \text{m/s}^2)(61.5\ \text{m})} = \boxed{34.7\ \text{m/s}}$$

(b) For the cannon at the base of the cliff, the maximum range is $R_{\max} = \frac{v_0^2}{g} = \frac{(34.7\ \text{m/s})^2}{9.81\ \text{m/s}^2} = \boxed{123\ \text{m}}$.

For the cannon at the top of the cliff, $R = v_{0x}t$, where $t = \sqrt{\frac{2\Delta y}{g}}$ since $v_{0y} = 0$.

$$R = v_{0x}\sqrt{\frac{2\Delta y}{g}} = (34.7\ \text{m/s})\sqrt{\frac{2(61.5\ \text{m})}{9.81\ \text{m/s}^2}} = \boxed{123\ \text{m}}$$

63. Determine $v_0 t$.

$$x = v_0 t \cos\theta$$

$$v_0 t = \frac{x}{\cos\theta} = \frac{23.12\ \text{m}}{\cos 42.0^\circ} = 31.11\ \text{m}$$

Determine v_0.

$$y = y_0 + v_0 t \sin\theta - \frac{1}{2}gt^2 = 0$$

$$0 = y_0 + (v_0 t)\sin\theta - \left(\frac{g}{2}\right)\left(\frac{v_0^{\ 2}t^2}{v_0^{\ 2}}\right)$$

$$= (6.00\ \text{ft})\left(\frac{0.3048\,\text{m}}{1\,\text{ft}}\right) + (31.11\ \text{m})\sin 42.0° - \left(\frac{9.81\ \frac{\text{m}}{\text{s}^2}}{2}\right)\frac{(31.1\ \text{m})^2}{v_0^{\ 2}}$$

$$= 22.6\ \text{m} - \frac{4747\ \frac{\text{m}^3}{\text{s}^2}}{v_0^{\ 2}}$$

$$v_0 = \sqrt{\frac{4747\ \frac{\text{m}^3}{\text{s}^2}}{22.65\ \text{m}}} = \boxed{14.5\ \text{m/s}}$$

64. Determine the speed of your snowball 5.0 m above the ground.

$$v^2 = v_x^{\ 2} + v_y^{\ 2}$$
$$= v_{0x}^{\ 2} + v_{0y}^{\ 2} - 2g\Delta y$$
$$= v_0^{\ 2}\cos^2\theta + v_0^{\ 2}\sin^2\theta - 2g\Delta y$$
$$= v_0^{\ 2}(\cos^2\theta + \sin^2\theta) - 2g\Delta y$$
$$= v_0^{\ 2} - 2g(y - y_0)$$
$$= \left(12\ \frac{\text{m}}{\text{s}}\right)^2 - 2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(5.0\ \text{m} - 15\ \text{m})$$
$$= 340\ \frac{\text{m}^2}{\text{s}^2}$$
$$v = \boxed{18\,\text{m/s}}$$

Since v is not a function of launch angle θ, your friend's snowball will have the same speed.

65. (a) To find the initial speed of the puck, eliminate t from the equations $x = (v_0 \cos\theta)t$ and $y = (v_0 \sin\theta)t - (1/2)gt^2$, then solve for v_0.

(b) $x = (v_0 \cos\theta)t$

$$t = \frac{x}{v_0 \cos\theta}$$

$$y = (v_0 \sin\theta)t - \frac{1}{2}gt^2$$

$$y = (v_0 \sin\theta)\left(\frac{x}{v_0 \cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{v_0 \cos\theta}\right)^2$$

$$= x\tan\theta - \frac{gx^2}{2v_0^2 \cos^2\theta}$$

$$x\tan\theta - y = \frac{gx^2}{2v_0^2 \cos^2\theta}$$

$$v_0 = \sqrt{\frac{gx^2}{2\cos^2\theta(x\tan\theta - y)}}$$

$$= \sqrt{\frac{\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(20.2\ \text{m})^2}{2(\cos^2 15.0°)[(20.2\ \text{m})\tan 15.0° - 2.00\ \text{m}]}}$$

$$= \boxed{25.1\ \text{m/s}}$$

66. $v_x = v_0 \cos\theta = \dfrac{x}{t}$

$$t = \frac{x}{v_0 \cos\theta}$$

$$y = (v_0 \sin\theta)t - \frac{1}{2}gt^2$$

$$= (v_0 \sin\theta)\left(\frac{x}{v_0 \cos\theta}\right) - \frac{1}{2}g\frac{x^2}{v_0^2 \cos^2\theta}$$

$$= x\tan\theta - \left(\frac{gx^2}{\cos^2\theta}\right)\frac{1}{v_0^2}$$

$$\left(\frac{2\cos^2\theta}{gx^2}\right)(x\tan\theta - y) = \frac{1}{v_0^2}$$

$$v_0 = \sqrt{\left(\frac{gx^2}{2\cos^2\theta}\right)\left(\frac{1}{x\tan\theta - y}\right)}$$

$$= \sqrt{\frac{\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(45\ \text{yd})^2\left(\frac{0.9144\ \text{m}}{\text{yd}}\right)^2}{(2\cos^2 40°)\left[(45\ \text{yd})\left(\frac{0.9144\ \text{m}}{\text{yd}}\right)\tan 40° - (-0.750\ \text{m})\right]}}$$

$$= \boxed{20\,\text{m/s}}$$

67. (a) Determine the time it takes for the stream of water to reach the insect.

$$y = v_{0y}t - \frac{1}{2}gt^2$$

$$0 = \frac{1}{2}gt^2 - v_{0y}t + y$$

Use the quadratic formula to solve for t.

$$t = \frac{v_{0y}}{g} \pm \frac{1}{g}\sqrt{{v_{0y}}^2 - 2gy}$$

$$= \frac{\left(2.00\ \frac{\text{m}}{\text{s}}\right)\sin 50.0°}{9.81\ \frac{\text{m}}{\text{s}^2}} \pm \frac{1}{9.81\ \frac{\text{m}}{\text{s}^2}}\sqrt{\left(2.00\ \frac{\text{m}}{\text{s}}\right)^2 \sin^2 50.0° - 2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.0300\ \text{m})}$$

$$= 0.156\ \text{s} \pm 0.135\ \text{s}$$

$$= 0.021\ \text{s},\ 0.291\ \text{s}$$

The shortest time it will take for the archerfish to hit its target is 0.021 s.
The horizontal distance is

$$x = v_{0x}t = (v_0 \cos\theta)t = \left(2.00\ \frac{\text{m}}{\text{s}}\right)(\cos 50.0°)(0.021\ \text{s}) = \boxed{2.7\ \text{cm}}$$

(b) The beetle will have $\boxed{0.021\ \text{s}}$ to react.

68. (a) Determine the time it takes for the stream of water to reach the insect.

$$y = v_{0y}t - \frac{1}{2}gt^2$$

$$0 = \frac{1}{2}gt^2 - v_{0y}t + y$$

Use the quadratic formula to solve for t.

$$t = \frac{v_{0y}}{g} \pm \frac{1}{g}\sqrt{{v_{0y}}^2 - 2gy}$$

$$= \frac{\left(2.00\ \frac{\text{m}}{\text{s}}\right)\sin 50°}{9.81\ \frac{\text{m}}{\text{s}^2}} \pm \frac{1}{9.81\ \frac{\text{m}}{\text{s}^2}}\sqrt{\left(2.00\ \frac{\text{m}}{\text{s}}\right)^2 \sin^2 50° - 2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.0300\ \text{m})}$$

$$= 0.156\ \text{s} \pm 0.135\ \text{s}$$

$$= 0.021\ \text{s},\ 0.291\ \text{s}$$

The greatest horizontal distance from which the archerfish can hit the beetle is

$$x = v_{0x}t = (v_0 \cos\theta)t = \left(2.00\ \frac{\text{m}}{\text{s}}\right)\cos 50°(0.291\ \text{s}) = \boxed{37.4\ \text{cm}}$$

(b) The beetle has $\boxed{0.291\ \text{s}}$ to react.

69. $R = \frac{v_0^2}{g}\sin 2\theta$

$$y_{\text{max}} = \frac{v_0^2 \sin^2\theta}{2g}$$

Set $R = y_{\text{max}}$.

$$\frac{v_0^2}{g}\sin 2\theta = \frac{v_0^2 \sin^2\theta}{2g}$$

$$2\sin\theta\cos\theta = \frac{1}{2}\sin^2\theta$$

$$4\cos\theta = \sin\theta$$

$$4 = \tan\theta$$

$$\theta = \tan^{-1} 4 = \boxed{76^\circ}$$

70. (a) $x = v_0\sqrt{\frac{2h}{g}} = W$

$$v_0 = \boxed{W\sqrt{\frac{g}{2h}}}$$

(b) $v_y^2 = 2gh$

$$v_y = -\sqrt{2gh}$$

$$\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{-\sqrt{2gh}}{v_0} = \tan^{-1}\frac{-\sqrt{2gh}}{W\sqrt{\frac{g}{2h}}} = \boxed{\tan^{-1}\left(\frac{-2h}{W}\right)}$$

71. The landing speed is given by

$$v^2 = v_x^2 + v_y^2$$

$$= v_{0x}^2 + v_{0y}^2 - 2g\Delta y$$

$$= v_0^2 - 2g\Delta y$$

$$\boxed{v = \sqrt{v_0^2 - 2g(y-h)} \neq f(\theta)}$$

The landing speed *v* is independent of launch angle θ for a given launch height *h*.

72. $\frac{H}{R} = \left(\frac{v_0^2\sin^2\theta}{2g}\right)\left(\frac{g}{v_0^2\sin 2\theta}\right)$

$$= \frac{\sin^2\theta}{2\sin 2\theta}$$

$$= \frac{\sin^2\theta}{4\sin\theta\cos\theta}$$

$$= \frac{\sin\theta}{4\cos\theta}$$

$$= \frac{1}{4}\tan\theta$$

73. Determine T_0.

$$y = v_{0y}t - \frac{1}{2}gt^2 = 0$$

$$t = \frac{2v_{0y}}{g} = T_0$$

Determine T.

$$y = v_{0y}T - \frac{1}{2}gT^2 = h$$

$$0 = \frac{1}{2}gT^2 - v_{0y}T + h$$

Use the quadratic formula to find T.

$$T = \frac{v_{0y}}{g} \pm \frac{1}{g}\sqrt{v_{0y}^2 - 2gh} = \frac{v_{0y}}{g} \pm \frac{v_{0y}}{g}\sqrt{1 - \frac{2gh}{v_{0y}^2}}$$

$H = \dfrac{v_{0y}^2}{2g}$ and $T_0 = \dfrac{2v_{0y}}{g}$, so

$$T = \frac{T_0}{2} \pm \frac{T_0}{2}\sqrt{1 - \frac{h}{H}} = \frac{1}{2}T_0\left(1 + \sqrt{1 - \frac{h}{H}}\right)$$

where the plus sign was chosen to make T later than the time of maximum height.

74. $v_y^2 = 2gh$

$$v_y = \pm\sqrt{2gh}$$

$$\tan\theta = \frac{v_y}{v_x} = \frac{\pm\sqrt{2gh}}{v_0}$$

$$h = \boxed{\frac{v_0^2 \tan^2\theta}{2g}}$$

75. (a) $\vec{\mathbf{v}}_{cg} = \vec{\mathbf{v}}_{ct} + \vec{\mathbf{v}}_{tg} = (-2.25 \text{ m/s})\hat{\mathbf{x}} + (4.38 \text{ m/s})\hat{\mathbf{y}} + (12.1 \text{ m/s})\hat{\mathbf{x}} = (9.9 \text{ m/s})\hat{\mathbf{x}} + (4.38 \text{ m/s})\hat{\mathbf{y}}$

At the $\boxed{\text{top}}$ of the motion the vertical component of the coin's velocity is zero. At that point its minimum speed is $\boxed{9.9 \text{ m/s}}$.

(b) $v_{cg} = \sqrt{(9.85 \text{ m/s})^2 + (4.38 \text{ m/s})^2} = \boxed{11 \text{ m/s}}$

$$\theta = \tan^{-1}\left(\frac{4.38 \text{ m/s}}{9.85 \text{ m/s}}\right) = \boxed{24°}$$

(c) $y_{max} = \dfrac{(v_0 \sin\theta)^2}{2g} = \dfrac{(10.78 \text{ m/s})^2(\sin 23.97)^2}{2(9.81 \text{ m/s}^2)} = \boxed{0.978 \text{ m}}$

(d) $\Delta y = \dfrac{v_{0y}^2}{2g} = \dfrac{(4.38 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = \boxed{0.978 \text{ m}}$

Other than rounding, the results are the same.

76. $r=\sqrt{x^2+y^2}$

$$=\sqrt{[v_0(\cos\theta)t]^2+\left[v_0(\sin\theta)t-\frac{1}{2}gt^2\right]^2}$$

$$=\sqrt{v_0^2t^2(\cos^2\theta+\sin^2\theta)-v_0(\sin\theta)gt^3+g^2t^4/4}$$

$$\boxed{r=\sqrt{v_0^2t^2-v_0(\sin\theta)gt^3+(g^2/4)t^4}}$$

The following graph was created using $v_0 = 20$ m/s and $\theta = 73.7°$. The graph shows r increasing, then decreasing, then increasing again.