Chapter 28

Physical Optics: Interference and Diffraction

Answers to Even-numbered Conceptual Questions

2. If the slit spacing, d, were less than the wavelength, λ, the condition for a bright fringe (Equation 28-1) could be satisfied only for the central bright fringe ($m = 0$). For nonzero values of m there are no solutions, because $\sin\theta$ cannot be greater than one. In addition, Equation 28-2 shows that if d is greater than $\lambda/2$, though still less than λ, there will be only one dark fringe on either side of the central bright fringe. If d is less than $\lambda/2$, no dark fringes will be observed.

4. The locations of bright and dark fringes depends on the wavelength of light. Therefore, if white light is used in a two-slit experiment, each bright fringe will show some separation into colors, giving a "rainbow" effect.

6. Submerging the two-slit experiment in water would reduce the wavelength of the light from λ to λ/n, where $n = 1.33$ is the index of refraction of water. Therefore, the angles to all the bright fringes would be reduced, as we can see from Equation 28-1. It follows that the two-slit pattern of bright fringes would be more tightly spaced in this case.

8. The soap film in the photograph is thinnest near the top (as one might expect) because in that region the film appears black. Specifically, light reflected from the front surface of the film has its phase changed by 180˚; light that reflects from the back surface of the film has no change in phase. Therefore, light from the front and back surfaces of the film will undergo destructive interference as the path length between the surfaces goes to zero. This is why the top of the film, where the film is thinnest, appears black in the photo.

10. One possible reason is that one of the films may have an index of refraction greater than that of glass, whereas the other may have an index of refraction that is less than that of glass. If this is the case, the phase change in reflection from the film-glass interface will be different for the two films. This, in turn, would result in different colors appearing in the reflected light.

12. Light reflected from the top of the film has a phase change of 180˚; light reflected from the film-water surface also has a phase change of 180˚, since the film's index of refraction is less than that of water. It follows that the film appears bright (constructive interference) where the film's thickness goes to zero.

14. The location of dark fringes in a single-slit diffraction pattern is given by Equation 28-12. Notice that if W is decreased, the angle θ must increase to compensate, and to maintain the equality. This is why the dark fringes move outward. As the angle θ approaches 90˚ for $m = \pm 1$, all of the dark fringes have moved outward to infinity. Clearly, this occurs in Equation 28-12 when the width W is equal to the wavelength, λ.

16. Reducing the index of refraction would increase the wavelength of light within the eye. We know from Equation 28-14, however, that resolution decreases as the wavelength increases; that is, increasing λ in Equation 28-14 implies that the required angle of separation, θ, must also increase. Therefore, the eye's resolution would decrease in this case.

18. A cat's eye would give greater resolution in the vertical direction, because the effective aperture is greater in that direction. As shown in Equation 28-14, the greater the aperture, *D*, the smaller the angle, θ, and the greater the resolution.

20. In general, the larger the aperture in an optical instrument, the greater the resolution. This follows directly from Equation 28-14, where we see that a large aperture *D* implies a small angle θ. If the angular separation that can be resolved is decreased, the resolution is increased.

Solutions to Problems

1. $\frac{78.0\text{ m}}{26.0\text{ m}} = 3.00$ wavelengths

$\frac{143\text{ m}}{26.0\text{ m}} = 5.50$ wavelengths

Since the path difference is $5.50 - 3.00 = 2.50$ wavelengths, the waves interfere $\boxed{\text{destructively}}$ at the observation point.

2. (a) $\frac{91.0\text{ m}}{26.0\text{ m}} = 3.50$ wavelengths

$\frac{221\text{ m}}{26.0\text{ m}} = 8.50$ wavelengths

Since the path difference is $8.50 - 3.50 = 5.00$ wavelengths, the waves interfere $\boxed{\text{constructively}}$ at the observation point.

(b) $\frac{44.0\text{ m}}{26.0\text{ m}} = 1.69$ wavelengths

$\frac{135\text{ m}}{26.0\text{ m}} = 5.19$ wavelengths

Since the path difference is $5.19 - 1.69 = 3.50$ wavelengths, the waves interfere $\boxed{\text{destructively}}$ at the observation point.

3. The largest wavelength that will give constructive interference equals the path difference of the two sources and the observation point.

$\lambda = 285\text{ m} - 181\text{ m} = \boxed{104\text{ m}}$

4. (a) $450\text{ m} - 150\text{ m} = \boxed{300\text{ m}}$

(b) $\ell_1 - \ell_2 = \dfrac{300\text{ m}}{2} = 150\text{ m}$

$$\ell_1 = \sqrt{(450\text{ m})^2 + v^2t^2}$$
$$\ell_2 = \sqrt{(150\text{ m})^2 + v^2t^2}$$
$$\sqrt{(450\text{ m})^2 + v^2t^2} - \sqrt{(150\text{ m})^2 + v^2t^2} = 150\text{ m}$$
$$\left[\sqrt{(450\text{ m})^2 + v^2t^2}\right]^2 = \left[150\text{ m} + \sqrt{(150\text{ m})^2 + v^2t^2}\right]^2$$
$$(450\text{ m})^2 + v^2t^2 = (150\text{ m})^2 + (300\text{ m})\sqrt{(150\text{ m})^2 + v^2t^2} + (150\text{ m})^2 + v^2t^2$$
$$\left[\sqrt{(150\text{ m})^2 + v^2t^2}\right]^2 = \left[\frac{(450\text{ m})^2 - 2(150\text{ m})^2}{300\text{ m}}\right]^2$$
$$(150\text{ m})^2 + v^2t^2 = \left[\frac{(450\text{ m})^2 - 2(150\text{ m})^2}{300\text{ m}}\right]^2$$
$$t = \frac{1}{17\,\frac{\text{m}}{\text{s}}}\sqrt{\left[\frac{(450\text{ m})^2 - 2(150\text{ m})^2}{300\text{ m}}\right]^2 - (150\text{ m})^2}$$
$$= \boxed{30\text{ s}}$$

5. The maximum wavelength corresponds to the lowest frequency. Student A is equidistant from each speaker, but student B is not. Find the difference of the distances from each speaker to student B. This difference is the maximum wavelength.

$$\sqrt{(1.5\text{ m})^2 + (3.0\text{ m})^2} - 1.5\text{ m} = \lambda$$
$$f = \frac{v}{\lambda} = \frac{343\,\frac{\text{m}}{\text{s}}}{\sqrt{(1.5\text{ m})^2 + (3.0\text{ m})^2} - 1.5\text{ m}} = \boxed{180\text{ Hz}}$$

6. $\lambda = \dfrac{v}{f} = \dfrac{343\,\frac{\text{m}}{\text{s}}}{185\text{ Hz}} = 1.85\text{ m}$ is the wavelength of the 185 Hz tone.

Determine the ratio of the path difference to the wavelength.

$$\frac{\sqrt{(1.5\text{ m})^2 + (3.0\text{ m})^2} - 1.5\text{ m}}{\frac{343\frac{\text{m}}{\text{s}}}{185\text{ Hz}}} = 1.0$$

Since the path difference is equal to one wavelength, the tone heard at location B is a $\boxed{\text{minimum}}$.

7. The path differences equal to one and two wavelengths correspond to the first and second lowest frequencies, respectively.

$$2.00\text{ m} + \frac{0.525\text{ m}}{2} - \left(2.00\text{ m} - \frac{0.525\text{ m}}{2}\right) = 0.525\text{ m}$$

Lowest frequency:

$$f = \frac{v}{\lambda} = \frac{343\,\frac{\text{m}}{\text{s}}}{0.525\text{ m}} = \boxed{653\text{ Hz}}$$

Next lowest frequency:

$$f = \frac{v}{\lambda} = \frac{343\,\frac{\text{m}}{\text{s}}}{\frac{0.525\text{ m}}{2}} = \boxed{1310\text{ Hz}}$$

8. The path differences equal to one-half and one and one-half wavelengths correspond to the first and second lowest frequencies, respectively.

$$2.00\text{ m}+\frac{0.525\text{ m}}{2}-\left(2.00\text{ m}-\frac{0.525\text{ m}}{2}\right)=0.525\text{ m}$$

Lowest frequency:

$$f=\frac{v}{\lambda}=\frac{343\,\frac{\text{m}}{\text{s}}}{2(0.525\text{ m})}=\boxed{327\text{ Hz}}$$

Next lowest frequency:

$$f=\frac{v}{\lambda}=\frac{343\,\frac{\text{m}}{\text{s}}}{\frac{2}{3}(0.525\text{ m})}=\boxed{980\text{ Hz}}$$

9. Find the path difference for Moe and Curly.

$$\sqrt{(1.40\text{ m})^2+(3.00\text{ m})^2}-\sqrt{(0.60\text{ m})^2+(3.00\text{ m})^2}=0.25\text{ m}$$

If this is one-half wavelength, then $\frac{3.00\text{ m}}{2(0.25\text{ m})}=6.0$ wavelengths is an integral number of wavelengths at Larry.

So, Larry will hear a loud tone and Moe and Curly will hear very little.

$$f=\frac{v}{\lambda}=\frac{343\,\frac{\text{m}}{\text{s}}}{2(0.25\text{ m})}=\boxed{690\text{ Hz}}\text{ is the lowest frequency.}$$

If 0.25 m is one and one-half wavelengths, then $\frac{3.00\text{ m}}{\frac{2}{3}(0.25\text{ m})}=18$ wavelengths is an integral number of wavelengths at Larry. So, Larry will again hear a loud tone and Moe and Curly will hear very little.

$$f=\frac{v}{\lambda}=\frac{343\,\frac{\text{m}}{\text{s}}}{\frac{2}{3}(0.25\text{ m})}=\boxed{2100\text{ Hz}}\text{ is the second lowest frequency.}$$

10. (a) Larry is equidistant from the two speakers. Waves leaving the speakers out of phase will arrive at Larry's position out of phase, so Larry hears a minimum. This is true for any frequency.

(b) Find the path difference for Moe and Curly.

$$\sqrt{(1.40\text{ m})^2+(3.00\text{ m})^2}-\sqrt{(0.60\text{ m})^2+(3.00\text{ m})^2}=0.251\text{ m}$$

To hear a maximum the waves from the two speakers must arrive in phase. Since they are out of phase at the sources, the path difference must be one-half wavelength or one and one-half wavelengths.

$$f=\frac{v}{\lambda}=\frac{343\text{ m/s}}{2(0.251\text{ m})}=\boxed{0.68\text{ kHz}}\text{ is the lowest frequency}$$

For one and one-half wavelengths, $1.5\lambda=0.251$ m.

$$f=\frac{v}{\lambda}=\frac{343\text{ m/s}}{(0.251\text{ m}/1.5)}=\boxed{2.0\text{ kHz}}\text{ is the second lowest}$$

11. (a) For a minimum signal the path difference of 450 m − 150 m = 300 m must be one-half wavelength. So, $\lambda=\boxed{600\text{ m}}$.

(b) Since the path difference will equal the wavelength, the waves will arrive in phase, producing a maximum.

(c) The next minimum signal occurs when the path difference equals one and one-half wavelengths.

$$1.5\lambda = 300\text{ m}$$

$$\lambda = \boxed{200\text{ m}}$$

12. $\sin\theta = m\dfrac{\lambda}{d} = (1)\dfrac{\lambda}{d} = \dfrac{\lambda}{d}$

$$d = \frac{\lambda}{\sin\theta} = \frac{670\times10^{-9}\text{m}}{\sin 25^\circ} = \boxed{1.6\ \mu\text{m}}$$

13. $\sin\theta = m\dfrac{\lambda}{d} = (3)\dfrac{\lambda}{d} = \dfrac{3\lambda}{d}$

$$\lambda = \frac{1}{3}d\sin\theta = \frac{1}{3}(0.0324\times10^{-3}\text{ m})\sin 3.51^\circ = \boxed{661\text{ nm}}$$

14. $\sin\theta = \left(m - \dfrac{1}{2}\right)\dfrac{\lambda}{d} = \left(1 - \dfrac{1}{2}\right)\dfrac{\lambda}{d} = \dfrac{\lambda}{2d}$

$$\frac{d}{\lambda} = \frac{1}{2\sin\theta} = \frac{1}{2\sin 0.29^\circ} = \boxed{99}$$

15. (a) $\sin\theta = m\dfrac{\lambda}{d} = (2)\dfrac{\lambda}{d} = \dfrac{2\lambda}{d}$

$$\lambda = \frac{1}{2}d\sin\theta = \frac{1}{2}(48.0\times10^{-5}\text{ m})\sin 0.0990^\circ = \boxed{415\text{ nm}}$$

(b) Since the wavelength is directly proportional to the separation, the wavelength will $\boxed{\text{increase}}$ if the separation is increased.

(c) $\lambda = \dfrac{1}{2}(68.0\times10^{-5}\text{ m})\sin 0.0990^\circ = \boxed{587\text{ nm}}$

16. Determine the angle of the third-order minima.

$$y = L\tan\theta_3$$

$$\theta_3 = \tan^{-1}\frac{y}{L}$$

Determine $\frac{\lambda}{d}$.

$$\sin\theta_3 = \left(m - \frac{1}{2}\right)\frac{\lambda}{d} = \left(3 - \frac{1}{2}\right)\frac{\lambda}{d} = \frac{5\lambda}{2d}$$

$$\frac{\lambda}{d} = \frac{2}{5}\sin\theta_3$$

Determine the angle of the first-order minima.

$$\sin\theta_1 = \left(m - \frac{1}{2}\right)\frac{\lambda}{d} = \left(1 - \frac{1}{2}\right)\frac{\lambda}{d} = \frac{\lambda}{2d}$$

$$\theta_1 = \sin^{-1}\frac{\lambda}{2d}$$

Calculate the width of the central bright fringe.

$$2y = 2L\tan\theta_1 = 2L\tan\left(\sin^{-1}\frac{\lambda}{2d}\right) = 2L\tan\left[\sin^{-1}\left(\frac{1}{5}\sin\theta_3\right)\right] = 2L\tan\left\{\sin^{-1}\left[\frac{1}{5}\sin\left(\tan^{-1}\frac{y}{L}\right)\right]\right\}$$

$$= 2(1.00\text{ m})\tan\left\{\sin^{-1}\left[\frac{1}{5}\sin\left(\tan^{-1}\frac{\frac{0.250\text{ m}}{2}}{1.00\text{ m}}\right)\right]\right\} = \boxed{4.96\text{ cm}}$$

17. Determine the angle to the center of the second-order maximum.

$$y = L\tan\theta$$

$$\theta = \tan^{-1}\frac{y}{L}$$

Calculate the slit separation.

$$\sin\theta = m\frac{\lambda}{d} = \frac{2\lambda}{d}$$

$$d = \frac{2\lambda}{\sin\theta} = \frac{2\lambda}{\sin\left(\tan^{-1}\frac{y}{L}\right)} = \frac{2(632.8\times10^{-9}\text{ m})}{\sin\left(\tan^{-1}\frac{\frac{23.0\times10^{-3}\text{ m}}{2}}{1.40\text{ m}}\right)} = \boxed{154\ \mu\text{m}}$$

18. Find the angle to the first bright fringe.

$$y = L\tan\theta$$

$$\theta = \tan^{-1}\frac{y}{L}$$

Calculate the slit separation.

$$\sin\theta = m\frac{\lambda}{d} = (1)\frac{\lambda}{d} = \frac{\lambda}{d}$$

$$d = \frac{\lambda}{\sin\theta} = \frac{\lambda}{\sin\left(\tan^{-1}\frac{y}{L}\right)} = \frac{546\times10^{-9}\text{ m}}{\sin\left(\tan^{-1}\frac{0.0516\text{ m}}{8.75\text{ m}}\right)} = \boxed{92.6\ \mu\text{m}}$$

19. $\sin\theta = m\dfrac{\lambda}{d}$

$$\lambda = \frac{d\sin\theta}{m}$$

$$= \frac{d\sin\left(\tan^{-1}\frac{y}{L}\right)}{m} \qquad (y = L\tan\theta)$$

$$= \frac{(0.230\times10^{-3}\text{ mm})\sin\left(\tan^{-1}\frac{7.15\times10^{-3}\text{ m}}{2.50\text{ m}}\right)}{1}$$

$$= \boxed{658\text{ nm}}$$

20. (a) Since the separations, and therefore the angles, between the fringes are less for the second color as compared to the first, and since the wavelength is proportional to $\sin\theta,$ the wavelength of the second color is $\boxed{\text{less than}}$ 505 nm. ($\sin\theta_1 < \sin\theta_2$ if $\theta_1 < \theta_2$ on $0 < \theta \le 90°$.)

(b) First color:

$$\frac{y}{L} = \tan\theta_1 \approx \sin\theta_1 = \left(m - \frac{1}{2}\right)\frac{\lambda_1}{d} = \left(5 - \frac{1}{2}\right)\frac{\lambda_1}{d} = \frac{9\lambda_1}{2d}$$

Second color:

$$\frac{y}{L} = \tan\theta_2 \approx \sin\theta_2 = m\frac{\lambda_2}{d} = (5)\frac{\lambda_2}{d} = \frac{5\lambda_2}{d}$$

Equate the results for each color and solve for λ_2.

$$\frac{5\lambda_2}{d} = \frac{9\lambda_1}{2d}$$

$$\lambda_2 = \frac{9\lambda_1}{10}$$

$$= \frac{9}{10}(505\text{ nm})$$

$$= \boxed{455\text{ nm}}$$

21. (a) $\sin\theta = \dfrac{m\lambda}{d}$

$$y = L\tan\theta \approx L\sin\theta = L\frac{m\lambda}{d} = (1.25\text{ m})\frac{4.5(505\times10^{-9}\text{ m})}{127\times10^{-6}\text{ m}} = \boxed{2.24\text{ cm}}$$

(b) The distance between the fringes is proportional to the wavelength, and so must be inversely proportional to the frequency. A more tightly packed pattern would require a shorter wavelength and a higher frequency. Frequency must be $\boxed{\text{increased}}$.

22. (a) $\lambda = \dfrac{d\sin\theta}{m} = \dfrac{d\sin\left(\tan^{-1}\frac{y}{L}\right)}{m}$

$$\lambda = \frac{(135\ \mu\text{m})\sin\left(\tan^{-1}\frac{0.0230\text{ m}}{1.40\text{ m}}\right)}{4}$$

$$= \boxed{554\text{ nm}}$$

(b) Increasing the frequency decreases the wavelength. With a smaller wavelength the bright spots will move $\boxed{\text{closer together}}$.

23. For the center fringe, $\theta = \sin^{-1}\frac{\lambda}{d}$. So,

$$L \geq \frac{y}{\tan\theta} = \frac{y}{\tan\theta} = \frac{y}{\tan\left(\sin^{-1}\frac{\lambda}{d}\right)} = \frac{0.0200\text{ m}}{\tan\left(\sin^{-1}\frac{632.8\times10^{-9}\text{ m}}{0.0220\times10^{-3}\text{ m}}\right)} = \boxed{69.5\text{ cm}}.$$

24. Find the wavelength for which destructive interference occurs.

$$\frac{2nt}{\lambda_{\text{vacuum}}} = m$$

$$\lambda_{\text{vacuum}} = \frac{2nt}{m} = \frac{2(1.33)(401\times10^{-9}\text{ m})}{m} = \frac{1070\text{ nm}}{m}$$

Since the visible spectrum spans the wavelength range 400 nm to 700 nm, $m = 2$ is the only valid choice.

$$\lambda_{\text{vacuum}} = \frac{1066.6\text{ nm}}{2} = \boxed{533\text{ nm}}$$

The missing color is $\boxed{\text{green}}$.

25. $$\frac{2nt}{\lambda_{\text{vacuum}}} - \frac{1}{2} = m$$

$$\frac{2nt}{\lambda_{\text{vacuum}}} = m + \frac{1}{2}$$

$$\lambda_{\text{vacuum}} = \frac{2nt}{m+\frac{1}{2}} = \frac{2(1.33)(815\text{ nm})}{m+\frac{1}{2}}$$

m	0	1	2	3	4	5
λ (nm)	4340	1450	867	619	482	394

The visible wavelengths that are constructively reflected are $\boxed{482\text{ nm and }619\text{ nm}}$.

26. $$\frac{2nt}{\lambda_{\text{vacuum}}} = m$$

$$t > \frac{m\lambda_{\text{vacuum}}}{2n} = \frac{(1)(670\times10^{-9}\text{ m})}{2(1.30)} = \boxed{260\text{ nm}}$$

27. Phase change for reflection at the air-film interface: $\frac{1}{2}$

Phase change for reflection at the film-glass interface: $\frac{1}{2} + \frac{2t}{\lambda_n}$

Phase difference: $\frac{1}{2}+\frac{2t}{\lambda_n}-\frac{1}{2}=\frac{2t}{\lambda_n}$

Set the phase difference equal to an integer.

$$\frac{2t}{\lambda_n}=m=\frac{2nt}{\lambda_{\text{vacuum}}}$$

$$\lambda_{\text{vacuum}}=\frac{2nt}{m}=\frac{2(1.33)(792\text{ nm})}{m}$$

m	1	2	3	4	5	6
λ (nm)	2110	1050	702	527	421	351

The visible wavelengths that are constructively reflected are $\boxed{421\text{ nm and }527\text{ nm}}$.

28. Since $n_{\text{oil}}>n_{\text{water}}$, there is no phase change at the oil-water interface, therefore, we can use Equation 28-10 to find the minimum thickness of the oil film.

$$\frac{2nt}{\lambda_{\text{vacuum}}}=m$$

$$t=\frac{m\lambda_{\text{vacuum}}}{2n}$$

$$t_{\text{min}}=\frac{(1)\lambda_{\text{vacuum}}}{2n}=\frac{521\text{ nm}}{2(1.38)}=\boxed{189\text{ nm}}$$

29. **(a)** Since the radio waves are reflected from a solid surface (the plane), there is a half-wavelength phase change of the wave. So, you observe $\boxed{\text{destructive}}$ interference.

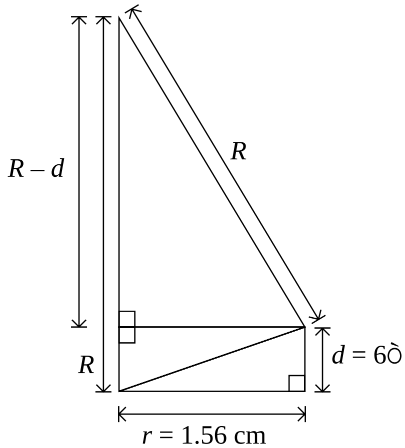
(b) $88.00\lambda+36.00\text{ km}=\sqrt{(12.00\text{ km})^2+(2.230\text{ km})^2}+\sqrt{(24.00\text{ km})^2+(2.230\text{ km})^2}$

$$\lambda=\frac{\sqrt{(12.00\text{ km})^2+(2.230\text{ km})^2}+\sqrt{(24.00\text{ km})^2+(2.230\text{ km})^2}-36.00\text{ km}}{88.00}=\boxed{3.5\text{ m}}$$

30. **(a)** $\frac{2d}{\lambda}=m=12$

$d=6\lambda$

The figure below illustrates the situation.



Find R.

$$R^2 = r^2 + (R - 6\lambda)^2$$
$$= r^2 + R^2 - 12\lambda R + 36\lambda^2$$
$$12\lambda R = r^2 + 36\lambda^2$$
$$R = \frac{r^2 + 36\lambda^2}{12\lambda}$$
$$= \frac{(0.0156\text{ m})^2 + 36(648\times10^{-9}\text{ m})^2}{12(648\times10^{-9}\text{ m})}$$
$$= \boxed{31.3\text{ m}}$$

(b) Since $r^2 >> 36\lambda^2$, $R \approx \dfrac{r^2}{12\lambda}$. So, if λ is increased, $r \approx \sqrt{12\lambda R}$ will be $\boxed{\text{greater}}$ than 1.56 cm.

31. <u>bottom surface of the upper plate</u>

phase change $= \dfrac{2nt}{\lambda_{\text{vacuum}}}$

<u>upper surface of the lower plate</u>

phase change $= \dfrac{2nt}{\lambda_{\text{vacuum}}} + \dfrac{2d}{\lambda_{\text{vacuum}}} + \dfrac{1}{2}$

difference in phase changes $= \dfrac{2nt}{\lambda_{\text{vacuum}}} + \dfrac{2d}{\lambda_{\text{vacuum}}} + \dfrac{1}{2} - \dfrac{2nt}{\lambda_{\text{vacuum}}} = \dfrac{2d}{\lambda_{\text{vacuum}}} + \dfrac{1}{2}$

$\dfrac{2d}{\lambda_{\text{vacuum}}} + \dfrac{1}{2}$ must be equal to an integer for constructive interference. Since 0 would result in either a negative d or λ, 1 is the lowest such integer.

$$\frac{2d}{\lambda_{\text{vacuum}}} + \frac{1}{2} = 1, 2, 3, \ldots$$

$$\frac{2d}{\lambda_{\text{vacuum}}} - \frac{1}{2} = 0, 1, 2, \ldots$$

So, for constructive interference, $\dfrac{2d}{\lambda_{\text{vacuum}}} - \dfrac{1}{2} = m$ where $m = 0, 1, 2, \ldots$.

$\dfrac{2d}{\lambda_{\text{vacuum}}} + \dfrac{1}{2}$ must be equal to $\dfrac{1}{2}$ or some odd multiple of $\dfrac{1}{2}$ for destructive interference.

$$\frac{2d}{\lambda_{\text{vacuum}}} + \frac{1}{2} = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$$

$$\frac{2d}{\lambda_{\text{vacuum}}} = 0, 1, 2, \ldots$$

So, for destructive interference, $\dfrac{2d}{\lambda_{\text{vacuum}}} = m$ where $m = 0, 1, 2, \ldots$.

λ (nm)	600.0	800.0	343.0
Constructive Interference $\dfrac{2d}{\lambda} - \dfrac{1}{2} = m$	1.50	1.00	3.00
Destructive Interference $\dfrac{2d}{\lambda} = m$	2.00	1.50	3.50

(a) Destructive interference

(b) Constructive interference

(c) Constructive interference

32. (a) $\frac{2nt}{\lambda}-\frac{1}{2}=m$

$$\frac{2nt}{\lambda}=m+\frac{1}{2}$$

$$t=\frac{\left(m+\frac{1}{2}\right)\lambda}{2n}$$

$$t_{\text{min}}=\frac{\lambda}{4n}=\frac{652\text{ nm}}{4(1.33)}=\boxed{123\text{ nm}}$$

(b) $\frac{2nt}{\lambda}=m$

$$\lambda=\frac{2nt}{m}=\frac{2(1.33)\left[\frac{652\text{ nm}}{4(1.33)}\right]}{m}=\frac{652\text{ nm}}{2m}=\frac{326\text{ nm}}{m}$$

m	1	2	3
λ(nm)	326	163	109

Since $\lambda<400$ nm for all m, no visible wavelengths will destructively interfere.

33. (a) Find the difference in phase changes of the light reflected from each interface.

air-magnesium fluoride interface

$$\ell_{\text{eff, 1}}=\frac{1}{2}\lambda$$

$$\frac{\ell_{\text{eff, 1}}}{\lambda}=\frac{1}{2}$$

magnesium fluoride-glass interface

$$\ell_{\text{eff, 2}}=2t+\frac{1}{2}\lambda_n$$

$$\frac{\ell_{\text{eff, 2}}}{\lambda_n}=\frac{2t}{\lambda_n}+\frac{1}{2}$$

$$\frac{\ell_{\text{eff, 2}}}{\lambda}=\frac{2nt}{\lambda}+\frac{1}{2}$$

difference in phase changes $=\frac{2nt}{\lambda}+\frac{1}{2}-\frac{1}{2}=\frac{2nt}{\lambda}$

For destructive interference:

$\frac{2nt}{\lambda}-\frac{1}{2}=m$ where $m=0, 1, 2, \ldots$.

Determine the minimum film thickness ($m = 0$).

$$\frac{2nt}{\lambda} = \frac{1}{2}$$
$$t = \frac{\lambda}{4n} = \frac{595\text{ nm}}{4(1.38)} = \boxed{108\text{ nm}}$$

(b) Higher frequency corresponds to lower wavelength. So, since $t \propto \lambda,$ the coating should be made $\boxed{\text{thinner}}$.

34. (a)
$$\frac{2nt}{\lambda} - \frac{1}{2} = m$$
$$\frac{2nt}{\lambda} = m + \frac{1}{2}$$
$$t = \frac{\left(m+\frac{1}{2}\right)\lambda}{2n} = \frac{\left(m+\frac{1}{2}\right)(590\text{ nm})}{2(1.33)}$$

m	0	1
t (nm)	110	330

The two minimum thicknesses are $\boxed{110\text{ nm and }330\text{ nm}}$.

(b)
$$\frac{2nt}{\lambda} = m$$
$$t = \frac{m\lambda}{2n} = \frac{m(590\text{ nm})}{2(1.33)}$$

m	1	2
t (nm)	220	440

The two minimum thicknesses are $\boxed{220\text{ nm and }440\text{ nm}}$.

35. (a) Since $n_{\text{glass}} > n_{\text{coating}}$, there is no phase change due to the glass lens.

$$\lambda_{\text{vacuum}} = \frac{2nt}{m} = \frac{2(1.480)(340.0\text{ nm})}{m}$$

m	1	2	3
λ (nm)	1006	503.2	335.5

$\lambda = \boxed{503.2\text{ nm}}$ will be absent.

(b) Since $n_{\text{glass}} > n_{\text{coating}}$, there is a phase change due to the glass lens. Adding $\frac{1}{2}$ to the left side of the equation for destructive interference will account for the reflection at the coating-glass interface.

$$\frac{2nt}{\lambda_{\text{vacuum}}} + \frac{1}{2} = m$$

$$\lambda_{\text{vacuum}} = \frac{2nt}{m - \frac{1}{2}} = \frac{2(1.480)(340.0 \text{ nm})}{m - \frac{1}{2}}$$

m	1	2	3	4
λ (nm)	2013	670.9	402.6	287.5

$\lambda = \boxed{670.9 \text{ nm and } 402.6 \text{ nm}}$ will be absent.

36. The situation is essentially part of an air wedge.

The upper glass plate has a slope of $\frac{0.0520 \text{ mm} - 0.0500 \text{ mm}}{70.0 \text{ mm}} = \frac{2.0}{7.00 \times 10^4}$.

So, the plates would touch at $x = 0$, so $d = 0.0520$ mm when $x = a$. Find a.

$$d = ma$$

$$a = \frac{d}{m} = \frac{0.0520 \times 10^{-3} \text{ m}}{\frac{2.0}{7.00 \times 10^4}} = 1.8 \text{ m}$$

Find the number of dark bands.

$$m = \frac{2d}{\lambda} = \frac{2(0.0520 \times 10^{-3} \text{ m})}{589 \times 10^{-9} \text{ m}} = 177$$

The distance between adjacent dark bands is

$$\Delta x = \frac{1.8 \text{ m}}{177} = \boxed{1.0 \text{ cm}}$$

37. $\sin\theta = m\frac{\lambda}{W}$

$$W = \frac{m\lambda}{\sin\theta} = \frac{(1)(670 \times 10^{-9} \text{ m})}{\sin 23^\circ} = \boxed{1.7\ \mu\text{m}}$$

38. $\sin\theta = m\dfrac{\lambda}{W}$

$$\theta = \sin^{-1}\frac{m\lambda}{W}$$

$$2\theta = 2\sin^{-1}\frac{m\lambda}{W}$$

$$= 2\sin^{-1}\frac{mv}{Wf}$$

$$= 2\sin^{-1}\frac{(1)\left(343\ \frac{\text{m}}{\text{s}}\right)}{(0.82\ \text{m})(1100\ \text{Hz})}$$

$$= \boxed{45^\circ}$$

39. For small angles, $\tan\theta \sim \sin\theta \sim \theta.$

$$\tan\theta = \frac{\frac{W_{\text{cf}}}{2}}{L} \sim \theta$$

$$2\theta = \frac{2\lambda}{W} = 2\left(\frac{\frac{W_{\text{cf}}}{2}}{L}\right) = \frac{W_{\text{cf}}}{L}$$

$$W = \frac{2\lambda L}{W_{\text{cf}}} = \frac{2(546\times10^{-9}\ \text{m})(1.60\ \text{m})}{0.0250\ \text{m}} = \boxed{69.9\ \mu\text{m}}$$

40. $\sin\theta = m\dfrac{\lambda}{W} = (1)\dfrac{\lambda}{W} = \dfrac{\lambda}{W}$

$$\theta = \sin^{-1}\frac{\lambda}{W}$$

$$y = L\tan\theta$$

$$= L\tan\left(\sin^{-1}\frac{\lambda}{W}\right)$$

$$= (1.60\ \text{m})\tan\left(\sin^{-1}\frac{626\times10^{-9}\ \text{m}}{7.64\times10^{-6}\ \text{m}}\right)$$

$$= \boxed{13.2\ \text{cm}}$$

41. $\sin\theta = m\dfrac{\lambda}{W} = (3)\dfrac{\lambda}{W} = \dfrac{3\lambda}{W}$

$$\theta = \sin^{-1}\frac{3\lambda}{W}$$

$$y = L\tan\theta$$

$$= L\tan\left(\sin^{-1}\frac{3\lambda}{W}\right)$$

$$= (1.60\ \text{m})\tan\left[\sin^{-1}\frac{3(626\times10^{-9}\ \text{m})}{7.64\times10^{-6}\ \text{m}}\right]$$

$$= \boxed{40.6\ \text{cm}}$$

42. **(a)** $\Delta y = y_2 - y_1 = L(\tan\theta_2 - \tan\theta_1)$

Since $\tan\theta \approx \theta$ for small angles and $\theta \approx \sin\theta = \dfrac{m\lambda}{W}$, we have the following:

$$\Delta y \approx L(\theta_2 - \theta_1) \approx L\left(\frac{2\lambda}{W} - \frac{\lambda}{W}\right) = \frac{\lambda L}{W}.$$

So, $W \approx \dfrac{\lambda L}{\Delta y} = \dfrac{(610\times 10^{-9}\text{ m})(2.3\text{ m})}{0.12\text{ m}} = \boxed{12\ \mu\text{m}}$.

(b) Since $\Delta y \approx \dfrac{\lambda L}{W}$, increasing the slit width, W, will $\boxed{\text{decrease}}$ the distance between the first and second dark fringes.

43. The last dark fringes theoretically occur where θ approaches $\pm 90°$.

$$\sin \pm 90° = \pm 1 = m\frac{\lambda}{W}$$

$$m = \pm\frac{W}{\lambda} = \pm\frac{8.00\times 10^{-6}\text{ m}}{553\times 10^{-9}\text{ m}} = \pm 14.5$$

So, there are $\boxed{14}$ dark fringes produced on either side of the central maximum.

44. **(a)** $\sin\theta = m\dfrac{\lambda}{W} = (2)\dfrac{\lambda}{W} = \dfrac{2\lambda}{W}$

$$W = \frac{2\lambda}{\sin\theta}$$

Find θ.

$$y = L\tan\theta$$

$$\theta = \tan^{-1}\frac{y}{L}$$

Substitute.

$$W = \frac{2\lambda}{\sin\left(\tan^{-1}\frac{y}{L}\right)} = \frac{2(632.8\times 10^{-9}\text{ m})}{\sin\left(\tan^{-1}\frac{\frac{0.152\text{ m}}{2}}{1.50\text{ m}}\right)} = \boxed{25.0\ \mu\text{m}}$$

(b) Since the angle a wave diffracts is greater the larger the wavelength of the wave, the distance indicated will be
$\boxed{\text{less than}}$ 15.2 cm (591 nm < 632.8 nm).

45. The approximate (angular) width of the central fringe is given by $2\theta_1 = 2\dfrac{\lambda}{W}$. The (metric) width is given by $2y_1 = 2L\tan\theta_1$.

$$1.60\text{ cm} = 2y_1 = 2L\tan\theta_1 \approx 2L\theta_1 = 2L\left(\frac{\lambda}{W}\right) = \frac{2L\lambda}{W}$$

So, $\dfrac{\lambda}{W} \approx \dfrac{2y_1}{2L} = \dfrac{0.0160\text{ m}}{2(1.00\text{ m})} = 0.00800.$

Find $2y_2$.

$$\begin{aligned}2y_2 &= 2L\tan\theta_2\\ &= 2L\tan\left(\sin^{-1}\frac{2\lambda}{W}\right)\\ &= 2(1.00\text{ m})\tan[\sin^{-1}2(0.00800)]\\ &= \boxed{3.20\text{ cm}}\end{aligned}$$

46. (a) $y = L\tan\theta_{\text{min}}$

$$\begin{aligned}L &= \frac{y}{\tan\theta_{\text{min}}}\\ &= \frac{y}{\tan\left(1.22\frac{\lambda}{D}\right)}\\ &= \frac{0.050\text{ m}}{\tan\frac{1.22(690\times10^{-9}\text{ m})}{12\times10^{-6}\text{ m}}}\\ &= \boxed{71\text{ cm}}\end{aligned}$$

(b) $L = \dfrac{0.050\text{ m}}{\tan\frac{1.22(420\times10^{-9}\text{ m})}{12\times10^{-6}\text{ m}}} = \boxed{1.2\text{ m}}$

47.

$$\begin{aligned}y &= L\tan\theta_{\text{min}}\\ &= L\tan\left(1.22\frac{\lambda}{D}\right)\end{aligned}$$

$$\tan^{-1}\frac{y}{L} = 1.22\frac{\lambda}{D}$$

$$\begin{aligned}D &= \frac{1.22\lambda}{\tan^{-1}\frac{y}{L}}\\ &= \frac{1.22(550\times10^{-9}\text{ m})}{\tan^{-1}\frac{0.050\text{ m}}{160\times10^{3}\text{ m}}}\\ &= \boxed{2.1\text{ m}}\end{aligned}$$

48. $\theta_{\text{min}} = 1.22\dfrac{\lambda}{D}$

$$\begin{aligned}D &= \frac{1.22\lambda}{\theta_{\text{min}}}\\ &= \frac{1.22(550\times10^{-9}\text{ m})}{\left(\frac{2.5}{3600}\right)^{\circ}\left(\frac{2\pi\text{ rad}}{360^{\circ}}\right)}\\ &= \boxed{5.5\text{ cm}}\end{aligned}$$

49.

$$
\begin{aligned}
y &= L\tan\theta_{\text{min}} \\
&= L\tan\left(1.22\frac{\lambda}{D}\right) \\
\tan^{-1}\frac{y}{L} &= \frac{1.22\lambda}{D} \\
D &= \frac{1.22\lambda}{\tan^{-1}\frac{y}{L}} \\
&= \frac{1.22(450\times10^{-9}\text{ m})}{\tan^{-1}\frac{2.0\text{ m}}{27\times10^{3}\text{ m}}} \\
&= \boxed{7.4\text{ mm}}
\end{aligned}
$$

50.

$$
\begin{aligned}
y &= L\tan\theta_{\text{min}} \\
&= L\tan\left(1.22\frac{\lambda}{D}\right) \\
&= (613\times10^{3}\text{ m})\tan\frac{1.22(550\times10^{-9}\text{ m})}{2.4\text{ m}} \\
&= \boxed{17\text{ cm}}
\end{aligned}
$$

51. (a) Assuming small angles, the width is given by $2\theta = 2(1.22)\frac{\lambda}{D}$.

$$2\theta = 2(1.22)\frac{540\times10^{-9}\text{ m}}{110\times10^{-3}\text{ m}} = \boxed{1.2\times10^{-5}\text{ rad}}$$

(b)

$$
\begin{aligned}
2y &= 2L\tan\theta \\
&= 2(640\times10^{-3}\text{ m})\tan\frac{1.2\times10^{-5}}{2} \\
&= \boxed{7.7\ \mu\text{m}}
\end{aligned}
$$

52. (a) $$\theta_{\text{min}} = 1.22\frac{\lambda}{D} = \frac{1.22(550\times10^{-9}\text{ m})}{(6.0\text{ in.})\left(\frac{0.0254\text{ m}}{1\text{ in.}}\right)}\left(\frac{360°}{2\pi\text{ rad}}\right)\left(\frac{1\text{ arc second}}{\frac{1}{3600}°}\right) = \boxed{0.91\text{ arc second}}$$

(b) $$y = L\tan\theta_{\text{min}} = (384,400\text{ km})\tan\frac{1.22(550\times10^{-9}\text{ m})}{(6.0\text{ in.})\left(\frac{0.0254\text{ m}}{1\text{ in.}}\right)} = \boxed{1.7\text{ km}}$$

53. (a) $$\theta_{\text{min}} = 1.22\frac{\lambda}{D} = 1.22\frac{520\times10^{-9}\text{ m}}{0.50\times10^{-3}\text{ m}} = \boxed{1.3\times10^{-3}\text{ rad}}$$

(b) $y = L\tan\theta_{\text{min}}$

$$L = \frac{y}{\tan\theta_{\text{min}}} = \frac{y}{\tan\left(1.22\frac{\lambda}{D}\right)} = \frac{0.15\text{ m}}{\tan\frac{1.22(520\times10^{-9}\text{ m})}{0.50\times10^{-3}\text{ m}}} = \boxed{120\text{ m}}$$

54. $$\sin\theta = m\frac{\lambda}{d} = m\lambda N$$

$$\theta = \sin^{-1}\lambda Nm = \sin^{-1}\left[(655\times10^{-9}\text{ m})(767\text{ cm}^{-1})\left(\frac{100\text{ cm}}{1\text{ m}}\right)m\right]$$

m	1	2	3
θ (°)	2.88	5.77	8.67

55. $$\sin\theta = m\frac{\lambda}{d}$$

$$\sin 14° = (1)\frac{\lambda}{d}$$

$$d = \frac{\lambda}{\sin 14°} = \frac{0.030\times10^{-9}\text{ m}}{\sin 14°} = \boxed{0.12\text{ nm}}$$

56. $$\sin\theta = m\frac{\lambda}{d} = (1)\frac{\lambda}{d} = \frac{\lambda}{d}$$

$$\theta = \sin^{-1}\frac{\lambda}{d} = \sin^{-1}N\lambda$$

$$\theta_{\text{red}} - \theta_{\text{blue}} = \sin^{-1}\left[(2200\text{ cm}^{-1})\left(\frac{100\text{ cm}}{1\text{ m}}\right)(680\times10^{-9}\text{ m})\right] - \sin^{-1}\left[(2200\text{ cm}^{-1})\left(\frac{100\text{ cm}}{1\text{ m}}\right)(410\times10^{-9}\text{ m})\right] = \boxed{3.4°}$$

57. $$\sin\theta = m\frac{\lambda}{d} = (1)\lambda N = \lambda N$$

$$\lambda = \frac{\sin\theta}{N} = \frac{\sin\left(\tan^{-1}\frac{y}{L}\right)}{N} = \frac{\sin\left(\tan^{-1}\frac{0.164\text{ m}}{1.00\text{ m}}\right)}{(355\text{ mm}^{-1})\left(\frac{1000\text{ mm}}{1\text{ m}}\right)} = \boxed{456\text{ nm}}$$

58.
$$\begin{aligned}\sin\theta &= m\frac{\lambda}{d}\\ &= m\lambda N\\ N &= \frac{\sin\theta}{m\lambda}\\ &= \frac{\sin 1.250°}{(1)(587.5\times10^{-9}\text{ m})\left(\frac{100\text{ cm}}{1\text{ m}}\right)}\\ &= \boxed{371.3\text{ cm}^{-1}}\end{aligned}$$

59. (a)
$$\begin{aligned}\sin\theta &= m\frac{\lambda}{d}\\ &= m\lambda N\\ \lambda &= \frac{\sin\theta}{mN}\\ &= \frac{\sin 3.1°}{(2)(560\text{ cm}^{-1})\left(\frac{100\text{ cm}}{1\text{ m}}\right)}\\ &= \boxed{480\text{ nm}}\end{aligned}$$

(b) Since $\sin\theta$ is proportional to the number of lines per centimeter of the grating, a larger number of lines per centimeter results in a larger second-order maximum angle. So, the angle is $\boxed{\text{greater than}}$ 3.1°. ($\sin\theta_1 < \sin\theta_2$ if $\theta_1 < \theta_2$ on $0 < \theta \leq 90°$.)

60. $\sin\theta = \dfrac{m\lambda}{d} = m\lambda N \leq 1$

So, $m = \dfrac{1}{N\lambda_{\text{visible max}}} = \dfrac{1}{(7400\text{ cm}^{-1})\left(\frac{100\text{ cm}}{1\text{ m}}\right)(7.00\times10^{-7}\text{ m})} = 1.9.$

$1.9 < 2,$ so only $\boxed{\text{one}}$ complete visible spectrum will be formed on either side of the central maximum.

61.
$$\begin{aligned}\sin\theta &= m\frac{\lambda}{d}\\ m &= \frac{d\sin\theta}{\lambda}\\ &= \frac{\sin\theta}{N\lambda}\end{aligned}$$

The smallest visible wavelength (400 nm) will result in the highest-order visible maximum.

$$m = \frac{\sin 90°}{(890\text{ mm}^{-1})\left(\frac{1000\text{ mm}}{1\text{ m}}\right)(400\times10^{-9}\text{ m})} = \boxed{2}$$

62. $\sin\theta = m\dfrac{\lambda}{d} = mN\lambda$

$$\lambda = \frac{\sin\theta}{mN} = \frac{\sin 90°}{2(760\text{ mm}^{-1})\left(\frac{1000\text{ mm}}{1\text{ m}}\right)} = \boxed{660\text{ nm}}$$

63. $\sin\theta = m\dfrac{\lambda}{D}$

$$\theta = \sin^{-1}\frac{m\lambda}{D} = \sin^{-1} m\lambda N$$

m	1	2	3	4	5
θ_1 (°)	11	23	35.5	51	75
θ_2 (°)	16	34.5	58	—	—

The sequence of colors is:
violet, orange, violet, orange, violet, violet, orange, violet.

64. (a) $\lambda_1 = 420\text{ nm}$

$\lambda_2 = 630\text{ nm}$

$$m\lambda = \frac{\sin\theta}{N}$$

$$m_{1\,\text{max}} = \frac{\sin 90°}{(420\times10^{-9}\text{ m})(450\text{ mm}^{-1})\left(\frac{1000\text{ mm}}{1\text{ m}}\right)} = 5$$

$$m_{2\,\text{max}} = \frac{\sin 90°}{(630\times10^{-9}\text{ m})(450\text{ mm}^{-1})\left(\frac{1000\text{ mm}}{1\text{ m}}\right)} = 3$$

m	1	2	3	4	5
$m\lambda_1$ (nm)	420	840	1260	1680	2100
$m\lambda_2$ (nm)	630	1260	1890	—	—

The orders of the overlapping lines are 3 for 420-nm light and 2 for 630-nm light.

(b) $\dfrac{\sin\theta}{N} = m\lambda$

$$\theta = \sin^{-1} mN\lambda = \sin^{-1}(3)(450\text{ mm}^{-1})\left(\frac{1000\text{ mm}}{1\text{ m}}\right)(420\times10^{-9}\text{ m}) = \boxed{35°}$$

65. (a) For $m > 1, \sin\theta = \dfrac{m\lambda}{d} > 1$, which is impossible because $\sin\theta \le 1$.

(b) $\dfrac{2\lambda}{d} > 1$

$d < 2\lambda$

$d < 2(420\text{ nm})$

$\boxed{d < 840\text{ nm}}$

66. Find the angle of the first-order maxima.

$$y = L\tan\theta$$
$$\theta = \tan^{-1}\frac{y}{L}$$
$$= \tan^{-1}\frac{\frac{1.53\text{ m}}{2}}{2.00\text{ m}}$$

Find the ratio $\frac{\lambda}{d}$.

$$m\lambda = d\sin\theta$$
$$\frac{(1)\lambda}{d} = \sin\theta$$
$$= \sin\tan^{-1}\frac{\frac{1.53\text{ m}}{2}}{2.00\text{ m}}$$

Find the distance between the second-order maxima.

$$2y = 2L\tan\theta$$
$$= 2L\tan\sin^{-1}\frac{2\lambda}{d}$$
$$= 2(2.00\text{ m})\tan\sin^{-1}\left(2\sin\tan^{-1}\frac{\frac{1.53\text{ m}}{2}}{2.00\text{ m}}\right)$$
$$= \boxed{4.09\text{ m}}$$

67. incident path difference $= d\sin\frac{\phi}{2}$

diffracted path difference $= d\sin\frac{\phi}{2}$

total path difference $= 2d\sin\frac{\phi}{2}$

For constructive interference, the path difference must be an integer multiple of the wavelength. So,

$m\lambda = 2d\sin\frac{\phi}{2}$, where $m = 0, \pm1, \pm2, \ldots$.

68.

θ (°)	θ (rad)	$\sin\theta$	$\tan\theta$	$\frac{\sin\theta}{\tan\theta}$
0.0100	0.000175	0.000175	0.000175	1.00
1.00°	0.0175	0.0175	0.0175	1.00
5.00°	0.0873	0.0872	0.0875	0.996
10.0°	0.175	0.174	0.176	0.985
20.0°	0.349	0.342	0.364	0.940
30.0°	0.524	0.500	0.577	0.866
40.0°	0.698	0.643	0.839	0.766

69. $y = L\tan\theta_{\text{min}} = L\tan\left(1.22\dfrac{\lambda}{D}\right) = (0.25\text{ m})\tan\dfrac{1.22(540\times10^{-9}\text{ m})}{0.0030\text{ m}} = \boxed{55\ \mu\text{m}}$

70. $y = L\tan\theta$

$$\begin{aligned} L &= \frac{y}{\tan\theta} \\ &= \frac{y}{\tan\frac{1.22\lambda}{D}} \\ &= \frac{1.32\text{ m}}{\tan\frac{1.22(555\times10^{-9}\text{ m})}{0.0125\text{ m}}} \\ &= \boxed{24.4\text{ km}} \end{aligned}$$

71. (a) phase change for $R_2 = \dfrac{1}{2}$

phase change for $R_4 = \dfrac{2nt}{\lambda_{\text{vacuum}}} + \dfrac{1}{2}$

difference of phase changes $= \dfrac{2nt}{\lambda_{\text{vacuum}}} + \dfrac{1}{2} - \dfrac{1}{2} = \dfrac{2nt}{\lambda_{\text{vacuum}}} = \dfrac{1}{2}$ for destructive interference

$t = \dfrac{\lambda}{4n} = \dfrac{517\text{ nm}}{4(1.38)} = \boxed{93.7\text{ nm}}$

(b) phase change for $R_1 = \dfrac{nt}{\lambda_{\text{vacuum}}}$

phase change for $R_3 = \dfrac{3nt}{\lambda_{\text{vacuum}}} + \dfrac{1}{2}$

difference of phase changes $= \dfrac{3nt}{\lambda_{\text{vacuum}}} + \dfrac{1}{2} - \dfrac{nt}{\lambda_{\text{vacuum}}}$

$= \dfrac{2nt}{\lambda_{\text{vacuum}}} + \dfrac{1}{2}$

$= 1$ for constructive interference

$t = \dfrac{\lambda}{4n} = \boxed{93.7\text{ nm}}$

72. (a) <u>constructive interference</u>

$$\frac{2nt}{\lambda_{\text{vacuum}}} - \frac{1}{2} = m$$

$$2nt = \left(m + \frac{1}{2}\right)\lambda_{\text{vacuum}}$$

$$t = \frac{\left(m + \frac{1}{2}\right)\lambda_{\text{vacuum}}}{2n}$$

destructive interference

$$\frac{2nt}{\lambda_{\text{vacuum}}} = m$$

$$t = \frac{m\lambda_{\text{vacuum}}}{2n}$$

Determine the minimum thickness of a soap bubble that will allow at least one maximum (minimum).

$$t_{\text{c}} = \frac{\left(0+\frac{1}{2}\right)\lambda_{\text{vacuum}}}{2n} = \frac{\lambda_{\text{vacuum}}}{4n} = \frac{575 \text{ nm}}{4(1.33)} = 108 \text{ nm}$$

$$t_{\text{d}} = \frac{(1)\lambda_{\text{vacuum}}}{2n} = \frac{575 \text{ nm}}{2(1.33)} = 216 \text{ nm}$$

An interference maximum may be generated by a film that is too thin to generate an interference minimum.

(b) $t_{\text{c,min}} = 108 \text{ nm}$

$t_{\text{d,min}} = 216 \text{ nm}$

possible thicknesses: $108 \text{ nm} \le t < 216 \text{ nm}$

73. phase change at air-oil interface: $\frac{1}{2}$

phase change at oil-water interface: $\frac{2nt}{\lambda_{\text{vacuum}}} + \frac{1}{2}$

difference of phase changes $= \frac{2nt}{\lambda_{\text{vacuum}}} + \frac{1}{2} - \frac{1}{2} = \frac{2nt}{\lambda_{\text{vacuum}}}$

constructive interference

$$\frac{2nt}{\lambda_{\text{vacuum}}} = m$$

$$t = \frac{m\lambda_{\text{vacuum}}}{2n}$$

$$\frac{\lambda_{\text{red}}}{\lambda_{\text{blue}}} = \frac{687 \text{ nm}}{458 \text{ nm}} = 1.50$$

$$3.00\lambda_{\text{blue}} = 2.00\lambda_{\text{red}}$$

$$t = \frac{3.00\lambda_{\text{blue}}}{2n} = \frac{3.00(458 \text{ nm})}{2(1.30)} = \boxed{528 \text{ nm}}$$

74.

$$\frac{2nt}{\lambda_{\text{vac}}'} - \frac{1}{2} = m$$

$$\frac{2nt}{\lambda_{\text{vac}}'} = m + \frac{1}{2}$$

$$\lambda_{\text{vac}}' = \frac{2nt}{m+\frac{1}{2}}$$

$$\lambda_{\text{vac}} \cos\theta_{\text{refr}} = \frac{2nt}{m+\frac{1}{2}}$$

Find θ_{refr}.

$$n_{\text{air}} \sin\theta_{\text{i}} = n\sin\theta_{\text{refr}}$$
$$\theta_{\text{refr}} = \sin^{-1}\frac{n_{\text{air}}\sin 45°}{n}$$
$$\lambda_{\text{vac}} = \frac{2nt}{\left(m+\frac{1}{2}\right)\cos\sin^{-1}\frac{n_{\text{air}}\sin 45°}{n}}$$
$$= \frac{2(1.33)(800.0\text{ nm})}{\left(m+\frac{1}{2}\right)\cos\sin^{-1}\frac{1.00\sin 45°}{1.33}}$$

m	0	1	2	3	4	5	6
λ (nm)	5000	1700	1000	720	560	460	390

Since the visible range is $400\text{ nm} \le \lambda \le 700\text{ nm}$, the visible wavelengths that are constructively reflected are $\boxed{460\text{ nm and }560\text{ nm}}$.

75. $m\lambda = d\sin\theta = \dfrac{\sin\theta}{N}$

$$\theta = \sin^{-1} m\lambda N = \sin^{-1}(1)\lambda N = \sin^{-1}\lambda N$$
$$y = L\tan\theta$$
$$y_2 - y_1 = L(\tan\theta_2 - \tan\theta_1)$$
$$= L[\tan(\sin^{-1}\lambda_2 N) - \tan(\sin^{-1}\lambda_1 N)]$$
$$= (1.80\text{ m})\left\{\tan\left[\sin^{-1}(589.59\times10^{-9}\text{ m})\left(474\ \frac{\text{lines}}{\text{cm}}\right)\left(\frac{100\text{ cm}}{1\text{ m}}\right)\right] - \tan\left[\sin^{-1}(588.99\times10^{-9}\text{ m})\left(474\ \frac{\text{lines}}{\text{cm}}\right)\left(\frac{100\text{ cm}}{1\text{ m}}\right)\right]\right\}$$
$$= \boxed{50\ \mu\text{m}}$$

76. $\lambda' = \lambda\cos\theta_{\text{refr}}$

$$\theta_{\text{refr}} = \cos^{-1}\frac{\lambda'}{\lambda}$$
$$= \cos^{-1}\frac{420\text{ nm}}{560\text{ nm}}$$
$$= 41°$$
$$n_{\text{air}}\sin\theta_{\text{i}} = n\sin\theta_{\text{refr}}$$
$$\theta_{\text{i}} = \sin^{-1}\frac{n\sin\theta_{\text{refr}}}{n_{\text{air}}}$$
$$= \sin^{-1}\frac{(1.33)\sin 41.4°}{1.00}$$
$$= \boxed{62°}$$

77. phase change at air-oil interface: $\frac{1}{2}$

phase change at oil-water interface: $\frac{2nt}{\lambda_{\text{vacuum}}}$

difference of phase changes $= \frac{2nt}{\lambda_{\text{vacuum}}} - \frac{1}{2}$

for destructive interference:

$$\frac{2nt}{\lambda_{\text{vacuum}}} - \frac{1}{2} = m - \frac{1}{2},\ m = 0, 1, 2, \ldots$$

$$t = \frac{m\lambda_{\text{vacuum}}}{2n}$$

$$t = \frac{m_{\text{red}}\lambda_{\text{red}}}{2n_{\text{oil}}} = \frac{m_{\text{blue}}\lambda_{\text{blue}}}{2n_{\text{oil}}}$$

$$\frac{m_{\text{red}}}{m_{\text{blue}}} = \frac{\lambda_{\text{blue}}}{\lambda_{\text{red}}} = \frac{458\text{ nm}}{687\text{ nm}} = \frac{2}{3}$$

Using $m_{\text{blue}} = 3$, we have

$$t = \frac{3\lambda_{\text{blue}}}{2n_{\text{oil}}} = \frac{3(458\text{ nm})}{2(1.40)} = \boxed{491\text{ nm}}$$

78. (a) A ray of light reflected from the lower surface of the lens has no phase change. A ray of light reflected from the top surface of the flat piece of glass undergoes a half-wavelength phase change. Near the center of the lens the path difference for these two rays goes to zero. As a result they are half a wavelength out of phase, resulting in destructive interference. Hence, the center of the pattern is a dark spot.

(b)

$$\frac{2d}{\lambda} = m = 10$$

$$d = 5\lambda$$

Using the same geometry as in the solution of Problem 30,

$$R^2 = r^2 + (R - 5\lambda)^2$$

$$r = \sqrt{R^2 - (R - 5\lambda)^2} = \sqrt{10R\lambda - 25\lambda^2} \approx \sqrt{10R\lambda}$$

$$= \sqrt{10(26.1\text{ m})(589 \times 10^{-9}\text{ m})}$$

$$= \boxed{1.24\text{ cm}}$$

79. (a)

$$\sin\theta = m\frac{\lambda}{W} = (2)\frac{\lambda}{W}$$

$$\lambda = \frac{W\sin\theta}{2} = \frac{W}{2}\sin\left(\tan^{-1}\frac{y}{L}\right)$$

$$= \frac{11.2\ \mu\text{m}}{2}\sin\left[\tan^{-1}\frac{\left(\frac{0.152\text{ m}}{2}\right)}{1.25\text{ m}}\right]$$

$$= \boxed{340\text{ nm}}$$

(b) Since $\sin\theta$ is inversely proportional to *W*, decreasing *W* increases $\sin\theta$, resulting in a distance that is greater than 15.2 cm.

80. **(a)** $I = I_0 \left[\frac{\sin(N\phi/2)}{\sin(\phi/2)} \right]^2$ where $\phi = (2\pi d/\lambda)\sin\theta$

In the limit $\theta \to 0$, $\sin\theta \to 0$, $\phi \to 0$.

For small ϕ, $\sin(N\phi/2) = N\phi/2$ and $\sin(\phi/2) = \phi/2$.

Thus for small ϕ

$$I = I_0 \left[\frac{(N\phi/2)}{\phi/2} \right]^2 = N^2 I_0$$

(b) $I = 0$ when $\sin(N\phi/2) = 0$. This first occurs when

$$N\phi/2 = \pm\pi$$
$$\phi = \pm\frac{2\pi}{N}$$

(c) Since ϕ is inversely proportional to *N*, increasing *N* decreases ϕ, which results in a decrease in $\sin\theta$ and θ at which zero intensity occurs.

$$\sin\theta = \frac{\phi\lambda}{2\pi d}$$

Therefore the pattern becomes $\boxed{\text{narrower}}$.

81. bottom of top plate

$$\text{phase change } = \frac{2nt}{\lambda_{\text{vacuum}}}$$

top of bottom plate

$$\text{phase change } = \frac{2nt}{\lambda_{\text{vacuum}}} + \frac{2d}{\lambda_{\text{vacuum}}} + \frac{1}{2}$$

$$\text{difference in phase changes} = \frac{2nt}{\lambda_{\text{vacuum}}} + \frac{2d}{\lambda_{\text{vacuum}}} + \frac{1}{2} - \frac{2nt}{\lambda_{\text{vacuum}}}$$
$$= \frac{2d}{\lambda_{\text{vacuum}}} + \frac{1}{2}$$

$\frac{2d}{\lambda_{\text{vacuum}}} + \frac{1}{2}$ must be equal to an integer for constructive interference. Since 0 would result in either a negative *d* or λ, 1 is the lowest such integer.

$$\frac{2d}{\lambda_{\text{vacuum}}} + \frac{1}{2} = 1, 2, 3, \ldots$$
$$\frac{2d}{\lambda_{\text{vacuum}}} - \frac{1}{2} = 0, 1, 2, \ldots$$

So, $\boxed{\frac{2d}{\lambda_{\text{vacuum}}} - \frac{1}{2} = m \text{ where } m = 0, 1, 2, \ldots}$

82. <u>curved surface</u>

$$\text{phase change} = \frac{2n_{\text{glass}}t}{\lambda}$$

<u>plate</u>

$$\text{phase change} = \frac{2n_{\text{glass}}t}{\lambda} + \frac{2d}{\lambda} + \frac{1}{2}$$

t is the thickness of the curved glass and d is the distance from the bottom of the curved glass to the top of the plate.

$$\begin{aligned}\text{difference in phase changes} &= \frac{2n_{\text{glass}}t}{\lambda} + \frac{2d}{\lambda} + \frac{1}{2} - \frac{2n_{\text{glass}}t}{\lambda}\\ &= \frac{2d}{\lambda} + \frac{1}{2}\end{aligned}$$

Destructive interference (dark rings) occurs when

$$\begin{aligned}\frac{2d}{\lambda} + \frac{1}{2} &= \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots\\ \frac{2d}{\lambda} &= 0, 1, 2, \dots\\ &= n \text{ where } n = 0, 1, 2, \dots\end{aligned}$$

So, $d_n = \frac{n\lambda}{2}$.

Relate r to d.

Let the distance from the center of the curved glass to the n^{th} dark ring be one leg of a right triangle with the radius of curvature as the hypotenuse. The distance from the center of curvature to the point of contact of the two pieces of glass equals the radius of curvature. This distance minus d_n is the other leg of the triangle. Use the Pythagorean theorem to solve for r_n.

$$\begin{aligned}R^2 &= r_n^2 + (R - d_n)^2\\ r_n^2 &= R^2 - R^2 + 2Rd_n - d_n^2\\ r_n &= \sqrt{2Rd_n - d_n^2}\\ &= \sqrt{2R\left(\frac{n\lambda}{2}\right) - \left(\frac{n\lambda}{2}\right)^2}\\ &= \sqrt{n\lambda R - \frac{n^2\lambda^2}{4}}\end{aligned}$$

83. diameter $= 2y = 2L\tan\theta_{\text{min}}$

$$\begin{aligned}\sin\theta_{\text{min}} &= 1.22\frac{\lambda}{D}\\ \theta_{\text{min}} &= \sin^{-1}\frac{1.22\lambda}{D}\\ &= \sin^{-1}\frac{1.22\lambda_{\text{vacuum}}}{nD}\end{aligned}$$

$$\begin{aligned}\text{diameter} &= 2L\tan\left(\sin^{-1}\frac{1.22\lambda_{\text{vacuum}}}{nD}\right)\\ &= 2(0.0254\text{ m})\tan\left[\sin^{-1}\frac{1.22(550\times10^{-9}\text{ m})}{(1.336)(0.00300\text{ m})}\right]\\ &= \boxed{8.5\ \mu\text{m}}\end{aligned}$$