Chapter 29

Relativity

Answers to Even-numbered Conceptual Questions

2. The second postulate of relativity specifically refers to the speed of light *in a vacuum*. The speed of light in other mediums will always be less than the speed of light in a vacuum.

4. Recall that the proper time is the time between two events that occur at the same location, as seen by a given observer. **(a)** Proper time. **(b)** Proper time. **(c)** Proper time. **(d)** Dilated time.

6. The clock still runs slow. The direction of motion does not matter in time dilation, only the relative *speed* between the observer and the clock.

8. If the speed of light were only 35 mi/h, we would experience relativistic effects everyday. For example, a commuter would age more slowly than a person who works at home; a moving car would be noticeably shorter and distorted; you wouldn't be able to drive faster than 35 mi/h, no matter how powerful your car was and no matter how long you held the "pedal to the metal."

10. The effects of length contraction – in fact, all relativistic effects – would be less than they are now if the speed of light were larger. In fact, in the limit of an infinite speed of light, there would be no relativistic effects at all.

12. No, in both cases. The theory of relativity imposes no limits on the energy or momentum an object can have.

14. In principal, the apple has a greater mass when near the top of its fall. The reason is that there is more gravitational potential energy in the Earth-apple system when the apple is at a greater height, and this increased energy is equivalent to an increased mass via the relation $E = mc^2$. See Conceptual Checkpoint 29-3 for a similar situation.

16. The warm tea has the greater mass. We can see this by noting that the warm tea has more internal energy than the cool tea. If the difference in energy is E, the corresponding difference in mass is E/c^2. See Conceptual Checkpoint 29-3 for a similar situation.

Solutions to Problems

1. Since the speed of light in vacuum is c, Isaac measures the speed of Albert's light beam as $\boxed{c}$.

2. $$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{5.0\text{ s}}{\sqrt{1-\left(\frac{0.84c}{c}\right)^2}} = \boxed{9.2\text{ s}}$$

3.
$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$1-\frac{v^2}{c^2} = \left(\frac{\Delta t_0}{\Delta t}\right)^2$$
$$v^2 = c^2\left[1-\left(\frac{\Delta t_0}{\Delta t}\right)^2\right]$$
$$v = c\sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2}$$
$$= c\sqrt{1-\left(\frac{7.5\text{ s}}{12\text{ s}}\right)^2}$$
$$= \boxed{0.78c}$$

4.
$$v = \frac{d}{\Delta t}$$
$$\Delta t = \frac{d}{v}$$
$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$\Delta t_0 = \Delta t\sqrt{1-\frac{v^2}{c^2}}$$
$$\Delta t_0 = \frac{d}{v}\sqrt{1-\frac{v^2}{c^2}}$$
$$= \frac{25.3\text{ ly}}{0.9995c}\sqrt{1-\left(\frac{0.9995c}{c}\right)^2}$$
$$= (0.800\text{ y})\left(\frac{12\text{ months}}{1\text{ y}}\right)$$
$$= \boxed{9.60\text{ months}}$$

5.
$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$1-\frac{v^2}{c^2} = \left(\frac{\Delta t_0}{\Delta t}\right)^2$$
$$v = c\sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2}$$
$$= c\sqrt{1-\left(\frac{1.0000\text{ s}}{60.000\text{ s}}\right)^2}$$
$$= \boxed{0.99986110c}$$

6. $$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\begin{aligned}\Delta t_0 &= \Delta t\sqrt{1-\frac{v^2}{c^2}}\\ &= (15.44\text{ s})\sqrt{1-\left(\frac{0.7705c}{c}\right)^2}\\ &= \boxed{9.842\text{ s}}\end{aligned}$$

7. $$\begin{aligned}d_{av} &= v\Delta t\\ &= v\frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}\\ &= \frac{0.750\left(3.00\times 10^8\ \frac{\text{m}}{\text{s}}\right)\left(2.20\times 10^{-6}\ \text{s}\right)}{\sqrt{1-\left(\frac{0.750c}{c}\right)^2}}\\ &= \boxed{748\text{ m}}\end{aligned}$$

8. (a) $$\begin{aligned}\Delta t &= \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}\\ &= \frac{2.6\times 10^{-8}\ \text{s}}{\sqrt{1-\left(\frac{0.99c}{c}\right)^2}}\\ &= \boxed{2\times 10^{-7}\ \text{s}}\end{aligned}$$

(b) $$\begin{aligned}d_{\text{av}} &= v\Delta t\\ &= v\frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}\\ &= \frac{0.99\left(3.00\times 10^8\ \frac{\text{m}}{\text{s}}\right)\left(2.6\times 10^{-8}\ \text{s}\right)}{\sqrt{1-\left(\frac{0.99c}{c}\right)^2}}\\ &= \boxed{50\text{ m}}\end{aligned}$$

(c) $$\begin{aligned}d_{\text{av}} &= v\Delta t\\ &= 0.99\left(3.00\times 10^8\ \frac{\text{m}}{\text{s}}\right)(2.6\times 10^{-8}\ \text{s})\\ &= \boxed{7.7\text{ m}}\end{aligned}$$

9. $$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$1-\frac{v^2}{c^2} = \left(\frac{\Delta t_0}{\Delta t}\right)^2$$

$$v = c\sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2}$$

$$= c\sqrt{1-\left(\frac{0.15\text{ ns}}{0.25\text{ ns}}\right)^2}$$

$$= \boxed{0.80c}$$

10. (a) $\boxed{\text{Yes}}$, because of time dilation.

(b) $$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= \frac{1\text{ y}}{\sqrt{1-\left(\frac{0.99c}{c}\right)^2}}$$

$$= \boxed{7\text{ y}}$$

(c) $22\text{ y} + 1\text{ y} = 23\text{ y}$

$17\text{ y} + 7\text{ y} = 24\text{ y}$

$\boxed{\text{You are 23 years old and your sister is 24.}}$

11. $$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\frac{2\pi}{\omega} = \frac{\frac{2\pi}{\omega_0}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\omega = \omega_0\sqrt{1-\frac{v^2}{c^2}}$$

$$= \left(0.22\ \frac{\text{rad}}{\text{s}}\right)\sqrt{1-\left(\frac{0.62c}{c}\right)^2}$$

$$= \boxed{0.17\text{ rad/s}}$$

12. $$\Delta t_1 = \frac{\Delta t_0}{\sqrt{1-\frac{v_1^2}{c^2}}}$$

$$\Delta t_0 = \Delta t_1\sqrt{1-\frac{v_1^2}{c^2}}$$

$$\begin{aligned}\Delta t_2 &= \frac{\Delta t_0}{\sqrt{1-\frac{v_2^2}{c^2}}}\\ &= \frac{\Delta t_1\sqrt{1-\frac{v_1^2}{c^2}}}{\sqrt{1-\frac{v_2^2}{c^2}}}\\ &= \Delta t_1\sqrt{\frac{1-\frac{v_1^2}{c^2}}{1-\frac{v_2^2}{c^2}}}\\ &= (5.0\text{ min})\sqrt{\frac{1-\left(\frac{0.95c}{c}\right)^2}{1-\left(\frac{0.80c}{c}\right)^2}}\\ &= \boxed{3\text{ min}}\end{aligned}$$

13. (a) Since the time between each heartbeat will appear longer to the Earth-based observer, the measured heart rate will be $\boxed{\text{less than}}$ 72 beats per minute.

(b) $$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\begin{aligned}\frac{1\text{ beat}}{\Delta t} &= \frac{\sqrt{1-\frac{v^2}{c^2}}}{\Delta t_0}(72\text{ beats})\\ &= \frac{\sqrt{1-\left(\frac{0.65c}{c}\right)^2}}{1\text{ min}}(72\text{ beats})\\ &= \boxed{55\text{ beats/min}}\end{aligned}$$

14. $$\text{number of days of growth on Earth} = \frac{2.0\text{ in}}{0.30\ \frac{\text{in}}{\text{d}}}$$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Delta t_0 = \Delta t\sqrt{1-\frac{v^2}{c^2}}$$

$$\begin{aligned}\left(0.30\ \frac{\text{in}}{\text{d}}\right)\Delta t_0 &= \frac{2.0\text{ in}}{0.30\ \frac{\text{in}}{\text{d}}}\sqrt{1-\left(\frac{0.94c}{c}\right)^2}\left(0.30\ \frac{\text{in}}{\text{d}}\right)\\ &= \boxed{0.68\text{ in}}\end{aligned}$$

15. $\Delta t = 30.0\text{ d}$

$$\Delta t_0 = \Delta t\sqrt{1-\frac{v^2}{c^2}}$$

$$\begin{aligned}\Delta t_{\text{Earth-ship}} &= \Delta t - \Delta t_0\\ &= \Delta t - \Delta t\sqrt{1-\frac{v^2}{c^2}}\\ &= \Delta t\left(1-\sqrt{1-\frac{v^2}{c^2}}\right)\\ &= (30.0\text{ d})\left[1-\sqrt{1-\left(\frac{8250\ \frac{\text{m}}{\text{s}}}{3.00\times 10^8\ \frac{\text{m}}{\text{s}}}\right)^2}\,\right]\left(\frac{86,400\text{ s}}{1\text{ d}}\right)\\ &= \boxed{0.980\text{ ms}}\end{aligned}$$

16.

$$\begin{aligned}\Delta t &= \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}\\ \frac{d}{v} &= \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}\\ \frac{d^2}{v^2} &= \frac{{\Delta t_0}^2}{1-\frac{v^2}{c^2}}\\ \frac{d^2}{v^2}\left(1-\frac{v^2}{c^2}\right) &= {\Delta t_0}^2\\ \frac{1}{v^2}-\frac{1}{c^2} &= \frac{{\Delta t_0}^2}{d^2}\\ \frac{1}{v^2} &= \frac{{\Delta t_0}^2}{d^2}+\frac{1}{c^2}\\ &= \frac{1}{c^2}\left(\frac{c^2{\Delta t_0}^2}{d^2}+1\right)\\ v &= \frac{c}{\sqrt{1+\left(\frac{c\Delta t_0}{d}\right)^2}}\\ &= \frac{c}{\sqrt{1+\left[\frac{\left(3.00\times 10^8\ \frac{\text{m}}{\text{s}}\right)(1.00\text{ s})}{424,000\times 10^3\text{ m}}\right]^2}}\\ &= \boxed{0.8163c}\end{aligned}$$

17. $\Delta t = \dfrac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}$

$= \dfrac{2\text{ h}}{\sqrt{1-\left(\frac{0.825c}{c}\right)^2}}$

$= 3\text{ h }32\text{ min}$

The Endor clocks read $\boxed{\text{4:32 P.M.}}$

18. (a) According to the ground-based clocks, the flight will take $\boxed{\text{slightly more}}$ than 2.00 hours because of time dilation.

(b) $\Delta t \approx \Delta t_0\left(1+\dfrac{v^2}{2c^2}\right) = \Delta t_0 + \dfrac{v^2}{2c^2}\Delta t_0$

$$\Delta t - \Delta t_0 \approx \frac{v^2 \Delta t_0}{2c^2} = \frac{\left(212\ \frac{\text{m}}{\text{s}}\right)^2 (2.00\text{ h})}{2\left(3.00\times 10^8\ \frac{\text{m}}{\text{s}}\right)^2}\left(\frac{3600\text{ s}}{1\text{ h}}\right) = \boxed{1.80\text{ ns}}$$

19. $L = L_0\sqrt{1-\dfrac{v^2}{c^2}}$

$\dfrac{L^2}{L_0^2} = 1-\dfrac{v^2}{c^2}$

$v = c\sqrt{1-\dfrac{L^2}{L_0^2}}$

$= c\sqrt{1-\left(\dfrac{150\text{ m}}{220\text{ m}}\right)^2}$

$= \boxed{0.73c}$

20. $L = L_0\sqrt{1-\dfrac{v^2}{c^2}} = (1.89\text{ m})\sqrt{1-\left(\dfrac{20.0\ \frac{\text{mi}}{\text{h}}}{25.0\ \frac{\text{mi}}{\text{h}}}\right)^2} = \boxed{1.13\text{ m}}$

21. $L = L_0\sqrt{1-\dfrac{v^2}{c^2}}$

$\dfrac{L^2}{L_0^2} = 1-\dfrac{v^2}{c^2}$

$v = c\sqrt{1-\dfrac{L^2}{L_0^2}}$

$= c\sqrt{1-\left(\dfrac{80.5\text{ cm}}{124\text{ cm}}\right)^2}$

$= \boxed{0.761c}$

22. $L = L_0\sqrt{1-\frac{v^2}{c^2}} = (64\text{ m})\sqrt{1-\left(\frac{0.65c}{c}\right)^2} = \boxed{49\text{ m}}$

23. **(a)** $L = L_0\sqrt{1-\frac{v^2}{c^2}} = (0.50\text{ m})\sqrt{1-\left(\frac{0.88c}{c}\right)^2} = 0.24\text{ m}$

The dimensions are $\boxed{0.24\text{ m}\times 0.50\text{ m}\times 0.50\text{ m}}$.

(b) $V = L_0{}^2 L = L_0{}^3\sqrt{1-\frac{v^2}{c^2}} = (0.50\text{ m})^3\sqrt{1-\left(\frac{0.88c}{c}\right)^2} = \boxed{0.059\text{ m}^3}$

24. **(a)**

$$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$

$$\frac{L^2}{L_0{}^2} = 1-\frac{v^2}{c^2}$$

$$v = c\sqrt{1-\frac{L^2}{L_0{}^2}}$$

$$= c\sqrt{1-\left(\frac{4.0\text{ m}}{5.0\text{ m}}\right)^2}$$

$$= \boxed{0.60c}$$

Your car would have to be moving at more than $\boxed{0.60c}$.

(b) $L = L_0\sqrt{1-\frac{v^2}{c^2}}$

$$= (4.0\text{ m})\sqrt{1-\left(\frac{0.60c}{c}\right)^2}$$

$$= \boxed{3.2\text{ m}}$$

25. Outgoing trip

$$L_1 = L_0\sqrt{1-\frac{v_1^2}{c^2}}$$

$$L_0 = \frac{L_1}{\sqrt{1-\frac{v_1^2}{c^2}}}$$

Return trip

$$\begin{aligned}L_2 &= L_0\sqrt{1-\frac{v_2^2}{c^2}}\\ &= \frac{L_1}{\sqrt{1-\frac{v_1^2}{c^2}}}\sqrt{1-\frac{v_2^2}{c^2}}\\ &= L_1\sqrt{\frac{1-\frac{v_2^2}{c^2}}{1-\frac{v_1^2}{c^2}}}\\ &= (7.5\text{ ly})\sqrt{\frac{1-\left(\frac{0.88c}{c}\right)^2}{1-\left(\frac{0.45c}{c}\right)^2}}\\ &= \boxed{4.0\text{ ly}}\end{aligned}$$

26. (a) Due to length contraction, the lab traveled a distance less than 3.50 cm.

(b) $v = \frac{L_0}{\Delta t} = \frac{0.0350\text{ m}}{0.200\times10^{-9}\text{ s}} = \boxed{1.75\times10^8\text{ m/s}}$

(c)

$$\begin{aligned}L &= L_0\sqrt{1-\frac{v^2}{c^2}}\\ &= (0.0350\text{ m})\sqrt{1-\left(\frac{1.75\times10^8\ \frac{\text{m}}{\text{s}}}{3.00\times10^8\ \frac{\text{m}}{\text{s}}}\right)^2}\\ &= \boxed{2.84\text{ cm}}\end{aligned}$$

27. (a) You and your friend are moving with the same relative speed to each other. So, from your point of view, your friend's ship is also 120 m.

(b) In the rest frame of your friend, he measured his ship to be 150 m. Since the ships are identical, you measure your ship in your rest frame as 150 m.

(c) $L = L_0\sqrt{1-\frac{v^2}{c^2}}$

$$\frac{L^2}{L_0{}^2} = 1-\frac{v^2}{c^2}$$

$$v = c\sqrt{1-\frac{L^2}{L_0{}^2}}$$

$$= c\sqrt{1-\left(\frac{120\text{ m}}{150\text{ m}}\right)^2}$$

$$= \boxed{0.60c}$$

28. $\tan\theta = \frac{y_0}{x} = \frac{\text{height}}{\text{base}}$

$$\theta = \tan^{-1}\frac{y}{x}$$

$$= \tan^{-1}\frac{y}{x_0\sqrt{1-\frac{v^2}{c^2}}}$$

$$= \tan^{-1}\frac{4.0\text{ m}}{(3.0\text{ m})\sqrt{1-\left(\frac{0.90c}{c}\right)^2}}$$

$$= \boxed{72°}$$

29. (a)

$$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$

$$\left(\frac{L}{L_0}\right)^2 = 1-\frac{v^2}{c^2}$$

$$v = c\sqrt{1-\left(\frac{L}{L_0}\right)^2}$$

$$v = c\sqrt{1-\left(\frac{L_1}{L_0}\right)^2}$$

$$2v = c\sqrt{1-\left(\frac{L_2}{L_0}\right)^2}$$

$$2c\sqrt{1-\left(\frac{L_1}{L_0}\right)^2} = c\sqrt{1-\left(\frac{L_2}{L_0}\right)^2}$$

$$4\left[1-\left(\frac{L_1}{L_0}\right)^2\right] = 1-\left(\frac{L_2}{L_0}\right)^2$$

$$3 = 4\left(\frac{L_1}{L_0}\right)^2 - \left(\frac{L_2}{L_0}\right)^2$$

$$3 = \frac{1}{L_0^2}(4L_1^2 - L_2^2)$$

$$L_0 = \sqrt{\frac{4L_1^2 - L_2^2}{3}}$$

$$v = c\sqrt{1-\frac{L_1^2}{\frac{4L_1^2-L_2^2}{3}}} = c\sqrt{1-\frac{3(8.00\text{ m})^2}{4(8.00\text{ m})^2-(5.00\text{ m})^2}} = \boxed{0.411c}$$

(b) $$L_0 = \sqrt{\frac{4L_1^2 - L_2^2}{3}} = \sqrt{\frac{4(8.00\text{ m})^2-(5.00\text{ m})^2}{3}} = \boxed{8.77\text{ m}}$$

30. (a) The length of the *Picard* is contracted with respect to the observer on the *LaForge*. So, since this observer measures both ships and finds that they have the same length, the *Picard* must be longer. The length of the *LaForge* is contracted with respect to the observer on the *Picard*. So, since the *Picard* is longer and the *LaForge* is contracted, this observer will find that (i) the *Picard* is longer.

(b) *LaForge's* rest frame:

$$L_{\text{P}} = L_{\text{P0}}\sqrt{1-\frac{v^2}{c^2}} = L_{\text{L0}}$$

Picard's rest frame:

$$L_{\text{L}} = L_{\text{L0}}\sqrt{1-\frac{v^2}{c^2}}$$

Calculate the ratio.

$$\frac{L_{\text{P0}}}{L_{\text{L0}}} = \frac{L_{\text{L0}}}{\sqrt{1-\frac{v^2}{c^2}}}\left(\frac{1}{L_{\text{L0}}}\right) = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\left(\frac{0.80c}{c}\right)^2}} = \boxed{1.7}$$

31. $v_1 = 0.90c$

$v_2 = 0.10c$

v = the speed of the probe relative to Earth

$$v = \frac{v_1+v_2}{1+\frac{v_1v_2}{c^2}} = \frac{0.90c+0.10c}{1+\frac{(0.90c)(0.10c)}{c^2}} = \boxed{0.917c}$$

32. $v_1 = 0.90c$

$v_2 = -0.10c$

v = the speed of the probe relative to Earth

$$v = \frac{v_1+v_2}{1+\frac{v_1v_2}{c^2}} = \frac{0.90c-0.10c}{1+\frac{(0.90c)(-0.10c)}{c^2}} = \boxed{0.88c}$$

33. $v_1 = 0.60c$

$v_2 = c$

v = the speed of the beam of light relative to the observer

$$v = \frac{v_1+v_2}{1+\frac{v_1v_2}{c^2}} = \frac{0.60c+c}{1+\frac{(0.60c)c}{c^2}} = \boxed{1.00c}$$

34. $v_1 = 22 \text{ mi/h}$
$v_2 = 19 \text{ mi/h}$
v = the speed of the paper relative to the ground

$$v = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}} = \frac{22 \frac{\text{mi}}{\text{h}} + 19 \frac{\text{mi}}{\text{h}}}{1 + \frac{\left(22 \frac{\text{mi}}{\text{h}}\right)\left(19 \frac{\text{mi}}{\text{h}}\right)}{\left(35 \frac{\text{mi}}{\text{h}}\right)^2}} = \boxed{31 \text{ mi/h}}$$

35. $v_1 = 0.60c$
v_2 = the speed of asteroid 1 relative to asteroid 2
$v = 0.80c$

$$v = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}$$

$$v + \frac{v v_1}{c^2} v_2 = v_1 + v_2$$

$$v - v_1 = \left(1 - \frac{v v_1}{c^2}\right) v_2$$

$$v_2 = \frac{v - v_1}{1 - \frac{v v_1}{c^2}} = \frac{0.80c - 0.60c}{1 - \frac{(0.80c)(0.60c)}{c^2}} = \boxed{0.38c}$$

36. $v_1 = -0.6c$
v_2 = the speed of one ship relative to the other
$v = 0.6c$

$$v = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}$$

$$v + \frac{v v_1}{c^2} v_2 = v_1 + v_2$$

$$v - v_1 = \left(1 - \frac{v v_1}{c^2}\right) v_2$$

$$v_2 = \frac{v - v_1}{1 - \frac{v v_1}{c^2}} = \frac{0.6c + 0.6c}{1 - \frac{(0.6c)(-0.6c)}{c^2}} = \boxed{0.88c}$$

37. $v_1 = 0.41c$

v_2 = the speed of the spaceship relative to the asteroid

$v = 0.77c$

$$v = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}$$

$$v + \frac{v v_1}{c^2} v_2 = v_1 + v_2$$

$$v - v_1 = \left(1 - \frac{v v_1}{c^2}\right) v_2$$

$$v_2 = \frac{v - v_1}{1 - \frac{v v_1}{c^2}} = \frac{0.77c - 0.41c}{1 - \frac{(0.77c)(0.41c)}{c^2}} = \boxed{0.53c}$$

38. $v_1 = 0.74c$

$v_2 = -0.43c$

v = the speed of the second electron relative to the lab

$$v = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}} = \frac{0.74c - 0.43c}{1 + \frac{(0.74c)(-0.43c)}{c^2}} = \boxed{0.45c}$$

39. (a) v_1 = the speed of ship B relative to Earth

$v_2 = 0.50c$

$v = 0.80c$

$$v_1 = \frac{v - v_2}{1 - \frac{v v_1}{c^2}} = \frac{0.80c - 0.50c}{1 - \frac{(0.80c)(0.50c)}{c^2}} = \boxed{0.50c}$$

(b) The relative speed increases by $\boxed{\text{more than}}$ $0.10c$ because increases in speed near the speed of light produce a smaller net increase than would be expected classically.

(c) $v_1 = 0.50c$

v_2 = the speed of ship A relative to ship B

$v = 0.80c + 0.10c = 0.90c$

$$v_2 = \frac{v - v_1}{1 - \frac{v v_1}{c^2}} = \frac{0.90c - 0.50c}{1 - \frac{(0.90c)(0.50c)}{c^2}} = \boxed{0.73c}$$

40. (a) $\boxed{\text{No}}$, since the inventor is using simple velocity addition, which is valid only for $v \ll c$.

(b) $v_1 = 0.80c$
$v_2 = 0.80c$
v = the speed of the object relative to the ground

$$v = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}} = \frac{0.80c + 0.80c}{1 + \frac{(0.80c)^2}{c^2}} = \boxed{0.976c}$$

41. (a) $p = mv = (4.5\times10^6\text{ kg})(0.65)\left(3.00\times10^8\ \frac{\text{m}}{\text{s}}\right) = \boxed{8.8\times10^{14}\text{ kg}\cdot\text{m/s}}$

(b)
$$p = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{(4.5\times10^6\text{ kg})(0.65)\left(3.00\times10^8\ \frac{\text{m}}{\text{s}}\right)}{\sqrt{1-\left(\frac{0.65c}{c}\right)^2}} = \boxed{1.2\times10^{15}\text{ kg}\cdot\text{m/s}}$$

42.
$$p = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$p^2\left(1-\frac{v^2}{c^2}\right) = m^2v^2$$
$$p^2 = \left(m^2 + \frac{p^2}{c^2}\right)v^2$$
$$v = \frac{p}{\sqrt{m^2+\frac{p^2}{c^2}}} = \frac{7.74\times10^{20}\text{ kg}\cdot\frac{\text{m}}{\text{s}}}{\sqrt{(8.2\times10^{11}\text{ kg})^2 + \left(\frac{7.74\times10^{20}\text{ kg}\cdot\frac{\text{m}}{\text{s}}}{3.00\times10^8\ \frac{\text{m}}{\text{s}}}\right)^2}} = \boxed{2.86\times10^8\text{ m/s}}$$

43.
$$\frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}=4mv$$
$$\frac{1}{4}=\sqrt{1-\frac{v^2}{c^2}}$$
$$\frac{1}{16}=1-\frac{v^2}{c^2}$$
$$\frac{v^2}{c^2}=1-\frac{1}{16}$$
$$v=\boxed{\frac{\sqrt{15}}{4}c}$$

44.
$$\begin{aligned}p_1+p_2&=p\\&=0\\p_1&=-p_2\\\frac{m_1v_1}{\sqrt{1-\frac{v_1^2}{c^2}}}&=-\frac{m_2v_2}{\sqrt{1-\frac{v_2^2}{c^2}}}\\\frac{m_1{}^2v_1{}^2}{1-\frac{v_1^2}{c^2}}&=\frac{m_2{}^2v_2{}^2}{1-\frac{v_2^2}{c^2}}\\m_1{}^2v_1{}^2-\frac{m_1{}^2v_1{}^2v_2{}^2}{c^2}&=m_2{}^2v_2{}^2-\frac{m_2{}^2v_2{}^2v_1{}^2}{c^2}\\m_1{}^2v_1{}^2&=v_2{}^2\left[m_2{}^2-(m_2{}^2-m_1{}^2)\frac{v_1{}^2}{c^2}\right]\\v_2&=\pm\frac{m_1v_1}{\sqrt{m_2{}^2-(m_2{}^2-m_1{}^2)\frac{v_1^2}{c^2}}}\\&=-\frac{(88\text{ kg})\left(2.0\ \frac{\text{m}}{\text{s}}\right)}{\sqrt{(120\text{ kg})^2-\left[(120\text{ kg})^2-(88\text{ kg})^2\right]\left(\frac{2.0\ \frac{\text{m}}{\text{s}}}{3.0\ \frac{\text{m}}{\text{s}}}\right)^2}}\quad(v_1>0\text{ and }v_1\text{ is opposite }v_2.)\\&=\boxed{-1.6\text{ m/s}}\end{aligned}$$

45.

$$p_1 + p_2 = p$$

$$\frac{m_1 v_1}{\sqrt{1-\frac{v_1^2}{c^2}}} + \frac{m_2 v_2}{\sqrt{1-\frac{v_2^2}{c^2}}} = \frac{(m_1+m_2)v}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\left(\frac{m_1 v_1}{\sqrt{1-\frac{v_1^2}{c^2}}} + \frac{m_2 v_2}{\sqrt{1-\frac{v_2^2}{c^2}}}\right)^2 = \frac{(m_1+m_2)^2 v^2}{1-\frac{v^2}{c^2}}$$

$$\left(\frac{m_1 v_1}{\sqrt{1-\frac{v_1^2}{c^2}}} + \frac{m_2 v_2}{\sqrt{1-\frac{v_2^2}{c^2}}}\right)^2 - \left(\frac{m_1 v_1}{\sqrt{1-\frac{v_1^2}{c^2}}} + \frac{m_2 v_2}{\sqrt{1-\frac{v_2^2}{c^2}}}\right)^2 \frac{v^2}{c^2} = (m_1+m_2)^2 v^2$$

$$\left(\frac{m_1 v_1}{\sqrt{1-\frac{v_1^2}{c^2}}} + \frac{m_2 v_2}{\sqrt{1-\frac{v_2^2}{c^2}}}\right)^2 = v^2\left[(m_1+m_2)^2 + \frac{1}{c^2}\left(\frac{m_1 v_1}{\sqrt{1-\frac{v_1^2}{c^2}}} + \frac{m_2 v_2}{\sqrt{1-\frac{v_2^2}{c^2}}}\right)^2\right]$$

$$v^2 = \frac{\left(\frac{m_1 v_1}{\sqrt{1-\frac{v_1^2}{c^2}}} + \frac{m_2 v_2}{\sqrt{1-\frac{v_2^2}{c^2}}}\right)^2}{(m_1+m_2)^2 + \frac{1}{c^2}\left(\frac{m_1 v_1}{\sqrt{1-\frac{v_1^2}{c^2}}} + \frac{m_2 v_2}{\sqrt{1-\frac{v_2^2}{c^2}}}\right)^2}$$

$$= \frac{c^2}{1 + \frac{c^2(m_1+m_2)^2}{\left(\frac{m_1 v_1}{\sqrt{1-\frac{v_1^2}{c^2}}} + \frac{m_2 v_2}{\sqrt{1-\frac{v_2^2}{c^2}}}\right)^2}}$$

$$v = \pm\frac{c}{\sqrt{1 + \frac{c^2(m_1+m_2)^2}{\left(\frac{m_1 v_1}{\sqrt{1-\frac{v_1^2}{c^2}}} + \frac{m_2 v_2}{\sqrt{1-\frac{v_2^2}{c^2}}}\right)^2}}}$$

$$= \frac{3.0\ \frac{\text{m}}{\text{s}}}{\sqrt{1 + \left[\frac{\left(3.0\ \frac{\text{m}}{\text{s}}\right)(88\text{ kg}+120\text{ kg})}{\frac{(88\text{ kg})\left(2.0\ \frac{\text{m}}{\text{s}}\right)}{\sqrt{1-\left(\frac{2.0\ \frac{\text{m}}{\text{s}}}{3.0\ \frac{\text{m}}{\text{s}}}\right)^2}} + \frac{(120\text{ kg})\left(-1.2\ \frac{\text{m}}{\text{s}}\right)}{\sqrt{1-\left(\frac{-1.2\ \frac{\text{m}}{\text{s}}}{3.0\ \frac{\text{m}}{\text{s}}}\right)^2}}}\right]^2}} \quad (v > 0 \text{ since } p_1 > p_2.)$$

$$= \boxed{0.4\text{ m/s}}$$

46.

$$p_1 + p_2 = p$$

$$\frac{m_1 v_1}{\sqrt{1-\frac{v_1^2}{c^2}}} + 0 = \frac{(m_1 + m_2)v}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\frac{m_1 v_1}{\sqrt{1-\frac{v_1^2}{c^2}}} - \frac{m_1 v}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_2 v}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$m_2 = \frac{\sqrt{1-\frac{v^2}{c^2}}}{v}\left(\frac{m_1 v_1}{\sqrt{1-\frac{v_1^2}{c^2}}} - \frac{m_1 v}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

$$= \left(\frac{v_1}{v}\sqrt{\frac{1-\frac{v^2}{c^2}}{1-\frac{v_1^2}{c^2}}} - 1\right) m_1$$

$$= \left[\frac{0.50c}{0.26c}\sqrt{\frac{1-\left(\frac{0.26c}{c}\right)^2}{1-\left(\frac{0.50c}{c}\right)^2}} - 1\right](8.2\times 10^7\ \text{kg})$$

$$= \boxed{9.4\times 10^7\ \text{kg}}$$

47.

$$f = \frac{p_r - p_c}{p_r}$$

$$= 1 - \frac{p_c}{p_r}$$

$$= 1 - \frac{mv}{\frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}}$$

$$= 1 - \sqrt{1-\frac{v^2}{c^2}}$$

$$\sqrt{1-\frac{v^2}{c^2}} = 1 - f$$

$$1 - \frac{v^2}{c^2} = (1-f)^2$$

$$\frac{v^2}{c^2} = 1 - (1-f)^2$$

$$v = c\sqrt{1-(1-f)^2}$$

(a) $v = c\sqrt{1-(1-0.0100)^2} = \boxed{0.141c}$

(b) $v = c\sqrt{1-(1-0.0500)^2} = \boxed{0.312c}$

48.
$$p_e = p_p$$
$$\frac{m_e v_e}{\sqrt{1-\frac{v_e^2}{c^2}}} = \frac{m_p v_p}{\sqrt{1-\frac{v_p^2}{c^2}}}$$
$$m_e v_e \sqrt{1-\frac{v_p^2}{c^2}} = 1836 m_e v_p \sqrt{1-\frac{v_e^2}{c^2}}$$
$$v_e^2\left(1-\frac{v_p^2}{c^2}\right) = 1836^2 v_p^2\left(1-\frac{v_e^2}{c^2}\right)$$
$$v_e^2\left(1-\frac{v_p^2}{c^2}+\frac{1836^2 v_p^2}{c^2}\right) = 1836^2 v_p^2$$
$$v_e = \frac{1836 v_p}{\sqrt{1+(1836^2-1)\frac{v_p^2}{c^2}}}$$
$$= \frac{1836(0.0100c)}{\sqrt{1+(1836^2-1)\left(\frac{0.0100c}{c}\right)^2}}$$
$$= \boxed{0.999c}$$

49.
$$W = \Delta K$$
$$= K_f - K_i$$
$$= \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0 c^2 - 0$$
$$= m_0 c^2\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}-1\right)$$
$$= (1.673\times10^{-27}\text{ kg})\left(3.00\times10^8\ \frac{\text{m}}{\text{s}}\right)^2\left[\frac{1}{\sqrt{1-\left(\frac{0.90c}{c}\right)^2}}-1\right]$$
$$= \boxed{0.19\text{ nJ}}$$

50. (a)
$$E = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$= \frac{(1.675\times10^{-27}\text{ kg})\left(3.00\times10^8\ \frac{\text{m}}{\text{s}}\right)^2}{\sqrt{1-\left(\frac{0.99c}{c}\right)^2}}$$
$$= \boxed{1\text{ nJ}}$$

(b) $m_0 c^2 = (1.675\times10^{-27}\text{ kg})\left(3.00\times10^8\ \frac{\text{m}}{\text{s}}\right)^2 = \boxed{0.151\text{ nJ}}$

(c) $K = m_0 c^2 \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1 \right)$

$= (1.675 \times 10^{-27} \text{ kg})\left(3.00 \times 10^8 \ \frac{\text{m}}{\text{s}}\right)^2 \left(\frac{1}{\sqrt{1-\left(\frac{0.99c}{c}\right)^2}} - 1 \right)$

$= \boxed{0.9 \text{ nJ}}$

51. $W = \frac{1}{2}kx^2 = \Delta mc^2$

$\Delta m = \frac{kx^2}{2c^2} = \frac{\left(544 \ \frac{\text{N}}{\text{m}}\right)(0.38 \text{ m})^2}{2\left(3.00 \times 10^8 \ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{4.4 \times 10^{-16} \text{ kg}}$

52. $E = \Delta mc^2 = (8.3 \times 10^{-16} \text{ kg})\left(3.00 \times 10^8 \ \frac{\text{m}}{\text{s}}\right)^2 = \boxed{75 \text{ J}}$

53. $E_{\text{min}} = m_{\text{electron}}c^2 + m_{\text{positron}}c^2$

$= 2m_{\text{e}}c^2$

$= 2(9.11 \times 10^{-31} \text{ kg})\left(3.00 \times 10^8 \ \frac{\text{m}}{\text{s}}\right)^2$

$= \boxed{1.64 \times 10^{-13} \text{ J}}$

54. $2E_\gamma = 2m_{\text{p}}c^2$

$E_\gamma = m_{\text{p}}c^2$

$= (1.673 \times 10^{-27} \text{ kg})\left(3.00 \times 10^8 \ \frac{\text{m}}{\text{s}}\right)^2$

$= \boxed{0.151 \text{ nJ}}$

55. $$K = m_0c^2\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}-1\right)$$

$$\frac{K}{m_0c^2}+1=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\sqrt{1-\frac{v^2}{c^2}}=\frac{1}{1+\frac{K}{m_0c^2}}$$

$$1-\frac{v^2}{c^2}=\left(\frac{1}{1+\frac{K}{m_0c^2}}\right)^2$$

$$v=c\sqrt{1-\left(\frac{1}{1+\frac{K}{m_0c^2}}\right)^2}$$

$$=c\sqrt{1-\left(\frac{1}{1+\frac{2.7\times10^{23}\text{ J}}{(2.7\times10^6\text{ kg})\left(3.00\times10^8\ \frac{\text{m}}{\text{s}}\right)^2}}\right)^2}$$

$$=\boxed{0.88c}$$

56. $$\frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}=3m_0c^2$$

$$\frac{1}{3}=\sqrt{1-\frac{v^2}{c^2}}$$

$$\frac{1}{9}=1-\frac{v^2}{c^2}$$

$$\frac{v^2}{c^2}=1-\frac{1}{9}$$

$$v^2=\frac{8}{9}c^2$$

$$v=\boxed{\frac{2\sqrt{2}}{3}c}$$

57. $E = \Delta mc^2$

$$\Delta m = \frac{E}{c^2} = \frac{P\Delta t}{c^2} = \frac{(1.0\times10^9\text{ W})(31{,}536{,}000\text{ s})}{\left(3.0\times10^8\ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{0.35\text{ kg}}$$

58. $W = \Delta mc^2$

$$= (2m_\text{p} + 2m_\text{n} + 2m_\text{e} - m_\text{He})c^2$$

$$= [2(1.007276\text{ u}) + 2(1.008665\text{ u}) + 2(0.000549\text{ u}) - 4.002603\text{ u}]\left(\frac{931.49\ \frac{\text{MeV}}{c^2}}{1\text{ u}}\right)c^2$$

$$= \boxed{28.296\text{ MeV}}$$

59. $$f = \frac{K - K_\text{Cl}}{K} = 1 - \frac{K_\text{Cl}}{K} = 1 - \frac{\frac{1}{2}m_0v^2}{m_0c^2\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right)}$$

(a) $$f = 1 - \frac{(0.10c)^2}{2c^2\left(\frac{1}{\sqrt{1-\left(\frac{0.10c}{c}\right)^2}} - 1\right)} = 0.008$$

The percent difference is $\boxed{0.8\%}$.

(b) $$f = 1 - \frac{(0.90c)^2}{2c^2\left(\frac{1}{\sqrt{1-\left(\frac{0.90c}{c}\right)^2}} - 1\right)} = 0.69$$

The percent difference is $\boxed{69\%}$.

60.

$$K_e = K_p$$

$$m_e c^2\left(\frac{1}{\sqrt{1-\frac{v_e^2}{c^2}}}-1\right) = m_p c^2\left(\frac{1}{\sqrt{1-\frac{v_p^2}{c^2}}}-1\right)$$

$$m_e\left(\frac{1}{\sqrt{1-\frac{v_e^2}{c^2}}}-1\right) = 1836 m_e\left(\frac{1}{\sqrt{1-\frac{v_p^2}{c^2}}}-1\right)$$

$$\frac{1}{\sqrt{1-\frac{v_e^2}{c^2}}} = \frac{1836}{\sqrt{1-\frac{v_p^2}{c^2}}}-1836$$

$$1-\frac{v_e^2}{c^2} = \left(\frac{1836}{\sqrt{1-\frac{v_p^2}{c^2}}}-1836\right)^{-2}$$

$$v_e = c\sqrt{1-\left(\frac{1836}{\sqrt{1-\frac{v_p^2}{c^2}}}-1836\right)^{-2}}$$

$$= c\sqrt{1-\left[\frac{1836}{\sqrt{1-\left(\frac{0.0100c}{c}\right)^2}}-1836\right]^{-2}}$$

$$= \boxed{0.4c}$$

61. (a)

$$W = \Delta K = K_f - K_i$$

$$= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$= \frac{1}{2}m(v_f^2 - v_i^2)$$

$$= \frac{1}{2}(0.145 \text{ kg})[(35.0 \text{ m/s})^2 - (25.0 \text{ m/s})^2]$$

$$= \boxed{43.5 \text{ J}}$$

(b) Because of the increase in mass at relativistic speeds, the work required is greater.

(c) $W = \Delta K = K_\text{f} - K_\text{i}$

$$= \left(\frac{m_0c^2}{\sqrt{1-\frac{v_\text{f}^2}{c^2}}} - m_0c^2\right) - \left(\frac{m_0c^2}{\sqrt{1-\frac{v_\text{i}^2}{c^2}}} - m_0c^2\right)$$

$$= m_0c^2\left(\frac{1}{\sqrt{1-\frac{v_\text{f}^2}{c^2}}} - \frac{1}{\sqrt{1-\frac{v_\text{i}^2}{c^2}}}\right)$$

$$= (0.145\text{ kg})(3.00\times10^8\text{ m/s})^2\left(\frac{1}{\sqrt{1-\left(\frac{2.00000035\times10^8\ \frac{\text{m}}{\text{s}}}{3.00\times10^8\ \frac{\text{m}}{\text{s}}}\right)^2}} - \frac{1}{\sqrt{1-\left(\frac{2.00000025\times10^8\ \frac{\text{m}}{\text{s}}}{3.00\times10^8\ \frac{\text{m}}{\text{s}}}\right)^2}}\right)$$

$$= \boxed{7.00\times10^8\text{ J}}$$

62. (a) $K = \left(\frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0c^2\right) = m_0c^2$

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 2$$

$$\frac{1}{4} = 1 - \frac{v^2}{c^2}$$

$$\boxed{v = 0.866c}$$

(b) Because of the relativistic increase in mass, the speed increases by $\boxed{\text{less}}$ than a factor of two.

(c) $K = \left(\frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0c^2\right) = 2m_0c^2$

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 3$$

$$\frac{1}{9} = 1 - \frac{v^2}{c^2}$$

$$\boxed{v = 0.943c}$$

63. $$E_\text{i} = E_\text{f}$$
$$\frac{2m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} = mc^2$$
$$m = \frac{2m_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{2(0.240\text{ kg})}{\sqrt{1-\left(\frac{0.980c}{c}\right)^2}} = \boxed{2.4\text{ kg}}$$

64. $$R = \frac{2GM}{c^2} = \frac{2\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(2.00\times10^{30}\text{ kg})}{\left(3.00\times10^8\ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{2.96\text{ km}}$$

65. $$R = \frac{2GM}{c^2} = \frac{2\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(2.00\times10^{36}\text{ kg})}{\left(3.00\times10^8\ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{2.96\times10^6\text{ km}}$$

66. (a) $$g = \frac{GM}{r^2} = \frac{GM}{R^2} = \frac{GM}{\left(\frac{2GM}{c^2}\right)^2}$$
$$g = \frac{c^4}{4GM} = \frac{(3.00\times10^8\text{ m/s})^4}{4\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.2\times10^{36}\text{ kg})} = \boxed{5.8\times10^6\text{ m/s}^2}$$

(b) Because the Schwarzchild radius doubles as well, and g depends directly on M but inversely on the square of the distance from the center, g $\boxed{\text{decreases by a factor of 2}}$.

67. (a)

$$K = m_0c^2\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}-1\right)$$

$$\frac{K}{m_0c^2}+1 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$1-\frac{v^2}{c^2} = \frac{1}{\left(\frac{K}{m_0c^2}+1\right)^2}$$

$$\frac{v}{c} = \sqrt{1-\frac{1}{\left(\frac{K}{m_0c^2}+1\right)^2}}$$

$$= \sqrt{1-\frac{1}{\left(\frac{1.0\times10^{20}\text{ eV}}{938.28\times10^6\text{ eV}}+1\right)^2}}$$

$$= \sqrt{1-8.8\times10^{-23}}$$

$$\approx 1-4.4\times10^{-23} \approx 1.00$$

$$\boxed{v \approx 1.00c}$$

(b) $$K = \frac{1}{2}(15\times10^{-3}\text{ kg})(8.8\times10^{-3}\text{ m/s})^2$$

$$= 5.8\times10^{-7}\text{ J}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right)$$

$$= \boxed{3.6\times10^{12}\text{ eV} << 1.0\times10^{20}\text{ eV}}$$

68. $$h = \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2h}{g}} = \Delta t_0$$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= \sqrt{\frac{2h}{g\left(1-\frac{v^2}{c^2}\right)}}$$

$$= \sqrt{\frac{2(3.0\text{ m})}{\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\left[1-\left(\frac{0.89c}{c}\right)^2\right]}}$$

$$= \boxed{1.7\text{ s}}$$

69.
$$K = m_0c^2\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}-1\right)$$
$$\frac{K}{m_0c^2}+1 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$\left(\frac{K}{m_0c^2}+1\right)^{-2} = 1-\frac{v^2}{c^2}$$
$$\begin{aligned} v &= c\sqrt{1-\left(1+\frac{K}{m_0c^2}\right)^{-2}} \\ &= c\sqrt{1-\left(1+\frac{1.50\times10^3\text{ MeV}}{938\text{ MeV}}\right)^{-2}} \\ &= 0.923c \\ &= 2.77\times10^8\text{ m/s} \end{aligned}$$
$$p = \frac{m_0v}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{(1.673\times10^{-27}\text{ kg})\left(2.77\times10^8\ \frac{\text{m}}{\text{s}}\right)}{\sqrt{1-\left(\frac{0.923c}{c}\right)^2}} = \boxed{1.20\times10^{-18}\text{ kg}\cdot\text{m/s}}$$

70. (a) Since $E = m_0c^2$, an increase in energy results in an $\boxed{\text{increase}}$ in mass.

(b)
$$\begin{aligned} \Delta m &= \frac{\Delta E}{c^2} \\ &= \frac{\Delta U}{c^2} \\ &= \frac{\frac{3}{2}nR\Delta T}{c^2} \\ &= \frac{3(2.00\text{ mol})\left(8.31\ \frac{\text{J}}{\text{mol}\cdot\text{K}}\right)\frac{5}{9}(112)\text{ K}}{2\left(3.00\times10^8\ \frac{\text{m}}{\text{s}}\right)^2} \\ &= \boxed{1.72\times10^{-14}\text{ kg}} \end{aligned}$$

71. (a)

$$K = m_0c^2\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}-1\right)$$

$$\frac{K}{m_0c^2}+1 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$1-\frac{v^2}{c^2} = \left(1+\frac{K}{m_0c^2}\right)^{-2}$$

$$v = c\sqrt{1-\left(1+\frac{K}{m_0c^2}\right)^{-2}}$$

$$= c\sqrt{1-\left(1+\frac{156\text{ keV}}{511\text{ keV}}\right)^{-2}}$$

$$= \boxed{0.6427c}$$

(b) $$p = \frac{m_0v}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{(9.11\times10^{-31}\text{ kg})(0.6427)\left(3.00\times10^8\ \frac{\text{m}}{\text{s}}\right)}{\sqrt{1-\left(\frac{0.6427c}{c}\right)^2}} = \boxed{2.29\times10^{-22}\text{ kg}\cdot\text{m/s}}$$

(c) $p_\beta + p_\text{C} = p_\text{i} = 0$

$$p_\text{C} = -p_\beta = \boxed{-2.29\times10^{-22}\text{ kg}\cdot\text{m/s}}$$

(d)

$$p = \frac{m_0v}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$1-\frac{v^2}{c^2} = \frac{m_0{}^2v^2}{p^2}$$

$$1 = \left(\frac{1}{c^2}+\frac{m_0{}^2}{p^2}\right)v^2$$

$$v = \left(\frac{1}{c^2}+\frac{m_0{}^2}{p^2}\right)^{-1/2}$$

$$= c\left(1+\frac{m_0{}^2c^2}{p^2}\right)^{-1/2}$$

$$= \left(3.00\times10^8\ \frac{\text{m}}{\text{s}}\right)\left[1+\frac{(14.003242\text{ u})^2\left(1.66\times10^{-27}\ \frac{\text{kg}}{\text{u}}\right)^2\left(3.00\times10^8\ \frac{\text{m}}{\text{s}}\right)^2}{\left(-2.29\times10^{-22}\text{ kg}\cdot\frac{\text{m}}{\text{s}}\right)^2}\right]^{-1/2}$$

$$= \boxed{9.85\text{ km/s}}$$

72. Find v.

$$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$

$$\frac{1}{2}L_0 = L_0\sqrt{1-\frac{v^2}{c^2}}$$

$$\frac{1}{4} = 1-\frac{v^2}{c^2}$$

$$v = \frac{\sqrt{3}}{2}c$$

Find Δt.

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1.0\text{ s}}{\sqrt{1-\left(\frac{\frac{\sqrt{3}}{2}c}{c}\right)^2}} = \boxed{2.0\text{ s}}$$

73. (a) $\frac{1}{2}mv^2 = eV$

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2(1.60\times10^{-19}\text{ C})(2.50\times10^{3}\text{ V})}{9.11\times10^{-31}\text{ kg}}} = \boxed{2.96\times10^{7}\text{ m/s}}$$

(b) Since the energy related to a given speed is greater in relativity theory than in classical theory, the speed should be $\boxed{\text{less than}}$ that found in part (a).

(c) $K = m_0c^2\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}-1\right) = eV$

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 1+\frac{eV}{m_0c^2}$$

$$1-\frac{v^2}{c^2} = \left(1+\frac{eV}{m_0c^2}\right)^{-2}$$

$$v = c\sqrt{1-\left(1+\frac{eV}{m_0c^2}\right)^{-2}}$$

$$= \left(3.00\times10^8\ \frac{\text{m}}{\text{s}}\right)\sqrt{1-\left[1+\frac{(1.60\times10^{-19}\ \text{C})(2.50\times10^3\ \text{V})}{(9.11\times10^{-31}\ \text{kg})\left(3.00\times10^8\ \frac{\text{m}}{\text{s}}\right)^2}\right]^{-2}}$$

$$= \boxed{2.95\times10^7\ \text{m/s}}$$

74. Find v.

$$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$

$$\left(\frac{L}{L_0}\right)^2 = 1-\frac{v^2}{c^2}$$

$$v = c\sqrt{1-\left(\frac{L}{L_0}\right)^2}$$

Find v_2.
$v_1 = 0.65c$
v_2 = the speed of the shuttle craft relative to the starship.
v = the speed of the shuttle craft relative to Earth.

$$v = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}$$

$$v + \frac{v v_1}{c^2} v_2 = v_1 + v_2$$

$$v - v_1 = \left(1 - \frac{v v_1}{c^2}\right) v_2$$

$$v_2 = \frac{v - v_1}{1 - \frac{v v_1}{c^2}}$$

$$= \frac{c\sqrt{1 - \left(\frac{L}{L_0}\right)^2} - v_1}{1 - \frac{v_1 c\sqrt{1 - \left(\frac{L}{L_0}\right)^2}}{c^2}}$$

$$= \frac{c\sqrt{1 - \left(\frac{6.50 \text{ m}}{12.5 \text{ m}}\right)^2} - 0.65c}{1 - \frac{0.65c^2\sqrt{1 - \left(\frac{6.50 \text{ m}}{12.5 \text{ m}}\right)^2}}{c^2}}$$

$$= \boxed{0.46c}$$

75. (a) $R = \frac{p}{qB} = \frac{mv}{qB} = \frac{(1.673 \times 10^{-27} \text{ kg})(0.99)\left(3.00 \times 10^8 \text{ }\frac{\text{m}}{\text{s}}\right)}{(1.60 \times 10^{-19} \text{ C})(0.20 \text{ T})} = \boxed{16 \text{ m}}$

(b) $R = \frac{mv}{qB\sqrt{1 - \frac{v^2}{c^2}}} = \frac{(1.673 \times 10^{-27} \text{ kg})(0.99)\left(3.00 \times 10^8 \text{ }\frac{\text{m}}{\text{s}}\right)}{(1.60 \times 10^{-19} \text{ C})(0.20 \text{ T})\sqrt{1 - \left(\frac{0.99c}{c}\right)^2}} = \boxed{100 \text{ m}}$

76. (a) $v_1 = 0.75c$
$v_2 = -0.40c$
v = the speed of the shuttle craft relative to Earth.

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}}$$

$$= L_0\sqrt{1 - \frac{1}{c^2}\left(\frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}\right)^2}$$

$$= (13 \text{ m})\sqrt{1 - \frac{1}{c^2}\left[\frac{0.75c - 0.40c}{1 + \frac{(0.75c)(-0.40c)}{c^2}}\right]^2}$$

$$= \boxed{11 \text{ m}}$$

(b) Since the shuttle would have a greater speed relative to Earth, the length contraction of the shuttle craft would be greater, so the length as measured by an observer on Earth would be $\boxed{\text{less than}}$ that found in part (a).

(c) $L = (13\text{ m})\sqrt{1-\frac{1}{c^2}\left[\frac{0.75c+0.40c}{1+\frac{(0.75c)(0.40c)}{c^2}}\right]^2} = \boxed{6.1\text{ m}}$

77. (a)
$$\begin{aligned}L &= \sqrt{\left(L_0\sqrt{1-\frac{v^2}{c^2}}\right)^2+(\ell\sin\theta)^2}\\ &= \sqrt{\left(\ell\cos\theta\sqrt{1-\frac{v^2}{c^2}}\right)^2+(\ell\sin\theta)^2}\\ &= \sqrt{\left[(2.5\text{ m})\cos 45°\sqrt{1-\left(\frac{0.95c}{c}\right)^2}\right]^2+[(2.5\text{ m})\sin 45°]^2}\\ &= \boxed{1.9\text{ m}}\end{aligned}$$

(b)
$$\begin{aligned}\tan\theta &= \frac{y}{x}\\ \theta &= \tan^{-1}\frac{y}{x}\\ &= \tan^{-1}\frac{\ell\sin\theta}{\ell\cos\theta\sqrt{1-\frac{v^2}{c^2}}}\\ &= \tan^{-1}\frac{\tan\theta}{\sqrt{1-\frac{v^2}{c^2}}}\\ &= \tan^{-1}\frac{\tan 45°}{\sqrt{1-\left(\frac{0.95c}{c}\right)^2}}\\ &= \boxed{70°}\end{aligned}$$

78. (a)
$$\begin{aligned}\frac{1}{2}mv^2 &= eV\\ v &= \sqrt{\frac{2eV}{m}}\\ &= \sqrt{\frac{2(1.60\times 10^{-19}\text{ C})(256{,}000\text{ V})}{9.11\times 10^{-31}\text{ kg}}}\\ &= \boxed{3.00\times 10^8\text{ m/s}}\end{aligned}$$

(b) $$m_0c^2\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}-1\right)=eV$$

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}=1+\frac{eV}{m_0c^2}$$

$$1-\frac{v^2}{c^2}=\left(1+\frac{eV}{m_0c^2}\right)^{-2}$$

$$v=c\sqrt{1-\left(1+\frac{eV}{m_0c^2}\right)^{-2}}$$

$$=\left(3.00\times10^8\ \tfrac{\text{m}}{\text{s}}\right)\sqrt{1-\left[1+\frac{(1.60\times10^{-19}\ \text{C})(256{,}000\ \text{V})}{(9.11\times10^{-31}\ \text{kg})\left(3.00\times10^8\ \tfrac{\text{m}}{\text{s}}\right)^2}\right]^{-2}}$$

$$=\boxed{2.24\times10^8\ \text{m/s}}$$

79. $$m_0c^2=\frac{(pc)^2-K^2}{2K}=\frac{[(105\ \text{MeV}/c)c]^2-(35.0\ \text{MeV})^2}{2(35.0\ \text{MeV})}=\boxed{140\ \text{MeV}}$$

80. (a) $$\frac{GMm}{(2R)^2}=\frac{mv^2}{2R}$$

$$v=\sqrt{\frac{GM}{2R}}$$

$$=\sqrt{\frac{GM}{2\left(\frac{2GM}{c^2}\right)}}$$

$$=\boxed{\frac{1}{2}c}$$

(b) $$\frac{GMm}{R^2}=\frac{mv^2}{R}$$

$$v=\sqrt{\frac{GM}{R}}$$

$$=\sqrt{\frac{GM}{\left(2\frac{GM}{c^2}\right)}}$$

$$=\boxed{\frac{1}{\sqrt{2}}c}$$

81. (a) Since momentum is conserved, and the total relativistic mass will be less after the collision than before, the final speed must be greater than the average of the two initial speeds.

(b) $\frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}+0=\frac{(2m_0)v_\text{f}}{\sqrt{1-\frac{v_\text{f}^2}{c^2}}}$ by conservation of momentum,

$$\sqrt{1-\frac{v_\text{f}^2}{c^2}}=2\frac{v_\text{f}}{v}\sqrt{1-\frac{v^2}{c^2}}$$

$$1-\frac{v_\text{f}^2}{c^2}=4\left(\frac{v_\text{f}}{v}\right)^2-4\left(\frac{v_\text{f}}{c}\right)^2$$

$$4\frac{v_\text{f}^2}{v^2}-3\frac{v_\text{f}^2}{c^2}=1$$

$$v_\text{f}^2\left(4-3\frac{v^2}{c^2}\right)=v^2$$

$$v_\text{f}=\sqrt{\frac{v^2}{4-3\frac{v^2}{c^2}}}=\sqrt{\frac{(0.650c)^2}{4-3\frac{(0.650c)^2}{c^2}}}=\boxed{0.393c}$$

82. (a) The *y* component of length remains $L_0\sin\theta_0$. The *x* component is relativistically contracted to $L_0\cos\theta_0\sqrt{1-\frac{v^2}{c^2}}$.

$$\text{Total length}=L=\sqrt{(L_0\cos\theta_0)^2\left(1-\frac{v^2}{c^2}\right)+(L_0\sin\theta)^2}$$

$$L=\sqrt{L_0^2(\cos^2\theta_0+\sin^2\theta_0)-L_0^2\cos^2\theta_0\frac{v^2}{c^2}}=\sqrt{L_0^2\left(1-\frac{v^2}{c^2}\cos^2\theta_0\right)}=L_0\sqrt{1-\left(\frac{v^2}{c^2}\right)\cos^2\theta_0}$$

(b) $\tan\theta=\frac{L_y}{L_x}=\frac{L_0\sin\theta_0}{L_0\cos\theta_0\sqrt{1-\frac{v^2}{c^2}}}$

$$\tan\theta=\frac{\tan\theta_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

83. Let f = the pulse frequency observed at Earth. Find f_0 = the (proper) pulse frequency at the pulsar.

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\frac{1}{f} = \frac{1}{f_0\sqrt{1-\frac{v^2}{c^2}}}$$

$$f_0 = \frac{f}{\sqrt{1-\frac{v^2}{c^2}}}$$

Find v_2.

$v_1 = 0.950c$

v_2 = the speed of the pulsar relative to the *Endeavor*

$v = 0.800c$

$$v = \frac{v_1 + v_2}{1+\frac{v_1 v_2}{c^2}}$$

$$v + \frac{v v_1}{c^2} v_2 = v_1 + v_2$$

$$v - v_1 = \left(1 - \frac{v v_1}{c^2}\right) v_2$$

$$v_2 = \frac{v - v_1}{1-\frac{v v_1}{c^2}}$$

Find $f' =$ the pulse frequency relative to the *Endeavor*.

$$\Delta t' = \frac{\Delta t_0}{\sqrt{1-\frac{v_2^2}{c^2}}}$$

$$\frac{1}{f'} = \frac{1}{f_0\sqrt{1-\frac{v_2^2}{c^2}}}$$

$$f' = f_0\sqrt{1-\frac{v_2^2}{c^2}}$$

$$= \frac{f\sqrt{1-\frac{v_2^2}{c^2}}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= f\sqrt{\frac{1-\frac{v_2^2}{c^2}}{1-\frac{v^2}{c^2}}}$$

$$= f\sqrt{\frac{1-\frac{1}{c^2}\left(\frac{v-v_1}{1-\frac{vv_1}{c^2}}\right)^2}{1-\frac{v^2}{c^2}}}$$

$$= \left(153\ \frac{\text{pulses}}{\text{s}}\right)\sqrt{\frac{1-\frac{1}{c^2}\left[\frac{0.800c-0.950c}{1-\frac{(0.800c)(0.950c)}{c^2}}\right]^2}{1-\left(\frac{0.800c}{c}\right)^2}}$$

$$= \boxed{199\ \text{pulses/s}}$$

84. $$E = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$E^2 = \frac{m_0^2c^4}{1-\frac{v^2}{c^2}}$$

$$= \frac{m_0^2c^4 + m_0^2v^2c^2 - m_0^2v^2c^2}{1-\frac{v^2}{c^2}}$$

$$= \frac{m_0^2v^2c^2}{1-\frac{v^2}{c^2}} + \frac{m_0^2c^4 - m_0^2v^2c^2}{1-\frac{v^2}{c^2}}$$

$$= \left(\frac{m_0^2v^2}{\sqrt{1-\frac{v^2}{c^2}}}\right)^2 c^2 + \frac{m_0^2c^4\left(1-\frac{v^2}{c^2}\right)}{1-\frac{v^2}{c^2}}$$

$$= p^2c^2 + m_0^2c^4$$

$$E^2 = p^2c^2 + (m_0c^2)^2$$

85. $v = \dfrac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}} = \dfrac{v_1\left(1 + \frac{v_2}{v_1}\right)}{1 + \frac{v_1 v_2}{c^2}} = \dfrac{v_1\left(1 + \frac{v_1 v_2}{c^2}\frac{c^2}{v_1^2}\right)}{1 + \frac{v_1 v_2}{c^2}} > \dfrac{v_1\left(1 + \frac{v_1 v_2}{c^2}\right)}{1 + \frac{v_1 v_2}{c^2}} = v_1$

So, $v > v_1$.

$$v = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}} = \frac{v_2\left(1 + \frac{v_1}{v_2}\right)}{1 + \frac{v_1 v_2}{c^2}} = \frac{v_2\left(1 + \frac{v_1 v_2}{c^2}\frac{c^2}{v_2^2}\right)}{1 + \frac{v_1 v_2}{c^2}} > \frac{v_2\left(1 + \frac{v_1 v_2}{c^2}\right)}{1 + \frac{v_1 v_2}{c^2}} = v_2$$

So, $v > v_2$.

Now,

$$\frac{v_1}{c}\left(1 - \frac{v_2}{c}\right) < 1 - \frac{v_2}{c} \qquad \text{for } v_1 < c$$

$$\frac{v_1}{c} - \frac{v_1 v_2}{c} < 1 - \frac{v_2}{c}$$

$$\frac{v_1}{c} + \frac{v_2}{c} < 1 + \frac{v_1 v_2}{c^2}$$

$$\frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}} < c$$

$$v < c$$

Therefore, $v_1 < v < c$ and $v_2 < v < c$.

86.

$$p = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$p^2 = \frac{m_0^2 v^2}{1 - \frac{v^2}{c^2}}$$

$$p^2 - \frac{p^2}{c^2}v^2 = m_0^2 v^2$$

$$p^2 = \left(m_0^2 + \frac{p^2}{c^2}\right)v^2$$

$$v^2 = \frac{p^2}{m_0^2 + \frac{p^2}{c^2}}$$

$$= \frac{p^2 c^2}{m_0^2 c^2 + p^2}$$

$$= \frac{c^2}{\frac{m_0^2 c^2}{p^2} + 1}$$

$$v = \boxed{\frac{c}{\sqrt{1 + \left(\frac{m_0 c}{p}\right)^2}}}$$

87. Lab frame

$L_0 = 3.0 \text{ cm} = v\Delta t$

Σ^- frame

$L = v\Delta t_0 = v(0.15 \text{ ns})$

Find v.

$$L = L_0\sqrt{1-\frac{v^2}{c^2}} = v\Delta t_0$$

$$L_0{}^2\left(1-\frac{v^2}{c^2}\right) = v^2\Delta t_0{}^2$$

$$L_0{}^2 = v^2\left(\Delta t_0{}^2 + \frac{L_0{}^2}{c^2}\right)$$

$$v = \sqrt{\frac{L_0{}^2}{\Delta t_0{}^2 + \frac{L_0{}^2}{c^2}}}$$

$$= \sqrt{\frac{c^2}{\frac{c^2\Delta t_0{}^2}{L_0{}^2}+1}}$$

$$= \frac{c}{\sqrt{1+\frac{c^2\Delta t_0{}^2}{L_0{}^2}}}$$

$$= \frac{c}{\sqrt{1+\frac{\left(3.00\times10^8 \text{ }\frac{\text{m}}{\text{s}}\right)^2(0.15\times10^{-9}\text{ s})^2}{(0.030\text{ m})^2}}}$$

$$= \boxed{0.55c}$$