Chapter 13

Oscillations About Equilibrium

Answers to Even-numbered Conceptual Questions

2. The person's shadow undergoes periodic motion, with the same period as the period of the Ferris wheel's rotation. In fact, if we take into account the connection between uniform circular motion and simple harmonic motion, we can say that the shadow exhibits simple harmonic motion as it moves back and forth on the ground.

4. The mass moves a distance 2A in the time T/2; it moves a distance 3A in the time 3T/4.

6. The total energy of this system increases by factor of two. This follows because doubling the mass causes the kinetic energy to be doubled at all times. But the total energy of the system is equal to the maximum kinetic energy; therefore, the total energy is also doubled.

8. The object moves through a distance of 6A in the time 3T/2. Therefore, the amplitude of motion is 2D.

10. Recall that the maximum speed of a mass on a spring is $v_{\text{max}} = \omega A$, where $\omega = \sqrt{k/m}$. It follows that the maximum kinetic energy is $K_{\text{max}} = \frac{1}{2} m v_{\text{max}}^{2} = \frac{1}{2} m\left(kA^2/m\right) = \frac{1}{2} kA^2$. Note that the mass cancels in our final expression for the maximum kinetic energy. Therefore, the larger mass moves more slowly by just the right amount so that the kinetic energy is unchanged.

12. **(a)** The angular frequency is halved. **(b)** The frequency is halved. **(c)** The maximum speed is halved. **(d)** The maximum acceleration is reduced by a factor of 4. **(e)** The total mechanical energy is reduced by a factor of 4.

14. The constant A represents the amplitude of motion; the constant B is the angular frequency. Noting that the angular frequency is $\omega = 2\pi f$, we have that the frequency is $f = \omega / 2\pi = B / 2\pi$.

16. Recalling that $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$, we see that adding a passenger (which increases the mass) results in a lower frequency of oscillation.

18. The period of a pendulum is independent of the mass of its bob. Therefore, the period should be unaffected.

20. The answer is (ii). The period of a pendulum, whether a physical pendulum or a simple pendulum, decreases with decreasing length. Therefore, a shorter person's leg has a period that is less than a taller person's leg.

22. A pendulum behaves the same in an elevator moving with constant speed as it does in an elevator at rest. Therefore, the period is still T in both cases (a) and (b).

24. A mass on a spring behaves the same in an elevator that moves with constant speed as it does in one that is at rest. Therefore, the period is still T in both cases (a) and (b).

26. The answer is (iii). Both blocks have the same period because they experience the same restoring force for a given displacement from equilibrium.

28. The answer is (i). The period of a pendulum depends inversely on the acceleration of gravity, as we see in Equation 13-20. On the Moon, the acceleration due to gravity is less than on the Earth, therefore the period of the pendulum is greater than *T*.

Solutions to Problems

1. One cycle covers the length of the track twice.

$$T = \frac{2(5.0\text{ m})}{0.75\ \frac{\text{m}}{\text{s}}} = \boxed{13\text{ s}}$$

$$f = \frac{1}{T} = \frac{1}{13.3\text{s}} = \boxed{0.075\text{ Hz}}$$

2. $$f = \frac{12\text{ cycles}}{21\text{ s}} = \boxed{0.57\text{ Hz}}$$

$$T = \frac{1}{f} = \frac{1}{0.571\text{ s}} = \boxed{1.8\text{ s}}$$

3. $$T = \frac{1}{f} = \frac{1}{2.8\text{ Hz}} = \boxed{0.36\text{ s}}$$

4. $$t = 12T = \frac{12}{f} = \frac{12}{1.77\text{ Hz}} = \boxed{6.78\text{ s}}$$

5. $$f = \left(74\ \frac{\text{beats}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = \boxed{1.2\text{ Hz}}$$

$$T = \frac{1}{f} = \frac{1}{1.23\text{ Hz}} = \boxed{0.81\text{ s}}$$

6. (a) $$\left(1.45\ \frac{\text{beats}}{\text{s}}\right)\left(\frac{60\text{ s}}{\text{min}}\right) = \boxed{87.0\text{ beats/min}}$$

(b) The number of beats per minute will $\boxed{\text{increase}}$, because more beats per second means more beats per minute.

(c) $$\left(1.55\ \frac{\text{beats}}{\text{s}}\right)\left(\frac{60\text{ s}}{\text{min}}\right) = \boxed{93.0\text{ beats/min}}$$

7. (a) $$f = \left(2500\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{60\text{ s}}{\text{min}}\right) = \boxed{42\text{ Hz}}$$

$$T = \frac{1}{f} = \frac{1}{41.67\text{ Hz}} = \boxed{0.024\text{ s}}$$

(b) $$\text{number of rpm} = f\left(\frac{60\text{ s}}{\text{min}}\right) = \frac{\frac{60\text{ s}}{\text{min}}}{T} = \frac{\frac{60\text{ s}}{\text{min}}}{0.034\text{ s}} = \boxed{1800\text{ rpm}}$$

8. (a) $x = A\cos\omega t$ and $\omega = \frac{2\pi}{T}$, so here $T = \boxed{0.54\text{ s}}$.

(b) The mass is at one extreme of its motion at $t = 0$ and is at $x = 0$ one quarter-period later, at $t = \frac{1}{4}(0.54\text{ s}) = \boxed{0.135\text{ s}}$.

9. (a) $x = A\cos\omega t$ and $f = \frac{\omega}{2\pi}$, so here $f = \frac{1}{0.68\text{ s}} = \boxed{1.5\text{ Hz}}$.

(b) The mass is first at $x = -7.8$ cm when $\cos\left(\frac{2\pi t}{0.68\text{ s}}\right) = -1$, or one half-period after $t = 0$, namely at

$$t = \frac{1}{2}T = \frac{1}{2}(0.68\text{ s}) = \boxed{0.34\text{ s}}.$$

10. $T = 0.83$ s, $A = 6.4$ cm, and $x = A\cos\omega t$.

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.83\text{ s}}$$

$$x = \boxed{(6.4\text{ cm})\cos\left(\frac{2\pi t}{0.83\text{ s}}\right)}$$

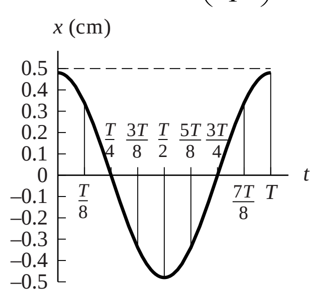
11. (a) $f = 2.00\times10^{14}$ Hz, $A = 3.50$ nm, and $x = A\cos\omega t$.

$$\omega = 2\pi f = 4.00\pi\times10^{14}\text{ s}^{-1}$$

$$x = \boxed{(3.50\text{ nm})\cos[(4.00\pi\times10^{14}\text{ s}^{-1})t]}$$

(b) $\boxed{\text{sine}}$ At $t = 0$, $\sin\omega t = 0$

12. $x = (0.48\text{ cm})\cos\left(\frac{2\pi t}{T}\right)$



13. (a) $x = A\cos\left[\left(\frac{2\pi}{T}\right)t\right]$, so here $T = \boxed{0.88\text{ s}}$.

(b) $x = (6.5\text{ cm})\cos\left[\left(\frac{2\pi}{0.88\text{ s}}\right)(0.25\text{ s})\right] = \boxed{-1.4\text{ cm}}$

(c) $A\cos\left[\left(\frac{2\pi}{T}\right)(0.25\text{ s} + T)\right] = A\cos\left[\left(\frac{2\pi}{T}\right)(0.25\text{ s}) + 2\pi\right] = A\cos\left[\left(\frac{2\pi}{T}\right)(0.25\text{ s})\right]$

14. (a) $x = (0.0440\text{ m})\cos\left(\dfrac{2\pi t}{3.15\text{ s}}\right)$. At $t = 6.37$ s,

$$x = (0.0440\text{ m})\cos\left(\frac{2\pi(6.37\text{ s})}{3.15\text{ s}}\right) = \boxed{0.0436\text{ m}}$$

(b) $\boxed{\text{negative } x \text{ direction}}$. At $t = 6.30$ s (= $2T$) the mass was at its largest displacement in the positive x direction. 0.07 s later it is headed toward the origin.

15. One time span begins and ends when

$$A\cos\left(\frac{2\pi t}{T}\right) = \frac{A}{2}$$
$$\frac{2\pi t}{T} = \cos^{-1}\left(\frac{1}{2}\right)$$
$$t = \frac{T}{2\pi}\cos^{-1}\left(\frac{1}{2}\right) = \pm\frac{T}{2\pi}\left(\frac{\pi}{3}\right) = \pm\frac{T}{6}$$

So, the time span is $\dfrac{T}{6} - \left(-\dfrac{T}{6}\right) = \dfrac{T}{3}$. The object's position is greater than $A/2$ for $\boxed{\text{one-third of a cycle}}$.

16. One time span begins and ends when

$$-A\left(\frac{2\pi}{T}\right)\sin\left(\frac{2\pi t}{T}\right) = \frac{1}{2}A\left(\frac{2\pi}{T}\right)$$
$$\frac{2\pi t}{T} = \sin^{-1}\left(-\frac{1}{2}\right)$$
$$t = \frac{T}{2\pi}\sin^{-1}\left(-\frac{1}{2}\right) = \frac{T}{2\pi}\left(\frac{7\pi}{6}\right) \text{ or } \frac{T}{2\pi}\left(\frac{11\pi}{6}\right) = \frac{7T}{12} \text{ or } \frac{11T}{12}$$

So, the time span is $\dfrac{11T}{12} - \dfrac{7T}{12} = \dfrac{T}{3}$.

The object's velocity is greater than $v_{\text{max}}/2$ for $\boxed{\text{one-third of a cycle}}$.

17. $v_{\text{max}} = A\omega$, $a_{\text{max}} = A\omega^2$

(a) $A = \dfrac{(A\omega)^2}{A\omega^2} = \boxed{\dfrac{v_{\text{max}}^{\;2}}{a_{\text{max}}}}$

(b) $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{\frac{a_{\text{max}}}{v_{\text{max}}}} = \boxed{\dfrac{2\pi v_{\text{max}}}{a_{\text{max}}}}$

18. $x = (0.50\text{ m})\cos\left[\left(1.3\ \frac{\text{rad}}{\text{s}}\right)t\right]$

At $t = 2.5$ s, $x = \boxed{-0.50\text{ m}}$, at $t = 5.0$ s, $x = \boxed{0.49\text{ m}}$, and at $t = 7.5$ s, $x = \boxed{-0.47\text{ m}}$.

19. $v_{\text{max}} = A\omega,\ a_{\text{max}} = A\omega^2$

(a) $A = \dfrac{(A\omega)^2}{A\omega^2} = \dfrac{{v_{\text{max}}}^2}{a_{\text{max}}} = \dfrac{\left(4.3\ \frac{\text{m}}{\text{s}}\right)^2}{\left(0.65\ \frac{\text{m}}{\text{s}^2}\right)} = \boxed{28\text{ m}}$

(b) $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{\frac{a_{\text{max}}}{v_{\text{max}}}} = \dfrac{2\pi v_{\text{max}}}{a_{\text{max}}} = \dfrac{2\pi\left(4.3\ \frac{\text{m}}{\text{s}}\right)}{\left(0.65\ \frac{\text{m}}{\text{s}^2}\right)} = \boxed{42\text{ s}}$

20. $v_{\text{max}} = A\omega = A\left(\dfrac{2\pi}{T}\right) = \dfrac{2\pi(0.204\text{ m})}{2.80\text{ s}} = \boxed{0.458\text{ m/s}}$

21. (a)
$$\begin{aligned} a_{\text{max}} &= A\omega^2 \\ &= A\left(\frac{2\pi}{T}\right)^2 \\ &= (0.0335\text{ m})\left(\frac{2\pi}{1.65\text{ s}}\right)^2 \\ &= 0.486\ \frac{\text{m}}{\text{s}^2} \\ &= \left(0.486\ \frac{\text{m}}{\text{s}^2}\right)\left(\frac{g}{9.81\ \frac{\text{m}}{\text{s}^2}}\right) \\ &= \boxed{0.0495g} \end{aligned}$$

(b) $v_{\text{max}} = A\omega = A\left(\dfrac{2\pi}{T}\right) = (0.0335\text{ m})\left(\dfrac{2\pi}{1.65\text{ s}}\right) = \boxed{0.128\text{ m/s}}$

(c) $\boxed{\text{The goldfinch mass does not enter into the calculation.}}$

22. (a) $T = \dfrac{C}{v} = \dfrac{2\pi r}{v} = \dfrac{2\pi(0.45\text{ m})}{0.67\ \frac{\text{m}}{\text{s}}} = \boxed{4.2\text{ s}}$

(b) $A = r = \boxed{0.45\text{ m}}$

(c) $v_{\text{max}} = A\omega = \dfrac{2\pi A}{T} = \dfrac{Av}{r} = v = \boxed{0.67\text{ m/s}}$

(d) $a_{\text{max}} = A\omega^2 = A\left(\dfrac{2\pi}{T}\right)^2 = A\left(\dfrac{v}{r}\right)^2 = \dfrac{v^2}{r} = \dfrac{\left(0.67\ \frac{\text{m}}{\text{s}}\right)^2}{0.45\text{ m}} = \boxed{1.0\text{ m/s}^2}$

23. $\left(1700\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 178\ \frac{\text{rad}}{\text{s}}$

(a) $a_{\text{max}} = A\omega^2 = (3.5\ \text{cm})(178\ \text{s}^{-1})^2 = \boxed{1100\ \text{m/s}^2}$

(b) $v_{\text{max}} = A\omega = (3.5\ \text{cm})(178\ \text{s}^{-1}) = \boxed{6.2\ \text{m/s}}$

24. $A = 10.0$ cm, $\omega = 2.00\ \text{s}^{-1}$. The "+ π'' does not affect the amplitude or frequency, but only where the cycle starts at $t = 0$.

(a) $K_{\text{max}} = \frac{1}{2}mv_{\text{max}}{}^2 = \frac{1}{2}m(A\omega)^2 = \frac{1}{2}(0.50\ \text{kg})(0.100\ \text{m})^2(2.00\ \text{s}^{-1})^2 = \boxed{0.10\ \text{J}}$

(b) $F_{\text{max}} = ma_{\text{max}} = m(A\omega^2) = (0.50\ \text{kg})(0.100\ \text{m})(2.00\ \text{s}^{-1})^2 = \boxed{0.20\ \text{N}}$

25. (a) $\boxed{\text{The rider must begin hanging on when } a_{\text{max}} \text{ (at the top of the cycle) equals } g.}$

(b)

$$\begin{aligned} a_{\text{max}} &= g \\ A\omega^2 &= g \\ A\left(\frac{2\pi}{T}\right)^2 &= g \\ A &= \left(\frac{T}{2\pi}\right)^2 g \\ &= \left(\frac{0.74\ \text{s}}{2\pi}\right)^2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) \\ &= \boxed{0.14\ \text{m}} \end{aligned}$$

26. k units: $\frac{\text{N}}{\text{m}} = \frac{\text{kg}\cdot\text{m}}{\text{s}^2\cdot\text{m}} = \frac{\text{kg}}{\text{s}^2}$. Mass has units kg. So, $\left(\frac{k}{m}\right)^{1/2}$ has units $\left(\frac{1}{\text{s}^2}\right)^{1/2} = \text{s}^{-1}$.

27.

$$\begin{aligned} T &= 2\pi\sqrt{\frac{m}{k}} \\ k &= \left(\frac{2\pi}{T}\right)^2 m \\ &= \left(\frac{2\pi}{0.65\ \text{s}}\right)^2 (0.32\ \text{kg}) \\ &= \boxed{30\ \text{N/m}} \end{aligned}$$

28. block 1

$$\sum F_x = ma = -kx - kx = -2kx$$

Since $a = -\omega^2 x$ for simple harmonic motion, $\omega^2 = 2k/m$. So, the period

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{2k}{m}}} = \pi\sqrt{\frac{2m}{k}} = \pi\sqrt{\frac{2(1.25\text{ kg})}{49.2\ \frac{\text{N}}{\text{m}}}} = \boxed{0.225\text{ s.}}$$

block 2

$$\sum F_x = ma = -kx - kx = -2kx$$

$$T = \pi\sqrt{\frac{2m}{k}} = \boxed{0.225\text{ s}}$$

The periods for block 1 and block 2 are the same. Why? A displacement of x will result in a force of $-2kx$ for each block.

29. $F = ky$

$$k = \frac{F}{y}$$

$$k = \frac{mg}{y}$$

$$= \frac{(0.50\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}{(15\times10^{-2}\text{ m})}$$

$$= 32.7\ \frac{\text{N}}{\text{m}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$m = \left(\frac{T}{2\pi}\right)^2 k$$

$$= \left(\frac{0.75\text{ s}}{2\pi}\right)^2 k$$

$$= \left(\frac{0.75\text{ s}}{2\pi}\right)^2 \left(32.7\ \frac{\text{N}}{\text{m}}\right)$$

$$= \boxed{0.47\text{ kg}}$$

30. $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.50\text{ kg}}{65\ \frac{\text{N}}{\text{m}}}} = 0.5511\text{ s}$

(a) $\omega = \frac{2\pi}{T} = \sqrt{\frac{65\ \frac{\text{N}}{\text{m}}}{0.50\text{ kg}}} = \boxed{11\text{ s}^{-1}}$

(b) $v_{\text{max}} = A\omega = (0.031\text{ m})(11.4\text{ s}^{-1}) = \boxed{0.35\text{ m/s}}$

(c) $T = \boxed{0.55\text{ s}}$

31. $F = ky$

$$k = \frac{F}{y}$$

$$k = \frac{mg}{y}$$

$$= \frac{(125\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}{0.0800\text{ m}}$$

$$= 15.33\ \frac{\text{kN}}{\text{m}}$$

(a) $$T = 2\pi\sqrt{\frac{\text{M}+\text{m}}{\text{k}}}$$

$$M + m = \left(\frac{T}{2\pi}\right)^2 k$$

$$= \left(\frac{1.65\text{ s}}{2\pi}\right)^2 \left(15.33\times10^3\ \frac{\text{N}}{\text{m}}\right)$$

$$= \boxed{1060\text{ kg}}$$

(b) $M = 1057\text{ kg} - 125\text{ kg} = \boxed{932\text{ kg}}$

32. (a) $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.85\text{ kg}}{150\ \frac{\text{N}}{\text{m}}}} = \boxed{0.47\text{ s}}$

(b) $A = \frac{v_{\text{max}}}{\omega} = \frac{Tv_{\text{max}}}{2\pi} = \frac{(0.4730\text{ s})\left(0.35\ \frac{\text{m}}{\text{s}}\right)}{2\pi} = \boxed{2.6\text{ cm}}$

(c) $a_{\text{max}} = A\omega^2 = A\left(\frac{2\pi}{T}\right)^2 = (0.02635\text{ m})\left(\frac{2\pi}{0.4730\text{ s}}\right)^2 = \boxed{4.6\ \text{m/s}^2}$

33. $k = \frac{F}{x}$

$k = \frac{mg}{d}$ and $T = 2\pi\sqrt{\frac{m}{k}}$, so

$$T = 2\pi\sqrt{\frac{d}{g}}$$

$$d = \left(\frac{T}{2\pi}\right)^2 g$$

$$= \left(\frac{0.5559\text{ s}}{2\pi}\right)^2 \left(9.81\ \frac{\text{m}}{\text{s}^2}\right)$$

$$= \boxed{0.0768\text{ m}}$$

34. **(a)** The period $\boxed{\text{increases,}}$ because the person's mass is added to the system and $T \propto \sqrt{m}$.

(b) $T = 2\pi\sqrt{\frac{m}{k}} \propto \sqrt{m}$, so if the mass increases by $\frac{112 \text{ kg}}{511 \text{ kg}} = 21.9\%$, the period $\boxed{\text{increases}}$ by $\sqrt{1.219} - 1$.

That is, $\frac{\sqrt{1.219m} - \sqrt{m}}{\sqrt{m}} = 0.104$, or $\boxed{10.4\%}$.

35. **(a)** The period is $\boxed{\text{more}}$, because the spring constant is less.

(b) $k' = \frac{1}{2}k$

$$T' = 2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{2m}{k}} = \boxed{\sqrt{2}T}$$

36. $\text{Work} = \frac{1}{2}kx^2 = \frac{1}{2}\left(9.87 \frac{\text{N}}{\text{m}}\right)(0.175 \text{ m})^2 = \boxed{0.151 \text{ J}}$

37.

$$K = E - U$$

$$\frac{1}{2}mv^2 = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$$

$$v = \sqrt{\frac{k(A^2 - x^2)}{m}} = \sqrt{\frac{\left(12.3 \frac{\text{N}}{\text{m}}\right)[(0.256 \text{ m})^2 - (0.128 \text{ m})^2]}{0.321 \text{ kg}}} = \boxed{1.37 \text{ m/s}}$$

38. $E = \frac{1}{2}kA^2 = \frac{1}{2}\left(12.3 \frac{\text{N}}{\text{m}}\right)(0.256 \text{ m})^2 = \boxed{0.403 \text{ J}}$

39. $T = 2\pi\sqrt{\frac{m}{k}} = \frac{1}{f}$

$$k = (2\pi f)^2 m$$

$$E = \frac{1}{2}kA^2 = 2(\pi f)^2 mA^2 = 2(2.6\pi \text{ Hz})^2(1.2 \text{ kg})(0.078 \text{ m})^2 = \boxed{0.97 \text{ J}}$$

40. (a) v is governed by $\frac{1}{2}mv^2 = K$, and $K + U = E$. So, write $K = E - U$ and then obtain v.

(b)
$$K = E - U$$
$$\frac{1}{2}mv^2 = \frac{1}{2}kA^2 - \frac{1}{2}k\left(\frac{A}{2}\right)^2$$
$$v = \sqrt{\frac{k\left[A^2 - \left(\frac{A}{2}\right)^2\right]}{m}}$$
$$= \sqrt{\frac{\left(26\ \frac{\text{N}}{\text{m}}\right)[(0.032\text{ m})^2 - (0.016\text{ m})^2]}{0.40\text{ kg}}}$$
$$= \boxed{0.22\text{ m/s}}$$

41. (a)
$$K_{\text{max}} = U_{\text{max}}$$
$$\frac{1}{2}m{v_{\text{max}}}^2 = \frac{1}{2}kA^2$$
$$v_{\text{max}} = \sqrt{\frac{k}{m}A^2}$$
$$= \sqrt{\left(\frac{26\ \frac{\text{N}}{\text{m}}}{0.40\text{ kg}}\right)(0.032\text{ m})^2}$$
$$= \boxed{0.26\text{ m/s}}$$

(b)
$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$
$$x = \sqrt{A^2 - \frac{mv^2}{k}}$$
$$= \sqrt{A^2 - \frac{m\left(\frac{1}{2}v_{\text{max}}\right)^2}{k}}$$
$$= \sqrt{(0.032\text{ m})^2 - \frac{(0.40\text{ kg})\left(0.129\ \frac{\text{m}}{\text{s}}\right)^2}{26\ \frac{\text{N}}{\text{m}}}}$$
$$= \boxed{2.8\text{ cm}}$$

42. (a)
$$T = 2\pi\sqrt{\frac{m}{k}}$$
$$m = \left(\frac{T}{2\pi}\right)^2 k$$
$$= \left(\frac{0.45\text{ s}}{2\pi}\right)^2\left(650\ \frac{\text{N}}{\text{m}}\right)$$
$$= \boxed{3.3\text{ kg}}$$

(b) $W = mg = (3.33\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) = \boxed{33\text{ N}}$

43. $K_{\text{max}} = U_{\text{max}}$, and $T = 2\pi\sqrt{\frac{m}{k}}$, so $\sqrt{\frac{k}{m}} = \frac{2\pi}{T}$.

$$\frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kA^2$$

$$v_{\text{max}} = A\sqrt{\frac{k}{m}} = A\frac{2\pi}{T} = (0.023\text{ m})\frac{2\pi}{0.45\text{ s}} = \boxed{0.32\text{ m/s}}$$

44. (a)

$$K_{\text{i}} = U_{\text{f}}$$

$$\frac{1}{2}mv_{\text{i}}^2 = \frac{1}{2}kA^2$$

$$k = m\left(\frac{v_{\text{i}}}{A}\right)^2$$

$$= (0.540\text{ kg})\left(\frac{1.13\ \frac{\text{m}}{\text{s}}}{0.25\text{ m}}\right)^2$$

$$= \boxed{11\text{ N/m}}$$

(b) The time in question is one-quarter period.

$$\frac{1}{4}T = \frac{\pi}{2}\sqrt{\frac{m}{k}} = \frac{\pi}{2}\sqrt{\frac{0.540\text{ kg}}{11.03\ \frac{\text{N}}{\text{m}}}} = \boxed{0.35\text{ s}}$$

(c) It $\boxed{\text{decreases}}$. A greater force constant means a stiffer spring and a greater stopping force.

45. Let $v_0 =$ the bullet speed and $v =$ the speed of the block and the bullet at the moment after impact.

$$K_{\text{i}} = U_{\text{f}}$$

$$\frac{1}{2}(M+m)v^2 = \frac{1}{2}kA^2$$

$$v = \sqrt{\frac{kA^2}{M+m}}$$

$$= \sqrt{\frac{\left(36.0\ \frac{\text{N}}{\text{m}}\right)(0.0150\text{ m})^2}{0.500\text{ kg} + 0.0100\text{ kg}}}$$

$$= 0.1260\ \frac{\text{m}}{\text{s}}$$

(a) By conservation of momentum,

$$mv_0 = (M+m)v$$

$$v_0 = \left(\frac{M+m}{m}\right)v$$

$$= \left(\frac{0.500\text{ kg} + 0.0100\text{ kg}}{0.0100\text{ kg}}\right)\left(0.1260\ \frac{\text{m}}{\text{s}}\right)$$

$$= \boxed{6.43\ \text{m/s}}$$

(b) Time to rest is one quarter-period:

$$\frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{M+m}{k}} = \frac{\pi}{2}\sqrt{\frac{0.510\text{ kg}}{36.0\ \frac{\text{N}}{\text{m}}}} = \boxed{0.187\text{ s}}$$

46. $T = 2\pi\sqrt{\frac{L}{g}}$

$$L = \left(\frac{T}{2\pi}\right)^2 g$$

$$= \left(\frac{\frac{60.0\text{ s}}{10}}{2\pi}\right)^2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)$$

$$= \boxed{8.95\text{ m}}$$

47. $T = 2\pi\sqrt{\frac{L}{g}}$

$$g = \left(\frac{2\pi}{T}\right)^2 L$$

$$= \left(\frac{2\pi}{\frac{16\text{ s}}{5.0}}\right)(2.5\text{ m})$$

$$= \boxed{9.6\ \text{m/s}^2}$$

48. The time is one quarter-period:

$$\frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{L}{g}} = \frac{\pi}{2}\sqrt{\frac{10.0\text{ m}}{9.81\ \frac{\text{m}}{\text{s}^2}}} = \boxed{1.59\text{ s}}$$

49. $T = 2\pi\sqrt{\frac{L}{g}}$

$$L = \left(\frac{T}{2\pi}\right)^2 g$$

$$= \left(\frac{1.00\text{ s}}{2\pi}\right)^2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)$$

$$= \boxed{24.8\text{ cm}}$$

50. (a) Since $T = 2\pi\sqrt{\frac{L}{g}}$, cutting g by a factor of $\frac{1}{6}$ means increasing T by a factor of $\sqrt{6}$. The period would $\boxed{\text{increase}}$.

(b) $T_\text{M} = 2\pi\sqrt{\frac{L}{\frac{g}{6}}} = \sqrt{6}\cdot 2\pi\sqrt{\frac{L}{g}} = \sqrt{6}\cdot T_\text{E} = \sqrt{6}(1.00\text{ s}) = \boxed{2.45\text{ s}}$

51. $T = 2\pi\sqrt{\frac{R}{g}}\sqrt{\frac{I}{mR^2}} = 2\pi\sqrt{\frac{R}{g}}\sqrt{\frac{2mR^2}{mR^2}} = \boxed{2\sqrt{2}\pi\sqrt{\frac{R}{g}}}$

52. Treat the hat as a pendulum.

$$T = 2\pi\sqrt{\frac{\ell}{g}}\sqrt{\frac{I}{m\ell^2}}$$

$$T^2 = \frac{4\pi^2\ell I}{gm\ell^2}$$

$$I = \frac{mg\ell T^2}{4\pi^2} = \frac{(0.88\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.082\text{ m})(0.75\text{ s})^2}{4\pi^2} = \boxed{0.010\text{ kg}\cdot\text{m}^2}$$

53. (a) $\boxed{\text{greater than}}$. The center of mass of the meter stick is only 50 cm from the pivot point, so its effective length is shorter than a 1 meter simple pendulum. The "longer" simple pendulum has a greater period.

(b) For a meter stick pivoted at one end, $I = \frac{1}{3}mL^2$.

$$T = 2\pi\sqrt{\frac{\frac{L}{2}}{g}}\left(\sqrt{\frac{\frac{1}{3}mL^2}{m\left(\frac{L}{2}\right)^2}}\right) = 2\pi\sqrt{\frac{\frac{2}{3}L}{g}}$$

which is the period of a simple pendulum $\frac{2}{3}L = \boxed{\frac{2}{3}\text{ m in length}}$.

54. For a bean suspended from one end, $I = \frac{1}{3}mL^2$.

$$T = 2\pi\sqrt{\frac{\frac{L}{2}}{g}}\left(\sqrt{\frac{\frac{1}{3}ml^2}{m\left(\frac{L}{2}\right)^2}}\right) = 2\pi\sqrt{\frac{\frac{2}{3}L}{g}}$$

$$L = \frac{3}{2}\left(\frac{T}{2\pi}\right)^2 g = \frac{3}{2}\left(\frac{2.00\text{ s}}{2\pi}\right)^2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) = \boxed{1.49\text{ m}}$$

55. For a uniform rod suspended at one end, $I = \frac{1}{3}mL^2$.

$$T = 2\pi\sqrt{\frac{\frac{L}{2}}{g}}\left(\sqrt{\frac{\frac{1}{3}mL^2}{m\left(\frac{L}{2}\right)^2}}\right) = 2\pi\sqrt{\frac{\frac{2}{3}L}{g}} = 2\pi\sqrt{\frac{\frac{2}{3}(0.55\text{ m})}{9.81\ \frac{\text{m}}{\text{s}^2}}} = \boxed{1.2\text{ s}}$$

Assuming the leg swings through about 1.0 radian, $d = L\theta = 0.55\text{ m}$ and $v = \frac{d}{\frac{T}{2}} = \frac{0.55\text{ m}}{0.607\text{ s}} = \boxed{0.91\text{ m/s}}$.

56. The pendulum responds to "felt gravity" $g + a$ or $g - a$.

(a) $T = \boxed{2\pi\sqrt{\frac{L}{g+a}}}$

(b) $T = \boxed{2\pi\sqrt{\frac{L}{g-a}}}$

57.

$$K_\text{i} + U_\text{i} = K_\text{f} + U_\text{f}$$

$$\frac{1}{2}mv_\text{i}^2 + \frac{1}{2}kx_\text{i}^2 = \frac{1}{2}mv_\text{f}^2 + \frac{1}{2}kx_\text{f}^2$$

$$mv_\text{f}^2 = mv_\text{i}^2 + k(x_\text{i}^2 - x_\text{f}^2)$$

$$v_\text{f} = \sqrt{v_\text{i}^2 + \frac{k}{m}(x_\text{i}^2 - x_\text{f}^2)}$$

$$= \sqrt{\left(0.25\ \frac{\text{m}}{\text{s}}\right)^2 + \frac{56\ \frac{\text{N}}{\text{m}}}{1.1\text{ kg}}[(0.084\text{ m})^2 - (0.042\text{ m})^2]}$$

$$= \boxed{0.58\text{ m/s}}$$

58. $m = \frac{kT^2}{4\pi^2} = \frac{\left(2600\ \frac{\text{N}}{\text{m}}\right)(0.85\text{ s})^2}{4\pi^2} = \boxed{56\text{ kg}}$

59.

$$a_\text{max} = A\omega^2$$

$$= A(2\pi f)^2$$

$$= (0.10\times10^{-10}\text{ m})(2\pi\times10^{12}\text{ s}^{-1})^2$$

$$= \boxed{3\times10^{14}\text{ m/s}^2}$$

$$= \left[\frac{3\times10^{14}\text{ m/s}^2}{9.81\ \frac{\text{m}}{\text{s}^2}}\right]g$$

$$= \boxed{(3\times10^{13})g}$$

60. $f = \left(\frac{23\text{ cycles}}{(2002-1749)\text{ yr}}\right)\left(\frac{\text{yr}}{3.156\times10^7\text{ s}}\right) = \boxed{2.9\times10^{-9}\text{ Hz}}$

61. (a) $f=\frac{\omega}{2\pi}=\frac{2.41\text{ rad/s}}{2\pi}=\boxed{0.384\text{ Hz}}$

(b) $v_{\text{max}}=A\omega$ and $a_{\text{max}}=A\omega^2$

$v_{\text{max}}=\frac{a_{\text{max}}}{\omega}=\frac{0.302\text{ m/s}^2}{2.41\text{ rad/s}}=\boxed{0.125\text{ m/s}}$

(c) $A=\frac{v_{\text{max}}}{\omega}=\frac{0.1253\text{ m/s}}{2.41\text{ rad/s}}=\boxed{0.0520\text{ m}}$

62. (a) $v_{\text{max}}=A\omega,\ a_{\text{max}}=A\omega^2=\frac{F_{\text{max}}}{m}$

$\omega=\frac{a_{\text{max}}}{v_{\text{max}}}=\frac{F_{\text{max}}}{mv_{\text{max}}}=\frac{13\text{ N}}{(3.1\text{ kg})(0.70\text{ m/s})}=5.99\text{ rad/s}$

$A=\frac{v_{\text{max}}}{\omega}=\frac{0.70\text{ m/s}}{5.99\text{ rad/s}}=\boxed{0.12\text{ m}}$

(b) $\omega=\sqrt{\frac{k}{m}}$

$k=m\omega^2=(3.1\text{ kg})(5.99\text{ rad/s})^2=\boxed{110\text{ N/m}}$

(c) $f=\frac{\omega}{2\pi}=\frac{5.99\text{ rad/s}}{2\pi}=\boxed{0.95\text{ Hz}}$

63. (a) $\boxed{\text{less than}}$. Kinetic energy is lost because the collision is completely inelastic.

(b) Use conservation of energy during the swing.

$\frac{1}{2}(M+m)v^2=(M+m)gh$

$v=\sqrt{2gh}=\sqrt{2(9.81\text{ m/s}^2)(0.124\text{ m})}=1.560\text{ m/s}$

Use conservation of momentum during the collision.

$mv_0=(M+m)v$

$v_0=\frac{(M+m)v}{m}=\frac{(1.45\text{ kg}+0.00950\text{ kg})}{0.00950\text{ kg}}(1.560\text{ m/s})=\boxed{240\text{ m/s}}$

(c) $t=\frac{1}{4}T=\frac{1}{4}(2\pi)\sqrt{\frac{l}{g}}=\frac{\pi}{2}\sqrt{\frac{0.745\text{ m}}{9.81\text{ m/s}^2}}=\boxed{0.433\text{ s}}$

64. $A = 0.900A_0 = A_0 e^{-bt/2m}$

$$0.900 = e^{-bt/2m}$$

$$t = -\frac{2m}{b}(\ln 0.900)$$

$$= -\frac{2(0.00134\text{ kg})}{3.30\times 10^{-5}\ \frac{\text{kg}}{\text{s}}}(\ln 0.900)$$

$$= \boxed{8.56\text{ s}}$$

65. $x = A\cos\left(\frac{2\pi t}{T}\right)$. At $t = 0$, $x = A$. To find t for $x = \frac{A}{2}$, solve

$$\frac{A}{2} = A\cos\left(\frac{2\pi t}{T}\right)$$

$$\cos^{-1}\left(\frac{1}{2}\right) = \frac{2\pi t}{T}$$

$$t = \frac{T}{2\pi}\cos^{-1}\left(\frac{1}{2}\right)$$

$$= \frac{T}{2\pi}\left(\frac{\pi}{3}\right)$$

$$= \boxed{\frac{T}{6}}$$

66. For a disk pivoting about a point at its rim, $I = \frac{3}{2}mr^2$.

$$T = 2\pi\sqrt{\frac{r}{g}}\left(\sqrt{\frac{I}{mr^2}}\right) = 2\pi\sqrt{\frac{r}{g}}\left(\sqrt{\frac{3}{2}}\right) = 2\pi\sqrt{\frac{3}{2}}\left(\sqrt{\frac{0.15\text{ m}}{9.81\ \frac{\text{m}}{\text{s}^2}}}\right) = \boxed{0.95\text{ s}}$$

67. (a) The period $\boxed{\text{increases}}$.

(b) $I = \frac{3}{2}mr^2 + m_p(2r)^2 = \frac{3}{2}(0.32\text{ kg})(0.15\text{ m})^2 + (0.0085\text{ kg})(0.30\text{ m})^2 = 0.011565\text{ kg}\cdot\text{m}^2$

$$\ell = \frac{mr + m_p(2r)}{m + m_p} = \frac{(0.32\text{ kg})(0.15\text{ m}) + (0.0085\text{ kg})(0.30\text{ m})}{0.32\text{ kg} + 0.0085\text{ kg}} = 0.15388\text{ m}$$

$$T = 2\pi\sqrt{\frac{\ell}{g}}\left(\sqrt{\frac{I}{m\ell^2}}\right) = 2\pi\sqrt{\frac{0.15388\text{ m}}{9.81\ \frac{\text{m}}{\text{s}^2}}}\left(\sqrt{\frac{0.011565\text{ kg}\cdot\text{m}^2}{(0.3285\text{ kg})(0.15388\text{ m})^2}}\right) = \boxed{0.96\text{ s}}$$

68. For the hanging mass, $ky_0 = mg$ and so $\frac{m}{k} = \frac{y_0}{g}$.

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{y_0}{g}} = 2\pi\sqrt{\frac{0.12\text{ m}}{9.81\ \frac{\text{m}}{\text{s}}}} = \boxed{0.69\text{ s}}$$

For the pendulum,

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{y_0}{g}} = \boxed{0.69\text{ s}}$$

Since $y_0 = L$, $\boxed{\text{the periods are the same.}}$

69. $E = \frac{1}{2}kA^2$

$$U = \frac{1}{2}k\left(\frac{A}{2}\right)^2 = \frac{1}{4}\left(\frac{1}{2}kA^2\right)$$

$$K = E - U = \frac{3}{4}\left(\frac{1}{2}kA^2\right)$$

$$\frac{K}{U} = \frac{\frac{3}{8}kA^2}{\frac{1}{8}kA^2} = \boxed{3}$$

70. (a)

$$K_\text{i} = U_\text{f}$$

$$\frac{1}{2}mv^2 = \frac{1}{2}kA^2$$

$$A = \sqrt{\frac{mv^2}{k}} = \sqrt{\frac{(0.30\text{ kg})\left(1.34\ \frac{\text{m}}{\text{s}}\right)^2}{45\ \frac{\text{N}}{\text{m}}}} = \boxed{0.11\text{ m}}$$

(b) It takes one quarter-period.

$$\frac{1}{4}T = \frac{\pi}{2}\sqrt{\frac{m}{k}} = \frac{\pi}{2}\sqrt{\frac{0.30\text{ kg}}{45\ \frac{\text{N}}{\text{m}}}} = \boxed{0.13\text{ s}}$$

71.

$$A = A_0e^{-bt/2m}$$

$$\ln\left(\frac{A}{A_0}\right) = -\frac{bt}{2m}$$

$$b = \left(\frac{2m}{t}\right)\ln\left(\frac{A_0}{A}\right) = \left[\frac{2(2.44\cdot10^5\text{ kg})}{3.0\times10^2\text{ s}}\right]\ln 2 = \boxed{1100\text{ kg/s}}$$

72. **(a)** $\boxed{\text{greater than}}$. $v_{max} = A\omega,\ \omega = \frac{2\pi}{T} = \frac{2\pi}{4.0\text{ s}} > 1$

(b) $T = 4.0\text{ s}$

$$\omega = \frac{2\pi}{T} = \frac{\pi}{2}\text{ s}^{-1}$$

$$v_{max} = A\omega = (0.5\text{ m})\left(\frac{\pi}{2}\text{ s}^{-1}\right) = \frac{\pi}{4}\frac{\text{m}}{\text{s}} \approx \boxed{0.79\text{ m/s}}$$

(c) $E = \frac{1}{2}mv_{max}^2 = \frac{1}{2}(3.0\text{ kg})\left(\frac{\pi}{4}\frac{\text{m}}{\text{s}}\right)^2 = \boxed{0.93\text{ J}}$

73. **(a)** $t = \boxed{1.0\text{ s, }3.0\text{ s, }5.0\text{ s}}$. The force is maximum when the displacement is maximum.

(b) $F_{max} = ma_{max} = mA\omega^2 = mA\left(\frac{2\pi}{T}\right)^2 = (3.0\text{ kg})(0.50\text{ m})\left(\frac{2\pi}{4.0\text{ s}}\right)^2 = \boxed{3.7\text{ N}}$

(c) $t = \boxed{0,\ 2.0\text{ s, }4.0\text{ s, }6.0\text{ s}}$ The force is zero when the displacement is zero.

(d) At $t = 0.5$ s, $x = A\sin(\omega t) = A\sin\left(\frac{2\pi t}{T}\right)$

$$x = A\sin\left(\frac{2\pi\times 0.5\text{ s}}{4.0\text{ s}}\right) = A\sin\left(\frac{\pi}{4}\right) = 0.707A$$

$$F = kx = 0.707kA = 0.707F_{max} = 0.707(3.7\text{ N}) = \boxed{2.6\text{ N}}$$

74. **(a)** $f = \frac{N}{t} = \frac{T-39}{t} = \frac{68-39}{13\text{ s}} = \boxed{2.2\text{ Hz}}$

(b) At 75°F $f_{75} = \frac{75-39}{13\text{ s}} = 2.77\text{ Hz}$

At 62°F $f_{62} = \frac{62-39}{13\text{ s}} = 1.77\text{ Hz}$

Average frequency is $\frac{2.77\text{ Hz}+1.77\text{ Hz}}{2} = 2.27\text{ Hz}$

Number of chirps in 12 min $= (2.27\text{ Hz})(12\text{ min})\left(\frac{60\text{ s}}{\text{min}}\right) = \boxed{1600}$

75. **(a)** $T = 2\pi\sqrt{\frac{m}{k}}$

$$k = \left(\frac{2\pi}{T}\right)^2 m$$
$$= \left(\frac{2\pi}{1.1\text{ s}}\right)^2 (0.86\text{ kg})$$
$$= \boxed{28\text{ N/m}}$$

(b) $m = \left(\frac{T}{2\pi}\right)^2 k = \left(\frac{0.52\text{ s}}{2\pi}\right)^2 \left(28\ \frac{\text{N}}{\text{m}}\right) = \boxed{0.19\text{ kg}}$

76. The remaining mass has, in effect, been released with an "initial" displacement *A*, and that is the amplitude of the resulting motion.

77. **(a)** The period is less than without the peg, since half the cycle is speeded up due to the peg's shortening of the pendulum.

(b) The period equals half the period of the shortened pendulum plus half the period of the long pendulum:

$$T = \boxed{\pi\sqrt{\frac{\ell}{g}} + \pi\sqrt{\frac{L}{g}}}$$

(c) $T = \pi\left(\sqrt{\frac{0.25\text{ m}}{9.81\ \frac{\text{m}}{\text{s}^2}}} + \sqrt{\frac{1.0\text{ m}}{9.81\ \frac{\text{m}}{\text{s}^2}}}\right) = \boxed{1.5\text{ s}}$

78. The center of gravity is located

$$\ell = \frac{\left(\frac{L}{2}\right)m_1 + \left(-\frac{L}{2}\right)m_2}{m_1 + m_2} = \frac{L(m_1 - m_2)}{2(m_1 + m_2)}$$

units below the middle, and $I = (m_1 + m_2)\left(\frac{L}{2}\right)^2$.

So,

$$\sqrt{\frac{I}{m\ell^2}} = \sqrt{\frac{(m_1 + m_2)\left(\frac{L}{2}\right)^2}{(m_1 + m_2)\left(\frac{L}{2}\right)^2 \frac{(m_1 - m_2)^2}{(m_1 + m_2)^2}}} = \frac{m_1 + m_2}{m_1 - m_2}$$

and

$$T = 2\pi\sqrt{\frac{\ell}{g}}\left(\sqrt{\frac{I}{m\ell^2}}\right) = 2\pi\sqrt{\frac{L(m_1 - m_2)}{2g(m_1 + m_2)}}\left(\frac{m_1 + m_2}{m_1 - m_2}\right) = \boxed{\pi\sqrt{\frac{2L(m_1 + m_2)}{g(m_1 - m_2)}}}$$

79. **(a)** The pencil begins to rattle when the maximum acceleration of the speaker begins to exceed the acceleration due to gravity.

(b)
$$a_{\text{max}} = g$$
$$A\omega^2 = g$$
$$A(2\pi f)^2 = g$$
$$f = \boxed{\frac{1}{2\pi}\sqrt{\frac{g}{A}}}$$

80. For the hanging mass, $ky_0 = mg$, so $\frac{m}{k} = \frac{y_0}{g}$.

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{y_0}{g}} = \boxed{2\pi\sqrt{\frac{L}{g}}}$$

For the pendulum, $T = \boxed{2\pi\sqrt{\frac{L}{g}}}$.

81.
$$K + U = E$$
$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$
$$mv^2 = k(A^2 - x^2)$$
$$v = \sqrt{\frac{k}{m}(A^2 - x^2)}$$

Since $\omega = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$, $v = \omega\sqrt{A^2 - x^2}$.