

## Chapter 30

### Quantum Physics

#### Answers to Even-numbered Conceptual Questions

2. If energy is quantized, as suggested by Planck, the amount of energy for even a single high-frequency photon can be arbitrarily large. The finite energy in a blackbody simply can't produce such high-frequency photons, and therefore the infinite energy implied by the "ultraviolet catastrophe" cannot occur. In classical physics, any amount of energy can be in the form of high-frequency light – the energy does not have to be supplied in discrete, large lumps as in Planck's theory. Therefore, classical physics implies that all frequencies of light have the same amount of energy, no matter how high the frequency. This is what leads to the "catastrophe."
4. Planck's theory of blackbody radiation implies a one-to-one relationship between the absolute temperature of a blackbody and the frequency of light at the peak of its radiated energy spectrum. This relationship is given by Wien's displacement law (Equation 30-1). Therefore, by measuring the peak in the radiated energy from a star, we can tell its temperature. In broad terms, a blue star is very hot, a red star much less so, and a yellowish star like our Sun is intermediate in temperature.
6. If two blackbody curves intersected, there would be a range of frequencies where the low-temperature blackbody gives off more energy than the high-temperature blackbody. In this frequency range, then, it would be possible for energy to be spontaneously transferred from the low-temperature body to the high-temperature body, in violation of the second law of thermodynamics.
8. A monochromatic source of light means – literally – that it emits light of a single color. This means that all the photons emitted by the source have the same frequency, and hence they also have the same energy.
10. A photon from a green light source always has less energy than a photon from a blue light source, and always has more energy than a photon from a red light source. The reason for this is that the energy of a photon depends linearly on the frequency of light; that is,  $E = hf$ .
12. The likely explanation is that the second metal has a greater work function than the first metal. In this case, a shorter-wavelength photon – that is, a photon with higher frequency and higher energy – would be required to supply the additional energy needed to eject an electron.
14. Note that increasing the intensity while holding the frequency constant simply means that more photons – each with the original energy – strike the metal surface per second. **(a)** Stays the same. **(b)** Stays the same. **(c)** Increases. **(d)** Increases.
16. Classically, it should be possible to eject electrons with light of any frequency – all that is required is to increase the intensity of the beam of light sufficiently. The fact that this is not the case means that the classical picture is incorrect. In addition, the fact that there is a lowest frequency that will eject electrons implies that the energy of the photon is proportional to its frequency, in agreement with  $E = hf$ .

18. The scattered photon moves in the opposite direction to the incoming photon. To see this, suppose the initial photon moves in the positive  $x$  direction. This means that the initial  $y$  component of momentum is zero. After the collision, we are given that the electron moves in the positive  $x$  direction – it has no  $y$  component of momentum. Therefore, the scattered photon must also have zero  $y$  component of momentum, which means it will propagate in the negative  $x$  direction.
20. As your car accelerates, its momentum increases. It follows from Equation 30-16,  $\lambda = h/p$ , that your de Broglie wavelength decreases.
22. To double the electron's wavelength we must halve its momentum. This means, in turn, that we must decrease its kinetic energy by a factor of four (recall,  $K = p^2/2m$ ). Therefore, the electron should be accelerated from rest through the potential difference  $V_0/4$ .
24. (a) Increasing the mass of the particles implies that their momentum increases as well. It follows from Equation 30-16 that their de Broglie wavelength decreases. Therefore, from  $2d \sin \theta = m\lambda$  we see that the interference maximum will be at a smaller angle. (b) Increasing the energy of the particles also implies that their momentum increases. Therefore, we again expect a smaller angle for the interference maximum.

### Solutions to Problems

1.  $f_{\text{peak}} = (5.88 \times 10^{10} \text{ s}^{-1} \cdot \text{K}^{-1})T$

$$T = \frac{3.09 \times 10^{14} \text{ Hz}}{5.88 \times 10^{10} \text{ s}^{-1} \cdot \text{K}^{-1}} = \boxed{5260 \text{ K}}$$

2.  $f_{\text{peak}} = (5.88 \times 10^{10} \text{ s}^{-1} \cdot \text{K}^{-1})T$

$$= (5.88 \times 10^{10} \text{ s}^{-1} \cdot \text{K}^{-1}) \left[ \frac{5}{9} (95 - 32) + 273.15 \right] \text{ K}$$

$$= \boxed{1.81 \times 10^{13} \text{ Hz}}$$

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \frac{\text{m}}{\text{s}}}{1.812 \times 10^{13} \text{ Hz}} = \boxed{16.6 \mu\text{m}}$$

3. (a)  $f_{\text{peak}} = (5.88 \times 10^{10} \text{ s}^{-1} \cdot \text{K}^{-1})T$

$$= (5.88 \times 10^{10} \text{ s}^{-1} \cdot \text{K}^{-1})(2.7 \text{ K})$$

$$= \boxed{1.6 \times 10^{11} \text{ Hz}}$$

(b)  $\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \frac{\text{m}}{\text{s}}}{1.6 \times 10^{11} \text{ Hz}} = \boxed{1.9 \text{ mm}}$

4.  $f_{\text{peak}} = (5.88 \times 10^{10} \text{ s}^{-1} \cdot \text{K}^{-1})T$

$$= (5.88 \times 10^{10} \text{ s}^{-1} \cdot \text{K}^{-1})(5800 \text{ K})$$

$$= \boxed{3.4 \times 10^{14} \text{ Hz}}$$

5. (a) Since  $f_{\text{peak}} \propto T$ , doubling  $T$  increases  $f_{\text{peak}}$  by a factor of 2.

(b)  $T_K = T_C + 273.15$

$$\frac{20.0 + 273.15}{10.0 + 273.15} = \boxed{1.035}$$

6. (a)  $f_{\text{peak}} = (5.88 \times 10^{10} \text{ s}^{-1} \cdot \text{K}^{-1})T$   
 $= (5.88 \times 10^{10} \text{ s}^{-1} \cdot \text{K}^{-1})(2900 \text{ K})$   
 $= \boxed{1.7 \times 10^{14} \text{ Hz}}$

- (b) Since the peak frequency is that of infrared electromagnetic radiation, the light bulb radiates more energy in the infrared than the visible part of the spectrum.

7. (a) Since  $f_{\text{peak}} \propto T$ , the halogen bulb has the higher peak frequency.

(b)  $\frac{f_{\text{hal}}}{f_{\text{std}}} = \frac{T_{\text{hal}}}{T_{\text{std}}} = \frac{3400 \text{ K}}{2900 \text{ K}} = \boxed{1.2}$

(c)  $f_{\text{hal}} = (5.88 \times 10^{10} \text{ s}^{-1} \cdot \text{K}^{-1})(3400 \text{ K}) = 2.0 \times 10^{14} \text{ Hz}$

$$f_{\text{std}} = (5.88 \times 10^{10} \text{ s}^{-1} \cdot \text{K}^{-1})(2900 \text{ K}) = 1.7 \times 10^{14} \text{ Hz}$$

The halogen bulb produces a peak frequency closer to  $5.5 \times 10^{14} \text{ Hz}$  than the standard incandescent bulb.

8. (a) Star B is the blue star. Since  $\lambda \propto \frac{1}{f_{\text{peak}}}$  and  $f_{\text{peak}} \propto T$ ,  $\lambda \propto \frac{1}{T}$ . Higher temperature implies a shorter wavelength, and  $\lambda_{\text{blue}} < \lambda_{\text{yellow}}$ .

(b)  $\frac{f_A}{f_B} = \frac{T_A}{T_B} = \frac{4700 \text{ K}}{13,000 \text{ K}} = \boxed{0.36}$

9. (a)  $\omega = \sqrt{\frac{k}{m}} = 2\pi f$

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{1215 \frac{\text{N}}{\text{m}}}{1.340 \times 10^{-26} \text{ kg}}} = \boxed{4.792 \times 10^{13} \text{ Hz}}$$

(b)  $E = hf = (6.63 \times 10^{-34} \text{ J} \cdot \text{s})(4.792 \times 10^{13} \text{ Hz}) = \boxed{3.18 \times 10^{-20} \text{ J}}$

10.  $E = hf = \frac{hc}{\lambda}$

$$f = \frac{E}{h} = \frac{6.5 \times 10^{-19} \text{ J}}{6.63 \times 10^{-34} \text{ J} \cdot \text{s}} = \boxed{9.8 \times 10^{14} \text{ Hz}}$$

$$\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) \left( 3.00 \times 10^8 \frac{\text{m}}{\text{s}} \right)}{6.5 \times 10^{-19} \text{ J}} = \boxed{310 \text{ nm}}$$

$$11. \frac{P}{E} = \frac{P}{hf} = \frac{250 \times 10^3 \text{ W}}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(970 \times 10^3 \text{ Hz})} = \boxed{3.9 \times 10^{32} \text{ photons/s}}$$

$$12. E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) \left( 3.00 \times 10^8 \frac{\text{m}}{\text{s}} \right)}{50.4 \times 10^{-9} \text{ m}} = \boxed{3.95 \times 10^{-18} \text{ J}}$$

$$13. \frac{P}{E} = \frac{P}{hf} = \frac{1.0 \text{ W}}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(5.2 \times 10^{14} \text{ Hz})} = \boxed{2.9 \times 10^{18} \text{ photons/s}}$$

$$14. \begin{aligned} K_{\max} &= hf - W_0 \\ W_0 &= hf - K_{\max} \\ &= (6.63 \times 10^{-34} \text{ J} \cdot \text{s})(9.95 \times 10^{14} \text{ Hz}) - 0.180 \times 10^{-19} \text{ J} \\ &= \boxed{6.42 \times 10^{-19} \text{ J}} \end{aligned}$$

$$15. \begin{aligned} K_{\max} &= hf - W_0 \\ f &= \frac{K_{\max} + W_0}{h} \\ &= \frac{6.48 \times 10^{-19} \text{ J} + (4.58 \text{ eV}) \left( 1.60 \times 10^{-19} \frac{\text{J}}{\text{eV}} \right)}{6.63 \times 10^{-34} \text{ J} \cdot \text{s}} \\ &= \boxed{2.08 \times 10^{15} \text{ Hz}} \end{aligned}$$

$$16. \text{(a)} E = \frac{hc}{\lambda}$$

$$\frac{E_{\text{total}}}{E_{\text{photon}}} = \frac{\lambda E_{\text{total}}}{hc} = \frac{(350 \times 10^{-9} \text{ m})(2.0 \text{ J})}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) \left( 3.00 \times 10^8 \frac{\text{m}}{\text{s}} \right)} = \boxed{3.5 \times 10^{18} \text{ photons}}$$

$$\text{(b)} \frac{(750 \times 10^{-9} \text{ m})(2.0 \text{ J})}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) \left( 3.00 \times 10^8 \frac{\text{m}}{\text{s}} \right)} = \boxed{7.5 \times 10^{18} \text{ photons}}$$

$$17. \text{(a)} \text{ Each photon has energy } E = hf = \frac{hc}{\lambda}.$$

$$\frac{P}{E} = \frac{\lambda P}{hc} = \frac{(650 \times 10^{-9} \text{ m})(25 \text{ W})}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) \left( 3.00 \times 10^8 \frac{\text{m}}{\text{s}} \right)} = \boxed{8.2 \times 10^{19} \text{ photons/s}}$$

(b) Multiply the rate of photons by the ratio of the area of your pupil to the area of a sphere with radius 15 m.

$$\left( 8.2 \times 10^{19} \frac{\text{photons}}{\text{s}} \right) \frac{\pi \left( \frac{0.0050 \text{ m}}{2} \right)^2}{4\pi (15 \text{ m})^2} = \boxed{5.7 \times 10^{11} \text{ photons/s}}$$

18. (a) The rate of emitted photons is determined by the ratio of the power to the energy of a photon. This ratio is inversely proportional to the frequency of the emitted photons. So, the station with the lower frequency has the highest rate of photon emission. Therefore, station A emits more photons per second than station B.

- (b) Since the energy per photon is directly proportional to the frequency of the photon, the station that broadcasts at the higher frequency emits the higher energy photons. So, **station B** emits higher energy photons than station A.

19. (a)  $E = hf = \frac{hc}{\lambda}$

$$f = \frac{E}{h} = \frac{\left(104.2 \frac{\text{kcal}}{\text{mol}}\right) \left(\frac{1}{6.02 \times 10^{23} \frac{\text{molecules}}{\text{mol}}}\right) \left(\frac{4186 \text{ J}}{1 \text{ kcal}}\right)}{6.63 \times 10^{-34} \text{ J} \cdot \text{s}} = \boxed{1.09 \times 10^{15} \text{ Hz}}$$

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \frac{\text{m}}{\text{s}}}{1.09 \times 10^{15} \text{ Hz}} = \boxed{275 \text{ nm}}$$

- (b) The photon lies in the **ultraviolet** region of the electromagnetic spectrum since  $\lambda_{\text{UV}} < 400 \text{ nm}$ .

20. (a)  $\frac{P}{E} = \frac{\lambda P}{hc} = \frac{(632.8 \times 10^{-9} \text{ m})(1.0 \times 10^{-3} \text{ W})}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) \left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)} = \boxed{3.2 \times 10^{15} \text{ photons/s}}$

(b)  $f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \frac{\text{m}}{\text{s}}}{632.8 \times 10^{-9} \text{ m}} = \boxed{4.74 \times 10^{14} \text{ Hz}}$

21. (a)  $\frac{\left(\frac{P}{E}\right)_{\text{red}}}{\left(\frac{P}{E}\right)_{\text{blue}}} = \frac{\frac{\lambda_{\text{red}} P_{\text{red}}}{hc}}{\frac{\lambda_{\text{blue}} P_{\text{blue}}}{hc}} = \frac{\lambda_{\text{red}} P_{\text{red}}}{\lambda_{\text{blue}} P_{\text{blue}}} = \frac{(650 \text{ nm})(150 \text{ W})}{(460 \text{ nm})(25 \text{ W})} = 8.5 > 1$

So,  $\left(\frac{P}{E}\right)_{\text{red}} > \left(\frac{P}{E}\right)_{\text{blue}}$ . The **red** bulb emits more photons per second than the blue bulb.

(b)  $\frac{E_{\text{red}}}{E_{\text{blue}}} = \frac{\frac{hc}{\lambda_{\text{red}}}}{\frac{hc}{\lambda_{\text{blue}}}} = \frac{\lambda_{\text{blue}}}{\lambda_{\text{red}}} = \frac{460 \text{ nm}}{650 \text{ nm}} < 1$

So,  $E_{\text{red}} < E_{\text{blue}}$ . The **blue** bulb emits photons of higher energy than the red bulb.

- (c) **Red bulb**

$$\frac{P}{E} = \frac{\lambda P}{hc} = \frac{(650 \times 10^{-9} \text{ m})(150 \text{ W})}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) \left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)} = \boxed{4.9 \times 10^{20} \text{ photons/s}}$$

**Blue bulb**

$$\frac{(460 \times 10^{-9} \text{ m})(25 \text{ W})}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) \left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)} = \boxed{5.8 \times 10^{19} \text{ photons/s}}$$

22.  $W_0 = E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) \left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{264 \times 10^{-9} \text{ m}} = \boxed{7.53 \times 10^{-19} \text{ J}}$

23. (a) Since higher frequency implies higher energy, and since  $W_{\text{Al}} > W_{\text{Ca}}$ , **aluminum** requires higher-frequency light to produce photoelectrons.

$$(b) f_{\text{Al}} = \frac{W_{\text{Al}}}{h} = \frac{(4.28 \text{ eV}) \left( 1.60 \times 10^{-19} \frac{\text{J}}{\text{eV}} \right)}{6.63 \times 10^{-34} \text{ J} \cdot \text{s}} = \boxed{1.03 \times 10^{15} \text{ Hz}}$$

$$f_{\text{Ca}} = \frac{W_{\text{Ca}}}{h} = \frac{(2.87 \text{ eV}) \left( 1.60 \times 10^{-19} \frac{\text{J}}{\text{eV}} \right)}{6.63 \times 10^{-34} \text{ J} \cdot \text{s}} = \boxed{6.93 \times 10^{14} \text{ Hz}}$$

24. (a) Since  $K_{\text{max}} = hf - W_0 = \frac{hc}{\lambda} - W_0$ , a longer wavelength implies a smaller kinetic energy. So, the beam with wavelength  $\lambda_{\text{B}}$  produces photoelectrons with greater kinetic energy than the beam with wavelength  $\lambda_{\text{A}}$ .

$$(b) K_{\text{max,A}} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) \left( 3.00 \times 10^8 \frac{\text{m}}{\text{s}} \right) \left( \frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}} \right)}{620 \times 10^{-9} \text{ m}} - 1.9 \text{ eV} = \boxed{0.1 \text{ eV}}$$

$$K_{\text{max,B}} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) \left( 3.00 \times 10^8 \frac{\text{m}}{\text{s}} \right) \left( \frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}} \right)}{410 \times 10^{-9} \text{ m}} - 1.9 \text{ eV} = \boxed{1.1 \text{ eV}}$$

25. (a) The difference in energy between the incident photons and the work function of the metal is the photoelectron's kinetic energy. So, if two different metals are illuminated by photons with the same energy, the metal with the smaller work function will emit photoelectrons with a greater maximum kinetic energy. Therefore, since  $W_{\text{Cd}} < W_{\text{Zn}}$ , **cadmium** emits photoelectrons with the greater maximum kinetic energy.

$$(b) K_{\text{max}} = \frac{hc}{\lambda} - W_0$$

$$K_{\text{max,Zn}} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) \left( 3.00 \times 10^8 \frac{\text{m}}{\text{s}} \right) \left( \frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}} \right)}{275 \times 10^{-9} \text{ m}} - 4.33 \text{ eV} = \boxed{0.19 \text{ eV}}$$

$$K_{\text{max,Cd}} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) \left( 3.00 \times 10^8 \frac{\text{m}}{\text{s}} \right) \left( \frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}} \right)}{275 \times 10^{-9} \text{ m}} - 4.22 \text{ eV} = \boxed{0.30 \text{ eV}}$$

26. (a)  $K_{\text{max}} = hf - W_0 = (6.63 \times 10^{-34} \text{ J} \cdot \text{s}) (7.90 \times 10^{14} \text{ Hz}) \left( \frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}} \right) - 2.24 \text{ eV} = \boxed{1.03 \text{ eV}}$

- (b) No electrons are ejected from the potassium surface at photon energies lower than  $W_0$ . Set  $K_{\text{max}} = 0$ .

$$0 = hf - W_0$$

$$hf = W_0$$

$$f = \frac{W_0}{h}$$

$$= \frac{2.24 \text{ eV}}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) \left( \frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}} \right)}$$

$$= 5.41 \times 10^{14} \text{ Hz}$$

The range of frequencies for which no electrons are ejected is  $\boxed{4.00 \times 10^{14} \text{ Hz} \leq f \leq 5.41 \times 10^{14} \text{ Hz}}$ .

27. (a)  $K_{\max} = hf - W_0 = (6.63 \times 10^{-34} \text{ J} \cdot \text{s})(9.00 \times 10^{16} \text{ Hz}) \left( \frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}} \right) - 4.28 \text{ eV} = \boxed{369 \text{ eV}}$

(b) No electrons are ejected from the aluminum at photon energies lower than  $W_0$ . Set  $K_{\max} = 0$ .

$$0 = hf - W_0$$

$$hf = W_0$$

$$f = \frac{W_0}{h}$$

$$= \frac{4.28 \text{ eV}}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) \left( \frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}} \right)}$$

$$= 1.03 \times 10^{15} \text{ Hz}$$

The range of frequencies for which no electrons are ejected is  $\boxed{4.00 \times 10^{14} \text{ Hz} \leq f \leq 1.03 \times 10^{15} \text{ Hz}}$ .

28. (a) Since  $K_{\max} = hf - W_0$ , the photoelectrons emitted by the metal with the smaller work function will have the greater kinetic energy. Since  $W_{\text{Pt}} > W_{\text{Fe}}$ , the electrons ejected from the iron surface will have the greater maximum kinetic energy.

(b)  $K_{\max, \text{Pt}} = (6.63 \times 10^{-34} \text{ J} \cdot \text{s})(1.88 \times 10^{15} \text{ Hz}) \left( \frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}} \right) - 6.35 \text{ eV} = \boxed{1.44 \text{ eV}}$

$$K_{\max, \text{Fe}} = (6.63 \times 10^{-34} \text{ J} \cdot \text{s})(1.88 \times 10^{15} \text{ Hz}) \left( \frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}} \right) - 4.50 \text{ eV} = \boxed{3.29 \text{ eV}}$$

29.  $K_{\max} = hf - W_0$

$$W_0 = hf - K_{\max}$$

$$hf_1 - K_{\max,1} = hf_2 - K_{\max,2}$$

$$h(f_1 - f_2) = K_{\max,1} - K_{\max,2}$$

$$h = \frac{K_{\max,1} - K_{\max,2}}{f_1 - f_2}$$

$$= \frac{1.260 \times 10^{-19} \text{ J} - 2.480 \times 10^{-19} \text{ J}}{547.5 \times 10^{12} \text{ Hz} - 738.8 \times 10^{12} \text{ Hz}}$$

$$= \boxed{6.377 \times 10^{-34} \text{ J} \cdot \text{s}}$$

30.  $\frac{IA}{E} = \frac{I\pi r^2}{hf} = \frac{(5.0 \times 10^{-13} \frac{\text{W}}{\text{m}^2}) \pi \left( \frac{0.0085 \text{ m}}{2} \right)^2}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(7.0 \times 10^{14} \text{ Hz})} = \boxed{61 \text{ photons/s}}$

31. (a)  $p = \frac{h}{\lambda}$

$$\lambda = \frac{h}{p}$$

$$= \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{5.1 \times 10^{-33} \text{ kg} \cdot \frac{\text{m}}{\text{s}}}$$

$$= \boxed{13 \text{ cm}}$$

(b) The wavelength is much larger than the size of the holes in the metal screen.

32. Set  $p_{\text{electron}} = p_{\text{photon}}$ .

$$\begin{aligned}
 \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}} &= \frac{h}{\lambda} \\
 \frac{m_0 \lambda}{h} v &= \sqrt{1 - \frac{v^2}{c^2}} \\
 \frac{m_0^2 \lambda^2}{h^2} v^2 &= 1 - \frac{v^2}{c^2} \\
 \left( \frac{1}{c^2} + \frac{m_0^2 \lambda^2}{h^2} \right) v^2 &= 1 \\
 v &= \left( \frac{1}{c^2} + \frac{m_0^2 \lambda^2}{h^2} \right)^{-1/2} \\
 &= c \left[ 1 + \frac{m_0^2 \lambda^2 c^2}{h^2} \right]^{-1/2} \\
 &= \left( 3.00 \times 10^8 \frac{\text{m}}{\text{s}} \right) \left[ 1 + \frac{(9.11 \times 10^{-31} \text{ kg})^2 (0.35 \times 10^{-9} \text{ m})^2 (3.00 \times 10^8 \frac{\text{m}}{\text{s}})^2}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})^2} \right]^{-1/2} \\
 &= \boxed{2.1 \times 10^6 \text{ m/s}}
 \end{aligned}$$

33. Set  $p_{\text{photon}} = p_{\text{electron}}$ .

$$\begin{aligned}
 \frac{h}{\lambda} &= \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}} \\
 \lambda &= \frac{h}{m_0 v} \sqrt{1 - \frac{v^2}{c^2}} \\
 &= \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg}) \left( 1200 \frac{\text{m}}{\text{s}} \right)} \sqrt{1 - \left( \frac{1200 \frac{\text{m}}{\text{s}}}{3.00 \times 10^8 \frac{\text{m}}{\text{s}}} \right)^2} \\
 &= \boxed{610 \text{ nm}}
 \end{aligned}$$



34. Set
- $p_{\text{photon}} = p_{\text{neutron}}$
- .

$$\begin{aligned}
 \frac{hf}{c} &= \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}} \\
 f &= \frac{cm_0 v}{h \sqrt{1 - \frac{v^2}{c^2}}} \\
 &= \frac{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right) \left(1.675 \times 10^{-27} \text{ kg}\right) \left(1500 \frac{\text{m}}{\text{s}}\right)}{\left(6.63 \times 10^{-34} \text{ J} \cdot \text{s}\right) \sqrt{1 - \left(\frac{1500 \frac{\text{m}}{\text{s}}}{3.00 \times 10^8 \frac{\text{m}}{\text{s}}}\right)^2}} \\
 &= \boxed{1.1 \times 10^{18} \text{ Hz}}
 \end{aligned}$$

- 35.

$$\begin{aligned}
 p_H &= p_{\text{photon}} \\
 \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}} &= \frac{h}{\lambda} \\
 \frac{\lambda m_0 v}{h} &= \sqrt{1 - \frac{v^2}{c^2}} \\
 \frac{\lambda^2 m_0^2 v^2}{h^2} &= 1 - \frac{v^2}{c^2} \\
 v^2 \left(1 + \frac{c^2 \lambda^2 m_0^2}{h^2}\right) &= c^2 \\
 v &= c \left(1 + \frac{c^2 \lambda^2 m_0^2}{h^2}\right)^{-1/2} \\
 &= \left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right) \left[1 + \frac{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)^2 (122 \times 10^{-9} \text{ m})^2 (1.674 \times 10^{-27} \text{ kg})^2}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})^2}\right]^{-1/2} \\
 &= \boxed{3.25 \text{ m/s}}
 \end{aligned}$$

36.

$$\begin{aligned}
 p_H &= p_{\text{photon}} \\
 \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}} &= \frac{h}{\lambda} \\
 \frac{\lambda^2 m_0^2 v^2}{h^2} &= 1 - \frac{v^2}{c^2} \\
 \left( \frac{1}{c^2} + \frac{\lambda^2 m_0^2}{h^2} \right) v^2 &= 1 \\
 v &= c \left( 1 + \frac{\lambda^2 m_0^2 c^2}{h^2} \right)^{-1/2} \\
 &= \left( 3.00 \times 10^8 \frac{\text{m}}{\text{s}} \right) \left[ 1 + \frac{(486 \times 10^{-9} \text{ m})^2 (1.674 \times 10^{-27} \text{ kg})^2 \left( 3.00 \times 10^8 \frac{\text{m}}{\text{s}} \right)^2}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})^2} \right]^{-1/2} \\
 &= \boxed{0.815 \text{ m/s}}
 \end{aligned}$$

37. (a) Since  $\lambda_{\text{red}} > \lambda_{\text{blue}}$ , and since  $p = \frac{h}{\lambda}$ ,  $p_{\text{blue}} > p_{\text{red}}$ . A photon of blue light has the greater momentum.

$$\begin{aligned}
 \text{(b)} \quad p_{\text{red}} &= \frac{hf}{c} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(4.0 \times 10^{14} \text{ Hz})}{3.00 \times 10^8 \frac{\text{m}}{\text{s}}} = \boxed{8.8 \times 10^{-28} \text{ kg} \cdot \text{m/s}} \\
 p_{\text{blue}} &= \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(7.9 \times 10^{14} \text{ Hz})}{3.00 \times 10^8 \frac{\text{m}}{\text{s}}} = \boxed{1.7 \times 10^{-27} \text{ kg} \cdot \text{m/s}}
 \end{aligned}$$

38. (a) Since a photon's momentum is inversely proportional to its wavelength, the photon with the smaller momentum has the longer wavelength. So, photon B has the longer wavelength.

$$\begin{aligned}
 \text{(b)} \quad 2p_B &= p_A \\
 \frac{2h}{\lambda_B} &= \frac{h}{\lambda_A} \\
 \lambda_B &= 2\lambda_A \\
 &= 2(333 \text{ nm}) \\
 &= \boxed{666 \text{ nm}}
 \end{aligned}$$

$$\text{39. (a)} \quad \frac{P}{E_{\text{photon}}} = \frac{\lambda P}{hc} = \frac{(632.8 \times 10^{-9} \text{ m})(5.00 \times 10^{-3} \text{ W})}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) \left( 3.00 \times 10^8 \frac{\text{m}}{\text{s}} \right)} = \boxed{1.59 \times 10^{16} \text{ photons/s}}$$

$$\begin{aligned}
 \text{(b)} \quad \Delta p &= p_f - p_i \\
 &= 0 - p_i \\
 &= -\frac{h}{\lambda} \\
 &= -\frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{632.8 \times 10^{-9} \text{ m}} \\
 &= \boxed{-1.05 \times 10^{-27} \text{ kg} \cdot \text{m/s}}
 \end{aligned}$$

(c)  $P = Fv = Fc$

$$F = \frac{P}{c} = \frac{5.00 \times 10^{-3} \text{ W}}{3.00 \times 10^8 \frac{\text{m}}{\text{s}}} = \boxed{1.67 \times 10^{-11} \text{ N}}$$

40. (a)  $\frac{P}{E} = \frac{\lambda P}{hc} = \frac{(632.8 \times 10^{-9} \text{ m})(5.00 \times 10^{-3} \text{ W})}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \frac{\text{m}}{\text{s}})} = \boxed{1.59 \times 10^{16} \text{ photons/s}}$

(b)  $\Delta p = p_f - p_i$   
 $= -p_i - p_i$   
 $= -2p_i$   
 $= -\frac{2h}{\lambda}$   
 $= \frac{-2(6.63 \times 10^{-34} \text{ J} \cdot \text{s})}{632.8 \times 10^{-9} \text{ m}}$   
 $= \boxed{-2.10 \times 10^{-27} \text{ kg} \cdot \text{m/s}}$

(c)  $P = Fv = Fc$

Since the beam is reflected, the force is  $2P/c$ .

$$F = \frac{2P}{c} = \frac{2(5.00 \times 10^{-3} \text{ W})}{3.00 \times 10^8 \frac{\text{m}}{\text{s}}} = \boxed{3.33 \times 10^{-11} \text{ N}}$$

41.  $K_{\text{electron}} = -\Delta K_{\text{photon}} = K_i - K_f = 36 \text{ keV} - 27 \text{ keV} = \boxed{9 \text{ keV}}$

42.  $\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c} (1 - \cos\theta)$

(a)  $\Delta\lambda = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \frac{\text{m}}{\text{s}})} (1 - \cos 30.0^\circ) = \boxed{3.25 \times 10^{-13} \text{ m}}$

(b)  $\Delta\lambda = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \frac{\text{m}}{\text{s}})} (1 - \cos 90.0^\circ) = \boxed{2.43 \times 10^{-12} \text{ m}}$

(c)  $\Delta\lambda = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \frac{\text{m}}{\text{s}})} (1 - \cos 180.0^\circ) = \boxed{4.85 \times 10^{-12} \text{ m}}$

43. 
$$\Delta\lambda = \frac{h}{m_e c} (1 - \cos\theta)$$

$$\frac{m_e c \Delta\lambda}{h} = 1 - \cos\theta$$

$$\cos\theta = 1 - \frac{m_e c \Delta\lambda}{h}$$

$$\theta = \cos^{-1} \left( 1 - \frac{m_e c \Delta\lambda}{h} \right)$$

$$= \cos^{-1} \left[ 1 - \frac{(9.11 \times 10^{-31} \text{ kg}) \left( 3.00 \times 10^8 \frac{\text{m}}{\text{s}} \right) (3.33 \times 10^{-12} \text{ m})}{6.63 \times 10^{-34} \text{ J} \cdot \text{s}} \right]$$

$$= \boxed{110^\circ}$$

44. 
$$\Delta\lambda = \frac{h}{m_e c} (1 - \cos\theta) = \frac{1}{4} \Delta\lambda_{\text{max}}$$

$$\Delta\lambda_{\text{max}} = \frac{2h}{m_e c}$$

$$\frac{h}{m_e c} (1 - \cos\theta) = \frac{2h}{4m_e c}$$

$$1 - \cos\theta = \frac{1}{2}$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \cos^{-1} \frac{1}{2}$$

$$= \boxed{60^\circ}$$

45. (a) Since the change in wavelength does not depend on the wavelength of the photon, the change in wavelength is the same for both photons.

- (b) Since the change in wavelength is the same, the photon with the shorter wavelength,  $\lambda = 0.030 \text{ nm}$ , will experience the greater percent change in wavelength. So, the X-ray photon experiences the greater percent change.

- (c) visible-light photon

$$\frac{100\Delta\lambda}{\lambda} = \frac{100h}{\lambda m_e c} (1 - \cos\theta)$$

$$= \frac{100(6.63 \times 10^{-34} \text{ J} \cdot \text{s})}{(520 \times 10^{-9} \text{ m})(9.11 \times 10^{-31} \text{ kg}) \left( 3.00 \times 10^8 \frac{\text{m}}{\text{s}} \right)} (1 - \cos 180^\circ)$$

$$= \boxed{9.3 \times 10^{-4} \%}$$

X-ray photon

$$\frac{100\Delta\lambda}{\lambda} = \frac{100(6.63 \times 10^{-34} \text{ J} \cdot \text{s})}{(0.030 \times 10^{-9} \text{ m})(9.11 \times 10^{-31} \text{ kg}) \left( 3.00 \times 10^8 \frac{\text{m}}{\text{s}} \right)} (1 - \cos 180^\circ)$$

$$= \boxed{16 \%}$$

$$46. \Delta\lambda = \lambda_f - \lambda_i = \frac{h}{m_e c} (1 - \cos\theta)$$

$$(a) \quad p_i = \frac{h}{\lambda_i} = \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{0.240 \times 10^{-9} \text{ m}} = \boxed{2.76 \times 10^{-24} \text{ kg}\cdot\text{m/s}}$$

$$(b) \quad \lambda_f = \lambda_i + \frac{h}{m_e c} (1 - \cos\theta)$$

$$= 0.240 \times 10^{-9} \text{ m} + \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{(9.11 \times 10^{-31} \text{ kg}) \left( 3.00 \times 10^8 \frac{\text{m}}{\text{s}} \right)} (1 - \cos 135^\circ)$$

$$= 0.244 \times 10^{-9} \text{ m}$$

$$p_f = \frac{h}{\lambda_f} = \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{0.244 \times 10^{-9} \text{ m}} = \boxed{2.72 \times 10^{-24} \text{ kg}\cdot\text{m/s}}$$

$$47. (a) \quad \Delta\lambda = \lambda_f - \lambda_i = \frac{h}{m_e c} (1 - \cos\theta)$$

$$\lambda_i = \lambda_f - \frac{h}{m_e c} (1 - \cos\theta)$$

$$= 0.320 \text{ nm} - \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{(9.11 \times 10^{-31} \text{ kg}) \left( 3.00 \times 10^8 \frac{\text{m}}{\text{s}} \right)} (1 - \cos 175^\circ)$$

$$= \boxed{0.315 \text{ nm}}$$

$$(b) \quad E_i = \frac{hc}{\lambda_i} = \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s}) \left( 3.00 \times 10^8 \frac{\text{m}}{\text{s}} \right)}{0.315 \times 10^{-9} \text{ m}} = \boxed{6.31 \times 10^{-16} \text{ J}}$$

$$E_f = \frac{hc}{\lambda_f} = \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s}) \left( 3.00 \times 10^8 \frac{\text{m}}{\text{s}} \right)}{0.320 \times 10^{-9} \text{ m}} = \boxed{6.22 \times 10^{-16} \text{ J}}$$

$$(c) \quad K = E_i - E_f = 6.31 \times 10^{-16} \text{ J} - 6.22 \times 10^{-16} \text{ J} = \boxed{9 \times 10^{-18} \text{ J}}$$

$$48. (a) \quad \text{Since } \Delta\lambda \propto \frac{1}{m} \text{ and } m_e < m_{\text{He}}, \text{ the change in wavelength of the X-ray is greater for the } \boxed{\text{electron}}.$$

$$(b) \quad \Delta\lambda_{\text{electron}} = \frac{2h}{m_e c} = \frac{2(6.63 \times 10^{-34} \text{ J}\cdot\text{s})}{(9.11 \times 10^{-31} \text{ kg}) \left( 3.00 \times 10^8 \frac{\text{m}}{\text{s}} \right)} = \boxed{4.85 \text{ pm}}$$

$$\Delta\lambda_{\text{He}} = \frac{2h}{m_{\text{He}} c} = \frac{2(6.63 \times 10^{-34} \text{ J}\cdot\text{s})}{(6.64 \times 10^{-27} \text{ kg}) \left( 3.00 \times 10^8 \frac{\text{m}}{\text{s}} \right)} = \boxed{0.666 \text{ fm}}$$

$$49. \Delta\lambda = \frac{h}{m_e c} (1 - \cos\theta) = \lambda_f - \lambda_i$$

$$\frac{\lambda_f}{\lambda_i} = 1.100$$

$$\lambda_f - \lambda_i = 1.100\lambda_i - \lambda_i$$

$$\frac{h}{m_e c} (1 - \cos\theta) = 0.100\lambda_i$$

$$\begin{aligned}\lambda_i &= \frac{h}{0.100m_e c} (1 - \cos\theta) \\ &= \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{0.100(9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \frac{\text{m}}{\text{s}})} (1 - \cos 135^\circ) \\ &= \boxed{41.4 \text{ pm}}\end{aligned}$$

$$E_i = \frac{hc}{\lambda_i} = \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(3.00 \times 10^8 \frac{\text{m}}{\text{s}})}{41.4 \times 10^{-12} \text{ m}} = \boxed{4.80 \text{ eV}}$$

50. Eliminate  $p_e$  from Equations 30–13 and 30–14, and solve for  $\phi$ .

$$\frac{h}{\lambda} = \frac{h}{\lambda'} \cos\theta + p_e \cos\phi$$

$$p_e = \frac{\frac{h}{\lambda} - \frac{h}{\lambda'} \cos\theta}{\cos\phi}$$

$$0 = \frac{h}{\lambda'} \sin\theta - p_e \sin\phi$$

$$p_e = \frac{h \sin\theta}{\lambda' \sin\phi}$$

$$\frac{\frac{h}{\lambda} - \frac{h}{\lambda'} \cos\theta}{\cos\phi} = \frac{h \sin\theta}{\lambda' \sin\phi}$$

$$\tan\phi = \frac{h \sin\theta}{\lambda' \left( \frac{h}{\lambda} - \frac{h}{\lambda'} \cos\theta \right)}$$

$$= \frac{\sin\theta}{\frac{\lambda'}{\lambda} - \cos\theta}$$

$$= \frac{\sin\theta}{\frac{\lambda + \Delta\lambda}{\lambda} - \cos\theta}$$

$$= \frac{\sin\theta}{1 + \frac{\Delta\lambda}{\lambda} - \cos\theta}$$

$$\phi = \tan^{-1} \frac{\sin\theta}{1 + \frac{\Delta\lambda}{\lambda} - \cos\theta}$$

From Problem 43,  $\theta = 110^\circ$  (two significant figures). To avoid rounding errors, we will use  $\theta = 111.9^\circ$ , which was the value before reduction to two significant figures.

$$\phi = \tan^{-1} \frac{\sin 111.9^\circ}{1 + \frac{3.33 \times 10^{-12} \text{ m}}{0.525 \times 10^{-9} \text{ m}} - \cos 111.9^\circ} = \boxed{34^\circ}$$

$$\begin{aligned}
 51. \quad \lambda &= \frac{h}{p} \\
 &= \frac{h}{mv} \\
 v &= \frac{h}{\lambda m} \\
 &= \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{(7.62 \times 10^{-12} \text{ m})(6.69 \times 10^{-27} \text{ kg})} \\
 &= \boxed{13.0 \text{ km/s}}
 \end{aligned}$$

$$\begin{aligned}
 52. \quad \lambda &= \frac{h}{p} \\
 &= \frac{h}{mv} \\
 v &= \frac{h}{\lambda m} \\
 &= \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{(0.282 \times 10^{-9} \text{ m})(1.675 \times 10^{-27} \text{ kg})} \\
 &= \boxed{1.40 \text{ km/s}}
 \end{aligned}$$

$$53. \quad \lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{(73 \text{ kg})(4.1 \frac{\text{m}}{\text{s}})} = \boxed{2.2 \times 10^{-36} \text{ m}}$$

$$54. \quad K = \frac{1}{2}mv^2 = \frac{m^2v^2}{2m} = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})^2}{2(9.11 \times 10^{-31} \text{ kg})(1 \times 10^{-10} \text{ m})^2} = \boxed{2 \times 10^{-17} \text{ J}}$$

$$\begin{aligned}
 55. \text{ (a)} \quad \lambda &= \frac{h}{p} \\
 &= \frac{h}{mv} \\
 v &= \frac{h}{\lambda m} \\
 &= \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{(0.250 \times 10^{-9} \text{ m})(1.675 \times 10^{-27} \text{ kg})} \\
 &= \boxed{1.58 \text{ km/s}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad 2d \sin \theta &= m\lambda \\
 \theta &= \sin^{-1} \frac{m\lambda}{2d} \\
 &= \sin^{-1} \frac{(2)(0.250 \text{ nm})}{2(0.282 \text{ nm})} \\
 &= \boxed{62.4^\circ}
 \end{aligned}$$

56. (a) Since  $\lambda = \frac{h}{mv}$  and  $m_e < m_p$ , for identical speeds, an electron has a longer de Broglie wavelength than a proton.

$$(b) \frac{\lambda_e}{\lambda_p} = \frac{\frac{h}{m_e v}}{\frac{h}{m_p v}} = \frac{m_p}{m_e} = \frac{1.673 \times 10^{-27} \text{ kg}}{9.11 \times 10^{-31} \text{ kg}} = \boxed{1840}$$

57. (a)  $K = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$

Since  $K = \frac{h^2}{2m\lambda^2}$  and  $m_e < m_p$ , for identical wavelengths, an electron has a greater kinetic energy than a proton.

$$(b) \frac{K_e}{K_p} = \frac{\frac{h^2}{2m_e\lambda^2}}{\frac{h^2}{2m_p\lambda^2}} = \frac{m_p}{m_e} = \frac{1.673 \times 10^{-27} \text{ kg}}{9.11 \times 10^{-31} \text{ kg}} = \boxed{1840}$$

58. (a)  $\lambda = \frac{h}{p}$   
 $= \frac{h}{mv}$   
 $v = \frac{h}{\lambda m}$   
 $= \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(0.76 \text{ m})(65 \text{ kg})}$   
 $= \boxed{1.3 \times 10^{-35} \text{ m/s}}$

(b)  $x = vt$   
 $t = \frac{x}{v}$   
 $= \frac{x\lambda m}{h}$   
 $= \frac{(0.0010 \text{ m})(0.76 \text{ m})(65 \text{ kg})}{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}$   
 $= \boxed{7.5 \times 10^{31} \text{ s}}$



$$59. \quad K = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$$

$$U = qV$$

$$\text{Set } K = U.$$

$$\frac{h^2}{2m\lambda^2} = qV$$

$$\lambda^2 = \frac{h^2}{2mqV}$$

$$\lambda = \boxed{\frac{h}{\sqrt{2mqV}}}$$

$$60. \quad \Delta p_y \Delta y \geq \frac{h}{2\pi}$$

$$\Delta y \geq \frac{h}{2\pi \Delta p_y}$$

$$\Delta y \geq \frac{h}{2\pi m \Delta v_y}$$

$$\Delta y_{\text{baseball}} \geq \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi(0.15 \text{ kg})(0.050) \left(43 \frac{\text{m}}{\text{s}}\right)} = \boxed{3.3 \times 10^{-34} \text{ m}}$$

$$\Delta y_{\text{electron}} \geq \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi(9.11 \times 10^{-31} \text{ kg})(0.050) \left(43 \frac{\text{m}}{\text{s}}\right)} = \boxed{54 \text{ } \mu\text{m}}$$

$$61. \quad \Delta p_y \Delta y \geq \frac{h}{2\pi}$$

$$\Delta p_y \geq \frac{h}{2\pi \Delta y}$$

$$\Delta p_y \geq \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi(7.5 \times 10^{-15} \text{ m})}$$

$$\Delta p_y \geq \boxed{1.4 \times 10^{-20} \text{ kg} \cdot \text{m/s}}$$

$$62. \quad \Delta p_y \Delta y \geq \frac{h}{2\pi}$$

$$m \Delta v_y \Delta y \geq \frac{h}{2\pi}$$

$$\Delta v_y \geq \frac{h}{2\pi m \Delta y}$$

$$\Delta v_y \geq \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi(0.20 \text{ kg})(0.0022 \text{ m})}$$

$$\Delta v_y \geq \boxed{2.4 \times 10^{-31} \text{ m/s}}$$

$$\begin{aligned}
 63. \quad \Delta E \Delta t &\geq \frac{h}{2\pi} \\
 \Delta E &\geq \frac{h}{2\pi \Delta t} \\
 \Delta E &\geq \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi(1.0 \times 10^{-8} \text{ s})} \\
 \Delta E &\geq 1.1 \times 10^{-26} \text{ J} \\
 \Delta E_{\min} &= \boxed{1.1 \times 10^{-26} \text{ J}}
 \end{aligned}$$

$$\begin{aligned}
 64. \quad \Delta E \Delta t &\geq \frac{h}{2\pi} \\
 \Delta t &\geq \frac{h}{2\pi \Delta E} \\
 \Delta t &\geq \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi(0.0010 \text{ eV}) \left(1.60 \times 10^{-19} \frac{\text{J}}{\text{eV}}\right)} \\
 \Delta t &\geq 6.6 \times 10^{-13} \text{ s} \\
 \Delta t_{\min} &= \boxed{6.6 \times 10^{-13} \text{ s}}
 \end{aligned}$$

$$\begin{aligned}
 65. \quad \Delta E \Delta t &\geq \frac{h}{2\pi} \\
 \Delta E &\geq \frac{h}{2\pi \Delta t} \\
 \Delta E &\geq \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi(0.60 \times 10^{-9} \text{ s})} \\
 \Delta E_{\min} &= \boxed{1.8 \times 10^{-25} \text{ J}}
 \end{aligned}$$

$$\begin{aligned}
 66. \quad \Delta E \Delta t &\geq \frac{h}{2\pi} \\
 \Delta E &\geq \frac{h}{2\pi \Delta t} \\
 \Delta E &\geq \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi(2.5 \times 10^{-10} \text{ s})} \\
 \Delta E &\geq 4.2 \times 10^{-25} \text{ J} \\
 \Delta E_{\min} &= \boxed{4.2 \times 10^{-25} \text{ J}}
 \end{aligned}$$

$$\begin{aligned}
 67. \text{ (a)} \quad \Delta p \Delta x &\geq \frac{h}{2\pi} \\
 \Delta p &\geq \frac{h}{2\pi \Delta x} \\
 \Delta p &\geq \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi(0.15 \times 10^{-9} \text{ m})} \\
 \Delta p &\geq \boxed{7.0 \times 10^{-25} \text{ kg} \cdot \text{m/s}}
 \end{aligned}$$

$$(b) \quad K = \frac{p^2}{2m} = \frac{\Delta p^2}{2m} = \frac{(7.0 \times 10^{-25} \text{ kg} \cdot \frac{\text{m}}{\text{s}})^2}{2(9.11 \times 10^{-31} \text{ kg})} = \boxed{2.7 \times 10^{-19} \text{ J}}$$

$$\begin{aligned} 68. (a) \quad \Delta p \Delta x &\geq \frac{h}{2\pi} \\ \Delta p &\geq \frac{h}{2\pi \Delta x} \\ \Delta p &\geq \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi(0.15 \times 10^{-9} \text{ m})} \\ \Delta p &\geq \boxed{7.0 \times 10^{-25} \text{ kg} \cdot \text{m/s}} \end{aligned}$$

$$(b) \quad K = \frac{p^2}{2m} = \frac{\Delta p^2}{2m} = \frac{(7.0 \times 10^{-25} \text{ kg} \cdot \frac{\text{m}}{\text{s}})^2}{2(1.673 \times 10^{-27} \text{ kg})} = \boxed{1.5 \times 10^{-22} \text{ J}}$$

$$\begin{aligned} 69. \quad \frac{\Delta p}{p} &= 0.010 \\ \Delta p \Delta x &= \frac{h}{2\pi} \\ \Delta x &\geq \frac{h}{2\pi \Delta p} \\ \Delta x &\geq \frac{h}{2\pi(0.010 p)} \\ \Delta x &\geq \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi(0.010)(1.7 \times 10^{-25} \text{ kg} \cdot \frac{\text{m}}{\text{s}})} \\ \Delta x &\geq 62 \text{ nm} \\ \Delta x_{\min} &= \boxed{62 \text{ nm}} \end{aligned}$$

70. Determine the maximum wavelength required to eject electrons from each metal surface.

$$\lambda_{\max} = \frac{hc}{W_0} \quad hc = (6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s}) \left( \frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}} \right) = 12.43 \times 10^{-7} \text{ eV} \cdot \text{m}$$

$$\text{Al: } \lambda_{\max} = \frac{12.43 \times 10^{-7} \text{ eV} \cdot \text{m}}{4.28 \text{ eV}} = 2.90 \times 10^{-7} \text{ m} = 290 \text{ nm}$$

$$\text{Pb: } \lambda_{\max} = \frac{12.43 \times 10^{-7} \text{ eV} \cdot \text{m}}{4.25 \text{ eV}} = 2.92 \times 10^{-7} \text{ m} = 292 \text{ nm}$$

$$\text{Cs: } \lambda_{\max} = \frac{12.43 \times 10^{-7} \text{ eV} \cdot \text{m}}{2.14 \text{ eV}} = 5.81 \times 10^{-7} \text{ m} = 581 \text{ nm}$$

Only 581 nm is in the visible range, so use cesium.

$$71. \quad P = \left( \frac{N}{t} \right) E = \left( \frac{N}{t} \right) \frac{hc}{\lambda} = \left( \frac{100 \text{ photons}}{1 \text{ s}} \right) \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{(545 \text{ nm}) \left( \frac{1 \text{ m}}{10^9 \text{ nm}} \right)}$$

$$P = 3.65 \times 10^{-17} \text{ J/s} = \boxed{3.65 \times 10^{-17} \text{ W}}$$

$$72. \text{ (a) } f = \frac{1}{2\pi} \sqrt{\frac{g}{L}} = \frac{1}{2\pi} \sqrt{\frac{9.81 \frac{\text{m}}{\text{s}^2}}{0.88 \text{ m}}} = \boxed{0.53 \text{ Hz}}$$

(b) Assume  $v \ll c$ .

$$\begin{aligned} E &= nhf \\ \frac{1}{2}mv^2 &= \frac{nh}{2\pi} \sqrt{\frac{g}{L}} \\ v^2 &= \frac{nh}{\pi m} \sqrt{\frac{g}{L}} \\ v &= \left( \frac{nh}{\pi m} \right)^{1/2} \left( \frac{g}{L} \right)^{1/4} \\ &= \left[ \frac{(1.0 \times 10^{33})(6.63 \times 10^{-34} \text{ J} \cdot \text{s})}{\pi(0.15 \text{ kg})} \right]^{1/2} \left( \frac{9.81 \frac{\text{m}}{\text{s}^2}}{0.88 \text{ m}} \right)^{1/4} \\ &= \boxed{2.2 \text{ m/s}} \end{aligned}$$

$$73. \text{ (a) } \frac{P}{E_{\text{photon}}} = \frac{P}{hf} = \frac{1.0 \times 10^{-10} \text{ W}}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(96 \times 10^6 \text{ Hz})} = \boxed{1.6 \times 10^{15} \text{ photons/s}}$$

(b)  $P = Fv = Fc$

$$F = \frac{P}{c} = \frac{1.0 \times 10^{-10} \text{ W}}{3.00 \times 10^8 \frac{\text{m}}{\text{s}}} = \boxed{3.3 \times 10^{-19} \text{ N}}$$

$$\begin{aligned} 74. \text{ (a) } NE_{\text{photon}} &= mL_f \\ N &= \frac{mL_f}{E_{\text{photon}}} \\ &= \frac{mL_f}{hf} \\ &= \frac{(1.0 \text{ kg}) \left( 80.0 \frac{\text{kcal}}{\text{kg}} \right) \left( \frac{4186 \text{ J}}{\text{kcal}} \right)}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(6.0 \times 10^{14} \text{ Hz})} \\ &= \boxed{8.4 \times 10^{23} \text{ photons}} \end{aligned}$$

$$\begin{aligned} \text{(b) } N_{\text{H}_2\text{O}} &= \frac{E_{\text{photon}}}{E_{f, \text{H}_2\text{O}}} \\ &= \frac{hf}{m_{\text{H}_2\text{O}}L_f} \\ &= \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(6.0 \times 10^{14} \text{ Hz})}{(3.0 \times 10^{-26} \text{ kg}) \left( 80.0 \frac{\text{kcal}}{\text{kg}} \right) \left( \frac{4186 \text{ J}}{\text{kcal}} \right)} \\ &= \boxed{40 \text{ molecules}} \end{aligned}$$

$$\begin{aligned}
 75. \quad NE_{\text{photon}} &= mc_w \Delta T \\
 N &= \frac{mc_w \Delta T}{E_{\text{photon}}} \\
 &= \frac{\lambda mc_w \Delta T}{hc} \\
 &= \frac{(520 \times 10^{-9} \text{ m})(1 \text{ g}) \left( 4.186 \frac{\text{J}}{\text{g} \cdot \text{K}} \right) (1.0 \text{ K})}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) \left( 3.00 \times 10^8 \frac{\text{m}}{\text{s}} \right)} \\
 &= \boxed{1 \times 10^{19} \text{ photons}}
 \end{aligned}$$

$$\begin{aligned}
 76. \quad NE_{\text{photon}} &= mc_w \Delta T \\
 N &= \frac{mc_w \Delta T}{E_{\text{photon}}} \\
 &= \frac{\lambda mc_w \Delta T}{hc} \\
 &= \frac{(0.122 \text{ m})(215 \text{ mL}) \left( \frac{1 \text{ g}}{1 \text{ mL}} \right) \left( 4.186 \frac{\text{J}}{\text{g} \cdot \text{K}} \right) (70.0 \text{ K})}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) \left( 3.00 \times 10^8 \frac{\text{m}}{\text{s}} \right)} \\
 &= \boxed{3.86 \times 10^{28} \text{ photons}}
 \end{aligned}$$

$$\begin{aligned}
 77. \quad K_{\text{max}} &= hf - W_0 \\
 K_{\text{de Broglie}} &= \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} \\
 K &= hf - W_0 \\
 \frac{h^2}{2m\lambda^2} &= hf - W_0 \\
 \frac{2m\lambda^2}{h^2} &= \frac{1}{hf - W_0} \\
 \lambda^2 &= \frac{h^2}{2m(hf - W_0)} \\
 \lambda &= \frac{h}{\sqrt{2m(hf - W_0)}} \\
 &= \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{\sqrt{2(9.11 \times 10^{-31} \text{ kg}) \left[ (6.63 \times 10^{-34} \text{ J} \cdot \text{s})(2.11 \times 10^{15} \text{ Hz}) - (4.25 \text{ eV}) \left( 1.60 \times 10^{-19} \frac{\text{J}}{\text{eV}} \right) \right]}} \\
 &= \boxed{0.58 \text{ nm}}
 \end{aligned}$$

$$78. \quad \text{(a)} \quad \lambda = \frac{h}{p_e} = \frac{h}{m_e v} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg}) \left( 2.7 \times 10^6 \frac{\text{m}}{\text{s}} \right)} = \boxed{0.27 \text{ nm}}$$

$$\text{(b)} \quad \lambda = \frac{h}{p} = \frac{h}{p_e} = \lambda_e = \boxed{0.27 \text{ nm}}$$

$$79. f_{\text{peak}} = (5.88 \times 10^{10} \text{ s}^{-1} \cdot \text{K}^{-1})T$$

$$\begin{aligned} T &= \frac{f_{\text{peak}}}{5.88 \times 10^{10} \text{ s}^{-1} \cdot \text{K}^{-1}} \\ &= \frac{5.4 \times 10^{14} \text{ Hz}}{5.88 \times 10^{10} \text{ s}^{-1} \cdot \text{K}^{-1}} \\ &= \boxed{9200 \text{ K}} \end{aligned}$$

Firefly radiation is definitely not well approximated by blackbody radiation. At 9200 K, the firefly would be completely destroyed.

$$80. \text{ (a) Since } v \ll c, \text{ we can use Classical Theory. Substitute } \frac{1}{2}mv_{\text{max}}^2 \text{ for } K_{\text{max}} \text{ in the equation}$$

$$K_{\text{max}} = \frac{hc}{\lambda} - W_0 \text{ and solve for } W_0. \text{ Then calculate the cutoff frequency using } f_0 = \frac{W_0}{h}.$$

$$\begin{aligned} \text{(b) } K_{\text{max}} &= \frac{hc}{\lambda} - W_0 \\ W_0 &= \frac{hc}{\lambda} - K_{\text{max}} \\ &= \frac{hc}{\lambda} - \frac{1}{2}mv_{\text{max}}^2 \\ &= \left[ \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) \left( 3.00 \times 10^8 \frac{\text{m}}{\text{s}} \right)}{545 \times 10^{-9} \text{ m}} - \frac{1}{2} (9.11 \times 10^{-31} \text{ kg}) \left( 3.10 \times 10^5 \frac{\text{m}}{\text{s}} \right)^2 \right] \left( \frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}} \right) \\ &= \boxed{2.01 \text{ eV}} \end{aligned}$$

$$f_0 = \frac{W_0}{h} = \frac{2.0074 \text{ eV}}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) \left( \frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}} \right)} = \boxed{4.84 \times 10^{14} \text{ Hz}}$$

$$81. \text{ (a) Photon energy is inversely proportional to wavelength, so the atom's net energy has } \boxed{\text{decreased}}.$$

$$\begin{aligned} \text{(b) } \Delta E &= E_{\text{photon absorbed}} - E_{\text{photon emitted}} \\ &= \frac{hc}{\lambda_{\text{abs}}} - \frac{hc}{\lambda_{\text{emit}}} \\ &= \frac{1.24 \text{ keV} \cdot \text{nm}}{486.2 \text{ nm}} - \frac{1.24 \text{ keV} \cdot \text{nm}}{97.23 \text{ nm}} \\ &= \boxed{-10.2 \text{ eV}} \end{aligned}$$

$$82. \text{ (a) } K_{\max} = \frac{hc}{\lambda} - W_0$$

Solve for  $hc$ .

$$hc = \lambda(W_0 + K_{\max})$$

$$\lambda_1(W_0 + K_{\max, 1}) = \lambda_2(W_0 + K_{\max, 2})$$

Solve for  $W_0$ .

$$\begin{aligned} W_0 &= \frac{\lambda_2 K_{\max, 2} - \lambda_1 K_{\max, 1}}{\lambda_1 - \lambda_2} \\ &= \frac{(253.5 \text{ nm})(2.57 \text{ eV}) - (433.9 \text{ nm})(0.550 \text{ eV})}{433.9 \text{ nm} - 253.5 \text{ nm}} \\ &= 2.289 \text{ eV} \\ &= \boxed{2.29 \text{ eV}} \end{aligned}$$

$$\text{(b) } hc = \lambda(W_0 + K_{\max})$$

$$\begin{aligned} h &= \frac{\lambda}{c}(W_0 + K_{\max}) \\ &= \frac{433.9 \times 10^{-9} \text{ m}}{3.00 \times 10^8 \text{ m/s}}(2.289 \text{ eV} + 0.550 \text{ eV}) \\ &= (4.106 \times 10^{-15} \text{ eV} \cdot \text{s})(1.60 \times 10^{-19} \text{ J/eV}) \\ &= \boxed{6.57 \times 10^{-34} \text{ J} \cdot \text{s}} \end{aligned}$$

$$\text{(c) } K_{\max} = \frac{(4.106 \times 10^{-15} \text{ eV} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{365.0 \times 10^{-9} \text{ m}} - 2.29 \text{ eV} = \boxed{1.08 \text{ eV}}$$

$$83. \quad \lambda = \frac{h}{mv} \qquad v = \frac{h}{m\lambda} = \frac{h}{m\left(\frac{h}{\sqrt{5mkT}}\right)} = \sqrt{\frac{5kT}{m}}$$

$$v = \sqrt{\frac{5(1.38 \times 10^{-23} \text{ J/K})(450 \text{ K})}{(1.008 \text{ u})(1.66 \times 10^{-27} \text{ kg})}} = \boxed{4.31 \text{ km/s}}$$

84. (a) The deBroglie wavelength is inversely proportional to momentum, and as kinetic energy increases so does momentum. Therefore, the deBroglie wavelength decreases.

$$\begin{aligned}
 \text{(b)} \quad \lambda &= \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \left( \frac{h}{\sqrt{2m}} \right) \frac{1}{\sqrt{K}} \\
 &= \left( \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{\sqrt{2} \sqrt{9.11 \times 10^{-31} \text{ kg}}} \right) \frac{1}{\sqrt{K \left( \frac{1.60 \times 10^{-19} \text{ J}}{1 \text{ eV}} \right)}} \\
 &= 1.23 \times 10^{-9} \frac{\text{J}\cdot\text{s} \cdot \sqrt{\text{eV}}}{\sqrt{\text{kg}\cdot\text{J}}} \frac{1}{\sqrt{K}} \\
 &= 1.23 \times 10^{-9} \sqrt{\frac{\text{J}^2 \cdot \text{s}^2 \cdot \text{eV}}{\text{kg}\cdot\text{J}}} \frac{1}{\sqrt{K}} \\
 &= \frac{1.23 \times 10^{-9} \text{ m} \cdot \sqrt{\text{eV}}}{\sqrt{K}} \\
 &= \frac{1.23 \text{ nm} \cdot \sqrt{\text{eV}}}{\sqrt{K}}
 \end{aligned}$$

$$\begin{aligned}
 85. \text{ (a)} \quad K_{\text{av}} &= \frac{3}{2} kT \\
 \lambda_{\text{av}} &\approx \frac{h}{\sqrt{2mK_{\text{av}}}} = \frac{h}{\sqrt{3mkT}} \\
 &= \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{\sqrt{3(6.65 \times 10^{-27} \text{ kg})(1.38 \times 10^{-23} \frac{\text{J}}{\text{K}})(295.15 \text{ K})}} \\
 &= \boxed{0.0735 \text{ nm}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad PV &= NkT \\
 \text{Volume per atom} &= \frac{V}{N} = \frac{kT}{P} \\
 \text{Avg. separation} &= \left( \frac{kT}{P} \right)^{1/3} \\
 &= \left( \frac{(1.38 \times 10^{-23} \frac{\text{J}}{\text{K}})(295.15 \text{ K})}{101 \times 10^3 \text{ Pa}} \right)^{1/3} \\
 &= \boxed{3.43 \text{ nm}}
 \end{aligned}$$

$$86. \text{ (a)} \quad \lambda_c = \frac{h}{m_p c} = \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{(1.673 \times 10^{-27} \text{ kg})(3.00 \times 10^8 \text{ m/s})} = 1.32 \times 10^{-15} \text{ m} = \boxed{1.32 \text{ fm}}$$

$$\text{(b)} \quad E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(3.00 \times 10^8 \text{ m/s})}{1.32 \times 10^{-15} \text{ m}} = \boxed{1.51 \times 10^{-10} \text{ J}}$$

$$\text{(c)} \quad E = \frac{hc}{\lambda_c} = \frac{hc}{\frac{h}{mc}} = mc^2 = \text{rest energy of the particle}$$



87. (a) The difference in maximum kinetic energy observed from the two surfaces is due to the difference in the two materials' work functions. Since the maximum kinetic energy depends linearly upon the work function, and since the work function does not depend upon the frequency of the light, the difference in maximum kinetic energy stays the same.

(b)  $K_{\max} = hf - W_0$

$$\begin{aligned} W_{0B} - W_{0A} &= (hf - K_{\max B}) - (hf - K_{\max A}) \\ &= K_{\max A} - K_{\max B} \\ &= \boxed{2.00 \times 10^{-19} \text{ J}} \end{aligned}$$