

Chapter 19

Electric Charges, Forces, and Fields

Answers to Even-numbered Conceptual Questions

2. Giving an object a negative charge means transferring electrons to the object. This, in turn, increases its mass.
4. No, the basic physics of electric charges would not have been affected at all by an opposite assignment of positive and negative labels. The use of + and – signs, as opposed to labels such as A and B, has the distinct advantage that it gives zero net charge to an object that contains equal amounts of positive and negative charge.
6. **(a)** Referring to Table 19-1, we see that rubbing rabbit fur against glass will result in a positive charge for the rabbit fur, and a negative charge for the glass. **(b)** For glass and rubber, we see that the rubber acquires a negative charge. Note that in this case, the charge on the glass is positive; hence, the charge acquired by a material depends not only on the material itself, but also on the material that it is rubbed against. **(c)** Noting that rabbit fur and glass are adjacent in Table 19-1, whereas glass and rubber are widely separated, we conclude that the magnitude of triboelectric charge is greater in the glass-rubber case.
8. Initially, the bits of paper are uncharged and are attracted to the comb by polarization effects. (See Figure 19-5 and the accompanying discussion.) When one of the bits of paper comes into contact with the comb, it acquires charge from the comb. Now the piece of paper and the comb have charge of the same sign, and hence we expect a repulsive force between them.
10. No. Even uncharged objects will be attracted to a charged rod, due to polarization effects. See Figure 19-5 and the accompanying discussion.
12. Both force laws depend on the product of specific properties of the objects involved; in the case of gravity it is the mass that is relevant, in the case of electrostatics it is the electric charge. In addition, both forces decrease with increasing distance as $1/r^2$. The extremely important difference between the forces, however, is that gravity is always attractive, whereas electrostatic forces can be attractive or repulsive.
14. The answer is (b), the center of the square. At the center of the square, the forces exerted by the charges $+q$ and $-q$ are in the same direction – regardless of the sign of the third charge – giving a large net force. At an empty corner of the square, the third charge is farther from the other charges, and the two forces acting on it are in different directions. Both of these effects tend to decrease the magnitude of the net force.
16. No. If the ball is displaced slightly upward from the equilibrium position, the attractive electrostatic force will be larger than the gravitational force, which will displace the ball farther upward. Similarly, if the ball is displaced slightly downward, the gravitational force is now stronger than the electrostatic force, and the ball will move farther downward. Therefore, the equilibrium is unstable.
18. To begin, assume the fifth charge has the same sign as the four charges at the corners of the square. If the fifth charge is constrained to stay in the plane of the square then, yes, the equilibrium is stable; any displacement within the plane results in a net force back toward the center of the square. On the other hand, if the fifth charge can move in a direction

perpendicular to the plane of the square, its equilibrium is unstable – a small displacement above or below the plane results in a force away from the plane. Thus, in principle, the equilibrium is stable. In any practical situation, however, the instability in the perpendicular direction will eventually come into play. If, on the other hand, the fifth charge is of opposite sign, the situation is reversed. In particular, a displacement within the plane of the square results in an instability; a displacement perpendicular to the plane is stable. Again, the system is unstable in any practical situation.

20. The force is greater at point B than at point A for exactly the same reasons given in Conceptual Checkpoint 19-3. The only difference is that the *direction* of the forces will be reversed.
22. The symmetry of the triangle shows immediately that the direction of the net electric field at the center of the triangle is toward the charge $-q$; that is, 30° below the horizontal. The magnitude of the net electric field is $E_{\text{net}} = E \cos 60^\circ + E \cos 60^\circ + E = 2E$. (Though not asked for, the value of E is readily determined by noting that the distance from any one charge to the center of the triangle is $a/\sqrt{3}$, where a is the length of a side of the triangle. Therefore, $E = 3kq/a^2$.)
24. The proton can be moving in any direction at all relative to the direction of the electric field. On the other hand, the direction of the proton's acceleration must be in the same direction as the electric field.
26. One difference is that when an object is charged by induction, there is no physical contact between the object being charged and the object used to do the charging. In contrast, charging by contact – as the name implies – involves direct physical contact to transfer charge from one object to another. The other main difference is that when an object is charged by induction, the sign of the charge the object acquires is opposite to that of the object used to do the charging. Charging by contact gives the object being charged the same sign of charge as the original charged object.
28. Electric fields can exist in a vacuum, just as light can propagate through a vacuum. In fact, we shall see in Chapter 25 that light is simply a wave of oscillating electric and magnetic fields. Therefore, it also follows that magnetic fields can exist in a vacuum as well.
30. By definition, electric field lines point in the direction of the electric force on a positive charge at any given location in space. This force can point in only one direction at any one location, however. Therefore, electric field lines cannot cross, because if they did, it would imply two different directions for the electric force at the same location.
32. First, the electric field on the line between two charges can vanish only if the two charges have the same sign. Second, the fact that the field vanishes closer to charge 1 than to charge 2 means that charge 1 has the smaller magnitude – it is necessary to get closer to it to have the same effect as from charge 2.
34. The electric flux through the Gaussian surface depends on q_1 ; in general, the electric flux through a surface depends on the charge that is enclosed by the surface. Because charge q_2 is outside the Gaussian surface, however, it has no effect whatsoever on the total electric flux through the surface.
36. No. The electric flux through a surface depends on the total charge enclosed by the surface, but is completely independent of the location of the enclosed charges.

38. Any surface you can draw that includes the charges $+2q$ and $+q$, but excludes the charge $-q$, will have an electric flux equal to $+3q/\epsilon_0$.
40. If the conducting shell had been uncharged, we know that the charge on its inner surface would have been $-Q$ (so that the electric field vanishes inside the shell) and the charge on its outer surface would have been $+Q$ (so that its total charge is zero). When the shell is given a net charge, all of the excess charge moves to the outer surface of the shell. These considerations give the following results: **(a)** Inner surface, $-Q$; outer surface, $+Q + (-2Q) = -Q$. **(b)** Inner surface, $-Q$; outer surface, $+Q + (-Q) = 0$. Though it “appears” that the excess charge ($-Q$) is on the inner surface rather than the outer surface, we see that the charge on the inner surface is in fact the inner induced charge. The excess charge is on the outer surface, as expected, where it combines with the outer induced charge ($+Q$) to give zero charge. **(c)** Inner surface, $-Q$; outer surface, $+Q + (+Q) = +2Q$.

Solutions to Problems

1. $Q_{\text{net}} = N_e(-e) = (3.9 \times 10^7)(-1.60 \times 10^{-19} \text{ C}) = \boxed{-6.2 \times 10^{-12} \text{ C}}$

2. **(a)** $Q_{\text{net}} = N_p e + N_e(-e) = e(N_p - N_e) = (1.60 \times 10^{-19} \text{ C})(7.44 \times 10^6 - 6.15 \times 10^6) = \boxed{2.06 \times 10^{-13}}$

(b) $Q_{\text{net}} = (1.60 \times 10^{-19} \text{ C})(165 - 212) = \boxed{-7.5 \times 10^{-18} \text{ C}}$

3. Neutral carbon has six electrons.

$$Q = -6enN_A = -6(1.60 \times 10^{-19} \text{ C})(2 \text{ mol})(6.022 \times 10^{23} \text{ mol}^{-1}) = \boxed{-1 \times 10^6 \text{ C}}$$

4. **(a)** $Q = \frac{1 \text{ kg}}{9.11 \times 10^{-31} \text{ kg}}(-1.60 \times 10^{-19} \text{ C}) = \boxed{-2 \times 10^{11} \text{ C}}$

(b) $Q = \frac{1 \text{ kg}}{1.673 \times 10^{-27} \text{ kg}}(1.60 \times 10^{-19} \text{ C}) = \boxed{1 \times 10^8 \text{ C}}$

5. $(1.50 \text{ mol})(6.022 \times 10^{23} \text{ mol}^{-1})\left(\frac{1}{10^6}\right)(1.60 \times 10^{-19} \text{ C}) = \boxed{0.145 \text{ C}}$

6. $\frac{(1.8 \times 10^{13})(1.60 \times 10^{-19} \text{ C})}{0.14 \times 10^{-6} \text{ C/cm}} = \boxed{21 \text{ cm}}$

7. **(a)** $N_p + N_e = 1525$

$$Q = e(N_p - N_e)$$

$$= e(1525 - N_e - N_e)$$

$$\frac{Q}{e} - 1525 = -2N_e$$

$$N_e = \frac{1}{2}\left(1525 - \frac{Q}{e}\right)$$

$$= \frac{1}{2}\left(1525 - \frac{-5.456 \times 10^{-17} \text{ C}}{1.60 \times 10^{-19} \text{ C}}\right)$$

$$= \boxed{933}$$

$$\begin{aligned}
 \text{(b)} \quad M_{\text{total}} &= N_e m_e + N_p m_p \\
 &= (933)(9.11 \times 10^{-31} \text{ kg}) + (1525 - 933)(1.673 \times 10^{-27} \text{ kg}) \\
 &= \boxed{9.91 \times 10^{-25} \text{ kg}}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad F &= k \frac{q_1 q_2}{r^2} \\
 r &= \sqrt{\frac{k q_1 q_2}{F}} = \sqrt{\frac{(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2})(11.2 \times 10^{-6} \text{ C})(29.1 \times 10^{-6} \text{ C})}{1.77 \text{ N}}} = \boxed{1.29 \text{ m}}
 \end{aligned}$$

$$\begin{aligned}
 9. \quad F &= -k \frac{qQ}{r^2} \\
 Q &= -\frac{r^2 F}{kq} = -\frac{(1.31 \text{ m})^2 (0.975 \text{ N})}{(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2})(8.44 \times 10^{-6} \text{ C})} = \boxed{-2.21 \times 10^{-5} \text{ C}}
 \end{aligned}$$

$$\begin{aligned}
 10. \quad \frac{ke^2}{r^2} &= \frac{m_e v^2}{r} \\
 r &= \frac{ke^2}{m_e v^2} = \frac{(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2})(1.60 \times 10^{-19} \text{ C})^2}{(9.11 \times 10^{-31} \text{ kg})(7.3 \times 10^5 \frac{\text{m}}{\text{s}})^2} = \boxed{4.7 \times 10^{-10} \text{ m}}
 \end{aligned}$$

$$11. \quad \text{(a)} \quad F = k \frac{q_1 q_2}{r^2} = \left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right) \frac{(3.13 \times 10^{-6} \text{ C})(4.47 \times 10^{-6} \text{ C})}{(0.255 \text{ m})^2} = \boxed{1.93 \text{ N}}$$

(b) The magnitude of the electrostatic force depends upon the product of the charges of both particles, so the negative charge experiences a force magnitude which is the same as that experienced by the positive charge.

$$\begin{aligned}
 12. \quad F &= k \frac{q^2}{r^2} \\
 q &= \sqrt{\frac{Fr^2}{k}} \\
 \text{number of missing electrons} &= \frac{q}{e} \\
 \frac{q}{e} &= \frac{r}{e} \sqrt{\frac{F}{k}} = \frac{6.2 \times 10^{-10} \text{ m}}{1.60 \times 10^{-19} \text{ C}} \sqrt{\frac{5.4 \times 10^{-9} \text{ N}}{8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}}} = \boxed{3.0}
 \end{aligned}$$

$$\begin{aligned}
 13. \quad F &= k \frac{qQ}{r^2} = k \frac{q(\sigma A)}{r^2} = k \frac{q\sigma(4\pi R^2)}{r^2} \\
 r &= \sqrt{\frac{4\pi k q \sigma R^2}{F}} = \sqrt{\frac{4\pi (8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2})(1.75 \times 10^{-6} \text{ C})(12.1 \times 10^{-6} \frac{\text{C}}{\text{m}^2})(0.0622 \text{ m})^2}{46.9 \times 10^{-3} \text{ N}}} = \boxed{44.4 \text{ cm}}
 \end{aligned}$$

14. Let the x-axis be along the line of the three charges with the positive direction pointing from q_2 to q_3 .

$$\vec{F}_{12} = k \frac{q_1 q_2}{d^2} \hat{x}$$

$$\vec{F}_{13} = -k \frac{q_1 q_3}{(2d)^2} \hat{x}$$

$$\begin{aligned} \vec{F}_1 &= \vec{F}_{12} + \vec{F}_{13} \\ &= \frac{k}{d^2} \left[q_1 q_2 - \frac{1}{4} q_1 q_3 \right] \hat{x} \\ &= \frac{k}{d^2} \left[q(2.0q) - \frac{1}{4} q(3.0q) \right] \hat{x} \\ &= \frac{\left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) (12 \times 10^{-6} \text{ C})^2}{(0.16 \text{ m})^2} \left(2.0 - \frac{3.0}{4} \right) \hat{x} \\ &= \boxed{(63 \text{ N}) \hat{x}} \end{aligned}$$

15. (a) Let the x-axis be along the line of the three charges with the positive direction pointing from q_2 to q_3 .

$$\vec{F}_{21} = -k \frac{q_2 q_1}{d^2} \hat{x}$$

$$\vec{F}_{23} = k \frac{q_2 q_3}{d^2} \hat{x}$$

$$\begin{aligned} \vec{F}_2 &= \vec{F}_{21} + \vec{F}_{23} \\ &= \frac{k}{d^2} [-q_2 q_1 + q_2 q_3] \hat{x} \\ &= \frac{k}{d^2} [-(2.0q)q + (2.0q)(3.0q)] \hat{x} \\ &= \frac{\left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) (12 \times 10^{-6} \text{ C})^2}{(0.16 \text{ m})^2} (-2.0 + 6.0) \hat{x} \\ &= (200 \text{ N}) \hat{x} \end{aligned}$$

The net electrostatic force exerted on q_2 is $\boxed{200 \text{ N towards } q_3}$.

- (b) $\boxed{\text{If } d \text{ were tripled, the answer to part (a) would decrease by a factor of } 1/9.}$

$$16. \quad \vec{F}_{21} = -k \frac{q_2 q_1}{r_{21}^2} \hat{x} = -k \frac{(2.0q)q}{x^2} \hat{x}$$

$$\vec{F}_{23} = k \frac{q_2 q_3}{r_{23}^2} = k \frac{(2.0q)(3.0q)}{(0.32 \text{ m} - x)^2} \hat{x}$$

We want $\vec{F}_{21} + \vec{F}_{23} = 0$.

$$-\frac{2.0kq^2}{x^2} \hat{x} + \frac{6.0kq^2}{(0.32 \text{ m} - x)^2} \hat{x} = 0$$

$$\frac{2.0}{x^2} = \frac{6.0}{(0.32 \text{ m} - x)^2}$$

$$6.0x^2 = 2.0[(0.32 \text{ m})^2 - 2(0.32 \text{ m})x + x^2]$$

$$3.0x^2 = (0.32 \text{ m})^2 - 2(0.32 \text{ m})x + x^2$$

$$2.0x^2 + (0.64 \text{ m})x - (0.32 \text{ m})^2 = 0$$

Use the quadratic formula.

$$x = \frac{-0.64 \text{ m} \pm \sqrt{(0.64 \text{ m})^2 - 4(2.0)[-(0.32 \text{ m})^2]}}{2(2.0)} = 0.12 \text{ m}, -0.44 \text{ m}$$

$x = -0.44 \text{ m}$ is extraneous. The net electrostatic force experienced by q_2 is zero when it is

12 cm to the right of q_1 .

$$17. \quad K = \frac{1}{2}mv^2$$

$$v^2 = \frac{2K}{m}$$

From Example 19-1, $v = e\sqrt{\frac{k}{mr}}$, so

$$v^2 = \frac{e^2 k}{mr} = \frac{2K}{m}$$

$$r = \frac{ke^2}{2K} = \frac{\left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(1.60 \times 10^{-19} \text{ C})^2}{2(1.51 \text{ eV})\left(\frac{1.60 \times 10^{-19} \text{ J}}{\text{eV}}\right)} = \boxed{4.76 \times 10^{-10} \text{ m}}$$

$$18. \quad \frac{kqe}{r^2} = mg$$

$$r^2 = \frac{kqe}{mg}$$

$$r = \sqrt{\frac{kqe}{mg}}$$

$$= \sqrt{\frac{\left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(0.35 \times 10^{-9} \text{ C})(1.60 \times 10^{-19} \text{ C})}{(1.673 \times 10^{-27} \text{ kg})(9.81 \frac{\text{m}}{\text{s}^2})}}$$

$$= 5.5 \text{ km}$$

The proton must be placed 5.5 km below q .

$$\begin{aligned}
 19. \quad \frac{kqe}{r^2} &= mg \\
 r^2 &= \frac{kqe}{mg} \\
 r &= \sqrt{\frac{kqe}{mg}} \\
 &= \sqrt{\frac{(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2})(0.35 \times 10^{-9} \text{ C})(1.60 \times 10^{-19} \text{ C})}{(9.11 \times 10^{-31} \text{ kg})(9.81 \frac{\text{m}}{\text{s}^2})}} \\
 &= 240 \text{ km}
 \end{aligned}$$

The electron must be placed 240 km above q .

20. Let q_2 be at the origin and q_3 be on the positive x -axis.

$$\begin{aligned}
 \vec{F}_1 &= -\frac{kq_1q_2}{d^2} \hat{y} = -\frac{kq(2.0q)}{d^2} \hat{y} = -\frac{2.0kq^2}{d^2} \hat{y} \\
 \vec{F}_3 &= -\frac{kq_2q_3}{d^2} \hat{x} = -\frac{k(2.0q)(3.0q)}{d^2} \hat{x} = -\frac{6.0kq^2}{d^2} \hat{x} \\
 \vec{F}_4 &= \frac{kq_2q_4}{(\sqrt{2}d)^2} \left(-\frac{\hat{x}}{\sqrt{2}} + \frac{\hat{y}}{\sqrt{2}} \right) = \frac{k(2.0q)(4.0q)}{2d^2} \left(-\frac{\hat{x}}{\sqrt{2}} + \frac{\hat{y}}{\sqrt{2}} \right) = \frac{2.0\sqrt{2}kq^2}{d^2} (-\hat{x} + \hat{y}) \\
 \vec{F}_{\text{net}} &= \left(-\frac{2.0\sqrt{2}kq^2}{d^2} - \frac{6.0kq^2}{d^2} \right) \hat{x} + \left(\frac{2.0\sqrt{2}kq^2}{d^2} - \frac{2.0kq^2}{d^2} \right) \hat{y} = \frac{2.0kq^2}{d^2} [-(\sqrt{2} + 3.0)\hat{x} + (\sqrt{2} - 1)\hat{y}] \\
 F_{\text{net}} &= \frac{2.0 \left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) (2.4 \times 10^{-6} \text{ C})^2}{(0.33 \text{ m})^2} \sqrt{(\sqrt{2} + 3.0)^2 + (\sqrt{2} - 1)^2} = \boxed{4.2 \text{ N}} \\
 \theta &= \tan^{-1} \frac{\sqrt{2} - 1}{-\sqrt{2} - 3.0} = -5.4^\circ + 180^\circ = \boxed{174.6^\circ}
 \end{aligned}$$

21. (a) Let q_1 be at the origin and q_4 be on the positive x -axis.

$$\begin{aligned}
 \vec{F}_1 &= \frac{kq_1q_3}{(\sqrt{2}d)^2} \left(-\frac{\hat{x}}{\sqrt{2}} - \frac{\hat{y}}{\sqrt{2}} \right) = \frac{3.0\sqrt{2}kq^2}{4d^2} (-\hat{x} - \hat{y}) \\
 \vec{F}_2 &= \frac{kq_2q_3}{d^2} \hat{x} = \frac{6.0kq^2}{d^2} \hat{x} \\
 \vec{F}_4 &= \frac{kq_4q_3}{d^2} \hat{y} = \frac{12kq^2}{d^2} \hat{y} \\
 \vec{F}_{\text{net}} &= \left(\frac{6.0kq^2}{d^2} - \frac{3.0\sqrt{2}kq^2}{4d^2} \right) \hat{x} + \left(\frac{12kq^2}{d^2} - \frac{3.0\sqrt{2}kq^2}{4d^2} \right) \hat{y} = \frac{3.0kq^2}{d^2} \left[\left(2.0 - \frac{\sqrt{2}}{4} \right) \hat{x} + \left(4.0 - \frac{\sqrt{2}}{4} \right) \hat{y} \right] \\
 F_{\text{net}} &= \frac{3.0 \left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) (2.4 \times 10^{-6} \text{ C})^2}{(0.33 \text{ m})^2} \sqrt{\left(2.0 - \frac{\sqrt{2}}{4} \right)^2 + \left(4.0 - \frac{\sqrt{2}}{4} \right)^2} = \boxed{5.7 \text{ N}} \\
 \theta &= \tan^{-1} \frac{4.0 - \frac{\sqrt{2}}{4}}{2.0 - \frac{\sqrt{2}}{4}} = \boxed{66^\circ}
 \end{aligned}$$

- (b) If the distance d were doubled, the magnitude of the electrostatic force would be decreased by a factor of 1/4 and the direction would be unchanged.

22. (a) q_3 feels a force due to q_1 of $\vec{F}_1 = \frac{kq_1q_3}{x^2}\hat{x}$ and a force due to q_2 of $\vec{F}_2 = -\frac{kq_2q_3}{(d-x)^2}\hat{x}$, where q_3 is a signed quantity. Find x such that $F_1 = F_2$.

$$\begin{aligned}\frac{kq_1q_3}{x^2} &= \frac{kq_2q_3}{(d-x)^2} \\ (d-x)^2 &= \frac{q_2}{q_1}x^2 \\ d-x &= \pm\sqrt{\frac{q_2}{q_1}}x \\ d &= x\left(1 \pm \sqrt{\frac{q_2}{q_1}}\right) \\ x &= \frac{d}{1 \pm \sqrt{\frac{q_2}{q_1}}} \\ &= \frac{10.0\text{ cm}}{1 \pm \sqrt{\frac{5.1 \times 10^{-6}\text{ C}}{9.9 \times 10^{-6}\text{ C}}}} \\ &= 5.82\text{ cm or }35\text{ cm}\end{aligned}$$

5.82 cm is extraneous (the net force is not zero), so $x = \boxed{35\text{ cm}}$.

- (b) The forces will balance at this point regardless of whether q_3 is positive or negative because the sign of q_3 only determines whether the forces due to q_1 and q_2 are attractive or repulsive.

23. $q_1 + q_2 = 62.0 \times 10^{-6}\text{ C} = Q$

$$F(d = 0.270\text{ m}) = 1.50\text{ N}$$

$$F = \frac{kq_1q_2}{r^2} = \frac{kq_1(Q - q_1)}{d^2}$$

$$\frac{d^2F}{k} = q_1Q - q_1^2$$

$$0 = q_1^2 - Qq_1 + \frac{d^2F}{k}$$

Use the quadratic formula to solve for q_1 .

$$\begin{aligned}q_1 &= \frac{Q \pm \sqrt{Q^2 - 4(1)\left(\frac{d^2F}{k}\right)}}{2(1)} \\ &= \frac{62.0 \times 10^{-6}\text{ C} \pm \sqrt{(62.0 \times 10^{-6}\text{ C})^2 - \frac{4(0.270\text{ m})^2(1.50\text{ N})}{8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}}}}{2} \\ &= 31.0 \times 10^{-6}\text{ C} \pm 30.8 \times 10^{-6}\text{ C} \\ &= 61.8\text{ }\mu\text{C or }0.2\text{ }\mu\text{C}\end{aligned}$$

Since $q_2 > q_1$, $\boxed{q_2 = 62.0\text{ }\mu\text{C} - 0.2\text{ }\mu\text{C} = 61.8\text{ }\mu\text{C} \text{ and } q_1 = 0.2\text{ }\mu\text{C}}$.

24. Let q_1 be at the origin and q_3 be on the positive x-axis.

$$\begin{aligned} \text{(a)} \quad \vec{F}_2 &= \frac{kq_1q_2}{d^2} [\hat{x} \cos(-120^\circ) + \hat{y} \sin(-120^\circ)] \\ &= \frac{kq_1q_2}{d^2} \left(-\frac{1}{2}\hat{x} - \frac{\sqrt{3}}{2}\hat{y} \right) \end{aligned}$$

$$\vec{F}_3 = \frac{kq_1q_3}{d^2} \hat{x}$$

$$\begin{aligned} \vec{F}_{\text{net}} &= \left(\frac{kq_1q_3}{d^2} - \frac{kq_1q_2}{2d^2} \right) \hat{x} - \frac{\sqrt{3}kq_1q_2}{2d^2} \hat{y} \\ &= \frac{kq_1}{d^2} \left[\left(q_3 - \frac{1}{2}q_2 \right) \hat{x} - \frac{\sqrt{3}}{2}q_2 \hat{y} \right] \end{aligned}$$

$$\begin{aligned} F_{\text{net}} &= \frac{\left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) (2.1 \times 10^{-6} \text{ C})}{(0.0435 \text{ m})^2} \sqrt{\left[0.89 \times 10^{-6} \text{ C} - \frac{1}{2}(6.3 \times 10^{-6} \text{ C}) \right]^2 + \frac{3}{4}(6.3 \times 10^{-6} \text{ C})^2} \\ &= \boxed{59 \text{ N}} \end{aligned}$$

$$\begin{aligned} \theta &= \tan^{-1} \left(\frac{-\frac{\sqrt{3}}{2}q_2}{q_3 - \frac{1}{2}q_2} \right) \\ &= \tan^{-1} \left[\frac{-\frac{\sqrt{3}}{2}(6.3 \times 10^{-6} \text{ C})}{0.89 \times 10^{-6} \text{ C} - \frac{1}{2}(6.3 \times 10^{-6} \text{ C})} \right] \\ &= \boxed{247^\circ} \end{aligned}$$

- (b) If the distance d were doubled, the magnitude of the electrostatic force would be decreased by a factor of 1/4 and the direction would be unchanged.

25. Let q_1 be at the origin and q_3 be on the positive x-axis.

$$\begin{aligned} \text{(a)} \quad \vec{F}_1 &= \frac{kq_1q_2}{d^2} (\hat{x} \cos 60^\circ + \hat{y} \sin 60^\circ) \\ &= \frac{kq_1q_2}{d^2} \left(\frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}\hat{y} \right) \end{aligned}$$

$$\begin{aligned} \vec{F}_3 &= \frac{kq_3q_2}{d^2} [\hat{x} \cos(-60^\circ) + \hat{y} \sin(-60^\circ)] \\ &= \frac{kq_3q_2}{d^2} \left(\frac{1}{2}\hat{x} - \frac{\sqrt{3}}{2}\hat{y} \right) \end{aligned}$$

$$\begin{aligned} \vec{F}_{\text{net}} &= \left(\frac{kq_1q_2}{2d^2} + \frac{kq_3q_2}{2d^2} \right) \hat{x} + \left(\frac{\sqrt{3}kq_1q_2}{2d^2} - \frac{\sqrt{3}kq_3q_2}{2d^2} \right) \hat{y} \\ &= \frac{kq_2}{2d^2} [(q_1 + q_3)\hat{x} + \sqrt{3}(q_1 - q_3)\hat{y}] \end{aligned}$$

$$\begin{aligned}
 F_{\text{net}} &= \frac{kq_2}{2d^2} \sqrt{(q_1 + q_3)^2 + 3(q_1 - q_3)^2} \\
 d &= \sqrt{\frac{kq_2}{2F_{\text{net}}} \sqrt{(q_1 + q_3)^2 + 3(q_1 - q_3)^2}} \\
 &= \sqrt{\frac{(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2})(6.3 \times 10^{-6} \text{ C})}{2(0.65 \text{ N})} \sqrt{(2.1 \times 10^{-6} \text{ C} + 0.89 \times 10^{-6} \text{ C})^2 + 3(2.1 \times 10^{-6} \text{ C} - 0.89 \times 10^{-6} \text{ C})^2}} \\
 &= \boxed{40 \text{ cm}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad \theta &= \tan^{-1} \frac{\sqrt{3}(q_1 - q_3)}{q_1 + q_3} \\
 &= \tan^{-1} \frac{\sqrt{3}(2.1 \times 10^{-6} \text{ C} - 0.89 \times 10^{-6} \text{ C})}{2.1 \times 10^{-6} \text{ C} + 0.89 \times 10^{-6} \text{ C}} \\
 &= \boxed{35^\circ}
 \end{aligned}$$

26. (a) The speed would be greater than that found in the example because the speed varies directly with the square root of the nuclear charge.

$$\begin{aligned}
 \text{(b)} \quad \frac{kq_1q_2}{r^2} &= m_e a_{\text{cp}} \\
 \frac{ke(2e)}{r^2} &= m_e \frac{v^2}{r} \\
 v &= e \sqrt{\frac{2k}{m_e r}} \\
 &= (1.60 \times 10^{-19} \text{ C}) \sqrt{\frac{2(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2})}{(9.11 \times 10^{-31} \text{ kg})(5.29 \times 10^{-11} \text{ m})}} \\
 &= \boxed{3.09 \times 10^6 \text{ m/s}}
 \end{aligned}$$

27. (a) Since Q is at the center of the square, it lies exactly between each pair of charges. Since each charge in a pair has the same sign and magnitude, their forces exactly cancel. So, $\boxed{\vec{F}_{\text{net}} = 0}$.

- (b) Due to symmetry, $\vec{F}_{x,\text{net}} = 0$ and $\vec{F}_{y,\text{net}} = 2\vec{F}_{+,y} + 2\vec{F}_{-,y}$

$$\begin{aligned}
 &= 2 \left[\frac{kqQ}{\left(\frac{a}{\sqrt{2}}\right)^2} \sin 45^\circ \right] (-\hat{y}) + 2 \left[\frac{kqQ}{\left(\frac{a}{\sqrt{2}}\right)^2} \sin 45^\circ \right] (-\hat{y}) \\
 &= \boxed{-\frac{4\sqrt{2}kqQ}{a^2} \hat{y}}
 \end{aligned}$$

$$\text{28. (a)} \quad \frac{93.0 \times 10^{-12} \text{ C}}{1.60 \times 10^{-19} \frac{\text{C}}{\text{electron}}} = \boxed{5.81 \times 10^8 \text{ electrons}}$$

$$\begin{aligned}
 \text{(b)} \quad F &= \frac{kq_1q_2}{r^2} \\
 &= \left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \frac{(93.0 \times 10^{-12} \text{ C})^2}{(1.20 \times 10^{-2} \text{ m})^2} \\
 &= \boxed{5.40 \times 10^{-7} \text{ N}}
 \end{aligned}$$

$$\text{weight} = (0.140 \times 10^{-3} \text{ kg})(9.81 \text{ m/s}^2) = 1.37 \times 10^{-3} \text{ N}$$

$$\frac{F}{mg} = \frac{5.40 \times 10^{-7} \text{ N}}{1.37 \times 10^{-3} \text{ N}} = 3.94 \times 10^{-4}$$

The force is approximately $\frac{4}{10,000}$ the weight.

$$\begin{aligned}
 29. \text{ (a)} \quad T &= \frac{kq^2}{r^2} \\
 q &= r \sqrt{\frac{T}{k}} = (0.066 \text{ m}) \sqrt{\frac{0.21 \text{ N}}{8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}}} = \boxed{3.2 \times 10^{-7} \text{ C}}
 \end{aligned}$$

(b) **No**, because the repulsive force which causes the tension in the string depends only upon the magnitude, not the sign, of the charges.

(c) Regardless of whether the charges are positive or negative, a transfer of $+1.0 \mu\text{C}$ will result in the charges having opposite signs. So, the force between the charges will be attractive and **the tension will be zero**.

$$\begin{aligned}
 30. \quad r_1 &= 6.2 \text{ cm} \\
 r_2 &= 4.7 \text{ cm} \\
 d &= 33 \text{ cm} \\
 Q &= 55 \times 10^{-6} \text{ C} = q_1 + q_2 \\
 F &= 0.75 \text{ N} \\
 q_1 &= \sigma_1 A_1 = 4\pi r_1^2 \sigma_1 \\
 q_2 &= \sigma_2 A_2 = 4\pi r_2^2 \sigma_2 \\
 F &= \frac{kq_1q_2}{d^2} = \frac{kq_1(Q - q_1)}{d^2}
 \end{aligned}$$

Solve for q_1 .

$$\begin{aligned}
 -q_1^2 + q_1Q &= \frac{d^2 F}{k} \\
 0 &= q_1^2 - Qq_1 + \frac{d^2 F}{k}
 \end{aligned}$$

Use the quadratic formula.

$$\begin{aligned}
 q_1 &= \frac{Q \pm \sqrt{Q^2 - \frac{4d^2 F}{k}}}{2} \\
 &= \frac{1}{2}Q \pm \frac{1}{2}\sqrt{Q^2 - \frac{4d^2 F}{k}}
 \end{aligned}$$

Substitute for q_1 .

$$\begin{aligned}
 4\pi r_1^2 \sigma_1 &= \frac{1}{2}Q \pm \frac{1}{2}\sqrt{Q^2 - \frac{4d^2 F}{k}} \\
 \sigma_1 &= \frac{1}{8\pi r_1^2} \left(Q \pm \sqrt{Q^2 - \frac{4d^2 F}{k}} \right) \\
 &= \frac{1}{8\pi(0.062 \text{ m})^2} \left[55 \times 10^{-6} \text{ C} \pm \sqrt{(55 \times 10^{-6} \text{ C})^2 - \frac{4(0.33 \text{ m})^2 (0.75 \text{ N})}{8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}}} \right] \\
 &= 1.1 \times 10^{-3} \frac{\text{C}}{\text{m}^2} \text{ or } 3.4 \times 10^{-6} \frac{\text{C}}{\text{m}^2}
 \end{aligned}$$

Solve for σ_2 .

$$\begin{aligned}
 q_2 &= Q - q_1 \\
 4\pi r_2^2 \sigma_2 &= Q - 4\pi r_1^2 \sigma_1 \\
 \sigma_2 &= \frac{Q - 4\pi r_1^2 \sigma_1}{4\pi r_2^2} \\
 &= \frac{55 \times 10^{-6} \text{ C} - 4\pi(0.062 \text{ m})^2 \sigma_1}{4\pi(0.047)^2} \\
 &= 6.0 \times 10^{-6} \frac{\text{C}}{\text{m}^2} \text{ or } 2.0 \times 10^{-3} \frac{\text{C}}{\text{m}^2}
 \end{aligned}$$

There are two possibilities:

$r_{\text{sphere}} \text{ (cm)}$	1 st case $\sigma \left(\frac{\text{C}}{\text{m}^2} \right)$	2 nd case $\sigma \left(\frac{\text{C}}{\text{m}^2} \right)$
6.2	1.1×10^{-3}	3.4×10^{-6}
4.7	6.0×10^{-6}	2.0×10^{-3}

31. First, note that q_1 and q_2 must have the same sign or $+Q$ would accelerate towards the negative one. Set the forces due to q_1 and q_2 equal.

$$\begin{aligned}
 \frac{kq_1 Q}{\left(\frac{3}{4}d\right)^2} &= \frac{kq_2 Q}{\left(d - \frac{3}{4}d\right)^2} \\
 \frac{16q_1}{9} &= \frac{16q_2}{1} \\
 \boxed{q_1 = 9q_2}
 \end{aligned}$$

32. (a) $E = \frac{kq}{r^2} = \frac{\left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(5.00 \times 10^{-6} \text{ C})}{(1.00 \text{ m})^2} = \boxed{4.50 \times 10^4 \text{ N/C}}$

$$(b) \quad E = \frac{kq}{r^2} = \frac{\left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(5.00 \times 10^{-6} \text{ C})}{(2.00 \text{ m})^2} = \boxed{1.12 \times 10^4 \text{ N/C}}$$

$$33. \quad \vec{F} = qE\hat{y}$$

$$E = \frac{F}{q} = \frac{0.20 \text{ N}}{5.0 \times 10^{-6} \text{ C}} = 4.0 \times 10^4 \text{ N/C}$$

$$\vec{F} = qE\hat{y} = (-2.7 \times 10^{-6} \text{ C})\left(4.0 \times 10^4 \frac{\text{N}}{\text{C}}\right) = \boxed{(-0.11 \text{ N})\hat{y}}$$

$$\begin{aligned} 34. (a) \quad \vec{E} &= \frac{kq_1}{r_1^2}(-\hat{x}) + \frac{kq_2}{r_2^2}\hat{x} \\ &= k\left(-\frac{q_1}{r_1^2} + \frac{q_2}{r_2^2}\right)\hat{x} \\ &= \left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)\left[-\frac{6.2 \times 10^{-6} \text{ C}}{(0.040 \text{ m})^2} + \frac{9.5 \times 10^{-6} \text{ C}}{(0.100 \text{ m} + 0.040 \text{ m})^2}\right]\hat{x} \\ &= \boxed{(-3.0 \times 10^7 \text{ N/C})\hat{x}} \end{aligned}$$

$$\begin{aligned} (b) \quad \vec{E} &= \frac{kq_1}{r_1^2}\hat{x} + \frac{kq_2}{r_2^2}\hat{x} \\ &= k\left(\frac{q_1}{r_1^2} + \frac{q_2}{r_2^2}\right)\hat{x} \\ &= \left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)\left[\frac{6.2 \times 10^{-6} \text{ C}}{(0.040 \text{ m})^2} + \frac{9.5 \times 10^{-6} \text{ C}}{(0.100 \text{ m} - 0.040 \text{ m})^2}\right]\hat{x} \\ &= \boxed{(5.9 \times 10^7 \text{ N/C})\hat{x}} \end{aligned}$$

35. (a) The forces balance. So, the force due to the electric field must be opposite to that due to gravity. And, since the charge is negative, the electric field must be directed downward.

$$qE = mg$$

$$E = \frac{mg}{q}$$

$$\vec{E} = \frac{mg}{q}(-\hat{y}) = -\frac{(0.012 \text{ kg})(9.81 \frac{\text{m}}{\text{s}^2})}{3.6 \times 10^{-6} \text{ C}}\hat{y} = \boxed{(-3.3 \times 10^4 \text{ N/C})\hat{y}}$$

- (b) Since the downward force due to gravity was balanced by the upward force due to the electric field, and since the charge on the object has now increased, the acceleration will be upward.

$$F_q - F_g = ma$$

$$2qE - mg = ma$$

$$2q\left(\frac{mg}{q}\right) - mg = ma$$

$$mg = ma$$

$$a = g$$

$$\text{So, the acceleration is } \boxed{(9.81 \text{ m/s}^2)\hat{y}}.$$

36. Let q_1 be at the origin and q_3 be on the positive x -axis.

- (a) At a point halfway between charges q_1 and q_2 the contributions to the electric field attributed to each of those charges cancel one another. The remaining contribution comes from q_3 .

$$E = \frac{kq_3}{r^2} = \frac{\left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(5.00 \times 10^{-6} \text{ C})}{\left[(0.0275 \text{ m})^2 - \left(\frac{0.0275 \text{ m}}{2}\right)^2\right]} = \boxed{7.93 \times 10^7 \text{ N/C}}$$

- (b) At this location, the electric fields of q_2 and q_3 add, and the resulting field points toward q_3 . The field due to q_1 will have the same magnitude as found in part (a), and will be perpendicular to the combined fields of q_2 and q_3 . The vector sum of the electric fields from all three charges will have a magnitude greater than that found in part (a).

$$(c) \quad \vec{E}_1 = \frac{k|q_1|}{d^2 - \left(\frac{d}{2}\right)^2} (\cos 30^\circ \hat{x} + \sin 30^\circ \hat{y}) = \frac{k|q_1|}{d^2} \left(\frac{2\sqrt{3}}{3} \hat{x} + \frac{2}{3} \hat{y}\right)$$

$$\vec{E}_2 = \frac{k|q_2|}{\left(\frac{d}{2}\right)^2} (\cos 60^\circ \hat{x} - \sin 60^\circ \hat{y}) = \frac{k|q_2|}{d^2} (2\hat{x} - 2\sqrt{3}\hat{y})$$

$$\vec{E}_3 = \frac{k|q_3|}{\left(\frac{d}{2}\right)^2} (\cos 60^\circ \hat{x} - \sin 60^\circ \hat{y}) = \frac{k|q_3|}{d^2} (2\hat{x} - 2\sqrt{3}\hat{y})$$

Since $|q_1| = |q_2| = |q_3|$, let $q = |q_1|$ and we have

$$\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$\vec{E}_{\text{net}} = \frac{kq}{d^2} \left[\left(4 + \frac{2\sqrt{3}}{3}\right) \hat{x} + \left(\frac{2}{3} - 4\sqrt{3}\right) \hat{y} \right]$$

$$\begin{aligned} E_{\text{net}} &= \frac{kq}{d^2} \sqrt{\left(4 + \frac{2\sqrt{3}}{3}\right)^2 + \left(\frac{2}{3} - 4\sqrt{3}\right)^2} \\ &= \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(5.00 \times 10^{-6} \text{ C})(8.110)}{(2.75 \times 10^{-2} \text{ m})^2} \\ &= \boxed{4.82 \times 10^8 \text{ N/C}} \end{aligned}$$

37. The charges must have opposite signs or their electric fields would cancel out at the point of concern.

$$\begin{aligned} E &= \frac{kq}{r^2} + \frac{kq}{r^2} = \frac{2kq}{r^2} \\ q &= \frac{r^2 E}{2k} = \frac{(0.040 \text{ m})^2 \left(45 \frac{\text{N}}{\text{C}}\right)}{2 \left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)} = \boxed{4.0 \times 10^{-12} \text{ C}} \end{aligned}$$

38. (a) At the midpoint of any of the three sides, the electric fields due to the charges at the endpoints of that side cancel out. So, the electric field is due only to the charge opposite the side. The distance from that charge to the midpoint is the height of the triangle. Use the Pythagorean Theorem.

$$b^2 = c^2 - a^2 = (0.21 \text{ m})^2 - \left(\frac{0.21 \text{ m}}{2}\right)^2$$

$$E = \frac{kq}{b^2} = \frac{\left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(4.7 \times 10^{-6} \text{ C})}{(0.21 \text{ m})^2 - \left(\frac{0.21 \text{ m}}{2}\right)^2} = \boxed{1.3 \times 10^6 \text{ N/C}}$$

- (b) Due to symmetry, the electric field at the center of the triangle is zero. So, the magnitude there is less than that at the midpoint of a side.
39. (a) The magnitude of the electric field will be greatest for the $(+q, +q, -q, -q)$ configuration (2) because the magnitude of the electric field is zero for $(+q, -q, +q, -q)$ configuration (1).

- (b) The distance from each corner to the center is $d = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \frac{\sqrt{2}}{2}a$.

For case (1):

Place the square such that $q_1 = +q$ is at the origin, $q_2 = -q$ is at $(a, 0)$, $q_3 = +q$ is at (a, a) , and $q_4 = -q$ is at $(0, a)$.

$$\hat{x}: E_{1x} + E_{2x} + E_{3x} + E_{4x} = \frac{kq}{\frac{1}{2}a^2} (\cos 45^\circ + \cos 315^\circ + \cos 225^\circ + \cos 135^\circ)$$

$$= \frac{2kq}{a^2} \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \right)$$

$$= 0$$

$$\hat{y}: E_{1y} + E_{2y} + E_{3y} + E_{4y} = \frac{kq}{\frac{1}{2}a^2} (\sin 45^\circ + \sin 315^\circ + \sin 225^\circ + \sin 135^\circ)$$

$$= \frac{2kq}{a^2} \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \right)$$

$$= 0$$

So, $\boxed{\vec{E} = 0}$.

For case (2):

Place the square such that $q_1 = -q$ is at the origin, $q_2 = -q$ is at $(a, 0)$, $q_3 = +q$ is at (a, a) , and $q_4 = +q$ is at $(0, a)$.

$$\hat{x}: E_{1x} + E_{2x} + E_{3x} + E_{4x} = \frac{kq}{\frac{1}{2}a^2} (\cos 225^\circ + \cos 315^\circ + \cos 225^\circ + \cos 315^\circ)$$

$$= \frac{2kq}{a^2} \left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \right)$$

$$= 0$$

$$\begin{aligned}
 \hat{\mathbf{y}}: E_{1y} + E_{2y} + E_{3y} + E_{4y} &= \frac{kq}{\frac{1}{2}a^2} (\sin 225^\circ + \sin 315^\circ + \sin 225^\circ + \sin 315^\circ) \\
 &= \frac{2kq}{a^2} \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \right) \\
 &= -\frac{4\sqrt{2}kq}{a^2}
 \end{aligned}$$

$$\text{So, } \vec{\mathbf{E}} = \boxed{-\frac{4\sqrt{2}kq}{a^2} \hat{\mathbf{y}}}.$$

40. (a) Since both measurements of the electric field point in the positive x -direction, the point charge must be on the x -axis. Also, since the magnitude of an electric field is larger the closer it is measured to its source, the position at the point charge must be greater than $x = 10.0$ cm.

$$E = \frac{kq}{(x_0 - 0.0500 \text{ m})^2} = 10.0 \frac{\text{N}}{\text{C}}$$

$$E = \frac{kq}{(x_0 - 0.100 \text{ m})^2} = 15.0 \frac{\text{N}}{\text{C}}$$

$$\left(10.0 \frac{\text{N}}{\text{C}} \right) (x_0 - 0.0500 \text{ m})^2 = \left(15.0 \frac{\text{N}}{\text{C}} \right) (x_0 - 0.100 \text{ m})^2$$

$$(x_0 - 0.0500 \text{ m})^2 = 1.50(x_0 - 0.100 \text{ m})^2$$

$$x_0^2 - (0.1000 \text{ m})x_0 + 0.00250 \text{ m}^2 = 1.50x_0^2 - (0.300 \text{ m})x_0 + 0.0150 \text{ m}^2$$

$$0 = 0.50x_0^2 - (0.200 \text{ m})x_0 + 0.0125 \text{ m}^2$$

Use the quadratic formula.

$$x_0 = \frac{0.200 \text{ m} \pm \sqrt{(-0.200 \text{ m})^2 - 4(0.50)(0.0125 \text{ m}^2)}}{2(0.50)}$$

$$= 0.20 \text{ m} \pm 0.12 \text{ m}$$

$$= 0.32 \text{ m or } 0.08 \text{ m (0.08 m is extraneous.)}$$

So, the point charge is located at $\boxed{x = 32 \text{ cm}}$.

- (b) Since the electric field is directed towards the point charge, its sign must be negative. Determine the magnitude of the point charge.

$$\begin{aligned}
 q &= \frac{\left(10.0 \frac{\text{N}}{\text{C}} \right) (x_0 - 0.0500 \text{ m})^2}{k} \\
 &= \frac{\left(10.0 \frac{\text{N}}{\text{C}} \right) (0.32247 \text{ m} - 0.0500 \text{ m})^2}{8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}}
 \end{aligned}$$

$$= 8.3 \times 10^{-11} \text{ C}$$

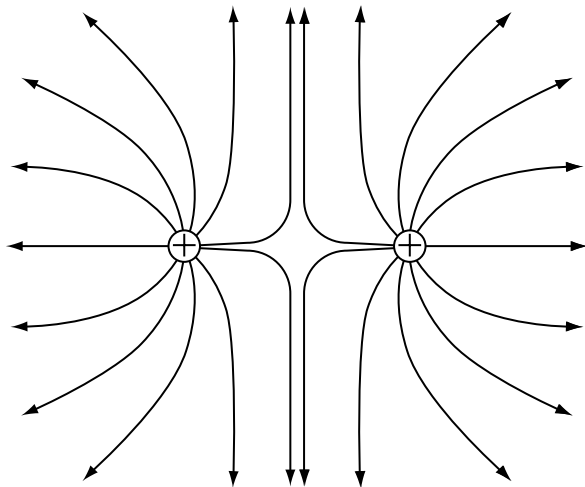
$$\text{So, } q = \boxed{-8.3 \times 10^{-11} \text{ C}}.$$

41. (a) The electric field lines begin at q_1 and q_3 and end at q_2 . Also, electric field lines start at positive charges or at infinity and end at negative charges or at infinity. So, since q_2 is negative, q_1 and q_3 must be $\boxed{\text{positive}}$.

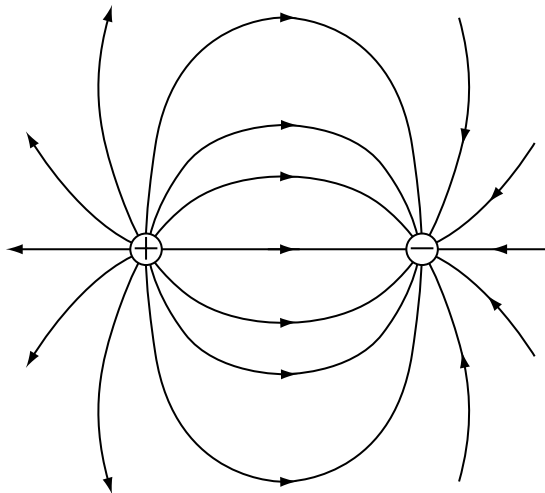
(b) q_1 has 8 lines leaving it. q_2 has 16 lines entering it. Since 8 is half of 16, and since the number of lines entering or leaving a charge is proportional to the magnitude of the charge, the magnitude of q_1 is one-half of q_2 , or $\boxed{5.00\mu\text{C}}$.

(c) By the reasoning of part (b), the magnitude of q_3 is $\boxed{5.00\mu\text{C}}$.

42.



43.



44. Since the number of lines entering or leaving a charge is proportional to the magnitude of the charge, and since the number of lines entering q_2 is twice the number exiting q_1 and q_3 , the magnitude of q_2 is twice that of q_1 and q_3 . Also, since lines enter q_2 and exit q_1 and q_3 , the signs of q_1 and q_3 are opposite to q_2 . Since $q_1 + q_2 < 0$ and $|q_2| > |q_1|$, $q_2 < 0$.

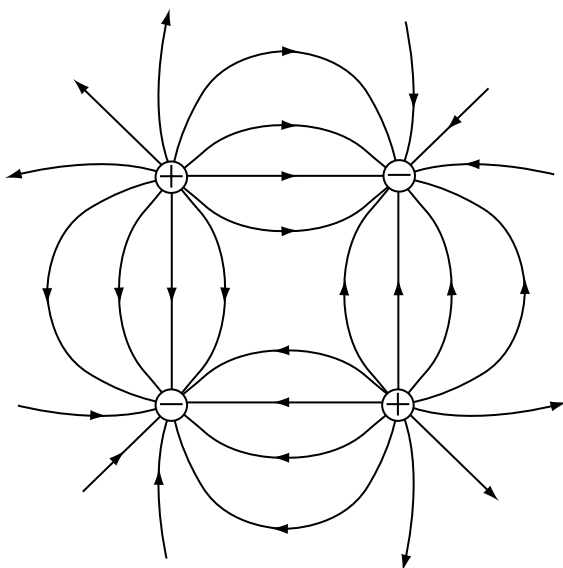
$$q_1 + q_2 = -2.5\mu\text{C}$$

$$-\frac{q_2}{2} + q_2 = -2.5\mu\text{C}$$

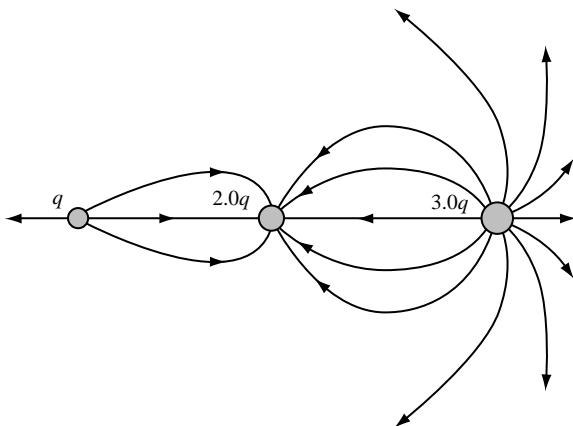
$$q_2 = \boxed{-5.0\mu\text{C}}$$

$$q_1 = -2.5\mu\text{C} + 5.0\mu\text{C} = \boxed{2.5\mu\text{C}} = q_3$$

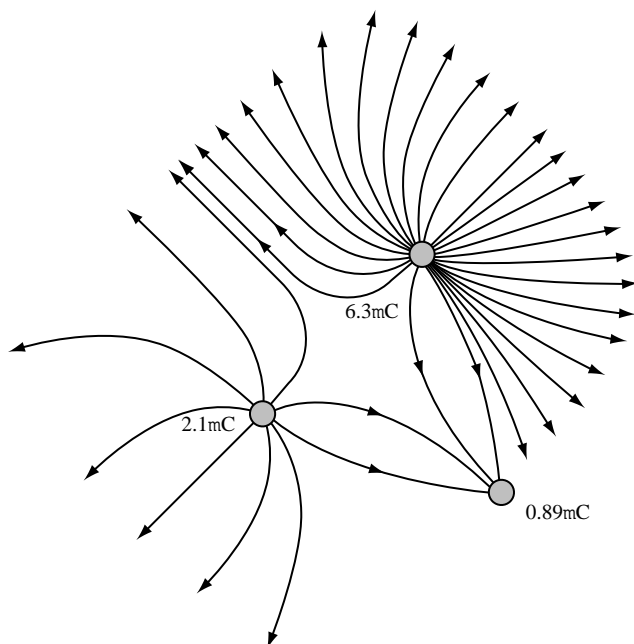
45.



46.



47.



48. Set $F_x = 0$.
 $-qE + T \sin \theta = 0$
 Set $F_y = 0$.
 $T \cos \theta - mg = 0$

$$(a) \quad m = \frac{T}{g} \cos \theta = \frac{0.350 \text{ N}}{9.81 \frac{\text{m}}{\text{s}^2}} \cos 12^\circ = \boxed{35 \text{ g}}$$

$$(b) \quad E = \frac{T}{q} \sin \theta = \frac{0.350 \text{ N}}{2.05 \times 10^{-6} \text{ C}} \sin 12^\circ = \boxed{3.5 \times 10^4 \text{ N/C}}$$

$$49. \quad \Phi = EA \cos \theta = \left(25,000 \frac{\text{N}}{\text{C}} \right) (0.0133 \text{ m}^2) \cos 55^\circ = \boxed{190 \text{ N} \cdot \text{m}^2/\text{C}}$$

$$50. \quad \Phi = \frac{q}{\epsilon_0} = \frac{q_1 + q_2 + q_3}{\epsilon_0} = \frac{(3.2 + 6.9 - 4.1) \times 10^{-6} \text{ C}}{8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}} = \boxed{6.8 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C}}$$

51. (a) The top has the greatest positive electric flux. The bottom has the greatest negative electric flux. The ends and sides have zero flux.

(b) Top:

$$\Phi = EA \cos \theta = \left(6.00 \times 10^3 \frac{\text{N}}{\text{C}} \right) (0.350 \text{ m})(0.250 \text{ m}) \cos 0^\circ = \boxed{525 \text{ N} \cdot \text{m}^2/\text{C}}$$

Bottom:

$$\Phi = \left(6.00 \times 10^3 \frac{\text{N}}{\text{C}} \right) (0.350 \text{ m})(0.250 \text{ m}) \cos 180^\circ = \boxed{-525 \text{ N} \cdot \text{m}^2/\text{C}}$$

Ends:

$$\Phi = EA \cos 90^\circ = \boxed{0}$$

Sides:

$$\Phi = EA \cos 90^\circ = \boxed{0}$$

$$52. \quad E = \frac{\sigma}{\epsilon_0}$$

$$\sigma = \epsilon_0 E = \left(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2} \right) \left(7.0 \times 10^5 \frac{\text{N}}{\text{C}} \right) = \boxed{6.2 \times 10^{-6} \text{ C/m}^2}$$

$$53. \quad q = \epsilon_0 \Phi = \left(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2} \right) (150.0 + 250.0 - 350.0 + 175.0 - 100.0 + 450.0) \frac{\text{N} \cdot \text{m}^2}{\text{C}} = \boxed{5.09 \times 10^{-9} \text{ C}}$$

$$54. \quad (\text{a}) \quad \Phi = \frac{q}{\epsilon_0} = \frac{q_1 + q_2}{\epsilon_0} = \frac{1.65 \mu\text{C} - 2.32 \mu\text{C}}{8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}} = \boxed{-7.6 \times 10^4 \text{ N} \cdot \text{m}^2/\text{C}}$$

$$(\text{b}) \quad \Phi = \frac{q_2 + q_3}{\epsilon_0} = \frac{-2.32 \mu\text{C} + 3.71 \mu\text{C}}{8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}} = \boxed{1.57 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C}}$$

$$(\text{c}) \quad \Phi = \frac{q_1 + q_2 + q_3}{\epsilon_0} = \frac{1.65 \mu\text{C} - 2.32 \mu\text{C} + 3.71 \mu\text{C}}{8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}} = \boxed{3.44 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C}}$$

$$(\text{d}) \quad \Phi = \frac{q_1 + q_2 + q_3 + Q}{\epsilon_0} = 0$$

$$Q = -q_1 - q_2 - q_3 = -1.65 \mu\text{C} + 2.32 \mu\text{C} - 3.71 \mu\text{C} = \boxed{-3.04 \mu\text{C}}$$

55. The Gaussian surface is chosen to be a cylinder with radius r and length L , and its axis is along the wire.

$$\Phi = EA = E(2\pi rL)$$

\vec{E} is assumed to be parallel to the end caps, so only the area of the curved surface is considered. The total charge enclosed by the Gaussian surface is $q = \lambda L$. Apply Gauss's law.

$$\Phi = \frac{q}{\epsilon_0} = \frac{\lambda L}{\epsilon_0} = E(2\pi rL)$$

Solve for E .

$$\boxed{E = \frac{\lambda}{2\pi\epsilon_0 r}}$$

56. $F = qE = ma$

$$a = \frac{q}{m} E$$

Since a , m , and E are all constant, so is a . Recall that for straight-line motion with constant acceleration, the following equation is true.

$$v_f^2 = v_i^2 + 2a\Delta x$$

For our case, $v_i = 0$ and $a = (q/m)E$.

$$v = \sqrt{2a\Delta x} = \sqrt{\frac{2qE}{m} \Delta x}$$

$$(a) \quad v = \sqrt{\frac{2(1.60 \times 10^{-19} \text{ C})(1.08 \times 10^5 \frac{\text{N}}{\text{C}})(0.0100 \text{ m})}{1.673 \times 10^{-27} \text{ kg}}} = \boxed{4.55 \times 10^5 \text{ m/s}}$$

$$(b) \quad v = \sqrt{\frac{2(1.60 \times 10^{-19} \text{ C})(1.08 \times 10^5 \frac{\text{N}}{\text{C}})(0.100 \text{ m})}{1.673 \times 10^{-27} \text{ kg}}} = \boxed{1.44 \times 10^6 \text{ m/s}}$$

57. $F = \frac{kq^2}{r^2}$

$$r = q\sqrt{\frac{k}{F}}$$

$$= (93 \times 10^{-12} \text{ C}) \sqrt{\frac{8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}}{4.0 \times 10^{-10} \text{ N}}}$$

$$= \boxed{0.44 \text{ m}}$$

58. $E = \frac{k|q|}{r^2}$

$$|q| = \frac{Er^2}{k} = \frac{(36,000 \text{ N/C})(0.50 \text{ m})^2}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} = 1.0 \times 10^{-6} \text{ C} = \boxed{1.0 \mu\text{C}}$$

Since the electric field points toward the charge, the charge at the origin is negative.

59. (a) $E = \frac{kq}{r^2} = \frac{(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2})(4.2 \times 10^{-6} \text{ C})}{(1.5 \text{ m})^2} = \boxed{1.7 \times 10^4 \text{ N/C}}$

(b) $\sum F_y = T + F_q - mg = 0$

$$T = mg - F_q$$

$$= mg - \frac{kq_1q_2}{r^2}$$

$$= (0.025 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) - \frac{(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2})(3.1 \times 10^{-6} \text{ C})(4.2 \times 10^{-6} \text{ C})}{(1.5 \text{ m})^2}$$

$$= \boxed{0.19 \text{ N}}$$

60. (a) If only q_2 and q_3 were present, the electric field would be zero at $x = 0.30$ m. Charge q_1 adds a field that points to the left for all positive x . This would cancel the combined fields from q_2 and q_3 just to the left of $x = 0.30$ m, where their combined field points to the right.

- (b) Between $x = 0.20$ m and $x = 0.40$ m the fields from q_1 and q_3 point to the left and the field from q_2 points to the right. Let x be a point on the x -axis between 0.20 m and 0.40 m. $|q_1| = |q_2| = |q_3| = q$

$$\vec{E} = \frac{-kq}{x^2} \hat{x} - \frac{kq}{(0.40 - x)^2} \hat{x} + \frac{kq}{(x - 0.20)^2} \hat{x}$$

$$\frac{E}{kq} = -\frac{1}{x^2} - \frac{1}{(0.40 - x)^2} + \frac{1}{(x - 0.20)^2}$$

Plotting the function $y = -\frac{1}{x^2} - \frac{1}{(0.40 - x)^2} + \frac{1}{(x - 0.20)^2}$ on a graphing calculator and determining the zero of the function gives $x = \underline{0.297 \text{ m}}$.

61. (a) The second point is located to the left of charge 1. To the left of q_1 the field from q_1 points to the right. The fields from both q_2 and q_3 point to the left, and, when combined, are strong enough to cancel the field from q_1 . To the right of q_3 the fields from both q_2 and q_3 point to the right, and the weaker field from q_1 points to the left.

- (b) Let x be the distance to a point to the left of q_1 .

$$\text{Let } |q_1| = |q_2| = |q_3| = q$$

$$\vec{E} = \frac{kq}{x^2} \hat{x} - \frac{kq}{(x + 0.20)^2} \hat{x} - \frac{kq}{(+0.40)^2} \hat{x}$$

$$\frac{E}{kq} = \frac{1}{x^2} - \frac{1}{(x + 0.20)^2} - \frac{1}{(x + 0.40)^2}$$

Plotting the function $y = \frac{1}{x^2} - \frac{1}{(x + 0.20)^2} - \frac{1}{(x + 0.40)^2}$ on a graphing calculator and finding the zeros of the function gives zeros at -0.0973 and $+0.688$. Since x represented the positive distance to the left of the origin, the correct answer is that the electric field is zero at $x = \underline{-0.688 \text{ m}}$.

62. (a) The Gaussian surface is chosen to be a cylinder with radius r and length L , and its axis is along the wire.

$$\Phi = EA = E(2\pi rL)$$

\vec{E} is assumed to be parallel to the end caps, so only the area of the curved surface is considered.

The total charge enclosed by the Gaussian surface is $q = \lambda L$, where λ is the charge per unit length of the wire. Apply Gauss's law.

$$\Phi = \frac{q}{\epsilon_0} = \frac{\lambda L}{\epsilon_0} = E(2\pi rL)$$

$$\lambda = 2\pi\epsilon_0 rE$$

$$\begin{aligned} &= 2\pi \left(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2} \right) (0.500 \text{ m}) \left(25,400 \frac{\text{N}}{\text{C}} \right) \\ &= \underline{7.06 \times 10^{-7} \text{ C/m}} \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad r &= \frac{\lambda}{2\pi\epsilon_0 E} \\
 &= \frac{7.06 \times 10^{-7} \frac{\text{C}}{\text{m}}}{2\pi \left(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N}\cdot\text{m}^2} \right) \left(\frac{1}{2} \right) \left(25,400 \frac{\text{N}}{\text{C}} \right)} \\
 &= \boxed{1.00 \text{ m}}
 \end{aligned}$$

63. number of protons = N_p
 charge of protons = $N_p e$
 mass of protons = $N_p m_p$
 number of electrons = N_e
 charge of electrons = $-N_e e$
 mass of electrons = $N_e m_e$
 $N_p e - N_e e = (N_p - N_e) e = Q = 1.84 \times 10^{-15} \text{ C}$
 $N_p m_p + N_e m_e = M = 4.56 \times 10^{-23} \text{ kg}$

$$\begin{aligned}
 \text{(a)} \quad N_p m_p &= M - N_e m_e \\
 N_p &= \frac{1}{m_p} (M - N_e m_e) \\
 (N_p - N_e) e &= Q \\
 \frac{M e}{m_p} - \frac{m_e}{m_p} N_e e - N_e e &= Q \\
 -N_e \left(\frac{m_e e}{m_p} + e \right) &= Q - \frac{M e}{m_p} \\
 N_e &= \frac{\frac{M e}{m_p} - Q}{\frac{m_e e}{m_p} + e} \\
 &= \frac{\frac{(4.56 \times 10^{-23} \text{ kg})(1.60 \times 10^{-19} \text{ C})}{1.673 \times 10^{-27} \text{ kg}} - 1.84 \times 10^{-15} \text{ C}}{\frac{(9.11 \times 10^{-31} \text{ kg})(1.60 \times 10^{-19} \text{ C})}{1.673 \times 10^{-27} \text{ kg}} + 1.60 \times 10^{-19} \text{ C}} \\
 &= \boxed{1.57 \times 10^4 \text{ electrons}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad N_p &= \frac{1}{m_p} (M - N_e m_e) \\
 &= \frac{4.56 \times 10^{-23} \text{ kg} - (15748)(9.11 \times 10^{-31} \text{ kg})}{1.673 \times 10^{-27} \text{ kg}} \\
 &= \boxed{2.73 \times 10^4 \text{ protons}}
 \end{aligned}$$

64. (a) $\vec{F}_{\text{net}} = \vec{F}_{31} + \vec{F}_{32}$

$$\begin{aligned}
 &= \frac{kq_1q_3}{a^2}(\cos\theta_1\hat{x} + \sin\theta_1\hat{y}) + \frac{kq_2q_3}{a^2}(\cos\theta_2\hat{x} + \sin\theta_2\hat{y}) \\
 &= \frac{kq_3}{a^2} \left[q_1 \left(\frac{\sqrt{a^2 - \left(\frac{a}{2}\right)^2}}{a}\hat{x} - \frac{\frac{a}{2}}{a}\hat{y} \right) + q_2 \left(-\frac{\sqrt{a^2 - \left(\frac{a}{2}\right)^2}}{a}\hat{x} - \frac{\frac{a}{2}}{a}\hat{y} \right) \right] \\
 &= \frac{kq_3}{a^2} \left[q_1 \left(\frac{\sqrt{3}}{2}\hat{x} - \frac{1}{2}\hat{y} \right) + q_2 \left(-\frac{\sqrt{3}}{2}\hat{x} - \frac{1}{2}\hat{y} \right) \right] \\
 &= \frac{kq_3}{a^2} \left[\frac{\sqrt{3}}{2}(q_1 - q_2)\hat{x} - \frac{1}{2}(q_1 + q_2)\hat{y} \right] \\
 F_{\text{net}} &= \frac{kq_3}{a^2} \sqrt{\frac{3}{4}(q_1 - q_2)^2 + \frac{1}{4}(q_1 + q_2)^2} \\
 &= \frac{\left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(7.3 \times 10^{-6} \text{ C})}{2(0.63 \text{ m})^2} \sqrt{3(7.3 \times 10^{-6} \text{ C} - 7.3 \times 10^{-6} \text{ C})^2 + (7.3 \times 10^{-6} \text{ C} + 7.3 \times 10^{-6} \text{ C})^2} \\
 &= \boxed{1.2 \text{ N}} \\
 \tan\theta &= \frac{-\frac{1}{2}(q_1 + q_2)}{\frac{\sqrt{3}}{2}(q_1 - q_2)} \rightarrow -\infty
 \end{aligned}$$

$\tan\theta$ approaches negative infinity when θ approaches $\frac{3\pi}{2}$.

So, $\theta = \frac{3\pi}{2}$ or $\boxed{270^\circ}$.

(b) In part (a), q_3 was farther away from q_1 and q_2 , and the x-components of the forces cancelled. Whereas, when q_3 is at the origin, the forces due to q_1 and q_2 add. So, the net force is greater than that found in part (a).

(c) $\vec{F}_{\text{net}} = \vec{F}_{31} + \vec{F}_{32}$

$$\begin{aligned}
 &= \frac{kq_1q_3}{\left(\frac{a}{2}\right)^2}(-\hat{y}) + \frac{kq_2q_3}{\left(\frac{a}{2}\right)^2}(-\hat{y}) \\
 &= -\frac{4kq_3}{a^2}(q_1 + q_2)\hat{y} \\
 &= -\frac{4\left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(7.3 \times 10^{-6} \text{ C})}{(0.63 \text{ m})^2}(7.3 \times 10^{-6} \text{ C} + 7.3 \times 10^{-6} \text{ C})\hat{y} \\
 &= \boxed{(-9.7 \text{ N})\hat{y}}
 \end{aligned}$$

65. (a) The net force on charge 1 will be less than the net force on charge 2. The net force acting on charge 1 is the vector sum of one attractive force and one repulsive force, which partially cancel; the net force acting on charge 2 is the vector sum of two attractive forces.

$$\begin{aligned}
 \text{(b)} \quad \vec{F}_{\text{net}} &= \vec{F}_{12} + \vec{F}_{13} \\
 &= -\frac{k|q_1||q_2|}{a^2}\hat{y} + \frac{k|q_1||q_3|}{a^2}(-\cos 30^\circ \hat{x} + \sin 30^\circ \hat{y}) \\
 &= -\frac{k|q_1||q_2|}{a^2}\hat{y} + \frac{k|q_1||q_3|}{a^2}\left(-\frac{\sqrt{3}}{2}\hat{x} + \frac{1}{2}\hat{y}\right)
 \end{aligned}$$

Let $|q_1| = |q_2| = |q_3| = q$. We have

$$\begin{aligned}
 \vec{F}_{\text{net}} &= -\frac{kq^2}{2a^2}(\sqrt{3}\hat{x} + \hat{y}) \\
 F_{\text{net}} &= \frac{kq^2}{2a^2}\sqrt{(\sqrt{3})^2 + 1^2} \\
 &= \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(7.3 \times 10^{-6} \text{ C})^2(2)}{2(0.63 \text{ m})^2} \\
 &= \boxed{1.2 \text{ N}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad \vec{F}_{\text{net}} &= \vec{F}_{21} + \vec{F}_{23} \\
 &= \frac{k|q_1||q_2|}{a^2}\hat{y} + \frac{k|q_2||q_3|}{a^2}(\cos 30^\circ \hat{x} + \sin 30^\circ \hat{y}) \\
 &= \frac{kq^2}{2a^2}(\sqrt{3}\hat{x} + 3\hat{y})
 \end{aligned}$$

Let $|q_1| = |q_2| = |q_3| = q$. We have

$$\begin{aligned}
 \vec{F}_{\text{net}} &= \frac{kq^2}{a^2}\left(\frac{\sqrt{3}}{2}\hat{x} + \frac{3}{2}\hat{y}\right) \\
 F_{\text{net}} &= \frac{kq^2}{2a^2}\sqrt{(\sqrt{3})^2 + 3^2} \\
 &= \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(7.3 \times 10^{-6} \text{ C})^2(\sqrt{12})}{2(0.63 \text{ m})^2} \\
 &= \boxed{2.1 \text{ N}}
 \end{aligned}$$

$$66. \text{ (a)} \quad E = \frac{\sigma}{\epsilon_0} = \frac{5.9 \times 10^{-6} \frac{\text{C}}{\text{m}^2}}{8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}} = \boxed{6.7 \times 10^5 \text{ N/C}}$$

- (b) Since E depends only on σ and not on the thickness of the membrane, the answer to part (a) would stay the same.

67. Place the square in the first quadrant with the $2.25\text{-}\mu\text{C}$ charges at $(0, 0)$ and (L, L) , and the others with charge Q at $(L, 0)$ and $(0, L)$. Let the $2.25\text{-}\mu\text{C}$ charges be called q . Note that the sign Q must be negative or all of the charges would fly away from each other. Because of the symmetry of the problem, if Q is found such that one of the $2.25\text{-}\mu\text{C}$ charges experiences zero net force, the other one will also. Set the net force on the charge at the origin equal to zero.

$$\sum F_x = 0 = \frac{kqQ}{L^2} + \frac{kq^2}{(\sqrt{2}L)^2} \cos 225^\circ$$

$$q = -\frac{1}{2}q \left(-\frac{\sqrt{2}}{2} \right) = \frac{\sqrt{2}}{4} q = \frac{\sqrt{2}}{4} (2.25 \times 10^{-6} \text{ C}) = 7.95 \times 10^{-7} \text{ C}$$

The sign of Q is negative and its magnitude is $7.95 \times 10^{-7} \text{ C}$.

68. (a)
$$\frac{GM_E M_M}{R^2} = \frac{kQ^2}{R^2}$$

$$Q = \sqrt{\frac{GM_E M_M}{k}}$$

$$= \sqrt{\frac{(6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2})(5.97 \times 10^{24} \text{ kg})(7.35 \times 10^{22} \text{ kg})}{8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}}}$$

$$= 5.71 \times 10^{13} \text{ C}$$

- (b) Since both the gravitational and electrical forces are inversely proportional to the square of the distance between the bodies, doubling the distance between the Earth and Moon would cause **no change** to the answer to part (a).
69. The magnitude of the electric force, the tension in each thread, and the magnitude of the charge on each ball are the same for both balls. So, we will solve for these quantities with respect to the ball on the right.

(a)
$$\sum F_x = T \sin 20.0^\circ - F = 0$$

$$\sum F_y = T \cos 20.0^\circ - mg = 0$$

$$F = T \sin 20.0^\circ = \left(\frac{mg}{\cos 20.0^\circ} \right) \sin 20.0^\circ = mg \tan 20.0^\circ = (1.2 \times 10^{-4} \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) \tan 20.0^\circ = 4.3 \times 10^{-4} \text{ N}$$

(b)
$$T = \frac{mg}{\cos 20.0^\circ} = \frac{(1.2 \times 10^{-4} \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right)}{\cos 20.0^\circ} = 1.3 \times 10^{-3} \text{ N}$$

(c) $F = T \sin 20.0^\circ$

$$\frac{kq^2}{d^2} = \frac{mg}{\cos 20.0^\circ} \sin 20.0^\circ$$

$$q = d \sqrt{\frac{mg \tan 20.0^\circ}{k}}$$

$$= (0.0205 \text{ m}) \sqrt{\frac{(1.2 \times 10^{-4} \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2}\right) \tan 20.0^\circ}{8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}}}$$

$$= \boxed{4.5 \times 10^{-9} \text{ C}}$$

70. Let $q = 2.44 \mu\text{C}$.

Set the forces in the x -direction equal to zero.

$$0 = -k_s x + \frac{kQq}{(d-x)^2}$$

Solve for d .

$$k_s x = \frac{kQq}{(d-x)^2}$$

$$(d-x)^2 = \frac{kQq}{k_s x}$$

$$d-x = \pm \sqrt{\frac{kQq}{k_s x}}$$

$$d = x \pm \sqrt{\frac{kQq}{k_s x}}$$

$$= 0.124 \text{ m} \pm \sqrt{\frac{(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2})(8.55 \times 10^{-6} \text{ C})(2.44 \times 10^{-6} \text{ C})}{(89.2 \frac{\text{N}}{\text{m}})(0.124 \text{ m})}}$$

$$= 0.254 \text{ m or } -0.006 \text{ m}$$

-0.006 m is extraneous, so $d = \boxed{0.254 \text{ m}}$.

71. (a) Since the charges are equally spaced around the circle, pairs of charges are opposite each other. The forces due to the charge pairs cancel at the center of the circle. So, the net force is due only to the charge on the negative x -axis. Thus, the direction of the repulsive force is along the positive x -axis.

(b) $F_{\text{net}} = \boxed{\frac{kq^2}{R^2}}$

72. $\Delta E = E_f - E_i = \frac{\sigma_f}{\epsilon_0} - \frac{\sigma_i}{\epsilon_0} = \frac{\Delta \sigma}{\epsilon_0}$

$$\Delta \sigma = \epsilon_0 \Delta E = \left(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}\right) \left[7.0 \times 10^5 \frac{\text{N}}{\text{C}} - \left(-3.0 \times 10^5 \frac{\text{N}}{\text{C}}\right)\right] = \boxed{8.85 \times 10^{-6} \text{ C/m}^2}$$

$$73. \text{ (a) } E = \frac{kQ}{R^2} = \frac{k\sigma A}{R^2} = \frac{k\sigma(4\pi R^2)}{R^2} = 4\pi k\sigma = \frac{\sigma}{\epsilon_0}$$

$$\sigma = \epsilon_0 E = \left(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}\right) \left(-110 \frac{\text{N}}{\text{C}}\right) = \boxed{-9.7 \times 10^{-10} \text{ C/m}^2}$$

$$\text{(b) } Q = \sigma A = \epsilon_0 E(4\pi R^2) = \frac{ER^2}{k} = \frac{\left(-110 \frac{\text{N}}{\text{C}}\right)(6.38 \times 10^6 \text{ m})^2}{8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}} = \boxed{-5.0 \times 10^5 \text{ C}}$$

- (c) Since the Moon's surface area is less than the Earth's for the same amount of charge, the surface charge density would be higher for the Moon than for the Earth. And, since the electric field is directly proportional to surface charge density, the electric field at the surface of the Moon would be greater than that for the Earth.

$$74. \text{ (a) } \sum F_x = 0 = QE \cos \theta - T$$

$$\sum F_y = 0 = QE \sin \theta - mg$$

$$E = \frac{mg}{Q \sin \theta} = \frac{(0.0037 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)}{(44 \times 10^{-6} \text{ C}) \sin 30.0^\circ} = \boxed{1.6 \times 10^3 \text{ N/C}}$$

$$\text{(b) } T = QE \cos \theta = Q \cos \theta \left(\frac{mg}{Q \sin \theta} \right) = \frac{mg}{\tan \theta} = \frac{(0.0037 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)}{\tan 30.0^\circ} = \boxed{6.3 \times 10^{-2} \text{ N}}$$

75. Place the square in the first quadrant such that the charges 1–4 lie at the points (0, 0), (a, 0), (a, a), and (0, a), respectively. Now, because the four charges at the corners of the square have the same sign and magnitude, the fifth charge must have the opposite sign and lie exactly in the middle of the square. Find the magnitude of the fifth charge q .

$$F_{1x} = F_{12} + F_{13} + F_{14} + F_{15} = 0$$

$$0 = \frac{kQ^2}{a^2} \cos 180^\circ + \frac{kQ^2}{(\sqrt{2}a)^2} \cos 225^\circ + \frac{kQ^2}{a^2} \cos 270^\circ + \frac{kqQ}{\left(\frac{a}{\sqrt{2}}\right)^2} \cos 45^\circ$$

$$0 = \frac{kQ}{a^2} \left(-Q - \frac{\sqrt{2}}{4}Q + 0 + \sqrt{2}q \right)$$

$$q = \frac{2\sqrt{2}+1}{4}Q$$

The fifth charge is located at the center of the square and has a charge of $-\frac{2\sqrt{2}+1}{4}Q$.

76. (a) Find the acceleration.

$$F = ma = qE$$

$$a = \frac{q}{m} E$$

Solve the equations of motion for a .

$$x = vt$$

$$\Delta y = \frac{1}{2} at^2 = \frac{1}{2} a \left(\frac{x}{v} \right)^2 = a \left(\frac{x^2}{2v^2} \right)$$

$$a = \frac{2v^2 \Delta y}{x^2} = \frac{q}{m} E$$

Solve for E .

$$E = \frac{2mv^2 \Delta y}{qx^2} = \frac{2(9.11 \times 10^{-31} \text{ kg}) \left(5.45 \times 10^6 \frac{\text{m}}{\text{s}} \right)^2 (0.00618 \text{ m})}{(1.60 \times 10^{-19} \text{ C})(0.0225 \text{ m})^2} = \boxed{4.13 \times 10^3 \text{ N/C}}$$

- (b)
- $v_x = v$

$$v_y = at = \left(\frac{2v^2 \Delta y}{x^2} \right) \left(\frac{x}{v} \right) = \frac{2v \Delta y}{x}$$

$$\begin{aligned} v_f &= \sqrt{v_x^2 + v_y^2} \\ &= \sqrt{v^2 + \frac{4v^2 \Delta y^2}{x^2}} \\ &= v \sqrt{1 + 4 \left(\frac{\Delta y}{x} \right)^2} \\ &= \left(5.45 \times 10^6 \frac{\text{m}}{\text{s}} \right) \sqrt{1 + 4 \left(\frac{0.00618 \text{ m}}{0.0225 \text{ m}} \right)^2} \\ &= \boxed{6.22 \times 10^6 \text{ m/s}} \end{aligned}$$

77. Initially, one sphere (1) has a positive charge and the other (2) has a negative charge. After they are connected by the wire, the charges equalize and the spheres are left with equal positive charges. So, $q_1 - |q_2| = 2q$ where $q_1 > |q_2| > 0$ and $q > 0$.

$$(a) \quad F_f = \frac{kq^2}{d^2}$$

$$q = \sqrt{\frac{d^2 F_f}{k}} = \sqrt{\frac{(0.45 \text{ m})^2 (0.032 \text{ N})}{8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}}} = \boxed{8.5 \times 10^{-7} \text{ C}}$$

$$\begin{aligned}
 \text{(b)} \quad F_i &= \frac{kq_1q_2}{d^2} \\
 |q_2| &= \frac{d^2F_i}{kq_1} \\
 q_1 - |q_2| &= q_1 - \frac{d^2F_i}{kq_1} = 2\sqrt{\frac{d^2F_f}{k}} \\
 0 &= q_1 - \frac{d^2F_i}{kq_1} - 2\sqrt{\frac{d^2F_f}{k}} \\
 0 &= q_1^2 + \left(-2\sqrt{\frac{d^2F_f}{k}}\right)q_1 + \left(-\frac{d^2F_i}{k}\right)
 \end{aligned}$$

Use the quadratic formula to find q_1 .

$$\begin{aligned}
 q_1 &= \frac{2}{2}\sqrt{\frac{d^2F_f}{k}} \pm \frac{1}{2}\sqrt{\frac{4d^2F_f}{k} - 4(1)\left(-\frac{d^2F_i}{k}\right)} \\
 &= \sqrt{\frac{d^2F_f}{k}} \pm \sqrt{\frac{d^2F_f}{k} + \frac{d^2F_i}{k}} \\
 &= \frac{d}{\sqrt{k}}(\sqrt{F_f} + \sqrt{F_f + F_i}) \\
 \frac{d}{\sqrt{k}}(\sqrt{F_f} - \sqrt{F_f + F_i}) &< 0, \text{ and since } q_1 > 0, \text{ it is extraneous.}
 \end{aligned}$$

$$\begin{aligned}
 q_1 &= \frac{0.45 \text{ m}}{\sqrt{8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}}}(\sqrt{0.032 \text{ N}} + \sqrt{0.032 \text{ N} + 0.095 \text{ N}}) \\
 &= \boxed{2.5 \times 10^{-6} \text{ C}}
 \end{aligned}$$

Find the charge on the other sphere.

$$\begin{aligned}
 -|q_2| &= 2q - q_1 \\
 q_2 &= \frac{2d}{\sqrt{k}}\sqrt{F_f} - \frac{d}{\sqrt{k}}(\sqrt{F_f} + \sqrt{F_f + F_i}) \\
 &= \frac{d}{\sqrt{k}}(\sqrt{F_f} - \sqrt{F_f + F_i}) \\
 &= \frac{0.45 \text{ m}}{\sqrt{8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}}}(\sqrt{0.032 \text{ N}} - \sqrt{0.032 \text{ N} + 0.095 \text{ N}}) \\
 &= \boxed{-8.4 \times 10^{-7} \text{ C}}
 \end{aligned}$$