Chapter 24

Alternating-Current Circuits

Answers to Even-numbered Conceptual Questions

- 2. Light will attain its maximum brightness 120 times per second; that is, twice per cycle. The reason for the factor of two per cycle is that the current reverses direction every cycle, and the bulb will be brightest when the current is a maximum in either direction.
- 4. The intensity of the light bulb will decrease if the frequency of the generator is increased. The reason is that inductors oppose any change in current, and the more rapidly the current changes the stronger the opposition. Thus, as the frequency is increased, the current in the circuit decreases, and the light bulb becomes dimmer.
- 6. Current and voltage are not always in phase in an ac circuit because capacitors and inductors respond not to the current itself as a resistor does but to the charge (capacitor) or to the rate of change of the current (inductor). The charge takes time to build up; therefore, a capacitor's voltage lags behind the current. The rate of change of current is greatest when the current is least; therefore, an inductor's voltage leads the current. Resistors, of course, are always in phase with the current.
- 8. As the frequency is increased, the inductive reactance increases as well. Therefore, at frequencies greater than the resonance frequency of an LC circuit, the inductive reactance is greater than the capacitive reactance. As a result, the inductor dominates, and the voltage leads the current. This means that the phase angle, ϕ , is positive.
- No. In a dc circuit the frequency is zero, which means that the inductive reactance is zero as well. Therefore, the inductor has no effect at all on the current in the circuit, nor on the brightness of the bulb. In a dc circuit, an ideal inductor is the same as a piece of zero-resistance wire.
- 12. At low frequency, the capacitor is essentially the same as a break in the circuit, whereas the inductor is essentially an ideal wire. It follows, then, that more current will be supplied by the generator if the inductor and the capacitor are connected in parallel.
- 14. Recall that charge in an *RLC* circuit is the analog of position in a mass-spring system. Therefore, the current which is the rate of charge of charge is analogous to the velocity which is the rate of charge of position. (See Table 24-2.)
- 16. Yes. All that is required for their resonance frequencies to be the same is for the product of *L* and *C* to be the same. (See Equation 24-18.)
- 18. At high frequency, we can replace the inductor with an open circuit. At low frequency, we can replace the capacitor with an open circuit. In either case, the effective resistance of the circuit is *R*; therefore, the current is the same.
- 20. At high frequency, the capacitor behaves essentially the same as an ideal wire, which means that the effective resistance of the circuit is R/2. At low frequency, the capacitor is basically a break in the circuit. In this case, the effective resistance of the circuit is R. It follows that the current in the circuit is greater at high frequency.

22. In a dc circuit, an ideal inductor behaves the same as an ideal wire, and a capacitor behaves the same as a break in the circuit. Therefore, the circuit in Figure 24-28, with an equivalent resistance of R/2, draws more current than the circuit in Figure 24-29, with an equivalent resistance of R.

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- 24. Adding a second capacitor in parallel with the first has the effect of increasing the effective capacitance. If the capacitance is increased, we can see from Equation 24-18 that the resonance frequency will decrease.
- 26. The value of the resistance does not affect the resonance frequency, as can be seen from Equation 24-18. The maximum current in the circuit is inversely proportional to the resistance, however. Therefore, the maximum current will be reduced by a factor of two.

Solutions to Problems

1.
$$V_{\text{rms}} = \frac{1}{\sqrt{2}} V_{\text{max}} = \frac{1}{\sqrt{2}} (45 \text{ V}) = \boxed{32 \text{ V}}$$

2.
$$V_{\text{max}} = \sqrt{2}V_{\text{rms}} = \sqrt{2}(240 \text{ V}) = \boxed{340 \text{ V}}$$

3.
$$R = \frac{V_{\text{max}}}{I_{\text{max}}} = \frac{\sqrt{2}V_{\text{rms}}}{I_{\text{max}}} = \frac{\sqrt{2}(120 \text{ V})}{2.2 \text{ A}} = \boxed{77 \Omega}$$

4. (a)
$$P_{\text{av}} = I_{\text{rms}}^2 R = (0.85 \text{ A})^2 (150 \Omega) = \boxed{110 \text{ W}}$$

(b)
$$P_{\text{max}} = I_{\text{max}}^2 R = (\sqrt{2}I_{\text{rms}})^2 R = 2(0.85 \text{ A})^2 (150 \Omega) = \boxed{220 \text{ W}}$$

5. (a)
$$P_{\text{av}} = \frac{V_{\text{rms}}^2}{R} = \frac{\left(\frac{101 \text{ V}}{\sqrt{2}}\right)^2}{3.33 \times 10^3 \Omega} = \boxed{1.53 \text{ W}}$$

(b)
$$P_{\text{max}} = \frac{V_{\text{max}}^2}{R} = \frac{(101 \text{ V})^2}{3.33 \times 10^3 \Omega} = \boxed{3.06 \text{ W}}$$

6. (a)
$$R = \frac{V_{\text{rms}}^2}{P_{\text{av}}} = \frac{(120 \text{ V})^2}{75 \text{ W}} = \boxed{190 \Omega}$$

(b)
$$I_{\text{max}} = \frac{V_{\text{max}}}{R} = \frac{\sqrt{2}V_{\text{rms}}}{R} = \frac{\sqrt{2}(120 \text{ V})}{192 \Omega} = \boxed{0.88 \text{ A}}$$

(c)
$$P_{\text{max}} = 2P_{\text{av}} = 2(75 \text{ W}) = 150 \text{ W}$$

7. Squaring the voltage gives 25 V^2 for both voltages of 5.0 V and $-5.0 \ V$.

So,
$$V_{\text{rms}} = \sqrt{(V^2)_{\text{av}}} = \sqrt{25 \text{ V}^2} = 5.0 \text{ V}$$
.

$$V_{\rm rms} = V_{\rm max}$$

8.
$$C = \frac{1}{\omega X_C} = \frac{1}{2\pi (57 \text{ s}^{-1})(65 \Omega)} = \boxed{43 \ \mu\text{F}}$$

9.
$$C = \frac{1}{\omega X_C} = \frac{1}{2\pi f X_C}$$

 $f' = \frac{1}{2\pi C X_C'} = \frac{2\pi f X_C}{2\pi X_C'} = f \frac{X_C}{X_C'} = (60.0 \text{ s}^{-1}) \frac{105 \Omega}{72.5 \Omega} = \boxed{86.9 \text{ Hz}}$

10.
$$I_{\text{rms}} = \omega C V_{\text{rms}} = 2\pi (100.0 \text{ s}^{-1})(105 \times 10^{-6} \text{ F})(20.0 \text{ V}) = \boxed{1.32 \text{ A}}$$

11. (a)
$$I_{\text{rms}} = \omega C V_{\text{rms}} = 2\pi (52 \text{ s}^{-1}) (0.010 \times 10^{-6} \text{ F}) (1.4 \text{ V}) = \boxed{4.6 \ \mu\text{A}}$$

(b)
$$I_{\text{max}} = \omega C V_{\text{max}} = 2\pi (52 \text{ s}^{-1}) (0.010 \times 10^{-6} \text{ F}) \sqrt{2} (1.4 \text{ V}) = \boxed{6.5 \ \mu\text{A}}$$

12. (a)
$$I_{\text{max}} = \frac{V_{\text{max}}}{X_C} = \sqrt{2}V_{\text{rms}}\omega C = \sqrt{2}(12.0 \text{ V})2\pi(30.0 \text{ s}^{-1})(45.5 \times 10^{-6} \text{ F}) = \boxed{146 \text{ mA}}$$

(b)
$$V = V_{\text{max}} \sin(\theta - 90^{\circ})$$

 $\theta = \sin^{-1} \frac{V}{V_{\text{max}}} + 90^{\circ}$
 $I = I_{\text{max}} \sin \theta = I_{\text{max}} \sin \left(\sin^{-1} \frac{V}{V_{\text{max}}} + 90^{\circ} \right) = (145.5 \times 10^{-3} \text{ A}) \sin \left(\sin^{-1} \frac{5.25 \text{ V}}{\sqrt{2}(12.0 \text{ V})} + 90^{\circ} \right) = \boxed{138 \text{ mA}}$

(c)
$$I = I_{\text{max}} \sin \theta' = (145.5 \times 10^{-3} \text{ A}) \sin \left(180^{\circ} - \sin^{-1} \frac{5.25 \text{ V}}{\sqrt{2}(12.0 \text{ V})} + 90^{\circ} \right) = \boxed{-138 \text{ mA}}$$

13. (a)
$$V_{\text{max}} = X_C I_{\text{max}} = \frac{I_{\text{max}}}{\omega C} = \frac{0.15 \text{ A}}{2\pi (120 \text{ s}^{-1})(22 \times 10^{-6} \text{ F})} = \boxed{9.0 \text{ V}}$$

(b)
$$V = V_{\text{max}} \sin(\theta - 90^{\circ}) = V_{\text{max}} \sin\left(\sin^{-1}\frac{I}{I_{\text{max}}} - 90^{\circ}\right) = (9.04 \text{ V})\sin\left(\sin^{-1}\frac{0.10 \text{ A}}{0.15 \text{ A}} - 90^{\circ}\right) = \boxed{-6.7 \text{ V}}$$

(c)
$$V = V_{\text{max}} \sin \theta' = V_{\text{max}} \sin \left(180^{\circ} - \sin^{-1} \frac{I}{I_{\text{max}}} - 90^{\circ} \right) = (9.04 \text{ V}) \sin \left(180^{\circ} - \sin^{-1} \frac{0.10 \text{ A}}{0.15 \text{ A}} - 90^{\circ} \right) = \boxed{6.7 \text{ V}}$$

14. (a)
$$I_{\text{rms}} = \omega C V_{\text{rms}} = 2\pi (1.00 \times 10^3 \text{ s}^{-1})(0.395 \times 10^{-6} \text{ F})(10.0 \text{ V}) = \boxed{24.8 \text{ mA}}$$

(b) Since the current is directly proportional to the frequency of the voltage, doubling the frequency doubles the current.

(c)
$$I_{\text{rms}} = 2\pi (2.00 \times 10^3 \text{ s}^{-1})(0.395 \times 10^{-6} \text{ F})(10.0 \text{ V}) = \boxed{49.6 \text{ mA}}$$

15. (a)
$$X_C = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{0.500 \text{ V}}{0.430 \times 10^{-3} \text{ A}} = \boxed{1.16 \text{ k}\Omega}$$

(b)
$$C = \frac{1}{\omega X_C} = \frac{1}{2\pi (1.00 \times 10^3 \text{ s}^{-1})(1.163 \times 10^3 \Omega)} = \boxed{0.137 \ \mu\text{F}}$$

(c)
$$I_{\text{rms}} = \omega C V_{\text{rms}} = 2\pi f (0.1369 \times 10^{-6} \text{ F})(0.500 \text{ V})$$

For $f = 2.00 \text{ kHz}$, $I_{\text{rms}} = 0.860 \text{ mA}$.
For $f = 10.0 \text{ kHz}$, $I_{\text{rms}} = 4.30 \text{ mA}$.

16. (a)
$$C = \frac{1}{\omega X_C} = \frac{I_{\text{rms}}}{\omega V_{\text{rms}}} = \frac{27 \times 10^{-3} \text{ A}}{2\pi (60.0 \text{ s}^{-1})(12 \text{ V})} = \boxed{6.0 \ \mu\text{F}}$$

(b) The current in the capacitor is directly proportional to the frequency, so the current will <u>increase</u> if the frequency increases.

(c)
$$I_{\text{rms}} = \omega C V_{\text{rms}} = 2\pi (410 \text{ s}^{-1})(5.97 \times 10^{-6} \text{ F})(12 \text{ V}) = 180 \text{ mA}$$

17.
$$I_{\text{max}} = \omega C V_{\text{max}} = 2\pi f_{\text{max}} C(\sqrt{2}V_{\text{rms}}) = 2\sqrt{2}\pi f_{\text{max}} C V_{\text{rms}} < 1.0 \text{ mA}$$

$$f_{\text{max}} < \frac{1.0 \times 10^{-3} \text{ A}}{2\sqrt{2}\pi (0.22 \times 10^{-6} \text{ F})(12 \text{ V})} = 43 \text{ Hz}$$

$$\boxed{0 \le f < 43 \text{ Hz}}$$

18.
$$I_{\text{rms}} = \omega C V_{\text{rms}} = 2\pi f C V_{\text{rms}}$$

$$f = \frac{I_{\text{rms}}}{2\pi C V_{\text{rms}}} = \frac{7.50 \times 10^{-3} \text{ A}}{2\pi (0.0150 \times 10^{-6} \text{ F})(504 \text{ V})} = \boxed{158 \text{ Hz}}$$

19.
$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} = \sqrt{(25.5 \ \Omega)^2 + \left[\frac{1}{2\pi (60.0 \ \text{s}^{-1})(95.0 \times 10^{-6} \ \text{F})}\right]^2} = \boxed{37.81 \ \Omega}$$

20.
$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} = \frac{20.0 \text{ V}}{\sqrt{(10.0 \times 10^3 \,\Omega)^2 + \left[\frac{1}{2\pi (100.0 \text{ s}^{-1})(0.250 \times 10^{-6} \text{ F})}\right]^2}} = \boxed{1.69 \text{ mA}}$$

21.
$$Z = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$R = \sqrt{\left(\frac{V_{\text{rms}}}{I_{\text{rms}}}\right)^2 - \left(\frac{1}{\omega C}\right)^2} = \sqrt{\left(\frac{95 \text{ V}}{0.72 \text{ A}}\right)^2 - \left[\frac{1}{2\pi (150 \text{ s}^{-1})(13 \times 10^{-6} \text{ F})}\right]^2} = \boxed{100 \Omega}$$

22. (a)
$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} = \frac{115 \text{ V}}{\sqrt{(3350 \ \Omega)^2 + \left[\frac{1}{2\pi (65.0 \text{ s}^{-1})(1.50 \times 10^{-6} \text{ F})}\right]^2}} = \boxed{30.9 \text{ mA}}$$

(b)
$$\phi = \cos^{-1} \frac{R}{Z} = \cos^{-1} \frac{R}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} = \cos^{-1} \frac{1}{\sqrt{1 + \left(\frac{1}{\omega RC}\right)^2}} = \cos^{-1} \frac{1}{\sqrt{1 + \left[\frac{1}{2\pi (65.0 \text{ s}^{-1})(3350 \Omega)(1.50 \times 10^{-6} \text{ F})}\right]^2}}$$

$$= \boxed{25.98^{\circ}}$$

23. (a)
$$Z = \frac{R}{\cos \phi}$$

$$\sqrt{R^2 + \frac{1}{\omega^2 C^2}} = \frac{R}{\cos \phi}$$

$$\frac{1}{\omega^2 C^2} = \frac{R^2}{\cos^2 \phi} - R^2$$

$$\omega^2 C^2 = [R^2 (\sec^2 \phi - 1)]^{-1}$$

$$\omega = \frac{1}{C} [R^2 \tan^2 \phi]^{-1/2}$$

$$f = \frac{1}{2\pi RC \sqrt{\tan^2 \phi}}$$

$$= \frac{1}{2\pi (3350 \ \Omega)(1.50 \times 10^{-6} \ \text{F}) \sqrt{\tan^2 (-23.0^\circ)}}$$

$$= \boxed{74.6 \ \text{Hz}}$$

(b)
$$P_{\text{av}} = I_{\text{rms}}^2 R = \left(\frac{V_{\text{rms}}}{Z}\right)^2 R = \frac{V_{\text{rms}}^2 R}{R^2 + \left(\frac{1}{\omega C}\right)^2} = \frac{(115 \text{ V})^2 (3350 \Omega)}{(3350 \Omega)^2 + \left[\frac{1}{2\pi (74.62 \text{ s}^{-1})(1.50 \times 10^{-6} \text{ F})}\right]^2} = \boxed{3.35 \text{ W}}$$

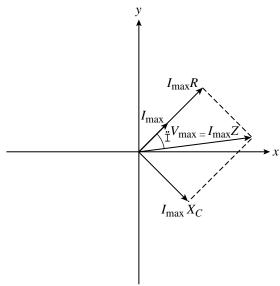
24. (a)
$$\phi = \cos^{-1} \frac{R}{Z}$$

$$= \cos^{-1} \frac{R}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$$

$$= \cos^{-1} \frac{1}{\sqrt{1 + \left(\frac{1}{\omega RC}\right)^2}}$$

$$= \cos^{-1} \frac{1}{\sqrt{1 + \left[\frac{1}{2\pi (60.0 \text{ s}^{-1})(105 \Omega)(32.2 \times 10^{-6} \text{ F})}\right]^2}}$$

$$= 38.12^{\circ}$$

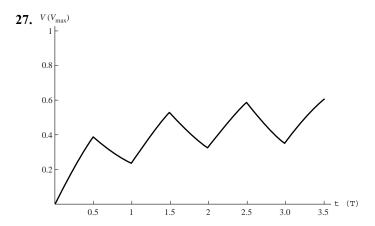


(b)
$$P_{\text{av}} = \frac{V_{\text{rms},R}^2}{R} = \frac{(V_{\text{rms}} \cos \phi)^2}{R} = \frac{[(120 \text{ V}) \cos 38.12^\circ]^2}{105 \Omega} = \boxed{84.9 \text{ W}}$$

25.
$$\cos \phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} = \frac{1}{\sqrt{1 + \left(\frac{1}{\omega RC}\right)^2}} = \frac{1}{\sqrt{1 + \left[\frac{1}{2\pi (70.0 \text{ s}^{-1})(105 \Omega)(82.4 \times 10^{-6} \text{ F})}\right]^2}} = \boxed{0.9672}$$

26. (a)
$$\cos \phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} = \frac{1}{\sqrt{1 + \left(\frac{1}{\omega RC}\right)^2}} = \frac{1}{\sqrt{1 + \left[\frac{1}{2\pi (150 \text{ s}^{-1})(4.0 \times 10^3 \Omega)(0.35 \times 10^{-6} \text{ F})}\right]^2}} = \boxed{0.797}$$

(b) Since $Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$, increasing ω decreases Z. The power factor is inversely proportional to Z, so if Z decreases, the power factor increases. Thus, if the frequency increases, the power factor increases.



28.
$$L = \frac{X_L}{\omega}$$

 $X_L' = \omega' L = \omega' \left(\frac{X_L}{\omega}\right) = \frac{\omega'}{\omega} X_L = \frac{2\pi (60.0 \text{ Hz})}{2\pi (75.0 \text{ Hz})} (56.5 \Omega) = \boxed{45.2 \Omega}$

29.
$$I_{\text{rms}} = \frac{V_{\text{rms}}}{\omega L} = \frac{115 \text{ V}}{2\pi (60.0 \text{ s}^{-1})(77.5 \times 10^{-3} \text{ H})} = \boxed{3.94 \text{ A}}$$

30.
$$V_{\text{rms}} = \omega L I_{\text{rms}} = 2\pi (25 \text{ s}^{-1})(66 \times 10^{-3} \text{ H})(2.1 \text{ A}) = 22 \text{ V}$$

31.
$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{\sqrt{R^2 + \omega^2 L^2}} = \frac{20.0 \text{ V}}{\sqrt{(525 \Omega)^2 + 4\pi^2 (60.0 \text{ s}^{-1})^2 (255 \times 10^{-3} \text{ H})^2}} = \boxed{37.5 \text{ mA}}$$

32. (a)
$$Z = \frac{V_{\text{rms}}}{I_{\text{rms}}}$$

$$R^2 + \omega^2 L^2 = \left(\frac{V_{\text{rms}}}{I_{\text{rms}}}\right)^2$$

$$R = \sqrt{\left(\frac{V_{\text{rms}}}{I_{\text{rms}}}\right)^2 - \omega^2 L^2}$$

$$= \sqrt{\left(\frac{25 \text{ V}}{0.26 \text{ A}}\right)^2 - 4\pi^2 (60.0 \text{ s}^{-1})^2 (145 \times 10^{-3} \text{ H})^2}$$

$$= \boxed{79 \Omega}$$

(b)
$$V_{\text{rms},R} = I_{\text{rms}}R = (0.26 \text{ A})(79 \Omega) = 21 \text{ V}$$

(c)
$$V_{\text{rms},L} = I_{\text{rms}} X_L = I_{\text{rms}} \omega L = (0.26 \text{ A}) 2\pi (60.0 \text{ s}^{-1}) (145 \times 10^{-3} \text{ H}) = \boxed{14 \text{ V}}$$

(d)
$$\sqrt{V_{\text{rms},R}^2 + V_{\text{rms},L}^2} = \sqrt{(21 \text{ V})^2 + (14 \text{ V})^2} = \boxed{25 \text{ V}}$$

33. (a)
$$\cos \phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (\omega L)^2}} = \frac{1}{\sqrt{1 + \left(\frac{\omega L}{R}\right)^2}} = \frac{1}{\sqrt{1 + \left[\frac{2\pi (1.00 \times 10^3 \text{ s}^{-1})(365 \times 10^{-3} \text{ H})}{2.00 \times 10^3 \Omega}\right]^2}} = \boxed{0.657}$$

(b)
$$P_{\text{av}} = \frac{V_{\text{rms}}^2}{\sqrt{R^2 + (\omega L)^2}} \cos \phi = \frac{(24 \text{ V})^2}{\sqrt{(2.00 \times 10^3 \text{ }\Omega)^2 + 4\pi^2 (1.00 \times 10^3 \text{ s}^{-1})^2 (365 \times 10^{-3})^2}} (0.657) = \boxed{0.12 \text{ W}}$$

34. (a)
$$I_{\text{rms}} = \frac{V_{\text{rms}}}{\omega L} = \frac{10.0 \text{ V}}{2\pi (1.00 \times 10^3 \text{ s}^{-1})(0.220 \times 10^{-3} \text{ H})} = \boxed{7.23 \text{ A}}$$

(b) Since *I* is inversely proportional to the frequency, doubling the frequency decreases *I* by a factor of $\frac{1}{2}$

(c)
$$I_{\text{rms}} = \frac{10.0 \text{ V}}{2\pi (2.00 \times 10^3 \text{ s}^{-1})(0.220 \times 10^{-3} \text{ H})} = \boxed{3.62 \text{ A}}$$

35.
$$\omega = \frac{V_{\text{rms}}}{I_{\text{rms}}L} = 2\pi f$$

$$f = \frac{12 \text{ V}}{2\pi (1.0 \times 10^{-3} \text{ A})(0.22 \times 10^{-6} \text{ H})} = 8.7 \text{ GHz}$$
As $I_{\text{rms}} \to 0, f \to \infty$, so
$$\boxed{8.7 \text{ GHz} < f < \infty.}$$

36.
$$\cos \phi = \frac{R}{\sqrt{R^2 + (\omega L)^2}}$$
$$\cos^2 \phi = \frac{R^2}{R^2 + \omega^2 L^2}$$
$$R^2 + \omega^2 L^2 = \frac{R^2}{\cos^2 \phi}$$
$$\omega^2 L^2 = R^2 \sec^2 \phi - R^2$$
$$L^2 = \frac{R^2}{\omega^2 \tan^2 \phi}$$
$$L = \frac{R}{\omega} |\tan \phi|$$
$$= \frac{2.7 \Omega}{2\pi (60.0 \text{ Hz})} \tan 76^\circ$$
$$= \boxed{29 \text{ mH}}$$

37. (a)
$$Z = \sqrt{R^2 + (\omega L)^2}$$

$$Z^2 - R^2 = \omega^2 L^2$$

$$\omega = \sqrt{\frac{Z^2 - R^2}{L^2}}$$
Since
$$Z = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{120 \text{ V}}{1.5 \text{ A}} = 80 \text{ }\Omega,$$

$$\omega = \sqrt{\frac{(80 \text{ }\Omega)^2 - (68 \text{ }\Omega)^2}{(31 \times 10^{-3} \text{ H})^2}}$$

$$= 1359 \frac{\text{rad}}{\text{s}}$$

$$f = \frac{1}{2\pi} \left(1359 \frac{\text{rad}}{\text{s}}\right) = \boxed{0.22 \text{ kHz}}$$

- (b) $V_{\text{rms},R} = I_{\text{rms}}R$ = (1.5 A)(68 Ω) = 102 V = $\boxed{0.10 \text{ kV}}$
- (c) $V_{\text{rms},L} = I_{\text{rms}} \omega L$ = $(1.5 \text{ A}) \left(1359 \frac{\text{rad}}{\text{s}} \right) (31 \times 10^{-3} \text{ H})$ = $\boxed{63 \text{ V}}$
- (d) $V_{\text{rms},R} + V_{\text{rms},L} = 102 \text{ V} + 63 \text{ V} = 165 \text{ V} > 120 \text{ V}$ $\sqrt{V_{\text{rms},R}^2 + V_{\text{rms},L}^2} = \sqrt{(102 \text{ V})^2 + (63 \text{ V})^2} = 120 \text{ V}$

38. (a)
$$\phi = \cos^{-1} \frac{R}{Z}$$

$$= \cos^{-1} \frac{R}{\sqrt{R^2 + (\omega L)^2}}$$

$$= \cos^{-1} \frac{1}{\sqrt{1 + \left(\frac{\omega L}{R}\right)^2}}$$

$$= \cos^{-1} \frac{1}{\sqrt{1 + \left[\frac{2\pi (60.0 \text{ s}^{-1})(22.5 \times 10^{-3} \text{ H})}{105 \Omega}\right]^2}}$$

$$V_{\text{max}} = I_{\text{max}} Z$$

$$I_{\text{max}} X_L$$

$$V_{\text{max}} = I_{\text{max}} Z$$

$$V_{\text{max}} = I_{\text{max}} Z$$

$$V_{\text{max}} = I_{\text{max}} Z$$

(b)
$$P_{\text{av}} = \frac{V_{\text{rms},R}^2}{R} = \frac{(V_{\text{rms}}\cos\phi)^2}{R} = \frac{[(120 \text{ V})\cos 4.62^\circ]^2}{105 \Omega} = \boxed{136 \text{ W}}$$

39. As F is increased, ωL increases while R remains constant. The maximum voltage phasor will swing toward the inductor's voltage phasor, and the angle ϕ will increase.

From Figure 24-18, we see that

$$\tan \phi = \frac{I_{\text{max}} X_L}{I_{\text{max}} R} = \frac{X_L}{R} = \frac{\omega L}{R}$$

so that
$$L = \frac{R \tan \phi}{\omega}$$
.

For Problem 36
$$L = \frac{(2.7 \Omega)(\tan 76^\circ)}{2\pi(60.0 \text{ Hz})} = 29 \text{ mH}.$$

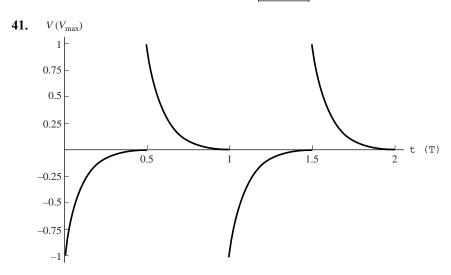
Solving
$$\tan \phi = \frac{\omega L}{R}$$
 for ϕ then gives

$$\phi = \tan^{-1} \left(\frac{\omega L}{R} \right) = \tan^{-1} \left(\frac{2\pi (70.0 \text{ Hz})(29 \times 10^{-3} \text{ H})}{2.7 \Omega} \right) = \boxed{78^{\circ}}$$

40. (a)
$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(7.0 \ \Omega)^2 + (15 \ \Omega)^2} = 16.6 \ \Omega = \boxed{17 \ \Omega}$$

(b)
$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{240 \text{ V}}{16.6 \Omega} = 14.46 \text{ A} = \boxed{14 \text{ A}}$$

(c)
$$P_{\text{av}} = I_{\text{rms}}^2 R = (14.46 \text{ A})^2 (7.0 \Omega) = \boxed{1.5 \text{ kW}}$$



42. Find
$$I_{\rm rms}$$
.

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{115 \text{ V}}{\sqrt{(9900 \ \Omega)^2 + \left[2\pi (60.0 \text{ s}^{-1})(0.025 \text{ H}) - \frac{1}{2\pi (60.0 \text{ s}^{-1})(0.15 \times 10^{-6} \text{ F})}\right]^2}} = 5.7 \text{ mA}$$

Determine the voltages.

$$V_{\text{rms, }R} = I_{\text{rms}}R = (5.7 \times 10^{-3} \text{ A})(9900 \Omega) = \boxed{56 \text{ V}}$$

$$V_{\text{rms, }L} = I_{\text{rms}} X_L = I_{\text{rms}} \omega L = (5.7 \times 10^{-3} \text{ A}) 2\pi (60.0 \text{ s}^{-1}) (0.025 \text{ H}) = \boxed{54 \text{ mV}}$$

$$V_{\text{rms, }C} = I_{\text{rms}} X_C = \frac{I_{\text{rms}}}{\omega C} = \frac{5.7 \times 10^{-3} \text{ A}}{2\pi (60.0 \text{ s}^{-1})(0.15 \times 10^{-6} \text{ F})} = \boxed{100 \text{ V}}$$

43.
$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{(1500 \ \Omega)^2 + \left[2\pi (60.0 \ s^{-1})(0.155 \ H) - \frac{1}{2\pi (60.0 \ s^{-1})(12.5 \times 10^{-6} \ F)}\right]^2} = \boxed{1.51 \ k\Omega}$$

44. (a) In the limit of high frequency, the inductor behaves like a very large resistor. So, nearly all of the current flows through the circuit with the lone resistor.

$$I_{\rm rms} = \frac{V_{\rm rms}}{R} = \frac{65 \text{ V}}{15 \Omega} = \boxed{4.3 \text{ A}}$$

(b) In the limit of low frequency, the reactance of the inductor approaches zero. So, the current flows through each resistor equally.

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R_{\text{eq}}} = \frac{2V_{\text{rms}}}{R} = \frac{2(65 \text{ V})}{15 \Omega} = \boxed{8.7 \text{ A}}$$

45. (a) In the limit of high frequency, the reactance of the capacitor approaches zero. So, the current flows through each resistor equally.

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R_{\text{eq}}} = \frac{2V_{\text{rms}}}{R} = \frac{2(65 \text{ V})}{15 \Omega} = \boxed{8.7 \text{ A}}$$

(b) In the limit of low frequency, the capacitor behaves like a very large resistor. So, nearly all of the current flows through the circuit with the lone resistor.

$$I_{\rm rms} = \frac{V_{\rm rms}}{R} = \frac{65 \text{ V}}{15 \Omega} = \boxed{4.3 \text{ A}}$$

46. $\tan \phi = \frac{X_L - X_C}{R} = \frac{\omega L - \frac{1}{\omega C}}{R}$

$$\phi = \tan^{-1} \frac{\omega L - \frac{1}{\omega C}}{R} = \tan^{-1} \frac{2\pi (60.0 \text{ s}^{-1})(0.250 \text{ H}) - \frac{1}{2\pi (60.0 \text{ s}^{-1})(1.5 \times 10^{-6} \text{ F})}}{9900 \Omega} = \boxed{-9.6^{\circ}}$$

- 47. (a) $\cos \phi = \frac{R}{\sqrt{R^2 + \left(\omega L \frac{1}{\omega C}\right)^2}} = \frac{105 \Omega}{\sqrt{(105 \Omega)^2 + \left[2\pi (125 \text{ s}^{-1})(0.0950 \text{ H}) \frac{1}{2\pi (125 \text{ s}^{-1})(10.0 \times 10^{-6} \text{ F})}\right]^2}} = \boxed{0.894}$
 - **(b)** $\cos \phi = \frac{R}{\sqrt{R^2 + (\omega L \frac{1}{\omega C})^2}} = \frac{1}{\sqrt{1 + \frac{1}{R^2} (\omega L \frac{1}{\omega C})^2}}$

As *R* increases, the denominator of the expression above decreases, and thus, the power factor increases.

(c)
$$\cos \phi = \frac{525 \Omega}{\sqrt{(525 \Omega)^2 + \left[2\pi (125 \text{ s}^{-1})(0.0950 \text{ H}) - \frac{1}{2\pi (125 \text{ s}^{-1})(10.0 \times 10^{-6} \text{ F})}\right]^2}} = \boxed{0.995}$$

48. Find $I_{\rm rms}$.

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$= \frac{6.00 \text{ V}}{\sqrt{(2.50 \Omega)^2 + \left[2\pi (30.0 \times 10^3 \text{ s}^{-1})(0.300 \times 10^{-3} \text{ H}) - \frac{1}{2\pi (30.0 \times 10^3 \text{ s}^{-1})(0.100 \times 10^{-6} \text{ F})}\right]^2}}$$

$$= 1.4 \text{ A}$$

(a)
$$V_{\text{rms},L} = I_{\text{rms}} X_L = I_{\text{rms}} \omega L = (1.4 \text{ A}) 2\pi (30.0 \times 10^3 \text{ s}^{-1}) (0.300 \times 10^{-3} \text{ H}) = \boxed{79 \text{ V}}$$

(b)
$$V_{\text{rms},C} = I_{\text{rms}} X_C = \frac{I_{\text{rms}}}{\omega C} = \frac{1.4 \text{ A}}{2\pi (30.0 \times 10^3 \text{ s}^{-1})(0.100 \times 10^{-6} \text{ F})} = \boxed{74 \text{ V}}$$

(c)
$$V_{\text{rms},LC} = I_{\text{rms}} X_L - I_{\text{rms}} X_C = 79 \text{ V} - 74 \text{ V} = \boxed{5 \text{ V}}$$

(d)
$$V_{\rm rms} = 6.00 \text{ V}$$

49. Again, find I_{rms} .

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$= \frac{6.00 \text{ V}}{\sqrt{(2.50 \Omega)^2 + \left[2\pi (60.0 \times 10^3 \text{ s}^{-1})(0.300 \times 10^{-3} \text{ H}) - \frac{1}{2\pi (60.0 \times 10^3 \text{ s}^{-1})(0.100 \times 10^{-6} \text{ F})}\right]^2}}$$

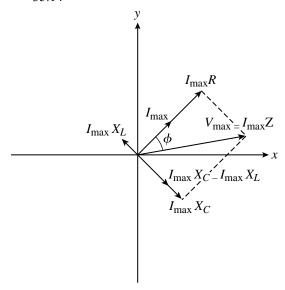
$$= 69.3 \text{ m/s}$$

(a)
$$V_{\text{rms},R} = I_{\text{rms}}R = (69.3 \text{ mA})(2.50 \Omega) = \boxed{0.173 \text{ V}}$$

(b)
$$V_{\text{rms},L} = I_{\text{rms}} \omega L = (69.3 \text{ mA}) 2\pi (60.0 \times 10^3 \text{ s}^{-1}) (0.300 \times 10^{-3} \text{ H}) = \boxed{7.84 \text{ V}}$$

(c)
$$V_{\text{rms},C} = \frac{I_{\text{rms}}}{\omega C} = \frac{69.3 \text{ mA}}{2\pi (60.0 \times 10^3 \text{ s}^{-1})(0.100 \times 10^{-6} \text{ F})} = \boxed{1.84 \text{ V}}$$

- (d) Since the rms voltages are not all in phase, their sum will be greater than the overall rms voltage of 6.00 V. (The sum of the magnitudes of out-of-phase phasors is greater than the magnitude of their vector sum.)
- 50. (a) $\phi = \cos^{-1} \frac{R}{Z}$ $= \cos^{-1} \frac{R}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$ $= \cos^{-1} \frac{1}{\sqrt{1 + (\frac{\omega L}{R} - \frac{1}{\omega RC})^2}}$ $= \cos^{-1} \frac{1}{\sqrt{1 + \left[\frac{2\pi (60.0 \text{ s}^{-1})(22.5 \times 10^{-3} \text{ H})}{105 \Omega} - \frac{1}{2\pi (60.0 \text{ s}^{-1})(105 \Omega)(32.2 \times 10^{-6} \text{ F})}\right]^2}}$ $= 35.14^{\circ}$



(b)
$$P_{\text{av}} = \frac{V_{\text{rms},R}^2}{R} = \frac{(V_{\text{rms}}\cos\phi)^2}{R} = \frac{[(120 \text{ V})\cos 35.14^\circ]^2}{105 \Omega} = \boxed{91.7 \text{ W}}$$

51.
$$V_{\text{rms}} = I_{\text{rms}} Z$$

$$Z = \frac{V_{\text{rms}}}{I_{\text{rms}}}$$

$$\sqrt{R^2 + (X_L - X_C)^2} = \frac{V_{\text{rms}}}{I_{\text{rms}}}$$

$$R^2 + (X_L - X_C)^2 = \left(\frac{V_{\text{rms}}}{I_{\text{rms}}}\right)^2$$

$$(X_L - X_C)^2 = \left(\frac{V_{\text{rms}}}{I_{\text{rms}}}\right)^2 - R^2$$

$$X_L - X_C = \pm \sqrt{\left(\frac{V_{\text{rms}}}{I_{\text{rms}}}\right)^2 - R^2}$$

$$X_L = X_C \pm \sqrt{\left(\frac{V_{\text{rms}}}{I_{\text{rms}}}\right)^2 - R^2}$$

$$= 6.6 \times 10^3 \ \Omega \pm \sqrt{\left(\frac{120 \text{ V}}{24 \times 10^{-3} \text{ A}}\right)^2 - (3.3 \times 10^3 \ \Omega)^2}$$

$$= \boxed{10.4 \text{ k}\Omega \text{ or } 2.8 \text{ k}\Omega}$$

52. (a)
$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(25.0 \ \Omega)^2 + (45.0 \ \Omega)^2} = \boxed{51.5 \ \Omega}$$

(b)
$$\cos \phi = \frac{R}{Z} = \frac{25.0 \ \Omega}{51.48 \ \Omega} = \boxed{0.486}$$

(c)
$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{485 \text{ V}}{51.48 \Omega} = \boxed{9.42 \text{ A}}$$

(d)
$$\cos \phi = 1 = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

 $R^2 + (X_L - X_C)^2 = R^2$
 $(X_L - X_C)^2 = 0$
 $X_C = X_L$
 $\frac{1}{\omega C} = X_L$
 $C = \frac{1}{\omega X_L}$
 $C = \frac{1}{2\pi (60.0 \text{ s}^{-1})(45.0 \Omega)}$
 $C = \frac{1}{2\pi (90.0 \text{ s}^{-1})(45.0 \Omega)}$

(e)
$$P_{\text{av}} = I_{\text{rms}} V_{\text{rms}} \cos \phi = I_{\text{rms}} V_{\text{rms}} (1)$$

 $I_{\text{rms}} = \frac{P_{\text{av}}}{V_{\text{rms}}} = \frac{2.22 \times 10^3 \text{ W}}{485 \text{ V}} = \boxed{4.58 \text{ A}}$
 $9.42 \text{ A} > 4.58 \text{ A}$

53.
$$\omega = \frac{1}{\sqrt{LC}}$$

$$L = \frac{1}{\omega^2 C} = \frac{1}{4\pi^2 (2.4 \times 10^3 \text{ s}^{-1})^2 (47 \times 10^{-6} \text{ F})} = 94 \ \mu\text{H}$$

54. At resonance
$$X_L = X_C$$
, so $Z = R$.
 $R = Z = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{120 \text{ V}}{1.5 \text{ A}} = \boxed{80 \Omega}$

55. (a) The current will be at its maximum when the frequency is the resonance frequency.

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.0576 \text{ H})(179 \times 10^{-6} \text{ F})}} = \boxed{49.6 \text{ Hz}}$$

(b) The impedance will be at its minimum when the frequency is the resonance frequency.

$$f = 49.6 \text{ Hz}$$

56. Determine the resonance frequency.

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.15\times10^{-3} \text{ H})(0.20\times10^{-3} \text{ F})}} = 920 \text{ Hz}$$

At resonance, the current is at its maximum. As the frequency is increased beyond or decreased below resonance, the current decreases. Since 1.0 kHz is above the resonance frequency, the frequency should be increased to reduce the current.

57. (a) Set
$$X_C = X_L$$
.

$$X_{C} = \frac{1}{\omega C} = \omega L = X_{L}$$

$$\omega = \frac{1}{\sqrt{LC}} = 2\pi f$$

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.033 \text{ H})(33 \times 10^{-6} \text{ F})}} = \boxed{150 \text{ Hz}}$$

(b)
$$f = \frac{1}{2\pi\sqrt{LC}} = 150 \text{ Hz}$$

58. (a)
$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.518 \text{ H})(0.200 \times 10^{-6} \text{ F})}} = \boxed{494 \text{ Hz}}$$

(b)
$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$= \sqrt{(105 \Omega)^2 + \left[2\pi (494.47 \text{ s}^{-1})(0.518 \text{ H}) - \frac{1}{2\pi (494.47 \text{ s}^{-1})(0.220 \times 10^{-6} \text{ F})}\right]^2}$$

$$= \boxed{180 \Omega}$$

(c)
$$\cos \phi = \frac{R}{Z} = \frac{105 \Omega}{180 \Omega} = \boxed{0.58}$$

59. (a) Since the resonance frequency is inversely proportional to the square root of the product of L and C, the resonance frequency will decrease if L and C are doubled.

(b)
$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

 $f = \frac{1}{2\pi\sqrt{(2L)(2C)}} = \frac{1}{2\pi\sqrt{4LC}} = \frac{1}{4\pi\sqrt{LC}} = \frac{f_0}{2} = \frac{125 \text{ Hz}}{2} = \boxed{62.5 \text{ Hz}}$

60. (a)
$$\omega = \frac{1}{\sqrt{LC}}$$

$$L = \frac{1}{\omega^2 C} = \frac{1}{4\pi^2 (95 \times 10^6 \text{ Hz})^2 (0.29 \times 10^{-6} \text{ F})} = \boxed{9.7 \text{ pH}}$$

(b) At resonance,
$$Z = R$$
.

Set
$$R = Z = Z_{11}/5$$
.

$$R = \frac{1}{5}\sqrt{R^2 + (X_L - X_C)^2}$$

$$25R^2 = R^2 + (X_L - X_C)^2$$

$$24R^2 = (X_L - X_C)^2$$

$$R = \frac{|X_L - X_C|}{\sqrt{24}}$$

$$= \frac{|\omega L - \frac{1}{\omega C}|}{2\sqrt{6}}$$

$$= \frac{2\pi (11 \times 10^3 \text{ s}^{-1})(9.7 \times 10^{-12} \text{ H}) - \frac{1}{2\pi (11 \times 10^3 \text{ s}^{-1})(0.29 \times 10^{-6} \text{ F})}}{2\sqrt{6}}$$

$$= \boxed{10 \Omega}$$

61.
$$I_{\text{rms}} = \frac{V_{\text{rms}}}{X_C} = \omega C_{\text{eq}} V_{\text{rms}} = 2\pi (60.0 \text{ s}^{-1}) (4.40 \times 10^{-6} \text{ F} + 8.80 \times 10^{-6} \text{ F}) (115 \text{ V}) = \boxed{572 \text{ mA}}$$

62.
$$I_{\text{rms}} = \frac{V_{\text{rms}}}{X_C} = \omega C_{\text{eq}} V_{\text{rms}} = 2\pi (60.0 \text{ s}^{-1}) \left(\frac{1}{4.40 \times 10^{-6} \text{ F}} + \frac{1}{8.80 \times 10^{-6} \text{ F}} \right)^{-1} (115 \text{ V}) = \boxed{127 \text{ mA}}$$

63.
$$X_C = \frac{1}{\omega C_{\text{eq}}} = \frac{1}{2\pi (60.0 \text{ s}^{-1})(10.0 \times 10^{-6} \text{ F} + 20.0 \times 10^{-6} \text{ F})} = \boxed{88.4 \Omega}$$

64. (a) $X_C > X_L$ means $\frac{1}{\omega C} > \omega L$, which implies $\omega_{\text{res}} = \frac{1}{\sqrt{LC}} > \omega$. The resonance frequency is greater than 60.0 Hz.

(b)
$$f = \frac{1}{2\pi\sqrt{LC}}$$

= $\frac{1}{2\pi\sqrt{(90.0 \times 10^{-3} \text{ H})(15.0 \times 10^{-6} \text{ F})}}$
= $\boxed{137 \text{ Hz}}$

(c) At resonance, $Z = R = \boxed{175 \ \Omega}$

Chapter 24: Alternating-Current Circuits

65. (a)
$$P_{\text{av}} = \frac{V_{\text{rms}}^2}{R} = \frac{V_{\text{max}}^2}{2R}$$

$$R = \frac{V_{\text{max}}^2}{2P_{\text{av}}} = \frac{(15 \text{ V})^2}{2(22 \text{ W})} = \boxed{5.1 \Omega}$$

(b)
$$I_{\text{rms}} = \frac{P_{\text{av}}}{V_{\text{rms}}} = \frac{\sqrt{2}P_{\text{av}}}{V_{\text{max}}} = \frac{\sqrt{2}(22 \text{ W})}{15 \text{ V}} = \boxed{2.1 \text{ A}}$$

- (c) Reducing R increases P_{av} , because V_{rms} is fixed, whereas I_{rms} increases when R decreases.
- 66. Since the voltage across a capacitor lags the current by 90° ($\pi/2$ radians), the earliest possible time that the voltage across the capacitor is a maximum is

$$t = \frac{\theta}{\omega} = \frac{\frac{\pi}{2}}{2\pi f} = \frac{1}{4f} = \frac{1}{4(5.0 \text{ s}^{-1})} = \boxed{50 \text{ ms}}.$$

67. Since the voltage across an inductor leads the current by 90° ($\pi/2$ radians), the lowest possible frequency at which the generator operates is

$$f = \frac{\theta}{2\pi t} = \frac{\frac{\pi}{2}}{2\pi t} = \frac{1}{4t} = \frac{1}{4(25 \times 10^{-3} \text{ s})} = \boxed{10 \text{ Hz}}$$

68.
$$P_{\text{av}} = I_{\text{rms}}^2 R = \frac{V_{\text{rms}}^2 R}{Z^2} = \frac{V_{\text{rms}}^2 R}{R^2 + \frac{1}{\omega^2 C^2}} = \frac{(112 \text{ V})^2 (3.30 \times 10^3 \Omega)}{(3.30 \times 10^3 \Omega)^2 + \frac{1}{4\pi^2 (60.0 \text{ s}^{-1})^2 (2.75 \times 10^{-6} \text{ F})^2}} = \boxed{3.50 \text{ W}}$$

69. (a)
$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{\sqrt{R^2 + \omega^2 L^2}} = \frac{12.0 \text{ V}}{\sqrt{(1150 \Omega)^2 + 4\pi^2 (1050 \text{ s}^{-1})^2 (0.525 \text{ H})^2}} = \boxed{3.29 \text{ mA}}$$

(b)
$$\frac{I_{\text{rms}}}{2} = \frac{V_{\text{rms}}}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

$$\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} = \frac{2V_{\text{rms}}}{I_{\text{rms}}}$$

$$R^2 + (\omega L - \frac{1}{\omega C})^2 = 4\left(\frac{V_{\text{rms}}}{I_{\text{rms}}}\right)^2 - R^2$$

$$(\omega L - \frac{1}{\omega C})^2 = 4\left(\frac{V_{\text{rms}}}{I_{\text{rms}}}\right)^2 - R^2$$

$$\omega L - \frac{1}{\omega C} = \pm \sqrt{4\left(\frac{V_{\text{rms}}}{I_{\text{rms}}}\right)^2 - R^2}$$

$$\frac{1}{\omega C} = \omega L \pm \sqrt{4\left(\frac{V_{\text{rms}}}{I_{\text{rms}}}\right)^2 - R^2}$$

$$C = \frac{1}{\omega^2 L \pm \omega \sqrt{4\left(\frac{V_{\text{rms}}}{I_{\text{rms}}}\right)^2 - R^2}}$$

$$= \frac{1}{4\pi^2 (1050 \text{ s}^{-1})^2 (0.525 \text{ H}) \pm 2\pi (1050 \text{ s}^{-1}) \sqrt{4\left(\frac{12.0 \text{ V}}{3.288 \times 10^{-3} \text{ A}}\right)^2 - (1150 \Omega)^2}}$$

$$= 14.2 \text{ nF} \text{ or } -40.5 \text{ nF}$$

The answer is 14.2 nF because capacitance cannot be less than zero.

70. (a)
$$\cos \phi = \frac{R}{Z}$$

$$Z = \frac{R}{\cos \phi} = \frac{525 \ \Omega}{\cos 30.0^{\circ}} = \boxed{606 \ \Omega}$$

(b) By inspection of the diagram, we see that $X_L > X_C$. If the frequency is increased, $(X_L - X_C)^2 = [\omega L - 1/(\omega C)]^2$ will increase. Since $Z = \sqrt{R^2 + (X_L - X_C)^2}$, the impedance will increase if the frequency is increased.

71. (a)
$$\cos \phi = \frac{R}{Z}$$

$$R = Z \cos \phi$$

$$= (337 \Omega) \cos 30.0^{\circ}$$

$$= \boxed{292 \Omega}$$

(b) V_{max} leads I_{max} , so $X_L > X_C$; that is, $\omega L > \frac{1}{\omega C}$. This implies $\omega > \frac{1}{\sqrt{LC}} = \omega_{\text{res}}$. The driving frequency is greater than the resonance frequency.

72. (a)
$$\cos \phi = \frac{R}{Z}$$

$$Z = \frac{R}{\cos \phi}$$

$$= \frac{25 \Omega}{\cos 30.0^{\circ}}$$

$$= \boxed{29 \Omega}$$

(b)
$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$Z^2 = R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2$$

$$Z^2 - R^2 = \left(\omega L - \frac{1}{\omega C}\right)^2$$

$$\sqrt{Z^2 - R^2} = \omega L - \frac{1}{\omega C} \quad \text{since } X_L > X_C$$

$$\frac{1}{\omega C} = \omega L - \sqrt{Z^2 - R^2}$$

$$C = \frac{1}{\omega \left(\omega L - \sqrt{Z^2 - R^2}\right)}$$

$$= \frac{1}{2\pi (55 \text{ s}^{-1}) \left[2\pi (55 \text{ s}^{-1}) (160 \times 10^{-3} \text{ H}) - \sqrt{(29 \Omega)^2 - (25 \Omega)^2}\right]}$$

$$= \sqrt{71 \mu F}$$

- (c) Because $X_L = \omega L$ is greater than $X_C = \frac{1}{\omega C}$, decreasing C will cause the quantity $\left(\omega L \frac{1}{\omega C}\right)^2$ to decrease, and so $Z = \sqrt{R^2 + \left(\omega L \frac{1}{\omega C}\right)^2}$ will decrease.
- 73. (a) $I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{\sqrt{R^2 + X^2}}$, where X is the reactance of the inductor or the capacitor. If X increases, I_{rms} decreases. If X decreases, I_{rms} increases. $X_L = \omega L$ increases with increasing frequency. $X_C = \frac{1}{\omega C}$ decreases with increasing frequency. So, the box must contain a capacitor.

(b)
$$I_{\text{rms}} = \frac{V_{\text{rms}}}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

$$\sqrt{R^2 + \frac{1}{\omega^2 C^2}} = \frac{V_{\text{rms}}}{I_{\text{rms}}}$$

$$R^2 + \frac{1}{\omega^2 C^2} = \left(\frac{V_{\text{rms}}}{I_{\text{rms}}}\right)^2 - R^2$$

$$\frac{1}{\omega^2 C^2} = \left(\frac{V_{\text{rms}}}{I_{\text{rms}}}\right)^2 - R^2$$

$$\omega^2 C^2 = \left[\left(\frac{V_{\text{rms}}}{I_{\text{rms}}}\right)^2 - R^2\right]^{-1}$$

$$C = \frac{1}{\omega} \left[\left(\frac{V_{\text{rms}}}{I_{\text{rms}}}\right)^2 - R^2\right]^{-1/2}$$

$$= \frac{1}{2\pi (25.0 \times 10^3 \text{ s}^{-1})} \left[\left(\frac{0.750 \text{ V}}{87.2 \times 10^{-3} \text{ A}}\right)^2 - (5.00 \Omega)^2\right]^{-1/2}$$

$$= \frac{0.910 \ \mu\text{F}}$$
74. (a)
$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$Z^2 = R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2$$

$$\left(\omega L - \frac{1}{\omega C}\right)^2 = Z^2 - R^2$$

$$\left(\omega^2 L - \frac{1}{C}\right)^2 = \omega^2 (Z^2 - R^2)$$

$$\omega^4 L^2 - \frac{2\omega^2 L}{C} + \frac{1}{C^2} = \omega^2 (Z^2 - R^2)$$

$$0 = L^2 \omega^4 + \left(R^2 - Z^2 - \frac{2L}{C}\right)\omega^2 + \frac{1}{C^2}$$

Use the quadratic formula to solve for ω^2 .

$$\omega^{2} = \frac{-\left(R^{2} - Z^{2} - \frac{2L}{C}\right) \pm \sqrt{\left(R^{2} - Z^{2} - \frac{2L}{C}\right)^{2} - 4(L^{2})\left(\frac{1}{C^{2}}\right)}}{2L^{2}}$$

$$\omega = \sqrt{\frac{1}{LC} + \frac{Z^{2} - R^{2}}{2L^{2}} \pm \sqrt{\left(\frac{1}{LC} + \frac{Z^{2} - R^{2}}{2L^{2}}\right)^{2} - \frac{1}{L^{2}C^{2}}}}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{\frac{1}{(0.515 \text{ H})(252 \times 10^{-6} \text{ F})} + \frac{(2.00 \times 10^{3} \Omega)^{2} - (1.00 \times 10^{3} \Omega)^{2}}{2(0.515 \text{ H})^{2}}}$$

$$\pm \sqrt{\left[\frac{1}{(0.515 \text{ H})(252 \times 10^{-6} \text{ F})} + \frac{(2.00 \times 10^{3} \Omega)^{2} - (1.00 \times 10^{3} \Omega)^{2}}{2(0.515 \text{ H})^{2}}\right]^{2} - \frac{1}{(0.515 \text{ H})^{2}(252 \times 10^{-6} \text{ F})^{2}}}$$

$$= \boxed{535.6 \text{ Hz or } 0.4 \text{ Hz}}$$

(b) Find the resonance frequency.

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.515 \text{ H})(252 \times 10^{-6} \text{ F})}} = 14.0 \text{ Hz}$$

At resonance, the impedance is at its minimum value. So, to decrease the impedance, the frequency must be changed so that it approaches the resonance frequency.

If f = 535.6 Hz, the frequency must be decreased, and if f = 0.4 Hz, the frequency must be increased.

75.
$$\omega = \frac{1}{\sqrt{LC}} = 2\pi f$$

$$P_{\text{av}} = \frac{V_{\text{rms}}^2}{Z^2} R$$

(a) At resonance, Z = R.

$$P_{\text{av}} = \frac{V_{\text{rms}}^2 R}{R^2} = \frac{V_{\text{rms}}^2}{R} = \frac{(24 \text{ V})^2}{25.0 \Omega} = \boxed{23 \text{ W}}$$

(b)
$$P_{\text{av}} = \frac{V_{\text{rms}}^2 R}{Z^2}$$

$$= \frac{V_{\text{rms}}^2 R}{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

$$= \frac{V_{\text{rms}}^2 R}{R^2 + \left[\left(\frac{2}{\sqrt{LC}}\right)L - \left(\frac{\sqrt{LC}}{2}\right)\frac{1}{C}\right]^2}$$

$$= \frac{V_{\text{rms}}^2 R}{R^2 + \left(2\sqrt{\frac{L}{C}} - \frac{1}{2}\sqrt{\frac{L}{C}}\right)^2}$$

$$= \frac{(24 \text{ V})^2 (25.0 \Omega)}{(25.0 \Omega)^2 + \frac{9(0.325 \text{ H})}{4(45.2 \times 10^{-6} \text{ F})}}$$

$$= \boxed{0.86 \text{ W}}$$

(c)
$$P_{\text{av}} = \frac{V_{\text{rms}}^2 R}{R^2 + \left[\left(\frac{1}{2\sqrt{LC}} \right) L - (2\sqrt{LC}) \frac{1}{C} \right]^2}$$
$$= \frac{V_{\text{rms}}^2 R}{R^2 + \left(\frac{1}{2} \sqrt{\frac{L}{C}} - 2\sqrt{\frac{L}{C}} \right)^2}$$
$$= \frac{(24 \text{ V})^2 (25.0 \Omega)}{(25.0 \Omega)^2 + \frac{9(0.325 \text{ H})}{4(45.2 \times 10^{-6} \text{ F})}}$$
$$= \boxed{0.86 \text{ W}}$$

76. (a)
$$P_{\text{av}} = \frac{V_{\text{rms}}^2 R}{Z^2} = \frac{V_{\text{rms}}^2 R}{R^2} = \frac{V_{\text{rms}}^2}{R} = \frac{(110 \text{ V})^2}{120 \Omega} = \boxed{100 \text{ W}}$$

(b)
$$\frac{P_{\text{av}}}{4} = \frac{V_{\text{rms}}^2}{4R} = \frac{V_{\text{rms}}^2 R}{R^2 + \omega^2 L_{\text{max}}^2}$$
$$\frac{1}{4R} = \frac{R}{R^2 + \omega^2 L_{\text{max}}^2}$$
$$R^2 + \omega^2 L_{\text{max}}^2 = 4R^2$$
$$\omega^2 L_{\text{max}}^2 = 3R^2$$
$$L_{\text{max}} = \sqrt{3} \frac{R}{\omega}$$
$$= \sqrt{3} \frac{120 \ \Omega}{2\pi (60.0 \ \text{s}^{-1})}$$
$$= \boxed{0.55 \ \text{H}}$$

77. (a)
$$\cos \phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + \omega^2 L^2}} = \frac{1}{\sqrt{1 + \left(\frac{\omega L}{R}\right)^2}} = \frac{1}{\sqrt{1 + \left[\frac{2\pi (60.0 \text{ s}^{-1})(0.053 \text{ H})}{15 \Omega}\right]^2}} = \boxed{0.60}$$

(b)
$$0.80 = \frac{R}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

$$R^2 + (\omega L - \frac{1}{\omega C})^2 = (\frac{R}{0.80})^2$$

$$(\omega L - \frac{1}{\omega C})^2 = R^2 (\frac{1}{0.80^2} - 1)$$

$$\omega L - \frac{1}{\omega C} = \pm R \sqrt{\frac{1}{0.80^2} - 1}$$

$$\frac{1}{\omega C} = \omega L \pm R \sqrt{\frac{1}{0.80^2} - 1}$$

$$C = (\omega^2 L \pm \omega R \sqrt{\frac{1}{0.80^2} - 1})^{-1}$$

$$= \left[4\pi^2 (60.0 \text{ s}^{-1})^2 (0.053 \text{ H}) \pm 2\pi (60.0 \text{ s}^{-1})(15 \Omega) \sqrt{\frac{1}{0.80^2} - 1} \right]^{-1}$$

$$= \left[85 \mu \text{F or } 300 \mu \text{F} \right]$$

78. (a)
$$\omega = \frac{1}{\sqrt{LC}}$$

$$C = \frac{1}{\omega^2 L} = \frac{1}{4\pi^2 (95 \times 10^6 \text{ Hz})^2 (2.8 \times 10^{-6} \text{ H})} = \boxed{1.0 \text{ pF}}$$

- **(b)** At resonance, the impedance is at a minimum. So, any change in the capacitance will increase the impedance.
- (c) $Z = R = 5.0 \Omega$

(d)
$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

 $= \sqrt{R^2 + \left[\omega L - \frac{1}{\omega}\left(\frac{100}{101}\omega^2 L\right)\right]^2}$
 $= \sqrt{R^2 + \left(\omega L - \frac{100}{101}\omega L\right)^2}$
 $= \sqrt{R^2 + \left(\frac{1}{101}\right)^2 \omega^2 L^2}$
 $= \sqrt{(5.0 \ \Omega)^2 + \frac{4\pi^2}{101^2}(95 \times 10^6 \ \text{Hz})^2(2.8 \times 10^{-6} \ \text{H})^2}$
 $= \boxed{17 \ \Omega}$

79. (a) Let
$$T = t_0$$
.
 $0 \le t < \frac{t_0}{2} : V = V_{\text{max}}$
 $\frac{t_0}{2} \le t < t_0 : V = 0$
 $V_{\text{av}} = \frac{V_{\text{max}} \left(\frac{t_0}{2}\right) + (0)\left(\frac{t_0}{2}\right)}{t_0} = \boxed{\frac{V_{\text{max}}}{2}}$

(b)
$$(V^2)_{av} = \frac{V_{max}^2}{2}$$

$$V_{rms} = \sqrt{(V^2)_{av}} = \sqrt{\frac{V_{max}^2}{2}} = \boxed{\frac{V_{max}}{\sqrt{2}}}$$

80.
$$Z = \frac{V_{\text{rms}}}{I_{\text{rms}}}$$

$$\sqrt{R^2 + X_C^2} = \frac{V_{\text{rms}}}{I_{\text{rms}}}$$

$$R^2 + \frac{1}{\omega^2 C^2} = \left(\frac{V_{\text{rms}}}{I_{\text{rms}}}\right)^2$$

$$I_{\text{rms1}} = 45.0 \text{ mA}$$

$$f_1 = 20.0 \text{ kHz}$$

$$I_{\text{rms2}} = 50.0 \text{ mA}$$

$$f_2 = 25.0 \text{ kHz}$$

$$R^2 + \frac{1}{\omega_1^2 C^2} = \frac{V_{\text{rms}}^2}{I_{\text{rms1}}^2}, R^2 + \frac{1}{\omega_2^2 C^2} = \frac{V_{\text{rms}}^2}{I_{\text{rms2}}^2}$$

$$\begin{split} \frac{V_{\rm rms}^2}{I_{\rm rms}^2} &- \frac{1}{\omega_{\rm l}^2 C^2} = \frac{V_{\rm rms}^2}{I_{\rm rms}^2} - \frac{1}{\omega_{\rm 2}^2 C^2} \\ V_{\rm rms}^2 \left(\frac{1}{I_{\rm rms}^2} - \frac{1}{I_{\rm rms}^2} \right) &= \frac{1}{C^2} \left(\frac{1}{\omega_{\rm l}^2} - \frac{1}{\omega_{\rm 2}^2} \right) \\ &= \frac{1}{V_{\rm rms}} \sqrt{\frac{1}{\omega_{\rm l}^2} - \frac{1}{\omega_{\rm 2}^2}} \left(\frac{1}{I_{\rm rms}^2} - \frac{1}{I_{\rm rms}^2} \right)^{-1/2} \\ &= \frac{1}{5.00 \text{ V}} \sqrt{\frac{1}{4\pi^2}} \left[\frac{1}{(20.0 \times 10^3 \text{ s}^{-1})^2} - \frac{1}{(25.0 \times 10^3 \text{ s}^{-1})^2} \right] \left[\frac{1}{(45.0 \times 10^{-3} \text{ A})^2} - \frac{1}{(50.0 \times 10^{-3} \text{ A})^2} \right]^{-1/2} \\ &= \boxed{99 \text{ nF}} \\ R &= \sqrt{\frac{V_{\rm rms}^2}{I_{\rm rms}^2} - \frac{1}{\omega_{\rm l}^2 C^2}} = \sqrt{\frac{(5.00 \text{ V})^2}{(45.0 \times 10^{-3} \text{ A})^2} - \frac{1}{4\pi^2 (20.0 \times 10^3 \text{ s}^{-1})^2 (98.6 \times 10^{-9} \text{ F})^2}} = \boxed{76 \Omega} \end{split}$$

81.
$$Z = \frac{V_{\text{rms}}}{I_{\text{rms}}}$$

$$\sqrt{R^2 + X_L^2} = \frac{V_{\text{rms}}}{I_{\text{rms}}}$$

$$R^2 + \omega^2 L^2 = \left(\frac{V_{\text{rms}}}{I_{\text{rms}}}\right)^2$$

$$I_{\text{rms}1} = 45.0 \text{ mA}$$

$$f_1 = 20.0 \text{ kHz}$$

$$I_{\text{rms}2} = 40.0 \text{ mA}$$

$$f_2 = 25.0 \text{ kHz}$$

$$R^2 + \omega_1^2 L^2 = \frac{V_{\text{rms}}^2}{I_{\text{rms}1}^2}$$

$$R^2 + \omega_2^2 L^2 = \frac{V_{\text{rms}}^2}{I_{\text{rms}2}^2}$$

$$\begin{split} \frac{V_{\text{rms}}^2}{I_{\text{rms}1}^2} - \omega_1^2 L^2 &= \frac{V_{\text{rms}}^2}{I_{\text{rms}2}^2} - \omega_2^2 L^2 \\ L^2 \left(\omega_2^2 - \omega_1^2 \right) &= V_{\text{rms}}^2 \left(\frac{1}{I_{\text{rms}2}^2} - \frac{1}{I_{\text{rms}1}^2} \right) \\ L &= \frac{V_{\text{rms}}}{\sqrt{\omega_2^2 - \omega_1^2}} \sqrt{\frac{1}{I_{\text{rms}2}^2} - \frac{1}{I_{\text{rms}1}^2}} \\ &= \frac{5.00 \text{ V}}{2\pi \sqrt{(25.0 \times 10^3 \text{ s}^{-1})^2 - (20.0 \times 10^3 \text{ s}^{-1})^2}} \sqrt{\frac{1}{(40.0 \times 10^{-3} \text{ A})^2} - \frac{1}{(45.0 \times 10^{-3} \text{ A})^2}} \\ &= \boxed{608 \ \mu\text{H}} \\ R &= \sqrt{\frac{V_{\text{rms}}^2}{I_{\text{rms}1}^2} - \omega_1^2 L^2} = \sqrt{\frac{(5.00 \text{ V})^2}{(45.0 \times 10^{-3} \text{ A})^2} - 4\pi^2 (20.0 \times 10^3 \text{ s}^{-1})^2 (608 \times 10^{-6} \text{ H})^2} = \boxed{81 \ \Omega} \end{split}$$

82. (a)
$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$Z^2 - R^2 = \frac{1}{\omega^2 C^2}$$

$$\omega = \frac{1}{c\sqrt{Z^2 - R^2}}$$
Since $Z = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{120}{2.9} \Omega$,
$$\omega = \frac{1}{(25 \times 10^{-6} \text{ F})\sqrt{\left(\frac{120}{2.9} \Omega\right)^2 - (32 \Omega)^2}}$$

$$= 1525 \frac{\text{rad}}{\text{s}}$$

$$f = \frac{1}{2\pi} \left(1525 \frac{\text{rad}}{\text{s}}\right) = \boxed{0.24 \text{ kHz}}$$

(b)
$$V_{\text{rms},R} = I_{\text{rms}}R = (2.9 \text{ A})(32 \Omega) = \boxed{93 \text{ V}}$$

(c)
$$V_{\text{rms},C} = I_{\text{rms}} \left(\frac{1}{\omega C} \right) = (2.9 \text{ A}) \left(\frac{1}{\left(1525 \frac{\text{rad}}{\text{s}} \right) (25 \times 10^{-6} \text{ F})} \right) = \boxed{76 \text{ V}}$$

(d)
$$V_{\text{rms},R} + V_{\text{rms},C} = 93 \text{ V} + 76 \text{ V} = 169 \text{ V} > 120 \text{ V}$$

$$\sqrt{V_{\text{rms},R}^2 + V_{\text{rms},C}^2} = \sqrt{(93 \text{ V})^2 + (76 \text{ V})^2} = 120 \text{ V}$$