Chapter 10

Rotational Kinematics and Energy

Answers to Even-numbered Conceptual Questions

2. The angular speed of an object with constant angular acceleration changes linearly with time. Therefore, at the time $t/2$ the angular speed of this object is $\omega/2$.

4. Jason and Betsy both complete one revolution in the same amount of time. Therefore, their periods of rotation are the same.

6. Yes. In fact, this is the situation whenever you drive in a circular path with constant speed.

8. Every point on Earth has the same angular speed. Therefore, the smallest linear speed occurs where the distance from the axis of rotation is smallest; namely, at the poles.

10. At the top of these building, your distance from the axis of rotation of the Earth is greater than it was on the ground floor. Therefore, your linear speed at the top of the building is greater than it is on the ground floor.

12. The moment of inertia is greater when the axis of rotation is on the rim of the wheel. The reason is that much of the wheel's mass is now at a significantly greater distance from the axis of rotation, compared with the case where the axis is at the center of the wheel.

14. The long, thin minute hand – with mass far from the axis of rotation – has the greater moment of inertia.

16. The moment of inertia of an object changes with the position of the axis of rotation because the distance from the axis to all the elements of mass have been changed. It is not just the shape of an object that matters, but the distribution of mass with respect to the axis of rotation.

18. Spin the two spheres with equal angular speeds. The one with the larger moment of inertia – the hollow sphere – has the greater kinetic energy, and hence will spin for a longer time before stopping.

20. The moment of inertia of the Earth was increased slightly, because mass (water in the dam) was moved from a lower elevation to a higher elevation as the dam was filled. Raising this mass to a higher elevation moves it farther from the axis of rotation, which increases the moment of inertia.

22. When the chunky stew is rolled down the aisle, all of the contents of the can roll together with approximately the same angular speed. This is because the chunky stew is thick and almost solid. The beef broth, however, is little more than water. Therefore, when the broth is rolled down the aisle, almost all that is actually rolling is the metal can itself. It follows that the stew has the greater initial kinetic energy, and hence it rolls a greater distance.

24. Assuming the ball starts spinning immediately on encountering the no-slip surface, with no loss of energy, it will rise to the same height from which it was released. However, some energy will be lost in a real system as the ball begins to spin; therefore, the ball should reach a height slightly less than its release height.

26. **(a)** What determines the winner of the race is the ratio I/mr^2, as we see in the discussion just before Conceptual Checkpoint 10-4. This ratio is $MR^2/MR^2 = 1$ for the first hoop and $(2M)R^2/[(2M)R^2] = 1$ for the second hoop. Therefore, the two hoops finish the race at the same time. **(b)** As in part (a), we can see that the ratio I/mr^2 is equal to 1 regardless of the radius. Thus, all hoops, regardless of their mass or radius, finish the race in the same time.

Solutions to Problems

1. $30°\left(\frac{\pi}{180°}\right) = \boxed{\frac{\pi}{6}}$

$45°\left(\frac{\pi}{180°}\right) = \boxed{\frac{\pi}{4}}$

$90°\left(\frac{\pi}{180°}\right) = \boxed{\frac{\pi}{2}}$

$180°\left(\frac{\pi}{180°}\right) = \boxed{\pi}$

2. $\frac{\pi}{6}\left(\frac{180°}{\pi}\right) = \boxed{30°}$

$0.70\pi\left(\frac{180°}{\pi}\right) = \boxed{126°}$

$1.5\pi\left(\frac{180°}{\pi}\right) = \boxed{270°}$

$5\pi\left(\frac{180°}{\pi}\right) = \boxed{900°}$

3. **(a)** $\boxed{\text{1 rev/min}}$

(b) $\boxed{\text{1 rev/h}}$

4. **(a)** $\left(\frac{1\text{ rev}}{60\text{ s}}\right)\left(\frac{3600\text{ s}}{\text{h}}\right) = \boxed{60\text{ rev/h}}$

(b) $\left(\frac{1\text{ rev}}{60\text{ s}}\right)\left(\frac{60\text{ s}}{\text{min}}\right)\left(\frac{360\text{ deg}}{\text{rev}}\right) = \boxed{360\text{ deg/min}}$

(c) $\left(\frac{1\text{ rev}}{60\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{\text{rev}}\right) = \boxed{\frac{\pi}{30}\text{rad/s}}$

5. tire: $\left(2.00\times10^3\ \frac{\text{deg}}{\text{s}}\right)\left(\frac{\pi}{180\text{ deg}}\right) = 34.9\text{ rad/s}$

drill: $\left(400.0\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi}{\text{rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 41.9\text{ rad/s}$

propeller: 40.0 rad/s

$\boxed{\text{tire, propeller, drill}}$

6. $\theta = \frac{s}{r} = \frac{1.50\text{ m}}{0.38\text{ m}} = \boxed{3.9\text{ rad}}$

7. $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.033\text{ s}} = \boxed{190\ \text{rad/s}}$

8. $\left(\frac{1\text{ rev}}{\text{day}}\right)\left(\frac{2\pi}{\text{rev}}\right)\left(\frac{1\text{ day}}{24\text{ h}}\right)\left(\frac{1\text{ h}}{60\text{ min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = \boxed{7.27\times10^{-5}\ \text{rad/s}}$

9. $\left(\frac{1\text{ rev}}{\text{yr}}\right)\left(\frac{1\text{ yr}}{365\text{ days}}\right)\left(\frac{1\text{ day}}{24\text{ h}}\right)\left(\frac{1\text{ h}}{60\text{ min}}\right) = \boxed{1.9\times10^{-6}\ \text{rev/min}}$

10. (a) $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.200\text{ s}} = \boxed{31.4\ \text{rad/s}}$

(b) $v_T = r\omega = \left(\frac{3.5\text{ in.}}{2}\right)\left(31.4\ \frac{\text{rad}}{\text{s}}\right) = \boxed{55\ \text{in./s}}$

(c) The $\boxed{\text{same}}$; angular speed does not depend on the distance from the axis of rotation.

11. (a) $\omega_{\text{av}} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_{0.010\text{ s}} - \theta_0}{0.010\text{ s}} = \frac{[(1.25\text{ rad/s})(0.010\text{ s}) + (42.5\text{ rad/s}^2)(0.010\text{ s})^2] - 0}{0.010\text{ s}} = \boxed{1.3\times10^2\ \text{rad/s}}$

(b) $\omega_{\text{av}} = \frac{\theta_{1.010\text{ s}} - \theta_{1.000\text{ s}}}{0.010\text{ s}} = \frac{[(125\text{ rad/s})(1.010\text{ s}) + (42.5\text{ rad/s}^2)(1.010\text{ s})^2] - (125\text{ rad} + 42.5\text{ rad})}{0.010\text{ s}}$

$= \boxed{2.1\times10^2\ \text{rad/s}}$

(c) $\omega_{\text{av}} = \frac{\theta_{2.010\text{ s}} - \theta_{2.000\text{ s}}}{0.010\text{ s}}$

$= \frac{[(125\text{ rad/s})(2.010\text{ s}) + (42.5\text{ rad/s}^2)(2.010\text{ s})^2] - [(125\text{ rad/s})(2.000\text{ s}) + (42.5\text{ rad/s})(2.000\text{ s})^2]}{0.010\text{ s}}$

$= \boxed{3.0\times10^2\ \text{rad/s}}$

(d) $\boxed{\text{positive}}$ The positive angular velocity is increasing.

(e) $\alpha_{\text{av}} = \frac{\omega_{1.00\text{ s}} - \omega_0}{1.00\text{ s}} = \frac{210.4\text{ rad/s} - 125.4\text{ rad/s}}{1.00\text{ s}} = \boxed{85\ \text{rad/s}^2}$

$\alpha_{\text{av}} = \frac{\omega_{2.00\text{ s}} - \omega_{1.00\text{ s}}}{1.00\text{ s}} = \frac{295.4\text{ rad/s} - 210.4\text{ rad/s}}{1.00\text{ s}} = \boxed{85\ \text{rad/s}^2}$

12. $\omega = \omega_0 + \alpha t$

$t = \frac{\omega - \omega_0}{\alpha} = \frac{-5.0\ \frac{\text{rad}}{\text{s}} - 5.40\ \frac{\text{rad}}{\text{s}}}{-2.10\ \frac{\text{rad}}{\text{s}^2}} = \boxed{5.0\text{ s}}$

13.
$$\alpha = \frac{\omega^2 - \omega_0{}^2}{2(\theta - \theta_0)}$$
$$\theta - \theta_0 = \frac{\omega^2 - \omega_0{}^2}{2\alpha} = \frac{\left(2.00\ \frac{\text{rad}}{\text{s}}\right)^2 - \left(3.40\ \frac{\text{rad}}{\text{s}}\right)^2}{2\left(-0.736\ \frac{\text{rad}}{\text{s}^2}\right)} = \boxed{5.14\ \text{rad}}$$

14. $$\alpha = \frac{\omega^2 - \omega_0^2}{2\Delta\theta} = \frac{(26\ \text{rad/s})^2 - (12\ \text{rad/s})^2}{2(2.5\ \text{rev})\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)} = \boxed{17\ \text{rad/s}^2}$$

15. $$\Delta\theta = \frac{1}{2}(\omega_0 + \omega)t = \frac{1}{2}(12\ \text{rad/s} + 26\ \text{rad/s})(2.5\ \text{s}) = \boxed{48\ \text{rad}}$$

16. (a) $$\omega^2 = \omega_0{}^2 + 2\alpha(\theta - \theta_0)$$
$$\alpha = \frac{\omega^2 - \omega_0{}^2}{2(\theta - \theta_0)} = \frac{0^2 - \left(6.15\ \frac{\text{rad}}{\text{s}}\right)^2}{2(13.2\ \text{rev})\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)} = \boxed{-0.228\ \text{rad/s}^2}$$

(b) $$\omega = \omega_0 + \alpha t$$
$$t = \frac{\omega - \omega_0}{\alpha} = \frac{0 - 6.15\ \frac{\text{rad}}{\text{s}}}{-0.228\ \frac{\text{rad}}{\text{s}^2}} = \boxed{27.0\ \text{s}}$$

17. (a) $$\omega = \omega_0 + \alpha t$$
$$\alpha = \frac{\omega - \omega_0}{t} = \frac{0 - 0.50\ \frac{\text{rev}}{\text{s}}}{12\ \text{s}} = -0.0417\ \frac{\text{rev}}{\text{s}^2}$$
$$\omega^2 = \omega_0{}^2 + 2\alpha(\theta - \theta_0)$$
$$\theta - \theta_0 = \frac{\omega^2 - \omega_0{}^2}{2\alpha} = \frac{0 - \left(0.50\ \frac{\text{rev}}{\text{s}}\right)^2}{2\left(-0.0417\ \frac{\text{rev}}{\text{s}^2}\right)} = \boxed{3.0\ \text{rev}}$$

(b) $$\theta - \theta_0 = \frac{\left(0.25\ \frac{\text{rev}}{\text{s}}\right)^2 - \left(0.50\ \frac{\text{rev}}{\text{s}}\right)^2}{2\left(-0.0417\ \frac{\text{rev}}{\text{s}^2}\right)} = \boxed{2.2\ \text{rev}}$$

18. (a) $\omega^2 = \omega_0{}^2 + 2\alpha(\theta - \theta_0)$

$$\theta - \theta_0 = \frac{\omega^2 - \omega_0{}^2}{2\alpha} = \frac{\left(6.3\ \frac{\text{rad}}{\text{s}}\right)^2 - 0^2}{2.2\ \frac{\text{rad}}{\text{s}^2}} = (18.0\text{ rad})\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right) = \boxed{2.9\text{ rev}}$$

(b) $\omega = \omega_0 + \alpha t$

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{6.3\ \frac{\text{rad}}{\text{s}} - 0}{2.2\ \frac{\text{rad}}{\text{s}^2}} = \boxed{2.9\text{ s}}$$

19. $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$$\theta_{\substack{\text{hour}\\\text{hand}}} = \frac{1}{4} + \left(\frac{1\text{ rev}}{12\text{ h}}\right)\left(\frac{1\text{ h}}{60\text{ min}}\right)t + \frac{1}{2}(0)t^2 = \frac{1}{4}\text{ rev} + \left(\frac{1\text{ rev}}{720\text{ min}}\right)t$$

$$\theta_{\substack{\text{minute}\\\text{hand}}} = 0 + \left(\frac{1\text{ rev}}{\text{h}}\right)\left(\frac{1\text{ h}}{60\text{ min}}\right)t + \frac{1}{2}(0)t^2 = \left(\frac{1\text{ rev}}{60\text{ min}}\right)t$$

$$\theta_{\substack{\text{hour}\\\text{hand}}} - \theta_{\substack{\text{minute}\\\text{hand}}} = \frac{1}{8}\text{ rev}$$

Substitute.

$$\frac{1}{4}\text{rev} + \left(\frac{1\text{ rev}}{720\text{ min}}\right)t - \left(\frac{1\text{ rev}}{60\text{ min}}\right)t = \frac{1}{8}\text{rev}$$

$$\frac{1}{8}\text{rev} = \left[\left(\frac{1\text{ rev}}{60\text{ min}}\right) - \left(\frac{1\text{ rev}}{720\text{ min}}\right)\right]t$$

$$\frac{1}{8}\text{rev} = \left[\frac{12\text{ rev}}{720\text{ min}} - \frac{1\text{ rev}}{720\text{ min}}\right]t$$

$$\frac{1}{8}\text{rev} = \left(\frac{11\text{ rev}}{720\text{ min}}\right)t$$

$$t = \left(\frac{1}{8}\text{rev}\right)\left(\frac{720\text{ min}}{11\text{ rev}}\right) = 8.18\text{ min}$$

The angle between the hands is 45.0° at $\boxed{\text{just past 3:08}}$.

20. (a) $\omega = \omega_0 + \alpha t$

$$|\alpha| = \left|\frac{\omega - \omega_0}{t}\right| = \left|\frac{0 - \left(3850\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)}{10.2\text{ s}}\right| = \boxed{6.29\ \text{rev/s}^2}$$

(b) $\omega^2 = \omega_0{}^2 + 2\alpha(\theta - \theta_0)$

$$\theta - \theta_0 = \frac{\omega^2 - \omega_0{}^2}{2\alpha} = \frac{0^2 - \left[\left(3850\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\right]^2}{2\left(-6.291\ \frac{\text{rev}}{\text{s}^2}\right)} = \boxed{327\ \text{rev}}$$

21. $\omega = \omega_0 + \alpha t$

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\left[\frac{1\text{ rev}}{(24\text{ h})\left(\frac{3600\text{ s}}{\text{h}}\right) + \left(\frac{0.548\text{ s}}{365\text{ rev}}\right)(1\text{ rev})} - \frac{1\text{ rev}}{(24\text{ h})\left(\frac{3600\text{ s}}{\text{h}}\right)}\right]}{(100\text{ yr})\left(\frac{365\text{ days}}{\text{yr}}\right)\left(\frac{24\text{ h}}{\text{day}}\right)\left(\frac{3600\text{ s}}{\text{h}}\right)} = \boxed{-4.01\times10^{-22}\ \text{rad/s}^2}$$

22. (a) Calculate the angular acceleration, then the angular displacement.

(b) $\omega = \omega_0 + \alpha t$

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{\left(310\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) - 0}{3.0\text{ s}} = 1.72\ \frac{\text{rev}}{\text{s}^2}$$

$$\omega^2 = \omega_0{}^2 + 2\alpha(\theta - \theta_0)$$

$$\theta - \theta_0 = \frac{\omega^2 - \omega_0{}^2}{2\alpha} = \frac{\left[\left(310\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\right]^2 - 0^2}{2\left(1.72\ \frac{\text{rev}}{\text{s}^2}\right)} = \boxed{7.8\ \text{rev}}$$

23. (a) $\omega = \omega_0 + \alpha t$

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{0 - \left(5820\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)}{2.00\text{ s}} = \boxed{-48.5\ \text{rev/s}^2}$$

(b) $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

$$\theta - \theta_0 = \frac{\omega^2 - \omega_0^2}{2\alpha} = \frac{0^2 - \left[\left(5820\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\right]^2}{2\left(-48.5\ \frac{\text{rev}}{\text{s}^2}\right)} = (97.0\text{ rev})\left(\frac{2\pi\text{ rad}}{\text{rev}}\right) = 609.5\text{ rad}$$

$$s = r(\theta - \theta_0) = (5.00\text{ in.})(609.5)\left(\frac{1\text{ ft}}{12\text{ in.}}\right) = \boxed{254\text{ feet}}$$

(c) $\Delta\theta = 97.0$ rev, so the blade returns to its initial angular position, and $d = \boxed{0}$.

24. (a) $\omega = \omega_0 + \alpha t$

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{\left(2.0\times 10^5\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{\text{rev}}\right) - 0}{750\ \frac{\text{rad}}{\text{s}^2}} = \boxed{28\text{ s}}$$

(b) $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

$$\theta - \theta_0 = \frac{\omega^2 - \omega_0^2}{2\alpha} = \frac{\left[\left(2.0\times 10^5\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\right]^2 - 0^2}{2\left(750\ \frac{\text{rad}}{\text{s}^2}\right)} = (2.92\times 10^5\text{ rad})\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right) = \boxed{4.7\times 10^4\text{ rev}}$$

25. $v_t = r\omega = (7.2\text{ cm})\left(\frac{1\text{ rev}}{12\text{ h}}\right)\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = \boxed{0.010\ \text{mm/s}}$

26. **(a)** $\omega_1 = \omega_2 = \dfrac{1\text{ rev}}{4.5\text{ s}} = \boxed{0.22\ \text{rev/s}}$

(b) $v_{t1} = r_1\omega_1 = (2.0\text{ m})\left(\dfrac{2\pi}{\text{rev}}\right)\left(\dfrac{1\text{ rev}}{4.5\text{ s}}\right) = \boxed{2.8\ \text{m/s}}$

$v_{t2} = r_2\omega_2 = (1.5\text{ m})\left(\dfrac{2\pi}{\text{rev}}\right)\left(\dfrac{1\text{ rev}}{4.5\text{ s}}\right) = \boxed{2.1\ \text{m/s}}$

27. $\omega = \dfrac{v_t}{r} = \dfrac{3.4\text{ m/s}}{\left(\frac{0.29\text{ m}}{2}\right)} = \boxed{23\text{ rad/s}}$

28. **(a)** $v_t = r\omega = (2.75\text{ m})\left(\dfrac{1\text{ rev}}{45\text{ s}}\right)\left(\dfrac{2\pi\text{ rad}}{\text{rev}}\right) = \boxed{0.38\text{ m/s}}$

(b) $v_t = (1.75\text{ m})\left(\dfrac{1\text{ rev}}{45\text{ s}}\right)\left(\dfrac{2\pi\text{ rad}}{\text{rev}}\right) = \boxed{0.24\text{ m/s}}$

29. **(a)** $\omega = \dfrac{v_t}{r} = \dfrac{8.50\ \frac{\text{m}}{\text{s}}}{7.20\text{ m}} = \boxed{1.18\ \text{rad/s}}$

(b) $a_{cp} = \dfrac{v_t^2}{r} = \dfrac{\left(8.50\ \frac{\text{m}}{\text{s}}\right)^2}{7.20\text{ m}} = \boxed{10.0\ \text{m/s}^2}$

(c) $\boxed{\text{The vine}}$ pulls upward on Jeff.

30. $a_{cp} = r\omega^2 = (7.20\text{ m})\left(0.850\ \dfrac{\text{rad}}{\text{s}}\right)^2 = \boxed{5.20\ \text{m/s}^2}$

$a_t = r\alpha = (7.20\text{ m})\left(0.620\ \dfrac{\text{rad}}{\text{s}^2}\right) = \boxed{4.46\ \text{m/s}^2}$

$a = \sqrt{a_{cp}^2 + a_t^2} = \sqrt{\left(5.202\ \dfrac{\text{m}}{\text{s}^2}\right)^2 + \left(4.464\ \dfrac{\text{m}}{\text{s}^2}\right)^2} = \boxed{6.85\ \text{m/s}^2}$

$\theta = \tan^{-1}\left(\dfrac{a_{cp}}{a_t}\right) = \tan^{-1}\left(\dfrac{5.202\ \frac{\text{m}}{\text{s}^2}}{4.464\ \frac{\text{m}}{\text{s}^2}}\right) = \boxed{0.862\text{ rad}}$

31. $\omega = \omega_0 + \alpha t$

$\alpha = \dfrac{\omega - \omega_0}{t} = \dfrac{4.00\ \frac{\text{rev}}{\text{s}} - 0}{3.00\text{ s}} = 1.333\ \text{rev/s}^2$

(a) $a_t = r\alpha = (6.00\text{ cm})\left(1.333\ \dfrac{\text{rev}}{\text{s}^2}\right)\left(\dfrac{2\pi}{\text{rev}}\right) = \boxed{0.503\ \text{m/s}^2}$

(b) $\boxed{0.503\ \text{m/s}^2}$

32. (a) $v_t = r\omega = (6.00\text{ cm})(5.25\text{ rad/s}) = \boxed{31.5\text{ cm/s}}$

(b) $a_{cp} = r\omega^2 = (6.00\text{ cm})(5.25\text{ rad/s})^2 = 165\text{ cm/s}^2 = \boxed{1.65\text{ m/s}^2}$

(c) $v_t = \dfrac{31.5\text{ cm/s}}{2} = \boxed{15.8\text{ cm/s}}$

$a_{cp} = \dfrac{1.654\text{ m/s}^2}{2} = \boxed{0.827\text{ m/s}}$

33. (a) $v_t = r\omega = (3.7\text{ cm})\left(\dfrac{3.0\text{ rev}}{\text{s}}\right)\left(\dfrac{2\pi}{\text{rev}}\right) = \boxed{69.7\text{ cm/s}}$

(b) $\boxed{\text{It would also double.}}$

34. $\omega = \left(\dfrac{1\text{ rev}}{32\text{ s}}\right)\left(\dfrac{2\pi}{\text{rev}}\right) = 0.196\text{ rad/s}$

(a) $a_{cp} = r\omega^2 = (9.5\text{ m})\left(0.196\ \dfrac{\text{rad}}{\text{s}}\right)^2 = \boxed{0.366\text{ m/s}^2}$

The acceleration is in the direction of the axis of rotation, which is $\boxed{\text{downward}}$.

(b) $a_{cp} = \boxed{0.366\text{ m/s}^2}$; the direction is $\boxed{\text{upward}}$.

35. $a_t = r\alpha = (9.5\text{ m})\left(-0.22\ \dfrac{\text{rad}}{\text{s}^2}\right) = -2.09\ \dfrac{\text{m}}{\text{s}^2}$

From Example 30, $a_{cp} = 0.366\text{ m/s}^2$ in the downward direction.

$a = \sqrt{a_{cp}{}^2 + a_t{}^2} = \sqrt{\left(0.366\ \dfrac{\text{m}}{\text{s}^2}\right)^2 + \left(-2.09\ \dfrac{\text{m}}{\text{s}^2}\right)^2} = \boxed{2.12\text{ m/s}^2}$

$\theta = \tan^{-1}\dfrac{a_{cp}}{a_t} = \tan^{-1}\left(\dfrac{0.366\ \frac{\text{m}}{\text{s}^2}}{-2.09\ \frac{\text{m}}{\text{s}^2}}\right) = \boxed{170°}$

36. (a)
$$\begin{aligned}
\sum F_x &= ma_{cp}\\
T\sin\theta &= mr\omega^2\\
T\sin\theta &= mL(\sin\theta)\omega^2\\
T &= mL\omega^2\\
\omega &= \sqrt{\frac{T}{mL}}\\
&= \sqrt{\frac{11\text{ N}}{(0.52\text{ kg})(4.5\text{ m})}}\\
&= \boxed{2.2\text{ rad/s}}
\end{aligned}$$

(b) Since ω is inversely proportional to L, decreasing L $\boxed{\text{increases}}$ ω.

37. (a) $v_t = r\omega = (3.20\text{ mm})\left(2.15\times10^4\ \frac{\text{rad}}{\text{s}}\right) = \boxed{68.8\ \text{m/s}}$

(b) $T = \frac{2\pi r}{v_t} = \frac{2\pi(3.20\times10^{-3}\ \text{m})}{275\ \frac{\text{m}}{\text{s}}} = \boxed{7.31\times10^{-5}\ \text{s}}$

38. $a_t = r\alpha = (3.20\text{ mm})\left(232\ \frac{\text{rad}}{\text{s}^2}\right) = \boxed{74.2\ \text{cm/s}^2}$

$a_{cp} = r\omega^2 = (3.20\text{ mm})\left(640\ \frac{\text{rad}}{\text{s}}\right)^2 = \boxed{1.3\ \text{km/s}^2}$

39. (a) $\omega = \frac{v_t}{r} = \frac{2.18\times10^6\ \frac{\text{m}}{\text{s}}}{5.29\times10^{-11}\ \text{m}} = \boxed{4.12\times10^{16}\ \text{rad/s}}$

(b) $\omega = \left(4.12\times10^{16}\ \frac{\text{rad}}{\text{s}}\right)\left(\frac{1\ \text{rev}}{2\pi\ \text{rad}}\right) = 6.56\times10^{15}\ \frac{\text{rev}}{\text{s}}$

$\Rightarrow \boxed{6.56\times10^{15}\ \text{orbits}}$

(c) $a_{cp} = r\omega^2 = (5.29\times10^{-11}\ \text{m})\left(4.12\times10^{16}\ \frac{\text{rad}}{\text{s}}\right)^2 = \boxed{8.98\times10^{22}\ \text{m/s}^2}$

40. $a_t = R\alpha$

$a_{cp} = R\omega^2 = R(\alpha t)^2 = R\alpha^2 t^2$

$a_{cp} = a_t$

$R\alpha^2 t^2 = R\alpha$

$t^2 = \frac{R\alpha}{R\alpha^2}$

$t^2 = \frac{1}{\alpha}$

$t = \boxed{\sqrt{\frac{1}{\alpha}}}$

41. $\omega = \frac{v_t}{r} = \frac{25\ \frac{\text{m}}{\text{s}}}{(31\ \text{cm})\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)} = \boxed{81\ \text{rad/s}}$

42. $v_t = r\omega = (0.260\ \text{m})\left(0.373\ \frac{\text{rev}}{\text{s}}\right)\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right) = \boxed{0.609\ \text{m/s}}$

43. $r = \frac{C}{2\pi} = \frac{70.0 \text{ cm}}{2\pi} = 11.14 \text{ cm}$

$v = \frac{\Delta x}{\Delta t} = \frac{(12.0 \text{ yd})\left(\frac{3 \text{ ft}}{\text{yd}}\right)\left(\frac{1 \text{ m}}{3.281 \text{ ft}}\right)}{3.45 \text{ s}} = 3.18 \ \frac{\text{m}}{\text{s}}$

$\omega = \frac{v}{r} = \frac{3.18 \ \frac{\text{m}}{\text{s}}}{0.1114 \text{ m}} = \boxed{28.5 \ \text{rad/s}}$

44. $\alpha = \frac{a}{r} = \frac{1.12 \ \frac{\text{m}}{\text{s}^2}}{(33 \text{ cm})\left(\frac{\text{m}}{100 \text{ cm}}\right)} = 3.394 \ \frac{\text{rad}}{\text{s}^2}$

$\omega_0 = \frac{v_0}{r} = \frac{17 \ \frac{\text{m}}{\text{s}}}{(33 \text{ cm})\left(\frac{\text{m}}{100 \text{ cm}}\right)} = 51.52 \ \frac{\text{rad}}{\text{s}}$

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = \left(51.52 \ \frac{\text{rad}}{\text{s}}\right)(0.65 \text{ s}) + \frac{1}{2}\left(3.394 \ \frac{\text{rad}}{\text{s}^2}\right)(0.65 \text{ s})^2 = \boxed{34 \text{ rad}}$

45. (a) $a = \frac{\Delta v}{\Delta t} = \frac{8.90 \ \frac{\text{m}}{\text{s}} - 0}{12.2 \text{ s}} = 0.7295 \ \text{m/s}^2$

$\alpha = \frac{a}{r} = \frac{0.7295 \ \frac{\text{m}}{\text{s}^2}}{(36.0 \text{ cm})\left(\frac{1 \text{ m}}{100 \text{ cm}}\right)} = \boxed{2.03 \ \text{rad/s}^2}$

(b) $\boxed{\text{greater than}}$

46. $I = mR^2$

$R = \sqrt{\frac{I}{m}} = \sqrt{\frac{0.13 \text{ kg}\cdot\text{m}^2}{0.98 \text{ kg}}} = \boxed{0.36 \text{ m}}$

47. $\omega = \frac{(1 \text{ rev})\left(\frac{2\pi \text{ rad}}{\text{rev}}\right)}{4.20 \text{ s}} = 1.496 \ \text{rad/s}$

$K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(4.30 \text{ kg}\cdot\text{m}^2\right)\left(1.496 \ \frac{\text{rad}}{\text{s}}\right)^2 = \boxed{4.81 \text{ J}}$

48. $K = \frac{1}{2}I\omega^2$

$I = \frac{2K}{\omega^2} = \frac{2(4.1 \text{ J})}{\left(12 \ \frac{\text{rad}}{\text{s}}\right)^2} = \boxed{0.057 \text{ kg}\cdot\text{m}^2}$

49. (a) $K_{\text{t}} = \frac{1}{2}mv^2 = \frac{1}{2}(1.20 \text{ kg})\left(1.41 \ \frac{\text{m}}{\text{s}}\right)^2 = \boxed{1.19 \text{ J}}$

(b) $K_{\text{r}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(mr^2\right)\omega^2 = \frac{1}{2}mr^2\left(\frac{v}{r}\right)^2 = \frac{1}{2}mv^2 = \boxed{1.19 \text{ J}}$

(c) $K = K_\text{t} + K_\text{r} = 1.193\text{ J} + 1.193\text{ J} = \boxed{2.39\text{ J}}$

50. (a) $K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}mR^2\right)\omega^2 = \frac{1}{4}(0.013\text{ kg})(0.060\text{ m})^2\left(32\ \frac{\text{rad}}{\text{s}}\right)^2 = \boxed{1.2\times10^{-2}\text{ J}}$

(b) $\omega_0{}^2 = \frac{2K_0}{I}$

$\omega_0 = \sqrt{\frac{2K_0}{I}}$

Double the kinetic energy.

$\omega = \sqrt{\frac{2(2K_0)}{I}} = \sqrt{2}\omega_0 = \sqrt{2}\left(32\ \frac{\text{rad}}{\text{s}}\right) = \boxed{45\text{ rad/s}}$

51. $K_\text{t} = \frac{1}{2}mv^2 = \frac{1}{2}(0.15\text{ kg})\left(46\ \frac{\text{m}}{\text{s}}\right)^2 = \boxed{160\text{ J}}$

$K_\text{r} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2 = \frac{1}{5}MR^2\omega^2 = \frac{1}{5}(0.15\text{ kg})(0.037\text{ m})^2\left(41\ \frac{\text{rad}}{\text{s}}\right)^2 = \boxed{0.069\text{ J}}$

52. $K_\text{r} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{3}MR^2\right)\left(\frac{v}{R}\right)^2 = \frac{1}{3}Mv^2$

$K_\text{t} = \frac{1}{2}Mv^2$

$K = K_\text{r} + K_\text{t} = \frac{1}{3}Mv^2 + \frac{1}{2}Mv^2 = \frac{5}{6}Mv^2$

(a) $\frac{\frac{1}{3}Mv^2}{\frac{5}{6}Mv^2} = \boxed{\frac{2}{5}}$

(b) It $\boxed{\text{stays the same}}$ since the fraction doesn't depend on v.

53. rate of decrease of $K = \dfrac{K_\text{f} - K_\text{i}}{\Delta t}$

$$= \frac{\frac{1}{2}I_\text{E}\omega_\text{f}^2 - \frac{1}{2}I_\text{E}\omega_\text{i}^2}{\Delta t}$$

$$= \frac{0.331 M_\text{E} R_\text{E}^2\left(\omega_\text{f}^2 - \omega_\text{i}^2\right)}{2\Delta t}$$

$$= \frac{0.331(5.97\times10^{24}\text{ kg})(6.38\times10^{6}\text{m})^2\left\{\left[\frac{(1\text{ rev})\left(\frac{2\pi}{\text{rev}}\right)}{(24\text{ h})\left(\frac{3600\text{ s}}{\text{h}}\right)+\frac{0.548\text{ s}}{365}}\right]^2 - \left[\frac{(1\text{ rev})\left(\frac{2\pi}{\text{rev}}\right)}{(24\text{ h})\left(\frac{3600\text{ s}}{\text{h}}\right)}\right]^2\right\}}{2(100\text{ yr})\left(\frac{365\text{ days}}{\text{yr}}\right)\left(\frac{24\text{ h}}{\text{day}}\right)\left(\frac{3600\text{ s}}{\text{h}}\right)}$$

$$= \boxed{-2.34\times10^{12}\text{ W}}$$

$$= (-2.34\times10^{12}\text{ W})\left(\frac{1.341\times10^{-3}\text{ hp}}{\text{W}}\right)$$

$$= \boxed{-3.14\times10^{9}\text{ hp}}$$

54. (a) $K_\text{r} = \dfrac{1}{2}I\omega^2$

$$= \frac{1}{2}\left(\frac{1}{12}ML^2\right)\omega^2$$

$$= \frac{1}{24}(0.58\text{ kg})(0.56\text{ m})^2\left[\left(\frac{3500\text{ rev}}{\text{min}}\right)\left(\frac{2\pi}{\text{rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\right]^2$$

$$= \boxed{1.0\text{ kJ}}$$

(b) $K_\text{r} = U = mgh$

$$h = \frac{K_\text{r}}{mg} = \frac{1018\text{ J}}{(0.58\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)} = \boxed{180\text{ m}}$$

55. $h = \left(\dfrac{v^2}{2g}\right)\left(1 + \dfrac{I}{mR^2}\right)$

Solve for I.

$$I = mR^2\left(\frac{2gh}{v^2} - 1\right) = (2.1\text{ kg})(0.080\text{ m})^2\left[\frac{2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.074\text{ m})}{\left(0.33\ \frac{\text{m}}{\text{s}}\right)^2} - 1\right] = \boxed{0.17\text{ kg}\cdot\text{m}^2}$$

56.

$$E_\text{i} = E_\text{f}$$

$$\begin{aligned} mgh &= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \\ &= \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{r}\right)^2 \\ &= \frac{1}{2}v^2\left(m + \frac{I}{r^2}\right) \\ h &= \frac{v^2}{2mg}\left(m + \frac{I}{r^2}\right) \\ &= \frac{\left(0.50\ \frac{\text{m}}{\text{s}}\right)^2}{2(0.056\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}\left[0.056\ \text{kg} + \frac{2.9\times10^{-5}\ \text{kg}\cdot\text{m}^2}{(0.0064\ \text{m})^2}\right] \\ &= \boxed{0.17\ \text{m}} \end{aligned}$$

57.

$$U_\text{i} + K_\text{i} = U_\text{f} + K_\text{f}$$

$$mgh + 0 = 0 + \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{r}\right)^2$$

(a)

$$\begin{aligned} mgh &= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 \\ gh &= \frac{1}{2}v^2 + \frac{1}{4}v^2 \\ &= \frac{3}{4}v^2 \\ v &= \sqrt{\frac{4gh}{3}} \\ &= \sqrt{\frac{4\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(1.1\ \text{m})}{3}} \\ &= \boxed{3.8\ \text{m/s}} \end{aligned}$$

(b)

$$\begin{aligned} mgh &= \frac{1}{2}mv^2 + \frac{1}{2}\left(mr^2\right)\left(\frac{v}{r}\right)^2 \\ gh &= \frac{1}{2}v^2 + \frac{1}{2}v^2 \\ &= v^2 \\ v &= \sqrt{gh} \\ &= \sqrt{\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(1.1\ \text{m})} \\ &= \boxed{3.3\ \text{m/s}} \end{aligned}$$

58. (a) Equate the initial and final energies, then solve for the mass of the pulley.

(b)
$$U_i + K_i = U_f + K_f$$
$$m_1gh + 0 + 0 = m_2gh + \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{1}{2}I_p\omega^2$$
$$m_1gh = m_2gh + \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}\left(\frac{1}{2}m_pR_p^{\ 2}\right)\left(\frac{v}{R_p}\right)^2$$
$$\frac{1}{4}m_pv^2 = m_1gh - m_2gh - \frac{1}{2}(m_1 + m_2)v^2$$
$$m_p = \frac{4\left[(m_1 - m_2)gh - \frac{1}{2}(m_1 + m_2)v^2\right]}{v^2}$$
$$m_p = \frac{4(5.0\text{ kg} - 3.0\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.75\text{ m}) - \frac{1}{2}(5.0\text{ kg} + 3.0\text{ kg})\left(0.22\ \frac{\text{m}}{\text{s}}\right)^2}{\left(0.22\ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{1200\text{ kg}}$$

59. Perform an energy balance between the initial state and the state when the ball reaches the lowest point on the surface.
$$U_i + K_i = U_f + K_f$$
$$mgh_i + 0 = 0 + \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{r}\right)^2$$
$$mgh_i = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2$$
$$mgh_i = \frac{1}{2}mv^2 + \frac{1}{5}mv^2$$
$$= \frac{7}{10}mv^2$$
$$v = \sqrt{\frac{10gh_i}{7}}$$

(a) $\omega = \frac{v}{r} = \frac{1}{r}\sqrt{\frac{10gh_i}{7}} = \left(\frac{1}{0.028\text{ m}}\right)\sqrt{\frac{10(9.81\text{ m/s}^2)(0.75\text{ m})}{7}} = \boxed{120\text{ rad/s}}$

(b) Now, perform an energy balance between the time the ball reaches the lowest point on the surface and when the ball reaches its maximum height.

$$U_\text{i} + K_\text{ri} + K_\text{ti} = U_\text{f} + K_\text{rf} + K_\text{tf}$$

Now, $K_\text{ri} = K_\text{rf}$, so

$$U_\text{i} + K_\text{ti} = U_\text{r} + K_\text{tf}$$

$$0 + \frac{1}{2}mv^2 = mgh_\text{f} + 0$$

$$h_\text{f} = \frac{v^2}{2g} = \frac{\left(\frac{10gh_\text{i}}{7}\right)}{2g} = \frac{5h_\text{i}}{7} = \frac{5(0.75\text{ m})}{7} = \boxed{0.54\text{ m}}$$

60. (a)

$$U_\text{i} + K_\text{i} = U_\text{f} + K_\text{f}$$

$$0 + \frac{1}{2}mv_\text{i}^2 + \frac{1}{2}I\omega_\text{i}^2 = mgh + \frac{1}{2}mv_\text{f}^2 + \frac{1}{2}I\omega_\text{f}^2$$

$$\frac{1}{2}mv_\text{i}^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v_\text{i}}{r}\right)^2 = mgh + \frac{1}{2}mv_\text{f}^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v_\text{f}}{r}\right)^2$$

$$\frac{1}{2}mv_\text{i}^2 + \frac{1}{5}mv_\text{i}^2 = mgh + \frac{1}{2}mv_\text{f}^2 + \frac{1}{5}mv_\text{f}^2$$

$$\frac{7}{10}v_\text{i}^2 = gh + \frac{7}{10}v_\text{f}^2$$

$$v_\text{f} = \sqrt{v_\text{i}^2 - \frac{10}{7}gh} = \sqrt{\left(2.85\ \frac{\text{m}}{\text{s}}\right)^2 - \frac{10}{7}\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.53\text{ m})} = \boxed{0.83\ \text{m/s}}$$

(b) The speed is independent of the ball's radius, and $\boxed{\text{stays the same}}$.

61. (a)

$$U_\text{i} + K_\text{i} = U_\text{f} + K_\text{f}$$

$$m_\text{b}gh + 0 = 0 + \frac{1}{2}m_\text{b}v^2 + \frac{1}{2}I_\text{p}\omega^2$$

$$m_\text{b}gh = \frac{1}{2}m_\text{b}v^2 + \frac{1}{2}\left(\frac{1}{2}m_\text{p}r^2\right)\left(\frac{v}{r}\right)^2$$

$$m_\text{b}gh = \frac{1}{2}m_\text{b}v^2 + \frac{1}{4}m_\text{p}v^2$$

$$m_\text{b}gh = v^2\left(\frac{m_\text{b}}{2} + \frac{m_\text{p}}{4}\right)$$

$$v = \sqrt{\frac{m_\text{b}gh}{\frac{m_\text{b}}{2} + \frac{m_\text{p}}{4}}}$$

$$= \sqrt{\frac{(1.3\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.50\text{ m})}{\frac{1.3\text{ kg}}{2} + \frac{0.31\text{ kg}}{4}}}$$

$$= \boxed{3.0\ \text{m/s}}$$

(b) The speed will decrease, because I_p is increased.

62. The initial height of the center of mass of the leg is (0.95 m)/2 = 0.475 m.

$$U_\text{i} + K_\text{i} = U_\text{f} + K_\text{f}$$

$$mgh + 0 = 0 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2$$

$$\omega = \frac{\sqrt{6gh}}{L}$$

$$= \frac{\sqrt{6\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.475\text{ m})}}{0.95\text{ m}}$$

$$= 5.6\ \text{rad/s}$$

$$v = r\omega = (0.95\text{ m})\left(5.57\ \frac{\text{rad}}{\text{s}}\right) = \boxed{5.3\ \text{m/s}}$$

63. (a)

$$U_\text{i} + K_\text{i} = U_\text{f} + K_\text{f}$$

$$mgh + 0 = 0 + K_\text{f}$$

$$K_\text{f} = mgh$$

$$= (2.0\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.75\text{ m})$$

$$= \boxed{15\text{ J}}$$

(b) $K_f = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{3}{4}mv^2$

$v = 2\sqrt{\frac{K_f}{3m}} = 2\sqrt{\frac{14.7\text{ J}}{3(2.0\text{ kg})}} = 3.13\text{ m/s}$

$K_r = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{1}{4}mv^2 = \frac{1}{4}(2.0\text{ kg})\left(3.13\ \frac{\text{m}}{\text{s}}\right)^2 = \boxed{4.9\text{ J}}$

(c) $K_t = \frac{1}{2}mv^2 = \frac{1}{2}(2.0\text{ kg})\left(3.13\ \frac{\text{m}}{\text{s}}\right)^2 = \boxed{9.8\text{ J}}$

64. **(a)** $U_i + K_i = U_f + K_f$

$mgh + 0 = 0 + K_f$

$K_f = mgh = (2.0\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.75\text{ m}) = \boxed{15\text{ J}}$

(b) $K_f = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{7}{10}mv^2$

$v = \sqrt{\frac{10K_f}{7m}} = \sqrt{\frac{10(14.7\text{ J})}{7(2.0\text{ kg})}} = 3.24\text{ m/s}$

$K_r = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{1}{5}mv^2 = \frac{1}{5}(2.0\text{ kg})\left(3.24\ \frac{\text{m}}{\text{s}}\right)^2 = \boxed{4.2\text{ J}}$

(c) $K_t = \frac{1}{2}mv^2 = \frac{1}{2}(2.0\text{ kg})\left(3.24\ \frac{\text{m}}{\text{s}}\right)^2 = \boxed{11\text{ J}}$

65. $\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\frac{1}{2}\text{ rev}}{2.1\text{ s}} = \boxed{1.2\text{ rev/s}}$

66. $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\left(mr^2\right)\left(\frac{v}{r}\right)^2 = mv^2$

$v = \sqrt{\frac{K}{m}} = \sqrt{\frac{0.10\text{ J}}{0.050\text{ kg}}} = \boxed{1.4\text{ m/s}}$

67. $a_{cp} = \frac{v^2}{r}$

$r = \frac{v^2}{a_{cp}} = \frac{\left(245\ \frac{\text{m}}{\text{s}}\right)^2}{7.00\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)} = \boxed{874\text{ m}}$

68. The disk has rotated about 20°.

$$t=\frac{\theta}{\omega}=\frac{(20°)\left(\frac{\pi\text{ rad}}{180°}\right)}{\left(50.4\ \frac{\text{rev}}{\text{s}}\right)\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)}=1.1\text{ ms}$$

The positions of the BB are about 41.5 cm and 64.0 cm. Estimate the speed of the BB.

$$v=\frac{\Delta x}{\Delta t}=\frac{64.0\text{ cm}-41.5\text{ cm}}{1.1\text{ ms}}=\boxed{200\text{ m/s}}$$

69. (a) $v_\text{t}=r\omega=(0.070\ \text{m})\left[\left(50.4\ \frac{\text{rev}}{\text{s}}\right)\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\right]=\boxed{22\ \text{m/s}}$

(b) $s=r\theta=(0.07\text{ m})(20°)\left(\frac{\pi\ \text{rad}}{180°}\right)=0.0244\text{ m}$

$$\frac{v_\text{BB}}{\Delta x_\text{BB}}=\frac{v_\text{t}}{s}$$

$$v_\text{BB}=\left(\frac{\Delta x_\text{BB}}{s}\right)v_\text{t}=\left(\frac{0.225\text{ m}}{0.0244\text{ m}}\right)\left(22\ \frac{\text{m}}{\text{s}}\right)=\boxed{200\ \text{m/s}}$$

(c) $r=\frac{v_\text{t}}{\omega}=\frac{200\ \frac{\text{m}}{\text{s}}}{\left(50.4\ \frac{\text{rev}}{\text{s}}\right)\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)}=\boxed{63\text{ cm}}$

(d) $F_\text{cp}=ma_\text{cp}=m\frac{{v_\text{t}}^2}{r}=\frac{(1.0\times10^{-3}\ \text{kg})\left(22\ \frac{\text{m}}{\text{s}}\right)^2}{0.070\text{ m}}=\boxed{6.9\text{ N}}$

70. (a) After 2:10 p.m. and before 2:15 p.m. At 2:10 p.m. the minute hand points to the 2 but the hour hand has moved toward the 3. At 2:15 p.m. the minute hand points to the 3 but the hour hand hasn't reached it yet.

(b) $\theta=\theta_0+\omega_0 t+\frac{1}{2}\alpha t^2$

Let 2 P.M. be the initial time.

$$\theta_{\substack{\text{hour}\\\text{hand}}}=\frac{1}{6}\ \text{rev}+\left(\frac{1\text{ rev}}{12\text{ h}}\right)\left(\frac{1\text{ h}}{60\text{ min}}\right)t+0=\frac{1}{6}\ \text{rev}+\left(\frac{1\text{ rev}}{720\text{ min}}\right)t$$

$$\theta_{\substack{\text{minute}\\\text{hand}}}=0+\left(\frac{1\text{ rev}}{60\text{ min}}\right)t+0=\left(\frac{1\text{ rev}}{60\text{ min}}\right)t$$

Set $\theta_{\substack{\text{hour}\\\text{hand}}}=\theta_{\substack{\text{minute}\\\text{hand}}}$.

$$\frac{1}{6}\ \text{rev}+\left(\frac{1\text{ rev}}{720\text{ min}}\right)t=\left(\frac{1\text{ rev}}{60\text{ min}}\right)t$$

$$\frac{1}{6}\ \text{rev}=\left(\frac{12\text{ rev}}{720\text{ min}}\right)t-\left(\frac{1\text{ rev}}{720\text{ min}}\right)t$$

$$\frac{1}{6}\ \text{rev}=\left(\frac{11\text{ rev}}{720\text{ min}}\right)t$$

$$t=\left(\frac{720\text{ min}}{11\text{ rev}}\right)\left(\frac{1\text{ rev}}{6}\right)=10.91\text{ min}$$

Meet your friend at about 2:11 P.M.

71. (a) After 2:40 p.m. and before 2:45 p.m. At 2:40 p.m. the minute hand points to the 8 but the hour hand has moved past the 2. At 2:45 p.m. the minute hand points to the 9 but the hour hand has not yet reached the 3.

(b) $\theta_{\substack{\text{minute}\\\text{hand}}} = \theta_{\substack{\text{hour}\\\text{hand}}} + \frac{1}{2}\text{ rev}$

Substitute from Example 66.

$$\left(\frac{1\text{ rev}}{60\text{ min}}\right)t = \frac{1}{6}\text{ rev} + \left(\frac{1\text{ rev}}{720\text{ min}}\right)t + \frac{1}{2}\text{ rev}$$

$$\left(\frac{11\text{ rev}}{720\text{ min}}\right)t = \frac{2}{3}\text{ rev}$$

$$t = \left(\frac{720\text{ min}}{11\text{ rev}}\right)\left(\frac{2}{3}\text{ rev}\right) = 43.6\text{ min}$$

Meet your friend at about 2:44 P.M.

72. $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.55\text{ s}} = 11.42\text{ rad/s}$

(a) $v = r\omega = (6.5\text{ cm})\left(11.42\ \frac{\text{rad}}{\text{s}}\right) = \boxed{74\text{ m/s}}$

(b) $a_{\text{cp}} = r\omega^2 = (6.5\text{ cm})\left(11.42\ \frac{\text{rad}}{\text{s}}\right)^2 = \boxed{8.5\text{ m/s}^2}$

(c) The linear speed will be halved; the centripetal acceleration will be 1/4 as large.

73. (a) $x = x_0 + v_0 t + \frac{1}{2}at^2$

$$0 = x_0 + 0 + \frac{1}{2}at^2$$

$$t = \sqrt{\frac{-2x_0}{a}} = \sqrt{\frac{-2(3.0\text{ m})}{-9.81\ \frac{\text{m}}{\text{s}^2}}} = 0.782\text{ s}$$

$$\begin{aligned}\theta &= \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2\\ &= 0 + \omega_0 t + 0\\ &= \omega_0 t\\ &= \left(2.4\ \frac{\text{rad}}{\text{s}}\right)(0.782\text{ s})\\ &= (1.88\text{ rad})\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right)\\ &= \boxed{0.30\text{ rev}}\end{aligned}$$

(b) It does not depend on her initial speed. Her initial speed determines how far she travels horizontally before hitting the water, but has no effect on the time she falls.

74. $y = y_0 + v_0 t + \frac{1}{2}at^2$

$0 = 14\text{ m} + 0 + \frac{1}{2}(-9.81\text{ m/s}^2)t^2$

$t = \sqrt{\frac{2(14\text{ m})}{9.81\text{ m/s}^2}} = 1.70\text{ s}$

$\Delta\theta = \omega t = \left(12\ \frac{\text{rad}}{\text{s}}\right)(1.70\text{ s})\left(\frac{\text{rev}}{2\pi\text{ rad}}\right) = \boxed{3.2\text{ rev}}$

75. (a) $\alpha = \frac{a}{r} = \frac{3.2\text{ m/s}^2}{\frac{0.018\text{ m}}{2}} = \boxed{360\text{ rad/s}^2}$

(b) $\omega = \omega_0 + \alpha t = 0 + (355.6\text{ rad/s}^2)(1.5\text{ s}) = \boxed{530\text{ rad/s}}$

76. $y = y_0 + v_0 t + \frac{1}{2}at^2$

$0 = y_0 + 0 + \frac{1}{2}at^2$

$t = \sqrt{\frac{-2y_0}{a}}$

$= \sqrt{\frac{-2(0.86\text{ m})}{-9.81\ \frac{\text{m}}{\text{s}^2}}}$

$= 0.419\text{ s}$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 = 0 + \frac{v}{r}t + 0$

$v = \frac{r\theta}{t} = \frac{(4.0\text{ cm})(0.77\text{ rev})\left(\frac{2\pi}{\text{rev}}\right)}{0.419\text{ s}} = \boxed{46\text{ cm/s}}$

77. (a) $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\theta_{\text{cw}} = 2\pi\text{ rad} - \left(0.050\ \frac{\text{rad}}{\text{s}}\right)t + 0$

$\theta_{\text{ccw}} = 0 + \left(0.037\ \frac{\text{rad}}{\text{s}}\right)t + 0$

Set $\theta_{\text{cw}} = \theta_{\text{ccw}}$.

$2\pi\text{ rad} - \left(0.050\ \frac{\text{rad}}{\text{s}}\right)t = \left(0.037\ \frac{\text{rad}}{\text{s}}\right)t$

$2\pi\text{ rad} = \left(0.087\ \frac{\text{rad}}{\text{s}}\right)t$

$t = \frac{2\pi}{0.087\ \frac{\text{rad}}{\text{s}}}$

$= \boxed{72\text{ s}}$

(b) $\theta_{\text{ccw}} = \left(0.037\ \frac{\text{rad}}{\text{s}}\right)(72.2\ \text{s}) = (2.67\ \text{rad})\left(\frac{360°}{2\pi\ \text{rad}}\right) = \boxed{153°\ \text{ccw from North}}$

78. **(a)** $\theta = \theta_0 + \frac{1}{2}(\omega_0 + \omega)t$

$$t = \frac{2(\theta - \theta_0)}{\omega_0 + \omega} = \frac{2(120\ \text{rev})\left(\frac{2\pi}{\text{rev}}\right)}{35\ \frac{\text{rad}}{\text{s}} + 25\ \frac{\text{rad}}{\text{s}}} = \boxed{25\ \text{s}}$$

(b) $\boxed{25\ \text{s}}$, because $\Delta\omega$ is the same as in part (a), and α is constant.

79. **(a)** $a = \frac{\Delta v}{\Delta t} = \frac{\left(45\ \frac{\text{mi}}{\text{hr}}\right)\left(\frac{1609\ \text{m}}{\text{mi}}\right)\left(\frac{1\ \text{hr}}{3600\ \text{s}}\right) - 0}{8.1\ \text{s}} = 2.48\ \text{m/s}^2$

$$\alpha = \frac{a}{r} = \frac{2.48\ \frac{\text{m}}{\text{s}^2}}{(32\ \text{cm})\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)} = \boxed{7.8\ \text{rad/s}^2}$$

(b) It doubles.

80. **(a)** The sphere, because more of its kinetic energy is translational.

(b) $U_\text{i} + K_\text{i} = U_\text{f} + K_\text{f}$

$$mgh + 0 = 0 + K_\text{f}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(Bmr^2\right)\left(\frac{v}{r}\right)^2$$

where $B = \begin{cases} \frac{1}{2} & \text{for a cylinder} \\ \frac{2}{5} & \text{for a sphere} \end{cases}$

$$gh = \frac{1}{2}v^2 + \frac{B}{2}v^2$$

$$= \frac{v^2}{2}(1 + B)$$

$$v = \sqrt{\frac{2gh}{1 + B}}$$

$$v_{\text{cyl}} = \sqrt{\frac{2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.75\ \text{m})}{1 + \frac{1}{2}}} = \boxed{3.1\ \text{m/s}}$$

$$v_{\text{sphere}} = \sqrt{\frac{2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.75\ \text{m})}{1 + \frac{2}{5}}} = \boxed{3.2\ \text{m/s}}$$

81. (a) $a_{cp} = r\omega^2$

$$r = \frac{a_{cp}}{\omega^2} = \frac{6840\left(9.81\ \frac{m}{s^2}\right)}{\left[\left(6050\ \frac{rev}{min}\right)\left(\frac{1\ min}{60\ s}\right)\left(\frac{2\pi}{rev}\right)\right]^2} = 0.167\ m$$

$$d = 2r = 2(0.167\ m) = \boxed{0.334\ m}$$

(b) $F = ma = (15.0\ g)\left(\frac{1\ kg}{1000\ g}\right)(6840)\left(9.81\ \frac{m}{s^2}\right) = \boxed{1.01\ kN}$

82. $U_i + K_i = U_f + K_f$

$$0 + \frac{1}{2}I\omega^2 = mgh + 0$$

$$\omega = \sqrt{\frac{2mgh}{I}} = \sqrt{\frac{2(0.11\ kg)\left(9.81\ \frac{m}{s^2}\right)(1.0\ m)}{7.4\times10^{-5}\ kg\cdot m^2}} = \boxed{170\ rad/s}$$

83. (a) $\alpha = \frac{\Delta\omega}{\Delta t} = \frac{550\ \frac{rad}{s} - 430\ \frac{rad}{s}}{8.2\ s} = \boxed{15\ rad/s^2}$

(b) $\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t = \frac{1}{2}\left(430\ \frac{rad}{s} + 550\ \frac{rad}{s}\right)(8.2\ s) = \boxed{4.0\times10^3\ rad}$

84. (a) Assume the angle through which a wing moves from top to bottom is $120° = \frac{2\pi}{3}$ rad. Then the total angle through which each wing moves in one second is $250(2)\left(\frac{2\pi}{3}\ rad\right) = \frac{1000}{3}\pi$ rad.

Assume the wing's angular speed is zero at each end of its up and down motion.

Then $\omega_{max} = 2\omega_{av} = 2\frac{\Delta\theta}{\Delta t} = 2\frac{\left(\frac{1000\pi}{3}\ rad\right)}{1\ s} = \boxed{2100\ rad/s}$.

(b) Assume a wing is 1.0 cm long.

$v = r\omega = (0.010\ m)(2100\ rad/s) = \boxed{21\ m/s}$

85. (a) $\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi\ rad}{(240\times10^6\ yr)\left(\frac{3.156\times10^7\ s}{yr}\right)} = \boxed{8.3\times10^{-16}\ rad/s}$

(b) $v = r\omega$

$$r = \frac{v}{\omega} = \frac{137\ \frac{mi}{s}}{8.30\times10^{-16}\ \frac{rad}{s}} = \boxed{1.7\times10^{17}\ mi}\left(\frac{1.609\ km}{mi}\right) = \boxed{2.7\times10^{17}\ km}$$

86. (a) $\alpha = \dfrac{\omega_f - \omega_i}{\Delta t} = \dfrac{\frac{2\pi}{T+\Delta T} - \frac{2\pi}{T}}{\Delta t} = \dfrac{\frac{2\pi}{33.0\times10^{-3}\text{ s} + 1.26\times10^{-5}\text{ s}} - \frac{2\pi}{33.0\times10^{-3}\text{ s}}}{(1\text{ yr})\left(\frac{365\text{ days}}{\text{yr}}\right)\left(\frac{24\text{ h}}{\text{day}}\right)\left(\frac{3600\text{ s}}{\text{h}}\right)} = \boxed{-2.30\times10^{-9}\text{ rad/s}^2}$

(b)

$$\begin{aligned}\omega &= \omega_0 + \alpha t\\ 0 &= \frac{2\pi}{T_0} + \alpha t\\ t &= -\frac{2\pi}{\alpha T_0}\\ &= -\frac{2\pi}{\left(-2.30\times10^{-9}\ \frac{\text{rad}}{\text{s}^2}\right)\left(33.0\times10^{-3}\text{ s}\right)}\\ &= (8.28\times10^{10}\text{ s})\left(\frac{1\text{ h}}{3600\text{ s}}\right)\left(\frac{1\text{ day}}{24\text{ h}}\right)\left(\frac{1\text{ yr}}{365\text{ days}}\right)\\ &= \boxed{2.62\times10^3\text{ yr}}\end{aligned}$$

(c)

$$\begin{aligned}\omega_0 &= \omega - \alpha t\\ &= \frac{2\pi}{T} - \alpha t\\ &= \frac{2\pi}{33.0\times10^{-3}\text{ s}} - \left(-2.30\times10^{-9}\ \frac{\text{rad}}{\text{s}}\right)(2000\text{ yr} - 1054\text{ yr})\left(\frac{365\text{ days}}{\text{yr}}\right)\left(\frac{24\text{ h}}{\text{day}}\right)\left(\frac{3600\text{ s}}{\text{h}}\right)\\ &= 259\text{ rad/s}\end{aligned}$$

$$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{259\ \frac{\text{rad}}{\text{s}}} = \boxed{2.43\times10^{-2}\text{ s}}$$

87. (a) $\Delta x = \dfrac{1.0\text{ m}}{2} = \boxed{0.50\text{ m}}$

(b) $\theta = \dfrac{s}{r} = \dfrac{1.0\text{ m}}{(6.5\text{ cm})\left(\frac{1\text{ m}}{100\text{ cm}}\right)} = (15.38\text{ rad})\left(\dfrac{1\text{ rev}}{2\pi}\right) = \boxed{2.4\text{ rev}}$

88. (a) Down is positive.

$$\begin{aligned}\sum F_y &= W - N = ma_{cp}\\ N &= W - ma_{cp}\\ &= mg - ma_{cp}\\ &= m\left(g - r\omega^2\right)\\ &= (65\text{ kg})\left\{9.81\ \frac{\text{m}}{\text{s}^2} - \left(\frac{12\text{ m}}{2}\right)\left[\left(8.1\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi}{\text{rev}}\right)\right]^2\right\}\\ &= \boxed{360\text{ N}},\ \boxed{\text{up}}\end{aligned}$$

(b) Up is positive.

$$\sum F_y = N - W = ma_{\text{cp}}$$

$$\begin{aligned} N &= W + ma_{\text{cp}} \\ &= mg + ma_{\text{cp}} \\ &= m\left(g + r\omega^2\right) \\ &= (65\text{ kg})\left\{9.81\ \frac{\text{m}}{\text{s}^2} + \left(\frac{12\text{ m}}{2}\right)\left[\left(8.1\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi}{\text{rev}}\right)\right]^2\right\} \\ &= \boxed{920\text{ N}},\ \boxed{\text{up}} \end{aligned}$$

(c) Up is positive.

$$\sum F_y = 0$$

$$N = W = mg = (65\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) = \boxed{640\text{ N, up}}$$

89. (a) $U_\text{i} + K_\text{i} = U_\text{f} + K_\text{f}$

$$mgh + 0 = 0 + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2$$

$$gh = \frac{7}{10}v^2$$

$$\begin{aligned} v &= \sqrt{\frac{10gh}{7}} \\ &= \sqrt{\frac{10\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.61\text{ m})}{7}} \\ &= 2.92\ \text{m/s} \end{aligned}$$

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

$$0 = 1.22\text{ m} + (0)t + \frac{1}{2}\left(-9.81\ \frac{\text{m}}{\text{s}^2}\right)t^2$$

$$t = \sqrt{\frac{2(-1.22\text{ m})}{-9.81\ \frac{\text{m}}{\text{s}^2}}} = 0.499\text{ s}$$

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 = 0 + \left(2.92\ \frac{\text{m}}{\text{s}}\right)(0.499\text{ s}) + 0 = \boxed{1.5\text{ m}}$$

(b) $\text{revolutions} = \dfrac{\theta}{2\pi} = \dfrac{\omega t}{2\pi} = \dfrac{1}{2\pi}\left(\dfrac{v}{r}\right)t = \dfrac{1}{2\pi}\left(\dfrac{v}{\frac{1}{2}d}\right)t = \dfrac{vt}{\pi d} = \dfrac{\left(2.92\ \frac{\text{m}}{\text{s}}\right)(0.499\text{ s})}{\pi(0.17\text{ m})} = \boxed{2.7\text{ rev}}$

(c) Increase A frictionless incline would not cause the ball to rotate. All the lost potential energy would go to translational kinetic energy, resulting in a larger horizontal velocity as the ball leaves the bottom of the ramp. So ϕ would increase.