Chapter 26

Geometrical Optics

Answers to Even-numbered Conceptual Questions

2. Three images are formed of object B. One extends from (–3 m, 1 m) to (–3 m, 2 m) to (–4 m, 2 m). Another image forms an "L" from (3 m, –1 m) to (3 m, –2 m) to (4 m, –2 m). Finally, the third image extends from (–3 m, –1 m) to (–3 m, –2 m) to (–4 m, –2 m).

4. As can be seen in the photo accompanying this question, the hands on the mirror-image clock rotate counterclockwise.

6. The main mirror in a telescope is always concave, because concave mirror focus parallel rays of light (as from the stars) to a point in front of the mirror. Convex mirrors, on the other hand, disperse parallel rays of light by sending them outward on divergent paths.

8. Looking at the front side of a spoon means we are looking at a concave mirror. In addition, holding the spoon at arm's length means that we are outside the focal point of the mirror – clearly, the focal length of the front side of a spoon is only a few centimeters. The situation, then, is like that illustrated in Figure 26-18 (a). It follows that our image is reduced, real, and inverted.

10. Referring to Figure 26-18 (a), we see that as the object is moved farther to the left, the image moves to the right – toward the focal point of the mirror.

12. The concave side of the dish collects the parallel rays coming from a geosynchronous satellite and focuses them at the focal point of the dish. The convex side of the dish would send the parallel rays outward on divergent paths. The situation is analogous to that of light in an optical telescope, as discussed in the answer to Question 6.

14. A three-dimensional corner reflector produces an image that is inverted. To see this, imagine a corner reflector at about waist level. Light from your head approaches the reflector moving downward. After reflecting, the light from your head moves on a parallel path but in the opposite direction; that is, it now moves upward. Similarly, light from your feet moves upward before reflection, but downward after reflection. Now, if the reflected light from your head moves upward, and the reflected light from your feet moves downward, it follows that the image of your head is below the image of your feet – your image is inverted.

16. No. Light bends toward the normal when it enters a medium in which its speed of propagation is less than it was in the first medium – as when light passes from air to water. On the other hand, light bends away from the normal if it enters a medium in which its speed is increased – as when light passes from water to air.

18. When light goes from air to glass it slows down; when it goes from glass to air it speeds up. In general, the speed of light is determined solely by the medium in which it propagates, irrespective of its past history.

20. In a real image, light passes through the location of the image before reaching the eye. In a virtual image, light propagates *as if* it were coming from the image – though the reflected or refracted light never actually passes through the image location.

22. The oil used in the bottle to the left has an index of refraction that is equal to the index of refraction of the glass in the eye dropper. Therefore, light is undeflected when it passes from the oil to the glass or from the glass to the oil. Since light propagates the same as if the eye dropper were not present, the dropper is invisible.

24. No. In order to see, a person's eyes must first bring light to a focus, and then must absorb the light to convert it to nervous impulses that can travel to the brain. Both the bending of the light and its absorption would give away the presence of the invisible man. Therefore, if the man were truly invisible, he would be unable to see.

26. You are actually seeing light from the sky, which has been bent upward by refraction in the low-density air near the hot ground. See Figure 26-23 for a case where one would see a tree reflected in the "pool of water."

28. When the mug is filled with water, light coming upward from the bottom of the mug is bent toward the horizontal when it passes from water to air. If the light had not been bent, it would have passed over your head – placing the bottom of the mug out of sight. With the bending, however, the light can now propagate to your eyes – making the bottom of the mug visible.

30. Filling a container with a liquid results in the container appearing to be shallower than it actually it. This phenomenon is referred to as the *apparent depth*. The cause of the apparent depth effect is illustrated in Figure 26-22, and discussed in Problem 48. It is clear from Figure 26-22 that the greater the index of refraction of the liquid, the greater the bending of light, and hence the smaller the apparent depth. We conclude, then, that liquid B (with its smaller apparent depth) has the greater index of refraction.

32. Referring to Figure 26-38, we see that the red part of the rainbow is at the top of the bow. Clearly, the infrared part of the rainbow will be adjacent to the red light in the bow, which means it will be higher in the sky. Therefore, the rainbow in the infrared picture is higher than the rainbow in the visible-light picture.

Solutions to Problems

1. $\theta_\text{i} = \theta_\text{r}$

$$\theta_\text{i} + \theta_\text{r} = 2\theta_\text{i} = 38° + 2(5.0°) = \boxed{48°}$$

2. The two mirrors and the ray form a triangle. The angle of incidence for mirror 2 is $90° - [180° - 120° - (90° - 55°)] = 65°.$ Since $\theta_\text{r} = \theta_\text{i}, \theta_\text{r}$ for mirror 2 is $\boxed{65°}$.

3. $\theta_\text{i}' = \theta_\text{i} + \theta = \theta_\text{r} + \theta = \theta_\text{r}'$

So, the reflected ray is rotated by θ with respect to the normal, which has also been rotated by θ, so the reflected ray is rotated by $\boxed{2\theta}$.

4. (a) Since the distance from the wall increased, the angle of incidence decreased. This implies that the sun was going down. So, the observations occurred in the $\boxed{\text{afternoon}}$.

(b) The angle of elevation is given by $\tan\theta = \dfrac{h}{d}$.

$$\Delta\theta = \theta_\text{f} - \theta_\text{i} = \tan^{-1}\frac{h}{d_\text{f}} - \tan^{-1}\frac{h}{d_\text{i}} = \tan^{-1}\frac{1.40\text{ m}}{3.75\text{ m}} - \tan^{-1}\frac{1.40\text{ m}}{2.50\text{ m}} = -8.8°$$

$\boxed{\text{The sun's angle of elevation decreased by 8.8°.}}$

5. $\Delta y = y_f - y_i = d\tan\theta_f - d\tan\theta_i = (2.0\text{ m})(\tan 37° - \tan 32°) = \boxed{26\text{ cm}}$

6. The mirror should be tilted by an angle equal to the angle of reflection of the light from the shoes.

$$\theta_r = \tan^{-1}\frac{h}{d} = \tan^{-1}\frac{\frac{1.75\text{m}}{2}}{1.50\text{m}} = \boxed{30.3°}$$

7. **(a)** Since $\theta_i = \theta_r$, the mirror must be midway between your eyes and belt buckle. So, the mirror is $(0.70\text{ m})/2 = \boxed{35\text{ cm below}}$ the level of your eyes.

(b) $\theta = \tan^{-1}\dfrac{0.35\text{ m}}{2.0\text{ m}} = \boxed{9.9°}$

(c) Since the vertical position of the mirror relative to your eyes is halfway between your eyes and belt buckle regardless of the distance you stand from the mirror, $\boxed{\text{you will still see the buckle}}$.

8. **(a)** d = horizontal distance between reflections = 2(68.0 cm)tan15.0°
The first reflection occurs at x = (68.0 cm)tan 15.0°.
n = number of reflections – 1

$$n[2(68.0\text{ cm})\tan 15.0°] \le 168\text{ cm} - (68.0\text{ cm})\tan 15.0°$$
$$n \le \frac{168\text{ cm} - (68.0\text{ cm})\tan 15.0°}{2(68.0\text{ cm})\tan 15.0°}$$
$$\le 4.11$$

The light beam reflects $\boxed{\text{five times}}$ from the top.

(b) n = number of reflections – 1

$$n[2(68.0\text{ cm})\tan 15.0°] \le 168\text{ cm} - 2(68.0\text{ cm})\tan 15.0°$$
$$n \le \frac{168\text{ cm} - 2(68.0\text{ cm})\tan 15.0°}{2(68.0\text{ cm})\tan 15.0°}$$
$$\le 3.61$$

The light beam reflects $\boxed{\text{four times}}$ from the bottom.

9.
$$2x + 13.0\text{ ft} = 27.0\text{ ft}$$
$$2x = 27.0\text{ ft} - 13.0\text{ ft}$$
$$= 14.0\text{ ft}$$
$$x = \boxed{7.00\text{ ft}}$$

10. **(a)** The distance decreases at twice the walking rate.

$$2\left(2.4\,\tfrac{\text{m}}{\text{s}}\right) = \boxed{4.8\text{ m/s}}$$

(b) $2\left(2.4\,\tfrac{\text{m}}{\text{s}}\right)\cos 33° = \boxed{4.0\text{ m/s}}$

11. h = height of mirror

h_p/x_p = height/distance of person

h_t/x_t = height/distance of table

$\theta = \theta_i = \theta_r$

$$\tan\theta = \frac{h - h_t}{x_t} = \frac{h_p - h}{x_p}$$

$$h - h_t = \frac{x_t}{x_p}(h_p - h)$$

$$h\left(1 + \frac{x_t}{x_p}\right) = \frac{x_t}{x_p}h_p + h_t$$

$$h = \frac{\frac{x_t}{x_p}h_p + h_t}{1 + \frac{x_t}{x_p}}$$

$$= \frac{\frac{1.5\text{ m}}{3.0\text{ m}}(1.8\text{ m}) + 0.80\text{ m}}{1 + \frac{1.5\text{ m}}{3.0\text{ m}}}$$

$$= \boxed{1.13\text{ m}}$$

12. h/w = height/width of mirror

h_w/w_w = height/width of window

x_e/x_w = distance of eye/window from mirror

$\theta = \theta_i = \theta_r$

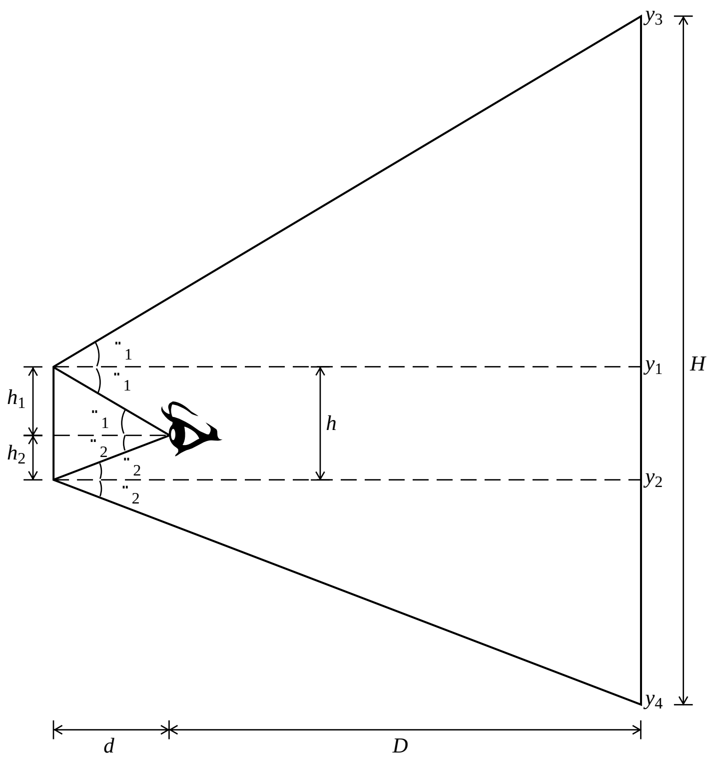
$$\tan\theta_{\text{vertical}} = \frac{h}{x_e} = \frac{h_w}{x_e + x_w}$$

$$h = h_w \frac{x_e}{x_e + x_w} = (0.30\text{ m})\frac{0.50\text{ m}}{0.50\text{ m} + 1.50\text{ m}} = \boxed{7.5\text{ cm}}$$

$$\tan\theta_{\text{horizontal}} = \frac{w}{x_e} = \frac{w_w}{x_e + x_w}$$

$$w = w_w \frac{x_e}{x_e + x_w} = (1.3\text{ m})\frac{0.50\text{ m}}{0.50\text{ m} + 1.50\text{ m}} = \boxed{33\text{ cm}}$$

13. (a)



$$\frac{h_1}{d} = \tan\theta_1 = \frac{y_3 - y_1}{D+d}$$
$$\frac{h_2}{d} = \tan\theta_2 = \frac{y_2 - y_4}{D+d}$$
$$\tan\theta_1 + \tan\theta_2 = \frac{h_1 + h_2}{d} = \frac{y_3 - y_4 - (y_1 - y_2)}{D+d}$$
$$h = h_1 + h_2 = y_1 - y_2$$
$$H = y_3 - y_4$$

Substitute.

$$\frac{h}{d} = \frac{H-h}{D+d}$$
$$\frac{H}{D+d} = \frac{h}{d} + \frac{h}{D+d}$$
$$H = h\left(1 + \frac{D+d}{d}\right)$$
$$= (0.32\text{ m})\left(1 + \frac{95\text{ m} + 0.50\text{ m}}{0.50\text{ m}}\right)$$
$$= \boxed{61\text{ m}}$$

(b) $H = h\left(1 + \frac{D+d}{d}\right) = h\left(2 + \frac{D}{d}\right)$

Decreasing d increases the ratio D/d, and thus, H. So, if the mirror is moved closer, the answer to part (a) will $\boxed{\text{increase}}$.

14. *d* will be greatest when the rays are incident at the edges of the mirror, 11 cm apart. The reflected rays and the diameter of the mirror form an icosceles triangle. The distance *d* bisects this triangle forming two right triangles where *d* is one leg, (11 cm)/2 is the other leg, the reflected ray from the mirror to the point of intersection is the hypotenuse, and 27°/2 is θ.

$$\tan\theta = \tan\frac{27°}{2} = \frac{y}{x} = \frac{\frac{11\text{ cm}}{2}}{d}$$

$$d = \frac{11\text{ cm}}{2}\cot\frac{27°}{2} = \boxed{23\text{ cm}}$$

15. Referring to the figure in Example 26–2, we see that the incident and reflected angles are $90.001° - 45.000° = 45.001°$. So, the laser beam deviates from the proper path by $2(45.001° - 45.000°) = 0.002°$. Let $\theta = 0.002°$, *x* = Earth-Moon distance, and *y* = the distance from the starting point.

$$\tan\theta = \frac{y}{x}$$

$$y = x\tan\theta = (3.84\times10^5\text{ km})\tan 0.002° = \boxed{10\text{ km}}$$

16. (a) $f = -\frac{1}{2}R = -\frac{1}{2}(0.96\text{ m}) = \boxed{-48\text{ cm}}$

(b) $f = \frac{1}{2}R = \frac{1}{2}(0.96\text{ m}) = \boxed{48\text{ cm}}$

17. $f = -\frac{1}{2}R = -\frac{1}{4}D = -\frac{1}{4}(33.5\text{ cm}) = \boxed{-8.38\text{ cm}}$

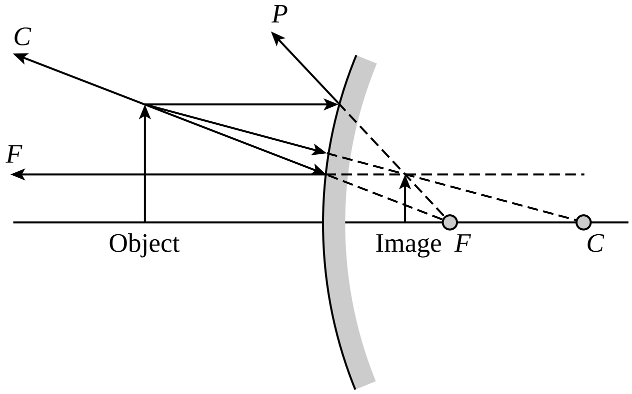
18. $R = 2f = 2(15\text{ cm}) = \boxed{30\text{ cm}}$

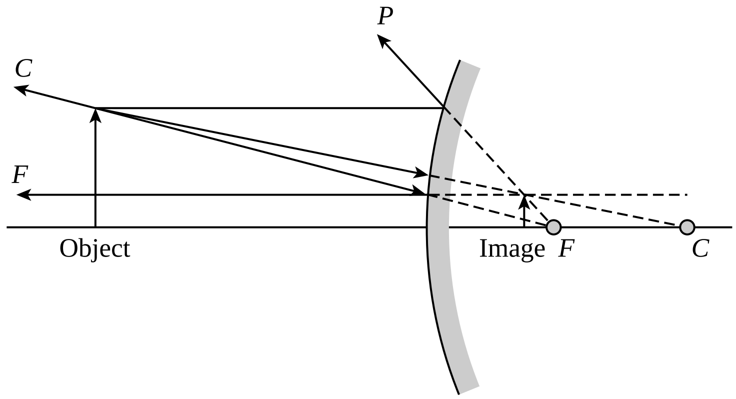
19. $\frac{1}{d_\text{i}} = \frac{1}{f} - \frac{1}{d_\text{o}}$

$$d_\text{i} = \left(\frac{1}{f} - \frac{1}{d_\text{o}}\right)^{-1} = \left(\frac{1}{\frac{40.0\text{ cm}}{2}} - \frac{1}{30.0\text{ cm}}\right)^{-1} = 60.0\text{ cm}$$

The object will be formed $\boxed{\text{60.0 cm in front of the mirror}}$.

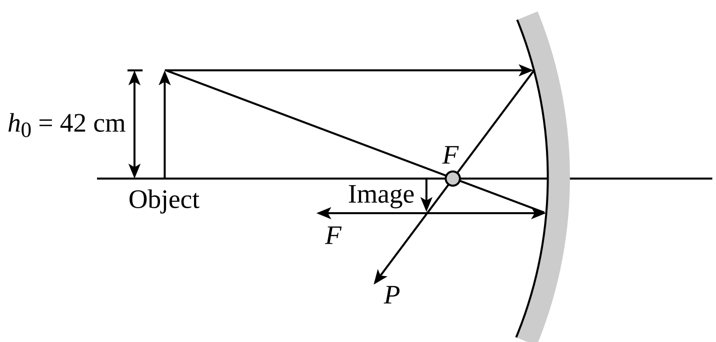
20.





From the ray diagrams, we see that the image increases in size as the object is brought closer to the mirror's surface.

21. (a)



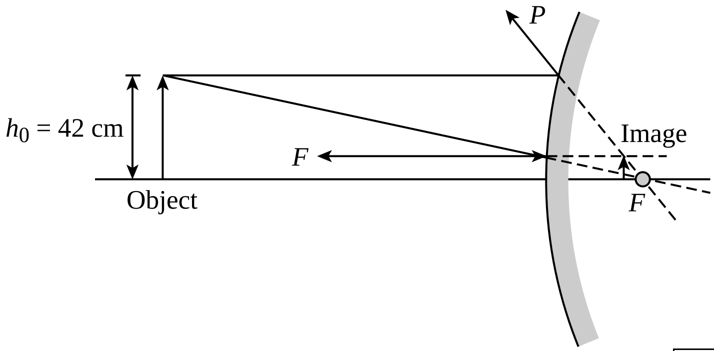
According to the diagram, the image is located approximately 63 cm in front of the mirror and is approximately 16 cm tall.

(b) The image is inverted.

22. $d_\text{i} = \left(\frac{1}{f} - \frac{1}{d_\text{o}}\right)^{-1} = \left(\frac{1}{0.50\text{ m}} - \frac{1}{2.0\text{ m}}\right)^{-1} = \boxed{0.67\text{ m}}$

$m = -\frac{d_\text{i}}{d_\text{o}} = -\frac{1}{2.0\text{ m}}\left(\frac{1}{0.50\text{ m}} - \frac{1}{2.0\text{ m}}\right)^{-1} = \boxed{-0.33}$

23. (a)



According to the diagram, the image is located approximately $\boxed{\text{40 cm behind the mirror}}$ and is approximately $\boxed{\text{10 cm tall}}$.

(b) The image is $\boxed{\text{upright}}$.

24. $d_\text{i} = \left(\frac{1}{f} - \frac{1}{d_\text{o}}\right)^{-1} = \left(\frac{1}{-0.50\text{ m}} - \frac{1}{2.0\text{ m}}\right)^{-1} = \boxed{-0.40\text{ m}}$

$m = -\frac{d_\text{i}}{d_\text{o}} = -\frac{1}{2.0\text{ m}}\left(\frac{1}{-0.50\text{ m}} - \frac{1}{2.0\text{ m}}\right)^{-1} = \boxed{0.20}$

25. Since $d_\text{o} >> d_\text{i}$, $\frac{1}{f} = \frac{1}{d_\text{o}} + \frac{1}{d_\text{i}} \approx 0 + \frac{1}{d_\text{i}}$, or $f = d_\text{i}$. Thus, the radius of curvature is

$R = -2f = -2d_\text{i} = -2(-4.6\text{ cm}) = \boxed{9.2\text{ cm}}$.

26. (a) To form a virtual, upright, and enlarged image, the mirror should be $\boxed{\text{concave}}$.

(b) $m = \frac{h_\text{i}}{h_\text{o}} = -\frac{d_\text{i}}{d_\text{o}}$

$$d_\text{i} = -d_\text{o}\frac{h_\text{i}}{h_\text{o}} = -d_\text{o}\frac{2.54\text{ m}}{0.500\text{ m}} = -5.08d_\text{o}$$

$$\frac{1}{f} = \frac{1}{d_\text{o}} + \frac{1}{d_\text{i}}$$

$$\frac{2}{R} = \frac{1}{d_\text{o}} + \frac{1}{-5.08d_\text{o}}$$

$$= \frac{1}{d_\text{o}}\left(1 - \frac{1}{5.08}\right)$$

$$R = 2d_\text{o}\left(1 - \frac{1}{5.08}\right)^{-1}$$

$$= 2(3.00\text{ m})\left(1 - \frac{1}{5.08}\right)^{-1}$$

$$= \boxed{7.47\text{ m}}$$

(c) $d_\text{i} = -5.08d_\text{o} = -5.08(3.00\text{ m}) = -15.2\text{ m}$

The image will be formed $\boxed{\text{15.2 m behind the mirror}}$.

27. (a) $d_i = \left(\frac{1}{f} - \frac{1}{d_o}\right)^{-1} = \left(-\frac{2}{R} - \frac{1}{d_o}\right)^{-1} = \left(-\frac{4}{D} - \frac{1}{d_o}\right)^{-1} = \left(-\frac{4}{0.18\text{ m}} - \frac{1}{3.2\text{ m}}\right)^{-1} = -4.4\text{ cm}$

The image is $\boxed{\text{4.4 cm behind the surface of the globe}}$.

(b) $h_i = mh_o = -\frac{d_i}{d_o}h_o = -\frac{-0.044\text{ m}}{3.2\text{ m}}(1.6\text{ m}) = \boxed{2.2\text{ cm}}$

28. (a) $m = -\frac{d_i}{d_o} = -\frac{3.5\text{ m}}{1.5\text{ m}} = \boxed{-2.3}$

(b) Since $m < 0$, the image is $\boxed{\text{inverted}}$.

(c) $f = \left(\frac{1}{d_o} + \frac{1}{d_i}\right)^{-1} = \left(\frac{1}{1.5\text{ m}} + \frac{1}{3.5\text{ m}}\right)^{-1} = \boxed{1.1\text{ m}}$

29. (a) $d_i = \left(\frac{1}{f} - \frac{1}{d_o}\right)^{-1} = \left(\frac{1}{16.9\text{ m}} - \frac{1}{20.0\text{ m}}\right)^{-1} = 110\text{ m}$

The astronomer's image is located $\boxed{\text{110 m in front of the mirror}}$.

(b) Her image is $\boxed{\text{real}}$ because light passes through it.

(c) $m = -\frac{d_i}{d_o} = -\frac{1}{20.0\text{ m}}\left(\frac{1}{16.9\text{ m}} - \frac{1}{20.0\text{ m}}\right)^{-1} = \boxed{-5.5}$

30. (a) $d_i = -md_o = -3(22\text{ cm}) = \boxed{-66\text{ cm}}$

(b) $f = \left(\frac{1}{d_o} + \frac{1}{d_i}\right)^{-1} = \left(\frac{1}{22\text{ cm}} + \frac{1}{-66\text{ cm}}\right)^{-1} = \boxed{33\text{ cm}}$

31. (a) $d_i = -md_o = -(-3)(22\text{ cm}) = \boxed{66\text{ cm}}$

(b) $f = \left(\frac{1}{d_o} + \frac{1}{d_i}\right)^{-1} = \left(\frac{1}{22\text{ cm}} + \frac{1}{66\text{ cm}}\right)^{-1} = \boxed{17\text{ cm}}$

32. (a) $d_i = -md_o = -\frac{1}{4}(32\text{ cm}) = \boxed{-8.0\text{ cm}}$

(b) $f = \left(\frac{1}{d_o} + \frac{1}{d_i}\right)^{-1} = \left(\frac{1}{32\text{ cm}} + \frac{1}{-8.0\text{ cm}}\right)^{-1} = \boxed{-11\text{ cm}}$

33. (a) Since convex mirrors always produce upright and reduced images, the shopper's image is $\boxed{\text{upright}}$.

(b) $\dfrac{1}{d_o}+\dfrac{1}{d_i}=\dfrac{1}{f}=-\dfrac{2}{R}$

$$R=-2\left(\frac{1}{d_o}+\frac{1}{d_i}\right)^{-1}=-2\left(\frac{1}{d_o}+\frac{1}{-md_o}\right)^{-1}=-2d_o\left(1-\frac{1}{m}\right)^{-1}=-2(19\text{ ft})\left[1-\frac{5.7\text{ ft}}{(6.4\text{ in.})\left(\frac{1\text{ ft}}{12\text{ in.}}\right)}\right]^{-1}=\boxed{4\text{ ft}}$$

34. $h_o=\dfrac{h_i}{m}=-\dfrac{d_oh_i}{d_i}=-\dfrac{(21\text{ m})(-3.5\text{ cm})}{7.0\text{ cm}}=\boxed{11\text{ m}}$

35. $\dfrac{1}{d_o}+\dfrac{1}{d_i}=\dfrac{1}{f}=\dfrac{2}{R}$

$$R=2\left(\frac{1}{d_o}+\frac{1}{d_i}\right)^{-1}=2\left(\frac{1}{d_o}+\frac{1}{-md_o}\right)^{-1}=2d_o\left(1-\frac{1}{m}\right)^{-1}=2(25\text{ cm})\left(1-\frac{1}{2.0}\right)^{-1}=\boxed{1.0\text{ m}}$$

36. $\dfrac{1}{f}=\dfrac{1}{d_o}+\dfrac{1}{d_i}=\dfrac{1}{d_o}+\dfrac{1}{\frac{d_o}{3}}=\dfrac{4}{d_o}$

$d_o=4f=4(36\text{ cm})=\boxed{1.4\text{ m}}$

$d_i=\dfrac{d_o}{3}=\dfrac{4}{3}f=\dfrac{4}{3}(36\text{ cm})=\boxed{48\text{ cm}}$

37. $n=\dfrac{c}{v}=\dfrac{3.00\times10^8\ \frac{\text{m}}{\text{s}}}{\frac{0.960\text{ m}}{4.00\times10^{-9}\text{ s}}}=\boxed{1.25}$

38. $\dfrac{v_w}{v_d}=\dfrac{\frac{c}{n_w}}{\frac{c}{n_d}}=\dfrac{n_d}{n_w}=\dfrac{2.42}{1.33}=\boxed{1.82}$

39. $n_{air}\sin\theta_i=n_w\sin\theta_{refr}$

$\theta_{refr}=\sin^{-1}\dfrac{n_{air}\sin\theta_i}{n_w}$

percent error $=\dfrac{\theta_{refr,Ptolemy}-\theta_{refr}}{\theta_{refr}}\times100\%$

$n_{air}=1.000293$
$n_w=1.33$

θ_i (°)	10.0	20.0
% Error	6.6	4

40. $n_{benzene}=n_{air}\dfrac{\sin\theta_i}{\sin\theta_{refr}}=(1.000)\dfrac{\sin 43°}{\sin 27°}=\boxed{1.5}$

41. $n_{\text{air}} \sin\theta_{\text{i}} = n_{\text{ice}} \sin\theta_{\text{refr}}$

$$\theta_{\text{i}} = \sin^{-1}\left(\frac{n_{\text{ice}}}{n_{\text{air}}}\sin\theta_{\text{refr}}\right) = \sin^{-1}\left(\frac{1.31}{1.000}\sin 35°\right) = \boxed{49°}$$

42. (a) Since $\sin\theta_{\text{i}}$ is proportional to n_{w} in Snell's Law, and since $\sin\theta_{\text{i}}$ increases as θ_{i} increases for $0 \le \theta \le 90°$, a greater index of refraction $(n_{\text{w}} > n_{\text{ice}})$ requires a $\boxed{\text{greater}}$ angle of incidence.

(b) $n_{\text{air}} \sin\theta_{\text{i}} = n_{\text{w}} \sin\theta_{\text{refr}}$

$$\theta_{\text{i}} = \sin^{-1}\left(\frac{n_{\text{w}}}{n_{\text{air}}}\sin\theta_{\text{refr}}\right) = \sin^{-1}\left(\frac{1.33}{1.000}\sin 35°\right) = \boxed{50°}$$

43. The diver sees θ_{refr} and the friend sees $90° - \theta_{\text{i}}$.

$n_{\text{air}} \sin\theta_{\text{i}} = n_{\text{w}} \sin\theta_{\text{refr}}$

$$90° - \theta_{\text{i}} = 90° - \sin^{-1}\left(\frac{n_{\text{w}}}{n_{\text{air}}}\sin\theta_{\text{refr}}\right) = 90° - \sin^{-1}\left(\frac{1.33}{1.000}\sin 35°\right) = \boxed{40°}$$

44. $y = vt = \frac{c}{n}t$

$$t = \frac{ny}{c}$$

$$t_{\text{total}} = \frac{n_{\text{ice}}y_{\text{ice}}}{c} + \frac{n_{\text{w}}y_{\text{w}}}{c} = \frac{1.31(0.12 \text{ m}) + 1.33(4.00 \text{ m} - 0.12 \text{ m})}{3.00 \times 10^8 \frac{\text{m}}{\text{s}}} = \boxed{17.7 \text{ ns}}$$

45. $x = vt = \frac{c}{n}t$

$$t = \frac{nx}{c}$$

$$t_{\text{total}} = \frac{n_1 x_1}{c} + \frac{n_2 x_2}{c} = \frac{1.33(3.31 \text{ m}) + 1.51(1.51 \text{ m})}{3.00 \times 10^8 \frac{\text{m}}{\text{s}}} = \boxed{22.3 \text{ ns}}$$

46. $\tan\theta_{\text{refr}} = \frac{y}{x}$

$n_{\text{air}} \sin\theta = n_{\text{g}} \sin\theta_{\text{refr}}$

$$\theta = \sin^{-1}\left(\frac{n_{\text{g}}}{n_{\text{air}}}\sin\theta_{\text{refr}}\right) = \sin^{-1}\left(\frac{n_{\text{g}}}{n_{\text{air}}}\sin\left[\tan^{-1}\frac{y}{x}\right]\right) = \sin^{-1}\left(\frac{1.52}{1.000}\sin\left[\tan^{-1}\frac{5.0 \text{ cm}}{20.0 \text{ cm}}\right]\right) = \boxed{22°}$$

47. $\sin\theta_i = \dfrac{W}{\sqrt{W^2+H^2}}$

$$\sin\theta_{refr} = \frac{\frac{W}{2}}{\sqrt{\left(\frac{W}{2}\right)^2+H^2}} = \frac{W}{\sqrt{W^2+4H^2}}$$

$$n_{air}\sin\theta_i = n_w\sin\theta_{refr}$$
$$n_{air}\frac{W}{\sqrt{W^2+H^2}} = n_w\frac{W}{\sqrt{W^2+4H^2}}$$
$$\frac{n_{air}{}^2}{W^2+H^2} = \frac{n_w{}^2}{W^2+4H^2}$$
$$n_{air}{}^2(W^2+4H^2) = n_w{}^2(W^2+H^2)$$
$$H^2(4n_{air}{}^2-n_w{}^2) = W^2(n_w{}^2-n_{air}{}^2)$$
$$H = W\sqrt{\frac{n_w{}^2-n_{air}{}^2}{4n_{air}{}^2-n_w{}^2}}$$
$$= (6.2\text{ cm})\sqrt{\frac{(1.33)^2-(1.00)^2}{4(1.00)^2-(1.33)^2}}$$
$$= \boxed{3.6\text{ cm}}$$

48. $n_{air}\sin\theta_2 = n_w\sin\theta_1$

Since $\dfrac{d_{coin}}{5.0\text{ ft}} << 1,$ we can use the small angle approximation $\sin\theta\approx\theta.$

$$n_{air}\theta_2 = n_w\theta_1$$
$$\frac{\theta_2}{\theta_1} = \frac{n_w}{n_{air}} = \frac{1.33}{1.00} = 1.33$$

The angular size of the coin appears larger, which results in an apparent depth smaller than the actual depth by the same factor.

$$d_{apparent} = \frac{d}{1.33} = \frac{5.0\text{ ft}}{1.33} = \boxed{3.8\text{ ft}}$$

49. The angles of incidence are both exactly 45°.

$$n = \frac{n_{air}}{\sin\theta_c} = \frac{1.000}{\sin 45°} = \boxed{1.414}$$

50. $n_{prism} = \dfrac{n}{\sin\theta_c}$

$$n_{prism1} = \frac{1.21}{\sin 45°} = 1.71$$
$$n_{prism2} = \frac{1.43}{\sin 45°} = 2.02$$
$$\boxed{1.71 \le n_{prism} < 2.02}$$

51. (a) From the figure, it is evident that $\sin\theta_c = \sin(90° - \theta_{refr}) = \cos\theta_{refr}$.
From Snell's Law,

$$n\sin\theta_{refr} = n_{air}\sin\theta$$
$$n = \frac{n_{air}\sin\theta}{\sin\theta_{refr}}.$$

From the definition of total internal reflection, $\sin\theta_c = \frac{n_{air}}{n} = \frac{n_{air}}{\frac{n_{air}\sin\theta}{\sin\theta_{refr}}} = \frac{\sin\theta_{refr}}{\sin\theta} = \cos\theta_{refr}$.

So, $\tan\theta_{refr} = \sin\theta$, and $\theta_{refr} = \tan^{-1}\sin\theta$.
Substitute.

$$n = \frac{n_{air}\sin\theta}{\sin(\tan^{-1}\sin\theta)} = \frac{1.000\sin 77°}{\sin(\tan^{-1}\sin 77°)} = \boxed{1.4}$$

(b) If θ is decreased, θ_{refr} decreases. If θ_{refr} decreases, θ_c increases, as does $\sin\theta_c$. Since *n* is inversely proportional to $\sin\theta_c$, increasing $\sin\theta_c$ decreases *n*. So, *n* is $\boxed{\text{decreased}}$.

52. (a) Given:

$$n_{air}\sin\theta = n_{glass}\sin\theta_{refr}$$
$$\sin\theta_c = \frac{n_{air}}{n_{glass}}$$
$$\sin\theta_{refr} = \cos\theta_c$$

Solve for θ:

$$\theta = \sin^{-1}\left[\frac{n_{glass}}{n_{air}}\cos\left(\sin^{-1}\frac{n_{air}}{n_{glass}}\right)\right]$$
$$= \sin^{-1}\left[\frac{1.40}{1.00}\cos\left(\sin^{-1}\frac{1.00}{1.40}\right)\right]$$
$$= \boxed{78.5°}$$

(b) $\boxed{\text{No.}}$ If θ is increased, the angle of incidence at the vertical surface will be less than the critical angle.

53. (a) $\sin\theta = \frac{n_{glass}}{n_{air}}\sin\theta_{refr}$

If n_{glass} is increased while θ_{refr} is held constant, θ must $\boxed{\text{increase}}$.

(b)

$$\theta = \sin^{-1}\left(\frac{n_{glass}}{n_{air}}\sin\theta_{refr}\right)$$
$$= \sin^{-1}\left[\frac{1.66}{1.00}\sin\left(\tan^{-1}\frac{5.00\text{ cm}}{20.0\text{ cm}}\right)\right]$$
$$= \boxed{23.7°}$$

54. $\theta_i = \theta_{refr} = \theta_B = \dfrac{110°}{2} = 55°$

$$\tan\theta_B = \frac{n_2}{n_1}$$
$$n_2 = n_1 \tan\theta_B = (1.00)\tan 55° = \boxed{1.4}$$

55. Use Snell's Law.

$n_{air} \sin 45° = n \sin\theta_{refr1}$

$n \sin\theta_{i2} = n_{air} \sin 34°$

Determine θ_{i2} or θ_{refr1} to find *n*.

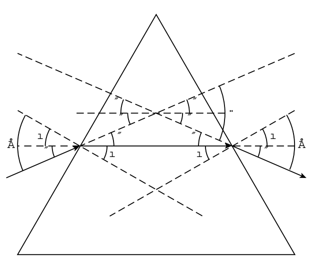
From the figure, we see that $\theta_{i2} + \theta_{refr1} = 45°$, or $\theta_{i2} = 45° - \theta_{refr1}$.

Find θ_{refr1}.

Eliminate *n* from the two equations derived from Snell's Law.

$$\frac{n_{air}\sin 45°}{\sin\theta_{refr1}} = \frac{n_{air}\sin 34°}{\sin\theta_{i2}}$$
$$\frac{\sin 34°}{\sin 45°}\sin\theta_{refr1} = \sin\theta_{i2} = \sin(45° - \theta_{refr1}) = \sin 45°\cos\theta_{refr1} - \cos 45°\sin\theta_{refr1}$$
$$\left(\frac{\sin 34°}{\sin 45°} + \cos 45°\right)\sin\theta_{refr1} = \sin 45°\cos\theta_{refr1}$$
$$\tan\theta_{refr1} = \frac{\sin 45°}{\frac{\sin 34°}{\sin 45°} + \cos 45°}$$
$$\theta_{refr1} = \tan^{-1}\frac{\sin 45°}{\frac{\sin 34°}{\sin 45°} + \cos 45°}$$
$$n = \frac{n_{air}\sin 45°}{\sin\theta_{refr1}} = \frac{1.000\sin 45°}{\sin\left(\tan^{-1}\frac{\sin 45°}{\frac{\sin 34°}{\sin 45°}+\cos 45°}\right)} = \boxed{1.7}$$

56.



The figure shows various angle relationships. From the figure, we see that $\gamma = \alpha - \beta$ and $\theta = 2\gamma$. So, $\theta = 2\alpha - 2\beta$. Now, $\alpha = \theta_{i1} = \theta_{refr2}$ and $\beta = \theta_{refr1} = \theta_{i2}$. Use Snell's law to find either θ_{i1} or θ_{refr2} and either θ_{refr1} or θ_{i2}. From the figure, we see that $60° + \beta = 90°$, so $\beta = 30° = \theta_{refr1} = \theta_{i2.}$ Now that we know β we can determine $\alpha = \theta_{i1} = \theta_{refr2}$.

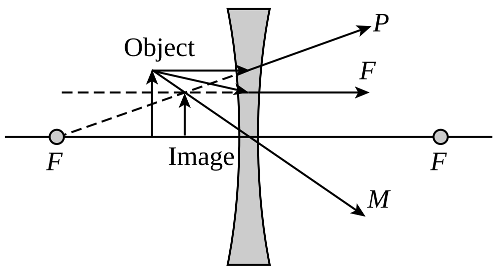
$n_{\text{air}} \sin\alpha = n\sin\beta$

$$\alpha = \sin^{-1}\frac{n\sin\beta}{n_{\text{air}}}$$

Calculate θ.

$$\theta = 2(\alpha-\beta) = 2\left(\sin^{-1}\frac{n\sin\beta}{n_{\text{air}}} - \beta\right) = 2\left[\sin^{-1}\frac{(1.42)\sin 30^\circ}{1.00} - 30^\circ\right] = \boxed{30.5^\circ}$$

57. **(a)**

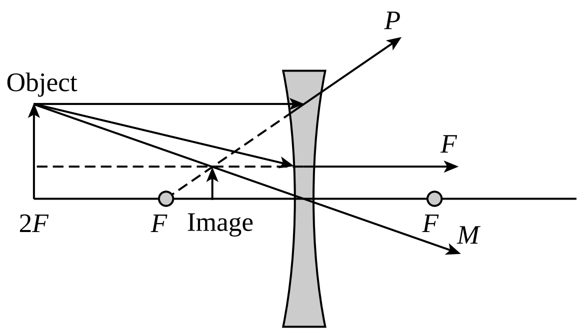


The image is located between the object and the lens, at about $\boxed{\frac{1}{3}|f|}$.

(b) The image is $\boxed{\text{upright}}$.

(c) The image is $\boxed{\text{virtual}}$, since it is on the same side of the lens as the object.

58. **(a)**

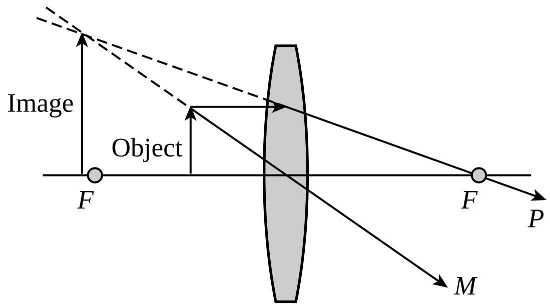


The image is located between the object and the lens, at about $\boxed{\frac{2}{3}|f|}$.

(b) The image is $\boxed{\text{upright}}$.

(c) The image is $\boxed{\text{virtual}}$, since it is on the same side of the lens as the object.

59. **(a)**

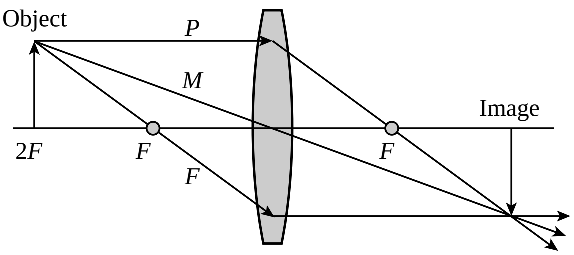


The image is located just to the left of f.

(b) The image is upright.

(c) The image is virtual, since it is on the same side of the lens as the object.

60. **(a)**

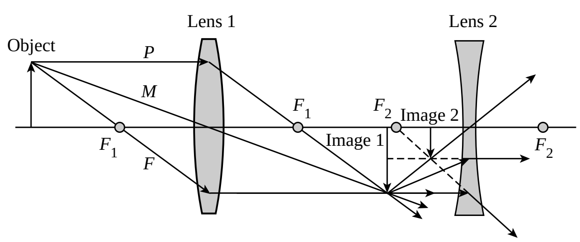


The image is located on the right side of the lens at about $2f$.

(b) The image is inverted.

(c) The image is real, because the image is on the opposite side of the lens from the object, and light passes through it.

61. **(a)**

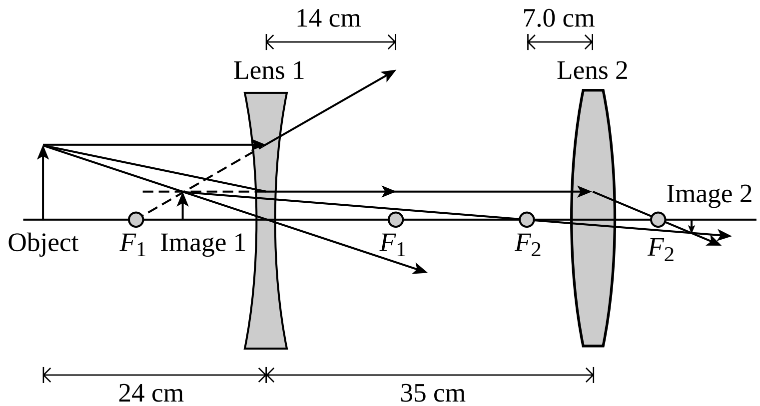


The image is located just to the left of Lens 2.

(b) The image is inverted.

(c) Since the final image is on the same side of Lens 2 as its object (the original image, which is real), it is virtual.

62. (a)



The final image is located to the right of Lens 2, just beyond F_2.

(b) The image is inverted.

(c) The image is real, because the image is on the opposite side of the lens from the object, and light passes through it.

63. Since the object (the sun) is so far away, $1/d_o \approx 0.$

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}\approx 0+\frac{1}{d_i}$$

So, $f \approx d_i = \boxed{22\text{ cm}}$.

64. $\frac{1}{d_i}+\frac{1}{d_o}=\frac{1}{f}$

$$d_i=\left(\frac{1}{f}-\frac{1}{d_o}\right)^{-1}=\left(\frac{1}{-32\text{ cm}}-\frac{1}{23\text{ cm}}\right)^{-1}=-13\text{ cm}$$

The image is located 13 cm in front of the lens.

$$m=-\frac{d_i}{d_o}=-\frac{1}{d_o}\left(\frac{1}{f}-\frac{1}{d_o}\right)^{-1}=-\frac{1}{23\text{ cm}}\left(\frac{1}{-32\text{ cm}}-\frac{1}{23\text{ cm}}\right)^{-1}=\boxed{0.58}$$

65. $\frac{1}{d_i}+\frac{1}{d_o}=\frac{1}{f}$

$$f=\left(\frac{1}{d_i}+\frac{1}{d_o}\right)^{-1}=\left(\frac{1}{15\text{ cm}}+\frac{1}{37\text{ cm}}\right)^{-1}=\boxed{11\text{ cm}}$$

66. (a) $\frac{1}{d_\text{i}}+\frac{1}{d_\text{o}}=\frac{1}{f}$

$$d_\text{i}=\left(\frac{1}{f}-\frac{1}{d_\text{o}}\right)^{-1}=\left(\frac{1}{35.0\text{ mm}}-\frac{1}{36.3\text{ mm}}\right)^{-1}=0.98\text{ m}$$

The image is located $\boxed{\text{0.98 m to the right of the lens}}$.

(b) $h_\text{i}=mh_\text{o}=-\frac{d_\text{i}h_\text{o}}{d_\text{o}}=-\frac{\left(\frac{1}{35.0\text{ mm}}-\frac{1}{36.3\text{ mm}}\right)^{-1}(2.54\text{ cm})}{36.3\text{ mm}}=-68\text{ cm}$

The image is inverted and $\boxed{68\text{ cm}}$ tall.

67. (a) $\frac{1}{d_\text{i}}+\frac{1}{d_\text{o}}=\frac{1}{f}$

$$d_\text{i}=\left(\frac{1}{f}-\frac{1}{d_\text{o}}\right)^{-1}=\left(\frac{1}{0.045\text{ m}}-\frac{1}{5.0\text{ m}}\right)^{-1}=\boxed{45\text{ mm}}$$

(b) $m=-\frac{d_\text{i}}{d_\text{o}}=-\frac{\left(\frac{1}{0.045\text{ m}}-\frac{1}{5.0\text{ m}}\right)^{-1}}{5.0\text{ m}}=\boxed{-0.0091}$

68. (a) $m=-\frac{d_\text{i}}{d_\text{o}}=-\frac{1}{\left(\frac{1}{f}-\frac{1}{d_\text{o}}\right)d_\text{o}}=\frac{1}{1-\frac{d_\text{o}}{f}}$

Since $m>0$, $1-\frac{d_\text{o}}{f}>0$. To increase m, $1-\frac{d_\text{o}}{f}$ must decrease. So, d_o must increase. The object should be moved $\boxed{\text{farther away}}$ from the lens.

(b) $m_\text{i}=3.0=\frac{1}{1-\frac{d_\text{oi}}{f}}$

$$1-\frac{d_\text{oi}}{f}=\frac{1}{3.0}$$

$$\frac{d_\text{oi}}{f}=\frac{2}{3.0}$$

$$d_\text{oi}=\frac{2}{3.0}f$$

$$m_\text{f}=4.0=\frac{1}{1-\frac{d_\text{oi}}{f}}$$

$$1-\frac{d_{\text{of}}}{f}=\frac{1}{4.0}$$
$$\frac{d_{\text{of}}}{f}=\frac{3}{4.0}$$
$$d_{\text{of}}=\frac{3}{4.0}f$$
$$\begin{aligned}\text{distance}&=\left|d_{\text{of}}-d_{\text{oi}}\right|\\&=\left|\frac{3}{4.0}f-\frac{2}{3.0}f\right|\\&=\left|\frac{f}{12}\right|\\&=\frac{|f|}{12}\\&=\frac{36\text{ cm}}{12}\\&=\boxed{3.0\text{ cm}}\end{aligned}$$

69. (a) To project a real image onto the wall, a convex lens should be used. Since convex lenses have positive focal lengths, $\boxed{\text{the lens with focal length } f_1}$ should be used.

(b)
$$\begin{aligned}\frac{1}{f}&=\frac{1}{d_{\text{o}}}-\frac{1}{d_{\text{i}}}\\&=-\frac{m}{d_{\text{i}}}+\frac{1}{d_{\text{i}}}\\&=\frac{1}{d_{\text{i}}}(1-m)\end{aligned}$$
$$\begin{aligned}d_{\text{i}}&=f(1-m)\\&=(0.400\text{ m})[1-(-2.00)]\\&=\boxed{1.20\text{ m}}\end{aligned}$$

70. (a)
$$d_{\text{i1}}=\left(\frac{1}{f_1}-\frac{1}{d_{\text{o1}}}\right)^{-1}=\left(\frac{1}{14\text{ cm}}-\frac{1}{24\text{ cm}}\right)^{-1}$$
$$d_{\text{i2}}=\left(\frac{1}{f_2}-\frac{1}{d_{\text{o2}}}\right)^{-1}=\left(\frac{1}{f_2}-\frac{1}{35\text{ cm}-d_{\text{i1}}}\right)^{-1}=\left[\frac{1}{-7.0\text{ cm}}-\frac{1}{35\text{ cm}-\left(\frac{1}{14\text{ cm}}-\frac{1}{24\text{ cm}}\right)^{-1}}\right]^{-1}=-1\text{ cm}$$
$$35\text{ cm}+d_{\text{i2}}=35\text{ cm}-1\text{ cm}=\boxed{34\text{ cm}}$$

(b)
$$m_1m_2=\left(-\frac{d_{\text{i1}}}{d_{\text{o1}}}\right)\left(-\frac{d_{\text{i2}}}{d_{\text{o2}}}\right)=\frac{d_{\text{i1}}d_{\text{i2}}}{d_{\text{o1}}(35\text{ cm}-d_{\text{i1}})}=\frac{\left(\frac{1}{0.14\text{ m}}-\frac{1}{0.24\text{ m}}\right)^{-1}\left[-\frac{1}{0.070\text{ m}}-\frac{1}{0.35\text{ m}-\left(\frac{1}{0.14\text{ m}}-\frac{1}{0.24\text{ m}}\right)^{-1}}\right]^{-1}}{(0.24\text{ m})\left[0.35\text{ m}-\left(\frac{1}{0.14\text{ m}}-\frac{1}{0.24\text{ m}}\right)^{-1}\right]}$$
$$=\boxed{-1}$$

71. (a) $d_{i1}=\left(\frac{1}{f_1}-\frac{1}{d_{o1}}\right)^{-1}=\left(\frac{1}{-14\text{ cm}}-\frac{1}{24\text{ cm}}\right)^{-1}=-8.84\text{ cm}$

$$d_{i2}=\left(\frac{1}{f_2}-\frac{1}{d_{o2}}\right)^{-1}=\left(\frac{1}{f_2}-\frac{1}{35\text{ cm}-d_{i1}}\right)^{-1}=\left[\frac{1}{7.0\text{ cm}}-\frac{1}{35\text{ cm}-\left(\frac{1}{-14\text{ cm}}-\frac{1}{24\text{ cm}}\right)^{-1}}\right]^{-1}=8.3\text{ cm}$$

$35\text{ cm}+d_{i2}=35\text{ cm}+8.3\text{ cm}=\boxed{43\text{ cm}}$

(b) $m=m_1m_2=\left(-\frac{d_{i1}}{d_{o1}}\right)\left(-\frac{d_{i2}}{d_{o2}}\right)=\frac{d_{i1}d_{i2}}{d_{o1}(35\text{ cm}-d_{i1})}$

$$=\frac{\left(\frac{1}{-0.14\text{ m}}-\frac{1}{0.24\text{ m}}\right)^{-1}\left[\frac{1}{0.070\text{ m}}-\frac{1}{0.35\text{ m}-\left(\frac{1}{-0.14\text{ m}}-\frac{1}{0.24\text{ m}}\right)^{-1}}\right]^{-1}}{(0.24\text{ m})\left[0.35\text{ m}-\left(\frac{1}{-0.14\text{ m}}-\frac{1}{0.24\text{ m}}\right)^{-1}\right]}=\boxed{-0.070}$$

72. (a) $m=-\frac{d_i}{d_o}=-\frac{1}{\left(\frac{1}{f}-\frac{1}{d_o}\right)d_o}=\frac{1}{1-\frac{d_o}{f}}=\frac{1}{1+\frac{d_o}{|f|}}$

Since $m\geq 0,\ 1+\frac{d_o}{|f|}>0.$ To decrease m, $1+\frac{d_o}{|f|}$ must increase. So, d_o must increase. The object should be moved $\boxed{\text{farther away}}$ from the lens.

(b) $m_i=\frac{1}{3}=\frac{1}{1-\frac{d_{oi}}{f}}$

$$1-\frac{d_{oi}}{f}=3$$

$$\frac{d_{oi}}{f}=-2$$

$$d_{oi}=-2f$$

$$m_f=\frac{1}{4}=\frac{1}{1-\frac{d_{of}}{f}}$$

$$1-\frac{d_{of}}{f}=4$$

$$\frac{d_{of}}{f}=-3$$

$$d_{of}=-3f$$

$\text{distance}=|d_{of}-d_{oi}|=|-3f+2f|=|-f|=\boxed{36\text{ cm}}$

73. (a) A concave lens can produce an image of a distant object within Albert's far point, which allows him to focus upon it. So, Albert's eyeglasses are $\boxed{\text{concave}}$.

(b) $\dfrac{1}{f}=\dfrac{1}{d_\text{o}}+\dfrac{1}{d_\text{i}}=\dfrac{1}{d_\text{i}}$ for $d_\text{o}\to\infty.$

$f=d_\text{i}=\boxed{-3.0\text{ m}}$

74. $m=-\dfrac{d_\text{i}}{d_\text{o}}=2$

$$\frac{1}{f}=\frac{1}{d_\text{i}}+\frac{1}{d_\text{o}}=\frac{1}{-2d_\text{o}}+\frac{1}{d_\text{o}}=\frac{1}{2d_\text{o}}$$

$f=2d_\text{o}=2(1.2\text{ cm})=\boxed{2.4\text{ cm}}$

75. (a) $m=-\dfrac{d_\text{i}}{d_\text{o}}$

$d_\text{i}=-md_\text{o}=-(0.67)(21\text{ cm})=-14\text{ cm}$

Since $d_\text{i}<0,$ the image appears on the same side of the lens as the object, which means it is a $\boxed{\text{virtual}}$ image.

(b) $\dfrac{1}{f}=\dfrac{1}{d_\text{i}}+\dfrac{1}{d_\text{o}}$

$$f=\left(\frac{1}{d_\text{i}}+\frac{1}{d_\text{o}}\right)^{-1}=\left(\frac{1}{-0.67(21\text{ cm})}+\frac{1}{21\text{ cm}}\right)^{-1}=\boxed{-43\text{ cm}}$$

(c) Since $f<0$, the lenses are $\boxed{\text{concave}}$.

76. (a) $m=-\dfrac{d_\text{i}}{d_\text{o}}$

$d_\text{i}=-md_\text{o}=-(1.5)(21\text{ cm})=-32\text{ cm}$

Since $d_\text{i}<0,$ the image appears on the same side of the lens as the object, which means it is a $\boxed{\text{virtual}}$ image.

(b) $\dfrac{1}{f}=\dfrac{1}{d_\text{i}}+\dfrac{1}{d_\text{o}}$

$$f=\left(\frac{1}{d_\text{i}}+\frac{1}{d_\text{o}}\right)^{-1}=\left(\frac{1}{-1.5(21\text{ cm})}+\frac{1}{21\text{ cm}}\right)^{-1}=\boxed{63\text{ cm}}$$

(c) Since $f>0$, the lenses are $\boxed{\text{convex}}$.

77. $n_1\sin\theta_1=n_2\sin\theta_2$

$$\theta_2=\sin^{-1}\frac{n_1\sin\theta_1}{n_2}$$

$$\theta_\text{v}-\theta_\text{r}=\sin^{-1}\frac{(1.000)\sin 45°}{1.332}-\sin^{-1}\frac{(1.000)\sin 45°}{1.320}=-0.33°$$

The dispersion is $\boxed{0.33°}$.

78. $\theta_{i1} = 90° - 60° = 30°$

$n_{air} \sin\theta_{i1} = n \sin\theta_{refr1}$

$$\theta_{refr1} = \sin^{-1}\frac{n_{air}\sin\theta_{i1}}{n}$$

$\theta_{i2} = 180° - 120° - \theta_{refr1} = 60° - \theta_{refr1}$

$n\sin\theta_{i2} = n_{air}\sin\theta_{refr2}$

$$\begin{aligned}\theta_{refr2} &= \sin^{-1}\frac{n\sin\theta_{i2}}{n_{air}}\\ &= \sin^{-1}\frac{n\sin(60° - \theta_{refr1})}{n_{air}}\\ &= \sin^{-1}\frac{n\sin\left(60° - \sin^{-1}\frac{n_{air}\sin\theta_{i1}}{n}\right)}{n_{air}}\end{aligned}$$

$$\begin{aligned}\theta_v - \theta_r &= \sin^{-1}\frac{1.505\sin\left(60° - \sin^{-1}\frac{1.000\sin 30°}{1.505}\right)}{1.000} - \sin^{-1}\frac{1.421\sin\left(60° - \sin^{-1}\frac{1.000\sin 30°}{1.421}\right)}{1.000}\\ &= \boxed{13.92°}\end{aligned}$$

79. $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \propto n - 1$

$$\frac{\frac{1}{d_o} + \frac{1}{d_{ir}}}{\frac{1}{d_o} + \frac{1}{d_{iv}}} = \frac{n_r - 1}{n_v - 1}$$

$$\left(\frac{1}{d_o} + \frac{1}{d_{ir}}\right)\frac{n_v - 1}{n_r - 1} = \frac{1}{d_o} + \frac{1}{d_{iv}}$$

$$\begin{aligned}d_{iv} &= \left[\left(\frac{1}{d_o} + \frac{1}{d_{ir}}\right)\frac{n_v - 1}{n_r - 1} - \frac{1}{d_o}\right]^{-1}\\ &= \left[\left(\frac{1}{0.240\text{ m}} + \frac{1}{0.550\text{ m}}\right)\frac{1.605 - 1}{1.572 - 1} - \frac{1}{0.240\text{ m}}\right]^{-1}\\ &= \boxed{46.2\text{ cm from the lens}}\end{aligned}$$

80. (a) Converging and diverging lenses have opposite-sign focal lengths. When $n_{fluid} > n_{lens}$, the denominator $n_{lens} - n_{fluid}$ is negative, which means that f_{fluid} and f_{air} have opposite signs.

(b) $f_{fluid} = \left[\frac{(1.52 - 1)1.33}{1.52 - 1.33}\right](25.0\text{ cm}) = \boxed{91\text{ cm}}$

81. (a) It will decrease the number of reflections (boxed), because the beam will travel farther to the right between successive reflections.

(b) d = horizontal distance between reflections = (105 cm) tan 15.0° = 28.13 cm

n = number of reflections

$$n \le \frac{168\text{ cm}}{28.13\text{ cm}} = 5.97$$

The light beam reflects five (boxed) times from the top and bottom combined.

82. Because $\theta_{\text{incident}} = \theta_{\text{reflected}}$, the bottom of the mirror is initially located $0.70 \text{ m}/2 = 0.35 \text{ m}$ below your eyes. After you climb onto the stool (with height *h*), the bottom of the mirror is located $0.35 \text{ m} + h = 1.2 \text{ m}/2 = 0.60 \text{ m}$ below your eyes. So, $h = 0.60 \text{ m} - 0.35 \text{ m} = \boxed{0.25 \text{ m}}$.

83. (a) $\frac{1}{d_\text{o}} + \frac{1}{d_\text{i}} = \frac{1}{f}$

$$d_\text{i} = \left(\frac{1}{f} - \frac{1}{d_\text{o}}\right)^{-1} = \left(\frac{1}{-0.75 \text{ m}} - \frac{1}{2.2 \text{ m}}\right)^{-1} = -56 \text{ cm}$$

The image is located $\boxed{\text{56 cm behind the mirror}}$.

(b) $m = -\frac{d_\text{i}}{d_\text{o}} = -\frac{-0.56 \text{ m}}{2.2 \text{ m}} = 0.25$

The image is $\boxed{\text{upright}}$.

(c) $h_\text{i} = \left(\frac{0.56}{2.2}\right)(1.7 \text{ m}) = \boxed{43 \text{ cm}}$

84. (a) $m = \pm 2 = -\frac{d_\text{i}}{d_\text{o}}$

$$d_\text{i} = \pm 2 d_\text{o}$$

$$\frac{1}{d_\text{o}} + \frac{1}{d_\text{i}} = \frac{1}{f} = \frac{1}{\frac{1}{2}R} = \frac{2}{R}$$

$$\frac{2}{R} = \frac{1}{d_\text{o}} + \frac{1}{\pm 2 d_\text{o}} = \frac{1}{d_\text{o}}\left(1 \pm \frac{1}{2}\right)$$

$$d_\text{o} = \frac{R}{2}\left(1 \pm \frac{1}{2}\right) = \left(\frac{1}{2} \pm \frac{1}{4}\right)R = \left(\frac{1}{2} \pm \frac{1}{4}\right)(31 \text{ cm}) = 7.8 \text{ cm, } 23 \text{ cm}$$

The object can be placed $\boxed{\text{7.8 cm or 23 cm}}$ in front of the mirror.

(b) $d_\text{o} = 7.8$ cm:

$$d_\text{i} = \left(\frac{1}{f} - \frac{1}{d_\text{o}}\right)^{-1} = \left(\frac{2}{R} - \frac{1}{d_\text{o}}\right)^{-1} = \left(\frac{2}{31 \text{ cm}} - \frac{1}{7.8 \text{ cm}}\right)^{-1} = -16 \text{ cm} < 0$$

$$m = -\frac{d_\text{i}}{d_\text{o}} = \frac{-1}{d_\text{o}\left(\frac{2}{R} - \frac{1}{d_\text{o}}\right)} = \frac{1}{1 - \frac{2d_\text{o}}{R}} = \frac{1}{1 - \frac{2(7.8 \text{ cm})}{31 \text{ cm}}} = 2.0 > 0$$

$d_\text{o} = 23$ cm:

$$d_\text{i} = \left(\frac{2}{31 \text{ cm}} - \frac{1}{23 \text{ cm}}\right)^{-1} = 48 \text{ cm} > 0$$

$$m = \frac{1}{1 - \frac{2(23 \text{ cm})}{31 \text{ cm}}} = -2 < 0$$

7.8 cm: virtual and upright 23 cm: real and inverted

85. $v_A = xv_B$

$$\frac{c}{n_A} = x\frac{c}{n_B}$$

$$\frac{n_A}{n_B} = \boxed{\frac{1}{x}}$$

86. (a) $n_a \sin\theta_a = n_o \sin\theta_o$

$n_o \sin\theta_o = n_w \sin\theta_w$

$$\theta_w = \sin^{-1}\left(\frac{n_a}{n_w}\sin\theta_a\right) = \sin^{-1}\left(\frac{1.00}{1.33}\sin 60.0°\right) = \boxed{40.6°}$$

(b) The answer to part (a) does not depend upon the thickness of the film, because θ_w depends only upon the original angle of incidence and the indices of refraction of air and water.

87. (a) $n_a \sin\theta_a = n_o \sin\theta_o$

$n_o \sin\theta_o = n_w \sin\theta_w$

$$\theta_w = \sin^{-1}\left(\frac{n_a}{n_w}\sin\theta_a\right)$$

θ_w is maximum when $\theta_a \approx 90°$:

$$\theta_w = \sin^{-1}\left(\frac{1.00}{1.33}\sin 90°\right) = \boxed{48.8°}$$

(b) The answer to part (a) does not depend on the refractive index of the oil, because θ_w depends only on the original angle of incidence and the indices of refraction of air and water.

88. (a) $n_w \sin\theta_w = n_o \sin\theta_o$

$n_o \sin\theta_o = n_a \sin\theta_a$

$$\theta_w = \sin^{-1}\left(\frac{n_a}{n_w}\sin\theta_a\right)$$

$\theta_w = \theta_c$ when $\theta_a = 90°$:

$$\theta_c = \sin^{-1}\left(\frac{1.00}{1.33}\sin 90°\right) = \boxed{48.8°}$$

(b) θ_c is the *smallest* θ_w that produces total internal reflection at the oil-air surface. If θ is decreased, there will not be total internal reflection.

89. (a) $n_{\text{air}}\sin\theta_{\text{i}} = n\sin(90°-\theta)$

$$90°-\theta = \sin^{-1}\frac{n_{\text{air}}\sin\theta_{\text{i}}}{n}$$

$$\theta = 90°-\sin^{-1}\frac{(1.00)\sin 50.0°}{1.62}$$

$$= \boxed{61.8°}$$

(b) $\sin\theta_{\text{c}} = \frac{n_2}{n_1}$

$$\theta_{\text{c}} = \sin^{-1}\frac{n_2}{n_1}$$

$$= \sin^{-1}\frac{1.00}{1.62}$$

$$= 38.1° < 61.8° = \theta$$

$\theta > \theta_{\text{c}}$, so the internal reflection is total.

90. (a) $\frac{1}{d_{\text{i}}}+\frac{1}{d_{\text{o}}}=\frac{1}{f}$

$$d_{\text{i}} = \left(\frac{1}{f}-\frac{1}{d_{\text{o}}}\right)^{-1}$$

$$\frac{h_{\text{i}}}{h_{\text{o}}} = m = -\frac{d_{\text{i}}}{d_{\text{o}}} = \frac{-1}{d_{\text{o}}\left(\frac{1}{f}-\frac{1}{d_{\text{o}}}\right)} = \frac{1}{1-\frac{d_{\text{o}}}{f}} = \frac{1}{1-\frac{75.0\text{ cm}}{30.0\text{ cm}}} = \boxed{-0.67}$$

(b) $d_{\text{i2}}-d_{\text{i1}} = \left(\frac{1}{f}-\frac{1}{d_{\text{o2}}}\right)^{-1}-\left(\frac{1}{f}-\frac{1}{d_{\text{o1}}}\right)^{-1}$

$$= \left(\frac{1}{30.0\text{ cm}}-\frac{1}{74.0\text{ cm}}\right)^{-1}-\left(\frac{1}{30.0\text{ cm}}-\frac{1}{76.0\text{ cm}}\right)^{-1}$$

$$= 0.89\text{ mm}$$

$$\frac{L_{\text{i}}}{L_{\text{o}}} = \frac{0.89\text{ cm}}{2.0\text{ cm}} = \boxed{0.45}$$

91. (a) $\frac{1}{d_{\text{i}}}+\frac{1}{d_{\text{o}}}=\frac{1}{f}$

$$d_{\text{i}} = \left(\frac{1}{f}-\frac{1}{d_{\text{o}}}\right)^{-1}$$

$$\frac{h_{\text{i}}}{h_{\text{o}}} = m = -\frac{d_{\text{i}}}{d_{\text{o}}} = \frac{-1}{d_{\text{o}}\left(\frac{1}{f}-\frac{1}{d_{\text{o}}}\right)} = \frac{1}{1-\frac{d_{\text{o}}}{f}} = \frac{1}{1-\frac{75.0\text{ cm}}{-30.0\text{ cm}}} = \boxed{0.29}$$

(b) $d_{i2} - d_{i1} = \left(\frac{1}{f} - \frac{1}{d_{o2}}\right)^{-1} - \left(\frac{1}{f} - \frac{1}{d_{o2}}\right)^{-1}$

$= \left(\frac{1}{-30.0\text{ cm}} - \frac{1}{74.0\text{ cm}}\right)^{-1} - \left(\frac{1}{-30.0\text{ cm}} - \frac{1}{76.0\text{ cm}}\right)^{-1}$

$= 0.163\text{ cm}$

$\frac{L_i}{L_o} = \frac{0.163\text{ cm}}{2.0\text{ cm}} = \boxed{0.082}$

92. (a) The sight line needs to cover a vertical distance of 19.6 m − 1.6 m = 18.0 m in a horizontal distance of 95 m + 2(0.50 m) = 96 m. The height of the mirror bottom is 19.6 m minus the vertical distance covered by the long segment of the sight line:

$h_1 = 19.6\text{ m} - (18.0\text{ m})\left(\frac{95.5\text{ m}}{96\text{ m}}\right) = \boxed{1.7\text{ m}}$

(b) The short segment of the sight line, in a horizontal distance of 0.50 m, covers a vertical distance of (1.694 m + 0.32 m) − 1.6 m = 0.414 m. The total vertical distance covered by both segments of the sight line is $h_2 = (0.414\text{ m})\left(\frac{96\text{ m}}{0.50\text{ m}}\right) = 79.5\text{ m}.$

The highest point on the building is located at $\boxed{79.5\text{ m} + 1.6\text{ m} = 81\text{ m}}$.

93. Convex lens:

$d_{i1} = \left(\frac{1}{f_1} - \frac{1}{d_{o1}}\right)^{-1} = \left(\frac{1}{20.0\text{ cm}} - \frac{1}{30.0\text{ cm}}\right)^{-1} = 60.0\text{ cm}$

The first image occurs 60.0 cm – 40.0 cm = 20.0 cm to the right of the concave lens.
Concave lens:

$f_2 = \left(\frac{1}{d_{o2}} + \frac{1}{d_{i2}}\right)^{-1} = \left(\frac{1}{-20.0\text{ cm}} + \frac{1}{60.0\text{ cm}}\right)^{-1} = \boxed{-30.0\text{ cm}}$

94. $\frac{1}{f_1} = \frac{1}{d_{o1}} + \frac{1}{d_{i1}} \qquad \frac{1}{f_2} = \frac{1}{d_{o2}} + \frac{1}{d_{i2}}$

$\frac{1}{d_{i1}} = \frac{1}{f_1} - \frac{1}{d_{o1}} = -\frac{1}{d_{o2}}$

$\frac{1}{f_2} = -\frac{1}{f_1} + \frac{1}{d_{o1}} + \frac{1}{d_{i2}}$

$\frac{1}{f_2} + \frac{1}{f_1} = \frac{1}{d_{o1}} + \frac{1}{d_{i2}}$

For f_{eff}, $d_{o1} = d_o$ and $d_{i2} = d_i$.

$\frac{1}{f_2} + \frac{1}{f_1} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f_{\text{eff}}}$

So, $f_{\text{eff}} = \boxed{\frac{f_1 f_2}{f_1 + f_2}}$.

95. $m = -\dfrac{d_\text{i}}{d_\text{o}}$

$$\frac{1}{f} = \frac{1}{d_\text{o}} + \frac{1}{d_\text{i}} = \frac{1}{d_\text{o}} + \frac{1}{-md_\text{o}} = \frac{1}{d_\text{o}}\left(1 - \frac{1}{m}\right) = \frac{m-1}{md_\text{o}}$$

$$\boxed{f = \left(\frac{m}{m-1}\right)d_\text{o}}$$

96. First interface:

$n_\text{air} \sin\theta = n \sin\theta_1$

$$\theta_1 = \sin^{-1}\frac{n_\text{air}\sin\theta}{n}$$

Second interface:

$n\sin\theta_1 = n_\text{air}\sin\theta_2$

$$\theta_2 = \sin^{-1}\frac{n\sin\theta_1}{n_\text{air}} = \sin^{-1}\left[\left(\frac{n}{n_\text{air}}\right)\left(\frac{n_\text{air}\sin\theta}{n}\right)\right] = \theta$$

The dashed line representing the original path, the ray through the glass, and a perpendicular from the end of the ray to the original path form a right triangle. The smallest angle is the difference of θ and θ_1. The side opposite this angle is *d*. If we find the length of the hypotenuse (ray in glass) in terms of known quantities, we can solve for *d*. Let *r* = the hypotenuse.

$$\sin(\theta - \theta_1) = \frac{d}{r}$$

Notice that *t*, *r*, and the horizontal distance between the two points of incidence form another right triangle.
Let *x* = the horizontal distance.
Find *x*.

$$\frac{x}{t} = \tan\theta_1$$

$$x = t\tan\theta_1$$

Find *r*.

$$r^2 = x^2 + t^2 = t^2\tan^2\theta_1 + t^2 = t^2(1 + \tan^2\theta_1)$$

$$r = t\sqrt{1+\tan^2\theta_1} = t\sqrt{\sec^2\theta_1} = t\sec\theta_1$$

Substitute.

$$\begin{aligned} d &= r\sin(\theta - \theta_1) \\ &= t\sec\theta_1 \sin(\theta - \theta_1) \\ &= \boxed{\frac{t\sin\left(\theta - \sin^{-1}\frac{1.000\sin\theta}{n}\right)}{\cos\sin^{-1}\frac{1.000\sin\theta}{n}}} \end{aligned}$$

97. For the extreme case, let $\theta_\text{i} = 90°$. This gives

$$1\sin 90° = n\sin\phi \Rightarrow \sin\phi = \frac{1}{n}$$

The angle of incidence on the curved surface is $\theta = 90° - \phi$, and the critical condition for total internal reflection is

$$\sin\theta = \frac{1}{n}$$

Combining these results, we have

$$\sin\theta = \frac{1}{n} = \sin(90° - \phi) = \cos\phi = \sqrt{1-\sin^2\phi} = \sqrt{1-\frac{1}{n^2}}$$

$$\frac{1}{n} = \sqrt{1-\frac{1}{n^2}} \Rightarrow n = \sqrt{2}$$

98. (a) Find the angle of refraction.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\begin{aligned}\theta_2 &= \sin^{-1}\frac{n_1\sin\theta_1}{n_2}\\ &= \sin^{-1}\frac{(1.00)\sin 45°}{1.40}\\ &= 30°\end{aligned}$$

Calculate the time to go from *A* to *B*.

$$\begin{aligned}t &= \frac{d_1}{v_1} + \frac{d_2}{v_2}\\ &= \frac{d_1}{\frac{c}{n_1}} + \frac{d_2}{\frac{c}{n_2}}\\ &= \frac{n_1 d_1 + n_2 d_2}{c}\\ &= \frac{(1.00)\sqrt{(50.0\text{ m})^2 + (50.0\text{ m})^2} + (1.40)\frac{50.0\text{ m}}{\cos 30.3°}}{3.00\times 10^8\ \frac{\text{m}}{\text{s}}}\\ &= \boxed{506\text{ ns}}\end{aligned}$$

(b)

$$\begin{aligned}t &= \frac{n_1 d_1 + n_2 d_2}{c}\\ &= \frac{(n_1 + n_2)d}{c}\\ &= \frac{(1.00+1.40)\frac{\sqrt{(100.0\text{ m})^2 + [50.0\text{ m} + (50.0\text{ m})\tan 30.3°]^2}}{2}}{3.00\times 10^8\ \frac{\text{m}}{\text{s}}}\\ &= \boxed{510\text{ ns}}\end{aligned}$$

99. Use Snell's law at each interface.

$n_1 \sin\theta_1 = n_2 \sin\theta_2$ (first interface)

and

$n_2 \sin\theta_2 = n_3 \sin\theta_3$, (second interface)

So, $n_1 \sin\theta_1 = n_3 \sin\theta_3$.

100. (a) Use Snell's Law.

$n_{\text{air}} \sin\theta_{\text{i1}} = n\sin\theta_{\text{refr1}}$

$n\sin\theta_{\text{i2}} = n_{\text{air}} \sin\theta_{\text{refr2}}$

From the figure, we see that $\theta_{\text{i2}} + \theta_{\text{refr1}} = 45°$, or $\theta_{\text{refr1}} = 45° - \theta_{\text{i2}}$.

We seek θ_{i1} so that $\theta_{\text{refr2}} = 90°$. Then

$$\theta_{\text{i2}} = \sin^{-1}\frac{n_{\text{air}}}{n}$$

$$\sin\theta_{\text{refr1}} = \sin\left(45° - \sin^{-1}\frac{n_{\text{air}}}{n}\right)$$

$$\theta_{\text{i1}} = \sin^{-1}\left[\frac{n}{n_{\text{air}}}\sin\left(45° - \sin^{-1}\frac{n_{\text{air}}}{n}\right)\right]$$

$$= \sin^{-1}\left[\frac{1.66}{1.00}\sin\left(45° - \sin^{-1}\frac{1.00}{1.66}\right)\right]$$

$$= \boxed{13°}$$

(b) Increasing the angle of incidence at the slanted surface will decrease the angle of incidence at the vertical surface. Reflection will no longer be total.

(c) Use $\theta_{\text{i1}} = 45°$, $\theta_{\text{refr2}} = 90°$. Then

$1.00\sin 45° = n\sin\theta_{\text{refr1}}$

$n\sin\theta_{\text{i2}} = 1.00\sin 90°$

so that

$$\frac{1}{\sqrt{2}} = n\sin\theta_{\text{refr1}}$$

$$n\sin\theta_{\text{i2}} = 1$$

And since $\theta_{\text{refr1}} = 45° - \theta_{\text{i2}}$,

$$\frac{1}{\sqrt{2}} = n(\sin 45°\cos\theta_{\text{i2}} - \cos 45°\sin\theta_{\text{i2}})$$

Note that $\sin\theta_{\text{i2}} = \dfrac{1}{n}$ and therefore $\cos\theta_{\text{i2}} = \dfrac{\sqrt{n^2-1}}{n}$.

$$\frac{1}{\sqrt{2}} = n\left(\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{n^2-1}}{n} - \frac{1}{\sqrt{2}}\cdot\frac{1}{n}\right)$$

$$1 = \sqrt{n^2-1} - 1$$

$$n = \sqrt{5} = \boxed{2.24}$$