

## Chapter 21

### Electric Current and Direct-Current Circuits

#### Answers to Even-numbered Conceptual Questions

2. No. An electric current is produced when a net charge moves. If your body is electrically neutral, no current is produced when you walk.
4. Yes. There is a net charge on the comb, and by moving it from one place to another you have created an electric current.
6. Car headlights are wired in parallel, as we can tell by the fact that some cars have only one working headlight.
8. The equivalent resistance decreases because there are now more paths through which the electric current can flow.
10. Material A satisfies Ohm's law because the relationship between current and voltage is linear.
12. Yes. Just connect two of these resistors in parallel and you will have an equivalent resistance of  $R/2$ .
14. Assuming the lights are connected to the same potential difference,  $V$ , the resistance of the lights can be compared using  $P = V^2 / R$ . Since light A has twice the power of light B, it must also have half the resistance of light B.
16. Resistors connected in parallel have the same potential difference across their terminals.
18. When the switch is closed, light 2 is shorted out and the equivalent resistance of the circuit drops from  $3R$  to  $2R$ . It follows that the current in the circuit increases.
20. Originally, light 3 is dark; after the switch is closed, light 3 is illuminated. The intensity of lights 1 and 2 are unaffected by closing the switch; after all, they still have the same potential difference and the same resistance. Therefore, it is clear from  $P = V^2 / R$  that they will continue to dissipate the same amount of power.
22. The total power dissipated in the circuit increases when the switch is closed. Before the switch was closed light 1 dissipated the power  $P = \mathcal{E}^2 / R$ , as did light 2. Now all three lights dissipate the power  $P = \mathcal{E}^2 / R$ .
24. Resistivity is an intrinsic property of a particular substance. In this sense it is similar to density, which has a particular value for each particular substance. Resistance, however, is a property associated with a given resistor. For example, the resistance of a given wire can be large because its resistivity is large, or because it is long. Similarly, the weight of a ball can be large because its density is large, or because it has a large radius.
26. If two heaters, each with resistance  $R$ , are connected in series, the equivalent resistance is now  $2R$ . The heaters are connected to the same potential, however, and therefore they draw half the original current. From  $P = I^2 R$  we can see that each heater now dissipates one quarter the power of the single heater, for a total power consumption of one half the original value.

28. A number of factors come into play here. First, the bottom of a bird's foot is tough, and definitely not a good conductor of electricity. Second, and more important, is the fact that a potential difference is required for there to be a flow of current. Just being in contact with a high-voltage wire isn't enough to cause a problem; somewhere else there must be contact with a lower voltage. But the bird is in contact with basically the same high voltage in two different places, which doesn't lead to a potential difference. The only potential difference the bird experiences is due to the very small voltage drop along the segment of wire between the bird's two feet.
30. Magnetic resonance imaging (MRI) machines would definitely benefit from room-temperature superconductivity. As it is, they must cool their magnets to low temperature. Similarly, electrical power transmission would benefit if the resistance of the wires could be eliminated. On the other hand, a toaster or an electric oven requires resistance to do its job; superconductivity would not be a help.
32. Because the bulbs operate on the same potential difference,  $V$ , we can compare their resistances by considering the following power expression,  $P = V^2 / R$ . It is clear that bulb B has a resistance that is four times greater than the resistance of bulb A.
34. Because the bulbs operate with the same current,  $I$ , we can compare their potential differences by considering the following power expression,  $P = IV$ . It is clear that bulb A has a potential difference that is four times greater than the potential difference of bulb B.
36. The fuse should be connected in series with the circuit it protects. Then, if the fuse burns out, no current will flow in the circuit.
38. (a) If the two capacitors are connected in series, they have the same charge on their plates. Therefore, we can compare their energies by considering the relation  $U = Q^2 / 2C$ . Clearly, the smaller capacitor stores twice as much energy as the larger capacitor. This seems counterintuitive at first, until you realize that it takes more work to put a given amount of charge on a small capacitor than it does to put the same charge on a large capacitor. (b) If the two capacitors are connected in parallel, they have the same potential difference between their plates. Therefore, we can compare their energies by considering the relation  $U = \frac{1}{2} CV^2$ . Clearly, in this case, the larger capacitor stores twice as much energy as the smaller capacitor.
40. The equivalent capacitance increases because now there is more plate area on which to store charge. As a result, more charge is stored for the same potential difference.
42. Capacitors connected in series all have the same magnitude of charge on their plates. This is illustrated in Figure 21-17 (a).

### Solutions to Problems

$$1. \Delta Q = I\Delta t = \left(1 \frac{\text{C}}{\text{s}}\right)(1 \text{ h})\left(\frac{3600 \text{ s}}{\text{h}}\right) = \boxed{3600 \text{ C}}$$

$$2. \Delta Q = I\Delta t = (0.12 \text{ A})(65 \text{ s}) = \boxed{7.8 \text{ C}}$$

$$\text{number of electrons} = \frac{7.8 \text{ C}}{1.6 \times 10^{-19} \text{ C}} = \boxed{4.9 \times 10^{19} \text{ electrons}}$$

$$3. \text{ electrons per second} = \frac{I}{e} = \frac{15 \text{ A}}{1.6 \times 10^{-19} \text{ C}} = \boxed{9.4 \times 10^{19} \text{ electrons/s}}$$

$$4. (a) \Delta Q = \frac{W}{\mathcal{E}} = \frac{260 \text{ J}}{12 \text{ V}} = \boxed{22 \text{ C}}$$

(b)  $\boxed{\text{decreases by a factor of 2}}$

$$5. \text{ electrons per second} = \frac{I}{e} = \frac{10.0 \times 10^{-15} \text{ A}}{1.60 \times 10^{-19} \text{ C}} = \boxed{62,500 \text{ electrons/s}}$$

$$6. (a) I = \frac{P}{\mathcal{E}} = \frac{85 \text{ W}}{120 \text{ V}} = \boxed{0.71 \text{ A}}$$

$$(b) \Delta t = \frac{\Delta Q}{I} = \left( \frac{\mathcal{E}}{P} \right) (1 \times 10^7 e) = \frac{1 \times 10^7 (120 \text{ V})(1.6 \times 10^{-19} \text{ C})}{85 \text{ W}} = \boxed{2 \times 10^{-12} \text{ s}}$$

$$7. (a) \Delta Q = (0.42 \text{ A} \cdot \text{h}) \left( \frac{3600 \text{ s}}{\text{h}} \right) = \boxed{1500 \text{ C}}$$

$$(b) \Delta t = \frac{\Delta Q}{I} = \frac{1512 \text{ C}}{5.6 \times 10^{-6} \text{ A}} = \boxed{8.6 \text{ y}}$$

$$8. R = \rho \frac{L}{A} = (1.59 \times 10^{-8} \Omega \cdot \text{m}) \frac{4.5 \text{ m}}{\pi \left( \frac{0.45 \times 10^{-3} \text{ m}}{2} \right)^2} = \boxed{0.45 \Omega}$$

$$9. R = \frac{V}{I} = \frac{18 \text{ V}}{0.35 \text{ A}} = \boxed{51 \Omega}$$

$$10. R = \rho \frac{L}{A} = \rho \frac{L}{\frac{1}{4} \pi d^2}$$

$$d = \sqrt{\frac{4 \rho L}{\pi R}} = \sqrt{\frac{4(5.6 \times 10^{-8} \Omega \cdot \text{m})(0.27 \text{ m})}{\pi(0.07 \Omega)}} = \boxed{0.5 \text{ mm}}$$

$$11. R = \rho \frac{L}{A} = (1.72 \times 10^{-8} \Omega \cdot \text{m}) \frac{(5 \text{ mi}) \left( \frac{1609 \text{ m}}{\text{mi}} \right)}{\pi \left( \frac{0.50 \times 10^{-3} \text{ m}}{2} \right)^2} = \boxed{700 \Omega}$$

$$12. (a) V = IR = I \rho \frac{L}{A} = (32 \text{ A})(1.72 \times 10^{-8} \Omega \cdot \text{m}) \frac{0.060 \text{ m}}{0.13 \times 10^{-4} \text{ m}^2} = \boxed{2.5 \text{ mV}}$$

(b) Since  $V$  is directly proportional to the separation of the bird's feet, the answer to part (a) will  $\boxed{\text{increase}}$ .

$$13. V = IR = I\rho \frac{L}{A} = I\rho \frac{L}{\frac{1}{4}\pi d^2}$$

$$L = \frac{\pi d^2 V}{4I\rho} = \frac{\pi(0.44 \times 10^{-3} \text{ m})^2 (15 \text{ V})}{4(0.76 \text{ A})(1.72 \times 10^{-8} \Omega \cdot \text{m})} = \boxed{170 \text{ m}}$$

$$14. (a) I = \frac{V}{R} = \frac{V}{\rho \frac{L}{A}} = \frac{VA}{\rho L} = \frac{(75 \times 10^{-3} \text{ V})(1.0 \times 10^{-6} \text{ m})^2}{(1.3 \times 10^7 \Omega \cdot \text{m})(8.0 \times 10^{-9} \text{ m})} = \boxed{7.2 \times 10^{-13} \text{ A}}$$

(b) decreases by a factor of 2

$$15. V = IR = I\rho \frac{L}{A} = I\rho \frac{L}{\frac{1}{4}\pi d^2}$$

$$\rho = \frac{\pi d^2 V}{4IL} = \frac{\pi(0.33 \times 10^{-3} \text{ m})^2 (12 \text{ V})}{4(2.1 \text{ A})(6.9 \text{ m})} = \boxed{7.1 \times 10^{-8} \Omega \cdot \text{m}}$$

$$16. (a) \frac{R}{L} = \frac{\rho}{A} = \frac{2.82 \times 10^{-8} \Omega \cdot \text{m}}{2.4 \times 10^{-7} \text{ m}^2} = \boxed{0.12 \Omega/\text{m}}$$

(b) The resistance per unit length is inversely proportional to the cross-sectional area, and the cross-sectional area is directly proportional to the square of the diameter. So, if the diameter were increased, the answer to part (a) would decrease.

$$(c) \frac{R}{L} = \frac{2.82 \times 10^{-8} \Omega \cdot \text{m}}{3.6 \times 10^{-7} \text{ m}^2} = \boxed{0.078 \Omega/\text{m}}$$

$$17. (a) R = \rho \frac{L}{A} = (0.15 \Omega \cdot \text{m}) \frac{10^{-1} \text{ m}}{\pi(10^{-2} \text{ m})^2} = \boxed{50 \Omega}$$

$$(b) V = IR = (15 \times 10^{-3} \text{ A})(50 \Omega) = \boxed{1 \text{ V}}$$

$$18. V = I_{AB}R = I_{AB}\rho \frac{L}{A} = I_{AB}\rho \frac{C}{AB}$$

$$I = \frac{V}{R} = \frac{I_{AB}\rho \frac{C}{AB}}{\rho \frac{A}{BC}} = \boxed{\left(\frac{C}{A}\right)^2 I_{AB}}$$

$$19. I = \frac{P}{V} = \frac{3.3 \times 10^3 \text{ W}}{75 \text{ V}} = \boxed{44 \text{ A}}$$

$$20. P = IV = (24 \times 10^{-3} \text{ A})(4.5 \text{ V}) = \boxed{0.11 \text{ W}}$$

$$21. P = \frac{V^2}{R} = \frac{(120 \text{ V})^2}{25 \Omega} = \boxed{580 \text{ W}}$$

$$22. \text{ cost} = \text{power} \times \text{time} \times \frac{\text{cost}}{\text{kWh}}$$

$$= (2.3 \text{ A})(120 \text{ V})(1 \text{ h}) \left( \frac{\$0.075}{10^3 \text{ Wh}} \right)$$

$$= \boxed{\$0.021}$$

$$23. \frac{\text{cost}}{\text{kWh}} = \frac{\text{cost}}{\text{power} \times \text{time}} = \frac{2.6 \text{ cents}}{(15 \text{ A})(12 \text{ V})(120 \text{ min}) \left( \frac{1 \text{ h}}{60 \text{ min}} \right)} \left( \frac{10^3}{\text{k}} \right) = \boxed{7.2 \text{ cents/kWh}}$$

$$24. \text{ (a) } I = \frac{P}{V} = \frac{75 \text{ W}}{95 \text{ V}} = \boxed{0.79 \text{ A}}$$

$$\text{ (b) } R = \frac{V^2}{P} = \frac{(95 \text{ V})^2}{75 \text{ W}} = \boxed{120 \Omega}$$

$$\text{ (c) } \boxed{\text{greater by a factor of 2}}$$

$$25. 905 \text{ cranking amps} = (905 \text{ A})(7.2 \text{ V})(30.0 \text{ s}) = 2.0 \times 10^5 \text{ J}$$

$$155\text{-minute reserve capacity} = (155 \text{ min})(25 \text{ A})(10.5 \text{ V}) \left( \frac{60 \text{ s}}{\text{min}} \right) = 2.4 \times 10^6 \text{ J}$$

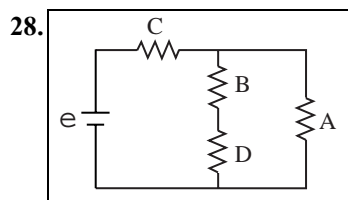
The 155-minute reserve capacity rating represents the greater amount of energy delivered by the battery.

$$26. \frac{1}{R} = \frac{1}{35 \Omega + 82 \Omega} + \frac{1}{45 \Omega}$$

$$R = \boxed{33 \Omega}$$

$$27. \frac{1}{11 \Omega} = \frac{n}{65 \Omega}$$

$$n = \boxed{6 \text{ resistors}}$$



29. Find the current.

$$I = \frac{V}{R_{\text{eq}}}$$

Find the power dissipated for each resistance.

$$\text{ (a) } P_{\text{cord}} = I^2 R_{\text{cord}} = \left( \frac{V}{R_{\text{eq}}} \right)^2 R_{\text{cord}} = \left( \frac{120 \text{ V}}{0.020 \Omega + 9.6 \Omega} \right)^2 (0.020 \Omega) = \boxed{3.1 \text{ W}}$$

$$(b) \quad P_{\text{he}} = I^2 R_{\text{he}} = \left( \frac{V}{R_{\text{eq}}} \right)^2 R_{\text{he}} = \left( \frac{120 \text{ V}}{0.020 \, \Omega + 9.6 \, \Omega} \right)^2 (9.6 \, \Omega) = \boxed{1.5 \text{ kW}}$$

30. Connect the 220- $\Omega$  and 79- $\Omega$  resistors in parallel. Then connect this pair in series with the 92- $\Omega$  resistor.

$$\left( \frac{1}{220 \, \Omega} + \frac{1}{79 \, \Omega} \right)^{-1} + 92 \, \Omega = 150 \, \Omega$$

$$31. (a) \quad I = \frac{V}{R_{\text{eq}}} = \frac{9.0 \text{ V}}{42 \, \Omega + 17 \, \Omega + 110 \, \Omega} = \boxed{53 \text{ mA}}$$

$$(b) \quad V_i = IR_i = \frac{V}{R_{\text{eq}}} R_i$$

$$V_{42} = \left( \frac{9.0 \text{ V}}{169 \, \Omega} \right) (42 \, \Omega) = \boxed{2.2 \text{ V}}$$

$$V_{17} = \left( \frac{9.0 \text{ V}}{169 \, \Omega} \right) (17 \, \Omega) = \boxed{0.91 \text{ V}}$$

$$V_{110} = \left( \frac{9.0 \text{ V}}{169 \, \Omega} \right) (110 \, \Omega) = \boxed{5.9 \text{ V}}$$

$$32. (a) \quad \frac{V}{I} = R_{\text{eq}} = R_1 + R_2 + R$$

$$R = \frac{V}{I} - R_1 - R_2 = \frac{18.0 \text{ V}}{0.16 \text{ A}} - 11 \, \Omega - 53 \, \Omega = \boxed{50 \, \Omega}$$

$$(b) \quad V_i = IR_i = \frac{V}{R_{\text{eq}}} R_i$$

$$V_{11} = \left( \frac{18.0 \text{ V}}{112.5 \, \Omega} \right) (11 \, \Omega) = \boxed{1.8 \text{ V}}$$

$$V_{53} = \left( \frac{18.0 \text{ V}}{112.5 \, \Omega} \right) (53 \, \Omega) = \boxed{8.5 \text{ V}}$$

$$V_R = \left( \frac{18.0 \text{ V}}{112.5 \, \Omega} \right) (48.5 \, \Omega) = \boxed{7.8 \text{ V}}$$

- (c) Since  $R \propto V$ , the answer to part (a) would have been larger if  $V > 18.0 \text{ V}$ .

$$33. (a) \quad \mathcal{E} = IR_{\text{eq}} = (1.3 \text{ A}) \left( \frac{1}{65 \, \Omega} + \frac{1}{25 \, \Omega} + \frac{1}{170 \, \Omega} \right)^{-1} = \boxed{21 \text{ V}}$$

$$(b) \quad I_i = \frac{V}{R_i}$$

$$I_{65} = \frac{21.22 \text{ V}}{65 \, \Omega} = \boxed{0.33 \text{ A}}$$

$$I_{25} = \frac{21.22 \text{ V}}{25 \, \Omega} = \boxed{0.85 \text{ A}}$$

$$I_{170} = \frac{21.22 \text{ V}}{170 \, \Omega} = \boxed{0.12 \text{ A}}$$

$$34. (a) \quad \frac{I}{\mathcal{E}} = \frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R}$$

$$R = \left( \frac{I}{\mathcal{E}} - \frac{1}{R_1} - \frac{1}{R_2} \right)^{-1} = \left( \frac{0.88 \text{ A}}{12.0 \text{ V}} - \frac{1}{22 \, \Omega} - \frac{1}{67 \, \Omega} \right)^{-1} = \boxed{77 \, \Omega}$$

$$(b) \quad I_i = \frac{\mathcal{E}}{R_i}$$

$$I_{22} = \frac{12.0 \text{ V}}{22 \, \Omega} = \boxed{0.55 \text{ A}}$$

$$I_{67} = \frac{12.0 \text{ V}}{67 \, \Omega} = \boxed{0.18 \text{ A}}$$

$$I_R = \frac{12.0 \text{ V}}{77 \, \Omega} = \boxed{0.16 \text{ A}}$$

(c) Since  $R \propto \frac{1}{I}$ , the answer to part (a) would have been **smaller** if  $I > 0.88 \text{ A}$ .

$$35. \quad \mathcal{E} = IR_{\text{eq}} = (0.62 \text{ A})(84 \, \Omega + 130 \, \Omega) = \boxed{130 \text{ V}}$$

$$36. \quad 26 \, \Omega = 12 \, \Omega + \left( \frac{1}{R} + \frac{1}{55 \, \Omega} \right)^{-1}$$

$$\frac{1}{14 \, \Omega} = \frac{1}{R} + \frac{1}{55 \, \Omega}$$

$$\frac{1}{R} = \frac{1}{14 \, \Omega} - \frac{1}{55 \, \Omega}$$

$$R = \left( \frac{1}{14 \, \Omega} - \frac{1}{55 \, \Omega} \right)^{-1}$$

$$= \boxed{19 \, \Omega}$$

$$37. \quad \frac{1}{R_{\text{eq}}} = \frac{1}{1.5 \, \Omega} + \frac{1}{2.5 \, \Omega} + \frac{1}{6.3 \, \Omega + \left( \frac{1}{4.8 \, \Omega} + \frac{1}{3.3 \, \Omega} + \frac{1}{8.1 \, \Omega} \right)^{-1}}$$

$$R_{\text{eq}} = \boxed{0.838 \, \Omega}$$

38. Find the resistance of one light bulb.

$$R = \frac{V^2}{P}$$

Find the minimum allowed equivalent resistance.

$$R_{\text{eq}} = \frac{V}{I_{\text{max}}}$$

Find the number of light bulbs.

$$\frac{1}{R_{\text{eq}}} = \frac{n}{R}$$

$$n = \frac{R}{R_{\text{eq}}} = \left( \frac{V^2}{P} \right) \left( \frac{I_{\text{max}}}{V} \right) = \frac{VI_{\text{max}}}{P} = \frac{(85 \text{ V})(2.1 \text{ A})}{65 \text{ W}} = 2.7$$

So, only **two** light bulbs can be connected in parallel before the total current exceeds 2.1 A.

39. Determine the current flowing through the entire circuit.

$$R_{\text{eq}} = 0.50 \, \Omega + 4.5 \, \Omega + 1.0 \, \Omega + \left( \frac{1}{3.2 \, \Omega} + \frac{1}{7.1 \, \Omega + 5.8 \, \Omega} \right)^{-1} = 8.6 \, \Omega$$

$$I = \frac{V}{R_{\text{eq}}} = \frac{12 \text{ V}}{8.564 \, \Omega} = 1.4 \text{ A}$$

- (a) The voltage where the circuits become parallel is  
 $12 \text{ V} - (1.40 \text{ A})(0.50 \, \Omega + 4.5 \, \Omega + 1.0 \, \Omega) = 3.6 \text{ V}$

$$I = \frac{3.6 \text{ V}}{3.2 \, \Omega} = \mathbf{1.1 \text{ A}}$$

The remaining current of  $1.4 \text{ A} - 1.1 \text{ A} = \mathbf{0.3 \text{ A}}$  flows through the branch containing the  $7.1 \, \Omega$  resistor.

- (b) So **1.4 A** flow through the battery.

(c)  $V_{\text{bat}} = \mathcal{E} - Ir = 12 \text{ V} - (1.40 \text{ A})(0.50 \, \Omega) = \mathbf{11.3 \text{ V}}$

40.  $R_{\text{eq}} = 12 \, \Omega + \left( \frac{1}{55 \, \Omega} + \frac{1}{75 \, \Omega} \right)^{-1} = 43.7 \, \Omega$

(a)  $I_{12} = \frac{12 \text{ V}}{43.7 \, \Omega} = \mathbf{0.27 \text{ A}}$

The voltage across the parallel part of the circuit is  $12 \text{ V} - (0.275 \text{ A})(12 \, \Omega) = 8.7 \text{ V}$ .

$$55 \, \Omega: I_{55} = \frac{8.7 \text{ V}}{55 \, \Omega} = \mathbf{0.16 \text{ A}}$$

$$75 \, \Omega: I_{75} = 0.275 \text{ A} - 0.158 \text{ A} = \mathbf{0.12 \text{ A}}$$

- (b) Increasing  $R$  causes the resistance of the parallel path and of the entire circuit to increase. As a result the total current decreases. When the current divides going through the parallel section the  $55 \, \Omega$  resistor gets an increased share, while the larger resistance gets a decreased share. Thus, **the current through the  $12 \, \Omega$  resistor and  $R$  decreases; the current through the  $55 \, \Omega$  resistor increases.**



41. (a)  $1.5\ \Omega: I_{1.5} = \frac{9.0\ \text{V}}{1.5\ \Omega} = \boxed{6.0\ \text{A}}$

$2.5\ \Omega: I_{2.5} = \frac{9.0\ \text{V}}{2.5\ \Omega} = \boxed{3.6\ \text{A}}$

$R_{\text{eq}} = \left[ \left( \frac{1}{4.8\ \Omega} + \frac{1}{3.3\ \Omega} + \frac{1}{8.1\ \Omega} \right)^{-1} + 6.3\ \Omega \right] = 7.88\ \Omega$

$6.3\ \Omega: I_{6.3} = \frac{9.0\ \text{V}}{7.88\ \Omega} = \boxed{1.1\ \text{A}}$

$V = 9.0\ \text{V} - (1.1\ \text{A})(6.3\ \Omega) = 1.82\ \text{V}$

$4.8\ \Omega: I_{4.8} = \frac{1.82\ \text{V}}{4.8\ \Omega} = \boxed{0.38\ \text{A}}$

$3.3\ \Omega: I_{3.3} = \frac{1.82\ \text{V}}{3.3\ \Omega} = \boxed{0.55\ \text{A}}$

$8.1\ \Omega: I_{8.1} = \frac{1.82\ \text{V}}{8.1\ \Omega} = \boxed{0.22\ \text{A}}$

- (b) A potential difference occurs across the 3 resistor parallel combination that is in series with the  $6.3\ \Omega$  resistor, resulting in a potential difference less than  $9.0\ \text{V}$  across the  $6.3\ \Omega$  resistor.

42. (a)  $R_{\text{eq}} = 0.25\ \Omega + 4.5\ \Omega + 1.0\ \Omega + \left[ \frac{1}{3.2\ \Omega} + \frac{1}{7.1\ \Omega + 5.8\ \Omega} \right]^{-1} = 8.31\ \Omega$

$I = \frac{12\ \text{V}}{8.31\ \Omega} = \boxed{1.4\ \text{A}}$

(b)  $V = 12\ \text{V} - (1.4\ \text{A})(0.25\ \Omega) = \boxed{11.6\ \text{V}}$

- (c) Increasing any resistance in the parallel section increases the resistance of the parallel section. That causes an increase in the equivalent resistance of circuit. The current will decrease.

43. (a) The  $13.8\ \Omega$ ,  $17.2\ \Omega$ , and  $(8.45\ \Omega + 4.11\ \Omega)$  paths are connected in parallel. Therefore the potential difference across each path is  $V = (1.22\ \text{A})(8.45\ \Omega + 4.11\ \Omega) = 15.32\ \text{V}$

$13.8\ \Omega: I = \frac{15.32\ \text{V}}{13.8\ \Omega} = 1.11\ \text{A}$

$17.2\ \Omega: I = \frac{15.32\ \text{V}}{17.2\ \Omega} = 0.891\ \text{A}$

Total current in the circuit is  $1.22\ \text{A} + 1.11\ \text{A} + 0.891\ \text{A} = 3.22\ \text{A}$

$15.0\ \Omega: V = (3.22\ \text{A})(15.0\ \Omega) = 48.3\ \text{V}$

$12.5\ \Omega: V = (3.22\ \text{A})(12.5\ \Omega) = 40.3\ \text{V}$

$\mathcal{E} = 48.3\ \text{V} + 40.3\ \text{V} + 15.3\ \text{V}$

$= \boxed{103.9\ \text{V}}$

- (b) Increasing the resistance of one of the parallel paths increases the total resistance of the parallel section. Since it is in series with the other two resistors, the equivalent resistance of the circuit increases, causing a decrease in the current in the circuit.

44. Find the current flowing through the  $17.2\text{-}\Omega$  resistor.

$$(0.750\text{ A})(13.8\text{ }\Omega) = I(17.2\text{ }\Omega)$$

$$I = \left( \frac{13.8\text{ }\Omega}{17.2\text{ }\Omega} \right) (0.750\text{ A}) = \boxed{0.602\text{ A}}$$

Find the current flowing through the  $8.45\text{-}\Omega$  and the  $4.11\text{-}\Omega$  resistors.

$$I(8.45\text{ }\Omega + 4.11\text{ }\Omega) = (13.8\text{ }\Omega)(0.750\text{ A})$$

$$I = \frac{(13.8\text{ }\Omega)(0.750\text{ A})}{8.45\text{ }\Omega + 4.11\text{ }\Omega}$$

$$= \boxed{0.824\text{ A}}$$

The current flowing through the entire circuit is  $0.750\text{ A} + 0.602\text{ A} + 0.824\text{ A} = 2.176\text{ A}$ . So, the current flowing through the  $15.0\text{-}\Omega$  and  $12.5\text{-}\Omega$  resistors is  $\boxed{2.176\text{ A}}$ .

45. (a) Because the resistors are identical, the potential is the same at both sides of the switch regardless of whether it is open or closed. So, the current through the battery will stay the same.

- (b) Switch open:

$$I_0 = \frac{\mathcal{E}}{R_{\text{eq}}} = \mathcal{E} \left( \frac{1}{2R} + \frac{1}{2R} \right) = \frac{\mathcal{E}}{R}$$

Switch closed:

$$I = \frac{\mathcal{E}}{R_{\text{eq}}} = \frac{\mathcal{E}}{\frac{R}{2} + \frac{R}{2}} = \frac{\mathcal{E}}{R} = \boxed{I_0}$$

46. Assume that  $I$  is clockwise.

$$0 = 11.5\text{ V} - I(6.22\text{ }\Omega) - I(15.1\text{ }\Omega) + 15.0\text{ V} - I(8.50\text{ }\Omega)$$

$$I = \frac{26.5\text{ V}}{29.82\text{ }\Omega} = \boxed{0.889\text{ A}}$$

The current flows clockwise.

47. (a) Since the two batteries now oppose each other, the net emf is less than before, and so the current will decrease.

- (b) Assume that  $I$  is clockwise.

$$0 = -11.5\text{ V} - I(6.22\text{ }\Omega) - I(15.1\text{ }\Omega) + 15.0\text{ V} - I(8.50\text{ }\Omega)$$

$$I = \boxed{0.12\text{ A}}$$

The current flows clockwise.

48. The batteries are oriented to drive the current clockwise.

- (a) The direction of current flow produces a potential drop from  $A$  to  $B$ , so the potential at  $B$  is less than zero.

- (b) The direction of current flow produces a potential drop from  $C$  to  $A$ , so the potential at  $C$  is greater than zero.

$$(c) \quad V_D = V_A - IR_{15.1} + V_{15.0} \quad \left( \text{with } I = \frac{\mathcal{E}_{\text{tot}}}{R_{\text{eq}}} \right)$$

$$= 0 - \left( \frac{15.0\text{ V} + 11.5\text{ V}}{8.50\text{ }\Omega + 6.22\text{ }\Omega + 15.1\text{ }\Omega} \right) (15.1\text{ }\Omega) + 15.0\text{ V}$$

$$= \boxed{1.6\text{ V}}$$

$$49. (a) I = \frac{V}{R_{eq}} = \frac{15 \text{ V}}{\left(\frac{1}{7.5 \Omega} + \frac{1}{6.2 \Omega + 12 \Omega}\right)^{-1} + 11 \Omega} = 0.92 \text{ A}$$

The current flowing through the  $11 \Omega$  resistor is  $\boxed{0.92 \text{ A}}$ .

$$(0.92 \text{ A} - I)(7.5 \Omega) = I(6.2 \Omega + 12 \Omega)$$

$$6.9 \text{ V} = I(25.7 \Omega)$$

$$I = 0.27 \text{ A}$$

The current flowing through the  $6.2 \Omega$  and  $12 \Omega$  resistors is  $\boxed{0.27 \text{ A}}$ .

The current flowing through the  $7.5 \Omega$  resistor is  $0.92 \text{ A} - 0.27 \text{ A} = \boxed{0.65 \text{ A}}$ .

$$(b) 0 = 15 \text{ V} - I_1(7.5 \Omega) - I(11 \Omega) \dots\dots\dots (I)$$

$$0 = 15 \text{ V} - I_2(6.2 \Omega) - I_2(12 \Omega) - I(11 \Omega) \dots\dots (II)$$

$$0 = I_2(7.5 \Omega) - I_2(6.2 \Omega) - I_2(12 \Omega) \dots\dots\dots (III)$$

Solve (III) for  $I_1$ .

$$I_1 = 2.4I_2$$

$$I = I_1 + I_2 = 3.4I_2$$

Substitute the result above into (II) and solve for  $I_2$ .

$$I_2 = \boxed{0.27 \text{ A}}$$

$$I_1 = 2.4(0.27 \text{ A}) = \boxed{0.65 \text{ A}}$$

$$I = 3.4(0.27 \text{ A}) = \boxed{0.92 \text{ A}}$$

$$50. I = \frac{V}{R_{eq}} = \frac{15 \text{ V}}{\left(\frac{1}{7.5 \Omega} + \frac{1}{6.2 \Omega + 12 \Omega}\right)^{-1} + 11 \Omega} = 0.92 \text{ A}$$

$$V_C = V_A - I(11 \Omega) = 0 - (0.92 \text{ A})(11 \Omega) = \boxed{-10 \text{ V}}$$

$$V_B = 15 \text{ V} - 10.1 \text{ V} = \boxed{4.9 \text{ V}}$$

$$51. (a) I = I_1 + I_2 \dots\dots\dots (I)$$

$$0 = 12 \text{ V} - I(3.9 \Omega) - I_1(1.2 \Omega) - I(9.8 \Omega)$$

$$I_1 = 10 \text{ A} - \frac{13.7}{1.2} I \dots\dots\dots (II)$$

$$0 = 12 \text{ V} - I(3.9 \Omega) - I_2(6.7 \Omega) - 9.0 \text{ V} - I(9.8 \Omega)$$

$$I_2 = \frac{3}{6.7} \text{ A} - \frac{13.7}{6.7} I \dots\dots\dots (III)$$

Substitute (II) and (III) into (I).

$$I = 10 \text{ A} - \frac{13.7}{1.2} I + \frac{3}{6.7} \text{ A} - \frac{13.7}{6.7} I$$

$$\left(1 + \frac{13.7}{1.2} + \frac{13.7}{6.7}\right) I = 10 \text{ A} + \frac{3}{6.7} \text{ A}$$

$$I = \frac{10 \text{ A} + \frac{3}{6.7} \text{ A}}{1 + \frac{13.7}{1.2} + \frac{13.7}{6.7}}$$

$$I = 0.72 \text{ A}$$

$$I_1 = 10 \text{ A} - \frac{13.7}{1.2} \left( \frac{10 \text{ A} + \frac{3}{6.7} \text{ A}}{1 + \frac{13.7}{1.2} + \frac{13.7}{6.7}} \right) = 2 \text{ A}$$

$$I_2 = \frac{3}{6.7} \text{ A} - \frac{13.7}{6.7} \left( \frac{10 \text{ A} + \frac{3}{6.7} \text{ A}}{1 + \frac{13.7}{1.2} + \frac{13.7}{6.7}} \right) = -1.0 \text{ A}$$

The currents through each resistor are as follows:

$$\boxed{3.9 \text{ } \Omega, 9.8 \text{ } \Omega : 0.72 \text{ A}; 1.2 \text{ } \Omega : 2 \text{ A}; 6.7 \text{ } \Omega : 1.0 \text{ A.}}$$

- (b) The potential at point  $A$  is greater than that at point  $B$  because the potential has been decreased by the  $1.2\text{-}\Omega$  resistor between  $A$  and  $B$ .

$$(c) \quad V_A - V_B = \left[ 10 \text{ A} - \frac{13.7}{1.2} \left( \frac{10 \text{ A} + \frac{3}{6.7} \text{ A}}{1 + \frac{13.7}{1.2} + \frac{13.7}{6.7}} \right) \right] (1.2 \text{ } \Omega) = \boxed{2 \text{ V}}$$

52. (a)  $I = I_1 + I_2$ ..... (I)

$$0 = 9.0 \text{ V} - I(5.0 \text{ } \Omega) - I_1(4.0 \text{ } \Omega)$$

$$I_1 = 2.25 \text{ A} - 1.25I$$
.....(II)

$$0 = 9.0 \text{ V} - 6.0 \text{ V} + I_2(2.0 \text{ } \Omega) - I_1(4.0 \text{ } \Omega)$$

$$I_2 = 2.0 I_1 - 1.5 \text{ A}$$
.....(III)

Substitute (II) into (III).

$$I_2 = 4.5 \text{ A} - 2.5I - 1.5 \text{ A} = 3.0 \text{ A} - 2.5I$$
.....(IV)

Substitute (II) and (IV) into (I).

$$I = 2.25 \text{ A} - 1.25I + 3.0 \text{ A} - 2.5I$$

$$4.75I = 5.25 \text{ A}$$

$$I = 1.1 \text{ A}$$

$$I_1 = 2.25 \text{ A} - 1.25(1.11 \text{ A}) = 0.9 \text{ A}$$

$$I_2 = 3.0 \text{ A} - 2.5(1.11 \text{ A}) = 0.2 \text{ A}$$

$$\boxed{\text{About } 0.9 \text{ A flows through the } 9.0 \text{ V battery and about } 0.2 \text{ A flows through the } 6.0 \text{ V battery.}}$$

(b)  $0 = 9.0 \text{ V} - 6.0 \text{ V} - I(2.0 \text{ } \Omega) - I(4.0 \text{ } \Omega)$

$$I = 0.50 \text{ A}$$

$$\boxed{0.50 \text{ A}} \text{ flows through both batteries.}$$

53.  $C_{\text{eq}} = 15 \text{ } \mu\text{F} + \left( \frac{1}{8.2 \text{ } \mu\text{F}} + \frac{1}{22 \text{ } \mu\text{F}} \right)^{-1} = \boxed{21 \text{ } \mu\text{F}}$

54.  $C_{\text{eq}} = \left( \frac{1}{4.5 \text{ } \mu\text{F}} + \frac{1}{12 \text{ } \mu\text{F}} + \frac{1}{32 \text{ } \mu\text{F}} \right)^{-1} = 3.0 \text{ } \mu\text{F}$

$$Q = C_{\text{eq}}V = (3.0 \times 10^{-6} \text{ F})(15 \text{ V}) = 45 \text{ } \mu\text{C}$$

$$V_{32} = \frac{Q}{C_{32}} = \frac{45 \text{ } \mu\text{C}}{32 \text{ } \mu\text{F}} = \boxed{1.4 \text{ V}}$$

$$55. U_{15} = \frac{1}{2} CV^2 = \frac{1}{2} (15 \times 10^{-6} \text{ F})(9.0 \text{ V})^2 = \boxed{6.1 \times 10^{-4} \text{ J}}$$

$$C_{\text{eq}} = \left( \frac{1}{8.2 \mu\text{F}} + \frac{1}{22 \mu\text{F}} \right)^{-1} = 6.0 \mu\text{F}$$

$$Q = C_{\text{eq}} V = (6.0 \times 10^{-6} \text{ F})(9.0 \text{ V}) = 54 \mu\text{C}$$

$$U_{8.2} = \frac{1}{2} CV^2 = \frac{1}{2} C \left( \frac{Q}{C} \right)^2 = \frac{Q^2}{2C} = \frac{(5.4 \times 10^{-5} \text{ C})^2}{2(8.2 \times 10^{-6} \text{ F})} = \boxed{1.8 \times 10^{-4} \text{ J}}$$

$$U_{22} = \frac{Q^2}{2C} = \frac{(5.4 \times 10^{-5} \text{ C})^2}{2(22 \times 10^{-6} \text{ F})} = \boxed{6.6 \times 10^{-5} \text{ J}}$$

$$56. \text{ (a) } C_{\text{eq}} = 7.5 \mu\text{F} + 15 \mu\text{F} = \boxed{23 \mu\text{F}}$$

(b) Since each capacitor has the same voltage across its plates, and since  $Q = CV$ , the 15- $\mu\text{F}$  capacitor stores more charge.

$$\text{(c) } Q_{7.5} = (7.5 \times 10^{-6} \text{ F})(12 \text{ V}) = \boxed{90 \mu\text{C}}$$

$$Q_{15} = (15 \times 10^{-6} \text{ F})(12 \text{ V}) = \boxed{180 \mu\text{C}}$$

$$57. \text{ (a) } C_{\text{eq}} = \left( \frac{1}{7.5 \mu\text{F}} + \frac{1}{15 \mu\text{F}} \right)^{-1} = \boxed{5.0 \mu\text{F}}$$

(b) The battery causes the first capacitor to acquire a positive charge on one plate, which in turn causes its other plate to acquire an equal negative charge. Since there was no net charge between the capacitors initially, a positive charge is acquired on the plate connected to the negatively charged plate of the first capacitor, which in turn causes its other plate to acquire an equal negative charge. All these charges are equal in magnitude, so neither capacitor stores more charge. Their charges are the same.

$$\text{(c) } Q = C_{\text{eq}} V = (5.0 \times 10^{-6} \text{ F})(12 \text{ V}) = \boxed{60 \mu\text{C}}$$

$$58. 9.22 \mu\text{F} = 7.22 \mu\text{F} + \left\{ \frac{1}{C} + \left[ 4.25 \mu\text{F} + \left( \frac{1}{12.0 \mu\text{F}} + \frac{1}{8.35 \mu\text{F}} \right)^{-1} \right]^{-1} \right\}^{-1}$$

$$\frac{1}{2.00 \mu\text{F}} = \frac{1}{C} + \frac{1}{9.17 \mu\text{F}}$$

$$C = \left( \frac{1}{2.00 \mu\text{F}} - \frac{1}{9.17 \mu\text{F}} \right)^{-1} = \boxed{2.56 \mu\text{F}}$$

$$59. U_1 + U_2 = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 = \frac{1}{2}C_1\left(\frac{Q}{C_1}\right)^2 + \frac{1}{2}C_2\left(\frac{Q}{C_2}\right)^2 = \frac{1}{2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)Q^2$$

$$U_{\text{eq}} = \frac{1}{2}C_{\text{eq}}V^2 = \frac{1}{2}C_{\text{eq}}\left(\frac{Q}{C_{\text{eq}}}\right)^2 = \frac{1}{2}\left(\frac{1}{C_{\text{eq}}}\right)Q^2 = \frac{1}{2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)Q^2$$

Therefore,  $U_1 + U_2 = U_{\text{eq}}$ .

60. Find the initial charge on the 11.2- $\mu\text{F}$  capacitor.

$$Q_1 = CV_1 = (11.2 \times 10^{-6} \text{ F})(12.0 \text{ V}) = 134 \mu\text{C}$$

After the switch is moved to position  $B$ ,  $Q_1$  will be shared among the two capacitors such that the voltage across each is the same.

$$V = \frac{Q_1 - Q_2}{C_1} = \frac{Q_2}{C_2}$$

$$\frac{C_1}{C_2}Q_2 = Q_1 - Q_2$$

$$\begin{aligned} Q_2 &= \frac{Q_1}{1 + \frac{C_1}{C_2}} \\ &= \frac{134.4 \mu\text{C}}{1 + \frac{11.2 \mu\text{F}}{9.50 \mu\text{F}}} \\ &= 61.7 \mu\text{C} \end{aligned}$$

$$V = \frac{Q_2}{C_2} = \frac{61.7 \mu\text{C}}{9.50 \mu\text{F}} = \boxed{6.49 \text{ V}}$$

61.  $q(t) = C\mathcal{E}(1 - e^{-t/\tau})$

$$\begin{aligned} q(4.2 \text{ ms}) &= (23 \times 10^{-6} \text{ F})(6.0 \text{ V}) \left\{ 1 - e^{-(4.2 \times 10^{-3} \text{ s})/[(150 \Omega)(23 \times 10^{-6} \text{ F})]} \right\} \\ &= \boxed{97 \mu\text{C}} \end{aligned}$$

62. (a)  $q(\tau) = C\mathcal{E}(1 - e^{-\tau/\tau})$

$$\begin{aligned} &= (45 \times 10^{-6} \text{ F})(9.0 \text{ V})(1 - e^{-1}) \\ &= \boxed{2.6 \times 10^{-4} \text{ C}} \end{aligned}$$

$$(b) I(\tau) = \frac{\mathcal{E}}{R} e^{-\tau/\tau} = \frac{9.0 \text{ V}}{120 \Omega} e^{-1} = \boxed{28 \text{ mA}}$$

63. (a)  $\tau = RC = (175 \Omega)(55.7 \times 10^{-6} \text{ F}) = \boxed{9.75 \text{ ms}}$

- (b)  $q(t) = C\mathcal{E}(1 - e^{-t/\tau})$

$q \rightarrow q_{\text{max}}$  as  $t \rightarrow \infty$ .

$$q_{\text{max}} = C\mathcal{E} = (55.7 \times 10^{-6} \text{ F})(12.0 \text{ V}) = \boxed{6.68 \times 10^{-4} \text{ C}}$$

$$(c) \quad I(t) = \frac{\mathcal{E}}{R} e^{-t/\tau}$$

$$\begin{aligned} I(0) &= \frac{\mathcal{E}}{R} \\ &= \frac{12.0 \text{ V}}{175 \, \Omega} \\ &= \boxed{68.6 \text{ mA}} \end{aligned}$$

$$64. (a) \quad C = \frac{\tau}{R} = \frac{2.5 \times 10^{-3} \text{ s}}{145 \, \Omega} = \boxed{17 \, \mu\text{F}}$$

$$\begin{aligned} (b) \quad I(t) &= \frac{\mathcal{E}}{R} e^{-t/\tau} \\ I(5.0 \text{ ms}) &= \frac{9.0 \text{ V}}{145 \, \Omega} e^{-(5.0 \text{ ms})/(2.5 \text{ ms})} \\ &= \boxed{8.4 \text{ mA}} \end{aligned}$$

$$\begin{aligned} 65. \quad q &= C\mathcal{E}(1 - e^{-t/RC}) \\ \frac{q}{C\mathcal{E}} &= 1 - e^{-t/RC} \\ 1 - \frac{q}{C\mathcal{E}} &= e^{-t/RC} \\ \ln\left(1 - \frac{q}{C\mathcal{E}}\right) &= -\frac{t}{RC} \\ R &= -\frac{t}{C \ln\left(1 - \frac{q}{C\mathcal{E}}\right)} \\ &= -\frac{25 \text{ s}}{(1200 \times 10^{-6} \text{ F}) \ln(1 - 0.90)} \\ &= \boxed{9.0 \text{ k}\Omega} \end{aligned}$$

$$\begin{aligned} 66. \quad q &= C\mathcal{E}(1 - e^{-t/RC}) \\ \frac{CV}{C\mathcal{E}} &= 1 - e^{-t/RC} \quad (q = CV) \\ e^{-t/RC} &= 1 - \frac{V}{\mathcal{E}} \\ -\frac{t}{RC} &= \ln\left(1 - \frac{V}{\mathcal{E}}\right) \\ t &= -RC \ln\left(1 - \frac{V}{\mathcal{E}}\right) \\ &= -(50.0 \times 10^3 \, \Omega)(140 \times 10^{-6} \text{ F}) \ln\left(1 - \frac{5.0 \text{ V}}{9.0 \text{ V}}\right) \\ &= \boxed{5.7 \text{ s}} \end{aligned}$$

67. (a)  $\tau = R_{\text{eq}} C = \left( \frac{1}{24 \, \Omega} + \frac{1}{13 \, \Omega + 6.5 \, \Omega} \right)^{-1} (62 \times 10^{-6} \text{ F}) = \boxed{6.7 \times 10^{-4} \text{ s}}$

(b)  $I(0) = \frac{\mathcal{E}}{R_{\text{eq}}} e^{-0/\tau} = (15 \text{ V}) \left( \frac{1}{24 \, \Omega} + \frac{1}{13 \, \Omega + 6.5 \, \Omega} \right) (1) = \boxed{1.4 \text{ A}}$

- (c) To increase the time constant, the equivalent resistance of the three resistors must be increased. Since increasing the resistance of the  $6.5 \, \Omega$  resistor increases the equivalent resistance, it should be increased to increase the time constant.

68. (a)  $0.50 C \mathcal{E} = C \mathcal{E} (1 - e^{-t/RC})$

$$0.50 = 1 - e^{-t/RC}$$

$$e^{-t/RC} = 1 - 0.50$$

$$-\frac{t}{RC} = \ln 0.50$$

$$t = -RC \ln 0.50$$

$$= \boxed{RC \ln 2.0}$$

(b)  $0.10 \frac{\mathcal{E}}{R} = \frac{\mathcal{E}}{R} e^{-t/RC}$

$$0.10 = e^{-t/RC}$$

$$\ln 0.10 = -\frac{t}{RC}$$

$$t = \boxed{RC \ln 10}$$

69.  $I = I_1 + I_2 \dots \dots \dots (I)$

$$0 = 12 \text{ V} - I(12 \, \Omega) - I_1(75 \, \Omega)$$

$$I_1 = \frac{12}{75} \text{ A} - \frac{12}{75} I \dots \dots \dots (II)$$

$$0 = 12 \text{ V} - I(12 \, \Omega) - I_2(55 \, \Omega)$$

$$I_2 = \frac{12}{55} \text{ A} - \frac{12}{55} I \dots \dots \dots (III)$$

Substitute (II) and (III) into (I).

$$I = \frac{12}{75} \text{ A} - \frac{12}{75} I + \frac{12}{55} \text{ A} - \frac{12}{55} I$$

$$I = 0.27 \text{ A}$$

$$I_1 = \frac{12}{75} \text{ A} - \frac{12}{75} (0.27 \text{ A}) = 0.12 \text{ A}$$

$$I_2 = \frac{12}{55} \text{ A} - \frac{12}{55} (0.27 \text{ A}) = 0.16 \text{ A}$$

$$P = I^2 R$$

$$P_{12} = (0.274 \text{ A})^2 (12 \, \Omega) = \boxed{0.90 \text{ W}}$$

$$P_{75} = (0.116 \text{ A})^2 (75 \, \Omega) = \boxed{1.0 \text{ W}}$$

$$P_{55} = (0.16)^2 (55 \, \Omega) = \boxed{1.4 \text{ W}}$$



70. Connect the  $521\text{-}\Omega$  and  $146\text{-}\Omega$  resistors in series. Then, connect this pair in parallel with the  $413\text{-}\Omega$  resistor.

$$R_{\text{eq}} = \left( \frac{1}{521\ \Omega + 146\ \Omega} + \frac{1}{413\ \Omega} \right)^{-1} = 255\ \Omega$$

71. Connect the  $7.2\text{-}\mu\text{F}$  and  $9.0\text{-}\mu\text{F}$  capacitors in series. Then, connect this pair in parallel with the  $18\text{-}\mu\text{F}$  capacitor.

$$C_{\text{eq}} = \left( \frac{1}{7.2\ \mu\text{F}} + \frac{1}{9.0\ \mu\text{F}} \right)^{-1} + 18\ \mu\text{F} = 22\ \mu\text{F}$$

72.  $I = \frac{65 \times 10^{-6}\ \text{C}}{0.65\ \text{h}} \left( \frac{1\ \text{h}}{3600\ \text{s}} \right) = \boxed{2.8 \times 10^{-8}\ \text{A}}$

73.  $R = \rho \frac{L}{A} = \rho \frac{L}{\frac{1}{4}\pi d^2}$

$$d = \sqrt{\frac{4\rho L}{\pi R}} = \sqrt{\frac{4(5.6 \times 10^{-8}\ \Omega \cdot \text{m})(2.0\ \text{m})}{\pi(5.0\ \Omega)}} = \boxed{0.17\ \text{mm}}$$

74.  $V = IR = I\rho \frac{L}{A} = (5.0\ \text{A})(1.72 \times 10^{-8}\ \Omega \cdot \text{m}) \frac{(24\ \text{ft})\left(\frac{0.3048\ \text{m}}{\text{ft}}\right)}{1.17 \times 10^{-6}\ \text{m}^2} = \boxed{0.54\ \text{V}}$

75. (a)  $R_1 = \frac{V^2}{P} = \frac{(120\ \text{V})^2}{75.0\ \text{W}} = \boxed{190\ \Omega}$

(b)  $R_{\text{eq}} = \frac{V^2}{P} = R_1 + R_2$

$$R_2 = \frac{V^2}{P} - R_1 = \frac{(120\ \text{V})^2}{50.0\ \text{W}} - \frac{(120\ \text{V})^2}{75.0\ \text{W}} = \boxed{96\ \Omega}$$

(c)  $P = \frac{V^2}{R_2} = \frac{(120\ \text{V})^2}{96\ \Omega} = \boxed{150\ \text{W}}$

76. (a)  $E = IV\Delta t = (7.5 \times 10^{-3}\ \text{A})(3.5\ \text{V})(35\ \text{s}) = \boxed{0.92\ \text{J}}$

(b)  $\Delta t = \frac{E}{P} = \frac{mgh}{IV} = \frac{(0.65\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(1\ \text{m})}{(7.5 \times 10^{-3}\ \text{A})(3.5\ \text{V})} = \boxed{4\ \text{min}}$

$$\begin{aligned}
 77. \quad mc\Delta T &= \frac{V^2}{R} \Delta t \\
 V &= \sqrt{\frac{mc\Delta TR}{\Delta t}} \\
 &= \sqrt{\frac{(4.2 \text{ kg})\left(4186 \frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(10 \text{ K})(250 \Omega)}{(3.5 \text{ min})\left(\frac{60 \text{ s}}{\text{min}}\right)}} \\
 &= \boxed{460 \text{ V}}
 \end{aligned}$$

78. (a) Immediately after the switch is thrown, the voltage across the capacitor is zero, so  $I_0 = \frac{9.0 \text{ V}}{11 \Omega}$ . Long after the switch is thrown, no current flows through the capacitor, so  $I = \frac{9.0 \text{ V}}{11 \Omega + 5.6 \Omega} < \frac{9.0 \text{ V}}{11 \Omega}$ . Therefore, the initial current is **greater than** the final current.

$$(b) \quad I = \frac{9.0 \text{ V}}{11 \Omega} = \boxed{0.82 \text{ A}}$$

$$(c) \quad I = \frac{9.0 \text{ V}}{11 \Omega + 5.6 \Omega} = \boxed{0.54 \text{ A}}$$

$$\begin{aligned}
 79. \quad V_s &= V_c \\
 \pi r_s^2 L_s &= \pi r_c^2 L_c \\
 L_s &= \left(\frac{r_c}{r_s}\right)^2 L_c \\
 R_s &= R_c \\
 \rho_s \frac{L_s}{\pi r_s^2} &= \rho_c \frac{L_c}{\pi r_c^2} \\
 \rho_s \frac{L_s}{r_s^2} &= \rho_c \frac{L_c}{r_c^2} \\
 \left(\frac{r_c}{r_s}\right)^2 L_s &= \frac{\rho_c}{\rho_s} L_c \\
 \left(\frac{r_c}{r_s}\right)^2 \left(\frac{r_c}{r_s}\right)^2 L_c &= \frac{\rho_c}{\rho_s} L_c \\
 \left(\frac{r_c}{r_s}\right)^4 &= \frac{\rho_c}{\rho_s} \\
 \frac{r_s}{r_c} &= \left(\frac{\rho_s}{\rho_c}\right)^{\frac{1}{4}} \\
 &= \left(\frac{1.59 \times 10^{-8} \Omega \cdot \text{m}}{1.72 \times 10^{-8} \Omega \cdot \text{m}}\right)^{\frac{1}{4}} \\
 &= \boxed{0.981}
 \end{aligned}$$

$$80. R_1 = \frac{2.2 \text{ V}}{0.15 \text{ A}} = \boxed{15 \Omega}$$

$$R_{\text{eq}} = \frac{V}{I} = R_1 + R_2$$

$$R_2 = \frac{V}{I} - R_1 = \frac{12 \text{ V}}{0.15 \text{ A}} - \frac{2.2 \text{ V}}{0.15 \text{ A}} = \boxed{65 \Omega}$$

$$81. \frac{q}{C} = \mathcal{E}(1 - e^{-t/RC})$$

$$\text{Set } \frac{q}{C} = V = 0.25 \text{ V.}$$

$$\frac{V}{\mathcal{E}} = 1 - e^{-t/RC}$$

$$e^{-t/RC} = 1 - \frac{V}{\mathcal{E}}$$

$$-\frac{t}{RC} = \ln\left(1 - \frac{V}{\mathcal{E}}\right)$$

$$\frac{1}{R} = -\frac{C}{t} \ln\left(1 - \frac{V}{\mathcal{E}}\right)$$

$$R = \left[ -\frac{C}{t} \ln\left(1 - \frac{V}{\mathcal{E}}\right) \right]^{-1}$$

$$= \left[ -(110 \times 10^{-6} \text{ F}) \left( \frac{75}{\text{min}} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) \ln\left(1 - \frac{0.25 \text{ V}}{9.0 \text{ V}}\right) \right]^{-1}$$

$$= \boxed{260 \text{ k}\Omega}$$

82. Since resistance is directly proportional to length, each of the three pieces has resistance  $\frac{R}{3}$ .

$$R_{\text{eq}} = \left( \frac{1}{\frac{R}{3}} + \frac{1}{\frac{R}{3}} + \frac{1}{\frac{R}{3}} \right)^{-1} = \boxed{\frac{R}{9}}$$

83. (a) Since  $P = I^2 R$ , and since  $I$  is the same for all three resistors, the resistance  $\boxed{2R}$  has the greatest rate of energy dissipation.

- (b) Since  $P = V^2/R$ , and since  $V$  is the same for all three resistors, the resistance  $\boxed{\frac{R}{2}}$  has the greatest rate of energy dissipation.

84. (a) Since  $R_{\text{eq}}$  for the upper two resistors in series is  $35 \Omega + 82 \Omega = 117 \Omega$ , which is greater than  $45 \Omega$ , the current in the  $45\text{-}\Omega$  resistor is greater than the current in the upper branch—that is, the current in the  $35\text{-}\Omega$  resistor.

- (b) The current through the  $45\text{-}\Omega$  resistor is

$$I_{45} = \frac{15\text{ V}}{45\text{ }\Omega} = \boxed{0.33\text{ A}}$$

The current through the  $35\text{-}\Omega$  resistor and the  $82\text{-}\Omega$  resistor is

$$I_{35} = \frac{15\text{ V}}{117\text{ }\Omega} = \boxed{0.13\text{ A}}$$

85. (a) With no current flowing, the current through the internal resistance is zero, and the voltage across the battery terminals is  $\boxed{9.0\text{ V}}$ .
- (b) When the switch is closed, current flows through the battery's internal resistance, producing a voltage drop. The voltage across the terminals decreases.
- (c) After a long time, the capacitor behaves like an open circuit. For the three resistances in series,  $R_{\text{eq}} = 11\text{ }\Omega + 5.6\text{ }\Omega + 0.73\text{ }\Omega = 17.33\text{ }\Omega$ . The potential difference across the terminals is

$$\begin{aligned} V &= \mathcal{E} - IR_i \quad \left( \text{where } I = \frac{\mathcal{E}}{R_{\text{eq}}} \right) \\ &= \mathcal{E} - \mathcal{E} \frac{R_i}{R_{\text{eq}}} \\ &= \mathcal{E} \left( 1 - \frac{R_i}{R_{\text{eq}}} \right) \\ &= (9.0\text{ V}) \left( 1 - \frac{0.73\text{ }\Omega}{17.33\text{ }\Omega} \right) \\ &= \boxed{8.6\text{ V}} \end{aligned}$$

86. (a)  $Q_{7.22} = (7.22 \times 10^{-6}\text{ F})(12.0\text{ V}) = \boxed{86.6\text{ }\mu\text{C}}$

For the  $4.25\text{-}\mu\text{F}$ ,  $12.0\text{-}\mu\text{F}$ , and  $8.35\text{-}\mu\text{F}$  capacitors in combination,

$$C_{\text{eq}} = 4.25\text{ }\mu\text{F} + \left( \frac{1}{12.0\text{ }\mu\text{F}} + \frac{1}{8.35\text{ }\mu\text{F}} \right)^{-1} = 9.17\text{ }\mu\text{F}$$

Putting this combination in series with  $C = 15.0\text{ }\mu\text{F}$  produces an equivalent capacitance of

$$\left( \frac{1}{9.17\text{ }\mu\text{F}} + \frac{1}{15.0\text{ }\mu\text{F}} \right)^{-1} = 5.69\text{ }\mu\text{F}$$

$$Q_{15.0} = (5.69 \times 10^{-6})(12.0\text{ V}) = \boxed{68.3\text{ }\mu\text{C}}$$

For the  $12.0\text{-}\mu\text{F}$  and  $8.35\text{-}\mu\text{F}$  capacitors in series,

$$C_{\text{eq}} = \left( \frac{1}{12.0\text{ }\mu\text{F}} + \frac{1}{8.35\text{ }\mu\text{F}} \right)^{-1} = 4.923\text{ }\mu\text{F}$$

$$\begin{aligned} Q_{4.25} &= Q_{15.0} \left( \frac{4.25\text{ }\mu\text{F}}{4.25\text{ }\mu\text{F} + 4.923\text{ }\mu\text{F}} \right) \\ &= (68.3\text{ }\mu\text{C}) \left( \frac{4.25\text{ }\mu\text{F}}{4.25\text{ }\mu\text{F} + 4.923\text{ }\mu\text{F}} \right) \\ &= \boxed{31.6\text{ }\mu\text{C}} \end{aligned}$$

$$Q_{12.0} = Q_{8.35} = 68.3\text{ }\mu\text{C} - 31.6\text{ }\mu\text{C} = \boxed{36.7\text{ }\mu\text{C}}$$

$$(b) U_{\text{Tot}} = \frac{1}{2} C_{\text{Tot}} V^2 = \frac{1}{2} (7.22 \times 10^{-6} \text{ F} + 5.69 \times 10^{-6} \text{ F}) (12.0 \text{ V})^2 = \boxed{9.30 \times 10^{-4} \text{ J}}$$

(c) Because  $C_{7.22}$  is in parallel with the other capacitors, increasing its value will increase the overall capacitance, and the total energy,  $U_{\text{Tot}} = \frac{1}{2} C_{\text{Tot}} V^2$ , will increase.

87. (a) Initially, the capacitor carries no charge and acts like a closed switch.

$$P_{24} = \frac{(15 \text{ V})^2}{24 \Omega} = \boxed{9.4 \text{ W}}$$

Since  $13 \Omega = 2(6.5 \Omega)$ ,

$$P_{13} = \frac{2}{3} \cdot \frac{(15 \text{ V})^2}{13 \Omega + 6.5 \Omega} = \boxed{7.7 \text{ W}}$$

$$P_{6.5} = \frac{1}{3} \cdot \frac{(15 \text{ V})^2}{13 \Omega + 6.5 \Omega} = \boxed{3.8 \text{ W}}$$

As  $t \rightarrow \infty$ , the capacitor acts like an open switch and  $I \rightarrow 0$ . Then  $P_{24} = P_{13} = P_{6.5} = \boxed{0}$ .

$$(b) R_{\text{eq}} = \left( \frac{1}{24 \Omega} + \frac{1}{13 \Omega + 6.5 \Omega} \right)^{-1} = 10.8 \Omega$$

$$\tau = R_{\text{eq}} C = (10.8 \Omega)(62 \times 10^{-6} \text{ F}) = 6.7 \times 10^{-4} \text{ s}$$

$$\begin{aligned} Q_{0.35} &= C \mathcal{E} (1 - e^{-t/\tau}) \\ &= (62 \times 10^{-6} \text{ F})(15 \text{ V})(1 - e^{-(0.35 \times 10^{-3} \text{ s})/(6.7 \times 10^{-4} \text{ s})}) \\ &= \boxed{3.8 \times 10^{-4} \text{ C}} \end{aligned}$$

$$(c) U_{\text{lim}} = \frac{1}{2} C \mathcal{E}^2 = \frac{1}{2} (62 \times 10^{-6} \text{ F})(15 \text{ V})^2 = \boxed{7.0 \times 10^{-3} \text{ J}}$$

(d) Doubling  $\mathcal{E}$  causes  $U_{\text{lim}} = \frac{1}{2} C \mathcal{E}^2$  to quadruple.

$$88. P_1 + P_2 = \frac{V^2}{R_1} + \frac{V^2}{R_2} = \left( \frac{1}{R_1} + \frac{1}{R_2} \right) V^2$$

$$P_{\text{total}} = \frac{V^2}{R_{\text{eq}}} = \left( \frac{1}{R_{\text{eq}}} \right) V^2 = \left( \frac{1}{R_1} + \frac{1}{R_2} \right) V^2$$

So,  $P_1 + P_2 = P_{\text{total}}$ .

$$89. 0 = \mathcal{E} - (0.65 \text{ A})(25 \Omega) - (0.65 \text{ A})r \dots\dots\dots \text{(I)}$$

$$0 = \mathcal{E} - (0.45 \text{ A})(55 \Omega) - (0.45 \text{ A})r \dots\dots\dots \text{(II)}$$

Subtract (II) from (I).

$$0 = 8.5 \text{ V} - (0.20 \text{ A})r$$

$$r = \boxed{43 \Omega}$$

$$\mathcal{E} = (0.65 \text{ A})(25 \Omega) + (0.65 \text{ A}) \left( \frac{8.5 \text{ V}}{0.20 \text{ A}} \right) = \boxed{44 \text{ V}}$$

90.  $0 = 6.0 \text{ V} - 4.0 \text{ V} - IR_2$  (series)

$$I = \frac{2.0 \text{ V}}{R_2}$$

$0 = 6.0 \text{ V} - I_2 R_2$  (parallel)

$$R_2 = \frac{6.0 \text{ V}}{I_2} = \frac{6.0 \text{ V}}{0.45 \text{ A}} = \frac{40}{3} \Omega = \boxed{13 \Omega}$$

$I = \frac{2.0 \text{ V}}{\frac{40}{3} \Omega} = 0.15 \text{ A}$  (series)

$$IR_1 = 4.0 \text{ V}$$

$$R_1 = \frac{4.0 \text{ V}}{0.15 \text{ A}} = \boxed{27 \Omega}$$

91.  $R_1 = 25.0 \Omega$

$R_2 = 15.0 \Omega$

$R_3 = 12.5 \Omega$

$R_4 = 85.0 \Omega$

$R_5 = 52.0 \Omega$

The currents are labeled according to the resistors through which they flow.

$$\mathcal{E} = I_1 R_1 + I_3 R_3 \dots \dots \dots \text{(I)}$$

$$\mathcal{E} = I_2 R_2 + I_5 R_5 \dots \dots \dots \text{(II)}$$

$$I_4 R_4 = I_2 R_2 - I_1 R_1 \dots \dots \dots \text{(III)}$$

$$I_4 R_4 = I_3 R_3 - I_5 R_5 \dots \dots \dots \text{(IV)}$$

$$I_3 = I_1 - I_4 \dots \dots \dots \text{(V)}$$

$$I_5 = I_2 + I_4 \dots \dots \dots \text{(VI)}$$

Substitute (V) into (I).

$$\mathcal{E} = I_1 R_1 + (I_1 - I_4) R_3$$

$$\mathcal{E} = I_1 (R_1 + R_3) - I_4 R_3 \dots \dots \dots \text{(VII)}$$

Substitute (VI) into (II).

$$\mathcal{E} = I_2 R_2 + (I_2 + I_4) R_5$$

$$\mathcal{E} = I_2 (R_2 + R_5) + I_4 R_5 \dots \dots \dots \text{(VIII)}$$

Multiply (VIII) by  $R_2$  and (III) by  $R_2 + R_5$  and add.

$$\mathcal{E} R_2 = I_2 (R_2 + R_5) R_2 + I_4 R_2 R_5$$

$$0 = -I_2 R_2 (R_2 + R_5) + I_1 R_1 (R_2 + R_5) + I_4 R_4 (R_2 + R_5)$$

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$$\mathcal{E} R_2 = I_1 R_1 (R_2 + R_5) + I_4 [R_2 R_5 + R_4 (R_2 + R_5)] \dots \dots \dots \text{(IX)}$$

Multiply (IX) by  $R_1 + R_3$  and (VII) by  $R_1 (R_2 + R_5)$  and subtract.

$$\mathcal{E} R_2 (R_1 + R_3) = I_1 R_1 (R_2 + R_5) (R_1 + R_3) + I_4 [R_2 R_5 + R_4 (R_2 + R_5)] (R_1 + R_3)$$

$$\mathcal{E} R_1 (R_2 + R_5) = I_1 (R_1 + R_3) R_1 (R_2 + R_5) - I_4 R_3 R_1 (R_2 + R_5)$$

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$$\mathcal{E} (R_2 R_3 - R_1 R_5) = I_4 \{ [R_2 R_5 + R_4 (R_2 + R_5)] (R_1 + R_3) + R_1 R_3 (R_2 + R_5) \}$$

Solve for  $I_4$ .

$$I_4 = \frac{(15.0 \text{ V})[(15.0 \Omega)(12.5 \Omega) - (25.0 \Omega)(52.0 \Omega)]}{[(15.0 \Omega)(52.0 \Omega) + (85.0 \Omega)(15.0 \Omega + 52.0 \Omega)](25.0 \Omega + 12.5 \Omega) + (25.0 \Omega)(12.5 \Omega)(15.0 \Omega + 52.0 \Omega)}$$

$$= -63.3 \text{ mA}$$

The current through the  $85.0\text{-}\Omega$  resistor is  $63.3 \text{ mA}$  and flows to the left.