Chapter 12

Gravity

Answers to Even-numbered Conceptual Questions

2. A person passing you on the street exerts a gravitational force on you, but it is so weak (about 10^{-7} N or less) that it is imperceptible.

4. **(a)** We can see from Equation 12-13 that if the radius of the Earth is decreased, with its mass remaining the same, the escape speed increases. The reason is that in this case the rocket starts closer to the center of the Earth, and therefore experiences a greater attractive force. It follows that a greater speed is required to overcome the increased force. **(b)** Satellites in orbit would not be affected. They would experience the same net force from the center of the Earth as before.

6. No. A satellite must be moving relative to the center of the Earth to maintain its orbit, but the North Pole is at rest relative to the center of the Earth. Therefore, a satellite cannot remain fixed above the North Pole.

8. More energy is required to go from the Earth to the Moon. To see this, note that you must essentially "escape" from the Earth to get to the Moon, and this takes much more energy than is required to "escape" from the Moon, with its much weaker gravity. This is why an enormous Saturn V rocket was required to get to the Moon, but only a small rocket on the lunar lander was required to lift off the lunar surface.

10. Yes. The rotational motion of the Earth is to the east, and therefore if you launch in that direction you are adding the speed of the Earth's rotation to the speed of your rocket.

12. Skylab's speed increased as its radius decreased. This can be seen by recalling that $T = (\text{constant})r^{3/2}$ (Kepler's third law) and that $v = 2\pi r / T$ (circular motion). It follows that $v = (\text{constant})r^{-1/2}$, and therefore the speed increases with decreasing radius. You might think that friction would slow Skylab – just like other objects are slowed by friction – but by dropping Skylab to a lower orbit, friction is ultimately responsible for an increase in speed.

14. More energy is required to put the satellite in orbit because, not only must you supply enough energy to get to the altitude *h*, you must also supply the kinetic energy the satellite will have in orbit.

16. As the astronauts approach a mascon, its increased gravitational attraction would increase the speed of the spacecraft. Similarly, as they pass the mascon, its gravitational attraction would now be in the backward direction, which would decrease their speed.

18. **(a)** The satellite drops into an elliptical orbit that brings it closer to the Earth. The situation is similar to that illustrated in Figure 12-13 (a). **(b)** The apogee distance remains the same. **(c)** The perigee distance is reduced.

20. As the tips of the fingers approach one another, we can think of them as like two small spheres (or we can replace the finger tips with two small marbles if we like). As we know, the net gravitational attraction outside a sphere of mass is the same as that of an equivalent point mass at its center. Therefore, the two fingers simply experience the finite force of two point masses separated by a finite distance.

22. It makes more sense to think of the Moon as orbiting the Sun, with the Earth providing a smaller force that makes the Moon "wobble" back and forth in its solar orbit.

24. The net force acting on the Moon is always directed toward the Sun, never away from the Sun. Therefore, the Moon's orbit must always curve toward the Sun. The path shown in the upper part of Figure 12-20, though it seems "intuitive", sometimes curves toward the Sun, sometimes away from the Sun. The correct path, shown in the lower part of Figure 12-20, curves sharply toward the Sun when the Earth is between the Moon and the Sun, and curves only slightly toward the Sun when the Moon is between the Sun and the Earth.

Solutions to Problems

1. $F = G\dfrac{m_1 m_2}{r^2}$

(a) $$F = \left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(0.20\text{ kg})(0.20\text{ kg})}{(0.25\text{ m})^2} = \boxed{4.3\times10^{-11}\text{ N}}$$

(b) $$F = \left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(0.20\text{ kg})(0.20\text{ kg})}{(0.50\text{ m})^2} = \boxed{1.1\times10^{-11}\text{ N}}$$

2. $F = G\dfrac{m_1 m_2}{r^2}$

(a) $$F = \left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(6.1\text{ kg})(7.2\text{ kg})}{(0.75\text{ m})^2} = \boxed{5.2\times10^{-9}\text{ N}}$$

(b) $$r = \sqrt{\frac{Gm_1m_2}{F}} = \sqrt{\frac{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(6.1\text{ kg})(7.2\text{ kg})}{2.0\times10^{-9}\text{ N}}} = \boxed{1.2\text{ m}}$$

3. (a) $$W_\text{s} = mg = (350\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) = \boxed{3.4\text{ kN}}$$

(b) $$F = G\frac{mM_\text{E}}{r^2} = \left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(350\text{ kg})(5.97\times10^{24}\text{ kg})}{(35\times10^6\text{ m})^2} = \boxed{110\text{ N}}$$

4. Use your mass ≈ 65 kg.

$$F = G\frac{m_1m_2}{r^2} = \left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(8.7\times10^{20}\text{ kg})(65\text{ kg})}{(12\times10^6\text{ m})^2} = \boxed{0.026\text{ N}}$$

5. The forces are equal in magnitude and opposite in direction.

$$F = G\frac{m_1 m_2}{r^2}$$

(a) $F = \left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(0.12\text{ kg})(0.20\text{ kg})}{(0.75\text{ m})^2} = \boxed{2.8\times10^{-12}\text{ N}}$

(b) $\boxed{2.8\times10^{-12}\text{ N}}$

6. (a) $F_\text{E} = G\frac{m_\text{s}M_\text{E}}{r_\text{E-s}^2} = 2G\frac{m_\text{s}M_\text{M}}{r_\text{M-s}^2} = 2F_\text{M}$

$$\frac{M_\text{E}}{r_\text{E-s}^2} = \frac{2M_\text{M}}{r_\text{M-s}^2}$$

$$\frac{r_\text{M-s}}{r_\text{E-s}} = \sqrt{\frac{2M_\text{M}}{M_\text{E}}} = \sqrt{\frac{2(7.35\times10^{22}\text{ kg})}{5.97\times10^{24}\text{ kg}}} = 0.157$$

Since $r_\text{E-s} + r_\text{M-s} = r_\text{E-M}$,

$$r_\text{E-s} + 0.157 r_\text{E-s} = 3.84\times10^8\text{ m}$$

$$r_\text{E-s} = \boxed{3.32\times10^8\text{ m}}$$

(b) It doesn't. The spaceship mass cancels out of the equation.

7. (a) $F = G\frac{M_\text{E}M_\text{S}}{r_\text{E-S}^2} + G\frac{M_\text{E}M_\text{M}}{r_\text{E-M}^2}$

$$= GM_\text{E}\left(\frac{M_\text{S}}{r_\text{E-M}^2} + \frac{M_\text{M}}{r_\text{E-M}^2}\right)$$

$$= \left(6.67\times10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.97\times10^{24}\text{ kg})\left(\frac{2.00\times10^{30}\text{ kg}}{(1.50\times10^{11}\text{ m})^2} + \frac{7.35\times10^{22}\text{ kg}}{(3.84\times10^8\text{ m})^2}\right)$$

$$\vec{\mathbf{F}} = \boxed{3.56\times10^{22}\text{ N, toward the Sun}}$$

(b) $F = GM_\text{M}\left(\frac{M_\text{S}}{r_\text{S-M}^2} - \frac{M_\text{E}}{r_\text{E-M}^2}\right)$

$$= \left(6.67\times10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(7.35\times10^{22}\text{ kg})\left(\frac{2.00\times10^{30}\text{ kg}}{(1.50\times10^{11}\text{ m} - 3.84\times10^8\text{ m})^2} - \frac{5.97\times10^{24}\text{ kg}}{(3.84\times10^8\text{ m})^2}\right)$$

$$\vec{\mathbf{F}} = \boxed{2.40\times10^{20}\text{ N, toward the Sun}}$$

(c) $F = GM_S \left(\frac{M_E}{r_{E\text{-}S}{}^2} + \frac{M_M}{r_{S\text{-}M}{}^2} \right)$

$$= \left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \right)(2.00 \times 10^{30} \text{ kg}) \left(\frac{5.97 \times 10^{24} \text{ kg}}{(1.50 \times 10^{11} \text{ m})^2} + \frac{7.35 \times 10^{22} \text{ kg}}{(1.50 \times 10^{11} \text{ m} - 3.84 \times 10^{8} \text{ m})^2} \right)$$

$$\vec{\mathbf{F}} = \boxed{3.58 \times 10^{22} \text{ N, toward the Earth-Moon system}}$$

8. $F_S = G\frac{M_E M_S}{r_{E\text{-}S}{}^2} = \left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \right) \frac{(5.97 \times 10^{24} \text{ kg})(2.00 \times 10^{30} \text{ kg})}{(1.50 \times 10^{11} \text{ m})^2} = 3.5395 \times 10^{22} \text{ N}$

$$F_M = G\frac{M_E M_M}{r_{E\text{-}M}{}^2} = \left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \right) \frac{(5.97 \times 10^{24} \text{ kg})(7.35 \times 10^{22} \text{ kg})}{(3.84 \times 10^{8} \text{ m})^2} = 1.9848 \times 10^{20} \text{ N}$$

$$F = \sqrt{F_x{}^2 + F_y{}^2} = \sqrt{(3.5395 \times 10^{22} \text{ N})^2 + (1.9848 \times 10^{20} \text{ N})^2} = \boxed{3.54 \times 10^{22} \text{ N}}$$

$$\theta = \tan^{-1} \frac{F_y}{F_x} = \tan^{-1} \frac{1.9848 \times 10^{20} \text{ N}}{3.5395 \times 10^{22} \text{ N}} = \boxed{0.321^\circ \text{ toward the Moon off the ray from the Earth to the Sun}}$$

9. $F_E = G\frac{M_S M_E}{r_{E\text{-}S}{}^2} = \left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \right) \frac{(2.00 \times 10^{30} \text{ kg})(5.97 \times 10^{24} \text{ kg})}{(1.50 \times 10^{11} \text{ m})^2} = 3.5395 \times 10^{22} \text{ N}$

$$F_M = G\frac{M_S M_M}{r_{S\text{-}M}{}^2} = \left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \right) \frac{(2.00 \times 10^{30} \text{ kg})(7.35 \times 10^{22} \text{ kg})}{(1.50 \times 10^{11} \text{ m})^2 + (3.84 \times 10^{8} \text{ m})^2} = 4.3577 \times 10^{20} \text{ N}$$

Seen from the Sun, the Earth and the Moon are separated by an angle ϕ such that

$$\sin\phi = \frac{3.84 \times 10^8}{\sqrt{(1.50 \times 10^{11})^2 + (3.84 \times 10^8)^2}} = 0.002560 \text{ and}$$

$$\cos\phi = \frac{1.50 \times 10^{11}}{\sqrt{(1.50 \times 10^{11})^2 + (3.84 \times 10^8)^2}} = 1.000$$

$$F = \sqrt{(F_E + F_M \cos\phi)^2 + (F_M \sin\phi)^2} = \boxed{3.58 \times 10^{22} \text{ N}}$$

$$\theta = \tan^{-1} \frac{F_M \sin\phi}{F_E + F_M \cos\phi}$$

$$= \tan^{-1} \frac{(4.3577 \times 10^{20} \text{ N})(0.002560)}{3.5395 \times 10^{22} \text{ N} + (4.3577 \times 10^{20} \text{ N})(1.000)}$$

$$= \boxed{0.00178^\circ \text{ toward the Moon off the ray from the Sun to the Earth}}$$

10. Each mass will be drawn toward a point halfway between the other two masses. Each of those other masses contributes to that attraction with a force component equal to $G\frac{m^2}{r^2}\cos 30^\circ$, where *r* is the length of a side of the triangle.

(a) $F = 2G\frac{m^2}{r^2}\cos 30^\circ = 2\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \right) \frac{(6.75 \text{ kg})^2}{(1.25 \text{ m})^2} \cos 30^\circ = \boxed{3.37 \times 10^{-9} \text{ N}}$

(b) Doubling $\boxed{\text{reduces } F \text{ by a factor of 4}}$: $F = \boxed{8.42 \times 10^{-10} \text{ N.}}$

11. (a) Let $m_1 = 1.0\text{ kg}$, $m_2 = 4.0\text{ kg}$, $m_3 = 3.0\text{ kg}$, and $m_4 = 2.0\text{ kg}$.

$$\begin{aligned}F_x &= G\frac{m_1m_4}{r_{14}^2} + G\frac{m_2m_4}{r_{24}^2}\cos\theta \\ &= G\frac{m_1m_4}{r_{14}^2} + G\frac{m_2m_4}{r_{24}^2}\left(\frac{r_{14}}{r_{24}}\right) \\ &= Gm_4\left(\frac{m_1}{r_{14}^2} + \frac{m_2r_{14}}{r_{24}^3}\right) \\ &= \left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(2.0\text{ kg})\left(\frac{1.0\text{ kg}}{(0.20\text{ m})^2} + \frac{(4.0\text{ kg})(0.20\text{ m})}{[(0.20\text{ m})^2 + (0.10\text{ m})^2]^{3/2}}\right) \\ &= 1.288\times10^{-8}\text{ N}\end{aligned}$$

$$\begin{aligned}F_y &= G\frac{m_2m_4}{r_{24}^2}\sin\theta + G\frac{m_3m_4}{r_{34}^2} \\ &= G\frac{m_2m_4}{r_{24}^2}\left(\frac{r_{34}}{r_{24}}\right) + G\frac{m_3m_4}{r_{34}^2} \\ &= Gm_4\left(\frac{m_2r_{34}}{r_{24}^3} + \frac{m_3}{r_{34}^2}\right) \\ &= \left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(2.0\text{ kg})\left(\frac{(4.0\text{ kg})(0.10\text{ m})}{[(0.20\text{ m})^2 + (0.10\text{ m})^2]^{3/2}} + \frac{3.0\text{ kg}}{(0.10\text{ m})^2}\right) \\ &= 4.479\times10^{-8}\text{ N}\end{aligned}$$

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(1.288\times10^{-8}\text{ N})^2 + (4.479\times10^{-8}\text{ N})^2} = \boxed{4.7\times10^{-8}\text{ N}}$$

$$\theta = \tan^{-1}\frac{F_y}{F_x} = \tan^{-1}\frac{4.479\times10^{-8}\text{ N}}{1.288\times10^{-8}\text{ N}} = \boxed{74°\text{ below horizontal, down and to the left}}$$

(b) All forces are reduced by a factor of $2^2 = 4$, including the net force on m_4. The directions of the forces are unchanged.

12. Let m_1 be an arbitrary unit of mass, then set $m_2 = \frac{1}{7}m_1$, $m_3 = \frac{1}{4}m_1$. With $r_{12} = x$ and $r_{23} = D - x$, the forces on m_2 cancel out when

$$\begin{aligned}G\frac{m_1m_2}{r_{12}^2} &= G\frac{m_2m_3}{r_{23}^2} \\ \frac{m_1}{x^2} &= \frac{m_3}{(D-x)^2} \\ m_1(D^2 - 2Dx + x^2) &= \frac{1}{4}m_1x^2 \\ 4x^2 - 8Dx + 4D^2 &= x^2 \\ 3x^2 - 8Dx + 4D^2 &= 0 \\ (3x - 2D)(x - 2D) &= 0\end{aligned}$$

$$x = \frac{2}{3}D \text{ or } x = 2D$$

$x = 2D$ is rejected because m_2 is not between m_1 and m_3.

So, $x = \boxed{(2/3)D.}$

13. $g_P = G\dfrac{M_P}{R_P^2}$

(a) $g_M = \left(6.67\times10^{-11}\ \dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\dfrac{0.0553(5.97\times10^{24}\ \text{kg})}{(2.44\times10^6\ \text{m})^2} = \boxed{3.70\ \text{m/s}^2}$

(b) $g_V = \left(6.67\times10^{-11}\ \dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\dfrac{0.816(5.97\times10^{24}\ \text{kg})}{(6.05\times10^6\ \text{m})^2} = \boxed{8.88\ \text{m/s}^2}$

14. $a = \dfrac{GM_E}{(R_E+h)^2} = \dfrac{1}{2}\left(\dfrac{GM_E}{R_E{}^2}\right)$

$$(R_E+h)^2 = 2R_E{}^2$$
$$h = (\sqrt{2}-1)R_E$$
$$= (\sqrt{2}-1)(6.37\times10^6\ \text{m})$$
$$= \boxed{2.64\times10^6\ \text{m}}$$

15. A sphere acts like a point mass at the sphere's center.

$$F = G\frac{m^2}{r^2} = \left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(6.3\ \text{kg})^2}{(0.22\ \text{m})^2} = \boxed{5.5\times10^{-8}\ \text{N}}$$

16. $g = G\dfrac{M_E}{r^2} = \left(6.67\times10^{-11}\ \dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\dfrac{5.97\times10^{24}\ \text{kg}}{(3.84\times10^8\ \text{m})^2} = \boxed{0.00270\ \text{m/s}^2}$

17. $g_T = \dfrac{GM_T}{R_T{}^2} = \dfrac{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(1.35\times10^{23}\ \text{kg})}{(2570\times10^3\ \text{m})^2} = \boxed{1.36\ \text{m/s}^2}$

18. (a) $F = G\dfrac{M_E m}{r^2}$

$$r = \sqrt{\frac{GM_E m}{F}}$$
$$= \sqrt{\frac{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.97\times10^{24}\ \text{kg})(4.0\ \text{kg})}{2.0\ \text{N}}}$$
$$= \boxed{2.8\times10^7\ \text{m}}$$

(b) $a = \dfrac{F}{m} = \dfrac{2.0\ \text{N}}{4.0\ \text{kg}} = \boxed{0.50\ \text{m/s}^2}$

(c) Since the gravitational force is inversely proportional to r^2, doubling r reduces F by a factor of 4. Its acceleration also reduces by a factor of 4 since the force has decreased by that factor and the mass has not changed.

19. $g = \dfrac{GM}{R^2}$

$$\frac{GM_\text{M}}{{R_\text{M}}^2} = \frac{1}{6}\left(\frac{GM_\text{E}}{{R_\text{E}}^2}\right)$$

$$M_\text{M} = \left(\frac{{R_\text{M}}^2}{6{R_\text{E}}^2}\right)M_\text{E} = \left(\frac{\left(\frac{1}{4}R_\text{E}\right)^2}{6{R_\text{E}}^2}\right)M_\text{E} = \boxed{\frac{1}{96}M_\text{E}}$$

20. (a) $\boxed{\text{Use } \frac{1}{2}mv_\text{i}^2 = mgh_\text{f} \text{ to find } g, \text{ and use } g = \frac{GM}{R^2} \text{ to find } M.}$

(b) $\dfrac{1}{2}m\left(134\ \dfrac{\text{m}}{\text{s}}\right)^2 = mg(5.00\times10^3\ \text{m})$

$$g = \frac{\left(134\ \frac{\text{m}}{\text{s}}\right)^2}{2(5.00\times10^3\ \text{m})} = 1.7956\ \frac{\text{m}}{\text{s}^2}$$

$$M = \frac{gR^2}{G} = \frac{\left(1.7956\ \frac{\text{m}}{\text{s}^2}\right)(1.82\times10^6\ \text{m})^2}{6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}} = \boxed{8.92\times10^{22}\ \text{kg}}$$

21. (a) $F_\text{E} = G\dfrac{m_\text{s}M_\text{E}}{r_{\text{E}-\text{s}}} = G\dfrac{m_\text{s}M_\text{M}}{{r_{\text{M}-\text{s}}}^2} = F_\text{M}$

$$\frac{M_\text{E}}{{r_{\text{E}-\text{s}}}^2} = \frac{M_\text{M}}{{r_{\text{M}-\text{s}}}^2}$$

$$\frac{r_{\text{M}-\text{s}}}{r_{\text{E}-\text{s}}} = \sqrt{\frac{M_\text{M}}{M_\text{E}}} = \sqrt{\frac{7.35\times10^{22}\ \text{kg}}{5.97\times10^{24}\ \text{kg}}} = 0.111$$

Since $r_{\text{E}-\text{s}} + r_{\text{M}-\text{s}} = r_{\text{E}-\text{M}}$

$$r_{\text{E}-\text{s}} + 0.1109r = 3.84\times10^8\ \text{m}$$

$$\text{r}_{\text{E}-\text{s}} = \boxed{3.46\times10^8\ \text{m}}$$

(b) The net gravitational force on the astronauts will steadily decrease, reaching zero at the location found in part (a), and then gradually increase in the opposite direction. However, since the astronauts and the spaceship have the same acceleration, the astronauts will appear to float inside the spaceship. They will not “walk” on the floor or ceiling.

22. (a) $g_A = G\dfrac{M_A}{R_A^2} = \left(6.67\times10^{-11}\ \dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\dfrac{(3.45\times10^{15}\ \text{kg})}{(12\times10^3\ \text{m})^2} = \boxed{1.6\times10^{-3}\ \text{m/s}^2}$

(b) $\omega = \dfrac{v_{esc}}{R_A} = \dfrac{\sqrt{\frac{2GM_A}{R_A}}}{R_A} = \sqrt{\dfrac{2g_A}{R_A}} = \sqrt{\dfrac{2(1.6\times10^{-3}\ \text{m/s}^2)}{12\times10^3\ \text{m}}} = \boxed{5.2\times10^{-4}\ \text{rad/s}}$

23. $T = \left(\dfrac{2\pi}{\sqrt{GM_M}}\right)r^{3/2}$

$= \left(\dfrac{2\pi}{\sqrt{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(7.35\times10^{22}\ \text{kg})}}\right)(1.74\times10^6\ \text{m}+110\times10^3\ \text{m})^{3/2}$

$= \boxed{1.98\ \text{h}}$

24. $T = \left(\dfrac{2\pi}{\sqrt{GM_E}}\right)r^{3/2}$

$v = \dfrac{2\pi r}{T} = \dfrac{2\pi r\sqrt{GM_E}}{2\pi r^{3/2}} = \sqrt{\dfrac{GM_E}{r}} = \sqrt{\dfrac{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.97\times10^{24}\ \text{kg})}{6.37\times10^6\ \text{m}+1700\times10^3\ \text{m}}} = \boxed{7.0\ \text{km/s}}$

25. $T = \left(\dfrac{2\pi}{\sqrt{GM}}\right)r^{3/2}$

$r = \left(\dfrac{T\sqrt{GM}}{2\pi}\right)^{2/3}$

$= \left(\dfrac{T}{2\pi}\right)^{2/3}(GM)^{1/3}$

$= \left(\dfrac{(320\ \text{days})\left(\frac{86{,}400\ \text{s}}{\text{day}}\right)}{2\pi}\right)^{2/3}\left[\left(6.67\times10^{-11}\ \dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(1.99\times10^{30}\ \text{kg})\right]^{1/3}$

$= \boxed{1.4\times10^{11}\ \text{m}}$

26. $T = \left(\dfrac{2\pi}{\sqrt{GM_M}}\right)r^{3/2}$

$= \left(\dfrac{2\pi}{\sqrt{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(0.108\times5.97\times10^{24}\ \text{kg})}}\right)(9378\times10^3\,\text{m})^{3/2}$

$= \boxed{7.64\ \text{h}}$

27. $T=\left(\dfrac{2\pi}{\sqrt{GM_\text{J}}}\right)r^{3/2}$

$$M_\text{J}=\left(\frac{2\pi}{T}\right)^2\frac{r^3}{G}$$

$$=\left(\frac{2\pi}{6.18\times10^5\text{ s}}\right)^2\frac{(1.07\times10^9\text{ m})^3}{6.67\times10^{-11}\,\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}}$$

$$=\boxed{1.90\times10^{27}\text{ kg}}$$

28. (a) $\boxed{\text{Use } T=\left(\dfrac{2\pi}{\sqrt{GM_{243\text{I}}}}\right)r^{3/2}\text{ solved for }M_{243\text{I}}.}$

(b) $M_{243\text{I}}=\left(\dfrac{2\pi}{T}\right)^2\dfrac{r^3}{G}=\left(\dfrac{2\pi}{(19\text{ h})\left(\frac{3600\text{ s}}{\text{h}}\right)}\right)^2\dfrac{(89\times10^3\text{ m})^3}{6.67\times10^{-11}\,\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}}=\boxed{8.9\times10^{16}\text{ kg}}$

29. (a) $T=\left(\dfrac{2\pi}{\sqrt{GM_\text{E}}}\right)r^{3/2}$

$$=\left(\frac{2\pi}{\sqrt{\left(6.67\times10^{-11}\,\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.97\times10^{24}\text{ kg})}}\right)(2.0\times10^7\text{ m}+6.38\times10^6\text{ m})^{3/2}$$

$$=\boxed{12\text{ h}}$$

(b) $v=\dfrac{2\pi r}{T}=\dfrac{2\pi(2.638\times10^7\text{ m})}{4.266\times10^4\text{ s}}=\boxed{3.9\text{ km/s}}$

30. (a) In a lower orbit the gravitational force is stronger, resulting in a larger centripetal acceleration and a higher orbital speed for $\boxed{\text{satellite 2}}$.

$$T=\left(\frac{2\pi}{\sqrt{GM_\text{E}}}\right)r^{3/2}$$

$$v=\frac{2\pi r}{T}$$

$$=\sqrt{\frac{GM_\text{E}}{r}}$$

(b) $v=\sqrt{\dfrac{GM_\text{E}}{2R_\text{E}}}=\sqrt{\dfrac{\left(6.67\times10^{-11}\,\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.97\times10^{24}\text{ kg})}{2(6.37\times10^6\text{ m})}}=\boxed{5.59\text{ km/s}}$

(c) $v=\sqrt{\frac{GM_E}{3R_E}}=\sqrt{\frac{2}{3}}\left(\sqrt{\frac{GM_E}{2R_E}}\right)=\sqrt{\frac{2}{3}}\left(5.59\times10^3\ \frac{\text{m}}{\text{s}}\right)=\boxed{4.56\text{ km/s}}$

31. $T=\left(\frac{2\pi}{\sqrt{GM_E}}\right)r^{3/2}$

(a) $T=\left(\frac{2\pi}{\sqrt{GM_E}}\right)(2R_E)^{3/2}$

$$=\left(\frac{2\pi}{\sqrt{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.97\times10^{24}\text{ kg})}}\right)(2\times6.37\times10^6\text{ m})^{3/2}$$

$$=\boxed{3.98\text{ h}}$$

(b) $T=\left(\frac{2\pi}{\sqrt{GM_E}}\right)(3R_E)^{3/2}$

$$=\left(\frac{3}{2}\right)^{3/2}\left(\frac{2\pi}{\sqrt{GM_E}}\right)(2R_E)^{3/2}$$

$$=\left(\frac{3}{2}\right)^{3/2}(14{,}318\text{ s})\left(\frac{1\text{ h}}{3600\text{ s}}\right)$$

$$=\boxed{7.31\text{ h}}$$

(c) [The periods do not depend on the mass of the satellite] because the satellite mass cancels out of the equation. [They depend inversely on the square root of the mass of the Earth.]

32. **(a)** Because $T\propto r^{3/2}$ and Diemos has the greater period, [Diemos is farther from Mars than Phobos is.]

(b) $T=\left(\frac{2\pi}{\sqrt{GM_M}}\right)r^{3/2}$

$$r=\left(\frac{T\sqrt{GM_M}}{2\pi}\right)^{2/3}$$

$$=\left(\frac{T}{2\pi}\right)^{2/3}(GM_M)^{1/3}$$

$$=\left(\frac{1.10\times10^5\text{ s}}{2\pi}\right)^{2/3}\left[\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(0.108\times5.97\times10^{24}\text{ kg})\right]^{1/3}$$

$$=\boxed{2.36\times10^7\text{ m}}$$

33. $m_A = m_B = m$

$d =$ separation

$r =$ orbital radius

The force between the two stars is given by Newton's Law of Universal Gravitation.

$$F = G\frac{m_A m_B}{d^2} = \frac{Gm^2}{d^2}$$

Assuming a circular orbit with radius *r*, the centripetal force is $F_{cp} = mr\omega^2$. Equate the gravitational force to the centripetal force, and substitute $2\pi/T$ for ω and $d/2$ for *r*.

$$\frac{Gm^2}{d^2} = m\left(\frac{d}{2}\right)\left(\frac{2\pi}{T}\right)^2$$

$$m = \frac{2\pi^2 d^3}{GT^2}$$

$$= \frac{2\pi^2(3.45\times10^{12}\text{ m})^3}{\left(6.67\times10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(2.52\times10^9\text{ s})^2}$$

$$= \boxed{1.91\times10^{30}\text{ kg}}$$

34. Assuming a circular orbit with circumference *C* and diameter *d*, the orbital speed is

$$v = \frac{C}{T} = \frac{\pi d}{T} = \frac{\pi(3.45\times10^{12}\text{ m})}{2.52\times10^9\text{ s}} = \boxed{4.30\text{ km/s}}$$

35. $\Delta U = U_2 - U_1$

$$= -G\frac{M_E m}{r_2} - \left(-G\frac{M_E m}{r_1}\right)$$

$$= GM_E m\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= \left(6.67\times10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.97\times10^{24}\text{ kg})(83.5\text{ kg})\left(\frac{1}{7330\times10^3\text{ m}} - \frac{1}{6610\times10^3\text{ m}}\right)$$

$$= \boxed{-4.94\times10^8\text{ J}}$$

36. (a) $U = -G\frac{m_1 m_2}{r}$

There are six contributions to the total potential energy of the system.

$$U = -\left(6.67\times10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left[\frac{(1.0\text{ kg})(2.0\text{ kg})}{0.20\text{ m}} + \frac{(1.0\text{ kg})(3.0\text{ kg})}{\sqrt{(0.20\text{ m})^2+(0.10\text{ m})^2}} + \frac{(1.0\text{ kg})(4.0\text{ kg})}{0.10\text{ m}} + \frac{(2.0\text{ kg})(3.0\text{ kg})}{0.10\text{ m}} + \frac{(2.0\text{ kg})(4.0\text{ kg})}{\sqrt{(0.20\text{ m})^2+(0.10\text{ m})^2}} + \frac{(3.0\text{ kg})(4.0\text{ kg})}{0.20\text{ m}}\right]$$

$$= \boxed{-1.5\times10^{-8}\text{ J}}$$

(b) It increases by a factor of 4 because each term involves the product of two masses.

(c) It increases by a factor of 2.

37. $U = -G\dfrac{M_E m}{r}$

(a) $U_1 = -\left(6.67\times10^{-11}\ \dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\dfrac{(5.97\times10^{24}\ \text{kg})(5.0\ \text{kg})}{6.37\times10^{6}\ \text{m}} = \boxed{-3.12558\times10^{8}\ \text{J}}$

(b) $U_2 = -\left(6.67\times10^{-11}\ \dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\dfrac{(5.97\times10^{24}\ \text{kg})(5.0\ \text{kg})}{6.37\times10^{6}\ \text{m}+1.0\times10^{3}\ \text{m}} = \boxed{-3.12509\times10^{8}\ \text{J}}$

(c) $U_2 - U_1 = \boxed{4.9\times10^{4}\ \text{J}}$

$mgh = (5.0\ \text{kg})\left(9.81\ \dfrac{\text{m}}{\text{s}^2}\right)(1.0\times10^{3}\ \text{m}) = \boxed{4.9\times10^{4}\ \text{J}}$

38. $\Delta U = U_2 - U_1 = -G\dfrac{m^2}{r_2} - \left(-G\dfrac{m^2}{r_1}\right) = Gm^2\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)$

(a) $\Delta U = \left(6.67\times10^{-11}\ \dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(0.59\ \text{kg})^2\left(\dfrac{1}{0.24\ \text{m}} - \dfrac{1}{1.0\ \text{m}}\right) = \boxed{7.4\times10^{-11}\ \text{J}}$

(b) $\Delta U = \left(6.67\times10^{-11}\ \dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(0.59\ \text{kg})^2\left(\dfrac{1}{0.24\ \text{m}} - \dfrac{1}{10.0\ \text{m}}\right) = \boxed{9.4\times10^{-11}\ \text{J}}$

39. (a)
$$\begin{aligned} K &= U_\infty - U_i \\ &= -U_i \\ &= G\frac{M_M m}{R_M} \\ &= \left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(7.35\times10^{22}\ \text{kg})(29,000\ \text{kg})}{(1.74\times10^{6}\ \text{m})} \\ &= \boxed{8.2\times10^{10}\ \text{J}} \end{aligned}$$

(b) $K = G\dfrac{M_E m}{R_E} = \left(6.67\times10^{-11}\ \dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\dfrac{(5.97\times10^{24}\ \text{kg})(29,000\ \text{kg})}{(6.37\times10^{6}\ \text{m})} = \boxed{1.8\times10^{12}\ \text{J}}$

40.
$$E_\text{p} = E_\text{a}$$
$$\frac{1}{2}mv_\text{p}^2 - G\frac{mM_\text{E}}{R_\text{p}} = \frac{1}{2}mv_\text{a}^2 - G\frac{mM_\text{E}}{R_\text{a}}$$
$$\frac{GM_\text{E}}{R_\text{a}} = \frac{1}{2}(v_\text{a}^2 - v_\text{p}^2) + \frac{GM_\text{E}}{R_\text{p}}$$
$$R_\text{a} = \frac{GM_\text{E}}{\frac{1}{2}(v_\text{a}^2 - v_\text{p}^2) + \frac{GM_\text{E}}{R_\text{p}}}$$
$$= \frac{1}{\frac{v_\text{a}^2 - v_\text{p}^2}{2GM_\text{E}} + \frac{1}{R_\text{p}}}$$
$$= \frac{1}{\frac{\left(3.64\times10^3\ \frac{\text{m}}{\text{s}}\right)^2 - \left(4.46\times10^3\ \frac{\text{m}}{\text{s}}\right)^2}{2\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.97\times10^{24}\ \text{kg})} + \frac{1}{2.00\times10^7\ \text{m}}}$$
$$= \boxed{2.40\times10^4\ \text{km}}$$

41. $$v_\text{esc} = \sqrt{\frac{2GM_\text{M}}{R_\text{M}}} = \sqrt{\frac{2\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(0.108\times5.97\times10^{24}\ \text{kg})}{3.39\times10^6\ \text{m}}} = \boxed{5.04\ \text{km/s}}$$

42. Its speed at B is determined by its kinetic energy at B, which equals the potential energy lost. The potential energy at a point is twice the potential energy due to a single asteroid. So,
$$\frac{1}{2}mv^2 = -2\Delta U$$
$$= -2\left[-G\frac{Mm}{r_\text{B}} - \left(-G\frac{Mm}{r_\text{A}}\right)\right]$$
$$= 2GMm\left(\frac{1}{r_\text{B}} - \frac{1}{r_\text{A}}\right)$$
$$v = \sqrt{4GM\left(\frac{1}{r_\text{B}} - \frac{1}{r_\text{A}}\right)}$$
$$= \sqrt{4\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(3.50\times10^{11}\ \text{kg})\left(\frac{1}{1500\ \text{m}} - \frac{1}{\sqrt{(1500\ \text{m})^2 + (3000\ \text{m})^2}}\right)}$$
$$= \boxed{0.186\ \text{m/s}}$$

43.

$$E_\text{i} = E_\text{f}$$

$$\frac{1}{2}mv^2 - G\frac{mM_\text{E}}{R_\text{E}} = -G\frac{mM_\text{E}}{2R_\text{E}}$$

$$\frac{1}{2}mv^2 = G\frac{mM_\text{E}}{2R_\text{E}}$$

$$v = \sqrt{\frac{GM_\text{E}}{R_\text{E}}}$$

$$= \sqrt{\frac{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.97\times10^{24}\ \text{kg})}{6.37\times10^6\ \text{m}}}$$

$$= \boxed{7.91\ \text{km/s}}$$

44. Energy balance:

$$\frac{1}{2}mv^2 = mgh$$

$$= m\left(\frac{GM_\text{M}}{R_\text{M}^{\,2}}\right)h$$

$$v = \sqrt{\frac{2GM_\text{M}h}{R_\text{M}^{\,2}}}$$

$$= \sqrt{\frac{2\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(7.35\times10^{22}\ \text{kg})(325\times10^3\ \text{m})}{(1.74\times10^6\ \text{m})^2}}$$

$$= \boxed{1.03\ \text{km/s}}$$

45. $v_\text{esc} = \sqrt{\frac{2GM}{R}}$

(a) $$v_\text{esc} = \sqrt{\frac{2\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(0.0553\times5.97\times10^{24}\ \text{kg})}{2.44\times10^6\ \text{m}}}$$

$$= \boxed{4.25\ \text{km/s}}$$

(b) $$v_\text{esc} = \sqrt{\frac{2\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(0.816\times5.97\times10^{24}\ \text{kg})}{6.05\times10^6\ \text{m}}}$$

$$= \boxed{10.4\ \text{km/s}}$$

46. (a) The speed of Halley's comet at aphelion is $\boxed{\text{less than}}$ its speed at perihelion because more of its energy is in the form of gravitational potential energy and, therefore, less is in the form of kinetic energy (energy conservation).

(b)

$$E_a = E_p$$

$$\frac{1}{2}mv_a{}^2 - G\frac{mM_S}{R_a} = \frac{1}{2}mv_p{}^2 - G\frac{mM_S}{R_p}$$

$$v_a = \sqrt{v_p{}^2 + 2GM_S\left(\frac{1}{R_a} - \frac{1}{R_p}\right)}$$

$$= \sqrt{\left(54.6\times10^3\ \frac{\text{m}}{\text{s}}\right)^2 + 2\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(2.00\times10^{30}\ \text{kg})\left(\frac{1}{5.270\times10^{12}\ \text{m}} - \frac{1}{8.823\times10^{10}\ \text{m}}\right)}$$

$$= \boxed{2.81\ \text{km/s}}$$

47.

$$E_i = E_f$$

$$\frac{1}{2}mv_i^2 - G\frac{M_M m}{R_M + h} = \frac{1}{2}mv_f^2 - G\frac{M_M m}{R_M}$$

$$v_i^2 - \frac{2GM_M}{R_M + h} = v_f^2 - \frac{2GM_M}{R_M}$$

$$v_f = \sqrt{v_i^2 + 2GM_M\left(\frac{1}{R_M} - \frac{1}{R_M + h}\right)}$$

$$= \sqrt{v_i^2 + \frac{2GM_M h}{R_M(R_M + h)}}$$

$$= \sqrt{\left(1630\ \frac{\text{m}}{\text{s}}\right)^2 + \frac{2\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(7.35\times10^{22}\ \text{kg})(110\times10^3\ \text{m})}{(1.74\times10^6\ \text{m})(1.85\times10^6\ \text{m})}}$$

$$= \boxed{1.73\ \text{km/s}}$$

48.

$$E_i = E_f$$

$$\frac{1}{2}mv_i^2 - G\frac{mM_E}{R_E} = \frac{1}{2}mv_f^2 - G\frac{mM_E}{h}$$

$$\frac{1}{2}m\left(\frac{2GM_E}{R_E}\right) - G\frac{mM_E}{R_E} = \frac{1}{2}m\left(\frac{GM_E}{2R_E}\right) - G\frac{mM_E}{h}$$

$$0 = G\frac{mM_E}{4R_E} - G\frac{mM_E}{h}$$

$$h = 4R_E$$

$$= 4(6.37\times10^6\ \text{m})$$

$$= \boxed{2.55\times10^7\ \text{m}}$$

49. $v_{esc} = \sqrt{\frac{2GM_P}{R_P}} = \sqrt{\frac{2G(10M_E)}{\frac{1}{10}R_E}} = 10\sqrt{\frac{2GM_E}{R_E}}$

The escape speed is $\boxed{\text{10 times that of the earth}}$.

50.
$$E_\text{i} = E_\text{f}$$
$$\frac{1}{2}mv_\text{i}^2 - G\frac{mM_\text{M}}{R_\text{M}} = \frac{1}{2}m\left(\frac{v_\text{i}}{2}\right)^2 - G\frac{mM_\text{M}}{R_\text{M}+h}$$
$$v_\text{i}^2 - \frac{1}{4}v_\text{i}^2 - 2G\frac{M_\text{M}}{R_\text{M}} = -2G\frac{M_\text{M}}{R_\text{M}+h}$$
$$h = \frac{2GM_\text{M}}{2G\frac{M_\text{M}}{R_\text{M}} - \frac{3}{4}v_\text{i}^2} - R_\text{M}$$
$$= \frac{2\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(7.35\times10^{22}\ \text{kg})}{2\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(7.35\times10^{22}\ \text{kg})}{(1.74\times10^{6}\ \text{m})} - \frac{3}{4}\left(1250\ \frac{\text{m}}{\text{s}}\right)^2} - 1.74\times10^6\ \text{m}$$
$$= \boxed{457\ \text{km}}$$

51. $v_\text{esc} = c = \sqrt{\frac{2GM_\text{S}}{R}}$
$$R = \frac{2GM_\text{S}}{c^2}$$
$$= \frac{2\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(2.00\times10^{30}\ \text{kg})}{\left(3.00\times10^8\ \frac{\text{m}}{\text{s}}\right)^2}$$
$$= \boxed{2.96\ \text{km}}$$

52. **(a)**
$$E_\text{i} = E_\text{f}$$
$$-G\frac{m^2}{r_\text{i}} = 2\left(\frac{1}{2}mv^2\right) - G\frac{m^2}{r_\text{f}}$$
$$mv^2 = Gm^2\left(\frac{1}{r_\text{f}} - \frac{1}{r_\text{i}}\right)$$
$$v = \sqrt{Gm\left(\frac{1}{r_\text{f}} - \frac{1}{r_\text{i}}\right)}$$
$$= \sqrt{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(0.148\ \text{kg})\left(\frac{1}{145\ \text{m}} - \frac{1}{355\ \text{m}}\right)}$$
$$= \boxed{2.01\times10^{-7}\ \text{m/s}}$$

(b) Increase, because v is proportional to $\sqrt{m}$.

53. On Earth, with $h << R_E$, $E_i = E_f \Rightarrow \frac{1}{2}mv^2 = mgh \Rightarrow v = \sqrt{2gh}$. So, on the asteroid, using the formula for escape velocity, $\sqrt{2gh} = \sqrt{\frac{2GM_A}{R_A}}$. And since $M_A = \frac{4}{3}\pi R_A^3 \rho$,

$$gh = \frac{4}{3}\pi G R_A{}^2 \rho$$

$$R_A = \sqrt{\frac{3gh}{4\pi G\rho}} = \sqrt{\frac{3(9.81\text{ m/s}^2)h}{4\pi\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(3500\ \frac{\text{kg}}{\text{m}^3}\right)}} = \boxed{\sqrt{(1.00\times10^7\text{ m})h}}$$

54. **(a)** $$F = 4\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(65\text{ kg})(10^6\times2.00\times10^{30}\text{ kg})(1.8\text{ m})}{\left[(10^6\text{ mi})\left(1609\ \frac{\text{m}}{\text{mi}}\right)\right]^3} = \boxed{15\text{ N}}$$

(b) $$r = \left[\frac{4GmMa}{F}\right]^{1/3} = \left[\frac{4GmMa}{10mg}\right]^{1/3} = \left[\frac{4GMa}{10g}\right]^{1/3}$$

$$= \left[\frac{4\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(2.00\times10^{36}\text{ kg})(1.8\text{ m})}{10(9.81\text{ m/s}^2)}\right]^{1/3}$$

$$= \boxed{2.1\times10^8\text{ m}}\left(\frac{\text{mi}}{1609\text{ m}}\right)$$

$$= \boxed{1.3\times10^5\text{ mi}}$$

55. For $r >> a$, $\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2} \approx \frac{4a}{r^3}$. So,

$$F_2 - F_1 = G\frac{mM_E}{(r-a)^2} - G\frac{mM_E}{(r+a)^2} = \frac{4GmM_E a}{r^3}$$

56. **(a)** $$F = G\frac{m^2}{(2a)^2} = G\frac{\left(\frac{4}{3}\pi a^3\rho\right)^2}{4a^2}$$

$$= \frac{4}{9}G\pi^2 a^4\rho^2$$

$$= \frac{4\pi^2}{9}\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\rho^2 a^4$$

$$= \boxed{\left(2.93\times10^{-10}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\rho^2 a^4}$$

(b) $G\dfrac{m^2}{4a^2}=\dfrac{4GmM_{\text{E}}a}{r^3}$

$$r=\left(\frac{16a^3M_{\text{E}}}{m}\right)^{1/3}=\left(\frac{16a^3M_{\text{E}}}{\frac{4}{3}\pi a^3\rho}\right)^{1/3}=\left(\frac{12M_{\text{E}}}{\pi\rho}\right)^{1/3}$$

(c) $r_{\text{E}}=\left(\dfrac{12M_{\text{E}}}{\pi\rho}\right)^{1/3}=\left(\dfrac{12(5.97\times10^{24}\text{ kg})}{\pi\left(3330\ \frac{\text{kg}}{\text{m}^3}\right)}\right)^{1/3}=\boxed{1.90\times10^{7}\text{ m}}$

$$r_{\text{S}}=\left(\frac{12M_{\text{S}}}{\pi\rho}\right)=\left(\frac{12(95.1)(5.97\times10^{24}\text{ kg})}{\pi\left(3330\ \frac{\text{kg}}{\text{m}^3}\right)}\right)^{1/3}=\boxed{8.67\times10^{7}\text{ m}}$$

57. $U=-G\dfrac{m_1m_2}{r}$

There are three contributions to the total potential energy of the system.

$$U=-\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left[\frac{(1.00\text{ kg})(2.00\text{ kg})}{1.00\text{ m}}+\frac{(1.00\text{ kg})(3.00\text{ kg})}{2.00\text{ m}}+\frac{(2.00\text{ kg})(3.00\text{ kg})}{1.00\text{ m}}\right]$$
$$=\boxed{-6.34\times10^{-10}\text{ J}}$$

58. With $h<<R$, $mgh=\dfrac{1}{2}mv^2$, where $g=\dfrac{GM}{R^2}$. So, $\dfrac{GMh}{R^2}=\dfrac{1}{2}v^2$.

$$M=\frac{(Rv)^2}{2Gh}=\frac{\left[(3560\times10^3\text{ m})\left(3.00\ \frac{\text{m}}{\text{s}}\right)\right]^2}{2\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(0.570\text{ m})}=\boxed{1.50\times10^{24}\text{ m}}$$

59. (a) $F=G\dfrac{M_{\text{M}}M_{\text{E}}}{{r_{\text{E-M}}}^2}=\left(6.67\times10^{-11}\ \dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\dfrac{(7.35\times10^{22}\text{ kg})(5.97\times10^{24}\text{ kg})}{(3.84\times10^{8}\text{ m})^2}=\boxed{1.98\times10^{20}\text{ N}}$

(b) $F=G\dfrac{M_{\text{M}}M_{\text{S}}}{{r_{\text{S-M}}}^2}=\left(6.67\times10^{-11}\ \dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\dfrac{(7.35\times10^{22}\text{ kg})(2.00\times10^{30}\text{ kg})}{(1.50\times10^{11}\text{ m})^2+(3.84\times10^{8}\text{ m})^2}=\boxed{4.36\times10^{20}\text{ N}}$

(c) It makes more sense to think of the moon as orbiting the Sun, with a small effect due to the Earth. The gravitational forces from the Sun and the Earth have the same order of magnitude, but the force due to the Earth is smaller by roughly a factor of two.

60. Force from the 2.00-kg mass:

$$F_2=G\frac{m_1m_2}{r^2}=\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(1.00\text{ kg})(2.00\text{ kg})}{(10.0\text{ m})^2}=1.334\times10^{-12}\text{ N}$$

Force from the 3.00-kg mass:

$$F_3=\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(1.00\text{ kg})(3.00\text{ kg})}{(10.0\text{ m})^2}=2.001\times10^{-12}\text{ N}$$

The vertical components add, the horizontal ones subtract:

$F_x = F_{3x} - F_{2x} = (2.001\times10^{-12}\text{ N})\cos 60° - (1.334\times10^{-12}\text{ N})\cos 60° = 3.335\times10^{-13}\text{ N (to the left)}$

$F_y = F_{3y} + F_{2y} = (2.001\times10^{-12}\text{ N})\sin 60° + (1.334\times10^{-12}\text{ N})\sin 60° = 2.888\times10^{-12}\text{ N (downward)}$

$$F = \sqrt{F_x{}^2 + F_y{}^2} = \sqrt{(3.335\times10^{-13}\text{ N})^2 + (2.888\times10^{-12}\text{ N})^2} = \boxed{2.91\times10^{-12}\text{ N}}$$

$$\theta = \tan^{-1}\frac{F_y}{F_x} = \tan^{-1}\left(\frac{2.888\times10^{-12}\text{ N}}{3.335\times10^{-13}\text{ N}}\right) = \boxed{83.4°\text{ below horizontal, to the left}}$$

61. $\Delta K + \Delta U = 0$

$\Delta K = -\Delta U = -(U_\text{f} - U_\text{i})$

$K_\text{i} = 0.$

$$U_\text{i} = 3\left(-\frac{Gm^2}{r}\right) = 3\left(-6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(25.0\text{ kg}^2)}{10.0\text{ m}} = -5.00\times10^{-10}\text{ J}$$

Upon arriving at the center, the center-to-center distance between pairs of spheres is 2(0.0726 m) = 0.1452 m.

$$U_\text{f} = 3\left(-6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(25\text{ kg}^2)}{0.1452\text{ m}} = -3.45\times10^{-8}\text{ J}$$

$$K_\text{f} = 3\left(\frac{1}{2}mv^2\right) = 3\left(\frac{1}{2}\right)(5.00\text{ kg})v^2 = -\left[-3.45\times10^{-8}\text{ J} - (-5.00\times10^{-10}\text{ J})\right] = 3.40\times10^{-8}\text{ J}$$

$$v = \sqrt{\frac{2(3.40\times10^{-8}\text{ J})}{3(5.00\text{ kg})}} = \boxed{6.73\times10^{-5}\text{ m/s}}$$

62. **(a)** $\Delta K + \Delta U = 0$

$(K_\text{f} - K_\text{i}) + (U_\text{f} - U_\text{i}) = 0$

$K_\text{i} = U_\text{i} = 0$

$K_\text{f} = -U_\text{f}$

$$\frac{1}{2}m_\text{A}v^2 = -\left(\frac{-Gm_\text{A}M_\text{E}}{r}\right)$$

$$v = \sqrt{\frac{2GM_\text{E}}{r}} = \sqrt{\frac{2\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.97\times10^{24}\text{ kg})}{(73{,}600\text{ mi})\left(\frac{1609\text{ m}}{\text{mi}}\right)}}$$

$$= \boxed{2.59\text{ km/s}}$$

(b) $$K = \frac{1}{2}mv^2 = \frac{1}{2}\rho V v^2 = \frac{1}{2}\rho\left(\frac{4}{3}\pi r^3\right)v^2$$

$$= \frac{1}{2}\left(3330\ \frac{\text{kg}}{\text{m}^3}\right)\left(\frac{4\pi}{3}\right)(1.0\times10^3\text{ m})^3(2.59\times10^3\text{ m/s})^2$$

$$= \boxed{4.7\times10^{19}\text{ J}}$$

63. **(a)** Since on any given planet $g = \dfrac{GM}{R^2} = \dfrac{G\left(\frac{4}{3}\pi R^3\right)\rho}{R^2} = \dfrac{4}{3}\pi G R\rho,$ reducing the radius to half its value will reduce g to half its value also.

(b) $g = \frac{1}{2}\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) = \boxed{4.91\ \text{m/s}^2}$

64. (a) $\boxed{\text{Since on any given planet } g = \frac{GM}{R^2}\text{, compressing the radius to half its value will quadruple } g.}$

(b) $g = 4\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) = \boxed{39.2\ \text{m/s}^2}$

65.

$$ma = F$$
$$m\left(\frac{v^2}{R_\text{E}+h}\right) = G\frac{M_\text{E}m}{(R_\text{E}+h)^2}$$
$$v^2 = \frac{GM_\text{E}}{R_\text{E}+h}$$
$$v = \sqrt{\frac{GM_\text{E}}{R_\text{E}+h}}$$

66.

$$m_1a_1 = m_2a_2$$
$$m_1\left(\frac{v_1^2}{r_1}\right) = m_2\left(\frac{v_2^2}{r_2}\right)$$
$$\frac{m_1\left(\frac{2\pi r_1}{T}\right)^2}{r_1} = \frac{m_2\left(\frac{2\pi r_2}{T}\right)^2}{r_2}$$
$$m_1r_1 = m_2r_2$$
$$\frac{m_2}{m_1} = \frac{r_1}{r_2}$$
$$= \boxed{\frac{1}{3}}$$

67.

$$m_1a_1 = F$$
$$m_1\left(\frac{v_1^2}{r_1}\right) = \frac{Gm_1m_2}{(r_1+r_2)^2}$$
$$\frac{\left(\frac{2\pi r_1}{T}\right)^2}{r_1} = \frac{Gm_2}{(r_1+3r_1)^2}$$
$$\frac{4\pi^2 r_1}{T^2} = \frac{G\left(\frac{1}{3}m_1\right)}{(4r_1)^2}$$
$$T = \boxed{\sqrt{\frac{192\pi^2 r_1^3}{Gm_1}}}$$

68. From Problem 46, $v_\text{p} = 54.6$ km/s, $v_\text{a} = 783\ \frac{\text{m}}{\text{s}}$, $R_\text{p} = 8.823\times10^{10}$ m, and $R_\text{a} = 6.152\times10^{12}$ m.

(a) $L_\text{p} = mv_\text{p}R_\text{p} = (9.8\times10^{14}\ \text{kg})\left(54.6\times10^{3}\ \frac{\text{m}}{\text{s}}\right)(8.823\times10^{10}\ \text{m}) = \boxed{4.7\times10^{30}\ \text{kg}\cdot\text{m}^2/\text{s}}$

(b) $L_\text{a} = mv_\text{a}R_\text{a} = (9.8\times10^{14}\ \text{kg})\left(783\ \frac{\text{m}}{\text{s}}\right)(6.152\times10^{12}\ \text{m}) = \boxed{4.7\times10^{30}\ \text{kg}\cdot\text{m}^2/\text{s}}$

69. $T^2 = \frac{4\pi^2 r^3}{GM}$

$$r = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} = \left\{\frac{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(0.108)(5.97\times10^{24}\ \text{kg})\left[(24.62\ \text{h})\left(\frac{3600\ \text{s}}{\text{h}}\right)\right]^2}{4\pi^2}\right\}^{1/3}$$

$$= 2.05\times10^{7}\ \text{m}$$

$$r = R + h$$

$$h = r - R = 2.05\times10^{7}\ \text{m} - 0.34\times10^{7}\ \text{m} = \boxed{1.71\times10^{7}\ \text{m}}$$

70. (a) Since its altitude is higher than geosynchronous, it moves slower, and so its period is $\boxed{\text{greater than 24 hours}}$.

(b) The satellite lags behind the Earth's eastward rotation and thus moves $\boxed{\text{westward}}$.

(c) $T = \left(\frac{2\pi}{\sqrt{GM_\text{E}}}\right)r^{3/2}$

$$= \left(\frac{2\pi}{\sqrt{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.97\times10^{24}\ \text{kg})}}\right)\left[(23{,}300\ \text{mi})\left(\frac{1609\ \text{m}}{\text{mi}}\right) + 6.37\times10^{6}\ \text{m}\right]^{3/2}$$

$$= \boxed{25.4\ \text{h}}$$

71. $E_\text{i} = E_\text{f}$, where the potential energy at a point is twice the potential due to a single asteroid. So,

$$\frac{1}{2}mv_\text{A}^2 - 2G\frac{Mm}{r_\text{A}} = \frac{1}{2}mv_\text{B}^2 - 2G\frac{Mm}{r_\text{B}}$$

$$v_\text{A} = \sqrt{v_\text{B}^2 + 4GM\left(\frac{1}{r_\text{A}} - \frac{1}{r_\text{B}}\right)}$$

$$= \sqrt{\left(0.953\ \frac{\text{m}}{\text{s}}\right)^2 + 4\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(3.50\times10^{11}\ \text{kg})\left(\frac{1}{\sqrt{(1500\ \text{m})^2 + (3000\ \text{m})^2}} - \frac{1}{1500\ \text{m}}\right)}$$

$$= \boxed{0.935\ \text{m/s}}$$

72. The force between the Moon and the Earth is

$$F_\text{E} = G\frac{M_\text{E}M_\text{M}}{{R_\text{E-M}}^2} = \left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(5.97\times10^{24}\ \text{kg})(7.35\times10^{22}\ \text{kg})}{(3.84\times10^{8}\ \text{m})^2} = 1.98\times10^{20}\ \text{N}$$

The force between the Moon and the Sun is weakest when the Moon is farthest from the Sun. Then,

$$F_\text{S} = G\frac{M_\text{S}M_\text{M}}{{R_\text{S-M}}^2} = \left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(2.00\times10^{30}\ \text{kg})(7.35\times10^{22}\ \text{kg})}{(1.50\times10^{11}\ \text{m}+3.84\times10^{8}\ \text{m})^2} = 4.34\times10^{20}\ \text{N}$$

So, even at its weakest, the Sun's pull on the Moon is greater than the Earth's.

73.

$$T = \left(\frac{2\pi}{\sqrt{GM_\text{S}}}\right)r^{3/2}$$

$$(T)\left(\frac{3.16\times10^{7}\ \text{s}}{\text{yr}}\right) = \left(\frac{2\pi}{\sqrt{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(1.99\times10^{30}\ \text{kg})}}\right)\left[(r)\left(\frac{1.50\times10^{11}\ \text{m}}{\text{AU}}\right)\right]^{3/2}$$

$$\left(\frac{3.16\times10^{7}\ \text{s}}{\text{yr}}\right)T = (3.16\times10^{7}\ \text{s}^{-1}\cdot\text{AU}^{-3/2})r^{3/2}$$

$$T = \left(1\ \frac{\text{yr}}{\text{AU}^{3/2}}\right)r^{3/2}$$

74. In a circular orbit,

$$ma = F$$

$$m\left(\frac{v^2}{r}\right) = G\frac{M_\text{E}m}{r^2}$$

$$v^2 = G\frac{M_\text{E}}{r}$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}G\frac{M_\text{E}m}{r}$$

(a) $$K = \frac{1}{2}\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(5.97\times10^{24}\ \text{kg})(1320\ \text{kg})}{(12,600\ \text{mi})\left(\frac{1609\ \text{m}}{\text{mi}}\right)} = \boxed{1.30\times10^{10}\ \text{J}}$$

(b) In a circular orbit, $E = K + U = \frac{1}{2}G\frac{Mm}{r} - G\frac{Mm}{r} = -\frac{1}{2}G\frac{Mm}{r}$.

$$\Delta E = -\frac{1}{2}GMm\left(\frac{1}{r_\text{f}} - \frac{1}{r_\text{i}}\right)$$

$$= -\frac{1}{2}\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.97\times10^{24}\ \text{kg})(1320\ \text{kg})\left[\left(\frac{1}{25,200\ \text{mi}} - \frac{1}{12,600\ \text{mi}}\right)\left(\frac{1}{\frac{1609\ \text{m}}{\text{mi}}}\right)\right]$$

$$= \boxed{6.48\times10^{9}\ \text{J}}$$

75. In a circular orbit,

$$ma = F$$
$$m\left(\frac{v^2}{r}\right) = G\frac{Mm}{r^2}$$
$$v = \sqrt{\frac{GM}{r}}$$

(a) $\boxed{\text{No}}$; *m* drops out of the expression for *v*.

(b) $$v = \sqrt{\frac{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.97\times10^{24}\ \text{kg})}{250\times10^3\ \text{m} + 6.37\times10^6\ \text{m}}} = \boxed{7.8\ \text{km/s}}$$

(c) $$T = \frac{2\pi r}{v} = \frac{2\pi(250\times10^3\ \text{m} + 6.37\times10^6\ \text{m})}{7.756\times10^3\ \frac{\text{m}}{\text{s}}} = \boxed{1.5\ \text{h}}$$

76. The smaller mass has an orbital radius of (2/3)*d*.

$$ma = F$$
$$m\left(\frac{v^2}{\frac{2}{3}d}\right) = G\frac{m(2m)}{d^2}$$
$$\frac{\left[\frac{2\pi\frac{2}{3}d}{T}\right]^2}{\frac{2}{3}d} = G\frac{2m}{d^2}$$
$$\left(\frac{2\pi}{T}\right)^2 = 3G\frac{m}{d^3}$$
$$T = \boxed{\frac{2\pi d^{3/2}}{\sqrt{3Gm}}}$$

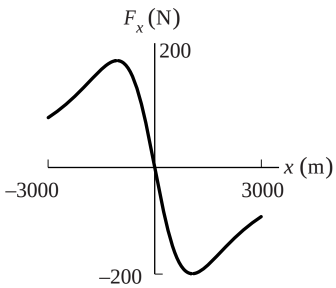
77.

$$Ma = F$$
$$M\frac{v^2}{R} = 2(\cos 30°)G\frac{M^2}{(2R\cos 30°)^2}$$
$$v^2 = \frac{GM}{2R\cos 30°}$$
$$T = \frac{2\pi R}{v} = 2\pi R\sqrt{\frac{2R\cos 30°}{GM}} = \boxed{2\sqrt[4]{3}\pi\sqrt{\frac{R^3}{GM}}}$$

78.

$$ma = F$$
$$m\left(\frac{v^2}{r}\right) = G\frac{Mm}{r^2}$$
$$v^2 = G\frac{M}{r}$$
$$K = \frac{1}{2}mv^2 = \boxed{\frac{GMm}{2r}}$$

79.
$$F_x = 2(\cos\theta)G\frac{Mm}{r^2}$$
$$= 2\left(-\frac{x}{\sqrt{x^2+(1500\text{ m})^2}}\right)G\frac{Mm}{x^2+(1500\text{ m})^2}$$
$$= -\frac{2GMmx}{[x^2+(1500\text{ m})^2]^{3/2}}$$
$$= -\frac{2\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(3.50\times10^{11}\text{ kg})(2.50\times10^{7}\text{ kg})x}{[x^2+(1500\text{ m})^2]^{3/2}}$$
$$= \boxed{-\frac{(1.17\times10^{9}\text{ N}\cdot\text{m}^2)x}{[x^2+(1500\text{ m})^2]^{3/2}}}$$



80.
$$E_1 = E_2$$
$$\frac{1}{2}mv_1^2 - G\frac{mM}{r_1} = \frac{1}{2}mv_2^2 - G\frac{mM}{r_2}$$
$$v_1^2 - 2G\frac{M}{r_1} = v_2^2 - 2G\frac{M}{r_2}$$
$$M = \frac{v_1^2 - v_2^2}{2G\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}$$
$$= \frac{\left(4280\ \frac{\text{m}}{\text{s}}\right)^2 - \left(3990\ \frac{\text{m}}{\text{s}}\right)^2}{2\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(\frac{1}{22{,}500\times10^3\text{ m}} - \frac{1}{24{,}100\times10^3\text{ m}}\right)}$$
$$= \boxed{5.97\times10^{24}\text{ kg}}$$

81. (a)
$$\frac{mv^2}{r} = \frac{GmM_\text{E}}{r^2}$$
$$v = \boxed{\sqrt{\frac{GM_\text{E}}{r}}}$$

(b)
$$\begin{aligned}E &= K + U\\ &= \frac{1}{2}mv^2 - G\frac{M_\text{E}m}{r}\\ &= \frac{1}{2}mv^2 - r(F_\text{cp})\\ &= \frac{1}{2}mv^2 - rma_\text{cp}\\ &= \frac{1}{2}mv^2 - rm\left(\frac{v^2}{r}\right)\\ &= -\frac{1}{2}mv^2\\ &= -K\end{aligned}$$

(c) This result applies to an object orbiting the Sun; it applies to any object in a circular orbit around a much larger object, and not significantly influenced by other objects.