

Chapter 11

Rotational Dynamics and Static Equilibrium

Answers to Even-numbered Conceptual Questions

2. As a car brakes, the forces responsible for braking are applied at ground level. The center of mass of the car is well above the ground, however. Therefore, the braking forces exert a torque about the center of mass that tends to rotate the front of the car downward. This, in turn, causes an increased upward force to be exerted by the front springs, until the net torque acting on the car returns to zero.
4. The force that accelerates a motorcycle is a forward force applied at ground level. The center of mass of the motorcycle, however, is above the ground. Therefore, the accelerating force exerts a torque on the cycle that tends to rotate the front wheel upward.
6. The moment of inertia is greatest when more mass is at a greater distance from the axis of rotation. Therefore, rotating the body about an axis through the hips results in the larger moment of inertia. Finally, since the angular acceleration is inversely proportional to the moment of inertia, it follows that a given torque produces the greater angular acceleration when the body rotates about an axis through the spine.
8. Consider an airplane propeller or a ceiling fan that is just starting to rotate. In these cases, the net force is zero – the center of mass is not accelerating – but the net torque is nonzero – the angular acceleration is nonzero.
10. As the person climbs higher on the ladder, the torque exerted about the base of the ladder increases. To counter this torque the wall must exert a greater horizontal force, and the floor must exert the same increased horizontal force in the opposite direction. Therefore, the ladder is more likely to slip as the person climbs higher.
12. A car accelerating from rest is not in static equilibrium – its center of mass is accelerating. Similarly, an airplane propeller that is just starting up is not in static equilibrium – it has an angular acceleration.
14. Initially, the center of mass of the glass is near its geometric center. As water is first added, the center of mass moves downward. Later, as the glass fills, the center of mass rises again to roughly its original position. Of course, the details depend on the precise shape of the glass.
16. The tail rotor on a helicopter has a horizontal axis of rotation, as opposed to the vertical axis of the main rotor. Therefore, the tail rotor produces a horizontal thrust that tends to rotate the helicopter about a vertical axis. As a result, if the angular speed of the main rotor is increased or decreased, the tail rotor can exert an opposing torque that prevents the entire helicopter from rotating in the opposite direction.
18. As the string is pulled downward it exerts a force on the puck that is directly through the axis of rotation. Therefore, the string exerts zero torque on the puck. It follows that the puck's angular momentum is conserved during this process. Now, from the relation $L = I\omega = (mr^2)\omega$, we see that the puck's angular speed must increase as $1/r^2$. Similarly, from $L = rp = rmv$ we see that the puck's linear speed must increase as $1/r$. Note that this latter conclusion is also consistent with the relation $v = r\omega$.

20. Yes. Imagine turning on a ceiling fan. This increases the fan's angular momentum, without changing its linear momentum.
22. The hard boiled egg spins faster. The reason is that as you spin the raw egg, some of the work you do is dissipated in the form of swirling motion within the egg – the egg doesn't spin as a whole, as does the hard boiled egg.
24. No. If the diver's initial angular momentum is zero, it must stay zero unless an external torque acts on her. A diver needs to start off with at least a small angular speed, which can then be increased by folding into a tucked position.
26. As the beetle begins to walk, it exerts a force and a torque on the turntable. The turntable exerts an equal but opposite force and torque on the beetle. Therefore, the system consisting of the beetle and turntable experiences no net change in its linear or angular momentum. If the turntable is much more massive than the beetle, it will barely rotate backward as the beetle moves forward. The beetle, then, will begin to circle around the perimeter of the turntable almost the same as if it were on solid ground. If the turntable is very light, however, it will rotate backward with a linear speed at the rim that is almost equal to the forward linear speed of the beetle. The beetle will progress very slowly relative to the ground in this case – though as far as it is concerned, it is running with its usual speed. In the limit of a massless turntable, the beetle remains in the same spot relative to the ground.
28. It is more difficult to do sit-ups with your hands behind your head, than with your arms stretched outward in front of you. The reason is that there is more mass farther from the axis of rotation in the former case.
30. The hollow sphere is harder to stop because it – with its greater moment of inertia – has more kinetic energy for a given speed. The more the kinetic energy, the more work that must be done to bring it to rest.

Solutions to Problems

1. $\tau = r(F \sin \theta)$

$$F = \frac{\tau}{r \sin \theta}$$

F is minimized when r and $\sin \theta$ are maximized. With $r = 0.25 \text{ m}$ and $\theta = 90^\circ$,

$$F = \frac{15 \text{ N} \cdot \text{m}}{(0.25 \text{ m}) \sin 90^\circ} = \boxed{60 \text{ N}}$$

2. Just before the weed comes out, the system is in static equilibrium:

$$\sum \tau = F_w(0.040 \text{ m}) - 1.23 \text{ N} \cdot \text{m} = 0$$

$$F_w = \frac{1.23 \text{ N} \cdot \text{m}}{0.040 \text{ m}} = \boxed{31 \text{ N}}$$

3. To hold the trophy in static equilibrium, the torque must be equal and opposite to the torque created by the weight of the trophy. (We neglect the weight of the arm.)

- (a) The moment arm is 0.505 m.

$$\tau = r_{\perp} mg = (0.505 \text{ m})(1.31 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) = \boxed{6.49 \text{ N} \cdot \text{m}}$$

- (b) The moment arm is $(0.505 \text{ m})(\cos 20.0^\circ) = 0.475 \text{ m}$.

$$\tau = r_{\perp} mg = (0.475 \text{ m})(1.31 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) = \boxed{6.10 \text{ N} \cdot \text{m}}$$

4. To counteract the torque created by the weight of the trap (and neglecting the weight of the arm and rope), the torque exerted about the shoulder must be

$$\tau = r_{\perp} mg = (0.70 \text{ m})(3.3 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) = \boxed{23 \text{ N} \cdot \text{m}}$$

5. (a) $\sum \tau = \tau_{\text{biceps}} - \tau_{\text{forearm}} - \tau_{\text{ball}}$

$$\begin{aligned} &= (12.6 \text{ N})(0.0275 \text{ m}) - (1.20 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) \left(\frac{0.340}{2} \text{ m} \right) - (1.42 \text{ N})(0.340 \text{ m}) \\ &= \boxed{-2.14 \text{ N} \cdot \text{m}} \end{aligned}$$

- (b) Negative net torque means clockwise motion: the forearm and hand will rotate downward.

- (c) Attaching the biceps farther from the elbow would increase the moment arm and increase the net torque.

6. $\sum \tau = r_{\text{child}} m_{\text{child}} g - r_{\text{adult}} F_{\text{adult}} = 0$

$$r_{\text{adult}} = \frac{r_{\text{child}} m_{\text{child}} g}{F_{\text{adult}}} = \frac{(1.5 \text{ m})(16 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right)}{95 \text{ N}} = 2.48 \text{ m}$$

- (a) $3.0 \text{ m} > 2.48 \text{ m}$; the adult has enough moment arm to push down and push the child up.

- (b) $2.5 \text{ m} > 2.48 \text{ m}$; the adult has just barely enough moment arm to push the child up.

- (c) $2.0 \text{ m} < 2.48 \text{ m}$; the adult does not have enough moment arm to hold the child up, and the child goes down.

7. For a hoop, $I = mr^2$.

$$\tau = I\alpha$$

$$\alpha = \frac{\tau}{I}$$

$$= \frac{\tau}{mr^2}$$

$$= x \frac{0.97 \text{ N} \cdot \text{m}}{(0.75 \text{ kg})(0.35 \text{ m})^2}$$

$$= \boxed{11 \text{ rad/s}^2}$$

8. $\Delta\omega = \alpha t$ and $\tau = I\alpha$, so

$$\Delta\omega = \left(\frac{\tau}{I} \right) t$$

$$I = \frac{\tau t}{\Delta\omega}$$

$$= \frac{(0.220 \text{ N} \cdot \text{m})(5.75 \text{ s})}{2.55 \frac{\text{rad}}{\text{s}}}$$

$$= \boxed{0.496 \text{ kg} \cdot \text{m}^2}$$

9. Treat the CD as a disk, with $I = \frac{1}{2}mr^2$.

$$\begin{aligned}\tau\Delta\theta &= \frac{1}{2}I\omega_f^2 \\ \tau &= \frac{I\omega_f^2}{2\Delta\theta} \\ &= \frac{1}{2}(0.017 \text{ kg})(0.12 \text{ m})^2 \frac{\left(450 \frac{\text{rev}}{\text{min}}\right)^2 \left(\frac{1}{60} \frac{\text{min}}{\text{s}}\right)^2}{2(3.0 \text{ rev})} \\ &= 0.00115 \text{ kg} \cdot \text{m}^2 \cdot \frac{\text{rev}}{\text{s}^2} \\ &= \left(0.00115 \text{ kg} \cdot \text{m}^2 \cdot \frac{\text{rev}}{\text{s}^2}\right) \left(\frac{2\pi \text{ rad}}{\text{rev}}\right) \\ &= \boxed{0.0072 \text{ N} \cdot \text{m}}\end{aligned}$$

10. For a uniform rod rotating about its center, $I = \frac{1}{12}mL^2$.

$$\tau = I\alpha = \frac{1}{12}mL^2\alpha = \frac{1}{12}(8.40 \text{ kg})(3.25 \text{ m})^2 \left(0.322 \frac{\text{rad}}{\text{s}^2}\right) = \boxed{2.38 \text{ N} \cdot \text{m}}$$

11. (a) For a disk, $I = \frac{1}{2}mr^2$.

$$\begin{aligned}-\tau\Delta\theta &= \text{Work} = -\frac{1}{2}I\omega_0^2 = -\frac{1}{4}mr^2\omega_0^2 = -\frac{1}{4}mr^2\omega_0^2 \\ \tau &= \frac{mr^2\omega_0^2}{4\Delta\theta} = \frac{(6.4 \text{ kg})(0.71 \text{ m})^2 \left(1.22 \frac{\text{rad}}{\text{s}}\right)^2}{4(0.75 \text{ rev}) \left(2\pi \frac{\text{rad}}{\text{rev}}\right)} = \boxed{0.25 \text{ N} \cdot \text{m}}\end{aligned}$$

- (b) Doubling the mass and halving the radius reduces I by a factor of 2 and therefore reduces the wheel's kinetic energy. The same torque brings the wheel to rest in a decreased angle of rotation.

12. $\tau = I\alpha$, $I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2$

(a) $I_x = (9.0 \text{ kg})(1.0 \text{ m})^2 + (2.5 \text{ kg})(0)^2 + (1.2 \text{ kg})(1.2 \text{ m})^2 = 9.0 \text{ kg} \cdot \text{m}^2$

$$\tau = \left(1.20 \frac{\text{rad}}{\text{s}^2}\right)(9.0 \text{ kg} \cdot \text{m}^2) = \boxed{11 \text{ N} \cdot \text{m}}$$

(b) $I_y = (9.0 \text{ kg})(0)^2 + (2.5 \text{ kg})(2 \text{ m})^2 + (1.2 \text{ kg})(0)^2 = 10 \text{ kg} \cdot \text{m}^2$

$$\tau = \left(1.20 \frac{\text{rad}}{\text{s}^2}\right)(10 \text{ kg} \cdot \text{m}^2) = \boxed{12 \text{ N} \cdot \text{m}}$$

(c) $I_z = (9.0 \text{ kg})(1.0 \text{ m})^2 + (2.5 \text{ kg})(2.0 \text{ m})^2 + (1.2 \text{ kg})(0)^2 = 19 \text{ kg} \cdot \text{m}^2$

$$\tau = \left(1.20 \frac{\text{rad}}{\text{s}^2}\right)(19 \text{ kg} \cdot \text{m}^2) = \boxed{23 \text{ N} \cdot \text{m}}$$

13. $\tau = I\alpha \Rightarrow \alpha = \frac{\tau}{I}$, where $I = \sum mr^2$.

- (a) For a constant torque, the angular acceleration is inversely proportional to the moment of inertia. The moment of inertia increases as the distance of the mass from the axis of rotation increases. The moment of inertia is greatest about the z axis and least about the x axis. Therefore, the angular acceleration is greatest about the x axis and least about the z axis.

(b) $\alpha_x = \frac{13 \text{ N} \cdot \text{m}}{(3.0 \text{ kg})(0.50 \text{ m})^2 + (4.0 \text{ kg})(0.50 \text{ m})^2 + (1.2 \text{ kg})(0)^2 + (2.5 \text{ kg})(0)^2} = \boxed{7.4 \text{ rad/s}^2}$

(c) $\alpha_y = \frac{13 \text{ N} \cdot \text{m}}{(3.0 \text{ kg})(0)^2 + (4.0 \text{ kg})(0.70 \text{ m})^2 + (1.2 \text{ kg})(0.70 \text{ m})^2 + (2.5 \text{ kg})(0)^2} = \boxed{5.1 \text{ rad/s}^2}$

(d) $\alpha_z = \frac{13 \text{ N} \cdot \text{m}}{(3.0 \text{ kg})(0.50 \text{ m})^2 + (4.0 \text{ kg})[(0.70 \text{ m})^2 + (0.50 \text{ m})^2] + (1.2 \text{ kg})(0.70 \text{ m})^2 + (2.5 \text{ kg})(0)^2}$
 $= \boxed{3.0 \text{ rad/s}^2}$

14. For the cylindrical reel, $I = \frac{1}{2}mr^2 = \frac{1}{2}(0.84 \text{ kg})(0.055 \text{ m})^2 = 0.00127 \text{ kg} \cdot \text{m}^2$.

Torque $\tau = rF = (0.055 \text{ m})(2.1 \text{ N}) = 0.1155 \text{ N} \cdot \text{m}$.

(a) $\alpha = \frac{\tau}{I} = \frac{0.1155 \text{ N} \cdot \text{m}}{0.00127 \text{ kg} \cdot \text{m}^2} = \boxed{91 \text{ rad/s}^2}$

(b) $\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}\left(90.9 \frac{\text{rad}}{\text{s}^2}\right)(0.25 \text{ s})^2 = 2.84 \text{ rad}$

Amount of line = $r\theta = (0.055 \text{ m})(2.84 \text{ rad}) = \boxed{0.16 \text{ m}}$

15. $I = 0.00127 \text{ kg} \cdot \text{m}^2$. New torque = $0.1155 \text{ N} \cdot \text{m} - 0.047 \text{ N} \cdot \text{m} = 0.068 \text{ N} \cdot \text{m}$.

(a) $\alpha = \frac{\tau}{I} = \frac{0.068 \text{ N} \cdot \text{m}}{0.00127 \text{ kg} \cdot \text{m}^2} = \boxed{54 \text{ rad/s}^2}$

(b) $\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}\left(53.5 \frac{\text{rad}}{\text{s}^2}\right)(0.25 \text{ s})^2 = 1.67 \text{ rad}$

Amount of line = $r\theta = (0.055 \text{ m})(1.67 \text{ rad}) = \boxed{0.092 \text{ m}}$

16. $\sum \tau = m_1 gr - m_2 gr = (m_1 - m_2)gr$
 $= (0.615 \text{ kg} - 0.351 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(0.0950 \text{ m})$
 $= \boxed{0.246 \text{ N} \cdot \text{m}}$

The magnitude of the frictional torque must equal this quantity for the system to be in static equilibrium.

17. $\tau = rF$

$$F = \frac{\tau}{r} = \frac{8.5 \text{ N} \cdot \text{m}}{0.15 \text{ m}} = \boxed{57 \text{ N}}$$

(Lid diameter does not enter into the solution.)

18. $\sum F_y = F_1 + F_2 - mg = 0$

$$\sum \tau = -F_1 \left(\frac{3}{4} L \right) + F_2 \left(\frac{1}{4} L \right) = 0$$

Multiply the second equation by 4 and subtract it from the first equation.

$$4F_1 - mg = 0$$

$$\boxed{F_1 = \frac{1}{4} mg}$$

From the first equation,

$$\frac{1}{4} mg + F_2 - mg = 0$$

$$\boxed{F_2 = \frac{3}{4} mg}$$

19. (a) The torque exerted by the biceps must balance the total torque exerted by the forearm, hand, and baseball. Since the moment arm for the force exerted by the biceps is much smaller than the moment arms for the forces exerted by the forearm, hand, and baseball, the biceps force must be **more** than the combined weight of the forearm, hand, and baseball.

(b) $\sum \tau = \tau_{\text{biceps}} - \tau_{\text{forearm}} - \tau_{\text{ball}} = 0$

$$F_{\text{biceps}} (0.0275 \text{ m}) - (1.20 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) \left(\frac{0.340}{2} \text{ m} \right) - (1.42 \text{ N})(0.340 \text{ m}) = 0$$

$$F_{\text{biceps}} (0.0275 \text{ m}) - 2.484 \text{ N} \cdot \text{m} = 0$$

$$F_{\text{biceps}} = \frac{2.484 \text{ N} \cdot \text{m}}{0.0275 \text{ m}}$$

$$= \boxed{90.3 \text{ N}}$$

20. (a) $\sum F_y = F_1 + F_2 - mg = 0$

$$m = \frac{F_1 + F_2}{g} = \frac{290 \text{ N} + 122 \text{ N}}{9.81 \frac{\text{m}}{\text{s}^2}} = \boxed{42 \text{ kg}}$$

(b) $\sum \tau = x_2 F_2 - x_{\text{cg}} mg = 0$

$$x_{\text{cg}} = \frac{x_2 F_2}{mg} = \frac{(2.50 \text{ m})(122 \text{ N})}{(42 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right)} = \boxed{0.74 \text{ m}}$$

21. $\sum F_y = F_{\text{front}} + F_{\text{rear}} - mg = 0$

$$mg = F_{\text{front}} + F_{\text{rear}} = 3F_{\text{front}}$$

Taking the rear feet as the center of rotation

$$\sum \tau = F_{\text{front}}(3.2 \text{ m}) - mg(d_{\text{cg}}) = 0$$

$$F_{\text{front}}(3.2 \text{ m}) = 3F_{\text{front}}(d_{\text{cg}})$$

$$d_{\text{cg}} = \frac{1}{3}(3.2 \text{ m})$$

$$= \boxed{1.1 \text{ m}}$$

22. The teeter-totter itself is balanced and contributes no net torque. $\sum \tau = 0$ requires

$$m_{\text{child}}gd_{\text{child}} = F_{\text{parent}}d_{\text{parent}}, \text{ so that } d_{\text{parent}} = \frac{m_{\text{child}}g}{F_{\text{parent}}}d_{\text{child}}.$$

(a) $d_{\text{parent}} = \frac{(36 \text{ kg})(9.81 \frac{\text{m}}{\text{s}^2})}{210 \text{ N}}(2.6 \text{ m}) = \boxed{4.4 \text{ N}}$

(b) $d_{\text{parent}} = \frac{(36 \text{ kg})(9.81 \frac{\text{m}}{\text{s}^2})}{310 \text{ N}}(2.6 \text{ m}) = \boxed{3.0 \text{ N}}$

(c) The answers would not change since only the teeter-totter's length enters into the calculations (the child sits half a length from the pivot point).

23. $\sum \tau = LF - m_{\text{rem}}g\left(\frac{L_{\text{rem}}}{2} - L\right) = 0$

$$LF + m_{\text{rem}}gL - \frac{1}{2}m_{\text{rem}}gL_{\text{rem}} = 0$$

$$L = \frac{m_{\text{rem}}gL_{\text{rem}}}{2(F + m_{\text{rem}}g)}$$

$$\begin{aligned} &= \frac{(0.110 \text{ kg})(9.81 \frac{\text{m}}{\text{s}^2})(0.210 \text{ m})}{2\left[0.365 \text{ N} + (0.110 \text{ kg})(9.81 \frac{\text{m}}{\text{s}^2})\right]} \\ &= \boxed{7.85 \text{ cm}} \end{aligned}$$

24. (a) With the stick-to-wall contact as the pivot,

$$\sum \tau = L_{\text{stick}}(T_{\text{string}} \sin \theta) - \left(\frac{1}{2}L_{\text{stick}}\right)(W_{\text{stick}}) = 0 \text{ where } \sin \theta = \frac{\sqrt{(2.5 \text{ m})^2 - (1 \text{ m})^2}}{2.5 \text{ m}} = 0.9165.$$

$$T_{\text{string}} = \frac{W_{\text{stick}}}{2 \sin \theta} = \frac{(0.13 \text{ kg})(9.81 \frac{\text{m}}{\text{s}^2})}{2(0.9165)} = \boxed{0.70 \text{ N}}$$

(b) A shorter string will make a smaller angle with the stick, and so have to have a greater tension to produce the same value for $T_{\text{string}} \sin \theta$.

$$(c) \sin \theta = \frac{\sqrt{(2.0 \text{ m})^2 - (1.0 \text{ m})^2}}{2.0 \text{ m}} = 0.8660$$

$$T_{\text{string}} = \frac{(0.13 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)}{2(0.8660)} = \boxed{0.74 \text{ N}}$$

$$25. \sum F_y = F_1 + F_2 - m_{\text{diver}}g - W_{\text{board}} = 0$$

$$\sum \tau = F_1(0) + F_2(d) - m_{\text{diver}}g(L) - W_{\text{board}}\left(\frac{1}{2}L\right) = 0$$

From the latter equation,

$$F_2 = \frac{(m_{\text{diver}}g + \frac{1}{2}W_{\text{board}})(L)}{d} = \frac{\left[(90.0 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right) + \frac{1}{2}(66 \text{ N})\right](5.00 \text{ m})}{1.50 \text{ m}} = \boxed{3.1 \text{ kN}}$$

$$F_1 = m_{\text{diver}}g + W_{\text{board}} - F_2 = (90.0 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right) + 66 \text{ N} - 3.053 \text{ kN} = \boxed{-2.1 \text{ kN}} \text{ (i.e., 2.10 kN downward)}$$

$$26. (a) \text{ Place the center of rotation at the shoulder. The } y\text{-axis is directed upward.}$$

$$\sum \tau = (22.5 \text{ cm})F_h - (67.0 \text{ cm} - 22.5 \text{ cm})mg = 0$$

$$F_h = \frac{(44.5 \text{ cm})(1.10 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)}{22.5 \text{ cm}} = 21.3 \text{ N}$$

$$\vec{F}_h = \boxed{(-21.3 \text{ N})\hat{y}}$$

$$(b) \sum F_y = -F_h + F_s - mg = 0$$

$$F_s = F_h + mg = 21.34 \text{ N} + (1.10 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right) = 32.1 \text{ N}$$

$$\vec{F}_s = \boxed{(32.1 \text{ N})\hat{y}}$$

$$27. (a) \sum \tau = r_{\perp\text{-wire}}T - r_{\perp\text{-weight}}mg = 0$$

$$T = \frac{r_{\perp\text{-weight}}}{r_{\perp\text{-wire}}}mg = \frac{\frac{1}{2}(1.2 \text{ m})\cos 25^\circ}{0.51 \text{ m}}(3.1 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right) = \boxed{32 \text{ N}}$$

(b) The horizontal component of the hinge force is equal to, and opposes, the horizontal wire tension: $\boxed{32 \text{ N}}$. The vertical component of the hinge force is equal to, and opposes, the rod weight: $(3.1 \text{ kg})(9.81 \text{ m/s}^2) = \boxed{30 \text{ N}}$.

28. (a) The tension will decrease, because $r_{\perp\text{-weight}}$ has decreased and $r_{\perp\text{-wire}}$ has increased.

$$(b) T = \frac{r_{\perp\text{-weight}}}{r_{\perp\text{-wire}}}mg = \frac{\frac{1}{2}(1.2 \text{ m})\cos 35^\circ}{(1.2 \text{ m})\sin 35^\circ}(3.1 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right) = \boxed{22 \text{ N}}$$

29. $\sum \tau = f_3 a - mgb - m_\ell gc = 0$, where c is the horizontal distance from the bottom of the ladder to the x - y value of the center of the mass of the ladder.

$$c = \sqrt{\left(\frac{4.0 \text{ m}}{2}\right)^2 - \left(\frac{3.8 \text{ m}}{2}\right)^2} = 0.6245 \text{ m}$$

$$f_3 = \frac{g(mb + m_\ell c)}{a} = \frac{\left(9.81 \frac{\text{m}}{\text{s}^2}\right)[(85 \text{ kg})(0.70 \text{ m}) + (7.2 \text{ kg})(0.6245 \text{ m})]}{3.8 \text{ m}} = 165.2 \text{ N} = \boxed{170 \text{ N}}$$

$$\sum F_x = f_2 - f_3 = 0$$

$$f_2 = f_3$$

$$= \boxed{170 \text{ N}}$$

$$\sum F_y = f_1 - mg - m_\ell g = 0$$

$$f_1 = (m + m_\ell)g$$

$$= (85 \text{ kg} + 7.2 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)$$

$$= \boxed{900 \text{ N}}$$

30. (a) Let L = the rod length.

$$\sum \tau = LT \sin 45^\circ - \frac{L}{2} F = 0$$

$$T = \frac{F}{2 \sin 45^\circ} = \boxed{\frac{F}{\sqrt{2}}}$$

- (b) The vertical component of force at the bolt is equal, and opposite to, the vertical component of the wire

tension: $\frac{F}{\sqrt{2}} \cos 45^\circ = \boxed{\frac{F}{2}}$. The horizontal component of force at the bolt can be found from $\sum \tau = 0$ with

the center of rotation at the *top* of the rod: $\frac{L}{2} F - L F_{x\text{-bolt}} = 0$ and $F_{x\text{-bolt}} = \boxed{\frac{F}{2}}$.

31. (a) N = the normal force exerted by the bowling ball on the meter stick
 L = the distance along the meter stick from the floor to the point of contact with the bowling ball
 a = radius of the bowling ball
 h = height from the floor to the point of contact between the bowling ball and the meter stick
 ℓ = length of the meter stick

Let the axis of rotation be at the point where the meter stick touches the floor.

$$\sum \tau = 0 = F_g r_{\text{cm}} - NL = mg \cos \theta \left(\frac{\ell}{2}\right) - N \left[\frac{a(1 + \cos \theta)}{\sin \theta}\right]$$

$$N = \frac{mg \ell \cos \theta \sin \theta}{2a(1 + \cos \theta)} = \frac{(0.214 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(1.00 \text{ m})(\cos 30.0^\circ)(\sin 30.0^\circ)}{2(0.108 \text{ m})(1 + \cos 30.0^\circ)} = 2.26 \text{ N}$$

$$\vec{N} = \boxed{2.26 \text{ N perpendicular to the meter stick and away from the bowling ball}}$$

- (b)
- F
- = the force exerted by the floor

$$\sum F_x = F_x - N \sin 30.0^\circ = 0$$

$$F_x = (2.26 \text{ N}) \sin 30.0^\circ = 1.1277 \text{ N}$$

$$\sum F_y = N \cos 30.0^\circ + F_y - mg = 0$$

$$F_y = mg - N \cos 30.0^\circ = (0.214 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) - (2.255 \text{ N}) \cos 30.0^\circ = 0.14616 \text{ N}$$

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(1.1277 \text{ N})^2 + (0.14616 \text{ N})^2} = \boxed{1.14 \text{ N}}$$

$$\theta = \tan^{-1} \left(\frac{F_y}{F_x} \right) = \tan^{-1} \left(\frac{0.14616 \text{ N}}{1.1277 \text{ N}} \right) = \boxed{7.38^\circ}$$

32. (a) At the top of the crate, the horizontal tipping force has a moment arm twice as long as the opposing crate weight, acting through the center of mass. So, the tipping force must be at least half as great as the weight:

$$\frac{1}{2} (14.2 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) = \boxed{69.7 \text{ N}}.$$

- (b) Halfway down the side of the crate, the tipping force has the same moment arm as the opposing crate weight and must equal the weight in magnitude. But because $\mu_s < 1$, the floor friction cannot sustain a static friction force equal to the crate weight, and so the crate slips before it tips.

33. (a) To keep the crate from sliding, the tipping force cannot exceed

$$f_{s,\max} = mg \mu_s = (14.2 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) (0.551) = 76.775 \text{ N}.$$

For this to be sufficient to tip the crate, the height h must provide a moment arm such that $hF = \frac{L}{2} mg$ and

$$h = \frac{mgL}{2F} = \frac{(14.2 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) (1.21 \text{ m})}{2(76.775 \text{ N})} = \boxed{1.10 \text{ m}}$$

- (b) $\boxed{76.8 \text{ N}}$

34. The orange juice should be placed on the lighter side, i.e., with the cereal. The distance d from the center should be such that

$$\sum \tau = \left(\frac{L}{2} \right) m_{\text{milk}} g - \left(\frac{L}{2} \right) m_{\text{cereal}} g - dm_{\text{juice}} g = 0$$

$$d = \frac{L(m_{\text{milk}} - m_{\text{cereal}})}{2m_{\text{juice}}} = \frac{(0.620 \text{ m})(1.81 \text{ kg} - 0.722 \text{ kg})}{2(1.80 \text{ kg})} = \boxed{18.7 \text{ cm}}$$

35. Since the board is just beginning to tip, there is no weight on the left sawhorse. With the right sawhorse as the center of rotation and d as the cat's distance from that sawhorse, $\sum \tau = Mg(0.500 \text{ m}) - mgd = 0$ and

$$d = \frac{M}{m} (0.500 \text{ m}) = \left(\frac{7.00 \text{ kg}}{2.5 \text{ kg}} \right) (0.500 \text{ m}) = 1.4 \text{ m}$$

So, the cat can get within $1.50 \text{ m} - 1.4 \text{ m} = \boxed{0.10 \text{ m}}$ of the right end of the board.

36. (a) The mass of the necklace is less than the meter stick's, because the moment arm for the necklace is greater than the moment arm for the mass of the meter stick.

- (b) With the new balance point as the center of rotation, M as the meter stick mass, and m as the necklace mass,
- $$\sum \tau = Mg(0.095 \text{ m}) - mg(0.500 \text{ m} - 0.095 \text{ m}) = 0.$$

$$m = M \left(\frac{0.095 \text{ m}}{0.500 \text{ m} - 0.095 \text{ m}} \right) = (0.34 \text{ kg}) \left(\frac{0.095 \text{ m}}{0.405 \text{ m}} \right) = \boxed{80 \text{ g}}$$

37. Start with two books. Place the bottom book such that its edge is under the center of mass of the book above. The center of mass of the system is

$$X_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{mL + m \frac{L}{2}}{2m} = \frac{3}{4}L$$

Place the third book such that its edge is under the center of mass of the system above.

The center of mass of the system is

$$X_{\text{cm,total}} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{m \left(\frac{L}{2} \right) + m \left(\frac{3L}{4} \right) + m \left(\frac{5L}{4} \right)}{m + m + m} = \frac{5}{6}L$$

Let x_R = position of right edge of the top book.

$$d = x_R - X_{\text{cm,total}} = \frac{7L}{4} - \frac{5L}{6} = \boxed{\frac{11}{12}L}$$

38. With the new balance point as the center of rotation, M as the bat mass, and m as the glove mass,

$$\sum \tau = Mg(0.247 \text{ m}) - mg(0.711 \text{ m} - 0.247 \text{ m}) = 0.$$

$$M = m \left(\frac{0.711 \text{ m} - 0.247 \text{ m}}{0.247 \text{ m}} \right) = (0.560 \text{ kg}) \left(\frac{0.464 \text{ m}}{0.247 \text{ m}} \right) = \boxed{1.05 \text{ kg}}$$

39. Let M = bucket mass, m = pulley mass, r = pulley radius, T = rope tension, and note that for the pulley,

$$I = \frac{1}{2}mr^2.$$

Downward force on bucket: $Ma = Mg - T$

Torque on pulley: $rT = \frac{1}{2}mr^2\alpha$

Bucket/pulley relationship: $a = r\alpha$

- (a) From the last two equations, $T = \frac{1}{2}ma$, and then from the first equation,

$$\begin{aligned} Ma &= Mg - \frac{1}{2}ma \\ a &= \frac{2Mg}{2M + m} \\ &= \frac{2(2.85 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right)}{2(2.85 \text{ kg}) + 0.742 \text{ kg}} \\ &= \boxed{8.68 \text{ m/s}^2} \end{aligned}$$

$$(b) \alpha = \frac{a}{r} = \frac{8.68 \frac{\text{m}}{\text{s}^2}}{0.121 \text{ m}} = \boxed{71.7 \text{ rad/s}^2}$$

$$(c) \Delta y = \frac{1}{2}at^2 = \frac{1}{2}\left(8.68 \frac{\text{m}}{\text{s}^2}\right)(1.50 \text{ s})^2 = \boxed{9.77 \text{ m}}$$

40. (a) Less, since if it were not, the bucket would not be subject to a net downward force and would not fall.

(b) From part (a) of the previous solution,

$$T = \frac{1}{2}ma = \frac{1}{2}(0.742 \text{ kg})\left(8.68 \frac{\text{m}}{\text{s}^2}\right) = \boxed{3.22 \text{ N}}$$

41. The torque is $rF = I\alpha$ where $I = \frac{1}{2}mr^2$ and

$$\begin{aligned} \alpha &= \frac{\Delta\omega}{\Delta t} \\ m &= \frac{2F\Delta t}{r\Delta\omega} \\ &= \frac{2(40.0 \text{ N})(3.50 \text{ s})}{(2.40 \text{ m})\left(0.0870 \frac{\text{rev}}{\text{s}}\right)\left(2\pi \frac{\text{rad}}{\text{rev}}\right)} \\ &= \boxed{213 \text{ kg}} \end{aligned}$$

42. (a) No; the side you are pulling on has the greater tension, because it is doing more work, accelerating not only the hanging mass but also the pulley.

(b) $T_1 = \boxed{25 \text{ N}}$

To find T_2 , let M = hanging mass, m = pulley mass, r = pulley radius, and note that for the pulley

$$I = \frac{1}{2}mr^2.$$

Upward force on hanging mass: $Ma = T_2 - Mg$

$$\text{Net torque on pulley: } rT_1 - rT_2 = \left(\frac{1}{2}mr^2\right)\alpha$$

Mass/pulley relationship: $a = r\alpha$

From the last two equations, $T_1 - T_2 = \frac{1}{2}ma$, so $a = \frac{2}{m}(T_1 - T_2)$, and from the first equation,

$$\frac{2M}{m}(T_1 - T_2) = T_2 - Mg. \text{ Solving for } T_2 \text{ yields}$$

$$T_2 = \frac{Mmg + 2MT_1}{2M + m} = \frac{(0.67 \text{ kg})(1.3 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right) + 2(0.67 \text{ kg})(25 \text{ N})}{2(0.67 \text{ kg}) + 1.3 \text{ kg}} = \boxed{16 \text{ N}}$$

43. From part (b) of the previous solution,

$$Ma = T_2 - Mg$$

$$a = \frac{T_2}{M} - g$$

$$= \frac{16 \text{ N}}{0.67 \text{ kg}} - 9.81 \frac{\text{m}}{\text{s}^2}$$

$$= \boxed{14 \text{ m/s}^2}$$

$$44. \sum F_y = 2.54 \text{ N} - Mg - mg = 0$$

$$\sum \tau = mg(0.200 \text{ m}) - Mg(0.300 \text{ m}) = 0$$

From the second equation, $m = \frac{3}{2}M$. From the first equation,

$$M = \frac{2.54 \text{ N}}{\frac{5}{2}g} = \frac{2.54 \text{ N}}{2.5\left(9.81 \frac{\text{m}}{\text{s}^2}\right)} = 0.1036 \text{ kg}$$

(a) $\boxed{0.104 \text{ kg}}$

(b) $\frac{3}{2}(0.1036 \text{ kg}) = \boxed{0.155 \text{ kg}}$

45. Take the direction from m_1 over the pulley to m_2 as positive.

Force on m_1 : $m_1 a = T_1 - m_1 g$

Force on m_2 : $m_2 a = m_2 g - T_2$

Net pulley torque: $R(T_2 - T_1) = \frac{1}{2}MR^2\alpha$

Mass/pulley relationship: $R\alpha = a$

Adding the first two equations and rearranging produces $T_2 - T_1 = (m_2 - m_1)g - (m_1 + m_2)a$,

which when substituted into the third equation yields $R[(m_2 - m_1)g - (m_1 + m_2)a] = \frac{1}{2}MR^2\alpha$.

Substitute using the fourth equation.

$$(m_2 - m_1)g - (m_1 + m_2)a = \frac{1}{2}Ma$$

$$a = \boxed{\left(\frac{m_2 - m_1}{m_1 + m_2 + \frac{1}{2}M}\right)g}$$

Which is the acceleration for both masses.

46. For a sphere,

$$L = I\omega$$

$$= \frac{2}{5}mr^2\omega$$

$$= \frac{2}{5}(5.97 \times 10^{24} \text{ kg})(6.38 \times 10^6 \text{ m})^2 \left(1 \frac{\text{rev}}{\text{day}}\right) \left(2\pi \frac{\text{rad}}{\text{rev}}\right) \left(\frac{1}{8.64 \times 10^4} \frac{\text{day}}{\text{s}}\right)$$

$$= \boxed{7.07 \times 10^{33} \text{ kg} \cdot \text{m}^2/\text{s}}$$

47. For a disk,

$$L = I\omega$$

$$= \frac{1}{2}mr^2\omega$$

$$= \frac{1}{2}(0.015 \text{ kg})(0.15 \text{ m})^2 \left(33 \frac{1}{3} \frac{\text{rev}}{\text{min}}\right) \left(2\pi \frac{\text{rad}}{\text{rev}}\right) \left(\frac{1}{60} \frac{\text{min}}{\text{s}}\right)$$

$$= \boxed{5.9 \times 10^{-4} \text{ kg} \cdot \text{m}^2/\text{s}}$$

48. For the fly,

$$\begin{aligned}
 L &= rmv \\
 &= rm(r\omega) \\
 &= r^2 m \omega \\
 &= (0.15 \text{ m})^2 (0.0011 \text{ kg}) \left(33 \frac{1}{3} \frac{\text{rev}}{\text{min}} \right) \left(\frac{2\pi}{\text{rev}} \right) \left(\frac{\text{min}}{60 \text{ s}} \right) \\
 &= \boxed{8.6 \times 10^{-5} \text{ kg} \cdot \text{m}^2/\text{s}}
 \end{aligned}$$

49. (a) $p = mv = (70.1 \text{ kg}) \left(3.35 \frac{\text{m}}{\text{s}} \right) = \boxed{235 \text{ kg} \cdot \text{m/s}}$

(b) $L = r_{\perp} p = (5.00 \text{ m}) \left(234.8 \frac{\text{kg} \cdot \text{m}}{\text{s}} \right) = \boxed{1.17 \times 10^3 \text{ kg} \cdot \text{m}^2/\text{s}}$

50. (a) $p = mv = (56.4 \text{ kg}) \left(2.68 \frac{\text{m}}{\text{s}} \right) = \boxed{151 \text{ kg} \cdot \text{m/s}}$

(b) $L = r_{\perp} p = (6.00 \text{ m}) \left(151 \frac{\text{kg} \cdot \text{m}}{\text{s}} \right) = \boxed{9.07 \times 10^2 \text{ kg} \cdot \text{m}^2/\text{s}}$

51. (a) Greater with respect to point B, because the moment arm is zero for point A.

- (b) Same, because the moment arm is the same for both points.

(c) $p = mv = (62.2 \text{ kg}) \left(5.85 \frac{\text{m}}{\text{s}} \right) = 363.9 \frac{\text{kg} \cdot \text{m}}{\text{s}}$

$$L_A = r_{\perp A} p = (0) \left(363.9 \frac{\text{kg} \cdot \text{m}}{\text{s}} \right) = \boxed{0}$$

$$L_B = r_{\perp B} p = (7.00 \text{ m}) \left(363.9 \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \right) = \boxed{2.55 \times 10^3 \text{ kg} \cdot \text{m}^2/\text{s}}$$

$$L_O = r_{\perp O} p = (7.00 \text{ m}) \left(363.9 \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \right) = \boxed{2.55 \times 10^3 \text{ kg} \cdot \text{m}^2/\text{s}}$$

52. (a) $L = \tau \Delta t = (0.12 \text{ N} \cdot \text{m})(0.50 \text{ s}) = \boxed{0.060 \text{ kg} \cdot \text{m}^2/\text{s}}$

(b) $\omega = \frac{L}{I} = \frac{0.060 \frac{\text{kg} \cdot \text{m}^2}{\text{s}}}{2.5 \times 10^{-3} \text{ kg} \cdot \text{m}^2} = \boxed{24 \text{ rad/s}}$

53. $\tau = \frac{L_2 - L_1}{\Delta t} = \frac{9700 \frac{\text{kg} \cdot \text{m}^2}{\text{s}} - 8500 \frac{\text{kg} \cdot \text{m}^2}{\text{s}}}{5.66 \text{ s}} = \boxed{210 \text{ N} \cdot \text{m}}$

54. Because the gerbils are running in place, their speed is zero and they contribute no angular momentum.

$$L = I \omega = (MR^2) \left(\frac{v}{R} \right) = MRv = (0.0050 \text{ kg})(0.095 \text{ m}) \left(0.55 \frac{\text{m}}{\text{s}} \right) = \boxed{2.6 \times 10^{-4} \text{ kg} \cdot \text{m}^2/\text{s}}$$

55. By conservation of angular momentum, $I_i \omega_i = I_f \omega_f$, so

$$\frac{I_f}{I_i} = \frac{\omega_i}{\omega_f} = \frac{3.28 \frac{\text{rad}}{\text{s}}}{5.72 \frac{\text{rad}}{\text{s}}} = \boxed{0.573}$$

56. Initial: $\frac{1}{2} m_{\text{child}} v_{\text{child}}^2 = \frac{1}{2} (34.0 \text{ kg}) \left(2.80 \frac{\text{m}}{\text{s}} \right)^2 = \boxed{133 \text{ J}}$

Final: $\frac{1}{2} I \omega^2 = \frac{1}{2} [510 \text{ kg} \cdot \text{m}^2 + (34.0 \text{ kg})(2.31 \text{ m})^2] \left(0.318 \frac{\text{rad}}{\text{s}} \right)^2 = \boxed{35.0 \text{ J}}$

57. By conservation of angular momentum, $I_i \omega_i = I_f \omega_f$, so

$$\frac{\omega_f}{\omega_i} = \frac{I_i}{I_f} = \boxed{2}$$

Her angular speed doubles.

58. (a) It increases, because the diver does work in going into her tuck.

(b) $\frac{\frac{1}{2} I_f \omega_f^2}{\frac{1}{2} I_i \omega_i^2} = \frac{\frac{1}{2} \left(\frac{1}{2} I_i \right) (2 \omega_i)^2}{\frac{1}{2} I_i \omega_i^2} = \boxed{2}$

59. $\left(0.641 \frac{\text{rev}}{\text{s}} \right) \left(2\pi \frac{\text{rad}}{\text{rev}} \right) = 4.0275 \frac{\text{rad}}{\text{s}}$

By conservation of angular momentum,

$$\begin{aligned} L_{\text{disk}} + L_{\text{person}} &= L_{\text{final}} \\ \left(\frac{1}{2} MR^2 \right) \omega_i + mvR &= \left(\frac{1}{2} MR^2 + mR^2 \right) \omega_f \\ \omega_f &= \frac{MR\omega_i + 2mv}{MR + 2mR} \\ &= \frac{(155 \text{ kg})(2.63 \text{ m}) \left(4.0275 \frac{\text{rad}}{\text{s}} \right) + 2(59.4 \text{ kg}) \left(3.41 \frac{\text{m}}{\text{s}} \right)}{(155 \text{ kg})(2.63 \text{ m}) + 2(59.4 \text{ kg})(2.63 \text{ m})} \\ &= \boxed{2.84 \text{ rad/s}} \end{aligned}$$

60. (a) It must decrease, because some energy will be dissipated in the “collision” between the person and the merry-go-round.

- (b) Initial:

$$\begin{aligned} \frac{1}{2} I_{\text{disk}} \omega_i^2 + \frac{1}{2} mv^2 &= \frac{1}{2} \left(\frac{1}{2} MR^2 \right) \omega_i^2 + \frac{1}{2} mv^2 \\ &= \frac{1}{4} (155 \text{ kg})(2.63 \text{ m})^2 \left(4.0275 \frac{\text{rad}}{\text{s}} \right)^2 + \frac{1}{2} (59.4 \text{ kg}) \left(3.41 \frac{\text{m}}{\text{s}} \right)^2 \\ &= \boxed{4.69 \text{ kJ}} \end{aligned}$$

Final:

$$\begin{aligned}
 \frac{1}{2} I_{\text{final}} \omega_f^2 &= \frac{1}{2} \left(\frac{1}{2} MR + mR^2 \right) \omega_f^2 \\
 &= \frac{1}{2} \left[\frac{1}{2} (155 \text{ kg})(2.63 \text{ m})^2 + (59.4 \text{ kg})(2.63 \text{ m})^2 \right] \left(3.018 \frac{\text{rad}}{\text{s}} \right)^2 \\
 &= \boxed{4.31 \text{ kJ}}
 \end{aligned}$$

61. By conservation of angular momentum,

$$mvr = I\omega$$

$$mvr = (I_{\text{student-stool}} + mr^2)\omega$$

$$\begin{aligned}
 \omega &= \frac{mvr}{I_{\text{student-stool}} + mr^2} \\
 &= \frac{(1.5 \text{ kg}) \left(2.7 \frac{\text{m}}{\text{s}} \right) (0.40 \text{ m})}{4.1 \text{ kg} \cdot \text{m}^2 + (1.5 \text{ kg})(0.40 \text{ m})^2} \\
 &= \boxed{0.37 \text{ rad/s}}
 \end{aligned}$$

62. (a) It
- decreases
- , because energy is dissipated in the collision between the mass and the student's hand.

(b) Initial:

$$\frac{1}{2} mv^2 = \frac{1}{2} (1.5 \text{ kg}) \left(2.7 \frac{\text{m}}{\text{s}} \right)^2 = \boxed{5.5 \text{ J}}$$

Final:

$$\frac{1}{2} I\omega^2 = \frac{1}{2} (I_{\text{student-stool}} + mr^2) \omega^2 = \frac{1}{2} [4.1 \text{ kg} \cdot \text{m}^2 + (1.5 \text{ kg})(0.40 \text{ m})^2] \left(0.373 \frac{\text{rad}}{\text{s}} \right)^2 = \boxed{0.30 \text{ J}}$$

63. (a) Angular momentum is conserved as the moment of inertia decreases, so the turntable rotates
- faster
- .

(b) From conservation of angular momentum,

$$(I + mr^2)\omega_i = I\omega_f$$

$$\begin{aligned}
 \omega_f &= \left(\frac{I + mr^2}{I} \right) \omega_i \\
 &= \frac{5.4 \times 10^{-3} \text{ kg} \cdot \text{m}^2 + (0.0013 \text{ kg})(0.15 \text{ m})^2}{5.4 \times 10^{-3} \text{ kg} \cdot \text{m}^2} \left(33 \frac{1}{3} \frac{\text{rev}}{\text{min}} \right) \\
 &= \left(33.514 \frac{\text{rev}}{\text{min}} \right) \left(\frac{2\pi \text{ rad}}{\text{rev}} \right) \left(\frac{1}{60} \frac{\text{min}}{\text{s}} \right) \\
 &= \boxed{3.5 \text{ rad/s}}
 \end{aligned}$$

64. (a) $I_i \omega_i = I_f \omega_f$

$$(I_s + 2mr_i^2)\omega_i = (I_s + 2mr_f^2)\omega_f$$

$$r_f = \sqrt{\frac{(I_s + 2mr_i^2) \frac{\omega_i}{\omega_f} - I_s}{2m}}$$

$$= \sqrt{\frac{[5.43 \text{ kg} \cdot \text{m}^2 + 2(1.25 \text{ kg})(0.759 \text{ m})^2] \left(\frac{2.95 \frac{\text{rev}}{\text{s}}}{3.54 \frac{\text{rev}}{\text{s}}} \right) - 5.43 \text{ kg} \cdot \text{m}^2}{2(1.25 \text{ kg})}}$$

$$= \boxed{0.344 \text{ m}}$$

(b) Initial:

$$\frac{1}{2} I_i \omega_i^2 = \frac{1}{2} (I_s + 2mr_i^2) \omega_i^2$$

$$= \frac{1}{2} [5.43 \text{ kg} \cdot \text{m}^2 + 2(1.25 \text{ kg})(0.759 \text{ m})^2] \left[\left(2.95 \frac{\text{rev}}{\text{s}} \right) \left(\frac{2\pi \text{ rad}}{\text{rev}} \right) \right]^2$$

$$= \boxed{1.18 \text{ kJ}}$$

Final:

$$\frac{1}{2} I_f \omega_f^2 = \frac{1}{2} (I_s + 2mr_f^2) \omega_f^2$$

$$= \frac{1}{2} [5.43 \text{ kg} \cdot \text{m}^2 + 2(1.25 \text{ kg})(0.344 \text{ m})^2] \left[\left(3.54 \frac{\text{rev}}{\text{s}} \right) \left(\frac{2\pi \text{ rad}}{\text{rev}} \right) \right]^2$$

$$= \boxed{1.42 \text{ kJ}}$$

65. (a) Total angular momentum must remain zero. With v_g as the child's ground-relative speed, then,

$Rmv_g - I\omega = 0$. But $v = v_g + R\omega$, so that $\omega = (v - v_g)/R$. Thus,

$$Rmv_g - I \left(\frac{v - v_g}{R} \right) = 0$$

$$(mR^2 + I)v_g - Iv = 0$$

$$v_g = \boxed{\frac{Iv}{I + mR^2}}$$

(b) $\boxed{\text{As } I \rightarrow 0, v_g \rightarrow 0.}$ This is correct, because an ultra-light merry-go-round would move easily beneath the child's feet and act like a slippery surface, preventing the child from generating forward motion relative to the ground. And $\boxed{\text{as } I \rightarrow \infty, v_g \rightarrow v.}$ This is also correct, since an ultra-massive merry-go-round would hardly budge, and then $v_g \approx v$.

66. For a uniform rod, $I = \frac{1}{12} m\ell^2$.

$$\text{Work} = \Delta K = \frac{1}{2} I \omega^2 = \frac{1}{2} \left(\frac{1}{12} m\ell^2 \right) \omega^2 = \frac{1}{24} (0.46 \text{ kg})(0.52 \text{ m})^2 \left(7.1 \frac{\text{rad}}{\text{s}} \right)^2 = \boxed{0.26 \text{ J}}$$

$$67. \Delta\theta = \left(\frac{1}{4} \text{ rev}\right) \left(\frac{2\pi \text{ rad}}{\text{rev}}\right) = \frac{\pi}{2} \text{ rad}$$

$$W = \tau\Delta\theta$$

$$\begin{aligned} \tau &= \frac{W}{\Delta\theta} \\ &= \frac{0.12 \text{ J}}{\frac{\pi}{2} \text{ rad}} \\ &= \boxed{0.076 \text{ N}\cdot\text{m}} \end{aligned}$$

$$68. rF = \tau, I = \frac{1}{2}mr^2, \text{ and } 60.0^\circ = \frac{\pi}{3} \text{ rad.}$$

$$W = \tau\Delta\theta = \frac{1}{2}I\omega_f^2$$

$$\omega_f = \sqrt{\frac{2\tau\Delta\theta}{I}} = \sqrt{\frac{2rF\Delta\theta}{\frac{1}{2}mr^2}} = 2\sqrt{\frac{F\Delta\theta}{mr}} = 2\sqrt{\frac{(36.1 \text{ N})\left(\frac{\pi}{3} \text{ rad}\right)}{(167 \text{ kg})(2.74 \text{ m})}} = \boxed{0.575 \text{ rad/s}}$$

$$69. \text{ One complete turn} = 2\pi \text{ rad}$$

$$W = \tau\Delta\theta = (3.8 \text{ N}\cdot\text{m})(2\pi \text{ rad}) = \boxed{24 \text{ J}}$$

$$70. \text{ For a hoop, } I = mr^2. \text{ Here, } \omega = \frac{v}{r}.$$

$$W = \Delta K = \frac{1}{2}I\omega^2 = \frac{1}{2}mr^2\left(\frac{v}{r}\right)^2 = \frac{1}{2}mv^2 = \frac{1}{2}(0.0065 \text{ kg})\left(1.4 \frac{\text{m}}{\text{s}}\right)^2 = \boxed{6.4 \times 10^{-3} \text{ J}}$$

$$71. \text{ From Problem 12, } I_x = 9.0 \text{ kg}\cdot\text{m}^2, I_y = 10 \text{ kg}\cdot\text{m}^2, \text{ and } I_z = 19 \text{ kg}\cdot\text{m}^2.$$

$$(a) W = \Delta K = \frac{1}{2}I_x\omega_x^2 = \frac{1}{2}(9.0 \text{ kg}\cdot\text{m}^2)\left(2.75 \frac{\text{rad}}{\text{s}}\right)^2 = \boxed{34 \text{ J}}$$

$$(b) W = \frac{1}{2}I_y\omega_y^2 = \frac{1}{2}(10 \text{ kg}\cdot\text{m}^2)\left(2.75 \frac{\text{rad}}{\text{s}}\right)^2 = \boxed{38 \text{ J}}$$

$$(c) W = \frac{1}{2}I_z\omega_z^2 = \frac{1}{2}(19 \text{ kg}\cdot\text{m}^2)\left(2.75 \frac{\text{rad}}{\text{s}}\right)^2 = \boxed{72 \text{ J}}$$

$$72. (a) I_x = (3.0 \text{ kg})(0.50 \text{ m})^2 + (4.0 \text{ kg})(0.50 \text{ m})^2 + (1.2 \text{ kg})(0)^2 + (2.5 \text{ kg})(0)^2 \\ = 1.75 \text{ kg}\cdot\text{m}^2$$

$$W = \Delta K = \frac{1}{2}I_x\omega_x^2 = \frac{1}{2}(1.75 \text{ kg}\cdot\text{m}^2)\left(2.75 \frac{\text{rad}}{\text{s}}\right)^2 = \boxed{6.6 \text{ J}}$$

$$(b) I_y = (3.0 \text{ kg})(0)^2 + (4.0 \text{ kg})(0.70 \text{ m})^2 + (1.2 \text{ kg})(0.70 \text{ m})^2 + (2.5 \text{ kg})(0)^2 = 2.55 \text{ kg}\cdot\text{m}^2$$

$$W = \frac{1}{2}I_y\omega_y^2 = \frac{1}{2}(2.55 \text{ kg}\cdot\text{m}^2)\left(2.75 \frac{\text{rad}}{\text{s}}\right)^2 = \boxed{9.6 \text{ J}}$$

$$\begin{aligned} \text{(c)} \quad I_z &= (3.0 \text{ kg})(0.50 \text{ m})^2 + (4.0 \text{ kg})[(0.7 \text{ m})^2 + (0.5 \text{ m})^2] + (1.2 \text{ kg})(0.70 \text{ m})^2 + (2.5 \text{ kg})(0)^2 \\ &= 5.02 \text{ kg} \cdot \text{m}^2 \end{aligned}$$

$$W = \frac{1}{2} I_z \omega_z^2 = \frac{1}{2} (5.02 \text{ kg} \cdot \text{m}^2) \left(2.75 \frac{\text{rad}}{\text{s}} \right)^2 = \boxed{19 \text{ J}}$$

$$73. \quad \left(3620 \frac{\text{rev}}{\text{min}} \right) \left(\frac{2\pi \text{ rad}}{\text{rev}} \right) \left(\frac{1}{60} \frac{\text{min}}{\text{s}} \right) = 379.09 \frac{\text{rad}}{\text{s}} = \omega$$

$$(6.30 \text{ rev}) \left(\frac{2\pi \text{ rad}}{\text{rev}} \right) = 39.58 \text{ rad} = \Delta\theta$$

$$\text{(a)} \quad W = \tau \Delta\theta = \frac{1}{2} I \omega^2 \text{ and } I = \frac{1}{2} m r^2.$$

$$\tau = \frac{\frac{1}{2} m r^2 \omega^2}{2 \Delta\theta} = \frac{\frac{1}{2} (0.755 \text{ kg})(0.152 \text{ m})^2 \left(379.09 \frac{\text{rad}}{\text{s}} \right)^2}{2(39.58 \text{ rad})} = \boxed{15.8 \text{ N} \cdot \text{m}}$$

(b) The time to rotate the first 3.15 revolutions is greater than the time to rotate the last 3.15 revolutions because the blade is speeding up. So more than half the time is spent in the first 3.15 revolutions. Therefore, the angular speed has increased to more than half of its final value. After 3.15 revolutions, the angular speed is greater than 1810 rpm.

$$\text{(c)} \quad (3.15 \text{ rev}) \left(\frac{2\pi \text{ rad}}{\text{rev}} \right) = 19.79 \text{ rad} = \Delta\theta$$

$$\begin{aligned} \tau \Delta\theta &= \frac{1}{2} I \omega^2 \\ &= \frac{1}{4} m r^2 \omega^2 \\ \omega &= \sqrt{\frac{4 \tau \Delta\theta}{m r^2}} \\ &= \sqrt{\frac{4(15.83 \text{ N} \cdot \text{m})(19.79 \text{ rad})}{(0.755 \text{ kg})(0.152 \text{ m})^2}} \\ &= (268 \text{ rad/s}) \left(\frac{60 \text{ s}}{\text{min}} \right) \left(\frac{1 \text{ rev}}{2\pi \text{ rad}} \right) \\ &= \boxed{2560 \text{ rpm}} \end{aligned}$$

74. (a) Taking the second pillar as the pivot point,

$$\begin{aligned} \sum \tau &= (828 \text{ N})(1.1 \text{ m}) - mgd = 0 \\ d &= \frac{(828 \text{ N})(1.10 \text{ m})}{mg} = \frac{(910.8 \text{ N} \cdot \text{m})}{(64.0 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right)} = \boxed{1.45 \text{ m}} \end{aligned}$$

$$\text{(b)} \quad \sum F_y = F_2 - F_1 - mg = 0$$

$$F_2 = F_1 + mg = 828 \text{ N} + (64.0 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) = \boxed{1.46 \text{ kN}}$$

75. ℓ = rod length, m_r = rod mass, m_p = person mass, and d = distance to wall.

$$\sum \tau = \ell T \sin \theta - m_r g \left(\frac{\ell}{2} \right) - m_p g d = 0$$

Setting $T = T_{\max} = 1400 \text{ N}$,

$$\begin{aligned} d &= \frac{\ell T_{\max} \sin \theta - m_r g \left(\frac{\ell}{2} \right)}{m_p g} \\ &= \frac{(4.25 \text{ m})(1400 \text{ N}) \sin 30.0^\circ - (47.0 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) \left(\frac{4.25 \text{ m}}{2} \right)}{(68.0 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right)} \\ &= \boxed{3.0 \text{ m}} \end{aligned}$$

76. (a) It increases, because the string does work on the puck. Alternatively, the speed increases because a center-directed force does not change the angular momentum, which must be conserved.

$$\begin{aligned} \text{(b)} \quad L_i &= L_f \\ mvr &= mv_f r_f \\ mvr &= mv_f \left(\frac{r}{2} \right) \\ v_f &= \boxed{2v} \end{aligned}$$

77. (a) The force from the index finger, since it has to counteract both the thumb's force and the pen's weight.

$$\begin{aligned} \text{(b)} \quad \text{Taking the thumb as the pivot point,} \\ \sum \tau &= F_f(0.035 \text{ m}) - mg(0.070 \text{ m}) = 0 \\ F_f &= \left(\frac{0.070 \text{ m}}{0.0035 \text{ m}} \right) mg = 2(0.025 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) = \boxed{0.49 \text{ N}} \\ \sum F_y &= F_f - F_t - mg = 0 \\ F_t &= F_f - mg = 0.4905 \text{ N} - (0.025 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) = \boxed{0.25 \text{ N}} \end{aligned}$$

78. $\sum \tau = f_3 a - mgb - m_\ell gc = 0$, where b and c are the horizontal distances from the bottom of the ladder to the x -values of the centers of mass of the person and the ladder, respectively.

$$c = \sqrt{\left(\frac{4.0 \text{ m}}{2} \right)^2 - \left(\frac{3.8 \text{ m}}{2} \right)^2} = 0.6245 \text{ m}$$

- (a) $b = c = 0.6245 \text{ m}$

$$f_3 = \frac{g(mb + m_\ell c)}{a} = \frac{gc(m + m_\ell)}{a} = \frac{\left(9.81 \frac{\text{m}}{\text{s}^2} \right) (0.6245 \text{ m}) \left(85 \text{ kg} + \frac{60.0 \text{ N}}{9.81 \frac{\text{m}}{\text{s}^2}} \right)}{3.8 \text{ m}} = \boxed{150 \text{ N}}$$

$$\begin{aligned}
 \sum F_x &= f_2 - f_3 = 0 \\
 f_2 &= f_3 \\
 &= \boxed{150 \text{ N}} \\
 \sum F_y &= f_1 - mg - m_\ell g = 0 \\
 f_1 &= (m + m_\ell)g \\
 &= \left(85 \text{ kg} + \frac{60.0 \text{ N}}{9.81 \frac{\text{m}}{\text{s}^2}} \right) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) \\
 &= \boxed{890 \text{ N}}
 \end{aligned}$$

$$(b) \quad b = \sqrt{\left[(4 \text{ m}) \left(\frac{3}{4} \right) \right]^2 + \left[(3.8 \text{ m}) \left(\frac{3}{4} \right) \right]^2} = 0.9367 \text{ m}$$

$$f_3 = \frac{g(mb + m_\ell c)}{a} = \frac{\left(9.81 \frac{\text{m}}{\text{s}^2} \right) \left[(85 \text{ kg})(0.9367 \text{ m}) + \left(\frac{60.0 \text{ N}}{9.81 \frac{\text{m}}{\text{s}^2}} \right) (0.6245 \text{ m}) \right]}{3.8 \text{ m}} = \boxed{220 \text{ N}}$$

$$f_2 = f_3 = \boxed{220 \text{ N}}$$

$$f_1 = (m + m_\ell)g = \boxed{890 \text{ N}}$$

79. This problem is similar to Problem 24. For a different solution approach here, note that from the sign's point of view the situation is symmetric: it has a movable support at each end and doesn't "know" whether the support is a wall or a wire. So, the force at the wall bolt is the same as it would be if the wall were instead a wire running up and to the right at 20.0° above horizontal. By symmetry, $f = T$ and $\sum F_y = 2T \sin \theta - mg = 0$.

$$(a) \quad T = \frac{mg}{2 \sin \theta} = \frac{(16.0 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right)}{2 \sin 20.0^\circ} = \boxed{229 \text{ N}}$$

$$(b) \quad F_x = F \cos \theta = T \cos \theta = (229.5 \text{ N}) \cos 20.0^\circ = \boxed{216 \text{ N}}$$

$$F_y = F \sin \theta = T \sin \theta = \frac{mg}{2} = \frac{(16.0 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right)}{2} = \boxed{78.5 \text{ N}}$$

80. The upper student exerts no vertical force, so for the lower student, $F_y = mg = 180 \text{ N}$. Since the only horizontal forces are those exerted by the students, they must be equal and opposite. Call the horizontal force of each student F_x . Using the couch's center of mass as the pivot point,

$$\sum \tau = F_y \cos \theta - 2F_x \sin \theta = 0$$

$$F_x = \frac{\cos \theta}{2 \sin \theta} F_y = \left(\frac{\cos 30.0^\circ}{2 \sin 30.0^\circ} \right) (180 \text{ N}) = 156 \text{ N}$$

$$\text{For the upper student, } F = \boxed{156 \text{ N}}.$$

$$\text{For the lower student, } F = \sqrt{(156 \text{ N})^2 + (180 \text{ N})^2} = \boxed{238 \text{ N}}.$$

$$81. \sum F_y = F_1 + F_2 - (m_{\text{swimmer}} + m_{\text{board}})g = 0$$

With the first pillar as the pivot point,

$$\sum \tau = F_2 d - m_{\text{board}} g \left(\frac{L}{2} \right) - m_{\text{person}} g x = 0$$

$$F_2 = \left(\frac{m_{\text{person}} g}{d} \right) x + \frac{m_{\text{board}} g L}{2d}$$

$$= \frac{(90.0 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right)}{1.50 \text{ m}} x + \frac{(85 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) (5.00 \text{ m})}{2(1.50 \text{ m})}$$

$$\vec{F}_2 = [(0.59 \text{ kN/m})x + 1.4 \text{ kN}] \hat{y}$$

$$F_1 = (m_{\text{swimmer}} + m_{\text{board}})g - F_2$$

$$= (90.0 \text{ kg} + 85 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) - \left(588.6 \frac{\text{N}}{\text{m}} \right) x - 1389.75 \text{ N}$$

$$\vec{F}_1 = [-(0.59 \text{ kN/m})x + 0.33 \text{ kN}] \hat{y}$$

82. (a) Start with two books. Place the bottom book such that its edge is under the center of mass of the book above. The center of mass of the system is

$$X_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{mL + m \frac{L}{2}}{2m} = \frac{3}{4}L$$

Place the third book such that its edge is under the center of mass of the system above.

The center of mass of the system is

$$X_{\text{cm, 3 books}} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{m \left(\frac{L}{2} \right) + m \left(\frac{3L}{4} \right) + m \left(\frac{5L}{4} \right)}{m + m + m} = \frac{5}{6}L$$

Place the fourth book such that its edge is under the center of mass of the other three.

The center of mass of the new system is

$$X_{\text{cm, 4 books}} = \frac{m_1 x_1 + 3m \left(X_{\text{cm, 3 books}} + \frac{L}{6} \right)}{m_1 + 3m} = \frac{m \left(\frac{L}{2} \right) + 3m \left(\frac{5L}{6} + \frac{L}{6} \right)}{4m} = \frac{7L}{8}$$

Let x_R = the position of the right edge of the top book.

$$d = x_R - X_{\text{cm, 4 books}} = \frac{23L}{12} - \frac{7L}{8} = \boxed{\frac{25}{24}L}$$

- (b) The distance is independent of the mass, as long as all books have the same mass. The answer to part (a) stays the same.

83. (a) With conservation of angular momentum, an increase in the moment of inertia leads to a decrease in the speed of rotation. The length of a day would increase.

$$(b) \quad I_i \omega_i = I_f \omega_f$$

$$\omega_f = \frac{I_i}{I_f} \omega_i$$

Also, with $T = \text{one period}$, $T = \frac{2\pi}{\omega}$, so

$$T_f = \frac{2\pi}{\omega_f} = \frac{2\pi}{\frac{I_i}{I_f} \omega_i} = \frac{I_f}{I_i} \left(\frac{2\pi}{\omega_i} \right) = \left(\frac{I_f}{I_i} \right) T_i$$

$$\Delta T = T_f - T_i = \left(\frac{I_f}{I_i} \right) T_i - T_i = \left(\frac{I_f}{I_i} - 1 \right) T_i = \left(\frac{0.332}{0.331} - 1 \right) (1 \text{ day}) \left(\frac{86,400 \text{ s}}{\text{day}} \right) = \boxed{261 \text{ s}}$$

84. Let the pivot point be at the bottom of the rod.

$$\sum \tau = (T \cos 45^\circ) L - F \left(\frac{L}{2} \right) = 0$$

$$\frac{T}{\sqrt{2}} = \frac{F}{2}$$

$$\sum F_y = 0 = N - T \sin 45^\circ - Mg = N - \frac{T}{\sqrt{2}} - Mg$$

$$N = \frac{T}{\sqrt{2}} + Mg$$

$$\sum F_x = 0 = -T \cos 45^\circ - f + F = \frac{-T}{\sqrt{2}} - \mu N + F = \frac{-F}{2} - \mu \left(\frac{T}{\sqrt{2}} + Mg \right) + F = \frac{F}{2} - \mu \left(\frac{F}{2} + Mg \right)$$

$$\frac{F}{2} (1 - \mu) = \mu Mg$$

$$F = \boxed{\frac{2\mu Mg}{1 - \mu}}$$

$$85. (a) \quad F = \frac{2\mu Mg}{1 - \mu} = \frac{2\left(\frac{1}{7}\right)(2.3 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)}{1 - \frac{1}{7}} = \boxed{7.5 \text{ N}}$$

$$(b) \quad \sum \tau = (T \cos 45^\circ) L - F \left(\frac{7}{8} L \right) = 0$$

$$T = \frac{7\sqrt{2}}{8} F$$

$$\sum F_y = 0 = N - T \sin 45^\circ - mg = N - \frac{T}{\sqrt{2}} - mg$$

$$N = \frac{T}{\sqrt{2}} + mg = \frac{1}{\sqrt{2}} \left(\frac{7\sqrt{2}}{8} F \right) + mg = \frac{7}{8} F + mg$$

$$\begin{aligned}
\sum F_x &= 0 \\
&= -T \cos 45^\circ - f + F \\
&= -\frac{T}{\sqrt{2}} - \mu N + F \\
&= -\frac{1}{\sqrt{2}} \left(\frac{7\sqrt{2}}{8} F \right) - \mu \left(\frac{7}{8} F + mg \right) + F \\
&= -\frac{7}{8} F - \left(\frac{1}{7} \right) \left(\frac{7}{8} F + mg \right) + F \\
&= \frac{1}{8} F - \frac{1}{8} F - \frac{1}{7} mg \\
&= -\frac{1}{7} mg
\end{aligned}$$

The applied force drops out of the equation, therefore the rod will never slip.

86. The net torque about the center of the cylinder is $\tau = I\alpha = Tr$, where T is the tension of the rope. So,

$$\begin{aligned}
\left(\frac{1}{2} mr^2 \right) \alpha &= Tr \\
\frac{1}{2} mr\alpha &= T
\end{aligned}$$

or, since $r\alpha = a$, $T = \frac{1}{2} ma$.

$$\begin{aligned}
\sum F_y &= mg - T = ma \\
ma &= mg - T \\
&= mg - \frac{1}{2} ma \\
a &= g - \frac{1}{2} a \\
a &= \frac{2}{3} g
\end{aligned}$$

87. The net torque about the center of the sphere is $\tau = I\alpha = Tr$, where T is the tension of the rope. So,

$$\begin{aligned}
\left(\frac{2}{5} mr^2 \right) \alpha &= Tr \\
\frac{2}{5} mr\alpha &= T
\end{aligned}$$

or, since $r\alpha = a$, $T = \frac{2}{5} ma$.

$$\begin{aligned}
\sum F &= mg - T = ma \\
ma &= mg - \frac{2}{5} ma \\
a &= g - \frac{2}{5} a \\
a &= \frac{5}{7} g
\end{aligned}$$

88. Upward force on the mass: $Ma = T - Mg$

$$\text{Net torque on the pulley: } rF - rT = \left(\frac{1}{2}mr^2\right)\alpha$$

Mass/pulley relationship: $a = r\alpha$

(a) From the last two equations, $F - T = \frac{1}{2}ma$ and $T = F - \frac{1}{2}ma$, and then from the first equation,

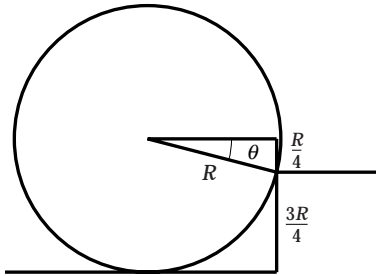
$$Ma = F - \frac{1}{2}ma - Mg. \text{ Solving for } a \text{ yields } a = \boxed{\frac{F - Mg}{M + \frac{1}{2}m}}.$$

(b) \boxed{F} = the tension on the pulled side

$$T = \text{the tension on the mass side} = F - \frac{1}{2}ma = F - \frac{Fm - Mmg}{2M + m} = \boxed{\frac{2FM + Mmg}{2M + m}}$$

(c) $\boxed{\text{As } m \rightarrow 0, T \rightarrow F, \text{ and as } m \rightarrow \infty, T \rightarrow Mg.}$

89.



$$\sin \theta = \frac{\frac{1}{4}R}{R} = \frac{1}{4}$$

$$\theta = \sin^{-1} \frac{1}{4} = 14.48^\circ$$

Taking the point of contact with the step as the pivot,

$$\sum \tau = MgR \cos \theta - F_{\min} \left(\frac{1}{4}R\right) = 0$$

$$F_{\min} = 4Mg \cos \theta = (4 \cos 14.40^\circ)Mg = \boxed{3.87Mg}$$

90. Forces: $T_1 - T_2 = Mg$

$$\text{Torques: } T_1 r_A = T_2 (5.60 r_A)$$

$$\text{Hanging mass: } T_2 = mg$$

$$\text{From the second equation, } T_1 = 5.60 T_2.$$

From the first equation,

$$5.60 T_2 - T_2 = Mg$$

$$T_2 = \frac{Mg}{4.60}$$

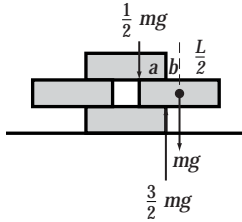
$$= \frac{(0.105 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2}\right)}{4.60}$$

$$= \boxed{0.224 \text{ N}}$$

$$T_1 = 5.60(0.2239 \text{ N}) = \boxed{1.25 \text{ N}}$$

$$m = \frac{T_2}{g} = \frac{Mg}{4.60} \left(\frac{1}{g} \right) = \frac{0.105 \text{ kg}}{4.60} = \boxed{22.8 \text{ g}}$$

91. By symmetry, each of the middle bricks supports half the weight of the top brick. The situation at the moment tipping begins looks like this:



Since $\sum \tau = 0$ implies $\left(\frac{1}{2}mg \right)a - (mg)b = 0$, it follows that $a = 2b$. And since $a + b = 3b = \frac{L}{2}$, $b = \frac{L}{6}$ and

$$x = \frac{L}{2} + \frac{L}{6} = \boxed{\frac{2}{3}L}.$$