

Chapter 6

Applications of Newton's Laws

Answers to Even-numbered Conceptual Questions

2. Spinning the wheels is likely to decrease the force exerted by the Jeep. The reason is that the force exerted by the spinning wheels is kinetic friction, and the coefficient of kinetic friction is generally less than the coefficient of static friction. The spinning wheels look better in the movie, however.
4. The maximum acceleration is determined by the normal force exerted on the drive wheels. If the engine of the car is in the front, and the drive wheels are in the rear, the normal force is less than it would be with front-wheel drive. During braking, however, all four wheels participate – including the wheels that sit under the engine.
6. Friction is beneficial whenever you want to start, stop, or turn a car. It is also beneficial when you tune a guitar or tie your shoes.
8. This is possible because if you spin the bucket rapidly enough, the force needed to produce circular motion is greater than the force of gravity. In this case, a force in addition to gravity must act at the top of the circle to keep the water moving in its circular path. This force is provided by the bottom of the bucket. Therefore, the bottom of the bucket pushes against the water, and the water pushes back against the bucket – this keeps the water from falling out of the bucket.
10. At the equator, you are moving in a circular path. Therefore, part of the force of gravity acting on you is providing your centripetal acceleration; the rest shows up as a reduced weight on the scale. At the poles, the scale reads your full weight. (In addition, the Earth bulges at the equator, due to the same effects just discussed. Therefore, you are farther from the center of the Earth when you are at the equator, and this too diminishes your weight, as we shall see in Chapter 12.)
12. Yes. Equilibrium simply means that the net force acting on an object is zero. Therefore, an object moving with constant velocity can be considered to be in equilibrium. In a frame of reference moving with the same velocity, the object would be at rest and would have zero net force acting on it – which is the way we usually think of equilibrium.
14. In principle, the reading on the scale would be greater than 900 N if the Earth were to stop rotating. See the discussion for Question 10 for further details.
16. Astronauts feel weightless because they are in constant free fall as they orbit, just as you would feel weightless inside an elevator that drops downward in free fall.
18. A passenger on this ride feels pushed against the wall, which means that the wall, in turn, exerts a normal force on the passenger. If the corresponding force of static friction is greater than the passenger's weight, the passenger stays put as the floor is lowered away. (Why do all passengers stay put at the same rotational speed, regardless of their weight?)
20. If the parking brake is applied while the car is in motion, the rear wheels begin to skid across the pavement. This means that the friction acting on the rear wheels is kinetic friction, which is smaller in magnitude than the static friction experienced by the front wheels. As a result, the rear wheels will overtake the front wheels, causing the car to spin around and begin moving rear wheels first. This is standard procedure for stunt drivers wishing to spin a car around in a chase scene.

22. As the basket within a washing machine rotates, the clothes collect on the rim of the basket. Here the basket exerts an inward force on the clothes causing them to follow a circular path. The water contained in the clothes, however, is able to pass through the holes of the basket where it can be drained from the machine.
24. (a) If the spring is cut in half, it stretches half as much for a given applied force as it stretched before being cut. Therefore, the force constant of the half spring is twice what it was for the full spring. (b) When two springs are connected end-to-end, they stretch twice as much for a given applied force. It follows that the force constant in this case is half what it was for a single spring.
26. People on the outer rim of a rotating space station must experience a force directed toward the center of the station in order to follow a circular path. This force is applied by the “floor” of the station, which is really its outermost wall. Because people feel an upward force acting on them from the floor, just as they would on Earth, the sensation is like an “artificial gravity.”
28. When a bicycle rider leans inward on a turn, the force applied to the wheels of the bicycle by the ground is both upward and inward. It is this inward force that produces the centripetal acceleration of the rider.
30. The physics of this scene is somewhere between “bad” and “ugly”. When the rope burns through, Robin is moving horizontally. This horizontal motion should continue as Robin falls, leading to a parabolic trajectory rather than the straight downward drop shown in the movie.

Solutions to Problems

1. Determine the deceleration caused by friction. If the player's initial velocity is in the positive direction, then the force of kinetic friction is negative.

$$f_k = -\mu_k N = -\mu_k mg$$

$$\frac{f_k}{m} = a = -\mu_k g = -0.41 \left(9.81 \frac{\text{m}}{\text{s}^2} \right) = -4.02 \frac{\text{m}}{\text{s}^2}$$

Calculate how far the baseball player slid.

$$\Delta x = \frac{v_f^2 - v_i^2}{2a} = \frac{0 - \left(7.90 \frac{\text{m}}{\text{s}} \right)^2}{2 \left(-4.02 \frac{\text{m}}{\text{s}^2} \right)} = 7.8 \text{ m}$$

The player slid 7.8 m.

2. Choose the x -axis along the direction of motion.

$$\sum F_x = W \sin \theta - f_k = ma$$

$$mg \sin \theta - \mu_k mg \cos \theta = ma$$

$$\mu_k = \tan \theta - \frac{a}{g \cos \theta} = \tan 35.0^\circ - \frac{1.05 \frac{\text{m}}{\text{s}^2}}{\left(9.81 \frac{\text{m}}{\text{s}^2} \right) \cos 35.0^\circ} = \boxed{0.570}$$

3. $F = ma = \mu_s mg = f_s$

$$\mu_s = \frac{a}{g} = \frac{12 \frac{\text{m}}{\text{s}^2}}{9.81 \frac{\text{m}}{\text{s}^2}} = \boxed{1.2}$$

4. $F_0 = f_s = \mu_s mg$

$$\mu_s = \frac{F_0}{mg} = \frac{2.25 \text{ N}}{(1.80 \text{ kg})(9.81 \frac{\text{m}}{\text{s}^2})} = \boxed{0.127}$$

$$F = f_k = \mu_k mg$$

$$\mu_k = \frac{F}{mg} = \frac{1.50 \text{ N}}{(1.80 \text{ kg})(9.81 \frac{\text{m}}{\text{s}^2})} = \boxed{0.0849}$$

5. The frictional force is $\boxed{0.75 \text{ N}}$ opposite the direction of the push.

6. (a) $\boxed{\text{When enough of the tie hangs over the edge, the force of gravity acting on the hanging mass of the tie overcomes the force of static friction acting on that portion of the tie lying on the table.}}$

(b) $\frac{1}{4}mg = \mu_s N = \mu_s \left(\frac{3}{4}m\right)g$

$$\mu_s = \boxed{\frac{1}{3}}$$

7. $F_x = \mu_s N$

$$F \cos \theta = \mu_s (mg + F \sin \theta)$$

$$\begin{aligned} F &= \frac{\mu_s mg}{\cos \theta - \mu_s \sin \theta} \\ &= \frac{0.57(32 \text{ kg})(9.81 \frac{\text{m}}{\text{s}^2})}{\cos 21^\circ - 0.57 \sin 21^\circ} \\ &= \boxed{250 \text{ N}} \end{aligned}$$

8. $F_x - \mu_k N = ma$

$$F \cos \theta - \mu_k (mg + F \sin \theta) = ma$$

$$a = \frac{F}{m} \cos \theta - \mu_k g - \frac{F \mu_k}{m} \sin \theta = \frac{330 \text{ N}}{32 \text{ kg}} \cos 21^\circ - 0.45 \left(9.81 \frac{\text{m}}{\text{s}^2}\right) - \frac{(330 \text{ N})(0.45)}{32 \text{ kg}} \sin 21^\circ = \boxed{3.5 \text{ m/s}^2}$$

9. Place the x -axis parallel to the ramp, pointing uphill.

(a) $\sum F_x = f_s - W_x = 0$

$$0 = \mu_s mg \cos \theta - mg \sin \theta$$

$$\mu_s = \tan \theta = \tan 23^\circ = \boxed{0.42}$$

(b) $\theta = \boxed{23^\circ}$; $\mu_s = f(\theta)$ only

10. (a) Determine the acceleration.

$$a = \frac{v_f^2 - v_i^2}{2\Delta x} = \frac{\left(12 \frac{\text{m}}{\text{s}}\right)^2 - 0}{2(20 \text{ m})} = 3.6 \frac{\text{m}}{\text{s}^2}$$

Calculate μ_s .

$$\sum F_x = f_s = ma$$

$$f_s = \mu_s mg = ma$$

$$\mu_s = \frac{a}{g} = \frac{3.6 \frac{\text{m}}{\text{s}^2}}{9.81 \frac{\text{m}}{\text{s}^2}} = \boxed{0.37}$$

- (b) First, determine the runner's acceleration from $v_f^2 = v_i^2 + 2a\Delta x$. Next, equate the force associated with this acceleration to the force of static friction between the runner's shoes and the track. Solve for μ_s .

11. (a) $F = ma = \mu_s N = \mu_s mg$

$$a = \mu_s g = (0.24) \left(9.81 \frac{\text{m}}{\text{s}^2}\right) = 2.3544 \frac{\text{m}}{\text{s}^2} = \boxed{2.4 \text{ m/s}^2}$$

- (b) $v_f = v_i + at$

$$t = \frac{v_f - v_i}{a} = \frac{(15 \frac{\text{m}}{\text{s}}) - 0}{2.3544 \frac{\text{m}}{\text{s}^2}} = \boxed{6.4 \text{ s}}$$

12. (a) $a = \frac{F}{m} = \frac{-\mu_k N}{m} = \frac{-\mu_k mg}{m} = -\mu_k g = -0.11 (9.81 \text{ m/s}^2) = -1.08 \text{ m/s}^2$

$$v^2 = v_0^2 + 2a\Delta x$$

$$\Delta x = \frac{v^2 - v_0^2}{2a} = \frac{0 - (5.3 \text{ m/s})^2}{2(-1.08 \text{ m/s}^2)} = \boxed{13 \text{ m}}$$

- (b) Doubling the mass doubles the normal force, causing the friction force to double. Therefore, the friction force increases.

- (c) Stay the same, since stopping distance is independent of mass.

- (d) $Fd = (-\mu_k mg)d = (-0.11)(0.12 \text{ kg})(9.81 \text{ m/s}^2)(13 \text{ m}) = \boxed{-1.7 \text{ N} \cdot \text{m}}$

$$\Delta \left(\frac{1}{2} mv^2 \right) = \frac{1}{2} mv^2 - \frac{1}{2} mv_0^2 = 0 - \frac{1}{2} (0.12 \text{ kg})(5.3 \text{ m/s})^2 = \boxed{-1.7 \text{ kg} \cdot \text{m}^2 / \text{s}^2}$$

13. (a) $a = \frac{F}{m} = \frac{-\mu_k N}{m} = \frac{-\mu_k mg}{m} = -\mu_k g = -(0.260)(9.81 \text{ m/s}^2) = \boxed{-2.55 \text{ m/s}^2}$

- (b) $v^2 = v_0^2 + 2a\Delta x = (4.33 \text{ m/s})^2 + 2(-2.55 \text{ m/s}^2)(0.125 \text{ m}) = 18.11 \text{ m}^2 / \text{s}^2$

$$v = \boxed{4.26 \text{ m/s}}$$

$$(c) -Fd = -\mu_k mgd = -(0.260)(1.95 \text{ kg})(9.81 \text{ m/s}^2)(0.125 \text{ m}) = \boxed{-0.622 \text{ N} \cdot \text{m}}$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}(1.95 \text{ kg})(18.11 \text{ m}^2/\text{s}^2) - \frac{1}{2}(1.95 \text{ kg})(4.33 \text{ m/s})^2 = \boxed{-0.622 \text{ kg} \cdot \text{m}^2/\text{s}^2}$$

14. The force of friction will cause the car to decelerate from v to 0.

$$(a) a = \frac{v_f^2 - v_i^2}{2\Delta x} = \frac{-v^2}{2\Delta x}$$

$$f_k = \mu N = \mu mg$$

$$f_k = -ma$$

$$\mu mg = \frac{mv^2}{2\Delta x}$$

$$\Delta x = \boxed{\frac{v^2}{2\mu g}}$$

- (b) The stopping distance quadruples.

$$(c) \Delta x = \boxed{\frac{v^2}{2\mu g}}; \Delta x \text{ does not depend on } m.$$

15. Place the y -axis along the direction of acceleration.

$$\sum F_y = T - W = ma$$

$$T = ma + W = ma + mg = m(a + g) = (4.25 \text{ kg})\left(1.80 \frac{\text{m}}{\text{s}^2} + 9.81 \frac{\text{m}}{\text{s}^2}\right) = \boxed{49.3 \text{ N}}$$

16. Place the y -axis along the axis of the spring with $y = 0$ at the top of the uncompressed spring.

$$\sum F_y = -ky - mg = 0$$

$$k = -\frac{mg}{y} = -\frac{(9.00 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)}{(-0.0450 \text{ m})} = \boxed{1.96 \text{ kN/m}}$$

17. Place the y -axis perpendicular to the road with $y = 0$ at the bumper before the box is loaded.

$$\sum F_y = -ky - mg = 0$$

$$k = -\frac{mg}{y} = -\frac{(110 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)}{-0.13 \text{ m}} = \boxed{8.3 \text{ kN/m}}$$

18. $\sum F_y = 2T_y - mg = 0$

$$2T \sin \theta = mg$$

$$T = \frac{mg}{2 \sin \theta}$$

$$= \frac{(50.0 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)}{2 \sin 15.0^\circ}$$

$$= \boxed{948 \text{ N}}$$

19. (a) Determine the magnitude of the force required to stretch the spring 2.00 cm.

$$F = -kx = -\left(150 \frac{\text{N}}{\text{m}}\right)(-0.0200 \text{ m}) = 3.0 \text{ N}$$

Calculate the magnitude of the force of static friction.

$$F - f_s = ma = 0$$

$$f_s = F = \boxed{3.0 \text{ N}}$$

- (b) No, the force was determined using only the spring constant and the extension of the spring.

20. Determine the magnitude of the force required to stretch the spring 2.50 cm.

$$F = -kx = -\left(150 \frac{\text{N}}{\text{m}}\right)(-0.0250 \text{ m}) = 3.75 \text{ N}$$

$$F - f_s = 0$$

$$F - \mu_s N = F - \mu_s W = 0$$

$$\mu_s = \frac{F}{W} = \frac{3.75 \text{ N}}{52.0 \text{ N}} = \boxed{0.072}$$

21. (a) $F = T_r$, and lifting at constant speed,

$$2T_r = W = mg. \text{ So,}$$

$$F = T_r = \frac{1}{2}mg = \frac{1}{2}(52 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right) = \boxed{260 \text{ N}}$$

- (b) For each chain,

$$T_c = 2T_r = 2(255.06 \text{ N}) = \boxed{510 \text{ N}}$$

22. $F = T_r$, and with acceleration, $2T_r = W + ma = m(g + a)$. So,

$$F = T_r = \frac{1}{2}m(g + a) = \frac{1}{2}(52 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2} + 0.23 \frac{\text{m}}{\text{s}^2}\right) = 261.04 \text{ N} = \boxed{260 \text{ N}}$$

$$T_c = 2T_r = 2(261.04 \text{ N}) = \boxed{520 \text{ N}}$$

23. (a) $\sum F_y = 0$

$$f_s - mg = 0$$

$$\mu_s F_s - mg = 0$$

$$\mu_s (-kx) - mg = 0$$

$$x = \frac{-mg}{\mu_s k}$$

$$x = \frac{-(0.27 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)}{(0.46)\left(120 \frac{\text{N}}{\text{m}}\right)} = \boxed{-4.8 \text{ cm}}$$

- (b) Yes; the spring displacement is proportional to the block's mass.

24. (a) When the car accelerates from the stop light, the point at which the tassel is attached to the car moves forward. The tassel only appears to move backward because as the car moves forward the bottom of the tassel remains for a moment where it originally was before it too must move with the car.

$$\begin{aligned}
 \text{(b)} \quad \sum F_y &= T \cos \theta - mg = 0 \\
 \sum F_x &= T \sin \theta - ma = 0 \\
 T &= \frac{mg}{\cos \theta} = \frac{ma}{\sin \theta} \\
 a &= g \tan \theta = \left(9.81 \frac{\text{m}}{\text{s}^2} \right) \tan 5.00^\circ = \boxed{0.858 \text{ m/s}^2}
 \end{aligned}$$

$$\begin{aligned}
 25. \quad \sum F_y &= T \cos \theta - mg = 0 \\
 T &= \frac{mg}{\cos \theta} = \frac{(0.0100 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right)}{\cos 5.00^\circ} = \boxed{0.0985 \text{ N}}
 \end{aligned}$$

$$\begin{aligned}
 26. \quad \text{(a)} \quad T &= mg + ma = 755 \text{ N} \\
 a &= \frac{755 \text{ N}}{m} - g = \frac{755 \text{ N}}{70.0 \text{ kg}} - \left(9.81 \frac{\text{m}}{\text{s}^2} \right) = 0.976 \frac{\text{m}}{\text{s}^2} \\
 x &= \frac{1}{2} at^2 \\
 t &= \sqrt{\frac{2x}{a}} \\
 &= \sqrt{\frac{2(3.40 \text{ m})}{0.976 \frac{\text{m}}{\text{s}^2}}} \\
 &= \boxed{2.64 \text{ s}}
 \end{aligned}$$

(b) For Jones to be pulled from the pit in less time would require a greater acceleration. This greater acceleration would cause the tension in the rope to exceed the 750 N limit and the rope would break.

$$27. \quad x_0 = 0.18 \text{ m}$$

$$\text{(a)} \quad F = -k(x - x_0) = -k(2x_0 - x_0) = -kx_0 = -\left(250 \frac{\text{N}}{\text{m}} \right) (0.18 \text{ m}) = -45 \text{ N}$$

The force required to stretch the spring to twice its equilibrium length is 45 N.

(b) No, the force required to compress the spring by $(1/2)x_0$ is only half the force found in (a). The force depends linearly on displacement.

28. (a) Less; horizontal components of the two string tensions must be equal, and $\cos \theta_2 > \cos \theta_1$.

$$\begin{aligned}
 \text{(b)} \quad T_1 \cos \theta_1 &= T_2 \cos \theta_2 \\
 T_2 &= T_1 \frac{\cos \theta_1}{\cos \theta_2} \\
 &= (1.7 \text{ N}) \frac{\cos 65^\circ}{\cos 32^\circ} \\
 &= \boxed{0.85 \text{ N}}
 \end{aligned}$$

$$\text{(c)} \quad W = T_{1y} + T_{2y} = T_1 \sin \theta_1 + T_2 \sin \theta_2 = (1.7 \text{ N}) \sin 65^\circ + (0.85 \text{ N}) \sin 32^\circ = \boxed{2.0 \text{ N}}$$

29. Place the coordinate system such that the y -axis points up and the x -axis is in the direction opposite of the force applied by the archer.

$$\sum F_y = 0$$

$$\sum F_x = T \cos 72.5^\circ + T \cos(-72.5^\circ) - F = 0$$

$$2T \cos 72.5^\circ = F$$

$$T = \frac{25.0 \text{ lb}}{2 \cos 72.5^\circ} = \boxed{41.6 \text{ lb}}$$

30. (a) Set the x -axis parallel to and pointing up the incline.

$$\sum F = 0$$

$$T - mg \sin 31^\circ = 0$$

$$T = mg \sin 31^\circ = (1.0 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) \sin 31^\circ = \boxed{5.1 \text{ N}}$$

(b) $\sum F = 0$

$$T - Mg \sin 31^\circ = 0$$

$$T = Mg \sin 31^\circ = (1.0 \text{ kg} + 2.0 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) \sin 31^\circ = \boxed{15 \text{ N}}$$

31. The rope tension equals the force needed to support the weight.

$$T_1 = T_2 = mg = (2.50 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) = 24.525 \text{ N}$$

$$F = T_1 \cos 30^\circ + T_2 \cos 30^\circ = 2(24.525 \text{ N}) \cos 30^\circ = \boxed{42.5 \text{ N}}$$

32. $M = 6.7 \text{ kg}$

m = hanging mass

$$\sum F = mg - Mg \sin \theta = 0$$

$$m = M \sin \theta = (6.7 \text{ kg}) \sin 42^\circ = \boxed{4.5 \text{ kg}}$$

33. Place the y -axis such that $\vec{W} = m\vec{g} = -mg\hat{y}$.

$$\sum F_y = -mg + F \sin\left(\frac{\theta}{2}\right) + F \sin\left(180^\circ - \frac{\theta}{2}\right) = 0$$

$$q = 120^\circ$$

$$F = \frac{mg}{\sin\left(\frac{\theta}{2}\right) + \sin\left(180^\circ - \frac{\theta}{2}\right)} = \frac{(0.15 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right)}{\sin 60^\circ + \sin 120^\circ} = \boxed{0.85 \text{ N}}$$

34. The direction of the y -axis is up and that of the x -axis is away from the wall.

$$\begin{aligned}
 \text{(a)} \quad \sum F_x &= N - F = 0 \\
 \sum F_y &= f_s - W = 0 \\
 f_s - W &= \mu_s N - mg \\
 N &= \frac{mg}{\mu_s} \\
 F = N &= \frac{mg}{\mu_s} = \frac{(1.3 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)}{0.86} = \boxed{15 \text{ N}}
 \end{aligned}$$

- (b) The force of friction stays the same (mg).

$$\begin{aligned}
 \text{35. } \vec{F}_{\text{total}} &= \vec{F}_1 + \vec{F}_2 + \vec{F}_3 \\
 |\vec{F}_1| &= |\vec{F}_2| = |\vec{F}_3| = T = \text{tension} \\
 F_{\text{total},x} &= T \cos \alpha + T \cos 160^\circ + T \cos(-160^\circ) \\
 &= T(\cos \alpha + 2 \cos 160^\circ) \\
 F_{\text{total},y} &= T \sin \alpha + T \sin 160^\circ + T \sin(-160^\circ) \\
 &= T \sin \alpha \\
 \alpha &\text{ was chosen such that } 180^\circ - \theta = \alpha.
 \end{aligned}$$

- (a) For \vec{F}_{total} to be aligned with the fractured femur, $F_{\text{total},y} / F_{\text{total},x} = (\tan 160^\circ)(180^\circ - 20^\circ)$.

$$\tan 160^\circ = \frac{T \sin \alpha}{T(\cos \alpha + 2 \cos 160^\circ)} = \frac{\sin \alpha}{\cos \alpha + 2 \cos 160^\circ}$$

Solve for α using a graphing calculator. Graph each side of the above equation and find their intersection.

$$\alpha = 120^\circ, \text{ so } \theta = 180^\circ - \alpha = \boxed{60^\circ}.$$

- (b) The tension is equal to the weight of the 4.25 kg mass, $T = mg$.

$$\begin{aligned}
 |\vec{F}| &= \sqrt{F_{\text{total},x}^2 + F_{\text{total},y}^2} \\
 &= \sqrt{T^2(\cos \alpha + 2 \cos 160^\circ)^2 + T^2 \sin^2 \alpha} \\
 &= mg \sqrt{(\cos \alpha + 2 \cos 160^\circ)^2 + \sin^2 \alpha} \\
 &= (4.25 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2}\right) \sqrt{(\cos 120^\circ + 2 \cos 160^\circ)^2 + \sin^2 120^\circ} \\
 &= \boxed{106 \text{ N}}
 \end{aligned}$$

36. Taking x as positive in the direction of each mass's motion,

$$\begin{aligned}
 a_1 &= a_2 = a_3 = a \\
 m_1 a &= T_1 \\
 m_2 a &= T_2 - T_1 \\
 m_3 a &= m_3 g - T_2 \\
 m_1 a + m_2 a + m_3 a &= T_1 + (T_2 - T_1) + (m_3 g - T_2) \\
 (m_1 + m_2 + m_3) a &= m_3 g
 \end{aligned}$$

So the masses move like a single mass $(m_1 + m_2 + m_3)$ under acceleration m_3g .

$$a = \frac{m_3g}{m_1 + m_2 + m_3} = \frac{(3.0\text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)}{1.0\text{ kg} + 2.0\text{ kg} + 3.0\text{ kg}} = \boxed{4.9 \text{ m/s}^2}$$

37. The positive axis is along the line of the string and points in the downward direction from the hanging mass.

M = large mass

m = small mass

$$\sum F = (T - Mg \sin \theta) + (mg - T) = (m + M)a$$

$$(m + M)a = mg - Mg \sin q$$

$$a = \frac{g(m - M \sin \theta)}{m + M} = \frac{\left(9.81 \frac{\text{m}}{\text{s}^2}\right)[3.2\text{ kg} - (5.7\text{ kg})\sin 35^\circ]}{3.2\text{ kg} + 5.7\text{ kg}} = -0.076 \text{ m/s}^2$$

The magnitude is $\boxed{0.076 \text{ m/s}^2}$; the direction is upward.

38. The x -axis is along the line of the string and points in the downward direction from the hanging mass.

M = large mass

m = small mass

$$\sum F_x = (T - Mg \sin \theta) + (mg - T) = (m + M)a$$

$$(m + M)a = mg - Mg \sin q$$

$$a = \frac{g(m - M \sin \theta)}{m + M} = \frac{\left(9.81 \frac{\text{m}}{\text{s}^2}\right)[4.2\text{ kg} - (5.7\text{ kg})\sin 30^\circ]}{4.2\text{ kg} + 5.7\text{ kg}}$$

$$\vec{a} = \boxed{1.3 \text{ m/s}^2 \text{ downward}}$$

39. Using the results from Problem 36,

$$a = \frac{m_3g}{m_1 + m_2 + m_3} = \frac{1}{2}g = 4.905 \frac{\text{m}}{\text{s}^2}. \text{ So,}$$

$$T_1 = m_1a = (1.0\text{ kg})\left(4.905 \frac{\text{m}}{\text{s}^2}\right) = \boxed{4.9 \text{ N}}$$

and

$$T_2 = m_2a + T_1 = (2.0\text{ kg})\left(4.905 \frac{\text{m}}{\text{s}^2}\right) + 4.905 \text{ N} = \boxed{15 \text{ N}}$$

40. For the mass on the table:

The x -axis is along the string.

The y -axis is perpendicular to the tabletop.

For the mass on the string:

The x -axis is along the string and points downward.

The y -axis points away from the table.

m_1 = mass on the table

m_2 = mass on the string

$$\begin{aligned}
 \sum F_{1x} &= T = m_1 a \\
 + \sum F_{2x} &= m_2 g - T = m_2 a \\
 \hline
 &= T + m_2 g - T = (m_1 + m_2) a \\
 a &= \left(\frac{m_2}{m_1 + m_2} \right) g = \left(\frac{2.80 \text{ kg}}{3.50 \text{ kg} + 2.80 \text{ kg}} \right) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) = 4.36 \frac{\text{m}}{\text{s}^2}
 \end{aligned}$$

$$(a) \quad T = m_1 a = (3.50 \text{ kg}) \left(4.36 \frac{\text{m}}{\text{s}^2} \right) = 15.3 \text{ N}$$

$$15.3 \text{ N} < m_2 g = (2.80 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) = 27.5 \text{ N}$$

The tension in the string is less than the weight of the hanging mass.

$$(b) \quad a = \boxed{4.36 \text{ m/s}^2}, \quad T = \boxed{15.3 \text{ N}}$$

41. The x-axis is in the direction of the force.

$$m_1 = 1.5 \text{ kg}$$

$$m_2 = 0.93 \text{ kg}$$

$$\begin{aligned}
 \sum F_{1x} &= F - T = m_1 a \\
 + \sum F_{2x} &= T = m_2 a \\
 \hline
 &F = (m_1 + m_2) a
 \end{aligned}$$

$$(a) \quad a = \frac{F}{m_1 + m_2} = \frac{6.4 \text{ N}}{1.5 \text{ kg} + 0.93 \text{ kg}} = \boxed{2.6 \text{ m/s}^2}$$

$$(b) \quad T = m_2 a = \left(\frac{m_2}{m_1 + m_2} \right) F = \left(\frac{0.93 \text{ kg}}{1.5 \text{ kg} + 0.93 \text{ kg}} \right) (6.4 \text{ N}) = \boxed{2.4 \text{ N}}$$

- (c) If the mass of block 1 is increased, the acceleration of the system, and hence of block 2, will decrease. Therefore, the tension will decrease.

42. (a) To keep the smaller bucket from moving, you must add 47 N of downward force to it. There is no acceleration. The tension in the rope is equal to the weight of the 110 N bucket of sand.

- (b) The y-axis points downward relative to the heavier bucket's position.

$$m_1 = \text{mass of heavier bucket}$$

$$m_2 = \text{mass of lighter bucket}$$

$$m_1 = \frac{W_1}{g} = \frac{110 \text{ N}}{9.81 \frac{\text{m}}{\text{s}^2}} = 11.2 \text{ kg}$$

$$m_2 = \frac{W_2}{g} = \frac{63 \text{ N}}{9.81 \frac{\text{m}}{\text{s}^2}} = 6.42 \text{ kg}$$

$$\begin{aligned}
 \sum F_1 &= -T + W_1 = m_1 a \\
 \sum F_2 &= T - W_2 = m_2 a \\
 \hline
 &W_1 - W_2 = (m_1 + m_2) a
 \end{aligned}$$

$$a = \frac{W_1 - W_2}{m_1 + m_2} = \frac{110 \text{ N} - 63 \text{ N}}{11.2 \text{ kg} + 6.42 \text{ kg}} = 2.67 \frac{\text{m}}{\text{s}^2}$$

$$T = m_2 a + W_2 = (6.42 \text{ kg}) \left(2.67 \frac{\text{m}}{\text{s}^2} \right) + 63 \text{ N} = \boxed{80 \text{ N}}$$

- (c) The heavier bucket is at rest on the ground. The lighter bucket is hanging in the air. The tension is equal to the weight of the hanging bucket, $\boxed{63 \text{ N}}$.

43. The centripetal acceleration is $a_{\text{cp}} = v^2 / r$.

The magnitude of the force of static friction is equal to the magnitude of the centripetal force.

$$f_s = F_{\text{cp}} = m a_{\text{cp}} = \frac{m v^2}{r} = \frac{(1200 \text{ kg}) \left(15 \frac{\text{m}}{\text{s}} \right)^2}{57 \text{ m}} = \boxed{4.7 \text{ kN}}$$

44. $a_{\text{cp}} = 52,000g = \frac{v^2}{r}$

$$v = \sqrt{r a_{\text{cp}}} = \sqrt{52,000 g r} = \sqrt{52,000 \left(9.81 \frac{\text{m}}{\text{s}^2} \right) (0.075 \text{ m})} = \boxed{200 \text{ m/s}}$$

45. $a_{\text{cp}} = 9g = \frac{v^2}{r}$

$$v = \sqrt{r a_{\text{cp}}} = \sqrt{9 g r} = \sqrt{9 \left(9.81 \frac{\text{m}}{\text{s}^2} \right) (15 \text{ m})} = \boxed{36 \text{ m/s}}$$

46. The y-axis points upward and the x-axis points toward the center of the curve.

$$\sum F_y = N \cos \theta - mg = 0$$

$$N = \frac{mg}{\cos \theta}$$

$$\sum F_x = N \sin \theta = m a_{\text{cp}}$$

$$a_{\text{cp}} = \frac{N \sin \theta}{m} = \left(\frac{mg}{\cos \theta} \right) \left(\frac{\sin \theta}{m} \right)$$

$$a_{\text{cp}} = g \tan \theta = \frac{v^2}{r}$$

$$r = \frac{v^2}{g \tan \theta} = \frac{\left(24.0 \frac{\text{m}}{\text{s}} \right)^2}{\left(9.81 \frac{\text{m}}{\text{s}^2} \right) \tan 30.0^\circ} = \boxed{102 \text{ m}}$$

$$\begin{aligned}
 47. \quad \sum F_y &= T - W = ma_{cp} \\
 T &= W + ma_{cp} \\
 &= mg + ma_{cp} \\
 &= m(g + a_{cp}) \\
 &= m \left(g + \frac{v^2}{r} \right) \\
 &= (61 \text{ kg}) \left[9.81 \frac{\text{m}}{\text{s}^2} + \frac{(2.4 \frac{\text{m}}{\text{s}})^2}{6.5 \text{ m}} \right] \\
 &= \boxed{650 \text{ N}}
 \end{aligned}$$

$$\begin{aligned}
 48. \quad \sum F_y &= T - W = ma_{cp} \\
 T &= m(g + a_{cp}) = m \left(g + \frac{v^2}{r} \right)
 \end{aligned}$$

If Jill's speed is doubled, the tension will increase by $m(3v^2/r) = 160 \text{ N}$. If Jill's mass is doubled, the tension will double.

49. (a) At the top of the Ferris wheel:

$$\begin{aligned}
 \sum F_y &= N - mg = ma_y = -m \frac{v^2}{r} \\
 N &= m \left(g - \frac{v^2}{r} \right)
 \end{aligned}$$

The normal force exerted on a rider is less than that rider's weight, which results in an apparent weight less than the rider's actual weight.

At the bottom of the Ferris wheel:

$$\begin{aligned}
 \sum F_y &= N - mg = ma_y = m \frac{v^2}{r} \\
 N &= m \left(g + \frac{v^2}{r} \right)
 \end{aligned}$$

The normal force exerted on a rider is greater than that rider's weight, which results in an apparent weight greater than the rider's actual weight.

$$(b) \quad v = \frac{C}{t} = \frac{2\pi r}{t} = \frac{2\pi(7.2 \text{ m})}{28 \text{ s}} = 1.616 \frac{\text{m}}{\text{s}}$$

$$W_{\text{top}} = m \left(g - \frac{v^2}{r} \right) = (55 \text{ kg}) \left[9.81 \frac{\text{m}}{\text{s}^2} - \frac{(1.616 \frac{\text{m}}{\text{s}})^2}{7.2 \text{ m}} \right] = \boxed{520 \text{ N}}$$

$$W_{\text{bot}} = m \left(g + \frac{v^2}{r} \right) = (55 \text{ kg}) \left[9.81 \frac{\text{m}}{\text{s}^2} + \frac{(1.616 \frac{\text{m}}{\text{s}})^2}{7.2 \text{ m}} \right] = \boxed{560 \text{ N}}$$

50. $\sum F_y = N - mg = -ma_{cp}$

$$N = m \left(g - \frac{v^2}{r} \right) = (67 \text{ kg}) \left[9.81 \frac{\text{m}}{\text{s}^2} - \frac{\left(12 \frac{\text{m}}{\text{s}} \right)^2}{35 \text{ m}} \right] = \boxed{380 \text{ N}} = W_{\text{apparent}}$$

51. $\sum F_y = N - mg = -ma_{cp}$

$$N = m \left(g - \frac{v^2}{r} \right) = 0$$

$$v^2 = gr$$

$$v = \sqrt{gr}$$

$$= \sqrt{\left(9.81 \frac{\text{m}}{\text{s}^2} \right) (35 \text{ m})}$$

$$= \boxed{19 \text{ m/s}}$$

52. (a) $\sum F = ma_{cp} - mg = 0$

$$a_{cp} = \frac{v^2}{r} = g$$

$$v = \sqrt{gr} = \sqrt{\left(9.81 \frac{\text{m}}{\text{s}^2} \right) (1.1 \text{ m})} = \boxed{3.3 \text{ m/s}}$$

(b) The answer is independent of mass.

53. From Problem 29,

$F = 2T \cos \theta$ and $T = mg$, so

$$F = 2mg \cos \theta$$

$$m = \frac{F}{2g \cos \theta} = \frac{32 \text{ N}}{2 \left(9.81 \frac{\text{m}}{\text{s}^2} \right) \cos 30^\circ} = \boxed{1.9 \text{ kg}}$$

54. $a_{cp} = \frac{v^2}{r} = \frac{\left(77 \frac{\text{m}}{\text{s}} \right)^2}{0.042 \text{ m}} = \boxed{1.4 \times 10^5 \text{ m/s}^2}$

55. The block is being pulled at constant speed. This implies that the acceleration is zero. The x -axis points opposite the direction of motion.

$$\sum F_x = f_k - F_s = 0$$

$$f_k = \mu_k N = \mu_k mg$$

$$F_s = kx = f_k$$

$$\mu_k = \frac{kx}{mg} = \frac{\left(85.0 \frac{\text{N}}{\text{m}} \right) (0.0620 \text{ m})}{(3.85 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right)} = \boxed{0.140}$$

56. The x -axis is along the direction of motion of the child.

$$\sum F_y = N - mg \cos \theta = 0$$

$$N = mg \cos \theta$$

$$\sum F_x = mg \sin \theta - f_k = ma$$

$$a = g \sin \theta - \frac{1}{m}(\mu_k N)$$

$$= g \sin \theta - \mu_k g \cos \theta$$

$$= g(\sin \theta - \mu_k \cos \theta)$$

$$= \left(9.81 \frac{\text{m}}{\text{s}^2}\right) [\sin 27.5^\circ - 0.415 \cos 27.5^\circ]$$

$$= \boxed{0.919 \text{ m/s}^2}$$

57. The x -axis points up and $x = 0$ is at the spring's equilibrium point.

$$\sum F_y = F_s - W = 0$$

$$0 = -kx - mg$$

$$m = \frac{-kx}{g}$$

$$= \frac{-(1800 \frac{\text{N}}{\text{m}})(-0.0375 \text{ m})}{9.81 \frac{\text{m}}{\text{s}^2}}$$

$$= \boxed{6.9 \text{ kg}}$$

58. The x -axis is along the direction of the applied force.

(a) $m_1 = 1.1 \text{ kg}$

$$m_2 = 1.92 \text{ kg}$$

$$\sum F_{1y} = N_1 - W_1 = 0$$

$$N_1 = m_1 g$$

$$\sum F_{2y} = N_2 - W_2 = 0$$

$$N_2 = m_2 g$$

$$\sum F_{1x} = F - T - f_1 = m_1 a$$

$$+ \sum F_{2x} = T - f_2 = m_2 a$$

$$F - f_1 - f_2 = (m_1 + m_2) a$$

$$a = \frac{F - \mu_k g(m_1 + m_2)}{m_1 + m_2} = \frac{9.4 \text{ N} - 0.24 \left(9.81 \frac{\text{m}}{\text{s}^2}\right)(1.1 \text{ kg} + 1.92 \text{ kg})}{1.1 \text{ kg} + 1.92 \text{ kg}} = \boxed{0.76 \text{ m/s}^2}$$

(b) $T = m_2 a + f_2 = m_2 a + \mu_k m_2 g = m_2 (a + \mu_k g) = (1.92 \text{ kg}) \left[0.76 \frac{\text{m}}{\text{s}^2} + 0.24 \left(9.81 \frac{\text{m}}{\text{s}^2}\right)\right] = \boxed{6.0 \text{ N}}$

59. The y -axis points up.

$$(a) \sum F_y = ma_{cp} = mg + T$$

$$T = m(a_{cp} - g) = m \left(\frac{v^2}{r} - g \right) = (3.25 \text{ kg}) \left[\frac{(3.23 \frac{\text{m}}{\text{s}})^2}{0.950 \text{ m}} - 9.81 \frac{\text{m}}{\text{s}^2} \right] = 3.81 \text{ N}$$

The magnitude of the tension at the top of the circle is 3.81 N.

$$(b) \sum F_y = ma_{cp} = T - mg$$

$$T = m(g + a_{cp}) = m \left(g + \frac{v^2}{r} \right) = (3.25 \text{ kg}) \left[9.81 \frac{\text{m}}{\text{s}^2} + \frac{(6.91 \frac{\text{m}}{\text{s}})^2}{0.950 \text{ m}} \right] = 195 \text{ N}$$

The magnitude of the tension at the bottom of the circle is 195 N.

60. The x -axis points up the incline, parallel to it.

$$(a) \sum F_y = N - mg \cos \theta = 0$$

$$N = mg \cos \theta$$

$$f_k = \mu_k N = \mu_k mg \cos \theta = 0.23(0.012 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) \cos 15^\circ = \boxed{0.026 \text{ N}}$$

The force of kinetic friction is directed down the incline.

$$(b) \sum F_x = f_s - mg \sin \theta = 0$$

$$f_s = mg \sin \theta = (0.012 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) \sin 15^\circ = \boxed{0.030 \text{ N}}$$

The force of static friction is directed up the incline.

61. (a) The x -axis points up the incline, parallel to it.

$$\sum F_y = N - mg \cos \theta = 0$$

$$N = mg \cos \theta$$

$$f_k = \mu_k N = 0.23(0.012 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) \cos 25^\circ = \boxed{0.025 \text{ N}}$$

The force of kinetic friction is directed down the incline.

(b) The force of kinetic friction has the same magnitude (0.025 N) and opposite direction (up the incline) from (a).

$$62. \sum F_x = F - f = ma$$

$$a = \frac{1}{m}(F - f)$$

$$f_{s, \max} = \mu_s N = (0.60)(22 \text{ N}) = 13.2 \text{ N}$$

$$f_k = \mu_k N = (0.40)(22 \text{ N}) = 8.8 \text{ N}$$

applied force	friction force	motion
0	0	at rest
5 N	5 N	at rest
10 N	10 N	at rest
15 N	8.8 N	accelerating
10 N	8.8 N	accelerating
8 N	8.8 N/8 N	decel./at rest
5 N	5 N	at rest

63. Place the x -axis up and to the right, parallel to the right wall of the wedge, and the y -axis up and to the left, parallel to the left wall of the wedge.

$$\sum F_x = F_1 - mg \sin 20^\circ = 0$$

$$\sum F_y = F_2 - mg \cos 20^\circ = 0$$

$$\begin{aligned} F_1 &= mg \sin 20^\circ \\ F_2 &= mg \cos 20^\circ \end{aligned}$$

64. The x -axis points upward, parallel to the plank, and the y -axis points up and to the left. $x = 0$ when the spring is unstretched. At maximum stretch with the box at rest,

$$\begin{aligned} \sum F_x &= -F_s + f_s - mg \sin 65^\circ = 0 \\ &= kx + \mu_s N - mg \sin 65^\circ = 0 \end{aligned}$$

$$\begin{aligned} \sum F_y &= N - mg \cos 65^\circ = 0 \\ N &= mg \cos 65^\circ \end{aligned}$$

$$kx + \mu_s mg \cos 65^\circ - mg \sin 65^\circ = 0$$

$$\begin{aligned} x &= \frac{mg \sin 65^\circ - \mu_s mg \cos 65^\circ}{k} \\ &= -\frac{mg(\sin 65^\circ - \mu_s \cos 65^\circ)}{k} \\ &= \frac{(2.0 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(\sin 65^\circ - 0.22 \cos 65^\circ)}{18 \frac{\text{N}}{\text{m}}} \\ &= \boxed{0.89 \text{ m}} \end{aligned}$$

65. (a) Let T be the tension in the slanting stretch of rope. Then $T \sin 45^\circ$ is the tension in the rope supporting mass B, and $T \cos 45^\circ$ is the tension in the rope pulling on mass A. But $\sin 45^\circ = \cos 45^\circ$, and so

$$f_{s \text{ on A}} = T \cos 45^\circ = T \sin 45^\circ = m_B g = (2.25 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right) = \boxed{22.1 \text{ N}}$$

which is below

$$f_{s, \max} = \mu_s m_A g = 0.320(8.50 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right) = 26.7 \text{ N}.$$

- (b) So long as mass A is heavy enough for $f_{s, \max} \geq 22.1 \text{ N}$, f_s is not affected by changes in mass A; f_s stays the same.

66. From the previous Problem, $f_{s \text{ on A}} = m_B g$ and

$$m_{B, \max} = \frac{f_{s, \max}}{g} = \frac{26.7 \text{ N}}{9.81 \frac{\text{m}}{\text{s}^2}} = \boxed{2.72 \text{ kg}}$$

67. (a) Greater; horizontal components of the two string tensions must be equal, and $\cos \theta_2 > \cos \theta_1$.

(b) $T_1 \cos \theta_1 = T_2 \cos \theta_2$

$$T_1 = T_2 \frac{\cos \theta_2}{\cos \theta_1} = (1.7 \text{ N}) \frac{\cos 32^\circ}{\cos 65^\circ} = \boxed{3.4 \text{ N}}$$

(c) $T_1 \sin \theta_1 + T_2 \sin \theta_2 = mg$

$$m = \frac{T_1 \sin \theta_1 + T_2 \sin \theta_2}{g} = \frac{(3.4 \text{ N}) \sin 65^\circ + (1.7 \text{ N}) \sin 32^\circ}{9.81 \text{ m/s}^2} = \boxed{0.41 \text{ kg}}$$

68. (a) At the top of the Ferris wheel

$$\sum F_y = N - mg = -m \frac{v^2}{r}$$

You feel “weightless” if $N = 0$; then $\frac{v^2}{r} = g$.

$$v = \frac{C}{t} = \frac{2\pi r}{t}$$

$$\frac{\left(\frac{2\pi r}{t}\right)^2}{r} = \frac{4\pi^2 r}{t^2} = g$$

$$t = \sqrt{\frac{4\pi^2 r}{g}} = \sqrt{\frac{4\pi^2 (7.2 \text{ m})}{9.81 \text{ m/s}^2}} = \boxed{5.4 \text{ s}}$$

- (b) The period is independent of the mass.

(c) $a = \frac{v^2}{r} = g = \boxed{9.81 \text{ m/s}^2}$. Direction is upward.

69. $\sum F_x = T \sin \theta = ma_{cp} = m \frac{v^2}{r}$

$$\sum F_y = T \cos \theta - mg = 0$$

$$T = \frac{mg}{\cos \theta} = \frac{mv^2}{r \sin \theta}$$

$$\tan \theta = \frac{v^2}{gr}$$

$$\theta = \tan^{-1} \frac{v^2}{gr} = \tan^{-1} \frac{\left(1.21 \frac{\text{m}}{\text{s}}\right)^2}{\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(0.44 \text{ m})} = \boxed{19^\circ}$$

$$T = \frac{mg}{\cos \theta} = \frac{(0.075 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)}{\cos 18.74^\circ} = \boxed{0.78 \text{ N}}$$

70. With constant speed, the cable tension is the same at both ends. So, for the cable,

$$\sum F_y = 2T \sin 22^\circ - mg = 0$$

$$T = \frac{mg}{2 \sin 22^\circ} = \frac{(14,000 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)}{2 \sin 22^\circ} = 1.83 \times 10^5 \text{ N}$$

Then, the force on the barge is

$$F = T \cos 22^\circ = (1.83 \times 10^5 \text{ N}) \cos 22^\circ = \boxed{170 \text{ kN}}$$

71. (a) For the top block to almost slip:

$$ma = f_{s, \max}$$

$$\sum F_y = N - mg = 0$$

$$a = \frac{f_{s, \max}}{m} = \frac{\mu_s mg}{m} = \mu_s g = 0.47 \left(9.81 \frac{\text{m}}{\text{s}^2}\right) = 4.6 \text{ m/s}^2$$

To give the system an acceleration a , $F = (m + M)a$. The acceleration that will cause the top block to slip is $a = \mu_s g$, so

$$F = (m + M)\mu_s g = (2.0 \text{ kg} + 5.0 \text{ kg})(0.47) \left(9.81 \frac{\text{m}}{\text{s}^2}\right) = \boxed{32 \text{ N}}$$

- (b) Since $F = (m + M)\mu_s g$, an increase in m will result in an increase in F .

72. Let the x -axis point in the direction of motion and the origin be at the end of the unstretched spring.

$$\sum F_y = N + F \sin \theta - W = 0$$

$$\sum F_x = F \cos \theta - f_k = 0$$

$$N = W - F \sin \theta$$

$$= W - (-kx) \sin \theta$$

$$= mg + kx \sin \theta$$

$$F \cos \theta = f_k = \mu_k N$$

$$\mu_k = \frac{F \cos \theta}{N}$$

$$\mu_k = \frac{-kx \cos \theta}{mg + kx \sin \theta}$$

$$= \frac{-(85 \frac{\text{N}}{\text{m}})(-0.021 \text{ m}) \cos 13^\circ}{(4.5 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right) + (85 \frac{\text{N}}{\text{m}})(-0.021 \text{ m}) \sin 13^\circ}$$

$$= \boxed{0.040}$$

73. $\sum F_y = T - W = Ma$

(a) $T = Ma + W = Ma + Mg = M(a + g) > W = Mg$

The tension is greater than the combined weight of the men.

(b) $T = M(a + g) = (172 \text{ kg})\left(1.10 \frac{\text{m}}{\text{s}^2} + 9.81 \frac{\text{m}}{\text{s}^2}\right) = \boxed{1.88 \text{ kN}}$

74. Place the x -axis along the direction of motion.

$$\sum F_y = N + T \sin \theta - W = 0$$

$$\sum F_x = T \cos \theta - f_k = 0$$

$$T \cos \theta = f_k = \mu_k N = \mu_k (W - T \sin \theta) = \mu_k W - T \mu_k \sin \theta$$

$$T(\cos \theta + \mu_k \sin \theta) = \mu_k W = \mu_k mg$$

$$T = \frac{\mu_k mg}{\cos \theta + \mu_k \sin \theta} = \frac{0.38(18 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)}{\cos 45^\circ + (0.38)\sin 45^\circ} = \boxed{69 \text{ N}}$$

$$N = W - T \sin \theta = mg - T \sin \theta = (18 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right) - (69 \text{ N})\sin 45^\circ = \boxed{130 \text{ N}}$$

75. (a) The coefficient of static friction can be determined from the information given. The force required to stretch the spring is equal in magnitude to the force of static friction between the wheel and the ground. If the wheel is not slipping, it is not moving relative to the point of contact between it and the ground.

- (b) Place the x -axis along the direction of motion of the toy bulldozer.

$$\sum F_y = N - W = 0$$

$$N = W = mg$$

$$\sum F_x = -kx + f_s = 0$$

$$f_s = \mu_s N = kx$$

$$\mu_s = \frac{kx}{N} = \frac{kx}{mg} = \frac{\left(13 \frac{\text{N}}{\text{m}}\right)(2.0 \text{ m})}{(3.0 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)} = \boxed{0.88}$$

76. (a) $\sum F_y = 2T \sin \theta - mg = 0$

$$T = \frac{mg}{2 \sin \theta} = \frac{(1.0 \times 10^{-4} \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)}{2 \sin 5.2^\circ} = \boxed{5.4 \text{ mN}}$$

- (b) The tension would be greater than that found in part (a) because the vertical component of the tension is less, thus requiring a larger total tension to compensate.

77. For the cereal box to just barely stay in place,

$$\sum F_y = f_s - W = 0$$

$$\mu_s N - mg = 0$$

$$\text{Also, } \sum F_x = N = ma, \text{ so that}$$

$$\mu_s ma - mg = 0$$

$$a = \frac{g}{\mu_s} = \frac{9.81 \frac{\text{m}}{\text{s}^2}}{0.38} = \boxed{26 \text{ m/s}^2}$$

78. (a) $\sum F_y = 2T \sin \theta - F = 0$

$$F = 2T \sin \theta = 2(2.7 \text{ N})\sin 4.1^\circ = \boxed{0.39 \text{ N}}$$

- (b) The required force would be less than that in (a) because the vertical component of the tension decreases with decreasing angle.

79. Convert mi/h to m/s.

$$\left(\frac{25 \text{ mi}}{\text{h}}\right)\left(\frac{1609 \text{ m}}{\text{mi}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 11.2 \frac{\text{m}}{\text{s}}$$

$$\sum F_y = T \cos \theta - W = 0$$

$$T = \frac{W}{\cos \theta} = \frac{mg}{\cos \theta}$$

$$F_{\text{cp}} = T \sin \theta$$

$$= \left(\frac{mg}{\cos \theta}\right) \sin \theta$$

$$= mg \tan \theta$$

$$F_{\text{cp}} = ma_{\text{cp}}$$

$$mg \tan \theta = m \frac{v^2}{r}$$

Solve for θ .

$$\tan \theta = \frac{v^2}{gr}$$

$$\theta = \tan^{-1} \left(\frac{v^2}{gr} \right) = \tan^{-1} \left[\frac{\left(11.2 \frac{\text{m}}{\text{s}} \right)^2}{\left(9.81 \frac{\text{m}}{\text{s}^2} \right) (95 \text{ m})} \right] = \boxed{7.7^\circ}$$

The mass of the dice drops out of the equations.

- 80.
- T_1
- = tension of the
- 15°
- rope

 T_2 = tension of the 30° rope

$$\sum F_y = T_1 \sin \theta_1 + T_2 \sin \theta_2 - W = 0$$

$$\sum F_x = T_1 \cos \theta_1 - T_2 \cos \theta_2 = 0$$

$$T_1 = T_2 \left(\frac{\cos \theta_2}{\cos \theta_1} \right)$$

$$T_1 \sin \theta_1 + T_2 \sin \theta_2 = mg$$

$$T_2 \left(\frac{\cos \theta_2}{\cos \theta_1} \right) \sin \theta_1 + T_2 \sin \theta_2 = mg$$

$$T_2 (\cos \theta_2 \tan \theta_1 + \sin \theta_2) = mg$$

$$T_2 = \frac{mg}{\cos \theta_2 \tan \theta_1 + \sin \theta_2} = \frac{(50.0 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right)}{\cos 35^\circ \tan 15^\circ + \sin 35^\circ} = \boxed{620 \text{ N}}$$

$$T_1 = T_2 \left(\frac{\cos \theta_2}{\cos \theta_1} \right) = (620 \text{ N}) \left(\frac{\cos 35^\circ}{\cos 15^\circ} \right) = \boxed{570 \text{ N}}$$

$$81. \sum F_y = F_{\text{lift}} \cos \theta - W = 0$$

$$\sum F_x = F_{\text{lift}} \sin \theta = ma_{\text{cp}}$$

$$F_{\text{lift}} = \frac{ma_{\text{cp}}}{\sin \theta}$$

$$0 = F_{\text{lift}} \cos \theta - mg = \left(\frac{ma_{\text{cp}}}{\sin \theta} \right) \cos \theta - mg = \frac{ma_{\text{cp}}}{\tan \theta} - mg$$

$$\tan \theta = \frac{ma_{\text{cp}}}{mg}$$

$$\theta = \tan^{-1} \left(\frac{a_{\text{cp}}}{g} \right) = \tan^{-1} \left(\frac{v^2}{gr} \right) = \tan^{-1} \left[\frac{\left(380 \times 10^3 \frac{\text{m}}{\text{h}} \right)^2 \left(\frac{1 \text{ h}}{3600 \text{ s}} \right)^2}{\left(9.81 \frac{\text{m}}{\text{s}^2} \right) (2100 \text{ m})} \right] = \boxed{28^\circ}$$

82. (a) F_{min} is the smallest value of F that prevents the block from sliding down the incline. F_{max} is the largest value of F that can act on the block without causing it to slide up the incline.

- (b) Let the x -axis be parallel to and up the incline.

$$\sum F_y = N - mg \cos \theta = 0, \text{ so } N = mg \cos \theta.$$

$$\sum F_x = F_{\text{min}} + f_s - mg \sin \theta = 0$$

$$\begin{aligned} F_{\text{min}} &= mg \sin \theta - f_s = mg \sin \theta - \mu_s N = mg \sin \theta - \mu_s mg \cos \theta \\ &= (3.1 \text{ kg})(9.81 \text{ m/s}^2)[0.707 - (0.50)(0.707)] \\ &= \boxed{11 \text{ N}} \end{aligned}$$

- (c) On the verge of sliding up the incline

$$\sum F_x = F_{\text{max}} - f_s - mg \sin \theta = 0$$

$$\begin{aligned} F_{\text{max}} &= f_s + mg \sin \theta = \mu_s N + mg \sin \theta = \mu_s mg \cos \theta + mg \sin \theta \\ &= (3.1 \text{ kg})(9.81 \text{ m/s}^2)[(0.50)(0.707) + (0.707)] \\ &= \boxed{32 \text{ N}} \end{aligned}$$

$$83. \sum F_x = T - W \sin \theta = 0$$

$$\boxed{T = mg \sin \theta}$$

With $\theta = 0^\circ$, the climber is lying on a level surface and does not hang from the rope at all. Therefore,

$$T = mg \sin 0^\circ = 0.$$

With $\theta = 90^\circ$, the climber is against a vertical wall and all the climber's weight is hanging from the rope.

Therefore, $T = mg \sin 90^\circ = mg$.

$$\begin{aligned}
 84. \quad \sum F_y &= N - W = 0 \\
 \sum F_x &= f_s = ma_{\text{cp}} \\
 N &= W = mg \\
 f_s &= \mu_s N \\
 \mu_s mg &= ma_{\text{cp}} \\
 \mu_s &= \frac{a_{\text{cp}}}{g} \\
 &= \frac{v^2}{rg} \\
 &= \frac{\left(1.9 \frac{\text{m}}{\text{s}}\right)^2}{(2.1 \text{ m})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)} \\
 &= \boxed{0.18}
 \end{aligned}$$

$$\begin{aligned}
 85. \quad &\text{Top block:} \\
 \sum F_y &= N_m - W_m = 0 \\
 N_m &= W_m = mg \\
 \sum F_x &= f_m = ma \\
 f_m &= \mu_s N_m = \mu_s mg \\
 &\text{Bottom block:} \\
 \sum F_y &= N_M - W_M - W_m = 0 \\
 N_M &= W_M + W_m = (M + m)g \\
 \sum F_x &= F - f_m - f_M = 0 \\
 F &= f_m + f_M = \mu_s mg + \mu_s (M + m)g = \boxed{\mu_s g(2m + M)}
 \end{aligned}$$

86. From Problem 37,

$$a = \frac{m_3 g}{m_1 + m_2 + m_3}$$

and so

$$\begin{aligned}
 T_1 &= m_1 a = \frac{m_1 m_3 g}{m_1 + m_2 + m_3} \\
 T_2 &= T_1 + m_2 a = \frac{(m_1 + m_2) m_3 g}{m_1 + m_2 + m_3}
 \end{aligned}$$

87. am = the mass of the rope on the table

bm = the mass of the rope hanging

$$am + bm = m$$

$$a + b = 1$$

The force pulling the hanging section of the rope is its weight, $W_\beta = \beta mg$.

The force of static friction acting on the section of rope on the table must be at least as great as W_β to prevent the rope from slipping.

$$f_s = \mu_s N$$

$$= \mu_s amg$$

$$\mu_s amg = \beta mg$$

$$\beta = \mu_s \alpha = \text{the hanging fraction of rope}$$

$$\beta = \mu_s (1 - \beta)$$

$$= \mu_s - \mu_s \beta$$

$$\beta(1 + \mu_s) = \mu_s$$

$$\beta = \boxed{\frac{\mu_s}{1 + \mu_s}}$$

88. At one end of the string, $T = Mg$; at the other end, $T = F_m$. And if M remains at rest, the motion of m is circular and $F_m = F_{cp}$.

$$Mg = F_{cp}$$

$$= ma_{cp}$$

$$= m \frac{v^2}{r}$$

$$M = \boxed{\frac{mv^2}{rg}}$$

89. $\sum F_y = N - W - F \sin \theta = 0$

$$\sum F_x = F \cos \theta - f_s = ma$$

$$f_s = \mu_s N = \mu_s (mg + F \sin \theta)$$

(a) $0 = F \cos \theta - f_s$
 $= F \cos \theta - \mu_s mg - \mu_s F \sin \theta$
 $= F(\cos \theta - \mu_s \sin \theta) - \mu_s mg$

$$F = \boxed{\frac{\mu_s mg}{\cos \theta - \mu_s \sin \theta}}$$

- (b) Set $\cos \theta - \mu_s \sin \theta > 0$ to determine the limit of μ_s .

$$\cos \theta - \mu_s \sin \theta > 0$$

$$\mu_s \sin \theta < \cos \theta$$

$$\mu_s < \frac{\cos \theta}{\sin \theta} = \frac{1}{\tan \theta}$$

If $\mu_s = \frac{1}{\tan \theta}$, $F = \infty$.

If $\mu_s > \frac{1}{\tan \theta}$, $F < 0$.

90. (a) $a_{cp} = \frac{v^2}{r} = \frac{\left[\left(\frac{25 \text{ mi}}{\text{h}} \right) \left(\frac{1609 \text{ m}}{\text{mi}} \right) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) \right]^2}{12 \text{ m}} = \boxed{10 \text{ m/s}^2}$

- (b) Taking
- T
- as the chain tension,

$$T \sin \theta = ma_{\text{cp}}$$

$$T \cos \theta = mg$$

$$\tan \theta = \frac{a_{\text{cp}}}{g}$$

$$\theta = \tan^{-1} \left(\frac{a_{\text{cp}}}{g} \right)$$

$$= \tan^{-1} \left(\frac{10.4 \frac{\text{m}}{\text{s}^2}}{9.81 \frac{\text{m}}{\text{s}^2}} \right)$$

$$= \boxed{47^\circ}$$

- (c)
- The angle corresponds to a ratio of vertical to horizontal force. Both of those forces are proportional to m , so that m drops out of the ratio.

91. The
- x
- axis is along the motion of the conveyor belt.

$$\sum F_y = N - W = 0$$

$$N = mg$$

$$\sum F_x = f_k = ma$$

$$a = \frac{f_k}{m} = \frac{\mu_k N}{m} = \mu_k g = 0.780 \left(9.81 \frac{\text{m}}{\text{s}^2} \right) = 7.65 \frac{\text{m}}{\text{s}^2}$$

$$(a) \quad v = at = 1.25 \frac{\text{m}}{\text{s}}$$

$$t = \frac{v}{a} = \frac{1.25 \frac{\text{m}}{\text{s}}}{7.65 \frac{\text{m}}{\text{s}^2}} = \boxed{0.163 \text{ s}}$$

$$(b) \quad x = \frac{1}{2} at^2 = \frac{1}{2} \left(7.65 \frac{\text{m}}{\text{s}^2} \right) (0.163 \text{ s})^2 = 0.102 \text{ m}$$

The box has moved 0.102 m.

92. The
- x
- axis is along the direction of motion.

$$\sum F_y = N - W = 0$$

$$N = mg$$

$$\sum F_x = F - f_k = ma$$

$$F - \mu_k mg = ma$$

$$F_1 = 75 \text{ N}$$

$$F_2 = 81 \text{ N}$$

$$a_1 = 0.50 \frac{\text{m}}{\text{s}^2}$$

$$a_2 = 0.75 \frac{\text{m}}{\text{s}^2}$$

There are two equations and two unknowns.

Subtract I from II.

$$F_2 - F_1 = m(a_2 - a_1)$$

$$m = \frac{F_2 - F_1}{a_2 - a_1} = \frac{81 \text{ N} - 75 \text{ N}}{0.75 \frac{\text{m}}{\text{s}^2} - 0.50 \frac{\text{m}}{\text{s}^2}} = \boxed{24 \text{ kg}}$$

$$\mu_k mg = F_1 - ma_1$$

$$\mu_k = \frac{F_1}{mg} - \frac{a_1}{g} = \frac{75 \text{ N}}{(24 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)} - \frac{0.50 \frac{\text{m}}{\text{s}^2}}{9.81 \frac{\text{m}}{\text{s}^2}} = \boxed{0.27}$$