SOLUTIONS MANUAL

CHAPTER 1

1. The energy contained in a volume dV is

$$U(\nu,T)dV = U(\nu,T)r^2 dr \sin\theta d\theta d\varphi$$

when the geometry is that shown in the figure. The energy from this source that emerges through a hole of area dA is

$$dE(\nu,T) = U(\nu,T)dV \frac{dA\cos\theta}{4\pi r^2}$$

The total energy emitted is

$$\begin{aligned} dE(\nu,T) &= \int_0^{c\Delta t} dr \int_0^{\pi/2} d\theta \int_0^{2\pi} d\varphi U(\nu,T)\sin\theta\cos\theta \frac{dA}{4\pi} \\ &= \frac{dA}{4\pi} 2\pi c\Delta t U(\nu,T) \int_0^{\pi/2} d\theta \sin\theta\cos\theta \\ &= \frac{1}{4} c\Delta t dA U(\nu,T) \end{aligned}$$

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By definition of the emissivity, this is equal to $E\Delta t dA$. Hence

$$E(\nu,T) = \frac{c}{4} U(\nu,T)$$

2. We have

$$w(\lambda,T) = U(\nu,T)\,|d\nu/d\lambda| = U(\frac{c}{\lambda})\frac{c}{\lambda^2} = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}$$

This density will be maximal when $dw(\lambda,T)/d\lambda = 0$. What we need is

$$\frac{d}{d\lambda}\left(\frac{1}{\lambda^5}\frac{1}{e^{A/\lambda}-1}\right) = (-5\frac{1}{\lambda^6} - \frac{1}{\lambda^5}\frac{e^{A/\lambda}}{e^{A/\lambda}-1}(-\frac{A}{\lambda^2}))\frac{1}{e^{A/\lambda}-1} = 0$$

Where $A = hc/kT$. The above implies that with $x = A/\lambda$, we must have

$$5 - x = 5e^{-x}$$

A solution of this is $x = 4.965$ so that

$$\lambda_{max} T = \frac{hc}{4.965k} = 2.898 \times 10^{-3} m$$

In example 1.1 we were given an estimate of the sun's surface temperature as 6000 K. From this we get

$$\lambda_{max}^{sun} = \frac{28.98 \times 10^{-4} mK}{6 \times 10^{3} K} = 4.83 \times 10^{-7} m = 483 nm$$

3. The relationship is

$$h\nu = K + W$$

where K is the electron kinetic energy and W is the work function. Here

$$h\nu = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34} J\,s)(3 \times 10^{8} m/s)}{350 \times 10^{-9} m} = 5.68 \times 10^{-19} J = 3.55 eV$$

With K = 1.60 eV, we get W = 1.95 eV

4. We use

$$\frac{hc}{\lambda_1} - \frac{hc}{\lambda_2} = K_1 - K_2$$

since W cancels. From ;this we get

$$h = \frac{1}{c} \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1} (K_1 - K_2) =$$
$$= \frac{(200 \times 10^{-9} m)(258 \times 10^{-9} m)}{(3 \times 10^{8} m/s)(58 \times 10^{-9} m)} \times (2.3 - 0.9) eV \times (1.60 \times 10^{-19}) J/eV$$
$$= 6.64 \times 10^{-34} J\,s$$

5. The maximum energy loss for the photon occurs in a head-on collision, with the photon scattered backwards. Let the incident photon energy be $h\nu$, and the backward-scattered photon energy be $h\nu'$. Let the energy of the recoiling proton be E. Then its recoil momentum is obtained from $E = \sqrt{p^2 c^2 + m^2 c^4}$. The energy conservation equation reads

$$h\nu + mc^2 = h\nu' + E$$

and the momentum conservation equation reads

$$\frac{h\nu}{c} = -\frac{h\nu'}{c} + p$$

that is

$$h\nu = -h\nu' + pc$$

We get $E + pc - mc^2 = 2h\nu$ from which it follows that

$$p^2c^2 + m^2c^4 = (2h\nu - pc + mc^2)^2$$

so that

$$pc = \frac{4h^2\nu^2 + 4h\nu mc^2}{4h\nu + 2mc^2}$$

The energy loss for the photon is the kinetic energy of the proton $K = E - mc^2$. Now $h\nu$ = 100 MeV and mc^2 = 938 MeV, so that

$$pc = 182 MeV$$

and

$$E - mc^2 = K = 17.6 MeV$$

6. Let $h\nu$ be the incident photon energy, $h\nu'$ the final photon energy and p the outgoing electron momentum. Energy conservation reads

$$h\nu + mc^2 = h\nu' + \sqrt{p^2c^2 + m^2c^4}$$

We write the equation for momentum conservation, assuming that the initial photon moves in the x –direction and the final photon in the y-direction. When multiplied by c it read

$$\mathbf{i}(h\nu) = \mathbf{j}(h\nu') + (\mathbf{i}p_xc + \mathbf{j}p_yc)$$

Hence $p_xc = h\nu; p_yc = -h\nu'$. We use this to rewrite the energy conservation equation as follows:

$$(h\nu + mc^2 - h\nu')^2 = m^2c^4 + c^2(p_x^2 + p_y^2) = m^2c^4 + (h\nu)^2 + (h\nu')^2$$

From this we get

$$h\nu' = h\nu\left(\frac{mc^2}{h\nu + mc^2}\right)$$

We may use this to calculate the kinetic energy of the electron

$$K = h\nu - h\nu' = h\nu\left(1 - \frac{mc^2}{h\nu + mc^2}\right) = h\nu\frac{h\nu}{h\nu + mc^2}$$

$$= \frac{(100keV)^2}{100keV + 510keV} = 16.4keV$$

Also

$$\mathbf{p}c = \mathbf{i}(100keV) + \mathbf{j}(-83.6keV)$$

which gives the direction of the recoiling electron.

7. The photon energy is

$$h\nu = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}J.s)(3\times10^8 m/s)}{3\times10^6\times10^{-9}m} = 6.63\times10^{-17}J$$

$$= \frac{6.63\times10^{-17}J}{1.60\times10^{-19}J/eV} = 4.14\times10^{-4}MeV$$

The momentum conservation for collinear motion (the collision is head on for maximum energy loss), when squared, reads

$$\left(\frac{h\nu}{c}\right)^2 + p^2 + 2\left(\frac{h\nu}{c}\right)p\eta_i = \left(\frac{h\nu'}{c}\right)^2 + p'^2 + 2\left(\frac{h\nu'}{c}\right)p'\eta_f$$

Here $\eta_i = \pm1$, with the upper sign corresponding to the photon and the electron moving in the same/opposite direction, and similarly for η_f. When this is multiplied by c^2 we get

$$(h\nu)^2 + (pc)^2 + 2(h\nu)pc\eta_i = (h\nu')^2 + (p'c)^2 + 2(h\nu')p'c\eta_f$$

The square of the energy conservation equation, with E expressed in terms of momentum and mass reads

$$(h\nu)^2 + (pc)^2 + m^2c^4 + 2Eh\nu = (h\nu')^2 + (p'c)^2 + m^2c^4 + 2E'h\nu'$$

After we cancel the mass terms and subtracting, we get

$$h\nu(E - \eta_i pc) = h\nu'(E' - \eta_f p'c)$$

From this can calculate $h\nu'$ and rewrite the energy conservation law in the form

$$E - E' = h\nu\left(\frac{E - \eta_i pc}{E' - p'c\eta_f} - 1\right)$$

The energy loss is largest if $\eta_i = -1; \eta_f = 1$. Assuming that the final electron momentum is not very close to zero, we can write $E + pc = 2E \quad and \quad E' - p'c = \frac{(mc^2)^2}{2E'}$ so that

$$E - E' = h\nu\left(\frac{2E \times 2E'}{(mc^2)^2}\right)$$

It follows that $\frac{1}{E'} = \frac{1}{E} + 16h\nu$ with everything expressed in MeV. This leads to *E'* =(100/1.64)=61 MeV and the energy loss is 39MeV.

8.We have λ' = 0.035 x 10^{-10} m, to be inserted into

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos 60^0) = \frac{h}{2m_e c} = \frac{6.63 \times 10^{-34} J.s}{2 \times (0.9 \times 10^{-30} kg)(3 \times 10^8 m/s)} = 1.23 \times 10^{-12} m$$

Therefore $\lambda = \lambda$' = (3.50-1.23) x 10^{-12} m = 2.3 x 10^{-12} m.

The energy of the X-ray photon is therefore

$$h\nu = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} J.s)(3 \times 10^8 m/s)}{(2.3 \times 10^{-12} m)(1.6 \times 10^{-19} J/eV)} = 5.4 \times 10^5 eV$$

9. With the nucleus initially at rest, the recoil momentum of the nucleus must be equal and opposite to that of the emitted photon. We therefore have its magnitude given by $p = h\nu / c$, where $h\nu = 6.2 \quad MeV$. The recoil energy is

$$E = \frac{p^2}{2M} = h\nu\frac{h\nu}{2Mc^2} = (6.2 MeV)\frac{6.2 MeV}{2 \times 14 \times (940 MeV)} = 1.5 \times 10^{-3} MeV$$

10. The formula $\lambda = 2a \sin\theta / n$ implies that $\lambda / \sin\theta \le 2a/3$. Since $\lambda = h/p$ this leads to $p \ge 3h / 2a \sin\theta$, which implies that the kinetic energy obeys

$$K = \frac{p^2}{2m} \ge \frac{9h^2}{8ma^2 \sin^2\theta}$$

Thus the minimum energy for electrons is

$$K = \frac{9(6.63 \times 10^{-34} J.s)^2}{8(0.9 \times 10^{-30} kg)(0.32 \times 10^{-9} m)^2 (1.6 \times 10^{-19} J/eV)} = 3.35 eV$$

For Helium atoms the mass is $4(1.67\times 10^{-27}kg)/(0.9\times 10^{-30}kg) = 7.42\times 10^{3}$ larger, so that

$$K = \frac{33.5eV}{7.42\times 10^{3}} = 4.5\times 10^{-3}eV$$

11. We use $K = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$ with $\lambda = 15 \times 10^{-9}$ m to get

$$K = \frac{(6.63\times 10^{-34}J.s)^2}{2(0.9\times 10^{-30}kg)(15\times 10^{-9}m)^2(1.6\times 10^{-19}J/eV)} = 6.78\times 10^{-3}eV$$

For $\lambda = 0.5$ nm, the wavelength is 30 times smaller, so that the energy is 900 times larger. Thus K =6.10 eV.

12. For a circular orbit of radius *r*, the circumference is $2\pi r$. If *n* wavelengths λ are to fit into the orbit, we must have $2\pi r = n\lambda = nh/p$. We therefore get the condition

$$pr = nh/2\pi = n\hbar$$

which is just the condition that the angular momentum in a circular orbit is an integer in units of $\hbar$.

13. We have $a = n\lambda/2\sin\theta$. For $n = 1$, $\lambda = 0.5 \times 10^{-10}$ m and $\theta = 5^{\circ}$. we get $a = 2.87 \times 10^{-10}$ m. For $n = 2$, we require $\sin\theta_2 = 2\sin\theta_1$. Since the angles are very small, $\theta_2 = 2\theta_1$. So that the angle is 10°.

14. The relation $F = ma$ leads to $mv^2/r = m\omega r$ that is, $v = \omega r$. The angular momentum quantization condition is $mvr = n\hbar$, which leads to $m\omega r^2 = n\hbar$. The total energy is therefore

$$E = \frac{1}{2}mv^2 + \frac{1}{2}m\omega^2 r^2 = m\omega^2 r^2 = n\hbar\omega$$

The analog of the Rydberg formula is

$$\nu(n \rightarrow n') = \frac{E_n - E_{n'}}{h} = \frac{\hbar\omega(n-n')}{h} = (n-n')\frac{\omega}{2\pi}$$

The frequency of radiation in the classical limit is just the frequency of rotation $\nu_{cl} = \omega/2\pi$ which agrees with the quantum frequency when $n - n' = 1$. When the selection rule $\Delta n = 1$ is satisfied, then the classical and quantum frequencies are the same *for all n*.

15. With $V(r) = V_0\,(r/a)^k$, the equation describing circular motion is

$$m\frac{v^2}{r} = \left|\frac{dV}{dr}\right| = \frac{1}{r}kV_0\left(\frac{r}{a}\right)^k$$

so that

$$v = \sqrt{\frac{kV_0}{m}}\left(\frac{r}{k}\right)^{k/2}$$

The angular momentum quantization condition $mvr = n\hbar$ reads

$$\sqrt{ma^2kV_0}\left(\frac{r}{a}\right)^{\frac{k+2}{2}} = n\hbar$$

We may use the result of this and the previous equation to calculate

$$E = \frac{1}{2}mv^2 + V_0\left(\frac{r}{a}\right)^k = (\frac{1}{2}k+1)V_0\left(\frac{r}{a}\right)^k = (\frac{1}{2}k+1)V_0\left[\frac{n^2\hbar^2}{ma^2kV_0}\right]^{\frac{k}{k+2}}$$

In the limit of $k \gg 1$, we get

$$E \to \frac{1}{2}(kV_0)^{\frac{2}{k+2}}\left[\frac{\hbar^2}{ma^2}\right]^{\frac{k}{k+2}}(n^2)^{\frac{k}{k+2}} \to \frac{\hbar^2}{2ma^2}n^2$$

Note that V_0 drops out of the result. This makes sense if one looks at a picture of the potential in the limit of large *k*. For $r < a$ the potential is effectively zero. For $r > a$ it is effectively infinite, simulating a box with infinite walls. The presence of V_0 is there to provide something with the dimensions of an energy. In the limit of the infinite box *with the quantum condition* there is no physical meaning to V_0 and the energy scale is provided by $\hbar^2/2ma^2$.

16. The condition $L = n\hbar$ implies that

$$E = \frac{n^2\hbar^2}{2I}$$

In a transition from n_1 to n_2 the Bohr rule implies that the frequency of the radiation is given

$$\nu_{12} = \frac{E_1 - E_2}{h} = \frac{\hbar^2}{2Ih}(n_1^2 - n_2^2) = \frac{\hbar}{4\pi I}(n_1^2 - n_2^2)$$

Let $n_1 = n_2 + \Delta n$. Then in the limit of large n we have $(n_1^2 - n_2^2) \rightarrow 2n_2\Delta n$, so that

$$\nu_{12} \rightarrow \frac{1}{2\pi}\frac{\hbar n_2}{I}\Delta n = \frac{1}{2\pi}\frac{L}{I}\Delta n$$

Classically the radiation frequency is the frequency of rotation which is $\omega = L/I$, i.e.

$$\nu_{cl} = \frac{\omega}{2\pi}\frac{L}{I}$$

We see that this is equal to ν_{12} when $\Delta n = 1$.

17. The energy gap between low-lying levels of rotational spectra is of the order of $\hbar^2 / I = (1/2\pi)h\hbar / MR^2$, where M is the reduced mass of the two nuclei, and R is their separation. (Equivalently we can take 2 x $m(R/2)^2 = MR^2$). Thus

$$h\nu = \frac{hc}{\lambda} = \frac{1}{2\pi}h\frac{\hbar}{MR^2}$$

This implies that

$$R = \sqrt{\frac{\hbar\lambda}{2\pi Mc}} = \sqrt{\frac{\hbar\lambda}{\pi mc}} = \sqrt{\frac{(1.05\times 10^{-34} J.s)(10^{-3} m)}{\pi(1.67\times 10^{-27} kg)(3\times 10^8 m/s)}} = 26 nm$$