

Chapter 9

Linear Momentum and Collisions

Answers to Even-numbered Conceptual Questions

2. Doubling an object's speed increases its kinetic energy by a factor of four, and its momentum by a factor of two.
4. No. For example, suppose object 2 has four times the mass of object 1. In this case, the two objects have the same kinetic energy if object 2 has half the speed of object 1. On the other hand, it follows that the momentum of object 2 is twice the momentum of object 1.
6. No. Consider, for example, a system of two particles. The total momentum of this system will be zero if the particles move in opposite directions with equal momentum. The kinetic energy of each particle is positive, however, and hence the total kinetic energy is also positive.
8. **(a)** The force due to braking – which ultimately comes from friction with the road – reduces the momentum of the car. The momentum lost by the car does not simply disappear, however. Instead, it shows up as an increase in the momentum of the Earth. **(b)** As with braking, the ultimate source of the force accelerating the car is the force of static friction between the tires and the road.
10. It is better if the collision is inelastic, because then the light pole gives your car only enough impulse to bring it to rest. If the collision is elastic, the impulse given to your car is about twice as much. This additional impulse – which acts over a very short period of time – could cause injury.
12. The rate of change in momentum is the same for both objects. As a result, the rate of change in velocity for the less massive object (the pebble) must be greater than it is for the more massive object (the boulder). Alternatively, we know that the acceleration (rate of change in velocity) of an object is proportional to the force acting on it and inversely proportional to its mass. These objects experience the same force, and therefore the less massive object has the greater acceleration.
14. Yes. Just point the fan to the rear of the boat. The resulting thrust will move the boat forward.
16. No. Any collision between cars will be at least partially inelastic, due to denting, sound production, heating, and other effects.
18. Yes. For example, we know that in a one-dimensional elastic collision of two objects of equal mass the objects “swap” speeds. Therefore, if one object was at rest before the collision, it is possible for one object to be at rest after the collision as well. See Figure 9-7 (a).
20. The speed of the ball when it leaves the tee is about twice the speed of the club. This follows for two reasons: (i) The collision is approximately elastic; and (ii) the mass of the club and the arms swinging the club is much greater than the mass of the ball. See Figure 9-7 (c).

22. Kinetic energy can be lost as objects rub against one another during a collision, doing negative, nonconservative work. These are internal forces, however, and hence they can have no effect on the total momentum of the system.
24. The speed of the ball after bouncing off the elephant will be greater than the speed it had before the collision. The situation is similar to that shown in Figure 9-7 (c), except that the small mass has a nonzero speed before the collision.
26. As this jumper clears the bar, a significant portion of his body extends below the bar due to the extreme arching of his back. Just as the center of mass of a donut can lie outside the donut, the center of mass of the jumper can be outside his body. In extreme cases, the center of mass can even be below the bar at all times during the jump.
28. The center of mass is higher than the midway point between the tip of the stalactite and the cave floor. The reason is that as the drops fall, their separations increase (see Conceptual Checkpoint 2-5). With the drops more closely spaced on the upper half of their fall, the center of mass is shifted above the halfway mark.
30. The center of mass of the hourglass starts at rest in the upper half of the glass and ends up at rest in the lower half. Therefore, the center of mass accelerates downward when the sand begins to fall – to get it moving downward – and then accelerates upward when most of the sand has fallen – to bring it to rest again. It follows from Equation 9-18 that the weight read by the scale is less than Mg when the sand begins falling, but is greater than Mg when most of the sand has fallen.
32. The scale supports the juggler and the three balls for an extended period of time. Therefore, we conclude that the *average* reading of the scale is equal to the weight of the juggler plus the weight of the three balls.
34. (a) Assuming a very thin base, we conclude that the center of mass of the glass is at its geometric center. (b) In the early stages of filling, the center of mass is below the center of the glass. When the glass is practically full, the center of mass is again at the geometric center of the glass. Thus, as water is added, the center of mass first moves downward, then turns around and moves back upward to its initial position.

Solutions to Problems

1. Setting the momentum of the ball equal to that of the car gives

$$P_b = m_b v_b = P_c$$

$$v_b = \frac{P_c}{m_b} = \frac{15,800 \text{ kg} \cdot \frac{\text{m}}{\text{s}}}{0.142 \text{ kg}} = 111.3 \frac{\text{km}}{\text{s}} = \left(\frac{111.3 \text{ km}}{\text{s}} \right) \left(\frac{3600 \text{ s}}{\text{h}} \right) \left(\frac{1 \text{ mi}}{1.609 \text{ km}} \right) = \boxed{2.49 \times 10^5 \text{ mi/h}}$$

2. $\vec{p}_{d1} = 4.40 \text{ kg} \cdot \frac{\text{m}}{\text{s}} \hat{x}$

$$\vec{p}_{d2} = -4.40 \text{ kg} \cdot \frac{\text{m}}{\text{s}} \hat{y}$$

$$\vec{p}_g = (9.00 \text{ kg}) \left(-1.30 \frac{\text{m}}{\text{s}} \right) \hat{y} = -11.7 \text{ kg} \cdot \frac{\text{m}}{\text{s}} \hat{y}$$

$$\vec{p}_{\text{total}} = \vec{p}_{d1} + \vec{p}_{d2} + \vec{p}_g = 4.40 \text{ kg} \cdot \frac{\text{m}}{\text{s}} \hat{x} - 4.40 \text{ kg} \cdot \frac{\text{m}}{\text{s}} \hat{y} - 11.7 \text{ kg} \cdot \frac{\text{m}}{\text{s}} \hat{y} = \boxed{4.40 \text{ kg} \cdot \text{m/s } \hat{x} - 16.1 \text{ kg} \cdot \text{m/s } \hat{y}}$$

3. Determine the total momentum of the dog and cat. Let north be in the y -direction.

$$\vec{p}_d + \vec{p}_c = m_d \vec{v}_d + m_c \vec{v}_c = \vec{p}_{\text{total}} = (20.0 \text{ kg}) \left(2.50 \frac{\text{m}}{\text{s}} \hat{y} \right) + (5.00 \text{ kg}) \left(3.00 \frac{\text{m}}{\text{s}} \hat{x} \right) = 15.0 \text{ kg} \cdot \frac{\text{m}}{\text{s}} \hat{x} + 50.0 \text{ kg} \cdot \frac{\text{m}}{\text{s}} \hat{y}$$

Set the total momentum of the dog and cat equal to that of the owner.

$$\vec{p}_0 = m_0 \vec{v}_0 = \vec{p}_{\text{total}}$$

$$\vec{v}_0 = \frac{\vec{p}_{\text{total}}}{m_0} = \frac{15.0 \text{ kg} \cdot \frac{\text{m}}{\text{s}} \hat{x} + 50.0 \text{ kg} \cdot \frac{\text{m}}{\text{s}} \hat{y}}{70.0 \text{ kg}} = 0.214 \frac{\text{m}}{\text{s}} \hat{x} + 0.714 \frac{\text{m}}{\text{s}} \hat{y}$$

$$\theta = \tan^{-1} \left(\frac{0.714}{0.214} \right) = \boxed{73.3^\circ}$$

$$v_0 = \sqrt{\left(0.214 \frac{\text{m}}{\text{s}} \right)^2 + \left(0.714 \frac{\text{m}}{\text{s}} \right)^2} = \boxed{0.745 \text{ m/s}}$$

4. (a) The carts must have equal and opposite momenta for the total momentum of the system to be zero.

$$\vec{p}_1 - \vec{p}_2 = 0 = m_1 \vec{v}_1 - m_2 \vec{v}_2$$

$$v_2 = \frac{m_1 v_1}{m_2} = \frac{(0.45 \text{ kg}) \left(1.1 \frac{\text{m}}{\text{s}} \right)}{0.65 \text{ kg}} = \boxed{0.76 \text{ m/s}}$$

- (b) $\boxed{\text{No}}$, kinetic energy is always greater than or equal to zero.

$$(c) \quad K_{\text{system}} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} (0.45 \text{ kg}) \left(1.1 \frac{\text{m}}{\text{s}} \right)^2 + \frac{1}{2} (0.65 \text{ kg}) \left(0.76 \frac{\text{m}}{\text{s}} \right)^2 = \boxed{0.46 \text{ J}}$$

5. Determine the speed of the baseball before it hits the ground.

$$v = \frac{p}{m} = \frac{0.780 \text{ kg} \cdot \frac{\text{m}}{\text{s}}}{0.150 \text{ kg}} = 5.20 \frac{\text{m}}{\text{s}}$$

Recall that $v^2 = v_0^2 - 2g(y - y_0)$.

$$y_0 - y = h = \frac{v^2 - v_0^2}{2g} = \frac{\left(5.20 \frac{\text{m}}{\text{s}} \right)^2 - (0)^2}{2 \left(9.81 \frac{\text{m}}{\text{s}^2} \right)} = \boxed{1.38 \text{ m}}$$

6. (a) $\Delta \vec{p} = \vec{p}_f - \vec{p}_i$

$$= m(\vec{v}_f - \vec{v}_i)$$

$$= (0.220 \text{ kg}) \left[2.0 \frac{\text{m}}{\text{s}} \hat{y} - \left(-2.5 \frac{\text{m}}{\text{s}} \hat{y} \right) \right]$$

$$= (0.220 \text{ kg}) \left(4.5 \frac{\text{m}}{\text{s}} \hat{y} \right)$$

$$= 0.99 \text{ kg} \cdot \frac{\text{m}}{\text{s}} \hat{y}$$

$$\Delta p = \boxed{0.99 \text{ kg} \cdot \text{m/s}}$$

$$(b) \quad p_f - p_i = m(v_f - v_i) = (0.220 \text{ kg}) \left(2.0 \frac{\text{m}}{\text{s}} - 2.5 \frac{\text{m}}{\text{s}} \right) = \boxed{-0.11 \text{ kg} \cdot \text{m/s}}$$

(c) The quantity found in **part (a)**.

$$7. \quad p_1 = p_{\text{total } x} = p_{\text{total}} \cos \theta = (17.6 \text{ kg} \cdot \text{m/s})(\cos 66.5^\circ) = 7.02 \text{ kg} \cdot \text{m/s}$$

$$m_1 = \frac{p_1}{v_1} = \frac{7.02 \text{ kg} \cdot \text{m/s}}{2.80 \text{ m/s}} = \boxed{2.51 \text{ kg}}$$

$$p_2 = p_{\text{total } y} = p_{\text{total}} \sin \theta = (17.6 \text{ kg} \cdot \text{m/s})(\sin 66.5^\circ) = 16.14 \text{ kg} \cdot \text{m/s}$$

$$m_2 = \frac{p_2}{v_2} = \frac{16.14 \text{ kg} \cdot \text{m/s}}{3.10 \text{ m/s}} = \boxed{5.21 \text{ kg}}$$

$$8. \quad I = F_{\text{av}} \Delta t = (1350 \text{ N})(6.20 \times 10^{-3} \text{ s}) = \boxed{8.37 \text{ kg} \cdot \text{m/s}}$$

9. To estimate the magnitude of the force, determine the magnitude of the average force.

$$F_{\text{av}} = \frac{\Delta p}{\Delta t} = \frac{m(v_f - v_i)}{\Delta t} = \frac{(0.045 \text{ kg})(67 \frac{\text{m}}{\text{s}} - 0)}{0.0010 \text{ s}} = \boxed{3.0 \text{ kN}}$$

$$10. \quad F_{\text{av}} = \frac{\Delta p}{\Delta t}$$

$$\Delta t = \frac{\Delta p}{F_{\text{av}}} = \frac{(0.50 \text{ kg})(3.2 \frac{\text{m}}{\text{s}} - 0)}{230 \text{ N}} = \boxed{7.0 \text{ ms}}$$

$$11. \quad I = \Delta p = m \Delta v$$

$$m = \frac{I}{\Delta v} = \frac{-9 \text{ kg} \cdot \frac{\text{m}}{\text{s}}}{-23 \frac{\text{m}}{\text{s}} - 4.5 \frac{\text{m}}{\text{s}}} = \boxed{0.3 \text{ kg}}$$

12. (a) Recall that $v = \sqrt{2gh}$. Just before the marble hits the floor, its speed is $v_i = -\sqrt{2gh_1}$ (negative downward motion). Just after it hits the floor, its speed is $v_f = \sqrt{2gh_2}$. So,

$$\begin{aligned} \vec{I} &= \Delta \vec{p} \\ &= m \Delta \vec{v} \\ &= m[\sqrt{2gh_2} \hat{y} - (-\sqrt{2gh_1} \hat{y})] \\ &= m\sqrt{2g}(\sqrt{h_2} + \sqrt{h_1}) \hat{y} \\ &= (0.0150 \text{ kg}) \sqrt{2 \left(9.81 \frac{\text{m}}{\text{s}^2} \right)} (\sqrt{0.640 \text{ m}} + \sqrt{1.44 \text{ m}}) \hat{y} \\ &= (0.133 \text{ kg} \cdot \text{m/s}) \hat{y} \end{aligned}$$

The impulse is **$0.133 \text{ kg} \cdot \text{m/s}$ in the positive y -direction**.

(b) Because the impulse is proportional to the sum of the square roots of the initial and final heights, the impulse would have been **greater** than that found in part (a).

13. The impulse is equal to the change in the y -component of the momentum of the ball.

$$I = \Delta p_y = m \Delta v_y = m[v_0 \cos 65^\circ - (-v_0 \cos 65^\circ)] = (0.60 \text{ kg}) \left(5.4 \frac{\text{m}}{\text{s}} \right) (2 \cos 65^\circ) = \boxed{2.7 \text{ kg} \cdot \text{m/s}}$$

14. (a) Let the initial motion of the ball be along the positive x -axis and the final along the positive y -axis.

$$\vec{I} = \Delta\vec{p} = m(\vec{v}_f - \vec{v}_i) = (0.14 \text{ kg}) \left[\left(18 \frac{\text{m}}{\text{s}} \right) \hat{y} - \left(-36 \frac{\text{m}}{\text{s}} \right) \hat{x} \right] = 5.04 \text{ kg} \cdot \frac{\text{m}}{\text{s}} \hat{x} + 2.52 \text{ kg} \cdot \frac{\text{m}}{\text{s}} \hat{y}$$

$$\theta = \tan^{-1} \left(\frac{2.52}{5.04} \right) = \boxed{27^\circ}$$

$$I = \sqrt{\left(5.04 \text{ kg} \cdot \frac{\text{m}}{\text{s}} \right)^2 + \left(2.52 \text{ kg} \cdot \frac{\text{m}}{\text{s}} \right)^2} = \boxed{5.6 \text{ kg} \cdot \text{m/s}}$$

- (b) Since $\Delta\vec{p}$ is directly proportional to the ball's mass, it would double in magnitude. There would be no change in the direction.

- (c) If the $\Delta\vec{p}$ of the ball is unchanged, then the impulse is unchanged.

15. $\vec{I} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = m(\vec{v}_f - \vec{v}_i)$
 $= (0.75 \text{ kg})[(5.2 \text{ m/s})\hat{x} + (3.7 \text{ m/s})\hat{y} - (8.8 \text{ m/s})\hat{x} - (-2.3 \text{ m/s})\hat{y}]$
 $= (-2.7 \text{ kg} \cdot \text{m/s})\hat{x} + (4.5 \text{ kg} \cdot \text{m/s})\hat{y}$

(a) $\theta = \tan^{-1} \left(\frac{I_y}{I_x} \right) = \tan^{-1} \left(\frac{4.5 \text{ kg} \cdot \text{m/s}}{-2.7 \text{ kg} \cdot \text{m/s}} \right) = -59^\circ + 180^\circ = \boxed{121^\circ \text{ from the } x\text{-axis}}$

(b) $I = \sqrt{(-2.7 \text{ kg} \cdot \text{m/s})^2 + (4.5 \text{ kg} \cdot \text{m/s})^2} = \boxed{5.2 \text{ kg} \cdot \text{m/s}}$

16. The sum of the canoes' momenta must be zero. The set up is similar to Example 9-3.

$$p_{1x} + p_{2x} = 0 = m_1 v_{1x} + m_2 v_{2x}$$

$$m_2 = \frac{-m_1 v_{1x}}{v_{2x}} = \frac{-(340 \text{ kg})(-0.52 \frac{\text{m}}{\text{s}})}{0.44 \frac{\text{m}}{\text{s}}} = \boxed{400 \text{ kg}}$$

17. This problem is similar to Problem 16, so we may use the above result for m_2 .

$$m_2 = \frac{-m_1 v_{1x}}{v_{2x}} = \frac{-(45 \text{ kg})(-0.62 \frac{\text{m}}{\text{s}})}{0.89 \frac{\text{m}}{\text{s}}} = \boxed{31 \text{ kg}}$$

18. Assume the bee's motion is in the negative direction.

$$p_{\text{bee}} + p_s = 0 = m_{\text{bee}} v_{\text{bee}} + m_s v_s$$

$$v_s = \frac{-m_{\text{bee}} v_{\text{bee}}}{m_s} = \frac{-(0.175 \text{ g})(-1.41 \frac{\text{cm}}{\text{s}})}{4.75 \text{ g}} = \boxed{51.9 \text{ mm/s}}$$

19. The sum of the pieces' momenta must be zero. Assume the motion is one-dimensional.

$$p_1 + p_2 = 0 = m_1 v_1 + m_2 v_2$$

$$m_1 v_1 = -m_2 v_2$$

$$\frac{m_1}{m_2} = \frac{-v_2}{v_1}$$

$$\left(\frac{m_1}{m_2} \right)^2 = \left(\frac{-v_2}{v_1} \right)^2 = \frac{v_2^2}{v_1^2}$$

Now, use the fact that $K_2 = 2K_1$.

$$K_2 = \frac{1}{2} m_2 v_2^2 = 2K_1 = 2 \left(\frac{1}{2} m_1 v_1^2 \right) = m_1 v_1^2$$

$$\frac{m_1}{m_2} = \frac{v_2^2}{2v_1^2} = \frac{1}{2} \left(\frac{m_1}{m_2} \right)^2$$

$$1 = \frac{1}{2} \left(\frac{m_1}{m_2} \right)$$

$$\frac{m_1}{m_2} = \boxed{2}$$

The piece with the smaller kinetic energy has the larger mass.

20. The motion is one-dimensional. Assume the satellite's motion is in the negative x -direction.

$$p_a + p_s = 0 = m_a v_a + m_s v_s$$

$$v_a = \frac{-m_s v_s}{m_a}$$

The initial distance is

$$x_0 = v_a t = \frac{-m_s v_s}{m_a} t = \frac{-(1100 \text{ kg}) \left(-0.13 \frac{\text{m}}{\text{s}} \right)}{(97 \text{ kg})} (7.5 \text{ s}) = \boxed{11 \text{ m}}$$

21. (a) v_L = speed of lumberjack relative to the shore

$v_{L,\log}$ = speed of lumberjack relative to the log

v_{\log} = speed of the log relative to the shore

Use conservation of momentum.

$$m_L v_L + m_{\log} v_{\log} = 0$$

$$m_L (v_{L,\log} + v_{\log}) + m_{\log} v_{\log} = 0$$

$$m_L v_{L,\log} + m_L v_{\log} + m_{\log} v_{\log} = 0$$

$$m_L v_{L,\log} = (-m_L - m_{\log}) v_{\log}$$

$$v_{\log} = \frac{m_L v_{L,\log}}{-m_L - m_{\log}} = \frac{(85 \text{ kg}) \left(2.7 \frac{\text{m}}{\text{s}} \right)}{-85 \text{ kg} - 380 \text{ kg}} = -0.494 \frac{\text{m}}{\text{s}}$$

$$v_L = v_{L,\log} + v_{\log} = 2.7 \frac{\text{m}}{\text{s}} + \left(-0.494 \frac{\text{m}}{\text{s}} \right) = \boxed{2.2 \text{ m/s}}$$

- (b) If the mass of the log had been greater, the lumberjack's speed relative to the shore would have been **greater than** that found in part (a), because $v_L + v_{\log} = v_L - |v_{\log}|$ = the speed of the lumberjack relative to the shore

and $|v_{\log}| \propto \frac{1}{m_{\log}}$, which decreases as m_{\log} increases.

$$(c) \quad v_{\log} = \frac{m_L v_{L,\log}}{-m_L - m_{\log}} = \frac{(85 \text{ kg}) \left(2.7 \frac{\text{m}}{\text{s}} \right)}{-85 \text{ kg} - 450 \text{ kg}} = -0.429 \frac{\text{m}}{\text{s}}$$

$$v_L = v_{L,\log} + v_{\log} = 2.7 \frac{\text{m}}{\text{s}} + \left(-0.429 \frac{\text{m}}{\text{s}} \right) = \boxed{2.3 \text{ m/s}}$$

22. Choose the motion of the two pieces at right angles to one another to be along the positive x - and y -axes. Use conservation of momentum.

$$p_1 + p_2 + p_3 = 0$$

$$= mv\hat{x} + mv\hat{y} + m\vec{v}_3$$

$$\vec{v}_3 = -v(\hat{x} + \hat{y})$$

$$v_3 = \sqrt{(-v)^2 + (-v)^2} = \boxed{\sqrt{2}v}$$

$$\theta = \tan^{-1}\left(\frac{-v}{-v}\right) = 225^\circ$$

$$\theta = \boxed{225^\circ \text{ if the two pieces with speed } v \text{ lie along the positive } x\text{- and } y\text{-axes.}}$$

23. The collision is completely inelastic because the two carts stick together. Assume the motion is one-dimensional. The initial momentum is equal to the final momentum.

$$p_i = mv + m(0) = 2mv_f = p_f$$

$$v_f = \frac{mv}{2m} = \frac{v}{2}$$

The final kinetic energy is

$$K_f = \frac{1}{2}(2m)v_f^2 = m\left(\frac{v}{2}\right)^2 = \boxed{\frac{1}{4}mv^2}$$

24. From Example 9-6,

$$\frac{m_2 v_2}{m_1 v_1} = \tan \theta$$

$$v_2 = \frac{m_1 v_1 \tan \theta}{m_2} = \frac{(950 \text{ kg})(20.0 \frac{\text{m}}{\text{s}}) \tan 40.0^\circ}{1300 \text{ kg}} = \boxed{12 \text{ m/s}}$$

and

$$v_f = \frac{m_1 v_1}{(m_1 + m_2) \cos \theta} = \frac{(950 \text{ kg})(20.0 \frac{\text{m}}{\text{s}})}{(950 \text{ kg} + 1300 \text{ kg}) \cos 40.0^\circ} = \boxed{11 \text{ m/s}}$$

25. Let the motion of player 1 be in the positive x -direction and the motion of player 2 be at an angle of 120° measured counterclockwise from the positive x -axis.

The initial momenta for the players are $\vec{p}_{1i} = mv\hat{x}$ and $\vec{p}_{2i} = mv(\cos \theta \hat{x} + \sin \theta \hat{y})$. The final momentum for the two-player system is $\vec{p}_f = 2m\vec{v}_f$. Using conservation of momentum, $mv\hat{x} + mv \cos \theta \hat{x} + mv \sin \theta \hat{y} = 2m\vec{v}_f$.

$$\vec{v}_f = \frac{1}{2}v[(1 + \cos \theta)\hat{x} + \sin \theta \hat{y}]$$

$$= \frac{1}{2}\left(5.75 \frac{\text{m}}{\text{s}}\right)[(1 + \cos 125^\circ)\hat{x} + \sin 125^\circ \hat{y}]$$

$$= \left(2.875 \frac{\text{m}}{\text{s}}\right)[(1 - 0.5736)\hat{x} + 0.8192\hat{y}]$$

$$= \boxed{(1.23 \text{ m/s}) \hat{x} + (2.36 \text{ m/s}) \hat{y}}$$

26. (a) The final kinetic energy of the car and truck together is less than the sum of their initial kinetic energies. Some of the kinetic energy is converted to sound and some to heat. Some of the energy creates the permanent deformations in the materials of the car and truck.

$$(b) \quad K_i = \frac{1}{2}m_c v_c^2 + \frac{1}{2}m_t v_t^2 = \frac{1}{2}(1200 \text{ kg})\left(2.5 \frac{\text{m}}{\text{s}}\right)^2 + \frac{1}{2}(2600 \text{ kg})\left(6.2 \frac{\text{m}}{\text{s}}\right)^2 = \boxed{54 \text{ kJ}}$$

$$K_f = \frac{1}{2}(m_c + m_t)v_f^2 = \frac{1}{2}(1200 \text{ kg} + 2600 \text{ kg})\left(5.0 \frac{\text{m}}{\text{s}}\right)^2 = \boxed{48 \text{ kJ}}$$

27. (a) Use momentum conservation. Let the subscripts b and B denote the bullet and the block, respectively.

$$m_b v_{bi} + m_B v_{Bi} = m_b v_{bf} + m_B v_{Bf}$$

$$m_b v_{bi} + 0 = m_b v_{bf} + m_B v_{Bf}$$

$$\begin{aligned} v_{bf} &= \frac{m_b v_{bi} - m_B v_{Bf}}{m_b} \\ &= \frac{(0.0040 \text{ kg})\left(650 \frac{\text{m}}{\text{s}}\right) - (0.095 \text{ kg})\left(23 \frac{\text{m}}{\text{s}}\right)}{0.004 \text{ kg}} \\ &= \boxed{1.0 \times 10^2 \text{ m/s}} \end{aligned}$$

- (b) The final kinetic energy is less than the initial kinetic energy because energy is lost to the heating and deformation of the bullet and block.

$$(c) \quad K_i = \frac{1}{2}m_b v_{bi}^2 = \frac{1}{2}(0.0040 \text{ kg})\left(650 \frac{\text{m}}{\text{s}}\right)^2 = \boxed{850 \text{ J}}$$

$$K_f = \frac{1}{2}m_b v_{bf}^2 + \frac{1}{2}m_B v_{Bf}^2 = \frac{1}{2}(0.0040 \text{ kg})\left(103.75 \frac{\text{m}}{\text{s}}\right)^2 + \frac{1}{2}(0.095 \text{ kg})\left(23 \frac{\text{m}}{\text{s}}\right)^2 = \boxed{47 \text{ J}}$$

28. (a) No

- (b) Use momentum conservation to determine the speed of the puddy-block system just after the collision.

$$m_b v_b + m_p v_p = (m_b + m_p)v_f$$

$$v_f = \frac{m_b(0) + m_p v_p}{m_b + m_p} = \left(\frac{m_p}{m_b + m_p}\right)v_p$$

Use v_f to determine K_f and equate K_f with the gravitational potential energy above the original position of the block.

$$\begin{aligned} \frac{1}{2}(m_b + m_p)\left(\frac{m_p}{m_b + m_p}\right)^2 v_p^2 &= (m_b + m_p)gh \\ h &= \left(\frac{m_p}{m_b + m_p}\right)^2 \left(\frac{v_p^2}{2g}\right) \\ &= \left(\frac{0.0700 \text{ kg}}{0.470 \text{ kg} + 0.0700 \text{ kg}}\right)^2 \left[\frac{\left(5.60 \frac{\text{m}}{\text{s}}\right)^2}{2\left(9.81 \frac{\text{m}}{\text{s}^2}\right)}\right] \\ &= \boxed{2.69 \text{ cm}} \end{aligned}$$

29. Use momentum conservation to determine the speed of the puddy-block system just after the collision.

$$m_b(0) + m_p v_p = (m_b + m_p) v_f$$

$$v_f = \left(\frac{m_p}{m_p + m_b} \right) v_p$$

Use v_f to determine K_f and equate K_f with the spring potential energy.

$$\frac{1}{2} (m_p + m_b) \left(\frac{m_p}{m_p + m_b} \right)^2 v_p^2 = \frac{1}{2} k \Delta x^2$$

$$\Delta x = \sqrt{\frac{m_p^2 v_p^2}{k(m_p + m_b)}} = \sqrt{\frac{(0.0500 \text{ kg})^2 \left(2.30 \frac{\text{m}}{\text{s}}\right)^2}{(20.0 \frac{\text{N}}{\text{m}})(0.0500 \text{ kg} + 0.430 \text{ kg})}} = \boxed{3.71 \text{ cm}}$$

30. (a) $K_i = \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2 = \frac{1}{2} (m_1 + m_2) v^2$

$$K_f = \frac{1}{2} (m_1 + m_2) \left(\frac{v}{4} \right)^2 = \frac{1}{32} (m_1 + m_2) v^2$$

$$\frac{K_f}{K_i} = \frac{\frac{1}{32} (m_1 + m_2) v^2}{\frac{1}{2} (m_1 + m_2) v^2} = \boxed{\frac{1}{16}}$$

- (b) Use momentum conservation.

$$m_1 v + m_2 (-v) = (m_1 + m_2) \frac{v}{4}$$

$$m_1 - m_2 = \frac{1}{4} (m_1 + m_2)$$

$$4m_1 - 4m_2 = m_1 + m_2$$

$$3m_1 = 5m_2$$

$$\frac{m_1}{m_2} = \boxed{\frac{5}{3}}$$

31. m_1 = the mass of the truck
 m_2 = the mass of the car
 v_0 = the initial speed of the truck

Use conservation of momentum to find an equation for the final speed of the truck.

$$m_1 v_0 = m_1 v_{1f} + m_2 v_{2f}$$

$$m_1 v_{1f} = m_1 v_0 - m_2 v_{2f}$$

$$v_{1f} = v_0 - \frac{m_2}{m_1} v_{2f}$$

There is one equation and two unknowns. Use conservation of energy to find a second equation relating v_{1f} and v_{2f} .

$$\frac{1}{2} m_1 v_0^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$m_1 v_{1f}^2 = m_1 v_0^2 - m_2 v_{2f}^2$$

Substitute for v_{1f} and solve for v_{2f} .

$$\begin{aligned}
m_1 \left(v_0 - \frac{m_2}{m_1} v_{2f} \right)^2 &= m_1 v_0^2 - m_2 v_{2f}^2 \\
m_1 v_0^2 - 2m_2 v_0 v_{2f} + \frac{m_2^2}{m_1} v_{2f}^2 &= m_1 v_0^2 - m_2 v_{2f}^2 \\
-2v_0 v_{2f} + \frac{m_2}{m_1} v_{2f}^2 &= -v_{2f}^2 \\
\left(\frac{m_1 + m_2}{m_1} \right) v_{2f}^2 &= 2v_0 v_{2f} \\
v_{2f} &= \left(\frac{2m_1}{m_1 + m_2} \right) v_0
\end{aligned}$$

Substitute for v_{2f} in the equation for v_{1f} .

$$v_{1f} = v_0 - \frac{m_2}{m_1} \left(\frac{2m_1}{m_1 + m_2} \right) v_0 = \left(1 - \frac{2m_2}{m_1 + m_2} \right) v_0 = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_0$$

Using the given information, $m_1 = 1620 \text{ kg}$,

$m_2 = 722 \text{ kg}$, and $v_0 = 14.5 \text{ m/s}$, the final speeds of the truck and car are: $v_{\text{truck}} = 5.56 \text{ m/s}$ and

$$v_{\text{car}} = 20.1 \text{ m/s}.$$

32. This problem is analogous to Problem 31. The hammer takes the place of the truck, and the nail takes the place of the car. Therefore, the kinetic energy acquired is given by

$$K_{2f} = \frac{1}{2} m_2 v_{2f}^2 = \frac{1}{2} m_2 \left(\frac{2m_1}{m_1 + m_2} \right)^2 v_0^2 = \frac{1}{2} (0.012 \text{ kg}) \left[\frac{2(0.550 \text{ kg})}{0.550 \text{ kg} + 0.010 \text{ kg}} \right]^2 \left(4.5 \frac{\text{m}}{\text{s}} \right)^2 = \boxed{0.47 \text{ J}}$$

33. From Example 9-7, $K_i = 0.197 \text{ J}$.

$$K_f = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 = K_i$$

Solve for v_{1f} .

$$v_{1f} = \sqrt{\frac{2K_i}{m_1} - \frac{m_2}{m_1} v_{2f}^2} = \sqrt{\frac{2(0.197 \text{ J})}{0.130 \text{ kg}} - \left(\frac{0.160 \text{ kg}}{0.130 \text{ kg}} \right) \left(1.03 \frac{\text{m}}{\text{s}} \right)^2} = \boxed{1.31 \text{ m/s}}$$

Set the final y -component of momentum equal to zero to determine θ .

$$\begin{aligned}
0 &= p_{1y} - p_{2y} \\
&= m_1 v_{1f} \sin \theta - m_2 v_{2f}
\end{aligned}$$

$$\begin{aligned}
 \sin \theta &= \frac{m_2 v_{2f}}{m_1 v_{1f}} \\
 \theta &= \sin^{-1} \left(\frac{m_2 v_{2f}}{m_1 v_{1f}} \right) \\
 &= \sin^{-1} \left[\frac{m_2 v_{2f}}{\sqrt{m_1 (2K_i - m_2 v_{2f}^2)}} \right] \\
 &= \sin^{-1} \left[\frac{(0.160 \text{ kg})(1.03 \frac{\text{m}}{\text{s}})}{\sqrt{(0.130 \text{ kg}) \left[2(0.197 \text{ J}) - (0.160 \text{ kg}) \left(1.03 \frac{\text{m}}{\text{s}} \right)^2 \right]}} \right] \\
 &= \boxed{74.8^\circ}
 \end{aligned}$$

34. For the neutron,

$$\frac{K_f}{K_i} = \frac{\frac{1}{2} m v_f^2}{\frac{1}{2} m v_i^2} = \left(\frac{v_f}{v_i} \right)^2.$$

Recall that

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_0, \text{ so}$$

$$v_f = \left(\frac{m - M}{m + M} \right) v_i.$$

$$\frac{K_f}{K_i} = \left(\frac{m - M}{m + M} \right)^2$$

$$(a) \quad \frac{K_f}{K_i} = \left(\frac{1.009u - 5.49 \times 10^{-4}u}{1.009u + 5.49 \times 10^{-4}u} \right)^2 = \boxed{0.998}$$

$$(b) \quad \frac{K_f}{K_i} = \left(\frac{1.009u - 1.007u}{1.009u + 1.007u} \right)^2 = \boxed{9.84 \times 10^{-7}}$$

$$(c) \quad \frac{K_f}{K_i} = \left(\frac{1.009u - 207.2u}{1.009u + 207.2u} \right)^2 = \boxed{0.9807}$$

35. (a) Let subscript 1 refer to the elephant and subscript 2 refer to the ball.
Use momentum conservation.

$$m_1 v_1 + m_2 v_2 = m_1 v_{1f} + m_2 v_{2f}$$

Use conservation of kinetic energy.

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2.$$

Rearranging the first equation gives

$$\frac{m_1 (v_1 - v_{1f})}{m_2 (v_{2f} - v_2)} = 1.$$

Rearranging the second equation gives

$$\frac{m_1(v_1^2 - v_{1f}^2)}{m_2(v_{2f}^2 - v_2^2)} = 1 = \frac{m_1(v_1 - v_{1f})(v_1 + v_{1f})}{m_2(v_{2f} - v_2)(v_{2f} + v_2)}.$$

Comparing these two equations implies that

$$\frac{(v_1 - v_{1f})}{(v_{2f} - v_2)} = 1, \text{ or } v_{2f} = v_1 + v_{1f} - v_2.$$

Substitute for v_{2f} in the first equation and solve for v_{1f} .

$$m_1 v_1 + m_2 v_2 = m_1 v_{1f} + m_2 (v_1 + v_{1f} - v_2)$$

$$(m_1 - m_2)v_1 + 2m_2 v_2 = (m_1 + m_2)v_{1f}$$

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_1 + \left(\frac{2m_2}{m_1 + m_2} \right) v_2$$

Since $v_{2f} = v_1 + v_{1f} - v_2$, $v_{1f} = v_{2f} + v_2 - v_1$.

Substitute for v_{1f} in the first equation and solve for v_{2f} .

$$m_1 v_1 + m_2 v_2 = m_1 (v_{2f} + v_2 - v_1) + m_2 v_{2f}$$

$$2m_1 v_1 + (m_2 - m_1)v_2 = (m_1 + m_2)v_{2f}$$

$$\begin{aligned} v_{2f} &= \left(\frac{2m_1}{m_1 + m_2} \right) v_1 + \left(\frac{m_2 - m_1}{m_1 + m_2} \right) v_2 \\ &= \left[\frac{2(5400 \text{ kg})}{5400 \text{ kg} + 0.150 \text{ kg}} \right] \left(-4.30 \frac{\text{m}}{\text{s}} \right) + \left[\frac{0.150 \text{ kg} - 5400 \text{ kg}}{5400 \text{ kg} + 0.150 \text{ kg}} \right] \left(8.11 \frac{\text{m}}{\text{s}} \right) \\ &= \boxed{-17 \text{ m/s}} \end{aligned}$$

(b) Kinetic energy has been transferred from the elephant to the ball.

36. Place the x -axis along the Earth-Moon center-to-center line with the origin at the center of the Earth.

$$\begin{aligned} X_{\text{cm}} &= \frac{\Sigma mx}{M} \\ &= \frac{m_E x_E + m_m x_m}{m_E + m_m} \\ &= \frac{m_E(0) + m_m x_m}{m_E + m_m} \\ &= \left(\frac{m_m}{m_E + m_m} \right) x_m \\ &= \left(\frac{7.35 \times 10^{22} \text{ kg}}{5.98 \times 10^{24} \text{ kg} + 7.35 \times 10^{22} \text{ kg}} \right) (3.85 \times 10^8 \text{ m}) \\ &= \boxed{4.67 \times 10^6 \text{ m}} \end{aligned}$$

The mean radius of the Earth is about $6.37 \times 10^6 \text{ m}$. Therefore, the center of mass is

$$6.37 \times 10^6 \text{ m} - 4.67 \times 10^6 \text{ m} = \boxed{1.70 \times 10^6 \text{ m below the surface of the Earth}}.$$

37. Place the origin at the center of the basket. Then, the two cartons of cereal are at $x_c = (-0.75 \text{ m})/2 = -0.375 \text{ m}$ (assuming the cartons are left of center).

Calculate the x -coordinate of the center of mass and set it equal to zero.

$$X_{\text{cm}} = \frac{\Sigma mx}{M} = \frac{2m_c x_c + m_m x_m}{2m_c + m_m} = 0$$

Solve for x_m .

$$2m_c x_c + m_m x_m = 0$$

$$\begin{aligned} x_m &= -\frac{2m_c x_c}{m_m} \\ &= -\frac{2(0.55 \text{ kg})(-0.375 \text{ m})}{1.8 \text{ kg}} \\ &= 0.23 \text{ m} \end{aligned}$$

The half gallon of milk should be placed 0.23 m from the center of the basket, opposite the cartons of cereal.

38. Calculate the x -coordinate of the center of mass. Assume that the mass of each brick is m and that the mass of each brick is distributed uniformly.

$$X_{\text{cm}} = \frac{\Sigma mx}{M} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{m\left(\frac{L}{2} + L + \frac{5L}{4}\right)}{3m} = \frac{1}{3}\left(\frac{11L}{4}\right) = \boxed{\frac{11}{12}L}$$

39. Place the origin at the center of the box with the plane of the missing top perpendicular to the positive z -axis. Due to symmetry, $X_{\text{cm}} = Y_{\text{cm}} = 0$.

$$Z_{\text{cm}} = \frac{\Sigma mz}{M} = \frac{mz_1 + mz_2 + mz_3 + mz_4 + mz_{\text{bottom}}}{5m} = \frac{m\left(0 + 0 + 0 + 0 - \frac{L}{2}\right)}{5m} = -\frac{L}{10}$$

The center of mass is $L/10$ units below the center of the box.

40. $X_{\text{cm}} = \frac{mx_2 + mx_3 + mx_4}{3m}$ (The subscript refers to the quadrant.)

By symmetry, $x_2 = x_3 = -x_4$.

$$X_{\text{cm}} = \frac{1}{3}(x_2 + x_2 - x_2) = \frac{x_2}{3}$$

$$x_2 = 3X_{\text{cm}} = 3(-1.2 \text{ in.}) = -3.6 \text{ in.}$$

$$Y_{\text{cm}} = \frac{my_2 + my_3 + my_4}{3m}$$

By symmetry, $y_2 = -y_3 = -y_4$.

$$Y_{\text{cm}} = \frac{1}{3}(y_2 - y_2 - y_2) = -\frac{y_2}{3}$$

$$y_2 = -3Y_{\text{cm}} = -3(-1.2 \text{ in.}) = 3.6 \text{ in.}$$

$$(x_2, y_2) = \boxed{(-3.6 \text{ in.}, 3.6 \text{ in.})}$$

41. Due to symmetry, $X_{\text{cm}} = 0$. Calculate Y_{cm} .

$$\begin{aligned}
 Y_{\text{cm}} &= \frac{\Sigma my}{M} \\
 &= \frac{my_1 + my_2 + m_s y_s}{2m + m_s} \\
 &= \frac{2my_1 + m_s y_s}{2m + m_s} \quad (y_1 = y_2) \\
 &= \frac{2(16u)(0.143 \text{ nm}) \sin 30^\circ + (32u)(0)}{2(16u) + 32u} \\
 &= 3.6 \times 10^{-11} \text{ m} \\
 (X_{\text{cm}}, Y_{\text{cm}}) &= \boxed{(0, 3.6 \times 10^{-11} \text{ m})}
 \end{aligned}$$

42. (a) Calculate X_{cm} .

$$X_{\text{cm}} = \frac{Mx_1 + Mx_2 + Mx_3}{3M} = \frac{1}{3}(0 + 0.50 \text{ m} + 1.5 \text{ m}) = 0.67 \text{ m}$$

Calculate Y_{cm} .

$$Y_{\text{cm}} = \frac{My_1 + My_2 + My_3}{3M} = \frac{1}{3}(0.50 \text{ m} + 0 + 0) = 0.17 \text{ m}$$

$$(X_{\text{cm}}, Y_{\text{cm}}) = \boxed{(0.67 \text{ m}, 0.17 \text{ m})}$$

- (b) The location of the center of mass would not be affected. The mass drops out of the equations.

43. We define the following subscripts:

f = floor

nf = not on floor

L = length of rope

Find the equation of motion of the top of the rope.

$$y_{\text{top}} = \left(0.910 \frac{\text{m}}{\text{s}} \right) t, 0 < t < \frac{2.00 \text{ m}}{0.910 \frac{\text{m}}{\text{s}}} = 2.20 \text{ s}$$

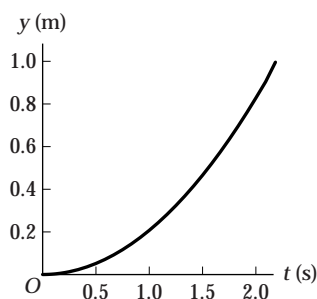
Set the origin at the floor.

$$y_{\text{nf}} = \frac{y_{\text{top}}}{2} \quad \text{and} \quad y_{\text{f}} = 0$$

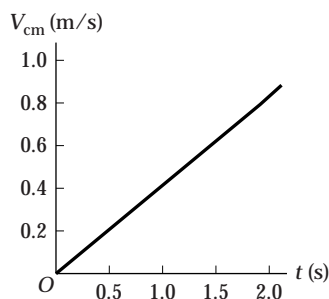
$$Y_{\text{cm}} = \frac{m_{\text{nf}} y_{\text{nf}} + m_{\text{f}} y_{\text{f}}}{m_{\text{nf}} + m_{\text{f}}}$$

Substitute.

$$Y_{\text{cm}} = \frac{\left(\frac{0.604 \text{ kg}}{2.00 \text{ m}} \right) y_{\text{top}} \left(\frac{y_{\text{top}}}{2} \right) + 0}{0.604 \text{ kg}} = \frac{1}{2.00 \text{ m}} \left(\frac{1}{2} \right) y_{\text{top}}^2 = \frac{1}{2(2.00 \text{ m})} \left[\left(0.910 \frac{\text{m}}{\text{s}} \right) t \right]^2 = \boxed{(0.207 \text{ m/s}^2) t^2}$$



$$V_{\text{cm}} = \frac{m_{\text{nf}} v_{\text{nf}} + m_{\text{f}} v_{\text{f}}}{m_{\text{nf}} + m_{\text{f}}} = \frac{\left[\left(\frac{m_{\text{nf}} + m_{\text{f}}}{L} \right) v_{\text{nf}} t \right] v_{\text{nf}} + m_{\text{f}} (0)}{m_{\text{nf}} + m_{\text{f}}} = \frac{v_{\text{nf}}^2}{L} t = \frac{\left(0.910 \frac{\text{m}}{\text{s}} \right)^2}{2.00 \text{ m}} t = \boxed{(0.414 \text{ m/s}^2) t}$$



44. We define the following subscripts:

f = floor

nf = not on floor

L = length of rope

Find the equation of motion of the top of the rope.

$$y_{\text{top}} = 2 \text{ m} - \left(0.910 \frac{\text{m}}{\text{s}} \right) t, 0 < t < \frac{2.00 \text{ m}}{0.910 \frac{\text{m}}{\text{s}}} = 2.20 \text{ s}$$

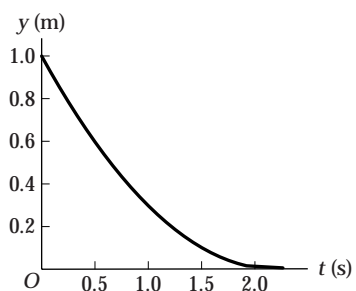
Set the origin at the floor.

$$y_{\text{nf}} = \frac{y_{\text{top}}}{2} \quad \text{and} \quad y_{\text{f}} = 0$$

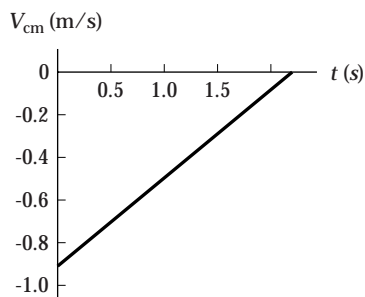
$$Y_{\text{cm}} = \frac{m_{\text{nf}} y_{\text{nf}} + m_{\text{f}} y_{\text{f}}}{m_{\text{total}}}$$

Substitute.

$$\begin{aligned} Y_{\text{cm}} &= \frac{\left(\frac{0.604 \text{ kg}}{2.00 \text{ m}} \right) y_{\text{top}} \left(\frac{y_{\text{top}}}{2} \right) + 0}{0.604 \text{ kg}} \\ &= \frac{1}{2.00 \text{ m}} \left(\frac{1}{2} \right) y_{\text{top}}^2 \\ &= \frac{1}{2(2.00 \text{ m})} \left[2 \text{ m} - \left(0.910 \frac{\text{m}}{\text{s}} \right) t \right]^2 \\ &= \frac{1}{2(2.00 \text{ m})} \left[4 \text{ m}^2 - \left(3.64 \frac{\text{m}^2}{\text{s}} \right) t + \left(0.828 \frac{\text{m}^2}{\text{s}^2} \right) t^2 \right] \\ Y_{\text{cm}} &= \boxed{1.00 \text{ m} - (0.910 \text{ m/s}) t + (0.207 \text{ m/s}^2) t^2} \end{aligned}$$



$$\begin{aligned}
 V_{\text{cm}} &= \frac{m_{\text{nf}} v_{\text{nf}} + m_{\text{f}} v_{\text{f}}}{m_{\text{nf}} + m_{\text{f}}} \\
 &= \frac{(m_{\text{nf}} + m_{\text{f}}) \left(1 + \frac{v_{\text{nf}}}{L} t\right) v_{\text{nf}} + m_{\text{f}} (0)}{m_{\text{nf}} + m_{\text{f}}} \\
 &= \left(1 + \frac{v_{\text{nf}}}{L} t\right) v_{\text{nf}} \\
 &= v_{\text{nf}} + \frac{v_{\text{nf}}^2}{L} t \\
 &= -0.910 \frac{\text{m}}{\text{s}} + \frac{\left(-0.910 \frac{\text{m}}{\text{s}}\right)^2}{2.00 \text{ m}} t \\
 V_{\text{cm}} &= \boxed{-0.910 \text{ m/s} + (0.414 \text{ m/s}^2) t}
 \end{aligned}$$



45. (a) Before the string breaks, the reading on the scale is the total weight of

$$Mg = (1.20 \text{ kg} + 0.150 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2}\right) = \boxed{13.2 \text{ N}}.$$

- (b) After the string breaks, the reading is $\boxed{13.2 \text{ N}}$. Because the ball is moving with constant speed, the center of mass of the system undergoes no net acceleration. Therefore, the reading will not change.

46. (a) Taking up to be positive, calculate the net external force acting on the cooking pot of water and the egg.

$$F_{\text{net, ext}} = F_{\text{s}} - m_{\text{p}} g - m_{\text{e}} g$$

Now, determine the acceleration of the center of mass.

$$\begin{aligned}
 A_{\text{cm}} &= \frac{m_{\text{p}}(0) + m_{\text{e}} \left(\frac{-g}{2}\right)}{M} = -\left(\frac{m_{\text{e}}}{m_{\text{e}} + m_{\text{p}}}\right) \frac{g}{2} = -\left(\frac{0.0460 \text{ kg}}{0.0460 \text{ kg} + 2.90 \text{ kg}}\right) \frac{\left(9.81 \frac{\text{m}}{\text{s}^2}\right)}{2} = \boxed{-0.0766 \text{ m/s}^2}
 \end{aligned}$$

- (b) Recall that $F_{\text{net,ext}} = MA_{\text{cm}}$, then

$$F_s - m_p g - m_e g = -\frac{m_e g}{2}.$$

$$F_s = m_p g + \frac{m_e g}{2} = g \left(m_p + \frac{m_e}{2} \right) = \left(9.81 \frac{\text{m}}{\text{s}^2} \right) \left(2.90 \text{ kg} + \frac{0.0460 \text{ kg}}{2} \right) = \boxed{28.7 \text{ N}}$$

- (c) After the egg comes to rest on the bottom of the pot, the reading is the total weight of

$$Mg = (2.90 \text{ kg} + 0.0460 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) = \boxed{28.9 \text{ N}}$$

47. (a) From Example 9-9, the velocity of the center of mass before the collision is

$$\begin{aligned} V_{\text{cm}} &= \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \\ &= \frac{m_1 v_1 + m_2 (0)}{m_1 + m_2} \\ &= \left(\frac{m_1}{m_1 + m_2} \right) v_1 \\ &= \left(\frac{0.750 \text{ kg}}{0.750 \text{ kg} + 0.275 \text{ kg}} \right) \left(0.455 \frac{\text{m}}{\text{s}} \right) \\ &= \boxed{0.333 \text{ m/s}} \end{aligned}$$

- (b) Use momentum conservation to find the speed of the carts after the collision.

$$\begin{aligned} m_1 v_1 &= m_1 v_f + m_2 v_f = (m_1 + m_2) v_f \\ v_f &= \left(\frac{m_1}{m_1 + m_2} \right) v_1 = V_{\text{cm}} = \boxed{0.333 \text{ m/s}} \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad K_i &= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \\ &= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 (0)^2 \\ &= \frac{1}{2} m_1 v_1^2 \\ &= \frac{1}{2} (0.750 \text{ kg}) \left(0.455 \frac{\text{m}}{\text{s}} \right)^2 \\ &= \boxed{0.0776 \text{ J}} \end{aligned}$$

$$K_f = \frac{1}{2} (m_1 + m_2) v_f^2 = \frac{1}{2} (0.750 \text{ kg} + 0.275 \text{ kg}) \left(0.333 \frac{\text{m}}{\text{s}} \right)^2 = \boxed{0.0568 \text{ J}}$$

48. (a) Before the string is cut, the force of gravity is countered by the force of the spring. Just after the string is cut, the upper block experiences a force of $F_s + F_g = 2mg - mg = mg$, and the lower block experiences a force of

$$F_g = -mg. \text{ The net force acting on the two-block system is } F_{\text{net,ext}} = mg + (-mg) = \boxed{0}.$$

- (b) Since $F_{\text{net,ext}} = MA_{\text{cm}} = 0$, $A_{\text{cm}} = \boxed{0}$.

49. The net force must be zero for the helicopter to hover.

$$F_{\text{net}} = 0 = \text{thrust} - mg = \left(\frac{\Delta m}{\Delta t} \right) v - mg$$

$$\frac{\Delta m}{\Delta t} = \frac{mg}{v} = \frac{(5500 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right)}{60.0 \frac{\text{m}}{\text{s}}} = \boxed{900 \text{ kg/s}}$$

50. $F_{\text{friction}} = 3.4 \text{ N} = \text{thrust} = \left(\frac{\Delta m}{\Delta t} \right) v$

$$\frac{\Delta m}{\Delta t} = \frac{3.4 \text{ N}}{v} = \frac{3.4 \text{ N}}{11 \frac{\text{m}}{\text{s}}} = 0.309 \frac{\text{kg}}{\text{s}}$$

Convert $\Delta m/\Delta t$ from kg/s to rocks/min.

$$\left(\frac{0.309 \text{ kg}}{\text{s}} \right) \left(\frac{60 \text{ s}}{\text{min}} \right) \left(\frac{1 \text{ rock}}{0.50 \text{ kg}} \right) = \boxed{37 \text{ rocks/min}}$$

51. Use conservation of momentum.

$$p_i = 0 = p_f = (m_p + m_s)v + 2m_b v_b$$

$$0 = (m_p + m_s)v + 2m_b \left(18.0 \frac{\text{m}}{\text{s}} + v \right)$$

$$0 = (m_p + m_s)v + 2m_b \left(18.0 \frac{\text{m}}{\text{s}} \right) + 2m_b v$$

$$-2m_b \left(18.0 \frac{\text{m}}{\text{s}} \right) = v(m_p + m_s + 2m_b)$$

$$v = \frac{-2m_b \left(18.0 \frac{\text{m}}{\text{s}} \right)}{m_p + m_s + 2m_b} = \frac{-2(0.850 \text{ kg}) \left(18.0 \frac{\text{m}}{\text{s}} \right)}{57.0 \text{ kg} + 2.10 \text{ kg} + 2(0.850 \text{ kg})} = -0.503 \frac{\text{m}}{\text{s}}$$

The person will recoil with a speed of $\boxed{0.503 \text{ m/s}}$.

52. Use conservation of momentum for each time the person throws a brick.

First brick

$$p_i = 0 = p_f = (m_p + m_s + m_b)v + m_b v_b$$

$$0 = (m_p + m_s + m_b)v + m_b \left(18.0 \frac{\text{m}}{\text{s}} + v \right)$$

$$0 = (m_p + m_s + m_b)v + m_b \left(18.0 \frac{\text{m}}{\text{s}} \right) + m_b v$$

$$v = \frac{-m_b \left(18.0 \frac{\text{m}}{\text{s}} \right)}{m_p + m_s + 2m_b} = \frac{-(0.850 \text{ kg}) \left(18.0 \frac{\text{m}}{\text{s}} \right)}{57.0 \text{ kg} + 2.10 \text{ kg} + 2(0.850 \text{ kg})} = -0.252 \frac{\text{m}}{\text{s}}$$

Second brick

$$p_i = (m_p + m_s + m_b)v = (m_p + m_s)v_p + m_b v_b = p_f$$

$$(m_p + m_s + m_b)v = (m_p + m_s)v_p + m_b \left(18.0 \frac{\text{m}}{\text{s}} + v_p \right)$$

$$\begin{aligned}
 v_p &= \frac{(m_p + m_s + m_b)v - m_b \left(18 \frac{\text{m}}{\text{s}}\right)}{m_p + m_s + m_b} \\
 &= \frac{(57.0 \text{ kg} + 2.10 \text{ kg} + 0.850 \text{ kg}) \left(-0.252 \frac{\text{m}}{\text{s}}\right) - (0.850 \text{ kg}) \left(18 \frac{\text{m}}{\text{s}}\right)}{57.0 \text{ kg} + 2.10 \text{ kg} + 0.850 \text{ kg}} \\
 &= -0.507 \frac{\text{m}}{\text{s}}
 \end{aligned}$$

The person will recoil with a speed of $\boxed{0.507 \text{ m/s}}$.

53. (a) The reading of the scale is given by the total weight of the sand and bucket plus the force of impact due to the pouring sand.

$$\begin{aligned}
 \text{Scale reading} &= (m_b + m_s)g + \frac{\Delta m}{\Delta t} v \\
 &= (0.540 \text{ kg} + 0.750 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2}\right) + \left(0.0560 \frac{\text{kg}}{\text{s}}\right) \left(3.20 \frac{\text{m}}{\text{s}}\right) \\
 &= \boxed{12.8 \text{ N}}
 \end{aligned}$$

$$(b) \quad W = (m_b + m_s)g = (0.540 \text{ kg} + 0.750 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2}\right) = \boxed{12.7 \text{ N}}$$

$$54. (a) \quad \text{thrust} = \frac{\Delta m}{\Delta t} v = (\text{mass per unit length})(\text{speed})v = \left(0.13 \frac{\text{kg}}{\text{m}}\right) \left(1.4 \frac{\text{m}}{\text{s}}\right)^2 = \boxed{0.25 \text{ N}}$$

- (b) The scale reads $\boxed{\text{more than } 2.5 \text{ N}}$. It reads the weight of the rope on the scale and the thrust due to the falling rope.

$$(c) \quad \text{Scale reading} = \frac{\Delta m}{\Delta t} v + mg = \left(0.13 \frac{\text{kg}}{\text{m}}\right) \left(1.4 \frac{\text{m}}{\text{s}}\right)^2 + (0.25 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2}\right) = \boxed{2.7 \text{ N}}$$

55. Place the origin at X_{cm} .

$$\begin{aligned}
 X_{\text{cm}} = 0 &= \frac{m_e x_e + m_t x_t}{m_e + m_t} \\
 x_e &= -\frac{m_t x_t}{m_e} \\
 &= -\frac{(72.5 \text{ kg})(555 \text{ ft})}{5.97 \times 10^{24} \text{ kg}} \\
 &= -6.74 \times 10^{-21} \text{ ft}
 \end{aligned}$$

The earth moves only $\boxed{6.74 \times 10^{-21} \text{ ft}}$.

56. (a) Use conservation of momentum.

$$\begin{aligned}
 mv + \frac{1}{2}m(0) &= m\left(\frac{v}{3}\right) + \frac{1}{2}mv_2 \\
 \frac{2}{3}mv &= \frac{1}{2}mv_2 \\
 v_2 &= \boxed{\frac{4}{3}v}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad K_i &= \frac{1}{2}mv^2 \\
 K_f &= \frac{1}{2}m\left(\frac{v}{3}\right)^2 + \frac{1}{2}\left(\frac{1}{2}m\right)\left(\frac{4}{3}v\right)^2 \\
 &= \frac{1}{18}mv^2 + \frac{4}{9}mv^2 = \frac{1}{2}mv^2 \\
 K_i &= K_f, \text{ therefore, the collision is } \boxed{\text{elastic}}.
 \end{aligned}$$

57. Use conservation of momentum to determine the horizontal speed of the bullet and block.

m = the mass of the bullet

M = the mass of the block

$$mv + M(0) = (m + M)v_f$$

$$v_f = \left(\frac{m}{m + M}\right)v$$

Recall that $d_y = h = (1/2)gt^2$ for a mass that is initially stationary. Find the horizontal distance.

$$x = v_f t = \left(\frac{m}{m + M}\right)v \sqrt{\frac{2h}{g}} = \left(\frac{0.0100 \text{ kg}}{0.0100 \text{ kg} + 1.30 \text{ kg}}\right)\left(725 \frac{\text{m}}{\text{s}}\right)\sqrt{\frac{2(0.750 \text{ m})}{9.81 \frac{\text{m}}{\text{s}^2}}} = \boxed{2.16 \text{ m}}$$

58. (a) Since egg 2 is farther from the center of mass than egg 1, the location of the center of mass will change more if egg 2 is removed.

$$\begin{aligned}
 \text{(b)} \quad X_{\text{cm}} &= X_{\text{cm,prev}} - \frac{mx_1}{12m} = 0 - \frac{3.0 \text{ cm}}{12} = -0.25 \text{ cm} \\
 Y_{\text{cm}} &= Y_{\text{cm,prev}} - \frac{my_1}{12m} = 0 - \frac{3.5 \text{ cm}}{12} = -0.29 \text{ cm} \\
 (X_{\text{cm}}, Y_{\text{cm}}) &= \boxed{(-0.25 \text{ cm}, -0.29 \text{ cm})}
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad X_{\text{cm}} &= X_{\text{cm,prev}} - \frac{mx_2}{12m} = 0 - \frac{15 \text{ cm}}{12} = -1.3 \text{ cm} \\
 Y_{\text{cm}} &= Y_{\text{cm,prev}} - \frac{my_2}{12m} = 0 - \frac{3.5 \text{ cm}}{12} = -0.29 \text{ cm} \\
 (X_{\text{cm}}, Y_{\text{cm}}) &= \boxed{(-1.3 \text{ cm}, -0.29 \text{ cm})}
 \end{aligned}$$

59. Use the thrust equation to estimate the force.

$$\text{thrust} = \frac{\Delta m}{\Delta t} v = \frac{\left(1000 \frac{\text{kg}}{\text{m}^3}\right)(1 \text{ m}^2)(31 \text{ in.})\left(0.0254 \frac{\text{m}}{\text{in.}}\right)\left(10 \frac{\text{m}}{\text{s}}\right)}{(9 \text{ h})\left(3600 \frac{\text{s}}{\text{h}}\right)} = \boxed{0.24 \text{ N}}$$

60. (a) The change in momentum per second is the weight of the apple.

$$\frac{\Delta p}{\Delta t} = F = mg = \boxed{3.0 \text{ N}}$$

$$\text{(b)} \quad \Delta p = F \Delta t = W \Delta t = W t_f = (3.0 \text{ N})(1.5 \text{ s}) = \boxed{4.5 \text{ kg} \cdot \text{m/s}}$$

61. Place the origin at the center of the wheel and the lead weight on the positive x -axis.

$$X_{\text{cm}} = 0 = \frac{m_{\text{lw}}x_{\text{lw}} + m_{\text{wh}}X_{\text{cm,prev}}}{m_{\text{lw}} + m_{\text{wh}}}$$

$$X_{\text{cm,prev}} = -\frac{m_{\text{lw}}}{m_{\text{wh}}}x_{\text{lw}} = -\left(\frac{0.0502 \text{ kg}}{35.5 \text{ kg}}\right)(25.0 \text{ cm}) = -3.54 \times 10^{-2} \text{ cm}$$

Before the lead weight was added, the center of mass was 0.354 mm from the center of the wheel.

62. Since there are no external forces acting on the system in the x -direction, X_{cm} of the system will not change. After the system comes to rest, the ball, hoop, and system have the same X_{cm} . Therefore, the x -coordinate of the ball will be X_{cm} .

$$X_{\text{cm}} = \frac{Mx_{\text{h}} + 2Mx_{\text{b}}}{3M} = \frac{1}{3}(x_{\text{h}} + 2x_{\text{b}}) = \frac{1}{3}\left[0 + 2\left(R - \frac{R}{4}\right)\right] = \left[\frac{R}{2}\right]$$

63. (a) When the canoeist walks toward the shore, the canoe will move away from the shore according to conservation of momentum. Therefore, her distance from the shore is greater than 2.5 m.

- (b) Place the origin at the center of the canoe before the canoeist walks toward the shore.

$X_{\text{cm}} = 0$ and will not change since there is no external force with an x -component acting on the system.

Assume the canoeist walks in the positive x -direction. After the canoeist walks to the end of the canoe, the distance between the canoeist and the canoe's center of mass is $x_M - x_m = 1.5 \text{ m}$, where x_M and x_m represent the x -component of the center of mass for the canoeist and the canoe, respectively.

$$X_{\text{cm}} = 0 = \frac{mx_m + Mx_M}{m + M}$$

$$\begin{aligned} 0 &= mx_m + Mx_M \\ &= mx_m + M(x_m + 1.5 \text{ m}) \\ &= (m + M)x_m + (1.5 \text{ m})M \end{aligned}$$

$$x_m = -(1.5 \text{ m})\left(\frac{M}{m + M}\right)$$

$$= -(1.5 \text{ m})\left(\frac{63 \text{ kg}}{22 \text{ kg} + 63 \text{ kg}}\right) = -1.1 \text{ m}$$

$$x_M = -1.1 \text{ m} + 1.5 \text{ m} = 0.4 \text{ m}$$

The distance between the system's center of mass and the shore is $\frac{3.0 \text{ m}}{2} + 2.5 \text{ m} = 4.0 \text{ m}$. So, the canoeist is

$4.0 \text{ m} - 0.4 \text{ m} = \underline{3.6 \text{ m}}$ from shore.

64. Place the origin at the center of the canoe before the canoeist walks toward the shore. $X_{\text{cm}} = 0$ and will not change since there is no external force with an x -component acting on the system. Assume the canoeist walks in the positive x -direction. After the canoeist walks to the end of the canoe, the distance between the canoeists and the canoe's center of mass is $x_M - x_m = 1.5 \text{ m}$, where x_M and x_m represent the x -component of the center of mass for the canoeist and the canoe, respectively.

$$X_{\text{cm}} = 0 = \frac{mx_m + Mx_M}{m + M}$$

$$\begin{aligned} 0 &= mx_m + Mx_M \\ &= m(x_M - 1.5 \text{ m}) + Mx_M \end{aligned}$$

$$m = -\frac{Mx_M}{x_M - 1.5 \text{ m}}$$

The distance between the system's center of mass and the shore is $(3.0 \text{ m})/2 + 2.5 \text{ m} = 4.0 \text{ m}$. So,

$$x_M = 4.0 \text{ m} - 3.4 \text{ m} = 0.6 \text{ m}.$$

$$m = -\frac{(63 \text{ kg})(0.6 \text{ m})}{0.6 \text{ m} - 1.5 \text{ m}} = \boxed{42 \text{ kg}}$$

65. (a) Scale reading $= W = (m_1 + m_2)g = (0.150 \text{ kg} + 1.20 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right) = \boxed{13.2 \text{ N}}$

(b) In the absence of the liquid, the ball would fall with an acceleration equal to g . The liquid is retarding the motion of the ball with a force of $m_1\left(g - \frac{g}{4}\right) = \left(\frac{3}{4}\right)m_1g$. So, the scale reading is

$$\frac{3}{4}m_1g + m_2g = \left(\frac{3}{4}m_1 + m_2\right)g = \left[\frac{3}{4}(0.150 \text{ kg}) + 1.20 \text{ kg}\right]\left(9.81 \frac{\text{m}}{\text{s}^2}\right) = \boxed{12.9 \text{ N}}$$

66. Use conservation of momentum.

$$0 = m_p v_p + m_h v_h$$

$$m_p = -\frac{m_h v_h}{v_p} = -\frac{(1.1 \text{ kg})\left(6.2 \frac{\text{m}}{\text{s}}\right)\cos 13^\circ}{-0.25 \frac{\text{m}}{\text{s}}} = \boxed{27 \text{ kg}}$$

67. Due to symmetry, $Y_{\text{cm}} = 0$.

$$X_{\text{cm}} = \frac{2(1.0u)(9.6 \times 10^{-11} \text{ m})\cos 52.25^\circ + (16u)(0)}{2(1.0u) + 16u} = 6.5 \times 10^{-12} \text{ m}$$

$$(X_{\text{cm}}, Y_{\text{cm}}) = \boxed{(6.5 \times 10^{-12} \text{ m}, 0)}$$

68. $m = \left(0.125 \frac{\text{kg}}{\text{m}}\right)(0.500 \text{ m}) = 0.0625 \text{ kg}$

$$\begin{aligned} F &= mg + \left(\frac{\Delta m}{\Delta t}\right)v \\ &= mg + \left[\left(\frac{m}{L}\right)v\right]v \\ &= (0.0625 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right) + \left[\left(0.125 \frac{\text{kg}}{\text{m}}\right)\left(1.33 \frac{\text{m}}{\text{s}}\right)\right]\left(1.33 \frac{\text{m}}{\text{s}}\right) \\ &= \boxed{0.834 \text{ N}} \end{aligned}$$

69. $V_{\text{cm}} = \frac{2}{3}v_0 = \frac{mv_0 + m_r\left(\frac{v_0}{2}\right)}{m + m_r}$

$$\frac{2}{3}(m + m_r) = m + \frac{1}{2}m_r$$

$$\left(\frac{2}{3} - \frac{1}{2}\right)m_r = \left(1 - \frac{2}{3}\right)m$$

$$m_r = \boxed{2m}$$

70. (a) $mv_0 = (2m)v_f$

$$v_f = \boxed{\frac{v_0}{2}}$$

(b) $\frac{1}{2}mv_0^2 = \frac{1}{2}(2m)v_f^2$

$$v_f = \boxed{\frac{v_0}{\sqrt{2}}}$$

71. Assume gravity is the only force acting on the rocket after it is launched. After rising for 2.5 s its speed is $v = 44.2 \text{ m/s} - (9.81 \text{ m/s}^2)(2.50 \text{ s}) = 19.7 \text{ m/s}$.

(a) Since the initial momentum is upward, each piece must have a momentum with a vertical component equal to half the initial momentum.

$$p_{1y} = p_{2y} = \frac{1}{2}(mv) = \left(\frac{m}{2}\right)v_f \sin 45^\circ$$

$$v_f = \frac{v}{\sin 45^\circ} = \frac{19.7 \text{ m/s}}{\sin 45^\circ} = \boxed{27.9 \text{ m/s}}$$

(b) Before the explosion $V_{\text{cm}} = \boxed{(19.7 \text{ m/s})\hat{y}}$.

Since the momentum of the system is the same after the explosion, and the total mass has not changed,

$V_{\text{cm}} = \boxed{(19.7 \text{ m/s})\hat{y}}$ after the explosion too.

(c) The only force acting on the system before and after the explosion is gravity. Therefore,

$$A_{\text{cm}} = \boxed{(-9.81 \text{ m/s}^2)\hat{y}}.$$

72. (a) $(11,000 \text{ kg} \cdot \text{m/s})\hat{x} + (-370 \text{ kg} \cdot \text{m/s})\hat{y} + \vec{p}_2 = (15,000 \text{ kg} \cdot \text{m/s})\hat{x} + (2100 \text{ kg} \cdot \text{m/s})\hat{y}$

$$\boxed{\vec{p}_2 = (4000 \text{ kg} \cdot \text{m/s})\hat{x} + (2470 \text{ kg} \cdot \text{m/s})\hat{y}}$$

(b) **No** Momentum depends only on mass and velocity. It is independent of position.

73. (a) Use momentum conservation.

$$m_1v_1 + m_2v_2 = m_1v_f + m_2v_f$$

$$(0.84 \text{ kg})(0) + (0.42 \text{ kg})(0.68 \text{ m/s}) = (0.84 \text{ kg} + 0.42 \text{ kg})v_f$$

$$v_f = \boxed{0.23 \text{ m/s}}$$

(b) The energy stored in the spring bumper is equal to the loss of kinetic energy at that time.

$$\Delta K = K_f - K_i = \frac{1}{2}(0.84 \text{ kg} + 0.42 \text{ kg})(0.227 \text{ m/s})^2 - \frac{1}{2}(0.42 \text{ kg})(0.68 \text{ m/s})^2 = -0.065 \text{ J}$$

Energy stored in the bumper is $\boxed{0.065 \text{ J}}$.

(c) Since this is a one-dimensional, head-on elastic collision, we can use the results of Problem 74.

$$v_{1,f} = \left(\frac{0.84 \text{ kg} - 0.42 \text{ kg}}{0.84 \text{ kg} + 0.42 \text{ kg}}\right)(0) + \left[\frac{2(0.42 \text{ kg})}{0.84 \text{ kg} + 0.42 \text{ kg}}\right](0.68 \text{ m/s}) = \boxed{0.45 \text{ m/s}}$$

$$v_{2,f} = \left[\frac{2(0.84 \text{ kg})}{0.84 \text{ kg} + 0.42 \text{ kg}}\right](0) + \left(\frac{0.42 \text{ kg} - 0.84 \text{ kg}}{0.84 \text{ kg} + 0.42 \text{ kg}}\right)(0.68 \text{ m/s}) = \boxed{-0.23 \text{ m/s}}$$

74. Use momentum conservation.

$$m_1 v_1 + m_2 v_2 = m_1 v_{1f} + m_2 v_{2f}$$

Use conservation of kinetic energy.

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2.$$

Rearranging the first equation gives

$$\frac{m_1 (v_1 - v_{1f})}{m_2 (v_{2f} - v_2)} = 1.$$

Rearranging the second equation gives

$$\frac{m_1 (v_1^2 - v_{1f}^2)}{m_2 (v_{2f}^2 - v_2^2)} = 1 = \frac{m_1 (v_1 - v_{1f})(v_1 + v_{1f})}{m_2 (v_{2f} - v_2)(v_{2f} + v_2)}.$$

Comparing these two equations implies that

$$\frac{v_1 + v_{1f}}{v_{2f} + v_2} = 1, \text{ or } v_{2f} = v_1 + v_{1f} - v_2.$$

Substitute for v_{2f} in the first equation and solve for v_{1f} .

$$m_1 v_1 + m_2 v_2 = m_1 v_{1f} + m_2 (v_1 + v_{1f} - v_2)$$

$$(m_1 - m_2) v_1 + 2m_2 v_2 = (m_1 + m_2) v_{1f}$$

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_1 + \left(\frac{2m_2}{m_1 + m_2} \right) v_2$$

Since $v_{2f} = v_1 + v_{1f} - v_2$, $v_{1f} = v_{2f} + v_2 - v_1$.Substitute for v_{1f} in the first equation and solve for v_{2f} .

$$m_1 v_1 + m_2 v_2 = m_1 (v_{2f} + v_2 - v_1) + m_2 v_{2f}$$

$$2m_1 v_1 + (m_2 - m_1) v_2 = (m_1 + m_2) v_{2f}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2} \right) v_1 + \left(\frac{m_2 - m_1}{m_1 + m_2} \right) v_2$$

$$\begin{aligned} 75. \quad v_{2f} - v_{1f} &= \left[\left(\frac{2m_1}{m_1 + m_2} \right) - \left(\frac{m_1 - m_2}{m_1 + m_2} \right) \right] v_{1i} + \left[\left(\frac{m_2 - m_1}{m_1 + m_2} \right) - \left(\frac{2m_2}{m_1 + m_2} \right) \right] v_{2i} \\ &= \left(\frac{m_1 + m_2}{m_1 + m_2} \right) v_{1i} + \left(\frac{-m_1 - m_2}{m_1 + m_2} \right) v_{2i} \\ &= v_{1i} - v_{2i} \end{aligned}$$

76. In each case, the potential energy of the spring is converted into the kinetic energy of the cart(s). So, the kinetic energy of the single cart is equal to the sum of the kinetic energies of the two carts.

$$\frac{1}{2} m v^2 = \frac{1}{2} m v_1^2 + \frac{1}{2} m v_2^2$$

Because of momentum conservation, $v_1 = -v_2$. So, we have $\frac{1}{2} m v^2 = \frac{1}{2} m v_f^2 + \frac{1}{2} m (-v_f)^2 = m v_f^2$.The final speed of each cart is $v_f = \boxed{\frac{\sqrt{2}}{2} v}$.

77. Assume v_0 is in the positive x -direction. Use conservation of momentum.

$$\begin{aligned} p_{xi} &= p_{xf} \\ mv_0 &= mv_{1x} + mv_{2x} \\ v_0 &= v_1 \cos \theta_1 + v_2 \cos \theta_2 \end{aligned} \quad (I)$$

$$\begin{aligned} p_{yi} &= p_{yf} \\ 0 &= v_1 \sin \theta_1 + v_2 \sin \theta_2 \end{aligned} \quad (II)$$

Use conservation of kinetic energy.

$$\begin{aligned} \frac{1}{2}mv_0^2 &= \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 \\ v_0^2 &= v_1^2 + v_2^2 \end{aligned} \quad (III)$$

Square (I) and subtract from (III).

$$\begin{aligned} v_0^2 &= v_1^2 + v_2^2 \\ -(v_0^2 &= v_1^2 \cos^2 \theta_1 + v_2^2 \cos^2 \theta_2 + 2v_1v_2 \cos \theta_1 \cos \theta_2) \\ \hline 0 &= v_1^2(1 - \cos^2 \theta_1) + v_2^2(1 - \cos^2 \theta_2) - 2v_1v_2 \cos \theta_1 \cos \theta_2 \\ &= v_1^2 \sin^2 \theta_1 + v_2^2 \sin^2 \theta_2 - 2v_1v_2 \cos \theta_1 \cos \theta_2 \end{aligned} \quad (IV)$$

From (II),

$$v_2 = -\frac{v_1 \sin \theta_1}{\sin \theta_2} \quad \text{and} \quad v_1^2 \sin^2 \theta_1 = v_2^2 \sin^2 \theta_2.$$

Substituting these results into (IV) gives

$$\begin{aligned} 0 &= 2v_1^2 \sin^2 \theta_1 - 2v_1 \left(\frac{v_1 \sin \theta_1}{\sin \theta_2} \right) \cos \theta_1 \cos \theta_2 \\ \sin^2 \theta_1 &= -\frac{\cos \theta_1 \cos \theta_2 \sin \theta_1}{\sin \theta_2} \\ 0 &= \cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2 \\ &= \cos(\theta_1 - \theta_2) \end{aligned}$$

So,

$$\theta_1 - \theta_2 = \cos^{-1} 0 = 90^\circ.$$

78. $F_{\text{net, ext}} = (m_c + m_s)A_{\text{cm}} = m_c a_c + m_s a_s$

$$a_c = \frac{F_{\text{net, ext}} - m_s a_s}{m_c} = \frac{40.0 \text{ N} - (9.50 \text{ kg})\left(2.32 \frac{\text{m}}{\text{s}^2}\right)}{21.0 \text{ kg}} = \boxed{0.855 \text{ m/s}^2}$$

79. Place the origin at the position of impact, and assume that the combined objects move away from the origin in the negative x -direction.

Use conservation of momentum.

$$\begin{aligned} p_{ix} &= p_{fx} \\ mv \cos \theta_1 + mv \cos \theta_2 &= (2m) \left(\frac{v}{3} \right) \\ \cos \theta_1 + \cos \theta_2 &= \frac{2}{3} \\ p_{iy} &= p_{fy} \\ mv \sin \theta_1 + mv \sin \theta_2 &= 0 \\ \sin \theta_1 &= -\sin \theta_2 = \sin(-\theta_2) \\ \theta_1 &= -\theta_2 \end{aligned}$$

Substitute this result into the previous result.

$$\begin{aligned}\cos \theta_1 + \cos(-\theta_1) &= \frac{2}{3} \\ 2 \cos \theta_1 &= \frac{2}{3} \\ \cos \theta_1 &= \frac{1}{3} \\ \theta_1 &= \cos^{-1}\left(\frac{1}{3}\right) \\ &= 70.5^\circ\end{aligned}$$

So, the initial angle was $2(70.5^\circ) = \boxed{141^\circ}$.

80. From Problem 74,

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1 + \left(\frac{2m_2}{m_1 + m_2}\right)v_2.$$

Choose the positive direction to be up.

$$v_{1f} = v_m \quad v_1 = -v$$

$$m_1 = m \quad v_2 = v$$

$$m_2 = M$$

$$v_m = \left(\frac{m - M}{m + M}\right)(-v) + \left(\frac{2M}{m + M}\right)v = \left(\frac{M - m + 2M}{m + M}\right)v = \left(\frac{3M - m}{m + M}\right)v$$

Recall that $v = \sqrt{2gh}$.

$$\begin{aligned}\sqrt{2gh_m} &= \left(\frac{3M - m}{m + M}\right)\sqrt{2gh} \\ h_m &= \left[\left(\frac{3M - m}{m + M}\right)^2 h\right]\end{aligned}$$

81. (a) If the rope's center of mass moves upward with constant acceleration, then the velocity of the rope's center of mass must be increasing linearly with time, since it is upward.

We define the following subscripts:

t = table

nt = not on table

$$V_{\text{cm}} = \frac{m_{\text{nt}}v_{\text{nt}} + m_{\text{t}}v_{\text{t}}}{M} = \frac{\left(\frac{M}{L}vt\right)v + m_{\text{t}}(0)}{M} = \left(\frac{v^2}{L}\right)t$$

V_{cm} is upward, and since both v and L are constant, V_{cm} is proportional to t . Hence A_{cm} is upward and

constant. $\boxed{A_{\text{cm}} = \frac{v^2}{L}}$, the slope of a graph of V_{cm} versus t .

(b) The rope being lowered has downward momentum. Its downward momentum is decreasing as more and more of its mass comes to rest. Therefore, there must be a net upward force acting on the rope, resulting in an upward acceleration of the rope's center of mass.

$$(c) \quad V_{\text{cm}} = \frac{m_{\text{nt}} v_{\text{nt}} + m_{\text{t}} v_{\text{t}}}{M} = \frac{\left(M + \frac{M}{L} vt\right)v + \left(\frac{M}{L} vt\right)(0)}{M} = v + \left(\frac{v^2}{L}\right)t$$

Even though v is negative if the rope is moving downward, the equation for V_{cm} is linear with positive slope. Therefore, A_{cm} , which is the slope of a velocity versus time graph, is positive and constant, having the same magnitude as in part (a), $\frac{v^2}{L}$.