Chapter 31

Atomic Physics

Answers to Even-numbered Conceptual Questions

2. There are many such reasons, but perhaps the most important is that orbiting electrons in Rutherford's model would radiate energy in the form of electromagnetic waves, with the result that atoms would collapse in a very small amount of time.

4. The observation that alpha particles are sometimes reversed in direction when they strike a thin gold foil led to the idea that there must be a great concentration of positive charge and mass within an atom. This became the nucleus in Rutherford's model.

6. In principle, there are an infinite number of spectral lines in any given series. The lines become more closely spaced as one moves higher in the series, which makes them hard to distinguish in practice.

8. Referring to Equation 31-8, we can see that doubling the electron's mass would double its ionization energy. This result follows from Equation 31-5, where we see that the radius of an electron's orbit is inversely proportional to the mass, and from Equation 31-6, where we see that the speed of the electron is independent of its mass. Combining these results shows that the total (negative) energy of the electron would double with its mass, as would the energy required to ionize the atom.

10. The lowest-energy photon this electron could absorb would be one that would raise it from the $n = 1$ orbit to the $n = 2$ orbit. Referring to Figure 31-9, we see that the energy of this photon is given by the following relation: $\Delta E = E_{\text{f}} - E_{\text{i}} =$ –3.40 eV – (–13.6 eV) = 10.2 eV.

12. First, note that the radius of a Bohr orbit depends on n^2, and that the potential energy of the atom depends on $1/r$. The potential energy, then, depends on $1/n^2$. It follows that the potential energy in the $(n + 1)^{\text{th}}$ Bohr orbit is $Un^2/(n+1)^2$.

14. If an electron in the ground state absorbs a photon with an energy of 13.6 eV it will be on the verge of dissociation.

16. There is no upper limit to the wavelength of lines in the spectrum of hydrogen. The reason is that the wavelength is inversely proportional to the energy difference between successive energy levels. The spacing between these levels goes to zero as one moves to higher levels, and therefore the corresponding wavelengths go to infinity. There is a lower limit to the wavelength, however, because there is an upper limit of 13.6 eV to the energy difference between any two energy levels.

18. Figure 31-18 shows that there are 10 states that can hold electrons in the 3*d* subshell, and a total of 8 electron states in the $n = 2$ shell.

20. This carbon atom is in an excited state. This is clear from the fact that there is room for one more electron in the 2*p* subshell, but instead, this electron is in the 3*s* subshell. Since the energy of the 3*s* subshell is greater than that of the 2*p* subshell, this atom has more energy than it would in its ground state.

22. All of these questions can be answered by referring to Figure 31-17 and Table 31-3. **(a)** Not allowed; there is no *d* subshell in the $n = 2$ shell. **(b)** Not allowed for two reasons. First, there is no *p* subshell in the $n = 1$ shell. Second, a *p* subshell cannot hold 7 electrons. **(c)** Allowed. **(d)** Not allowed; the $n = 4$ shell does not have a *g* subshell.

24. No. Atoms in their ground states can emit no radiation. Even if an electron dropped from a highly excited state to the ground state in one of these atoms, the result would not be an X ray. The reason is that the binding energy of these atoms is simply much lower than the energy of a typical X-ray photon.

26. These elements all have similar configurations of their outermost electrons. In fact, the outermost electrons or fluorine, chlorine, and bromine are $2p^5$, $3p^5$, and $4p^5$, respectively. Therefore, each of these atoms is one electron shy of completing the *p* subshell. This accounts for their similar chemical behavior.

28. The radiation that is absorbed by a fluorescent material has a smaller wavelength than the radiation it emits. See Figure 31-29 for a specific example. Note that a smaller wavelength implies a higher frequency, and hence a higher energy for the corresponding photon. This, in turn, is in agreement with the physical mechanism illustrated in Figure 31-28.

Solutions to Problems

1.
$$\begin{aligned}\frac{V_{\text{nucleus}}}{V_{\text{atom}}} &= \frac{\frac{4}{3}\pi r_{\text{n}}^{3}}{\frac{4}{3}\pi r_{\text{a}}^{3}}\\ &= \left(\frac{r_{\text{n}}}{r_{\text{a}}}\right)^3\\ &= \left(\frac{0.50\times10^{-15}\text{ m}}{5.3\times10^{-11}\text{ m}}\right)^3\\ &= \boxed{8.4\times10^{-16}}\end{aligned}$$

2. Use a ratio of radii.
$$\begin{aligned}\frac{r_{\text{n}}}{r_{\text{a}}} &= \frac{r_{\text{baseball}}}{r_{\text{electron}}}\\ r_{\text{electron}} &= \left(\frac{r_{\text{a}}}{r_{\text{n}}}\right)r_{\text{baseball}}\\ &= \left(\frac{5.3\times10^{-11}\text{ m}}{0.50\times10^{-15}\text{ m}}\right)\left(\frac{7.3\times10^{-2}\text{ m}}{2}\right)\\ &= \boxed{3.9\text{ km}}\end{aligned}$$

3.
$$\begin{aligned} W &= -\Delta U \\ &= \frac{kqQ}{r} \\ &= \frac{k(2e)(29e)}{r} \\ &= \frac{58ke^2}{r} \\ &= \frac{58\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(1.60\times10^{-19}\ \text{C})^2}{\frac{4.8\times10^{-15}\ \text{m}}{2}} \\ &= \boxed{5.6\ \text{pJ}} \end{aligned}$$

4.
$$\begin{aligned} \Delta K &= -\Delta U \\ K &= \frac{kqQ}{d} \\ d &= \frac{kqQ}{K} \\ &= \frac{k(2e)(79e)}{K} \\ &= \frac{158ke^2}{K} \\ &= \frac{158\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(1.60\times10^{-19}\ \text{C})^2}{(3.0\times10^6\,\text{eV})\left(\frac{1.60\times10^{-19}\ \text{J}}{\text{eV}}\right)} \\ &= \boxed{76\ \text{fm}} \end{aligned}$$

5.
$$\begin{aligned} \frac{1}{\lambda} &= R\left(\frac{1}{2^2}-\frac{1}{n^2}\right) \\ \lambda &= \frac{1}{R\left(\frac{1}{2^2}-\frac{1}{15^2}\right)} \\ &= \frac{1}{(1.097\times10^7\,\text{m}^{-1})\left(\frac{1}{4}-\frac{1}{225}\right)} \\ &= \boxed{371.2\ \text{nm}} \end{aligned}$$

6. $\lambda = \dfrac{1}{R\left(\frac{1}{2^2}-\frac{1}{n^2}\right)} < 400\text{ nm}$

$$\frac{1}{R(400\text{ nm})} < \frac{1}{4}-\frac{1}{n^2}$$

$$\frac{1}{n^2} < \frac{1}{4}-\frac{1}{R(400\text{ nm})}$$

$$n > \frac{1}{\sqrt{\frac{1}{4}-\frac{1}{R(400\text{ nm})}}}$$

$$n > \frac{1}{\sqrt{\frac{1}{4}-\frac{1}{(1.097\times 10^7\text{ m}^{-1})(400\text{ nm})}}}$$

$$n > \boxed{7}$$

7. $\dfrac{1}{\lambda} = R\left(\dfrac{1}{1^2}-\dfrac{1}{n^2}\right)$

$$\lambda = \frac{1}{R\left(1-\frac{1}{n^2}\right)}$$

The longest wavelength corresponds to the smallest *n*.

$R = 1.097\times 10^7\text{ m}^{-1}$, so

$$\lambda_2 = \frac{1}{R\left(1-\frac{1}{2^2}\right)} = \boxed{121.5\text{ nm}}$$

$$\lambda_3 = \frac{1}{R\left(1-\frac{1}{3^2}\right)} = \boxed{102.6\text{ nm}}$$

$$\lambda_4 = \frac{1}{R\left(1-\frac{1}{4^2}\right)} = \boxed{97.23\text{ nm}}$$

8. $\dfrac{1}{\lambda} = R\left(\dfrac{1}{3^2}-\dfrac{1}{n^2}\right)$

$$\lambda = \frac{1}{R\left(\frac{1}{9}-\frac{1}{n^2}\right)}$$

The longest wavelength corresponds to the smallest *n*. $R = 1.097\times 10^7\text{ m}^{-1}$, so

$$\lambda_4 = \frac{1}{R\left(\frac{1}{9}-\frac{1}{4^2}\right)} = \boxed{1875\text{ nm}}$$

$$\lambda_5 = \frac{1}{R\left(\frac{1}{9}-\frac{1}{5^2}\right)} = \boxed{1282\text{ nm}}$$

$$\lambda_6 = \frac{1}{R\left(\frac{1}{9}-\frac{1}{6^2}\right)} = \boxed{1094\text{ nm}}$$

9. (a) The longest wavelength in the Lyman series corresponds to $n = 2$. $R = 1.097\times10^7\ \text{m}^{-1}$.

$$\lambda_2 = \frac{1}{R\left(\frac{1}{1^2}-\frac{1}{2^2}\right)} = \boxed{121.5\ \text{nm}}$$

(b) The shortest wavelength in the Paschen series corresponds to $n\to\infty$. $R = 1.097\times10^7\ \text{m}^{-1}$.

$$\lambda_\infty = \frac{1}{R\left(\frac{1}{3^2}-0\right)} = \boxed{820.4\ \text{nm}}$$

10. $$\lambda = \left[R\left(\frac{1}{n'^2}-\frac{1}{n^2}\right)\right]^{-1} = \left[\left(1.097\times10^7\ \text{m}^{-1}\right)\left(\frac{1}{n'^2}-\frac{1}{n^2}\right)\right]^{-1}$$

(a) Paschen Series
$n' = 3$
For $n = 4$, $\lambda = 1.875\ \mu\text{m}$.
For $n\to\infty$, $\lambda = 0.8204\ \mu\text{m}$.
$0.8204\ \mu\text{m} \le \lambda_\text{P} \le 1.875\ \mu\text{m}$

Brackett Series
$n' = 4$
For $n = 5$, $\lambda = 4.051\ \mu\text{m}$.
For $n\to\infty$, $\lambda = 1.459\ \mu\text{m}$.
$1.459\ \mu\text{m} \le \lambda_\text{B} \le 4.051\ \mu\text{m}$

$\boxed{\text{The series overlap in the range } 1.459\ \mu\text{m} \le \lambda \le 1.875\ \mu\text{m}.}$

(b) Balmer Series
$n' = 2$
For $n = 3$, $\lambda = 0.6563\ \mu\text{m}$.
For $n\to\infty$, $\lambda = 0.3646\ \mu\text{m}$.

Paschen Series
$0.8204\ \mu\text{m} \le \lambda_\text{P} \le 1.875\ \mu\text{m}$

$\boxed{\text{No}}$, there is no overlap of the Balmer series with the Paschen series.

11. $$v_n = \frac{2\pi ke^2}{nh}$$
$$v_2 = \frac{\pi ke^2}{h}$$
$$\frac{v_2}{c} = \frac{\pi ke^2}{ch} = \frac{\pi\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(1.60\times10^{-19}\ \text{C})^2}{\left(3.00\times10^8\ \frac{\text{m}}{\text{s}}\right)(6.63\times10^{-34}\ \text{J}\cdot\text{s})} = \boxed{3.64\times10^{-3}}$$

12. $$F = \frac{ke^2}{r_1^2} = \frac{\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(1.60\times10^{-19}\ \text{C})^2}{(5.29\times10^{-11}\ \text{m})^2} = \boxed{8.22\times10^{-8}\ \text{N}}$$

13. $$|E_n| = \frac{13.6\ \text{eV}}{n^2}$$
$$E_4 = \frac{13.6\ \text{eV}}{4^2} = \boxed{0.850\ \text{eV}}$$

14. $\Delta E = E_5 - E_2 = (-13.6\text{ eV})\left(\frac{1}{5^2} - \frac{1}{2^2}\right) = \boxed{2.86\text{ eV}}$

15. (a) $p_n = mv_n = m\left(\dfrac{2\pi ke^2}{nh}\right)$

$$p_3 = \frac{2\pi mke^2}{3h} = \frac{2\pi(9.11\times10^{-31}\text{ kg})\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(1.60\times10^{-19}\text{ C})^2}{3(6.63\times10^{-34}\text{ J}\cdot\text{s})} = \boxed{6.62\times10^{-25}\text{ kg}\cdot\text{m/s}}$$

(b) $L_n = \dfrac{nh}{2\pi}$

$$L_3 = \frac{3h}{2\pi} = \frac{3(6.63\times10^{-34}\text{ J}\cdot\text{s})}{2\pi} = \boxed{3.17\times10^{-34}\text{ J}\cdot\text{s}}$$

16. (a) $K_3 = \dfrac{1}{2}mv_3^{\,2}$

$$= \frac{1}{2}m\left(\frac{2\pi ke^2}{3h}\right)^2 = \frac{2\pi^2mk^2e^4}{9h^2} = \frac{2\pi^2(9.11\times10^{-31}\text{ kg})\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)^2(1.60\times10^{-19}\text{ C})^4}{9(6.63\times10^{-34}\text{ J}\cdot\text{s})^2}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = \boxed{1.50\text{ eV}}$$

(b) $U_n = -\dfrac{ke^2}{r_n}$

$$U_3 = -\frac{ke^2}{r_3} = -\frac{ke^2}{9r_1} = -\frac{(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2})(1.60\times10^{-19}\text{ C})^2}{9(5.29\times10^{-11}\text{ m})}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = \boxed{-3.02\text{ eV}}$$

(c) $E_3 = K_3 + U_3 = 1.50\text{ eV} - 3.02\text{ eV} = \boxed{-1.52\text{ eV}}$

17.
$$E_n = E_3 + 1.23\text{ eV}$$
$$\frac{-13.6\text{ eV}}{n^2} = \frac{-13.6\text{ eV}}{3^2} + 1.23\text{ eV}$$
$$\frac{1}{n^2} = \frac{1}{9} - \frac{1.23}{13.6}$$
$$n = \left(\frac{1}{9} - \frac{1.23}{13.6}\right)^{-1/2} = 6.96 = \boxed{7}$$

18.
$$\frac{1}{\lambda} = R\left(\frac{1}{n_\mathrm{f}^2} - \frac{1}{n_\mathrm{i}^2}\right)$$
$$\frac{1}{\lambda R} = \frac{1}{n_\mathrm{f}^2} - \frac{1}{n_\mathrm{i}^2}$$

Solving by trial and error, we find that $\boxed{n_\mathrm{i} = 3 \text{ and } n_\mathrm{f} = 2}$.
$$\frac{1}{(656\text{ nm})(1.097\times10^7\text{ m}^{-1})} = 0.139 = \frac{1}{2^2} - \frac{1}{3^2}$$

19. (a)
$$|\Delta E| = \left|Rhc\left(\frac{1}{n_\mathrm{f}^2} - \frac{1}{n_\mathrm{i}^2}\right)\right|$$
$$= (1.097\times10^7\text{ m}^{-1})(6.63\times10^{-34}\text{ J}\cdot\text{s})\left(3.00\times10^8\ \frac{\text{m}}{\text{s}}\right)\left(\frac{1}{3^2} - \frac{1}{5^2}\right)$$
$$= \boxed{1.55\times10^{-19}\text{ J}}$$

(b) The energy of the photon would be $\boxed{\text{less than}}$ that found in part (a), since the absolute values of the lower *n* energy state differences are greater than those of the higher *n* energy state differences, and in both cases $\Delta n = 2$.

(c) $|\Delta E| = (1.097\times10^7\text{ m}^{-1})(6.63\times10^{-34}\text{ J}\cdot\text{s})\left(3.00\times10^8\ \frac{\text{m}}{\text{s}}\right)\left(\frac{1}{5^2} - \frac{1}{7^2}\right) = \boxed{4.27\times10^{-20}\text{ J}}$

20. (a) The longest wavelength photon is emitted by the transition with the smallest $|\Delta E|$. The smallest $|\Delta E|$ results from transition (iii) because $\Delta n = 1$ is the smallest Δn and (iii) involves the highest states of the four transitions.
$$\lambda = \frac{1}{R\left(\frac{1}{n_\mathrm{i}^2} - \frac{1}{n_\mathrm{f}^2}\right)} = \frac{1}{(1.097\times10^7\text{ m}^{-1})\left(\frac{1}{7^2} - \frac{1}{8^2}\right)} = \boxed{1.906\times10^{-5}\text{ m}}$$

(b) The shortest wavelength photon is emitted by the transition with the largest $|\Delta E|$. The largest $|\Delta E|$ results from transition (ii) because $\Delta n = 6$ is the largest Δn and (ii) involves the lowest state, $n = 2$, of the four transitions.
$$\lambda = \frac{1}{R\left(\frac{1}{n_\mathrm{i}^2} - \frac{1}{n_\mathrm{f}^2}\right)} = \frac{1}{(1.097\times10^7\text{ m}^{-1})\left(\frac{1}{2^2} - \frac{1}{8^2}\right)} = \boxed{3.889\times10^{-7}\text{ m}}$$

(c) The atom loses energy in transition $\boxed{\text{(iv)}}$ because the electron drops from a higher to a lower energy state, thus emitting a photon.

21. (a) $r_1 = \dfrac{h^2}{4\pi^2 mke^2}$

$$= \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2}{4\pi^2(207)(9.11\times10^{-31}\ \text{kg})\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(1.60\times10^{-19}\ \text{C})^2}$$

$$= \boxed{2.57\times10^{-13}\ \text{m}}$$

(b) The wavelengths in the Balmer series of muonium will be $\boxed{\text{less than}}$ those for hydrogen because the wavelength is inversely proportional to the particle's mass.

(c) The longest wavelength photon results when $n = 3$.

$$\lambda = \frac{1}{207R\left(\frac{1}{2^2}-\frac{1}{n^2}\right)} = \frac{1}{207(1.097\times10^7\ \text{m}^{-1})\left(\frac{1}{4}-\frac{1}{3^2}\right)} = \boxed{3.17\ \text{nm}}$$

22. (a) $r_n = \dfrac{n^2h^2}{4\pi^2 mkZe^2}$

$$r_4 = \frac{4^2(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2}{4\pi^2(9.11\times10^{-31}\ \text{kg})\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(3)(1.60\times10^{-19}\ \text{C})^2}$$

$$= \boxed{2.83\times10^{-10}\ \text{m}}$$

(b) The energy required to raise an electron from the $n = 4$ state to the $n = 5$ state in Li^{2+} is $\boxed{\text{greater than}}$ that for hydrogen because the force on the electron due to the three protons in the Li^{2+} nucleus is stronger than that due to the single proton in the hydrogen nucleus.

(c) <u>hydrogen</u>

$$|\Delta E| = hcR\left(\frac{1}{n_\text{i}^2}-\frac{1}{n_\text{f}^2}\right)$$

$$= (6.63\times10^{-34}\ \text{J}\cdot\text{s})\left(3.00\times10^8\ \tfrac{\text{m}}{\text{s}}\right)(1.097\times10^7\ \text{m}^{-1})\left(\frac{1}{4^2}-\frac{1}{5^2}\right)$$

$$= \boxed{4.91\times10^{-20}\ \text{J}}$$

<u>Li^{2+}</u>

$$|\Delta E| = hcRZ^2\left(\frac{1}{n_\text{i}^2}-\frac{1}{n_\text{f}^2}\right)$$

$$= (6.63\times10^{-34}\ \text{J}\cdot\text{s})\left(3.00\times10^8\ \tfrac{\text{m}}{\text{s}}\right)(1.097\times10^7\ \text{m}^{-1})(3^2)\left(\frac{1}{4^2}-\frac{1}{5^2}\right)$$

$$= \boxed{4.42\times10^{-19}\ \text{J}}$$

23. (a) $\lambda = \dfrac{1}{Z^2 R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)}$

The shortest wavelength occurs when $n \to \infty$.

$$\lambda = \frac{1}{4^2(1.097\times 10^7\ \text{m}^{-1})(1-0)} = \boxed{5.697\ \text{nm}}$$

(b) $|\Delta E| = hcZ^2 R\left(\dfrac{1}{1^2} - 0\right)$

$$= (6.63\times 10^{-34}\ \text{J}\cdot\text{s})\left(3.00\times 10^8\ \tfrac{\text{m}}{\text{s}}\right)(4^2)(1.097\times 10^7\ \text{m}^{-1})$$

$$= \boxed{3.49\times 10^{-17}\ \text{J}}$$

24. (a) The circumference of the orbit is $C = 2\pi r_n = 2\pi r_2$.

$C = v_n t = v_2 t$

$$t = \frac{C}{v_2}$$

$$= \frac{2\pi r_2}{v_2}$$

$$= 2\pi\left(\frac{2^2 h^2}{4\pi^2 mke^2}\right)\left(\frac{2h}{2\pi ke^2}\right)$$

$$= \frac{2h^3}{\pi^2 mk^2 e^4}$$

$$= \frac{2(6.63\times 10^{-34}\ \text{J}\cdot\text{s})^3}{\pi^2(9.11\times 10^{-31}\ \text{kg})\left(8.99\times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)^2(1.60\times 10^{-19}\ \text{C})^4}$$

$$= \boxed{1.22\times 10^{-15}\ \text{s}}$$

(b) number of orbits $= \dfrac{10^{-8}\ \text{s}}{1.22\times 10^{-15}\ \text{s}} = \boxed{8\times 10^6\ \text{orbits}}$

25. (a) $K_n = \frac{1}{2}mv_n^2$

$$= \frac{1}{2}m\left(\frac{2\pi ke^2}{nh}\right)^2$$

$$n = \left(\frac{m2\pi^2k^2e^4}{K_nh^2}\right)^{1/2}$$

$$= \left[\frac{(9.11\times10^{-31}\text{ kg})(2\pi^2)\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)^2(1.60\times10^{-19}\text{C})^4}{(1.35\times10^{-19}\text{ J})(6.63\times10^{-34}\text{ J}\cdot\text{s})^2}\right]^{1/2}$$

$$= \boxed{4}$$

(b) It $\boxed{\text{decreases}}$, because K is inversely proportional to orbit number.

(c) $K_5 = \frac{1}{2}mv_5^2 = \frac{1}{2}m\left(\frac{2\pi ke^2}{5h}\right)^2 = \frac{1}{25}K_1$, but $K_4 = \frac{1}{16}K_1$, so $K_1 = 16K_4$.

Thus, $K_5 = \frac{16}{25}K_4 = \frac{16}{25}(1.35\times10^{-19}\text{ J}) = \boxed{8.64\times10^{-20}\text{ J}}$.

26. (a) $U_n = -\frac{ke^2}{r_n} = -\frac{ke^2}{n^2r_1}$

$$n = \sqrt{\frac{-ke^2}{r_1U_n}} = \sqrt{\frac{-(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})^2}{(5.29\times10^{-11}\text{ m})(-1.20\times10^{-19}\text{ J})}} = \boxed{6}$$

(b) It becomes a smaller negative number and thus $\boxed{\text{increases}}$, since positive work is done on the electron to move it farther away from the nucleus.

(c) $U_7 = -\frac{ke^2}{r_7} = -\frac{ke^2}{49r_1} = -\frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})^2}{49(5.29\times10^{-11}\text{ m})} = \boxed{-8.88\times10^{-20}\text{ J}}$

27. $\frac{1}{2}mv^2 = hcR\left(1-\frac{1}{2^2}\right)$

$$v^2 = \frac{2}{m}\left(\frac{3}{4}hcR\right)$$

$$v = \sqrt{\frac{3hcR}{2m}}$$

$$= \sqrt{\frac{3(6.63\times10^{-34}\text{ J}\cdot\text{s})\left(3.00\times10^8\ \frac{\text{m}}{\text{s}}\right)(1.097\times10^7\text{ m}^{-1})}{2(1.674\times10^{-27}\text{ kg})}}$$

$$= \boxed{44.2\text{ km/s}}$$

28. (a) $|\Delta E| = \dfrac{2\pi^2 mk^2 e^4}{h^2}\left(\dfrac{1}{n_f^{\,2}} - \dfrac{1}{n_i^{\,2}}\right) = hf$

$$f = \boxed{\frac{2\pi^2 mk^2 e^4}{h^3}\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]}$$

(b) $f = \dfrac{1}{T} = \dfrac{v_n}{2\pi r_n} = \left(\dfrac{2\pi ke^2}{nh}\right)\left(\dfrac{1}{2\pi}\right)\left(\dfrac{4\pi^2 mke^2}{n^2h^2}\right) = \boxed{\dfrac{4\pi^2 mk^2 e^4}{n^3h^3}}$

(c) $\dfrac{1}{(n-1)^2} - \dfrac{1}{n^2} = \dfrac{n^2 - (n-1)^2}{n^2(n-1)^2} = \dfrac{n^2 - n^2 + 2n - 1}{n^2(n^2 - 2n + 1)} = \dfrac{2n-1}{n^4 - 2n^3 + n^2}$

As n becomes very large,

$$\frac{2n-1}{n^4 - 2n^3 + n^2} \to \frac{2}{n^3}.$$

Substitute $2/n^3$ into the frequency from part (a) and compare to that from part (b).

$$f_p = \frac{2\pi^2 mk^2 e^4}{h^3}\left(\frac{2}{n^3}\right) = \frac{4\pi^2 mk^2 e^4}{n^3h^3} = f_e$$

For very large n, the frequency of the emitted photon is the same as that of the electron's orbital motion.

29. $\lambda_n = \dfrac{2\pi r_n}{n}$

$\lambda_1 = \dfrac{2\pi r_1}{1} = 2\pi(5.29\times10^{-11}\text{ m}) = \boxed{3.32\times10^{-10}\text{ m}}$

30. $n\lambda = 2\pi r$

$n\lambda_n = 2\pi r_n$

$= 2\pi\left(\dfrac{n^2h^2}{4\pi^2 mke^2}\right)$

$\lambda = \boxed{\dfrac{nh^2}{2\pi mke^2}}$

31. There are five wavelengths, which corresponds to the $n = 5$ state of hydrogen.

$r_5 = 5^2 r_1 = 25(5.29\times10^{-11}\text{ m}) = \boxed{1.32\times10^{-9}\text{ m}}$

32. (a) $K = \dfrac{p^2}{2m} = \dfrac{1}{2m}\left(\dfrac{h}{\lambda}\right)^2 = \dfrac{h^2}{2m\lambda^2} = \dfrac{(6.63\times10^{-34}\text{ J}\cdot\text{s})^2}{2(9.11\times10^{-31}\text{ kg})(0.5\times10^{-10}\text{ m})^2}\left(\dfrac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = \boxed{0.6\text{ keV}}$

(b) $K = \dfrac{h^2}{2m\lambda^2} = \dfrac{(6.63\times10^{-34}\text{ J}\cdot\text{s})^2}{2(9.11\times10^{-31}\text{ kg})(10^{-15}\text{m})^2}\left(\dfrac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = \boxed{2\times10^{12}\text{ eV}}$

33. $\ell = 0, 1, 2, \ldots, (n-1)$

$n - 1 = 5 - 1 = 4$

The allowed values are $\ell = \boxed{0, 1, 2, 3, 4.}$

34. $m_\ell = -\ell,\ -\ell+1, -\ell+2, \ldots, -1, 0, 1, \ldots, \ell-2, \ell-1, \ell$
$n - 1 = 4 - 1 = 3 = \max\ \ell$
$m_\ell = -3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3$
There are $\boxed{7}$ possible values of m_ℓ.

35. $L = \sqrt{\ell(\ell+1)}\ (h/2\pi)$

(a) $\sqrt{6} = \sqrt{\ell(\ell+1)}$ yields $\ell = \boxed{2}$

(b) $\sqrt{15} = \sqrt{\ell(\ell+1)} \rightarrow \boxed{\text{no integer } \ell \text{ exists}}$

(c) $\sqrt{30} = \sqrt{\ell(\ell+1)}$ yields $\ell = \boxed{5}$

(d) $\sqrt{36} = \sqrt{\ell(\ell+1)} \rightarrow \boxed{\text{no integer } \ell \text{ exists}}$

36. (a) $E_4 = -\dfrac{(13.6\text{ eV})}{4^2} = \boxed{-0.850\text{ eV}}$

(b) $L = \sqrt{3(4)}\ (h/2\pi) = \dfrac{\sqrt{12}(6.63\times10^{-34}\text{ J}\cdot\text{s})}{2\pi} = \boxed{3.66\times10^{-34}\text{ J}\cdot\text{s}}$

(c) It is $\boxed{\text{greater}}$, because an $n = 5$ state is farther from the nucleus and has a smaller negative energy.

(d) It is $\boxed{\text{greater}}$, because $\ell = 3$ for an *f* sublevel and $\ell = 2$ for a *d* sublevel.

37. (a)

$$10\sqrt{57} = \sqrt{\ell(\ell+1)}$$
$$5700 = \ell(\ell+1)$$
$$\ell^2 + \ell - 5700 = 0$$

Using the quadratic formula, $\ell = \dfrac{-1 \pm \sqrt{1-4(1)(-5700)}}{2}$.

$\ell = \dfrac{-1\pm151}{2} = \boxed{75}$, -76. Reject -76 since $\ell \ge 0$.

(b) Because $\ell_{\max} = n-1$ and $n > \ell$, $n_{\min} = \ell_{\max} + 1 = \boxed{76}$.

(c) $E_{76} = -\dfrac{13.6\text{ eV}}{(76)^2} = \boxed{-0.00235\text{ eV}}$

38. (a) $E_n = -\dfrac{13.6\text{ eV}}{n^2}$

$$n = \sqrt{\frac{-13.6\text{ eV}}{-0.544\text{ eV}}} = \boxed{5}$$

(b) $\ell_{\max} = n-1 = 5-1 = 4$, so $L = \sqrt{4(5)}\ (h/2\pi) = \dfrac{\sqrt{20}(6.63\times10^{-34}\text{ J}\cdot\text{s})}{2\pi} = \boxed{4.72\times10^{-34}\text{ J}\cdot\text{s}}$

(c) For a given value of ℓ there are $2\ell+1$ possible values of m_ℓ and for each value of m_ℓ there are 2 possible values of m_s. Thus, there are $2(2\ell+1)$ states in each ℓ sublevel: For $\ell=3$, $2(2\ell+1)=\boxed{14}$.

39. (a) Because the maximum magnetic quantum number in state I is greater than that in state II, ℓ_{I} is greater than ℓ_{II} and, therefore, L_{I} is $\boxed{\text{greater than}}$ L_{II}.

(b) maximum $m_{\ell\text{I}}=3 \rightarrow \ell_{\text{I}}=3$
maximum $m_{\ell\text{II}}=2 \rightarrow \ell_{\text{II}}=2$

$$\frac{L_{\text{I}}}{L_{\text{II}}}=\frac{\sqrt{\ell_{\text{I}}(\ell_{\text{I}}+1)}\,\frac{h}{2\pi}}{\sqrt{\ell_{\text{II}}(\ell_{\text{II}}+1)}\,\frac{h}{2\pi}}=\frac{\sqrt{3(3+1)}}{\sqrt{2(2+1)}}=\boxed{\sqrt{2}}$$

40. Carbon has six protons ($Z = 6$) and six electrons.
$\boxed{1s^2 2s^2 2p^2}$

41.

	n	ℓ	m_ℓ	m_s
$1s^1$	1	0	0	$-\frac{1}{2}$
$1s^2$	1	0	0	$\frac{1}{2}$
$2s^1$	2	0	0	$-\frac{1}{2}$
$2s^2$	2	0	0	$\frac{1}{2}$
$2p^1$	2	1	–1	$-\frac{1}{2}$
$2p^2$	2	1	–1	$\frac{1}{2}$
$2p^3$	2	1	0	$-\frac{1}{2}$
$2p^4$	2	1	0	$\frac{1}{2}$
$2p^5$	2	1	1	$-\frac{1}{2}$
$2p^6$	2	1	1	$\frac{1}{2}$

42. Nitrogen has seven protons ($Z = 7$) and seven electrons.
$\boxed{1s^2 2s^2 2p^3}$

43. In the 3*s* subshell, $n = 3,\ \ell = 0,\ m_\ell = 0$, and $m_\mathrm{s} = \pm\frac{1}{2}$.

n	ℓ	m_ℓ	m_s
3	0	0	$-\frac{1}{2}$
3	0	0	$\frac{1}{2}$

44. In the 3*p* subshell, $n = 3,\ \ell = 1,\ m_\ell = -1,\ 0,\ 1$, and $m_\mathrm{s} = \pm\frac{1}{2}$.

n	ℓ	m_ℓ	m_s
3	1	−1	$-\frac{1}{2}$
3	1	−1	$\frac{1}{2}$
3	1	0	$-\frac{1}{2}$
3	1	0	$\frac{1}{2}$
3	1	1	$-\frac{1}{2}$
3	1	1	$\frac{1}{2}$

45.

	n	ℓ	m_ℓ	m_s
$1s^1$	1	0	0	$-\frac{1}{2}$
$1s^2$	1	0	0	$\frac{1}{2}$
$2s^1$	2	0	0	$-\frac{1}{2}$
$2s^2$	2	0	0	$\frac{1}{2}$
$2p^1$	2	1	–1	$-\frac{1}{2}$
$2p^2$	2	1	–1	$\frac{1}{2}$
$2p^3$	2	1	0	$-\frac{1}{2}$
$2p^4$	2	1	0	$\frac{1}{2}$
$2p^5$	2	1	1	$-\frac{1}{2}$
$2p^6$	2	1	1	$\frac{1}{2}$
$3s^1$	3	0	0	$-\frac{1}{2}$
$3s^2$	3	0	0	$\frac{1}{2}$

46. Nickel has 28 protons ($Z = 28$) and 28 electrons.

$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 3d^8 4s^2}$

47. For each ℓ, there are $2(2\ell + 1)$ states.

(a) $n = 2$
$\ell = 0,\ 1$

total number of states $= 2[2(0)+1] + 2[2(1)+1] = 2 + 6 = \boxed{8}$

(b) $n = 3$
$\ell = 0,\ 1,\ 2$

total number of states $= 2[2(0)+1] + 2[2(1)+1] + 2[2(2)+1] = 2 + 6 + 10 = \boxed{18}$

(c) $n = 4$
$\ell = 0,\ 1,\ 2,\ 3$

total number of states $= 18 + 2[2(3)+1] = 18 + 14 = \boxed{32}$

48. The states are given by

$$\sum_{\ell=0}^{n-1} 2(2\ell+1) = 2\sum_{\ell=0}^{n-1}(2\ell+1) = 2\{1+3+5+\ldots+[2(n-1)+1]\} = 2[1+3+5+\ldots+(2n-1)]$$

The sum in the brackets is the arithmetic series

$1+3+5+\ldots+(2n-1)=n^2$. So, $\sum_{\ell=0}^{n-1} 2(2\ell+1) = 2n^2$.

49.

	n	ℓ	m_ℓ	m_s
$5d^1$	5	2	–2	$-\frac{1}{2}$
$5d^2$	5	2	–2	$\frac{1}{2}$
$5d^3$	5	2	–1	$-\frac{1}{2}$
$5d^4$	5	2	–1	$\frac{1}{2}$
$5d^5$	5	2	0	$-\frac{1}{2}$
$5d^6$	5	2	0	$\frac{1}{2}$
$5d^7$	5	2	1	$-\frac{1}{2}$
$5d^8$	5	2	1	$\frac{1}{2}$
$5d^9$	5	2	2	$-\frac{1}{2}$
$5d^{10}$	5	2	2	$\frac{1}{2}$

50. $|\Delta E| = \frac{hc}{\lambda}$

$$\lambda = \frac{hc}{|\Delta E|}$$

$$\Delta E = E_K - E_L = -(13.6 \text{ eV})\left[\frac{(28-1)^2}{1^2} - \frac{(28-1)^2}{2^2}\right] = -7.44\,\text{keV}$$

$$\lambda = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})\left(3.00\times10^8 \ \frac{\text{m}}{\text{s}}\right)}{7435.8 \text{ eV}}\left(\frac{1 \text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = \boxed{0.167 \text{ nm}}$$

51. $|\Delta E| = E_L - E_K = -(13.6 \text{ eV})(82-1)^2\left(\frac{1}{2^2}-\frac{1}{1^2}\right) = \boxed{66.9 \text{ keV}}$

52. $\lambda = \frac{hc}{|\Delta E|} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})\left(3.00\times10^8 \ \frac{\text{m}}{\text{s}}\right)}{8500 \text{ eV}-2125 \text{ eV}}\left(\frac{1 \text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = \boxed{0.195 \text{ nm}}$

53. $$|\Delta E| = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})\left(3.00\times10^{8}\ \frac{\text{m}}{\text{s}}\right)}{0.0205\times10^{-9}\text{ m}} = 9.70\times10^{-15}\text{ J}$$

$$|\Delta E| = E_{\text{L}} - E_{\text{K}} = -(13.6\text{ eV})(Z-1)^2\left(\tfrac{1}{4}-1\right) = 9.70\times10^{-15}\text{ J}$$

$$(Z-1)^2 = 5940$$
$$Z-1 = 77$$
$$Z = \boxed{78}$$

54. **(a)** $$K_{\text{min}} = |E_{\text{K}}| = (13.6\text{ eV})\frac{(Z-1)^2}{1^2} = (13.6\text{ eV})(78-1)^2 = \boxed{80.6\text{ keV}}$$

(b) $V = \boxed{80.6\text{ kV}}$

55. **(a)** $$|\Delta E| = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})\left(3.00\times10^{8}\ \frac{\text{m}}{\text{s}}\right)}{193\times10^{-9}\text{ m}} = \boxed{1.03\times10^{-18}\text{ J}}$$

(b) $$\frac{E_{\text{total}}}{E_{\text{photon}}} = \frac{\lambda E_{\text{total}}}{hc} = \frac{(193\times10^{-9}\text{ m})(1.58\times10^{-13}\text{ J})}{(6.63\times10^{-34}\text{ J}\cdot\text{s})\left(3.00\times10^{8}\ \frac{\text{m}}{\text{s}}\right)} = \boxed{1.53\times10^{5}\text{ photons}}$$

56. **(a)** P = (number of photons per second)(energy per photon)

$$= (1.36\times10^{19}\text{ s}^{-1})\frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})\left(3.00\times10^{8}\ \frac{\text{m}}{\text{s}}\right)}{514\times10^{-9}\text{ m}}$$
$$= \boxed{5.26\text{ W}}$$

(b) Since $P \propto \frac{1}{\lambda}$, and since 414 nm < 514 nm, the power output of the second laser is $\boxed{\text{greater than}}$ that of the first.

(c) $$P = (1.36\times10^{19}\text{ s}^{-1})\frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})\left(3.00\times10^{8}\ \frac{\text{m}}{\text{s}}\right)}{414\times10^{-9}\text{ m}} = \boxed{6.53\text{ W}}$$

57. $E = hf = E_1$

$$f = \frac{(13.6\text{ eV})\left(1.60\times10^{-19}\ \frac{\text{J}}{\text{eV}}\right)}{6.63\times10^{-34}\text{ J}\cdot\text{s}} = \boxed{3.28\times10^{15}\text{ Hz}}$$

58. (a) $p = \frac{E}{c}$

$= \frac{1}{c}Rhc\left(\frac{1}{2^2} - \frac{1}{4^2}\right)$

$= (1.097\times10^7\ \text{m}^{-1})(6.63\times10^{-34}\ \text{J}\cdot\text{s})\left(\frac{1}{4} - \frac{1}{16}\right)$

$= \boxed{1.36\times10^{-27}\ \text{kg}\cdot\text{m/s}}$

(b) $mv = p$

$v = \frac{p}{m}$

$= \frac{1.364\times10^{-27}\ \text{kg}\cdot\frac{\text{m}}{\text{s}}}{1.674\times10^{-27}\ \text{kg}}$

$= \boxed{81.5\ \text{cm/s}}$

59. (a) $E = Rhc\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = (1.097\times10^7\ \text{m}^{-1})(6.63\times10^{-34}\ \text{J}\cdot\text{s})\left(3.00\times10^8\ \frac{\text{m}}{\text{s}}\right)\left(\frac{1}{4} - \frac{1}{16}\right) = \boxed{4.09\times10^{-19}\ \text{J}}$

(b) $p_{\text{photon}} = \frac{E}{c}$

The magnitude of the atom's momentum must equal that of the photon.

$K = \frac{p^2}{2m} = \frac{E^2}{2mc^2} = \frac{(4.09\times10^{-19}\ \text{J})^2}{2(1.674\times10^{-27}\ \text{kg})\left(3.00\times10^8\ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{5.55\times10^{-28}\ \text{J}}$

(c) They are $\boxed{\text{the same}}$ because of conservation of energy.

60. $\frac{3}{2}kT = E = Rhc\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$

$T = \frac{Rhc}{2k} = \frac{(1.097\times10^7\ \text{m}^{-1})(6.63\times10^{-34}\ \text{J}\cdot\text{s})\left(3.00\times10^8\ \frac{\text{m}}{\text{s}}\right)}{2\left(1.38\times10^{-23}\ \frac{\text{J}}{\text{K}}\right)} = \boxed{79{,}100\ \text{K}}$

61. (a) $\frac{E}{\frac{1}{4}\pi d^2\Delta t} = \frac{4(2.75\times10^{-3}\ \text{J})}{\pi(32.0\times10^{-6}\ \text{m})^2(1.25\times10^{-9}\ \text{s})} = \boxed{2.74\times10^{15}\ \text{W/m}^2}$

(b) $E_{\text{mol}} = \frac{4E}{\pi d^2}\left(\frac{1}{4}\pi d_{\text{mol}}{}^2\right) = E\left(\frac{d_{\text{mol}}}{d}\right)^2 = (2.75\times10^{-3}\ \text{J})\left(\frac{0.650\times10^{-9}\ \text{m}}{32.0\times10^{-6}\ \text{m}}\right)^2 = \boxed{1.13\times10^{-12}\ \text{J}}$

62. (a) $t = \frac{C}{v} = \frac{2\pi r_1}{\sqrt{\frac{2K}{m}}} = \pi r_1\sqrt{\frac{2m}{K}} = \pi(5.29\times10^{-11}\ \text{m})\sqrt{\frac{2(9.11\times10^{-31}\ \text{kg})}{(13.6\ \text{eV})\left(\frac{1.60\times10^{-19}\ \text{J}}{1\ \text{eV}}\right)}} = \boxed{1.52\times10^{-16}\ \text{s}}$

(b) $I = \dfrac{1.60\times10^{-19}\text{ C}}{1.52\times10^{-16}\text{ s}} = \boxed{1.05\text{ mA}}$

63. (a) The total and kinetic energies of hydrogen atom Bohr orbits are given by $-ke^2/(2r)$ and $ke^2/(2r)$, respectively.
So, $K = \boxed{0.85\text{ eV}}$.

(b) $U = E - K = -0.85\text{ eV} - 0.85\text{ eV} = \boxed{-1.70\text{ eV}}$

64. (a) The energy required to remove the last electron from a singly ionized helium atom is
$E = (13.6\text{ eV})(2)^2 = 54.4\text{ eV}.$
This is the largest transition energy associated with helium. Since energy is inversely related to wavelength, this energy corresponds to the smallest wavelength observed due to helium.

$$\lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})\left(3.00\times10^{8}\ \frac{\text{m}}{\text{s}}\right)}{(54.4\text{ eV})\left(\frac{1.60\times10^{-19}\text{ J}}{1\text{ eV}}\right)} = \boxed{22.9\text{ nm}}$$

(b) $|E| = \dfrac{hc}{\lambda} = 4hcR\left(\dfrac{1}{n_\text{f}^2} - \dfrac{1}{n_\text{i}^2}\right)$

$$\frac{1}{n_\text{f}^2} - \frac{1}{n_\text{i}^2} = \frac{1}{4\lambda R}$$

By trial-and-error, we find that $\boxed{n_\text{i} = 16 \text{ and } n_\text{f} = 4}$.

$$\frac{1}{4^2} - \frac{1}{16^2} = 0.0586 = \frac{1}{4(388.9\times10^{-9}\text{ m})(1.097\times10^{7}\text{ m}^{-1})}$$

65. (a) $r_6 = \dfrac{36h^2}{4\pi^2 mkZe^2}$

$$Z = \frac{36h^2}{4\pi^2 mke^2 r_6}$$

$$= \frac{36(6.63\times10^{-34}\text{ J}\cdot\text{s})^2}{4\pi^2(9.11\times10^{-31}\text{ kg})\left(8.99\times10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(1.60\times10^{-19}\text{ C})^2(2.72\times10^{-10}\text{ m})}$$

$$= 7.03$$

$$\approx \boxed{7}$$

(b) $E_3 = -(13.6\text{ eV})\dfrac{Z^2}{3^2} = -(13.6\text{ eV})\dfrac{(7)^2}{9} = \boxed{-74.0\text{ eV}}$

66. $E_{\text{K}} = -(13.6\text{ eV})\dfrac{(Z-1)^2}{1^2} = -(13.6\text{ eV})(42-1)^2 = -1681(13.6\text{ eV})$

$$E_{\text{M}} = -(13.6\text{ eV})\frac{(Z-7)^2}{3^2} = -(13.6\text{ eV})\frac{(42-9)^2}{9} = \frac{-1089}{9}(13.6\text{ eV})$$

$$\lambda = \frac{hc}{|\Delta E|} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})\left(3.00\times10^{8}\ \frac{\text{m}}{\text{s}}\right)}{\left|(13.6\text{ eV})\left[-1681-\left(\frac{-1089}{9}\right)\right]\left(\frac{1.60\times10^{-19}\text{ J}}{1\text{ eV}}\right)\right|} = \boxed{0.0586\text{ nm}}$$

67. $E_{\text{L}} = -(13.6\text{ eV})\dfrac{(Z-1)^2}{2^2} = -(13.6\text{ eV})\dfrac{(42-2)^2}{4} = -\dfrac{1600}{4}(13.6\text{ eV})$

$$E_{\text{M}} = -(13.6\text{ eV})\frac{(Z-7)^2}{3^2} = -(13.6\text{ eV})\frac{(42-9)^2}{9} = -\frac{1089}{9}(13.6\text{ eV})$$

$$\lambda = \frac{hc}{|\Delta E|} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})\left(3.00\times10^{8}\ \frac{\text{m}}{\text{s}}\right)}{\left|(13.6\text{ eV})\left[-\frac{1600}{4}-\left(-\frac{1089}{9}\right)\right]\left(\frac{1.60\times10^{-19}\text{ J}}{1\text{ eV}}\right)\right|} = \boxed{0.328\text{ nm}}$$

68. (a) $hf = E_{\text{photon}} = -\Delta E_{\text{atom}} = -(E_{\text{f}} - E_{\text{i}})$

$$\frac{hc}{\lambda} = E_{\text{i}} - E_{\text{f}}$$

For He^+, $E_{\text{n}} = -\dfrac{(13.6\text{ eV})(4)}{n^2}$ since $Z = 2$.

$$\frac{1}{\lambda} = \frac{1}{hc}\left[-\frac{54.4\text{ eV}}{n_{\text{i}}^2} - \frac{(-54.4\text{ eV})}{n_{\text{f}}^2}\right]$$

$$= \left(\frac{54.4\text{ eV}}{hc}\right)\left(\frac{1}{n_{\text{f}}^2} - \frac{1}{n_{\text{i}}^2}\right)$$

Since 54.4 eV > 13.6 eV, C is $\boxed{\text{greater than}}$ R.

(b) $C = \dfrac{(54.4\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})} = \boxed{4.38\times10^{7}\text{ m}^{-1}}$

(c) Pickering: $\dfrac{1}{\lambda} = (4.38\times10^{7}\text{ m}^{-1})\left(\dfrac{1}{4^2} - \dfrac{1}{6^2}\right) = 0.152\times10^{7}\text{ m}^{-1}$

$$\lambda = 6.58\times10^{-7}\text{ m} = 658\text{ nm}$$

Balmer:

$$\frac{1}{\lambda} = (1.097\times10^{7}\text{ m}^{-1})\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 0.152\times10^{7}\text{ m}^{-1}$$

$\lambda = 658$ nm

For $n = 6$, the assertion is verified.

69. (a) $r_n = (5.29\times10^{-11}\text{ m})n^2$

$$n = \sqrt{\frac{r_n}{5.29\times10^{-11}\text{ m}}} = \sqrt{\frac{8.0\times10^{-6}\text{ m}}{5.29\times10^{-11}\text{ m}}} = \boxed{389}$$

(b) $\frac{1}{\lambda} = (1.097\times10^{7}\text{ m}^{-1})\left(\frac{1}{388^2}-\frac{1}{389^2}\right) = 0.374\text{ m}^{-1}$

$\lambda = \boxed{2.67\text{ m}}$

(c) As n decreases, the energy levels are farther apart, so as n decreases, the energy released is greater. Since $\lambda \propto \frac{1}{\Delta E}$, the wavelength is $\boxed{\text{less than}}$ that found in part (b).

70. (a) $L = rmv$, so $L_n = r_n m v_n = n\hbar$, and $r_n = \frac{n\hbar}{mv_n}$.

Now,

$F_{\text{cp}} = \frac{mv^2}{r} = qBv$. So, $v_n = \frac{r_n qB}{m}$.

Substitute.

$$r_n = \frac{n\hbar}{mv_n}$$

$$r_n = \frac{n\hbar}{m}\left(\frac{m}{r_n qB}\right)$$

$$r_n{}^2 = \frac{n\hbar}{qB}$$

$$r_n = \sqrt{\frac{n\hbar}{qB}}$$

(b) $v_n = \frac{r_n qB}{m} = \frac{qB}{m}\sqrt{\frac{n\hbar}{qB}} = \boxed{\frac{1}{m}\sqrt{nqB\hbar}}$

71. (a) $p_n = \frac{h}{\lambda_n} = \frac{h}{\frac{2L}{n}} = \frac{nh}{2L}$ where $n = 1, 2, 3, \ldots$

(b) $E_n = \frac{p_n{}^2}{2m} = \frac{1}{2m}\left(\frac{nh}{2L}\right)^2 = n^2\left(\frac{h^2}{8mL^2}\right)$ where $n = 1, 2, 3, \ldots$

72. $T = \frac{C_n}{v_n} = \frac{2\pi r_n}{v_n} = 2\pi r_n\left(\frac{2\pi m r_n}{nh}\right) = \frac{4\pi^2 m r_n{}^2}{nh} = \frac{4\pi^2 m}{nh}\left(\frac{n^4h^4}{16\pi^4 m^2k^2e^4}\right) = \frac{n^3h^3}{4\pi^2 mk^2e^4} = T_1 n^3$