Chapter 20

Electric Potential and Electric Potential Energy

Answers to Even-numbered Conceptual Questions

2. The electric potential energy of the system decreases. In fact, it is converted into the kinetic energy gained by the electron.

4. The electric potential energy of the system decreases. In fact, the kinetic energy gained by the proton is equal to the decrease in the electric potential energy.

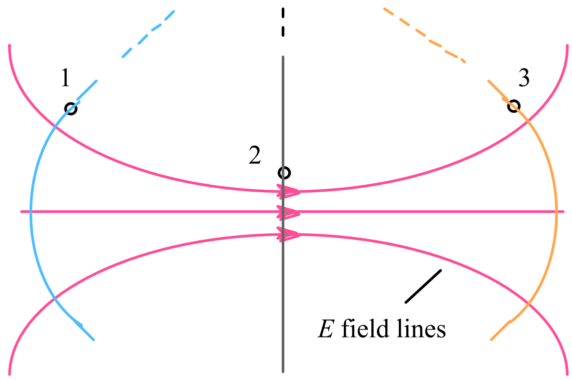
6. The two like charges, if released, will move away from one another to infinite separation, converting the positive electric potential energy into kinetic energy. The two unlike charges, however, attract one another – if their separation is to be increased, a positive work must be done. In fact, the minimum amount of work that must be done to create an infinite separation between the charges is equal to the magnitude of the original negative electric potential energy.

8. The electric potential energy of the proton-electron system is negative, as required to produce a bound atom. **(a)** If the electron is replaced with a proton, the two protons will repel one another and move off to infinite separation. This means that the initial electric potential energy of this system is positive, and that this positive potential energy is converted to kinetic energy. Therefore, the electric potential energy of the system increased by changing the electron to a proton. **(b)** If the proton is replaced with an electron the electric potential energy of the system increases, for exactly the same reason given in part (a).

10. Point 2 is closer to q_2 than to q_1 by a factor of the square root of 2. Therefore, if the electric potential is to be zero at point 2, it is necessary that q_2 be negative, and have a magnitude that is less than the magnitude of q_1 by a factor of $\sqrt{2}$. We conclude, then, that $q_2 = -Q/\sqrt{2}$. Notice that point 1 is closer to the positive charge +Q than to the negative charge $-Q/\sqrt{2}$, and that the negative charge has a smaller magnitude. It follows that the electric potential at point 1 is positive.

12. Not necessarily. The electric field is related to the rate of change of electric potential, not to its value. Therefore, if the electric field is zero in some region of space, it follows that the electric potential is constant in that region. The constant value of the electric potential may be zero, but it may also be positive or negative.

14. An equipotential surface must always cross an electric field line at right angles. Therefore, the equipotential surfaces in this system must have the shapes indicated here:



16. We know that the value of the electric potential must decrease as we move in the direction of the electric field (see, for example, Figure 20-3.) It follows that the electric potential decreases in value as we move from point 1 to point 2 to point 3.

18. The electric field is either zero on this surface, or it is nonzero and perpendicular to the surface. If the electric field had a nonzero component parallel to the surface, the electric potential would decrease as one moved in the direction of the parallel component.

20. Capacitors store electrical energy in the form of an electric field between their plates. This stored energy can remain in a capacitor even when a device such as a television is turned off. Accidentally touching the terminals can cause the sudden and potentially dangerous release of this energy.

22. A capacitor stores charge of opposite sign in two different locations – though, of course, the net charge is zero. We can think of a capacitor, then, as storing a "charge separation", along with the energy required to cause the charge separation in the first place.

24. **(a)** The electric field, which is related to how much the electric potential changes with distance, is decreased by the increased plate separation. **(b)** Since the electric field is decreased, the amount of charge on the plates must decrease as well. After all, it is the charge on the plates that is responsible for creating the electric field. **(c)** The capacitance of the capacitor decreases. This follows from parts (a) and (b); after all, if a capacitor stores less charge for the same potential difference, it has a smaller capacitance. **(d)** Suppose the plate separation is doubled. In this case, the electric field is reduced by a factor of two and the energy density is reduced by a factor of $2^2 = 4$. The volume of the capacitor is only doubled, however. Therefore, the total energy in the capacitor decreases by a factor of 2. In general, the energy stored in a capacitor decreases as its plate separation is increased at constant potential.

26. No. As an example, note that the volume of a milk container is not zero just because the container happens to be empty of milk. The same can be said about the capacitance of a capacitor that happens to be uncharged.

28. **(a)** The electric field between the plates remains the same, because both the potential difference and the plate separation are the same. It follows that the rate of change of potential with distance (the magnitude of the electric field) is unchanged. **(b)** There is more charge on the plates now, because more charge is required to produce the same electric field in the presence of the polarizing effect of the dielectric. **(c)** The capacitor stores more charge for the same potential difference; therefore, its capacitance is increased. **(d)** Recall that the energy stored in a capacitor with the potential difference V between its plates is $U = \frac{1}{2}QV = \frac{1}{2}CV^2$. We know that the dielectric increases the charge [part (b)] and also increases the capacitance [part (c)]. It follows that the stored energy is increased as well.

Solutions to Problems

1. $\Delta U = q_0 Ed$

(a) $\Delta U = q_0 E(0) = \boxed{0}$

(b) $\Delta U = -q_0 Ed = -(4.5\times10^{-6}\text{ C})\left(4.1\times10^5\ \frac{\text{N}}{\text{C}}\right)(6.0\text{ m}) = \boxed{-11\text{ J}}$

(c) $\Delta U = -q_0 Ed = -(4.5\times10^{-6}\text{ C})\left(4.1\times10^5\ \frac{\text{N}}{\text{C}}\right)(6.0\text{ m}) = \boxed{-11\text{ J}}$

2. $\Delta V = Ed$

(a) $\Delta V = E(0) = \boxed{0}$

(b) $\Delta V = -Ed = -\left(6.1\times10^5\ \frac{\text{N}}{\text{C}}\right)(6.0\text{ m}) = \boxed{-3.7\times10^6\text{ V}}$

(c) $\Delta V = -Ed = -\left(6.1\times10^5\ \frac{\text{N}}{\text{C}}\right)(6.0\text{ m}) = \boxed{-3.7\times10^6\text{ V}}$

3. $E = \frac{\Delta V}{d} = \frac{0.070\text{ V}}{0.10\times10^{-6}\text{ m}} = 7.0\times10^5\text{ V/m}$

The electric field points from higher to lower potential, so the electric field is $\boxed{7.0\times10^5\text{ V/m directed inward}}$ toward the cell.

4. $E = \frac{\Delta V}{d} = \frac{25{,}000\text{ V}}{0.012\text{ m}} = \boxed{2.1\times10^6\text{ V/m}}$

5. $\Delta U = -e\Delta V = -e(25{,}000\text{ V}) = \boxed{-25{,}000\text{ eV}}$

6. **(a)** $\Delta V = Ed = \left(1.2\times10^5\ \frac{\text{V}}{\text{m}}\right)(0.75\times10^{-3}\text{ m}) = \boxed{90\text{ V}}$

(b) $\Delta V = \left(2.4\times10^4\ \frac{\text{N}}{\text{C}}\right)(0.75\times10^{-3}\text{ m}) = \boxed{18\text{ V}}$

7. $q = \frac{\Delta U}{\Delta V} = \frac{-1.37\times10^{-15}\text{ J}}{2850\text{ V}} = -4.81\times10^{-19}\text{ C} = \boxed{-3.00e}$

8. $\Delta V = Ed = \left(100\ \frac{\text{V}}{\text{m}}\right)(555\text{ ft})\left(\frac{1\text{ m}}{3.281\text{ ft}}\right) = \boxed{2\times10^4\text{ V}}$

9. (a) $\Delta U = -qEd = -(12.5\times10^{-6}\text{ C})\left(6250\frac{\text{N}}{\text{C}}\right)(0.0550\text{ m}) = \boxed{-4.30\text{ mJ}}$

(b) $\Delta U = qEd = (12.5\times10^{-6}\text{ C})\left(6250\frac{\text{N}}{\text{C}}\right)(0.0550\text{ m}) = \boxed{4.30\text{ mJ}}$

(c) $\Delta U = qE(0) = \boxed{0}$

10. (a) $\Delta V = Ed = \left(3.0\times10^6\ \frac{\text{V}}{\text{m}}\right)(0.025\text{ in.})\left(\frac{0.0254\text{ m}}{1\text{ in.}}\right) = \boxed{1.9\text{ kV}}$

(b) The electric potential difference varies directly with the separation of the electrodes, so the required potential difference $\boxed{\text{increases}}$.

(c) $\Delta V = Ed = \left(3.0\times10^6\ \frac{\text{V}}{\text{m}}\right)(0.050\text{ in.})\left(\frac{0.0254\text{ m}}{1\text{ in.}}\right) = \boxed{3.8\text{ kV}}$

11. (a) $\Delta V = V_B - V_A = \boxed{0}$ because $V_B = V_A$.

(b) $\Delta V = V_B - V_C = -Ed = -\left(1200\frac{\text{N}}{\text{C}}\right)(0.040\text{ m}) = \boxed{-48\text{ V}}$

(c) $\Delta V = V_C - V_A = Ed = \left(1200\frac{\text{N}}{\text{C}}\right)(0.040\text{ m}) = \boxed{48\text{ V}}$

(d) $\boxed{\text{No}}$, it is not possible to determine V_A. We only know the potential differences. We need to know either V_B or V_C to determine V_A.

12.

$$\Delta K = -\Delta U$$
$$\frac{1}{2}mv^2 = q\Delta V$$
$$v = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{2(7.5\times10^5\text{ C})(12\text{ V})}{1200\text{ kg}}} = \boxed{120\text{ m/s}}$$

13. (a) $W = q\Delta V = e\Delta V = (1.60\times10^{-19}\text{ C})(0.070\text{ V}) = \boxed{1.1\times10^{-20}\text{ J}}$

(b) The work required to "pump" a Na^+ ion out of the cell is independent of the thickness of the cell wall, so the answer to part (a) $\boxed{\text{stays the same}}$.

14. (a) Since E equals the opposite of the slope, E has its greatest value and its greatest magnitude in $\boxed{\text{region 4}}$, where the slope has its greatest negative value.

(b) region 1:

$$E = -\frac{\Delta V}{\Delta s} = -\frac{(6.0\text{ V} - 8.0\text{ V})}{0.15\text{ m} - 0.00\text{ m}} = \boxed{13\text{ V/m}}$$

region 2:

$$E = -\frac{(6.0\text{ V} - 6.0\text{ V})}{0.25\text{ m} - 0.15\text{ m}} = \boxed{0}$$

region 3:

$$E = -\frac{(7.8\text{ V} - 6.0\text{ V})}{0.60\text{ m} - 0.25\text{ m}} = \boxed{-5.1\text{ V/m}}$$

region 4:

$$E = -\frac{(1.0\text{ V} - 7.8\text{ V})}{0.70\text{ m} - 0.60\text{ m}} = \boxed{68\text{ V/m}}$$

15. (a) $\Delta K = -\Delta U = -(U_\text{C} - U_\text{A}) = K_\text{A}$

$$U_\text{A} - U_\text{C} = K_\text{A}$$
$$U_\text{C} = U_\text{A} - K_\text{A}$$
$$\Delta K = -(U_\text{C} - U_\text{B}) = 2K_\text{A}$$
$$U_\text{B} - U_\text{C} = 2K_\text{A}$$
$$U_\text{C} = U_\text{B} - 2K_\text{A}$$

So, $U_\text{B} - 2K_\text{A} = U_\text{A} - K_\text{A}$.

Find K_A.

$$U_\text{B} - 2K_\text{A} = U_\text{A} - K_\text{A}$$
$$U_\text{B} - U_\text{A} = K_\text{A}$$

Find V_C.

$$\begin{aligned} U_\text{C} &= U_\text{A} - K_\text{A} \\ &= U_\text{A} - (U_\text{B} - U_\text{A}) \\ &= 2U_\text{A} - U_\text{B} \\ V_\text{C} &= 2V_\text{A} - V_\text{B} \\ &= 2(352\text{ V}) - 129\text{ V} \\ &= \boxed{575\text{ V}} \end{aligned}$$

(b) $K_\text{A} = U_\text{B} - U_\text{A} = -eV_\text{B} - (-eV_\text{A}) = e(V_\text{A} - V_\text{B}) = (1.60\times10^{-19}\text{ C})(352\text{ V} - 129\text{ V}) = \boxed{3.57\times10^{-17}\text{ J}}$

16. $\frac{1}{2}mv^2 = e\Delta V$

$$v = \sqrt{\frac{2e\Delta V}{m}}$$

(a) $v = \sqrt{\dfrac{2(1.60\times10^{-19}\text{ C})(190\text{ V})}{1.673\times10^{-27}\text{ kg}}} = \boxed{1.9\times10^5\text{ m/s}}$

(b) $v=\sqrt{\dfrac{2(1.60\times10^{-19}\text{ C})(190\text{ V})}{9.11\times10^{-31}\text{ kg}}}=\boxed{8.2\times10^{6}\text{ m/s}}$

17. $\frac{1}{2}mv^2=e\Delta V$

$v=\sqrt{\dfrac{2e\Delta V}{m}}=\sqrt{\dfrac{2(1.60\times10^{-19}\text{ C})(25{,}000\text{ V})}{9.11\times10^{-31}\text{ kg}}}=\boxed{9.4\times10^{7}\text{ m/s}}$

Note that relativistic effects have been ignored.

18. $\Delta V=\dfrac{mv^2}{2e}=\dfrac{m(0.1c)^2}{2e}=\dfrac{(1.67\times10^{-27}\text{ kg})(0.1)^2\left(3\times10^{8}\text{ }\frac{\text{m}}{\text{s}}\right)^2}{2(1.6\times10^{-19}\text{ C})}=\boxed{5\text{ MV}}$

19. (a) The particle has a positive charge, so it will move in the direction of the electric field, which is the $\boxed{\text{negative }x\text{-direction}}$.

(b) $\frac{1}{2}mv^2=-q\Delta V=qEd$

$v=\sqrt{\dfrac{2qEd}{m}}=\sqrt{\dfrac{2(0.045\times10^{-6}\text{ C})\left(1200\text{ }\frac{\text{N}}{\text{C}}\right)(0.050\text{ m})}{0.0035\text{ kg}}}=\boxed{3.9\text{ cm/s}}$

(c) Its increase in speed will be $\boxed{\text{less than}}$ its increase in speed in the first 5.0 cm because *v* is proportional to the square root of the distance traveled.

20. (a)

$$\frac{1}{2}mv_\text{i}^2+eV_\text{i}=\frac{1}{2}mv_\text{f}^2+eV_\text{f}$$

$$\frac{1}{2}mv_\text{i}^2-\frac{1}{2}mv_\text{f}^2=e(V_\text{f}-V_\text{i})$$

$$\begin{aligned}\Delta V&=\frac{mv_\text{i}^2-mv_\text{f}^2}{2e}\\&=\frac{(1.673\times10^{-27}\text{ kg})\left(4.0\times10^{5}\text{ }\frac{\text{m}}{\text{s}}\right)^2-0}{2(1.60\times10^{-19}\text{ C})}\\&=\boxed{840\text{ V}}\end{aligned}$$

(b)

$$\begin{aligned}\Delta V&=\frac{m}{2e}\left[v_\text{i}^2-\left(\frac{v_\text{i}}{2}\right)^2\right]\\&=\frac{mv_\text{i}^2}{2e}\left(1-\frac{1}{4}\right)\\&=\frac{3(1.673\times10^{-27}\text{ kg})\left(4.0\times10^{5}\text{ }\frac{\text{m}}{\text{s}}\right)^2}{8(1.60\times10^{-19}\text{ C})}\\&=\boxed{630\text{ V}}\end{aligned}$$

(c)
$$K_\text{i} + eV_\text{i} = K_\text{f} + eV_\text{f}$$
$$K_\text{i} - K_\text{f} = e(V_\text{f} - V_\text{i})$$
$$\Delta V = \frac{K_\text{i} - K_\text{f}}{e} = \frac{K_\text{i} - \frac{K_\text{i}}{2}}{e} = \frac{K_\text{i}}{2e} = \frac{mv_\text{i}^2}{4e} = \frac{(1.673\times10^{-27}\text{ kg})\left(4.0\times10^5\ \frac{\text{m}}{\text{s}}\right)^2}{4(1.60\times10^{-19}\text{ C})} = \boxed{420\text{ V}}$$

21. $V = \dfrac{kq}{r}$

$$q = \frac{rV}{k} = \frac{(1.5\text{ m})(2.8\times10^4\text{ V})}{8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}} = \boxed{4.7\ \mu\text{C}}$$

22. (a) $$V = \frac{kq}{r} = \frac{\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(-7.2\times10^{-6}\text{ C})}{3.0\text{ m}} = \boxed{-2.2\times10^4\text{ V}}$$

(b) $$V = \frac{kq}{r} = \frac{\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(-7.2\times10^{-6}\text{ C})}{3.0\text{ m}} = \boxed{-2.2\times10^4\text{ V}}$$

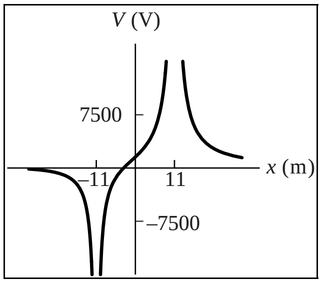
(c) $$V = \frac{kq}{r} = \frac{\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(-7.2\times10^{-6}\text{ C})}{\sqrt{(3.0\text{ m})^2 + (-3.0\text{ m})^2}} = \boxed{-1.5\times10^4\text{ V}}$$

23. $$V = \frac{kq}{r} = \frac{\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(1.60\times10^{-19}\text{ C})}{0.529\times10^{-10}\text{ m}} = \boxed{27.2\text{ V}}$$

24. $U = \dfrac{kq_1q_2}{r}$

$$r = \frac{kq_1q_2}{U} = \frac{\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(7.22\times10^{-6}\text{ C})(-26.1\times10^{-6}\text{ C})}{-126\text{ J}} = \boxed{1.34\text{ cm}}$$

25. (a)



(b) Since the magnitude of the positive charge is larger than the magnitude of the negative charge, the point at which the potential vanishes is closer to the negative charge.

(c) The point between the charges at which the potential vanishes is in the domain $-11\text{ m} < x < 0$. On this domain, the potential is given by

$$V = \frac{kq_1}{11\text{m} - x} + \frac{kq_2}{x + 11\text{m}}.$$

Set $V = 0$.

$$\frac{kq_1}{x - 11\text{m}} = \frac{kq_2}{x + 11\text{m}}$$

$$\frac{x - 11\text{m}}{q_1} = \frac{x + 11\text{m}}{q_2}$$

$$x\left(1 - \frac{q_1}{q_2}\right) = 11\text{m} + \frac{q_1}{q_2}(11\text{m})$$

$$x = \frac{11\text{m} + \frac{4.1\,\mu\text{C}}{-2.2\,\mu\text{C}}(11\text{m})}{1 - \frac{4.1\,\mu\text{C}}{-2.2\,\mu\text{C}}}$$

$$= -3\text{ m}$$

The point is $x = -3$ m.

26. (a) The point to the left of the negative charge at which the potential vanishes is in the domain $-\infty < x < -11$ m. On this domain, the potential is given by

$$V = \frac{kq_1}{11\text{m} - x} + \frac{kq_2}{-x - 11\text{m}}.$$

Set $V = 0$.

$$\frac{kq_1}{11\text{m} - x} = \frac{kq_2}{x + 11\text{m}}$$

$$\frac{11\text{m} - x}{q_1} = \frac{x + 11\text{m}}{q_2}$$

$$x\left(1 + \frac{q_2}{q_1}\right) = (11\text{m})\left(\frac{q_2}{q_1} - 1\right)$$

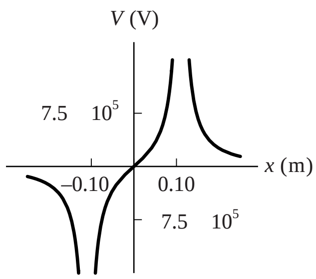
$$x = \frac{(11\text{m})\left(\frac{-2.2\,\mu\text{C}}{4.1\,\mu\text{C}} - 1\right)}{1 + \frac{-2.2\,\mu\text{C}}{4.1\,\mu\text{C}}}$$

$$= -36\text{ m}$$

The point is $x = -36$ m.

(b) The electric field at the point $x = -36$ m is $\boxed{\text{positive}}$. Why? Looking at the graph of $V(x)$ at $x = -36$ m, we see that the slope is negative. Since the electric field is equal to the negative slope of $V(x)$, and since the slope is negative, the electric field is positive.

27. (a) $V = kq\left(\frac{1}{|x - 0.10\text{ m}|} - \frac{1}{|x + 0.10\text{ m}|}\right)$



(b) For $x > 0.10$ m:

$$V = kq\left(\frac{1}{x - 0.10\text{ m}} - \frac{1}{x + 0.10\text{ m}}\right)$$

$$\frac{V}{kq} = \frac{0.20\text{ m}}{x^2 - (0.10\text{ m})^2}$$

$$x = \sqrt{\frac{(0.20\text{ m})kq}{V} + (0.10\text{ m})^2}$$

$$= \sqrt{\frac{(0.20\text{ m})\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(3.6\times10^{-6}\text{ C})}{1.3\times10^4\text{ V}} + (0.10\text{ m})^2}$$

$$= 0.71\text{ m}$$

For $-0.10\text{ m} < x < 0.10$ m:

$$V = kq\left(\frac{1}{0.10\text{ m} - x} - \frac{1}{x + 0.10\text{ m}}\right)$$

$$-\frac{V}{kq} = \frac{2x}{x^2 - (0.10\text{ m})^2}$$

$$-\frac{2kq}{V}x = x^2 - (0.10\text{ m})^2$$

$$0 = x^2 + \frac{2kq}{V}x - (0.10\text{m})^2$$

Use the quadratic formula to solve for x.

$$x = -\frac{kq}{V} \pm \frac{1}{2}\sqrt{\left(\frac{2kq}{V}\right)^2 - 4(1)[-(0.10\text{m})^2]}$$

$$= 0.002\text{ m or } -5.0\text{ m}$$

$x = -5.0$ m is extraneous because it does not satisfy $-0.10\text{ m} < x < 0.10$ m.

For $x < -0.10$ m:

$$V = kq\left(\frac{1}{0.10\text{ m} - x} + \frac{1}{x + 0.10\text{ m}}\right)$$

$$\frac{V}{kq} = \frac{-0.20\text{ m}}{x^2 - (0.10\text{ m})^2}$$

$$x = \sqrt{-(0.20\text{ m})\frac{kq}{V} + (0.10\text{ m})^2}$$

$$= \sqrt{-0.49\text{ m}^2}$$

There are no real solutions for $x < -0.10$ m.

So, $V = 1.3\times10^4$ V when $\boxed{x = 0.71\text{ m or } x = 0.002\text{ m}}$.

28. (a) $U_\text{i} + K_\text{i} = U_\text{f} + K_\text{f}$

$$\frac{kq^2}{r_\text{i}} + 0 = 0 + \frac{1}{2}mv_\text{f}^2$$

$$v_\text{f} = \sqrt{\frac{2kq^2}{mr_\text{i}}}$$

$$= \sqrt{\frac{2\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(3.05\times10^{-6}\text{ C})^2}{(0.00230\text{ kg})\sqrt{(1.15\text{ m})^2 + (0.550\text{ m})^2}}}$$

$$= \boxed{7.55\text{ m/s}}$$

(b) $U_\text{i} + K_\text{i} = U_\text{f} + K_\text{f}$

$$\frac{kq^2}{r_\text{i}} + 0 = \frac{kq^2}{r} + \frac{1}{2}m\left(\frac{v_\text{f}}{2}\right)^2$$

$$\frac{kq^2}{r_\text{i}} - \frac{mv_\text{f}^2}{8} = \frac{kq^2}{r}$$

$$r = \left(\frac{1}{r_\text{i}} - \frac{mv_\text{f}^2}{8kq^2}\right)^{-1}$$

$$= \left[\frac{1}{\sqrt{(1.15\text{ m})^2 + (0.550\text{ m})^2}} - \frac{(0.00230\text{ kg})\left(7.553\ \frac{\text{m}}{\text{s}}\right)^2}{8\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(3.05\times10^{-6}\text{ C})^2}\right]^{-1}$$

$$= \boxed{1.70\text{ m}}$$

29. (a)

$$U_i + K_i = U_f + K_f$$

$$\frac{kq_1q_2}{r_i} + 0 = \frac{kq_1q_2}{r_f} + \frac{1}{2}mv_f^2$$

$$\begin{aligned} v_f &= \sqrt{\frac{2kq_1q_2}{m}\left(\frac{1}{r_i} - \frac{1}{r_f}\right)} \\ &= \sqrt{\frac{2kq_1q_2}{m}\left(\frac{1}{r_i} - \frac{2}{r_i}\right)} \\ &= \sqrt{-\frac{2kq_1q_2}{mr_i}} \\ &= \sqrt{-\frac{2\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(20.2\times10^{-6}\text{ C})(-5.25\times10^{-6}\text{ C})}{(0.00320\text{ kg})\sqrt{(0.925\text{ m})^2 + (1.17\text{ m})^2}}} \\ &= \boxed{20.0\text{ m/s}} \end{aligned}$$

(b)

$$\Delta U_a = -\frac{kq_1q_2}{r_{ai}}$$

$$\Delta U_b = -\frac{kq_1q_2}{r_{bi}}$$

$$\frac{\Delta U_b}{\Delta U_a} = \frac{r_{ai}}{r_{bi}} > 1$$

Since the change in potential energy is greater than that for part (a), the change in kinetic energy is also greater. So, the speed is greater than that found in part (a).

(c)

$$\begin{aligned} v_f &= \sqrt{\frac{2kq_1q_2}{m}\left(\frac{1}{r_i} - \frac{1}{r_f}\right)} \\ &= \sqrt{\frac{2kq_1q_2}{m}\left(\frac{1}{r_i} - \frac{2}{r_i}\right)} \\ &= \sqrt{-\frac{2kq_1q_2}{mr_i}} \\ &= \sqrt{-\frac{2\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(20.2\times10^{-6}\text{ C})(-5.25\times10^{-6}\text{ C})}{(0.00320\text{ kg})\sqrt{\left(\frac{0.925\text{ m}}{2}\right)^2 + \left(\frac{1.17\text{ m}}{2}\right)^2}}} \\ &= \boxed{28.3\text{ m/s}} \end{aligned}$$

30. (a)

$$\begin{aligned} V &= \frac{kq_1}{r_1} + \frac{kq_2}{r_2} \\ &= \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left[\frac{-2.205\times10^{-6}\text{ C}}{\sqrt{(3.055\text{ m})^2 + (4.501\text{ m})^2}} + \frac{1.800\times10^{-6}\text{ C}}{\sqrt{(-2.533\text{ m})^2 + (0)^2}}\right] \\ &= \boxed{2.74\text{ kV}} \end{aligned}$$

(b) Find the distance between the two charges.

$$d=\sqrt{[3.055\text{ m}-(-2.533\text{ m})]^2+(4.501\text{ m}-0)^2}$$
$$=\sqrt{(5.588\text{ m})^2+(4.501\text{ m})^2}$$

Solve the equivalent one-dimensional problem.

$$V=0=\frac{q_1}{d-x'}+\frac{q_2}{x'}$$
$$x'=\frac{q_2 d}{q_2-q_1}=\frac{(1.800\times10^{-6}\text{ C})\sqrt{(5.588\text{ m})^2+(4.501\text{ m})^2}}{1.800\times10^{-6}\text{ C}-(-2.205\times10^{-6}\text{ C})}$$

Return to the original coordinates.

$$\theta=\tan^{-1}\frac{4.501\text{ m}}{3.055\text{ m}-(-2.533\text{ m})}$$
$$x=x'\cos\theta-2.533\text{ m}=-0.022\text{ m}$$
$$y=x'\sin\theta=2.023\text{ m}$$

The point at which the potential is zero is (–0.022 m, 2.023 m).

31. (a) $q_1=-6.1\ \mu\text{C}$
$q_2=+2.7\ \mu\text{C}$
$q_3=-3.3\ \mu\text{C}$

$$W=-\Delta U=-\frac{kq_2q_1}{r_{21}}-\frac{kq_2q_3}{r_{23}}$$
$$=-kq_2\left(\frac{q_1}{r_{21}}+\frac{q_3}{r_{23}}\right)$$
$$=-\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(2.7\times10^{-6}\text{ C})\left[\frac{-6.1\times10^{-6}\text{ C}}{0.25\text{ m}}+\frac{-3.3\times10^{-6}\text{ C}}{\sqrt{(0.25\text{ m})^2+(0.16\text{ m})^2}}\right]$$
$$=\boxed{0.86\text{ J}}$$

(b) The $-6.1\ \mu\text{C}$ charge is repelled by the $-3.3\ \mu\text{C}$ charge more than it is attracted by the $2.7\ \mu\text{C}$ charge. The work required will be negative, which is less than the work required in part (a).

(c)
$$W=-\Delta U=-\frac{kq_1q_2}{r_{12}}-\frac{kq_1q_3}{r_{13}}$$
$$=-kq_1\left(\frac{q_2}{r_{12}}+\frac{q_3}{r_{13}}\right)$$
$$=-\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(-6.1\times10^{-6}\text{ C})\left[\frac{2.7\times10^{-6}\text{ C}}{0.25\text{ m}}+\frac{-3.3\times10^{-6}\text{ C}}{0.16\text{ m}}\right]$$
$$=\boxed{-0.54\text{ J}}$$

32. $$W=-\Delta U=-\frac{kq_1q_2}{r_{12}}-\frac{kq_1q_3}{r_{13}}-\frac{kq_2q_3}{r_{23}}$$
$$=-k\left(\frac{q_1q_2}{r_{12}}+\frac{q_1q_3}{r_{13}}+\frac{q_2q_3}{r_{23}}\right)$$
$$=-\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left[\frac{(-6.1\times10^{-6}\ \text{C})(2.7\times10^{-6}\ \text{C})}{0.25\ \text{m}}+\frac{(-6.1\times10^{-6}\ \text{C})(-3.3\times10^{-6}\ \text{C})}{0.16\ \text{m}}\right.$$
$$\left.+\frac{(2.7\times10^{-6}\ \text{C})(-3.3\times10^{-6}\ \text{C})}{\sqrt{(0.25\ \text{m})^2+(0.16\ \text{m})^2}}\right]$$
$$=\boxed{-0.3\ \text{J}}$$

33. (a) $$V=\frac{kq_1}{r_1}+\frac{kq_2}{r_2}+\frac{kq_3}{r_3}$$
$$=k\left(\frac{q_1}{r_1}+\frac{q_2}{r_2}+\frac{q_3}{r_3}\right)$$
$$=\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{2.75\times10^{-6}\ \text{C}}{\frac{1.25\ \text{m}}{2}}+\frac{7.45\times10^{-6}\ \text{C}}{\sqrt{(1.25\ \text{m})^2-\left(\frac{1.25\ \text{m}}{2}\right)^2}}+\frac{-1.72\times10^{-6}\ \text{C}}{\frac{1.25\ \text{m}}{2}}\right)$$
$$=\boxed{76.7\ \text{kV}}$$

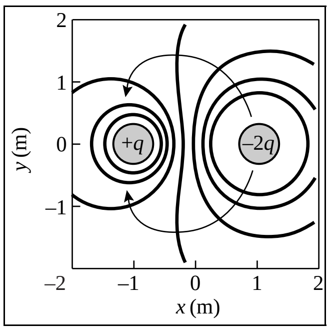
(b) $$U_\text{i}+K_\text{i}=U_\text{f}+K_\text{f}$$
$$qV+0=0+\frac{1}{2}mv^2$$
$$v=\sqrt{\frac{2qV}{m}}$$
$$v=\sqrt{\frac{2qk}{m}\left(\frac{q_1}{r_1}+\frac{q_2}{r_2}+\frac{q_3}{r_3}\right)}$$
$$=\sqrt{\frac{2(6.11\times10^{-6}\ \text{C})\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)}{0.00471\ \text{kg}}\left(\frac{2.75\times10^{-6}\ \text{C}}{\frac{1.25\ \text{m}}{2}}+\frac{7.45\times10^{-6}\ \text{C}}{\sqrt{(1.25\ \text{m})^2-\left(\frac{1.25\ \text{m}}{2}\right)^2}}+\frac{-1.72\times10^{-6}\ \text{C}}{\frac{1.25\ \text{m}}{2}}\right)}$$
$$=\boxed{14.1\ \text{m/s}}$$

34. Let the charges be numbered 1–4 beginning with the top left and counting clockwise.
$$U=kQ^2\left(\frac{1}{r_{12}}+\frac{1}{r_{13}}+\frac{1}{r_{14}}+\frac{1}{r_{23}}+\frac{1}{r_{24}}+\frac{1}{r_{34}}\right)$$
$$=kQ^2\left(\frac{1}{a}+\frac{1}{\sqrt{2}a}+\frac{1}{a}+\frac{1}{a}+\frac{1}{\sqrt{2}a}+\frac{1}{a}\right)$$
$$=\frac{kQ^2}{a}\left(4+\frac{2}{\sqrt{2}}\right)$$
$$=\boxed{\left(4+\sqrt{2}\right)\frac{kQ^2}{a}}$$

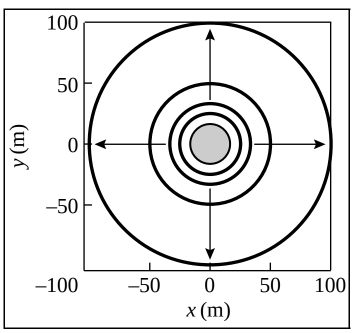
35. Let the charges be numbered 1–4 beginning with the top left and counting clockwise.

$$U = kQ^2\left(-\frac{1}{r_{12}}+\frac{1}{r_{13}}-\frac{1}{r_{14}}-\frac{1}{r_{23}}+\frac{1}{r_{24}}-\frac{1}{r_{34}}\right)$$
$$= kQ^2\left(-\frac{1}{a}+\frac{1}{\sqrt{2}a}-\frac{1}{a}-\frac{1}{a}+\frac{1}{\sqrt{2}a}-\frac{1}{a}\right)$$
$$= \frac{kQ^2}{a}\left(-4+\frac{2}{\sqrt{2}}\right)$$
$$= \boxed{-\left(4-\sqrt{2}\right)\frac{kQ^2}{a}}$$

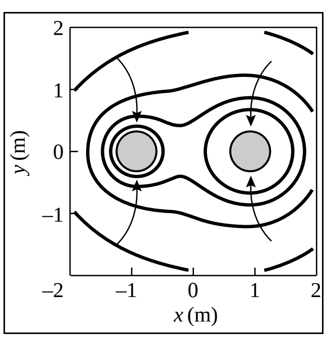
36. (a)



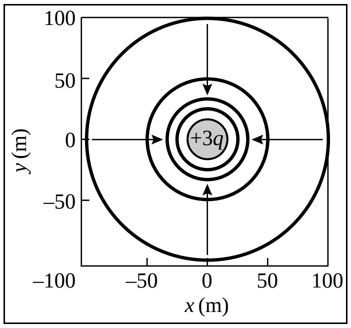
(b)



37. (a)



(b)



38. (a) Equipotential surfaces are perpendicular to the electric field, so the surfaces are oriented $\boxed{\text{parallel to the } yz\text{-plane}}$.

(b) Since the electric field points in the direction of decreasing potential, the electric potential $\boxed{\text{increases}}$ as you move in the direction opposite the electric field.

(c) $|E| = \dfrac{|\Delta V|}{|\Delta s|}$

$$|\Delta s| = \frac{|\Delta V|}{|E|} = \frac{|16\text{ V} - 14\text{ V}|}{7500\,\frac{\text{N}}{\text{C}}} = \boxed{3\times10^{-4}\text{ m}}$$

39. (a) $|E_x| = \left|-\dfrac{\Delta V}{\Delta x}\right| = \dfrac{10.0\text{ V}}{4\text{ cm}} = 250\text{ V/m}$

$|E_y| = \left|-\dfrac{\Delta V}{\Delta y}\right| = \dfrac{10.0\text{ V}}{2\text{ cm}} = 500\text{ V/m}$

$$E = \sqrt{\left(250\,\frac{\text{V}}{\text{m}}\right)^2 + \left(500\,\frac{\text{V}}{\text{m}}\right)^2} = \boxed{559\text{ V/m}}$$

The slope of each equipotential surface is $m = \dfrac{2\text{ cm} - 0\text{ cm}}{0\text{ cm} - 4\text{ cm}} = -\dfrac{1}{2}$. So, the slope of an electric field line is

$-\dfrac{1}{m} = 2$ because the electric field is perpendicular to equipotential surfaces.

$\tan\theta = 2$

$\theta = \tan^{-1} 2 = 63°$

But, because the electric field points in the direction of decreasing electric potential, $\theta = \boxed{243°}$.

(b) $\Delta s = \left|\dfrac{\Delta V}{E}\right| = \dfrac{5.00\text{ V}}{559\,\frac{\text{V}}{\text{m}}} = \boxed{8.94\text{ mm}}$

40. $Q = CV = (0.40\times10^{-6}\text{ F})(9.0\text{ V}) = 3.6\,\mu\text{C}$

$\boxed{\text{There is } 3.6\,\mu\text{C on one plate and } -3.6\,\mu\text{C on the other.}}$

41. $V = \dfrac{Q}{C} = \dfrac{4.8\,\mu\text{C}}{3.2\,\mu\text{F}} = \boxed{1.5\text{ V}}$

42. $C = \frac{Q}{V} = \frac{32\ \mu\text{C}}{3.0\ \text{V}} = \boxed{11\ \mu\text{F}}$

43. $C = \frac{\kappa\varepsilon_0 A}{d} = \frac{2.1\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(0.040\ \text{m})(5.0\ \text{m})}{0.025\times10^{-3}\ \text{m}} = \boxed{0.15\ \mu\text{F}}$

44. $C = \frac{\kappa\varepsilon_0 A}{d}$

$$\kappa = \frac{dC}{\varepsilon_0 A} = \frac{d}{\varepsilon_0 A}\left(\frac{Q}{V}\right) = \frac{(0.25\times10^{-3}\ \text{m})(1.2\times10^{-6}\ \text{C})}{\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\pi(0.056\ \text{m})^2(750\ \text{V})} = \boxed{4.6}$$

45. (a) $V = \frac{Q}{C} = \frac{Qd}{\kappa\varepsilon_0 A} = \frac{(4.7\times10^{-6}\ \text{C})(0.88\times10^{-3}\ \text{m})}{2.0\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(0.012\,\text{m}^2)} = \boxed{19\ \text{kV}}$

(b) The answer to part (a) will $\boxed{\text{decrease}}$ because V is inversely proportional to κ.

(c) $V = \frac{(4.7\times10^{-6}\ \text{C})(0.88\times10^{-3}\ \text{m})}{4.0\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(0.012\,\text{m}^2)} = \boxed{9.7\ \text{kV}}$

46. (a) $C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0(\pi R^2)}{d}$

$$R = \sqrt{\frac{dC}{\varepsilon_0\pi}} = \sqrt{\frac{(1.5\times10^{-3}\ \text{m})(1.0\times10^{-6}\ \text{F})}{\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\pi}} = \boxed{7.3\ \text{m}}$$

(b) Since increasing the separation of the plates decreases the capacitance, and since capacitance is directly proportional to the area of the plates, the radius of the plates should be $\boxed{\text{increased}}$, thereby increasing the area, which will maintain the capacitance.

(c) $R = \sqrt{\frac{(3.0\times10^{-3}\ \text{m})(1.0\times10^{-6}\ \text{F})}{\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\pi}} = \boxed{10\ \text{m}}$

47. (a) $C = \frac{\kappa\varepsilon_0 A}{d}$

$$d = \frac{\kappa\varepsilon_0 A}{C} = \frac{1.00059\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(3.45\times10^{-4}\ \text{m}^2)}{1330\times10^{-12}\ \text{F}} = \boxed{2.30\times10^{-6}\ \text{m}}$$

(b) $d = \frac{3.7\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(3.45\times10^{-4}\ \text{m}^2)}{1330\times10^{-12}\ \text{F}} = \boxed{8.5\times10^{-6}\ \text{m}}$

48. (a) $Q = CV = \frac{\kappa\varepsilon_0 AV}{d} = \frac{1.00059\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(0.0066\ \text{m}^2)(12\ \text{V})}{0.45\times10^{-3}\ \text{m}} = \boxed{1.6\times10^{-9}\ \text{C}}$

(b) Since Q is inversely proportional to d, the answer to part (a) will $\boxed{\text{decrease}}$.

(c) $$Q = \frac{1.00059\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(0.0066\ \text{m}^2)(12\ \text{V})}{0.90\times10^{-3}\ \text{m}} = \boxed{7.8\times10^{-10}\ \text{C}}$$

49. $$\Delta V = Ed = \left(3.0\times10^6\ \frac{\text{V}}{\text{m}}\right)(0.0050\ \text{m}) = \boxed{15\ \text{kV}}$$

50. (a) $$A = \frac{dC}{\kappa\varepsilon_0} = \frac{(2.6\times10^{-3}\ \text{m})(22\times10^{-12}\ \text{F})}{1.00059\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)} = \boxed{0.0065\ \text{m}^2}$$

(b) $$\Delta V = Ed = \left(3.0\times10^6\ \frac{\text{V}}{\text{m}}\right)(2.6\times10^{-3}\ \text{m}) = \boxed{7.8\ \text{kV}}$$

51. (a) $$C = \frac{\kappa\varepsilon_0 A}{d} = \frac{1.00059\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(0.50\times10^3\ \text{m})^2}{550\ \text{m}} = \boxed{4.0\ \text{nF}}$$

(b) $$Q = CV = CEd = (4.025\times10^{-9}\ \text{F})\left(3.0\times10^6\ \frac{\text{V}}{\text{m}}\right)(550\ \text{m}) = \boxed{6.6\ \text{C}}$$

52. $$Q = CV = \left(\frac{\kappa\varepsilon_0 A}{d}\right)(Ed) = \kappa\varepsilon_0 AE = 5.4\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(0.0300\ \text{m})(10.0\ \text{m})\left(1.00\times10^8\ \frac{\text{V}}{\text{m}}\right)$$
$$= \boxed{1.4\ \text{mC}}$$

53. $$W = U = \frac{1}{2}CV^2 = \frac{1}{2}(8.0\times10^{-6}\ \text{F})(3.0\ \text{V})^2 = \boxed{36\ \mu\text{J}}$$

54. $$U = \frac{1}{2}CV^2$$
$$C = \frac{2U}{V^2} = \frac{2(125\ \text{J})}{(1050\ \text{V})^2} = \boxed{227\ \mu\text{F}}$$

55. (a) $$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{\kappa\varepsilon_0 A}{d}\right)V^2 = \frac{4.5\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(4.75\times10^{-9}\ \text{m}^2)(0.0725\ \text{V})^2}{2(8.5\times10^{-9}\ \text{m})} = \boxed{5.8\times10^{-14}\ \text{J}}$$

(b) Since $U \propto \frac{1}{d}$, the answer to part (a) would $\boxed{\text{decrease}}$ if the thickness of the cell membrane is increased.

56. (a) $$mgh = \frac{1}{2}CV^2$$
$$h = \frac{CV^2}{2mg} = \frac{(0.22\times10^{-6}\ \text{F})(1.5\ \text{V})^2}{2(0.0050\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)} = \boxed{5.0\times10^{-6}\ \text{m}}$$

(b) $V=\sqrt{\frac{2mgh}{C}}=\sqrt{\frac{2(0.0050\text{ kg})\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)(0.010\text{ m})}{0.22\times10^{-6}\text{ F}}}=\boxed{67\text{ V}}$

57. energy density $=\frac{1}{2}\varepsilon_0 E^2$

$$=\frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2$$
$$=\frac{\varepsilon_0 V^2}{2d^2}$$
$$=\frac{\left(8.85\times10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(315\text{ V})^2}{2(0.200\times10^{-3}\text{ m})^2}$$
$$=\boxed{11.0\text{ J/m}^3}$$

58. energy density $=\frac{1}{2}\varepsilon_0 E^2$

$$E=\sqrt{\frac{2(\text{energy density})}{\varepsilon_0}}$$
$$=\sqrt{\frac{2(10.0\text{ J})}{\left(8.85\times10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(1.00\times10^{-9}\text{ m}^3)}}$$
$$=\boxed{4.75\times10^{10}\text{ N/C}}$$

59. **(a)** $Q=CV=(850\times10^{-6}\text{ F})(330\text{ V})=\boxed{0.28\text{ C}}$

(b) $U=\frac{1}{2}CV^2=\frac{1}{2}(850\times10^{-6}\text{ F})(330\text{ V})^2=\boxed{46\text{ J}}$

60. **(a)** $U=\frac{1}{2}CV^2$

$$=\frac{1}{2}\left(\frac{\kappa\varepsilon_0 A}{d}\right)V^2$$
$$=\frac{1.00059\left(8.85\times10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(405\times10^{-4}\text{ m}^2)(575\text{ V})^2}{2(2.25\times10^{-3}\text{ m})}$$
$$=\boxed{2.63\times10^{-5}\text{ J}}$$

(b) By doubling the separation, C is reduced by one half, but Q stays the same, so V is doubled because

$V=\frac{Q}{C}=2\frac{Q_0}{C_0}=2V_0$.

$U=\frac{1}{2}QV=\frac{1}{2}Q_0(2V_0)=Q_0V_0=2E_0=\boxed{5.27\times10^{-5}\text{ J}}$

(c) Since the energy stored in the capacitor increased, $W=\Delta U=\boxed{2.63\times10^{-5}\text{ J}}$.

61. **(a)** $\Delta V = V_B - V_A = -\frac{W}{q_0} = -\frac{0.052 \text{ J}}{5.7\times10^{-6} \text{ C}} = \boxed{-9.1 \text{ kV}}$

(b) $\Delta V = V_B - V_A = -\frac{W}{q_0} = -\frac{-0.052 \text{ J}}{-5.7\times10^{-6} \text{ C}} = \boxed{-9.1 \text{ kV}}$

(c) $\Delta V = V_B - V_A = \frac{W}{q_0} = \frac{0.052 \text{ J}}{5.7\times10^{-6} \text{ C}} = \boxed{9.1 \text{ kV}}$

62. Since the capacitance of a parallel-plate capacitor is directly proportional to the area of the plates and inversely proportional to the separation between them, the capacitance is $\boxed{\text{decreased by a factor of } \frac{\frac{1}{2}}{2} = \frac{1}{4}}$.

63. For a constant potential difference, the charge on the plates varies only with capacitance. The charge is directly proportional to the capacitance, and the capacitance is inversely proportional to the spacing. Therefore, the charge is inversely proportional to the spacing. So, if the spacing is doubled, the magnitude of the charge is $\boxed{\text{reduced by half}}$.

64. $V = 0 = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{Q}{r}\right)$

$$0 = \frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{Q}{r}$$

$$Q = -r\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$= -\sqrt{(2.23 \text{ m})^2 + (3.01 \text{ m})^2}\left(\frac{24.5\times10^{-6} \text{ C}}{\sqrt{(4.40 \text{ m})^2 + (6.02 \text{ m})^2}} + \frac{-11.2\times10^{-6} \text{ C}}{\sqrt{(-4.50 \text{ m})^2 + (6.75 \text{ m})^2}}\right)$$

$$= \boxed{-7.14 \ \mu\text{C}}$$

65. $F = \frac{mv^2}{r} = \frac{ke^2}{r^2}$

$$K = \frac{1}{2}mv^2 = \frac{ke^2}{2r}$$

$$W = (K+U)_f - (K+U)_i$$

$$= 0 - \left[\frac{ke^2}{2r} + \left(-\frac{ke^2}{r}\right)\right]$$

$$W = \frac{ke^2}{2r} = \frac{\left(8.99\times10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(1.60\times10^{-19} \text{ C})^2}{2(0.529\times10^{-10} \text{ m})} = \boxed{2.18\times10^{-18} \text{ J}}$$

66. (a) Due to symmetry, the electric potential is negative in quadrants 2 and 3, positive in quadrants 1 and 4, and zero along the y-axis. So, since the potential is greater than zero at points B, C, and D and zero at point A, the potential is $\boxed{\text{smallest at } A}$. Since point C is farthest from the negative charge, and since all four points are equally distant from the positive charge, the potential is $\boxed{\text{greatest at } C}$.

(b) Let $q = 1.2\ \mu\text{C}$ and $d = 0.50$ m.

$$V_\text{A} = \frac{kq}{d} + \frac{k(-q)}{d} = \boxed{0}$$

Due to symmetry, $V_\text{B} = V_\text{D}$.

$$V_\text{B} = V_\text{D} = \frac{kq}{d} + \frac{k(-q)}{\sqrt{d^2 + (2d)^2}} = \frac{kq}{d}\left(1 - \frac{1}{\sqrt{5}}\right) = \boxed{12\text{ kV}}$$

$$V_\text{C} = \frac{kq}{d} + \frac{k(-q)}{3d} = \frac{kq}{d}\left(1 - \frac{1}{3}\right) = \frac{2kq}{3d} = \boxed{14\text{ kV}}$$

67. (a) Since all four points are equally distant from the charge on the positive x-axis, its contribution to the total potential is the same for all four points. So, we can concentrate on the contribution due to the other charge. Since point C is farthest from the charge at $x = -0.50$ m, the potential is $\boxed{\text{smallest at } C}$. And, since point A is closest to this charge, the potential is $\boxed{\text{greatest at } A}$.

(b) Let $q = 1.2\ \mu\text{C}$ and $d = 0.50$ m.

$$V_\text{A} = \frac{kq}{d} + \frac{kq}{d} = \frac{2kq}{d} = \boxed{43\text{ kV}}$$

Due to symmetry, $V_\text{B} = V_\text{D}$.

$$V_\text{B} = V_\text{D} = \frac{kq}{d} + \frac{kq}{\sqrt{d^2 + (2d)^2}} = \frac{kq}{d}\left(1 + \frac{1}{\sqrt{5}}\right) = \boxed{31\text{ kV}}$$

$$V_\text{C} = \frac{kq}{d} + \frac{kq}{3d} = \frac{4kq}{3d} = \boxed{29\text{ kV}}$$

68. Initially the potential energy of the system is zero. After the three protons have been brought together, the potential energy of the system is equal to the work done to bring the protons together.

$$W = \Delta U = U_{12} + U_{13} + U_{23} = 3\left(\frac{ke^2}{r}\right) = \frac{3\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(1.60\times10^{-19}\text{ C})^2}{1.5\times10^{-15}\text{ m}} = \boxed{4.6\times10^{-13}\text{ J}}$$

69. (a)

$$\begin{aligned} U &= \frac{kQq}{r} \\ &= \frac{\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(87.1\times10^{-6}\text{ C})(-2.37\times10^{-6}\text{ C})}{0.323\text{ m}} \\ &= \boxed{-5.75\text{ J}} \end{aligned}$$

(b) $K_i + U_i = K_f + U_f$

$$0 + \frac{kQq}{r_i} = \frac{1}{2}mv^2 + \frac{kQq}{r_f}$$

$$\frac{1}{2}mv^2 = kQq\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

$$v = \sqrt{\frac{2kQq}{m}\left(\frac{1}{r_i} - \frac{1}{r_f}\right)}$$

$$= \sqrt{\frac{2\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(87.1\times10^{-6}\ \text{C})(-2.37\times10^{-6}\ \text{C})}{0.0526\ \text{kg}}\left(\frac{1}{0.323\ \text{m}} - \frac{1}{0.121\ \text{m}}\right)}$$

$$= \boxed{19.1\ \text{m/s}}$$

70. $W = \frac{1}{2}QV = \frac{1}{2}NeV = \frac{1}{2}(3.75\times10^{16})(1.60\times10^{-19}\ \text{C})(325\ \text{V}) = \boxed{0.975\ \text{J}}$

71. (a) There is a net repulsive force exerted on q by the three charges as it is moved into position. Therefore $\boxed{\text{positive}}$ work is done by the outside force used to move q into position.

(b) $W = \Delta U = (U_{14} + U_{24} + U_{34})_f - 0$

$$W = \left(\frac{kq_1q}{r_{14}} + \frac{kq_2q}{r_{24}} + \frac{kq_3q}{r_{34}}\right)$$

$$-1.3\times10^{-11}\ \text{J} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)q\left[\frac{2.75\times10^{-6}\ \text{C}}{\frac{1.25\ \text{m}}{2}} + \frac{7.45\times10^{-6}\ \text{C}}{\sqrt{(1.25\ \text{m})^2 - \left(\frac{1.25\ \text{m}}{2}\right)^2}} + \frac{(-1.72\times10^{-6}\ \text{C})}{\frac{1.25\ \text{m}}{2}}\right]$$

$$q = \boxed{-1.7\times10^{-16}\ \text{C}}$$

72. (a) $E_x = -\frac{\Delta V}{\Delta x} = \left(-\frac{10.0\ \text{V}}{0.0400\ \text{m}}\right) = \boxed{-2.50\times10^2\ \frac{\text{V}}{\text{m}}}$

(b) $E_y = -\frac{\Delta V}{\Delta y} = \left(-\frac{10.0\ \text{V}}{0.0200\ \text{m}}\right) = \boxed{-5.00\times10^2\ \frac{\text{V}}{\text{m}}}$

(c) $E = \sqrt{(-250\ \text{V/m})^2 + (-500\ \text{V/m})^2} = \boxed{559\ \text{V/m}}$

$\theta = \tan^{-1}\left(\frac{-500\ \text{V/m}}{-250\ \text{V/m}}\right) = 63.4° + 180.0° = \boxed{243.4°}$ since $\vec{\mathbf{E}}$ points in the direction of decreasing potential.

73. (a) $V = \frac{Q}{C} = \frac{Qd}{\varepsilon_0 A\kappa}$

Side-to-side A is large and d is small. End-to-end A is small and d is large. So $\boxed{\text{end-to-end}}$ generates a higher voltage.

(b) $Q = VC = V\dfrac{\varepsilon_0 A\kappa}{d} = (350\text{ V})\left(8.85\times10^{-12}\,\dfrac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\dfrac{(1.8\times10^{-2}\text{ m}^2)(95)}{1.0\text{ m}}$

$= \boxed{5.3\times10^{-9}\text{ C}}$

74. (a) $V = \dfrac{U}{q_0} = \dfrac{2.6\times10^{-5}\text{ J}}{6.2\times10^{-6}\text{ C}} = 4.2\text{ V}$, which occurs at approximately $x = \boxed{0.66\text{ m}}$.

(b) $V = \dfrac{U}{q_0} = \dfrac{4.3\times10^{-5}\text{ J}}{6.2\times10^{-6}\text{ C}} = 6.9\text{ V}$, which occurs at three values of x; approximately $\boxed{0.08\text{ m, }0.41\text{ m, and }0.62\text{ m}}$.

75. (a) Since the capacitance is inversely proportional to the separation, depressing the key $\boxed{\text{increases}}$ the capacitance by decreasing the separation.

(b) $\Delta C = C_\text{f} - C_\text{i} = \dfrac{\kappa\varepsilon_0 A}{d_\text{f}} - \dfrac{\kappa\varepsilon_0 A}{d_\text{i}} = \kappa\varepsilon_0 A\left(\dfrac{1}{d_\text{f}} - \dfrac{1}{d_\text{i}}\right)$

$\dfrac{\Delta C}{\kappa\varepsilon_0 A} + \dfrac{1}{d_\text{i}} = \dfrac{1}{d_\text{f}}$

$$d_\text{f} = \left(\frac{\Delta C}{\kappa\varepsilon_0 A} + \frac{1}{d_\text{i}}\right)^{-1}$$

$$= \left[\frac{0.425\times10^{-12}\text{ F}}{3.75\left(8.85\times10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(47.5\times10^{-6}\text{ m}^2)} + \frac{1}{0.550\times10^{-3}\text{ m}}\right]^{-1}$$

$= 0.479\text{ mm}$

$d = |d_\text{f} - d_\text{i}| = |0.479\text{ mm} - 0.550\text{ mm}| = \boxed{0.071\text{ mm}}$

76. (a) Since a positive charge would move in the direction of the electric field, and since the electric field points in the direction of high to low potential, $\boxed{\text{the left plate}}$ must be at the higher electric potential.

(b) Find E.

$T\sin\theta - qE = 0$

$T\cos\theta - mg = 0$

$E = \dfrac{T\sin\theta}{q} = \left(\dfrac{mg}{\cos\theta}\right)\dfrac{\sin\theta}{q} = \dfrac{mg}{q}\tan\theta$

Find ΔV.

$$\Delta V = Ed = \frac{mgd}{q}\tan\theta = \frac{(0.071\text{ kg})\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)(0.025\text{ m})}{6.77\times10^{-6}\text{ C}}\tan 22^\circ = \boxed{1.0\text{ kV}}$$

77. (a) $E = \dfrac{\sigma}{\varepsilon_0\kappa} = \dfrac{0.62\times10^{-3}\,\frac{\text{C}}{\text{m}^2}}{\left(8.85\times10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(5.5)} = \boxed{1.3\times10^{7}\text{ N/C}}$

Electric field lines point away from positive charges and point towards negative charges, so the electric field is $\boxed{\text{directed into the cell}}$.

(b) The $\boxed{\text{outer wall}}$ has the positive charge density, so it is at the higher potential.

$$\Delta V = Ed = \left(1.274\times 10^{7}\ \frac{\text{N}}{\text{C}}\right)(7.5\times 10^{-9}\ \text{m}) = \boxed{96\ \text{mV}}$$

78. (a) $(K+U)_{\text{i}} = (K+U)_{\text{f}} = K_{\text{f}}$, since $U = 0$ at ∞.

$$\frac{1}{2}mv_0^2 + \frac{kQ(-Q)}{a} + \frac{kQ(-Q)}{a} + \frac{k(-Q)^2}{\sqrt{2}a} = \frac{1}{2}mv^2$$

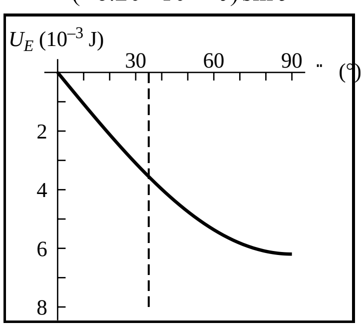
$$v_0^2 + \frac{2kQ^2}{ma}\left(-2 + \frac{1}{\sqrt{2}}\right) = v^2$$

$$v = \boxed{\sqrt{v_0^2 + \frac{kQ^2}{ma}\left(-4+\sqrt{2}\right)}}$$

(b) The second $-Q$ charge is acted upon by a stronger net attractive force as it moves away from the two $+Q$ charges, so it loses more energy and slows down more. Its eventual speed is $\boxed{\text{less}}$ than that of the first $-Q$ charge.

79. (a) $U_{\text{E}} = 0$ midway between the plates. As θ increases,

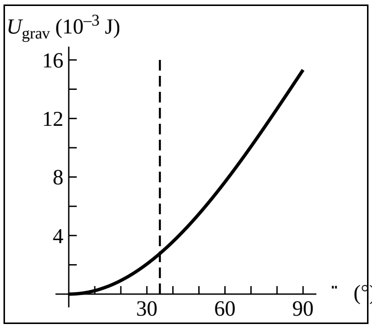
$$U_{\text{E}} = -qE(L\sin\theta)$$
$$= -(6.77\times 10^{-6}\ \text{C})\left(4.16\times 10^{4}\ \frac{\text{V}}{\text{m}}\right)(0.022\ \text{m})\sin\theta$$
$$= (-6.20\times 10^{-3}\ \text{J})\sin\theta$$



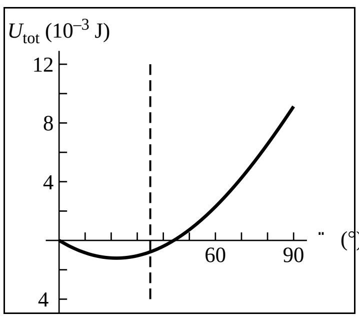
A full 90° swing is shown. The dotted line indicates where the swing will be stopped by the right capacitor plate.

(b) Let $U_{\text{grav}} = 0$ at the straight-down position. As θ increases,

$$U_{\text{grav}} = mgL(1-\cos\theta)$$
$$= (0.071\text{ kg})\left(9.81\ \frac{\text{kg}\cdot\text{m}}{\text{s}^2}\right)(0.022\text{ m})(1-\cos\theta)$$
$$= (15.3\times10^{-3}\text{ J})(1-\cos\theta)$$



(c) $U_{\text{tot}} = U_{\text{E}} + U_{\text{grav}}$



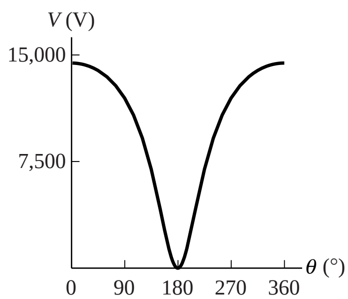
U_{tot} is at a minimum when $\theta = 22°$.

80. $V_1 = \dfrac{kq}{r_1}$ $\qquad V_2 = \dfrac{kq}{r_2}$

$$V_1 r_1 = V_2 r_2$$
$$V_1 r = V_2(r + 1.00\text{ m})$$
$$r(V_1 - V_2) = V_2(1.00\text{ m})$$
$$r = \frac{V_2(1.00\text{ m})}{V_1 - V_2}$$
$$= \frac{(6220\text{ V})(1.00\text{ m})}{2.70\times10^4\text{ V} - 6220\text{ V}}$$
$$= \boxed{0.299\text{ m}}$$

$$q = \frac{r_1 V_1}{k} = \frac{(0.2993\text{ m})(2.70\times10^4\text{ V})}{8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}} = \boxed{8.99\times10^{-7}\text{ C}}$$

81. $$V(\theta) = V_+ + V_-$$
$$= \frac{kq}{d} + \frac{k(-q)}{r}$$
$$= \frac{kq}{d}\left(1 - \frac{d}{r}\right)$$
$$= \frac{kq}{d}\left(1 - \frac{d}{\sqrt{x^2 + y^2}}\right)$$
$$= \frac{kq}{d}\left[1 - \frac{d}{\sqrt{(2d + d\cos\theta)^2 + (d\sin\theta)^2}}\right]$$
$$= \frac{kq}{d}\left[1 - \frac{1}{\sqrt{(2 + \cos\theta)^2 + \sin^2\theta}}\right]$$
$$= \frac{kq}{d}\left(1 - \frac{1}{\sqrt{4 + 4\cos\theta + \cos^2\theta + \sin^2\theta}}\right)$$
$$= \frac{kq}{d}\left(1 - \frac{1}{\sqrt{5 + 4\cos\theta}}\right)$$
$$= \frac{\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(1.2\times10^{-6}\ \text{C})}{0.50\ \text{m}}\left(1 - \frac{1}{\sqrt{5 + 4\cos\theta}}\right)$$
$$= \boxed{(22\ \text{kV})\left(1 - \frac{1}{\sqrt{5 + 4\cos\theta}}\right)}$$



82. $$C = \frac{\Delta Q}{\Delta V} = \frac{13.5\times10^{-6}\ \text{C}}{3.25\ \text{V}} = \boxed{4.15\ \mu\text{F}}$$

83. $$V = \frac{kq}{r} \qquad E = \frac{kq}{r^2}$$
$$\frac{V}{r} = \frac{kq}{r^2} = E$$
$$r = \frac{V}{E} = \frac{155\ \text{V}}{2240\ \frac{\text{N}}{\text{C}}} = \boxed{6.92\ \text{cm}}$$
$$q = \frac{rV}{k} = \frac{V^2}{kE} = \frac{(155\ \text{V})^2}{\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(2240\ \frac{\text{N}}{\text{C}}\right)} = \boxed{1.19\times10^{-9}\ \text{C}}$$