

Chapter 8

Potential Energy and Conservative Forces

Answers to Even-numbered Conceptual Questions

2. As water vapor rises, there is an increase in the gravitational potential energy of the system. Part of this potential energy is released as snow falls onto the mountain. If an avalanche occurs, the snow on the mountain accelerates down slope, converting more gravitational potential energy to kinetic energy.
4. As the ball falls, gravitational potential energy is converted to kinetic energy. When the ball hits the floor, some of the kinetic energy is converted to sound energy and heat, some to a compression of the ball – like compressing a spring. The ball now rebounds, converting the potential energy of compression back to kinetic energy. Finally, the kinetic energy of the ball is converted back to gravitational potential energy as the ball rises. The final height of the ball is less than its initial height because some energy has left the system in the form of sound and heat.
6. The work done in stretching the spring through a doubled distance is the force times the distance. Both of these quantities increase by a factor of two, and therefore the potential energy of the spring increases by a factor of four. We arrive at the same from the form of the spring potential energy, $U = \frac{1}{2} kx^2$, which depends on the square of the amount of stretch.
8. The initial mechanical energy of the system is the gravitational potential energy of the mass-Earth system. As the mass moves downward, the gravitational potential energy of the system decreases. At the same time, the potential energy of the spring increases, as it is compressed. Initially, the decrease in gravitational potential energy is greater than the increase in spring potential energy, which means that the mass gains kinetic energy. Eventually, the increase in spring energy equals the decrease in gravitational energy and the mass comes to rest.
10. If a spring is permanently deformed, it will not return to its original length. As a result, the work that was done to stretch the spring is not fully recovered – some of it goes into the energy of deformation. For this reason, the spring force is not conservative during the deformation. If the spring is now stretched or compressed by a small amount about its new equilibrium position, its force is again conservative – though the force constant will be different.
12. **(a)** The object's kinetic energy is a maximum when it is released, and a minimum when it reaches its greatest height. **(b)** The gravitational potential of the system is a minimum when the object is released, and a maximum when the object reaches its greatest height.
14. When the term “energy conservation” is used in everyday language, it doesn't refer to the total amount of energy in the universe. Instead, it refers to using energy wisely, especially when a particular source of energy – like oil or natural gas – is finite and nonrenewable.
16. The dive begins with the diver climbing the ladder to the diving board, which converts chemical energy in his muscles into an increased gravitational potential energy. Next, by jumping or pumping his legs on the diving board, the diver causes the board to flex and store potential energy. As the board rebounds, the diver springs into the air, using the kinetic energy derived from his leg muscles and the potential energy released by the board. The diver's kinetic energy is then converted into an increased gravitational

potential energy until the highest point of the dive is reached. After that, gravitational potential energy is converted back to kinetic energy as the diver moves downward. Finally, the kinetic energy of the diver is converted to heat, sound, and flowing water as he enters the pool.

18. (a) The potential energy of the system – which is gravitational potential energy – decreases as you move down the hill. (b) Your kinetic energy remains the same, since your speed is constant. (c) Yes. In order for your speed and kinetic energy to remain constant as you pedal down the hill, a nonconservative force must have done negative work on you and your bicycle. For example, you may have applied the brakes to control your speed, or the ground may be soft or muddy.
20. The distance covered by the ball is the same on the way down as it is on the way up, and hence the amount of time will be determined by the average speed of the ball on the two portions of its trip. Note that air resistance does negative, nonconservative work continuously on the ball as it moves. Therefore, its total mechanical energy is less on the way down than it is on the way up, which means that its speed at any given elevation is less on the way down. It follows that more time is required for the downward portion of the trip.
22. (a) The source of energy responsible for the heating is the gravitational potential energy that is released as the shuttle loses altitude. (b) The mechanical energy of the shuttle when it lands is considerably less than when it is in orbit. First, its speed on landing is low compared to its speed in orbit. Second, the energy that goes into heating the tiles is mechanical energy that has been converted to other forms.
24. Zero force implies a zero rate of change in the potential energy – but the value of the potential can be anything at all. Similarly, if the potential energy is zero, it does not mean that the force is zero. Again, what matters is the rate of change of the potential energy.
26. When the toy frog is pressed downward, work is done to compress the spring. This work is stored in the spring as potential energy. Later, when the suction cup releases the spring, the stored potential energy is converted into enough kinetic energy to lift the frog into the air.

Solutions to Problems

1. $W = mgy$

$$W_1 = -(2.6 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(4.0 \text{ m}) + 0 + (2.6 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(1.0 \text{ m}) + 0 + (2.6 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(1.0 \text{ m})$$

$$= \boxed{-51 \text{ J}}$$

$$W_2 = 0 - (2.6 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(2.0 \text{ m}) + 0 = \boxed{-51 \text{ J}}$$

$$W_3 = (2.6 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(1.0 \text{ m}) + 0 - (2.6 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(3.0 \text{ m}) = \boxed{-51 \text{ J}}$$

2. $W = \mu_k mgd$

$$W_1 = -0.23(2.6 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(4.0 \text{ m}) - 0.23(2.6 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(4.0 \text{ m}) - 0.23(2.6 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(1.0 \text{ m})$$

$$- 0.23(2.6 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(1.0 \text{ m}) - 0.23(2.6 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(1.0 \text{ m})$$

$$= \boxed{-65 \text{ J}}$$

$$W_2 = -0.23(2.6 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(2.0 \text{ m}) - 0.23(2.6 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(2.0 \text{ m}) - 0.23(2.6 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(1.0 \text{ m})$$

$$= \boxed{-29 \text{ J}}$$

$$W_3 = -0.23(2.6 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(1.0 \text{ m}) - 0.23(2.6 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(3.0 \text{ m}) - 0.23(2.6 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(3.0 \text{ m})$$

$$= \boxed{-41 \text{ J}}$$

3. (a) $W_1 = -\frac{1}{2}\left(550 \frac{\text{N}}{\text{m}}\right)(0.040 \text{ m})^2 + \left[\frac{1}{2}\left(550 \frac{\text{N}}{\text{m}}\right)(0.040 \text{ m})^2 - \frac{1}{2}\left(550 \frac{\text{N}}{\text{m}}\right)(0.020 \text{ m})^2\right] f = \boxed{-0.11 \text{ J}}$

$$W_2 = -\frac{1}{2}\left(550 \frac{\text{N}}{\text{m}}\right)(-0.020 \text{ m})^2 - \frac{1}{2}\left(550 \frac{\text{N}}{\text{m}}\right)(0.020 \text{ m})^2 + \frac{1}{2}\left(550 \frac{\text{N}}{\text{m}}\right)(0.020 \text{ m})^2 = \boxed{-0.11 \text{ J}}$$

(b) The results have no dependence on the mass of the block.

4. (a) $W_1 = 0 + (5.2 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(1.0 \text{ m}) = \boxed{51 \text{ J}}$

$$W_2 = (5.2 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(1.0 \text{ m}) + 0 = \boxed{51 \text{ J}}$$

(b) The results depend linearly on the mass.

5. (a) $W_{\text{sp}} = -\frac{1}{2}(480 \text{ N/m})(0.020 \text{ m})^2 + \frac{1}{2}(480 \text{ N/m})(0.020 \text{ m})^2 - \frac{1}{2}(480 \text{ N/m})(0.020 \text{ m})^2 = \boxed{-0.096 \text{ J}}$

$$W_f = -\mu_k Nd = -\mu_k mgd = -(0.16)(2.7 \text{ kg})(9.81 \text{ m/s}^2)(0.020 \text{ m} + 0.040 \text{ m}) = \boxed{-0.25 \text{ J}}$$

(b) $W_{\text{sp}} = -\frac{1}{2}(480 \text{ N/m})(0.020 \text{ m})^2 = \boxed{-0.096 \text{ J}}$

$$W_f = -(0.16)(2.7 \text{ kg})(9.81 \text{ m/s}^2)(0.020 \text{ m}) = \boxed{-0.085 \text{ J}}$$

6. $\Delta U = mg\Delta y$

$$mg = \frac{\Delta U}{\Delta y}$$

$$= \frac{-25,000 \text{ J}}{-40.0 \text{ m}}$$

$$= \boxed{630 \text{ N}}$$

$$7. U = mgy = (80.0 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) (8848 \text{ m}) = \boxed{6.94 \text{ MJ}}$$

8. (a) A doubling of the mass will double the extension of the spring. A doubling of the extension of the spring will increase the potential energy by a factor of 4.

$$(b) \sum F_y = F - mg = 0$$

$$F = mg = -k(-y) = ky$$

$$k = \frac{mg}{y}$$

$$U = \frac{1}{2}ky^2 = \frac{1}{2} \left(\frac{mg}{y} \right) y^2 = \frac{1}{2}mgy$$

$$y = \frac{2U}{mg} = \frac{2(0.962 \text{ J})}{(3.0 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right)} = 0.06538 \text{ m}$$

$$k = \frac{mg}{y} = \frac{(3.0 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right)}{0.06538 \text{ m}} = 450 \frac{\text{N}}{\text{m}}$$

Double the mass.

$$y = \frac{2mg}{k} = \frac{(6.0 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right)}{450 \frac{\text{N}}{\text{m}}} = 0.1308 \text{ m}$$

$$U = \frac{1}{2} \left(450 \frac{\text{N}}{\text{m}} \right) (0.1308 \text{ m})^2 = \boxed{3.85 \text{ J}}$$

$$\frac{3.85 \text{ J}}{0.962 \text{ J}} \approx \boxed{4}$$

$$9. U = \frac{1}{2}ky^2$$

$$k = \frac{2U}{y^2}$$

$$= \frac{2(0.0025 \text{ J})}{(0.0050 \text{ m})^2}$$

$$= 200 \frac{\text{N}}{\text{m}}$$

$$y = \sqrt{\frac{2U}{k}} = \sqrt{\frac{2(0.0084 \text{ J})}{200 \frac{\text{N}}{\text{m}}}} = \boxed{0.92 \text{ cm}}$$

$$10. (a) k = \frac{F}{x} = \frac{4.7 \text{ N}}{1.3 \times 10^{-2} \text{ m}} = 362 \frac{\text{N}}{\text{m}}$$

$$U = \frac{1}{2}kx^2$$

$$x = \sqrt{\frac{2U}{k}} = \sqrt{\frac{2(0.020 \text{ J})}{362 \text{ N/m}}} = 0.011 \text{ m} = \boxed{1.1 \text{ cm}}$$

- (b) Since U is proportional to x^2 , the stretch must double to quadruple the potential energy.
 $x = 2(0.0105 \text{ m}) = 0.021 \text{ m} = \boxed{2.1 \text{ cm}}$

$$\begin{aligned}
 11. \quad W &= U_2 - U_1 \\
 &= \frac{1}{2} kx_2^2 - \frac{1}{2} kx_1^2 \\
 &= \frac{1}{2} k(x_2^2 - x_1^2) \\
 k &= \frac{2W}{x_2^2 - x_1^2} \\
 &= \frac{2(30.0 \text{ J})}{(5.00 \times 10^{-2} \text{ m})^2 - (4.00 \times 10^{-2} \text{ m})^2} \\
 &= 6.67 \times 10^4 \frac{\text{N}}{\text{m}} \\
 W &= \frac{1}{2} k(x_2^2 - x_1^2) = \frac{1}{2} \left(6.67 \times 10^4 \frac{\text{N}}{\text{m}} \right) \left[(6.00 \times 10^{-2} \text{ m})^2 - (5.00 \times 10^{-2} \text{ m})^2 \right] = \boxed{36.7 \text{ J}}
 \end{aligned}$$

$$12. \quad \Delta U = mg\Delta y = (0.33 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) [1.2 \text{ m} - (1.2 \text{ m}) \cos 35^\circ] = \boxed{0.70 \text{ J}}$$

$$\begin{aligned}
 13. \quad \Delta K &= -\Delta U \\
 \frac{1}{2} mv_f^2 - 0 &= -(0 - mgh) \\
 v_f &= \sqrt{2gh} = \sqrt{2 \left(9.81 \frac{\text{m}}{\text{s}^2} \right) (2.61 \text{ m})} = \boxed{7.16 \text{ m/s}}
 \end{aligned}$$

$$\begin{aligned}
 14. \quad \Delta K &= -\Delta U \\
 K_f &= U_i - U_f + K_i = \frac{1}{2} mv_f^2 \\
 v_f^2 &= \frac{2}{m} \left(mgh - 0 + \frac{1}{2} mv_i^2 \right) \\
 v_f &= \sqrt{2gh + v_i^2} \\
 &= \sqrt{2 \left(9.81 \frac{\text{m}}{\text{s}^2} \right) (2.61 \text{ m}) + \left(0.840 \frac{\text{m}}{\text{s}} \right)^2} \\
 &= \boxed{7.21 \text{ m/s}}
 \end{aligned}$$

$$\begin{aligned}
 15. \quad (a) \quad \Delta U &= -\Delta K \\
 mg\Delta y &= \frac{1}{2} m(v_i^2 - v_f^2) \\
 \Delta y &= \frac{v_i^2 - v_f^2}{2g} \\
 &= \frac{\left(8.30 \frac{\text{m}}{\text{s}} \right)^2 - \left(7.10 \frac{\text{m}}{\text{s}} \right)^2}{2 \left(9.81 \frac{\text{m}}{\text{s}^2} \right)} \\
 &= \boxed{0.942 \text{ m}}
 \end{aligned}$$

(b) Doubling the ball's mass would cause no change to (a).

16. (a) $\Delta U = -\Delta K$

$$mg\Delta y = \frac{1}{2}m(v_i^2 - v_f^2)$$

$$\Delta y = \frac{v_i^2 - v_f^2}{2g}$$

$$= \frac{\left(16 \frac{\text{m}}{\text{s}}\right)^2 - \left(12 \frac{\text{m}}{\text{s}}\right)^2}{2\left(9.81 \frac{\text{m}}{\text{s}^2}\right)} = 5.7 \text{ m}$$

(b) The equation for Δy is independent of the mass.

17. $U = mgy$ $\Delta U = -\Delta K$

$$K = \frac{1}{2}mv^2$$

$$E = K + U$$

y (m)	4.0	3.0	2.0	1.0	0
U (J)	8.2	6.2	4.1	2.1	0
K (J)	0	2.1	4.1	6.2	8.2
E (J)	8.2	8.2	8.2	8.2	8.2

18. (a) $\Delta K = -\Delta U$

$$\frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}k(x_i^2 - x_f^2)$$

$$k = \frac{m(v_f^2 - v_i^2)}{x_i^2 - x_f^2}$$

$$= \frac{(2.7 \text{ kg})\left[0 - \left(1.1 \frac{\text{m}}{\text{s}}\right)^2\right]}{0 - (6.0 \times 10^{-2} \text{ m})^2}$$

$$= 910 \text{ N/m}$$

(b) $v_i = \sqrt{v_f^2 + \frac{k}{m}(x_f^2 - x_i^2)} = \sqrt{0 + \frac{(907.5 \frac{\text{N}}{\text{m}})}{2.7 \text{ kg}}\left[(1.5 \times 10^{-2} \text{ m})^2 - 0\right]} = 0.28 \text{ m/s}$

19. (a) $K_i + U_i = K_f + U_f$

$$\frac{1}{2}mv_i^2 + mgh = \frac{1}{2}mv_f^2 + 0$$

$$v_i = \sqrt{v_f^2 - 2gh}$$

$$= \sqrt{\left(29 \frac{\text{m}}{\text{s}}\right)^2 - 2\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(32 \text{ m})}$$

$$= 15 \text{ m/s}$$

$$(b) \quad K_i + U_i = K_f + U_f$$

$$0 + mgh = \frac{1}{2}mv^2 + 0$$

$$h = \frac{v^2}{2g} = \frac{\left(29 \frac{\text{m}}{\text{s}}\right)^2}{2\left(9.81 \frac{\text{m}}{\text{s}^2}\right)} = \boxed{43 \text{ m}}$$

$$20. \quad E = U + K = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

for $x = 0$

$$U = \frac{1}{2}k(0)^2 = \boxed{0}$$

$$K = \frac{1}{2}(1.60 \text{ kg})\left(0.950 \frac{\text{m}}{\text{s}}\right)^2 = \boxed{0.722 \text{ J}}$$

$$E = \boxed{0.722 \text{ J}}$$

for $x = 1.00 \text{ cm}$

$$U = \frac{1}{2}\left(902 \frac{\text{N}}{\text{m}}\right)\left(1.00 \times 10^{-2} \text{ m}\right)^2 = \boxed{0.0451 \text{ J}}$$

$$K = E - U = 0.722 \text{ J} - 0.0451 \text{ J} = \boxed{0.677 \text{ J}}$$

$$E = \boxed{0.722 \text{ J}}$$

for $x = 2.00 \text{ cm}$

$$U = \frac{1}{2}\left(902 \frac{\text{N}}{\text{m}}\right)\left(2.00 \times 10^{-2} \text{ m}\right)^2 = \boxed{0.180 \text{ J}}$$

$$K = 0.722 \text{ J} - 0.180 \text{ J} = \boxed{0.542 \text{ J}}$$

$$E = \boxed{0.722 \text{ J}}$$

for $x = 3.00 \text{ cm}$

$$U = \frac{1}{2}\left(902 \frac{\text{N}}{\text{m}}\right)\left(3.00 \times 10^{-2} \text{ m}\right)^2 = \boxed{0.406 \text{ J}}$$

$$K = 0.722 \text{ J} - 0.406 \text{ J} = \boxed{0.316 \text{ J}}$$

$$E = \boxed{0.722 \text{ J}}$$

for $x = 4.00 \text{ cm}$

$$U = \frac{1}{2}\left(902 \frac{\text{N}}{\text{m}}\right)\left(4.00 \times 10^{-2} \text{ m}\right)^2 = \boxed{0.722 \text{ J}}$$

$$K = 0.722 \text{ J} - 0.722 \text{ J} = \boxed{0 \text{ J}}$$

$$E = \boxed{0.722 \text{ J}}$$

$$21. (a) K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}m(0)^2 = \boxed{0}$$

$$K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}m(-2g\Delta y) = \frac{1}{2}(5.00 \text{ kg})(-2)\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(-2.00 \text{ m} - 0) = \boxed{98.1 \text{ J}}$$

$$\Delta K = K_f - K_i = 98.1 \text{ J} - 0 \text{ J} = \boxed{98.1 \text{ J}}$$

$$(b) K_i = \boxed{98.1 \text{ J}}$$

$$K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}m(-2g\Delta y) = \frac{1}{2}(5.00 \text{ kg})(-2)\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(-4.00 \text{ m} - 0) = \boxed{196.2 \text{ J}}$$

$$\Delta K = K_f - K_i = 196.2 \text{ J} - 98.1 \text{ J} = \boxed{98.1 \text{ J}}$$

$$22. (a) \Delta U = mg\Delta y = (0.33 \text{ kg})(9.81 \text{ m/s}^2)[1.2 \text{ m} - (1.2 \text{ m})\cos 35^\circ] = \boxed{0.70 \text{ J}}$$

$$(b) \Delta K = -\Delta U = -0.703 \text{ J}$$

$$\Delta K = K_A - K_B$$

$$K_A = K_B + \Delta K = \frac{1}{2}(0.33 \text{ kg})(2.4 \text{ m/s})^2 + (-0.703 \text{ J}) = 0.247 \text{ J}$$

$$v_A = \sqrt{\frac{2K_A}{m}} = \sqrt{\frac{2(0.247 \text{ J})}{0.33 \text{ kg}}} = \boxed{1.2 \text{ m/s}}$$

- (c) Change in gravitational potential energy is directly proportional to mass. But since both kinetic energy and gravitational potential energy are directly proportional to mass, the mass cancels out of the conservation of energy relation and speed is independent of mass.

$$23. (a) K_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(0.33 \text{ kg})(2.4 \text{ m/s})^2 = \boxed{0.95 \text{ J}}$$

$$(b) \Delta U = -\Delta K = -(K_F - K_B) = -(0 - 0.95 \text{ J}) = \boxed{0.95 \text{ J}}$$

$$(c) \Delta U = mg\Delta y = mgl(1 - \cos \theta)$$

$$\cos \theta = 1 - \frac{\Delta U}{mgl}$$

$$\theta = \cos^{-1}\left(1 - \frac{\Delta U}{mgl}\right) = \cos^{-1}\left[1 - \frac{0.95 \text{ J}}{(0.33 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(1.2 \text{ m})}\right] = \boxed{41^\circ}$$

24. (a) The kinetic energy of the two mass system is given by

$$\Delta K = \frac{1}{2}(m_1 + m_2)v_f^2 = m_2gh - m_1gh = -\Delta U$$

$$v_f = \sqrt{2gh\left(\frac{m_2 - m_1}{m_1 + m_2}\right)}$$

$$(b) v_f = \sqrt{2\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(1.2 \text{ m})\left(\frac{4.1 \text{ kg} - 3.7 \text{ kg}}{3.7 \text{ kg} + 4.1 \text{ kg}}\right)} = \boxed{1.1 \text{ m/s}}$$

25. $\Delta K = -\Delta U$

$$\frac{1}{2}(m_1 + m_2)v_i^2 = m_2 gh - m_1 gh = gh(m_2 - m_1)$$

$$h = \frac{v_i^2}{2g} \left(\frac{m_1 + m_2}{m_2 - m_1} \right) = \frac{\left(0.20 \frac{\text{m}}{\text{s}}\right)^2}{2\left(9.81 \frac{\text{m}}{\text{s}^2}\right)} \left(\frac{3.7 \text{ kg} + 4.1 \text{ kg}}{4.1 \text{ kg} - 3.7 \text{ kg}} \right) = \boxed{4.0 \text{ cm}}$$

26. $W_{\text{total}} = \Delta K = W_c + W_{\text{nc}}$

$$W_{\text{nc}} = \Delta K - W_c$$

$$= \frac{1}{2}m(v_f^2 - v_i^2) - (-\Delta U)$$

$$= \frac{1}{2}m(v_f^2 - v_i^2) + mg(y_f - y_i)$$

$$= \frac{1}{2}(72 \text{ kg}) \left[\left(8.2 \frac{\text{m}}{\text{s}}\right)^2 - \left(1.3 \frac{\text{m}}{\text{s}}\right)^2 \right] + (72 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2}\right)(0 - 1.75 \text{ m})$$

$$= \boxed{1.1 \text{ kJ}}$$

27. $W_{\text{total}} = \Delta K = W_c + W_{\text{nc}} = -\Delta U + W_{\text{nc}}$

$$\frac{1}{2}m(v_f^2 - v_i^2) = -mg(y_f - y_i) + W_{\text{nc}}$$

$$v_f = \sqrt{2g(y_i - y_f) + \frac{2}{m}W_{\text{nc}}} = \sqrt{2\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(2.2 \text{ m} - 0) + \frac{2}{18 \text{ kg}}(-373 \text{ J})} = \boxed{1.3 \text{ m/s}}$$

28. $W_{\text{total}} = \Delta K = W_c + W_{\text{nc1}} + W_{\text{nc2}}$

$$W_{\text{nc2}} = \frac{1}{2}m(v_f^2 - v_i^2) - W_c - W_{\text{nc1}} = \frac{1}{2}(72.0 \text{ kg}) \left(1.20 \frac{\text{m}}{\text{s}}\right)^2 - 0 - 160 \text{ J} = \boxed{-108 \text{ J}}$$

29. $\Delta K = W_c + W_{\text{nc}}$

$$W_{\text{nc}} = \frac{1}{2}m(v_f^2 - v_i^2) - W_c$$

$$= \frac{1}{2}(17,000 \text{ kg}) \left[(0)^2 - \left(82 \frac{\text{m}}{\text{s}}\right)^2 \right] - 0$$

$$= \boxed{57 \text{ MJ}}$$

30. (a) $\Delta K = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}(1100 \text{ kg}) \left[\left(12 \frac{\text{m}}{\text{s}}\right)^2 - \left(17 \frac{\text{m}}{\text{s}}\right)^2 \right] = \boxed{-80 \text{ kJ}}$

(b) The “missing” kinetic energy has been converted into heat energy via friction.

31. $\Delta K = -\Delta U + W_{\text{nc}}$

$$W_{\text{nc}} = -f_k d = -\mu_k m_1 g d$$

$$\Delta U = -m_2 g d$$

$$\frac{1}{2}(m_1 + m_2)(v_f^2 - v_i^2) = m_2 g d - \mu_k m_1 g d$$

$$\begin{aligned} v_f &= \sqrt{v_i^2 + \left(\frac{2}{m_1 + m_2}\right)(m_2 - \mu_k m_1) g d} \\ &= \sqrt{\left(1.3 \frac{\text{m}}{\text{s}}\right)^2 + \left(\frac{2}{2.40 \text{ kg} + 1.80 \text{ kg}}\right)[1.80 \text{ kg} - (0.450)(2.40 \text{ kg})]\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(0.500 \text{ m})} \\ &= \boxed{1.8 \text{ m/s}} \end{aligned}$$

32. (a) $U_i + K_i + W = U_f + K_f$

$$mgh + 0 + W = 0 + \frac{1}{2}mv^2$$

$$W = m\left(\frac{v^2}{2} - gh\right)$$

$$\begin{aligned} &= (42.0 \text{ kg})\left[\frac{\left(4.40 \frac{\text{m}}{\text{s}}\right)^2}{2} - \left(9.81 \frac{\text{m}}{\text{s}^2}\right)(1.75 \text{ m})\right] \\ &= \boxed{-314 \text{ J}} \end{aligned}$$

(b) Apply Newton's 2nd law perpendicular to the ramp.

$$\sum F = 0$$

$$N - mg \cos \theta = 0$$

$$N = mg \cos \theta$$

$$W = fd$$

$$= (\mu N)d$$

$$= \mu(mg \cos \theta)d$$

$$\mu = \frac{W}{mg(\cos \theta)d}$$

$$\begin{aligned} &= \frac{-314 \text{ J}}{(42.0 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(\cos 35.0^\circ)\left(\frac{-1.75 \text{ m}}{\sin 35.0^\circ}\right)} \\ &= \boxed{0.305} \end{aligned}$$

33. $W_{\text{nc}} = Fd$
 $U = mgh$
 $K = W_{\text{nc}} - \Delta U$
 $E = U + K$

for $d = 0$

$$W_{\text{nc}} = (-4.10 \text{ N})(0) = \boxed{0}$$

$$U = (1.75 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(1.00) = \boxed{17.2 \text{ J}}$$

$$K = 0 - 0 = \boxed{0}$$

$$E = 17.2 \text{ J} + 0 = \boxed{17.2 \text{ J}}$$

for $d = 0.500 \text{ m}$

$$W_{\text{nc}} = (-4.10 \text{ N})(0.500 \text{ m}) = \boxed{-2.05 \text{ J}}$$

$$U = (1.75 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(0.500) = \boxed{8.58 \text{ J}}$$

$$K = -2.05 \text{ J} - (8.58 \text{ J} - 17.17 \text{ J}) = \boxed{6.54 \text{ J}}$$

$$E = 8.58 \text{ J} + 6.54 \text{ J} = \boxed{15.1 \text{ J}}$$

for $d = 1.00 \text{ m}$

$$W_{\text{nc}} = (-4.10 \text{ N})(1.00 \text{ m}) = \boxed{-4.10 \text{ J}}$$

$$U = (1.75 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(0) = \boxed{0}$$

$$K = -4.10 \text{ J} - (0 - 17.2 \text{ J}) = \boxed{13.1 \text{ J}}$$

$$E = 0 + 13.1 \text{ J} = \boxed{13.1 \text{ J}}$$

34. $\Delta K = -\Delta U + W_{\text{nc}}$
 $= -mgh + W_{\text{nc}}$
 $= -(1300 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(17.0 \text{ m}) - 3.31 \times 10^5 \text{ J} + 6.34 \times 10^5 \text{ J}$
 $= \boxed{86 \text{ kJ}}$

35. (a) The skater has gone uphill because the work done by the skater is larger than that done by friction and the final speed of the skater is less than the initial speed.

$$(b) \Delta K = -\Delta U + W_{nc}$$

$$\frac{1}{2}m(v_f^2 - v_i^2) = -mgh + W_{nc}$$

$$h = \frac{\frac{1}{2}m(v_f^2 - v_i^2 - W_{nc})}{-mg}$$

$$= \frac{\frac{1}{2}(81.0 \text{ kg})\left[\left(1.60 \frac{\text{m}}{\text{s}}\right)^2 - \left(2.50 \frac{\text{m}}{\text{s}}\right)^2\right] - (3500 \text{ J} - 710 \text{ J})}{-(81.0 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)}$$

$$= \boxed{3.70 \text{ m}}$$

$$36. (a) \Delta K = -\Delta U + W_{nc}$$

$$\Delta U = -m_2gd$$

$$W_{nc} = -\mu_k m_1gd$$

$$\Delta K = m_2gd - \mu_k m_1gd$$

$$= (m_2 - \mu_k m_1)gd$$

$$d = \frac{\frac{1}{2}(m_1 + m_2)v_f^2}{g(m_2 - \mu_k m_1)} = \frac{(2.40 \text{ kg} + 1.80 \text{ kg})\left(2.05 \frac{\text{m}}{\text{s}}\right)^2}{2\left(9.81 \frac{\text{m}}{\text{s}^2}\right)[1.80 \text{ kg} - 0.350(2.40 \text{ kg})]} = \boxed{93.7 \text{ cm}}$$

$$(b) W_c = -\Delta U = -m_2g\Delta y = -(1.80 \text{ kg})(9.81 \text{ m/s}^2)(-0.937 \text{ m}) = \boxed{16.5 \text{ J}}$$

$$(c) W_{nc} = -\mu m_1gd = -(0.350)(2.40 \text{ kg})(9.81 \text{ m/s}^2)(0.937 \text{ m}) = \boxed{-7.72 \text{ J}}$$

$$(d) \Delta K = \frac{1}{2}(m_1 + m_2)[v_f^2 - v_0^2] = \frac{1}{2}(2.40 \text{ kg} + 1.80 \text{ kg})[(2.05 \text{ m/s})^2 - 0] = 8.83 \text{ J}$$

$$\Delta E = \Delta K + \Delta U = 8.83 \text{ J} + (-16.5 \text{ J}) = -7.7 \text{ J} = W_{nc}$$

$$W_{\text{total}} = W_c + W_{nc} = 16.5 \text{ J} + (-7.72 \text{ J}) = 8.8 \text{ J} = \Delta K$$

$$37. (a) \Delta U = mg(y_f - y_i) = (15,800 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(1440 \text{ m} - 1630 \text{ m}) = \boxed{-29.4 \text{ MJ}}$$

$$(b) \Delta K = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}(15,800 \text{ kg})\left[\left(29.0 \frac{\text{m}}{\text{s}}\right)^2 - \left(12.0 \frac{\text{m}}{\text{s}}\right)^2\right] = \boxed{5.51 \text{ MJ}}$$

$$(c) \boxed{\text{No.}} \text{ The total mechanical energy changes by } \Delta E = \Delta K + \Delta U = -23.9 \text{ MJ}.$$

38. $\Delta K = -\Delta U + W_{nc}$

$$\Delta U = W_{nc} - \Delta K$$

$$\frac{1}{2}kx^2 = -\mu_k mgx - \left(-\frac{1}{2}mv_1^2\right)$$

$$k = \frac{m(-2\mu_k gx + v_1^2)}{x^2}$$

$$= \frac{(1.80 \text{ kg}) \left[(-2)(0.560) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) (0.110 \text{ m}) + \left(2.00 \frac{\text{m}}{\text{s}} \right)^2 \right]}{(0.110 \text{ m})^2}$$

$$= \boxed{415 \text{ N/m}}$$

39. At point A, the object is at rest. As the object travels from point A to point B, some of its potential energy is converted into kinetic energy and the object's speed increases. As the object travels from point B to point C, some of its kinetic energy is converted back into potential energy and its speed decreases. From point C to point D, the speed increases again, and from point D to point E, the speed decreases.

40. (a) $\Delta K = -\Delta U$

$$\frac{1}{2}mv_B^2 = -\Delta U$$

$$v_B = \sqrt{\frac{-2\Delta U}{m}}$$

$$= \sqrt{\frac{(-2)(2.0 \text{ J} - 10.0 \text{ J})}{1.1 \text{ kg}}} = \boxed{3.8 \text{ m/s}}$$

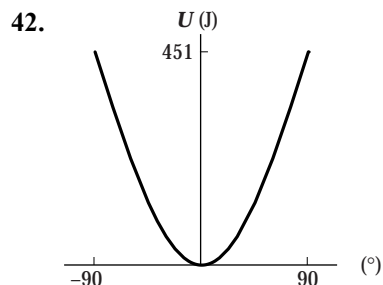
$$v_C = \sqrt{\frac{(-2)(6.0 \text{ J} - 10.0 \text{ J})}{1.1 \text{ kg}}} = \boxed{2.7 \text{ m/s}}$$

$$v_D = \sqrt{\frac{(-2)(5.0 \text{ J} - 10.0 \text{ J})}{1.1 \text{ kg}}} = \boxed{3.0 \text{ m/s}}$$

- (b) Points **A** and **E** are the turning points.

41. $E = \frac{1}{2}mv^2 + U = \frac{1}{2}(1.3 \text{ kg}) \left(1.65 \frac{\text{m}}{\text{s}} \right)^2 + 6.0 \text{ J} = 7.8 \text{ J}$

Just to the right of point A and just to the left of point E.



43. $U = mgl(1 - \cos \theta)$

where l is the length of the chains (2 m)

$$K = \frac{1}{2}mv^2 = E \text{ when the chains are vertical}$$

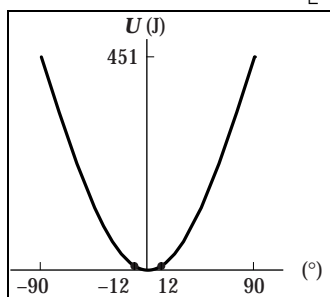
Find θ for $U = E$.

$$\frac{1}{2}mv^2 = mgl(1 - \cos \theta)$$

$$1 - \cos \theta = \frac{v^2}{2gl}$$

$$\cos \theta = 1 - \frac{v^2}{2gl}$$

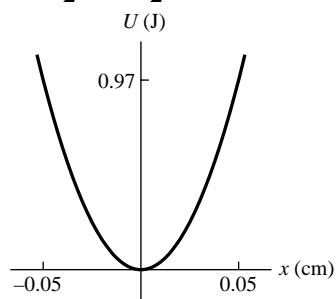
$$\theta = \cos^{-1} \left(1 - \frac{v^2}{2gl} \right) = \cos^{-1} \left[1 - \frac{\left(0.89 \frac{\text{m}}{\text{s}} \right)^2}{2 \left(9.81 \frac{\text{m}}{\text{s}^2} \right) (2.0 \text{ m})} \right] = \boxed{\pm 12^\circ}$$



44. (a) total mechanical energy $= U + K = 5.0 \text{ J} + 3.0 \text{ J} = \boxed{8.0 \text{ J}}$

(b) $\boxed{0.4 \text{ m} \leq x \leq 4.7 \text{ m}}$

45. (a) $U = \frac{1}{2}kx^2 = \frac{1}{2}(775 \text{ N/m})x^2 = (388 \text{ N/m})x^2$



(b) At turning points $U = K_0$

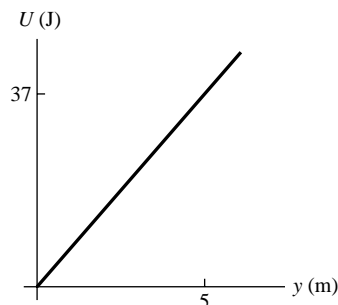
$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

$$x = \pm \sqrt{\frac{m}{k}}v = \pm \sqrt{\frac{0.95 \text{ kg}}{775 \text{ N/m}}}(1.3 \text{ m/s})$$

$$= \pm 0.046 \text{ m}$$

$$= \boxed{\pm 4.6 \text{ cm}}$$

46. (a) $U = mgy = (0.75 \text{ kg})(9.81 \text{ m/s}^2)y = (7.4 \text{ N})y$



(b) At turning point $U = K_0$

$$mgy = \frac{1}{2}mv^2$$

$$y = \frac{v^2}{2g} = \frac{(8.9 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)}$$

$$= \boxed{4.0 \text{ m}}$$

47. Separation = $L \pm x$

$$U_{\text{max}} = K_{\text{max}}$$

$$\frac{1}{2}kx^2 = \frac{1}{2}(2m)v_{\text{max}}^2$$

$$x = \sqrt{\frac{2m}{k}}v_{\text{max}}$$

$$\text{Separation} = \boxed{L \pm v_{\text{max}} \sqrt{\frac{2m}{k}}}$$

48. (a) The final speed is determined by the square root of the sum of the potential energy and the squared initial speed. Therefore, the initial speed contributes less than its value to the final speed. The final speed is less than 10.0 m/s.

(b) $\Delta K = -\Delta U$

$$\frac{1}{2}m(v_f^2 - v_i^2) = -mg(y_f - y_i) = mgh$$

$$v_f = \sqrt{2gh + v_i^2} = \sqrt{\left(8.50 \frac{\text{m}}{\text{s}}\right)^2 + \left(1.50 \frac{\text{m}}{\text{s}}\right)^2} = \boxed{8.63 \text{ m/s}}$$

49. $\Delta K = -\Delta U = mgh$

$$h = \frac{\Delta K}{mg} = \frac{\frac{1}{2}mv_f^2}{mg} = \frac{v_f^2}{2g} = \frac{\left(8.50 \frac{\text{m}}{\text{s}}\right)^2}{2\left(9.81 \frac{\text{m}}{\text{s}^2}\right)} = \boxed{3.68 \text{ m}}$$

50. Find the speed at point B.

$$\Delta K = -\Delta U$$

$$\frac{1}{2}m(v_f^2 - v_i^2) = mgh$$

$$v_f^2 = 2gh + v_i^2$$

Determine the centripetal acceleration.

$$a_{cp} = \frac{v_f^2}{r} = \frac{2gh + v_i^2}{r}$$

$$N = m(g + a_{cp}) = m \left(g + \frac{2gh + v_i^2}{r} \right) = (61 \text{ kg}) \left[9.81 \frac{\text{m}}{\text{s}^2} + \frac{2 \left(9.81 \frac{\text{m}}{\text{s}^2} \right) (1.75 \text{ m}) + \left(8.0 \frac{\text{m}}{\text{s}} \right)^2}{12 \text{ m}} \right] = \boxed{1.1 \text{ kN}}$$

- 51.
- $x = 0.40 \text{ cm} = 0.0040 \text{ m}$

$$u = 0.0046 \text{ J}$$

$$\frac{1}{2}kx^2 = U$$

$$k = \frac{2U}{x^2}$$

$$\frac{1}{2}k(2x)^2 = U + u$$

$$k = \frac{U + u}{2x^2} = \frac{2U}{x^2}$$

$$U + u = 4U$$

$$U = \frac{u}{3}$$

$$k = \frac{2 \left(\frac{u}{3} \right)}{x^2} = \frac{2(0.0046 \text{ J})}{3(0.0040 \text{ m})^2} = \boxed{190 \text{ N/m}}$$

- 52.
- $T = mg + ma_{cp}$

$$a_{cp} = \frac{v^2}{r}$$

$$\frac{1}{2}mv^2 = mgr$$

$$v^2 = 2gr$$

$$T = m \left(g + \frac{2gr}{r} \right) = m(g + 2g) = 3mg = 3(60.0 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) = \boxed{1.77 \text{ kN}}$$

53. (a)
- $U_{45} = U_{90} + \Delta U = 0 + mg\Delta y = (0.13 \text{ kg})(9.81 \text{ m/s}^2)[-(0.95 \text{ m})\cos 45^\circ] = \boxed{-0.86 \text{ J}}$

(b) **Greater**, since the change in height is greater for the first 45° decrease than for the second 45° decrease.

$$(c) \quad U_0 = U_{90} + \Delta U = 0 + mg\Delta y = (0.13 \text{ kg})(9.81 \text{ m/s}^2)(-0.95 \text{ m}) = \boxed{-1.2 \text{ J}}$$

54. (a)
- $W = -\Delta U = -mg\Delta y = -(0.25 \text{ kg})(9.81 \text{ m/s}^2)[-(1.2 \text{ m})(1 - \cos 35^\circ)] = \boxed{0.53 \text{ J}}$

$$W_{B \text{ to } A} = -W_{A \text{ to } B} = \boxed{-0.53 \text{ J}}$$

(b) Zero, since the force exerted by the string is perpendicular to the displacement of the bob.

55. (a) $\Delta E = \Delta U + \Delta K$

$$\begin{aligned}
 &= mg(y_f - y_i) + \frac{1}{2}m(v_f^2 - v_i^2) \\
 &= (865 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(2420 \text{ m}) + \frac{1}{2}(865 \text{ kg})\left(90.0 \frac{\text{m}}{\text{s}}\right)^2 \\
 &= \boxed{24.0 \text{ MJ}}
 \end{aligned}$$

(b) $\Delta K = \Delta U$

$$\begin{aligned}
 \frac{1}{2}mv^2 &= mgh \\
 v &= \sqrt{2gh} \\
 &= \sqrt{2\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(2420 \text{ m})} \\
 &= \boxed{218 \text{ m/s}}
 \end{aligned}$$

56. (a) $mgh = \frac{1}{2}mv^2$

$$\begin{aligned}
 h &= \frac{v^2}{2g} \\
 &= \frac{\left(2.12 \frac{\text{m}}{\text{s}}\right)^2}{2\left(9.81 \frac{\text{m}}{\text{s}^2}\right)} \\
 &= \boxed{0.229 \text{ m}}
 \end{aligned}$$

(b) If the child's initial speed is halved, the result would be reduced by a factor of four.

57. The landing site of a projectile launched horizontally is

$$\begin{aligned}
 x &= v\sqrt{\frac{2y}{g}} \\
 v^2 &= \frac{x^2 g}{2y} \\
 mgh &= \frac{1}{2}mv^2 \\
 h &= \frac{v^2}{2g} = \frac{1}{2g}\left(\frac{x^2 g}{2y}\right) = \frac{x^2}{4y} = \frac{(2.50 \text{ m})^2}{4(1.50 \text{ m})} = \boxed{1.04 \text{ m}}
 \end{aligned}$$

58. The landing site of a projectile launched horizontally is

$$x = v \sqrt{\frac{2y}{g}}$$

Determine v .

$$\Delta K = -\Delta U$$

$$\frac{1}{2}m(v^2 - v_i^2) = -mg(y_f - y_i) = mgh$$

$$v = \sqrt{2gh + v_i^2} = \sqrt{2\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(3.2 \text{ m}) + \left(0.54 \frac{\text{m}}{\text{s}}\right)^2} = 7.94 \frac{\text{m}}{\text{s}}$$

$$x = \left(7.94 \frac{\text{m}}{\text{s}}\right) \sqrt{\frac{2(1.5 \text{ m})}{9.81 \frac{\text{m}}{\text{s}^2}}} = \boxed{4.4 \text{ m}}$$

59. (a) $T_{\text{max}} = 355 \text{ N}$

$$m(g + a_{\text{cp}}) \leq 2T_{\text{max}}$$

$$a_{\text{cp}} = \frac{v^2}{r}$$

Determine v^2 .

$$\frac{1}{2}mv^2 = mgr(1 - \cos \theta)$$

$$v^2 = 2gr(1 - \cos \theta)$$

$$a_{\text{cp}} = \frac{2gr(1 - \cos \theta)}{r} = 2g(1 - \cos \theta)$$

$$m[g + 2g(1 - \cos \theta)] = mg(3 - 2\cos \theta) \leq 2T_{\text{max}}$$

$$mg \leq \frac{2T_{\text{max}}}{3 - 2\cos \theta} = \frac{2(355 \text{ N})}{3 - 2\cos 20.0^\circ} = \boxed{634 \text{ N}}$$

- (b) The maximum weight will decrease because a_{cp} will increase.

60. (a) $U_i + K_i = U_f + K_f$

$$0 + \frac{1}{2}mv_0^2 = mgh + \frac{1}{2}mv_f^2$$

$$v_f^2 = v_0^2 - 2gh$$

$$\sum F = ma_{\text{cp}}$$

$$mg - N = m \frac{v^2}{r}$$

$$v^2 = gr$$

Substituting

$$v_0^2 - 2gh = gr$$

$$v_0 = \boxed{\sqrt{2gh + gr}}$$

- (b) The car leaves the surface of the roadway.

$$61. \quad K_i = \frac{1}{2}mv^2$$

$$U_f = mgh$$

$$K_i = U_f$$

$$v = \sqrt{2gh}$$

$$= \sqrt{2 \left(9.81 \frac{\text{m}}{\text{s}^2} \right) (2.64 \text{ m})} = \boxed{7.20 \text{ m/s}}$$

62. Apply energy balance to find the velocity of the masses when m_2 hits the floor.

$$U_i + K_i = U_f + K_f$$

$$0 + 0 = m_1 gh + m_2 g(-h) + \frac{1}{2}(m_1 + m_2)v^2$$

$$v^2 = \frac{2gh(m_2 - m_1)}{m_1 + m_2}$$

Now apply an energy balance between the time m_2 has landed and m_1 reaches its maximum height.

$$U_i + K_i = U_f + K_f$$

$$m_1 gh + \frac{1}{2}m_1 v^2 = m_1 gh_f + 0$$

$$gh + \frac{1}{2} \left[\frac{2gh(m_2 - m_1)}{m_1 + m_2} \right] = gh_f$$

$$h_f = h \left(1 + \frac{m_2 - m_1}{m_1 + m_2} \right)$$

$$= (1.2 \text{ m}) \left(1 + \frac{4.1 \text{ kg} - 3.7 \text{ kg}}{3.7 \text{ kg} + 4.1 \text{ kg}} \right)$$

$$= 1.26 \text{ m}$$

$$\Delta h = 1.26 \text{ m} - 1.2 \text{ m} = \boxed{0.06 \text{ m}}$$

$$63. \quad U_i + K_i + W = U_f + K_f$$

$$0 + \frac{1}{2}mv_i^2 - fd = mgh + 0$$

$$\frac{1}{2}mv^2 - (\mu mg \cos \theta)d = mgd \sin \theta$$

$$d = \frac{v^2}{2g(\sin \theta + \mu \cos \theta)}$$

$$= \frac{\left(1.66 \frac{\text{m}}{\text{s}} \right)^2}{2 \left(9.81 \frac{\text{m}}{\text{s}^2} \right) (\sin 27.4^\circ + 0.62 \cos 27.4^\circ)}$$

$$= \boxed{0.14 \text{ m}}$$

64.

$$\begin{aligned}
 U_i + K_i + W &= U_f + K_f \\
 mgh + \frac{1}{2}mv_i^2 - fd &= 0 + 0 \\
 mgd \sin \theta + \frac{1}{2}mv_i^2 - (\mu mg \cos \theta)d &= 0 \\
 d &= \frac{v_i^2}{2g(\mu \cos \theta - \sin \theta)} \\
 &= \frac{\left(1.66 \frac{\text{m}}{\text{s}}\right)^2}{2\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(0.62 \cos 27.4^\circ - \sin 27.4^\circ)} \\
 &= \boxed{1.6 \text{ m}}
 \end{aligned}$$

65.

$$\begin{aligned}
 U_i + K_i &= U_f + K_f \\
 mgh + 0 &= 0 + \frac{1}{2}mv_f^2 \\
 v_f^2 &= 2gh \\
 \sum F_y &= ma_{\text{cp}} \\
 T - mg &= m \frac{v^2}{r} \\
 T &= mg + m \frac{v^2}{r} \\
 &= mg + m \frac{2gh}{r} \\
 T &= mg \left[1 + \frac{2h}{r} \right] \\
 &= mg \left[1 + \frac{2}{r}(r - r \cos \theta) \right] \\
 &= mg[1 + 2(1 - \cos \theta)] \\
 &= (73.0 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) [1 + 2(1 - \cos 35.0^\circ)] \\
 &= \boxed{975 \text{ N}}
 \end{aligned}$$

$$66. \quad U_i + K_i = U_f + K_f$$

$$mgh + 0 = 0 + \frac{1}{2}mv^2$$

$$v^2 = 2gh$$

$$\sum F_y = ma_{cp}$$

$$N - mg = m \frac{v^2}{r}$$

$$N = mg + m \frac{(2gh)}{r}$$

$$= mg \left(1 + \frac{2h}{r} \right)$$

$$= (57.0 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) \left[1 + \frac{2(4.00 \text{ m})}{4.00 \text{ m}} \right]$$

$$= \boxed{1.68 \text{ kN}}$$

$$67. \quad (a) \quad U_i + K_i = U_f + K_f$$

$$0 + \frac{1}{2}mv_i^2 = \frac{1}{2}kx^2 + 0$$

$$x = \pm v_i \sqrt{\frac{m}{k}} = \pm \left(1.7 \frac{\text{m}}{\text{s}} \right) \sqrt{\frac{2.2 \text{ kg}}{560 \frac{\text{N}}{\text{m}}}} = \boxed{\pm 0.11 \text{ m}}$$

$$(b) \quad x_1 = \pm v_i \sqrt{\frac{m}{k}}, \quad x_2 = \pm v_i \sqrt{\frac{m}{2k}}$$

$$\frac{x_2}{x_1} = \boxed{\frac{1}{\sqrt{2}}}$$

$$68. \quad U_i + K_i = U_f + K_f$$

$$mgh + 0 = 0 + \frac{1}{2}mv^2$$

$$v = \sqrt{2gh}$$

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

$$0 = y_0 + 0(t) + \frac{1}{2}a_y t^2$$

$$t = \sqrt{\frac{-2y_0}{a_y}} = \sqrt{\frac{-2(0.25 \text{ m})}{-9.81 \frac{\text{m}}{\text{s}^2}}} = 0.226 \text{ s}$$

$$d = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 = 0 + \sqrt{2gh}t + 0 = \sqrt{2 \left(9.81 \frac{\text{m}}{\text{s}^2} \right) (1.5 \text{ m} - 0.25 \text{ m}) (0.226 \text{ s})} = \boxed{1.1 \text{ m}}$$

69. $U_i + K_i + W_{nc} = U_f + K_f$

$$mgh + 0 + W_{nc} = 0 + \frac{1}{2}mv^2$$

$$(1.9 \text{ kg})(9.81 \text{ m/s}^2)(1.25 \text{ m}) + (-9.7 \text{ J}) = \frac{1}{2}(1.9 \text{ kg})v^2$$

$$v = 3.78 \text{ m/s}$$

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

$$0 = y_0 + 0(t) + \frac{1}{2}a_y t^2$$

$$t = \sqrt{\frac{-2y_0}{a_y}} = \sqrt{\frac{-2(0.25 \text{ m})}{-9.81 \text{ m/s}^2}} = 0.226 \text{ s}$$

$$d = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 = 0 + (3.78 \text{ m/s})(0.226 \text{ s}) + 0 = \boxed{0.85 \text{ m}}$$

70. $U_i + K_i = U_f + K_f$

$$0 + \frac{1}{2}mv^2 = \frac{1}{2}kx^2 + \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2$$

$$k = \frac{mv^2\left(1 - \frac{1}{2}\right)}{x^2} = \frac{mv^2}{2x^2}$$

$$k = \frac{(0.25 \text{ kg})(1.2 \text{ m/s})^2}{2(0.018 \text{ m})^2} = \boxed{560 \text{ N/m}}$$

71. $U_i + K_i = U_f + K_f$

$$\frac{1}{2}kx^2 + 0 = 0 + \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{k}{m}}x = \sqrt{\frac{650 \frac{\text{N}}{\text{m}}}{1.4 \text{ kg}}}(0.061 \text{ m})$$

$$= 1.31 \frac{\text{m}}{\text{s}}$$

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

$$0 = y_0 + (0)t + \frac{1}{2}a_y t^2$$

$$t = \sqrt{\frac{-2y_0}{a_y}} = \sqrt{\frac{-2(0.65 \text{ m})}{-9.81 \frac{\text{m}}{\text{s}^2}}} = 0.364 \text{ s}$$

$$d = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 = 0 + \sqrt{\frac{k}{m}}xt + 0 = \sqrt{\frac{650 \frac{\text{N}}{\text{m}}}{1.4 \text{ kg}}}(0.061 \text{ m})(0.364 \text{ s}) = \boxed{48 \text{ cm}}$$

72. (a) $U_i + K_i = U_f + K_f$

$$mgh + 0 = 0 + \frac{1}{2}mv_f^2$$

$$v_f^2 = 2gh$$

$$\sum F_y = ma_{cp}$$

$$T - mg = m \frac{v^2}{L}$$

$$T = mg + m \frac{v^2}{L}$$

$$= mg + m \frac{2gh}{L}$$

$$= mg \left(1 + \frac{2h}{L} \right)$$

$$= mg \left[1 + \frac{2}{L}(L - L \cos \theta) \right]$$

$$= \boxed{mg[1 + 2(1 - \cos \theta)]}$$

- (b) The longer L is, the greater the change in PE but the smaller the centripetal acceleration, so the effect of L cancels out.

73. (a) $U_i + K_i = U_f + K_f$

$$mg\ell + 0 = 0 + \frac{1}{2}mv^2$$

$$v^2 = 2g\ell$$

$$\sum F_y = ma_{cp}$$

$$T - mg = \frac{mv^2}{r}$$

$$T = m \left(g + \frac{v^2}{\ell} \right)$$

$$= m \left[g + \frac{2g\ell}{\ell} \right]$$

$$= 3mg$$

- (b) Tension depends upon a_{cp} , which is proportional to v^2 and inversely proportional to the radius r . Since both v^2 and r are proportional to ℓ , ℓ cancels out.

74. (a) $U_i + K_i = U_f + K_f$

$$mgl + 0 = mgy + 0$$

$$y = l$$

$$L \cos \theta = L - l$$

$$\theta = \cos^{-1} \left(1 - \frac{l}{L} \right)$$

$$\theta = \cos^{-1} \left(1 - \frac{0.325 \text{ m}}{0.652 \text{ m}} \right)$$

$$= \boxed{59.9^\circ}$$

(b) $y = l$. The mass rises to the same height from which it started, because energy is conserved.

(c) $\theta = \cos^{-1} \left(1 - \frac{l}{L} \right)$

75. $-N + mg \cos \theta = \frac{mv^2}{r}$

Determine v^2 .

$$\Delta K = -\Delta U$$

$$\frac{1}{2}mv^2 = -mg(y_f - y_i)$$

$$\begin{aligned} v^2 &= 2g(y_i - y_f) \\ &= 2g(r - r \cos \theta) \\ &= 2gr(1 - \cos \theta) \end{aligned}$$

Set $N = 0$ and solve for θ .

$$N = mg \cos \theta - \frac{m}{r}[2gr(1 - \cos \theta)] = 0$$

$$\begin{aligned} 0 &= mg \cos \theta - 2mg(1 - \cos \theta) \\ &= \cos \theta - 2 + 2 \cos \theta \\ &= 3 \cos \theta - 2 \end{aligned}$$

$$\cos \theta = \frac{2}{3}$$

$$\theta = \cos^{-1} \left(\frac{2}{3} \right)$$

$$= \boxed{48.2^\circ}$$

76. (a) $K_i = \frac{1}{2}(m_1 + m_2)v^2$

$$U_i = 0$$

$$K_f = 0$$

$$U_f = m_2gd = K_i$$

$$d = \frac{K_i}{m_2g} = \boxed{\frac{(m_1 + m_2)v^2}{2m_2g}}$$

- (b) The force the rope exerts on the block is in the same direction as the block's upward displacement. The work is positive.

- (c) The work done by the rope is non-conservative.

$$W_{\text{nc}} = \Delta E = \Delta K + \Delta U = \left(0 - \frac{1}{2}m_2v^2\right) + (m_2gd - 0) = m_2gd - \frac{1}{2}m_2v^2 = \boxed{\frac{1}{2}m_1v^2}$$

77. (a) $U_i + K_i + W_{\text{nc}} = U_f + K_f$

$$0 + \frac{1}{2}(m_1 + m_2)v^2 + (-\mu_k m_1gd) = m_2gd + 0$$

$$v = \sqrt{\frac{2(m_2 + \mu_k m_1)gd}{m_1 + m_2}} = \sqrt{\frac{2[1.1 \text{ kg} + (0.25)(2.4 \text{ kg})](9.81 \text{ m/s}^2)(0.065 \text{ m})}{2.4 \text{ kg} + 1.1 \text{ kg}}} = \boxed{0.79 \text{ m/s}}$$

- (b) The direction of the block's displacement and of the force exerted by the rope on the block are the same. The work is positive.

- (c) $U_i + K_i + W_{\text{nc}} = U_f + K_f$

$$0 + \frac{1}{2}m_2v^2 + W_{\text{nc}} = m_2gd + 0$$

$$W_{\text{nc}} = m_2gd - \frac{1}{2}m_2v^2 = (1.1 \text{ kg})(9.81 \text{ m/s}^2)(0.065 \text{ m}) - \frac{1}{2}(1.1 \text{ kg})(0.787 \text{ m/s})^2 = \boxed{0.36 \text{ J}}$$

78. (a) The lowest normal force will occur at the top of the loop, so we perform an energy balance between the start of the track and this point.

$$U_i + K_i = U_f + K_f$$

$$mgh + 0 = mg(2r) + \frac{1}{2}mv^2$$

$$v^2 = 2g(h - 2r)$$

Now we apply Newton's 2nd law to the block at the top of the loop.

$$\sum F = ma_{\text{cp}}$$

$$N + mg = m\frac{v^2}{r}$$

$$0 + mg = \frac{m[2g(h - 2r)]}{r}$$

$$h = \boxed{\frac{5}{2}r}$$

- (b) Because the speed of the block is independent of its mass and centripetal acceleration is independent of mass.

79. $U_i + K_i - W = U_f + K_f$

$$mgh + 0 - fd = 0 + \frac{1}{2}mv^2$$

$$mgh - \mu mgd = \frac{1}{2}mv^2$$

$$h = \frac{\frac{v^2}{2} + \mu gd}{g}$$

$$= \frac{v^2}{2g} + \mu d$$

$$= \frac{\left(3.50 \frac{\text{m}}{\text{s}}\right)^2}{2\left(9.81 \frac{\text{m}}{\text{s}^2}\right)} + 0.640(0.100 \text{ m})$$

$$= \boxed{68.8 \text{ cm}}$$

80. $U_i + K_i - W = U_f + K_f$

$$\frac{1}{2}kx^2 + 0 - fs = 0 + \frac{1}{2}mv^2$$

$$\frac{1}{2}kd^2 - \mu mgs = \frac{1}{2}mv^2$$

$$d = \sqrt{\frac{m(v^2 + 2\mu gs)}{k}}$$

$$= \sqrt{\frac{(1.2 \text{ kg}) \left[\left(2.3 \frac{\text{m}}{\text{s}}\right)^2 + 2(0.44) \left(9.81 \frac{\text{m}}{\text{s}^2}\right) (0.050 \text{ m}) \right]}{730 \frac{\text{N}}{\text{m}}}}$$

$$= \boxed{9.7 \text{ cm}}$$