Chapter 23

Magnetic Flux and Faraday's Law of Induction

Answers to Even-numbered Conceptual Questions

2. The magnetic field indicates the strength and direction of the magnetic force that a charged particle moving with a certain velocity would experience at a given point in space. The magnetic flux, on the other hand, can be thought of as a measure of the "amount" of magnetic field that passes through a given area.

4. The magnetic flux through the loop of wire is greatest when its normal points vertically downward, because in this case the normal points in almost the same direction as the magnetic field. There is little magnetic flux through the loop if its normal is horizontal; that is, when its normal is essentially at right angles to the field.

6. When the disk is at its maximum displacement to the right, it is well within the region of uniform magnetic field. Therefore, the magnetic flux through the disk is not changing. It follows, then, that the induced current at this point is a minimum; namely, zero.

8. The magnetic field will have little apparent effect, because the break in the ring will prevent a current from flowing around its circumference. What the magnetic field will do, however, is produce a nonzero emf between the two sides of the break.

10. Nothing. In this case, the break prevents a current from circulating around the ring. This, in turn, prevents the ring from experiencing a magnetic force that would propel it into the air.

12. No. The fact that the two wires are not connected means that no current can flow through the rod. As a result, the magnetic field exerts zero force on the rod. If the system is frictionless, no force will be required to keep the rod moving with a constant speed.

14. As the penny begins to tip over, there is a large change in the magnetic flux through its surface, due to the great intensity of the MRI magnetic field. This change in magnetic flux generates an induced current in the penny that opposes its motion. As a result, the penny falls over slowly, as if it were immersed in molasses.

16. Initially, the rod accelerates to the left, due to the downward current it carries. As it speeds up, however, the motional emf it generates will begin to counteract the emf of the battery. Eventually the two emfs balance one another, and current stops flowing in the rod. From this point on, the rod continues to move with constant speed.

18. When the angular speed of the coil in an electric generator is increased, the rate at which the magnetic flux changes increases as well. As a result, the magnitude of the induced emf produced by the generator increases. Of course, the frequency of the induced emf increases as well.

20. The energy stored in the inductor remains the same. Doubling the number of turns per length quadruples the inductance of the solenoid, as we can see from Equation 23-14. The energy stored in an inductor, however, depends on both the inductance of the inductor and the current it carries, as we see in Equation 23-19. In fact, the energy stored in an inductor depends on the square of the current. Therefore, halving the current reduces the stored energy by a factor of four. The net effect, then, is no change in the energy.

22. The secondary current in transformer 2 is 6 I_s. The factor is 6 is the result of a factor of 3 in the primary current, times a factor of 2 due to the doubling of the number of turns in the primary coil.

Solutions to Problems

1. $\Phi = BA\cos\theta = (0.055\text{ T})\pi(0.021\text{ m})^2\cos 12° = \boxed{7.5\times 10^{-5}\text{ Wb}}$

2. The sides of the box are parallel to the field, so the magnetic flux through the sides is $\boxed{\text{zero}}$. The magnetic flux through the bottom is $\Phi = BA\cos\theta = (0.0250\text{ T})(0.325\text{ m})(0.120\text{ m})\cos 0° = \boxed{9.75\times 10^{-4}\text{ Wb}}$.

3. $B = \dfrac{\Phi}{A\cos\theta} = \dfrac{4.8\times 10^{-5}\text{ T}\cdot\text{m}^2}{(0.055\text{ m})(0.072\text{ m})\cos 32°} = \boxed{14\text{ mT}}$

4. The horizontal component of the magnetic field is parallel to the floor, so it does not contribute to the flux.

$\Phi = BA\cos\theta = (4.2\times 10^{-5}\text{ T})(22\text{ m})(18\text{ m})\cos 0° = \boxed{1.7\times 10^{-2}\text{ Wb}}$

5. $\Phi = BA\cos\theta = (1.7\text{ T})\pi\left(\dfrac{1.2\text{ m}}{2}\right)^2\cos 0° = \boxed{1.9\text{ Wb}}$

6. $\Phi = BA\cos\theta = (5.9\times 10^{-5}\text{ T})(1.1\text{ m})(0.62\text{ m})\cos(90° - 70°) = \boxed{4\times 10^{-5}\text{ Wb}}$

7. (a) $B = \dfrac{\Phi}{A\cos\theta} = \mu_0 nI$

$$I = \frac{\Phi}{\mu_0 nA\cos\theta} = \frac{12.8\times 10^{-4}\text{ T}\cdot\text{m}^2}{\left(4\pi\times 10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)(375\text{ m}^{-1})\pi\left(\frac{0.150\text{ m}}{2}\right)^2\cos 0°} = \boxed{154\text{ A}}$$

(b) Since the current is inversely proportional to the square of the diameter of the solenoid, doubling the diameter would decrease the answer to part (a) by a factor of one fourth.

8. $\Phi = BA\cos\theta = \mu_0 nIL^2\cos 0° = \mu_0 nIL^2$

(a) $\Phi = \left(4\pi\times 10^{-7}\ \dfrac{\text{T}\cdot\text{m}}{\text{A}}\right)(1250\text{ m}^{-1})(2.50\text{ A})(0.0300\text{ m})^2 = \boxed{3.53\times 10^{-6}\text{ Wb}}$ $(L < d)$

(b) $\Phi = \left(4\pi\times 10^{-7}\ \dfrac{\text{T}\cdot\text{m}}{\text{A}}\right)(1250\text{ m}^{-1})(2.50\text{ A})\pi\left(\dfrac{0.0600\text{ m}}{2}\right)^2 = \boxed{1.11\times 10^{-5}\text{ Wb}}$ $(L = d)$

(c) Φ is the same as that found in part (b), $\boxed{1.11\times 10^{-5}\text{ Wb}}$ $(L > d)$.

9. $|\mathcal{E}| = N\left|\dfrac{\Delta\Phi}{\Delta t}\right| = N\left|\dfrac{BA}{\Delta t}\right| = \dfrac{(50)(0.25\text{ T})\pi(0.15\text{ m})^2}{0.12\text{ s}} = \boxed{7.4\text{ V}}$

10. (a) The magnetic flux has its greatest magnitude at $t = \boxed{0 \text{ s}, 0.2 \text{ s}, 0.4 \text{ s, and } 0.6 \text{ s}}$.

(b) The magnitude of the induced emf is greatest when the magnitude of the slope of the plot is greatest, which occurs at $t = \boxed{0.1 \text{ s}, 0.3 \text{ s, and } 0.5 \text{ s}}$.

11. (a) $\mathcal{E} = -N\dfrac{\Delta\Phi}{\Delta t} = -(1)\dfrac{10 \text{ Wb} - 0}{0.1 \text{ s}} = \boxed{-0.1 \text{ kV}}$

(b) $\mathcal{E} = \boxed{0}$ because $\dfrac{\Delta\Phi}{\Delta t} = 0$.

(c) $\mathcal{E} = -(1)\dfrac{-5 \text{ Wb} - 10 \text{ Wb}}{0.6 \text{ s} - 0.2 \text{ s}} = \boxed{0.04 \text{ kV}}$

12. (a) The flux at $t = 0.25$ s is about 8 Wb. This is $\boxed{\text{greater}}$ (in both a signed and an unsigned sense) than the flux at $t = 0.55$ s, which is about -3 Wb.

(b) The two emfs are the $\boxed{\text{same}}$, because at those two times the flux is changing at the same rate, namely $\dfrac{\Delta\Phi}{\Delta t} = \dfrac{-5 \text{ Wb} - 10 \text{ Wb}}{0.6 \text{ s} - 0.2 \text{ s}} = -37.5 \dfrac{\text{Wb}}{\text{s}}$

(c) $\mathcal{E} = -N\dfrac{\Delta\Phi}{\Delta t} = -(1)\left(-37.5 \dfrac{\text{Wb}}{\text{s}}\right) = \boxed{0.04 \text{ kV}}$

13. (a) The induced emf's magnitude is greater $\boxed{\text{near } t = 0.5 \text{ s}}$, because Φ is changing more rapidly there. (In fact, it is not changing at all at the instant $t = 0.4$ s.)

(b) The induced emf will have maximum magnitude where Φ is changing most rapidly: $\boxed{0.1 \text{ s}, 0.3 \text{ s}, 0.5 \text{ s}, \ldots}$.

(c) At its steepest points, the graph appears to have a slope $\Delta\Phi/\Delta t$ of about $\pm(6 \text{ Wb})/(0.1 \text{ s}) = \pm 60 \text{ Wb/s}$.

Near $t = 0.3$ s: $\mathcal{E} = -N\dfrac{\Delta\Phi}{\Delta t} \approx -(1)(60 \text{ Wb/s}) = \boxed{-0.06 \text{ kV}}$

Near $t = 0.4$ s: $\mathcal{E} = \boxed{0}$ because $\dfrac{\Delta\Phi}{\Delta t} = 0$

Near $t = 0.5$ s: $\mathcal{E} = -N\dfrac{\Delta\Phi}{\Delta t} \approx -(1)(-60 \text{ Wb/s}) = \boxed{0.06 \text{ kV}}$

14. $|\mathcal{E}| = |IR| = N\left|\dfrac{\Delta\Phi}{\Delta t}\right| = (1)\left|\dfrac{A\Delta B}{\Delta t}\right|$

$$\left|\frac{\Delta B}{\Delta t}\right| = \left|\frac{IR}{A}\right| = \frac{(0.22 \text{ A})(110\ \Omega)}{7.4\times10^{-2} \text{ m}^2} = \boxed{3.3\times10^{2} \text{ T/s}}$$

15. $|\mathcal{E}| = N\left|\dfrac{\Delta\Phi}{\Delta t}\right| = N\left|\dfrac{BA-(-BA)}{\Delta t}\right| = (100)\dfrac{2(0.20 \text{ T})(0.050 \text{ m}^2)}{0.30 \text{ s}} = \boxed{6.7 \text{ V}}$

16. $|\mathcal{E}| = N\left|\dfrac{\Delta\Phi}{\Delta t}\right| = N\left|\dfrac{B\Delta A}{\Delta t}\right| = N\left|\dfrac{B(s^2 - \pi r^2)}{\Delta t}\right| = (1)\dfrac{(0.105 \text{ T})\left|\left(\frac{1.12 \text{ m}}{4}\right)^2 - \pi\left(\frac{1.12 \text{ m}}{2\pi}\right)^2\right|}{4.25 \text{ s}} = \boxed{5.29\times10^{-4} \text{ V}}$

17. $|\mathcal{E}| = N\left|\dfrac{\Delta\Phi}{\Delta t}\right|$

$$N = |\mathcal{E}|\left|\frac{\Delta\Phi}{\Delta t}\right|^{-1}$$
$$= |\mathcal{E}|\left|\frac{BA}{\Delta t}\right|^{-1}$$
$$= (6.0\text{ V})\frac{1.5\text{ s}}{(0.20\text{ T})\pi\left(\frac{0.12\text{ m}}{2}\right)^2}$$
$$= \boxed{4000}$$

18. The magnetic field due to the bar magnet points downward as viewed from above. The field is increasing, so according to Lenz's law, the current in the loop will flow such that it opposes the increasing magnetic field. So, the current in the loop flows $\boxed{\text{counterclockwise}}$ as viewed from above.

19. **(a)** When the loop is above the magnet, the magnetic field is increasing and directed out of the page. The current in the loop will oppose the increasing field by flowing $\boxed{\text{clockwise}}$.

(b) When the loop is below the magnet, the magnetic field is decreasing and is directed out of the page. The current in the loop will oppose the decreasing field by flowing $\boxed{\text{counterclockwise}}$.

20. The current in the loop opposes the loop's change in position.

(a) The loop resists moving downward toward the magnet. (The poles of the loop's field line up with the magnet poles north-to-north and south-to-south, causing repulsion.) So the string tension is $\boxed{\text{less}}$ than the loop's weight.

(b) The loop resists moving downward away from the magnet. (The poles of the loop's field line up with the magnet poles north-to-south and south-to-north, causing attraction.) So the string tension is again $\boxed{\text{less}}$ than the loop's weight.

21. The current in the loop opposes the loop's change in position.

(a) The loop resists moving upward toward the magnet. (The poles of the loop's field line up with the magnet poles north-to-north and south-to-south, causing repulsion.) So the string tension is $\boxed{\text{greater}}$ than the loop's weight.

(b) The loop resists moving upward away from the magnet. (The poles of the loop's field line up with the magnet poles north-to-south and south-to-north, causing attraction.) So the string tension is again $\boxed{\text{greater}}$ than the loop's weight.

22. **(a)** Since the current in the wire is constant, the magnetic field through the circuit does not vary with time, so the induced current is $\boxed{\text{zero}}$.

(b) Since the current in the wire is increasing, the magnetic field through the circuit is increasing. And, since the magnetic field is directed out of the page, the induced current in the circuit will induce a magnetic field into the page. So, the current in the circuit flows $\boxed{\text{clockwise}}$.

23. If the current in the wire changes direction, the direction of the magnetic field is reversed, changing its direction from out of the page to into the page. According to Lenz's law, the current induced in the circuit will oppose this change by flowing $\boxed{\text{counterclockwise}}$, generating a field which is directed out of the page.

24. Since the field is increasing and is directed into the page, the current in the circuit will flow counterclockwise to generate a field directed out of the page to oppose it. So, the $\boxed{\text{bottom}}$ plate will become positively charged.

25. If the magnetic field changes direction, the current in the circuit will flow clockwise to generate a field to oppose the change. So, the $\boxed{\text{top}}$ plate will become positively charged.

26. **(a)** The induced emf is $\boxed{\text{zero}}$ because the magnetic field is parallel to the plane of the loop.

(b) The induced emf is still $\boxed{\text{zero}}$ because, although the field is varying in time, it is still parallel to the plane of the loop, so there is no time-varying flux through the loop to generate an emf.

(c) The answer to part (b) $\boxed{\text{does not change}}$ because the field is still parallel to the plane of the loop.

27. The increasing current in the wire generates an increasing magnetic field which is directed out of the page above the wire and into the page below.

Loop A: The induced emf is $\boxed{\text{clockwise}}$. The field generated is into the page and opposes the outwardly directed field generated by the current in the wire.

Loop B: Although the field through the loop is varying with time, the net flux through the plane of the loop is zero, so the induced emf is $\boxed{\text{zero}}$.

Loop C: The induced emf is $\boxed{\text{counterclockwise}}$. The field generated is out of the page and opposes the inwardly

directed field generated by the current in the wire.

28. $\mathcal{E} = vB\ell$

$$B = \frac{\mathcal{E}}{v\ell} = \frac{0.75 \text{ V}}{\left(2.0 \frac{\text{m}}{\text{s}}\right)(0.50 \text{ m})} = \boxed{0.75 \text{ T}}$$

29. $$\mathcal{E} = vB\ell = \left(850\times10^3 \frac{\text{m}}{\text{h}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)(5.0\times10^{-6} \text{ T})(39.9 \text{ m}) = \boxed{47 \text{ mV}}$$

30. **(a)** $|\mathcal{E}| = IR = vBL$

$$v = \frac{IR}{BL} = \frac{(0.125 \text{ A})(12.5\ \Omega)}{(0.750 \text{ T})(0.45 \text{ m})} = \boxed{4.6 \text{ m/s}}$$

(b) $\boxed{\text{The equation for } v \text{ is independent of the direction of motion of the bar, so the answer to part (a) would not change.}}$

31. The current flows clockwise, so the magnetic force is directed to the left. The external force must be equal and opposite the magnetic force to maintain the rod's constant speed. So, it is directed to the right.

(a) The magnitude of the force is $F = \frac{B^2vL^2}{R}$. Now $|\mathcal{E}| = IR = vBL$, so $v = \frac{IR}{BL}$, and

$$F = \frac{B^2L^2}{R}\left(\frac{IR}{BL}\right) = IBL = (0.125 \text{ A})(0.750 \text{ T})(0.45 \text{ m}) = \boxed{42 \text{ mN}}.$$

(b) $P = I^2R = (0.125 \text{ A})^2(12.5\ \Omega) = \boxed{195 \text{ mW}}$

(c) $P = Fv = (IBL)\left(\frac{IR}{BL}\right) = I^2R = \boxed{195 \text{ mW}}$

32. (a) $I=\sqrt{\dfrac{P}{R}}=\sqrt{\dfrac{5.0\text{ W}}{12\ \Omega}}=\boxed{0.65\text{ A}}$

(b) The magnetic field strength is $B=\dfrac{\sqrt{P_0R}}{v_0\ell}$.

$$I=\frac{|\mathcal{E}|}{R}=\frac{Bv\ell}{R}=\left(\frac{\sqrt{P_0R}}{v_0\ell}\right)\frac{v\ell}{R}=\frac{v}{v_0}\sqrt{\frac{P_0}{R}}$$

$$v=v_0I\sqrt{\frac{R}{P_0}}=\left(3.1\ \frac{\text{m}}{\text{s}}\right)(0.85\text{ A})\sqrt{\frac{12\ \Omega}{5.0\text{ W}}}=\boxed{4.1\text{ m/s}}$$

33. (a) The mechanical power is equal to the electrical power dissipated by the light bulb.

$$I=\sqrt{\frac{P}{R}}=\sqrt{\frac{7.5\text{ W}}{12\ \Omega}}=\boxed{0.79\text{ A}}$$

(b) The magnetic field strength is $B=\dfrac{\sqrt{P_0R}}{v_0\ell}$.

$$I=\frac{|\mathcal{E}|}{R}=\frac{Bv\ell}{R}=\left(\frac{\sqrt{P_0R}}{v_0\ell}\right)\frac{v\ell}{R}=\frac{v}{v_0}\sqrt{\frac{P_0}{R}}$$

$$v=v_0I\sqrt{\frac{R}{P_0}}=\left(3.1\ \frac{\text{m}}{\text{s}}\right)(0.79\text{ A})\sqrt{\frac{12\ \Omega}{5.0\text{ W}}}=\boxed{3.8\text{ m/s}}$$

34. $\mathcal{E}_{\text{max}\,1}=NBA\omega_1 \qquad \mathcal{E}_{\text{max}\,2}=NBA\omega_2$

$$\frac{\mathcal{E}_{\text{max}\,1}}{\omega_1}=NBA=\frac{\mathcal{E}_{\text{max}\,2}}{\omega_2}$$

$$\omega_2=\frac{\mathcal{E}_{\text{max}\,2}}{\mathcal{E}_{\text{max}\,1}}\omega_1=\frac{55\text{ V}}{45\text{ V}}(210\text{ rpm})=\boxed{260\text{ rpm}}$$

35. $B=\dfrac{\mathcal{E}_{\text{max}}}{NA\omega}=\dfrac{65\text{ V}}{(120)(0.25\text{ m})(0.35\text{ m})\left(190\ \frac{\text{rad}}{\text{s}}\right)}=\boxed{33\text{ mT}}$

36. $\mathcal{E}_{\text{max}}=NBA\omega=\dfrac{L}{2\pi r}BA\omega=\dfrac{1.6\text{ m}}{2\pi(0.032\text{ m})}(0.070\text{ T})\pi(0.032\text{ m})^2\left(95\ \dfrac{\text{rev}}{\text{min}}\right)\left(\dfrac{2\pi\text{ rad}}{\text{rev}}\right)\left(\dfrac{1\text{ min}}{60\text{ s}}\right)=\boxed{18\text{ mV}}$

37. (a) Only the $\boxed{\text{horizontal}}$ component of the magnetic field is important. The vertical component is parallel to the plane of the circular coil at all times. Thus, it does not contribute to the flux through the coil.

(b) $\mathcal{E}_{\text{max}}=NBA\omega=(155)(3.80\times10^{-5}\text{ T})\pi\left(\dfrac{0.220\text{ m}}{2}\right)^2\left(1250\ \dfrac{\text{rev}}{\text{min}}\right)\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\dfrac{1\text{ min}}{60\text{ s}}\right)=\boxed{29.3\text{ mV}}$

38. $N=\dfrac{\mathcal{E}_{\text{max}}}{BA\omega}=\dfrac{170\text{ V}}{(0.050\text{ T})(0.016\text{ m}^2)\left(3600\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)}=\boxed{560\text{ turns}}$

39. $\mathcal{E} = -L\frac{\Delta I}{\Delta t} = -(45.0\times10^{-3}\text{ H})\frac{515\times10^{-3}\text{ A}}{16.5\times10^{-3}\text{ s}} = \boxed{-1.40\text{ V}}$

40. $L = \mu_0\left(\frac{N^2}{\ell}\right)A$

$$N = \sqrt{\frac{\ell L}{\mu_0 A}} = \sqrt{\frac{(0.22\text{ m})(45\times10^{-3}\text{ H})}{\left(4\pi\times10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)(0.035\text{ m}^2)}} = \boxed{470\text{ turns}}$$

41. (a) $A = \frac{\ell L}{\mu_0 N^2} = \frac{(0.24\text{ m})(7.3\times10^{-3}\text{ H})}{\left(4\pi\times10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)(450)^2} = \boxed{6.9\times10^{-3}\text{ m}^2}$

(b) $\mathcal{E} = -L\frac{\Delta I}{\Delta t} = -(7.3\times10^{-3}\text{ H})\left(\frac{-3.2\text{ A}}{55\times10^{-3}\text{ s}}\right) = \boxed{0.42\text{ V}}$

42. $L = \mu_0\frac{N^2}{\ell}A = \left(4\pi\times10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)\frac{600^2}{0.25\text{ m}}\pi(0.043\text{ m})^2 = \boxed{10\text{ mH}}$

43.
$$\begin{aligned}\mathcal{E} &= -L\frac{\Delta I}{\Delta t}\\ &= -\mu_0 n^2 A\ell\frac{\Delta I}{\Delta t}\\ &= -\left(4\pi\times10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)(455\text{ m}^{-1})^2(1.81\times10^{-3}\text{ m}^2)(0.750\text{ m})\frac{2.00\text{ A}}{45.5\times10^{-3}\text{ s}}\\ &= \boxed{-15.5\text{ mV}}\end{aligned}$$

44. (a) $L = \frac{|\mathcal{E}|}{\left|\frac{\Delta I}{\Delta t}\right|} = \frac{75\times10^{-3}\text{ V}}{2.0\ \frac{\text{A}}{\text{s}}} = \boxed{38\text{ mH}}$

(b) Since the induced emf is directly proportional to the inductance, and since the inductance of the solenoid is inversely proportional to its length, the induced emf is inversely proportional to the length of the solenoid. So, if the length of the solenoid is doubled, the induced emf will be $\boxed{\text{less than}}$ 75 mV.

(c) $L \propto 1/\ell$, so $L_2 = L_1/2$.

$$\frac{|\mathcal{E}_2|}{L_2} = \left|\frac{\Delta I}{\Delta t}\right| = \frac{|\mathcal{E}_1|}{L_1}$$

$$|\mathcal{E}_2| = \frac{L_2}{L_1}|\mathcal{E}_1| = \frac{\frac{1}{2}L_1}{L_1}|\mathcal{E}_1| = \frac{1}{2}|\mathcal{E}_1| = \frac{1}{2}(75\text{ mV}) = \boxed{38\text{ mV}}$$

45. $I=\frac{1}{2}I_{\text{final}}=\frac{\mathcal{E}}{2R}=\frac{\mathcal{E}}{R}(1-e^{-tR/L})$

$$e^{-tR/L}=\frac{1}{2}$$

$$t\frac{R}{L}=-\ln\frac{1}{2}$$

$$t=\frac{L}{R}\ln 2$$

$$=\frac{63\times10^{-3}\text{ H}}{130\ \Omega}\ln 2$$

$$=\boxed{3.4\times10^{-4}\text{ s}}$$

46. (a) $R_{\text{eq}}=R+\left(\frac{1}{R}+\frac{1}{2R}\right)^{-1}=R+\frac{2R}{3}=\frac{5}{3}R$

$$\tau=\frac{L}{R_{\text{eq}}}=\frac{3L}{5R}=\frac{3(37\times10^{-3}\text{ H})}{5(50\ \Omega)}=\boxed{4\times10^{-4}\text{ s}}$$

(b) $I=\frac{\mathcal{E}}{R_{\text{eq}}}(1-e^{-2\tau/\tau})=\frac{6.0\text{ V}}{\frac{5}{3}(50\ \Omega)}(1-e^{-2})=\boxed{60\text{ mA}}$

(c) $I=\frac{\mathcal{E}}{R_{\text{eq}}}=\frac{6.0\text{ V}}{\frac{5}{3}(50\ \Omega)}=\boxed{70\text{ mA}}$

47. (a) $0.95I_{\text{final}}=0.95\frac{\mathcal{E}}{R}=\frac{\mathcal{E}}{R}[1-e^{-(2.00\text{ s})/\tau}]$

$$e^{-(2.00\text{ s})/\tau}=0.05$$

$$-\frac{2.00\text{ s}}{\tau}=\ln 0.05$$

$$\tau=-\frac{2.00\text{ s}}{\ln 0.05}$$

$$=\boxed{0.7\text{ s}}$$

(b) $R=\frac{L}{\tau}=\frac{0.275\text{ H}}{-\frac{2.00\text{ s}}{\ln 0.05}}=\boxed{0.4\ \Omega}$

48. (a)

$$I = \frac{\mathcal{E}}{R}(1 - e^{-tR/L})$$

$$\frac{RI}{\mathcal{E}} = 1 - e^{-tR/L}$$

$$e^{-tR/L} = 1 - \frac{RI}{\mathcal{E}}$$

$$\frac{-tR}{L} = \ln\left(1 - \frac{RI}{\mathcal{E}}\right)$$

$$L = \frac{-tR}{\ln\left(1 - \frac{RI}{\mathcal{E}}\right)} = -\frac{(0.15\text{ s})(5.5\ \Omega)}{\ln\left[1 - \frac{(5.5\ \Omega)(0.22\text{ A})}{9.0\text{ V}}\right]} = \boxed{5.7\text{ H}}$$

(b) $t = -\frac{L}{R}\ln\left(1 - \frac{RI}{\mathcal{E}}\right) = -\frac{5.7\text{ H}}{5.5\ \Omega}\ln\left[1 - \frac{(5.5\ \Omega)(0.40\text{ A})}{9.0\text{ V}}\right] = \boxed{0.29\text{ s}}$

(c) $I_{\text{max}} = \frac{\mathcal{E}}{R} = \frac{9.0\text{ V}}{5.5\ \Omega} = \boxed{1.6\text{ A}}$

49. $U = \frac{1}{2}LI^2 = \frac{1}{2}L\left(\frac{\mathcal{E}}{R}\right)^2 = \frac{1}{2}(6.1\times10^{-3}\text{ H})\left(\frac{9.0\text{ V}}{5.5\ \Omega}\right)^2 = \boxed{8.2\times10^{-3}\text{ J}}$

50. $U = \frac{1}{2}\mu_0 n^2 A\ell I^2$

$$A = \frac{2E}{\mu_0 n^2 \ell I^2} = \frac{2(0.31\text{ J})}{\left(4\pi\times10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)(490\text{ m}^{-1})^2(1.5\text{ m})(12\text{ A})^2} = \boxed{9.5\times10^{-3}\text{ m}^2}$$

51. (a) $u_B = \frac{B^2}{2\mu_0} = \frac{(50.0\text{ T})^2}{2\left(4\pi\times10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)} = \boxed{9.95\times10^8\text{ J/m}^3}$

(b) Set $u_E = u_B$.

$$\frac{\varepsilon_0 E^2}{2} = \frac{B^2}{2\mu_0}$$

$$E = \frac{B}{\sqrt{\varepsilon_0\mu_0}} = \frac{50.0\text{ T}}{\sqrt{\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(4\pi\times10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)}} = \boxed{1.50\times10^{10}\text{ V/m}}$$

52. (a) $U=\frac{1}{2}LI^2=\frac{1}{2}L\left(\frac{\mathcal{E}}{R_{\text{eq}}}\right)^2=\frac{1}{2}L\mathcal{E}^2\left(\frac{1}{7.5\ \Omega}+\frac{1}{R}\right)^2$

$$\frac{1}{R}+\frac{1}{7.5\ \Omega}=\sqrt{\frac{2U}{L\mathcal{E}^2}}$$

$$R=\left(\sqrt{\frac{2U}{L\mathcal{E}^2}}-\frac{1}{7.5\ \Omega}\right)^{-1}$$

$$=\left[\sqrt{\frac{2(0.11\text{ J})}{(62\times10^{-3}\text{ H})(12\text{ V})^2}}-\frac{1}{7.5\ \Omega}\right]^{-1}$$

$$=\boxed{40\ \Omega}$$

(b) The energy stored in the inductor is inversely proportional to the square of the equivalent resistance. If the equivalent resistance is decreased, the energy stored is increased. If *R* is decreased, the equivalent resistance is decreased. So, the value of *R* should be $\boxed{\text{less than}}$ the value found in part (a).

53. The two resistors in parallel have an overall resistance R_{eq} given by

$$\frac{1}{R_{\text{eq}}}=\frac{1}{7.5\ \Omega}+\frac{1}{14\ \Omega}$$

$$R_{\text{eq}}=4.88\ \Omega$$

(a) At $t=\tau$, $I=\frac{\mathcal{E}}{R_{\text{eq}}}(1-e^{-\tau/\tau})$ and the energy stored in the inductor is

$$U=\frac{1}{2}LI^2=\frac{L\mathcal{E}^2}{2R_{\text{eq}}^2}(1-e^{-\tau/\tau})^2$$

$$=\frac{(62\times10^{-3}\text{ H})(12\text{ V})^2}{2(4.88\ \Omega)^2}(1-e^{-1})^2$$

$$=\boxed{0.075\text{ J}}$$

(b) At $t=2\tau$,

$$U=\frac{L\mathcal{E}^2}{2R_{\text{eq}}^2}(1-e^{-2\tau/\tau})^2$$

$$=\frac{(62\times10^{-3}\text{ H})(12\text{ V})^2}{2(4.88\ \Omega)^2}(1-e^{-2})^2$$

$$=\boxed{0.14\text{ J}}$$

(c) Because $\tau=L/R_{\text{eq}}$ and R_{eq} goes up when *R* goes up, increasing *R* causes τ to $\boxed{\text{decrease}}$.

54. (a) The current through the inductor starts at zero and rises. Since energy stored in the inductor is proportional to the square of the current, more energy is stored $\boxed{\text{long after}}$ the switch is closed.

(b) For the three resistors on the right,

$$\frac{1}{R_{\text{eq-right}}} = \frac{1}{R} + \frac{1}{R+R} = \frac{3}{2R}$$

$$R_{\text{eq-right}} = \frac{2}{3}R$$

Then $R_{\text{eq}} = \frac{2}{3}R + R = \frac{5}{3}R.$ At $t = \tau,$ $I = \frac{\mathcal{E}}{R_{\text{eq}}}(1-e^{-1})$ and the energy stored in the inductor is

$$U = \frac{1}{2}LI^2 = \frac{L\mathcal{E}^2}{2R_{\text{eq}}^2}(1-e^{-1})^2 = \boxed{\frac{9L\mathcal{E}^2}{50R^2}(1-e^{-1})^2}$$

(c) After many characteristic time intervals, $I = \frac{\mathcal{E}}{R_{\text{eq}}}$ and $U = \boxed{\frac{9L\mathcal{E}^2}{50R^2}}.$

55. (a) $I = \sqrt{\frac{2U\ell}{\mu_0 N^2 A}} = \sqrt{\frac{2(8.3\text{ J})(0.31\text{ m})}{\left(4\pi\times10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)(500)^2\pi\left(\frac{0.070\text{ m}}{2}\right)^2}} = 65.24\text{ A} = \boxed{65\text{ A}}$

(b) $B = \mu_0 nI = \left(4\pi\times10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)\left(\frac{500}{0.31\text{ m}}\right)(65.24\text{ A}) = \boxed{0.13\text{ T}}$

(c) $u_{\text{B}} = \frac{B^2}{2\mu_0} = \frac{\mu_0{}^2n^2I^2}{2\mu_0} = \frac{1}{2}\mu_0 n^2I^2 = \frac{1}{2}\left(4\pi\times10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)\left(\frac{500}{0.31\text{ m}}\right)^2(65.24\text{ A})^2 = \boxed{7.0\text{ kJ/m}^3}$

56. $\frac{N_{\text{p}}}{N_{\text{s}}} = \frac{V_{\text{p}}}{V_{\text{s}}} = \frac{110\text{ V}}{3.0\text{ V}} = \boxed{37}$

57. (a) The voltage needs to be stepped down. To step down a primary voltage to a secondary voltage, the number of turns on the secondary coil must be less than the number of turns on the primary coil. So, the number of turns on the secondary coil must be $\boxed{\text{less than}}$ 125.

(b) $N_{\text{s}} = N_{\text{p}}\left(\frac{V_{\text{s}}}{V_{\text{p}}}\right) = (125\text{ turns})\left(\frac{9.0\text{ V}}{120\text{ V}}\right) = \boxed{9.4\text{ turns}}$

58. $V_{\text{s}} = V_{\text{p}}\left(\frac{N_{\text{s}}}{N_{\text{p}}}\right) = (120\text{ V})\left(\frac{1}{13}\right) = \boxed{9.2\text{ V}}$

59. $\frac{N_{\text{s}}}{N_{\text{p}}} = \frac{V_{\text{s}}}{V_{\text{p}}} = \frac{11{,}000\text{ V}}{120\text{ V}} = \boxed{92}$

60. $\frac{N_p}{N_s} = \frac{V_p}{V_s} = \frac{120 \text{ V}}{6.0 \text{ V}} = 20$

$V_p' = V_s\left(\frac{N_p}{N_s}\right) = (120 \text{ V})(20) = \boxed{2.4 \text{ kV}}$

61. $I_p = I_s\left(\frac{N_s}{N_p}\right) = (12\times10^{-3} \text{ A})\left(\frac{500}{25}\right) = \boxed{0.2 \text{ A}}$

$V_p = V_s\left(\frac{N_p}{N_s}\right) = (4800 \text{ V})\left(\frac{25}{500}\right) = \boxed{200 \text{ V}}$

62. $|\mathcal{E}| = Bv\ell = (2.0\times10^{-10} \text{ T})\left(8.0\times10^3 \frac{\text{m}}{\text{s}}\right)(5.0 \text{ m}) = \boxed{8.0 \ \mu\text{V}}$

63. $\Phi_{\text{max}} = BA = (0.15\times10^{-3} \text{ T})\pi\left(\frac{2.0\times10^{-3} \text{ m}}{2}\right)^2 = \boxed{4.7\times10^{-10} \text{ Wb}}$

64. $|\mathcal{E}| = N\left|\frac{\Delta\Phi}{\Delta t}\right| = N\left|\frac{BA(\cos 0° - \cos 90°)}{\Delta t}\right| = (80)\frac{(0.15\times10^{-3} \text{ T})\pi\left(\frac{0.0020 \text{ m}}{2}\right)^2(1-0)}{32\times10^{-3} \text{ s}} = \boxed{1.2 \ \mu\text{V}}$

65. $\Phi = B(A_{xy}\cos\theta_{xy} + A_{xz}\cos\theta_{xz})$

$= B(A\cos\theta_{xy} + A\cos\theta_{xz})$

$= (0.035 \text{ T})(0.24 \text{ m})(0.36 \text{ m})(\cos 65° + \cos 25°)$

$= \boxed{4.0 \text{ mWb}}$

66. (a) Only the vertical component of $\vec{\mathbf{B}}$ contributes flux.

$\Phi = B_z A = B_z \pi r^2$

$= (0.52 \text{ T})\pi(0.037 \text{ m})^2$

$= \boxed{2.2 \text{ mWb}}$

(b) Since the *x* component does not contribute to the flux, the flux stays the same.

(c) Since the *z* component is the source of all the flux, the magnitude of the flux increases.

67. $\mathcal{E}_{\text{av}} = N\left|\frac{\Delta\Phi}{\Delta t}\right| = (1)\left|\frac{BA-0}{\Delta t}\right| = \frac{(1.3 \text{ T})(0.060 \text{ m})(0.080 \text{ m})}{21\times10^{-3} \text{ s}} = \boxed{0.30 \text{ V}}$

68. (a) According to the RHR, the induced electric field points generally upward. Therefore, the bottom of the antenna is initially at the higher potential. Once the mobile negative charges (electrons) in the antenna have migrated to the bottom, they produce a downward-pointing E-field that counteracts the magnetically induced one, with the result that the antenna is at a constant potential along its entire length.

(b) $|\mathcal{E}| = (B\cos\theta)v\ell = (5.9\times10^{-5} \text{ T})\cos 72°\left(25 \frac{\text{m}}{\text{s}}\right)(0.85 \text{ m}) = \boxed{0.39 \text{ mV}}$

69. $\mathcal{E}_{max} = NBA\omega = (305)(0.85\text{ T})(0.14\text{ m})(0.17\text{ m})\left(525\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = \boxed{340\text{ V}}$

70. Since all magnetic field lines that enter the box also exit, the net magnetic flux through the box is $\boxed{\text{zero}}$.

71.
$$|\mathcal{E}| = \left|\frac{\Delta\Phi}{\Delta t}\right| = \left|\frac{A\Delta B}{\Delta t}\right| = A\left|\frac{\Delta B}{\Delta t}\right|$$
$$= (1.13\times10^{-2}\text{ m}^2)\left(3.00\times10^4\ \frac{\text{T}}{\text{s}}\right)$$
$$= \boxed{339\text{ V}}$$

72. **(a)** $\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t} = -NA\frac{\Delta B}{\Delta t} = -(155)\pi\left(\frac{0.0375\text{ m}}{2}\right)^2(0) = \boxed{0}$

(b) $\mathcal{E} = -(155)\pi\left(\frac{0.0375\text{ m}}{2}\right)^2\left(\frac{-0.01\text{ T} - 0.02\text{ T}}{5\times10^{-3}\text{ s}}\right) = \boxed{1\text{ V}}$

(c) $\mathcal{E} = -(155)\pi\left(\frac{0.0375\text{ m}}{2}\right)^2(0) = \boxed{0}$

(d) $\mathcal{E} = -(155)\pi\left(\frac{0.0375\text{ m}}{2}\right)^2\left[\frac{0.01\text{ T} - (-0.01\text{ T})}{10\times10^{-3}\text{ s}}\right] = \boxed{-0.3\text{ V}}$

73. Determine the length of the tube by solving for ℓ in the equation for the inductance of a solenoid.

$$L = \mu_0\left(\frac{N^2}{\ell}\right)A$$
$$\ell = \frac{\mu_0 N^2 A}{L}$$

The number of turns required is found by dividing the length of the tube by the diameter of the wire.

$$N = \frac{\ell}{d} = \frac{\mu_0 N^2 A}{dL}$$

Solve for N.

$$N = \frac{dL}{\mu_0 A}$$

The length of the wire is found my multiplying N by the circumference of the tube.

$$\text{length} = 2\pi rN = 2\pi r\frac{dL}{\mu_0\pi r^2} = \frac{2dL}{\mu_0 r} = \frac{2(3.32\times10^{-4}\text{ m})(50.0\times10^{-3}\text{ H})}{\left(4\pi\times10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)(0.0267\text{ m})} = \boxed{990\text{ m}}$$

74. $\tau = \frac{L}{R} = 2RC$

(a) $R = \sqrt{\frac{L}{2C}} = \sqrt{\frac{25\times10^{-3}\text{ H}}{2(45\times10^{-6}\text{ F})}} = \boxed{17\ \Omega}$

(b) $\tau = \frac{L}{R} = L\sqrt{\frac{2C}{L}} = \sqrt{2CL} = \sqrt{2(45\times10^{-6}\text{ F})(25\times10^{-3}\text{ H})} = \boxed{1.5\text{ ms}}$

75. (a)

$$\begin{aligned} I &= \frac{\mathcal{E}}{R}(1-e^{-tR/L}) \\ \frac{IR}{\mathcal{E}} &= 1-e^{-tR/L} \\ e^{-tR/L} &= 1-\frac{IR}{\mathcal{E}} \\ -\frac{tR}{L} &= \ln\left(1-\frac{IR}{\mathcal{E}}\right) \\ t &= -\frac{L}{R}\ln\left(1-\frac{IR}{\mathcal{E}}\right) \\ &= -\frac{35\times10^{-3}\text{ H}}{110\ \Omega}\ln\left[1-\frac{(0.22\times10^{-3}\text{ A})(110\ \Omega)}{3.0\text{ V}}\right] \\ &= \boxed{2.6\ \mu\text{s}} \end{aligned}$$

(b) $U = \frac{1}{2}LI^2 = \frac{1}{2}L\left(\frac{\mathcal{E}}{R}\right)^2 = \frac{1}{2}(35\times10^{-3}\text{ H})\left(\frac{3.0\text{ V}}{110\ \Omega}\right)^2 = \boxed{13\ \mu\text{J}}$

76. (a) $I = \frac{\mathcal{E}}{R}(1-e^{-tR/L}) = \frac{9.0\text{ V}}{170\ \Omega}[1-e^{-(0.150\times10^{-3}\text{ s})(170\ \Omega)/(32\times10^{-3}\text{ H})}] = \boxed{29\text{ mA}}$

(b) $E = \frac{1}{2}LI^2 = \frac{1}{2}(32\times10^{-3}\text{ H})(29.1\times10^{-3}\text{ A})^2 = \boxed{14\ \mu\text{J}}$

77. (a) Initially, the rod falls with the acceleration of gravity. As the rod falls, the induced current begins to flow, and the rod is acted upon by an upward magnetic force. This causes the acceleration to decrease. When the acceleration reaches zero, the rod falls with constant speed.

(b) As the rod falls, the magnetic flux increases. The induced emf opposes the increase, resulting in a clockwise flowing current.

(c) $mg = ILB$

$$I = \frac{mg}{LB}$$

$$v = \frac{|\mathcal{E}|}{LB} = \frac{IR}{LB} = \left(\frac{mg}{LB}\right)\frac{R}{LB} = \boxed{\frac{mgR}{L^2B^2}}$$

78. (a) Since the magnetic field is uniform, the rate of change of the magnetic flux is zero.

(b) $\frac{\Delta\Phi}{\Delta t} = \frac{0-BA}{\Delta t} = -\frac{BWL}{\frac{L}{v}} = \boxed{-vWB}$

(c) Since there is no magnetic field, the rate of change of the magnetic flux is zero.

(d) In part (a): $\Delta\Phi/\Delta t = 0$, so $I_{\text{ind}} = \boxed{0}$.

In part (b): the induced current flows $\boxed{\text{counterclockwise}}$, to generate an out-of-the page field that compensates for the change in flux.

In part (c): $\Delta\Phi/\Delta t = 0$, so $I_{\text{ind}} = \boxed{0}$.

79. (a) $I = \dfrac{\mathcal{E}}{R_{\text{eq}}} = \boxed{\dfrac{\mathcal{E}}{R_1}}$

(b) Just after the switch is closed, the current begins to increase. The current flows through the light bulb, as well as the inductor. As time goes by, the light bulb begins to dim as the resistance due to the inductor decreases. After "a long time," the inductor behaves as an ideal wire with no resistance, and the maximum current flows entirely through the inductor, bypassing the light bulb.

(c) After the switch is opened, the current continues to flow for a finite time due to the inductor. The current flows through the light bulb, causing it to flash and then go out.

(d) $V_{\text{before}} = IR_2 = (0)R_2 = \boxed{0}$

$V_{\text{after}} = IR_2 = \left(\dfrac{\mathcal{E}}{R_1}\right)R_2 = \boxed{\mathcal{E}\dfrac{R_2}{R_1}}$

80. (a) $\dfrac{u_{\text{E}}}{u_{\text{B}}} = 1 = \dfrac{\frac{\varepsilon_0 E^2}{2}}{\frac{B^2}{2\mu_0}} = \dfrac{\varepsilon_0\mu_0 E^2}{B^2}$

$\dfrac{E}{B} = \boxed{\dfrac{1}{\sqrt{\varepsilon_0\mu_0}}}$

(b) $\dfrac{E}{B} = \dfrac{1}{\sqrt{\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(4\pi\times10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)}} = \boxed{3.00\times10^8\ \text{m/s} = c}$