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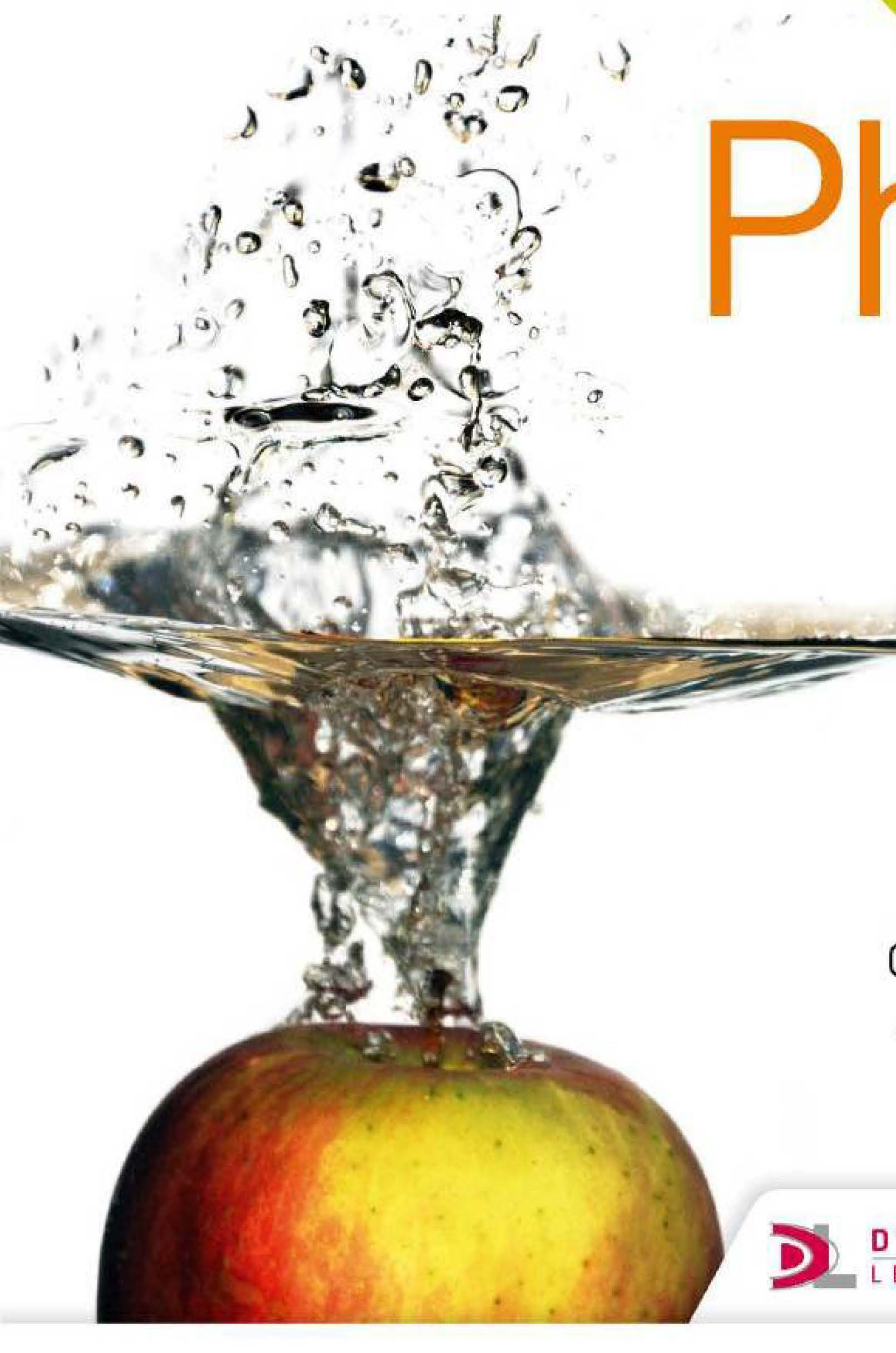
AQA A-level Physics 2



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Physics 2

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Contents

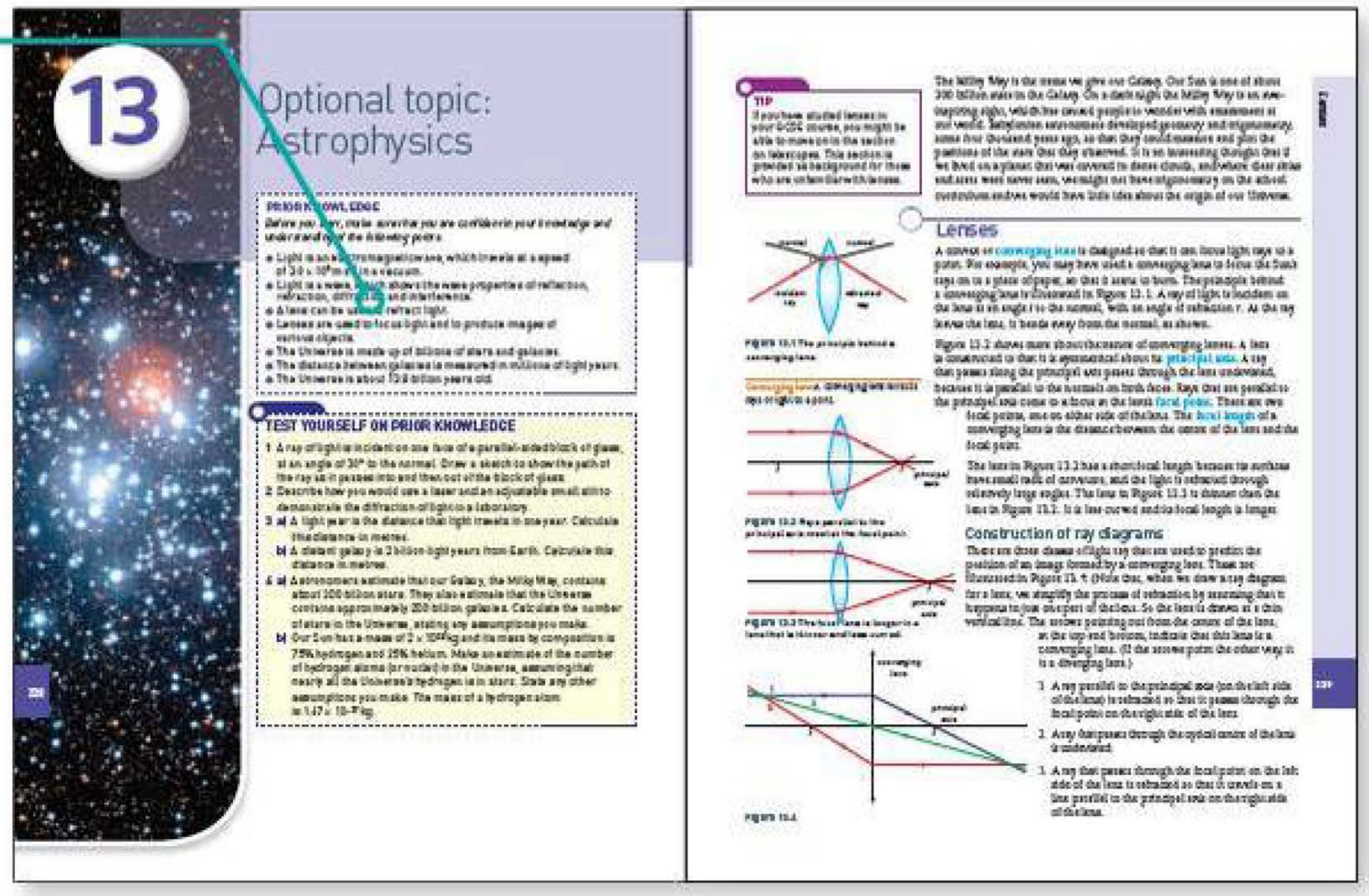
Get the most from this book

Welcome to the **AQA A-level Physics Year 2 Student's Book**. This book covers Year 2 of the AQA A-level Physics specification.

The following features have been included to help you get the most from this book.

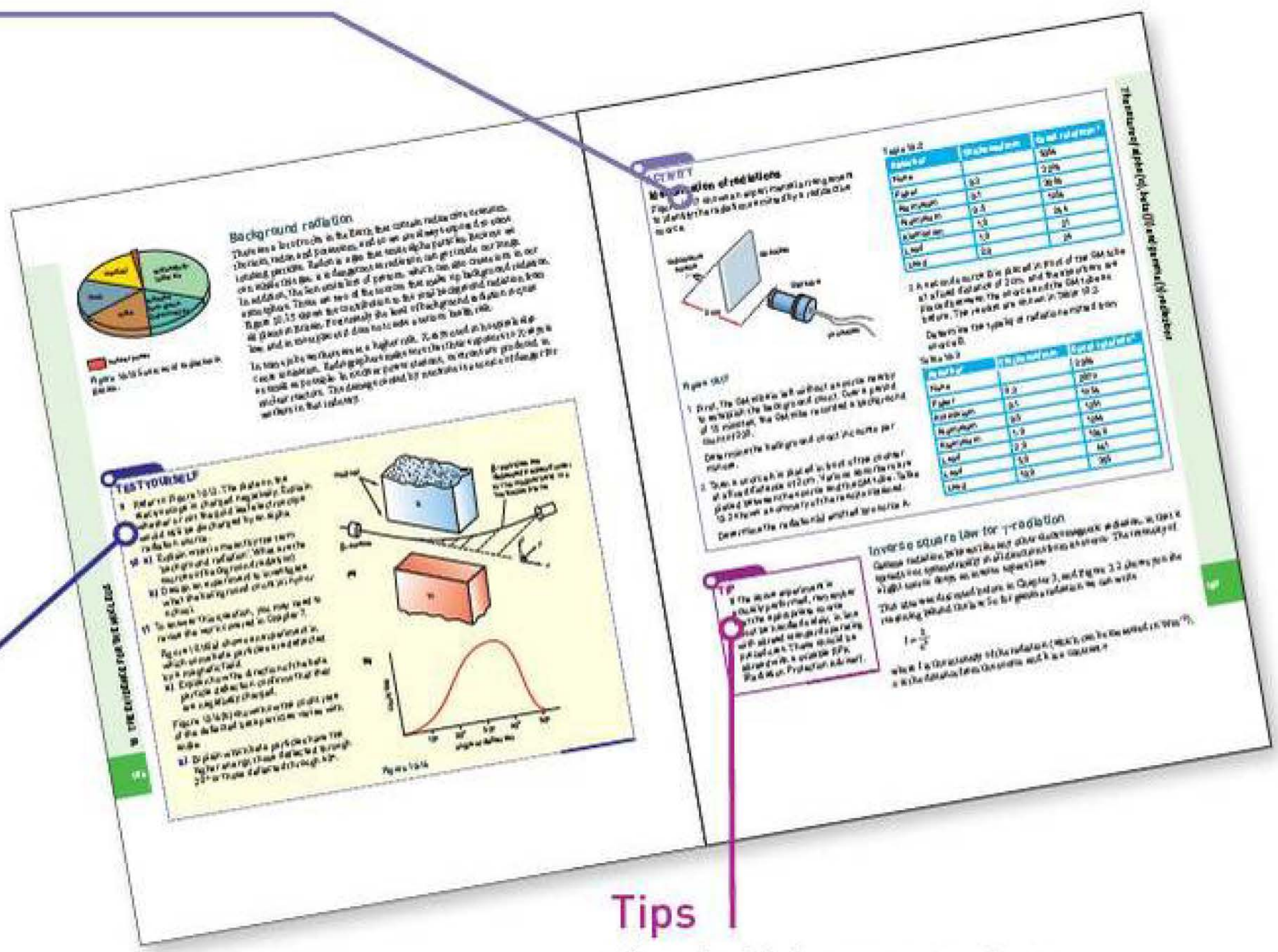
Prior knowledge

This is a short list of topics that you should be familiar with before starting a chapter. The questions will help to test your understanding.



Activities and Required practicals

These practical-based activities will help consolidate your learning and test your practical skills. AQA's required practicals are clearly highlighted.



Test yourself questions

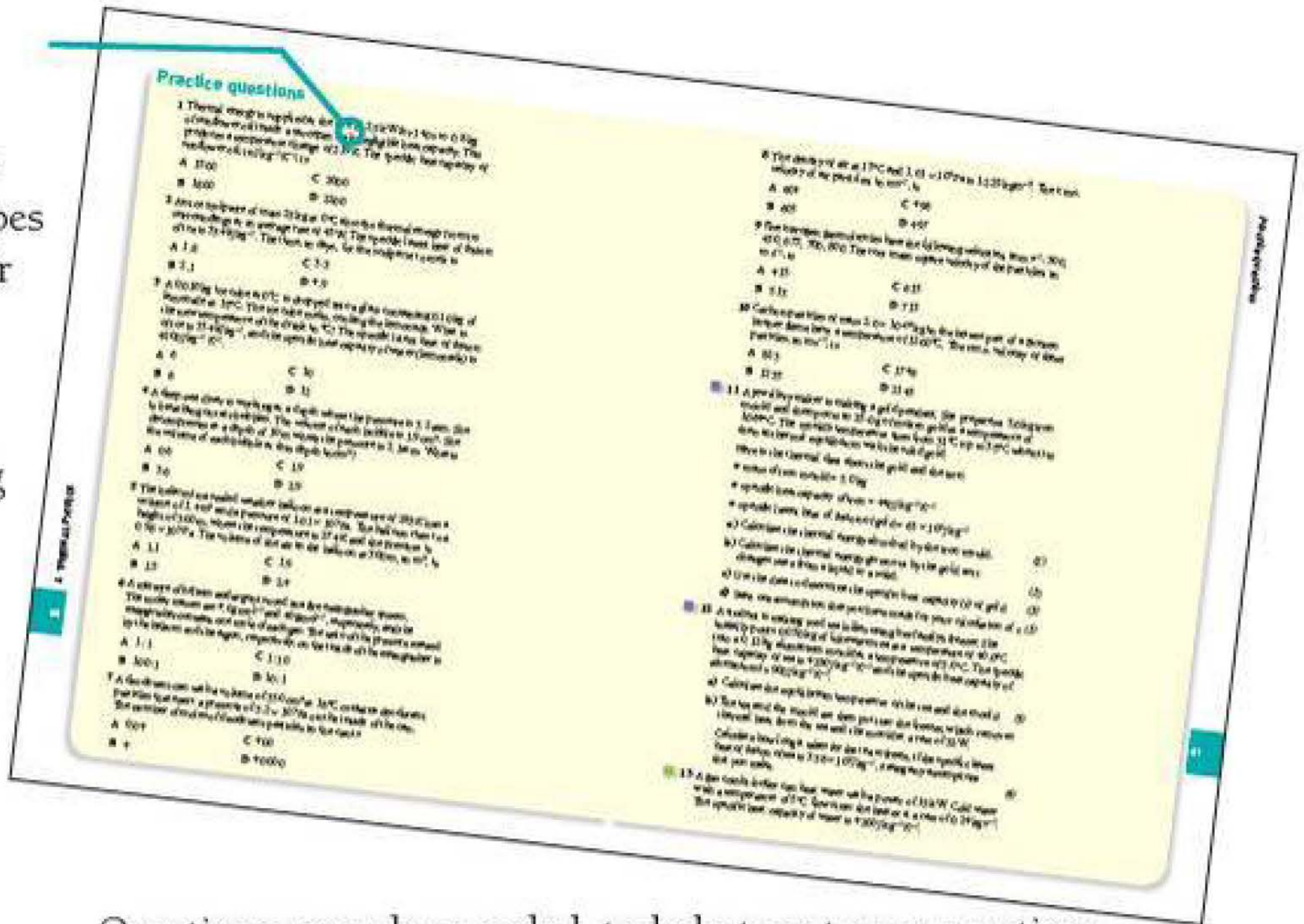
These short questions, found throughout each chapter, are useful for checking your understanding as you progress through a topic.

Tips

These highlight important facts, common misconceptions and signpost you towards other relevant topics.

Practice questions

You will find Practice questions at the end of every chapter. These follow the style of the different types of questions you might see in your examination, including multiple-choice questions, and are colour coded to highlight the level of difficulty. Test your understanding even further with Stretch and challenge questions.



Questions are colour-coded, to help target your practice:

- Green – Basic questions that everyone should be able to answer without difficulty.
- Orange – Questions that are a regular feature of exams and that all competent candidates should be able to handle.
- Purple – More demanding questions which the best candidates should be able to do.
- Stretch and challenge – Questions for the most able candidates to test their full understanding and sometimes their ability to use ideas in a novel situation.

Key terms and formulae

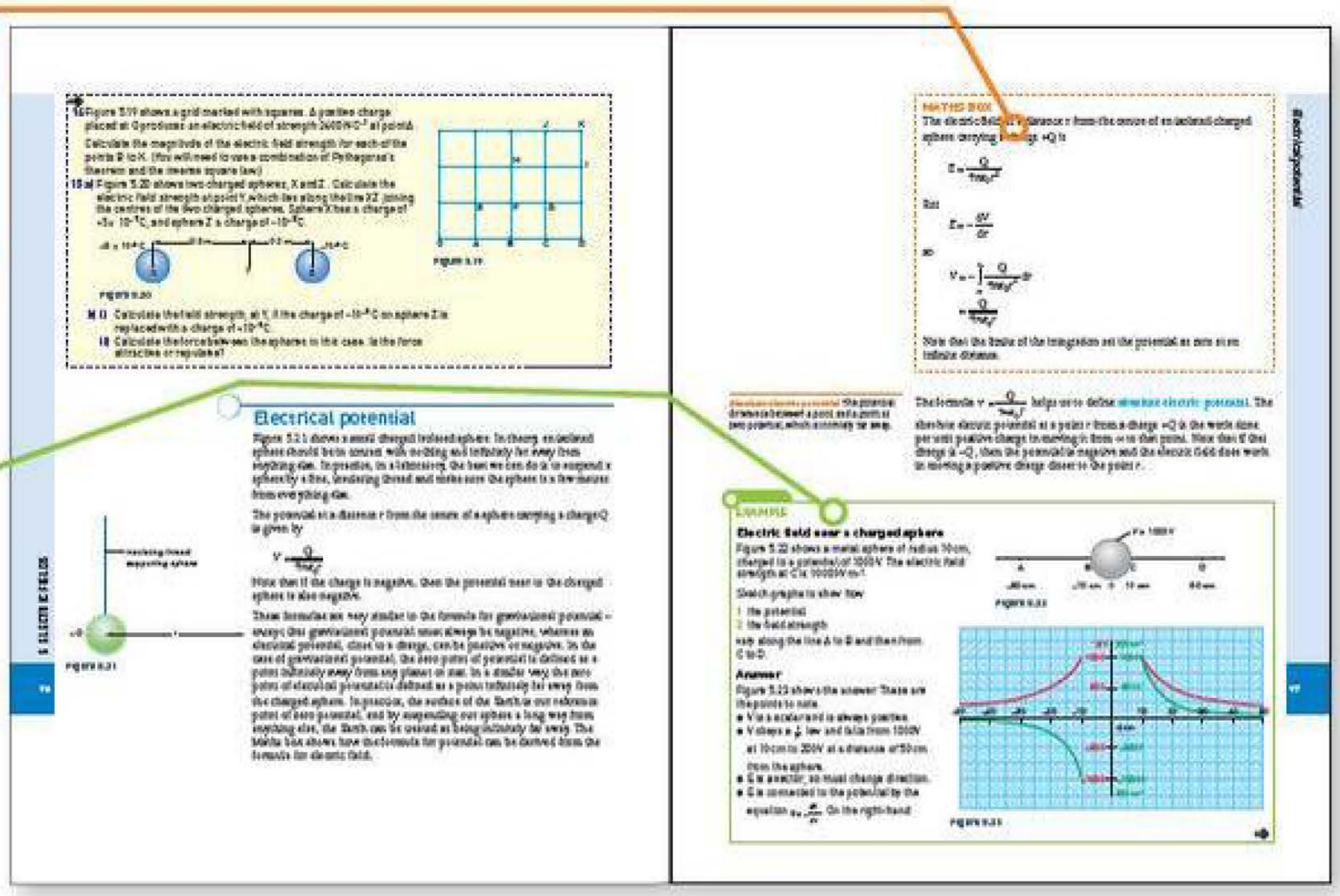
These are highlighted in the text and definitions are given in the margin to help you pick out and learn these important concepts.

Maths boxes

These provide additional material for the more mathematical physicists.

Examples

Examples of questions and calculations feature full workings and sample answers.



AQA has provided five optional topics as part of the full A-level course so students can focus on their areas of interest: *Astrophysics, Medical physics, Engineering physics, Turning points in physics, and Electronics*.

A chapter covering the first optional topic, *Astrophysics*, has been included in this book (Chapter 13), as well as a dedicated chapter for developing your *Maths in physics* (Chapter 14). Additional chapters covering the other optional topics can be accessed online, as well as further chapters focusing on *Developing practical skills in physics*, and *Preparing for written assessments*. More information on how to access these can be found in the *Free online resources* section at the back of this book.

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t = top, *b* = bottom, l = left, *r* = right

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1 Circular motion

PRIOR KNOWLEDGE

Before you start, make sure that you are confident in your knowledge and understanding of the following points:

$$\text{Acceleration} = \frac{\text{change of velocity}}{\text{time}}$$

- Resultant force = mass × acceleration.
- You need to recall that a vector quantity has magnitude and direction: force, velocity and acceleration are vectors.
- Resolving a vector into components.
- Circumference of a circle = 2π × radius of circle, $c = 2\pi r$.
- Newton's first law of motion: a body remains at rest or continues to move in a straight line at a constant speed unless acted on by an unbalanced force.

TEST YOURSELF ON PRIOR KNOWLEDGE

1 a) Explain the difference between speed and velocity.
  b) Explain why acceleration is a vector quantity.

2 The three diagrams in Figure 1.1 show three separate examples of how a vehicle's velocity changes from v_1 to v_2 over a time of 10 s. Use the equation $a = \frac{v_2 - v_1}{t}$ to calculate the magnitude and direction of the acceleration in each case.

3 The vehicle in question 2 has a mass of 2 kg. In each case shown in Figure 1.1, calculate the average resultant force that caused the acceleration of the vehicle.

4 An unbalanced force acts on a moving vehicle. Explain three changes that could occur to the vehicle's velocity.

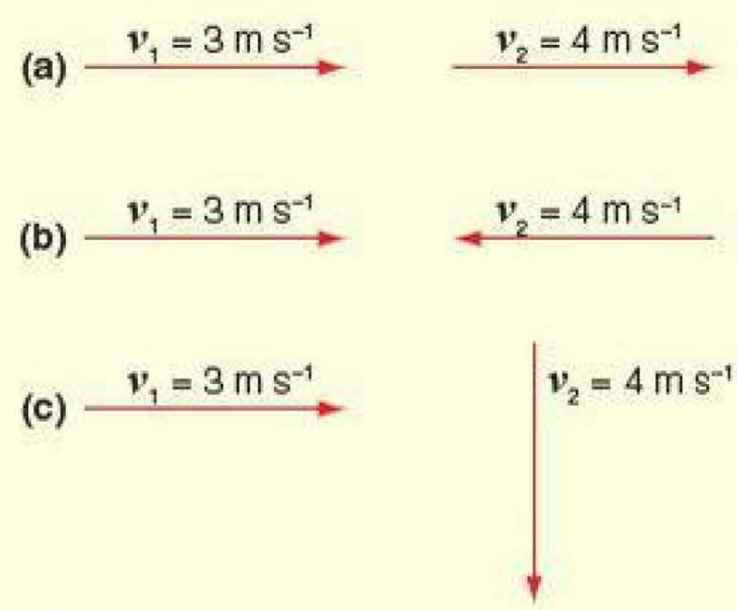


Figure 1.1

5 You walk a quarter of the way round a circle of diameter 20 m.
  a) Calculate the distance you have walked.
  b) Calculate your displacement if you started at the north of the circle and walked to the eastern side of the circle.

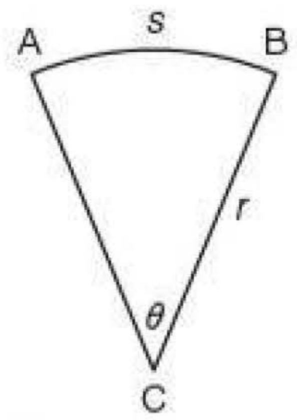


Figure 1.2

Radian The radian measure of a central angle of a circle, θ, is defined as the ratio of the arc length, s, subtended by the angle θ, to the radius r:

$$\theta = \frac{s}{r}$$

Angular displacement The angle (measured in radians) through which a line rotates about a fixed point.

Angular velocity The rate of change of angular displacement (measured in radians per second).

Circular measure

You are used to measuring angles in degrees, but in physics problems involving rotations we use a different measure.

In Figure 1.2, an arc AB is shown. The length of the arc is s, and the radius of the circle is r. We define the angle θ as

$$\theta = \frac{s}{r}$$

The advantage of this measure is that θ is a ratio of lengths, so it has no unit. However, to avoid the confusion that the angle might be measured in degrees, we give this measure the unit **radian**, abbreviated to rad.

Since the circumference of a circle is $2\pi r$, it follows that 2π radians is the equivalent of 360°:

$$2\pi\,\text{rad} = 360°$$

so

$$1\,\text{rad} = \frac{360°}{2\pi} = 57.3°$$

Equations of rotation

When something rotates about a fixed point we use the term **angular displacement** to measure how far the object has rotated. For example, in Figure 1.2, when an object rotates from A to B, its angular displacement is θ radians.

The term **angular velocity**, ω, is used to measure the rate of angular rotation. Angular velocity has units of radians per second or rad s^{-1}:

$$\omega = \frac{\theta}{t}$$

or

$$\omega = \frac{\Delta\theta}{\Delta t}$$

where $\Delta\theta$ is the small angle turned into a small time Δt.

In general, there is a useful relationship connecting the time period of one complete rotation, T, and angular velocity, ω, because after one full rotation the angular displacement is 2π:

$$\omega = \frac{2\pi}{T}$$

or

$$\omega = 2\pi f$$

where f is the frequency of rotation. There is a further useful equation, which connects angular velocity with the velocity of rotation. Since

$$s = \theta r$$

EXAMPLE

The 'big wheel'

A 'big wheel' at a funfair takes its passengers for a ride, completing six complete revolutions in 120 s.

1 Calculate the angular displacement of the wheel.

Answer

$$\theta = 6 \times 2\pi = 12\pi\,\text{rad} = 37.7\,\text{rad}$$

2 Calculate the average angular velocity during the ride.

Answer

$$\omega = \frac{12\pi}{120\,\text{s}} = 0.1\pi = 0.31\,\text{rad s}^{-1}$$

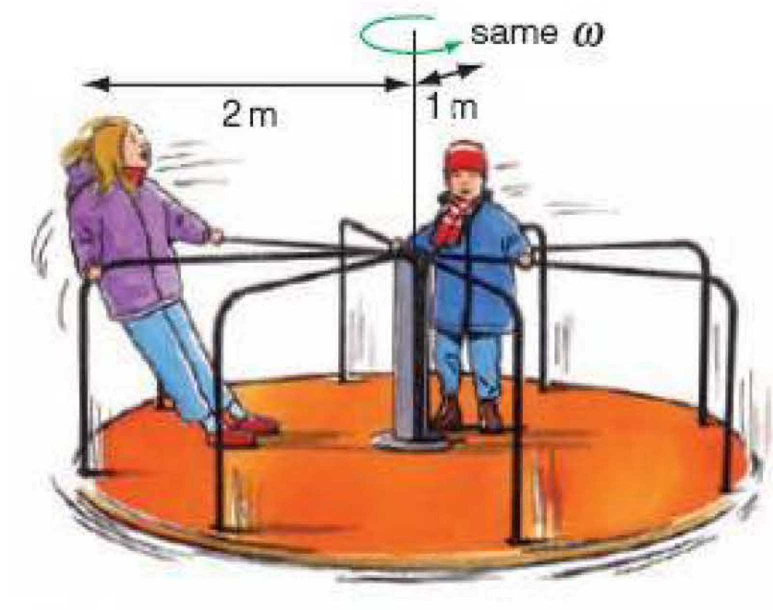


Figure 1.3

and

$$v = \frac{\Delta s}{\Delta t} = \frac{\Delta \theta}{\Delta t} r$$

then

$$v = \omega r$$

This equation shows that the rotational speed of something is faster further away from the centre. For example, all the children on a roundabout in a playground have the same angular velocity ω, but the ones near the edge are moving faster (Figure 1.3).

TEST YOURSELF

1 The Earth has a radius of 6400 km. The Shetland Isles are at latitude of 60°.
   a) Calculate the angular velocity of the Earth.
   b) Calculate the velocity of rotation of a point on the equator.
   c) Calculate the velocity of rotation of the Shetland Isles.

2 A proton in a synchrotron travels round a circular path of radius 85 m at a speed of close to $3.0 \times 10^8\,\mathrm{m\,s^{-1}}$.
   a) Calculate the time taken for one revolution of the synchrotron.
   b) Calculate the frequency of rotation of the protons.
   c) Calculate the proton's angular velocity.

3 The Sun rotates around the centre of our Galaxy, the Milky Way, once every 220 million years, in an orbit of about 30 000 light years.
   a) Calculate the angular velocity of the Sun about the centre of the Milky Way.
   Calculate the velocity of the Sun relative to the centre of the Galaxy.
   [1 light year = 9.47×10^{15} m; 1 year = 3.16×10^7 s]

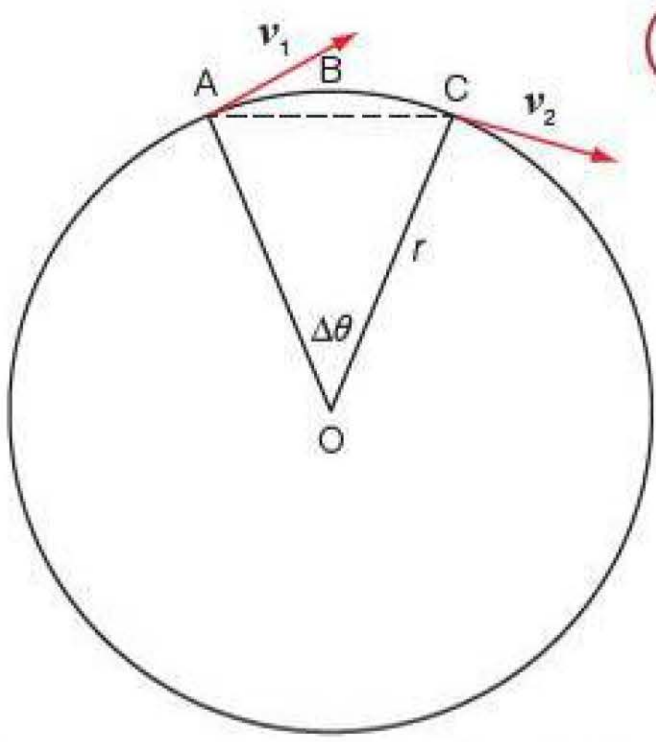


Figure 1.4 A particle moving round a circular path with a constant speed is always accelerating.

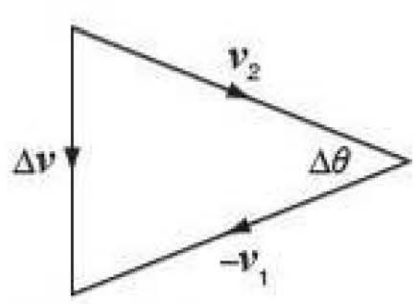


Figure 1.5

Centripetal acceleration

In Figure 1.4 a particle is moving round a circular path at a constant speed v, and because it is continually changing direction the particle is always accelerating.

It is easier to understand the acceleration when you recall the formula:

$$\text{acceleration} = \frac{\text{change of velocity}}{\text{time}}$$

Velocity is a vector quantity, so if the direction of the motion changes, even though there is no change of speed, there must be an acceleration.

Figure 1.5 shows the direction of the acceleration. In going from position A to position C, the particle's velocity changes from v_1 to v_2. So the change in velocity, $\Delta \mathbf{v}$, is the vector sum $\mathbf{v}_2 - \mathbf{v}_1$.

The diagram shows the change in velocity, $\Delta \mathbf{v}$, which is directed along the line BO, towards the centre of the circle. So, as the particle moves around the circular path, there is an acceleration towards the centre of the circle. This is called the centripetal acceleration. Because this acceleration is at right angles to the motion, there is no speeding up of the particle, just a change of direction.

Centripetal acceleration When a particle moves in a circular path of radius r, at a constant speed v, there must be a centripetal acceleration towards the centre of the circle, given by

$$a = \frac{v^2}{r}$$

The size of the acceleration, a, is calculated using this formula:

$$a = \frac{v^2}{r}$$

or because $v = \omega r$

$$a = \omega^2 r$$

Here v is the constant speed of the particle, ω is its angular velocity, and r is the radius of the path.

MATHS BOX

You are not expected to be able to derive the formula for centripetal acceleration, but it is given here for those who want to know where the formula comes from.

In Figure 1.4, the particle moves from A to C in a small time Δt. We now look at the instantaneous acceleration at the point B, by considering a very small angle $\Delta\theta$. The distance travelled round the arc AC, Δs, is given by

$$\Delta s = v\Delta t \quad \text{and} \quad \Delta s = r\Delta\theta$$

so

$$r\Delta\theta = v\Delta t \quad \text{and} \quad \Delta\theta = \frac{v}{r}\Delta t \qquad \text{(i)}$$

In Figure 1.5 the angle θ is given by

$$\Delta\theta = \frac{\Delta v}{v} \qquad \text{(ii)}$$

provided $\Delta\theta$ is very small. Then by combining equations (i) and (ii), it follows that

$$\frac{\Delta v}{v} = \frac{v}{r}\Delta t$$

or

$$a = \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

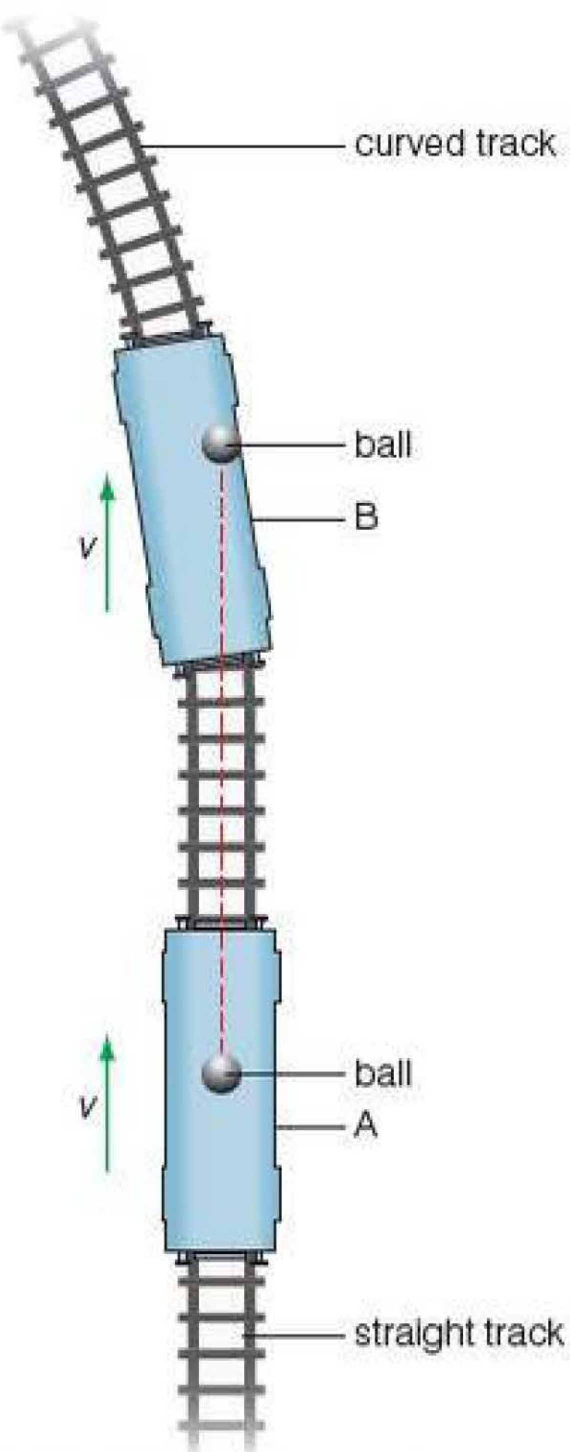


Figure 1.6 A train carriage turns a corner, but a ball on the floor of the carriage keeps on moving in a straight line.

Centripetal force

Figure 1.6 illustrates the path of a railway carriage as it turns round a corner (part of a circle), moving from A to B at a constant speed v. The rails provide a force to change the direction of the carriage. However, a ball that is placed on the floor behaves differently. The ball carries on moving in a straight line until it meets the side of the carriage. The ball experiences no force, so, as predicted by Newton's first law of motion, it carries on moving in a straight line at a constant speed, until the side of the carriage exerts a force on it.

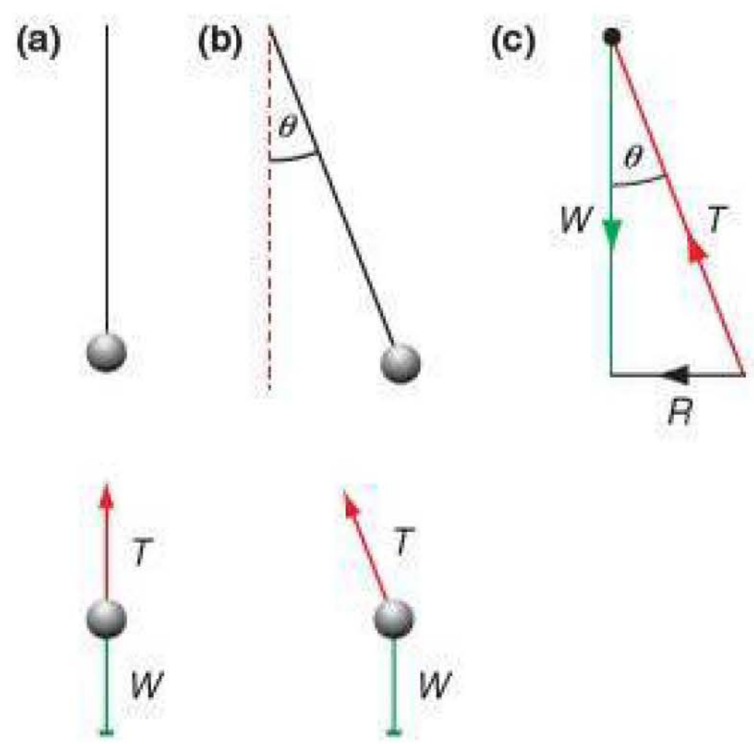


Figure 1.7 Views of a ball suspended from the back of the train carriage looking forwards. (a) When the train moves along a straight track, the ball hangs straight down. (b) When the train moves around the curved track, as in Figure 1.6, the ball is displaced to the right. (c) There is a resultant unbalanced force *R* acting on the ball.

Centripetal force When an object moves around a circular path, there must be a centripetal force acting towards the centre of the circle. Something must provide this force, such as a pull from a string or a push from the road.

Now suppose that the ball is suspended from the ceiling of the carriage and the experiment is repeated. Figure 1.7 illustrates what happens now as the carriage moves from a straight track to a curved track. In Figure 1.7(a) the carriage moves along a straight track at a constant speed. The ball hangs straight down and the forces acting on it balance: the tension of the string, *T*, upwards, balances the ball's weight, *W*, downwards.

In Figure 1.7(b) the train turns the corner. The ball keeps moving in a straight line until tension in the string acts to pull the ball round the corner. Now the forces acting on the ball do *not* balance. The vector sum of the tension *T* and the weight *W* provides an unbalanced force *R*, which acts towards the centre of the circle (Figure 1.7c).

This unbalanced force *R* provides the centripetal acceleration. So we can write

$$R = \frac{mv^2}{r}$$

where *R* is the unbalanced **centripetal force**, *m* is the mass of the ball, *v* is the ball's forward speed, and *r* is the radius of the (circular) bend it is going round.

It is important to understand that a centripetal force does not exist because something is moving round a curved path. It is the other way around – according to Newton's second law of motion, to make something change direction a force is required to make the object accelerate. In the example you have seen here, the tension in the string provides the centripetal force, which is necessary to make the ball move in a circular path. When a car turns a corner, the frictional force from the road provides the centripetal force to change the car's direction. When a satellite orbits the Earth, the gravitational pull of the Earth provides the centripetal force to make the satellite orbit the Earth – there is no force acting on the satellite other than gravity.

A common misunderstanding

Figure 1.8 shows the same ball discussed earlier held hanging, at rest, at an angle in the laboratory. Now it is kept in place by the balance of three forces: the tension in the string, *T*, its weight, *W*, and a sideways push, *P*, from a student's finger.

If the student removes his finger, the ball will accelerate and begin moving to the left, because there is now an unbalanced force acting on it, exactly as there was in Figure 1.7.

However, the situations are different. In Figure 1.8 the ball is stationary until the finger is removed, and it begins to accelerate and move in the direction of the unbalanced force. In Figure 1.7 the ball is moving forwards and the action of the unbalanced force is to change the direction of the ball.

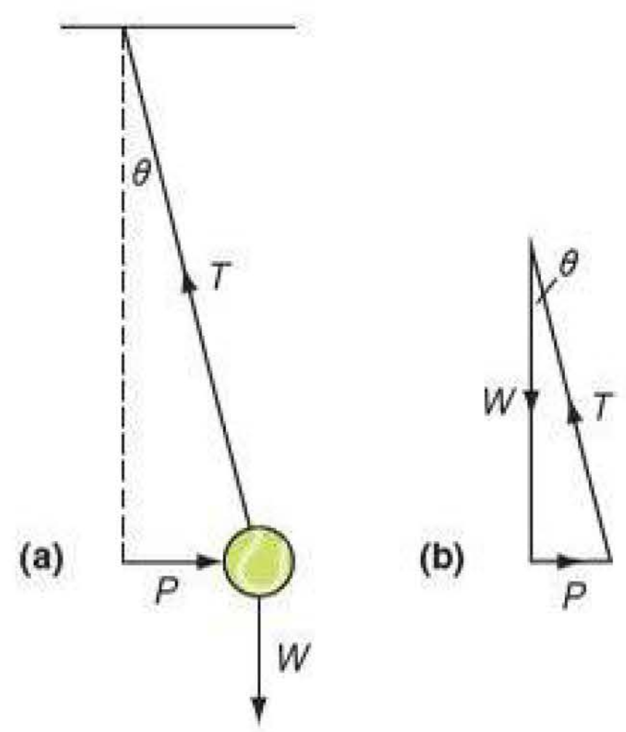


Figure 1.8

TEST YOURSELF

4 Explain how a force can change the velocity of a body without increasing its speed.

5 The force of gravity makes things fall towards the ground. Explain why the Earth's gravity does not make the Moon fall towards the Earth.

6 A satellite is in orbit around the Earth, at a distance of 7000 km from the Earth's centre. The mass of the satellite is 560 kg and the gravitational field strength at that height is $8.2\,\text{N kg}^{-1}$.
   a) Draw a diagram to show the direction and magnitude of the force (or forces) that act(s) on it.
   b) Calculate the centripetal acceleration of the satellite.
   c) Calculate
      i) the speed of the satellite
      ii) the time period of its orbit.

7 This question refers to the suspended ball in the train, illustrated in Figure 1.7. The ball has a mass of 0.15 kg.
   a) The train accelerates forwards out of the station along a straight track at a rate of $2\,\text{m s}^{-2}$.
      i) Explain why the ball is displaced backwards.
      ii) Calculate the resultant force on the ball.
      iii) Show that the angle at which the ball hangs to the vertical is about 11.5°.
   b) The train reaches a speed of $55\,\text{m s}^{-1}$ and travels round a curved piece of the track. At this moment, the ball is deflected sideways by about 11.5°.
      i) State and explain the direction and magnitude of the resultant force on the ball.
      ii) Explain why the ball is accelerating. In which direction is the acceleration? Calculate the magnitude of the acceleration.
      iii) Explain why the ball's speed remains constant.
      iv) Calculate the radius of the bend the train is going round.
   c) The train carriage that carries the ball has a mass of 40 tonnes.
      i) Calculate the centripetal force that acts on the carriage as it turns the corner. What provides this force?
      ii) Explain why trains go round tight bends at reduced speeds.

ACTIVITY

Investigating centripetal forces

Figure 1.9 shows a way in which you can investigate centripetal forces. The idea is that you whirl a rubber bung around your head in a horizontal circle. The bung is attached by a thin string to a plastic tube, which is held vertically. A weight is hung on the bottom of the string. This causes the tension to provide the necessary centripetal force to keep the bung moving in its circular path.

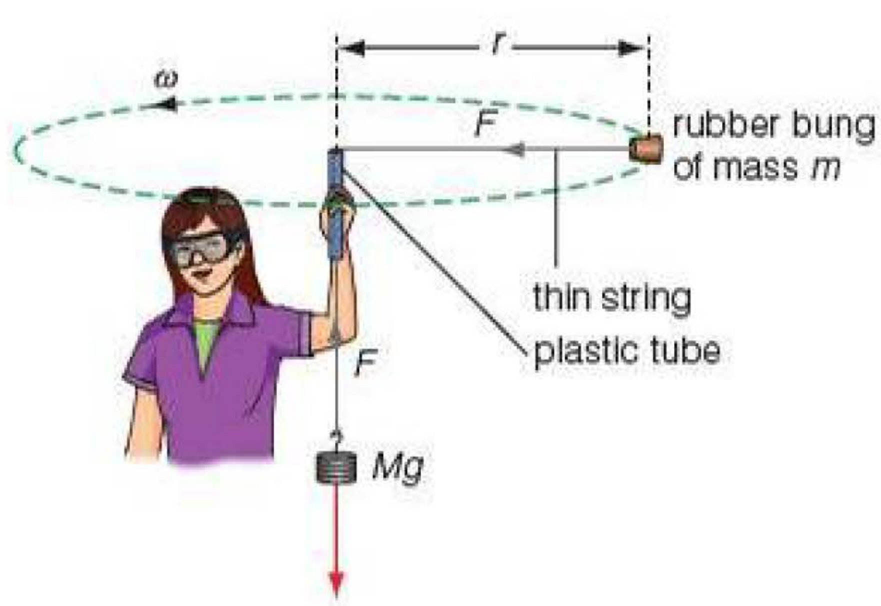


Figure 1.9

- Use a bung, of mass *m*, of about 50 g to 100 g.
- Wear safety glasses (useful to protect yourself from others doing the same experiment).
- A suitable plastic tube is an old case from a plastic ballpoint pen.
- The time of rotation, *T*, can be calculated by measuring the time for 10 rotations, 10*T*.
- The radius *r* can be measured after you have finished 10 rotations by pinching the string with your finger, then measuring the length from the top of the tube to the centre of the bung.
- We assume that there is no friction between the plastic tube and the string.
- It is assumed that the string is horizontal, although this will not be entirely possible, so it is important to try to meet this condition as far as you can.

Table 1.1 shows some data measured by a student doing this experiment.

1 Copy and complete Table 1.1 by filling in the gaps. Comment on how well the results support the hypothesis that the weight on the end of the string causes the centripetal force to keep the bung in its circular path. In this experiment the bung has a mass of 0.09 kg.

⇨

Table 1.1

M/kg	F/N	10T/s	ω/rad s^{-1}	r/m	$m\omega^2r$/N
0.1		18.8		0.94	
0.2		11.5		0.78	
0.2		10.2		0.56	
0.3		8.6		0.61	
0.3		7.9		0.52	
0.4		5.4		0.31	

2 Discuss the sources of error in this experiment. Suggest how the errors can be minimised.

3 To improve the reliability of the data, it might be helpful to plot a graph.

a) Plot a graph of F against $m\omega^2r$.

b) Explain why this should be a straight line. What gradient do you expect to get when you measure it?

EXAMPLE

Round in circles

1 A physics teacher, shown in Figure 1.10, demonstrates a well-known trick. She puts a beaker of water on a tray, suspended by four strings at its corners. Then she whirls the tray round in a vertical circle, so that the beaker is upside down at the top. She then asks why the water does not fall at the top of the swing. A student (who has not been paying attention) says 'the pull of gravity is balanced by an outwards force'. Explain why this is *not* correct.

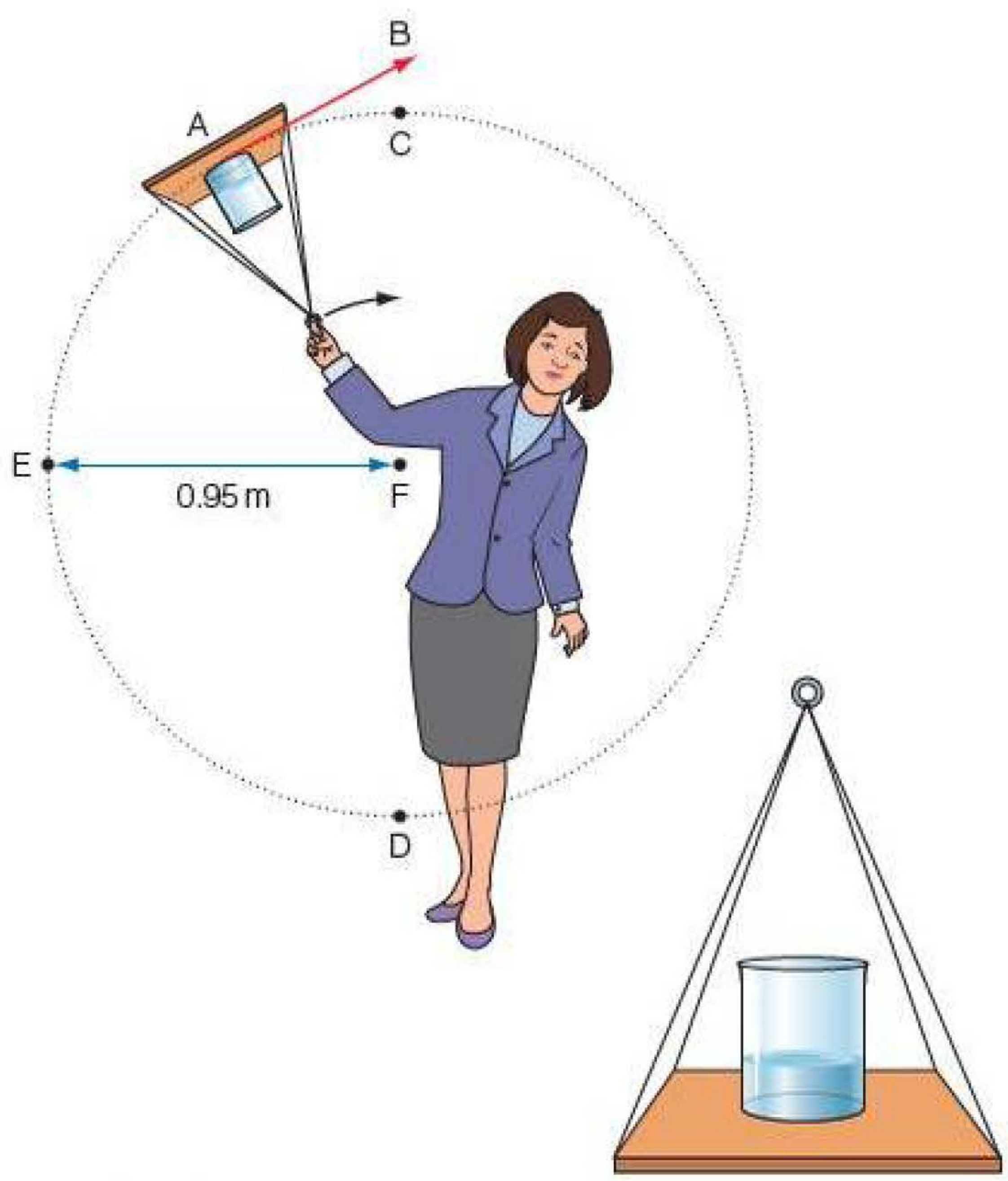


Figure 1.10

Answer

The teacher gives this explanation: At the top, the water and the beaker are falling together. Look at Figure 1.10. At point A, the beaker is travelling along the direction AB. The string pulls the beaker down in the direction BC so at the top it has fallen to point C.

The teacher repeated the demonstration and asked the students to time the revolutions. The students determined that the tray completed 10 revolutions in 8.3 s. They measure the radius of the circle to be 0.95 m.

The speed of the tray is

$$v = \frac{2\pi r}{T}$$

$$= \frac{2\pi \times 0.95\,\text{m}}{0.83\text{s}}$$

$$= 7.2\,\text{m s}^{-1}$$

So, while the beaker rotates, it has a centripetal acceleration of

$$a = \frac{v^2}{r}$$

$$= \frac{(7.2\,\text{m s}^{-1})^2}{0.95\ \text{m}}$$

$$= 55\,\text{m s}^{-2}$$

This tells us that the water is accelerating all the time at 55 m s^{-2} (more than five times the gravitational acceleration). So the water does not fall out ofthe beaker at the top, because it is already falling with an acceleration greater than gravitational acceleration.

2 A cyclist is cycling at $14.5\,m\,s^{-1}$ in a velodrome where the track is bankedat an angle of 40° to the horizontal (Figure 1.11). The track is curved so that the cyclist is turning in a horizontal circle of radius 25 m. The cyclist and bicycle together have a mass of 110 kg.

a) Calculate the centripetal force acting on the cyclist.

Answer

$$F = \frac{mv^2}{r}$$

$$= \frac{110\,kg \times (14.5\,m\,s^{-1})^2}{25\,m}$$

$$= 930\,N\,(2\,s.f.)$$

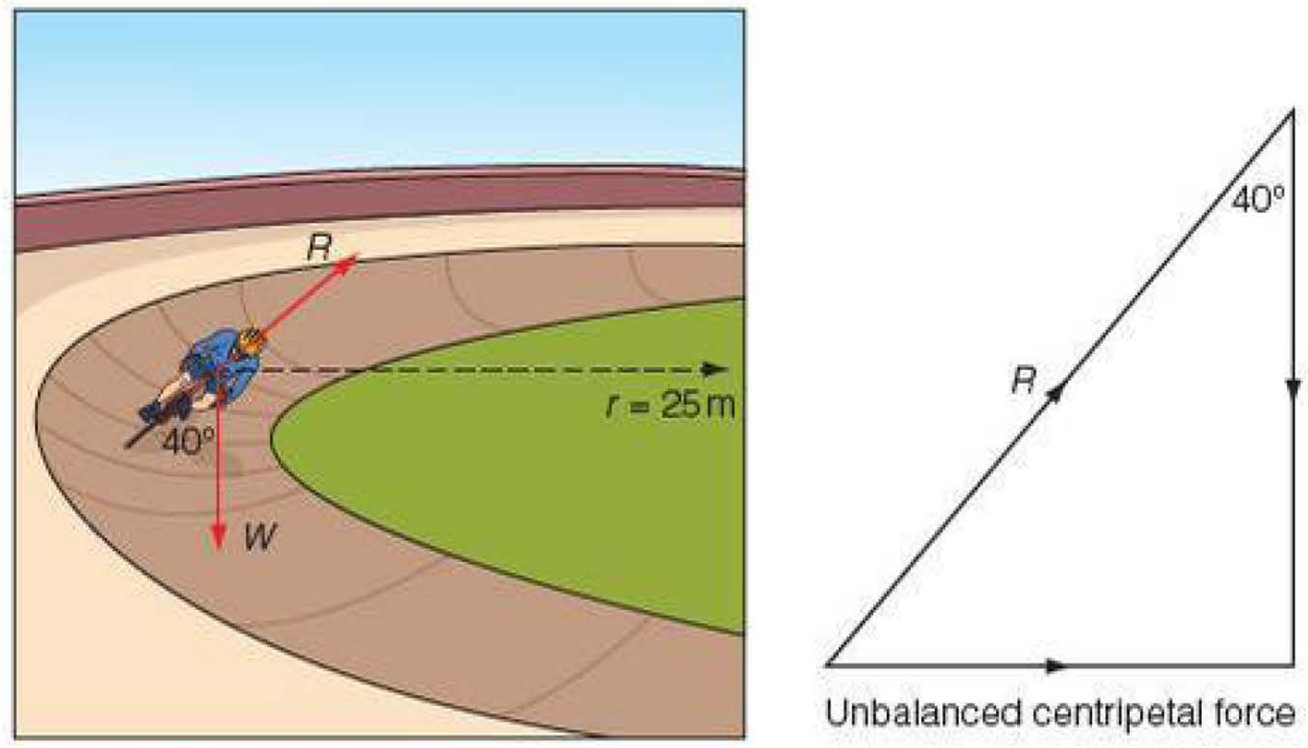


Figure 1.11

b) Calculate the contact force R from the track on the bicycle.

Answer

Figure 1.11 shows the two forces acting on the bicycle and cyclist: the contact force R and the weight W. The forces combine to produce the unbalanced centripetal force, which keeps the cyclist moving round her horizontal circular path.

Force R may be resolved horizontally and vertically as follows:

$$R_v = R\cos 40°$$

$$R_h = R\sin 40°$$

The vertical component R_v balances the weight, and the horizontal component provides the unbalanced centripetal force. (It is important to realise that the cyclist can only lean her bicycle as shown because she is accelerating towards the centre of the circle. She would fall over if she were stationary.) So

$$R\sin 40° = 930\,N$$

$$R = \frac{930\,N}{\sin 40°}$$

$$= 1440\,N$$

TEST YOURSELF

8 This question refers to the teacher's demonstration with the beaker of water shown in Figure 1.10.

a) The beaker and water have a combined mass of 0.1 kg. Use this information, together with the information in the text, to calculate the centripetal force required to keep the beaker in the circular path that the teacher used.

b) The only two forces that act on the beaker and water are their weight, W, and the contact force, R, from the tray. Calculate the size and direction of R at the following points shown in Figure 1.10:

i) C ii) D iii) E.

c) The water will fall out of the beaker at point C if the beaker moves so slowly that the required centripetal acceleration is less than g.

Assuming the teacher still rotates the beaker in a circle of 0.95 m radius, calculate the minimum speed at which the water does not fall out of the beaker at point C.

9 Formula 1 (F1) racing cars are designed to enable them to corner at high speeds. Traction between the tyres and the road surface is increased by using soft rubber tyres, which provide a large frictional force, and by using wings to increase the down force on the car.

The tyres of an F1 car can provide a maximum frictional force to resist sideways movement of 15 500 N. The car's mass (including the driver) is 620 kg.

Calculate the maximum cornering speed of the car going round a bend of

a) radius 30 m b) radius 120 m.

Practice questions

1 The orbit of an electron in a hydrogen atom may be considered to be a circle of radius 5×10^{-11} m. The period of rotation of the electron is 1.5×10^{-16} s. The speed of rotation of the electron is

A $2 \times 10^{5}\,\text{m s}^{-1}$ **C** $2 \times 10^{6}\,\text{m s}^{-1}$

B $4 \times 10^{5}\,\text{m s}^{-1}$ **D** $4 \times 10^{7}\,\text{m s}^{-1}$

2 From the information in question 1, the centripetal acceleration of the electron is

A $3 \times 10^{22}\,\text{m s}^{-2}$ **C** $12 \times 10^{22}\,\text{m s}^{-2}$

B $9 \times 10^{22}\,\text{m s}^{-2}$ **D** $30 \times 10^{22}\,\text{m s}^{-2}$

3 The Moon orbits the Earth once every 29 days with a radius of orbit of 380 000 km. The angular velocity of the Moon is

A $2.5 \times 10^{-6}\,\text{rad s}^{-1}$ **C** $8.5 \times 10^{-6}\,\text{rad s}^{-1}$

B $5.0 \times 10^{-6}\,\text{rad s}^{-1}$ **D** $25 \times 10^{-6}\,\text{rad s}^{-1}$

4 From the information in question 3, the Moon's centripetal acceleration is

A $2.4\,\text{mm s}^{-2}$ **C** $7.6\,\text{mm s}^{-2}$

B $4.0\,\text{mm s}^{-2}$ **D** $24\,\text{mm s}^{-2}$

5 The centripetal acceleration of a car moving at a speed of $30\,\text{m s}^{-1}$ round a bend of radius 0.45 km is

A $1.0\,\text{m s}^{-2}$ **C** $100\,\text{m s}^{-2}$

B $2.0\,\text{m s}^{-2}$ **D** $200\,\text{m s}^{-2}$

6 A satellite is in orbit around the Earth in a circular orbit of radius 10 000 km. The angular velocity of the satellite is $6.4 \times 10^{-4}\,\text{rad s}^{-1}$. The time of orbit of the satellite is

A 4800 s **C** 8400 s

B 6800 s **D** 9800 s

7 From the information in question 6, the centripetal acceleration of the satellite is

A $2\,\text{m s}^{-2}$ **C** $4\,\text{m s}^{-2}$

B $3\,\text{m s}^{-2}$ **D** $8\,\text{m s}^{-2}$

8 A student swings a bucket of water in a vertical circle of radius 1.3 m. The bucket and water have a mass of 2.5 kg. The bucket rotates once every 1.4 s. When the bucket is upside down, the water does not fall out. Which of the following gives a correct explanation of why the water stays in the bucket.

A The weight of the water is balanced by a centrifugal force.

B The centripetal force and the weight of the water balance.

C The water and bucket are falling at the same rate.

D The bucket moves so fast that the water has no time to fall.

9 From the information in question 8, the centripetal force on the swinging bucket is

A 6 N **C** 52 N

B 36 N **D** 65 N

10 From the information in question 8, for the swinging bucket, at the bottom of the swing the rope exerts a force on the student's hand of

A 90 N **C** 52 N

B 63 N **D** 36 N

11 An astronaut undergoes some training to test his tolerance to acceleration. He is placed in a rotor, which carries him in a circle of radius 7.0 m. The rotor completes 10 revolutions in 24.3 s, moving at a constant speed.

a) Explain why the astronaut is accelerating, although his speed is constant. (2)

b) Calculate the size of the astronaut's acceleration. (3)

12 (Synoptic question: you need to think about energy transformations to help solve this question.)

A large steel ball of mass 2100 kg is used to demolish buildings. The ball is suspended on a cable of length 8 m, and is pulled back to a height of 4 m above its lowest point, before being released to hit a building.

a) Calculate the maximum speed of the ball just prior to hitting the building. (3)

b) Calculate the tension in the cable when the ball is at its lowest point. (4)

13 Figure 1.12 shows an aircraft propeller that is undergoing tests in a laboratory.

The propeller is made out of high-strength, low-density carbon-fibre-reinforced plastic (CFRP). In a test, it is rotating at a rate of 960 times per minute.

a) Calculate the angular velocity of the propeller. (2)

b) Calculate the speed of the propeller blade at these two positions. (2)

i) A

ii) B

c) Explain why the propeller blade is made of CFRP. (2)

d) At which point is the blade more likely to fracture, A or B? Explain your answer. (2)

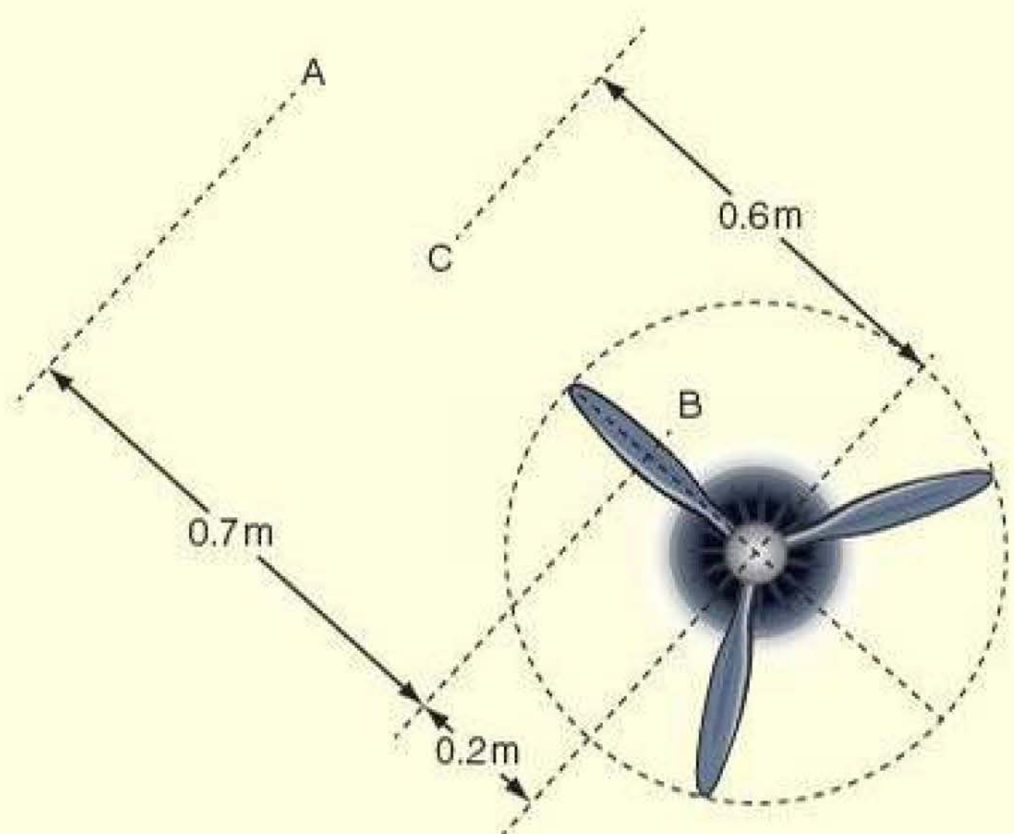


Figure 1.12

e) Estimate the centripetal force required to keep a propeller blade rotating at a rate of 960 times per second, if its centre of mass is 0.6 m from the centre of rotation and the mass of the blade is 3.5 kg. *(3)*

Stretch and challenge

14 This question is about apparent weight. Your weight is the pull of gravity on you. But what gives you the sensation of weight is the reaction force from the floor you are standing on.

a) A man has a mass of 80 kg. Calculate his apparent weight (the reaction from the floor) when he is in a lift that is

i) moving at a constant speed of $3\,m\,s^{-1}$

ii) accelerating upwards at $1.5\,m\,s^{-2}$

iii) accelerating downwards at $1.5\,m\,s^{-2}$.

b) A designer plans the funfair ride shown in Figure 1.13. A vehicle in an inverting roller coaster leaves point A with a very low speed before reaching point B, the bottom of the inverting circle. It then climbs to point C, 14 m above B, before leaving the loop and travelling to point D.

Assuming that no energy is transferred to other forms due to frictional forces, show that

i) the speed of the vehicle at B is $20\,m\,s^{-1}$

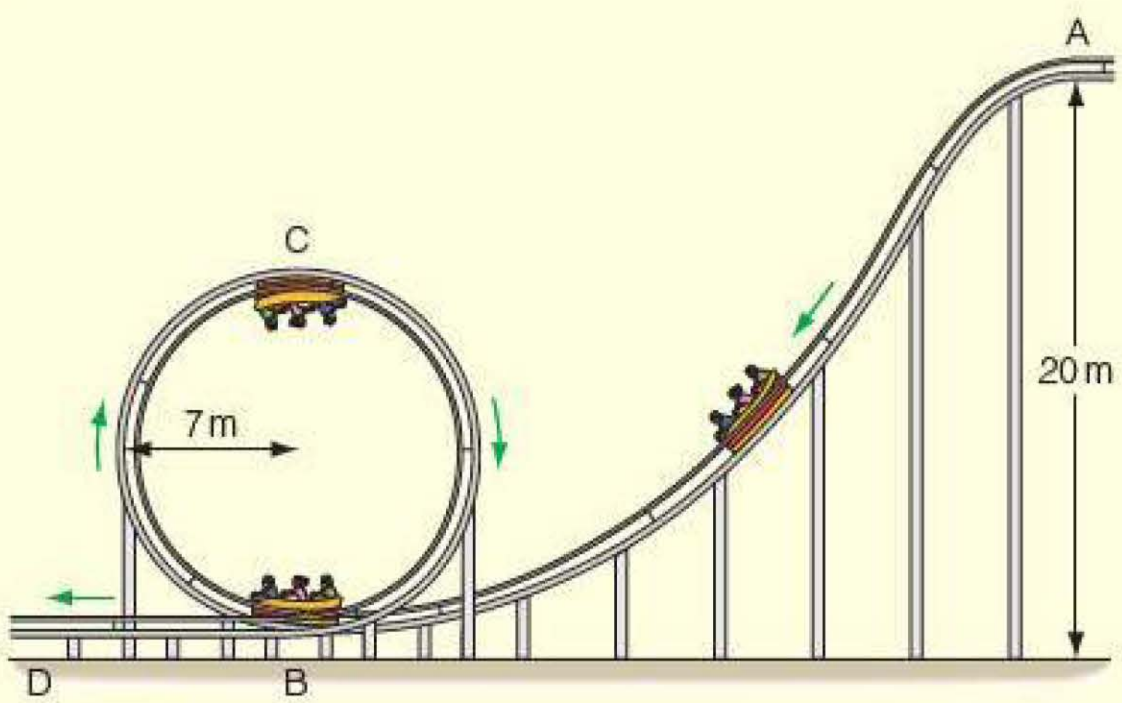


Figure 1.13

ii) the speed of the vehicle at C is $11\,m\,s^{-1}$.

c) Use your answers to (b) to calculate the centripetal acceleration required to keep the vehicle in its circular path

i) at B **ii)** at C.

d) Now calculate the apparent weight of a passenger of mass 70 kg

i) at B **ii)** at C.

In the light of your answers, discuss whether or not this is a safe ride.

e) Figure 1.14 shows the design of a space station. It rotates so that it produces an artificial gravity. The reaction force from the outer surface provides a force to keep people in their circular path.

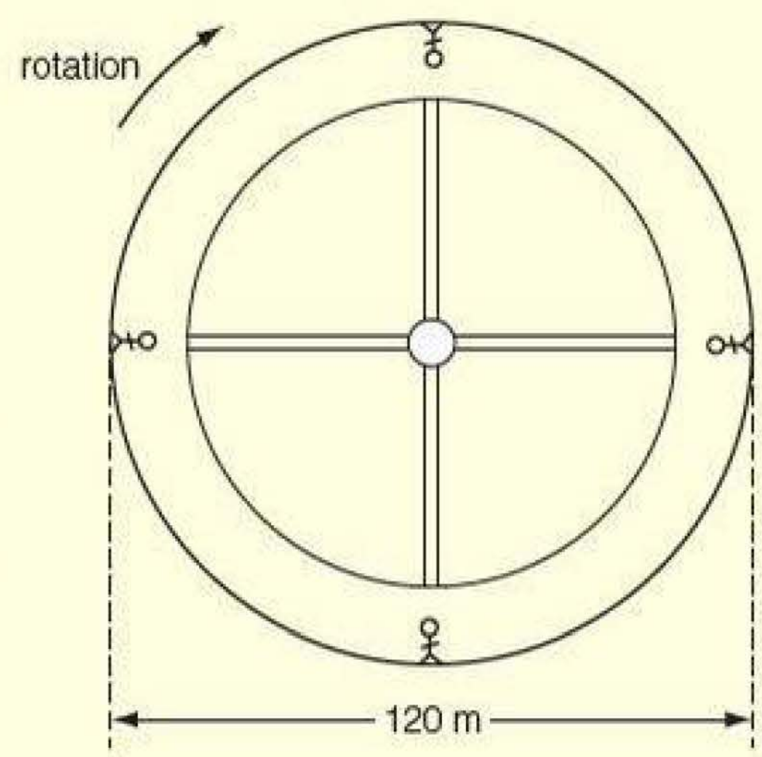


Figure 1.14

Use the information in the diagram to calculate the angular velocity required to provide an apparent gravity of $9.8\,m\,s^{-2}$.

2 Simple harmonic motion

PRIOR KNOWLEDGE

Before you start, make sure that you are confident in your knowledge and understanding of the following points:

- Displacement, velocity, force and acceleration are all vector quantities.
- Acceleration = $\frac{\text{change of velocity}}{\text{time}}$
- Time period = $\frac{1}{\text{frequency}}$
- Frequency = number of oscillations per second.
- The natural measure of angle is the radian; 2π radians = 360°.
- Resultant force = mass × acceleration.

TEST YOURSELF ON PRIOR KNOWLEDGE

1 a) An electromagnetic wave has a frequency of 2.6 GHz. Calculate the time period of the wave.

b) A boat that is anchored at sea lifts up six times in 30 s as waves pass it. What is the frequency of the waves?

2 A car is travelling with a velocity of $20\,\text{m}\,\text{s}^{-1}$ due north. Two minutes later the car has travelled round a large bend and is travelling with a velocity of $15\,\text{m}\,\text{s}^{-1}$ due south. Calculate the car's average acceleration over this time.

3 Calculate the values of these trigonometric functions, where the angle has been expressed in radians.

a) $\tan 0.01$

b) $\sin \pi$

c) $\cos \pi$

Simple harmonic motion

Last year, when you studied wave motion, you learnt that all types of waves require a vibrating source to produce them. For example, vibrating or oscillating electric and magnetic fields are responsible for the production of electromagnetic waves. There are also many examples of mechanical waves – sound waves, water waves, waves on strings or wires, and shock waves from earthquakes. All these waves are caused by a vibrating source.

In this chapter, you are going to be studying oscillations about a fixed point. Figure 2.1 shows three examples of mechanical oscillations – a clamped ruler, a mass on a spring and a pendulum. In each of these examples, we observe that the motion is repetitive about a fixed point. The oscillating object is stationary at each end of the motion, and is moving with its maximum speed, in either direction, at the midpoint.

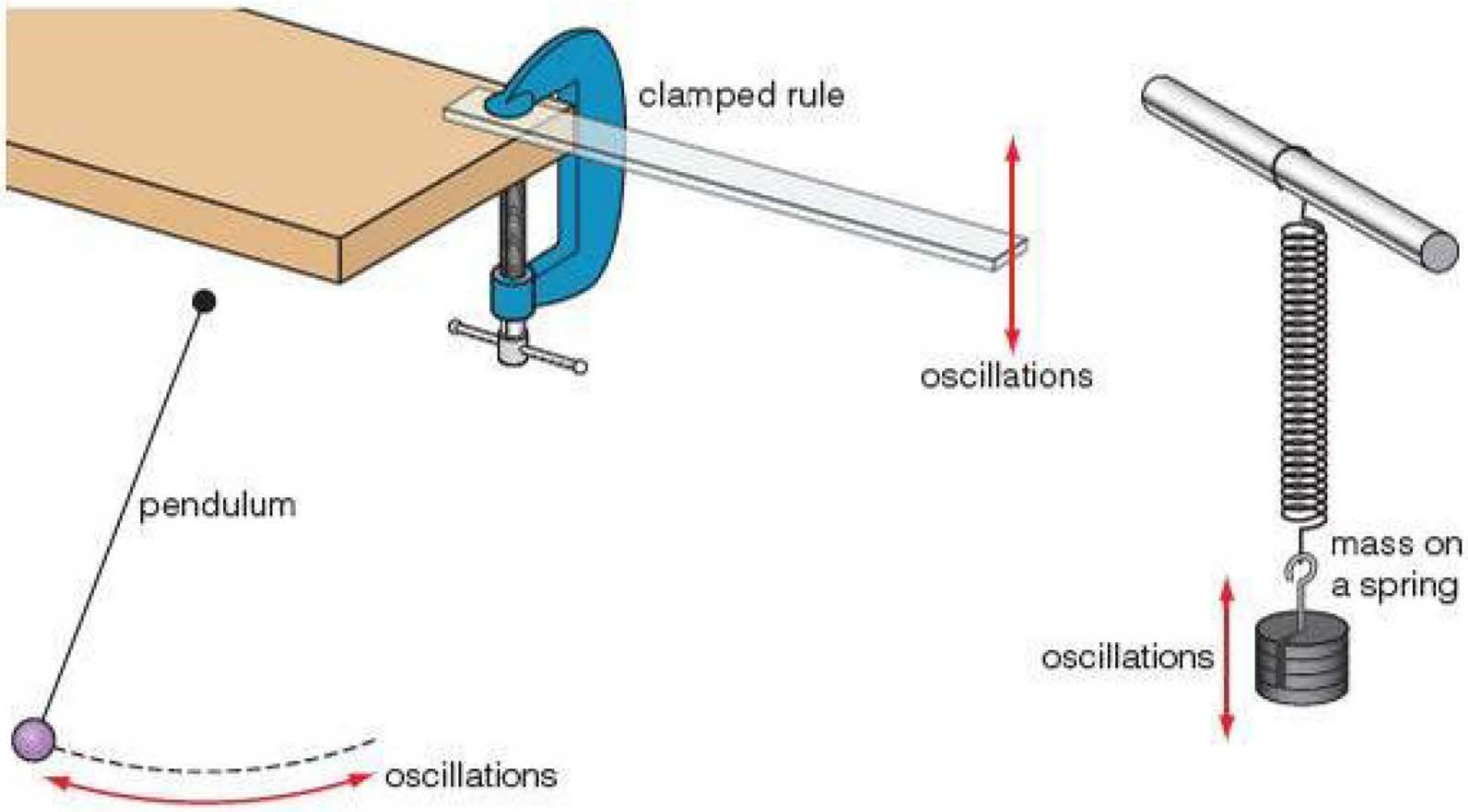


Figure 2.1

To a good approximation, these objects have these features in common:

- The force acting on the body always acts towards the equilibrium position.
- The force acting on the body is proportional to its displacement from the equilibrium position.

Simple harmonic motion A repetitive motion about an equilibrium position. The equation that describes this motion is $F = -kx$.

An oscillating body that satisfies both these conditions is said to be moving with **simple harmonic motion** or SHM. The two features of the motion above may be summarised in the equation:

$$F = -kx$$

or

$$ma = -kx$$

or

$$a = -\frac{k}{m}x \qquad \text{(i)}$$

Here k is a constant (which can be called the spring constant or the force per unit displacement). The significance of the minus sign is that it shows that the force (and acceleration) are in the opposite direction to the displacement. Force, acceleration and displacement are vectors, so we must define the direction of the displacement and motion.

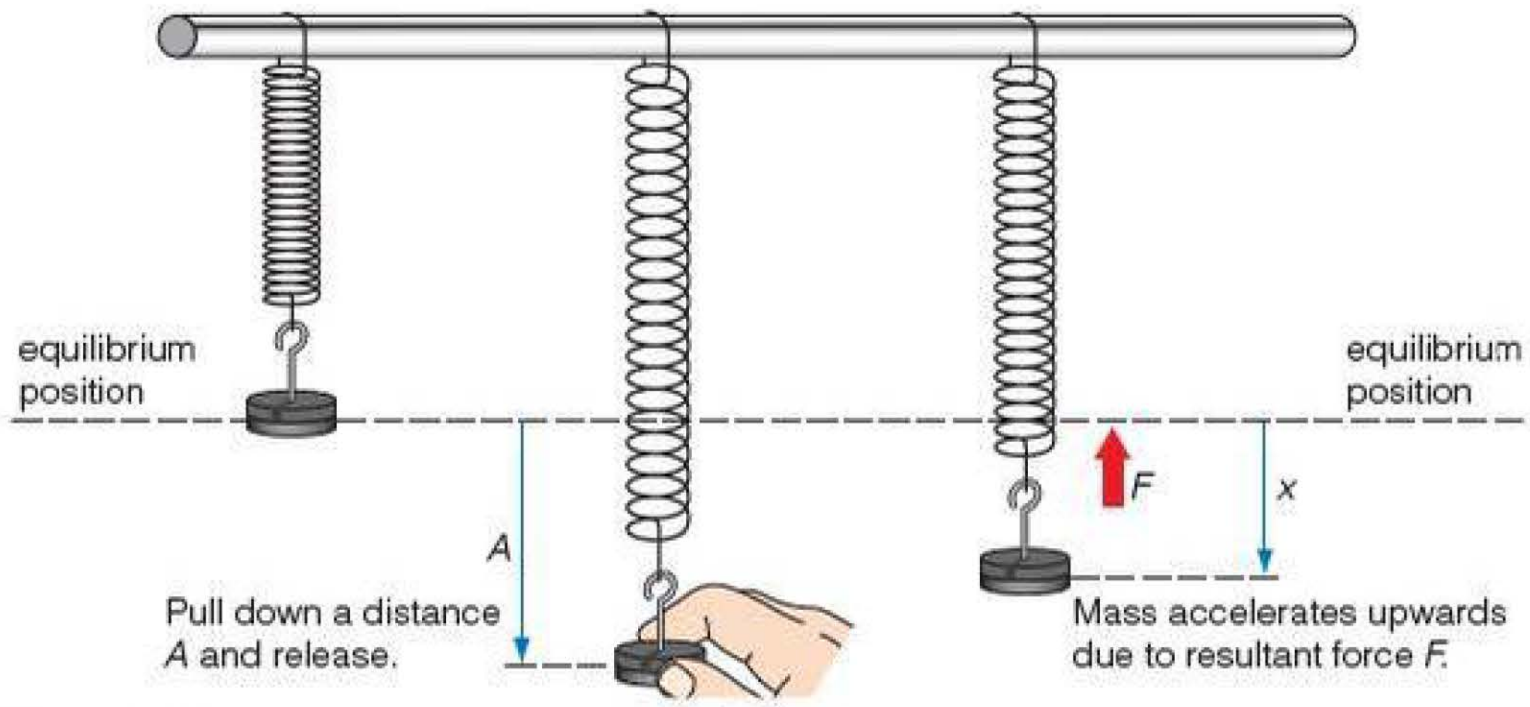


Figure 2.2 A mass on a spring is a simple harmonic oscillator.

Figure 2.2 shows some important features of a simple harmonic oscillator. When at rest, the mass hangs in its *equilibrium position*. A is the *amplitude* of the oscillation – this is the greatest displacement of the oscillator from its equilibrium position. When the mass is displaced downwards by x, the force acts upwards on the mass towards the equilibrium position.

If you investigate the time period of a simple harmonic oscillator, you will discover that the time period does *not* depend on the amplitude of the oscillations, provided the amplitude is small. If you overstretch a spring or swing a pendulum through a large angle, the motion ceases to be simple harmonic.

Mathematical description of SHM

Figure 2.3

The question we want to answer is this: How do the displacement, velocity and acceleration of a simple harmonic oscillator vary with time?

Figure 2.3 gives us some insight. Here a mass is oscillating up and down on a spring. The mass has been stroboscopically photographed by a camera, moving horizontally at a constant speed. The shape of the curve we see is sinusoidal. Figure 2.4 shows how the displacement of the mass varies with time if it is released from rest with an amplitude A.

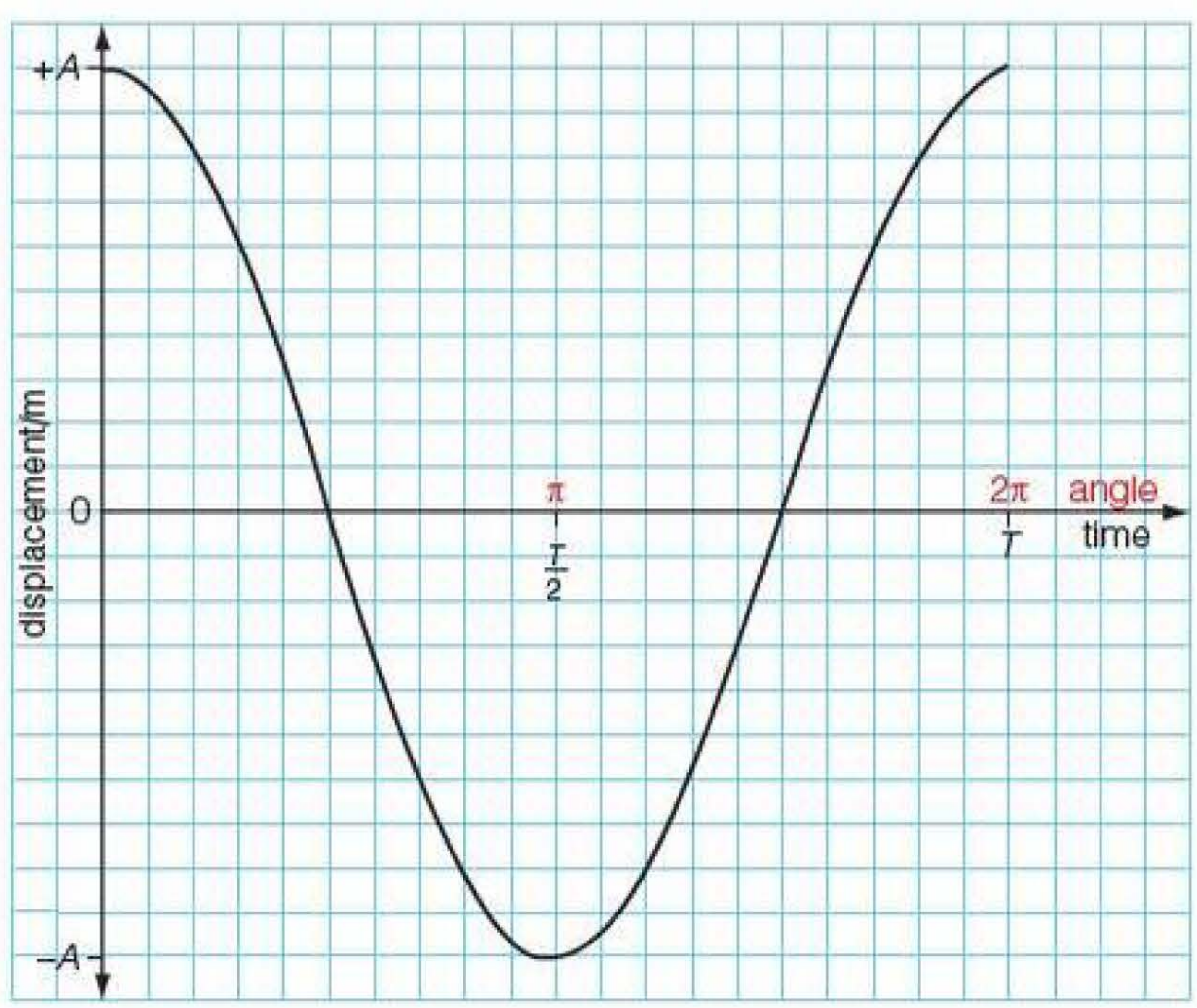


Figure 2.4

TIP

NOTE

In the AQA specification the symbol ω is used to represent $2\pi f$ or $\frac{2\pi}{T}$, so you will also meet equations in this form:

$x = A\cos\omega t$

$a = -\omega^2 x$

$v_{max} = \omega A$

$a_{max} = \omega^2 A$

$v = \pm\omega\sqrt{A^2 - x^2}$

The graph has the shape of a cosine function, which can be written as

$$x = A\cos\theta$$

But the value of θ is 2π after one complete cycle so, at the end of the cycle,

$$x = A\cos(2\pi)$$

However, we know that the oscillation is a function of t. The function that fits the equation is

$$x = A\cos\left(\frac{2\pi t}{T}\right)$$

or

$$x = A\cos(2\pi ft) \qquad \text{(ii)}$$

where T is the time period for one oscillation. Remember that $T = \frac{1}{f}$ where f is the frequency of the oscillation. This function solves the equation because after one oscillation $t = T$, so the inside of the bracket has the value 2π.

Once we have an equation that connects displacement with time, we can also produce equations that link velocity with time, and then also acceleration with time. These are shown below:

$$x = A\cos(2\pi ft)$$

$$v = -2\pi fA\sin(2\pi ft) \qquad \text{(iii)}$$

$$a = -(2\pi f)^2 A\cos(2\pi ft) \qquad \text{(iv)}$$

and since $x = A\cos(2\pi ft)$

$$a = -(2\pi f)^2 x \qquad \text{(v)}$$

We derive this assuming $x = A$ when $t = 0$. However, the same equation would have been obtained whatever the starting condition.

(Mathematicians will see that the velocity equation is the derivative of the displacement equation, and that the acceleration equation is the derivative of the velocity equation.)

Since the maximum value of a sine or cosine function is 1, we can write the maximum values for x, v and a as follows:

$$x_{max} = A \qquad \text{(vi)}$$

$$v_{max} = 2\pi fA \qquad \text{(vii)}$$

$$a_{max} = (2\pi f)^2 A \qquad \text{(viii)}$$

We also write down one further useful equation now, which allows us to calculate the velocity v of an oscillating particle at any displacement x:

$$v = \pm 2\pi f\sqrt{A^2 - x^2} \qquad \text{(ix)}$$

This will be proved later when we consider the energy of an oscillating system.

Figure 2.5 shows graphically the relationship between x, v and a. These graphs are related to each other.

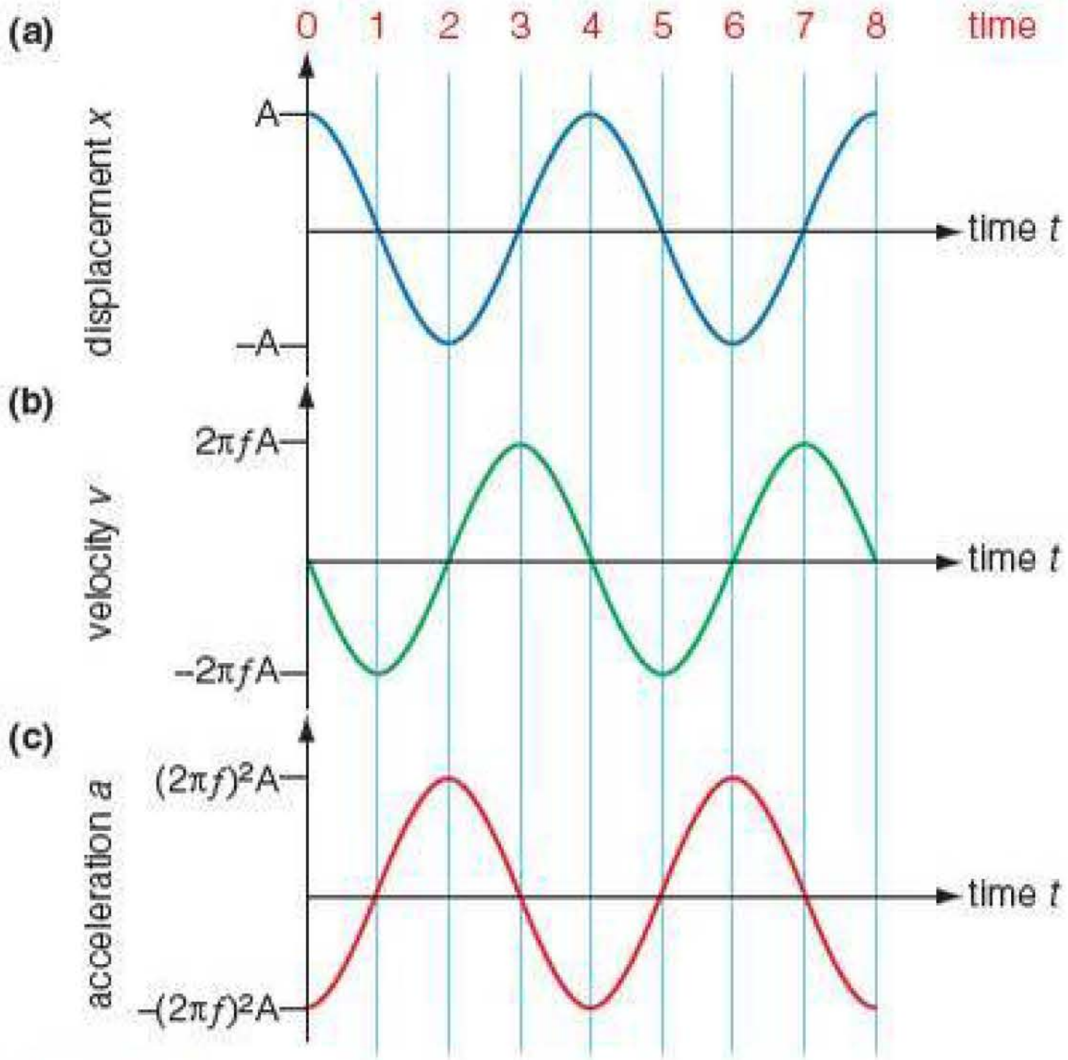


Figure 2.5

- The graph of velocity v against time t links to the gradient of the displacement–time (x–t) graph because

  $$v = \frac{\Delta x}{\Delta t}$$

  For example, at time 0 (in Figure 2.5), the gradient of the x–t graph (a) is zero, so the velocity is zero. At time 1, the gradient of the x–t graph (a) is at its highest and is negative, so the velocity is at its maximum negative value.
- The graph of acceleration a against time t (c) links to the gradient of the velocity–time (v–t) graph (b) because

  $$a = \frac{\Delta v}{\Delta t}$$

  For example, at time 1 (in Figure 2.5), the gradient of the v–t graph (b) is zero, so the acceleration is zero. At time 2, the gradient of the v–t graph (b) is positive and at its largest value, so the acceleration has its largest value.

EXAMPLE

SHM of a mass on a spring

A mass hanging on a spring oscillates with simple harmonic motion. The amplitude of the oscillation is 4.0 cm, and the frequency of the oscillation is 0.5 Hz. The spring is released from rest at its lowest position, 4 cm below its equilibrium position (Figure 2.6).

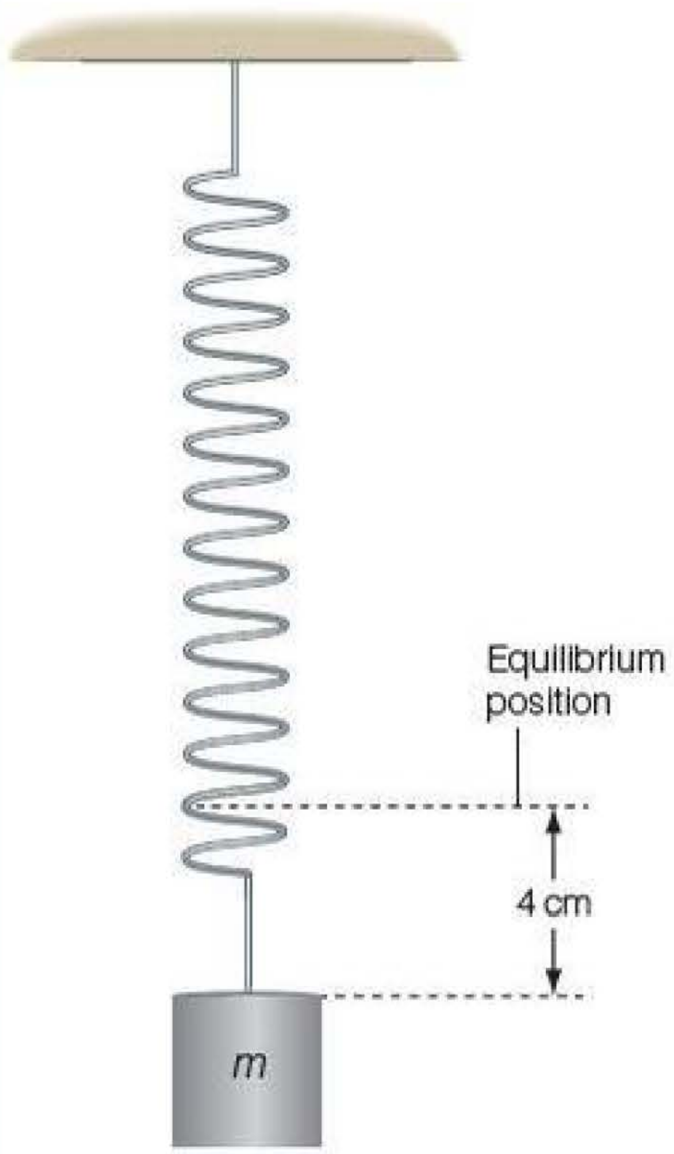


Figure 2.6

1 Calculate the maximum velocity of the mass.

Answer

$$v_{max} = 2\pi f A$$
$$= 2\pi \times 0.5\ s^{-1} \times 0.04\ m$$
$$= 0.13\ m\ s^{-1}$$

2 Calculate the maximum acceleration of the mass.

Answer

$$a_{max} = (2\pi f)^2 A$$
$$= (2\pi \times 0.5\ s^{-1})^2 \times 0.04\ m$$
$$= 0.39\ m\ s^{-2}$$

3 Calculate the acceleration of the mass 1.2 s after release.

Answer

We need to define a direction before applying our formulae. In Figure 2.6 we define positive as our downwards direction. (This is arbitrary. You will get the same answer if you choose this direction to be negative.)

$$a = -(2\pi f)^2 A\cos(2\pi ft)$$
$$= -(2\pi \times 0.5\ s^{-1})^2 \times 0.04\ m \times \cos(2\pi \times 0.5 \times 1.2)$$
$$= -9.9\ s^{-2} \times 0.04\ m \times (-0.81)$$
$$= 0.32\ m\ s^{-2}$$

Since this is positive, the acceleration is downwards.

4 Calculate the velocity of the mass when it is displaced 2 cm from itsequilibrium position.

$$v = \pm 2\pi f\sqrt{A^2 - x^2}$$

$$= \pm 2\pi \times 0.5\,\text{s}^{-1}(0.04^2 - 0.02^2)^{\frac{1}{2}}\,\text{m}$$

$$= \pm \pi\,\text{s}^{-1} \times 0.035\,\text{m}$$

$$= \pm 0.11\,\text{m}\,\text{s}^{-1}$$

TEST YOURSELF

1 A pendulum is released from point A in Figure 2.7. It swings from A to C and back with SHM. The distance AC is 24 cm, and the time taken to travel from A to B is 0.8 s.
   a) State the frequency of the oscillation.
   b) State the amplitude of the oscillation.
   c) i) Calculate the speed of the pendulum as it passes B.
      ii) Calculate the velocity of the pendulum when it is displaced 4 cm from B.
   d) Calculate the acceleration of the pendulum when it is displaced 6 cm to the right of B.

2 A ruler is clamped to a bench. When the free end is displaced, the ruler oscillates with SHM, at a frequency of 100 Hz. The amplitude of the oscillations is 1.8 mm.
   a) Calculate the highest velocity of the ruler.
   b) Calculate the highest acceleration of the ruler. State where this is.
   c) State the point in the oscillation where
      i) the acceleration of the ruler is zero
      ii) the velocity of the ruler is zero.

3 A marker buoy is oscillating in a vertical line with SHM. The buoy takes 2.8 s for one oscillation and is seen to fall a distance of 1.8 m from its highest to its lowest point.
   a) Calculate the buoy's maximum velocity.
   b) Calculate the buoy's acceleration when it is 1.4 m below its highest point.

4 Figure 2.8 shows the displacement of a particle oscillating with SHM. To represent its motion, the equation $x = A\sin(2\pi ft)$ is used, where $x = 0$ when $t = 0$.
   Copy the diagram and add sketches, using the same time axis, to show the variations of v and a with time.

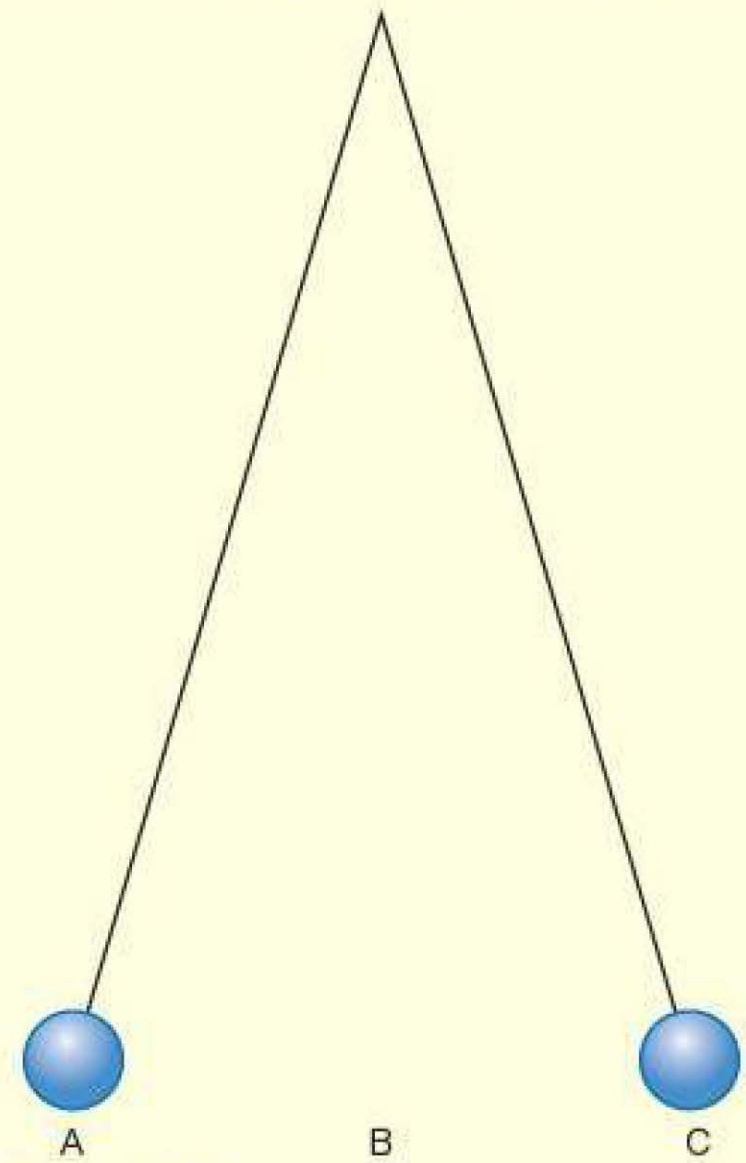


Figure 2.7

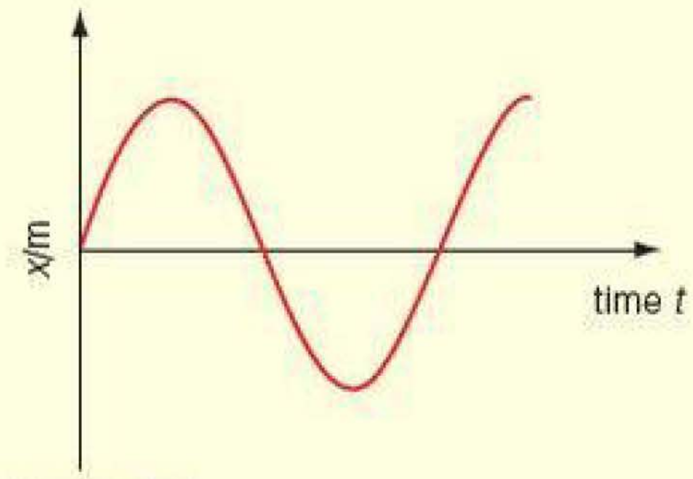


Figure 2.8

Time period of oscillations

We can combine two of the equations that we used in the previous sections to produce a further equation that links the time period of oscillation, T, to the mass of the oscillating particle, m, and the force per unit displacement, k.

EXAMPLE

Calculating the time period

1 A mass of 400 g hangs on a steel spring, which has a spring constant of 0.20 N cm^{-1}. Calculate the time period for one oscillation.

Answer

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2\pi\sqrt{\frac{0.4\text{ kg}}{20\text{ N m}^{-1}}}$$

$$= 0.89\text{ s} \approx 0.9\text{ s}$$

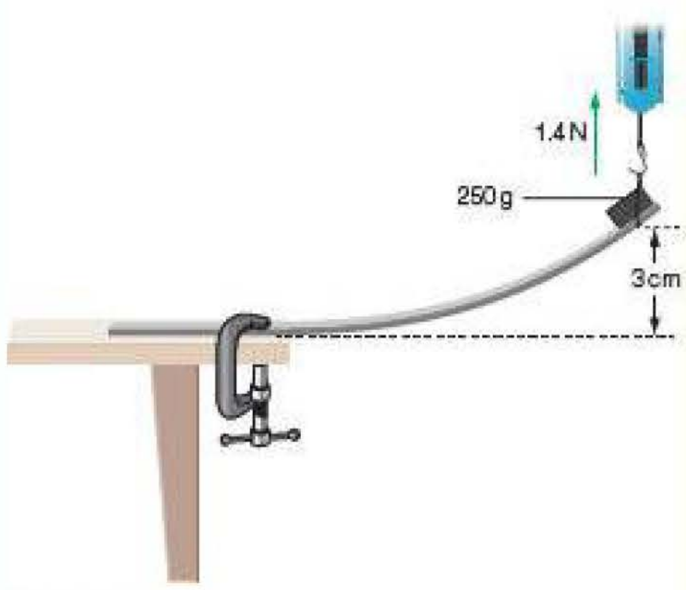


Figure 2.9

2 In Figure 2.9, a light ruler is clamped to a desk and a mass of 250 g is attached securely to the free end. When a newtonmeter is attached to the end of the ruler, a force of 1.4 N displaces the ruler by 3 cm. Calculate the time period of the oscillations, stating any assumptions you make.

Answer

The force per unit displacement is

$$k = \frac{1.4\text{ N}}{0.03\text{ m}} = 47\text{ N m}^{-1}$$

so

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2\pi\sqrt{\frac{0.25\text{ kg}}{47\text{ N m}^{-1}}}$$

$$= 0.46\text{ s} \approx 0.5\text{ s}$$

The two equations that define the motion of a simple harmonic oscillator that we need from the earlier sections are equations (i) and (v):

$$a = -\frac{k}{m}x \quad \text{and} \quad a = -(2\pi f)^2 x$$

Combining these gives

$$\frac{k}{m} = (2\pi f)^2$$

or

$$\frac{k}{m} = \left(\frac{2\pi}{T}\right)^2$$

or

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad \text{(x)}$$

Once you recognise that a particle is oscillating with SHM, you can use this general solution to calculate the time period of any oscillator.

The simple pendulum

Figure 2.10(a) shows a pendulum held at rest by a small sideways force F. Figure 2.10(b) shows the three forces acting on the pendulum bob to keep it in equilibrium.

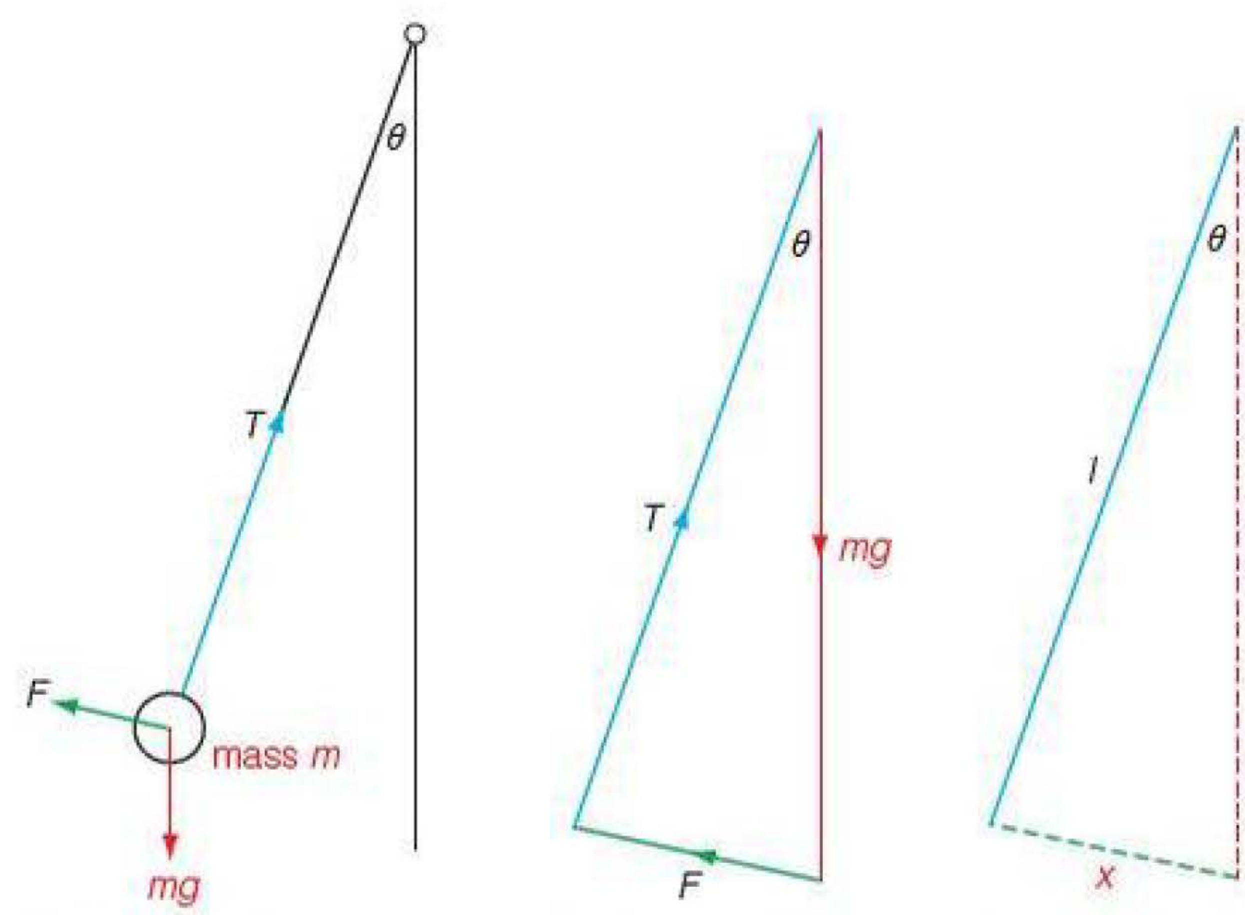


Figure 2.10(a) Figure 2.10(b) Figure 2.10(c)

The force $F = mg\sin\theta$. For small angles we have $\theta \approx \sin\theta$, and therefore

$$F = mg\theta. \qquad \text{(xi)}$$

Figure 2.10(c) shows that x can be related to the length of the pendulum, l, by

$$x = l\tan\theta.$$

For small angles, we also have $\theta \approx \tan\theta$, and therefore

$$x = l\theta$$

and

$$\theta = \frac{x}{l} \qquad \text{(xii)}$$

Combining equations (xi) and (xii) gives

$$F = mg\frac{x}{l}$$

When the pendulum is released, the restoring force now acts in the opposite direction. So

$$ma = -mg\frac{x}{l}$$

and

$$a = -g\frac{x}{l} = -\frac{g}{l}x$$

This is the defining equation for SHM because the acceleration is proportional to, and in the opposite direction to, the displacement.

Therefore

$$(2\pi f)^2 = \frac{g}{l}$$

$$\left(\frac{2\pi}{T}\right)^2 = \frac{g}{l}$$

and

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad \text{(xiii)}$$

REQUIRED PRACTICAL 7

Investigation into simple harmonic motion using a mass–spring system and a simple pendulum.

Note: This is just one example of how you might tackle this required practical.

Make a simple pendulum using a small mass hanging on a piece of string about 1.5 m long.

- Investigate whether the time period of the pendulum depends on the amplitude of the swings.
- Investigate how the time period, *T*, of the pendulum varies with length between 1.5 m and 0.2 m.
- Plot a suitable straight line graph to investigate whether: $T^2 \alpha l$. Use the gradient of the graph to find a value for *g*.

ACTIVITY

Oscillation of a tethered trolley

The purpose of this activity is to investigate how the time period of a tethered trolley depends on its mass. The mass of the trolley is changed by putting additional weights on top of it.

In Figure 2.11, the identical springs A and B are both under tension, but in the trolley's equilibrium position the forces from the two springs balance.

⇨

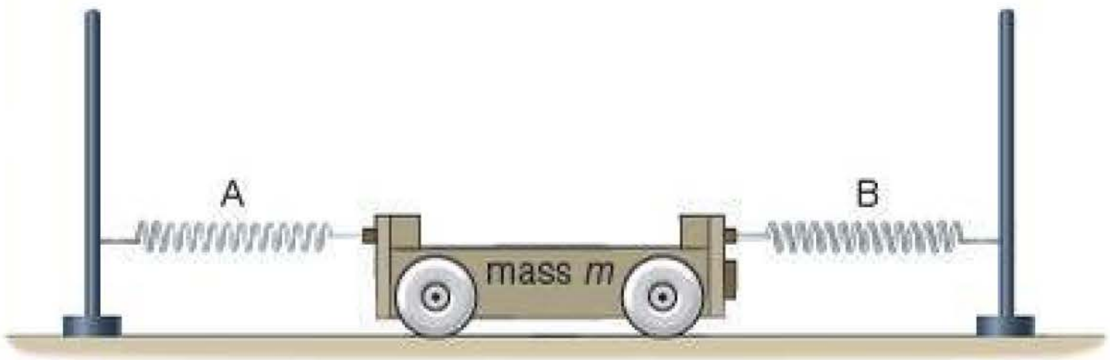


Figure 2.11 A tethered trolley oscillates with SHM. Each spring has a spring constant k.

Each spring has a spring constant k. What is the resultant force acting on the trolley when it is displaced a distance x to the right?

- Spring A exerts an extra force of kx to the left.
- Spring B exerts a force *reduced* by kx to the right.

So the resultant force on the trolley is $2kx$ to the left.

1 Explain why the time period of the tethered trolley is given by

$$T = 2\pi\sqrt{\frac{m}{2k}}$$

2 In an experiment, a student determines the spring constant of her springs to be 17.8 N m^{-1}. She then recorded the set of data shown in Table 2.1 for the oscillation of her trolley, as she varied its mass.

Table 2.1

Mass of trolley/kg	1.0	1.5	2.0	2.5	3.0	4.0
Time for five oscillations/s	5.3	6.4	7.4	8.3	9.1	10.5

Plot a graph of T^2 against m.

a) Discuss whether or not your graph is consistent with the formula quoted in part 1.

b) Determine the gradient of your graph. Comment on this result.

ACTIVITY

Other systems that might show SHM

Figure 2.12 shows two systems to investigate. It is suggested that both are simple harmonic oscillators.

1 A U-tube is partly filled with water, as shown in (a). The total length of the water in the tube is L.

a) Investigate whether or not the amplitude of the oscillations affects their time period.

b) Check to see if the time period of the oscillations agrees with the formula:

$$T = 2\pi\sqrt{\frac{L}{2g}}$$

2 A weighted boiling tube is allowed to oscillate up and down in a large beaker of water (b). In its equilibrium position, a length L of the tube is submerged.

a) Investigate whether or not the amplitude of the oscillations affects their time period.

b) Check to see if the time period of the oscillations agrees with the formula:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

(Both formulae for the time periods are derived in the on-line material.)
Note: Make sure to wash your hands after handling lead shot

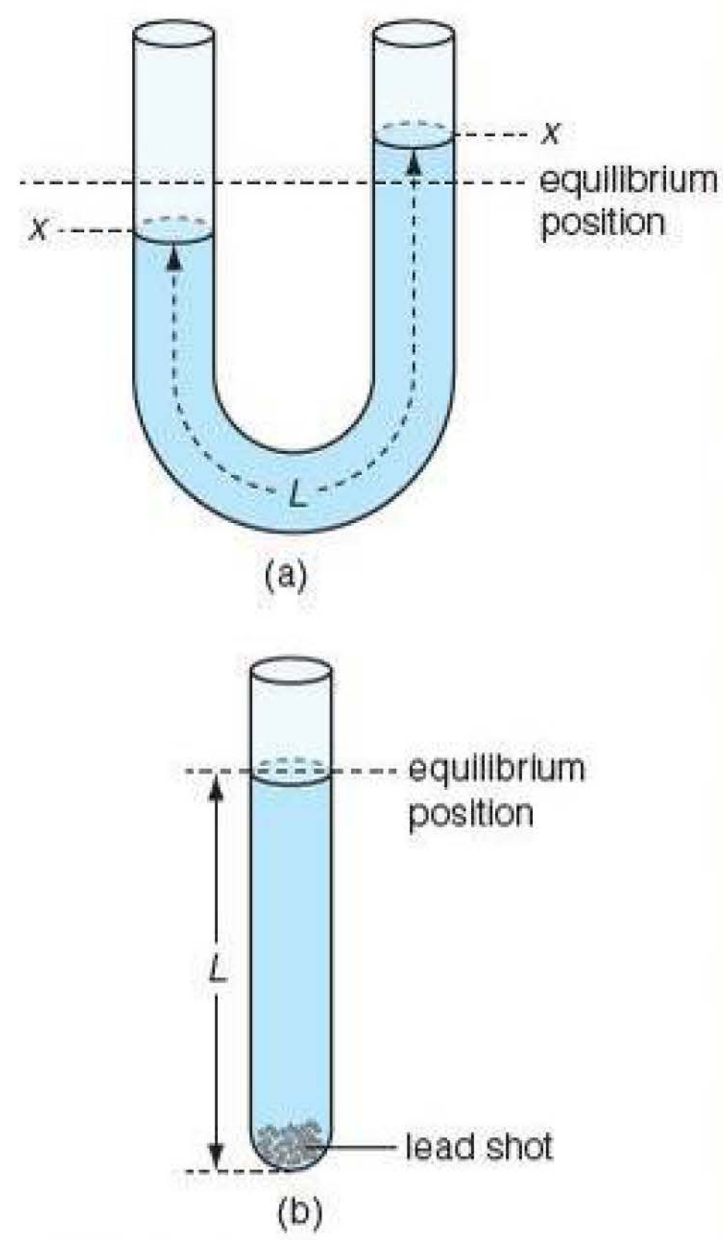


Figure 2.12

TEST YOURSELF

5 A pendulum has a length of 2 m. Calculate its time period of oscillation in each of these places:

a) on Earth

b) on the Moon, where $g = 1.6\,N\,kg^{-1}$

c) on a comet, where $g = 0.0006\,N\,kg^{-1}$.

6 A 'baby bouncer' is a harness that can be used to amuse and exercise a baby before the baby can walk. Figure 2.13 shows a baby enjoying this experience. This is a simplified model, in fact, the baby's feet will touch the floor.

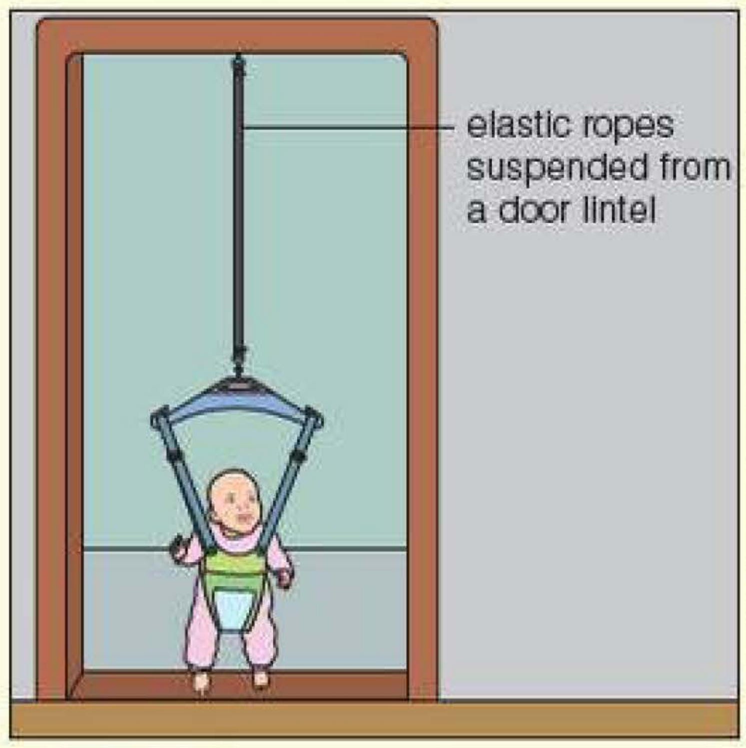


Figure 2.13

The suspension ropes for a bouncer are 1.30 m long and stretch to 1.48 m when a baby of mass 9.5 kg is put in it.

a) i) Determine the spring constant for the baby bouncer.

ii) Determine the time period of the baby's simple harmonic motion.

iii) Determine the baby's maximum speed, when released from 10 cm above the equilibrium position.

b) When the baby was bouncing three months later, the baby's father noticed that the time period of the oscillations had increased to 1.0 s. He is delighted that his baby has put on weight (or mass, as baby's mother correctly points out). What is baby's mass now?

c) Explain what is meant by the terms

i) weight

ii) mass.

7 Figure 2.14 shows a graph of displacement against time for a mass of 0.5 kg oscillating on a spring.

a) Use the graph to estimate the speed of the mass between points A and B.

b) Use your knowledge of SHM equations to calculate the theoretical maximum speed from the graph, using the information shown on the axes.

c) Calculate the spring constant of the spring.

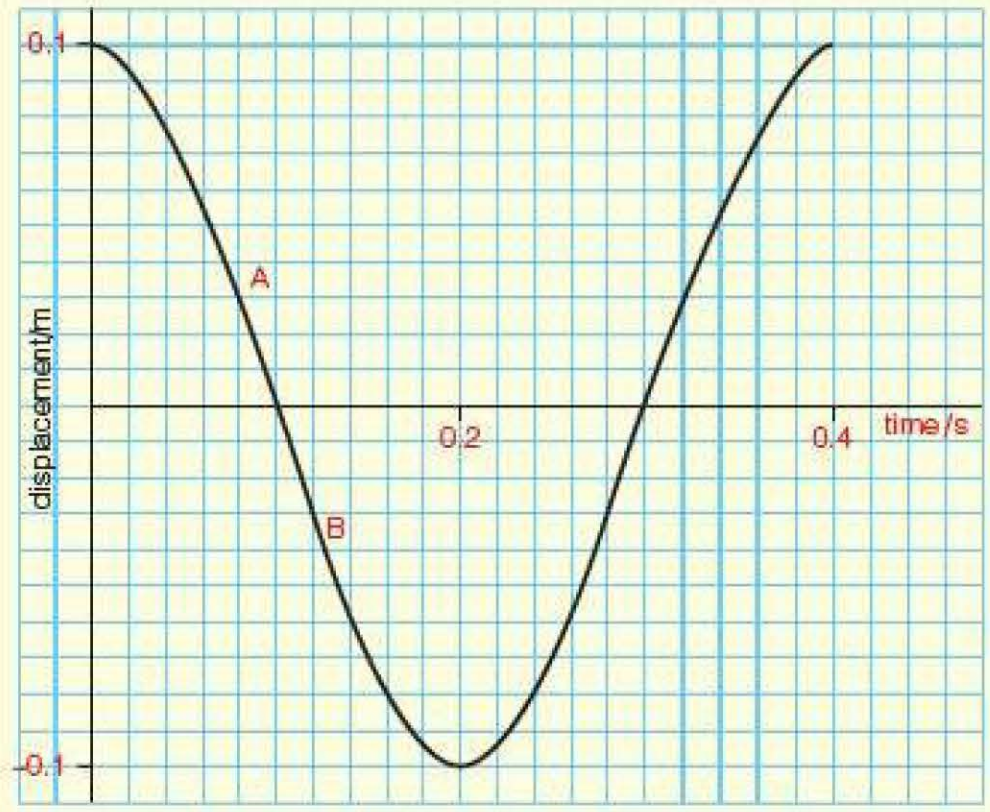


Figure 2.14

Energy in simple harmonic motion

Figure 2.15 shows a pendulum swinging backwards and forwards from A to B to C, and then back to B and A. As the pendulum moves, there is a continuing transfer of energy from one form to another.

- At A, the velocity of the pendulum bob is zero. Here the kinetic energy, E_k, is zero, but the bob has its maximum potential energy, E_p.
- At B, the velocity of the pendulum is at its maximum value, and the bob is at its lowest height. Therefore, E_k is at its maximum, and E_p is at its minimum value. This can be defined as the system's zero point of potential energy.

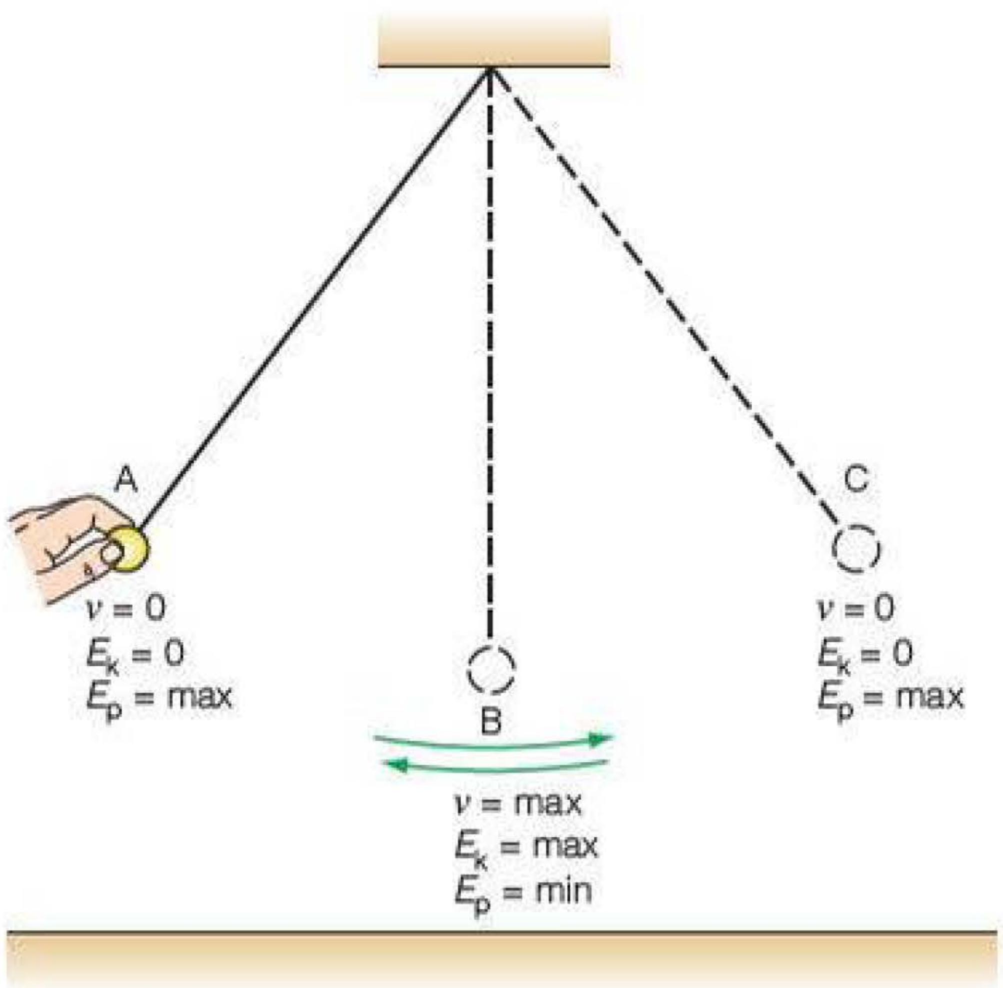


Figure 2.15

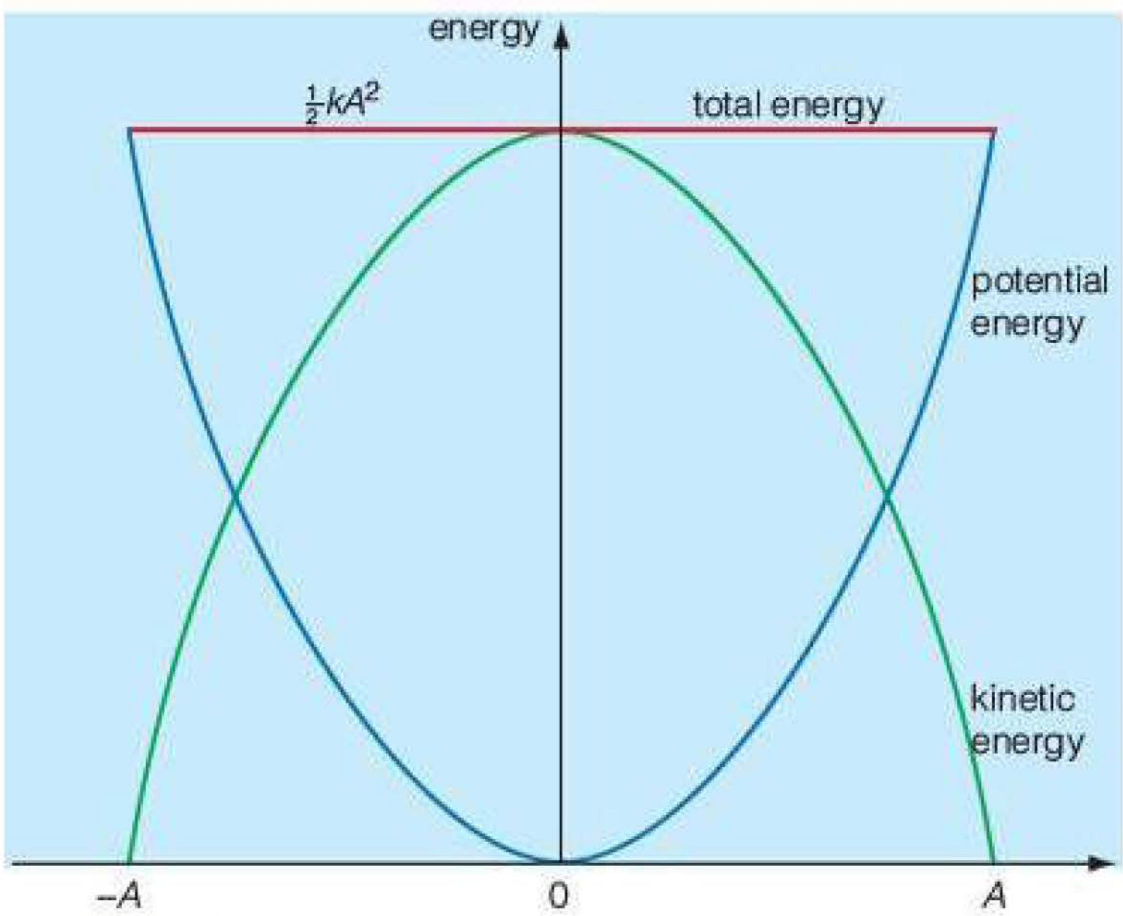


Figure 2.16

- At C, the velocity is once more zero. So the bob has zero kinetic energy and its maximum value of potential energy.

The potential energy can be calculated as follows. The force acting on the pendulum along its line of motion is $-kx$ when it has been displaced by x (where k is the force per unit displacement).

The work done to take the mass to x is

$$W = \text{average force} \times \text{distance}$$
$$= -\frac{1}{2}kx \times (-x)$$
$$= \frac{1}{2}kx^2$$

or the potential energy is given by

$$E_p = \frac{1}{2}kx^2 \qquad \text{(xiv)}$$

So the maximum potential energy of any simple harmonic oscillator is $\frac{1}{2}kA^2$, where A is the amplitude of the displacement.

The kinetic energy of the oscillator at a velocity v is

$$E_k = \frac{1}{2}mv^2$$

Figure 2.16 shows how the potential energy E_p and the kinetic energy E_k change with displacement for a simple harmonic oscillator. The total energy of the system remains constant (assuming there are no energy transfers out of the system.)

Figure 2.17 shows how the potential, kinetic and total energies change with time as the pendulum oscillates. In one oscillation, the potential energy and the kinetic energy both reach a maximum twice. The total energy remains constant.

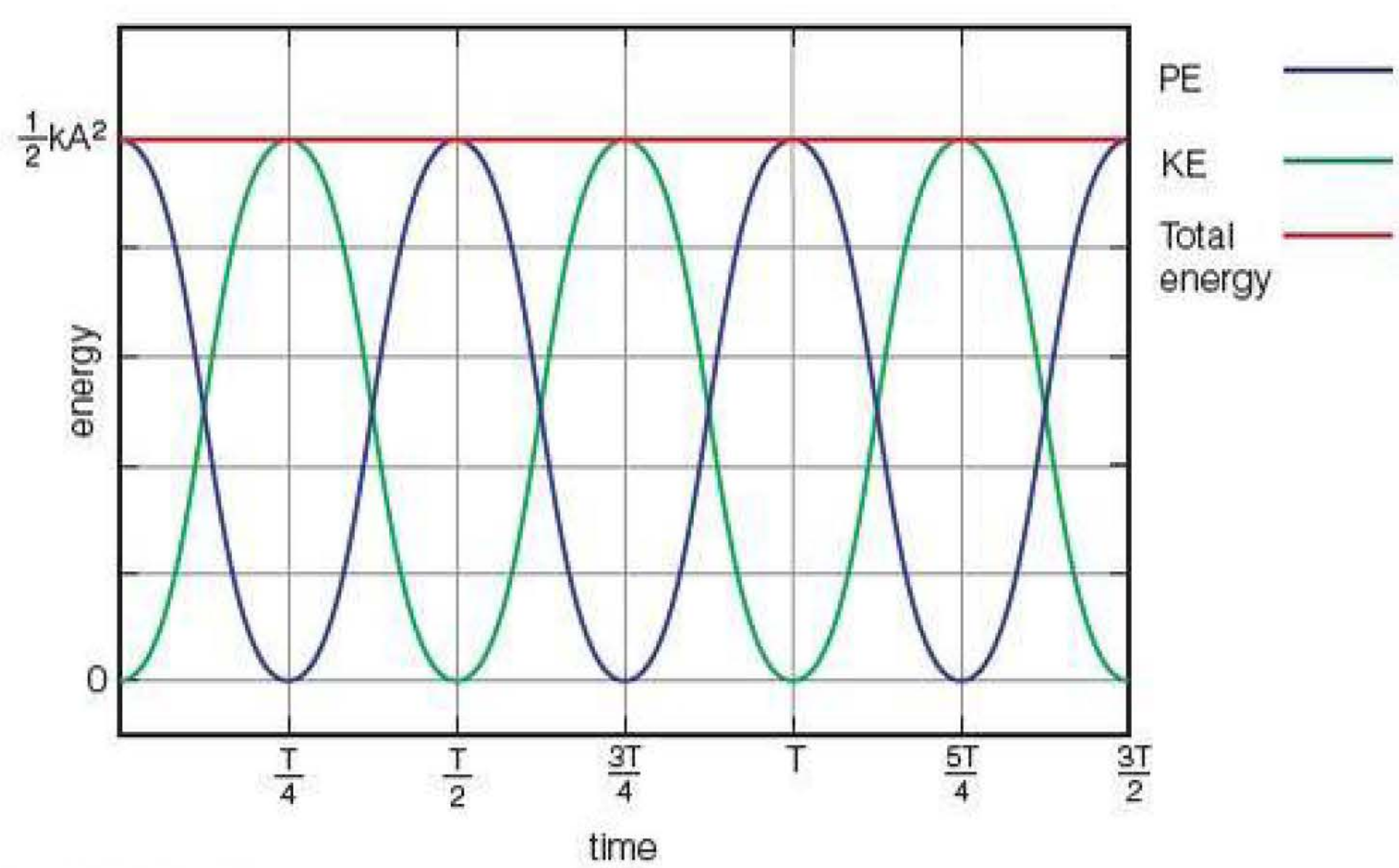


Figure 2.17

We can now write an equation to link these three energies at a displacement x:

total energy = kinetic energy + potential energy

$$\frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \qquad \text{(xv)}$$

So

$$mv^2 = k(A^2 - x^2)$$

and

$$v = \pm\left(\frac{k}{m}\right)^{\frac{1}{2}}(A^2 - x^2)^{\frac{1}{2}} \qquad \text{(xvi)}$$

(Remember that when you take the square root of a function, there is a positive and a negative root.) But

$$T = 2\pi\sqrt{\frac{m}{k}}$$

or

$$\frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}$$

and so

$$2\pi f = \sqrt{\frac{k}{m}} \qquad \text{(xvii)}$$

When equation (xvii) is substituted into equation (xvi), we get the familiar equation for velocity:

$$v = \pm 2\pi f(A^2 - x^2)^{\frac{1}{2}}$$

or

$$v = \pm 2\pi f\sqrt{A^2 - x^2} \qquad \text{(xviii)}$$

TEST YOURSELF

8 A simple model of a diatomic gas molecule treats the two atoms as small masses, which are connected by an atomic bond that behaves like a tiny spring. The atoms in a particular molecule vibrate with SHM at a frequency of 10^{13} Hz and amplitude 2×10^{-12} m. The mass of each atom is about 10^{-25} kg.

a) What fraction of a typical atomic separation does this amplitude represent?

b) Calculate the approximate force constant of the interatomic bond.

c) Calculate the total energy of vibration of the two atoms:

i) in joules (J)

ii) in electronvolts (eV).

9 Draw a sketch to show how the potential energy, kinetic energy and total energy of a particle, oscillating with SHM, vary with the particle's displacement.

10 A pendulum with a mass of 0.1 kg oscillates with an amplitude of 0.2 m. When the displacement of the pendulum is at its maximum from the equilibrium ⇨

point, the pendulum has a potential energy of 0.08 J, and when the displacement is 0.1 m, the potential energy is 0.02 J.

a) Calculate the speed of the pendulum at displacements of

i) 0 ii) 0.1 m.

b) i) Use the maximum speed of the pendulum and the amplitude of the swing to calculate the time period of the pendulum.

ii) Now calculate the length of the pendulum.

Free, clamped and forced oscillations

So far, we have only dealt with free oscillations. These are oscillations that (in theory) carry on indefinitely because there are no forces acting to stop the oscillation. A close approximation to a free oscillator is a very heavy pendulum supported by a very fine wire, which is attached to a rigid support. Under these circumstances, the energy transfers from the pendulum are very low, and the pendulum keeps swinging for a long time. Of course, in the end any pendulum stops swinging because frictional forces transfer the pendulum's energy to the surroundings as heat. A pendulum clock can run for a week, because energy from a slowly falling weight gives the pendulum a little energy every time it swings.

Damped oscillation A damped oscillation occurs when friction or wind resistance takes energy out of the oscillation.

In practice, all mechanical oscillations are **damped oscillations**. In such oscillations, the oscillator transfers energy to the surroundings. When the damping is light, energy is transferred slowly. When the damping is heavy, energy is transferred more quickly and the oscillations stop after a few swings. The best way for you to investigate the effects of damping on an oscillator is to use a motion sensor and a data logger. In this way, small changes in amplitude can be recorded, which you could not do by eye.

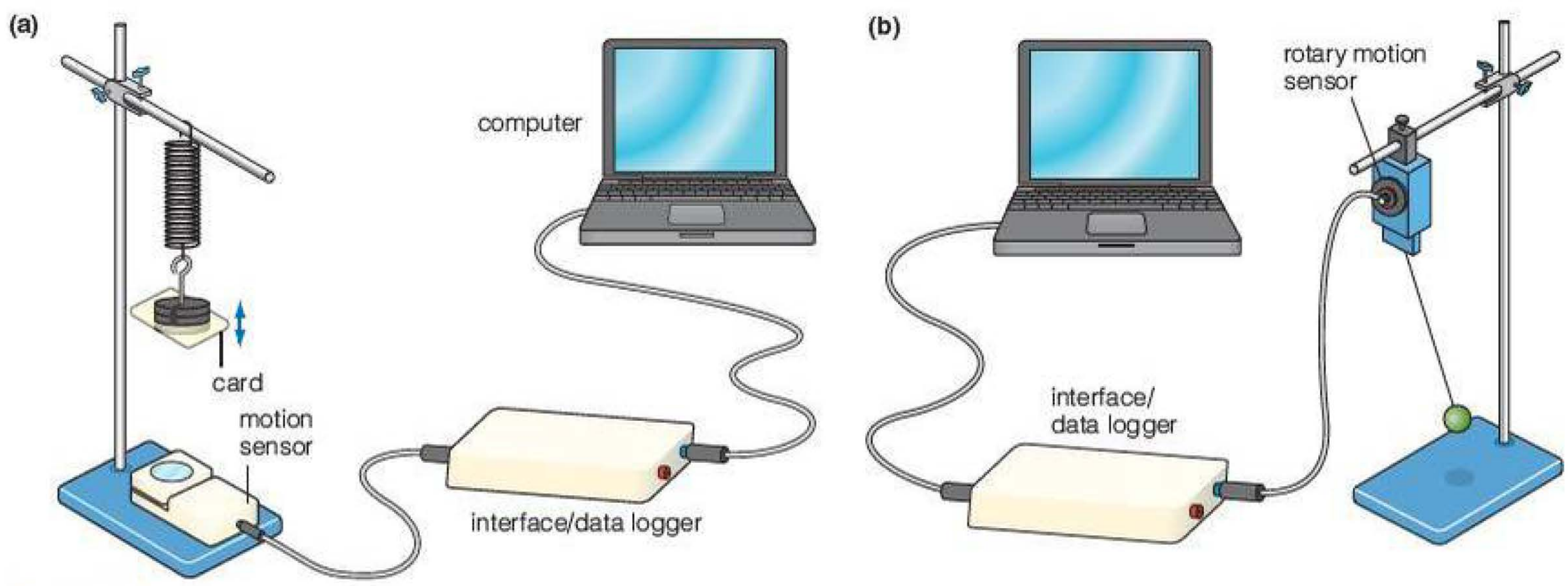


Figure 2.18

Figure 2.18 shows experimental set-ups to investigate damping in two oscillating systems. In Figure 2.18(a) the motion sensor records the displacement against time for a mass on a spring. The card on the bottom has two functions: first, to act as a good reflector for the motion sensor; second, to act as a 'damper'. It causes drag to dampen the oscillations. Increasing the size of the card will increase the damping of the oscillator.

In Figure 2.18(b) a rotary sensor records the motion of a pendulum. The computer records how the angle of rotation varies with time. By attaching cards to the pendulum, the motion of the pendulum can be damped. Figure 2.19 shows how the angular displacement of a pendulum varies with time, for different amounts of damping.

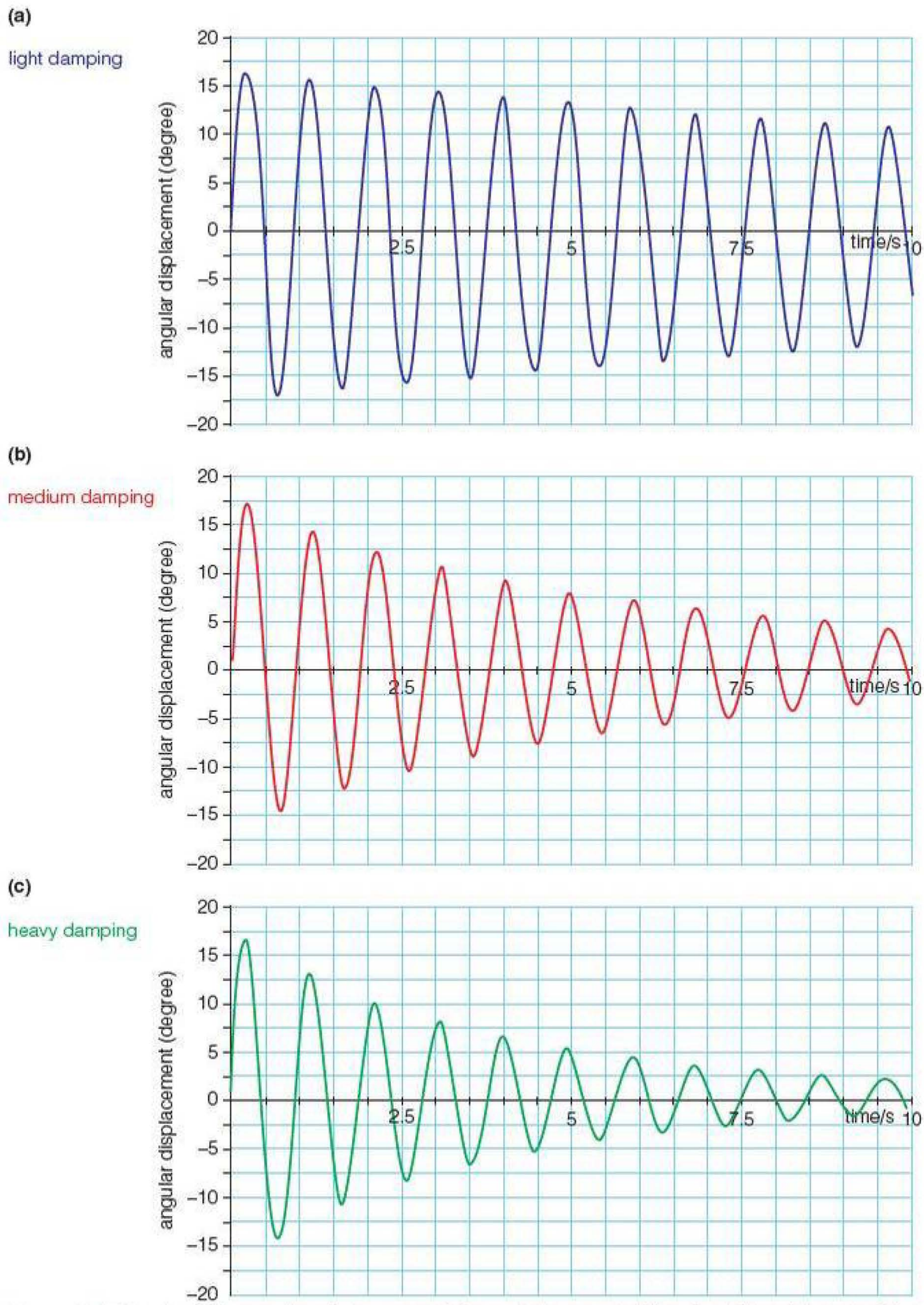


Figure 2.19 Graphs of the angular displacement of a pendulum against time for different levels of damping: (a) light damping, (b) medium damping and (c) heavier damping. Data provided by Data Harvest Group Ltd.

You may be able to use data logging equipment to investigate damping for yourself.

ACTIVITY

A damped oscillator

A teacher suggests that the amplitude of a damped oscillator decays exponentially with time. This means that the amplitude can be described by the following equation:

$$A = A_0 e^{-\lambda t}$$

where A is the amplitude at time t and A_0 is the amplitude at $t = 0$ (the start of the swings).

1 Investigate this relationship, using the graphs in Figure 2.19. Work in teams of three, so that each person can analyse one of the graphs. Rather than working in seconds, work in 'swings'. Then copy and complete Table 2.2. Measure the amplitude after each complete swing.

Table 2.2

Number of swings	Amplitude of graph 2.19(a)/degrees	Amplitude of graph 2.19(b)/degrees	Amplitude of graph 2.19(c)/degrees
0			
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

2 To investigate if the amplitude decays exponentially, we can plot a graph of the natural logarithm of the amplitude against the number of swings. Since it is suggested that the amplitude obeys the law

$$A = A_0 e^{-\lambda t} \text{ (where } t \text{ is the number of swings)}$$

then

$$\ln A = \ln A_0 - \lambda t$$

a) Using your data, plot a graph of $\ln A$ against the number of swings.

b) Discuss whether or not your data follows an exponential law.

c) Determine the 'half-life' $T_{\frac{1}{2}}$ for your pendulum using the expression

$$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

where you determine λ from your graph. (You will find a similar expression derived in Chapter 11.) Here $T_{\frac{1}{2}}$ is the number of swings it takes your pendulum's amplitude to reduce by a half.

Forced oscillations

When you let a pendulum swing freely, it swings at its natural frequency, which is determined by its length. Gradually the pendulum transfers its energy and slows down. It is also possible to force a pendulum to oscillate at a different frequency by pushing it at regular time intervals. This is demonstrated in Figure 2.20. In Figure 2.20(a) a pendulum is driven by hand at a frequency below its natural frequency. The amplitude of the oscillations is low, and the pendulum bob moves in phase with the hand. In Figure 2.20(b) the hand moves backwards and forwards at a high frequency, above the natural frequency of the pendulum. Now the pendulum bob moves out of phase with the hand, and the amplitude of the oscillations is still small. In Figure 2.20(c) the pendulum is given a push at its natural

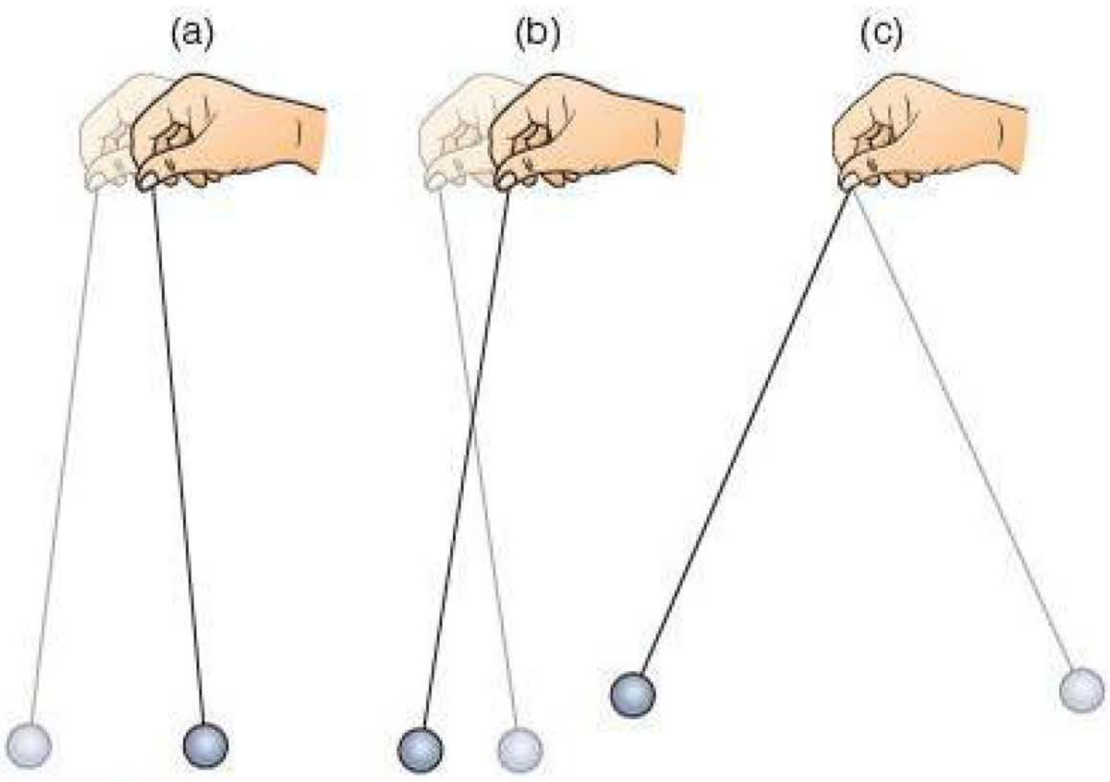


Figure 2.20

Resonant frequency The resonant frequency of a structure (or oscillator) is the same as its natural frequency. When an oscillator is driven or pushed at its natural frequency, the amplitude of the oscillations grows large.

Resonance An oscillator undergoes high-amplitude oscillations (resonance) when the driving frequency is the same as the natural frequency.

frequency. Just as the pendulum stops moving, the hand gives it a small nudge. Now the oscillations of the pendulum become very large. The pendulum is said to be driven at its **resonant frequency**. When you push a child on a swing, you push at the resonant frequency. Just as the child reaches the maximum displacement from the centre, the swing momentarily stops. After that point, you give the swing a push and the amplitude of the swing builds up.

The idea of **resonance** is demonstrated by Barton's pendulums (see Figure 2.21). Here, a number of light pendulums (A–E) are suspended from a string. Also attached to the string is one heavy pendulum (X), which is the 'driving' pendulum. When the driving pendulum is released, it pushes the string as it swings. These pushes then begin to drive the other pendulums. Most of them swing with low amplitude, but the pendulum that has the same length (L) as the driver swings with a large amplitude. This is because its natural frequency is the same as the driving frequency.

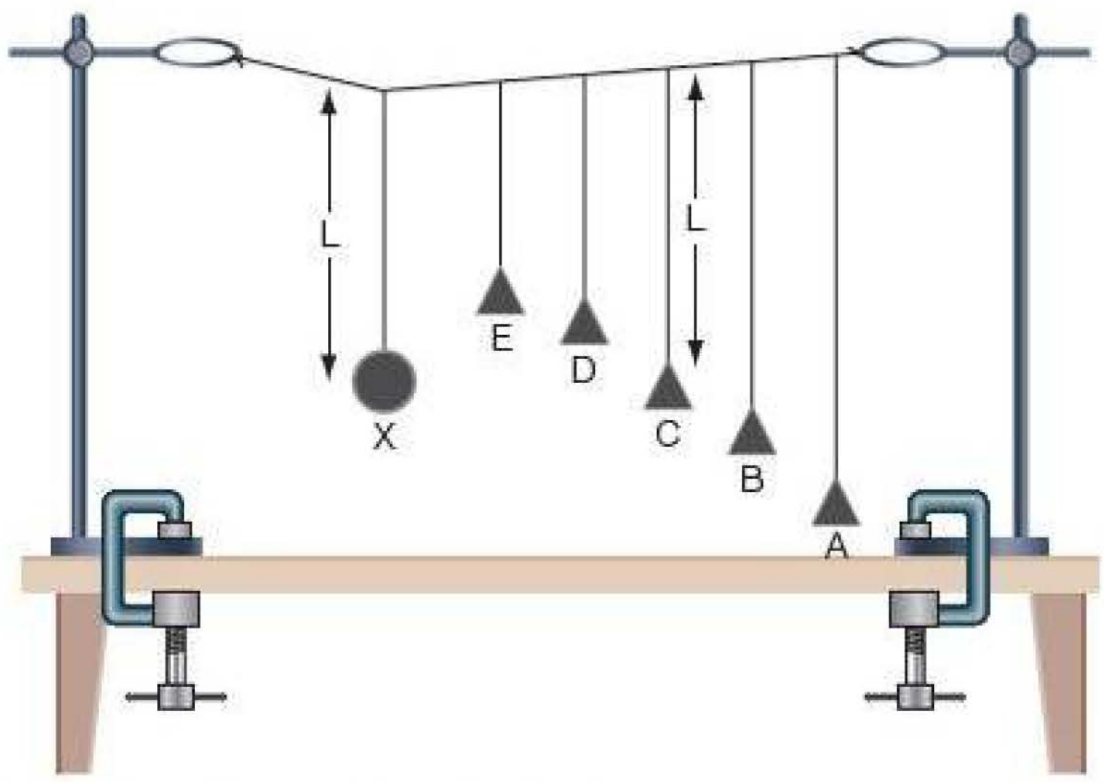


Figure 2.21 Barton's pendulums.

ACTIVITY

Investigating resonance

You can use the apparatus in Figure 2.22 to help you understand how the amplitude of a driven oscillator changes with the driving frequency. The oscillator pulls the string up and down. The string is attached to a mass on a spring, which oscillates up and down at the same frequency as the driving oscillator.

Proceed as follows.

- Choose a spring (or springs) and masses so that the natural frequency of the system is about 1–2 Hz. Measure this frequency.
- Vary the frequency of the oscillator in small steps from about a third of the natural frequency to about three times the natural frequency. Record the amplitude of the oscillations in each case.

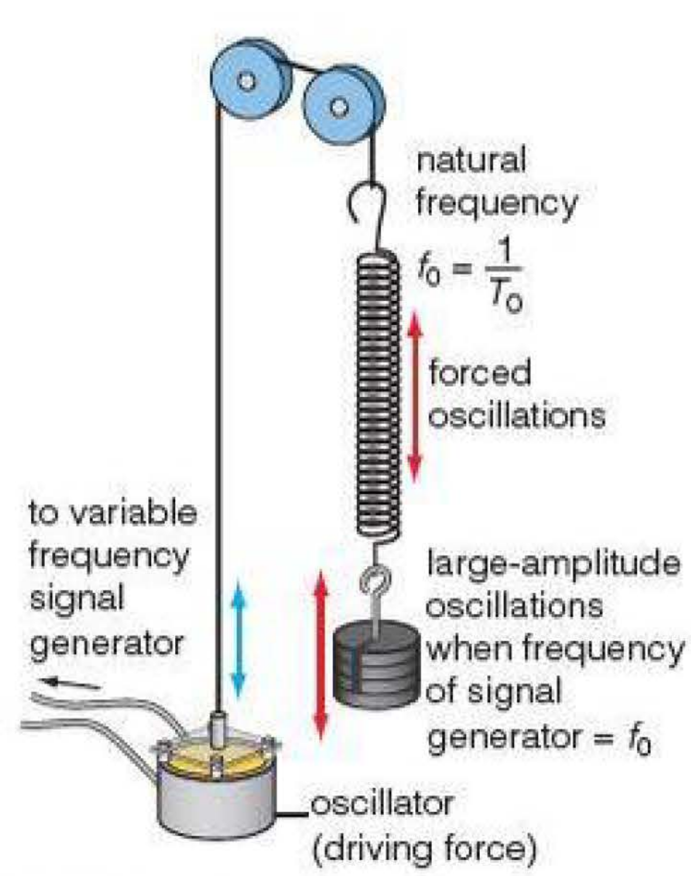


Figure 2.22

⇨

(This can be difficult because the oscillations do not always settle into a steady pattern.)

- Plot a graph of the amplitude of the oscillations against frequency.
- Repeat the experiment with a piece of card attached to the masses to increase damping.

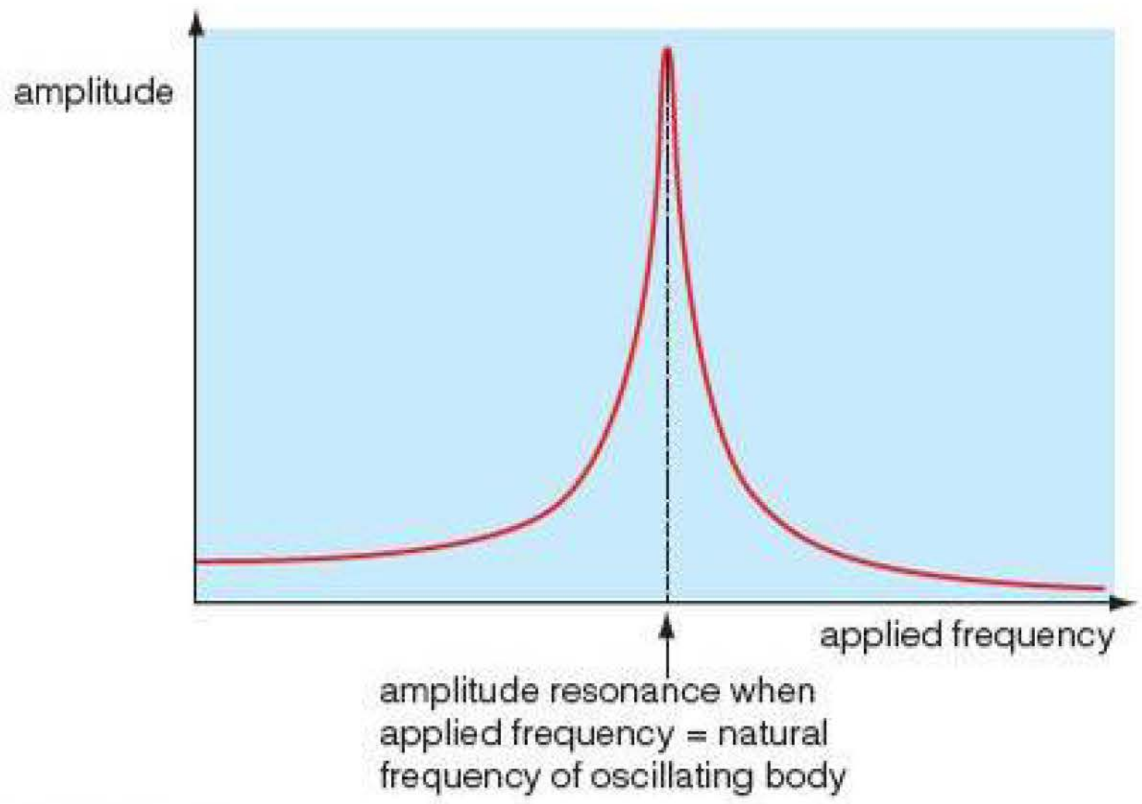


Figure 2.23

Figure 2.23 shows an idealised resonance curve that you might get when you try the activity. The amplitude of the driven oscillations peaks sharply at the natural frequency of the system. The sharpness of the peak depends on the amount of damping. When a system is heavily damped, the peak is not so sharp because energy is being lost from the system and the amplitude does not build up so far.

Figure 2.24 shows the effect of increasing damping on a resonance curve:

- the peak of the amplitude is lower
- the peak is broader
- the peak of the amplitude occurs at a frequency slightly lower than the natural frequency of the system.

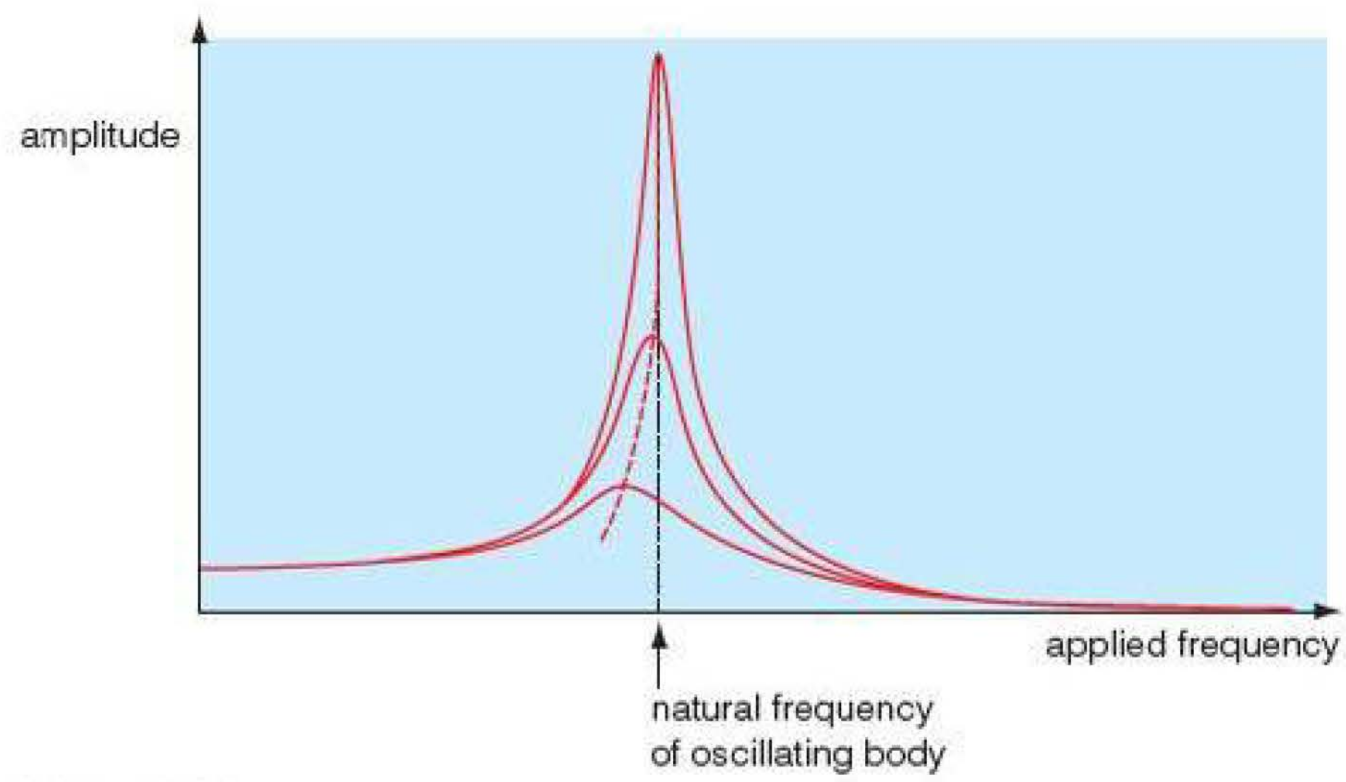


Figure 2.24

Examples of resonance

Musical instruments provide a good example of resonance. If the air in a wind instrument oscillates at the natural frequency of the instrument, a loud note is produced. In a stringed instrument, when a string is plucked it vibrates at its natural frequency.

Figure 2.25 shows how you can demonstrate resonance in the laboratory. Here a tuning fork is held above a column of air. The length of the column can be adjusted by moving the reservoir of water, on the right, up or down. When the tuning fork is made to vibrate, the column of air vibrates. However, the amplitude of the oscillations is small when the driving frequency does not match the natural frequency of oscillations in the air column. When the length of the column is adjusted so that its natural frequency is the same as the tuning fork's frequency, a loud sound is heard as the air resonates.

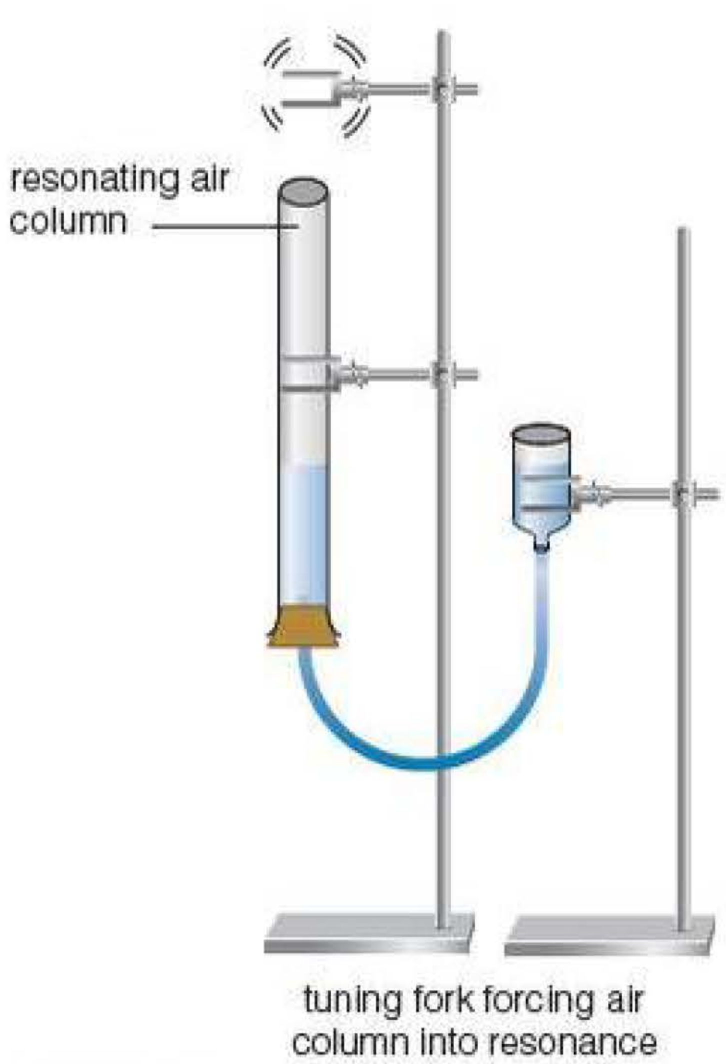


Figure 2.25

A microwave oven takes advantage of the resonance of water molecules. The frequency of the microwaves is matched to the natural frequency of oscillation of water molecules. So when something is cooked in the microwave oven, water molecules absorb energy from the microwaves. The water molecules start to vibrate. This energy is then dissipated as random vibrational energy among all the molecules in the food. Random vibrational energy is heat energy.

Resonance can cause serious problems in any mechanical structure, because all structures have a natural frequency of oscillation. Even a large structure such as a chimney or a bridge can be set oscillating by eddies of wind. And if the wind causes vortices of just the right frequency, large oscillations can build up. Famous examples of bridges being made to oscillate by the wind include the Millennium Bridge in London in 2000, and the Tacoma Narrows Bridge in the USA in 1940. The decks of large boats can also be made to oscillate if the boat hits waves with the same frequency as the natural frequency of part of the deck. The Broughton Suspension Bridge was an iron suspension bridge built in 1826 to span the River Irwell in Manchester. In 1831, the bridge collapsed due to the mechanical resonance caused by a troop of soldiers marching in step. Unfortunately for them, the frequency of their steps caused the bridge to oscillate so much that it collapsed. As a result of the accident, the British Army issued an order that troops should 'break step' when crossing any bridge.

Reducing resonance

The best way to avoid resonance in structures is to design them so that their natural frequencies lie well outside the range of frequencies likely to be caused by wind blowing across them. However, if that is not possible then the amplitude of oscillations can be reduced by damping the motion. In the case of the Millennium Bridge, the oscillations were reduced by applying fluid dampers (see Figure 2.26).

Figure 2.26 Fluid dampers were used to reduce the lateral oscillations of the Millennium Bridge.

Figure 2.27 The spring and the piston-cylinder on a shock absorber form the suspension system for a wheel in a car.

Pistons are able to move inside a cylinder containing fluid and this causes the energy to be dissipated rapidly. Car suspension systems use the same idea. A car uses springs and shock absorbers to make the ride more comfortable for passengers. When a car goes into a pothole (for example) a strong spring allows the wheel to drop into the hole. With no shock absorber, the car would then oscillate up and down. But the shock absorber removes the energy. It consists of a piston that moves inside an oil-filled cylinder. The shock absorbers are *critically damped*. This means that the wheel only oscillates once before returning to its normal position relative to the car (see Figure 2.27).

TEST YOURSELF

11 Figure 2.28 shows a system of pendulums suspended from a string. A heavy pendulum on the right is set in motion, which then sets the other pendulums in motion too. These light pendulums are made using small paper cones.

a) Describe the motions of the eight light pendulums.

b) The small cones are replaced by larger paper cones. Describe the changes you see in the motion of the pendulums.

12 a) Explain what is meant by the term 'resonance'.

b) Give and explain an example of how resonance can be useful in a mechanical system.

c) Give and explain an example of how resonance can cause problems in a mechanical system.

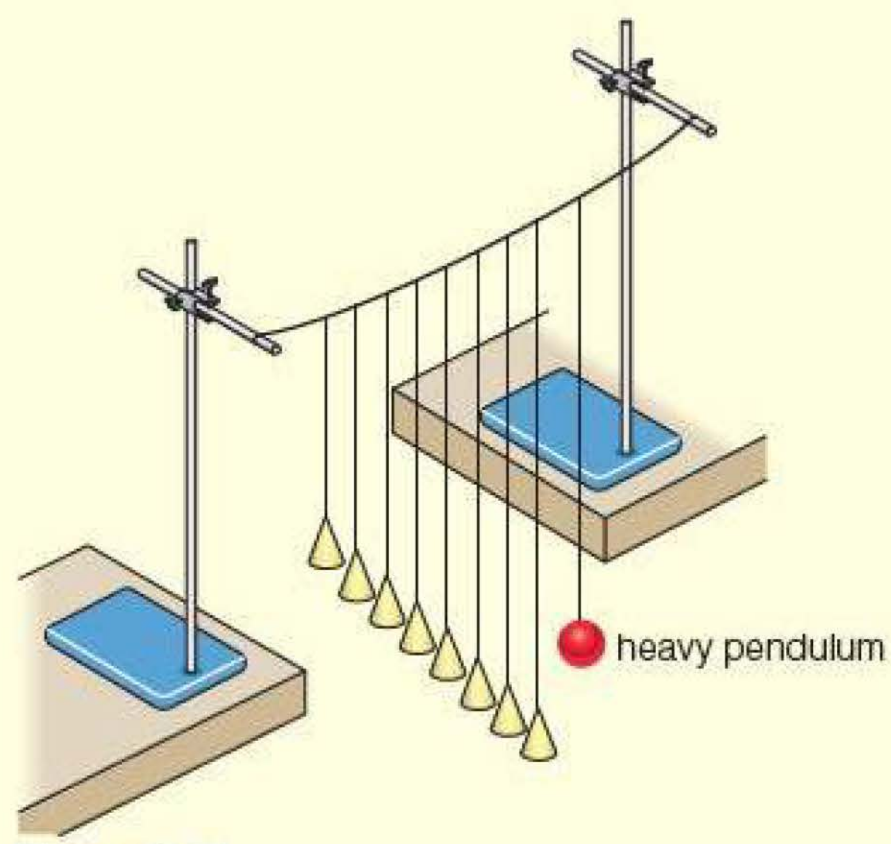


Figure 2.28

Practice questions

1 A metal ruler is clamped to the desk and a mass of 80 g is fixed securely to the end (Figure 2.29). A force of 12 N is applied to displace the end of the ruler by 3 mm upwards. When the ruler is released, it oscillates with simple harmonic motion.

The frequency of the oscillation is

A 1 Hz **C** 36 Hz

B 12 Hz **D** 360 Hz

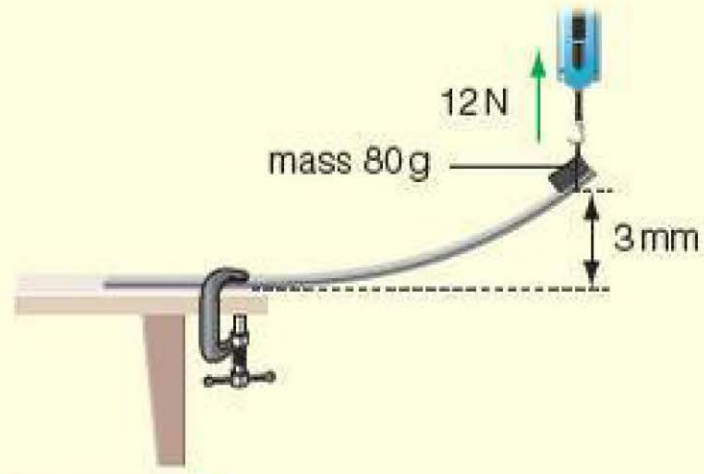


Figure 2.29

2 The maximum speed of the ruler in Figure 2.29 when released with an amplitude of 3 mm is

A $0.36\,\mathrm{m\,s^{-1}}$ **C** $3.6\,\mathrm{m\,s^{-1}}$

B $0.67\,\mathrm{m\,s^{-1}}$ **D** $6.7\,\mathrm{m\,s^{-1}}$

3 In Figure 2.30(a) a mass *m* oscillates vertically on a spring with time period *T*. The same mass is now attached to two springs, as shown in Figure 2.30(b). All the springs are identical.

The mass now oscillates with time period

A $2T$ **C** $\dfrac{T}{\sqrt{2}}$

B $\sqrt{2}T$ **D** $\dfrac{T}{2}$

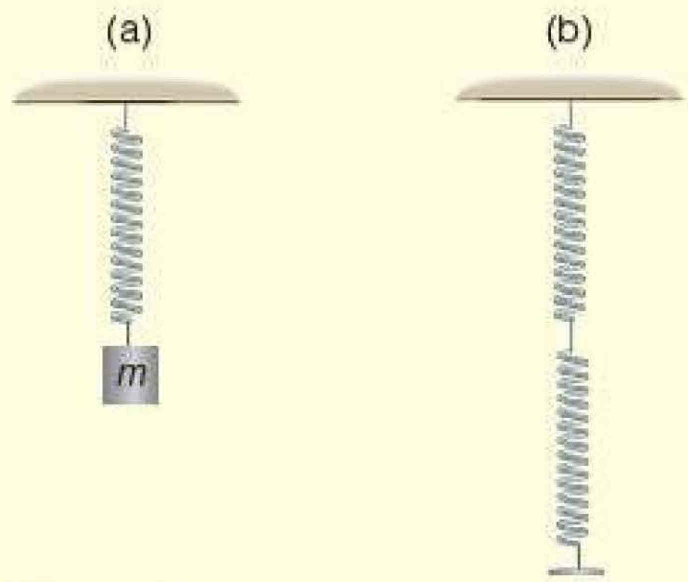


Figure 2.30

4 A pendulum of length 12 m is suspended from the ceiling of a tall room. The time period of the pendulum is

A 2 s **C** 7 s

B 5 s **D** 12 s

5 A mass *m* hangs on a spring with spring constant *k*. The mass oscillates with simple harmonic motion. The amplitude of the oscillations is *A*. The highest speed of the mass is

A $\sqrt{\dfrac{k}{m}}A$ **C** $\dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}A$

B $2\pi\sqrt{\dfrac{k}{m}}A$ **D** $2\pi\dfrac{k}{m}A$

6 An astronaut lands on a planet and decides to calculate the gravitational field strength, *g*, using a pendulum of length 1.5 m. For his pendulum, he discovers that the time period is 1.4 s. The gravitational field strength is

A $15\,\mathrm{N\,kg^{-1}}$ **C** $45\,\mathrm{N\,kg^{-1}}$

B $30\,\mathrm{N\,kg^{-1}}$ **D** $60\,\mathrm{N\,kg^{-1}}$

7 A copper atom in a lattice can be modelled as a mass, *m*, tethered by two springs, *k*, as shown in Figure 2.31, where *m* is 10^{-25} kg and *k* for each spring is $40\,\mathrm{N\,m^{-1}}$.

The frequency of the oscillations of the atom is given by $f = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$. The frequency of the oscillations is

A 3.5×10^{11} Hz **C** 4.5×10^{12} Hz

B 6.6×10^{11} Hz **D** 8.5×10^{12} Hz

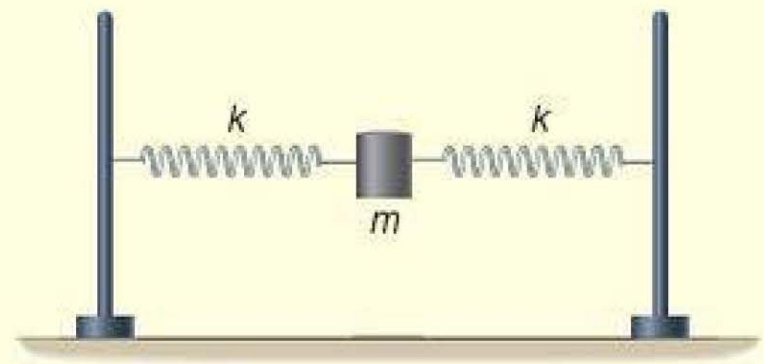


Figure 2.31

8 The oscillating copper atom in Figure 2.31 has oscillations with an amplitude of 5×10^{-11} m. The energy of the oscillations is

A 0.1 eV **C** 0.5 eV

B 0.3 eV **D** 0.6 eV

9 A pendulum swings with an amplitude of 10 cm. The time period of the swings is 0.8 s. When the pendulum is displaced 6 cm from its equilibrium position, its speed is

A $0.6\,\text{m s}^{-1}$ **C** $1.0\,\text{m s}^{-1}$

B $0.8\,\text{m s}^{-1}$ **D** $2.0\,\text{m s}^{-1}$

10 A particle oscillates with simple harmonic motion. Its displacement is given by the equation $x = A\cos(2\pi ft)$, where A is the amplitude of the oscillation, 2.5 cm, and f is the frequency of the oscillation, 10 Hz. At a time of 0.025 s, the displacement of the oscillator is

A −2.5 cm **C** 0

B −1.25 cm **D** +1.25 cm

11 a) What condition is necessary for a body to exist with simple harmonic motion? *(1)*

b) A buoy of mass 65 kg oscillates up and down in the sea with simple harmonic motion (Figure 2.32). It takes an additional downwards force of 28 N to push the buoy an extra 5 cm into the water.

Calculate the force per unit displacement, k, in N m^{-1}, for the buoy. *(1)*

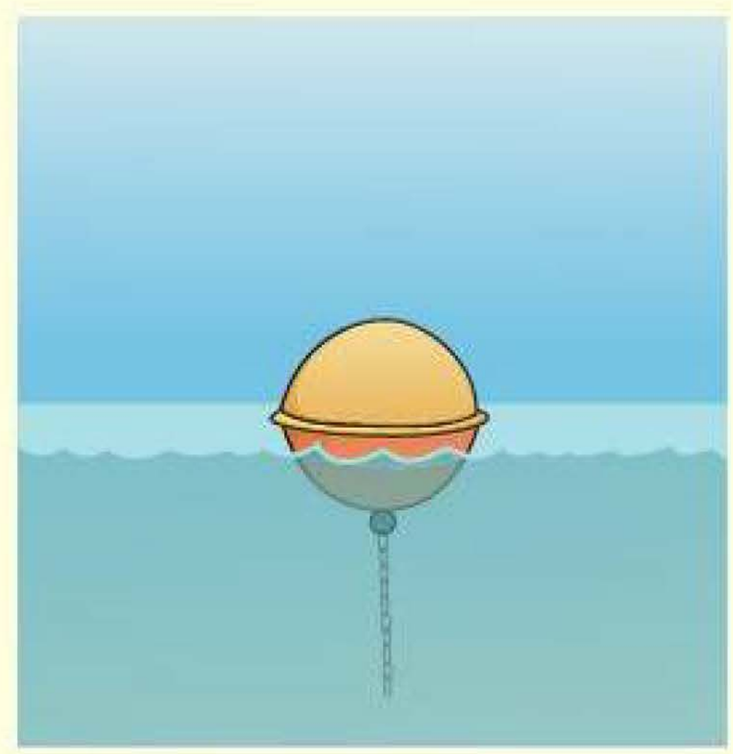


Figure 2.32

c) The time period of the oscillation is given by $T = 2\pi\sqrt{\frac{m}{k}}$, where m is the mass of the buoy and k is the force per unit displacement.

Use the equation above to calculate the time period of oscillations. *(2)*

d) At $t = 0$ the buoy is at rest. Copy the axes of the graph in Figure 2.33 and sketch how the kinetic energy of the buoy changes over one complete oscillation, time T. *(2)*

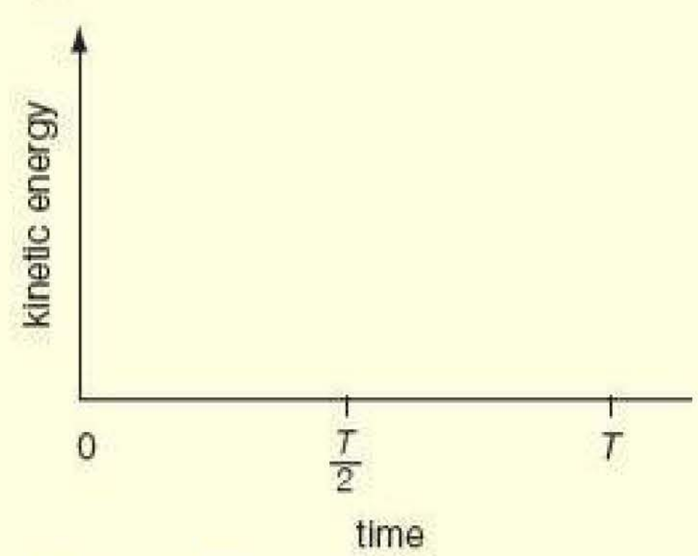


Figure 2.33

12 A girl sits on a swing. She takes 39 s to complete 14 swings. The girl has a mass of 42 kg.

a) Calculate the distance between the girl's centre of gravity and the suspension point of the swing. (3)

The girl is travelling with a speed of $3.5\,m\,s^{-1}$ at the lowest point of her swing.

b) Calculate the height between the lowest and highest points of her swing. Assume that she is swinging freely. (2)

c) i) Calculate the centripetal force that acts on the girl at the bottom of her swing when she is travelling at $3.5\,m\,s^{-1}$. (2)

ii) Calculate the total upwards force exerted on her by the swing at this point. (1)

13 In Figure 2.34, a trolley of mass 0.7 kg is moving to the left with velocity $0.1\,m\,s^{-1}$. A stiff compressible spring is attached to the trolley, which acts as a buffer. The trolley collides with a solid barrier and rebounds elastically. The spring obeys Hooke's law and has a spring constant k.

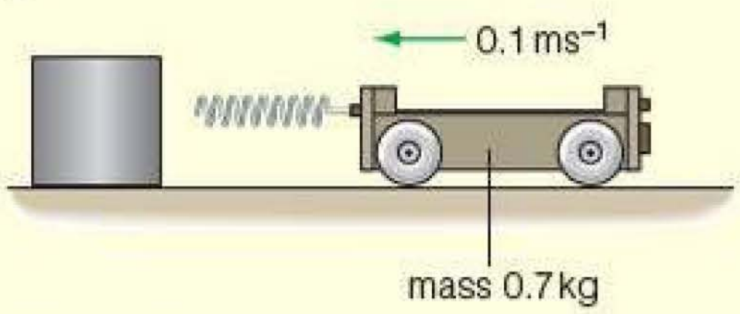


Figure 2.34

a) Write an equation to describe how the force, F, that the spring exerts on the trolley changes with the compression of the spring, x. Hence explain why the motion of the trolley, while the spring is in contact with the block, is simple harmonic. (2)

Figure 2.35 shows graphs of how (i) the velocity and (ii) the acceleration of the trolley change with time while the trolley is in contact with the block.

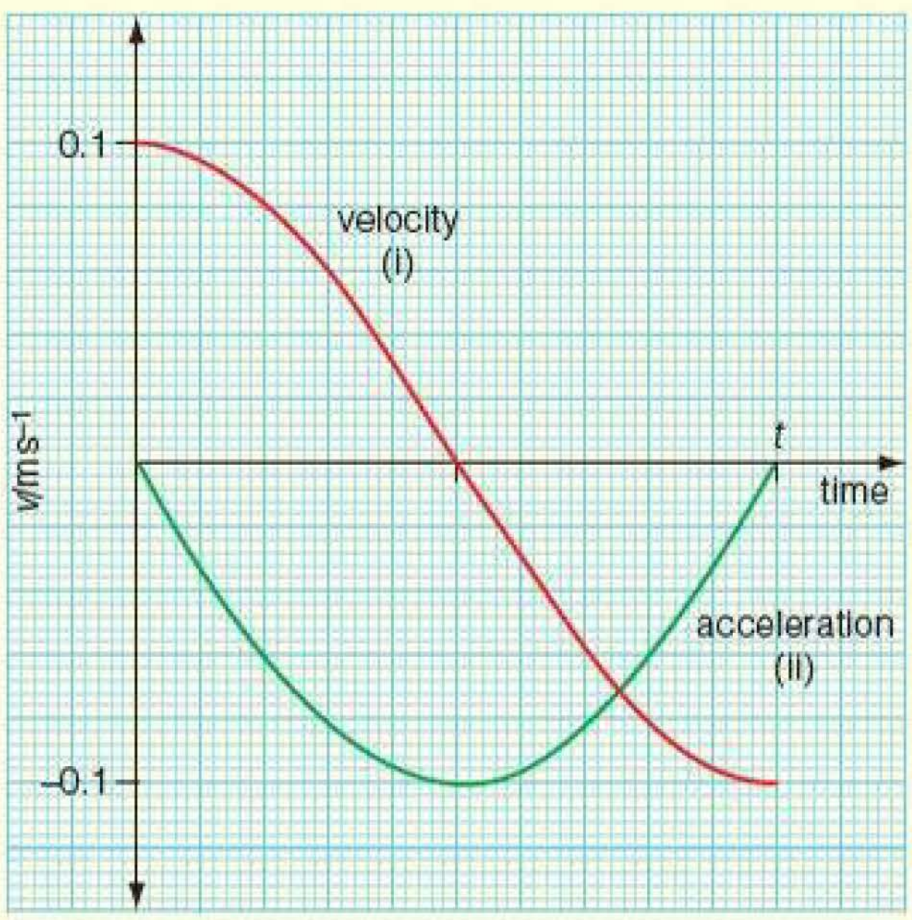


Figure 2.35

b) Use your knowledge of the equations of motion to explain the relationship between graphs (i) and (ii). (3)

c) The trolley has a mass of 0.7 kg and the spring has a spring constant of $25\,N\,m^{-1}$. Calculate the time, t, that the trolley is in contact with the block. (3)

d) i) Calculate the maximum compression of the spring. (2)

ii) Calculate the maximum acceleration of the trolley. (2)

e) The trolley now approaches the block with a speed of $0.2\,m\,s^{-1}$. Describe what effect this has on the following: (2)

i) the maximum acceleration of the trolley

ii) the time of contact with the block.

f) Explain what would happen to the time of contact, t, if the trolley was now made more massive by adding weight to it. (2)

14 Describe how you would design and set up some mechanical apparatus to demonstrate resonance. (6)

Stretch and challenge

15 A particle of mass m is connected by two pieces of elastic, each of unextended length l, to two fixed points A and B, one vertically above the other, and separated by a distance $3l$.

a) If the elastic is such that a force F causes an extension e given by

$$F = \frac{ke}{l}$$

find the equilibrium position of the particle.

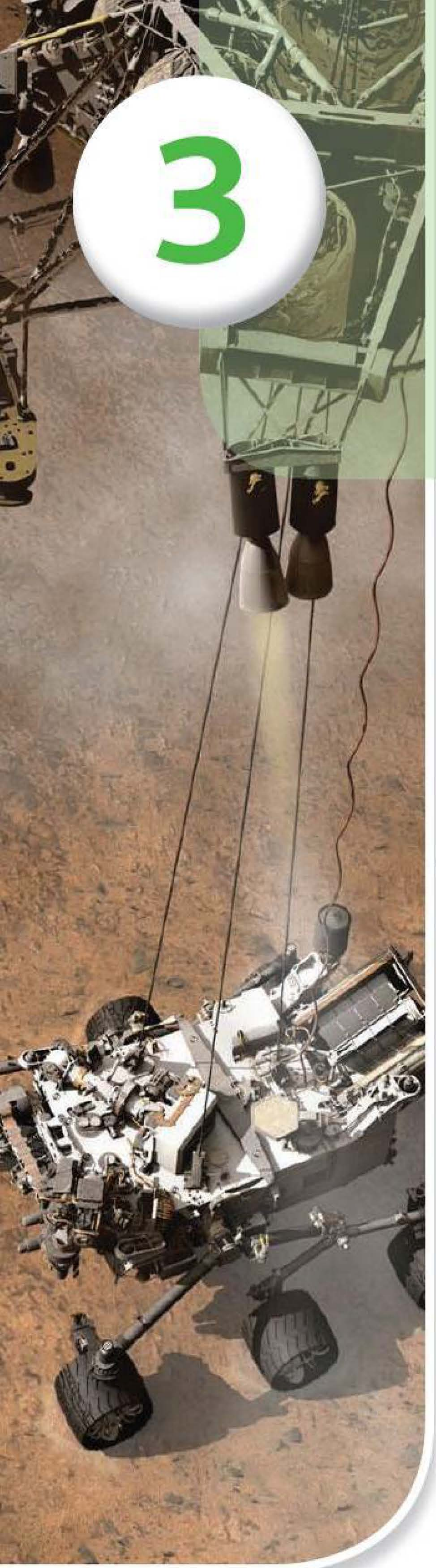
b) The particle is now depressed a small distance z_0 from its equilibrium position, and then released. Assuming that it can move only vertically up or down, and that damping effects can be neglected, derive an equation describing its subsequent motion. Then solve the equation to find the vertical motion z of the particle as a function of time. Find the time period of the subsequent oscillations, and the maximum velocity of the particle.

c) A piece of graph paper has x and y coordinates drawn on it. On these axes is plotted a point whose x coordinate is the vertical displacement of the particle at time t during its motion, and whose y coordinate is the vertical velocity at the same instant. Explain what would be seen if a whole series of points were drawn corresponding to positions and velocities of the particle over an extended time period.

d) Describe qualitatively the effects on the motion of the particle in the following *separate* situations:

i) the initial displacement z_0 is not small

ii) the damping effects cannot be neglected.



3 Gravitation

PRIOR KNOWLEDGE

Before you start, make sure that you are confident in your knowledge and understanding of the following points:

- The Earth, other planets and stars produce gravitational fields, which exert a force on other massive objects.
- A gravitational field exerts a 'non-contact' force, which acts over very long distances.
- Gravitational field strength, g, is defined as the force that acts on a mass of 1 kg. On the Earth, $g = 9.81\,\mathrm{N\,kg^{-1}}$.
- A planet's or star's gravitational field strength, at its surface, depends on its mass and radius.
- The gravitational potential energy, E_p (in J) gained by a mass m (in kg) lifted through a height h (in m) in a uniform gravitational field g (in $\mathrm{N\,kg^{-1}}$) is given by $\Delta E_p = mgh$.
- The kinetic energy (E_k) of an object with mass m (in kg) and velocity v (in $\mathrm{m\,s^{-1}}$) is given by $E_k = \frac{1}{2}mv^2$.

TEST YOURSELF ON PRIOR KNOWLEDGE

1 Listed below are four bodies in our Solar System, and four possible values of gravitational field strengths at their surfaces. Match the field strengths to the bodies, explaining your choices.
Four bodies in our Solar System:
Sun, Earth, Mercury, Ceres (a dwarf planet).
Possible values of surface gravity:
$0.3\,\mathrm{N\,kg^{-1}}$, $3.7\,\mathrm{N\,kg^{-1}}$, $9.8\,\mathrm{N\,kg^{-1}}$, $270\,\mathrm{N\,kg^{-1}}$.

2 Olympus Mons is the tallest mountain on Mars and stands a height of 22.0 km above the Martian plain. Mount Everest stands at a height of 8.8 km above sea level. Compare the energy expended by a mountaineer climbing these two mountains. The gravitational field strength on the surface of Mars is $3.7\,\mathrm{N\,kg^{-1}}$. (Assume that the mountaineer has a mass of 120 kg, including equipment and/or spacesuits.)

3 a) An object is dropped on Earth from a height of 5 m. Calculate its speed when it hits the ground.

b) An object is dropped from a height of 31 m on the Moon, and it reaches a speed of $10\,\mathrm{m\,s^{-1}}$ when it hits the surface of the Moon. Calculate the Moon's gravitational field strength.

c) On a planet, an object is dropped from a height h and it hits the ground with a speed of $10\,\mathrm{m\,s^{-1}}$. Calculate the object's speed when it hits the ground if it is dropped from a height of $2h$.

Newton's law of gravity

We live in an age in which humans have travelled to the Moon and space probes have been sent to investigate all the planets in our Solar System. From our own experience and this exploration of space, we know that, on Earth, gravity exerts a larger force on more massive objects. We have learnt that the Moon has a smaller surface gravity than the Earth because it is a less massive body. We have understood that a planet's pull of gravity gets weaker further away from that planet's surface.

These facts seem obvious to us now, but that is because we have seen experimental proof of them. It is a mark of Isaac Newton's genius that he had enough insight to produce a law of gravitation based on the observations of astronomers before him.

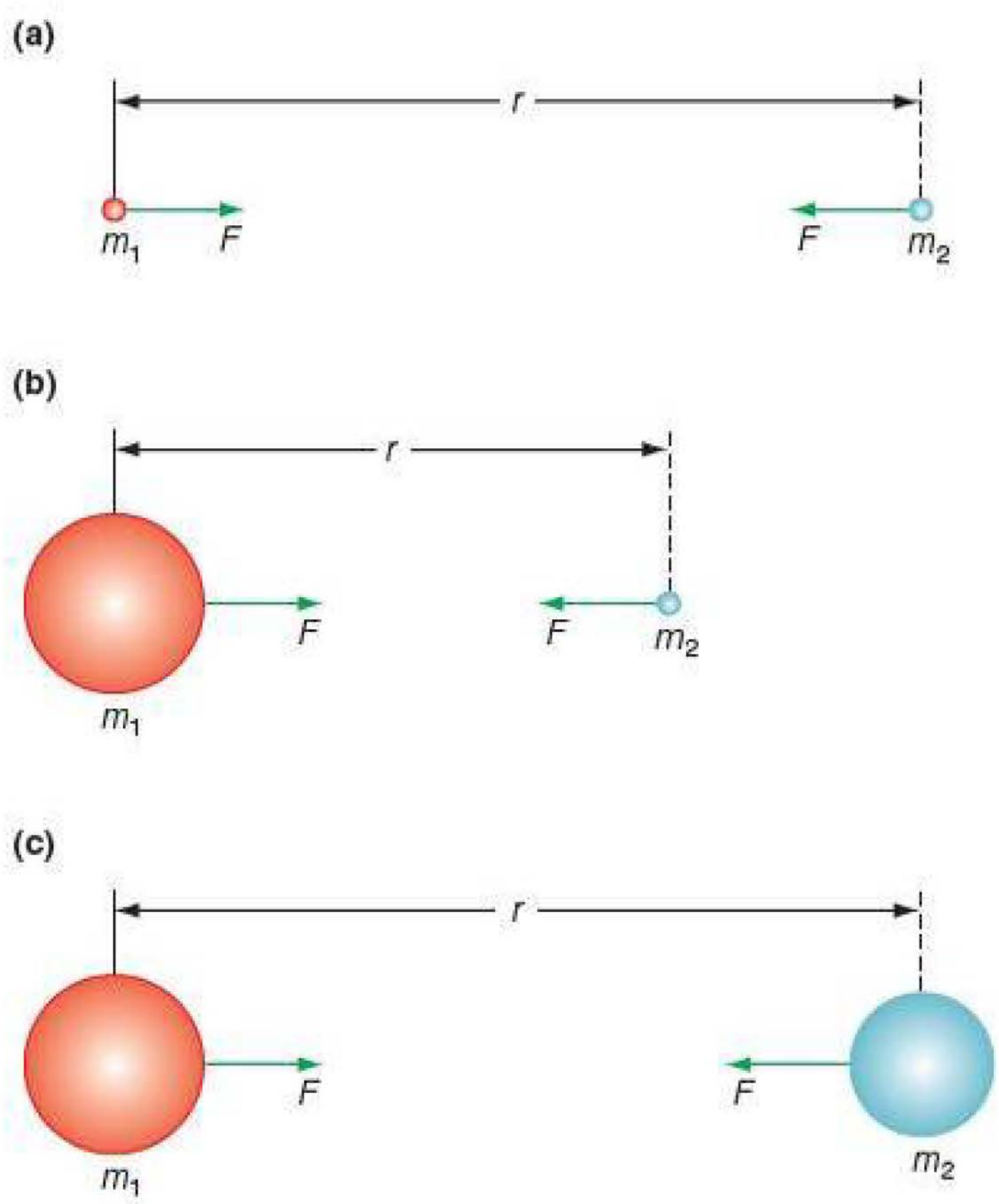


Figure 3.1 In each of these examples, Newton's law can be used to calculate the gravitational force of attraction that the objects exert on each other.

Newton's law of gravitation states that the gravitational force of attraction between two point masses m_1 and m_2 (measured in kg) separated by a distance r (in m) is given by

$$F = \frac{Gm_1m_2}{r^2}$$

where G is the universal constant of gravitation:

$$G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$$

This constant can only be accurately measured by careful laboratory experiment.

Although Newton's law only applies to point masses, it can also be used to calculate the force of attraction between large spherical objects (such as planets and stars) because a sphere behaves as if all the mass were concentrated at its centre. So Newton's law can be correctly used to calculate the force of attraction in each of the cases in Figure 3.1: (a) the force between two point masses; (b) the force between a planet and a small mass; and (c) the force between two planets or stars. Newton's law cannot be used to calculate the force between two irregularly shaped objects, unless a complicated summation of the forces is made.

EXAMPLE

Gravitational force

1 Calculate the gravitational force between the Sun, mass 2×10^{30} kg, and Halley's comet, mass 3×10^{14} kg, when separated by a distance of 5×10^{9} km. This is its furthest distance from the Sun, which it will reach in 2023.

Answer

$$F = \frac{Gm_1m_2}{r^2}$$

$$= \frac{(6.67 \times 10^{-11}\,\mathrm{Nm^2kg^{-2}}) \times (2 \times 10^{30}\,\mathrm{kg}) \times (3 \times 10^{14}\,\mathrm{kg})}{(5 \times 10^{12}\,\mathrm{m})^2}$$

$$= 2 \times 10^{9}\,\mathrm{N}\ (1\ \mathrm{s.f.})$$

It is important to remember to convert the distance into metres in the calculation.

2 Calculate the gravitational attraction between you (assume you have a mass of 65 kg) and the Earth, which has a mass of 6.0×10^{24} kg. The Earth's radius is 6400 km.

Answer

$$F = \frac{Gm_1m_2}{r^2}$$

$$= \frac{(6.67 \times 10^{-11}\,\text{N}\,\text{m}^2\text{kg}^{-2}) \times (6.0 \times 10^{24}\,\text{kg}) \times (65\,\text{kg})}{(6.4 \times 10^6\,\text{m})^2}$$

$$= 637\,\text{N}$$

This is what you would expect when you remember that this is your weight, which can also be calculated using:

$$W = mg = 65\,\text{kg} \times 9.8\,\text{N}\,\text{kg}^{-1} = 637\,\text{N}$$

You should also remember, from Newton's third law of motion, that you exert a gravitational force of 637 N on the Earth too.

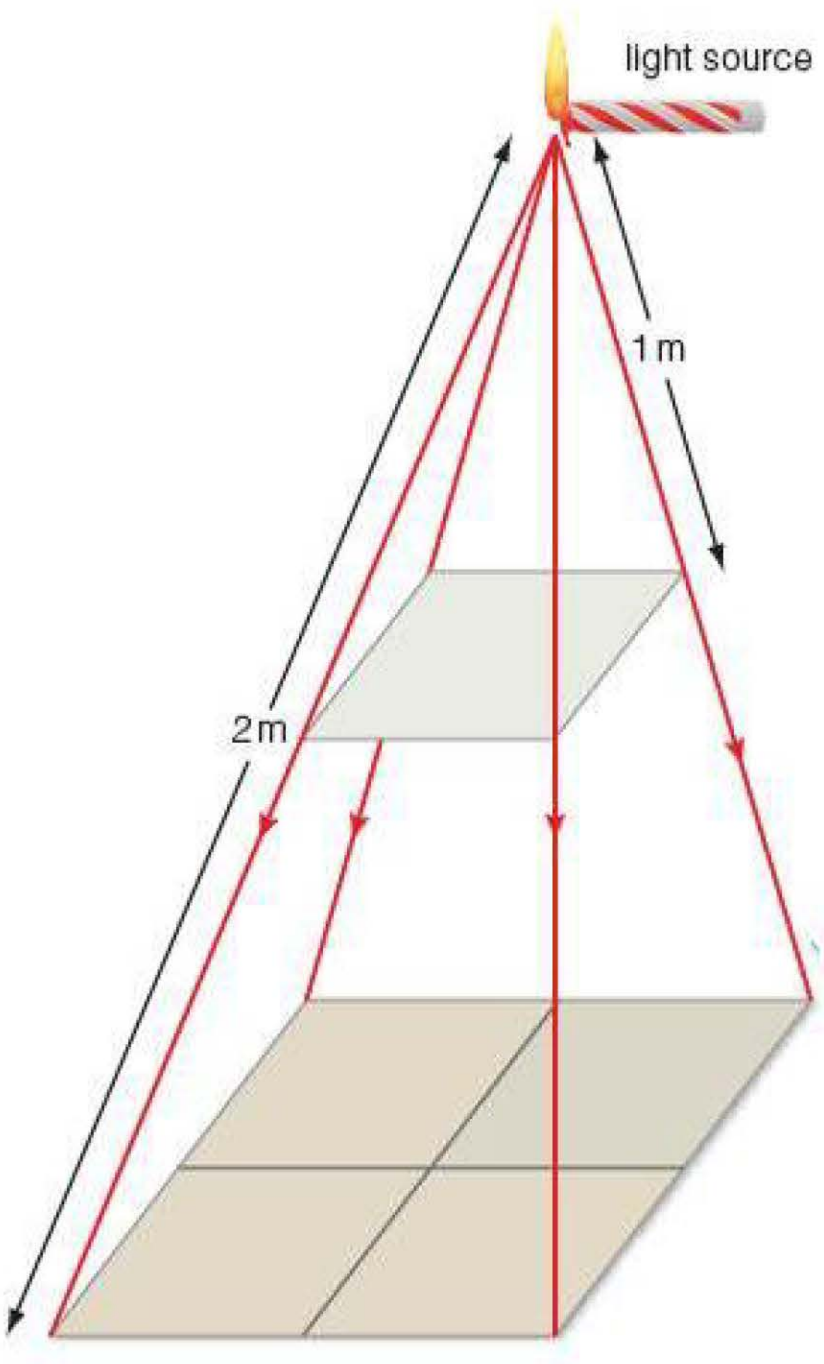


Figure 3.2 The intensity of light 2 m away from a light source is a quarter of the intensity 1 m away from the source, because the light spreads across four times the area.

The inverse square law

When Newton formulated his law of gravity, he imagined that the force of gravity spreads out in the same way as light spreads out from a candle.

Figure 3.2 shows his idea. When you hold a card at a distance of 1 m from the candle, you see a particular intensity of light, I. When you move a distance of 2 m from the candle, the same amount of light now spreads out over four cards of the same area. So the light intensity is now a quarter of its original value, $\frac{1}{4}I$.

Light intensity obeys an inverse square law:

$$I = \frac{L}{4\pi r^2}$$

Here I is the intensity of light in $\text{W}\,\text{m}^{-2}$, L is the luminosity of the light source (the amount of energy emitted per second) in W, and r is the distance away from the source in m. The factor 4π comes into the equation because the light spreads out into a sphere, and the surface area of a sphere of radius r is $4\pi r^2$.

TEST YOURSELF

1 An astronaut of mass 100 kg (including his spacesuit) stands on a planet of mass 4×10^{25} kg, with a radius of 8400 km.
   a) Calculate his weight.
   b) Calculate the gravitational field strength at the surface of the planet.

2 a) Make an estimate of the gravitational attraction between two people, each of mass 80 kg, standing about 10 m apart. What assumption do you make in this calculation?
   b) Explain why we do not notice gravitational forces between objects on the Earth.
   c) You can charge a balloon, by rubbing it, and get it to stick to a wall. What does this tell you about the size of electrostatic forces compared with gravitational forces?

3 Jupiter is approximately five times further from the Sun than the Earth is, and Jupiter has a mass approximately 300 times larger than that of the Earth. How many times larger is the Sun's gravitational pull on Jupiter than the Sun's gravitational pull on the Earth?

4 a) A spherical light bulb is fitted in a darkened room. The light intensity at a distance 1 m away from it is $0.2\,\mathrm{W\,m^{-2}}$. Calculate the light intensity at a distance of 3 m away from it.
   b) Calculate the electrical power of the light bulb assuming it is 20% efficient in transferring electrical energy into light energy.

5 Estimate the gravitational attraction between our Galaxy, the Milky Way, and the Andromeda Galaxy. The galaxies are separated by a distance of about 2.4×10^{19} km and each galaxy has a mass of about 7×10^{11} solar masses. The mass of the Sun (one solar mass) is 2×10^{30} kg.

Gravitational fields

Gravitational field A region in space in which a massive object experiences a gravitational force.

Field strength The strength of the gravitational field measured in $\mathrm{N\,kg^{-1}}$. Field lines represent the direction and strength of the field.

A **gravitational field** is a region in which a massive object experiences a gravitational force. Any object with mass produces a gravitational field, but we usually use the term to describe the region of space around large celestial objects such as galaxies, stars, planets and moons.

The gravitational **field strength** in a region of space is defined by

$$g = \frac{F}{m}$$

where g is the field strength measured in $\mathrm{N\,kg^{-1}}$ and F is the gravitational force in N acting on a mass m in kg.

Uniform fields

Near the surface of a planet, the gravitational field is very nearly *uniform*, which means that the field is of the same strength and direction everywhere. Figure 3.3(a) illustrates a uniform field. The field lines show the direction of the gravitational force on an object, and the spacing of the lines gives a measure of the strength of the field. Figure 3.3(b) shows another gravitational field, which is half the strength of the field in Figure 3.3(a). You should remember that the spacing of the lines is chosen just for illustrative purposes – another person might have represented the field strengths in these diagrams with a different separation of the field lines.

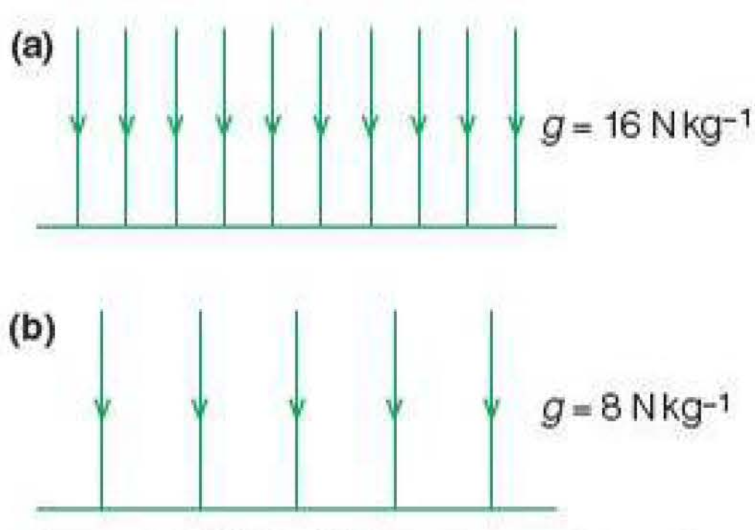


Figure 3.3 (a) Field lines of equal spacing in the same direction show a uniform gravitational field. (b) This shows another uniform field with a lower strength.

Radial fields

Figure 3.4 shows the shape of a gravitational field near to Earth; this is a radial field. Here the field lines all point towards the centre of the planet. This is why we can use Newton's law of gravity to calculate the gravitational forces between two planets. The field is exactly the same shape as it would have been if all of the mass of the planet were concentrated at its centre C.

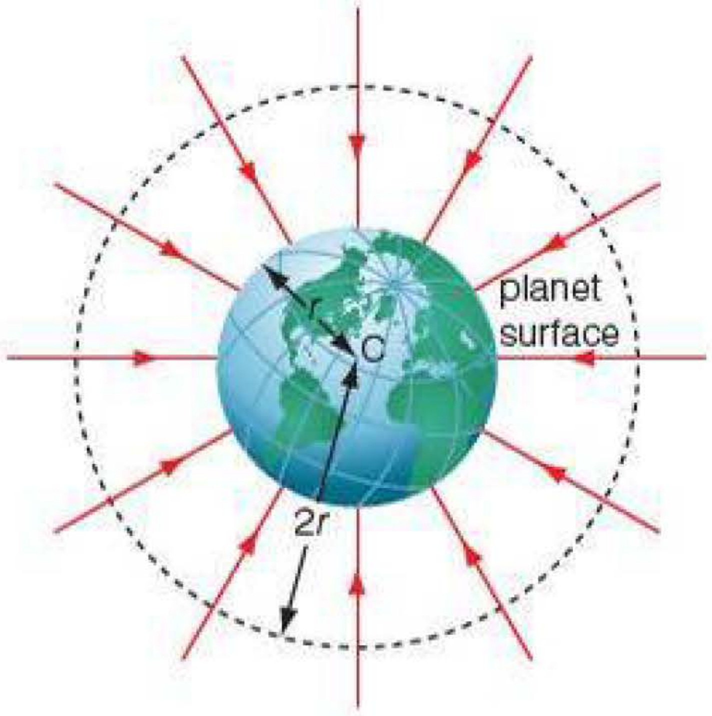


Figure 3.4 The radial gravitational field decreases with distance away from a planet.

You can also see from Figure 3.4 that the gravitational field decreases in strength with increasing distance from the centre of the planet. The field lines at a distance of $2r$ from the centre of the planet are further apart than they are at distance r, which is the planet's surface. (Remember that the planet is a sphere, so that the lines spread out in three dimensions. The diagram shows the field lines in just one plane.)

We can also produce a formula to describe the field strength close to a planet. From Newton's law of gravity, we know that

$$F = \frac{Gm_1m_2}{r^2}$$

Also if m_2 is the mass of a small object close to the surface of the planet, we know that

$$g = \frac{F}{m_2}$$

Therefore

$$g = \frac{Gm_1}{r^2}$$

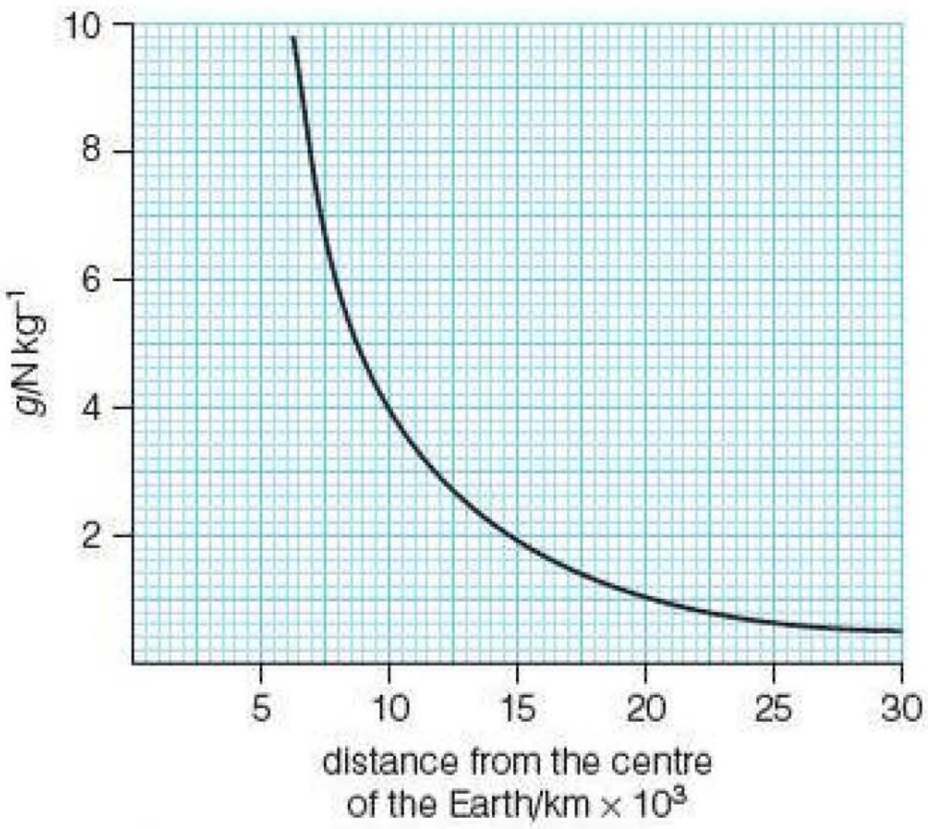


Figure 3.5 A graph of g as a function of distance from the Earth's centre.

Note that we often use a capital M to describe the mass of a large object such as a star or planet. So using M as the mass of a planet, and r as the distance away from the centre of the planet, we have

$$g = \frac{GM}{r^2}$$

Figure 3.5 shows how the gravitational field strength varies with height above the Earth.

For most planets, treating them as uniform spheres works as a good approximation. For most objects with a mass larger than 10^{21} kg, the forces of gravity overcome the massive forces of the rocks to turn the planet into a sphere. However, smaller moons and minor planets can have irregular shapes, so Newton's law of gravity cannot be used to simply predict fields near them, but it can be used accurately at large distances.

EXAMPLE

Planets and stars

1 A minor planet has a mass of 2×10^{22} kg, and it has a radius of 1200 km. Calculate the gravitational field strength at its surface.

Answer

$$g = \frac{GM}{r^2}$$

$$= \frac{(6.67 \times 10^{-11}\,\text{N m}^2\text{kg}^{-2}) \times (2 \times 10^{22}\,\text{kg})}{(1.2 \times 10^6\,\text{m})^2}$$

$$= 0.9\,\text{N kg}^{-1}$$

2 A star has a gravitational field strength at its surface of 300 N kg^{-1}. Another star has the same mass but 10 times the radius of the first star. Calculate the gravitational field strength at the surface of the second star.

Answer

$$g_1 = \frac{GM}{r_1^2}$$

$$g_2 = \frac{GM}{r_2^2}$$

$$\frac{g_2}{g_1} = \frac{GM}{r_2^2} \times \frac{r_1^2}{GM} = \left(\frac{r_1}{r_2}\right)^2$$

So

$$g_2 = g_1\left(\frac{r_1}{r_2}\right)^2 = 300\ \text{N kg}^{-1} \times \left(\frac{1}{10}\right)^2 = 3\ \text{N kg}^{-1}$$

3 Show that the gravitational field strength near to the surface of a planet or star is given by $g = \frac{4}{3}\pi G\rho r$, where ρ is the density of the body and r its radius.

Answer

$$g = \frac{GM}{r^2} = \frac{G \times \frac{4}{3}\pi\rho r^3}{r^2} = \frac{4}{3}\pi G\rho r$$

TIP

The volume V of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.

TEST YOURSELF

In these questions use $G = 6.7 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$.

6 A planet has a mass of 4.6×10^{23} kg and a radius of 3200 km.

a) Calculate the gravitational field strength at the planet's surface.

b) Calculate the gravitational field at a height of 6400 km above the planet's surface.

7 A star has a gravitational field strength of 400 N kg^{-1} and a radius of 8.4×10^5 km. At the end of its life, assume that it collapses to become a neutron star, of the same mass but with a radius of 14 km.

a) Calculate the ratio of the star's initial radius to the radius of the neutron star it eventually becomes.

b) Calculate the gravitational field strength on the surface of the neutron star.

8 Show that there is a negligible difference between the Earth's gravitational field strength at sea level and at the top of Mount Everest. The Earth's radius at sea level is about 6400 km, and the height of Mount Everest is 8.8 km. The mass of the Earth is 6.0×10^{24} kg.

9 This question requires you to think about gravitational fields and also to recall last year's work on v–t and s–t graphs.

A spacecraft is flying away from the centre of the Earth, at a height of 10 000 km. However, it is travelling too slowly to escape from the Earth. At a height of 20 000 km above the Earth's surface, it stops moving and begins to fall back to Earth. Use Figure 3.5 to help you to sketch:

a) a velocity–time graph for the spacecraft

b) a displacement–time graph for the spacecraft.

In both cases, start the graph from the initial height of 20 000 km, and finish the graphs as the spacecraft crashes into the Earth.

10 Express the unit for G in SI base units.

Gravitational potential

You will be familiar with the equation that we use to calculate the increase in gravitational potential energy when a mass is lifted in a gravitational field. We calculate the change of gravitational potential energy, ΔE_p, using the equation

$$\Delta E_p = mg\Delta h$$

where m is the mass lifted in kg, g is the gravitational field strength in $N\,kg^{-1}$, and Δh is the increase in height.

Figure 3.6 shows the gravitational field lines (in green) close to the surface of a planet, where the gravitational field strength is $5\,N\,kg^{-1}$. When a 1 kg mass is lifted through a height of 20 m in this field, the equation above tells us that the increase in gravitational potential energy of the mass is 100 J ($\Delta E_p = 1\,kg \times 5\,N\,kg^{-1} \times 20\,m = 100\,J$). When the mass is lifted through 40 m, the increase in potential energy becomes 200 J.

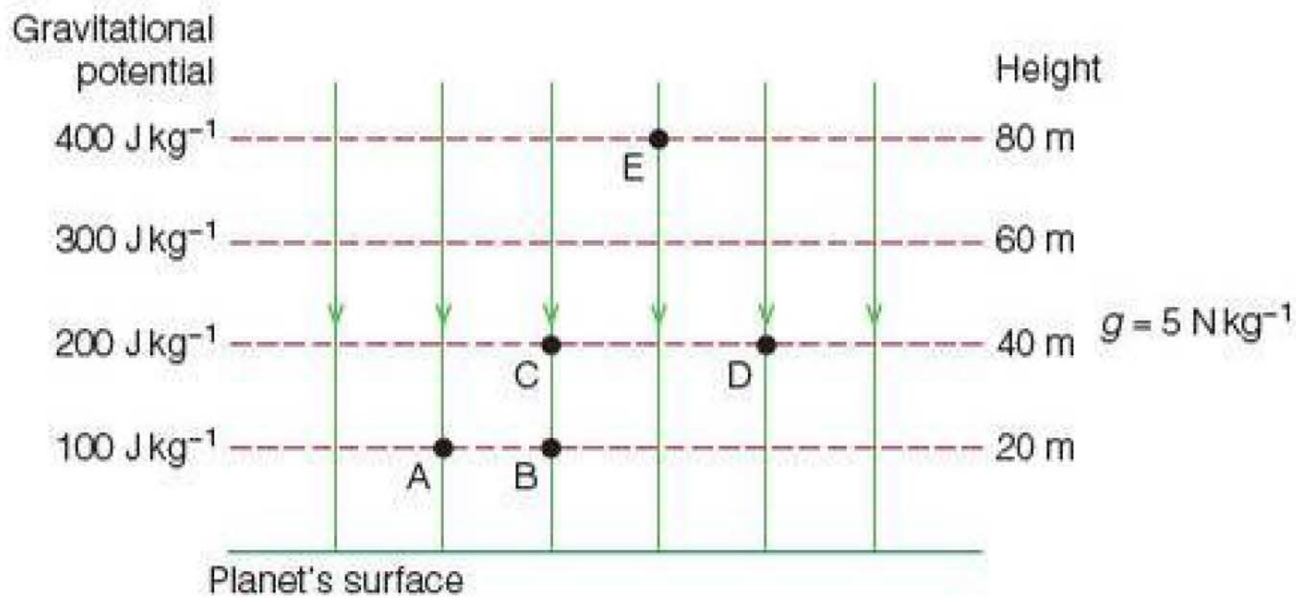


Figure 3.6 Gravitational field lines and equipotentials close to the surface of a planet, where $g = 5\,N\,kg^{-1}$.

Gravitational potential difference The gravitational potential energy difference per kilogram. Gravitational potential, and potential difference, have units of $J\,kg^{-1}$.

These calculations lead us to the idea of **gravitational potential difference**, which can be defined as the change in gravitational potential energy per kg.

Gravitational potential is given the symbol V, and gravitational potential difference is given the symbol ΔV. Since $\Delta E_p = mg\Delta h$, it follows that

$$\Delta V = \frac{\Delta E_p}{m} = g\Delta h$$

so

$$\Delta V = g\Delta h$$

Gravitational potential has units of $J\,kg^{-1}$.

EXAMPLE

Gravitational potential difference

The equation in the main text can be used to calculate the magnitude of potential changes. In Figure 3.6, what is the gravitational potential difference between being on the ground and being at a height of 80 m?

Answer

$\Delta V = g\Delta h = 5\,N\,kg^{-1} \times 80\,m$
$= 400\,J\,kg^{-1}$

Equipotential lines

Figure 3.6 also shows equipotentials close to the surface of the planet. In the diagram these look like lines, but in three dimensions they are surfaces. On the diagram, equipotential surfaces have been drawn at intervals of $100\,J\,kg^{-1}$. When an object moves along an equipotential, it means that the potential (and therefore the potential energy) stays the same.

Equipotential surfaces are always at right angles to the gravitational field. When something moves at right angles to the field (and hence along an equipotential), no work is done by or against the gravitational field, so there is no potential energy change. When something moves along a field line, there is a change of gravitational potential energy.

To be exact, in the link between **gravitational field** and potential, we should link them with this equation:

$$g = -\frac{\Delta V}{\Delta h}$$

Gravitational field A gravitational field g is linked to the gravitational potential gradient by the equation

$$g = -\frac{\Delta V}{\Delta h}$$

The significance of the minus sign is that the potential gradient $\frac{\Delta V}{\Delta h}$ is in a positive direction upwards, because the potential increases as the height above the planet increases. The gravitational field direction is downwards.

EXAMPLE

Gravitational potential energy change

Refer to Figure 3.6.

1 What is the gravitational potential energy change in moving a 2 kg mass from A to B?

Answer

The mass moves along an equipotential, so the change is 0.

2 What is the gravitational potential energy change in moving a 2 kg mass from A to C?

Answer

It does not matter which path the mass takes, the potential change from A to C is 100 J kg^{-1}. So the potential energy change is

$\Delta E_p = m\Delta V = 2\,\text{kg} \times 100\,\text{J kg}^{-1}$
$= 200\,\text{J}$

TEST YOURSELF

11 a) Explain the term 'gravitational potential difference'.
b) Give the unit of gravitational potential.
c) Explain the term equipotential.

12 Refer to Figure 3.6. Calculate the work done in moving a 5 kg mass through these distances:
a) A to D
b) C to D
c) B to E.

13 Calculate the gravitational potential gradient in Figure 3.6. Comment on your answer.

Gravitational potential in radial fields

In this section we calculate gravitational potential energy changes over large distances in gravitational fields, which change in strength. Figure 3.7 shows how the gravitational force on an object of mass *m* changes near to a planet of mass *M*. (Note the common use of *M* for the planet and *m* for a small mass near the planet).

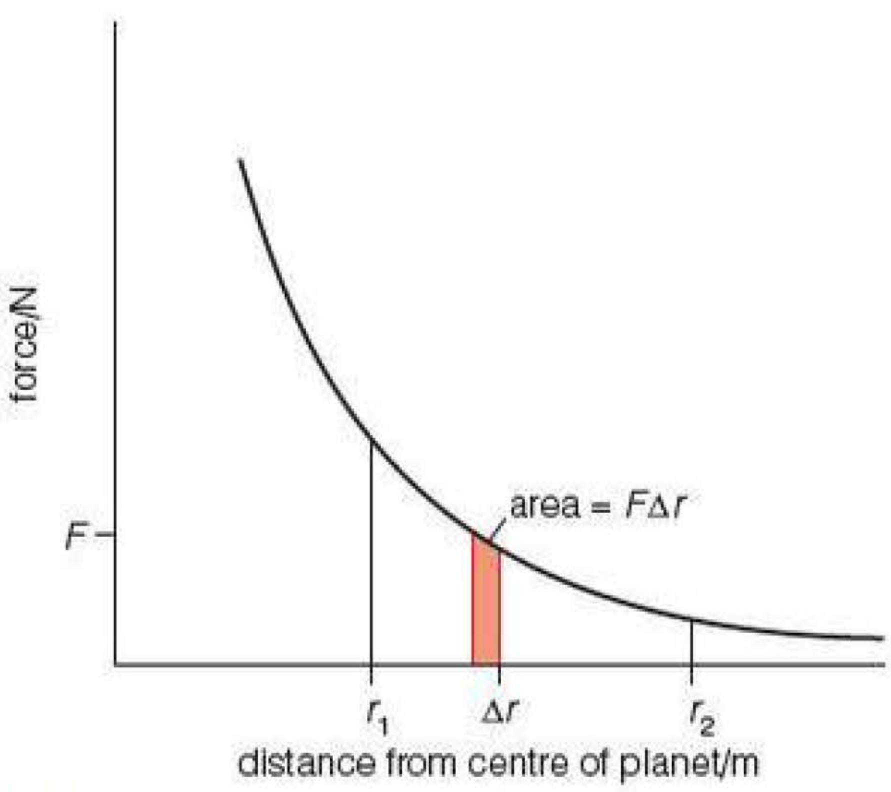


Figure 3.7 Graph of gravitational force acting on an object in the vicinity of a planet.

The graph shows that a force, *F*, acts on the object at a distance *r* from the centre of the planet. If the object is moved a small distance, Δr, further away from the planet, we can calculate the increase in the object's gravitational potential energy as

$$\Delta E_p = \text{work done} = F\Delta r$$

$F\Delta r$ is more usually written as $mg\Delta h$ because the force acting on the mass is equal to its weight, mg.

How do we calculate the increase in gravitational potential energy if the mass is moved from r_1 to r_2? This is more complicated because the force changes as we move from r_1 to r_2.

In the earlier calculation, $F\Delta r$ represents a small area under the graph. So the work done on the mass (or the increase in gravitational potential energy) in moving from r_1 to r_2 is the area under the graph.

The formula to calculate the increase in potential energy is given as equation (i) (the Maths box shows how the formula is derived):

$$\Delta E_p = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right] \qquad \text{(i)}$$

From this, we can also derive a formula for the increase in potential ΔV, because

$$\Delta V = \frac{\Delta E_p}{m} = GM\left[\frac{1}{r_1} - \frac{1}{r_2}\right] \qquad \text{(ii)}$$

> **MATHS BOX**
>
> Using Newton's law of gravitation, the work done on m is
>
> $$\text{work done} = F\Delta r = \frac{GMm}{r^2}\Delta r$$
>
> So the increase in gravitational potential energy in moving m from r_1 to r_2 is
>
> $$\Delta E_p = \int_{r_1}^{r_2} \frac{GMm}{r^2}\,dr$$
>
> $$= \left[-\frac{GMm}{r}\right]_{r_1}^{r_2}$$
>
> $$= GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

Equation (ii) allows us to think about defining gravitational potential close to a planet. We can calculate the potential change in moving from a distance r_1 to a point infinitely far away from the planet. When $r_2 = \infty, \frac{1}{r_2} = 0$. So the potential change in moving from r_1 to ∞ is

$$\Delta V = \frac{GM}{r_1} \qquad \text{(iii)}$$

However, we choose to define ∞ as the point of zero potential for all planets and stars. If we chose any other point as zero, such as the surface of the Earth, we would get a more complicated set of equations when we deal with potentials near to other planets.

So, since we define the potential as zero at infinity, it means the potential near to any planet is a negative quantity, because potential energy decreases as something falls towards a planet. This leads to the following definition of potential V at a distance r from the centre of a planet of mass M:

$$V = -\frac{GM}{r}$$

Figure 3.8 shows the gravitational field lines and equipotentials near to a planet. The equipotentials are shown in equal steps of $1 \times 10^7\,\mathrm{J\,kg^{-1}}$ from the surface G, where the potential is $-8 \times 10^7\,\mathrm{J\,kg^{-1}}$, to A, where the potential is $-2 \times 10^7\,\mathrm{J\,kg^{-1}}$.

Position	Potential (V) $\mathrm{J\,kg^{-1}}$
A	-2×10^7
B	-3×10^7
C	-4×10^7
D	-5×10^7
E	-6×10^7
F	-7×10^7
G	-8×10^7

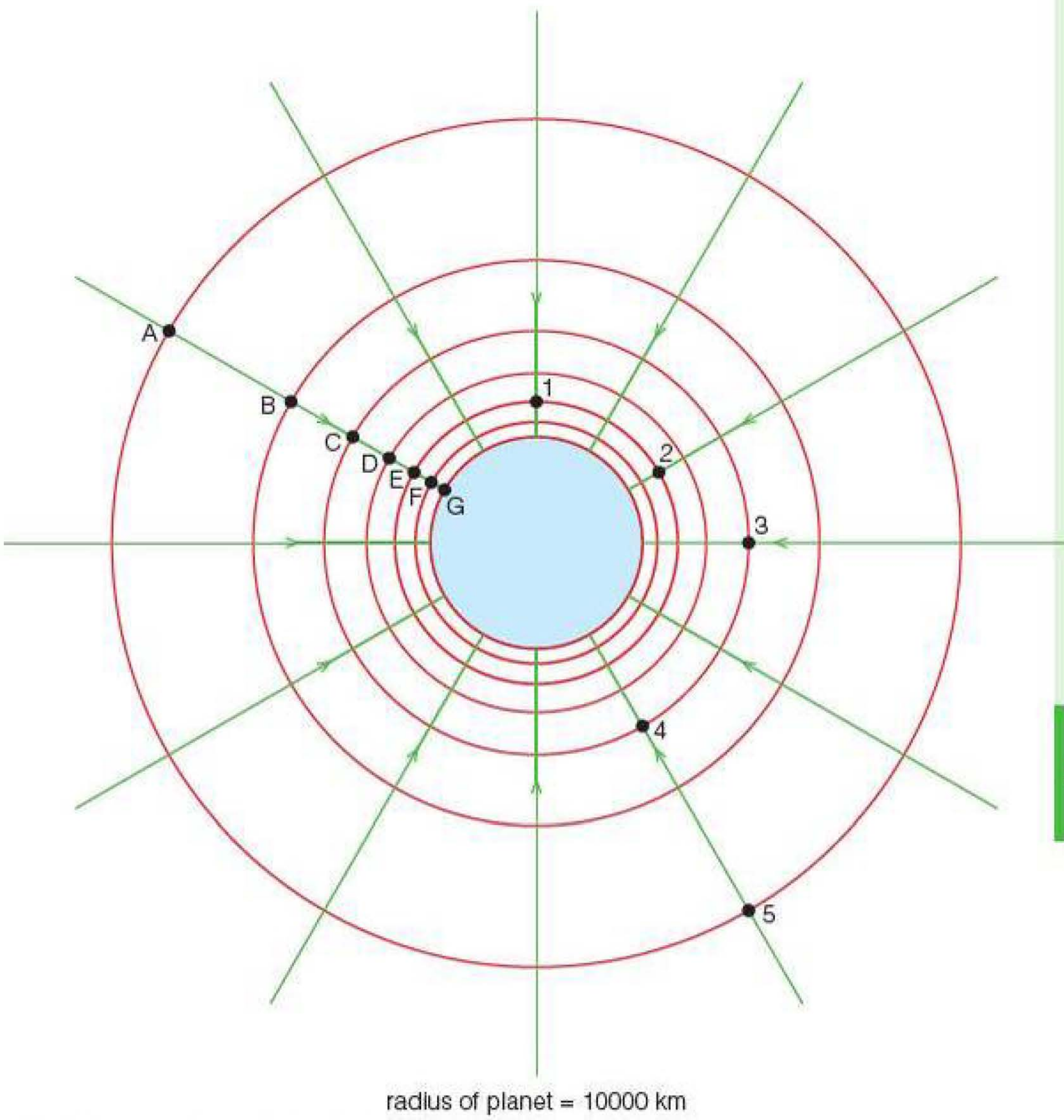


Figure 3.8 This diagram shows gravitational field lines and equipotentials near to a planet.

ACTIVITY

Equipotentials and variation of potential with distance

Using the information in Figure 3.8, copy and complete Table 3.1 linking potential and distance from the centre of the planet.You will need a ruler to measure the distance of the equipotentials from the planet centre.

Table 3.1

Potential/ 10^7 J kg^{-1}	r/ 10^7 m	$\frac{1}{r}$/ 10^{-7} m^{-1}
−8	1.00	1.00
−7	1.14	0.88
−6	1.33	0.75
−5		
−4		
−3		
−2		

1 Plot a graph of potential against r.

2 Use the gradient of the graph to determine the planet's gravitational field at a distance of
   a) 20 000 km from the centre
   b) 40 000 km from the centre.

3 Plot a graph of potential against $\frac{1}{r}$. Use the graph to determine the mass of the planet.

This diagram shows two important linked points:

- The potential gradient is steeper close to the planet's surface, because the equipotentials are closer together.
- The field lines are closer together near the surface, because the gravitational field strength g is stronger.

These two statements are linked through the equation you met earlier:

$$g = -\frac{\Delta V}{\Delta h}$$

or

$$g = -\frac{\Delta V}{\Delta r}$$

These equations are exactly the same, except that Δh has been used for a change in height, and Δr has been used for a change in distance from the centre of a planet.

Escape velocity

The Earth has its atmosphere because the molecules of gas, moving in our atmosphere, do not have enough kinetic energy to escape from the pull of gravity at the Earth's surface. So how fast does something have to move to escape from the Earth's surface?

When a fast-moving object leaves the surface of a planet, we can write that:

decrease in kinetic energy = increase in gravitational potential energy

$$\Delta E_k = \Delta E_p$$

This assumes that the object is not affected by an atmosphere, and is in free fall – this is not a spacecraft with a rocket.

The equation above can be written as

$$\frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2 = m\Delta V$$

If the object is to just escape the pull of the planet, its speed, v_2, will just reach zero at an infinite distance from the planet. This leads to the idea of **escape velocity**, which is the minimum velocity that an object must have at the surface of a planet in order to escape the pull of gravity of the planet using its own kinetic energy.

The gravitational potential at the surface of a planet is given by

$$V = -\frac{GM}{r}$$

so the change in potential is

$$\Delta V = \frac{GM}{r}$$

So the escape velocity for a planet can be calculated using

$$\frac{1}{2}mv^2 = m\Delta V = \frac{GMm}{r}$$

Escape velocity The minimum velocity an object must have at the surface of a planet to escape the pull of gravity using its own kinetic energy.

or

$$v^2 = \frac{2GM}{r} \qquad \text{(iv)}$$

For the Earth, $M = 6 \times 10^{24}$ kg and $r = 6400$ km, so

$$v^2 = \frac{2 \times (6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}) \times (6 \times 10^{24}\ \mathrm{kg})}{6.4 \times 10^6\ \mathrm{m}}$$

$$v = 11\,200\ \mathrm{m\,s^{-1}}$$

Since air molecules travel at approximately $500\ \mathrm{m\,s^{-1}}$ on the surface of the Earth, they travel well below the Earth's escape velocity.

TEST YOURSELF

14 Explain why we choose to define the zero point of gravitational potential at an infinite distance from any planet.

15 Calculate the gravitational potential at the surface of Jupiter. The mass of the planet is 1.9×10^{27} kg and its radius is 70000 km.

16 This question is based on the information in Figure 3.8.

a) Calculate the work done in taking a 1200 kg spacecraft from

i) point 1 to point 2

ii) point 2 to point 3

iii) point 3 to point 4

iv) point 4 to point 5.

b) Use your answer to part (a) to explain why a spacecraft can stay in a circular orbit round a planet indefinitely.

c) The spacecraft returns to the planet. It passes point 5 travelling at a speed of $5200\ \mathrm{m\,s^{-1}}$, and falls freely to point 2. How fast is it travelling as it passes point 2?

17 The radius of the planet shown in Figure 3.8 is 10000 km. Either by estimating the distance between the equipotentials near the planet's surface, or otherwise, calculate the value of g near the planet's surface.

18 When a large star collapses at the end of its life, it can collapse into a black hole. The pull of gravity at its surface is so strong that not even light can escape. Einstein's theory of general relativity predicts that black holes are 'singularities', which means they have collapsed into a tiny space. However, we can use Newton's theory to calculate the maximum size of such a hole.

a) Use the equation for escape velocity (marked (iv) in the text) to calculate the maximum radius for a black hole formed by a star of mass 10^{31} kg. The speed of light is $3 \times 10^8\ \mathrm{m\,s^{-1}}$.

b) Calculate:

i) the gravitational field at this surface

ii) the gravitational potential at this surface.

Orbits

In Figure 3.9, a planet of mass m is in a circular orbit around a star of mass M. You have already studied circular motion. Now we can combine the equations of circular motion and gravitation to link the speed or time period of a planet's orbit to its distance from the Sun.

The pull of gravity provides the necessary centripetal force to keep the planet in orbit. So we can write

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\frac{GM}{r^2} = \frac{v^2}{r}$$

$$v^2 = \frac{GM}{r}$$

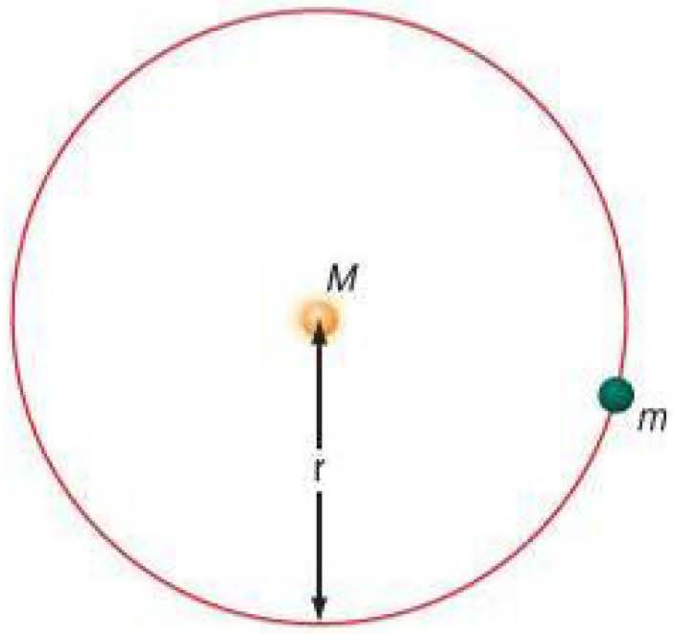


Figure 3.9

From this, you can see that the speed of the orbit is faster for small orbits. Figure 3.10 shows the link between orbital speed and the distance of our eight planets from the Sun.

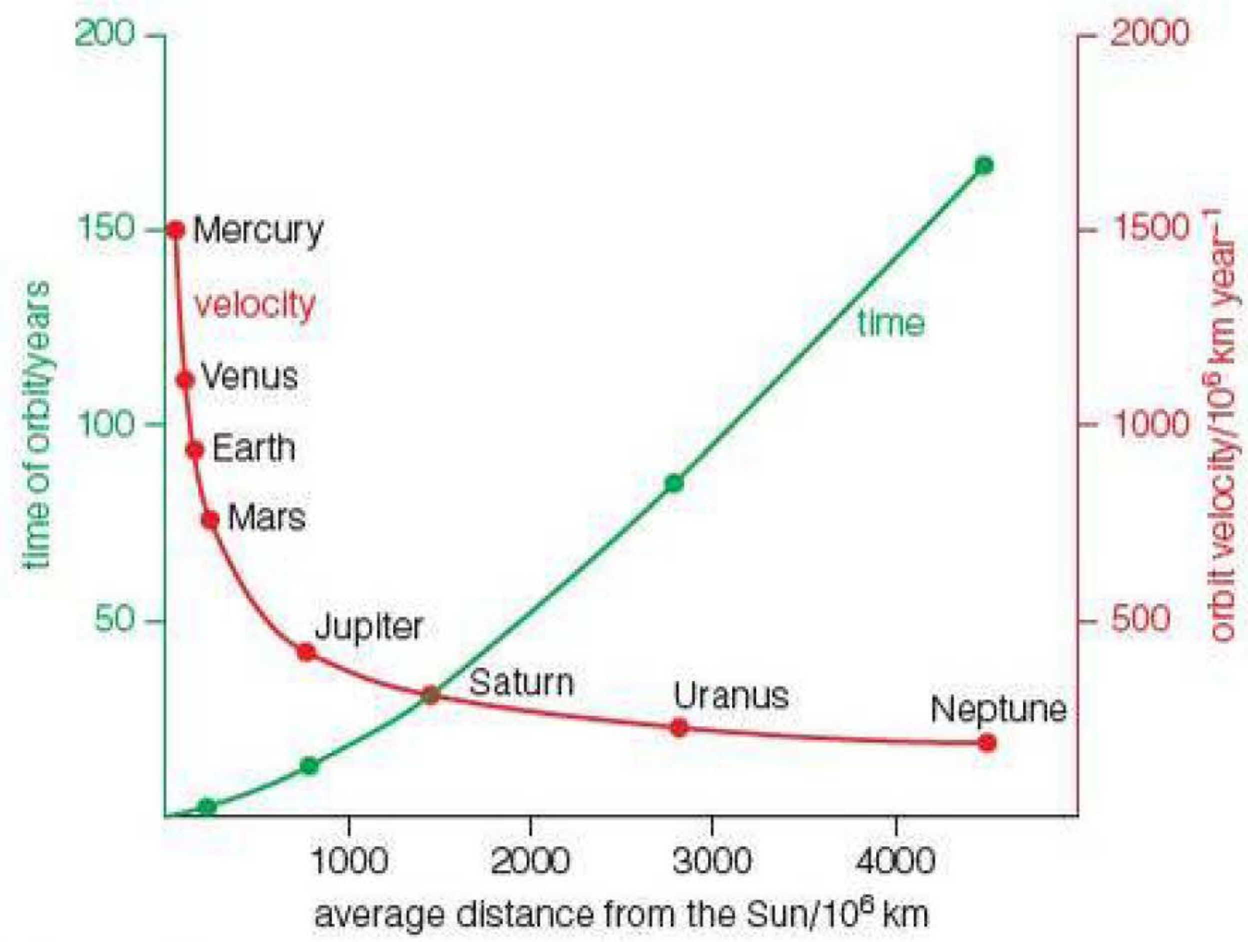


Figure 3.10

We can also link the time period of the orbit (1 year for the Earth) to the radius of the orbit. The speed of the orbit is linked to the circumference, $2\pi r$, and the time period of the orbit, T, through the equation

$$v = \omega r = \frac{2\pi r}{T}$$

so

$$v^2 = \frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

Figure 3.10 also shows the relationship between time period and orbital radius of the planet – the green curve.

Elliptical orbits

In the previous section, we treated all orbits as if they were circular – this is because it is relatively easy to cope with the mathematics of circular orbits. In practice, very few orbits are circular – most orbits have an elliptical shape. Figure 3.11 shows a possible elliptical orbit for a comet (black dot) travelling round the Sun. Most planetary orbits are nearly circular. For example, the Earth's closest approach to the Sun (perihelion) is 147×10^6 km and its furthest distance (aphelion) is 152×10^6 km. However, comets and some minor planets have elongated (or eccentric) elliptical orbits.

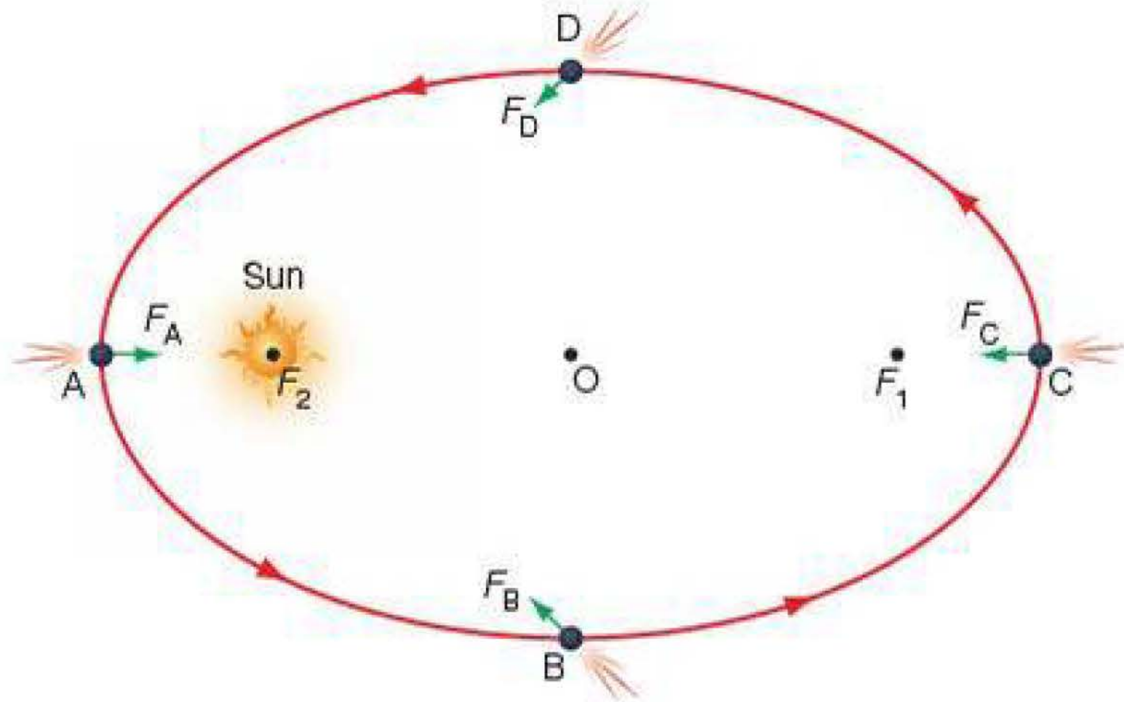


Figure 3.11 Comets move in elongated or eccentric elliptical orbits round the Sun. Note that the Sun's solar wind always blows the comet's tail away from the Sun.

You should note that the relationship derived in the last section linking the time period of an orbit to the radius of the orbit is still valid for elliptical orbits, provided r is taken to be half the *major axis* of the ellipse. The major axis in Figure 3.11 is the distance AC, and the minor axis is the distance BD. So, for the elliptical orbits we can write

$$T^2 = \left(\frac{4\pi^2}{GM}\right)(r_{sma})^3$$

where r_{sma} is the semi-major axis, equal to AO or OC.

When a planet or comet moves in an elliptical orbit, its speed changes, but the total energy of the body stays the same.

At point A in Figure 3.11, the comet is moving at its fastest orbital speed, but it has the smallest gravitational potential energy, because it is closest to the Sun.

At point B, there is a component of the Sun's gravitational pull on the comet, which slows it down.

When the comet reaches point C, it is at its furthest point from the Sun. It has its lowest kinetic energy at this point, but its highest gravitational potential energy.

At point D, the comet is falling back towards the Sun. There is a component of the Sun's gravitational pull, which speeds it up. The comet's potential energy is being transferred into kinetic energy, and the comet reaches its maximum speed again at A.

Satellite orbits

There are many satellites in orbit round the Earth, which are used for a range of purposes. Two of the most common uses of satellites are communications and observation.

Satellites placed in low orbits are able to take photographs of the world below. We are used to seeing images of mountain ranges, lakes and cities taken from space, and we use weather images on a daily basis.

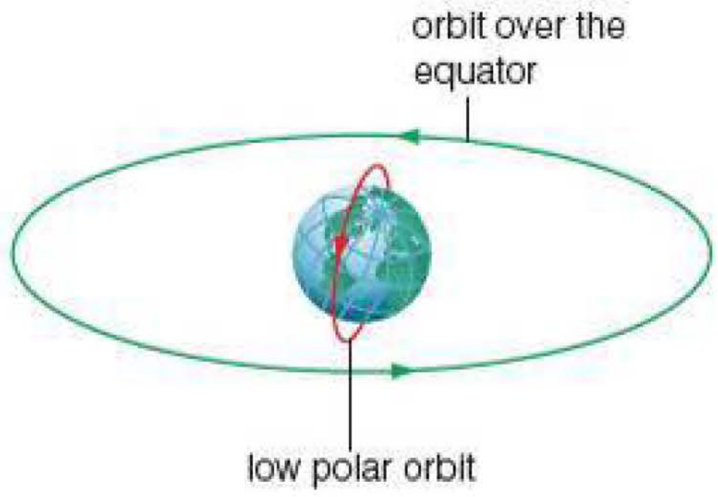


Figure 3.12

Satellites placed in higher orbits are useful for communications, because messages may be sent from one part of the world to another via the satellite. Different types of orbit are illustrated in Figure 3.12.

One of the most useful orbits for satellites is the geosynchronous orbit. In this case the satellite is placed in an orbit above the Earth's equator, at such a height that it takes exactly one day to complete an orbit. The orbit is synchronised with the Earth's rotation, so that it remains in the same place above the Earth's surface. This means that our satellite dishes, for example, can be aligned with a satellite, which always lies in the same position relative to Earth.

EXAMPLE

Geosynchronous satellite

Calculate the height of a geosynchronous satellite in orbit above the Earth's surface.

Answer

We can use the equation we derived earlier in this chapter:

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

or

$$r^3 = \left(\frac{GM}{4\pi^2}\right) T^2$$

$$r^3 = \frac{(6.67 \times 10^{-11}\,\text{N m}^2\,\text{kg}^{-3}) \times (6.0 \times 10^{24}\,\text{kg}) \times (24 \times 3\,600\,\text{s})^2}{4\pi^2}$$

$$r = 42.4 \times 10^6\ \text{m}$$

Since the radius of the Earth is 6.4×10^6 m, the height of the orbit is about 36×10^6 m or 36 000 km.

When a satellite is launched, it requires more energy to place it in a higher orbit, even though it travels more slowly. This is because we have to increase the potential energy of the satellite more to place it in a high orbit.

MATHS BOX

How much energy has to be supplied to a satellite of mass m to lift it into an orbit of radius r_2 above the Earth?

Answer

At the Earth's surface, radius r_1, the satellite's gravitational potential energy is $-\frac{GMm}{r_1}$ and its kinetic energy is zero.

In its orbit of radius r_2, the satellite's potential energy is $-\frac{GMm}{r_2}$. The satellite's kinetic energy in orbit may be calculated as follows.

In a circular orbit we can write

$$\frac{mv^2}{r_2} = \frac{GMm}{r_2^2}$$

⇨

so

$$E_k = \frac{1}{2}mv^2 = \frac{GMm}{2r_2}$$

The satellite's total energy is the sum of its kinetic energy and potential energy:

$$E_k + E_p = \frac{GMm}{2r_2} - \frac{GMm}{r_2} = -\frac{GMm}{2r_2}$$

This shows us that the total energy approaches zero as r_2 tends to infinity, and it is more negative for small value of r_2. You will remember that we chose to define the zero point of potential energy at infinity.

So the work done to put a satellite in orbit is the difference between its energy in its orbit and its energy on the Earth's surface:

$$\text{work done} = -\frac{GMm}{2r_2} - \left(-\frac{GMm}{r_1}\right)$$

$$= GMm\left[\frac{1}{r_1} - \frac{1}{2r_2}\right]$$

In practice, the kinetic energy of the satellite at the Earth's surface is not zero, as the Earth is rotating. This kinetic energy reduces the work done necessary to put a satellite in orbit and is one reason why launch sites are near the equator where the speed of rotation is greatest.

ACTIVITY

The moons of Saturn

Saturn is thought to have 62 moons in orbit around it. Table 3.2 shows information about six of Saturn's inner moons.

Table 3.2

Moon	Orbital radius/10^3 km	Orbital period/ days
Atlas	137	0.6
Mimas	185	0.9
Methone	194	
Enceladus	238	1.4
Tethys	295	1.9
Dione	377	2.7

The orbital period and orbital radius are linked by the equation:

$$T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$$

1 Draw up a table of T^2 and r^3 values (T should be calculated in seconds, and r in metres). Plot a graph of T^2 against r^3. From your graph, determine:
   a) Methone's orbital period
   b) Saturn's mass.

2 Dione has the same orbital radius about Saturn as our Moon does about Earth. Our Moon has an orbital period of about 27 days, which is 10 times longer than Dione's. Use the expression above to deduce the ratio of Saturn's mass to the Earth's mass.

3 You could also check the relationship between T and r by plotting a graph of $\log_{10} T$ against $\log_{10} r$.

$$T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$$

By taking the log of both sides we get

$$\log T^2 = \log\left[\left(\frac{4\pi^2}{GM}\right)r^3\right]$$

Therefore

$$\log T^2 = \log\left(\frac{4\pi^2}{GM}\right) + \log r^3$$

and then

$$2\log T = \log\left(\frac{4\pi^2}{GM}\right) + 3\log r$$

$$\log T = \frac{1}{2}\log\left(\frac{4\pi^2}{GM}\right) + \frac{3}{2}\log r$$

When you plot a graph of $\log T$ against $\log r$, you should find

that its gradient is $\frac{3}{2}$, which confirms the relationship that T^2 is proportional to r^3.

TEST YOURSELF

19 A satellite is in a low orbit over the poles of the Earth. It is 300 km above the surface of the Earth. The gravitational field strength at the surface of the Earth is $9.8\,N\,kg^{-1}$, and the radius of the Earth is 6400 km.

a) i) Explain how this satellite might be used.

ii) State another use of satellites, and explain what orbit you would use for the satellite you have chosen.

b) i) Calculate the gravitational field strength at a height of 300 km.

ii) Calculate the orbital period of the satellite.

20 **a)** Explain the difference between a planet's orbit and a comet's orbit.

b) Explain why a comet's orbital speed changes throughout its orbit.

21 A satellite takes 120 minutes to orbit the Earth.

a) Calculate the satellite's angular velocity.

b) Calculate the radius of the satellite's orbit. Take GM_E to equal $4.0 \times 10^{14}\,N\,kg^{-1}\,m^2$.

22 The Sun has a mass of 2×10^{30} kg; the radius of the Earth's orbit is 1.5×10^8 km; the Earth's mass is 6×10^{24} kg.

a) i) Calculate the angular velocity of the Earth.

ii) Calculate the Earth's orbital speed.

b) i) Calculate the centripetal force on the Earth.

ii) Show that the centripetal force on the Earth is equal to the Sun's gravitational pull on the Earth.

c) Calculate the Sun's gravitational field strength at a distance equal to the Earth's orbit round it.

Practice questions

1 The gravitational field strength at the surface of a planet, of radius 8000 km, is $15\,\mathrm{N\,kg^{-1}}$. The gravitational field strength at a height of 4000 km above the planet is

A $10\,\mathrm{N\,kg^{-1}}$ **C** $6.7\,\mathrm{N\,kg^{-1}}$

B $8.0\,\mathrm{N\,kg^{-1}}$ **D** $4.4\,\mathrm{N\,kg^{-1}}$

2 The gravitational field strength on the surface of the Earth is g. The gravitational field strength on a planet with twice the mass of the Earth and twice the radius of the Earth is

A $g/8$ **C** $g/2$

B $g/4$ **D** g

3 The gravitational potential on the surface of a planet with mass M and radius R is $-V$. The potential on a second planet with mass $2M$ and radius $R/3$ is

A $-2V/3$ **C** $-2V$

B $-3V/2$ **D** $-6V$

4 A satellite is in a circular orbit round a planet of radius 5200 km, at a height of 1800 km. At this height the gravitational field strength is $4.2\,\mathrm{N\,kg^{-1}}$. The speed of the satellite is

A $5.4\,\mathrm{km\,s^{-1}}$ **C** $2.4\,\mathrm{km\,s^{-1}}$

B $4.5\,\mathrm{km\,s^{-1}}$ **D** $0.2\,\mathrm{km\,s^{-1}}$

5 The centres of two planets, each of mass M, are separated by a distance r.

Which of the following correctly gives the gravitational field strength and the gravitational potential, at a point halfway between the centres of the planets?

	Gravitational field	Gravitational potential
A	$8GM/r^2$	0
B	$4GM/r^2$	$-2GM/r$
C	0	0
D	0	$-4GM/r$

6 The gravitational field strength at the surface of the Earth is $9.8\,\mathrm{N\,kg^{-1}}$. At the surface of the Moon the field strength is $1.7\,\mathrm{N\,kg^{-1}}$. The Earth has a mass 81 times that of the Moon. The ratio of the Earth's radius to the Moon's radius is

A 2.9 **C** 4.9

B 3.7 **D** 7.6

7 Two stars of mass M and $4M$ are a distance r apart (Figure 3.13).

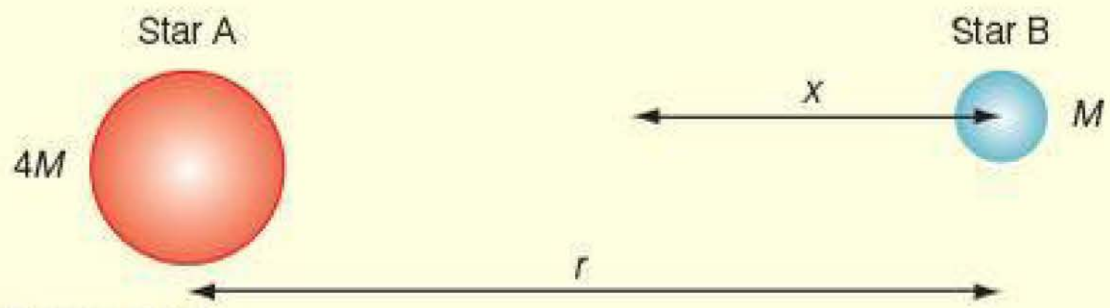


Figure 3.13

The resultant gravitational field strength is zero along the line between their centres at a distance x from the centre of the star with mass M. The ratio of x/r is

A 3/4 **C** 1/2

B 2/3 **D** 1/3

8 The diameter of the Earth is twice that of Mars and the mass of the Earth is 10 times that of Mars. The gravitational potential at the surface of Mars is $-13\,\text{MJ kg}^{-1}$. The gravitational potential at the surface of Earth is

A $-290\,\text{MJ kg}^{-1}$ **C** $-65\,\text{MJ kg}^{-1}$

B $-120\,\text{MJ kg}^{-1}$ **D** $-28\,\text{MJ kg}^{-1}$

9 A satellite is in orbit above the Earth at a distance of 9000 km from the Earth's centre. At this height the gravitational field strength is $5.0\,\text{N kg}^{-1}$. The time period of the orbit of the satellite is

A 90 minutes **C** 180 minutes

B 140 minutes **D** 270 minutes

10 The time period, T, of a body orbiting the Sun is given by the formula

$$T^2 = \frac{4\pi R^3}{GM}$$

where M is the mass of the Sun and R is the radius of the orbit. Halley's comet takes 76 years to orbit the Sun. The ratio

$$\frac{\text{average radius of Halley's comet orbit}}{\text{average radius of the Earth's orbit}}$$ is

A 9 **C** 76

B 18 **D** 660

11 a) Work out the correct unit, expressed in SI base units, for $\left[\frac{g^2}{G}\right]$. (2)

b) The gravitational field strength at the surface of the Earth is six times the gravitational field strength on the surface of the Moon. The mean density of the Moon is 0.6 times the mean density of the Earth. Calculate the ratio: $\left[\frac{\text{radius of Earth}}{\text{radius of Moon}}\right]$. (3)

12 In Figure 3.14, the gravitational potential at A is $-16\,\text{MJ}\,\text{kg}^{-1}$ and the gravitational field strength at A is $4\,\text{N}\,\text{kg}^{-1}$.

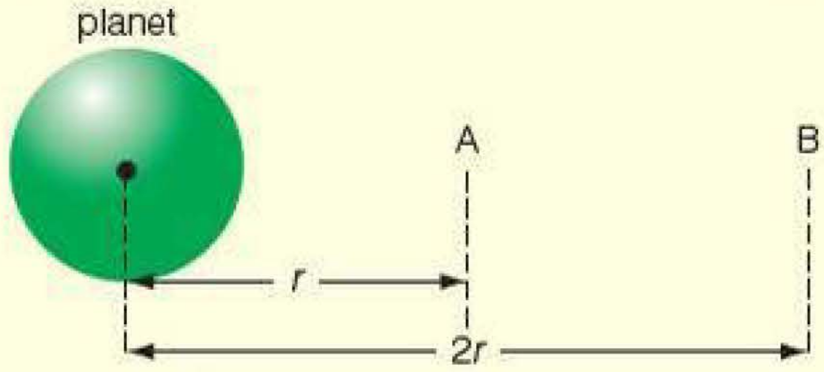


Figure 3.14

a) Calculate the work done in taking a 120 kg mass from A to B. *(3)*

b) Calculate the gravitational field strength at B. *(2)*

13 The gravitational field strength at the surface of the Sun is $270\,\text{N}\,\text{kg}^{-1}$. Betelgeuse is a red giant star, which has a density of approximately 0.01 times that of the Sun and a radius about 1000 times that of the Sun.

Calculate the gravitational field strength on the surface of Betelgeuse. *(3)*

14 At point P in Figure 3.15, the gravitational field strength is zero, and the gravitational potential is $-8.0 \times 10^7\,\text{J}\,\text{kg}^{-1}$.

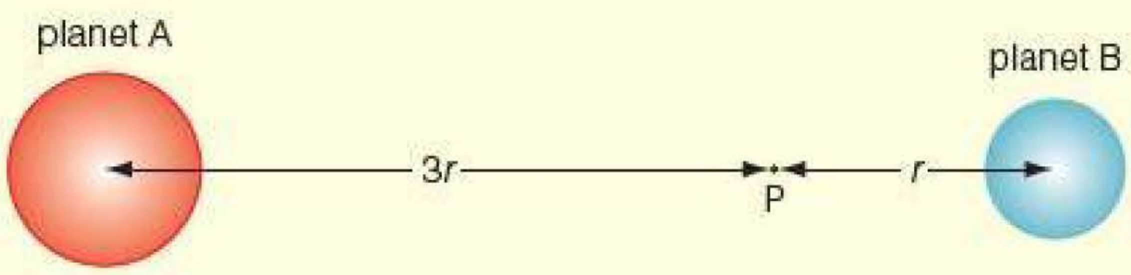


Figure 3.15

a) Calculate the work done to remove a spacecraft of mass 600 kg to a point infinitely far away from these planets. *(3)*

b) Calculate the ratio: $\dfrac{\text{mass of planet A}}{\text{mass of planet B}}$. *(3)*

15 Io is a moon of Jupiter. Io rotates around Jupiter once every 42 hours, in an orbit of radius of 420 000 km.

a) Calculate the angular velocity of Io. *(3)*

b) Use the data above to calculate the mass of the planet Jupiter. *(3)*

c) The radius of the orbit of a moon is proportional to $T^{\frac{2}{3}}$, where T is the time period of the orbit. Ganymede is another moon of Jupiter that takes 168 hours to rotate around the planet.

Calculate the radius of Ganymede's orbit. *(4)*

16 Two identical spheres exert a gravitational force F on each other.

a) What gravitational force do two spheres, each twice the mass of the original spheres, exert on each other when the separation of their centres is four times the original separation? *(2)*

b) The gravitational force between two uniform spheres is $3.7 \times 10^{-9}\,\text{N}$ when the distance between their centres is

200 mm. The mass of one sphere is 3.0 kg; calculate the mass of the second sphere. *(3)*

c) The gravitational potential difference between the surface of a planet and a point 20 m above the surface is 800 J kg^{-1}. Calculate the gravitational field strength in the region close to the planet's surface. *(3)*

17 a) Calculate the time period of the Earth's rotation, if you were to be made to feel weightless at the equator. The radius of the Earth is 6.4×10^6 m. *(3)*

b) A satellite is in orbit, of radius r, around a planet of mass M. Write down expressions for

i) its orbital speed (2)

ii) the time period of its orbit. (2)

18 Figure 3.16 shows a sketch of the Earth–Moon system. The gravitational potential at the surface of the Earth is –62.8 MJ kg^{-1}; the gravitational potential at the surface of the Moon is –2.3 MJ kg^{-1}; the gravitational potential at point N is –1.3 MJ kg^{-1}. Point N is the neutral point between the Earth and Moon where the gravitational field is zero.

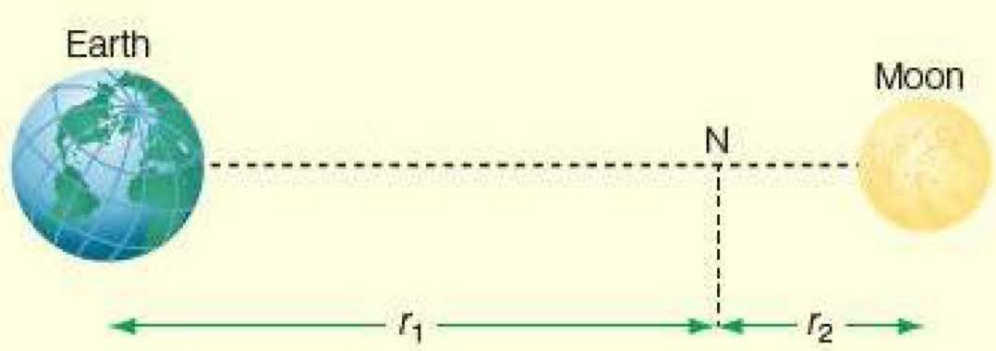


Figure 3.16

a) The Earth is 81 times as massive as the Moon. Calculate the ratio $\frac{r_1}{r_2}$. (2)

b) i) Calculate the minimum amount of energy required to move a space probe of mass 2.0×10^4 kg from the Earth to point N. *(3)*

ii) Explain why no more fuel is required to take the space probe from point N to the Moon. *(1)*

c) The amount of fuel required to take a spacecraft to the Moon is much higher than that required to return it to Earth. Explain why this is so referring to the forces involved – gravitational field strength and gravitational potential. *(6)*

Stretch and challenge

19 M87 (Messier Catalogue number 87), is a giant galaxy, and is about 6×10^{12} times as massive as our Sun. The gravitational pull of this galaxy keeps star clusters in orbit around it. In the centre of this galaxy is a giant black hole of about 5×10^9 solar masses.

a) The event horizon of the black hole is the maximum radius from which something can just escape the black hole travelling at the speed of light.

i) Write down an expression for the gravitational potential at this point, in terms of the mass of the black hole, M, and the radius of the event horizon, r.

ii) Write an expression for the kinetic energy of a kilogram mass travelling at the speed of light, c.

iii) Calculate the radius of the event horizon. Take the mass of the Sun to be 2×10^{30} kg, the speed of light to be $3 \times 10^8\,\text{m s}^{-1}$, and $G = 6.7 \times 10^{-11}\,\text{N kg}^{-2}\,\text{m}^2$.

b) i) A globular cluster of stars orbits M87 at a distance of 300 000 light years. Calculate the time period of the orbit.

[1 light year = 9.5×10^{15} m]

ii) What happens to any stars or clusters of stars near the galaxy if they rotate too slowly to stay in an orbit?

20 Figure 3.17 shows the orbit of a comet as it falls in towards the Sun and then leaves the inner Solar System again. The gravitational potential due to the Sun has the following values at these distances from the Sun: at Saturn's orbit, $-93\,\text{MJ kg}^{-1}$; at Jupiter's orbit, $-172\,\text{MJ kg}^{-1}$; at the Earth's orbit, $-893\,\text{MJ kg}^{-1}$.

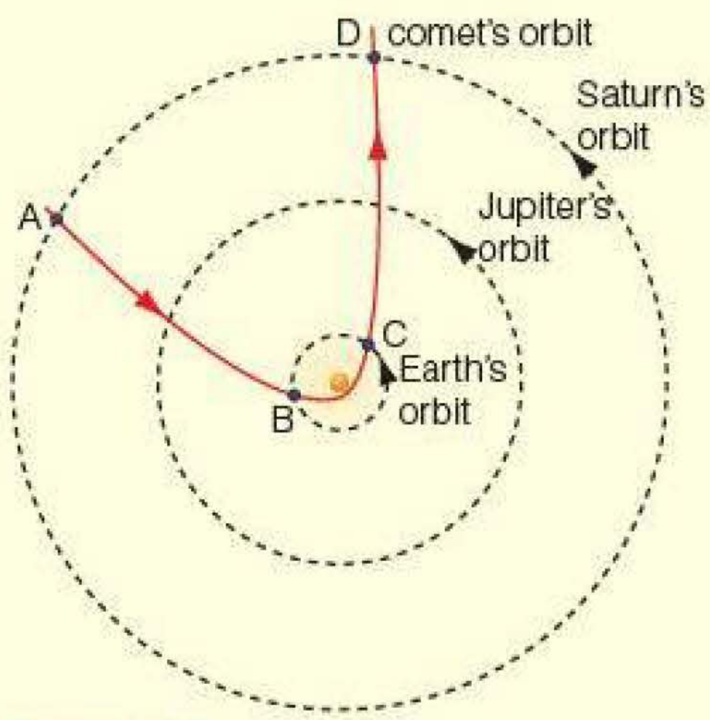


Figure 3.17

a) From this information calculate this ratio:

$$\frac{\text{radius of Saturn's orbit}}{\text{radius of Jupiter's orbit}}$$

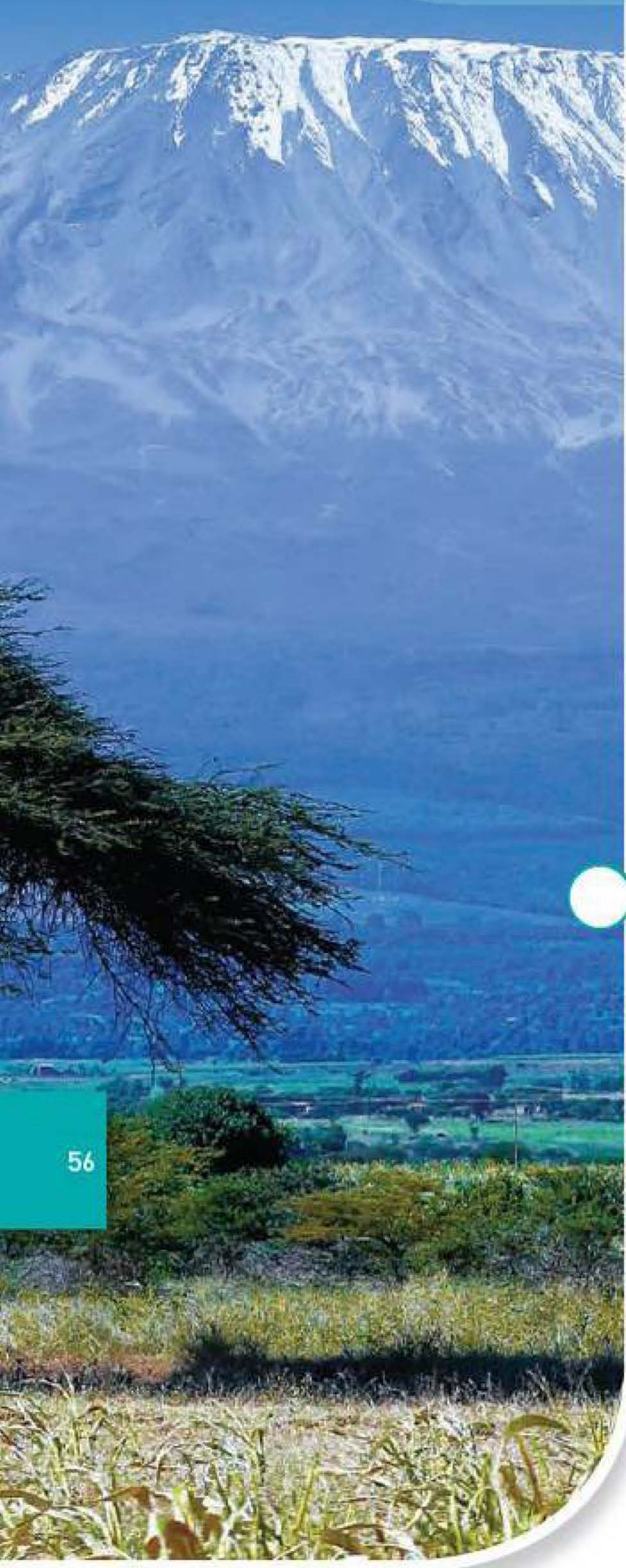
b) As the comet moves from point A to point B, it increases its speed. Explain why.

c) i) The comet has a mass m. At point A its speed is v_A and at point B its speed is v_B. Write an equation to link the comet's increase in kinetic energy to its decrease in gravitational potential energy.

ii) At point A the comet's speed is $3 \times 10^4\,\text{m s}^{-1}$. Calculate the comet's speed at point B.

iii) State the comet's speed at points C and D.

4 Thermal physics



PRIOR KNOWLEDGE

Before you start, make sure that you are confident in your knowledge and understanding of the following points:

- The kinetic theory model of solids, liquids and gases assumes that particles are incompressible spheres.
- Solids have a close-packed, regular particle structure – the particles vibrate about fixed points.
- Liquids have a close-packed, random, irregular particle structure – the particles are free to move.
- Gases have a widely spaced, irregular particle structure – the particles move at high speed in random directions.
- Thermal energy can be transferred from somewhere hot (at a high temperature) to somewhere cooler (at lower temperature) by the processes of conduction, convection, radiation and evaporation.

TEST YOURSELF ON PRIOR KNOWLEDGE

1 Draw simple diagrams showing the arrangements of particles inside a solid, a liquid and a gas.
2 Explain the difference between the transfer of thermal energy, from a hot body to a cold body, through conduction and through convection.
3 Explain how evaporation transfers thermal energy away from a hot cup of tea.

Thermodynamics

During the late 18th and early 19th centuries scientists, inventors and engineers began to develop steam engine technology, such as the giant steam-powered beam-engine pumps used to pump water out of deep Cornish tin mines.

Development of the engines required a systematic and fundamental understanding of the nature of heat energy; its relationship to the behaviour of steam and the other materials making up the engines; and the work done by the engine. This study became known as thermodynamics and Britain led the world, not only in the development of the new engines, but also in the fundamental physics of thermodynamics.

Thermodynamics deals with the macroscopic (large-scale) behaviour of a system, but it is complemented by the kinetic theory of matter, which deals with the microscopic, particle-scale behaviour of matter. Some aspects of thermal physics are best explained in terms of macroscopic thermodynamics, such as the behaviour of engines, but other aspects are best explained using microscopic kinetic theory, such as Brownian motion (the tiny random motion of pollen or smoke particles seen under a microscope).

Internal energy

Internal energy The sum of the randomly distributed kinetic and potential energies of the particles in a body.

One of the most fundamental properties of thermodynamics is the concept of **internal energy**, U, which is the sum (sometimes called an *ensemble* in thermodynamics) of the randomly distributed kinetic energies and potential energies of the particles in a body:

$$U = \sum(\text{kinetic energies}) + \sum(\text{potential energies})$$

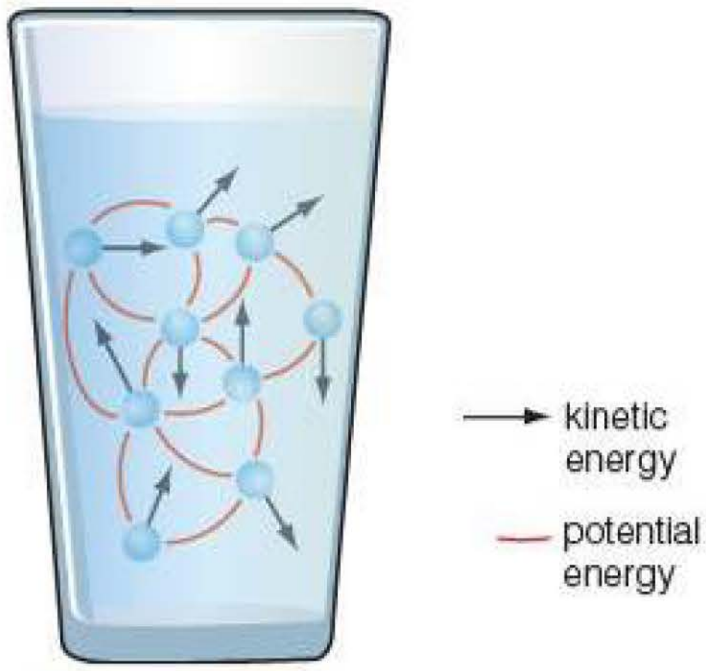


Figure 4.1 Glass of water, showing internal energy.

Consider a glass of water (Figure 4.1). The water particles have two types of energy – kinetic energy associated with their movement (the faster they move, or vibrate or rotate, the higher their kinetic energy) and potential energy associated with any forces or interactions between the particles (such as any electrostatic attraction or repulsion).

The kinetic energies of the particles depend on their temperature, and the potential energies depend on any intermolecular forces between the particles. For ideal gases, in which there are no intermolecular forces, the internal energy is dependent on only the kinetic energies.

The first law of thermodynamics

Fluid A substance that can flow – i.e. a gas or a liquid.

The physicists working on the theories of how engines worked quickly realised that there was an interplay between the changes in heat energy and the work being done on or by the **fluids** in the engines. This was formalised by the first law of thermodynamics, written by Rudolf Clausius in 1850. A modern version of his law can be stated as follows:

> The increase in internal energy of a system is equal to the heat added to the system minus the work done by the system.

In terms of symbols, this can be written:

$$\Delta U = \Delta Q - \Delta W$$

where ΔU is the increase in internal energy of the system (usually a gas), ΔQ is the thermal energy added to the system and ΔW is the work done by the system.

Work done by an expanding gas

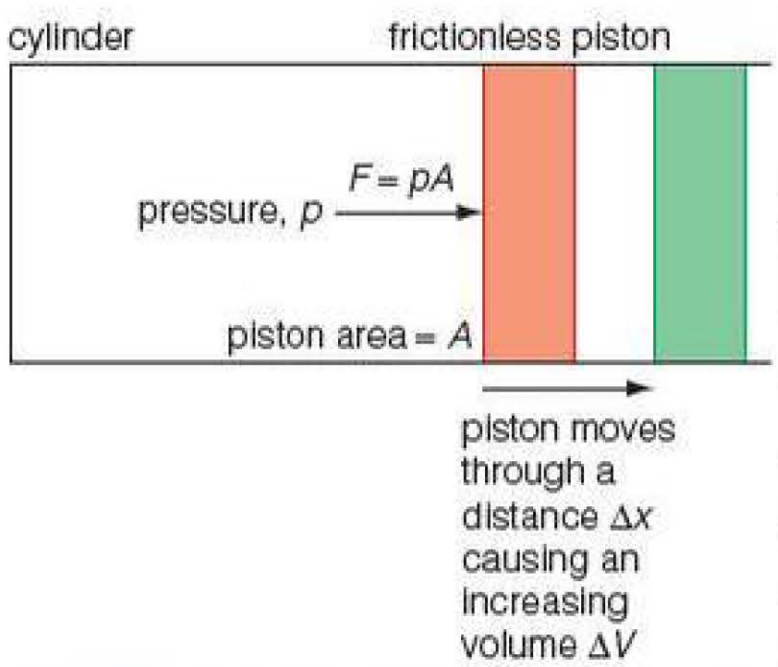


Figure 4.2 A gas expanding in a cylinder.

When a gas expands, it exerts a force on the surroundings, causing them to move – the gas does *work* on the surroundings. We can use the first law of thermodynamics to determine the work done, ΔW, by an expanding gas at constant temperature (called an *isothermal* change). Consider a gas enclosed in a cylinder by a frictionless piston, as shown in Figure 4.2.

The gas of volume V exerts a pressure p on the walls of the cylinder. This in turn exerts a force F on the frictionless piston of area A, where

$$F = pA$$

This causes an increase in the volume, ΔV. We assume that ΔV is very small and that the force moves the piston at a slow but steady rate such that the

TEST YOURSELF

1 What is thermodynamics?
2 State two ways in which the internal energy of a gas inside a bicycle tyre pump can be increased.
3 Calculate the work done on a gas when its internal energy increases by 1864 kJ as it is heated, causing its thermal energy to increase by 1247 kJ.

external force exerted on the piston is equal to the force exerted by the pressure p of the gas in the cylinder. This effectively makes the pressure exerted by the gas constant during the expansion. The gas does work, and so ΔW is positive. The force on the piston moves it through a distance, Δx, such that:

$$\Delta W = -F\Delta x$$

substituting for $F = pA$ gives

$$\Delta W = pA\Delta x$$

But $A\Delta x = \Delta V$, the change in volume of the gas, so

$$\Delta W = p\Delta V$$

Heating up substances and changes of state

When substances are heated, thermal energy is supplied to the particles of the substance, increasing their internal energy U, and therefore the average kinetic energy of the particles. Increasing the average kinetic energy of the particles causes the temperature of the particles to rise. The size of temperature change, $\Delta\theta$, is dictated by several macroscopic, measurable factors: the amount of thermal heat energy supplied, Q; the mass of the substance, m; and a quantity called the specific heat capacity of the substance, c, which is unique to each substance; and its state. These factors are related to each other by the equation:

$$Q = mc\Delta\theta$$

The thermal energy Q is measured in joules (J), the mass m is measured in kilograms (kg), and the temperature change $\Delta\theta$ is measured in kelvin (K), so the units of specific heat capacity, c, are $\text{J kg}^{-1}\,\text{K}^{-1}$.

The specific heat capacity of a material is a fundamental property of the material and is particularly important to engineers and scientists designing engines and insulation systems. A specific heat capacity dictates how easy it is for a material to change its temperature. Materials with very high specific heat capacities, such as water, $c_w = 4186\,\text{J kg}^{-1}\,\text{K}^{-1}$ (usually rounded up to $4200\,\text{J kg}^{-1}\,\text{K}^{-1}$), require a great deal of thermal energy to increase the temperature of 1 kg of the material by 1 K, whereas materials such as gold with quite low specific heat capacities, $c_{Au} = 126\,\text{J kg}^{-1}\,\text{K}^{-1}$, require only a small quantity of thermal energy to increase the temperature of 1 kg of the material by 1 K.

Water has a particularly high specific heat capacity. Other common materials on Earth have substantially lower values: granite rock, for example, has a specific heat capacity of $790\,\text{J kg}^{-1}\,\text{K}^{-1}$, less than one fifth that of water. Without this property, life may not have been possible on Earth, because water would almost alway be in the gaseous state.

The specific heat capacity of a material enables us to measure the change in temperature of a material following a change in thermal energy.

EXAMPLE

Warming water

An aluminium saucepan is used to warm 1.50 kg of tap water (at 18°C) for a hot-water bottle by heating it on a 3.0 kW electric hob for 4.0 minutes. Assuming that 60% of the electrical energy is absorbed by the water, and that there are no subsequent heat losses, calculate the final temperature of the warm water. The specific heat capacity of water is $c_w = 4186\ \mathrm{J\,kg^{-1}\,K^{-1}}$.

Answer

Total electrical energy produced by the electric hob is

$$E = 3.0 \times 10^3\,\mathrm{W} \times 4.0 \times 60\,\mathrm{s}$$

$$= 7.2 \times 10^5\,J$$

Thermal energy supplied to the water is

$$Q = \frac{60}{100} \times 7.2 \times 10^5\,\mathrm{J}$$

$$= 4.32 \times 10^5\,\mathrm{J}$$

But $Q = mc\Delta\theta$ so

$$\Delta\theta = \frac{Q}{mc}$$

$$= \frac{4.32 \times 10^5\,\mathrm{J}}{1.50\,\mathrm{kg} \times 4186\,\mathrm{J\,kg^{-1}\,K^{-1}}}$$

$$= 68.8\,\mathrm{K} = 69\,\mathrm{K}\ (2\ \mathrm{s.f.})$$

Because a temperature change of 1 K is equal to a temperature change of 1°C, the final temperature of the water in the saucepan is 18°C + 69°C = 87°C.

EXAMPLE

Falling lead shot

The specific heat capacity of lead can be determined by letting lead shot fall inside a long tube. The lead shot heats up as gravitational potential energy is transferred to thermal energy of the shot. The experiment is shown in Figure 4.3.

A student tipped some lead shot up and down in the tube and found that after 20 turns the temperature of the lead had risen by 1.5°C. Estimate the specific heat capacity of lead. You may assume that the tube itself does not warm up.

Note: If you are handling lead shot, make sure to wash your hands afterwards.

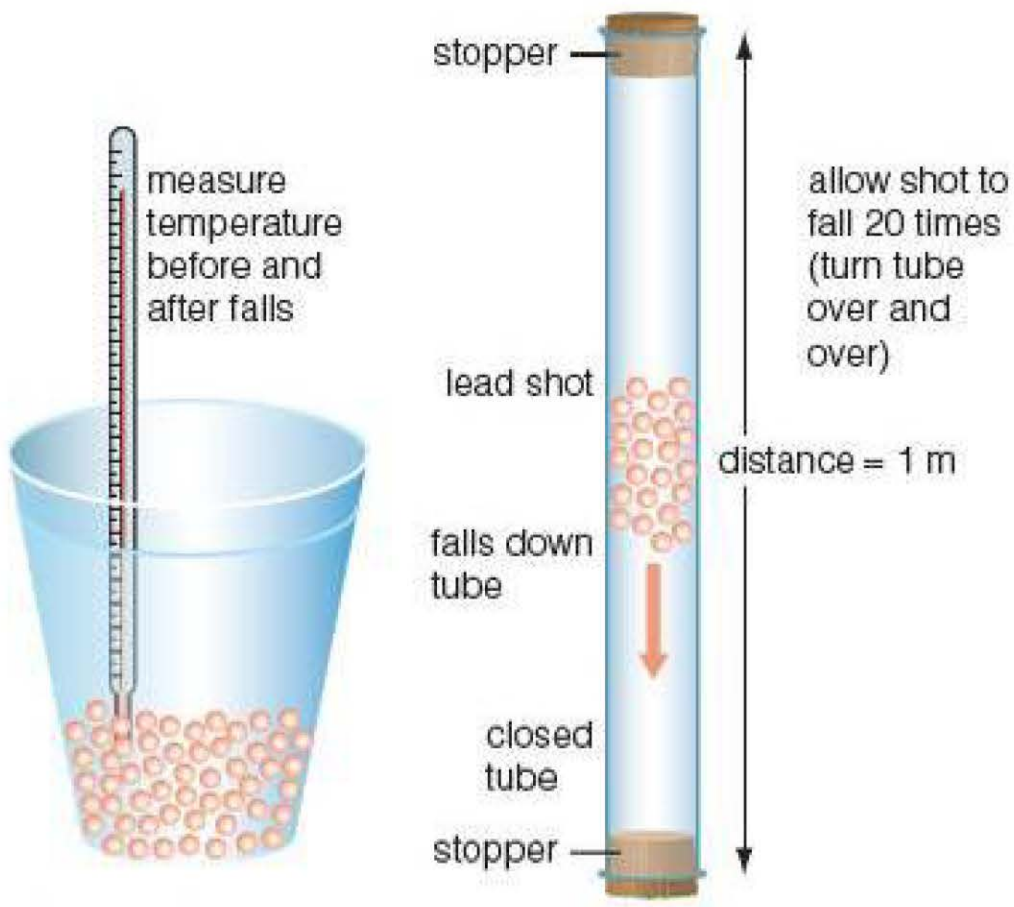


Figure 4.3 Lead shot experiment.

Answer

The gravitational potential energy of the falling lead is transferred to heat in the lead. So

$mgh = mc\Delta\theta$

and (because a temperature change of 1°C is equal to a temperature change of 1 K) we obtain

$$c = \frac{gh}{\Delta\theta}$$

$$= \frac{9.8\,\mathrm{N\,kg^{-1}} \times 20\,\mathrm{m}}{1.5\,\mathrm{K}}$$

$$= 130\,\mathrm{J\,kg^{-1}\,K^{-1}}$$

Mixing hot with cold

If a hot liquid or a solid is placed into a cold liquid, the internal energy transferred from the hot object when it cools down is equal to the thermal energy gained by the cold liquid and its container, plus the thermal energy lost to its surroundings. In the example shown in Figure 4.4, the thermal energy lost to the surroundings is assumed to be negligible.

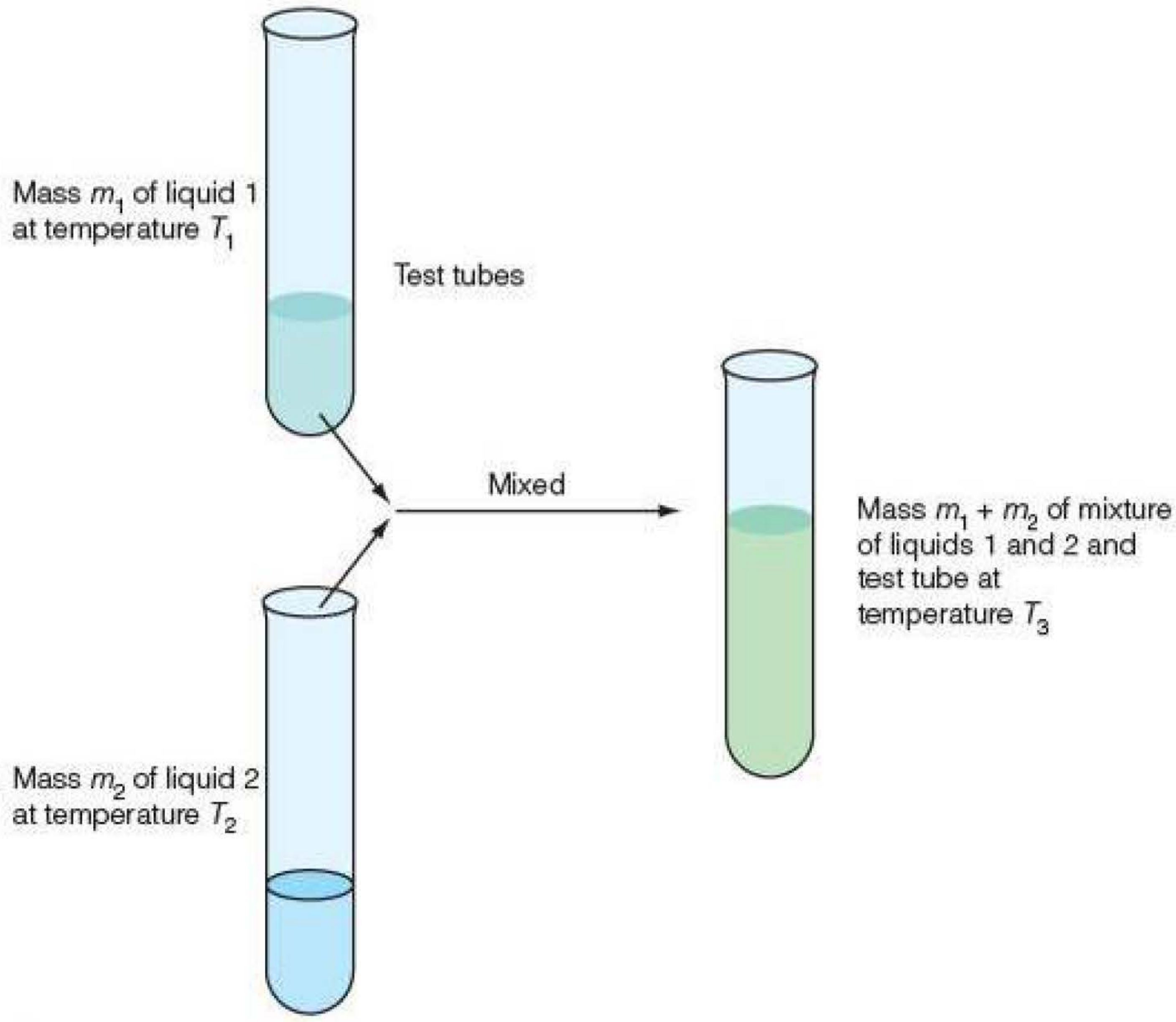


Figure 4.4 Specific heat capacity and mixtures.

EXAMPLE

Mixing hot and cold liquids

In an experiment, 20.0 g of hot seawater at 65°C is mixed with 80.0 g of tap water at 12.0°C inside a copper calorimeter of mass 75.0 g also at 12°C. If the thermal energy lost to the surroundings is negligible, calculate the new temperature of the mixture and the calorimeter. The specific heat capacities of seawater, tap water and copper are $3990\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$, $4200\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$ and $386\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$, respectively.

Answer

The new (unknown) final temperature of the water mixture and the calorimeter we will call T (°C). So (because a temperature change in °C is equal to a temperature change in K) the thermal energy Q lost by the hot seawater is:

$$Q = m_{\text{seawater}} \times c_{\text{seawater}} \times \Delta\theta_{\text{seawater}}$$

$$= 20.0 \times 10^{-3}\,\text{kg} \times 3990\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1} \times (65 - T)\,\text{K}$$

$$= (5187 - 79.8T)\,\text{J}$$

This is equal to the thermal energy Q gained by the tap water and the copper calorimeter:

$$Q = [80.0 \times 10^{-3}\,\text{kg} \times 4200\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1} \times (T - 12)\,\text{K}]\,\text{J} + [75.0 \times 10^{-3}\,\text{kg} \times 386\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1} \times (T - 12)\,\text{K}]\,\text{J}$$

$$= [336T - 4032]\,\text{J} + [28.95T - 347.4]\,\text{J}$$

$$= (364.95T - 4379.4)\,\text{J}$$

Equating these two values and rearranging gives

$$5187 - 79.8T = 364.95T - 4379.4$$

$$444.75T = 9566.4$$

$$T = 21.5°\text{C}$$

ACTIVITY

Measuring the specific heat capacity of a metal block

The specific heat capacity of a solid material can be measured (with reasonable certainty in the laboratory) by heating a known mass of the material with a known quantity of thermal energy, usually supplied via an electrical heater. One such experiment involving a copper block is shown in Figure 4.5.

In this experiment, the mass of the copper block was measured with an electronic balance and was found to be 0.814 kg. The block is heated using a stabilised 12.0 V dc power supply delivering 4.0 A of current to the electric heater in the block. The temperature of the block was measured every 20 s for 2 minutes while all the apparatus came to thermal equilibrium. After 2 minutes the heater was switched on and the temperature recorded every 20 s again for a further 3 minutes, before it was switched off. The temperature of the block continued to be measured every 20 s for a further 2 minutes, during which time the block started to cool down.

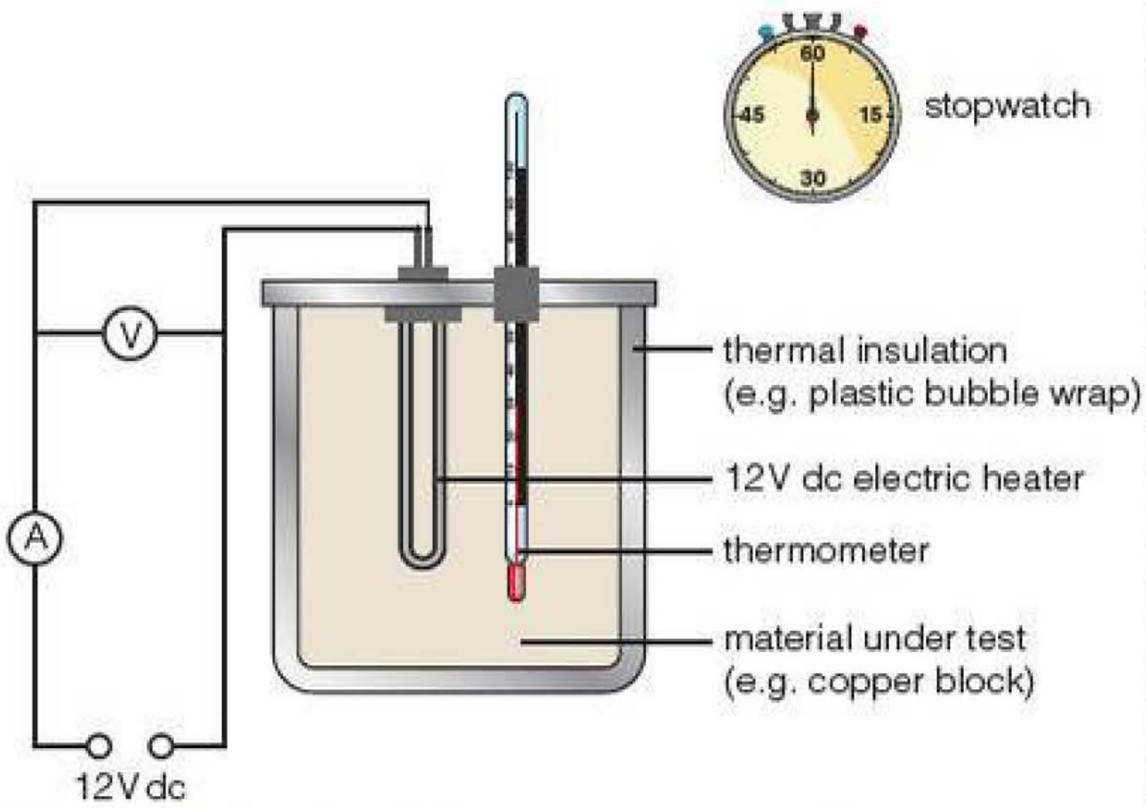


Figure 4.5 Experiment to measure the specific heat capacity of a metal block.

The results of the experiment are shown in Table 4.1.

Table 4.1

Time, *t*/s	Temperature, *T*/°C
0	16.4
20	16.5
40	16.4
60	16.4
80	16.3
100	16.4
120	16.4
140	19.1
160	21.9
180	24.6
200	27.4
220	30.1
240	32.9
260	35.6
280	38.4
300	41.1
320	40.8
340	40.5
360	40.2
380	39.9
400	39.6
420	39.3

1 Plot a graph of the results and draw a smooth best-fitting line through the points.

2 A calorimetric technique is used to determine the temperature change of the block. The cooling part of the graph is used to take into account the heat still present in the electric heater when it was turned off but had not transferred into the block. A sketch of how to use this technique is shown in Figure 4.6.

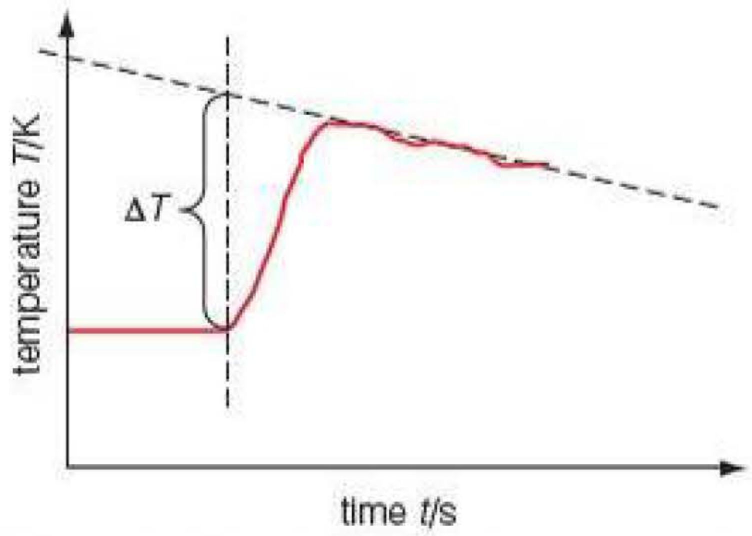


Figure 4.6 Graph showing how to calculate the temperature change.

The change in temperature of the block, ΔT, will always be slightly higher than the highest temperature reached minus the starting temperature – this accounts for the extra thermal energy left in the heater when it is switched off. In other words, ΔT gives the temperature the block would have reached if all the energy could be transferred instantly to the block, without any heat being lost. Use this graphical calorimetric

⇨

technique to determine the temperature change of the copper block in this experiment.

3 Use the rest of the data to calculate the specific heat capacity of the copper in the block.

4 The given specific heat capacity of copper at room temperature is about 386 J kg^{-1} K^{-1}. Suggest reasons why your calculated value may be different from the given value.

5 Explain how repeating the experiment would lead to determining the uncertainty in the measurement of the specific heat capacity of copper using this technique.

6 This experiment suffers from a collection of random and systematic errors. Identify these errors, state whether they are random or systematic in nature, and suggest ways in which they could be minimised.

TEST YOURSELF

4 A student uses a microwave oven to warm up a cup of cold tea.

a) Thermal energy is supplied to the tea at a rate of 750 W. The tea has a mass of 0.42 kg and an initial temperature of 17°C. Calculate the final temperature of the tea. Assume that the specific heat capacity of the tea is 4200 J kg^{-1} K^{-1}.

b) In reality, some of the thermal energy goes into the cup and some is used by (water) particles to evaporate. What is the effect of this evaporation on the final temperature of the tea?

c) Following re-heating the tea, the student decides that the tea is too strong and adds milk from the fridge at a temperature of 5.5°C, and the temperature of the tea drops to 71.0°C. During this time the student assumes that no thermal energy is lost to the surroundings. Calculate the decrease in thermal energy of the tea.

d) If all the thermal energy transferred by the tea is used to heat up the milk, calculate the mass of the milk added by the student to the tea. Take the specific heat capacity of milk to be 4000 J kg^{-1} K^{-1}.

5 A college sports dome has an internal air volume of 24 000 m^3.

a) If the air inside the dome has a density of 1.2 kg m^{-3}, calculate the mass of air inside the dome.

b) During winter, the dome is kept inflated with air pumped in from outside with a temperature of 5.0°C. Overnight, the heater in the dome is turned off, and the average temperature of the air in the dome falls to 5.0°C. Calculate the thermal energy required to heat the air in the dome to a more pleasant 16°C in the morning. The specific heat capacity of air is 1000 J kg^{-1} K^{-1}.

c) The dome contains four industrial space heaters rated at 14.7 kW. If the space heaters are 100% efficient, how long will it take for the heaters to warm the air up in the dome to 16°C?

d) Explain why the actual energy value required to heat up the dome will be larger than that calculated in (b).

6 A large tropical fish tank has dimensions of 240 cm × 60 cm × 60 cm.

a) If the density of water is 1.0 g cm^{-3}, calculate the mass of water in the tank (in kg).

b) The tank is set up using water from an outside water butt with a temperature of 9.5°C. The thermostat on the heater is set to 25.5°C. Calculate the thermal energy needed to warm up the water in the tank to the desired temperature. The specific heat capacity of water is 4200 J kg^{-1} K^{-1}.

c) The tank is kept at its optimum temperature of 25.5°C by a 100 W heater and thermostat. If the heater should develop a fault and fail, show that the initial rate of fall of temperature in the tank will be about 0.1°C per hour.

d) The rate of fall of temperature of the water in the fish tank, $\Delta\theta/\Delta t$, can be described using Newton's law of cooling:

$$\frac{\Delta\theta}{\Delta t} = -k(\theta_w - \theta_s)$$

where θ_w is the temperature of the water in the tank and θ_s is the temperature of the surroundings. If the temperature of the water when the heater failed was 25.5°C and the temperature of the room it was in was 15.0°C, use your answer to (c) to determine the value of the constant k.

e) Use your value of k to determine the rate of cooling of the water in the fish tank if the temperature of the room was to fall to 8°C.

Measuring the specific heat capacity of water using a continuous flow method

The specific heat capacity of a fluid can be measured using a continuous flow method (Figure 4.7), where the fluid moves over an electric heater at a constant rate. It is assumed that the thermal energy transferred from the apparatus to the surroundings is constant. The experiment is carried out and then the flow rate of the fluid is changed, and a second set of readings is taken. The heat loss can then be eliminated from the calculations.

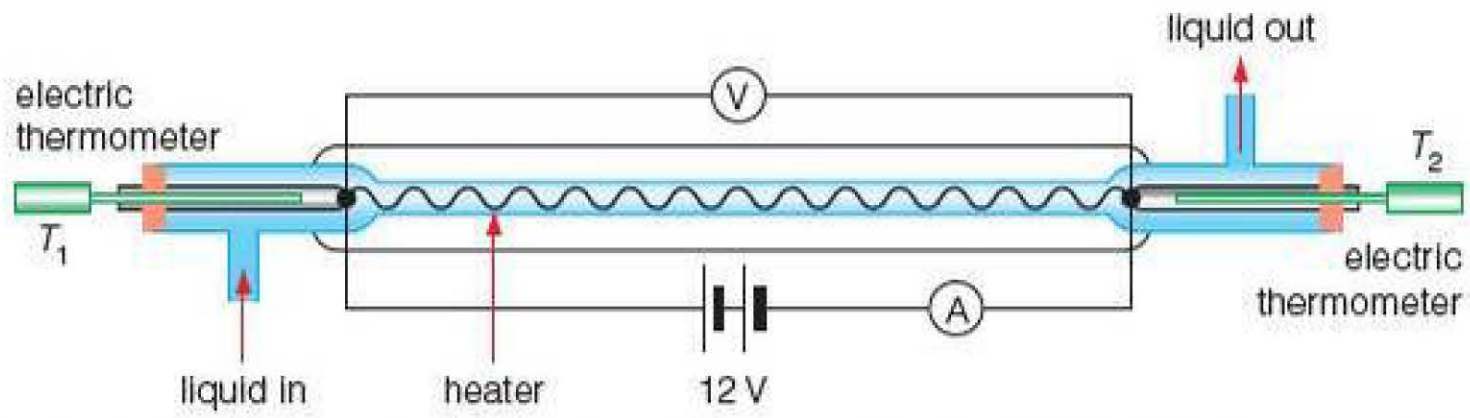


Figure 4.7 Measurement of specific heat capacity by the continuous flow method.

A fluid flows through an insulated tube containing an electric heating wire, as shown in Figure 4.8. The rise in temperature of the fluid is measured by the two electronic thermometers and calculated by $\Delta\theta = T_2 - T_1$. The mass of the fluid that flows through the apparatus in a time t_1 is m_1, and is measured using a beaker on a balance and a stopwatch. The flow rate of the fluid is then altered to give another value, m_2, and the heater controls are changed to give the same temperature difference $\Delta\theta$. The specific heat capacity of the fluid can then be determined by assuming that the thermal losses to the surroundings are constant for both flow rates.

For the first flow rate, the electrical energy supplied to the fluid in time t_1 is given by

$$I_1V_1t_1 = m_1c\Delta\theta + E_{lost} \qquad \text{(i)}$$

where I_1 and V_1 are the initial current and p.d. of the heater and E_{lost} is the thermal energy lost to the surroundings. For the second flow rate:

$$I_2V_2t_2 = m_2c\Delta\theta + E_{lost} \qquad \text{(ii)}$$

E_{lost} can be assumed to be the same in each experiment, so subtracting equation (ii) from equation (i) gives

$$I_1V_1t_1 - I_2V_2t_2 = m_1c\Delta\theta - m_2c\Delta\theta$$
$$= c\Delta\theta\,(m_1 - m_2)$$

If the experiments are both run for the same time t, then

$$c = \frac{(I_1V_1 - I_2V_2)t}{(m_1 - m_2)\Delta\theta}$$

EXAMPLE

Continuous flow method

Work out the specific heat capacity of water using the following data measured during one such experiment involving water in a continuous flow method:

- time of each experiment, t = 60 s
- temperature difference in both experiments, $\Delta\theta$ = 10.0°C
- p.d. across the heater, $V_1 = V_2$ = 12.0V
- current through heater in experiment 1, I_1 = 6.0 A
- current through heater in experiment 2, I_2 = 2.0 A
- mass of water flowing in experiment 1 for 60 s, m_1 = 126.0 g
- mass of water flowing in experiment 2 for 60 s, m_2 = 56.0 g

Answer

The specific heat capacity is worked out by substituting values in the equation:

$$c = \frac{(I_1V_1 - I_2V_2)t}{(m_1 - m_2)\Delta\theta}$$

$$= \frac{[(6\,\text{A} \times 12\,\text{V}) - (2\,\text{A} \times 12\,\text{V})] \times 60\,\text{s}}{(0.126\,\text{kg} - 0.056\,\text{kg}) \times 10\,\text{K}}$$

$$= 4114\,\text{J kg}^{-1}\,\text{K}^{-1}$$

Specific latent heat The specific latent heat of a material is the amount of thermal energy required to change the state of 1 kg of material, without a change in temperature, at a specified ambient pressure (normally atmospheric pressure, p = 1 atm).

Changing state

When liquids are heated up to their boiling point, the thermal energy is used to increase the internal energy of the molecules of the liquid. We measure this as a temperature change. However, at the boiling point, the temperature change stops and all the thermal energy input is used to overcome the intermolecular forces between the particles of the liquid, converting it into a gas.

The amount of thermal energy required to change the state of a substance, without a change in temperature, Q (in J), is given by

$$Q = ml$$

where m is the mass of the substance (in kg) and l is the **specific latent heat** ('latent' means 'hidden') of the substance (in J kg^{-1}). This equation applies to all the phase changes involved with changes of state. So water, for example, has a specific latent heat of vaporisation, l_v, which deals with the phase change from liquid to gas (and vice versa), and a specific latent heat of fusion, l_f, which deals with the phase change from solid to liquid (and vice versa).

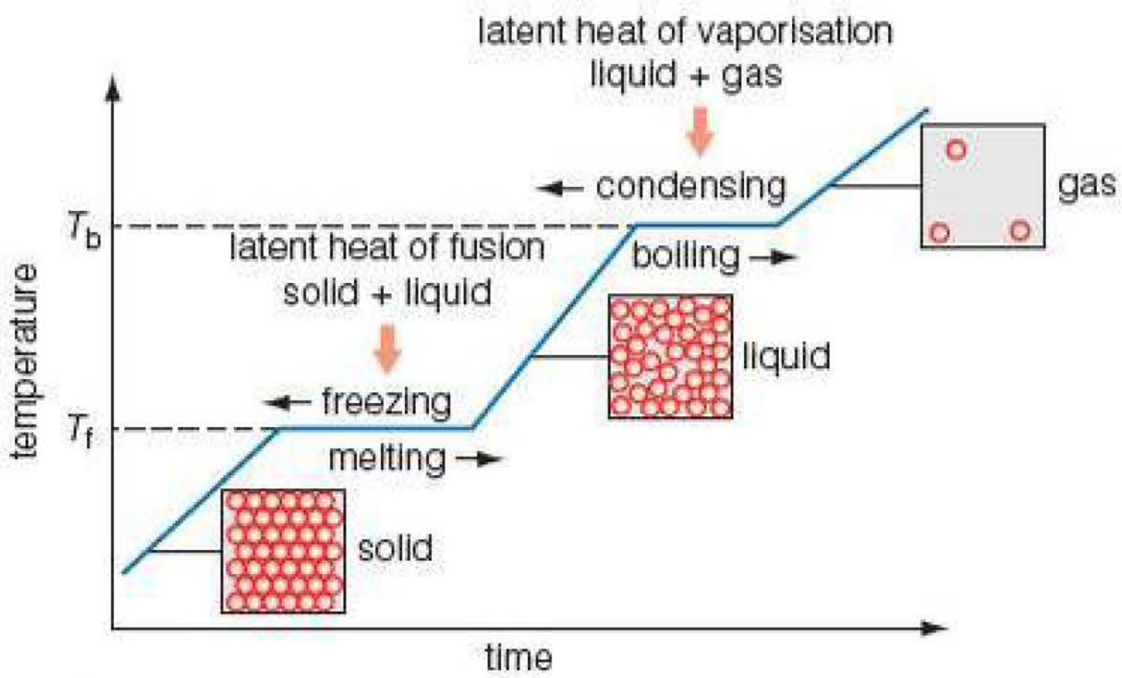


Figure 4.8 Kinetic theory graph.

The relationship between the kinetic theory models of solids, liquids and gases and the concept of latent heat is illustrated by Figure 4.8. Thermal energy supplied to a substance that is changing state is used to loosen the intermolecular bonds holding the particles together (completely in the case of a liquid turning into a gas). The thermal energy is called a *latent heat* because during the change of state the temperature does not change, despite thermal energy being supplied to the substance.

The values of l_v and l_f for a few selected materials are shown in Table 4.2. Once again, the high values for water mean that a high proportion of the water on planet Earth is in the liquid state, and our ambient temperature is kept within a relatively small range.

Table 4.2

Material	Specific latent heat of vaporisation, l_v/kJ kg^{-1}	Specific latent heat of fusion, l_f/kJ kg^{-1}
Water	2260	334
Carbon dioxide	574	184
Nitrogen	200	26
Oxygen	213	14
Lead	871	23

EXAMPLE

Evaporating water

A Bunsen burner delivers heat energy at a rate of 900 W to water inside a glass beaker. The water is at its boiling point, and the 900 W of thermal energy is used to turn 0.50 kg of water into steam. Calculate how long it will take for the water to turn to steam. The specific latent heat of vaporisation of water is 2260 kJ kg^{-1}.

Answer

Using the equation from the main text

$Q = ml_v = 0.50\,\text{kg} \times 2260 \times 10^3\,\text{J kg}^{-1} = 1.13 \times 10^6\,\text{J}$

If the power supplied as thermal heat to the water is 900 W, the time required to boil the water is:

$$t = \frac{Q}{P} = \frac{1.13\times10^6\,\text{J}}{900\,\text{W}} = 1256\,\text{s} \approx 21\,\text{minutes}$$

TEST YOURSELF

7 A 12.5 g ice cube melts in the sunshine. Calculate the thermal energy from the Sun absorbed by the ice during melting. The specific latent heat of fusion of water is $334\,\text{kJ}\,\text{kg}^{-1}$.

8 Lead is a major component of the solder used to construct integrated circuits. A soldering iron delivers 18 W of thermal energy to a small 4.2 g block of lead. Calculate the time taken for all the lead to melt. The specific latent heat of fusion of lead is $23\,\text{kJ}\,\text{kg}^{-1}$.

9 A range oven rated at 3 kW actually delivers 2.7 kW of thermal power to 1.5 kg of water inside a whistling kettle. The specific heat capacity of water, c, is $4200\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$, and the specific latent heat of vaporisation of water, l_v, is $2260\,\text{J}\,\text{kg}^{-1}$.

a) How much thermal energy is required to heat the water from 8°C to 100°C?

b) How long does it take the kettle to heat the water from 8°C to 100°C?

c) The water starts to boil and the whistle on the kettle starts to blow and keeps blowing for 25 s until the kettle is removed from the heat. What mass of water is converted into steam during the 25 s of boiling?

10 The specific latent heat of vaporisation of a liquid (such as water) can be measured using the apparatus shown in Figure 4.9.

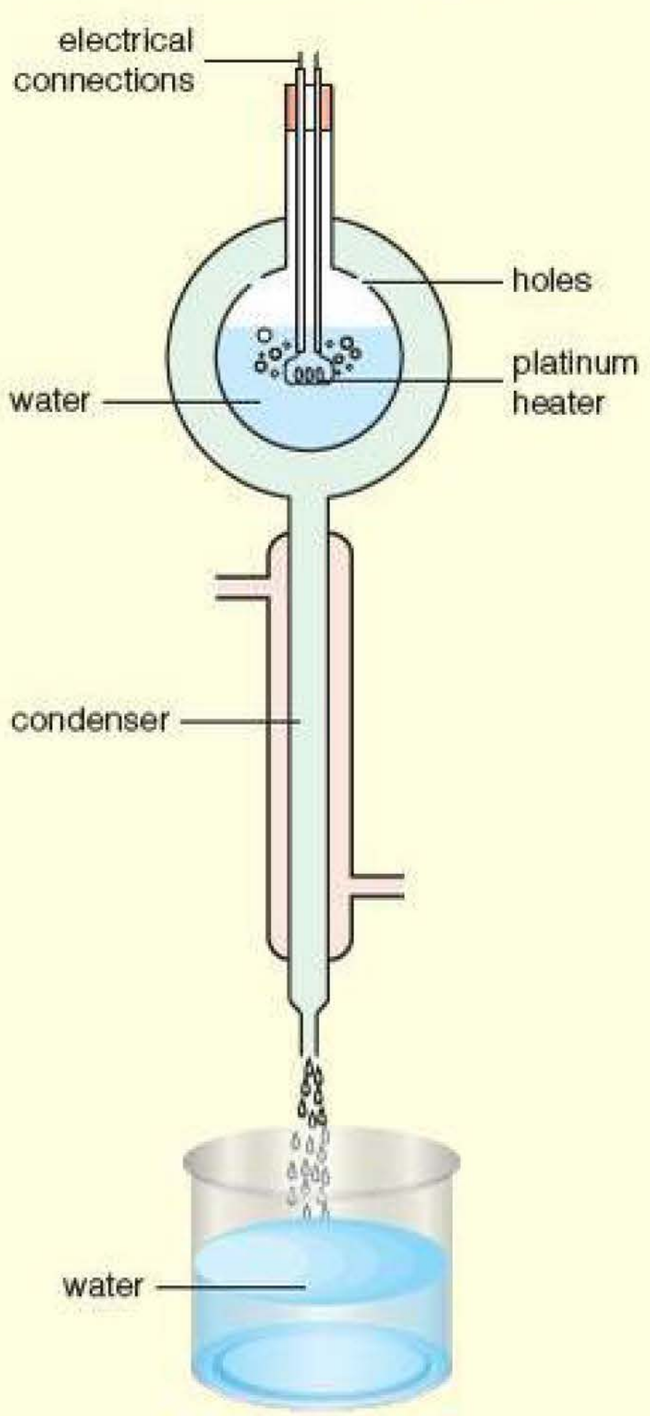


Figure 4.9 Apparatus for measuring the specific latent heat of vaporisation of water.

The heat supplied to the water is given by

$$V_1 I_1 t = m_1 l_v + E$$

where V_1 is the p.d. across the heater, I_1 is the current supplied to the heater, t is the time that the experiment is left to run, boiling a mass of water m_1 in the time t, l_v is the specific latent heat of vaporisation of water and E is the thermal energy lost to the surroundings. The experiment is then repeated with a different p.d., V_2, across the heater and a different current, I_2, flowing through it, boiling a different mass of water, m_2, in the same time t. In this case:

$$V_2 I_2 t = m_2 l_v + E$$

(The thermal energy lost in each experiment, E, will be the same.)

a) Use both equations to derive an expression for the specific latent heat of vaporisation of water.

b) In one such experiment, the following data was obtained in $t = 600\,\text{s}$.

Quantity	Value	Quantity	Value
V_1	8.00V	V_2	12.00V
I_1	2.41A	I_2	3.00V
m_1	5.8g	m_2	10.3g

Use this data and your answer to (a) to calculate a value for the specific heat capacity of water.

11 Stearic acid, a common chemical found in soaps, is frequently used to show the phase change of a material. A student set up an experiment using a test tube with 4 g of initially liquid stearic acid contained in a small water bath containing 25 g of water set at 95°C. She puts thermometer probes connected to a data logger into the stearic acid and the water in the water bath and then she turns off the temperature control. The data logger measures and records the temperatures over the course of ten minutes as the water and the stearic acid cool down. Her results are shown in Figure 4.10.

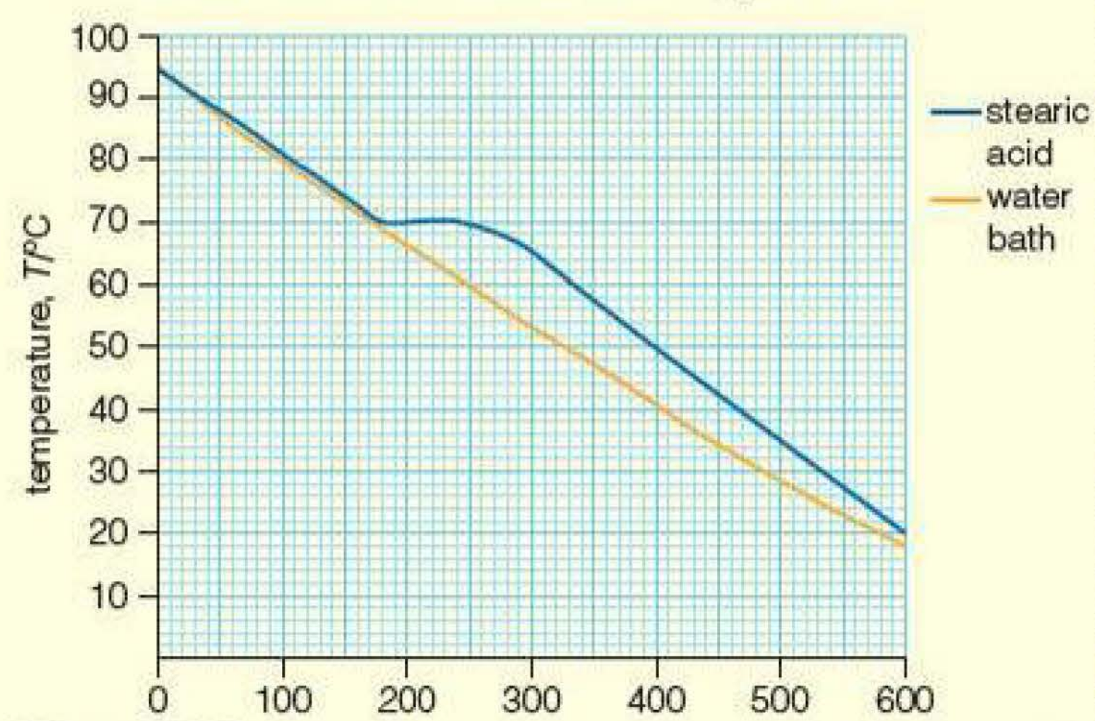


Figure 4.10

⇨

a) Draw a diagram of the experimental set-up.

b) The specific heat capacity of water is $4200\,J\,kg^{-1}\,K^{-1}$. Neglecting the effect of the glass test tube, estimate the rate of thermal energy transfer from the water (and the stearic acid).

c) Use the graph and the rate of thermal heat transfer to estimate:

i) the specific heat capacity of solid stearic acid

ii) the specific latent heat of fusion of stearic acid.

The gas laws

Between 1662 and 1802 three laws were discovered by a collection of European physicists that seemed to describe the behaviour of gases in response to changes in their pressure, volume and temperature. The laws themselves are all empirical, which means that they describe the mathematical relationships between the three variables purely based on experiments.

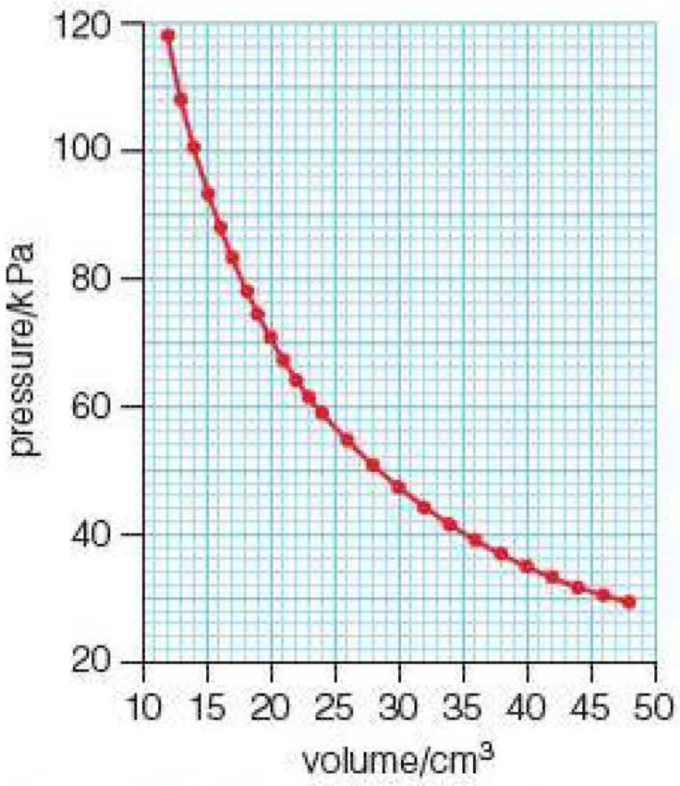


Figure 4.11 Boyle's law graph.

Boyle's law

The first gas law to be discovered was Boyle's law, the relationship between the pressure and volume of a gas. The experiments were carried out by Robert Boyle and his research student Robert Hooke in 1662, involving J-shaped tubes of sealed glass and mercury. Boyle quickly realised that there was a relationship between the volume of the air trapped behind the mercury and the weight of the mercury acting across the cross-sectional area of the tube causing increased pressure.

Boyle realised that the pressure acting on the gas and the volume occupied by the gas were inversely proportional to each other. Boyle would have obtained results similar to those shown in Figure 4.11. A modern version of his law states:

> For a fixed mass of an ideal gas at constant temperature, the pressure of the gas is inversely proportional to its volume.

Writing this mathematically:

$$p \propto \frac{1}{V} \quad \text{or} \quad p \times V = \text{constant}$$

where p is the pressure acting on a gas of volume V. A more useful version of this equation involves the same mass of gas at the same temperature, but different pressures and volumes, where

$$p_1V_1 = p_2V_2$$

REQUIRED PRACTICAL 8

Investigating Boyle's (constant temperature) law

Note: This is just one example of how you might tackle this required practical.

A student uses the standard Boyle's law apparatus shown in Figure 4.12 to determine the value of the constant involved in the equation

$$pV = \text{constant}$$

In the experiment, performed at 17°C, the foot pump is used to pressurise the oil inside the cylinder, which compresses the air column above it,reducing its volume. The pump valve is closed when the pressure is at a maximum and the air column volume is at a minimum.

Figure 4.12 Apparatus used to investigate Boyle's law.

⇨

The apparatus is then left to come to (thermal) equilibrium as the oil drains back down the sides of the column. The pressure and the volume of the air column then

measured and recorded. The student then opens the pump valve very slightly and the pressure is reduced slightly, expanding the air in the column. The valve is shut, the apparatus is allowed to come to thermal equilibrium again and the pressure and volume are measured and recorded. This process is repeated until the pressure returns fully to its atmospheric value. The student's results are shown in Table 4.3.

Table 4.3

Pressure, $p/10^5$ Pa ($\pm 0.01 \times 10^5$ Pa)	Volume, V/cm^3 (± 0.5 cm^3)
3.5	9.0
3.0	10.0
2.5	12.0
2.0	15.5
1.5	20.0
1.0	30.5

The student estimates that she can measure the pressure readings from the pressure gauge with an uncertainty of $\pm 0.01 \times 10^5$ Pa, and the volume from the measuring scale with an uncertainty of ± 0.5 cm^3.

1 Make a copy of the table and add two further columns: $1/V$ (in cm^{-3}) and $p \times V$ (in 10^5 Pa cm^3) – calculate the values and enter them in the table.
2 Plot graphs of the following:
   - V against p
   - $1/V$ against p.
3 For each graph, include error bars and a best-fitting line.
4 Use your graphs to measure a value for the constant, where pV is constant. Use your graph to estimate an uncertainty in the value of the constant.
5 Explain why the student allowed the experiment to come to thermal equilibrium before measuring the pressure and volume in the apparatus.

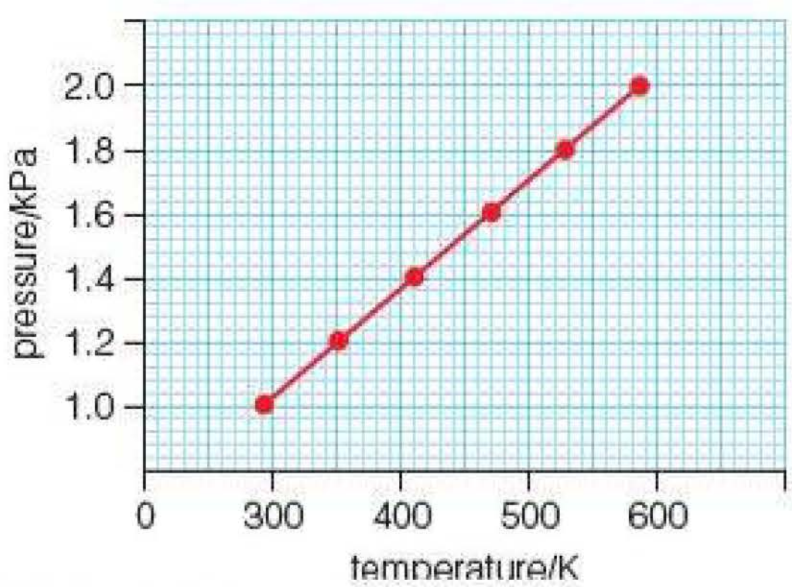


Figure 4.13 Amonton's law (pressure–temperature law).

The pressure–temperature law (Amonton's law) and absolute zero temperature

In 1702 Guillaume Amonton discovered the empirical relationship between the pressure and temperature of a gas as a result of his efforts to design and build air thermometers. Amonton realised empirically that there was a linear relationship between the two variables, provided that the mass and the volume of the gas were kept constant. Amonton struggled to build accurate thermometers and, although his ideas were published, they lacked basic quantitative data. A modern version of his graph is shown in Figure 4.13.

A modern version of Amonton's law can be written more formally as:

> The pressure of a fixed mass and fixed volume of gas is directly proportional to the absolute temperature of the gas.

Writing this mathematically:

$$p \propto T$$

or

$$\frac{p}{T} = \text{constant}$$

This relationship has a third, more useful, form that is used to compare the same gas under different pressures and temperatures. This can be written as:

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

Amonton realised at the time of his experiments that if he extrapolated his data back through lower and lower temperatures there would be a temperature where the pressure of a gas dropped to zero. At this

Absolute zero The temperature when all molecular motion ceases, and the pressure of a gas drops to zero. The accepted value is the zero of the Kelvin temperature scale and is defined as −273.15°C.

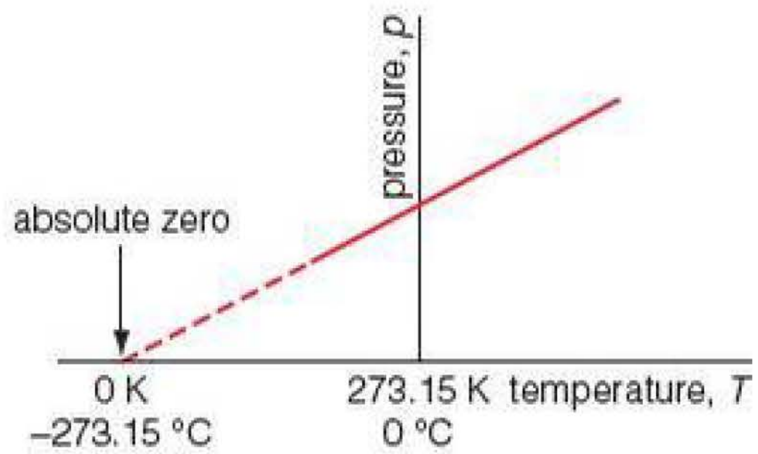


Figure 4.14 Absolute zero.

TIP
Always use the absolute (Kelvin) temperature scale when doing problems and calculations involving the gas laws.

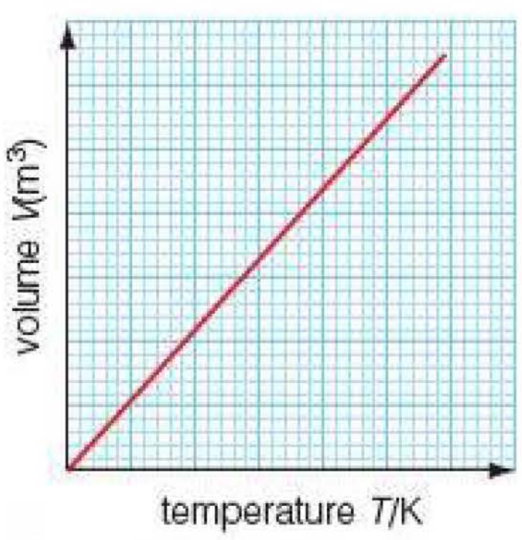


Figure 4.15 Charles' law.

temperature, the molecules would stop moving and so they could not exert a pressure by hitting anything else. This temperature became known as **absolute zero** and Amonton calculated it to be −240°C, which, considering the thermometer technology available to him at the time (air thermometers), was a pretty remarkable achievement – he was only about 35°C out. In 1848 William Thomson (Lord Kelvin) used the concept of absolute zero to construct a temperature scale with absolute zero as the 'zero' of his scale. Using the better thermometer technology of the day, Kelvin predicted that absolute zero would be at a temperature of −273°C or 0 K – only 0.15°C away from today's defined value of −273.15°C (Figure 4.14).

Kelvin's absolute scale of temperature became the SI unit of temperature, and is defined in terms of two fixed temperature points – absolute zero (0 K) and the triple point of water (0.01°C or 273.16 K – the temperature and pressure values where ice, liquid water and water vapour can coexist). Converting a temperature from °C into an absolute temperature measured in K involves using the equation:

$$T \text{ (in K)} = T \text{ (in °C)} + 273.15$$

You must note that the magnitude (size) of 1°C is equivalent to 1 K. In other words: $|1°C| = |1\text{ K}|$.

Charles' law

In 1802, the French chemist Joseph Gay-Lussac published a paper showing the experimental link between the volume and the temperature for a gas. Gay-Lussac named the law after his balloonist friend, Jacques Charles, who produced an unpublished version of the law following his observations of the behaviour of balloons. The empirical law is illustrated by the graph in Figure 4.15.

A modern version of Charles' law states:

> At constant pressure the volume of a fixed mass of an ideal gas is directly proportional to its absolute temperature.

Writing this mathematically:

$$V \propto T$$

or

$$\frac{V}{T} = \text{constant}$$

Once again, the temperature T is an absolute temperature using the Kelvin scale ($T/K = T/°C + 273.15$). Another useful form of this relationship (similar to the other gas laws) is

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

REQUIRED PRACTICAL 8

Investigation of Charles' law for a gas

Note: This is just one example of how you might tackle this required practical.

Charles' law can be used to make an estimate for the value of absolute zero. This can be achieved by measuring the volume of a gas at different temperatures and then extrapolating the graph back to a volume of zero. In reality, non-ideal gases do not end up with zero volume but, for the purposes of this experiment, the difference is so small that it will not affect the outcome.

In this experiment, two small sealed gas syringes, one with a total volume of 10 cm^3 and the second with a volume of 30 cm^3, are put, one at a time, into a freezer cabinet at −15°C, and then into a beaker of iced water at 0°C (Figure 4.16). The iced water is then gradually warmed using a Bunsen burner. The volume of the air trapped inside the gas syringes is recorded at temperatures of −15, 0, 20, 40 and 80°C. The results of the experiment are shown in Table 4.4.

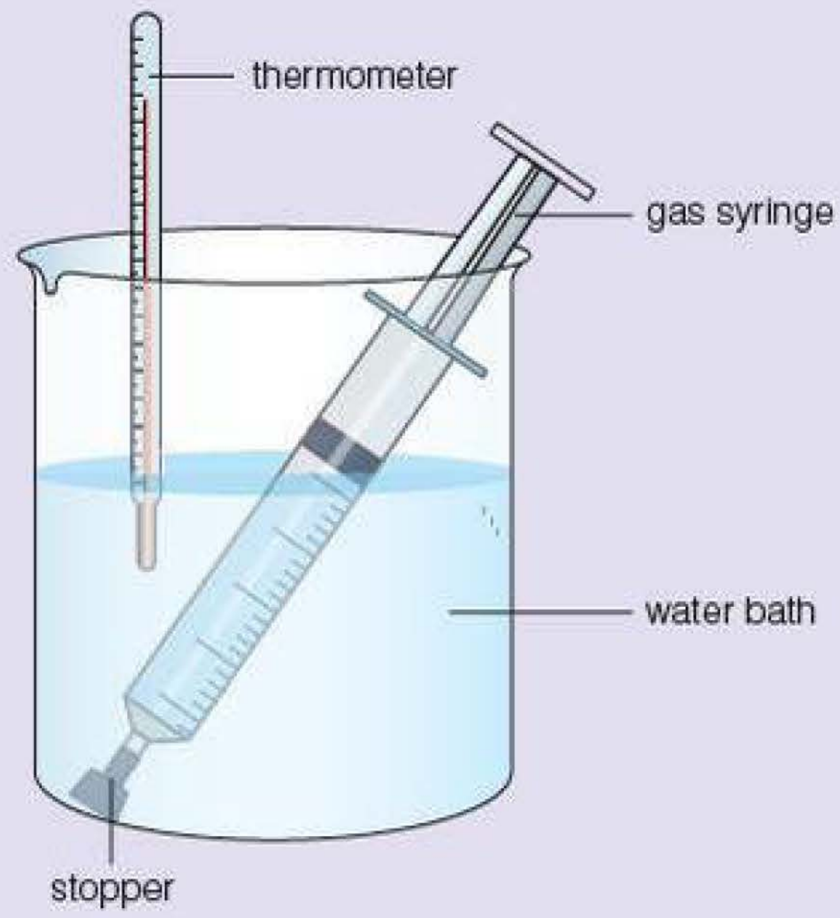


Figure 4.16 Measuring absolute zero.

Table 4.4

Temperature, T/°C	Volume of air in syringe, V/ml (±0.2 cm^3)	
	10 cm^3 syringe	30 cm^3 syringe
−15	4.3	13.5
0	4.6	14.2
20	4.9	15.1
40	5.2	16.4
80	5.9	18.3

1 Plot a graph of this data. Include error bars on the volume measurements and best-fitting lines.
2 Extrapolate each best-fitting line back so that it crosses the temperature axis. Use the temperature-axis intercepts to determine a range of values for the absolute zero temperature in °C.

Combining the gas laws

The three gas laws – namely Boyle's law, Amonton's law and Charles' law – can be combined into one expression linking the pressure, volume and temperature of a gas, in a combined gas law, which is expressed mathematically as:

$$\frac{pV}{T} = \text{constant}$$

A more useful form of this equation can be written as

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

where p_1, V_1 and T_1 describe the initial pressure, volume and temperature of a gas, and p_2, V_2 and T_2 describe the final pressure, volume and temperature of the same gas after a change has been applied to it.

TIP

Remember that, when using the combined gas law, absolute temperature *must* be used and temperatures in degrees Celsius must be converted to kelvins.

In questions on the combined gas law, pressure is usually measured in pascals (Pa) or kilopascals (kPa), where a pressure of 1 Pa is equivalent to a force of 1 N acting over an area of 1 m^2. Pressure is also measured in atmospheres (atm), where one standard atmosphere is defined as a pressure equivalent to 101 325 Pa, and is the average value for atmospheric pressure at sea level. The volume of a gas is normally measured in m^3, but cm^3, litres and ml are also commonly used. Temperatures may be given in kelvin or degrees Celsius. The latter must be converted to kelvin.

Which of these units should you use in your work? The answer is generally to use the units included in the question, converting temperatures to kelvin. State any calculated values using the units from the question.

Also, you should be aware that questions sometimes refer to **standard temperature and pressure** (STP), which are 0°C (273.15 K) and 1.01×10^5 Pa (1 atm), and **room temperature and pressure** (RTP), which are 25°C (278.15 K) and 1.01×10^5 Pa (1 atm).

Standard temperature and pressure This refers to 0°C (273.15 K) and 1.01×10^5 Pa (1 atm).

Room temperature and pressure This refers to 25°C (278.15 K) and 1.01×10^5 Pa (1 atm).

EXAMPLE

Changing volume of a balloon

The volume of a party balloon at a room temperature of 15°C and an atmospheric pressure of 1.0 atm at sea level is 2400 cm^3. The balloon is taken up a mountain, where the temperature is 1.4°C and the atmospheric pressure has dropped to a value of 0.80 atm. Calculate the new volume of the balloon at the top of the mountain.

Answer

Use

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

Substituting numbers:

$$\frac{1\,\text{atm} \times 2400\,\text{cm}^3}{(15 + 273.15)\,\text{K}} = \frac{0.80\,\text{atm} \times V_2}{(1.4 + 273.15)\,\text{K}}$$

Rearranging gives:

$$V_2 = \frac{1\,\text{atm} \times 2400\,\text{cm}^3 \times (1.4 + 273.15)\,\text{K}}{(15 + 273.15)\,\text{K} \times 0.80\,\text{atm}}$$

$$= 2858.4\,\text{cm}^3 \approx 2900\,\text{cm}^3 \text{ (2 s.f.)}$$

TEST YOURSELF

12 A small patio heater gas bottle has a volume of 6.0 litres (6.0×10^{-3} m^3) and contains butane gas at a temperature of 5.0°C and a pressure of 2.5 MPa. What would be the volume of the gas if it were let out of the canister into an inflating balloon on a warm day at 20°C and atmospheric pressure (0.1 MPa)?

13 A closed gas syringe contains a fixed mass of air at 24°C. To what temperature must the gas be heated so that its volume doubles, when the pressure remains constant?

14 An empty treacle tin contains air at a temperature of 16°C and a pressure of 1.5×10^5 Pa. The lid will blow off the tin if the pressure inside the tin rises beyond 2.4×10^5 Pa.

a) At what temperature will the top blow off if the air is heated evenly with a Bunsen burner?

b) Why will this trick work better if you put some water in the tin?

15 A large car tyre has a volume of 22×10^{-3} m^3, and the air inside is pumped to a pressure of 2.5 atm *above* atmospheric pressure (1 atm). Calculate the volume that the air inside the tyre would occupy at atmospheric pressure. You should assume that the temperature remains constant.

16 A steam cleaner has a steam tank with a volume of 150 cm^3 in which the steam is kept at a temperature of 100°C and a pressure of 1.5×10^6 Pa. If the steam cleaner is used to clean windows outside on a cold day where the air is at atmospheric pressure and the temperature is 6.5°C, calculate the volume of steam generated if all the steam is let out of the tank.

Avogadro's law, the ideal gas equation and moles

A fourth experimental gas law was hypothesised and published by Amadeo Avogadro in 1811. In this law he suggested that equal volumes of gases at the same temperature and pressure contained the same number of molecules.

Mathematically, this means that:

$$V \propto n \quad \text{or} \quad \frac{V}{n} = \text{constant}$$

where n is the number of moles of the gas. Once again, as with the other gas laws, this is more usefully written as:

$$\frac{V_1}{n_1} = \frac{V_2}{n_2}$$

In 1834, the French physicist Emile Clapeyron combined all four gas laws and produced the ideal gas equation:

$$pV = nRT$$

where R is a constant now known as the molar gas constant, $R = 8.31\,\text{J}\,\text{mol}^{-1}\,\text{K}^{-1}$. The ideal gas equation is a tremendously powerful equation that models the behaviour of gases extremely well, particularly when the gases are at relatively low pressures and high temperatures. At conditions close to liquefaction, it works less well because the gases behave much less like ideal gases.

The Avogadro constant, N_A, was proposed by Jean Baptiste Perrin in 1909, to represent the number of particles (usually molecules or atoms) present in one mole (1 mol) of a substance. Perrin named it in honour of Amadeo Avogadro. Perrin went on to win the Nobel Prize in Physics for his attempts to measure the Avogadro constant accurately. The Avogadro constant is today defined as $6.022\,141\,29 \times 10^{23}\,\text{mol}^{-1}$ (rounded to $6.02 \times 10^{23}\,\text{mol}^{-1}$ on the AQA Physics Datasheet).

Dividing the molar gas constant, R, by the Avogadro constant, N_A, yields another fundamental constant in physics, the Boltzmann constant, k:

$$k = \frac{R}{N_A}$$

and it has the value:

$$k = \frac{R}{N_A} = \frac{8.31\,\text{J}\,\text{mol}^{-1}\,\text{K}^{-1}}{6.02 \times 10^{23}\,\text{mol}^{-1}} = 1.38 \times 10^{-23}\,\text{J}\,\text{K}^{-1}$$

The Boltzmann constant is the fundamental constant that links the macroscopic measurements of pressure, volume and temperature to the microscopic behaviour of particles in a gas, and has a fundamental position in the model of an ideal gas. The constant enables the microscopic model to make predictions about the way that ideal gases behave on a macroscopic scale where they can be measured empirically by the gas laws.

The above equation can be rearranged to give:

$$R = kN_A$$

which can be substituted into the ideal gas equation, giving:

$$pV = nN_AkT$$

But nN_A is the number of particles in the gas and is given the symbol, N, so:

$$pV = NkT$$

(This implies that the pressure of an ideal gas is independent of the mass of the particles.)

Molar mass and molecular mass

Counting particles is not a good way to gauge the amount of substance present in a gas (or liquid or solid). It is incredibly difficult to observe individual atoms or molecules, let alone to count them. A better way to work out the amount of matter in a substance is to use the mass of particles and weigh large collections of them using an electronic balance. If the mass of one particle is known, then a measurement of the mass of a large number of them will yield the number of particles present. We therefore define two quantities: the molecular mass, m, which is the mass of one molecule of substance; and the molar mass, M_m, which is the mass of one mole (N_A) of molecules of the substance. These two quantities are related to each other by:

$$M_m = N_A m$$

If the mass of a known gas is measured, M_g, then dividing this value by the molar mass gives the number of moles, n, and dividing it by the molecular mass, m, gives the number of molecules, N:

$$n = \frac{M_g}{M_m} \text{ and } N = \frac{M_g}{m}$$

Both of these can then be substituted into the ideal gas equation, allowing all quantities to be measured macroscopically:

$$pV = \frac{M_g}{M_m}RT \text{ and } pV = \frac{M_g}{m}kT$$

EXAMPLE

Propane gas cylinder

A propane gas cylinder has a volume of $0.14\,m^3$ and the pressure of the gas inside the cylinder is 2.0×10^6 Pa *above* atmospheric pressure at 300 K.

1 Calculate the number of moles of propane gas inside the cylinder.

Answer

Using the ideal gas equation, $pV = nRT$:

$$n = \frac{pV}{RT}$$

$$= \frac{(2.0\times10^6 + 1.0\times10^5)\,Pa \times 0.14\,m^3}{8.31\,J\,mol^{-1}\,K^{-1} \times 300\,K} = 118\,mol$$

2 Calculate the number of propane molecules inside the cylinder.

Answer

One mole of an ideal gas contains 6.02×10^{23} molecules of gas (the Avogadro constant, N_A). The total number of molecules of gas in the container is therefore $6.02 \times 1023 \times 118 = 7.1 \times 10^{25}$ molecules.

3 Calculate the mass of gas inside the cylinder if the molar mass of propane is $44.1\,g\,mol^{-1}$.

Answer

If the molar mass of propane is $44.1\,g\,mol^{-1}$, and there are 118 mol of gas, then the mass of propane inside the cylinder is $44.1\,g\,mol^{-1} \times 118 = 5204\,g = 5.2\,kg$ (2 s.f.).

4 Calculate the mass of one molecule of propane.

Answer

The mass of one molecule of propane is

$$m = \frac{\text{molar mass}}{N_A}$$

$$= \frac{44.1\times10^{-3}\,kg\,mol^{-1}}{6.02\times10^{23}\,mol^{-1}} = 7.3\times10^{-26}\,kg$$

TEST YOURSELF

17 1.5 moles of an ideal gas at a temperature of 312 K is kept at a pressure of 1.7×10^5 Pa. Calculate the volume of the gas under these conditions.

18 A weather balloon contains helium gas and occupies a volume of $0.85\,m^3$. At a particular weather station, the pressure is 1.2×10^5 Pa and the temperature of the surrounding air is 18°C. Assuming that helium behaves as an ideal gas, show that the balloon contains about 42 mol of helium gas.

19 A tyre on a cycle in the Tour de France contains 0.15 mol of air at a temperature of 293 K and has a volume of $8.2 \times 10^{-4}\,m^3$. It is assumed that the air behaves as an ideal gas.

a) Calculate the pressure of the air inside the tyre.

b) At the end of a stage, the pressure in the tyre has risen to 5.45×10^5 Pa. Use this information to estimate the temperature of the air in the tyre at the end of the stage. (Assume that the volume does not change.)

20 Look at the graphs (labelled A, B, C and D) in Figure 4.17 showing the behaviour of an ideal gas.

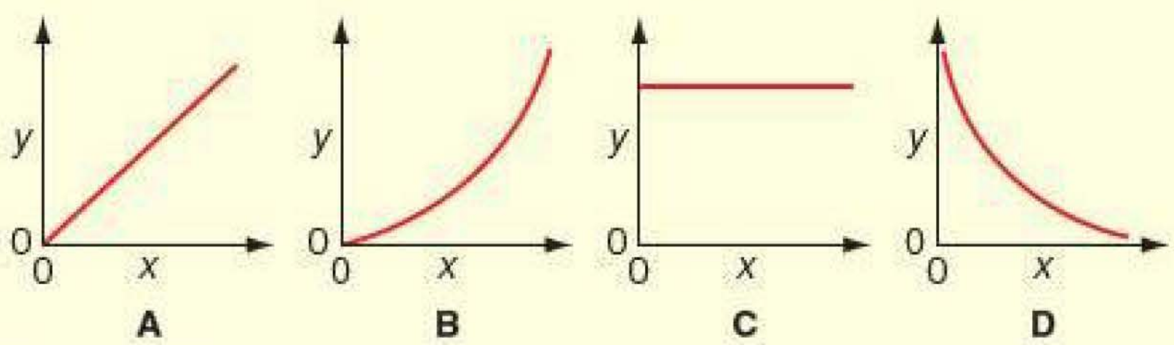


Figure 4.17

a) Which graph shows the variation of the volume of a fixed mass of the gas (*y*-axis) at constant temperature with the pressure (*x*-axis) of the gas?

b) Which graph shows the variation in pressure (*y*-axis) of a fixed mass of gas at constant volume with absolute temperature (*x*-axis)?

c) Which graph shows the variation of the product ($p \times V$) of a fixed mass of the gas (*y*-axis) at constant temperature with the pressure (*x*-axis) of the gas?

21 This question is about the two standard conditions of an ideal gas – standard temperature and pressure (STP) and room temperature and pressure (RTP). The table below gives some data for 1 mol of an ideal gas under each of these conditions.

Quantity	Standard temperature and pressure (STP)	Room temperature and pressure (RTP)
Temperature, T/K	273	
Pressure, $p/10^5$ Pa	1.01	1.01
Volume, V/m^3		2.45×10^{-2}

Copy and complete the table.

ACTIVITY

The ideal gas equation and Mount Kilimanjaro

Mount Kilimanjaro is the highest mountain in Africa (Figure 4.18), and scientists have found that about half of all the climbers who attempt to scale its height suffer from altitude sickness before they reach the summit as a result of ascending the mountain too quickly. Every year approximately 1000 climbers are evacuated from the mountain suffering from acute altitude sickness, and on average 10 climbers die. Kilimanjaro is a deceptively dangerous place.

Figure 4.18 Mt Kilimanjaro, the highest mountain in Africa.

⇨

Here is some data about Mount Kilimanjaro on the mountain itself and on the plains below:

- summit elevation 5895 m above sea level
- summit air pressure 50 kPa
- summit average air temperature −6.8 °C
- plains elevation 1018 m above sea level
- plains air pressure 90 kPa
- plains average air temperature 30 °C
- plains air density 1.03 kg m^{-3}

1 Use this data to calculate the density of air at the summit of Mount Kilimanjaro.

2 The proportions of oxygen and nitrogen in the air on the surrounding plains and the summit is constant (21% oxygen and 79% nitrogen). The average adult lung capacity is about 6 litres. Calculate the number of oxygen molecules in a person's lungs

a) on the surrounding plains

b) at the summit of Mount Kilimanjaro.

Ideal gases

There are many occasions in physics where we use a simplified model to explain the behaviour of a system. Models use basic first principles and then usually add in more complexity to fine-tune the behaviour of the model so that it better reflects reality. One good example of a simple model is the model of an ideal gas, which is used to explain the behaviour of gases subject to the changes in their temperature, pressure and volume. Real gases do not behave exactly like ideal gases but their general behaviour is sufficiently close that the model predicts and explains most of the common patterns in the behaviour of the real thing.

The ideal gas model assumes the following:

- An ideal gas consists of a large number of identical, small, hard spherical molecules.
- The volume of the molecules is very much smaller than the volume of the container.
- All the collisions between the molecules themselves and the container are elastic and all motion is frictionless (i.e. no energy loss in motion or collision).
- The movements of the molecules obey Newton's laws of motion.
- The average distance between molecules is very much larger than the size of the molecules.
- The molecules are constantly moving in random directions with a distribution of velocities about a mean velocity.
- There are no attractive or repulsive intermolecular forces apart from those that occur during their collisions.
- The only forces between the gas molecules and the surroundings are those that determine the collisions of the molecules with the walls.
- There are no long-range forces between the gas molecules and their surroundings.
- The time spent between collisions is very much larger than the time spent colliding.

Molecular motion

One of the most important properties of an ideal gas is the idea that they move in random directions. This is important because if this was not true then gases would exert more pressure on one surface of their container than they would on another, i.e. the direction that the particles travel in would be important, and the theories would be different in different directions.

The fact that gases (and all fluid particles) have a random molecular motion was first observed and described by the botanist Robert Brown in 1827, as a result of his observations of pollen grains floating on water. Brown saw the grains moving in random directions as he observed them through a light microscope but he was unable to explain why they moved – this was left to Albert Einstein during his 'annus mirabilis' (miracle year) of 1905, during which he published his ideas about the photoelectric effect, special relativity, mass–energy equivalence ($E = mc^2$) as well as Brownian motion. Einstein explained that the pollen grains were moving in random directions as the result of the cumulative effect of the water molecules randomly hitting the pollen grains. At different times the pollen grains are hit by water molecules more on one side than they are on the other sides, resulting in a motion in that direction that appears random in nature. Einstein's theory of Brownian motion was confirmed experimentally in 1908 by Jean Baptiste Perrin (of Avogadro constant fame). Perrin produced a series of positional plots showing this random motion by observing the motion of 0.5 μm carbon particles on a grid of 3 μm × 3 μm squares and recording their positions every 30 s. His plots would have looked like the modern version in Figure 4.19.

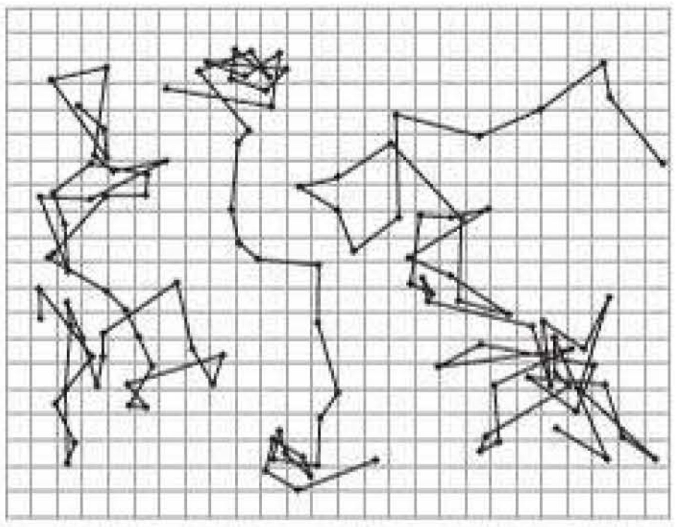


Figure 4.19 A modern version of Jean Baptiste Perrin's plot of the Brownian motion of carbon particles.

Perrin analysed the motion of the particles and concluded that the motion was truly random, in line with Einstein's theory. Both Perrin and Einstein were (separately) awarded Nobel Prizes partly because of their work on Brownian motion.

Pressure, volume, temperature and molecular motion

The importance of Brownian motion and the properties of an ideal gas should not be underestimated. The observation of the random motion of fluid particles and the subsequent theory proposed by Einstein provide a way of explaining the macroscopic gas law quantities, and hence the gas laws themselves in terms of a microscopic molecular model.

Pressure

Macroscopic pressure is defined in terms of a force acting over a given area. The kinetic theory model of an ideal gas shows us that the force is due to the collisions of the molecules with the walls of the container. The molecules are moving in random directions with a mean average velocity. The particles hit the walls of the container and rebound off at the same speed (all the collisions are elastic). This produces a change of momentum, and the cumulative effect of all the particles colliding over the total inside surface area of the container per second causes a force per unit area, which exerts a pressure acting in all directions (as the motion is random).

Volume

The motion of molecules inside a container is random in direction. This means that there is no preferred direction, so the molecules will spread out throughout the container filling its volume. Gases take the volume of their container. If the dimensions of the container are changed, the motion of the molecules will react to the change and will continue to fill the available volume. The behaviour of real gases is closest to that of an ideal gas at low pressures, well away from their phase boundary where they change into a liquid.

Temperature

For an ideal gas, because there are no intermolecular forces, increasing the temperature of the gas only increases the kinetic energy of the particles. This increases the average velocity of the particles. The particles still move in random directions, and they fill the container. Increasing the temperature for a fixed volume increases the pressure because the particles' average speed is higher and therefore the change of momentum during collisions with the walls is greater, and the particles hit the walls more often. This leads to higher forces and therefore higher pressures. Allowing the pressure to remain constant requires the volume to change.

A molecular kinetic theory model

Although the gas laws are empirical in nature (they were developed as a result of analysis of experimental data), the kinetic theory model is derived from theoretical first principles. However, they both produce the same results when observing the macroscopic behaviour of gases, but only the kinetic theory version can explain the behaviour of gases on a microscopic, molecular scale.

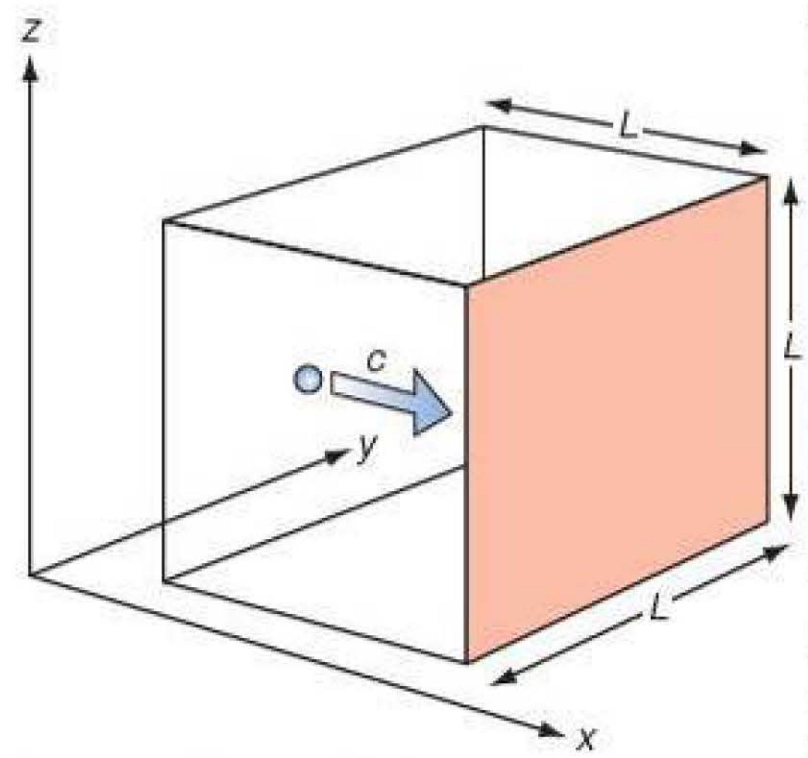


Figure 4.20 Particle in a box.

In 1860 James Clerk Maxwell and Ludwig Boltzmann both (independently) used the assumptions of the ideal gas model to link the pressure and density of a gas, connecting for the first time the molecular behaviour of a gas to one of its mechanical properties. Their theories started with the motion of one gas particle inside a cubic box (Figure 4.20).

The gas has a volume V and density ρ and is enclosed inside a cubic box of side L. Inside the box there are N identical particles with the same mass, m, and the gas particles have a range of different velocities c_1, c_2, $c_3 \ldots c_N$. It is assumed that the volume of the particles is negligible compared to the volume of the box.

Consider one particle moving parallel to the x-axis with a velocity c_1. The particle collides with the shaded wall in the diagram. The ideal gas theory assumes that the collision is totally elastic and so the particle rebounds back off the wall with a velocity of $-c_1$. The particle therefore experiences a total change in momentum equal to $2mc_1$, during the collision. If the totally elastic collision assumption was untrue then the particles would gradually loose energy during the collisions and the average velocity of the particles in the box would decrease, resulting in a drop in overall gas pressure. Experimental evidence tells us that this does not happen.

The particle then travels back across the box, collides with the opposite face before returning to the shaded wall in a time interval $\Delta t = \frac{2L}{c_1}$. This means that in the time interval Δt, the particle makes one collision with the wall and exerts a force on it. If the particle obeys Newton's Second Law of motion then,

$$F = \frac{\textit{change in momentum}}{\textit{time for change}} = \frac{2mc_1}{(2L/c_1)} = \frac{mc_1^2}{L}$$

The shaded wall has an area, $A = L^2$, so the pressure exerted by the one particle is:

$$p = \frac{F}{A} = \frac{mc_1^2}{L^3}$$

There are N particles in the box, and if they were all travelling parallel to the x-axis: total pressure on the shaded wall would be

$$= \left(\frac{m}{L^3}\right) \times (c_1^2 + c_2^2 + c_3^2 + \cdots + c_N^2)$$

but in reality, particles are moving in random directions, with a velocity, c, comprising components at right angles to each other in the x, y, and z

directions (c_x, c_y and c_z). Using three dimensional Pythagoras Theorem, $c^2 = c_x^2 + c_y^2 + c_z^2$, but as on average, $c_x^2 = c_y^2 = c_z^2$, so $c_x^2 = \frac{1}{3}c^2$. As there are N particles in the box, the pressure, P, parallel to tae x axis is therefore

$$= \frac{1}{3} \times \left(\frac{m}{L^3}\right) \times (c_1^2 + c_2^2 + c_3^2 + \cdots + c_N^2)$$

We now define a quantity called the root mean square velocity, (c_{rms}), (the square root of the average of the square velocities) where:

$$c_{rms} = \sqrt{\frac{(c_1^2 + c_2^2 + c_3^2 + \cdots + c_N^2)}{N}} \text{ so } N(c_{rms})^2 = (c_1^2 + c_2^2 + c_3^2 + \cdots + c_N^2)$$

Substituting and replacing $L^3 = V$, gives:

$$pV = \frac{1}{3} Nm(c_{rms})^2$$

Nm is the total mass of the gas inside the box so the density of the gas inside the box is given by:

$$\rho = \frac{Nm}{V}$$

so:

$$p = \frac{1}{3} \rho (c_{rms})^2$$

Comparing two models of the behaviour of gases

We now have two models that describe the behaviour of a gas. The first model, the ideal gas equation, describes the experimental, macroscopic behaviour of the gas:

$$pV = nRT$$

The second model, involving the kinetic theory model, describes the behaviour from a theoretical point of view in terms of a microscopic, mechanical model of the particles:

$$p = \frac{1}{3}\rho(c_{\text{rms}})^2 = \frac{1}{3}\frac{Nm}{V}(c_{\text{rms}})^2$$

If the average molecular kinetic energy is $\overline{E_k}$ (the bar above the quantity means 'mean average'), then $E_{\text{k}} = \frac{1}{2}m(c_{\text{rms}})^2$ and so

$$p = \frac{2}{3} \times \frac{N}{V} \times \frac{1}{2}m(c_{\text{rms}})^2 \quad \text{or} \quad pV = \frac{2}{3} \times N \times \overline{E_{\text{k}}}$$

If this equation is compared to the ideal gas equation, then

$$nRT = \frac{2}{3} \times N \times \overline{E_{\text{k}}} \quad \text{or} \quad \overline{E_{\text{k}}} = \frac{3}{2} \times \frac{n}{N} \times RT$$

Because n is the number of moles of the gas and N is the number of particles of the gas, then

$$N = n \times N_{\text{A}} \quad \text{or} \quad \frac{n}{N} = \frac{1}{N_{\text{A}}}$$

Substituting this for n/N gives

$$\overline{E_{\text{k}}} = \frac{3}{2} \times \frac{R}{N_{\text{A}}} \times T$$

But R/N_A has already been defined as equal to k, the Boltzmann constant, which effectively is the gas constant *per particle of gas*. So

$$E_k = \frac{3}{2}kT$$

where E_k is the kinetic energy of one particle of the gas. This is truly a remarkable end point. We started with three macroscopic, easily measureable properties of a gas, and we end up with a simple equation that allows us to measure the kinetic energy of a particle of gas by measuring only its temperature.

EXAMPLE

Molecules in a gas syringe

A collection of 50 ideal gas molecules are observed inside a gas syringe. At a particular time, the distribution of their molecular speeds is:

Answer

Speed, c/km s^{-1}	1.80	1.90	2.00	2.10	2.20	2.30	2.40
Number of particles	6	8	12	10	7	4	3

Calculate the root mean square speed of the particles.

Answer

The first step is to calculate the square speeds of the particles:

Speed, c/km s^{-1}	1.80	1.90	2.00	2.10	2.20	2.30	2.40
c^2/km^2 s^{-2}	3.24	3.61	4.00	4.41	4.84	5.29	5.76
Number of particles	6	8	12	10	7	4	3

The next step is to calculate the mean square speed, which is the average of all the square speeds:

$$\overline{c^2} = \frac{(3.24\times6)+(3.61\times8)+(4.00\times12)+(4.41\times10)+(4.84\times7)+(5.29\times4)+(5.76\times3)}{50}$$

$$= 4.25\,\text{km}^2\,\text{s}^{-2}$$

To calculate the root mean square (r.m.s.) speed c_{rms}, we need to take the square root of this number:

$$(c_{rms}) = \sqrt{\overline{c^2}} = \sqrt{4.25\,\text{km}^2\,\text{s}^{-2}}$$

$$= 2.06\,\text{km}\,\text{s}^{-1}$$

EXAMPLE

Air particles in a room

Calculate the r.m.s. speed of the air particles in a room. The density of air is 1.3 kg m^{-3} and the room pressure of the air is 1.01×10^5 Pa.

Answer

We start with

$$p = \frac{1}{3}\rho(c_{rms})^2$$

Rearrange to make c_{rms} the subject:

$$c_{rms} = \sqrt{\frac{3p}{\rho}} = \sqrt{\frac{3\times1.01\times10^5\,\text{Pa}}{1.3\,\text{kg}\,\text{m}^{-3}}}$$

$$= 482.8\,\text{m}\,\text{s}^{-1} = 480\,\text{m}\,\text{s}^{-1}\ \text{(2 s.f.)}$$

EXAMPLE

Neon particles in a bulb

A neon-filled lamp bulb used in advertising signs contains neon particles at a pressure of 1.03×10^5 Pa and a temperature of 60°C. The molar mass of neon is $20.2\,\mathrm{g\,mol^{-1}}$. Calculate the density of the neon in the bulb.

Answer

First calculate the kinetic energy of the individual neon particles:

$$E_k = \frac{3}{2}kT = \frac{3}{2} \times 1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}} \times (273 + 60)\,\mathrm{K}$$

$$= 6.89 \times 10^{-21}\ \mathrm{J}$$

This can then be used to calculate the value of $(c_{rms})^2$. We have

$$\overline{E_k} = \frac{1}{2}m(c_{rms})^2$$

so

$$(c_{rms})^2 = \frac{2E_k}{m} = \frac{2E_k}{M_m/N_A} = \frac{2E_kN_A}{M_m}$$

$$= \frac{2 \times 6.89 \times 10^{-21}\ \mathrm{J} \times 6.02 \times 10^{23}\ \mathrm{mol^{-1}}}{20.2 \times 10^{-3}\ \mathrm{kg\,mol^{-1}}}$$

$$= 408\,648\ \mathrm{m^2\,s^{-2}} = 4.1 \times 10^5\ \mathrm{m^2\,s^{-2}}\ \text{(2 s.f.)}$$

Substituting into

$$p = \frac{1}{3}\rho\,(c_{rms})^2$$

gives

$$\rho = \frac{3p}{(c_{rms})^2}$$

$$= \frac{3 \times 1.03 \times 10^5\ \mathrm{Pa}}{408\,648\ \mathrm{m^2\,s^{-2}}}$$

$$= 0.756\ \mathrm{kg\,m^{-3}} = 0.76\ \mathrm{kg\,m^{-3}}\ \text{(2 s.f.)}$$

TEST YOURSELF

22 The sealed gas syringe shown in Figure 4.21 is filled with argon gas.

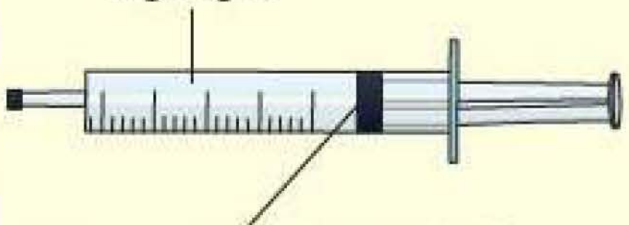


Figure 4.21

When an argon atom makes an elastic collision with the plunger, it undergoes a total momentum change of $1.33 \times 10^{-22}\,\mathrm{kg\,m\,s^{-1}}$.

a) If we assume that, every second, 2.6×10^{24} argon atoms collide perpendicularly with the plunger, calculate the force exerted by the argon on the plunger.

b) Calculate the pressure of the argon gas inside the syringe.

23 An official FIFA size 5 football has a circumference of 70 cm and an internal air pressure of 6.9×10^5 Pa at 300 K. The molar mass of air is $29\,\mathrm{g\,mol^{-1}}$.

a) Calculate the mass of air inside the football.

b) Calculate the root mean square velocity of the air particles inside the football.

24 At STP, 3 mol of an ideal gas occupy a volume of $0.067\,\mathrm{m^3}$. The Avogadro constant, $N_A = 6.02 \times 10^{23}$. Calculate:

a) a value for the molar gas constant, *R*

b) a value for the Boltzmann constant, *k*

c) the average kinetic energy of one molecule of the ideal gas.

25 An ideal gas can be modelled as many molecules in continuous motion enclosed in a container. Use the ideas of the kinetic theory to explain why the pressure of a fixed mass of an ideal gas at constant volume increases as the temperature of the gas rises. State any assumptions that you need to make.

26 A cylinder of helium gas, used to inflate party balloons, has a volume of $3.0 \times 10^{-3}\,\mathrm{m^3}$ and contains 42 g of helium gas at a room temperature of 20°C. The molar mass of helium is $4.0\,\mathrm{g\,mol^{-1}}$. Calculate:

a) the pressure inside the cylinder

b) the number of helium atoms inside the cylinder

c) the root mean square speed of the helium atoms inside the cylinder.

The cylinder is now stored outside, where the temperature is close to 0°C.

d) State and explain (without the aid of calculations) the effect of this temperature change on the values that you have calculated in parts (a), (b) and (c).

Practice questions

1 Thermal energy is supplied at the rate of 2.5 kW for 140 s to 0.8 kg of sunflower oil inside a saucepan with negligible heat capacity. This produces a temperature change of 219 K. The specific heat capacity of sunflower oil, in $J\,kg^{-1}\,K^{-1}$, is

A 1500 **C** 2000

B 1800 **D** 2200

2 An ice sculpture of mass 25 kg at 0 °C absorbs thermal energy from its surroundings at an average rate of 45 W. The specific latent heat of fusion of ice is $334\,kJ\,kg^{-1}$. The time, in days, for the sculpture to melt is

A 1.8 **C** 3.3

B 2.1 **D** 4.9

3 A 0.010 kg ice cube at 0°C is dropped into a glass containing 0.10 kg of lemonade at 15°C. The ice cube melts, cooling the lemonade. What is the new temperature of the drink in °C? The specific latent heat of fusion of ice is $334\,kJ\,kg^{-1}$, and the specific heat capacity of water (lemonade) is $4200\,J\,kg^{-1}\,K^{-1}$.

A 6 **C** 10

B 8 **D** 12

4 A deep-sea diver is working at a depth where the pressure is 3.2 atm. She is breathing out air bubbles. The volume of each bubble is $1.9\,cm^3$. She decompresses at a depth of 10 m where the pressure is 2.1 atm. What is the volume of each bubble at this depth in cm^3?

A 0.6 **C** 1.9

B 3.6 **D** 2.9

5 The helium in a sealed weather balloon at a temperature of 283 K has a volume of $1.4\,m^3$ and a pressure of 1.01×10^5 Pa. The balloon rises to a height of 300 m, where the temperature is 274 K and the pressure is 0.98×10^5 Pa. The volume of the air in the balloon at 300 m, in m^3, is

A 1.1 **C** 1.9

B 1.5 **D** 2.4

6 A mixture of helium and argon is used in a fire extinguisher system. The molar masses are $4.0\,g\,mol^{-1}$ and $40\,g\,mol^{-1}$, respectively, and the extinguisher contains one mole of each gas. The ratio of the pressure exerted by the helium and the argon, respectively, on the inside of the extinguisher is

A 1:1 **C** 1:10

B 100:1 **D** 10:1

7 A deodorant can with a volume of $330\,cm^3$ at 18°C contains deodorant particles that exert a pressure of 3.2×10^5 Pa on the inside of the can. The number of moles of deodorant particles in the can is

A 0.04 **C** 400

B 4 **D** 40 000

8 The density of air at 15°C and 1.01×10^5 Pa is 1.225 kg m^{-3}. The r.m.s. velocity of air particles, in m s^{-1}, is

A 604 **C** 498

B 603 **D** 497

9 Five nitrogen gas molecules have the following velocities, in m s^{-1}: 300, 450, 675, 700, 800. The root mean square velocity of the particles, in m s^{-1}, is

A 413 **C** 613

B 513 **D** 713

10 Carbon particles of mass 2.0×10^{-26} kg in the hottest part of a Bunsen burner flame have a temperature of 1200°C. The r.m.s. velocity of these particles, in m s^{-1}, is

A 823 **C** 1746

B 1235 **D** 2143

11 A jewellery maker is making a gold pendant. She prepares a 3.0 kg iron mould and then pours in 25.0 g of molten gold at a temperature of 1064°C. The mould's temperature rises from 31°C up to 35°C when it is then in thermal equilibrium with the solid gold.

Here is the thermal data about the gold and the iron:

- mass of iron mould = 3.0 kg
- specific heat capacity of iron = 440 J kg^{-1} K^{-1}
- specific latent heat of fusion of gold = 63×10^3 J kg^{-1}

a) Calculate the thermal energy absorbed by the iron mould. *(2)*

b) Calculate the thermal energy given out by the gold as it changes state from a liquid to a solid. *(1)*

c) Use the data to determine the specific heat capacity (c) of gold. *(3)*

d) State one assumption that you have made for your calculation of c. *(1)*

12 A student is making iced tea lollies using her family's freezer. She initially pours 0.050 kg of lukewarm tea at a temperature of 40.0°C into a 0.12 kg aluminium mould at a temperature of 5.0°C. The specific heat capacity of tea is 4250 J kg^{-1} K^{-1} and the specific heat capacity of aluminium is 900 J kg^{-1} K^{-1}.

a) Calculate the equilibrium temperature of the tea and the mould. *(3)*

b) The tea and the mould are then put into the freezer, which removes thermal heat from the tea and the mould at a rate of 32 W.

Calculate how long it takes for the tea to freeze, if the specific latent heat of fusion of tea is 3.38×10^5 J kg^{-1}, stating any assumptions that you make. *(4)*

13 A gas combi-boiler can heat water with a power of 15 kW. Cold water with a temperature of 5°C flows into the heater at a rate of 0.24 kg s^{-1}. The specific heat capacity of water is 4200 J kg^{-1} K^{-1}.

a) Combi-boilers are highly efficient and you can assume that all the thermal energy from the heater is transferred to the water. Calculate the output temperature of the water. (2)

b) The water supply to the heater fails and 0.24 kg of water is trapped inside the heating compartment of the heater. The water inside the compartment has an average temperature of 35°C and the heater continues to heat the water. How long will it take before the water reaches 80°C, when the emergency cut-out valve turns off the gas supply? (2)

14 Formula 1 tyres have a volume of $0.09\,m^3$ and are filled with nitrogen to a pressure of 1.4×10^5 Pa at 285 K.

a) Calculate the number of moles of nitrogen in the tyre. (1)

b) F1 tyres are designed to work at an optimum racing temperature of 363 K. Calculate the racing pressure in the tyre. You can assume that the tyre does not expand when heated. (2)

c) Calculate the root mean square (r.m.s.) velocity of the nitrogen molecules in the tyre when it is at racing pressure. The molar mass of nitrogen is $0.028\,kg\,mol^{-1}$. (3)

d) Describe one similarity and one difference in the way that the nitrogen molecules behave in the tyre at the different pressures. (2)

15 A fixed mass of helium gas is enclosed in a container with a volume of $0.055\,m^3$. The gas is cooled and a student measures and records the pressure of the gas, in atm, for different temperatures. The table shows the results:

Temperature, *T*/K	320	300	280	260	240
Pressure, *p*/atm	1.30	1.22	1.17	1.08	0.95

a) Use the data to plot a graph of the results, with temperature on the *x*-axis and pressure on the *y*-axis. Start both axes at zero. (3)

b) Use your graph to calculate the number of moles of helium gas present in the container. (3)

c) The pressure inside the container is reduced to 0.50 atm by cooling the container. Use your graph to determine the temperature of the gas at this pressure. (1)

d) Use your answer to (c) to calculate the average kinetic energy of a helium atom at a pressure of 0.5 atm. (2)

e) Hence calculate the total internal energy, *U*, of the helium. (2)

16 This question is about ideal gases.

a) State what is meant by an 'ideal gas'. (2)

b) An ideal gas at 300 K is enclosed inside a gas canister of volume $3.3 \times 10^{-4}\,m^3$ at a pressure of 2.02×10^5 Pa. Calculate the number of moles of gas enclosed inside the canister. (2)

c) The molar mass of the gas is $0.084\,kg\,mol^{-1}$. Calculate the density of the gas inside the canister. (3)

d) The canister is taken to the top of Mount Kilimanjaro, where it is used to inflate an air-mat. If the temperature at the top of the mountain is 266 K and the air pressure is 0.50×10^{-5} Pa, what is the combined volume of the canister and air-mat that could be inflated by the gas in the canister at this pressure. *(1)*

Stretch and challenge

The questions that follow here are British Physics Olympiad questions.

17 a) State Boyle's law.

Figure 4.23(a) shows a length of capillary tubing in which a column of air is trapped by a mercury column of length 100 mm. The length of the air column is 400 mm. The bottom of the tubing is sealed and the top is open to the atmosphere.

b) The tubing is now inverted, as shown in Figure 4.23(b), and the air column is seen to increase in length to 520 mm. Use this observation to calculate a value for atmospheric pressure, expressed in mm of mercury.

c) A typical value for atmospheric pressure, expressed in SI units, is 101 kPa. The surface area, A, of the Earth is related to its mean radius by the expression, $A = 4\pi R^2$, where R has the value 6400 km. Calculate:

i) the sum of the magnitudes of the forces exerted by the atmosphere on the surface of the Earth

ii) the mass of the Earth's atmosphere, assuming that g does not vary with height above the Earth's surface

iii) the number of molecules in the atmosphere, assuming that the molar mass of air is $30\,\text{g}\,\text{mol}^{-1}$

iv) the height of the atmosphere if the density $\rho = 1.2\,\text{kg}\,\text{m}^{-3}$.

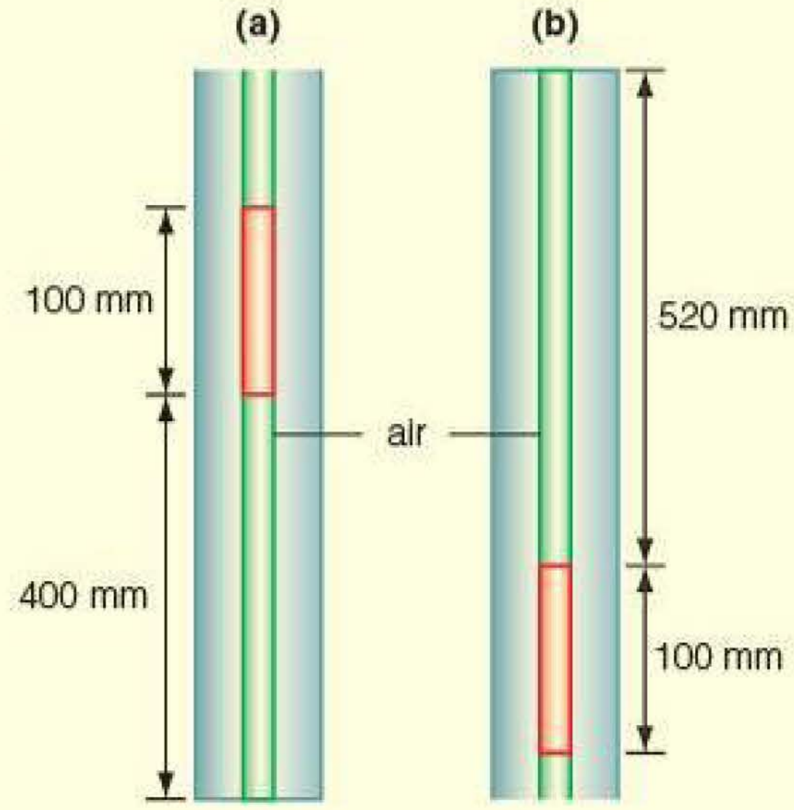


Figure 4.22

d) The height of the atmosphere calculated in c) iv) is less than the height at which many aircraft fly. Explain why our calculation gives a low result for the height.

e) The height of the atmosphere is typically given as 200 km. Does this mean that our calculation of the mass is completely wrong (by a significant factor)?

(BPhO A2-2005 Q2; and A2-2011 Q4)

18 An accurate thermometer, of heat capacity $20.0\,\text{J}\,\text{K}^{-1}$, reads 18.0°C. It is then placed in 0.250 kg of water and both reach the same final temperature of 50°C. Calculate the temperature of the water before the thermometer was placed in it. The specific heat capacity of water is $4200\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$.

(BPhO R1-2005 Q1(a))

19 Wet clothing at 0°C is hung out to dry. The air temperature is 0°C and there is a dry wind blowing. After some time it is found that some of the water has evaporated and the water remaining on the clothes has frozen. The specific heat of fusion of ice is $333\,\text{kJ}\,\text{kg}^{-1}$ and the specific latent heat of evaporation of water is $2500\,\text{kJ}\,\text{kg}^{-1}$.

a) What is the source of energy required to evaporate the water? Explain the mechanism of evaporation.

b) Estimate the fraction, by mass, of water originally in the clothes that freezes.

(BPhO R1-2005 Q1(e))

20 A lead bullet at 320 K is stopped by a sheet of steel so that it reaches its melting point of 600 K and completely melts. If 80% of the kinetic energy of the bullet is converted into internal energy, calculate the speed with which the bullet hit the steel sheet. The specific heat capacity of lead is $0.12\,\text{kJ}\,\text{kg}^{-1}\,\text{K}^{-1}$ and its specific latent heat of fusion is $21\,\text{kJ}\,\text{kg}^{-1}$.

(BPhO R1-2007 Q1(f))

21 a) Water in an electric kettle is brought to the boil in 180 s by raising its temperature from 20°C to 100°C. It then takes a further 1200 s to boil the kettle dry. Calculate the specific latent heat of vaporisation of water, l_v, at 100°C, stating any assumptions made.

b) A cylinder, with a weightless piston, has an internal diameter of 0.24 m. The cylinder contains water and steam at 100°C. It is situated in a constant-temperature water bath at 100°C, as shown in Figure 4.24. Atmospheric pressure is 1.01×10^5 Pa. The steam in the cylinder occupies a length of 0.20 m and has a mass of 0.37 g.

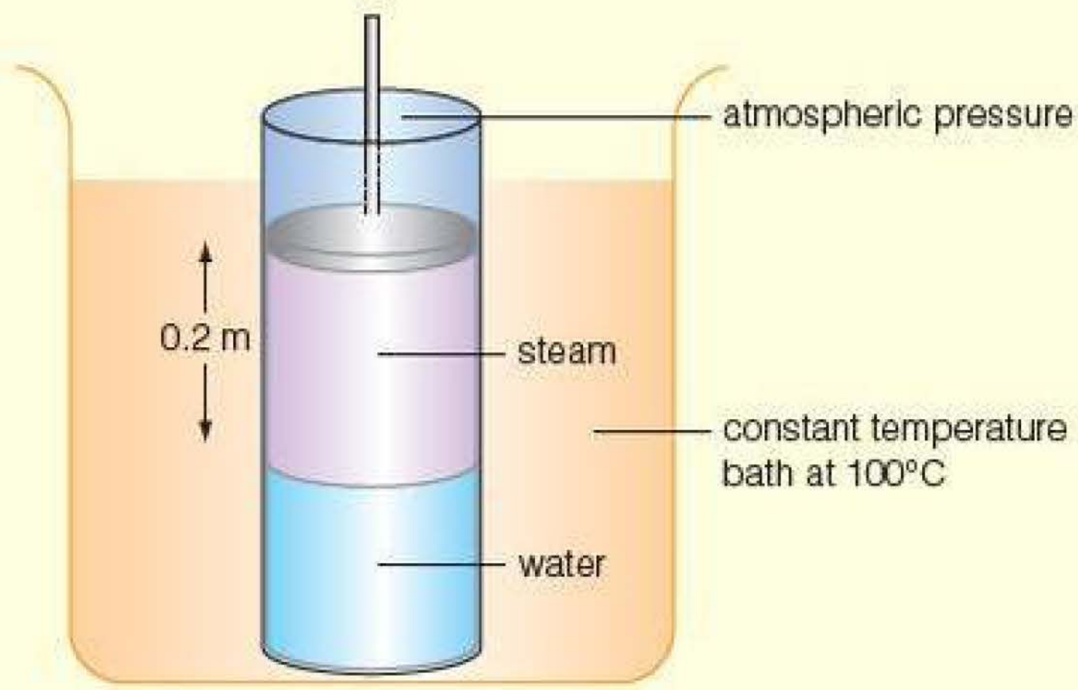


Figure 4.23

i) What is the pressure p of the steam in the cylinder?

ii) If the piston moves very slowly down a distance 0.10 m, how much work, W, will be done in reducing the volume of the steam?

iii) What is the final temperature, T_f, in the cylinder?

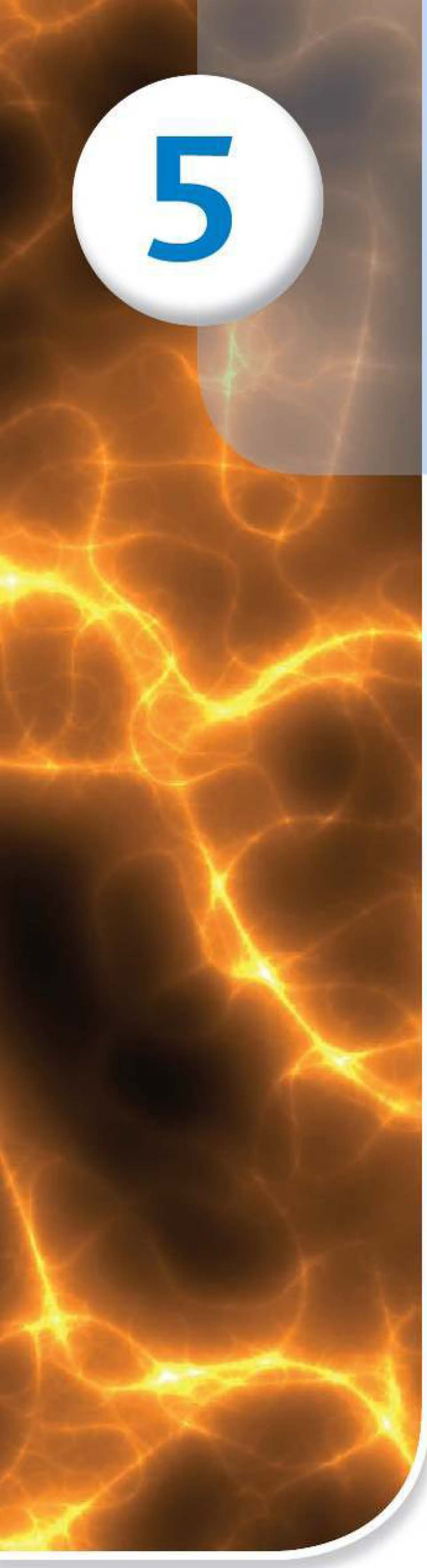
iv) Determine the heat, Q_c, produced in the cylinder.

c) A molecule of oxygen near the surface of the Earth has a velocity vertically upwards equal in magnitude to the root mean square (r.m.s.) value. If it does not encounter another molecule, calculate:

i) the height H reached if the surface temperature is 283 K

ii) the surface temperature, T_s, required for the molecule to escape from the Earth's gravitational field if the potential energy per unit mass at the Earth's surface is $\left(-G\frac{M_E}{R_E}\right)$. The oxygen molecule has a molar mass of $0.032\,\text{kg}\,\text{mol}^{-1}$.

(BPhO R1-2002 Q2)



5 Electric fields

PRIOR KNOWLEDGE

Before you start, make sure that you are confident in your knowledge and understanding of the following points:

- Atoms and molecules contain protons and electrons, which carry positive and negative charges, respectively. These charges are equal in size. An atom is neutral because there are as many positively charged protons as there are negatively charged electrons.
- Some materials, such as plastic, can become charged by rubbing with a cloth. If the plastic is charged positively, then electrons have been removed from the plastic and transferred to the cloth, which now carries a negative charge. Another type of plastic might be charged negatively when rubbed by a cloth – electrons have been transferred to the plastic and the cloth will be charged positive.
- Like charges repel each other, and unlike charges attract each other (Figure 5.1).

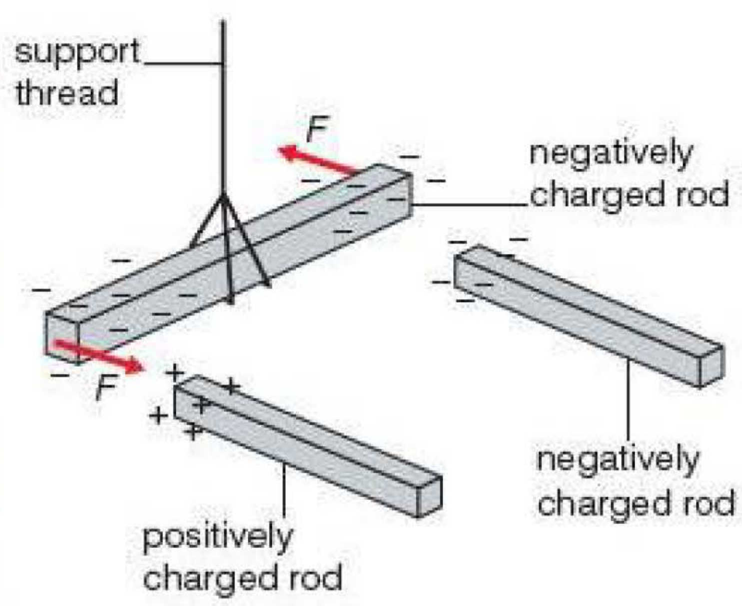


Figure 5.1

- Electric charges exert a force on each other over a distance. For example, a charged comb can pick up pieces of paper (Figure 5.2).

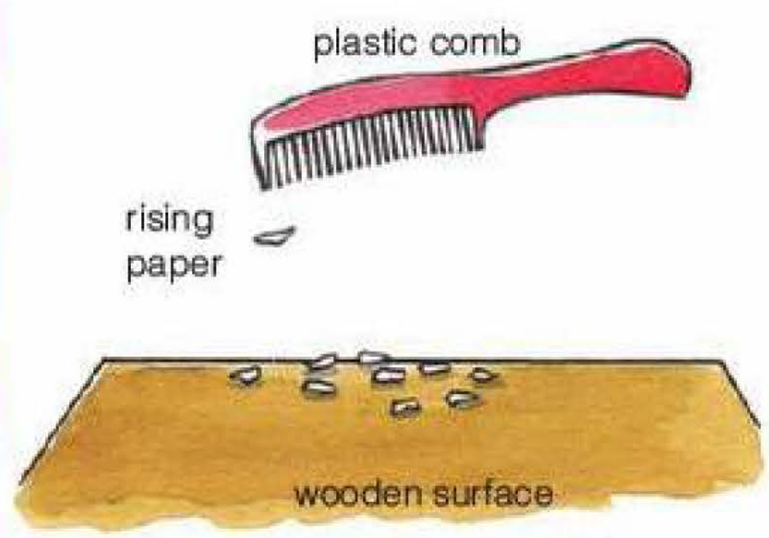


Figure 5.2

- Charges produce an electric field.
- An electric field is a region in space where a charged object experiences a force.
- Forces between charges are stronger when they are closer together. The forces are weaker when the charges are further apart.

TEST YOURSELF ON PRIOR KNOWLEDGE

1 Explain why an atom of magnesium, which has 12 protons in its nucleus, is neutral.
2 Explain how ions are formed.
3 Explain what is meant by the term 'electric field'. Name two other types of force field.
4 Draw a diagram to explain how a comb, which is positively charged, can lift up a piece of paper, which is neutral. (This takes some explaining, in terms of electron movement in the paper and attractive and repulsive forces.)

Volcanic ash thrust into the atmosphere produces ideal conditions for lightning. The enormous quantity of pulverised material and gases ejected into the atmosphere creates a dense plume of charged particles. The friction of particles moving past each other transfers charge in the same way as a balloon can be charged by rubbing. Potential differences of millions of volts exist within the plume, which are sufficient to drive large currents, which discharge the clouds of ash.

Coulomb's law

The starting point for work in electrostatics is Coulomb's law, which states that the force between two point charges Q_1 and Q_2 separated by a distance r (Figure 5.3a) is given by

$$F = \frac{Q_1Q_2}{4\pi\varepsilon_0 r^2}$$

Here F is measured in newtons (N), r is measured in metres (m), Q_1 and Q_2 are measured in coulombs (C), and ε_0 is a constant, the permittivity of free space: $\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$. (The Maths box on p. 87 shows how the units are derived, for the interested reader.)

Sometimes you will find the constant $\frac{1}{(4\pi\varepsilon_0)}$ quoted as $9.0 \times 10^9\,\mathrm{F^{-1}\,m}$. You might also see Coulomb's law written in the form

$$F = \frac{kQ_1Q_2}{r^2}$$

where $k = \frac{1}{(4\pi\varepsilon_0)}$.

The value of ε_0 quoted refers to the permittivity of free space – which means a vacuum. The value of permittivity varies from one medium to another. However, the permittivity of air is very close to that of a vacuum, so we shall use the value of ε_0 quoted in those calculations.

The equation for Coulomb's law is very similar to that for Newton's law of gravitation, except that the force between two charges can be repulsive if the two charges have the same sign, or attractive if the two charges have the opposite sign. The force between two masses is always attractive. Coulomb's law may also be used to calculate the electrostatic force between two charged spheres carrying charges Q_1 and Q_2. In this case the distance used is the separation of the centres of the two spheres, as shown in Figure 5.3(b).

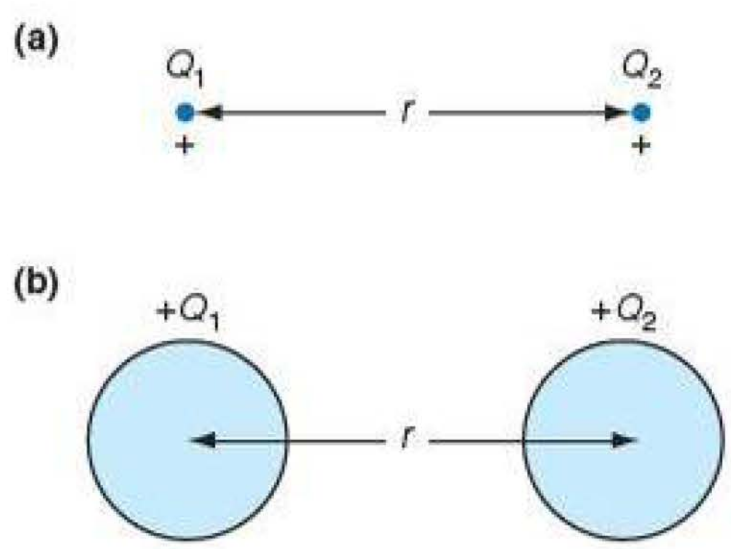


Figure 5.3

MATHS BOX

Where does the unit of permittivity, ε_0, quoted as $\mathrm{F\,m^{-1}}$ or farads per metre, come from? The reasoning below explains this.

Coulomb's law states that

$$F = \frac{Q_1Q_2}{4\pi\varepsilon_0 r^2}$$

so

$$\varepsilon_0 = \frac{Q_1Q_2}{4\pi F r^2}$$

The units of ε_0 are therefore

$$[\varepsilon_0] = \frac{[\mathrm{C}]\times[\mathrm{C}]}{[\mathrm{N}]\times[\mathrm{m}^2]}$$

$$= \frac{[\mathrm{C}]\times[\mathrm{C}]}{[\mathrm{N\,m}]\times[\mathrm{m}]}$$

However, a joule = newton × metre, and volt $= \frac{\text{joule}}{\text{coulomb}}$. Therefore the units of ε_0 are

$$[\varepsilon_0] = \frac{[\mathrm{C}]\times[\mathrm{C}]}{[\mathrm{J}]\times[\mathrm{m}]}$$

$$= \frac{[\mathrm{C}]}{[\mathrm{V}]\times[\mathrm{m}]}$$

But the definition of capacitance tells us that farad $= \frac{\text{coulomb}}{\text{volt}}$. Therefore ε_0 has units $\mathrm{F\,m^{-1}}$.

EXAMPLE

Coulomb's law

1 Calculate the force of attraction between two point charges A and B separated by a distance of 0.2 m. The charge at A is +2 μC and the charge at B is −1 μC.

Answer

$$F = \frac{Q_1Q_2}{4\pi\varepsilon_0 r^2}$$

$$= \frac{(2\times10^{-6}\,\mathrm{C})\times(-10^{-6}\,\mathrm{C})}{(4\pi\times8.85\times10^{-12}\,\mathrm{F\,m^{-1}})\times(0.2\,\mathrm{m})^2}$$

$$= -0.45\,\mathrm{N}$$

The significance of the minus sign is to remind us that the force is attractive, but it is not really necessary to include it.

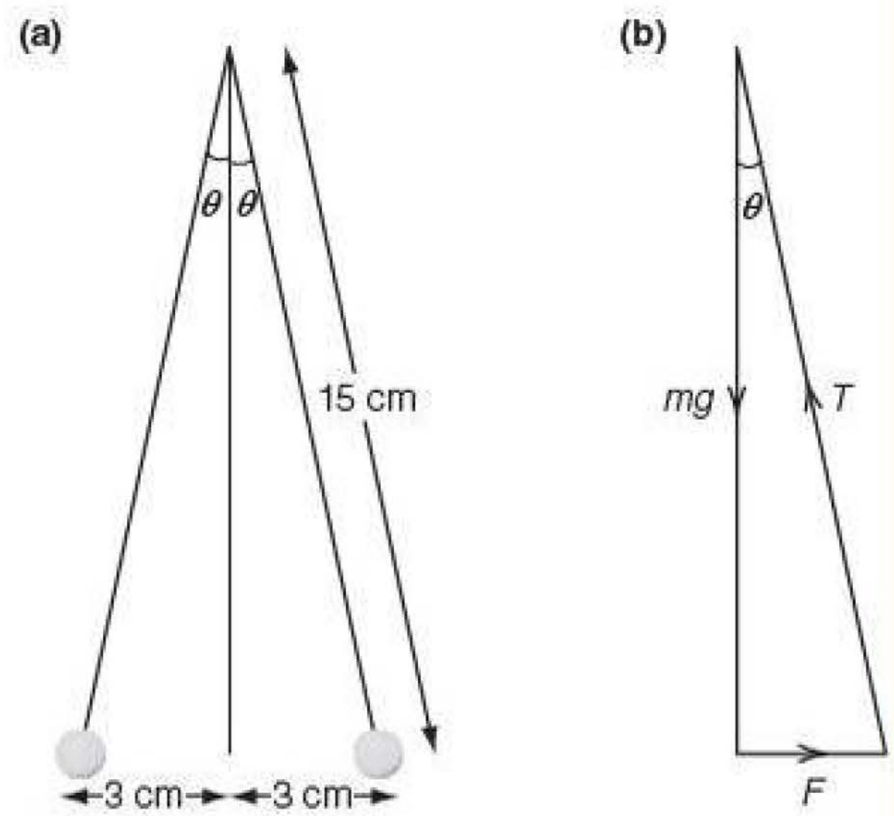


Figure 5.4

2 Figure 5.4(a) shows two light polystyrene spheres, which have been coated in a conducting metallic paint. Each has been charged positively by a high-voltage supply to about 3 kV. They are suspended by pieces of cotton 15 cm long, and they are pushed apart by the repulsive electrostatic force between them. The mass of each sphere is 0.08 g.
Use the information in the diagram to calculate the charge on each sphere.

Answer

Figure 5.4(b) shows the forces acting on the right-hand sphere. The tension in the cotton, T, is balanced by the electrostatic force, F, and the weight of the ball, mg. From the triangle of forces we can see that

$$\tan\theta = \frac{F}{mg}$$

From the triangle showing the displacement of the two spheres we can see that

$$\sin\theta = \frac{3\,\text{cm}}{15\,\text{cm}} = 0.2$$

However, when θ is small, $\sin\theta \approx \tan\theta$. Therefore

$\tan\theta \approx \sin\theta$ and $\frac{F}{mg} = 0.2$. So

$$F = 0.2\,mg$$

$$= 0.2 \times (0.08 \times 10^{-3}\,\text{kg}) \times 9.8\,\text{N kg}^{-1}$$

$$= 1.6 \times 10^{-4}\,\text{N}$$

From Coulomb's law,

$$F = \frac{Q_1Q_2}{4\pi\varepsilon_0 r^2} = \frac{Q^2}{4\pi\varepsilon_0 r^2}$$

since $Q_1 = Q_2$. Therefore

$$Q^2 = 4\pi\varepsilon_0 r^2 \times F$$

$$Q^2 = 4\pi \times (8.85 \times 10^{-12}\,\text{F m}^{-1}) \times (0.06\,\text{m})^2 \times 1.6 \times 10^{-4}\,\text{N}$$

$$Q = 8.0 \times 10^{-9}\,\text{C}$$

ACTIVITY

Testing Coulomb's law

Figure 5.5 shows an experimental arrangement for investigating Coulomb's law. The two polystyrene spheres are charged by a high-voltage supply. Sphere A is held in a fixed position and sphere B is free to move. A light bulb is used to cast a shadow of the spheres onto graph paper, so that their separation, and also the deflection of sphere B, can be measured more easily.

Table 5.1 shows the results for six different separations of the spheres' shadows. Between each set of measurements the spheres were recharged.

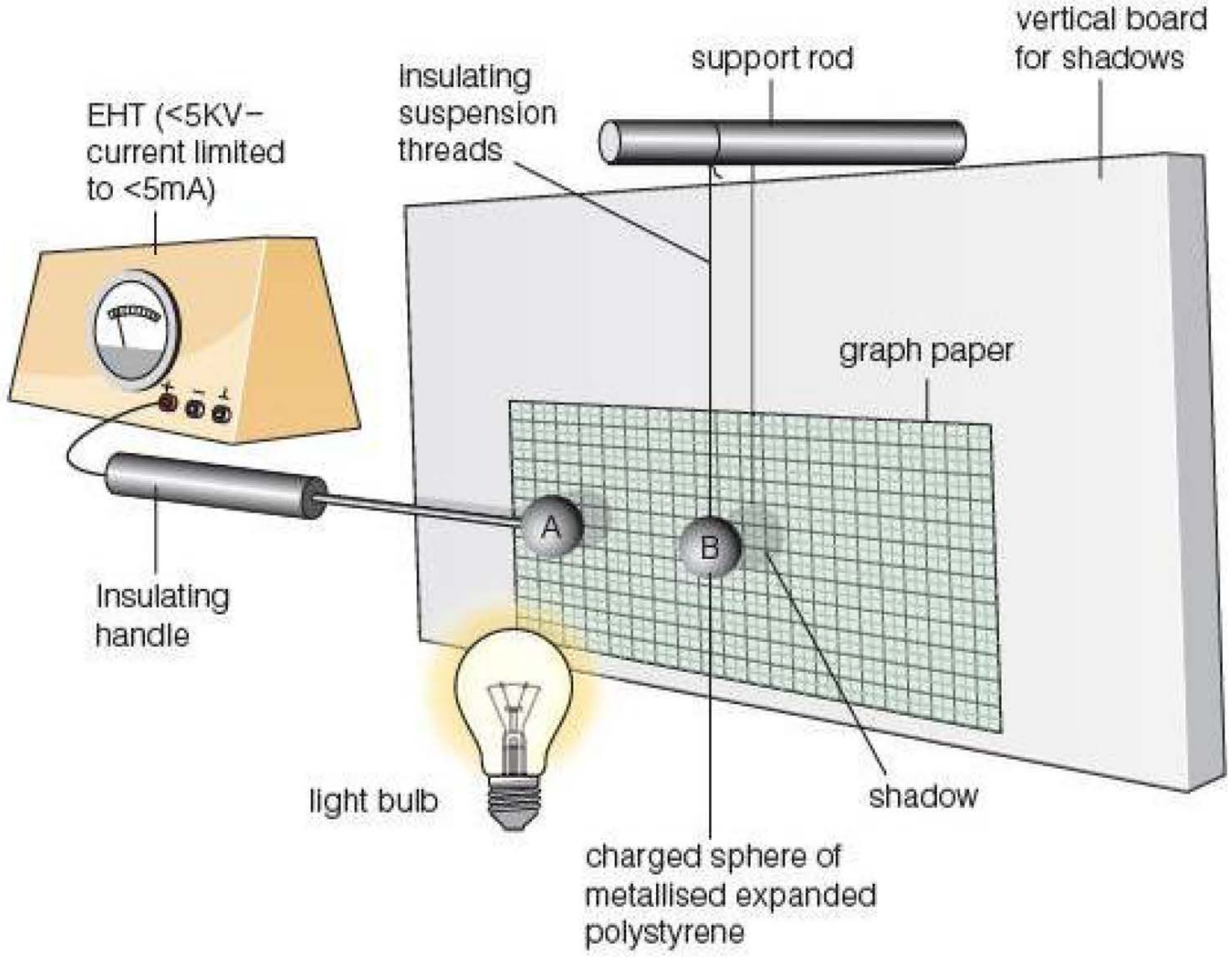


Figure 5.5

Table 5.1

Distance between centres of spheres' shadows/mm	100	70	50	40	30	20
Deflection of sphere B's shadow/mm	1	3	5	8	15	29

1 Plot a graph of the deflection of sphere B's shadow against $\frac{1}{r^2}$, where r is the separation of the centres of the spheres' shadows.

Discuss whether or not your graph supports Coulomb's law.

2 Explain why the deflection of sphere B is proportional to the force between the balls.

3 Discuss the sources of error in this experiment. You should consider both systematic and random errors.

4 Discuss how you could calculate the real separation of the spheres, rather than their shadows.

TEST YOURSELF

For these questions, use the following values: mass of an electron = 9.1×10^{-31} kg, charge on an electron = 1.6×10^{-19} C, $\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$.

1 The centre of a small sphere carrying a charge of +2.0 nC is placed at a distance of 240 mm from the centre of a second small sphere carrying a charge of −5.0 nC.
  a) Calculate the force of attraction between them.
  b) Calculate the size of the force between the spheres when they are separated by each of the following distances:
    i) 120 mm
    ii) 80 mm
    iii) 60 mm
    iv) 48 mm.

2 The electron and proton in a hydrogen atom are on average about a distance of 5×10^{-11} m apart. Calculate the force the proton exerts on the electron. How big a force does the electron exert on the proton?

3 A uranium nucleus contains 92 protons; the nucleus has a radius of 8.0×10^{-15} m. Calculate the force on an alpha particle at the surface of the uranium nucleus. Comment on the size of this force.

4 Two charges, one three times the size of the other, experience a repulsive force of 80 mN when they are separated by a distance of 10 cm. Calculate the size of the larger charge.

5 A small polystyrene sphere of mass 1 g is suspended near to another larger sphere so that their centres lie along the same horizontal line. Both spheres are charged negatively. The small sphere is now deflected so that the thread holding it is deflected to an angle of 36° to the vertical.
  a) Draw a free-body diagram to show the forces acting on the sphere.
  b) Calculate the size of the repulsive force between the spheres.

6 a) The average distance of electrons from the nucleus, in the lowest energy state of a gold atom, is 7×10^{-13} m. Calculate the force between such an electron and the gold nucleus. Gold has an atomic number of 79.
  b) By making the assumption that the electron moves in a circular orbit of radius 7×10^{-13} m. calculate the speed of the electron. How does this speed compare with the speed of light?

Electric field strength

In the last section you met the idea of two point charges exerting a force on each other. A charge produces an electric field around it, which exerts a force on another charged object. This idea is similar to a magnetic field close to a magnet, or a gravitational field around a planet.

An electric field strength is defined by the equation

$$E = \frac{F}{Q}$$

where F is the force, in newtons, which acts on a charge, Q, in coulombs. So electric field strength is measured in newtons per coulomb, $\mathrm{N\,C^{-1}}$. The direction of the electric field is defined as the direction of the force on a positive charge. Electric field is a vector quantity because it has both magnitude and direction.

We represent electric fields by drawing lines. Figure 5.6 shows two uniform electric fields. The stronger field in Figure 5.6(a) is represented by field lines that are closer together. The fields are uniform because in all places the field has the same strength and the same direction. Note that the field lines start on a positive charge and end on a negative charge. The positive charge in Figure 5.6(a) experiences an electrostatic force downwards.

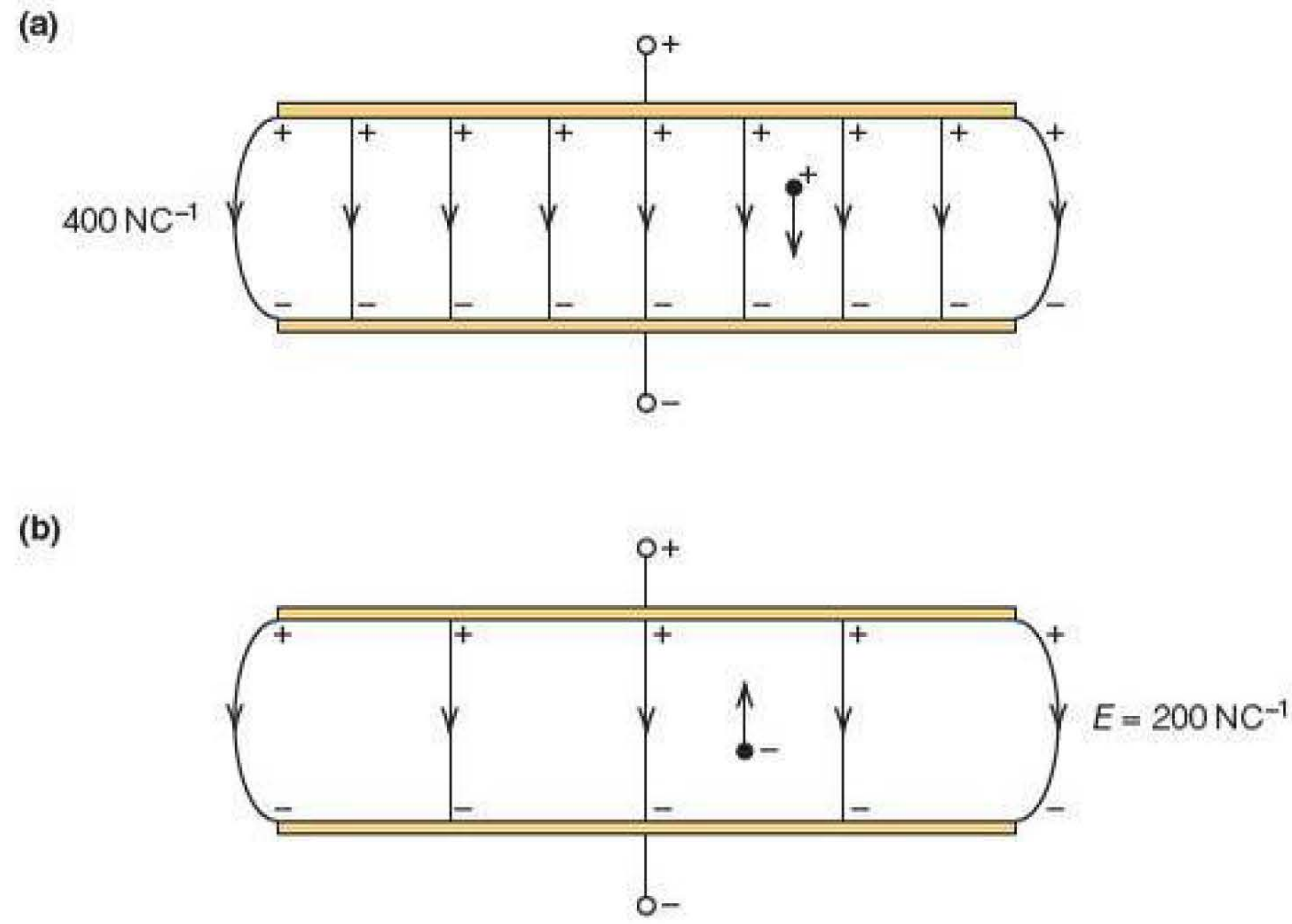


Figure 5.6

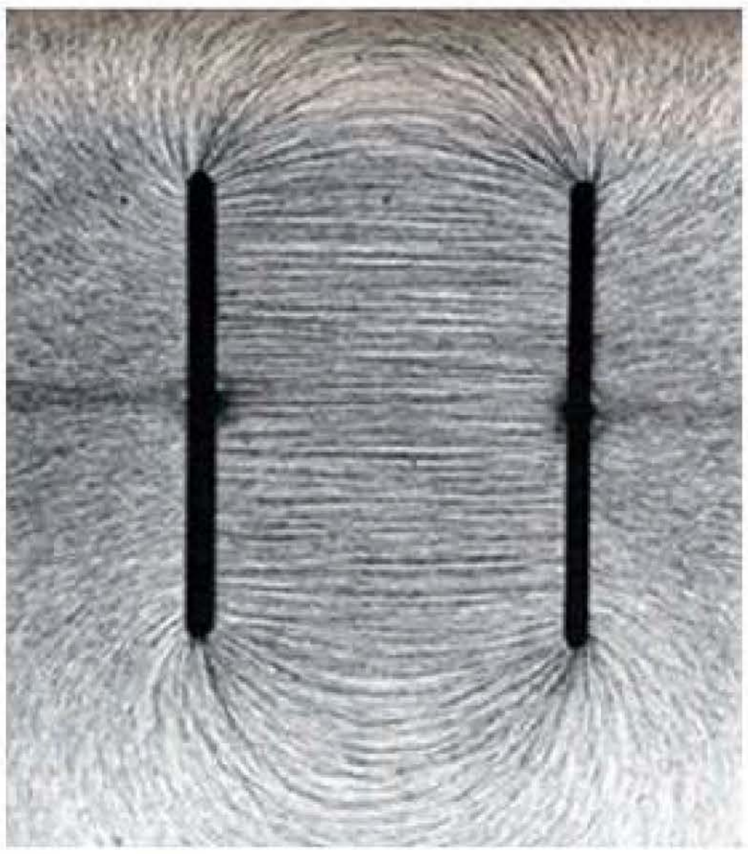


Figure 5.7

Electric field lines are a model to help us visualise a field, but a direct way of showing an electric field is shown in Figure 5.7. In this photograph, a potential difference has been applied to two metal plates, which have been placed into an insulating liquid. Then short pieces of a fine thread have been sprinkled on top of the liquid. When the electric field is applied, the pieces of thread line up along the field lines, in the same way that iron filings follow magnetic field lines.

> **EXAMPLE**
>
> ### Force on a small charge
>
> A small charge of $+2\,\mu C$ is placed in the electric field in Figure 5.7(a). What force does it experience?
>
> **Answer**
>
> $$E = \frac{F}{Q}$$
>
> So
>
> $$F = EQ$$
>
> $$= 400\,\mathrm{N\,C^{-1}} \times 2 \times 10^{-6}\,\mathrm{C}$$
>
> $$= 8 \times 10^{-4}\,\mathrm{N}\text{ downwards}$$

Electric field strength and potential gradient

Figure 5.8 shows a charge $+Q$ placed in an electric field between two parallel plates. The plates have a potential difference of V between them, and their separation is d (in m). How much work is done if the charge is moved from A to B? This question can be answered in two ways.

First, the work done = $F \times d$, but the force to move the charge must be equal in magnitude to the force on the charge, due to the electric field, $E \times Q$. So

$$\text{work done} = EQd$$

Secondly, the work done is also equal to the energy gained by the charge in moving through a potential difference V. This is VQ – you should remember that a volt is defined as a joule per coulomb. Therefore

$$EQd = VQ$$

and

$$E = \frac{V}{d}$$

This equation allows us to calculate the magnitude of a uniform electric field between two parallel plates. Note that the electric field strength can also be measured in $\mathrm{V\,m^{-1}}$.

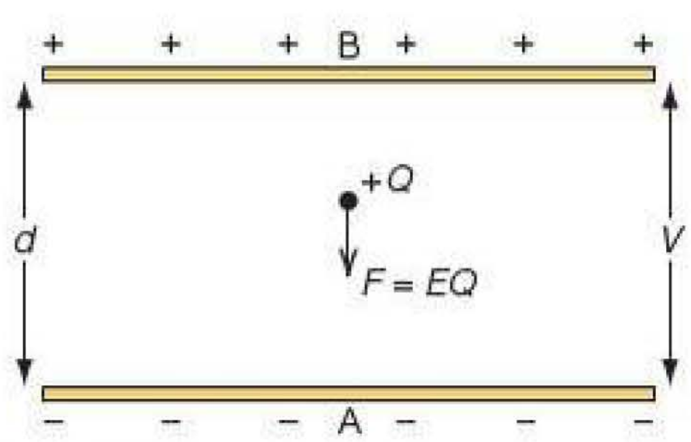


Figure 5.8

Figure 5.9 allows us to produce a more general formula to link electric field and potential gradient. In the formula above, we just concentrated on the magnitude of the field, but strictly speaking we should have included its direction.

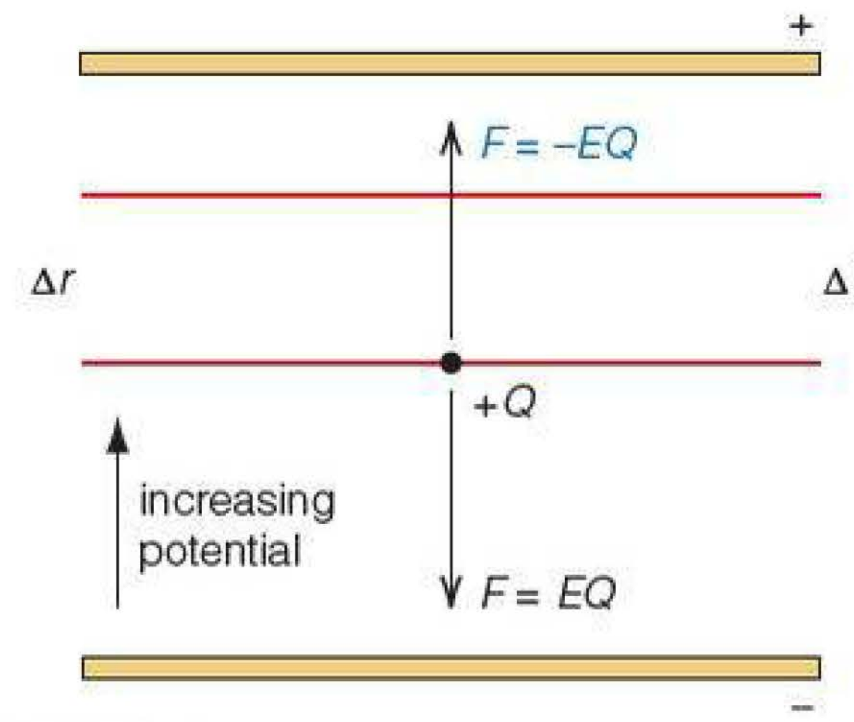


Figure 5.9

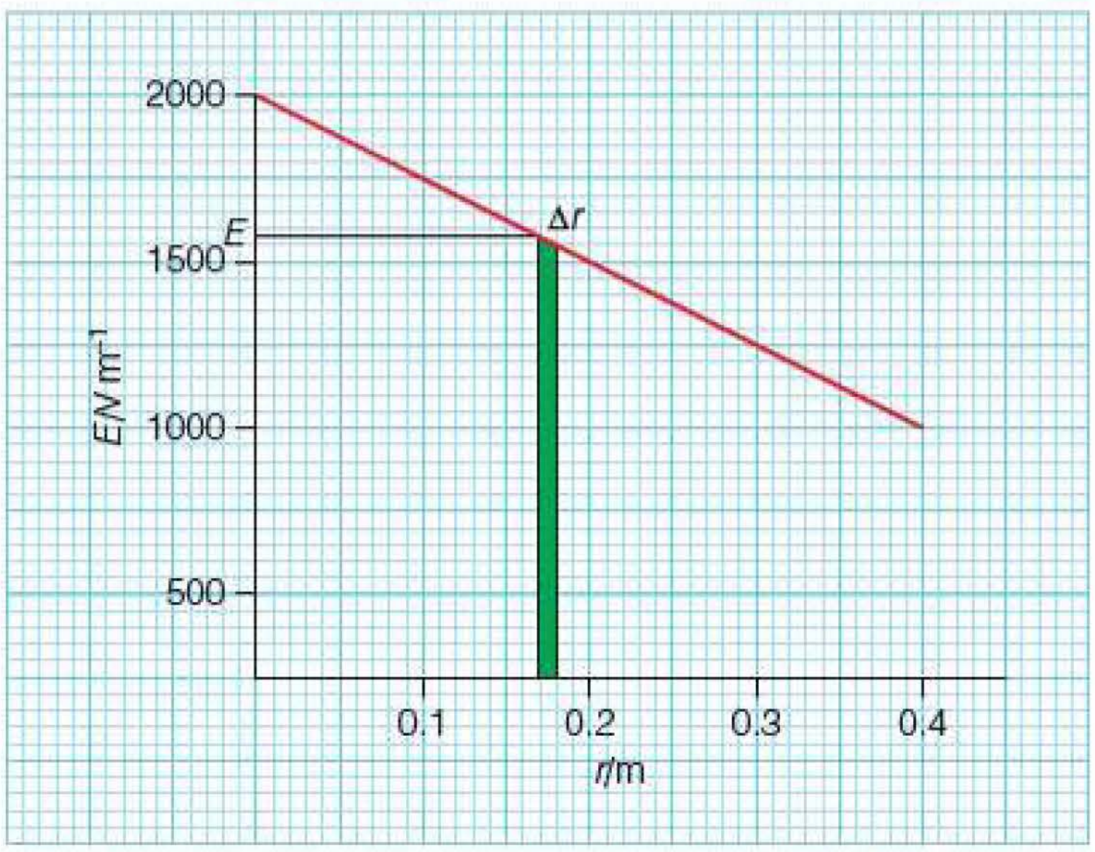


Figure 5.10

Electric field strength is defined by

$$E = \frac{F}{Q}$$

In a uniform field

$$E = \frac{V}{d}$$

and generally

$$E = -\frac{\Delta V}{\Delta r}$$

Units of electric field strength are $N\,C^{-1}$ or $V\,m^{-1}$.

The area under an E against r graph is ΔV, the change in potential.

In Figure 5.9 the charge Q is moved through a potential ΔV through a small distance Δr by the force $F = -EQ$. (This is marked in blue in the diagram and is in the opposite direction to the force from the electric field.)

So we now get

$$\text{work done} = -EQ\Delta r = \Delta V Q$$

or

$$E = -\frac{\Delta V}{\Delta r}$$

So the **electric field strength** is equal in magnitude to the potential gradient, but it is in the opposite direction.

Figure 5.10 shows a graph of electric field strength in a region. The area under the graph may be used to calculate the potential difference between two points. A shaded section (green) under the graph has an area $E\Delta r$, which has a value ΔV.

In general, the area under a graph of E against r gives the change of potential ΔV.

EXAMPLE

Change in electric potential

Use Figure 5.10 to calculate the change in potential in moving from position $r = 0$ to $r = 0.2\,m$.

Answer

$$\Delta V = \text{area under the graph}$$
$$= \frac{1}{2}(2000\,V\,m^{-1} + 1500\,V\,m^{-1}) \times 0.2\,m$$
$$= 350\,V$$

Deflection of charged particles by electric fields

Any charged particle experiences a force in an electric field. So when a moving charged particle enters an electric field, it will change direction. (Only when a charged particle moves *parallel* to an electric field does it keep moving in the *same* direction.) Figure 5.11 shows a photograph of an electron beam tube, which can be used to deflect electrons.

Figure 5.12 shows the principle behind the electron beam tube. Electrons are accelerated by an 'electron gun' on the left-hand side. They then travel across a fluorescent screen, which shows the electron path. The electrons travel from P to Q. The electrons are deflected by applying a potential difference between A and B. When a potential difference is applied so that the top plate is positive, the electrons are deflected upwards along a path such as PR.

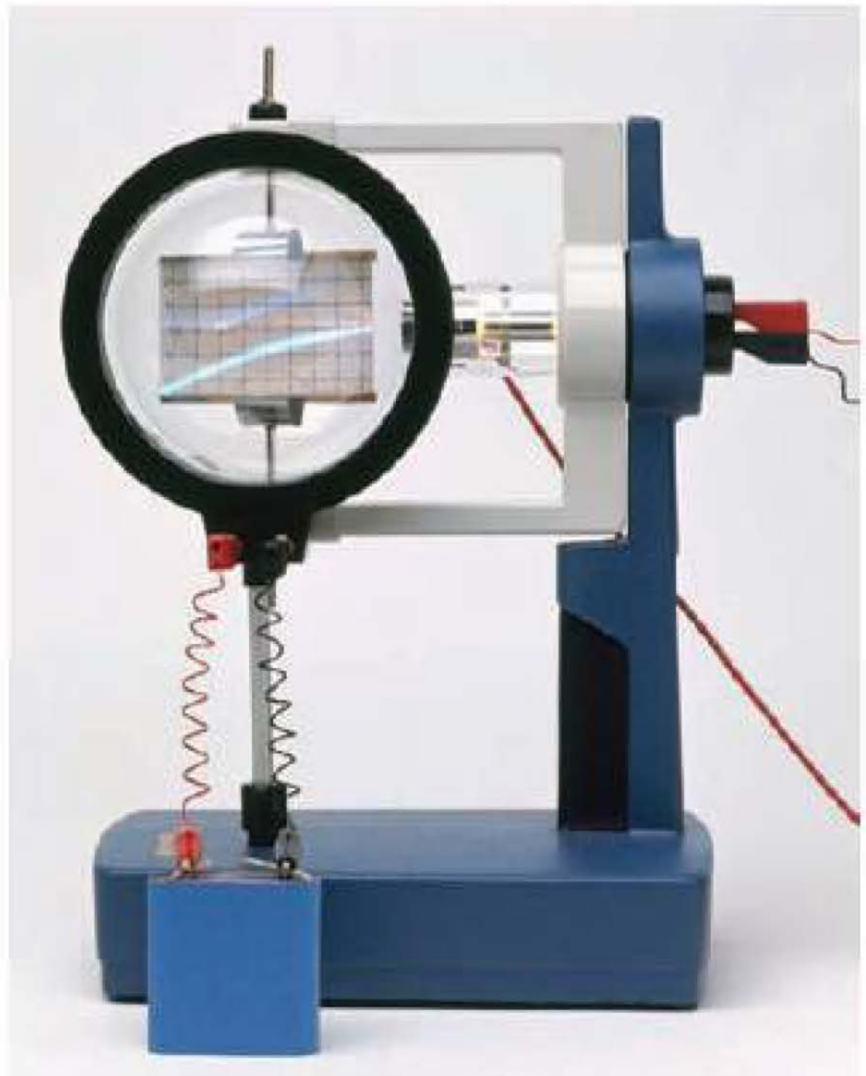


Figure 5.11 Electrons can be deflected and observed inside this evacuated tube.

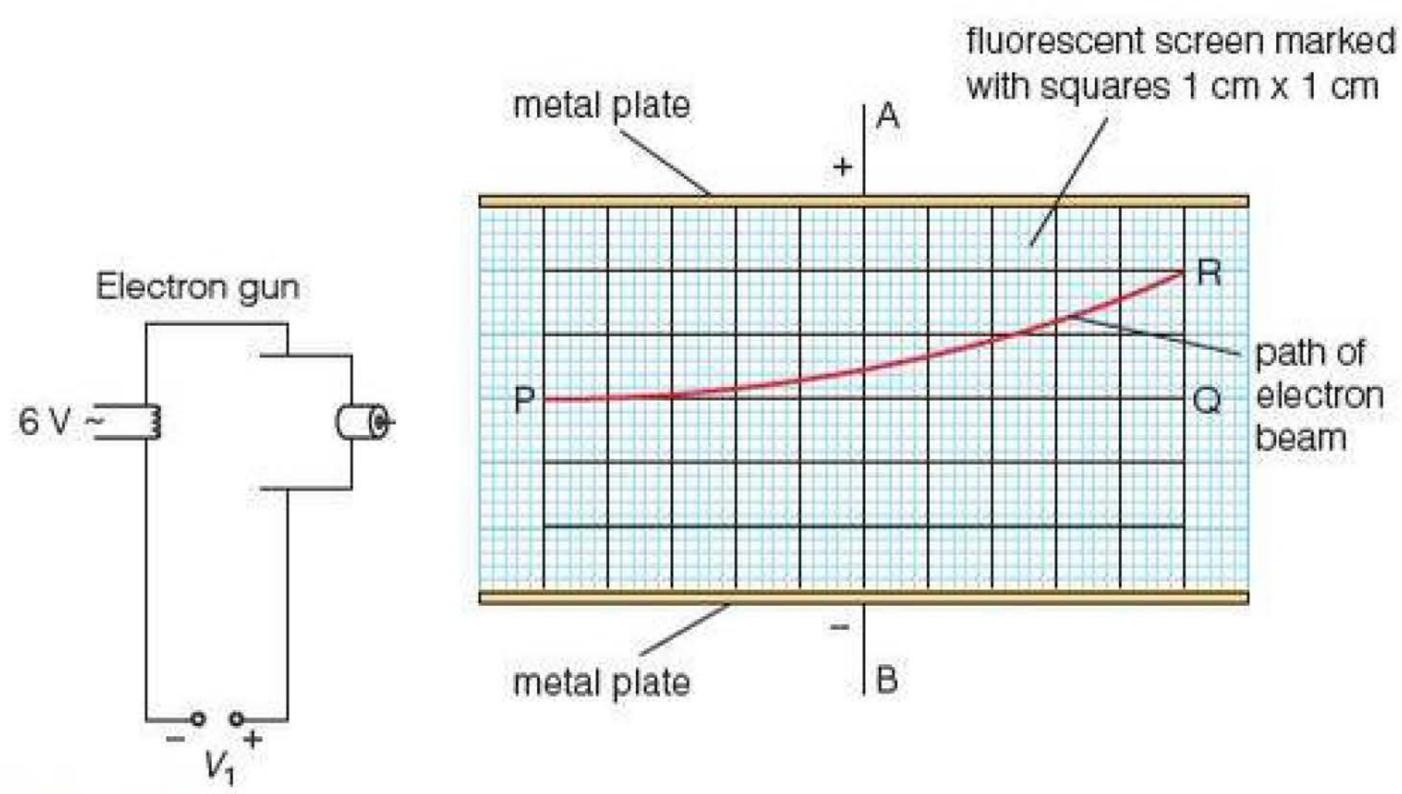


Figure 5.12

The path PR is a parabola, which can be explained as follows. The electrons are travelling in a vacuum, so their velocity in the direction PQ remains unchanged. While the electrons are in the electric field, they experience a constant acceleration upwards due to the electric field. This is rather like throwing a ball sideways – the ball's horizontal velocity remains constant, but gravity gives the ball a constant downwards acceleration. The ball falls along a parabolic path.

EXAMPLE

Electron beam tube

This example refers to Figure 5.12. In an experiment, a beam of electrons is directed along the line PQ. The electrons arrive at P with a velocity of $4.0 \times 10^7\,\mathrm{m\,s^{-1}}$ travelling in the direction PQ. The squares on the grid measure 1 cm × 1 cm.

1 Calculate the time taken for the electrons to travel from P to Q.

Answer

$$d = v \times t$$

$$t = \frac{d}{v} = \frac{0.1\,\mathrm{m}}{4.0\times10^{7}\,\mathrm{s}} = 2.5\times10^{-9}\,\mathrm{s}$$

Now a potential difference of 2200V is applied between A and B, so that the beam deflects upwards.

2 Calculate the acceleration of an electron in this electric field.

Answer

The electric field strength is

$$E = \frac{V}{d} = \frac{2200\,\mathrm{V}}{0.06\,\mathrm{m}} = 36.6\,\mathrm{kV\,m^{-1}}$$

The acceleration is given by

$$a = \frac{F}{m} = \frac{EQ}{m}$$

where Q is the charge on an electron and m is its mass. This gives

$$a = \frac{3.66\times10^{4}\,\mathrm{V\,m^{-1}}\times1.6\times10^{-19}\,\mathrm{C}}{9.1\times10^{-31}\,\mathrm{kg}} = 6.4\times10^{15}\,\mathrm{m\,s^{-2}}$$

3 Show that the electron beam is deflected upwards to point R, which is about 2 cm above point Q.

Answer

To calculate the upwards displacement of the beam, we use the equation of motion:

$$s = ut + \frac{1}{2}at^2 = \frac{1}{2}at^2$$

since the initial upward velocity u = 0. So

$$s = \left(\frac{1}{2}\times 6.4\times10^{15}\,\mathrm{m\,s^{-2}}\right)\times(2.5\times10^{-9}\,\mathrm{s})^2 = 0.02\,\mathrm{m}\text{ or }2\,\mathrm{cm}$$

TEST YOURSELF

7 The strength of an electric field may be expressed in units of either $\mathrm{N\,C^{-1}}$ or $\mathrm{V\,m^{-1}}$. By considering the definitions of the volt and the joule, show that these two quantities are the same.

8 Figure 5.13 shows a charged particle placed between two charged parallel plates. The potential difference between the plates is 1500V and their separation is 7.5 cm.

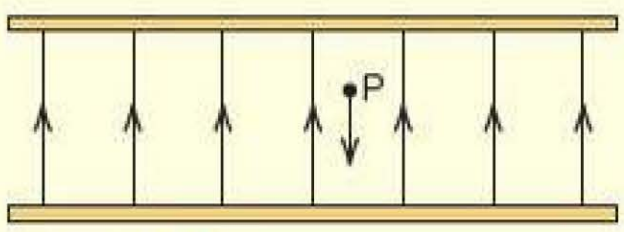


Figure 5.13

a) Calculate the magnitude of the electric field strength between the plates.

b) A particle P experiences a force, from the electric field, of $1.5 \times 10^{-7}\,\mathrm{N}$ in the direction shown. Calculate the charge on the particle.

c) Calculate the work done by the electric field on the particle in taking it from the top plate to the bottom place.

9 This question refers to the deflection of electrons shown in Figure 5.12. Explain what will happen to the path of the electrons when, separately,

a) the potential difference between A and B is increased

b) the potential difference V_1 is increased so that the electrons travel faster as they enter the deflecting area.

10 A small polystyrene ball is dropped between a pair of parallel plates as shown in Figure 5.14. As it enters the plates, it has reached its terminal speed, which is $1.0\,\mathrm{m\,s^{-1}}$. Initially, the plates are uncharged.

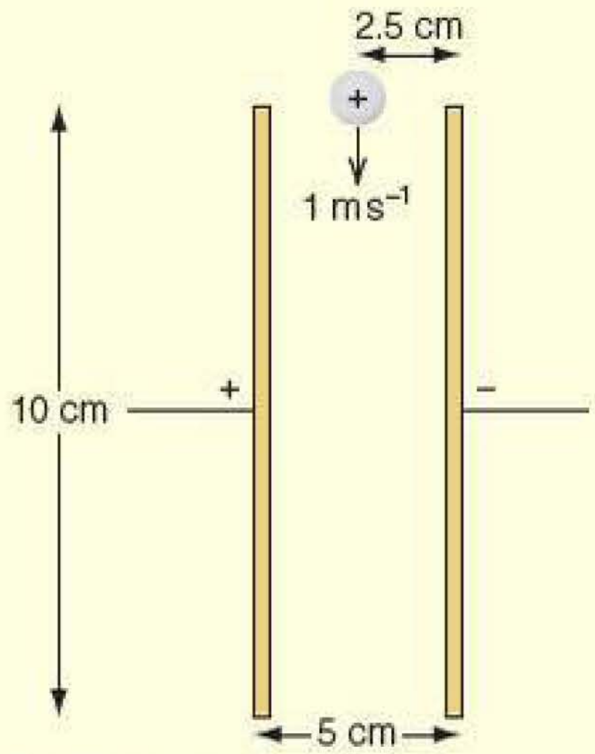


Figure 5.14

a) Explain why a falling object has a terminal speed. (You may need to refer back to Chapter 8 in Book 1.)

b) Calculate the time taken for the ball to fall through the plates.

The experiment is repeated when the plates are charged as shown in Figure 5.14, and the potential difference between them is 3000 V. In this second experiment, the ball has been given a positive charge of 1.2×10^{-8} C. It enters the plates at the top at its terminal velocity of $1.0\,\mathrm{m\,s^{-1}}$.

c) Sketch the path of the ball as it falls between the plates.

d) **i)** Calculate the electric field between the plates.

ii) Calculate the force on the ball due to the electric field.

e) The ball has a mass of 0.2 g. Calculate its sideways acceleration as it enters the field.

f) Calculate the ball's maximum sideways deflection in the field. Why might the deflection be less than this?

11 Calculate the potential difference between the points $r = 0$ and $r = 0.4$ m in Figure 5.10.

12 A small oil drop, of mass 1.7×10^{-15} kg, carries a negative charge. It is suspended between two parallel plates as shown in Figure 5.15. The p.d. between the plates is adjusted to 312 V so that the drop remains stationary.

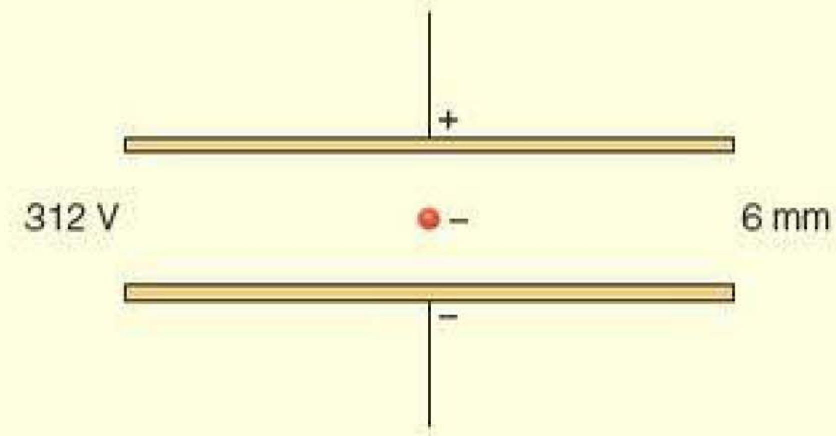


Figure 5.15

a) Draw a free-body diagram to show the forces on the drop.

b) Use the information above and in the diagram to calculate the charge on the drop.

c) The charge on the drop is changed by the use of ionising radiation. A p.d. of 208 V is now used to balance the drop. Calculate the charge on the drop now.

d) What is the smallest charge the drop can carry? For a drop that carries this charge, calculate the p.d. that would hold the drop in a stationary position.

e) Find out who first did this experiment to determine the charge on the electron.

Radial electric fields

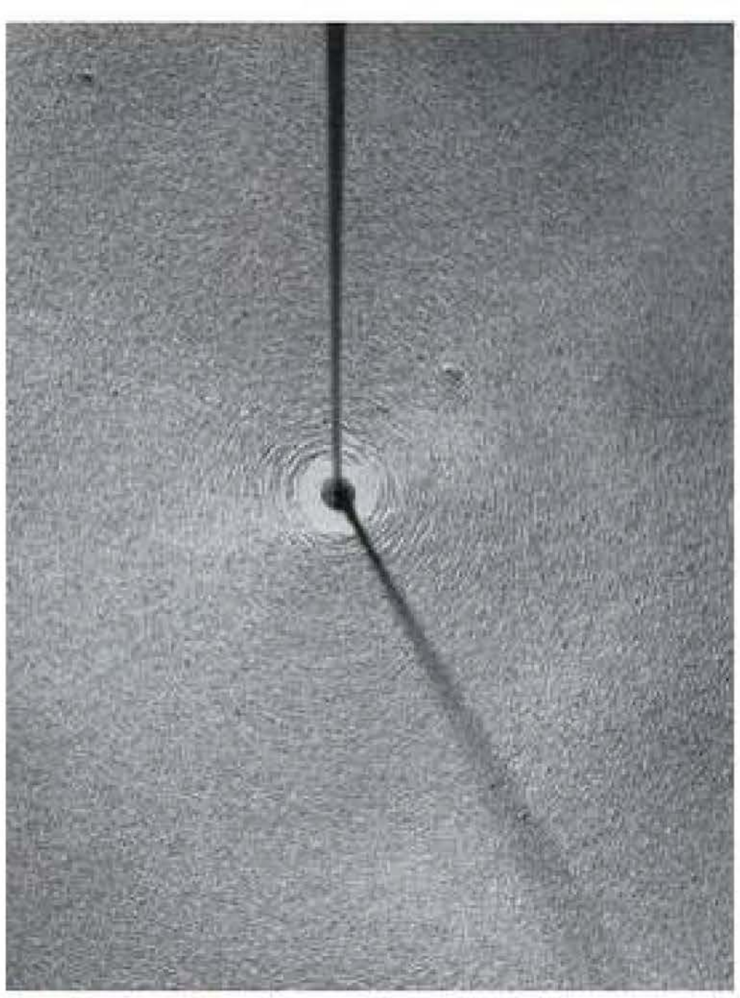


Figure 5.16

Figure 5.16 shows a photograph of the shape of an electric field close to a small point charge. This photograph is obtained in the same way as that shown in Figure 5.7. The electric field has a symmetrical radial shape near to a small point charge.

Figure 5.17 shows how we can represent the electric field lines close to a positively charged sphere. The lines point outwards symmetrically from the sphere as if they had come from the centre of the sphere. You can also see that the lines spread out. This means that the field gets weaker as the distance increases from the sphere. This is very similar to the shape of the gravitational field near to a planet, except that the gravitational field lines must always point towards the planet. (Under what circumstances do the field lines point towards a charged sphere?)

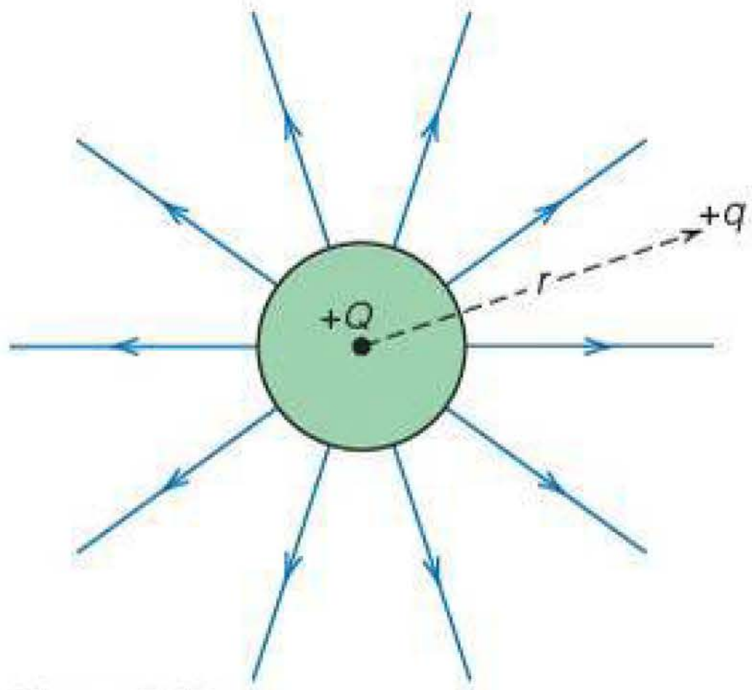


Figure 5.17

We can produce a formula for the electric field close to a sphere as follows. We know from Coulomb's law that the force between a sphere, carrying charge Q, and a small charge q at a distance r from the centre is

$$F = \frac{Qq}{4\pi\varepsilon_0 r^2}$$

We also know that $F = Eq$. It follows that the electric field close to the sphere is given by the formula

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

So the strength of the electric field obeys an inverse square law.

As you can see from Figure 5.17, the electric field is a vector quantity. So when we consider the field close to two or more point charges, we must take account of the direction of the electric field.

EXAMPLE

Resultant electric fields

Figure 5.18 shows two small charges, one with a charge of $+4Q$, the other with charge $+Q$. The magnitude of the electric field at C due to the charge $+4Q$ is $40\,\mathrm{N\,C^{-1}}$.

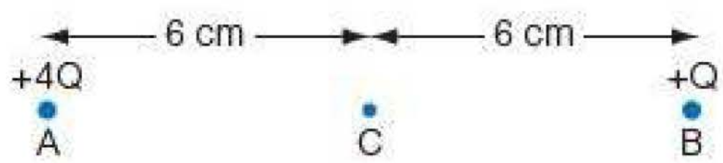


Figure 5.18

1 Calculate the magnitude of the electric field at C due to the charge $+Q$ alone.

Answer

The field at C due to the charge +Q is $10\,\mathrm{N\,C^{-1}}$, because the charge is $\frac{1}{4}$. But the field is in the opposite direction (right to left).

2 Calculate the magnitude of the electric field at C due to the two charges together.

Answer

The resultant field is $40\,\mathrm{N\,C^{-1}} - 10\,\mathrm{N\,C^{-1}} = 30\,\mathrm{N\,C^{-1}}$ to the right.

3 Show that the position of point O, where the electric field is zero along the line AB, lies 4 cm from B.

Answer

The field due to A is

$$E_A = \frac{k \times 4Q}{0.08^2} = \frac{kQ}{0.0016} \quad \text{where } k = \frac{1}{4\pi\varepsilon_0}$$

The field due to B is

$$E_B = \frac{kQ}{0.04^2} = \frac{kQ}{0.0016}$$

So at a point 4 cm from B the two fields cancel each other out.

TEST YOURSELF

13 a) The electric field due to a point charge is $300\,\mathrm{N\,C^{-1}}$ at a distance of 100 mm away from it. Calculate the strength of the field at distances of

i) 50 mm
ii) 200 mm
iii) 250 mm.

b) Sketch a graph to show how the field strength varies with distance away from the charge.

14 Figure 5.19 shows a grid marked with squares. A positive charge placed at O produces an electric field of strength 3600 $N\,C^{-1}$ at point A.

Calculate the magnitude of the electric field strength for each of the points B to K. (You will need to use a combination of Pythagoras's theorem and the inverse square law.)

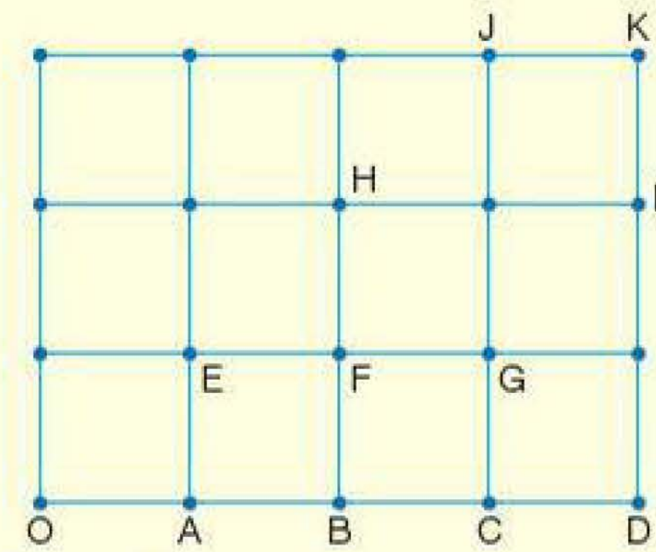


Figure 5.19

15 a) Figure 5.20 shows two charged spheres, X and Z. Calculate the electric field strength at point Y, which lies along the line XZ joining the centres of the two charged spheres. Sphere X has a charge of $+5 \times 10^{-9}$ C, and sphere Z a charge of -10^{-8} C.

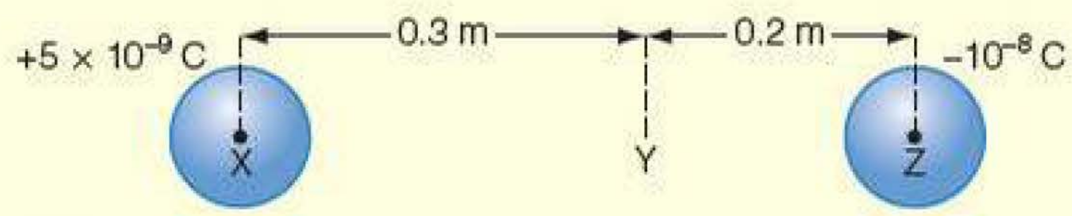


Figure 5.20

b) i) Calculate the field strength, at Y, if the charge of -10^{-8} C on sphere Z is replaced with a charge of $+10^{-8}$ C.

ii) Calculate the force between the spheres in this case. Is the force attractive or repulsive?

Electrical potential

Figure 5.21 shows a small charged isolated sphere. In theory, an isolated sphere should be in contact with nothing and infinitely far away from anything else. In practice, in a laboratory, the best we can do is to suspend a sphere by a fine, insulating thread and make sure the sphere is a few metres from everything else.

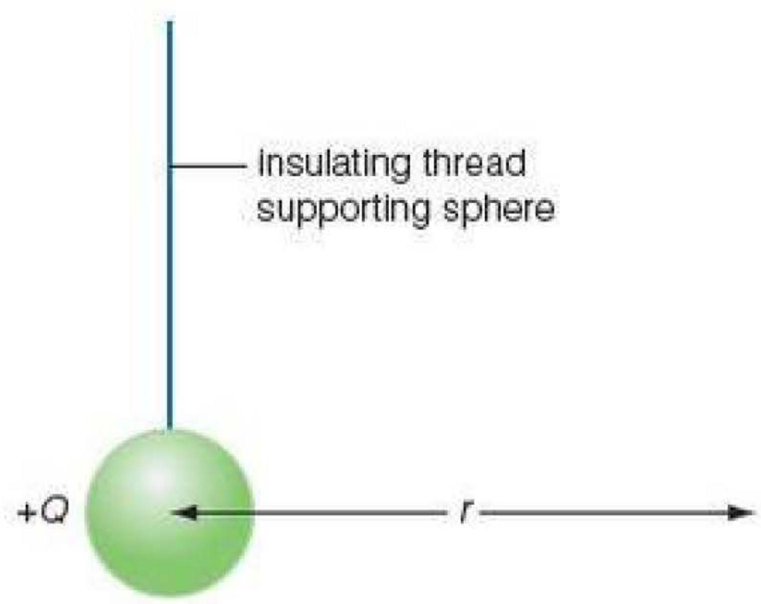


Figure 5.21

The potential at a distance r from the centre of a sphere carrying a charge Q is given by

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

Note that if the charge is negative, then the potential near to the charged sphere is also negative.

These formulae are very similar to the formula for gravitational potential – except that gravitational potential must always be negative, whereas an electrical potential, close to a charge, can be positive or negative. In the case of gravitational potential, the zero point of potential is defined as a point infinitely away from any planet or star. In a similar way, the zero point of electrical potential is defined as a point infinitely far away from the charged sphere. In practice, the surface of the Earth is our reference point of zero potential, and by suspending our sphere a long way from anything else, the Earth can be treated as being infinitely far away. The Maths box shows how the formula for potential can be derived from the formula for electric field.

MATHS BOX

The electric field at a distance r from the centre of an isolated charged sphere carrying a charge +Q is

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

But

$$E = -\frac{dV}{dr}$$

so

$$V = -\int_{\infty}^{r} \frac{Q}{4\pi\varepsilon_0 r^2} dr$$
$$= \frac{Q}{4\pi\varepsilon_0 r}$$

Note that the limits of the integration set the potential as zero at an infinite distance.

Absolute electric potential The potential difference between a point and a point at zero potential, which is infinitely far away.

The formula $V = \frac{Q}{4\pi\varepsilon_0 r}$ helps us to define **absolute electric potential**. The absolute electric potential at a point r from a charge $+Q$ is the work done per unit positive charge in moving it from ∞ to that point. Note that if that charge is $-Q$, then the potential is negative and the electric field does work in moving a positive charge closer to the point r.

EXAMPLE

Electric field near a charged sphere

Figure 5.22 shows a metal sphere of radius 10 cm, charged to a potential of 1000 V. The electric field strength at C is 10 000 V m^{-1}.

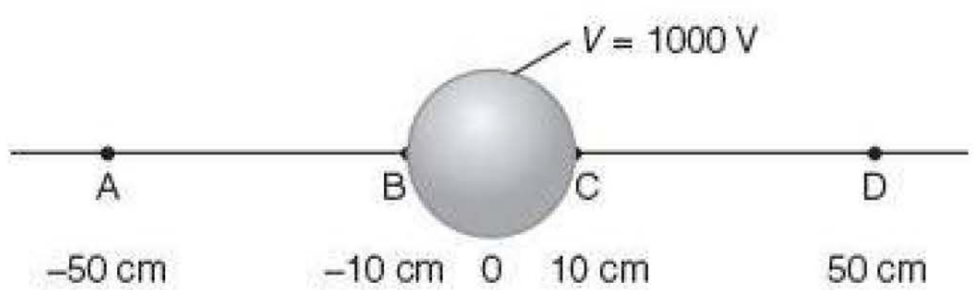


Figure 5.22

Sketch graphs to show how

1 the potential
2 the field strength

vary along the line A to B and then from C to D.

Answer

Figure 5.23 shows the answer. These are the points to note.

- V is a scalar and is always positive.
- V obeys a $\frac{1}{r}$ law and falls from 1000 V at 10 cm to 200 V at a distance of 50 cm from the sphere.
- E is a vector, so must change direction.
- E is connected to the potential by the equation $E = -\frac{dV}{dr}$. On the right-hand

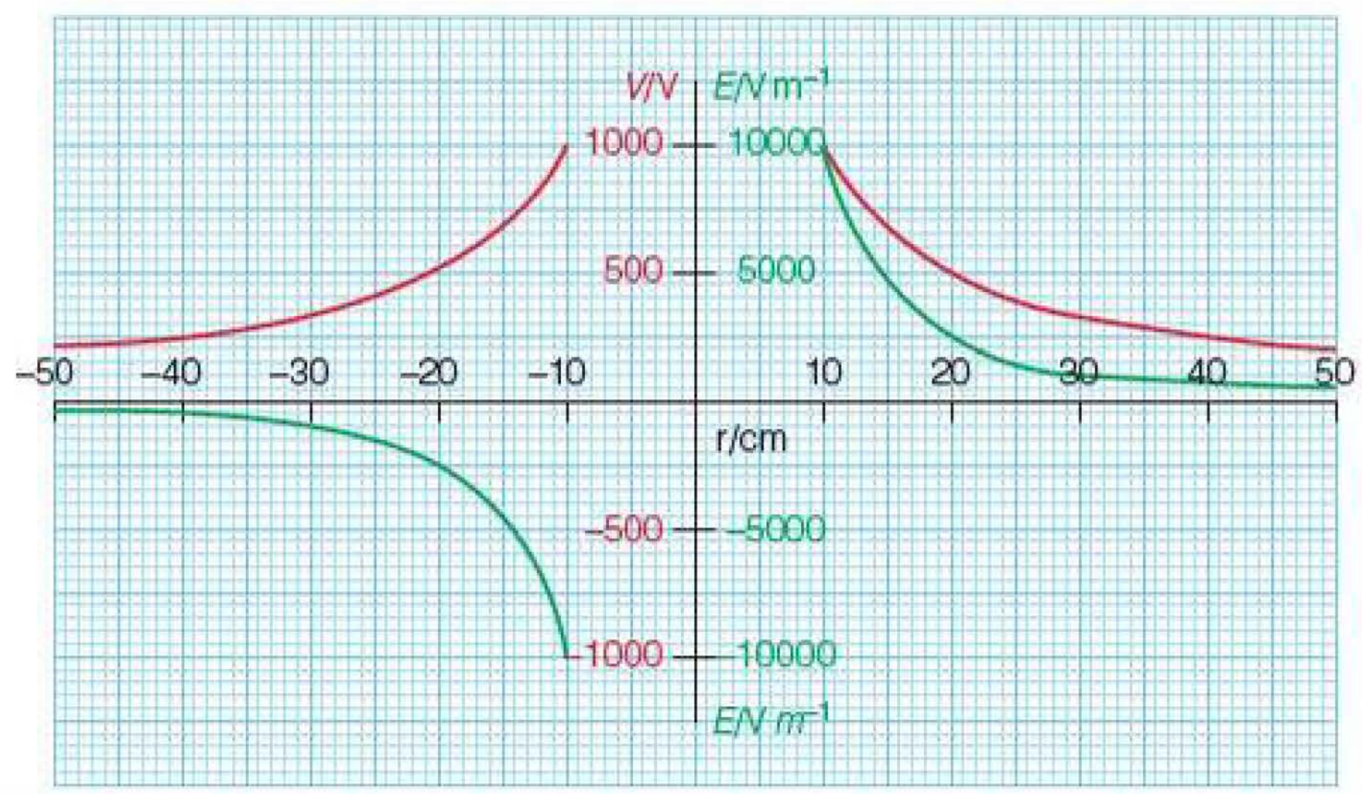


Figure 5.23

side of the sphere, the potential gradient is negative, so E is a positive quantity. On the left-hand side of the sphere, the potential gradient is positive, so E is a negative quantity.
(More simply, the electric field must be in opposite directions on either side of the sphere, because a positive charge is always repelled by it.)

- E obeys a $\frac{1}{r^2}$ law and falls from 10 000 V m^{-1} at 10 cm to 400 V m^{-1} at a distance of 50 cm from the sphere.

Potential difference The work done, against an electric field, in moving unit charge from one point to a second point at a higher potential. If a charge moves from a point of higher potential to a lower potential, work is done by the electric field.

Potential difference

Electric **potential difference** is the difference in electrical potential between two points. When the potential difference is ΔV, the work done in moving a charge Q between the two points is

$$\Delta W = Q\Delta V$$

EXAMPLE

Work done in moving a charge

How much work is done in taking a charge of 1.00×10^{-9} C from the position r_2 to r_1, in Figure 5.24?

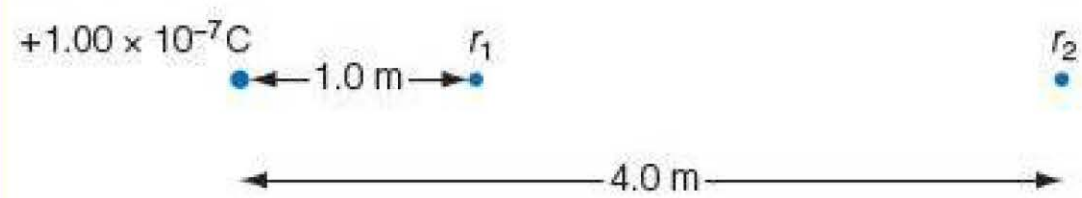


Figure 5.24

Answer

The potential at a point is

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

so the potential difference between r_1 and r_2 is

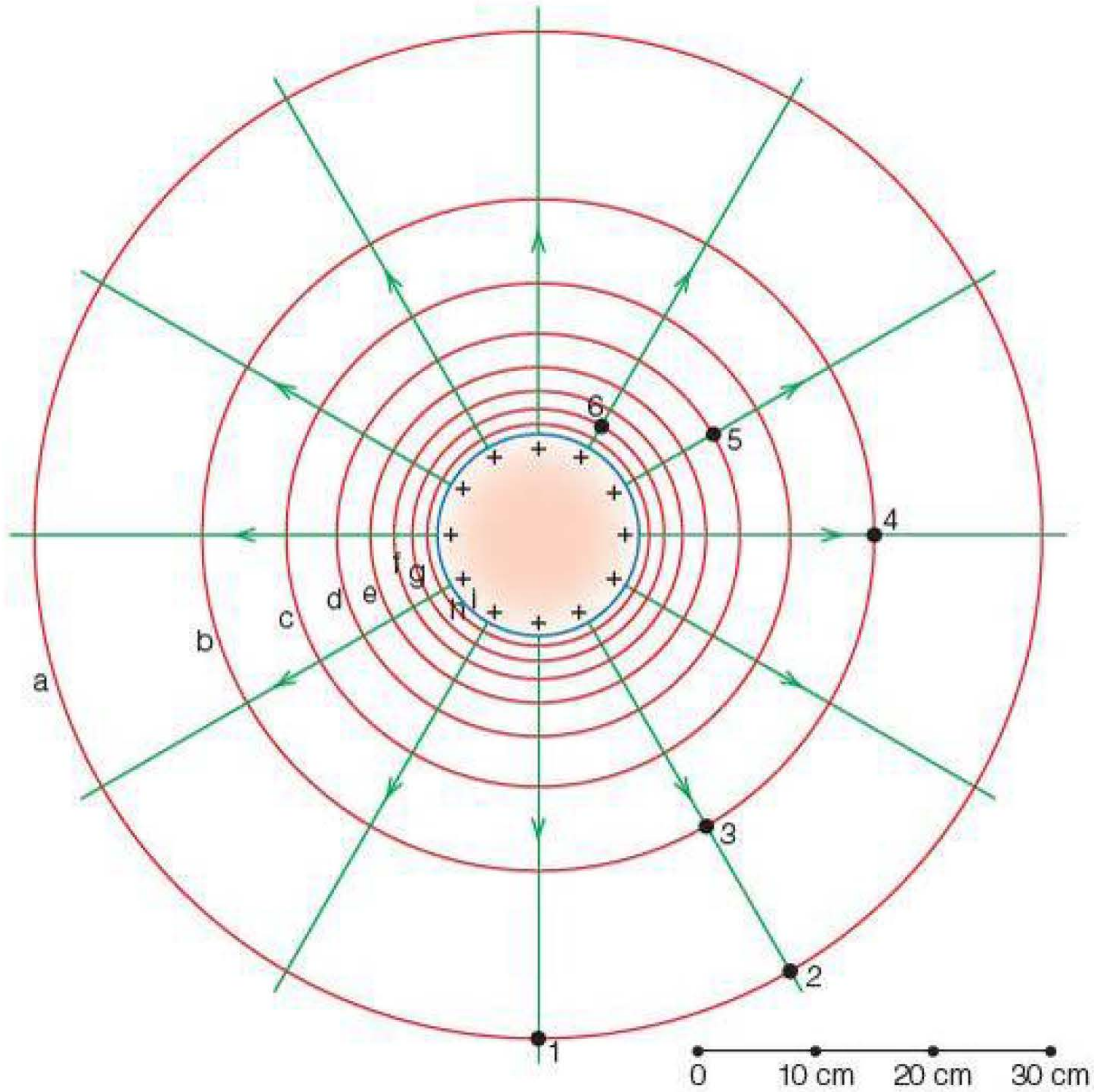
$$\Delta V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$
$$= \frac{1.00 \times 10^{-7}\,\text{C}}{4\pi \times 8.85 \times 10^{-12}\,\text{F m}^{-1}}\left[\frac{1}{1\,\text{m}} - \frac{1}{4\,\text{m}}\right]$$
$$= 675\,\text{V}$$

So the work done is

$$\Delta W = Q\Delta V$$
$$= 1.00 \times 10^{-9}\,\text{C} \times 675\,\text{V}$$
$$= 675\,\text{nJ}$$

Equipotential surfaces

Figure 5.25 shows a metal sphere that is charged to a potential of 1000 V. The red circles drawn round the sphere show equipotential surfaces, where the potential is the same, e.g. 900 V and 800 V. Although the diagram shows circles, this is because the diagram can only be drawn in two dimensions. The sphere is surrounded by spherical equipotential surfaces.



Position	Potential
a	200 V
b	300 V
c	400 V
d	500 V
e	600 V
f	700 V
g	800 V
h	900 V
i	1000 V

Figure 5.25

The equipotential surfaces may be linked to the electric field strength through the equation

$$E = -\frac{\Delta V}{\Delta r}$$

Figure 5.25 shows that, near the surface of the sphere, the equipotential surfaces are closer together. This means that both the potential gradient and the electric field strength are higher near the sphere's surface than they are further away. The green lines in the diagram represent electric field lines, which point radially away from the positive sphere. These lines get further apart with distance from the sphere, which also shows a field diminishing with distance away from the sphere's centre.

The field lines are always at right angles to the equipotential surfaces. So, when a charged particle moves along an equipotential surface, no work is done by the electric field. This can be explained using two ideas.

First, work done is defined by

$$\Delta W = \Delta V Q$$

or, in words, the work done on a charge is the change in potential multiplied by the charge. When $\Delta V = 0$ (moving along an equipotential surface) the work done is zero.

Secondly, work is also defined by

$$\Delta W = F\Delta r$$

or, in words, the work done on an object is the force acting on the object multiplied by the distance moved in the *direction* of the force. When a charge is moved along an equipotential, it does not move in the direction of the force or the electric field, which acts on it, but at right angles to the force.

EXAMPLE

Moving a charge on equipotential surfaces

In Figure 5.25, a charge of 1.0×10^{-6} C is moved first from point 1 to point 2, and then from point 2 to point 3. How much work is done in each case?

Answer

No work is done in moving the charge from point 1 to point 2, that is $\Delta V = 0$.

In moving from point 2 to point 3

$$\Delta W = Q\Delta V$$
$$= 1.0 \times 10^{-6}\ \text{C} \times (300\ \text{V} - 200\ \text{V})$$
$$= 1.0 \times 10^{-4}\ \text{J}$$

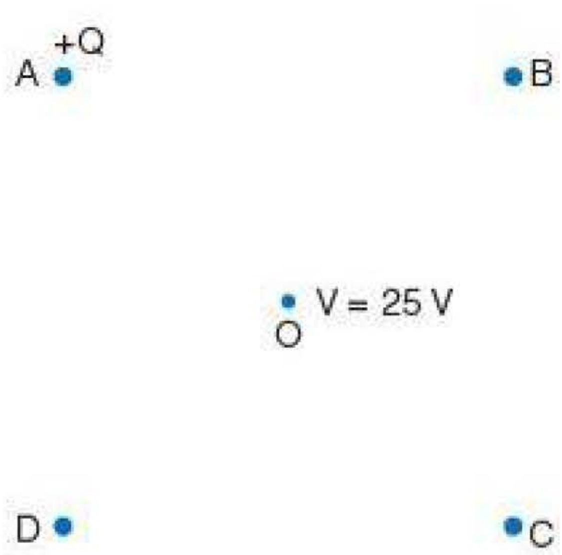


Figure 5.26

Further examples of field and potential

Figure 5.26 shows a square ABCD. At point A there is a charge $+Q$; the potential at point O (the centre of the square) due to this charge is 25 V. What is the potential at O when a charge of $+Q$ is placed at each of the points A, B, C and D?

The answer to this is 100 V. To move a positive charge to point O, work has to be done against each of the four charges. Therefore the potential at O is four times larger. (The potential at a point is the work done per unit charge to take it from infinity to that point.)

Since electric potential is a scalar quantity, we can calculate the potential at a point close to two or more charges by adding the potential due to each charge. Figure 5.27 shows the electric potential (calculated by computer) near to a positive and negative ion, which are separated by a distance of 10^{-9} m.

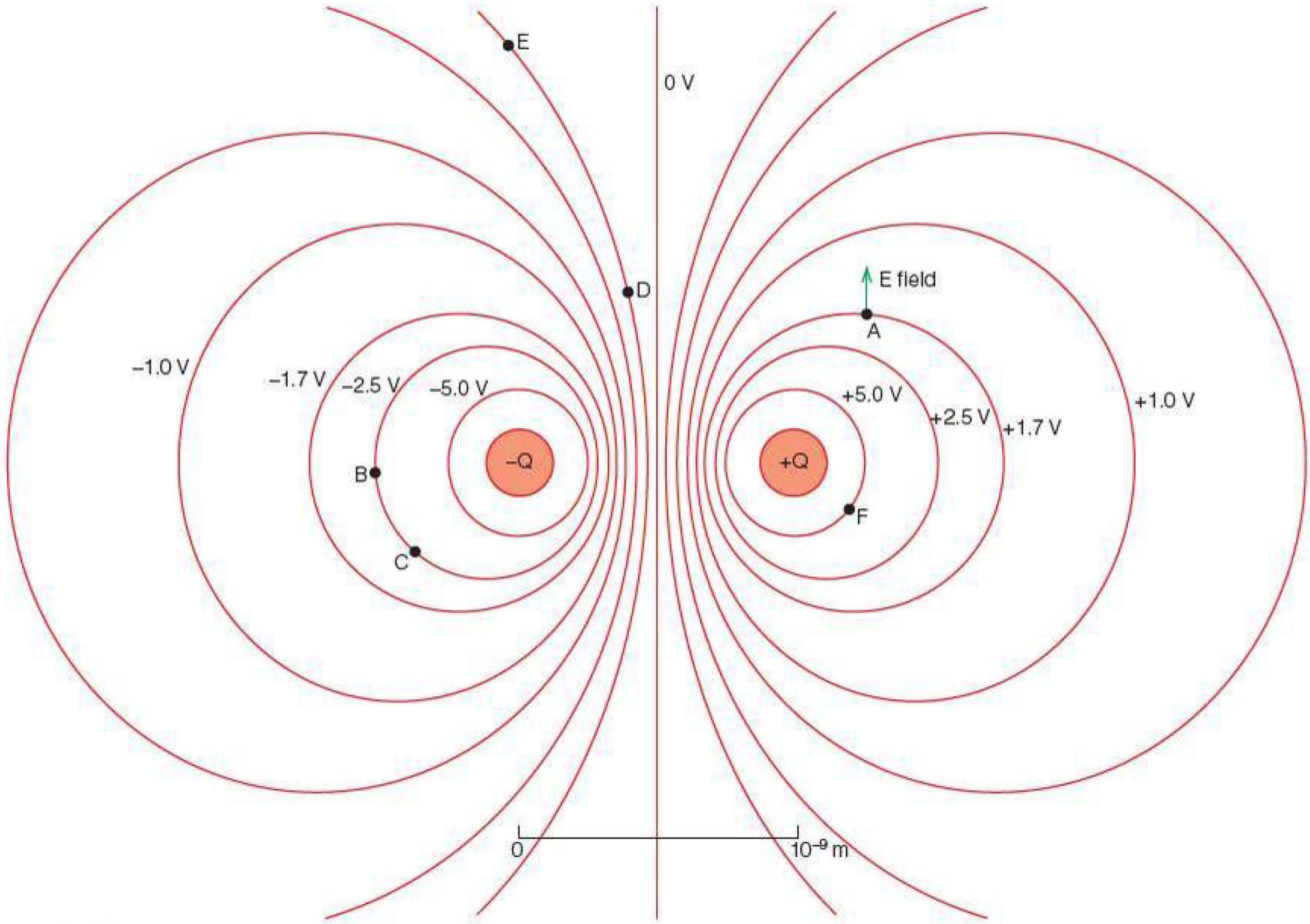


Figure 5.27

TEST YOURSELF

16 An isolated metal sphere is charged with a positive charge of 1.5×10^{-7} C.

a) i) Calculate the potential at point A, a distance of 0.25 m from the sphere's centre.

ii) Calculate the potential at point B, a distance of 0.75 m from the sphere's centre.

b) Calculate the work done in moving a charge of 2.0×10^{-8} C from B to A.

17 This question refers to Figure 5.25. Calculate the work done in moving a positive charge of 2.0×10^{-7} C from

a) point 3 to point 4

b) point 4 to point 5

c) point 3 to point 6.

18 This question refers to possible arrangements of charges in Figure 5.26.

a) Calculate the potential at O, for each of the following arrangements of charges:

i) A, $+Q$; B, $+Q$; C, $-Q$; D, $+Q$

ii) A, $+Q$; B, $-Q$; C, $+Q$; D, $-Q$

iii) A, $+2Q$; B, $-3Q$; C, $+Q$; D, $-Q$.

b) For which of the above arrangements is the electric field strength zero at O? Explain your answer.

19 An isolated charge of $+Q$ is placed at point A in Figure 5.28. The potential at point B is 120 V.

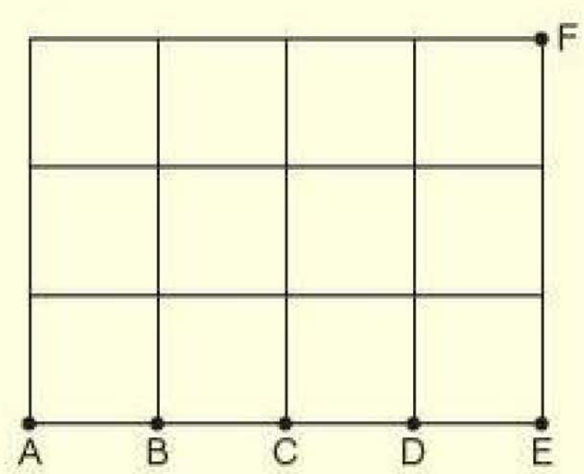


Figure 5.28

⇨

a) Calculate the potential at points
   i) C
   ii) D
   iii) E
   iv) F.

b) A second charge of $+Q$ is now placed at E. Calculate the potential at points
   i) B
   ii) C
   iii) D
   iv) F.

c) The charge at E is now replaced with a charge of $-Q$. Calculate the potential at points
   i) B
   ii) C
   iii) D
   iv) F.

20 This question refers to the equipotential surfaces shown near to the ions in Figure 5.27.

a) Which of the following statements is/are true? Explain your reasoning.
   i) The direction of the electric field at point A is correctly drawn.
   ii) The strength of the electric field is stronger at D than at E.

b) Calculate the gain in electrical potential energy when an electron is moved from
   i) B to C
   ii) B to F.

   Express your answers in eV.

c) Sketch the shape of the electric field close to the two ions.

Similarities between electricity and gravitation

A comparison between Newton's law of gravitation and Coulomb's law shows that there are many similarities between the actions of the two types of force. There are also some important differences.

The main similarities are as follows:

- Both electric and gravitational forces are non-contact; forces are exerted over a distance without direct contact.
- Both forces are of infinite range.
- Both forces obey inverse square laws.

The main differences are as follows:

- Gravitational forces between masses are always attractive; electric forces can be attractive or repulsive.
- An electric force is much stronger than the gravitational force.
- It is possible to shield an electric force, but the gravitational force acts on all objects.
- An electric force only acts on charged objects; the gravitational force acts on all objects.

Table 5.2 shows a useful summary of the similarities and differences.

Table 5.2

Characteristic	Gravitation	Electricity
Acts on	Mass (positive only)	Charge (positive or negative)
Force (N)	$F = \frac{GMm}{r^2}$	$F = \frac{Q_1Q_2}{4\pi\varepsilon_0 r^2}$
	attractive only	attractive or repulsive
	infinite range	infinite range
relative strength	1	10^{36}
Field	$g = \frac{F}{m}$	$E = \frac{F}{Q}$
units	$N\,kg^{-1}$	$N\,C^{-1}$ or $V\,m^{-1}$
radial field	$g = \frac{GM}{r^2}$	$E = \frac{Q}{4\pi\varepsilon_0 r^2}$
Potential difference	$\Delta V = \frac{\Delta W}{m}$	$\Delta V = \frac{\Delta W}{Q}$
units	$J\,kg^{-1}$	$J\,C^{-1}$
Potential gradient	$g = -\frac{\Delta V}{\Delta r}$	$E = -\frac{\Delta V}{\Delta r}$
Potential in radial fields	$V = -\frac{GM}{r}$	$V = \frac{Q}{4\pi\varepsilon_0 r}$
	$V = 0$ at ∞	$V = 0$ at ∞
Potential energy	$E_p = \frac{-GMm}{r}$	$E_p = \frac{Q_1Q_2}{4\pi\varepsilon_0 r}$

TEST YOURSELF

21 a) Calculate the gravitational force between two protons separated by a distance of 10^{-10} m.

b) Calculate the electrostatic force between the protons separated by a distance of 10^{-10} m.

c) Calculate the ratio of the two forces calculated above.

d) What will the ratio of the forces be when the protons are separated by 10^{-12} m?

(Look up data for these calculations.)

22 The strong nuclear force binds nucleons together in a nucleus. It is thought that the force acts over a range of about 10^{-15} m, and that the force is 137 times stronger than the electric force. Comment on the information in the previous sentence. Does it make sense?

23 Explain how it is possible to shield a region from an electric field.

Practice questions

1 A proton and an electron are separated by a distance of 5×10^{-11} m. The size of the electrostatic force between them is

A 18×10^{-8} N **C** 6×10^{-8} N

B 9×10^{-8} N **D** 1.8×10^{-8} N

2 The potential energy of a proton–electron pair separated by 5×10^{-11} m is

A 288 eV **C** 28.8 eV

B 92.2 eV **D** 5.6 eV

3 An electron starts at rest and is accelerated through a potential difference of 1200 V. The speed of the electron after it has accelerated though this p.d. is

A $8 \times 10^{6}\,\mathrm{m\,s^{-1}}$ **C** $4 \times 10^{7}\,\mathrm{m\,s^{-1}}$

B $2 \times 10^{6}\,\mathrm{m\,s^{-1}}$ **D** $2 \times 10^{7}\,\mathrm{m\,s^{-1}}$

4 On a dry day the electric field near the surface of the Earth is $140\,\mathrm{V\,m^{-1}}$ downwards. A drop of water of mass 4.2 mg is suspended in the electric field. The charge that the drop carries is

A 3×10^{-4} C **C** 6×10^{-7} C

B -3×10^{-7} C **D** -3×10^{-4} C

Use this information to answer questions 5 and 6:

Figure 5.29 shows two equal positive charges that are placed at B and C, along a line AD. Four graphs show the possible variation of quantities along the line AD.

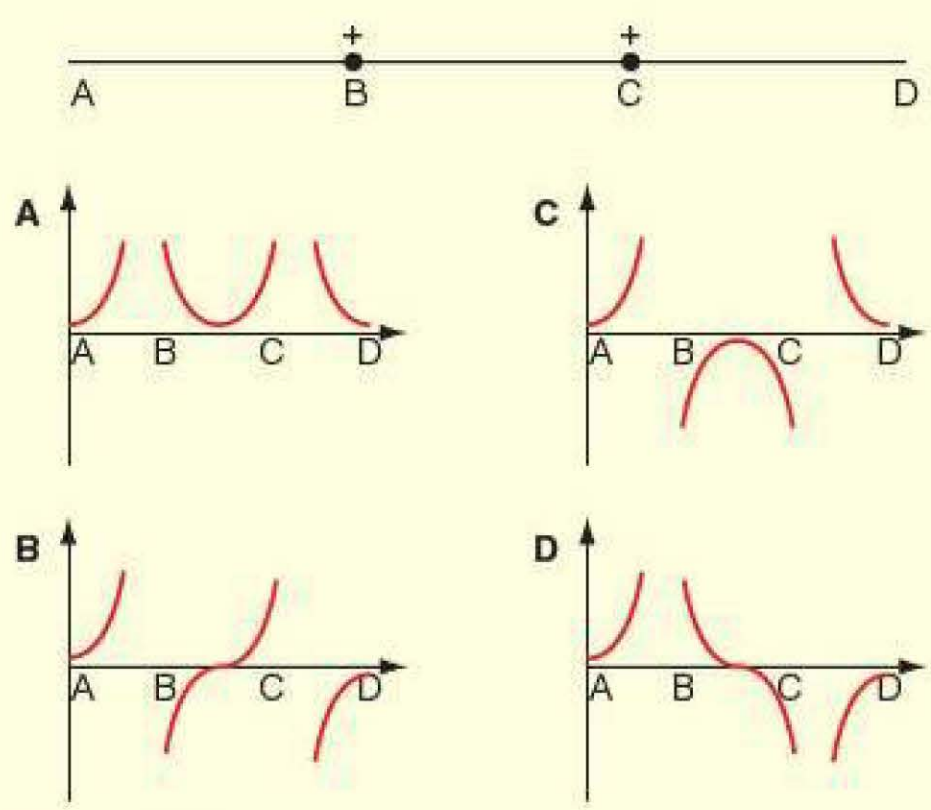


Figure 5.29

5 Which graph shows the variation of electric field along AD?

6 Which graph shows the variation of electric potential along AD?

Use this information to answer questions 7 and 8:

The charge at C is now replaced with a negative charge of the same size.

7 Which graph now shows the variation of electric field along AD?

8 Which graph now shows the variation of electric potential along AD?

9 Figure 5.30 shows a series of equipotentials. Which of the following statements is not true?

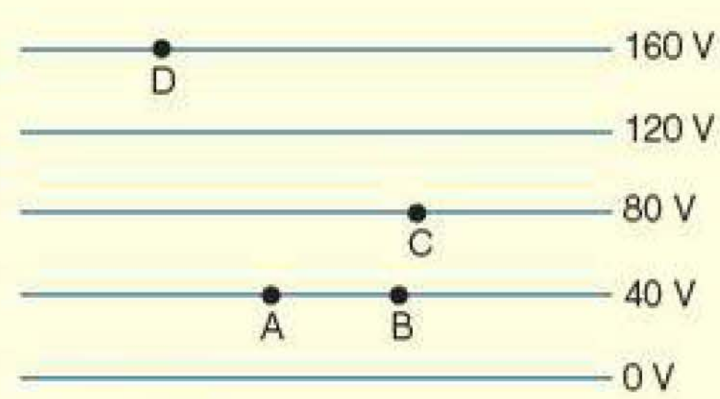


Figure 5.30

A The work done in taking a charge of 0.1 C from A to B is zero.

B The work done in taking a charge of 0.1 C from A to C is 4 J.

C The electric field at D is stronger than the electric field at B.

D The direction of the electric field is downwards.

10 The electric field strength at a distance of 10 cm from the surface of a metal sphere is 900 N C^{-1}. The sphere has a radius of 20 cm. What is the electric field strength at a distance of 70 cm from the surface of the sphere?

A 450 N C^{-1}

B 225 N C^{-1}

C 100 N C^{-1}

D 50 N C^{-1}

11 In Figure 5.31(a) an electron is placed at P in an electric field, which is represented by the field lines shown.

a) i) In which direction will the electron accelerate? *(1)*

ii) Describe how the electron's acceleration changes with its position in the field. Explain your answer. *(2)*

b) An electron is now placed in another electric field, at Q, as shown in Figure 5.31(b).

i) Describe how the electron's acceleration changes with its position in the field now. *(1)*

ii) The electron, at Q, is replaced by a proton. Compare the proton's acceleration with the electron's acceleration. *(3)*

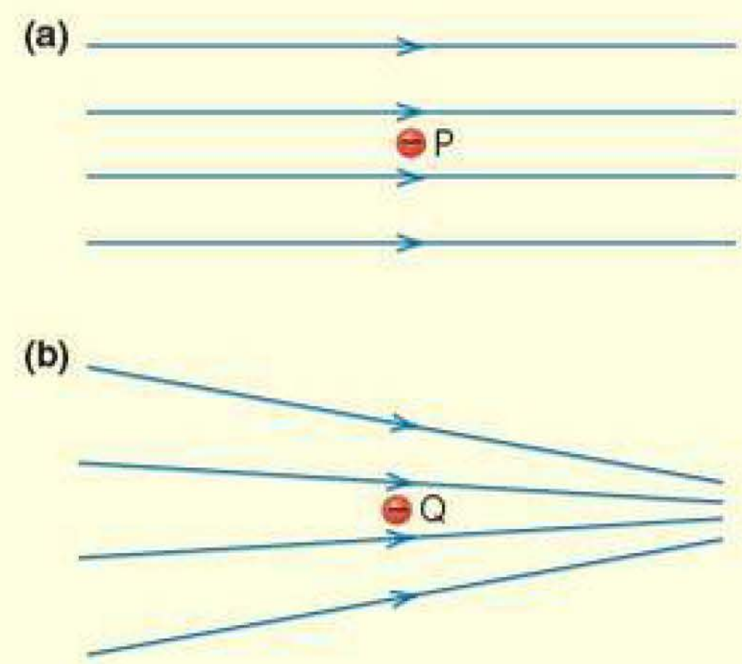


Figure 5.31

12 Figure 5.32 shows two parallel plates, which are connected to a low-voltage supply. The plates are in a region where there is a vacuum. A small polystyrene sphere is placed at X between the plates. The sphere carries an electric charge of $+4.0 \times 10^{-18}$ C, and it has a mass of 2.6×10^{-15} kg.

a) Calculate the size of the electric force acting on the sphere. *(3)*

b) Draw a free-body diagram to show the forces acting on the sphere. *(2)*

c) Calculate the magnitude and direction of the sphere's acceleration after its release. *(4)*

d) A different sphere is now introduced into the field at point X. It carries twice as much charge as the first sphere, and it is twice as massive. Compare the magnitude and direction of this second sphere's acceleration with the first sphere. *(2)*

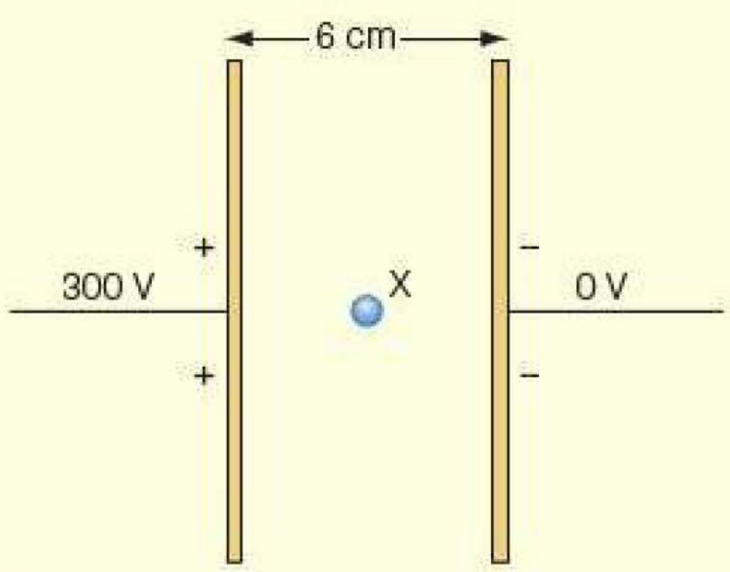


Figure 5.32

13 A small plastic ball is suspended on a fine glass spring as shown in Figure 5.33. It carries a negative electric charge. When a potential difference of 500 V is applied to the plates, the ball moves upwards by a deflection of 9 mm. The spring constant of the spring is 0.12 N m^{-1}.

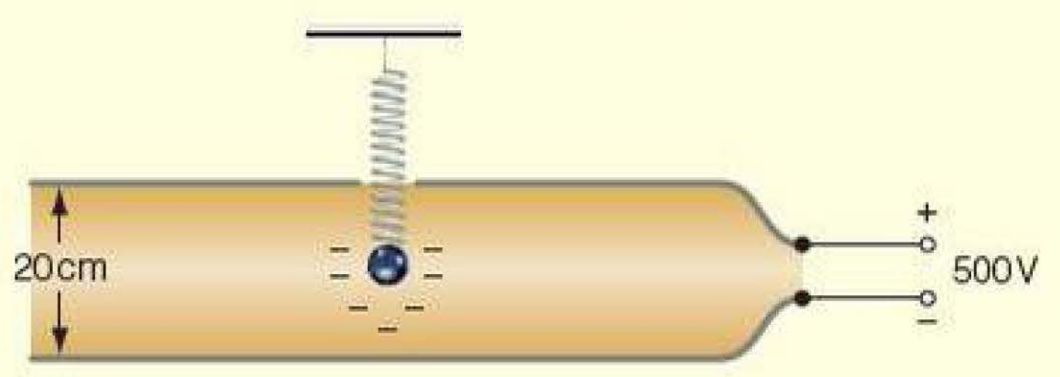


Figure 5.33

a) Use the deflection of the ball to calculate the electrostatic the force acting on it. *(2)*

b) Use the information in the diagram to calculate the electric field strength. *(2)*

c) Deduce the charge on the sphere. (2)

d) The electric field is now switched off. Explain why the sphere oscillates with simple harmonic motion. Calculate the time period of the motion; the mass of the sphere is 15 g. (3)

(You will only be able to do this part of the question if you have studied simple harmonic motion in Chapter 2.)

14 A small sphere of mass 2.4 g is charged and suspended in an electric field (Figure 5.34). It is deflected from the vertical at an angle of 10°.

a) Use the information in Figure 5.34 to calculate the strength of the electric field between the plates. (1)

b) Show that the force due to the electric field acting on the sphere is about 4×10^{-3} N. (3)

c) Calculate the charge on the sphere. (2)

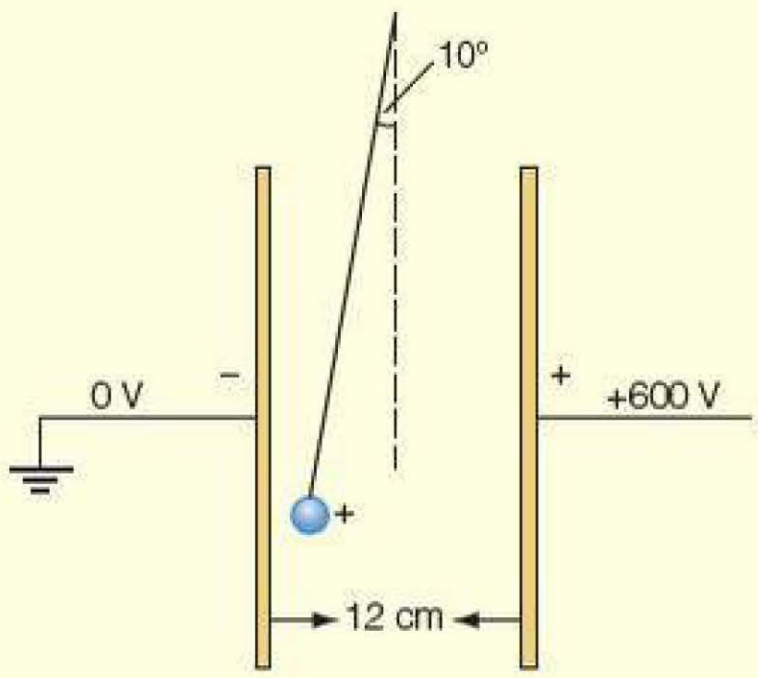


Figure 5.34

15 Figure 5.35 shows the arrangement of two protons that form a hydrogen molecule. They are separated by a distance of 3.0×10^{-10} m.

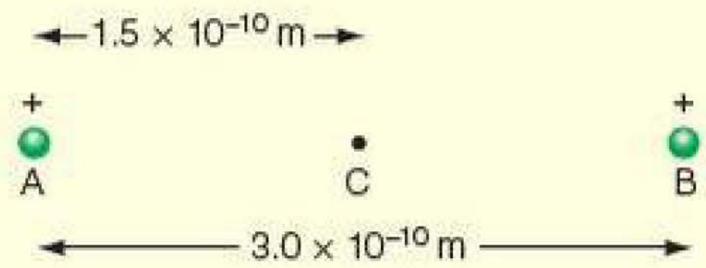


Figure 5.35

a) What is the electric field strength at a point C, midway between the two protons? (1)

b) i) Show that the electric potential at point C, due to the proton at A only, is 9.6 V. (3)

ii) State the potential at point C due to both protons at A and B. (1)

c) An electron, in its ground state, has 3.7 eV of kinetic energy at point C. Show that the total energy of the electron at this point is –15.5 eV. (2)

d) State the ionisation energy of the hydrogen molecule. (1)

16 In Figure 5.36 two charges are placed at points A and B, which are 1.0 m apart. At A there is a charge of +6 nC, and at B a charge of –6 nC.

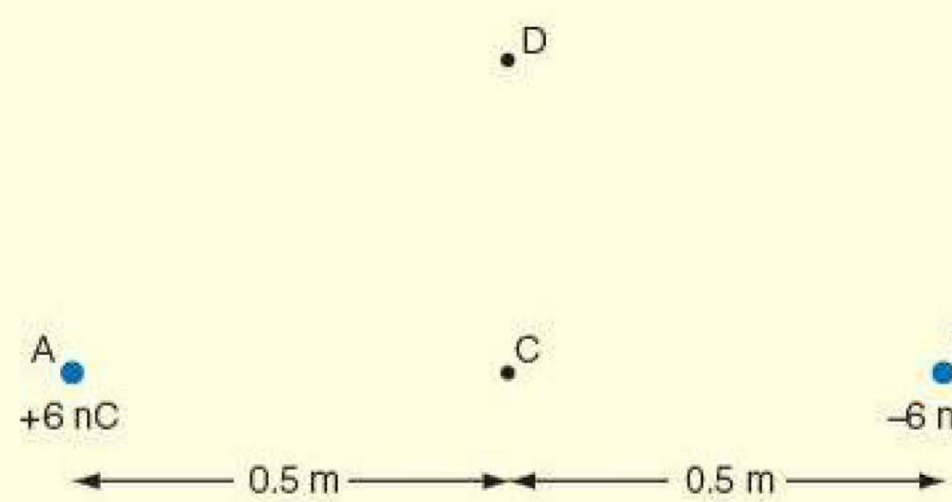


Figure 5.36

a) Calculate the strength of the electric field at point C, which is halfway between the charges. (3)

b) i) Draw a vector diagram to show the two electric fields due to the charges A and B at point D. (2)

ii) Use the diagram to show the direction of the electric field at D. (1)

17 Figure 5.37 shows equipotentials around a positive charge.

a) Explain how the equipotentials show that the electric field is stronger at point A than it is at point C. (2)

b) A charge of –2.0 nC is moved from

i) B to C

ii) C to D.

Calculate the work done against the electric field in each case. (3)

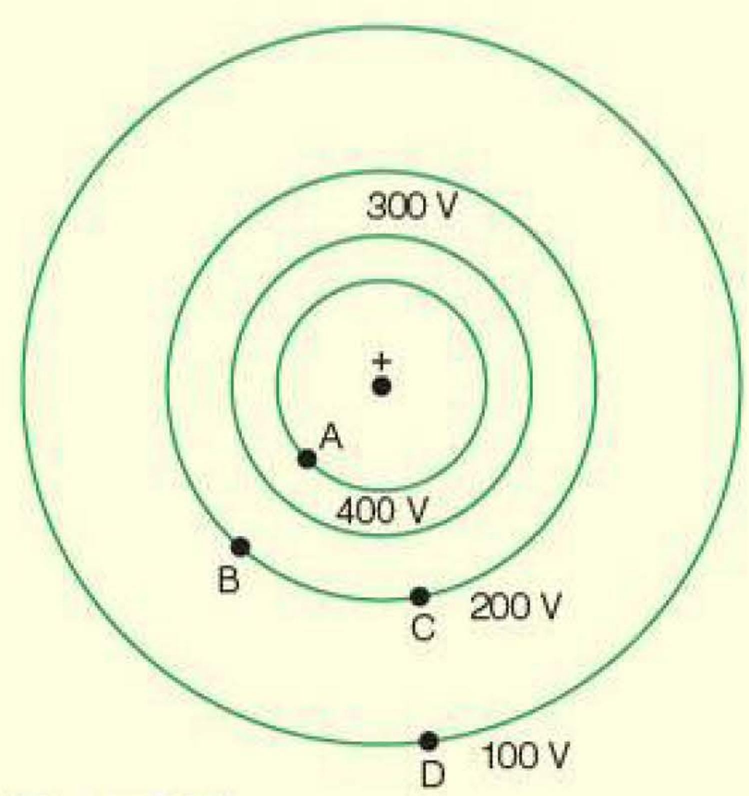


Figure 5.37

18 When the electric field strength reaches about $3.0 \times 10^6\,\mathrm{V\,m^{-1}}$, air can become ionised. In strong electric fields, free electrons gain sufficient energy to ionise air molecules.

Figure 5.38 shows electric potentials close to an isolated tree.

a) Explain why the electric field strength is stronger over the top of the tree. *(2)*

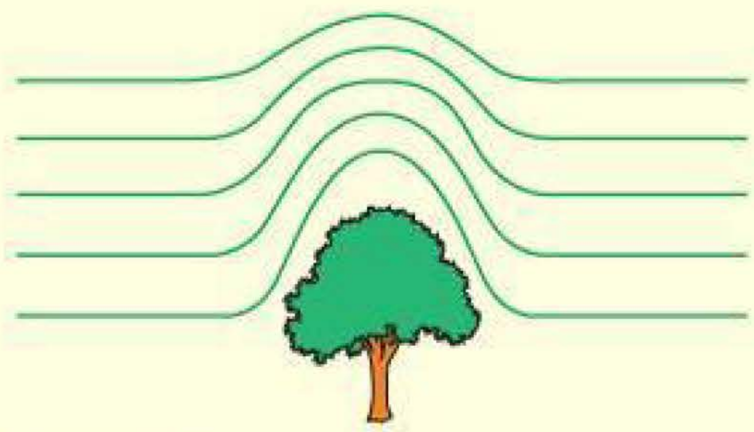


Figure 5.38

b) A free electron can ionise a molecule if it has sufficient energy to dislodge an electron that is attached to an atom or molecule. In air, at atmospheric pressure, an electron travels an average distance of 0.5 µm between collisions (this is called the mean free path).

i) Calculate the energy gained by an electron that accelerates a distance of 0.5 µm through an electric field of strength $46\,\mathrm{MV\,m^{-1}}$. Express your answer in eV. *(3)*

ii) Explain why gases may be ionised with weaker electric field when the gas pressure is low. *(2)*

c) A thundercloud at a height of 300 m above the ground is charged to a potential of $-7 \times 10^8\,\mathrm{V}$ relative to the Earth.

i) Sketch a diagram to show the electric field between the cloud and the ground. *(1)*

ii) Calculate the field strength under the cloud. *(2)*

The cloud is discharged by a flash of lightning, which carries a charge of 4.5 C, in a time of 0.024 s.

iii) Calculate the average current during the discharge. *(2)*

iv) Calculate the energy dissipated during the lightning strike. *(2)*

Stretch and challenge

19 Figure 5.39 shows two positive charges, $+q$, separated by a distance $2a$.

a) Show that the magnitude of the electric field, E_C, at C is given by

$$E_c = \frac{qx}{2\pi\varepsilon_0 (x^2 + a^2)^{\frac{3}{2}}}$$

and that the direction of the field is along the line OC.

b) Show that the electric field is a maximum for $x = \pm\frac{a}{\sqrt{2}}$.

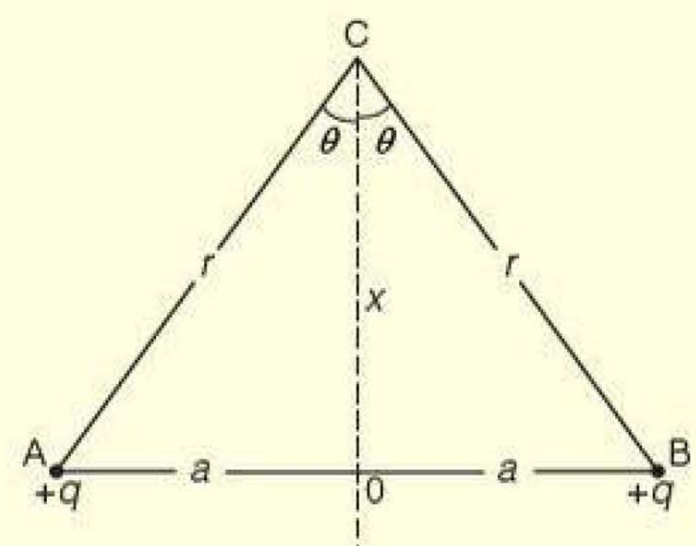


Figure 5.39

20 This question also refers to Figure 5.39. A particle with charge $+q$ (the same size as the two charges at A and B) is directed along the line CO. The particle starts a very long way from the charges. What is the minimum initial speed the particle must have if it is just to reach point O?

6 Capacitance

PRIOR KNOWLEDGE

Before you start, make sure that you are confident in your knowledge and understanding of the following points:

- Electric current, I, is the rate of flow of charge, $I = \frac{\Delta Q}{\Delta t}$.
- Potential difference (p.d.), V, is the amount of electrical work done per unit charge, $V = \frac{W}{Q}$.
- Electrical resistance, R, is defined by $R = \frac{V}{I}$.
- Electrical power, P, is the rate of doing electrical work, $P = VI = I^2R = \frac{V^2}{R}$.
- Electrical energy, $E = VIt$.
- Kirchhoff's second circuit law states that (in a complete loop of the circuit) sum of e.m.f.$_s$ = sum of p.d.$_s$.

TEST YOURSELF ON PRIOR KNOWLEDGE

1. A current of 25 mA flows through a fixed resistor for 1 minute. Calculate the total charge that flows through the resistor.
2. Calculate the work done in accelerating electrons with a charge of -1.6×10^{-19} C through a p.d. of 4800 V.
3. Calculate the current flowing through a 1.1 kΩ resistor with a p.d. of 7.7 V across it.
4. Calculate the electrical energy supplied to a 39 Ω heating element if a current of 1.2 A flows through it for 3 minutes.
5. A 12 V car battery supplies potential differences across a fixed 5.7 V GPS unit and a mobile phone charger connected in series. Calculate the p.d. across the charger.

Capacitors

Capacitors are components of electrical circuits that temporarily store electric charge. The addition of a capacitor into a circuit has two possible effects: either introducing a time delay into the circuit; or storing electrical energy for a short period of time. Capacitors are used extensively in electrical and electronic timing circuits, in power circuits, for smoothing electrical signals and as part of the signal-receiving circuits found in radios.

Modern capacitors consist of two parallel conducting plates (usually made of metal foils, films or coatings) separated by a thin insulating layer known as a dielectric (generally made from thin plastic films, electrolytes, ceramics

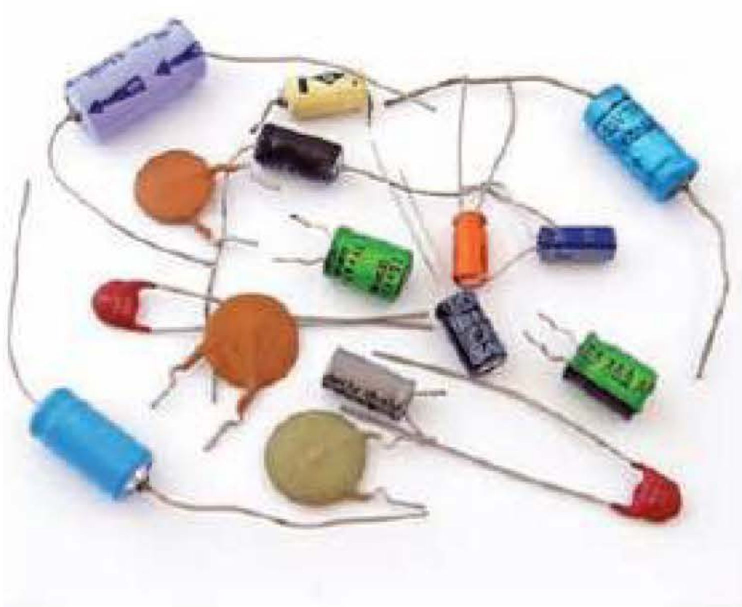


Figure 6.1 Different types of capacitor

or metal oxides). Most capacitors are then encased in a metal or plastic housing. Figure 6.1 shows a selection of different capacitors.

There are several different circuit symbols for capacitors depending on their type, although they are all based on the same simple pattern shown in Figure 6.2.

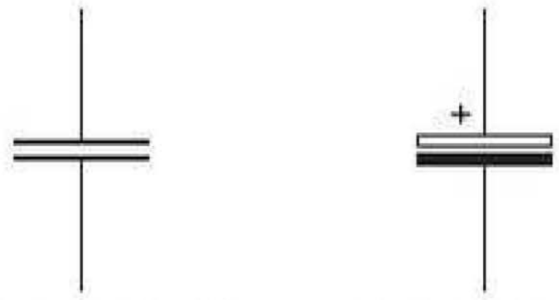


Figure 6.2 The main circuit symbols for capacitors.

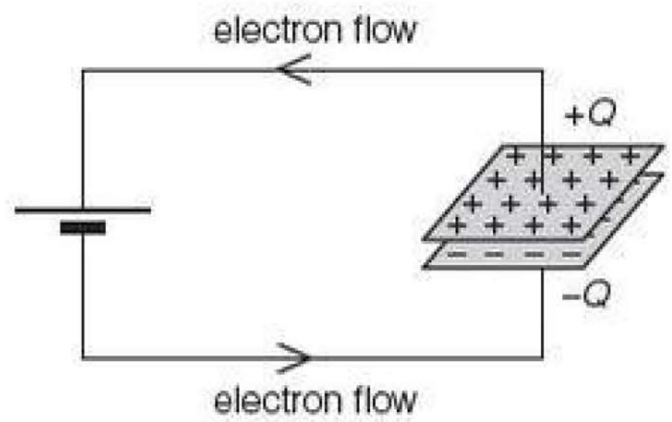


Figure 6.3 Capacitor plates.

A potential difference from a battery or a power supply connected across the metal plates causes electrons to flow off one plate, back through the battery and onto the second plate (Figure 6.3).

One plate becomes positively charged (where electrons are removed), while the plate with the excess of electrons becomes negatively charged. If the capacitor is then disconnected from the source of potential difference, the charge will stay on the plates until a conducting pathway allows the excess electrons to flow off the negatively charged plate and back onto the positive plate, until the two plates have equal charge again (Figure 6.4). The conducting pathway could be a different part of the circuit (controlled by a switch) or the charge could gradually leak away to the surroundings.

Capacitance The capacitance of a capacitor is the ability of the capacitor to store charge per unit potential difference.

Farad The unit of capacitance is the farad (F), where 1 F is equal to 1 $C V^{-1}$ (one coulomb per volt).

The ability of any object to store charge is called **capacitance**. Capacitance is given the symbol *C*, and the SI unit is the **farad** (F). The capacitance of a capacitor depends on the area of the metal plates, the distance between the plates and the electrical properties of the material separating the plates.

The amount of charge, Q, that can be stored on a capacitor depends on the size of the capacitance, *C*, and the potential difference, *V*, across the capacitor causing the separation of the charge:

$$Q = VC$$

The capacitance of a capacitor can then defined by

$$C = \frac{Q}{V}$$

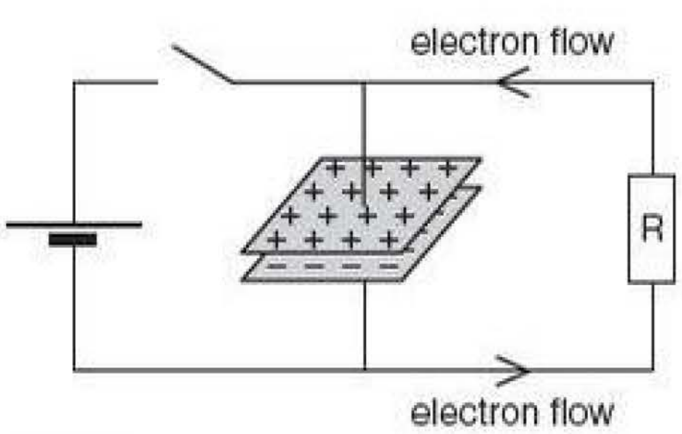


Figure 6.4 Capacitor plates discharging.

So one farad is equal to one coulomb per volt. Actually 1 F is quite a large capacitance, and useful 'real-life' capacitors have capacitances measured in microfarads (μF), nanofarads (nF) or picofarads (pF).

TIP

A capacitance of 1 F will store a charge of 1 C with a potential difference of 1 V across it.

ACTIVITY

Measuring the capacitance of a capacitor

A fully discharged capacitor is connected to a variable d.c. power supply and is then gradually charged to different potential differences. A digital coulombmeter is then used to measure the charge stored on the capacitor at each potential difference (Figure 6.5).

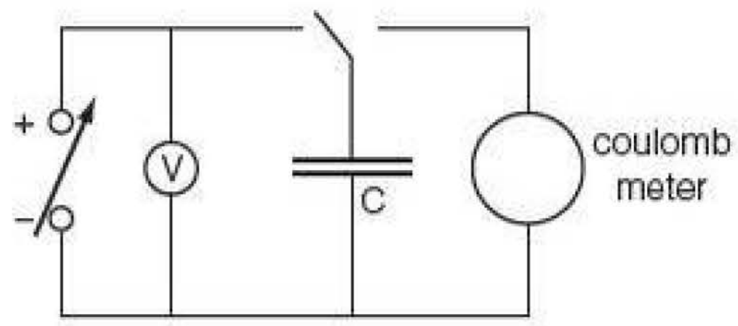


Figure 6.5 Digital coulombmeter measuring charge.

The data from this experiment is shown in Table 6.1.

Table 6.1

Potential difference, V/V	Charge stored, Q/nC
0.0	0
1.0	331
2.0	664
3.0	1023
4.0	1328
5.0	1670
6.0	1996

The data sheet supplied with the coulombmeter states that the tolerance of the charge readings is ±10%.

1 Plot a graph of Q against V.
2 Plot suitable error bars on your data points.
3 Use your graph to show that Q is proportional to V.
4 Calculate a value for C, the capacitance of the capacitor, where $Q = CV$, and use the error bars to determine an uncertainty in this value.
5 Another capacitor, with a capacitance of 2C, is connected into the same circuit **in place of the first capacitor**. Sketch a line on your graph illustrating the variation of Q with V for this new capacitor.

TEST YOURSELF

1 Define what is meant by the 'capacitance' of a capacitor.
2 Copy this table and correctly match the units in the first row to the quantities in the second row.

Units	C	A	F	V
Quantities	p.d.	capacitance	current	charge

3 List three factors that dictate the ability of a capacitor to store charge.
4 A 4200 μF capacitor is connected to a 6.0 V battery. Calculate the charge stored on the capacitor.
5 A capacitor stores a charge of 3.2 mC at a p.d. of 6.0 V. Calculate the value of the capacitance.

6 Five different capacitors are connected one at a time to the same battery, and a coulombmeter is used to measure the charge stored on each capacitor. The measurements are shown graphically in Figure 6.6. Use the graph to calculate the output p.d. from the battery.

7 Copy and complete the table.

Q	V	C
	6.0V	440μF
0.03C	12.0V	
30μC		10000μF
250nC	5.0V	
	9.0V	120nF

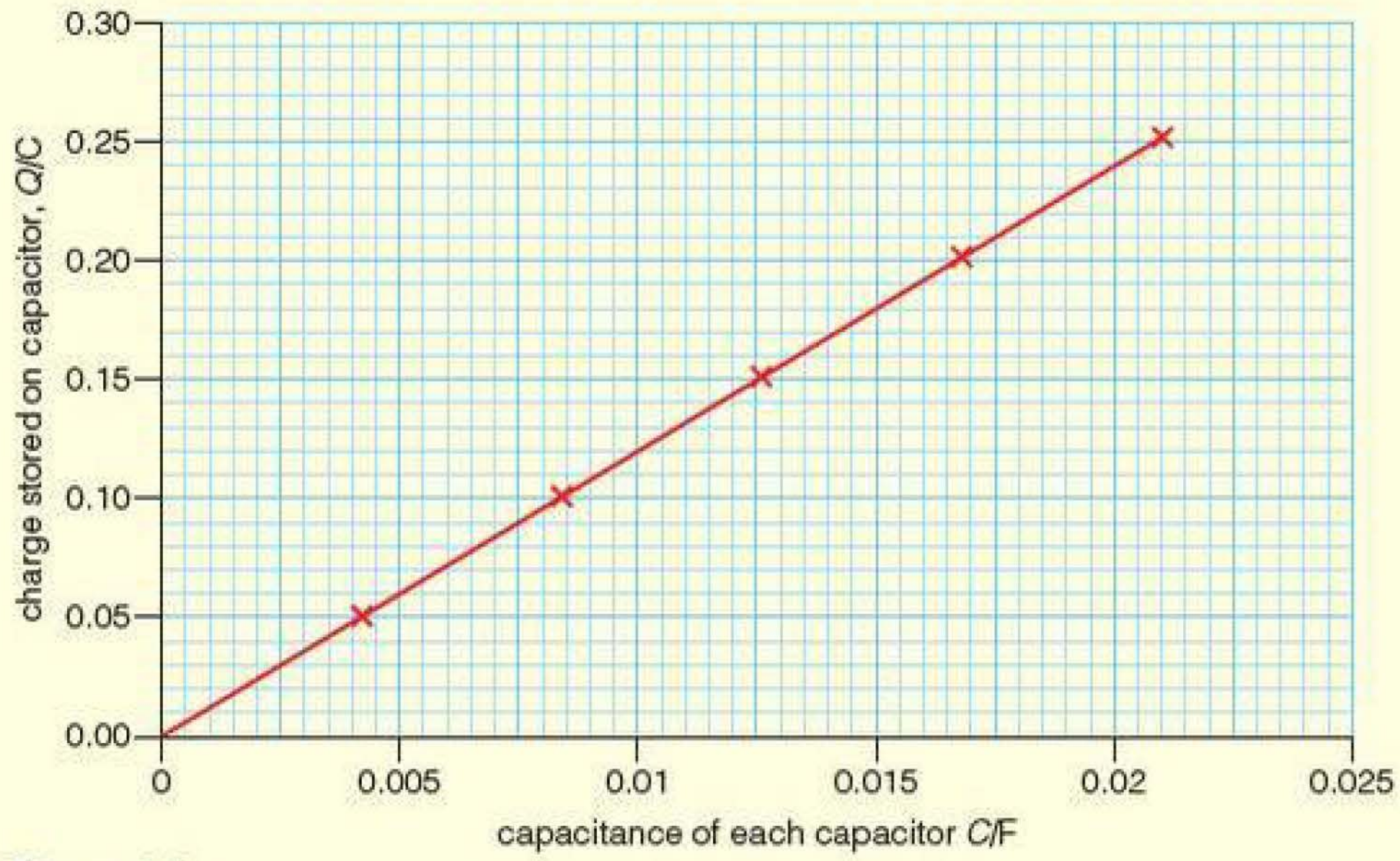


Figure 6.6

Permittivity The permittivity of a material is the resistance of the material to an electric field passing through it.

Dielectric constant Another term for the relative permittivity of an insulating material. It describes the relative resistance of the material to the propagation of electric field through it and describes the absolute permittivity of the material in terms of multiples of the permittivity of free space, $\varepsilon_r\varepsilon_0$.

Parallel-plate capacitors

The capacitance of a parallel-plate capacitor depends on the area of the plates, their distance apart and the ability of the insulating material between the plates to separate the charge, a property known as **permittivity**.

The permittivity of a material is the resistance of the material to an electric field passing through it. If the permittivity is high, then a larger charge can be stored on the plates for any given potential difference across them. The permittivity of capacitor insulating materials is always measured relative to the permittivity of free space (vacuum), ε_0, using a relative permittivity, ε_r, sometimes called the **dielectric constant** of the material. The total absolute permittivity of an insulator is therefore given by the product $\varepsilon_r\varepsilon_0$. The permittivity of free space $\varepsilon_0 = 8.854 \times 10^{-12}\,\mathrm{F\,m^{-1}}$. Table 6.2 gives the relative permittivity of a selection of materials commonly used in the construction of capacitors.

TIP

The permittivity of a material to an electric field is analogous to the resistance of a material to electric current flowing through it. In the case of permittivity, current is replaced by electric field.

Table 6.2

Material	Relative permittivity (dielectric constant)
Ceramic (ZnMg)TiO_2	32
Polyester	2.8–4.5
Polystyrene	2.5–2.7
Aluminium oxide (electrolyte)	9.8

The capacitance of a parallel-plate capacitor is given by

$$C = \frac{\varepsilon_r \varepsilon_0 A}{d}$$

where A is the area of the plates and d is their separation. Since the capacitance is also given by $C = Q/V$, it follows that

$$\frac{Q}{V} = \frac{\varepsilon_r \varepsilon_0 A}{d} \quad \text{or} \quad \frac{Q}{A} = \frac{\varepsilon_r \varepsilon_0 V}{d}$$

so the charge density on each plate is proportional to $\frac{V}{d}$ – which is the electric field E.

TEST YOURSELF

8 A capacitor is constructed from two sheets of aluminium foil, 45 cm × 95 cm, separated by a thin layer of polythene cling-film, 12.5 μm thick. The relative permittivity of polythene is 2.25 and the permittivity of free space $\varepsilon_0 = 8.854 \times 10^{-12}\,\text{F m}^{-1}$. Calculate the capacitance of the capacitor.

9 A 6.0 V battery is connected to a 20 nF capacitor. The area of the capacitor plates is $0.0016\,\text{m}^2$ and they are separated by a ceramic dielectric layer 5 μm thick.

a) Calculate the charge stored on the capacitor.

b) Calculate the relative permittivity of the ceramic dielectric.

10 A 29–520 pF air-filled variable capacitor is shown in Figure 6.7.
The five moving capacitor plates behave as five independent capacitors arranged in parallel, effectively multiplying the capacitance of one set of plates by 5. Each moving plate is separated from its static plate by an air gap 0.5 mm wide. The relative permittivity of air at room temperature is 1.00.

a) Calculate the maximum area of overlap of the plates.

b) Calculate the minimum area of overlap between the plates.

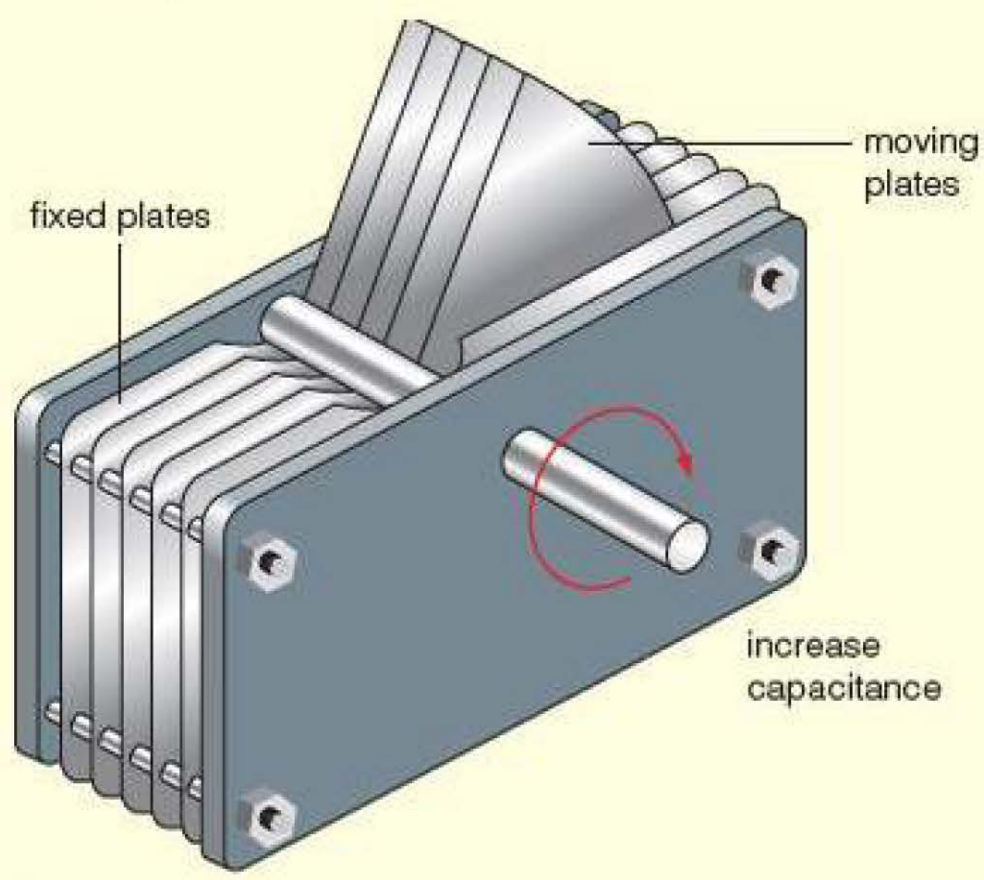


Figure 6.7 An air-filled variable capacitor.

ACTIVITY

Measuring the relative permittivity of a dielectric material

Figure 6.8 shows a digital multimeter being used to measure the capacitance of a pair of square capacitor plates separated by a thin layer of material from a supermarket shopping bag. Some digital multimeters can measure capacitance directly by charging and discharging the capacitor under test with a known current and then measuring the rate of rise of the subsequent potential difference. The faster the rate of rise, the smaller is the capacitance.

The *X* and *Y* dimensions of the plates are measured using a standard ruler to be 30.0 ± 0.1 cm × 30.0 ± 0.1 cm. The thickness of the shopping bag dielectric between the plates is measured randomly across the dielectric 10 times using a micrometer. The results are shown (in mm) in Table 6.3.

Figure 6.8 Digital multimeter measuring capacitance.

Table 6.3

0.13	0.13	0.14	0.14	0.13	0.13	0.13	0.12	0.13	0.13

The area of overlap of the plates is varied by moving the top plate diagonally relative to the bottom plate. The plates can be positioned with an uncertainty of 2 mm (*X* dimension) × 2 mm (*Y* dimension) and the digital multimeter can measure a capacitance to ±5%.

1 Use the data given and the data in Table 6.4 to determine the relative permittivity of the shopping bag material with a suitable value of uncertainty.

Table 6.4

X dimension of plates in overlap/cm (±0.2 cm)	*Y* dimension of plates in overlap/cm (±0.2 cm)	Capacitance, *C*/nF (±5%)
30.0	30.0	14.1
28.3	28.3	12.5
26.5	26.5	11.0
24.5	24.5	9.4
22.4	22.4	7.9
20.0	20.0	6.3
17.3	17.3	4.7
14.1	14.1	3.1
10.0	10.0	1.6

Dielectric heating

Some molecules, such as water, are called *polar molecules* because the opposite ends of the molecule have opposite charges (Figure 6.9). When water molecules form, the hydrogen atoms become slightly positively charged, and the oxygen atom becomes slightly negatively charged. (There is a covalent bond between the hydrogen and oxygen atoms in a water molecule, but the shared electrons in the bond are attracted more towards the oxygen atom than they are towards the hydrogen atoms.) In a polar molecule, the overall charge of the molecule is zero, but different 'ends' of the molecule may have opposite charges. Polar molecules easily

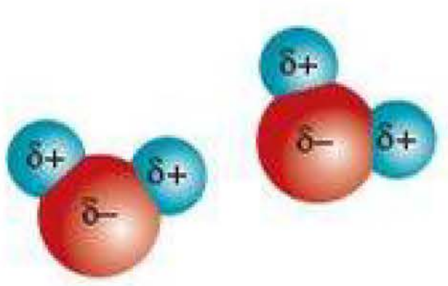


Figure 6.9 Water molecules are polar molecules.

stick to metal surfaces, where one end of the polar molecule induces an opposite charge in the metal, causing the molecule to be attracted to the metal surface.

Dielectric material An insulating material where the molecules that make up the material can be polarised inside an electric field. Electric charges do not move through the material, but the polar molecules align themselves with the field.

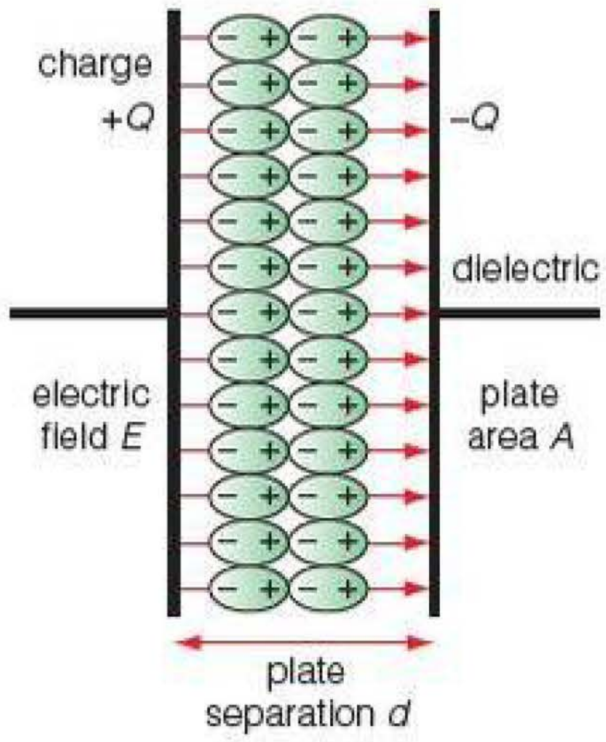


Figure 6.10 The polar molecules in a dielectric material align themselves with the electric field.

Most of the **dielectric materials** used to construct capacitors are solids, and the atoms and molecules are fixed within the structure. But what happens when the solid is replaced by a liquid (such as water) or a gas, where the particles are free to move between the metal plates? The electric field produced between the two plates of a capacitor will cause the charged particles in a liquid or a gas to align themselves in the direction of the field (Figure 6.10). The separated charge in a polar molecule is particularly able to align itself with the field between two capacitor plates.

If the electric field between the plates is suddenly reversed, the polar molecule will rotate and align itself with the direction of the electric field again (Figure 6.11). Alternating the electric field between the two plates will cause a polar molecule, such as water, to continuously rotate between them (called dipole rotation). This increases its kinetic energy, and causes it to collide with other adjacent molecules and atoms. These then acquire more kinetic energy and move in random directions, increasing their temperature and so dissipating the energy as heat.

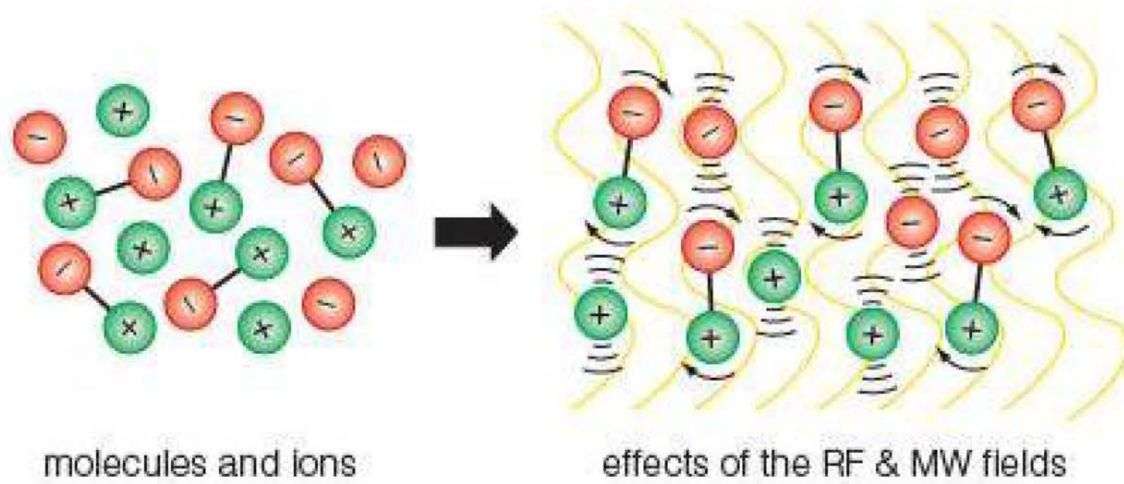


Figure 6.11 Polar molecules rotating in an alternating microwave field.

TIP

There is a common misconception that microwave ovens cook food from the 'inside out'. This is not the case. The microwaves cause water molecules near the surface (a millimetre or so) to rotate. These hit nearby molecules, forcing them into faster, more energetic motion, and the molecular vibration is dispersed into the centre of the food. This process is quicker than conventional conduction.

The alternating field between the plates can be produced by a microwave emitter, such as the magnetron inside a microwave oven. The frequency of the microwaves is tuned so that it rotates water molecules within food – causing the food to heat up rapidly. The optimum frequency for the rotation of water molecules in food is about 10 GHz, but if the frequency of the microwaves was set to this value then the water molecules in the outer layers of the food would absorb all the microwave energy, leaving a cool uncooked inner region and an outer superheated layer. As a result, domestic microwave ovens have a frequency of 2.45 GHz, which allows the outer layers to heat up more slowly and then conduct heat deeper into the food.

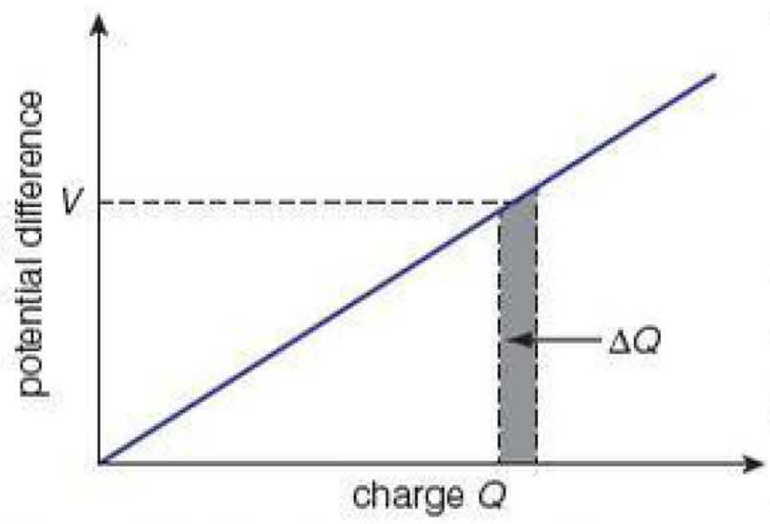


Figure 6.12 Graph of Q versus V for a capacitor.

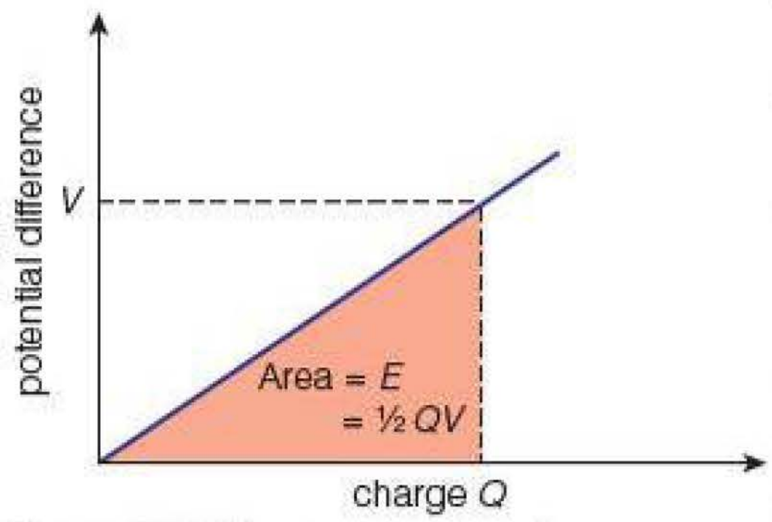


Figure 6.13 The energy stored on a capacitor is equivalent to the area under a Q against V graph.

The energy stored by a capacitor

When a capacitor is charged up, the p.d. from the electricity supply (or the energy per unit charge) causes electrons to flow off one plate, through the external circuit and onto the other plate. This separation of charge is kept steady provided that the p.d. is continuously applied, and that there is no leakage of charge. Once the p.d. is removed, and a complete discharging circuit is connected to the capacitor, the electrical energy stored by the separated charge can be released as the electrons flow back off the negatively charged plate and back onto the positively charged plate – this was shown in Figure 6.4. If the p.d. applied to the plates is increased, more charge and therefore energy is stored on the plates. A graph of potential difference against charge for a capacitor is shown in Figure 6.12.

You will remember from the definition of potential difference that $V = \frac{W}{Q}$, where W is the amount of work done per unit charge, Q. In the context of a capacitor, the potential difference V is the amount of work done in moving unit charge off one plate and onto another plate. At any potential difference V, the work done moving an amount of charge ΔQ is therefore $W = V\Delta Q$. This is represented by the shaded area in the graph in Figure 6.12. The total energy E stored on the capacitor, charging the capacitor from empty up to a charge of Q at a potential difference V, is calculated by adding up all the similar shaped areas from Q = 0 up to a charge Q. In other words, this is the whole area under the graph up to Q. Because the shape is a triangle (Figure 6.13), $E = \frac{1}{2}QV$. But $Q = CV$, so $E = \frac{1}{2}CV^2$ and $E = \frac{1}{2}\frac{Q^2}{C}$.

TIP

Use the correct version of the capacitor–energy equation depending on the question. Use the version containing the data given unless you are told to do otherwise. If you use calculated values, you are more likely to make an error.

TEST YOURSELF

11 Explain what is meant by a 'dielectric material'.

12 Explain how a water molecule can be heated by an alternating electric field.

13 A 3300 μF capacitor is charged by a 9.0 V battery. Calculate the energy stored in the capacitor.

14 Calculate the capacitance of a capacitor that stores 0.25 J of electrical energy when a p.d. of 24 V is connected across it.

15 Calculate the charge on a 220 μF capacitor storing an energy of 0.12 J.

⇨

16 The graphs X, Y and Z in Figure 6.14 show possible relationships between electrical quantities associated with a capacitor discharging through a resistor.

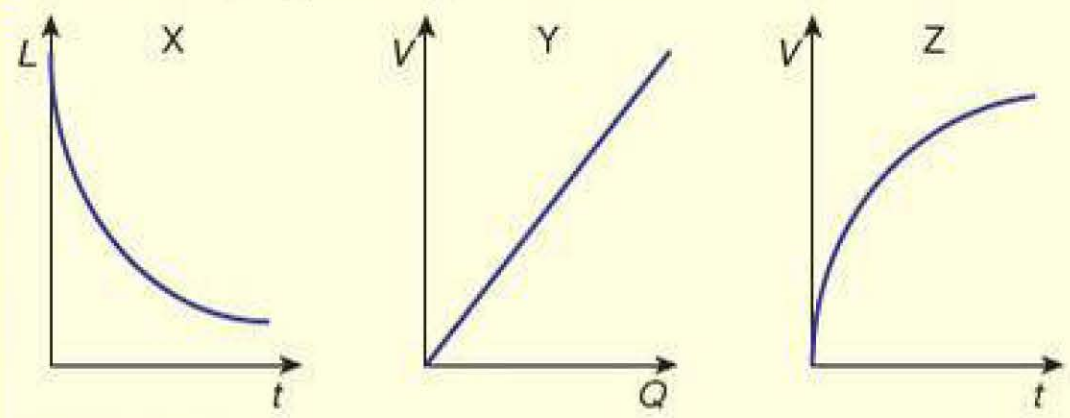


Figure 6.14

a) Which graph shows the gradient equal to $\frac{1}{C}$?

b) Which graph could be used to determine the total charge stored in the capacitor?

c) In which graph does the area under the line represent the energy stored in the capacitor?

17 The graph in Figure 6.15 shows how the charge stored on a capacitor varies with the p.d. applied across it.

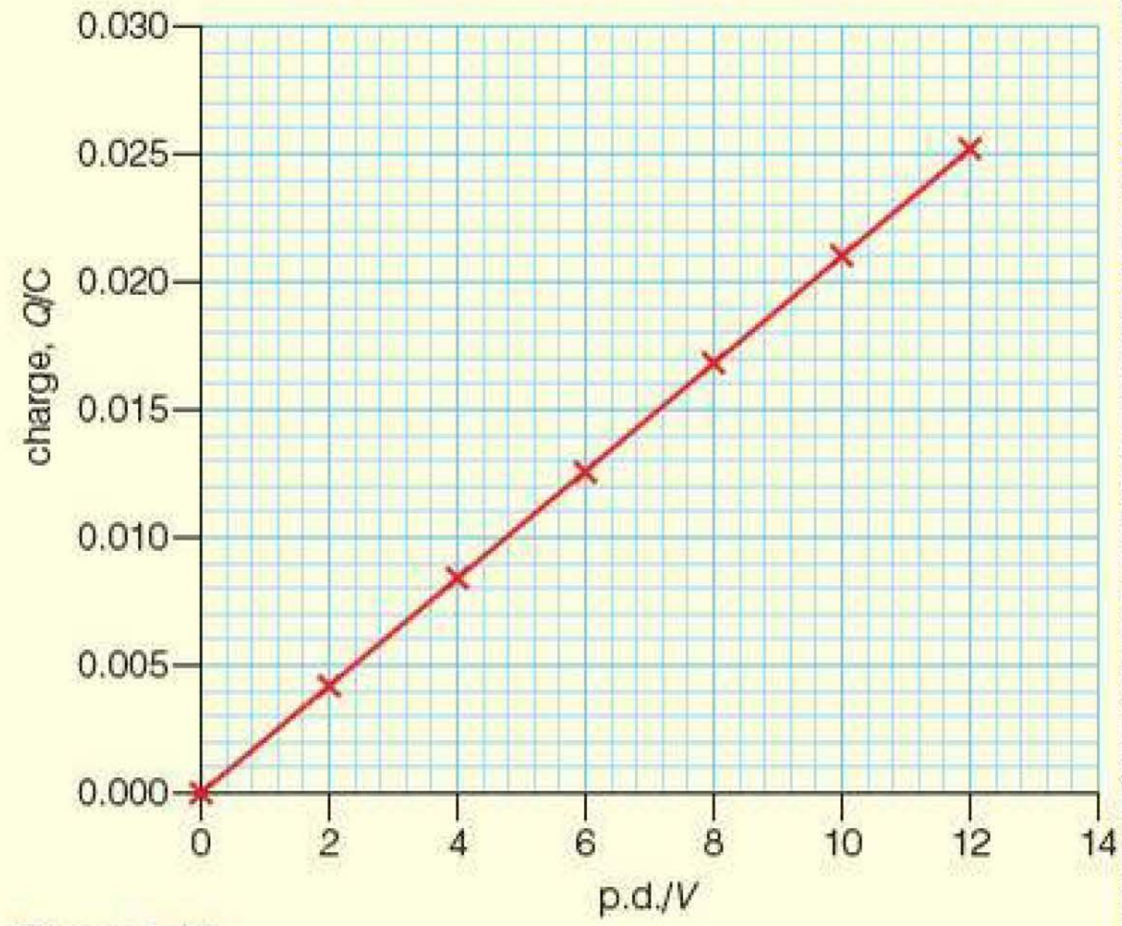


Figure 6.15

a) Use the graph to calculate the capacitance of the capacitor.

b) Use the graph to calculate the energy stored on the capacitor, when 8.0V is applied across it.

Capacitor charge and discharge

The circuits in Figure 6.16 show a battery, a switch and a fixed resistor (circuit A), and then the same battery, switch and resistor in series with a capacitor (circuit B). The capacitor is initially uncharged.

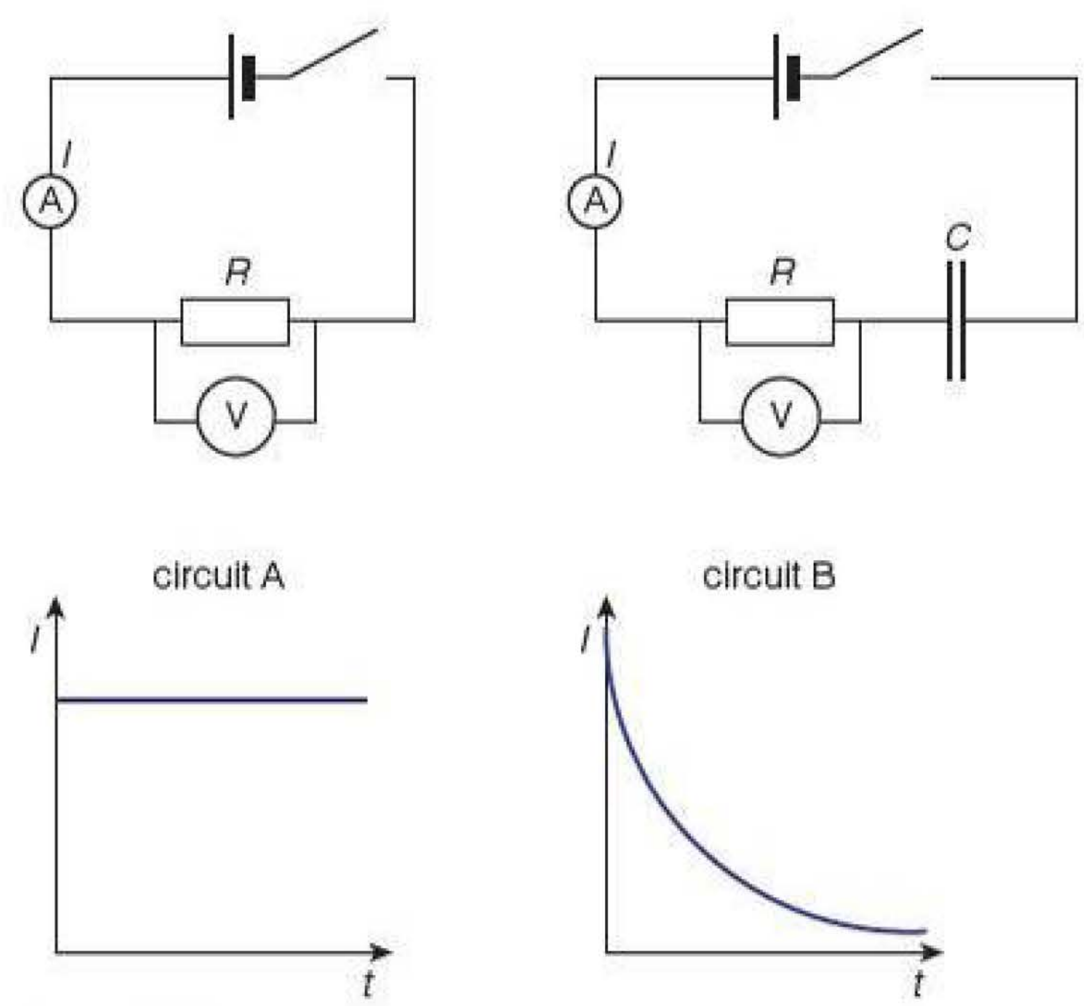


Figure 6.16 Circuit diagrams for a battery, resistor and capacitor network.

The graphs underneath the circuit diagrams show how the current varies with time from the moment that the switches are closed. In the case of the resistor alone (circuit A), the current immediately jumps to a value I, where $I = V/R$, and stays at that value regardless of how long it has been since the switch was closed. Time is not a factor in this circuit. In the case of circuit B, where an initially uncharged capacitor is connected in the circuit, the current also immediately rises to the same value, I, determined by $I = V/R$, but it then starts to decay away with time, eventually reaching zero. The series capacitor limits the way that current flows through the resistor.

If the capacitor is initially uncharged, the amount of charge that can be stored on it per second, $\frac{\Delta Q}{\Delta t} = I$, is initially determined by $I = \frac{V}{R}$. As the capacitor starts to store charge, so a p.d. is developed across the capacitor, $V_C = \frac{Q}{C}$. As the e.m.f. of the battery, ε, remains constant, then the potential difference, V_R, across the fixed resistor, R, reduces because

$$\varepsilon = V_R + V_C$$

Reducing V_R reduces the current, I, flowing. The initial current flowing onto the capacitor gradually decays away as the capacitor stores more charge, increasing V_C.

Graphs of charge Q stored on the capacitor with time are shown in Figure 6.17, one representing the capacitor charging, and one discharging.

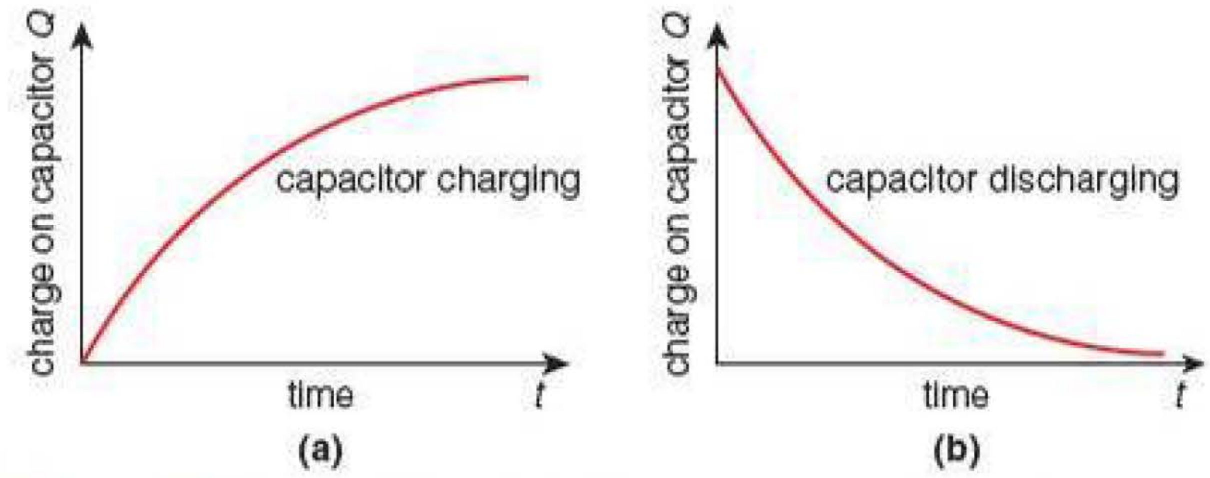


Figure 6.17 Graph of Q against t for a capacitor (a) charging and (b) discharging.

TIP

Remember that the gradient of a charge–time graph is the electric current.

The charging graph (Figure 6.17(a)) shows that, initially, the capacitor is uncharged, and the gradient of the graph, $\frac{\Delta Q}{\Delta t}$ (equal to the current, I), is at a maximum and is determined by $I = \frac{V_R}{R}$. As more charge is stored on the capacitor, so the gradient (and therefore the current) drops, until the capacitor is fully charged and the gradient is zero. As the capacitor discharges (Figure 6.17(b)), the amount of charge is initially at a maximum, as is the gradient (or current). The amount of charge then drops, as does the gradient of the graph. This is described by

$$\frac{\Delta Q}{\Delta t} \propto Q$$

The shape of the discharging graph is an exponential decay, meaning that the rate of decay of the charge (or the gradient or the current) depends on the amount of charge stored at any given time. For a discharging capacitor, the current is directly proportional to the amount of charge stored on the capacitor at time t.

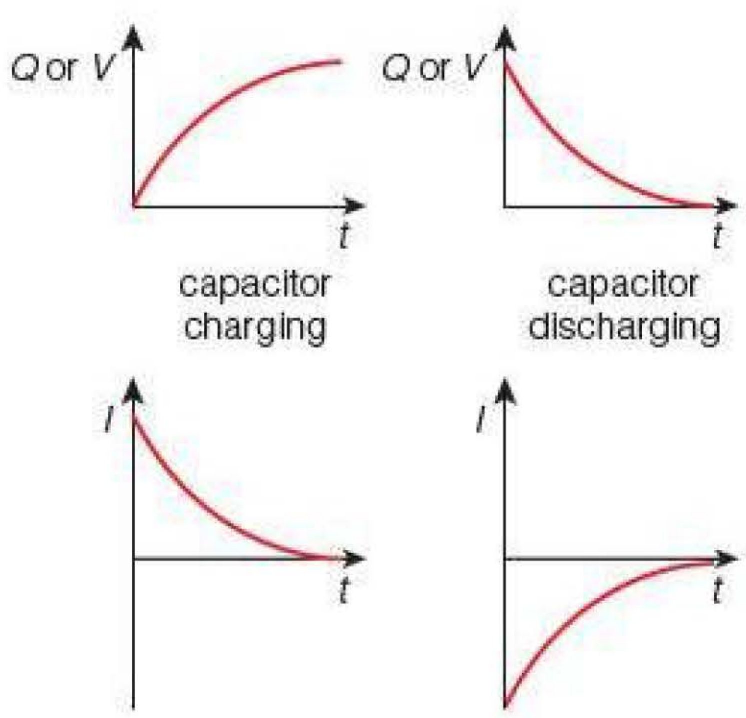


Figure 6.18 Graphs of Q or V and I against t, for charging and discharging capacitor.

Graphs of V (the p.d. across the capacitor) against t follow the same pattern as the graph of Q against t, because $Q \propto V$ (from $Q = VC$). When current–time graphs are plotted, you should remember that current can change direction and will flow one way on charging the capacitor and in the other direction when the capacitor is discharging. The size of the current is always at a maximum immediately after the switch is closed in the charging or discharging circuit, because the charging current will be highest when the capacitor is empty of charge, and the discharging current will be highest when the capacitor is full of charge. This is shown in the graphs in Figure 6.18.

The gradient of the Q against t graph, $\frac{\Delta Q}{\Delta t}$, is the current, I, as shown in Figure 6.19.

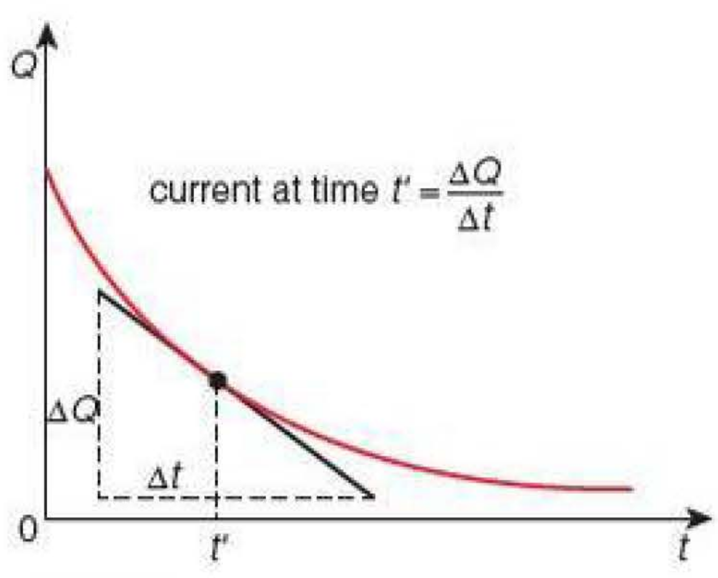


Figure 6.19 Graph illustrating how the gradient of a Q against t graph is the current.

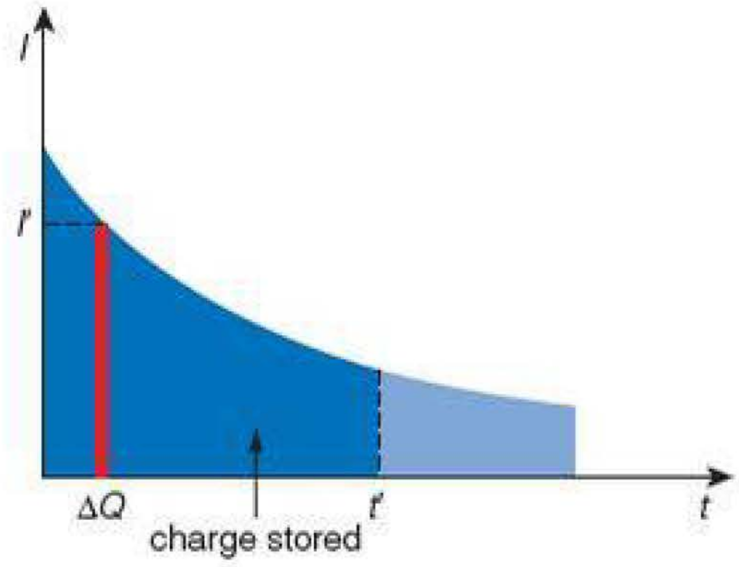


Figure 6.20 Relationship between charge and the area under a graph of I against t.

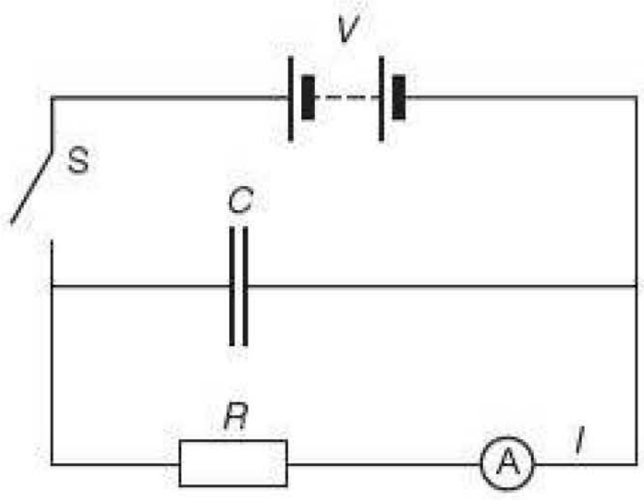


Figure 6.21 A capacitor discharging circuit.

During discharge the area under an I against t graph, up to time, t, is the charge transferred off the capacitor, as shown in Figure 6.20.

When the capacitor is discharging, during a small time interval Δt, the element of charge transferred is ΔQ, and this is calculated by $\Delta Q = I'\Delta t$. The total charge transferred, Q, during the discharge from $t = 0$ to $t = t'$ is the sum of all the charge elements during this period – this is the area under the graph.

Discharging a capacitor

Consider the circuit shown in Figure 6.21. When switch S is closed, the capacitor C immediately charges to a maximum value given by $Q = CV$. As switch S is opened, the capacitor starts to discharge through the resistor R and the ammeter. At any time t, the p.d. V across the capacitor, the charge stored on it and the current, I, flowing through the circuit and the ammeter are all related to each other by two equations.

Applying Kirchhoff's second circuit law around the capacitor–resistor loop:

$$V_C + V_R = 0$$

where V_C and V_R are the p.d.$_s$ across the capacitor and the resistor, respectively. Substituting for both using $Q = VC$ and $V = IR$ gives

$$\frac{Q}{C} + IR = 0$$

But $$I = \frac{\Delta Q}{\Delta t}$$

so $$\frac{Q}{C} = -\frac{\Delta Q}{\Delta t}R$$

or $$\Delta Q = -\frac{Q}{RC}\Delta t$$

This is a differential equation and requires calculus to provide the solution (for those interested, see the Maths box):

$$Q = Q_0\, e^{-\frac{t}{RC}}$$

As $Q = VC$ and $V = IR$, at any time during the discharge, $Q \propto V$ and $V \propto I$, so there are corresponding equations for the p.d. across the capacitor and the current flowing in the circuit during discharge:

$$V = V_0\, e^{-\frac{t}{RC}}$$

and

$$I = I_0\, e^{-\frac{t}{RC}}$$

MATHS BOX

This shows how to use calculus to solve the differential equation

$$\Delta Q = -\frac{Q}{RC}\Delta t$$

When integrating a differential equation, such as the one here, we let $\Delta t \rightarrow 0$, and then change to calculus notation (Δ to d) and group like terms on each side. Here this gives

$$\frac{dQ}{Q} = -\frac{1}{RC}dt$$

(showing a constant ratio), and

$$\int_{Q_0}^{Q} \frac{dQ}{Q} = -\int_{0}^{t} \frac{1}{RC}dt$$

Integrating gives

$$[\ln Q]_{Q_0}^{Q} = -\left[\frac{t}{RC}\right]_0^t$$

and so

$$\ln Q - \ln Q_0 = -\frac{t}{RC}$$

Then using the rules of logs

$$\ln\frac{Q}{Q_0} = -\frac{t}{RC}$$

so finally,

$$Q = Q_0 e^{-\frac{t}{RC}}$$

TIP

Remember that $t_{\frac{1}{2}} = 0.693RC$.

The time constant

The quantity RC is called the time constant of a capacitor circuit. The time constant is related to the half-life of the decay of charge off the capacitor, and is analogous to the half-life of radioactive decay. We define the half-life of capacitor discharge as the time taken for the charge stored on the capacitor (or the current or the voltage) to halve.

When $t = t_{\frac{1}{2}}$, $Q = \frac{Q_0}{2}$, so

$$\frac{Q_0}{2} = Q_0 e^{-\frac{t_{1/2}}{RC}}$$

Cancelling Q_0 from each side gives

$$\frac{1}{2} = e^{-\frac{t_{1/2}}{RC}}$$

and taking the natural logarithm of both sides of the equation leads to

$$\ln 0.5 = -\frac{t_{\frac{1}{2}}}{RC}$$

$$t_{\frac{1}{2}} = 0.693RC$$

Also when $t = RC$, we have

$$Q = Q_0 e^{-\frac{RC}{RC}} = Q_0 e^{-1} = 0.37Q_0$$

This means that one time constant, RC, is the time for the charge stored on the capacitor to drop to 37% of the initial value, Q_0.

Graphical analysis

The equations of exponential decay can be rewritten in the form of a straight line, so that a graph can be drawn and the gradient and y-intercept measured. This allows you to calculate the time constant of the circuit, if it is unknown.

Consider the current–time equation:

$$I = I_0 e^{-\frac{t}{RC}}$$

Rearranging and taking logarithms of both sides gives

$$\ln\left(\frac{I}{I_0}\right) = \ln(e^{-\frac{t}{RC}})$$

So

$$\ln I - \ln I_0 = -\frac{t}{RC}$$

or

$$\ln I = \ln I_0 - \frac{1}{RC}t \text{ or } \ln I = -\frac{1}{RC}t + \ln I_A$$

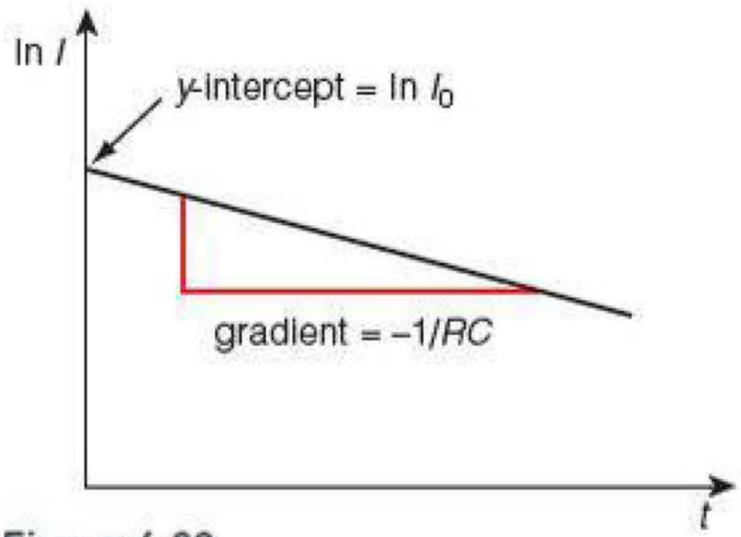


Figure 6.22

This is the equation of a straight line of the form

$$y = mx + c$$

where m is the gradient and c is the y-axis intercept of the line. Plotting $\ln I$ on the y-axis against t on the x-axis produces a gradient of $-\frac{1}{RC}$ and a y-axis intercept equal to $\ln I_0$, as illustrated in Figure 6.22.

TEST YOURSELF

18 The flash unit on a small disposable camera consists of a 2.0nF capacitor charged from a 1.5V battery, discharging through a 3.9MΩ resistance flash bulb.

a) Calculate the energy stored in the capacitor before discharge.

b) Calculate the time constant of the circuit.

The capacitor needs to discharge by 75% (two half-lives) before it can be recharged from the battery circuit.

c) Calculate the minimum recharge time for the capacitor.

19 A 2800μF capacitor and a variable resistor are used as part of the timing controls in a traffic light. Part of the timing circuit is shown in Figure 6.23.

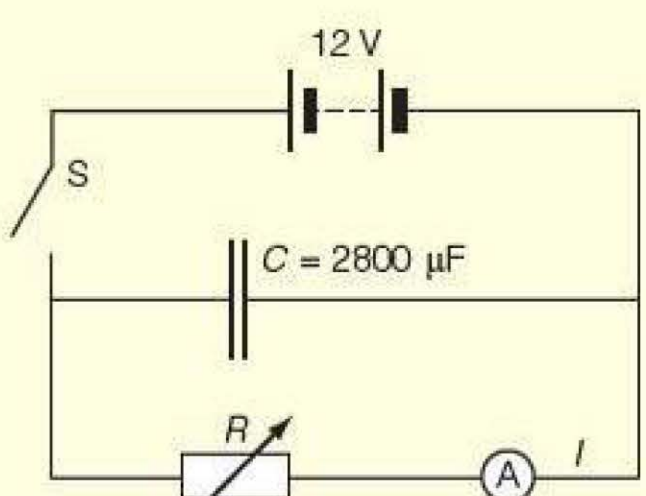


Figure 6.23

The initial value of the variable resistor R is set to 8.0kΩ and the timing circuit is controlled by closing and opening switch S, which is initially closed, completing the charging circuit. Calculate:

a) the initial charge stored in the capacitor

b) the total energy stored by the capacitor

c) the initial current flowing through variable resistor R.

Switch S is then opened.

d) Explain why the current flowing through the variable resistor reverses and starts to decrease.

e) Calculate the time constant of the discharge circuit.

f) Switch S is closed again and the variable resistor is adjusted so that the resistance is now halved to a value of 4.0kΩ. Use the words

halves doubles stays the same reduces to zero

to determine what happens to the quantities below when switch S is opened:

i) the initial charge stored on the capacitor

ii) the total initial energy stored in the capacitor

iii) the initial current flowing through the variable resistor

iv) the time constant of the discharge circuit.

20 A capacitor of capacitance C is fully charged by connecting it directly to a 3.0V battery. The capacitor is then disconnected from the battery and connected to a 12kΩ resistor in series with an ammeter. The graph in Figure 6.24 shows how the discharge current from the capacitor varies with time following its connection to the resistor.

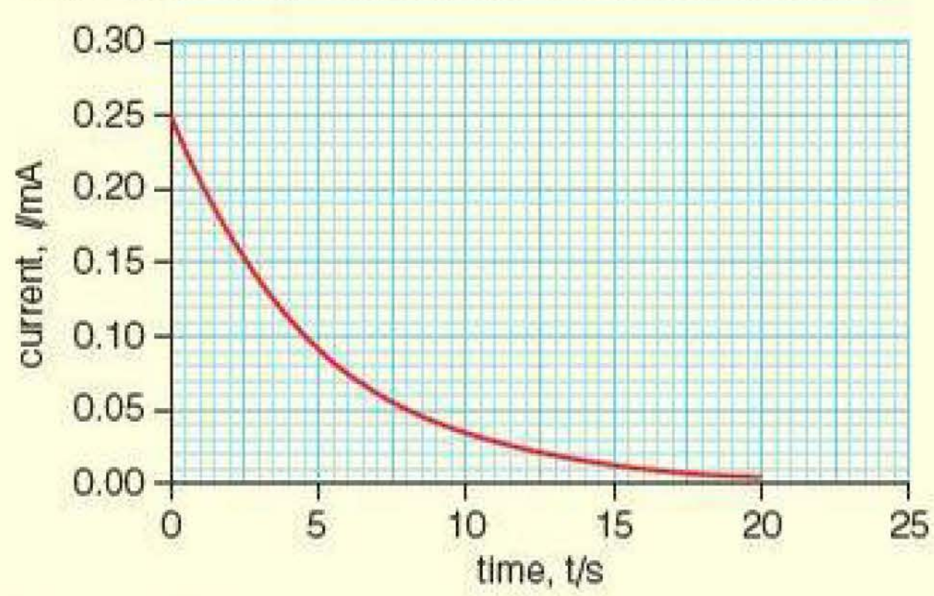


Figure 6.24

a) Explain why the quantity represented by the area under the graph is the initial charge stored on the capacitor.

b) Use the graph to estimate the initial charge stored on the capacitor.

c) Calculate the capacitance of the capacitor.

d) Calculate the initial energy stored on the capacitor.

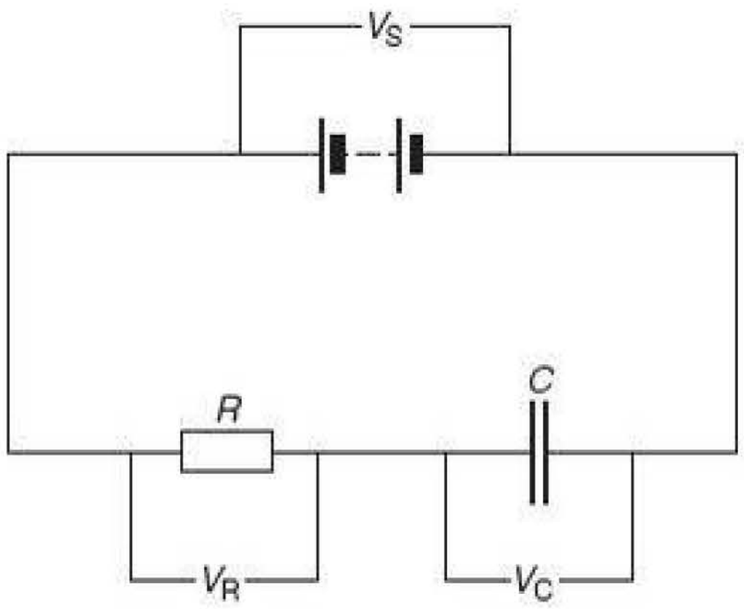


Figure 6.25 Circuit for charging a capacitor.

Charging a capacitor

In many cases, the charging of a capacitor is designed to be as quick as possible, and so the resistance of the charging part of a capacitor circuit is kept as low as possible (Figure 6.25). However, if the charging process is part of a circuit that requires a higher resistance, the charging time must be taken into account. There is no need for you to know how to derive the formula for charging a capacitor. The Maths box is included here for interested mathematicians.

Summarising the equations for charging a capacitor (from the Maths box):

$$Q = Q_0(1 - e^{-\frac{t}{RC}})$$

and therefore

$$V = V_0(1 - e^{-\frac{t}{RC}})$$

and

$$I = I_0 e^{-\frac{t}{RC}}$$

You do not need to be able to derive these formulae, but you do need to understand how to use them.

MATHS BOX

Consider the circuit shown in Figure 6.25. Kirchhoff's second circuit law tells us that the sum of the e.m.f.$_s$ in the circuit equals the sum of the p.d.$_s$ in the circuit, so

$$V_S = V_R + V_C$$

where V_S is the e.m.f. of the source, V_R is the p.d. across the resistor and V_C is the p.d. across the capacitor. Using Ohm's law and the basic capacitor equation, this becomes

$$V_S = IR + \frac{Q}{C}$$

During a small time interval Δt when the capacitor is charging, V_S and C do not change, $\frac{\Delta V_S}{\Delta t} = 0$, unlike I and Q, which do change. So in the small time interval we have

$$0 = \frac{\Delta I}{\Delta t}R + \frac{I}{C}\frac{\Delta Q}{\Delta t}$$

But $\frac{\Delta Q}{\Delta t} = I$, so

$$0 = \frac{\Delta I}{\Delta t}R + \frac{I}{C}$$

or

$$\frac{\Delta I}{\Delta t} = -\frac{I}{RC}$$

Rearranging and replacing with calculus notation gives

$$\frac{dI}{I} = -\frac{1}{RC}dt$$

Integrating in a similar way to before gives

$$I = I_0 e^{-\frac{t}{RC}}$$

At $t = 0$, $V_C = 0$ and $I = I_0 = \frac{V_S}{R}$, and so at any time, t, we can write

$$I = \frac{V_S}{R}e^{-\frac{t}{RC}}$$

or

$$V_R = V_S e^{-\frac{t}{RC}}$$

Going back to Kirchhoff's second circuit law,

$$V_C = V_S - V_R$$

and substituting for V_R gives

$$V_C = V_S - V_S e^{-\frac{t}{RC}}$$

Factorising this gives

$$V_C = V_S(1 - e^{-\frac{t}{RC}})$$

and then using the capacitor equation we obtain

$$Q = CV_S(1 - e^{-\frac{t}{RC}})$$

But CV_S is the maximum charge that can be stored on the capacitor when it is fully charged. That is equal to Q_0, the initial charge on the capacitor when it is about to discharge. So finally we have

$$Q = Q_0(1 - e^{-\frac{t}{RC}})$$

EXAMPLE

Charging a capacitor

A 3.0V battery charges an initially uncharged 10000μF capacitor through a 1000Ω resistor, as shown by the circuit diagram in Figure 6.26.

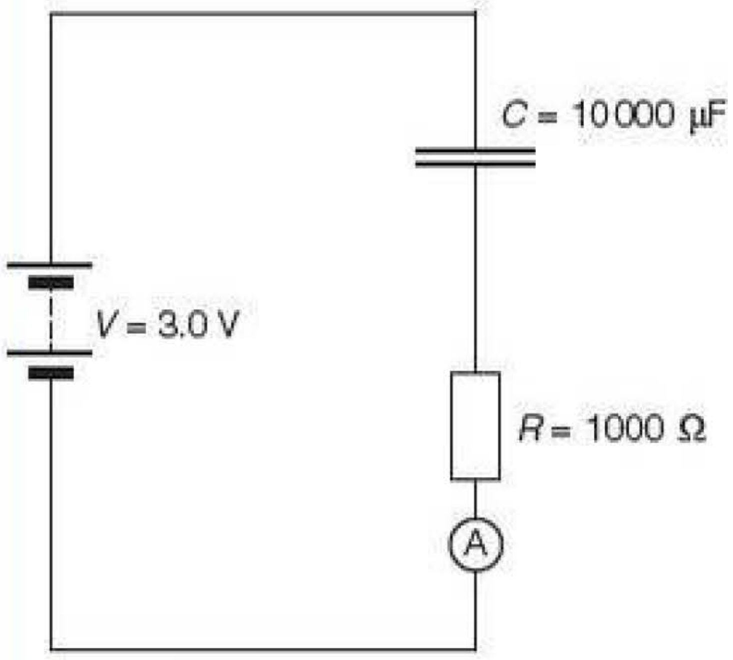


Figure 6.26

1 Calculate the voltage across the capacitor 25s after the switch is closed to charge the capacitor, which was initially uncharged.

Answer

The charging voltage is given by

$$V = V_0(1 - e^{-\frac{t}{RC}})$$

so

$$V = 3.0\,\text{V} \times (1 - e^{-\frac{25\,\text{s}}{1000\,\Omega \times 0.01\,\text{F}}})$$
$$= 2.75\,\text{V} = 2.8\,\text{V} \ \text{(2 s.f.)}$$

2 Calculate the charging current after 15s.

Answer

The current is given by

$$I = I_0 e^{-\frac{t}{RC}}$$

where I_0 is the initial charging current given by $I_0 = \frac{V_0}{R}$, so

$$I = \frac{V_0}{R} e^{-\frac{t}{RC}} = \frac{3.0\,\text{V}}{1000\ \Omega} e^{-\frac{15\,\text{s}}{1000\,\Omega \times 0.01\,\text{F}}}$$
$$= 6.7 \times 10^{-4}\ \text{A}$$

REQUIRED PRACTICAL 9

Investigating the charging and discharging of *capacitors*

Note: This is just one example of how you might tackle this required practical.

A student carries out an experiment to determine the capacitance of an unknown capacitor that she has recovered from a large stage amplifier. She connects up the circuit in Figure 6.27 using a battery pack, a variable resistor, a two-way switch and a data logger set to measure the current in mA as a function of time.

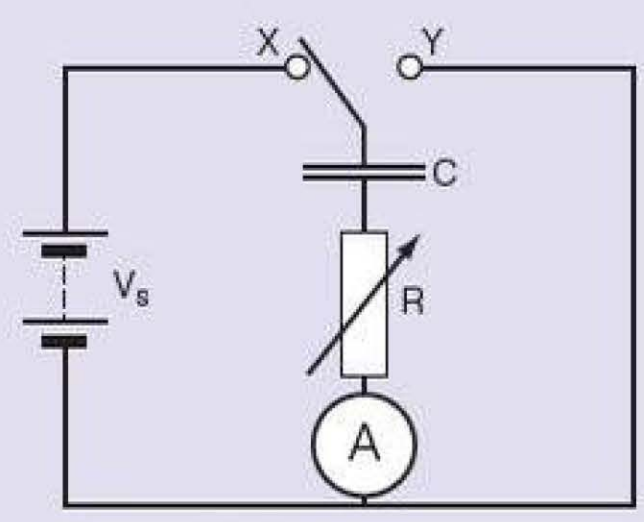


Figure 6.27

She sets the battery pack to 6.0V and the variable resistor to 880Ω. She ensures that the capacitor is completely discharged by earthing its connections before reconnecting into the circuit and moving the switch to position X. She then records the charging current (in mA) every 10s for 100s from the data logger, before moving the switch immediately to position Y and recording the discharging current every

⇨

10 s for a further 100 s. Her results are shown in Table 6.5. The data logger acting as an ammeter is able to measure the charging current as a positive current and the discharging current as a negative current.

Table 6.5

Status	Time, *t*/s	Current, *I*/mA
Charge	0	6.81
	10	3.97
	20	2.31
	30	1.34
	40	0.78
	50	0.46
	60	0.27
	70	0.15
	80	0.09
	90	0.05
	100	0.03
Discharge	100	−6.81
	110	−3.50
	120	−2.20
	130	−1.40
	140	−0.65
	150	−0.46
	160	−0.30
	170	−0.15
	180	−0.08
	190	−0.06
	200	−0.03

1 Plot a graph of the current (*y*-axis) against time (*x*-axis) – show the charging and discharging phases on the same graph.
2 Use your graph to estimate the total charge stored on the capacitor when it is fully charged. (Remember, the currents are in mA.)
3 Use your graph to estimate a value for the time constant, *RC*, of the circuit. Hence make an estimate of the capacitance of the capacitor.
4 Make a copy of the part of Table 6.5 showing the capacitor discharging. Add a further column to your table showing the natural logarithm of the modulus (magnitude) of the discharging current – for example, the first value, at 100 s, is the natural logarithm of 6.81 (= 1.92), followed by ln(3.50) = 1.25.
5 Plot a second graph of ln *I* (*y*-axis) against *t* (*x*-axis).
6 Use your log graph to calculate a value for the time constant, RC, of the circuit and hence another value for C, the unknown capacitance. including an estimate of the uncertainty of the value.
7 Compare your values from parts 3 and 6.

Practice questions

1 What is the charge stored on a 220 μF capacitor with a 6 V p.d. across it?

A 36.6 mC
B 2.70 mC
C 1.32 mC
D 3.96 mC

2 What is the energy stored in a 4200 μF capacitor with a 3 V p.d. across it?

A 13 mJ
B 0.93 mJ
C 1.3 mJ
D 19 mJ

3 A constant current of 208 μA is used to charge an initially uncharged capacitor or with a capacitance of 24 μF. How long will it take for the p.d. across the capacitor to rise to 2600 V?

A 54 s
B 108 s
C 166 s
D 300 s

4 Two capacitors, P and Q, are both charged by the same 12 V battery. The capacitance of P is 4200 μF, and the capacitance of Q is 420 μF. Which row of the table gives the correct ratios of energy and charge stored by each capacitor?

	$\frac{\text{energy stored in P}}{\text{energy stored in Q}}$	$\frac{\text{charge stored in P}}{\text{charge stored in Q}}$
A	1	1
B	10	1
C	10	10
D	1	10

5 A 2000 μF capacitor has been fully charged by a 6.0 V battery, before it is discharged through a 10 kΩ resistor. What is the charge stored by the capacitor 20 s after the discharge begins?

A 4.4 mC
B 2.2 mC
C 8.8 mC
D 1.1 mC

6 A capacitor of capacitance *C* is charged through a resistor of resistance *R* to a p.d. of 10 V. The capacitor is then allowed to discharge back through the same resistor, *R*, as shown in Figure 6.28, until the p.d. across the capacitor is 5 V, before being recharged again back up to 10 V.

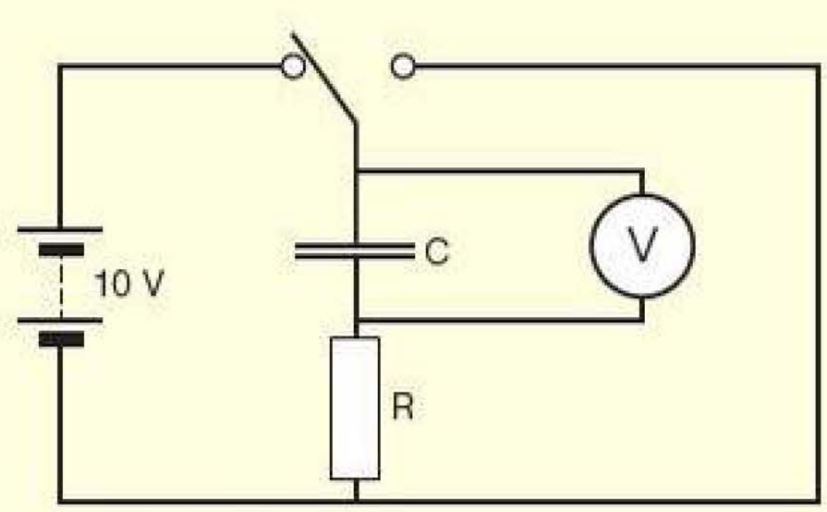


Figure 6.28

Which of the graphs in Figure 6.29 best shows how the p.d. across the capacitor varies with time during the discharge and the subsequent recharging?

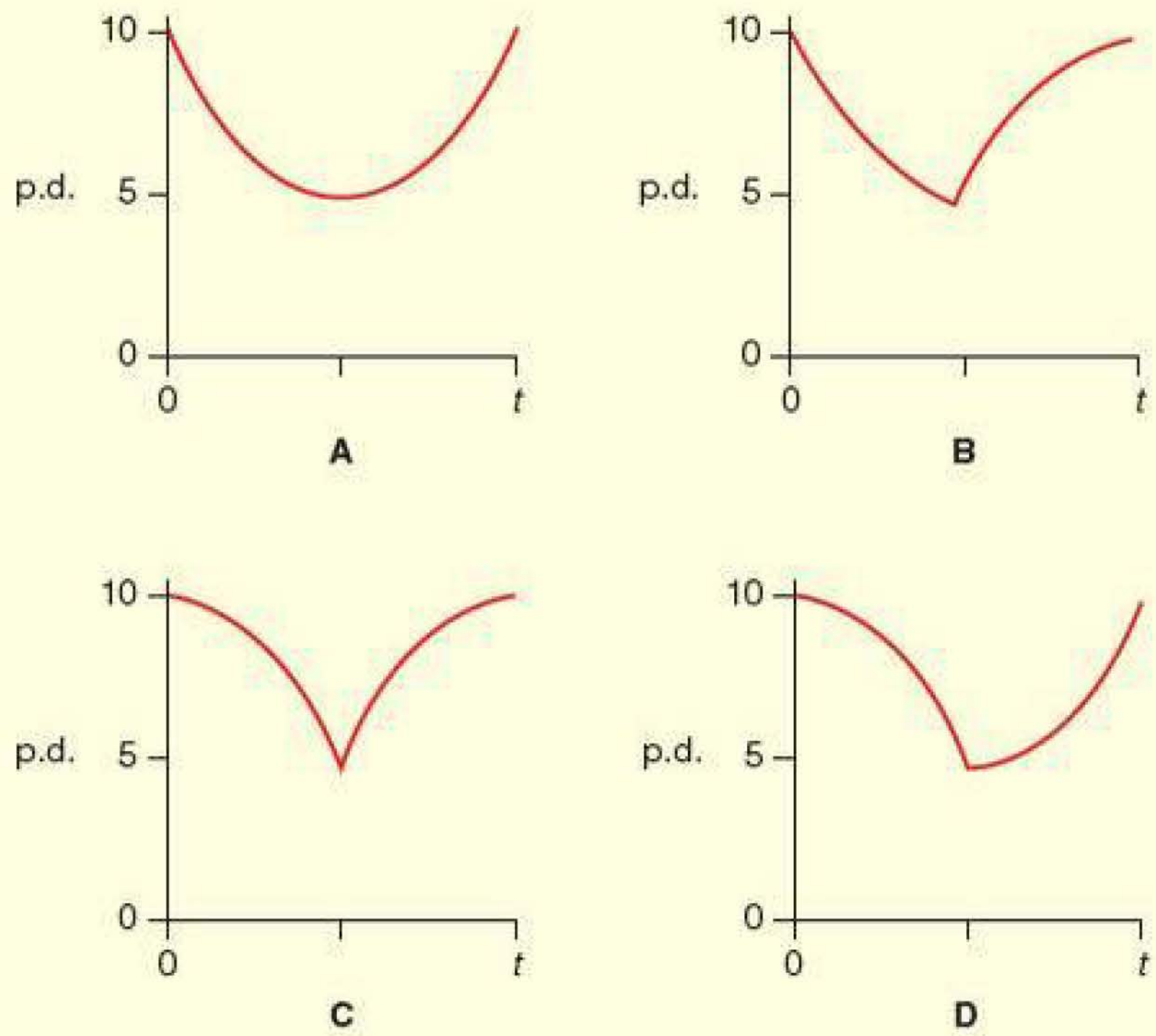


Figure 6.29

7 An initially fully charged capacitor, *C*, discharges through a resistor, *R*, and loses half its charge in 20 s. The time constant, *RC*, of the circuit is

A 19 s

B 9 s

C 39 s

D 29 s

8 Figure 6.30 shows how the p.d. across a capacitor varies with the charge stored on it.

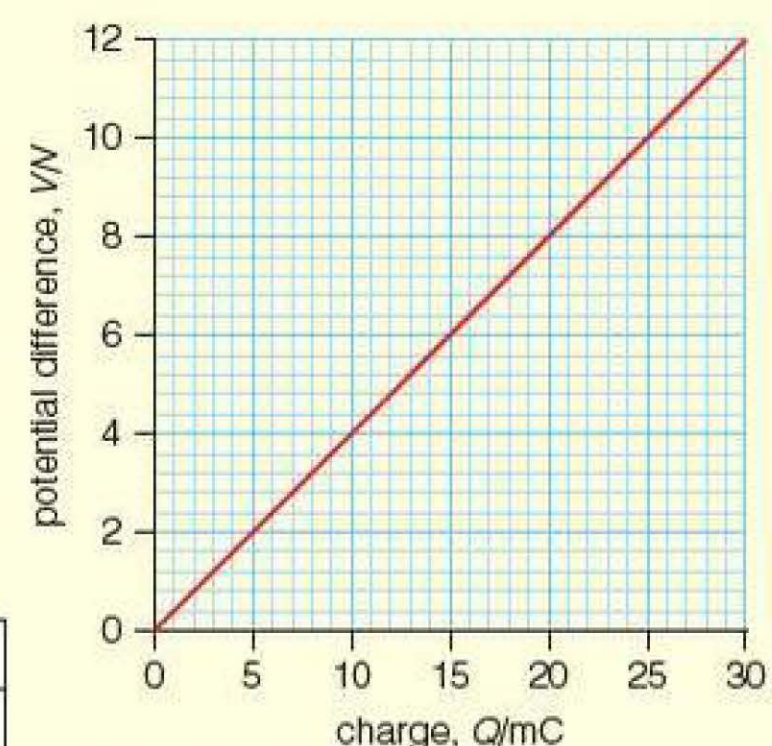


Figure 6.30

The table shows possible values of the capacitance and the energy stored by the capacitor when the p.d. across it is 12 V. Which row gives the correct values?

	Capacitance, *C*/mF	Energy stored, *E*/mJ
A	2.5	90
B	2.5	180
C	0.4	90
D	0.4	180

9 A 12 V car battery is used to fully charge an 18 mF capacitor. The capacitor then fully discharges through a small electric motor, which lifts a 200 g mass stack. If the motor lifts the mass stack with a 10% efficiency, through what height will the mass stack be lifted?

A 3 cm

B 6 cm

C 30 cm

D 60 cm

10 Figure 6.31 shows how the charge stored on a capacitor, of capacitance *C*, varies with time as it discharges through a resistor, *R*.

What is the time constant *RC* of this circuit?

A 6.3 s **C** 4.4 s

B 4.2 s **D** 3.0 s

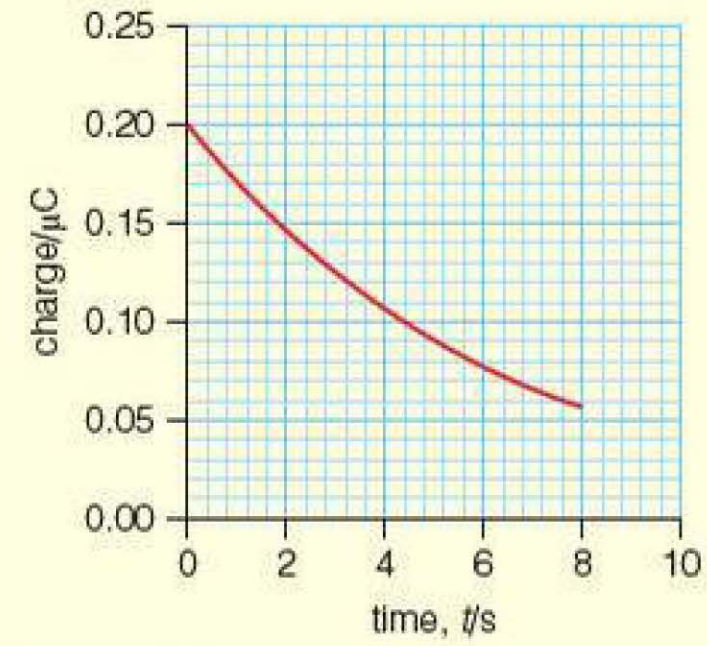


Figure 6.31

11 Figure 6.32 shows how the charge, *Q*, stored on a capacitor varies with the p.d. across it, *V*.

Which of the following statements is not correct?

A The energy stored on the capacitor can be calculated by measuring the area under the graph.

B The gradient of the graph is numerically equivalent to the capacitance of the capacitor.

C If the charge stored on the capacitor was doubled, the energy stored would quadruple.

D Doubling the capacitance would halve the gradient of the graph.

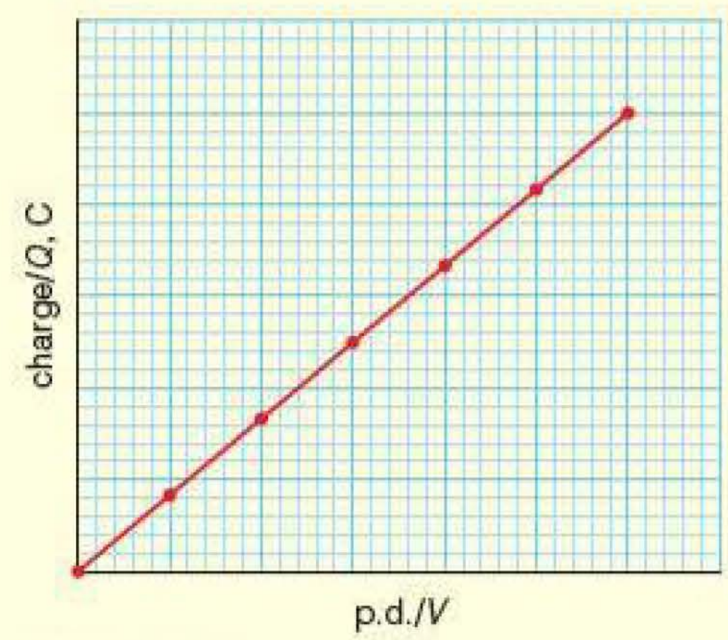


Figure 6.32

12 A capacitor, *C*, is charged with a potential difference of 12.0 V, and then discharges through a resistor, *R*, where $R = 50\,k\Omega$. A student measured the p.d. across the capacitor every 5 s using a data logger with graph-plotting software. The graph in Figure 6.33 shows his results.

a) Use the graph to calculate:

i) the initial discharge current flowing through the resistor *(1)*

ii) the time constant of the circuit, including the correct unit *(4)*

iii) the capacitance, *C*, of the capacitor *(2)*

iv) the charge stored on the capacitor after 30 s. *(3)*

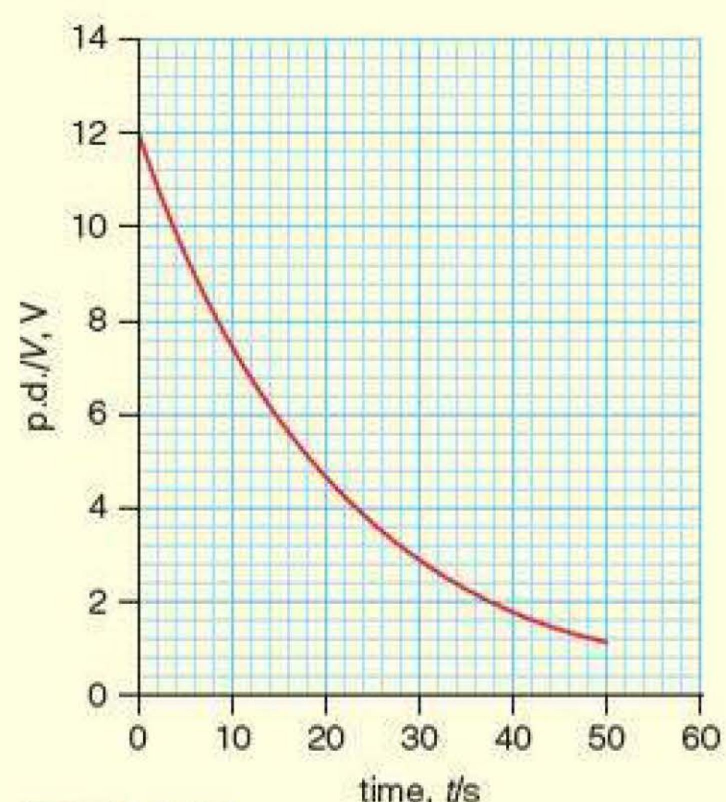


Figure 6.33

b) A garden automatic sprinkler system contains a time delay circuit using an identical capacitor to that used in part (a). The capacitor is charged using a 6 V battery and discharges through a similar $50\,k\Omega$ resistor.

i) The smaller 6 V p.d. from the battery changes the energy stored by the capacitor. State and explain how the energy stored on the capacitor changes compared to the value calculated in a)iv). *(2)*

ii) State and explain the effect of this change in p.d. on the time constant of the circuit. *(1)*

13 A resistor-capacitor circuit is used as a timing mechanism for an experiment to measure the acceleration due to gravity. The experimental set-up is shown in Figure 6.34.

Switch 1 is initially closed, keeping the capacitor *C* charged at 6.0 V. Switch 2 is also initially closed. The steel ball is dropped, opening switch 1, disconnecting the capacitor from the battery. The capacitor then starts to discharge through resistor R. The ball falls, opening switch 2, stopping the discharge through the resistor.

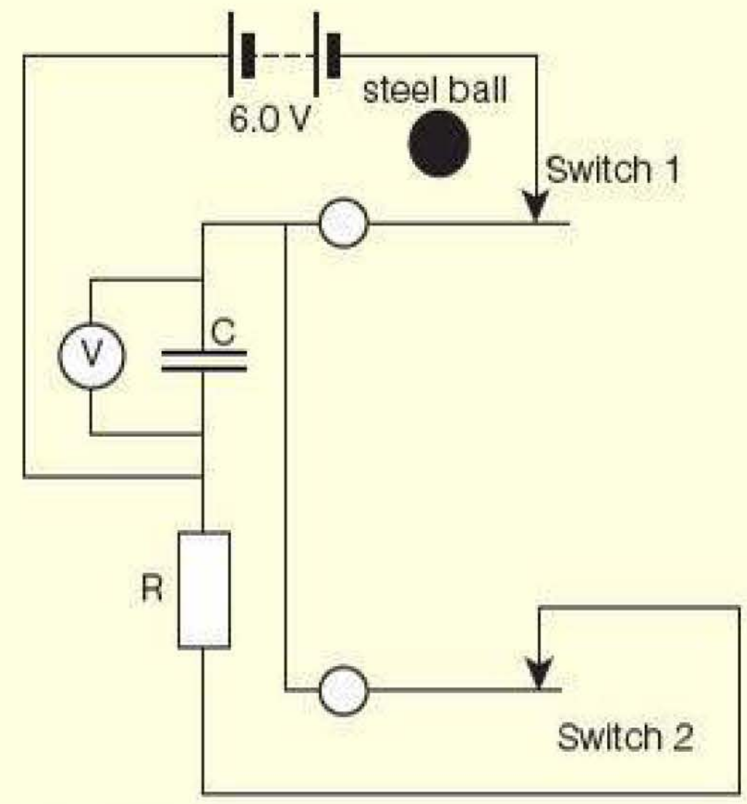


Figure 6.34

a) Describe the measurements that need to be made in this experiment, and explain how these measurements could be used to calculate the acceleration due to gravity. The quality of your written communication will also be assessed in this question. *(6)*

b) In one such experiment, the value of *C* was 440 μF and *R* was 10 kΩ. The p.d. of the battery was 6.0 V and the distance between the switches was 1.0 m. The voltmeter reading was initially 6.0 V and dropped to 5.4 V. Using this information, calculate the time for the ball to drop between the two switches. *(3)*

c) Use your answer to part b) to calculate the acceleration due to gravity. *(2)*

14 Many tablet computers use a capacitor as well as a rechargeable battery to store electrical energy. The rechargeable battery provides most of this electrical energy, but the capacitor is used as an emergency back-up, if the battery is suddenly removed or fails during operation. The capacitor stores just enough electrical energy to shut down the tablet safely.

a) Calculate the electrical energy stored in a capacitor of capacitance 12 800 μF found in a tablet computer operating with a lithium-ion 3.6 V battery. *(2)*

b) The 3.6 V lithium-ion battery can deliver a steady current of 0.84 mA for 3 hours. Show that the battery can store about 400 times more electrical energy than the capacitor. *(2)*

c) State two reasons why a capacitor would be unsuitable as the main energy store for a tablet computer. *(2)*

15 a) Explain what is meant by the 'capacitance' of a capacitor. *(2)*

b) A capacitor of 'capacitance' *C* is charged from a 9 V battery through a fixed resistor of resistance *R*. The graph in Figure 6.35 shows how the charge *Q* varies with time after the capacitor and resistor are connected in series with the battery.

Using the maximum charge stored on the capacitor, as determined from the graph, calculate the capacitance of the capacitor. *(3)*

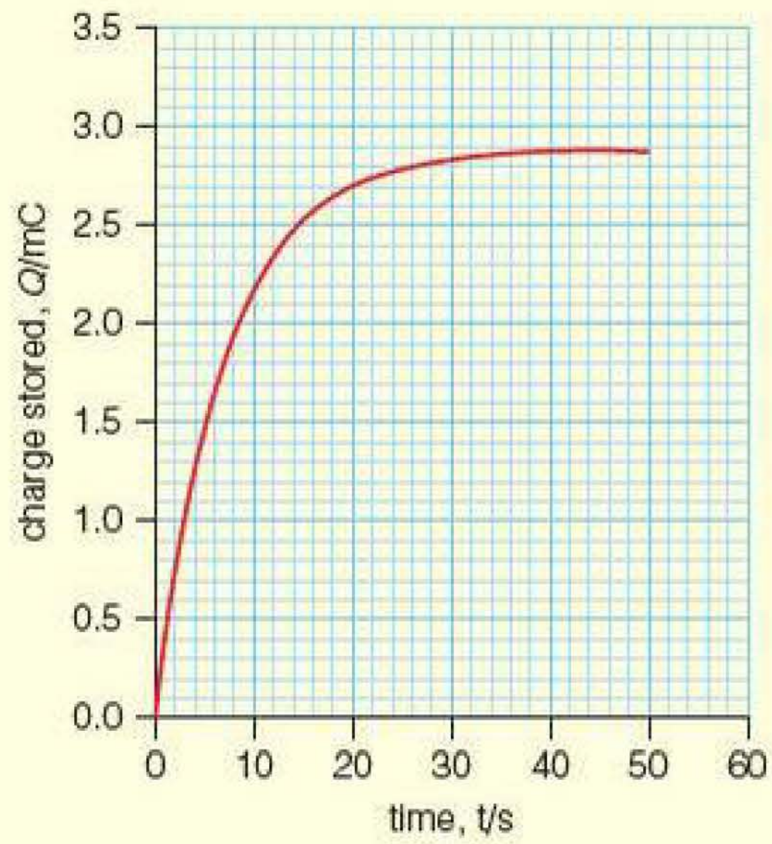


Figure 6.35

c) The time constant for a capacitor-charging circuit is the time taken for the charge to rise to 63% of its maximum value. Use the graph to determine the time constant of this circuit. (2)

d) Calculate the resistance of the resistor. (2)

e) State what value is represented by the gradient of the graph. (1)

f) Calculate the initial current flowing through the resistor during charging. (1)

g) Sketch a graph to show how the current flowing though the resistor varies with time for the 50 s following connection to the battery. (2)

16 A 640 μF capacitor is initially fully charged from a 12 V battery. The capacitor is then discharged through a 48 kΩ resistor.

a) Use this data to calculate:

i) the time constant of this circuit (1)

ii) the initial discharge current through the resistor (1)

iii) the initial charge stored by the capacitor (1)

iv) the initial energy stored by the capacitor. (1)

b) The capacitor is disconnected from the resistor after 40 s. Without losing any of its charge, it is connected to a second resistor of resistance 24 kΩ. Calculate:

i) the charge stored by the capacitor at the start of the discharge through the 24 kΩ resistor (2)

ii) the initial p.d. across the 24 kΩ resistor (1)

iii) the total energy transferred to the 24 kΩ resistor. (1)

17 A physics technician finds an unlabelled capacitor in a drawer and sets up the circuit shown in Figure 6.36 with a data-logging ammeter probe together with graph plotting software, to measure the capacitance of the capacitor.

The technician closes the switch, which charges the capacitor. She then opens the switch, allowing the capacitor to discharge through the data-logging ammeter and the resistor. The data logger records the current every 5 s after she opens the switch, and then draws a graph of the results, which is shown in Figure 6.37.

a) Use the graph to show that the resistance of R is 120 kΩ. (2)

b) Use the graph to determine the 'half-time' of the decay, and hence calculate a value for the time constant of the circuit. (2)

c) Calculate the capacitance, C, of the capacitor. (1)

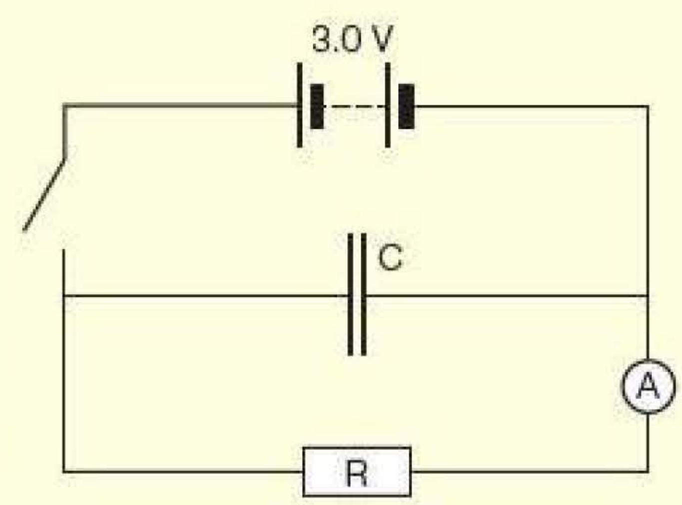


Figure 6.36

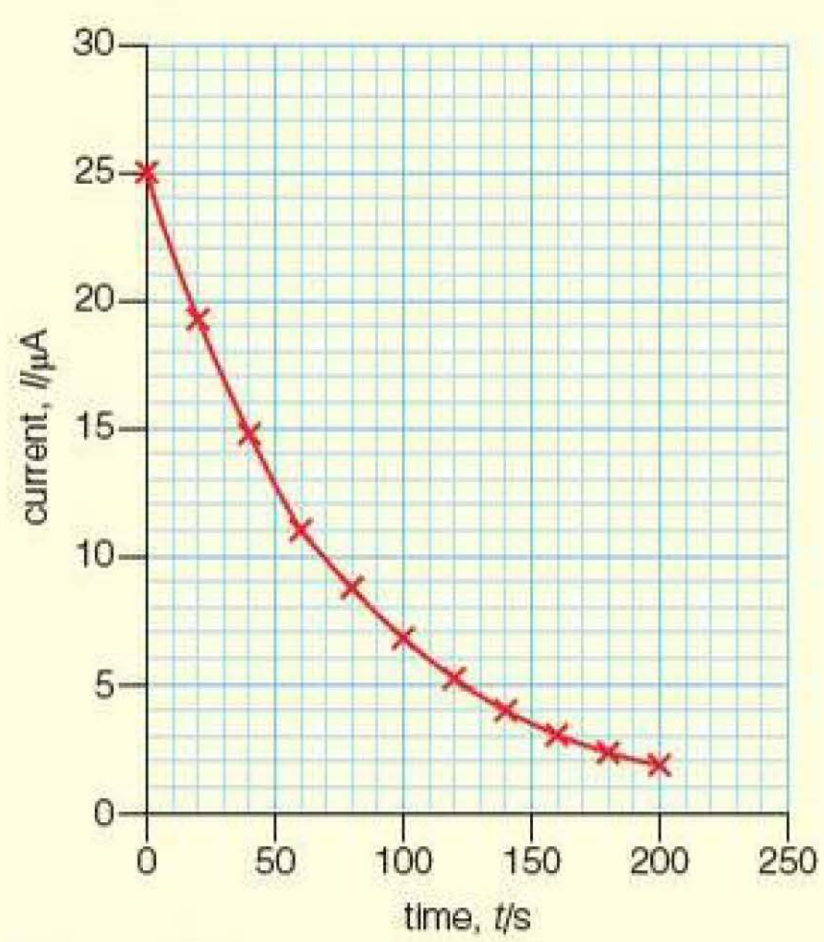


Figure 6.37

Stretch and challenge

The questions that follow are British Physics Olympiad questions.

18 A thundercloud has a horizontal lower surface area of 25.0 km^2, 750 m above the surface of the Earth.

a) Using a capacitor as a model, calculate the electrical energy, E stored when its potential is 1.00×10^5 V above the earth potential (0 V).

b) The cloud rises to 1250 m.

i) Explain whether the energy stored, has increased or decreased.

ii) What is the change in electrical energy, ΔE?

(BPhO R1-2012 Q1(h))

19 Two uncharged capacitors C_1 and C_2, with capacitances C_1 and C_2, are connected in series with a battery and a switch S. When the switch is closed there is a charge Q_1 on C_1 and Q_2 on C_2.

a) What is the relation between Q_1 and Q_2?

b) Give an expression for the potential difference across each capacitor.

c) Derive an expression for the capacitance C of a single capacitor equivalent to C_1 in series with C_2.

d) Calculate the total energy stored in the capacitors.

(BPhO R1-2007 Q5)

20 Two capacitors, of capacitance 2.0 μF and 4.0 μF, are each given a charge of 120 μC. The positive plates are now connected together as are the negative plates. Calculate:

a) the new p.d. between the plates of the capacitors

b) the change in energy. Explain this change.

(BPhO R1-2010 Q5)

7 Magnetic fields

PRIOR KNOWLEDGE

Before you start, make sure that you are confident in your knowledge and understanding of the following points:

- Magnets have two poles: a north-seeking pole (N, or north pole) and a south-seeking pole (S, or south pole).
- A magnetic field is a region where the magnet exerts a force on other magnets or magnetic materials, even if they are not in contact.
- A magnet attracts magnetic materials (e.g. iron) placed in its magnetic field provided the field is non-uniform. The magnet attracts another magnet if unlike poles are facing (north–south), and repels another magnet if like poles are facing (north–north or south–south).
- A current-carrying wire creates a magnetic field, which encircles the wire carrying the field.
- An electromagnet can be made by making a coil of wire and passing a current through it. The strength of an electromagnet increases if the current in the wire increases, if more turns are added to the coil, and if a magnetic core is placed inside the coil.
- Electromagnets made using a soft magnetic core (e.g. iron) lose their magnetism if the current is turned off. If a hard magnetic material is used (e.g. steel), the electromagnet retains its magnetism when the current is turned off.

TEST YOURSELF ON PRIOR KNOWLEDGE

1. Magnetism causes a non-contact force. Give examples of two other non-contact forces, and what these forces affect.
2. Describe how to make a strong, temporary magnet.
3. A compass needle is magnetic. Explain why the north pole of a compass needle is attracted to the Earth's geographic North Pole.
4. Iron can be magnetised if it is stroked repeatedly in one direction using a magnet. Iron atoms behave like mini-magnets. Magnetic domains are regions where groups of atoms line up in one direction. Explain how iron can be magnetised in terms of magnetic domains.

Experiments using radiation up to 10 billion times brighter than sunlight are carried out at the Diamond Light Source facility in Oxfordshire (Figure 7.1). Diamond is a synchrotron which accelerates beams of electrons to the speed of light. Very strong magnets direct the electrons through a pipe 56 Zm circumsference and under ultra-high vacuum. The electrons lose energy, emitted as synchrotron light, as they change direction in the magnetic field. Synchrotron light ranges from infrared radiation to x-rays.

Electrons also travel through magnets set out in arrays called insertion devices. These are even stronger magnets, some of which are super-conducting. Since the direction of the magnetic field repeatedly changes, the electrons are forced to wiggle through the device. The electrons release energy when they change directions, either as extremely intense electromagnetic radiation turned into specific frequencies, or a broad spectrum of radiation, depending on the arrangements of the magnets.

Figure 7.1 Diamond in Oxfordshire.

Magnetic flux lines

Magnets have a north pole and a south pole, so we call magnets dipoles. A *magnetic field* is the region where a magnet exerts a force on objects made from magnetic materials, or on other magnets. We represent the magnetic field using arrows, or *flux lines*, to indicate the direction and strength of the field in the region surrounding the magnet. The rules for drawing magnetic flux lines, electric field lines and gravitational field lines are similar. For magnets, these rules are as follows:

- Flux lines represent the direction of the force experienced by the north pole of a magnet at any point in the magnetic field. They run from the north pole to the south pole.
- A magnetic field is strongest where its flux density is highest, and this is shown as flux lines closest together (Figure 7.2).
- A magnetic field with twice the strength is drawn with twice the number of flux lines per unit area in the same region.
- The magnetic field of more than one magnet is the combined field of the individual magnets.
- Flux lines never cross.
- If there is more than one magnet, the magnetic fields cancel out in some places and there is a neutral point (Figure 7.3).

A magnet freely suspended in a magnetic field will align itself with the field.

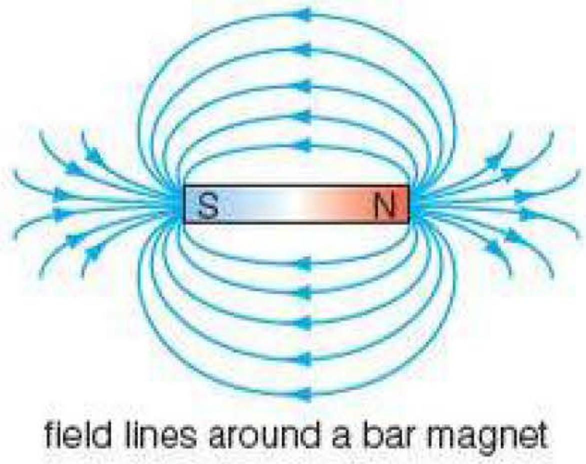


Figure 7.2 Flux lines for a bar magnet. The flux density is highest at the poles, where the field is strongest.

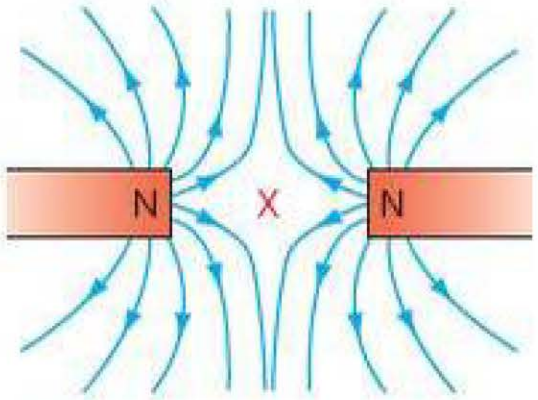


Figure 7.3 The combined field from two bar magnets. Neutral points are marked X.

Magnetic flux density The number of magnetic flux lines that pass through an area of 1 m^2

Tesla The flux density that causes a force of 1 N on a 1 m wire carrying a current of 1 A at right angles to the flux.

The strength of a magnetic field

The strength, or intensity, of a magnetic field is its **magnetic flux density**, *B*, also known as a *B*-field. Magnetic flux density is measured in **teslas** (T). Magnetic field or flux lines are a model that helps us to visualise the field. You will learn more about magnetic flux, ϕ, in Chapter 8.

If you look at a diagram of a magnetic field, you can see that the flux lines are closer together in places (for example, at the poles of a bar magnet as shown in Figure 7.2). You can tell that the magnetic field and the force produced by the magnet are stronger at the poles because this is where the flux lines are closer together.

Magnetic fields from current-carrying wires

Moving charges cause a magnetic field, which we describe using flux lines. The magnetic flux around a current-carrying wire is shown as concentric circles, indicating the magnitude and direction of the flux pattern. Moving away from the wire, flux lines are further apart because the field gets weaker. If you look at the wire with the conventional current flowing away from you, the flux lines circle the wire in a clockwise direction. Symbols inside the wire indicate the current direction: ⊗ indicates a current flowing away from you (Figure 7.4a) and ⊙ indicates a current flowing towards you (Figure 7.4b). The combination of flux lines for a loop of wire is shown in Figure 7.5.

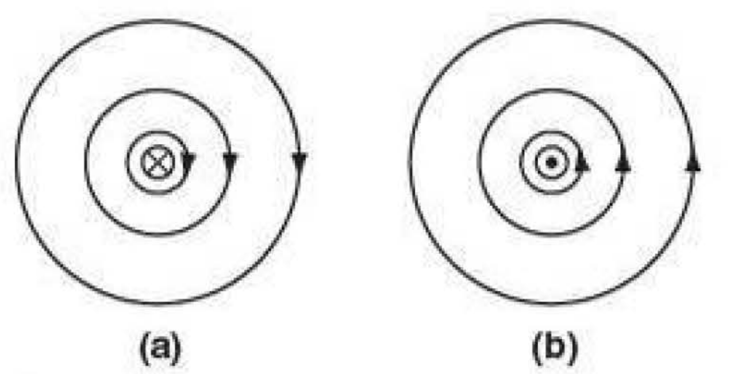


Figure 7.4 The magnetic flux of a current-carrying wire: (a) with the current flowing into the page and (b) with the current coming out of the page.

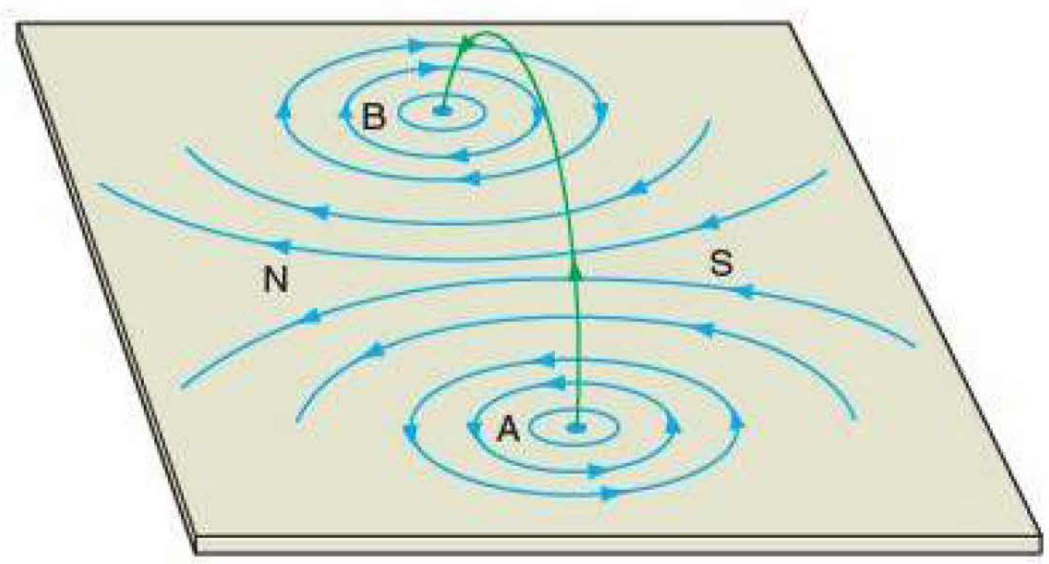


Figure 7.5 A magnetic field circles a current-carrying wire.

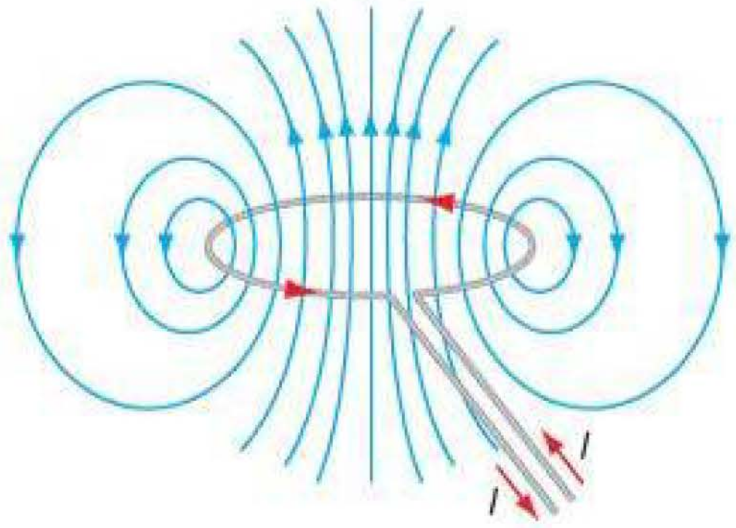


Figure 7.6 The magnetic field for a current-carrying loop of wire.

A *solenoid* is a current-carrying coil of wire that produces magnetic flux (Figure 7.7) – this is also described as an electromagnet formed from a coil of wire. The magnetic flux outside a solenoid is similar to the magnetic flux for a bar magnet, with the north pole at one end of the coil and the south pole at the other end, depending on the current direction.

A current-carrying wire in a magnetic field moves because a force acts on it. The magnetic field making the wire move is called a *catapult field*. The catapult field is due to the combined effect of the current-carrying wire's flux and the static flux. Figure 7.8 shows the separate fluxes, and how they combine to form a catapult field.

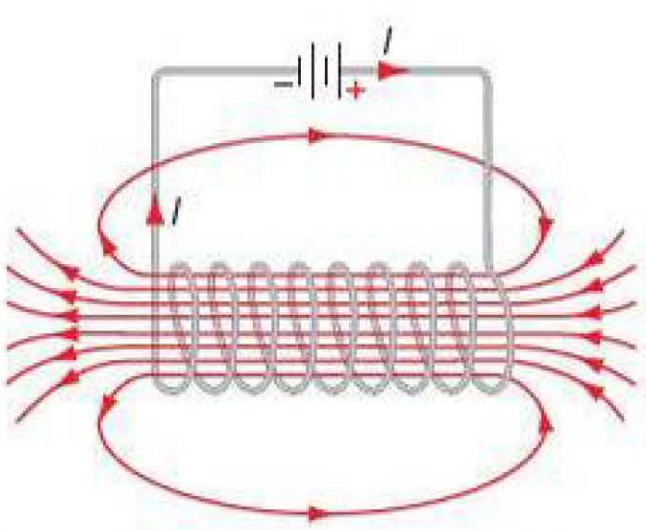


Figure 7.7 The magnetic field for a solenoid.

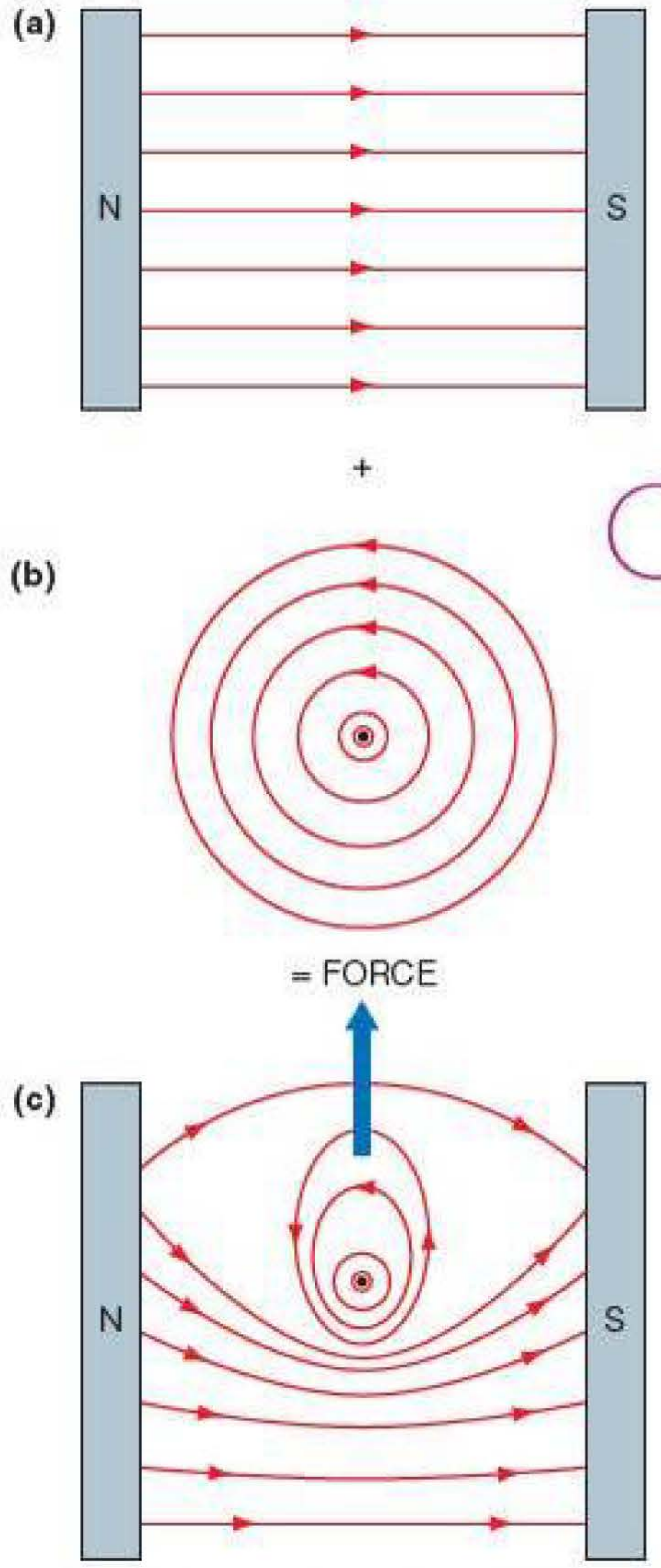


Figure 7.8 (a) A uniform magnetic field. (b) The field around a current-carrying wire. (c) The catapult field. The force is from the strong to the weak field.

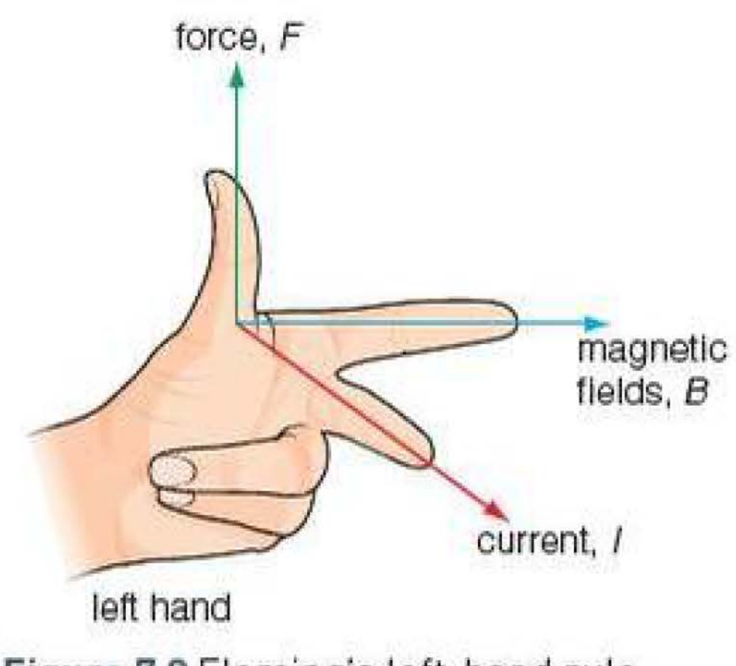


Figure 7.9 Fleming's left-hand rule.

Measuring magnetic field strength

A magnetic field runs through a coil of wire as well as outside it, as shown in Figures 7.6 and 7.7. You can measure the flux density of the field using a magnetic field sensor called a Hall probe. The probe contains a slice of semiconducting material. If a current flows in the semiconductor when it is perpendicular to the magnetic flux, a potential difference is generated across the sides of the semiconductor. This potential difference is directly proportional to the flux density.

Forces on current-carrying wires

You need to know how to calculate the size and direction of a force on a current-carrying wire in a magnetic field.

Calculating the direction of the force

Magnetic flux density is a vector quantity. When the directions of the magnetic flux, the current in the conductor and the force are all at right angles to each other, Fleming's left-hand motor rule, shown in Figure 7.9, helps you see the three-dimensional arrangement of these vectors.

Hold your thumb, first finger and second finger of your left hand at right angles to each other. The thuMb represents the direction of the force (Motion), the First finger represents the direction of the magnetic Field, and the seCond finger represents the direction of the Current.

EXAMPLE

Fleming's left-hand rule

A current-carrying wire is held so that the current is into the page, and the magnetic field direction is from the bottom to the top of the page. Use Fleming's left-hand rule to find the direction of the force on the wire.

Answer

With your second finger (current) pointing into the page, and your first finger (field) pointing from bottom to top of the page, you should find that the direction of the force is from left to right.

MATHS BOX

If the wire is at an angle θ to the flux, the force on the wire is calculated using $F = BIl \sin \theta$, where θ is the angle between the wire, carrying the current, and the flux lines (Figure 7.10). When the wire lies parallel to the flux lines ($\theta = 0$), there is no force on the wire.

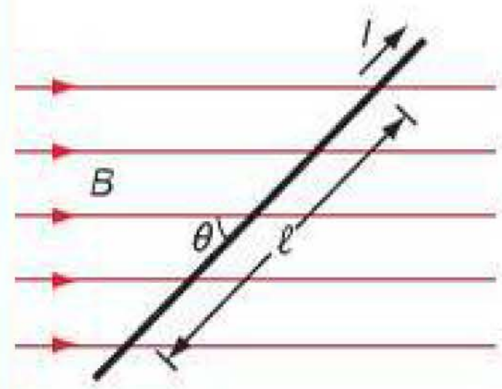


Figure 7.10

Calculating the size of the force

We can measure the flux density at any point by measuring the force exerted on a current-carrying wire at that point. The tesla is defined as the flux density that causes a force of 1 N on 1 m of a wire carrying a current of 1 A at right angles to the magnetic field. Written as an equation, this becomes

$$F = BIl$$

where F is force (N), B is magnetic flux density (T), I is current (A) and l is the length of the conductor in the field (m)

For the interested student, the Maths box gives more information about the size of the force.

EXAMPLE

Horseshoe magnet

Figure 7.11 shows a current-carrying wire held perpendicular to the field between the two poles of a horseshoe magnet. Assume there is 2 cm of the wire in the field.

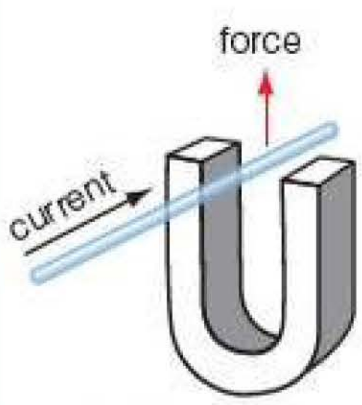


Figure 7.11

1 Calculate the magnetic flux density (B-field) for the wire if the current is 2.1 A and it experiences an upward force of 0.03 N.

Answer

The length of the wire in the field is 0.02 m. Substitute known values into the equation:

$$B = \frac{F}{Il} = \frac{0.03\,\text{N}}{2.1\,\text{A} \times 0.02\,\text{m}} = 0.71\,\text{T}$$

2 What current is required for the wire to experience an upward force of 0.09 N?

Answer

Rearrange the same equation to give I on the left-hand side, and substitute for the flux density, B, from the answer to part (a):

$$I = \frac{F}{Bl} = \frac{0.09\,\text{N}}{0.71\,\text{T} \times 0.02\,\text{m}} = 6.3\,\text{A}$$

3 The current flows into the page. State whether the direction of the magnetic field is from left to right, or right to left.

Answer

Using Fleming's left-hand rule, the field is from the right side to the left side of the page.

TEST YOURSELF

1 A 50 cm wire carries a current of 0.2 A. Calculate the force due to the magnetic field on the wire if it is placed:
   a) perpendicular to the Earth's magnetic field of flux density 50 μT
   b) perpendicular to the magnetic field of an electromagnet of flux density 1.5 T.

2 A wire is placed at right angles to a magnetic field of flux density 6 mT.
   a) If the force per metre of wire is 0.03 N, calculate the current in the wire.
   b) State the value of the angle, θ, if the magnitude of the force is zero.

3 Copper has a density of 8960 kg m^{-3}.
   a) Calculate the mass of an insulated piece of copper wire, 10 cm long with a cross-sectional area of 2×10^{-6} m^2. Ignore the mass of the insulation. Two such pieces of wire are used as weights in Figure 7.12.

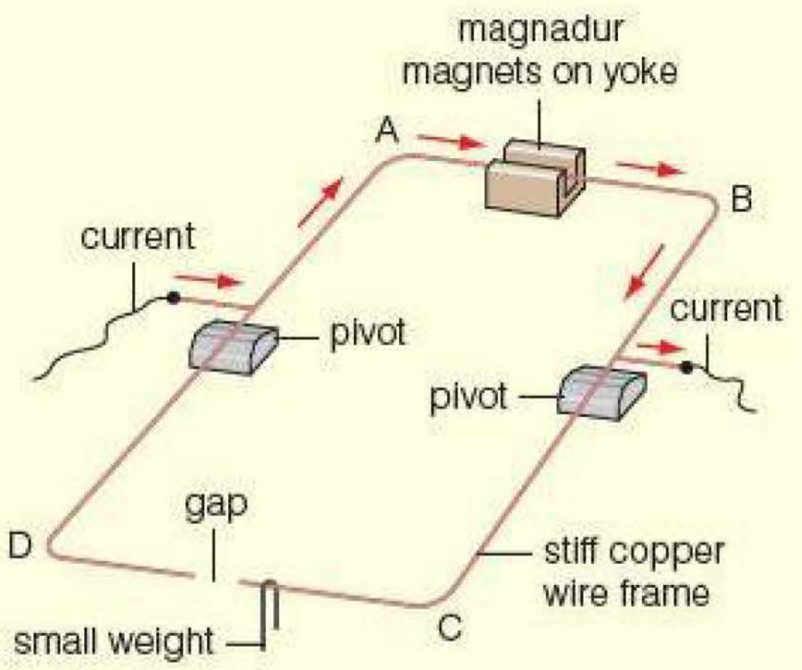


Figure 7.12

Figure 7.12 shows a current balance formed from a rectangular frame balanced on two pivots halfway along the sides AD and BC. Part of side AB lies at right angles to a uniform magnetic field. Sides AD and BC are 240 mm long. There is a small gap in side CD a small weight hangs on the wire hear the gap.

The frame balances when there is no current in the circuit. When a current of 3.2 A flows through the circuit, the weight must be moved 25 mm closer to the pivot to balance the frame.
   b) Calculate the change in the moment of the copper wire when the current is off, and when the current is on.
   c) Use your answer to part (b) to calculate the force on side AB when the current is on.
   d) The length of the wire frame in the field is 10 cm. Calculate the flux density.

4 Use Fleming's left-hand rule to find the direction of the force, magnetic field or current in the four situations shown in Figure 7.13.

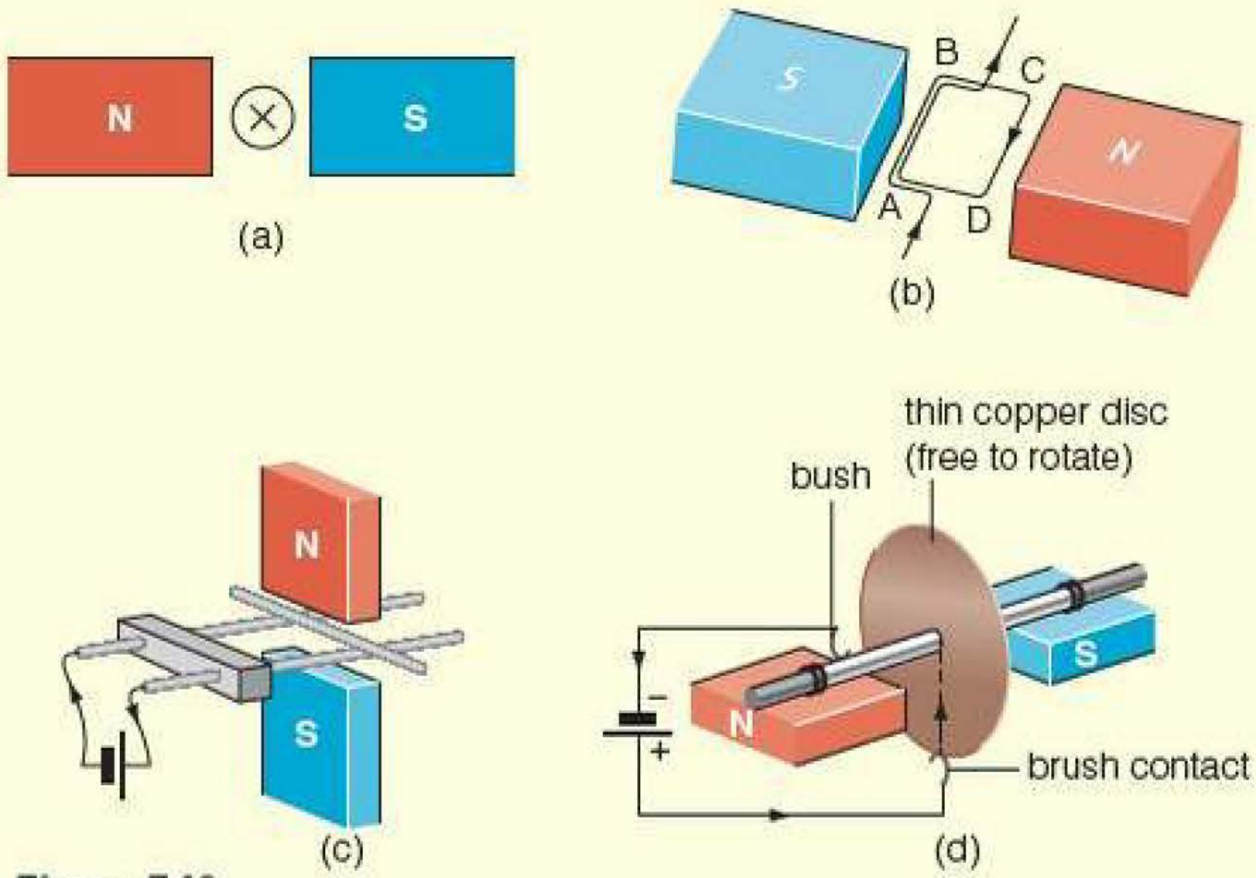


Figure 7.13

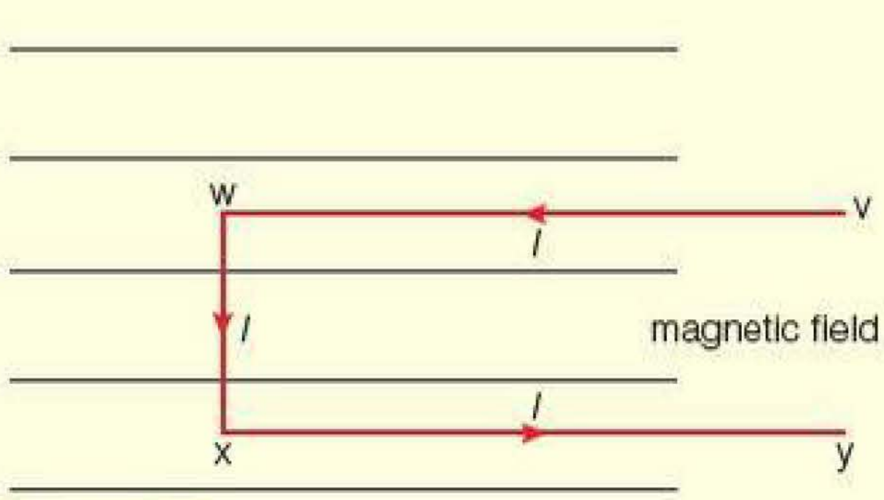


Figure 7.14

5 Part of a circuit, VWXY, lies in the same horizontal plane as a uniform magnetic field of flux density B. Two sides of the circuit are parallel to the flux, as shown in Figure 7.14.

⇨

a) When the current I flows through the wire, the side WX is forced upwards. State the direction of the magnetic field.

b) The length of wire, WX, in the field is 8.0 cm and this stem, experiences a force of 0.241 N when the current I is 4.6 A.

Calculate the flux density of the field, B, giving your answer to an appropriate number of significant figures.

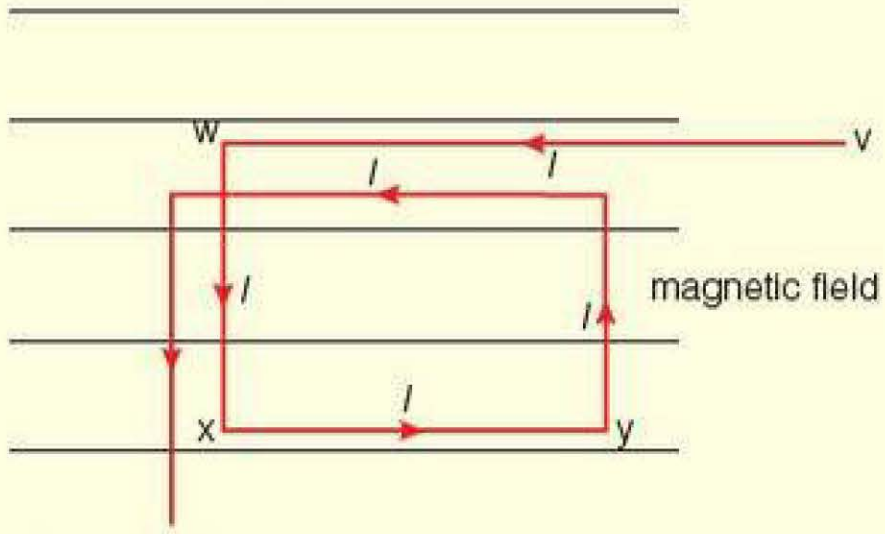


Figure 7.15

c) An extra twist of wire is made so that the wire loops and now leaves the field at X, as shown in Figure 7.15. Explain how the magnitude of the force varies in different sections of the circuit, VW, WX, XY and YV.

REQUIRED PRACTICAL 10

The force on a current-carrying conductor

Note: this is just one example of how you might tackle this required practical.

A sensitive electronic balance can be used to investigate how magnetic flux density, current and length of wire are related.

A current-carrying conductor experiences a force when it is perpendicular to a magnetic field. When magnets and a conductor are set up as shown in Figure 7.16, the electronic balance reading changes when current flows in the wire. Newton's third law means that the magnetic force on the wire is equal in size but acts in the opposite direction to the magnetic force on the magnet. The change of reading of the balance shows the magnetic force on the magnet.

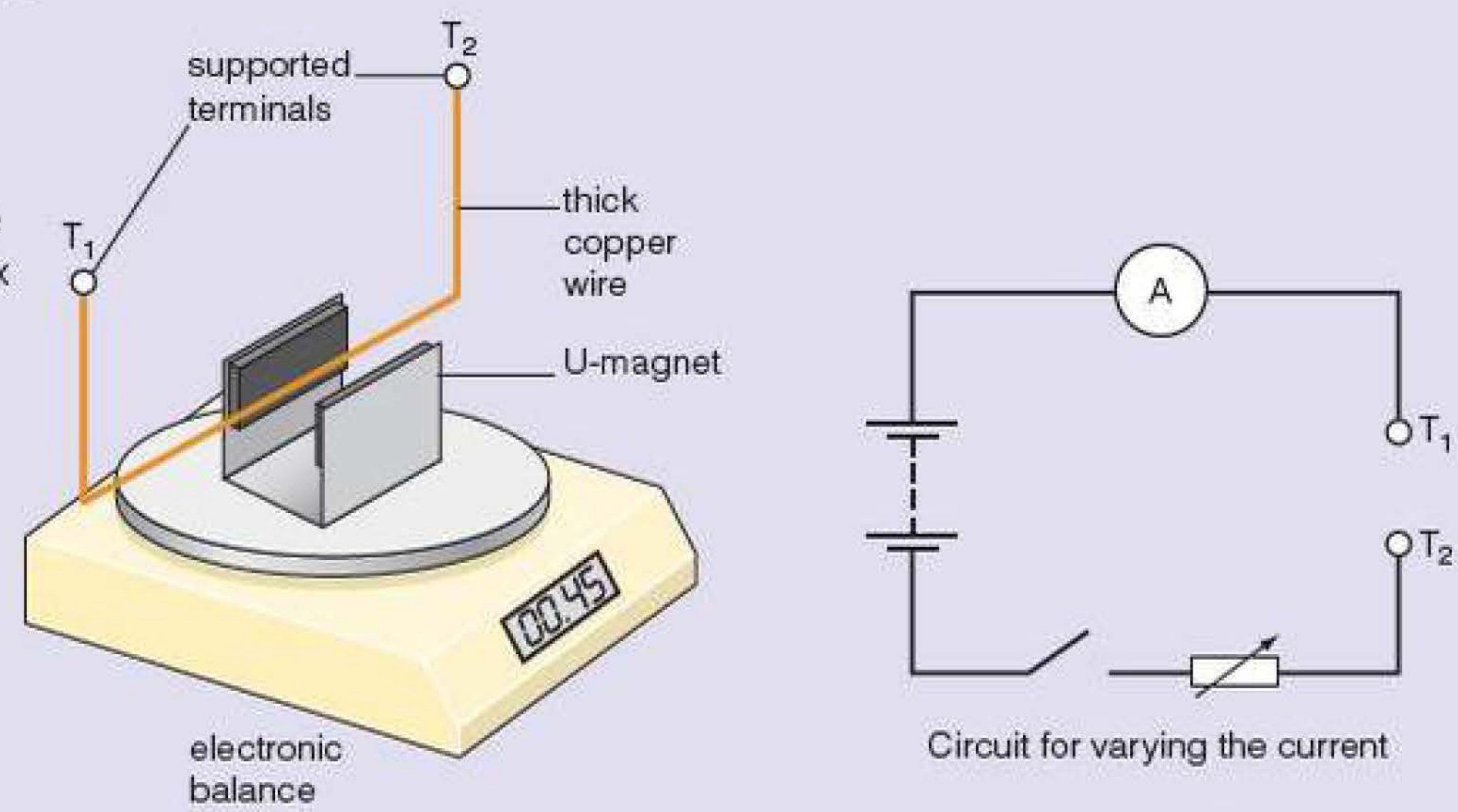


Figure 7.16 Experimental set-up for the activity, including the circuit.

Two ceramic magnets are fixed inside a steel yoke to create a uniform field. The yoke is placed on the electronic balance. The circuit is set up as shown in Figure 7.16, and the copper wire is supported horizontally between the magnets, perpendicular to the field. The balance is zeroed, and then the circuit is switched on. Readings are taken of the current (in amps) and electronic balance reading (in grams). At least six sets of readings are taken for different current values. The effect of changing the length of wire in the field can be measured by adding an extra magnet and then re-zeroing the balance before taking readings.

TIP

If this experiment is actually performed, remember that the wire will heat up, so turn the circuit off between readings. A heavy-duty rheostat is also required.

⇨

You are investigating the relationship

$$F = BIl$$

Force F is calculated by converting readings in grams from the electronic balance to force. The magnetic flux density B (T) is constant if you use the same magnets and same arrangement. The length of wire between the magnets is l, measured in metres. Some sample results are shown in Table 7.1. The length of the magnets is 4.5 cm.

Current/A	Reading from balance/g	Force/N
1.0	0.46	
2.0	0.91	
3.0	1.35	
4.0	1.83	
5.0	2.30	

TIP
Tare (zero) the balance between readings as necessary.

1 Copy and complete the table by filling in the missing values in the force column.
2 Explain why it is acceptable to have a different number of decimal places for the calculations of force.
3 Explain why it is acceptable to have a different number of significant figures for the balance readings.
4 Using a graph, or otherwise, show that the magnetic flux density B is 100 mT.

Forces on a charged particle moving in a magnetic field

Charged particles moving in a magnetic field also experience a force. Old-style televisions and computer monitors use electron guns to produce beams of rapidly moving electrons in evacuated tubes, and their direction is controlled using a varying magnetic field. You can calculate the force on a single charge, Q, travelling perpendicular to a magnetic field, with flux density B.

If charge Q travels a distance l in t seconds, then the charge has a velocity $v = \frac{l}{t}$. But $I = \frac{Q}{t}$ and we can substitute for I in

$$F = BIl$$

This gives

$$F = \frac{BQl}{t}$$

Since the velocity of the charge is $v = \frac{l}{t}$, this gives

$$F = BQv$$

As before, you can use your left hand to predict the direction of the force. The thumb represents force, the first finger represents the magnetic field and the second finger represents the direction of a moving positive charge. The sign of the charge is important – a positively charged particle and a negatively charged particle will move in *opposite* directions in the same field. This is because if a negative charge moves to the left (for example), the conventional current flows to the right.

TIP

Conventional current – in many circuits the charge carriers are electrons but this is not always the case. In an electrolyte, for example, positive and negative ions move in opposite directions when a current flows.

By convention, we always who the direction of movement of positive charge carries and so, in a simple wire circuit, this is in the opposite direction to electron flow.

Work done

When a charged particle moves at right angles to a uniform flux, the charged particle moves in a circle (Figure 7.17), because the force is always perpendicular to the motion, provided no energy is lost (for example, when the charge is in a vacuum). Work is force multiplied by distance moved in the direction of the force. Since the force is perpendicular to the motion, no work is done by the magnetic field on the charge, so the kinetic energy of the charge does not change.

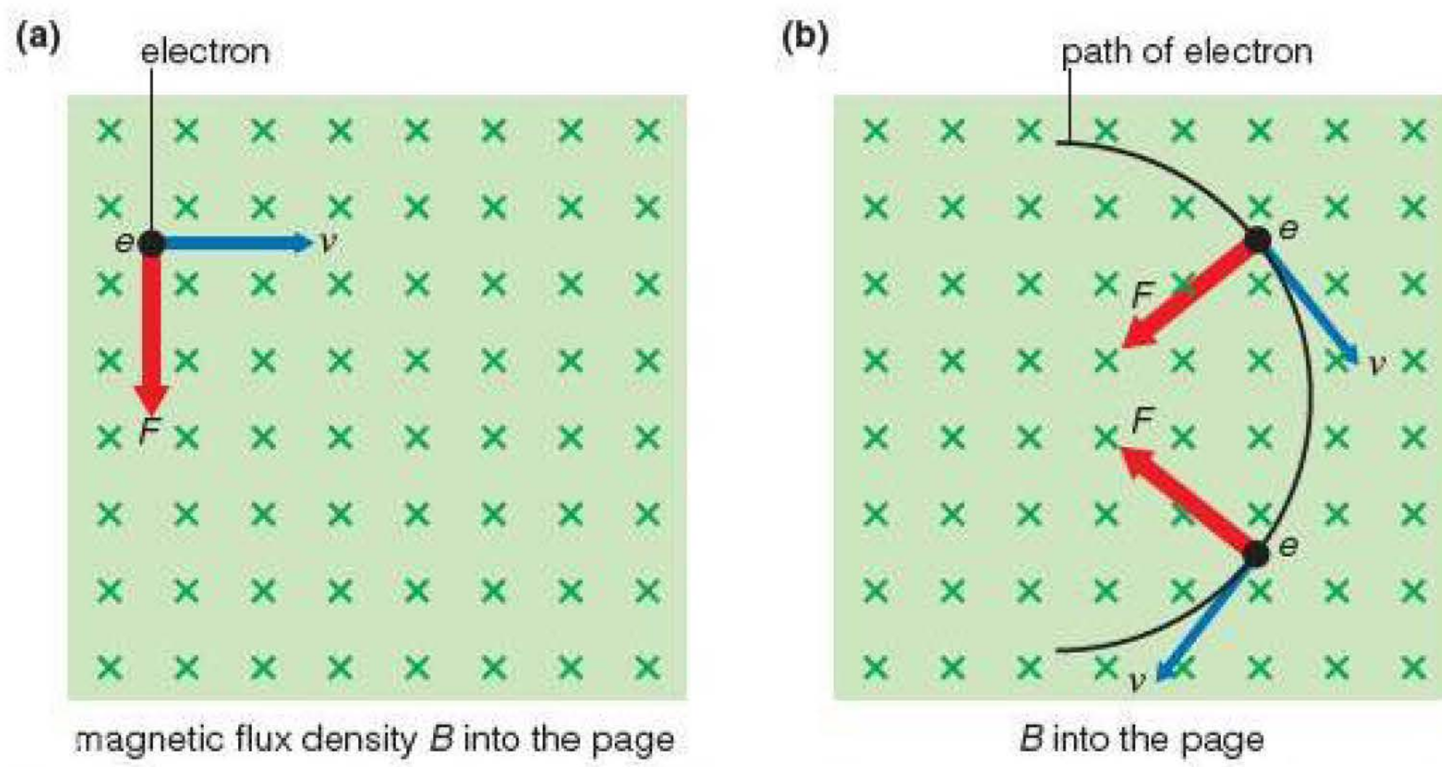


Figure 7.17 An electron travels in a circuit path when it moves perpendicular to a magnetic field because the force is perpendicular to its motion.

Applying ideas about circular motion

You can link the ideas of circular motion covered in Chapter 1 with the motion of a charged particle in a magnetic field.

Since the charged particle follows a circular path in a magnetic field (Figure 7.17), we know it experiences a centripetal force. For circular motion, the centripetal force must equal the force exerted by the magnetic field. If we link the equations for centripetal force and the movement of a particle in the magnetic field, we find that

$$F = \frac{mv^2}{r} = BQv$$

Dividing both sides by v gives

$$\frac{mv}{r} = BQ$$

This equation has many applications. For situations where B and Q are constant, the radius is proportional to the momentum of the particle:

$$mv = BQr$$

EXAMPLE

Paths of ionising radiation

Figure 7.18 shows ionising radiation travelling through a uniform magnetic field in a vacuum, at right angles to the lines of magnetic flux.

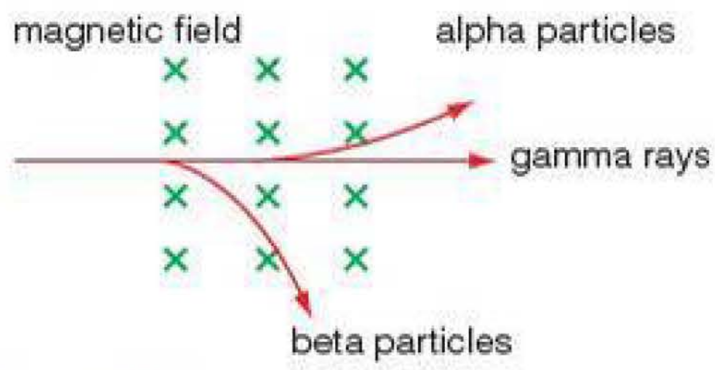


Figure 7.18

1 Compare the directions of motion of an alpha particle, a beta particle and a gamma ray if they move through the magnetic field as shown.

Answer

Using Fleming's left-hand rule, the magnetic field is into the page. The alpha particle is positively charged, so the force on it is upwards, and the alpha particle travels in a circular path and clockwise.

The gamma ray has no charge, so it continues to move in a straight line.

The beta particle has a negative charge, so the force on it is initially downwards, and it follows a clockwise circular path.

2 The diagram in Figure 7.18 is not to scale. Assuming the particles travel at the same speed, calculate the ratio of the radius of the paths for alpha particles and beta particles.

Answer

From the text,

$$\frac{mv}{r} = BQ$$

But v and B are constant, so r is proportional to m/Q. The alpha particle has a charge of 2e, and a mass of 8000me. The beta particle is an electron, with charge e and mass me. So the ratio is

$$\frac{\text{radius(alpha)}}{\text{radius(beta)}} = \frac{m\text{(alpha)}}{Q\text{(alpha)}}\frac{Q\text{(beta)}}{m\text{(beta)}}$$

$$= \frac{8000m_e}{2e}\frac{e}{m_e} = 4000$$

In reality, beta particles travel much faster than alpha particles, and relativistic effects increase the mass of beta particles. This means that the observed ratio is likely to be smaller. However, it shows that it is extremely difficult to deflect alpha particles using a typical school magnet.

3 Describe how the path will be different if the radiation travels in air, rather than in a vacuum.

Answer

In a vacuum, the charged particles travel in a circle. In air, charged particles in a magnetic field travel in a spiral, because they lose energy and slow down. Because r is proportional to v, as v decreases so does r.

You can also use the equation to find the radius r of the circle in which the charged particle travels, since $r = \frac{mv}{BQ}$. This idea is used in mass spectrometers, since charged particles with different mass/charge ratios travelling at the same speed will follow different paths.

TEST YOURSELF

6 Describe two situations in which a charged particle experiences no magnetic force when it is in a magnetic field.

7 An alpha particle travelling at 1% of the speed of light enters a field of flux density 1×10^{-3} T. Calculate the force experienced by the particle if it travels

a) parallel to the direction of the magnetic flux

b) perpendicular to the direction of the magnetic flux.

8 Calculate the flux density when an electron, travelling at right angles to the field direction, at a velocity of $1 \times 10^7\,\text{m s}^{-1}$, experiences a force of 1×10^{-15} N.

9 An electron enters a uniform magnetic field of flux density 0.036 T, travelling at right angles to the lines of flux. Calculate the speed of the electron if the radius of its path is 20 mm.

10 Figure 7.19 shows the paths of three particles in a magnetic field coming out of the page.

Compare the relative charges and speeds of the different particles A and B.

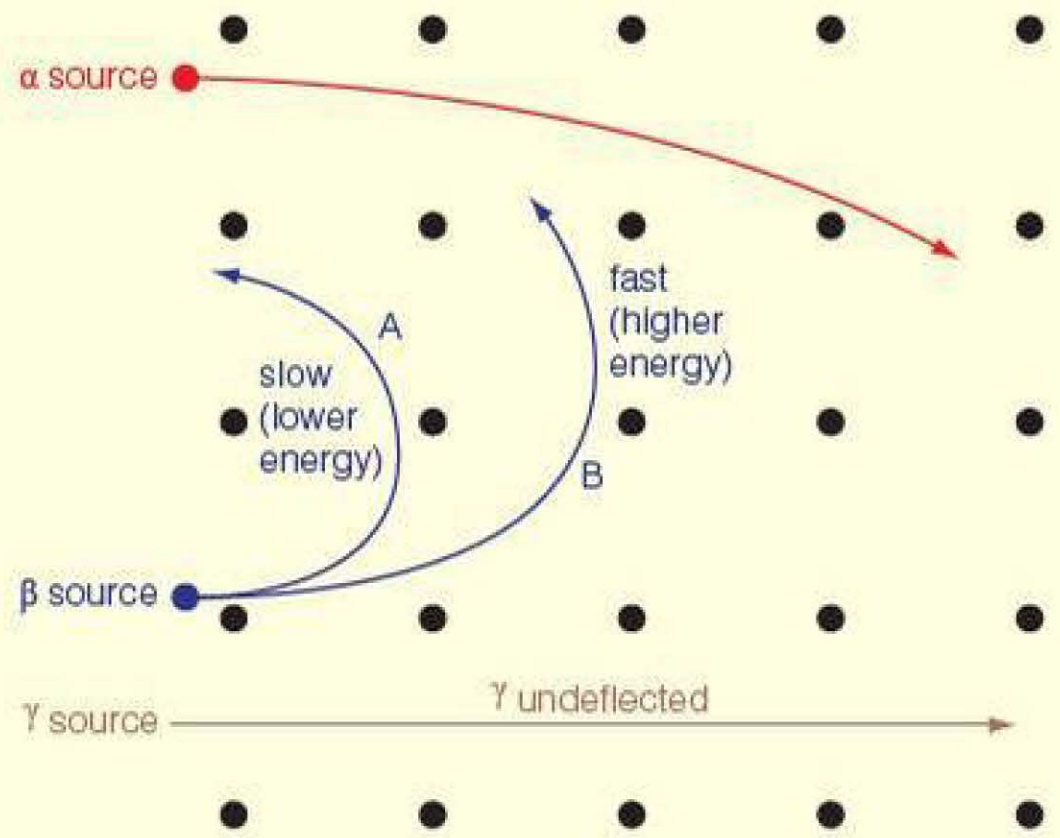


Figure 7.19

11 An electron travels in a circular path of radius 74 mm at right angles to a uniform magnetic field of flux density 0.43 mT.

a) Write down an expression for the centripetal force acting on the electron.

b) Show that the electron's speed is $5.6 \times 10^6\,\mathrm{m\,s^{-1}}$.

c) How does the radius of its path change if

i) the kinetic energy of the electron doubles

ii) allowance is made for the relativistic mass of the electron?

12 The mass of positively charged particles can be measured using a mass spectrometer. Charged particles are accelerated in a vacuum, and a velocity selector is used to select particles travelling at a specific velocity. These particles pass into a magnetic field applied perpendicular to their path and their position on a detector is recorded.

a) Explain why particles of the same charge but different masses (e.g. M, $2M$ and $3M$) have different paths in a magnetic field.

b) Explain how the paths would be different for the following three particles travelling in the same field: mass M and charge Q; mass $2M$ and charge $2Q$; mass $3M$ and charge $2Q$.

Cyclotrons

Cyclotron A particle accelerator that accelerates charged particles through a spiral path using a fixed magnetic field and an alternating potential difference.

A **cyclotron** is used to force charged particles into a circular path that accelerates them to very high speeds. Cyclotrons are often used with heavier particles like alpha particles and protons. Experiments using particle accelerators investigate the structures of complex molecules like proteins, as well as sub-nuclear structures.

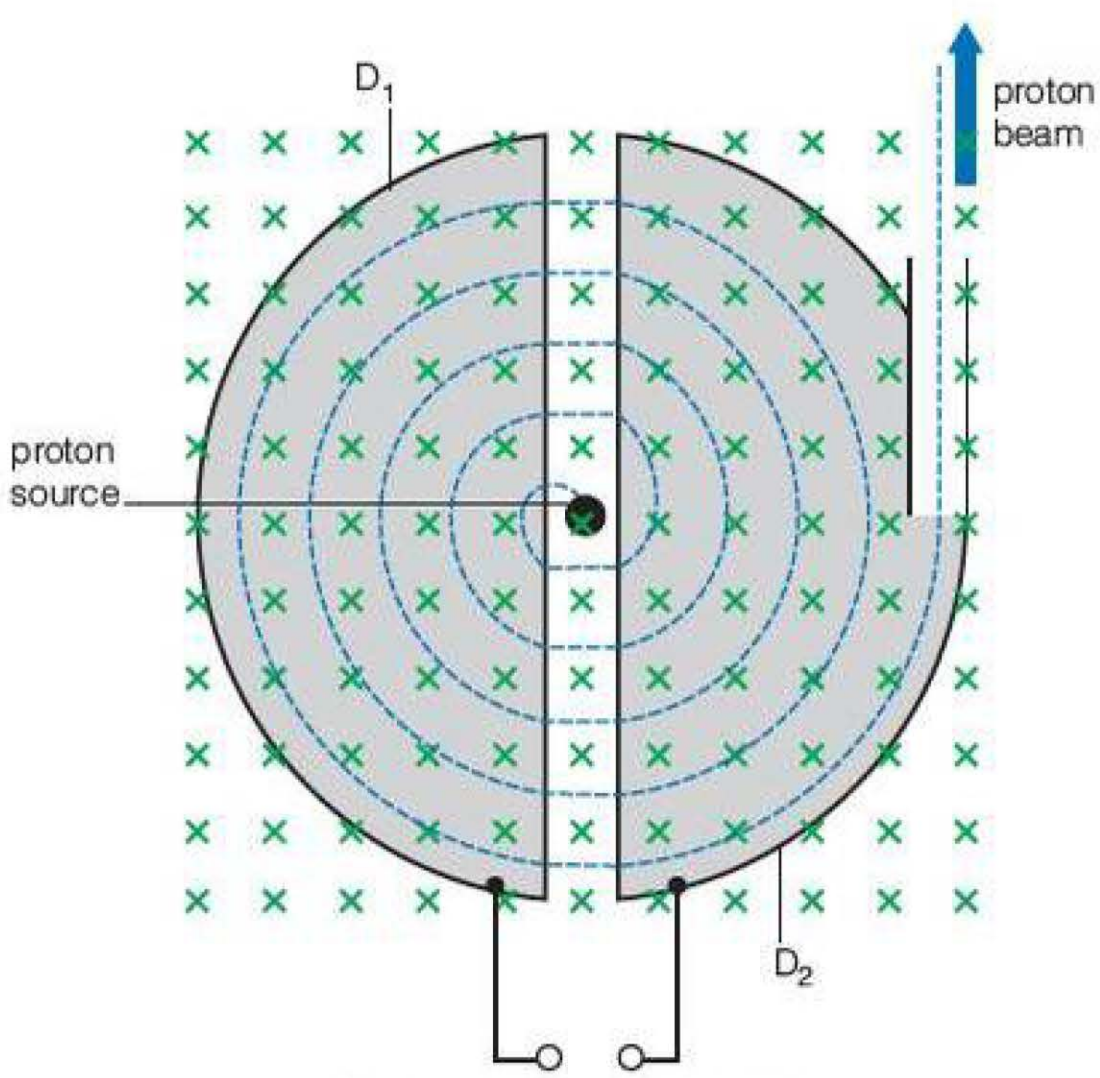


Figure 7.20 Structure of a cyclotron, a proton accelerator.

The cyclotron is formed from two semi-circular 'dees', separated by a small gap and connected to a high-frequency alternating potential difference (Figure 7.20). A strong magnetic field is applied perpendicular to the dees. The perpendicular magnetic field forces charged particles to move in a circular path inside the dees.

The particles experience a potential difference when they travel across the gap, and gain energy equal to QV (where Q is the particle's charge in coulombs, and V is the potential difference in volts). Since the particles have more kinetic energy, they move faster and accelerate to the next dee.

The ac voltage is timed to change direction every time the particles reach the gap between the dees. It must alternate to accelerate the particles each time they reach a gap. If the voltage did not alternate, the particles would follow a cycle of accelerate–decelerate–accelerate.

Particles spend the same time inside each dee, but the radius of their path increases after each gap and they travel further in the same time.

Remember that the centripetal force acting on the charged particle equals the magnetic force on the charged particle, so

$$\frac{mv^2}{r} = BQv$$

or

$$v = \frac{BQr}{m} \qquad \text{(i)}$$

Because the radius is proportional to the speed of the charged particle, the particles spiral outwards as they accelerate through the cyclotron.

The time, t, spent in each dee is given by

$$t = \frac{\pi r}{v} \qquad \text{(ii)}$$

because πr is half the circumference of the circle. Substituting for v in equation (ii) using equation (i) gives t, the time spent in one dee:

$$t = \frac{\pi r}{BQr/m} = \frac{\pi m}{BQ}$$

which does not depend on either radius or speed.

The effect of special relativity limits a particle's speed in a cyclotron. Particles get more massive as they travel close to the speed of light. As particles move faster and their mass increases, the time spent in each dee increases and the more massive particles get out of step with the alternating potential difference. A **synchrotron** overcomes this problem by increasing the magnetic field as the speed of the particles increases. The radius of their path remains constant even though the particles travel faster.

Synchrotron A particle accelerator that accelerates charged particles through a circular path using a varying magnetic field.

EXAMPLE

Alpha particles in a cyclotron

Alpha particles are accelerating in a cyclotron. The magnetic flux density is 0.8 T and the voltage across the gaps between the dees is 20 kV. The mass of an alpha particle is 6.64×10^{-27} kg and its charge is 3.2×10^{-19} C. Ignore relativistic effects.

1 Show that the frequency needed for the voltage supply to synchronise with the arrival of protons at the gaps is 6.14 MHz.

Answer

The time spent in one dee is $t = \frac{\pi m}{BQ}$; the period T for one complete circle (circling through two dees) is 2t.

The frequency for a complete circle is $f = \frac{1}{2t} = \frac{BQ}{2\pi m}$

So

$$f = \frac{0.8\,\text{T} \times 3.2 \times 10^{-19}\,\text{C}}{2\pi \times 6.64 \times 10^{-27}\,\text{kg}} = 6.14 \times 10^6\,\text{Hz}$$

2 How many circles should the alpha particles make to reach an energy of 10 MeV?

Answer

The energy gained when an alpha particle (charge 2e) crosses the gap is 2×20 keV = 40 keV. The alpha particle crosses two gaps per cycle, so it gains 80 keV (80×10^3 eV) per cycle. To reach an energy of 10 MeV (10×10^6 eV), the alpha particle must make $\frac{10 \times 10^6}{80 \times 10^3}$ circles, which is 125 circles.

TEST YOURSELF

13 In a cyclotron, explain the role of

a) alternating electric fields

b) fixed flux density.

14 Charged particles move at constant speed as they travel around a single dee. These particles are also accelerating. Explain how these two statements can both be true.

15 Protons are accelerated in a cyclotron. If the voltage supply alternates at a frequency of 4 MHz, calculate the magnetic flux density required.
The mass of a proton is 1.67×10^{-27} kg.

16 Explain why identical particles with different energies can be accelerated in a cyclotron together.

17 Find an expression for the maximum kinetic energy for a proton in a cyclotron of radius R.

Practice questions

1 Charged particles enter a magnetic field of flux density T at right angles to the field. Which one of these would decrease the radius of the circular path of the charged particles?

A decrease in charge Q

B decrease in mass m

C increase in velocity v

D decrease in flux density B

2 A positively charged particle travels north at a steady speed, v. A magnetic field is applied in the horizontal plane in which the particle is travelling. The flux is directed from east to west. Which of the following describes the motion of the particle after it enters the field?

A The particle accelerates upwards in the vertical plane.

B The particle's motion is unchanged.

C The particle accelerates downwards in the vertical plane.

D The particle accelerates in the westerly direction.

3 An electron enters a uniform magnetic field, travelling at a steady speed at right angles to the field. The shape of the electron's path in the field is

A a circle

B a straight line

C a parabola

D an ellipse

4 The speed of an electron of charge e and mass m travelling in a circular path of radius r in a magnetic field B is given by

A $Bemr$

B $\frac{Bem}{2\pi r}$

C $\frac{mer}{B}$

D $\frac{Ber}{m}$

5 A 50 cm wire carries a current of 1.2 A. The force the wire experiences if it is placed in a flux density 0.3 mT is

A 0.18 N

B 1.8×10^{-4} N

C 18 N

D 0.018 N

6 A wire carrying a current is placed parallel to a magnetic field. When the wire is gradually turned so it is perpendicular to the field, the force experienced by the wire

A stays at a constant value

B decreases to zero from a maximum value

C increases from zero to a maximum value

D remains zero

7 A magnetic field is from south to north. A current-carrying wire in the field experiences an upwards force. The direction of the current is

A into the page

B from east to west

C from west to east

D downwards

8 When a proton in a cyclotron travels through a dee, which of these statements is true?

A The proton is constantly accelerating and its speed is increasing.

B The proton travels at a constant speed and is not accelerating.

C The proton travels at a constant velocity and is constantly accelerating.

D The proton travels at a constant speed but is constantly accelerating.

9 The magnetic field of a cyclotron is

A constant in magnitude and applied perpendicular to the plane of the dees

B varying in magnitude and applied perpendicular to the plane of the dees

C constant in magnitude and applied parallel to the plane of the dees

D varying in magnitude and applied parallel to the plane of the dees

10 A beta particle travels from east to west across a magnetic field of strength 0.6 mT, which is directed northwards. The beta particle travels at $4 \times 10^5\,\text{m s}^{-1}$. The force the beta particle experiences in the field is

A 3.8×10^{-14} N downwards

B 3.8×10^{-17} N downwards

C 3.8×10^{-14} N upwards

D 3.8×10^{-17} N upwards

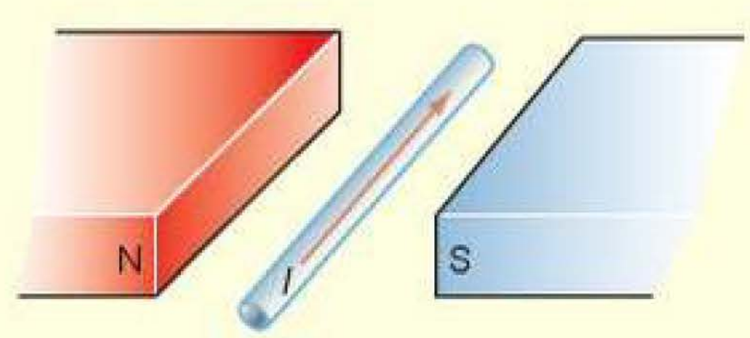


Figure 7.21

11 A current-carrying wire of length 10 cm is placed in a magnetic field as shown in Figure 7.21.

a) Predict the direction of the force acting on the wire. *(1)*

b) Calculate the flux density if the force on the wire is 2 mN when the current is 0.4 A. *(3)*

c) A student wants to use the magnetic field to lift the wire up. The wire's mass per unit length is $0.5\,g\,cm^{-1}$. Calculate the minimum current required to allow this piece of wire to lift. *(4)*

d) Explain why a larger current would be needed in reality. *(1)*

e) Explain what happens if an alternating current is used instead. *(4)*

12 A hospital cyclotron uses magnetic fields to accelerate protons around a circular path. The diameter of the cyclotron is 2.0 m.

a) If the field is in a suitable direction, the protons move in a circular path of constant radius. By referring to the force acting on the protons, explain how this happens. *(4)*

b) Calculate the centripetal force acting on a proton when travelling around the cyclotron at a speed of $6 \times 10^7\,m\,s^{-1}$. *(3)*

c) Calculate the flux density of the magnetic field needed to produce this force. *(3)*

d) A cyclotron accelerates the protons to their final speed by applying a varying potential difference at the gaps between the two dees in which the protons travel. Explain how the potential difference is used to accelerate the protons. *(4)*

13 Electrons travel around a tube placed in a magnetic field of flux density 0.3 mT (see Figure 7.22).

a) State and explain the direction of the magnetic field that forces the electrons to travel in this path. *(2)*

b) If the radius of the orbit is 15 cm, calculate the flux density producing this motion. *(4)*

c) Explain why the electrons are accelerating without speeding up. *(2)*

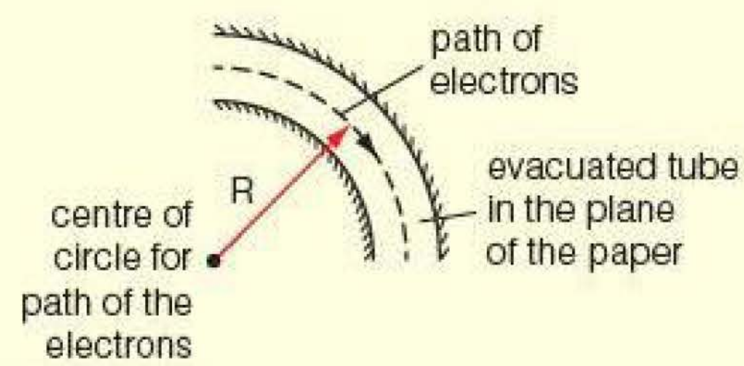


Figure 7.22

d) Predict the approximate speed required for protons to travel in the same orbit in the same flux density. *(3)*

14 An experiment is set up in which particles travel from left to right across a uniform magnetic field, directed into the plane of the page. The particles are travelling at the same speed. Describe in detail how their path through the magnetic field could be used to identify *(8)*

a) the sign of the charge of electrons, protons, alpha particles and neutrons

b) the relative masses of electrons, protons and alpha particles.

Stretch and challenge

15 A stream of charged particles is originally moving at velocity *v* and directed perpendicular to a uniform magnetic field. The particles follow a circular path in a plane perpendicular to the field and the original motion.

a) Describe how this path changes if the magnetic field varies continuously, becoming weaker then stronger.

The Aurora Borealis is the appearance of coloured lights in the sky usually seen near the poles of the Earth. This is caused by charged particles that spiral along the Earth's magnetic field lines, and are channelled to the poles (Figure 7.23). As the particles interact with gases in the atmosphere, they cause coloured light with the emission of electron transitions.

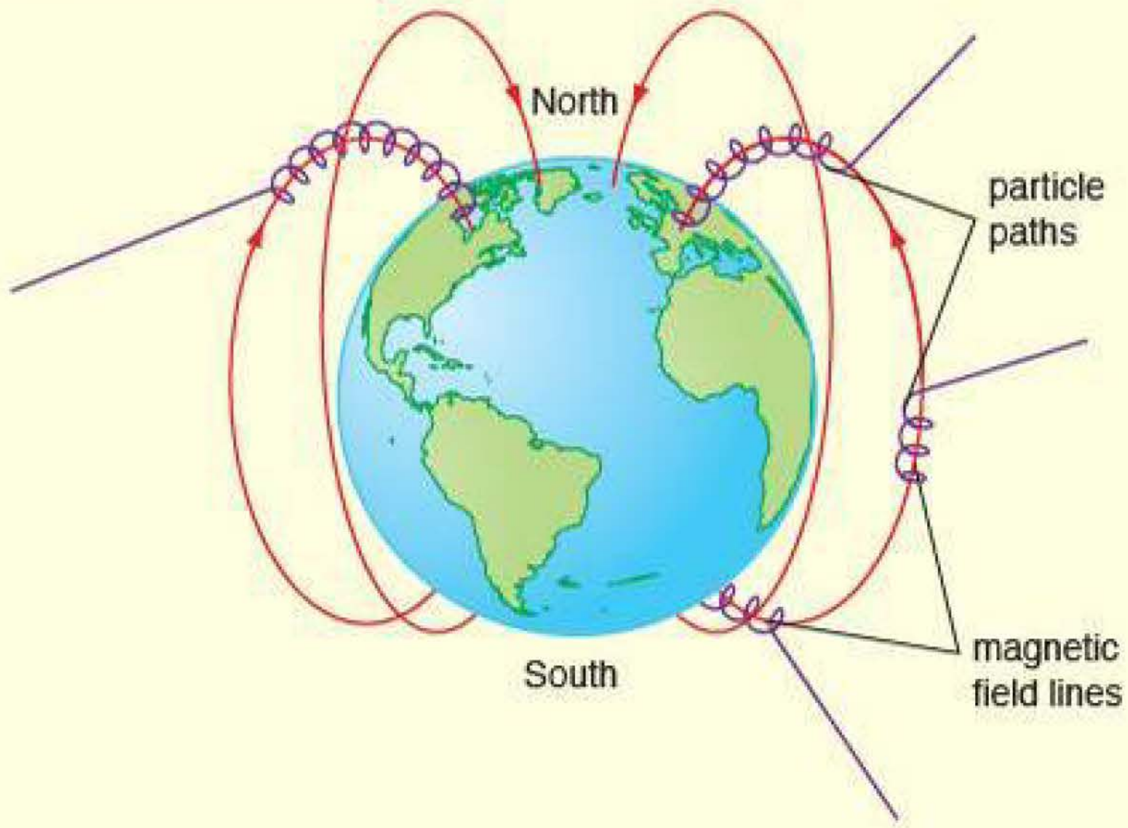


Figure 7.23

b) Describe how spiral motion is caused when a charged particle travels close to, but not quite parallel to a magnetic field line, by explaining:

i) what is meant by 'spiral motion'

ii) how the field affects components of velocity that are parallel and perpendicular to it

iii) how these components of velocity combine.

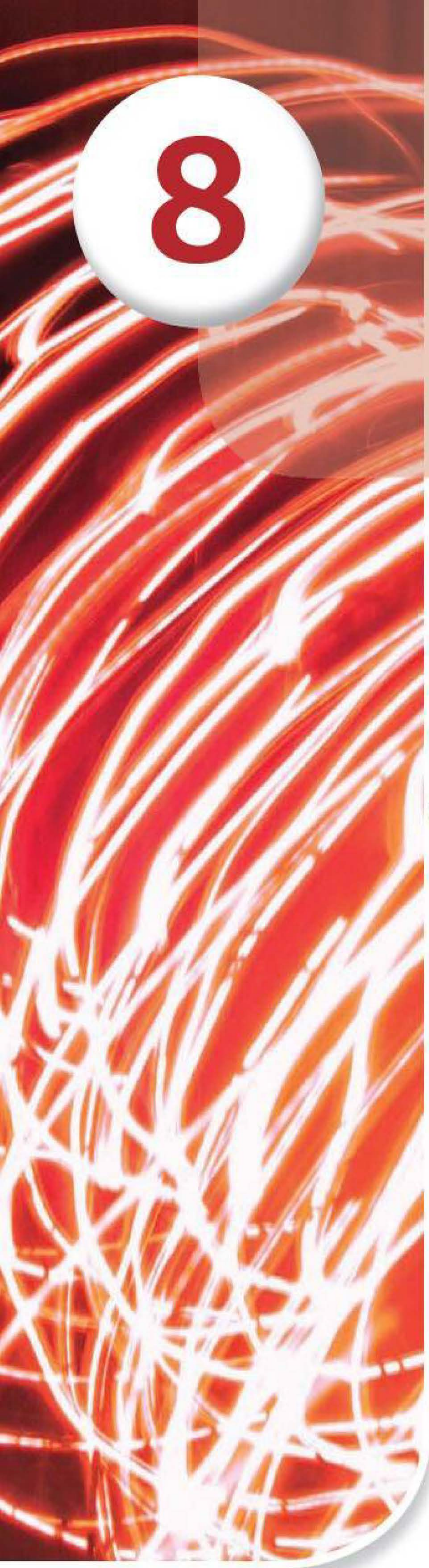
Use a diagram if this is helpful.

c) A charged particle is travelling at speed v at an angle θ to a magnetic field B.

i) State the components of velocity perpendicular and parallel to the field B.

ii) Calculate the radius of the spiral for a particle of charge Q, and the forward distance travelled while the particle circles the field once.

iii) Prove that these expressions are consistent with the expression $\tan\theta = \frac{2\pi r}{d}$.



Magnetic flux

PRIOR KNOWLEDGE

Before you start, make sure that you are confident in your knowledge and understanding of the following points:

- Magnetic flux density B is measured in teslas (T).
- The force on a current-carrying wire perpendicular to a magnetic field is given by $F = BIl$, where I is the current (A) and l is the length of wire in the field (m).
- The force on a charged particle moving perpendicular to a magnetic field is given by $F = BQv$, where Q is the charge (C) and v is its velocity ($\mathrm{m\,s^{-1}}$).
- Fleming's left-hand rule is used to determine the directions of force, magnetic field and current or velocity.
- The angular speed ω (also called angular frequency) is calculated as $\omega = 2\pi f$, where f is the frequency (Hz).

TEST YOURSELF ON PRIOR KNOWLEDGE

1 a) Work out the direction of the force acting on the current-carrying wire shown in Figure 8.1.

b) If 3 cm of the wire is in the field and experiences a force of 2 mN when the current flowing in the wire is 2 A, calculate the magnetic flux density in teslas.

2 a) A proton travels at 1% of the speed of light perpendicular to a magnetic field of flux density 1 mT. Calculate the force acting on the proton.

b) How does the force change if it is travelling parallel to the field?

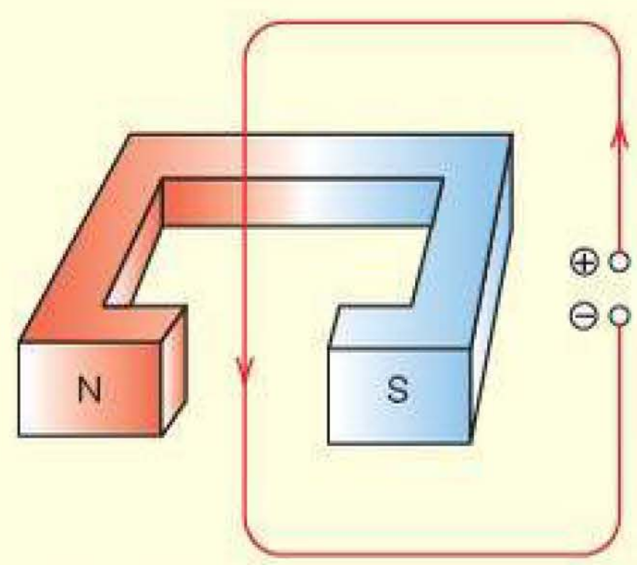


Figure 8.1

In this chapter, you will learn how we create an e.m.f. using electromagnetic induction. We use electromagnetic induction frequently, for example when generating mains electricity in power stations. Electromagnetic induction is also used in induction loops, for example to detect vehicles approaching traffic lights. A conducting loop buried under the road surface carries an alternating current, creating an alternating magnetic field, which is monitored constantly. When ferromagnetic material, such as a car, passes or remains over the loop, the change of flux causes a change in current, so the presence of a car is detected.

Magnetic flux

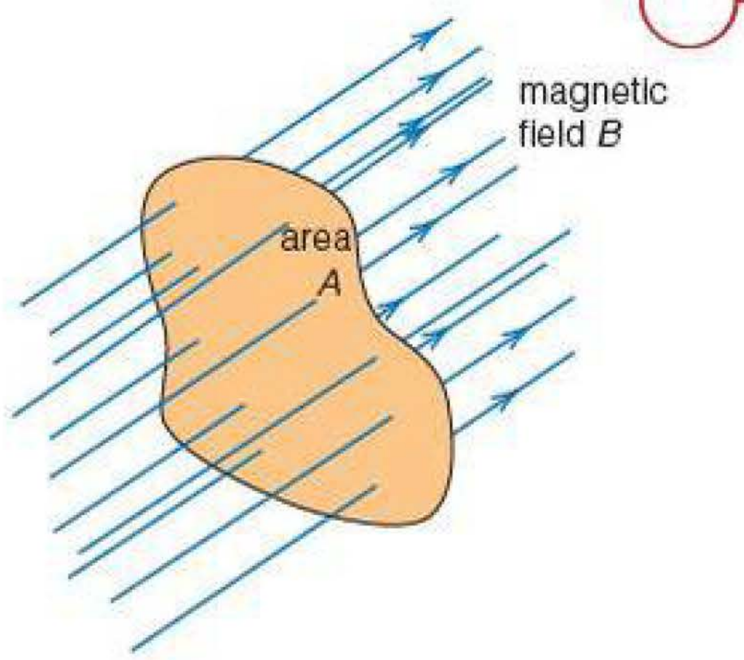


Figure 8.2

In Chapter 7, we came across the idea of *magnetic flux density*, which measures the strength of a magnetic field B, or B-field, in teslas (T). A diagram of a magnetic field indicates the density of magnetic flux by showing the number of flux lines per square metre (Figure 8.2).

Magnetic flux is defined as magnetic flux density B (in teslas, T) multiplied by the area of the surface, A (in m^2), where the area A is *perpendicular* to the lines of flux (Figure 8.2). Written as an equation, this becomes $\phi = BA$. Magnetic flux is measured in **webers** (Wb), where 1 Wb equals $1\,T\,m^2$.

Magnetic flux Magnetic flux ϕ is magnetic flux density × cross-sectional area perpendicular to field direction (measured in webers).

Weber The weber (Wb) is the unit of magnetic flux, equal to 1 tesla metre² ($1\,T\,m^2$)

When an area is not perpendicular to the lines of magnetic flux, as shown in Figure 8.3, the flux through the area A is now the component

$$\phi = BA\cos\theta$$

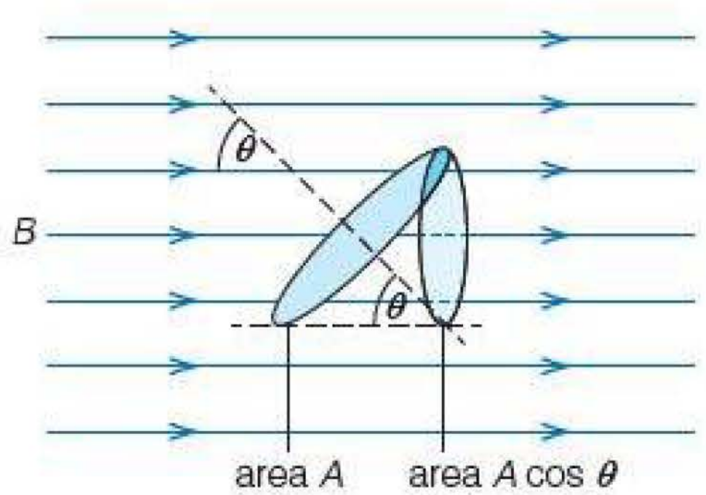


Figure 8.3 $A\cos\theta$ is the area perpendicular to the flux lines.

Figure 8.3 shows the area of a loop perpendicular to a field if the loop itself is at angle θ to the flux lines.

Cutting flux lines

When an object passes through a magnetic field, we can say that it 'cuts' the magnetic flux lines. Figure 8.4 shows a wire of length l moving downwards in a magnetic field with horizontal field lines. You can see that the wire cuts across the flux lines as long as it moves perpendicular to them. The wire cuts through more flux lines each second if

- length l is longer
- the wire moves faster
- the magnetic flux density is stronger.

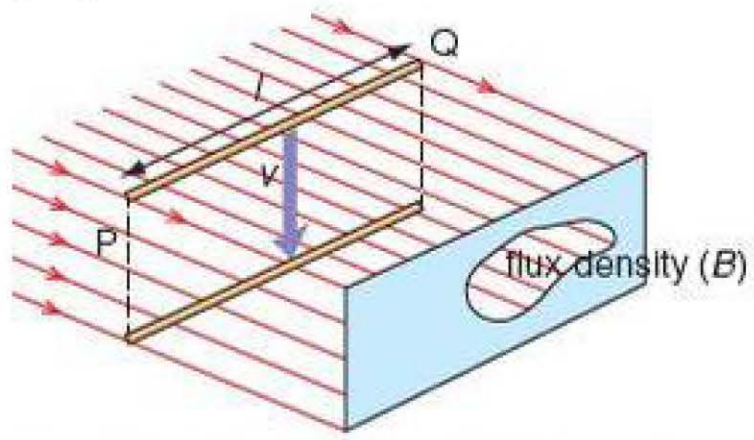


Figure 8.4 A wire 'cuts' field lines when it moves perpendicular to them.

If a conductor moves perpendicular to field lines, it 'cuts' the flux lines. But if the conductor moves parallel to field lines, they are not cut.

Magnetic flux linkage

Magnetic flux linkage is defined as $N\phi$, where ϕ is the number of flux lines that pass through , or link, with each of the turns of a coil of N turns. Since flux $\phi = BA$ for a single loop of wire, then the flux linkage is $N\phi = BAN$ if the coil of wire has N turns that are perpendicular to the lines of flux. Flux linkage is measured in **weber-turns**.

Magnetic flux linkage Magnetic flux linked by a coil, calculated as magnetic flux ϕ × number of turns N of the coil (measured in weber-turns).

Weber-turns The unit of magnetic flux linkage.

Flux linkage depends on several factors, as shown in Figure 8.5:

(a) the flux density

(b) the orientation of the coil and flux lines

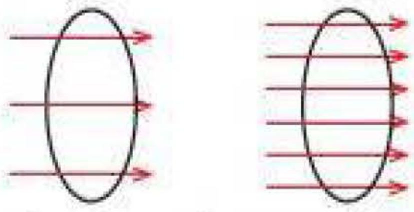
(c) the coil's cross-sectional area

(d) the number of turns on the coil.

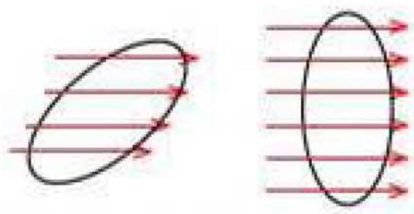
Flux linkage is important because an e.m.f. is induced in a coil, in which the flux linkage changes. You will learn more about this in the next section.

Figure 8.6 shows a coil being turned in a magnetic field. As the coil turns in the magnetic field, the area of the coil perpendicular to the field is given by $A\cos\theta$, and the magnetic flux linkage is given as

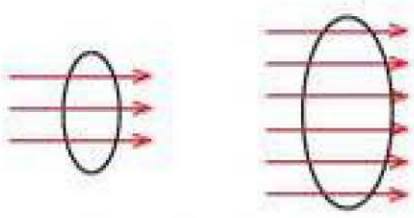
$$N\phi = BAN\cos\theta$$



(a) The flux through an area is proportional to the flux density.



(b) Flux varies depending on the relationship between the boundary faces and the direction of flux.



(c) Flux is proportional to the area within the boundary.

Figure 8.5

where N is the number of turns on the coil, ϕ is the magnetic flux (in Wb), B is the magnetic flux density (in T), A is the cross-sectional area of the coil (in m^2) and θ is the angle between the axis of the coil and the flux lines. The flux linkage changes as shown in Figure 8.7.

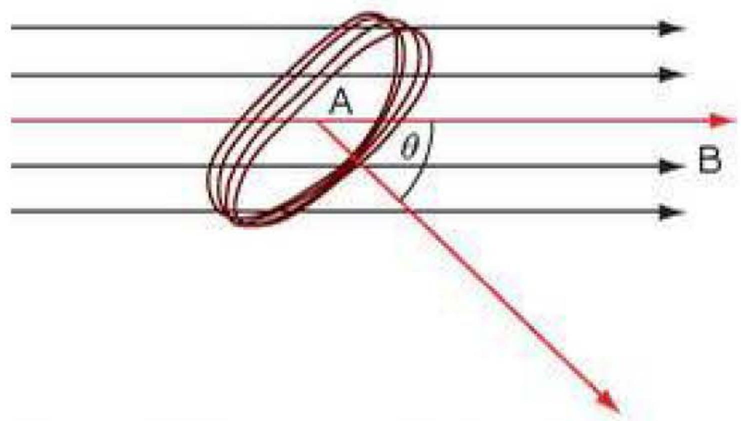


Figure 8.6 The magnetic flux linkage changes as a coil of cross-sectional area *A* and with *N* turns rotates in a flux density *B*.

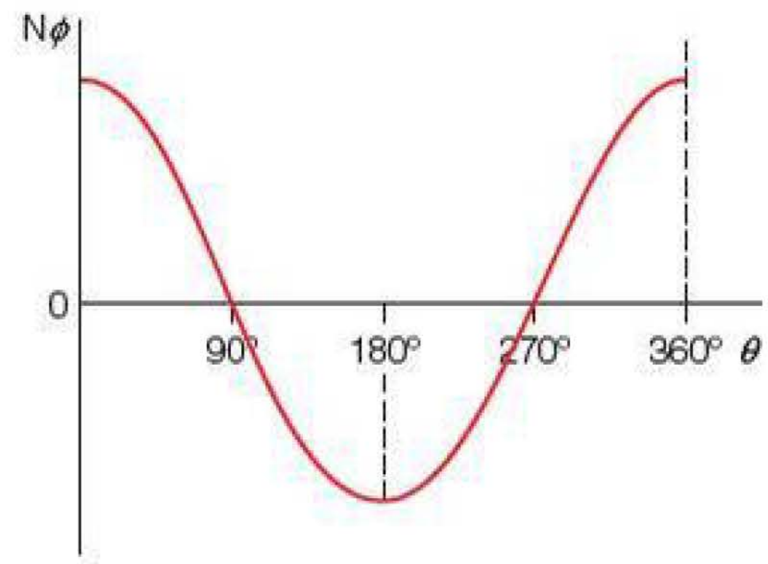


Figure 8.7 How flux linkage changes when a coil of *N* turns rotates in a field.

EXAMPLE

Magnetic flux linkage through a coil

Figure 8.8 shows a coil of wire formed as a 60° triangle with sides of length 30 cm. The coil has 50 turns. Calculate the magnetic flux linkage with the coil when it is placed with the axis at 40° to a vertical in a uniform horizontal flux of 0.02 T.

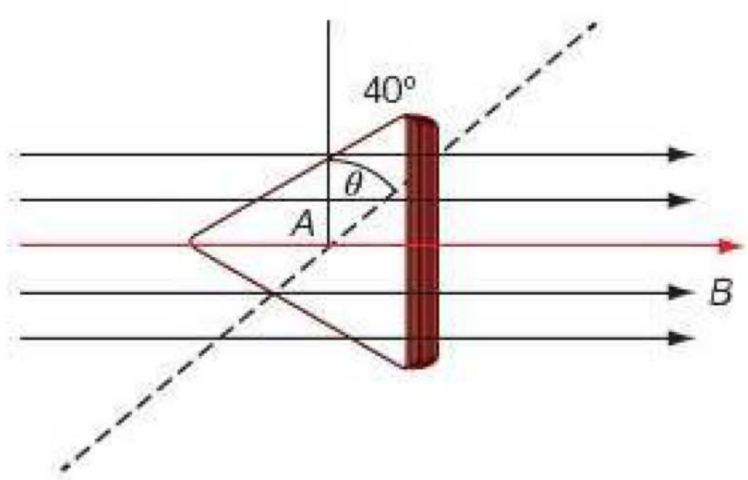


Figure 8.8

Answer

The area of the coil is

$$\left(\frac{1}{2} \times \text{base} \times \text{height}\right) = \frac{1}{2} \times 0.30\,\text{m} \times 0.30\,\text{m} \times \sin 60° = 0.039\,\text{m}^2$$

The flux linkage is

$$BAN\cos\theta = 0.02\,\text{T} \times 0.039\,\text{m}^2 \times 50 \times \cos 40°$$

$$= 0.03\,\text{Wb turns}$$

TEST YOURSELF

1 Calculate the magnetic flux through the face of a magnet, if the face measures 2.0 cm by 6.0 cm and the magnetic flux density of the magnet is 0.03 T.

2 Calculate the flux through the horizontal surface of the British Isles. The average flux density in the region is 53 μT at 20° to the vertical and the area of the British Isles is $3.0 \times 10^{11}\,\text{m}^2$.

3 **a)** Calculate the magnetic flux passing through a copper sphere of radius 3.0 m placed in a region of uniform magnetic flux density 2.0 T.

b) Explain whether or not the amount of magnetic flux changes if the sphere is sliced in half, along the axis perpendicular to the field.

4 A square coil of wire has sides 12 cm long, and 250 turns. Calculate the magnetic flux linkage when:

a) the cross-sectional area of the coil is perpendicular to a field of flux density 0.08 T.

b) the face of the coil makes an angle of 60° to the magnetic flux lines.

c) Sketch a diagram showing how the flux linkage changes if the coil is initially perpendicular to the field and is turned until it is parallel and then perpendicular again.

Electromagnetic induction

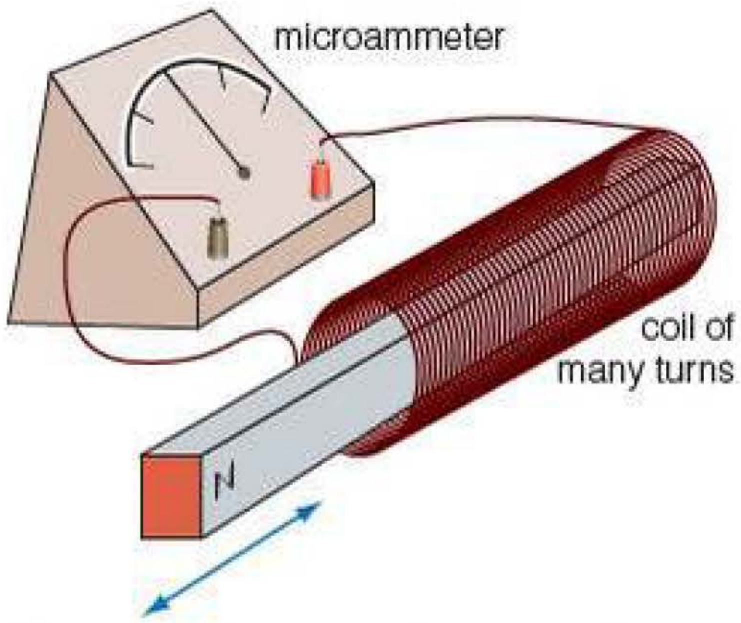


Figure 8.9 Using a moving magnet to induce an e.m.f.

Our lives would be completely different without Michael Faraday's discovery of electromagnetic induction in 1831 using insulated coils of wire and changing magnetic fields.

You can easily demonstrate electromagnetic induction using a coil of wire connected to a microammeter, as shown in Figure 8.9. The microammeter flicks one way when a bar magnet is moved into the coil, and the other way when the magnet is pulled out. It is zero when the magnet is stationary inside the coil. An e.m.f. is induced if there is relative movement between the coil and a magnetic field (either the magnet or the coil moves) or the magnetic flux linkage changes (for example, the strength of an electromagnet changes).

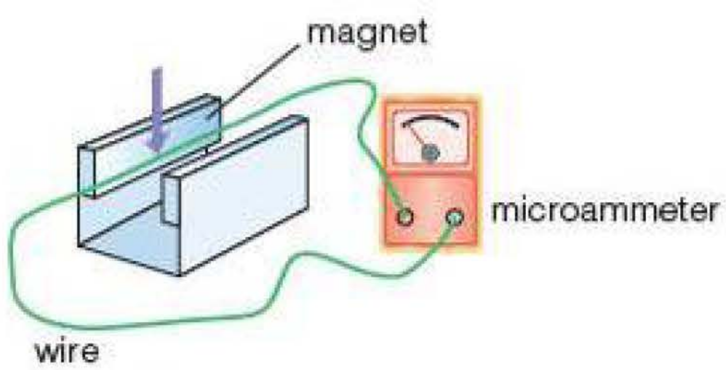


Figure 8.10 Moving a wire into a magnetic field induces an e.m.f.

Figure 8.10 shows electromagnetic induction caused by a length of wire moving between two magnets. The wire is connected to the microammeter, which flicks one way when the wire moves down, and flicks in the opposite direction when the wire moves up.

An e.m.f. is induced in the wire because an electric charge moving perpendicular to a magnetic field experiences a force, BQv (see Chapter 7). Using Fleming's left-hand rule, you can see that electrons in a wire move towards one end of the wire when the wire moves perpendicular to the magnetic field. This leaves one end of the wire negatively charged overall and the other end positively charged, creating a potential difference across the wire. A current can flow if the wire is part of a complete circuit – for example, when the wire is connected to a microammeter.

Faraday's law

Faraday's law The induced e.m.f. equals the rate of change of (magnetic) flux.

We can calculate the magnitude of the induced e.m.f. in a coil using **Faraday's law**. This states that the magnitude of the induced e.m.f. equals the rate of change of magnetic flux linkage, and is written as

$$\varepsilon = \frac{\Delta\,(N\phi)}{\Delta\,t}$$

where $\Delta(N\phi)$ is the change in flux linkage and Δt is the time over which that changes takes place. Since a coil of wire has a fixed number of turns, this becomes

$$\varepsilon = N\frac{\Delta\,\phi}{\Delta\,t}$$

We can use this law to help us to understand some earlier observations:

- Relative movement between a magnet and a coil changes the flux linkage in the coil (Figure 8.11). This generates an e.m.f.
- Rotating a coil in the plane perpendicular to the field changes the cross-sectional area *through which the flux passes*. This changes the flux linkage, and generates an e.m.f.
- Increasing the relative motion, or the speed at which the coil rotates, increases the rate of change of the flux linkage, which increases the induced e.m.f.
- If there is no relative movement or rotation, the flux linkage does not change, so no e.m.f. is generated.

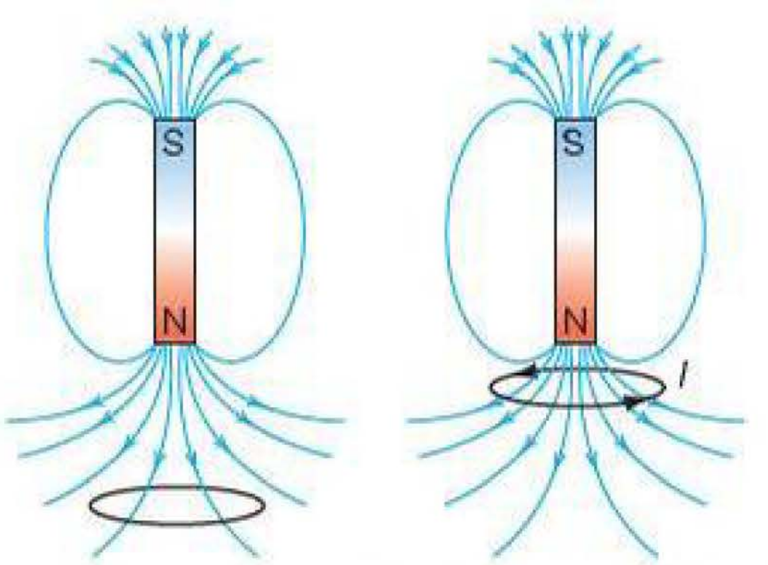


Figure 8.11 Flux linkage in the coil increases as the coil moves closer to the magnet.

Lenz's law

Faraday's law calculates the *magnitude* of the induced e.m.f. and is often combined with Lenz's law, which indicates the *direction* of the induced e.m.f. Lenz's law states that the direction of the induced e.m.f. opposes the changes causing it.

Lenz's law The direction of the induced e.m.f. causes effects that oppose the change producing it.

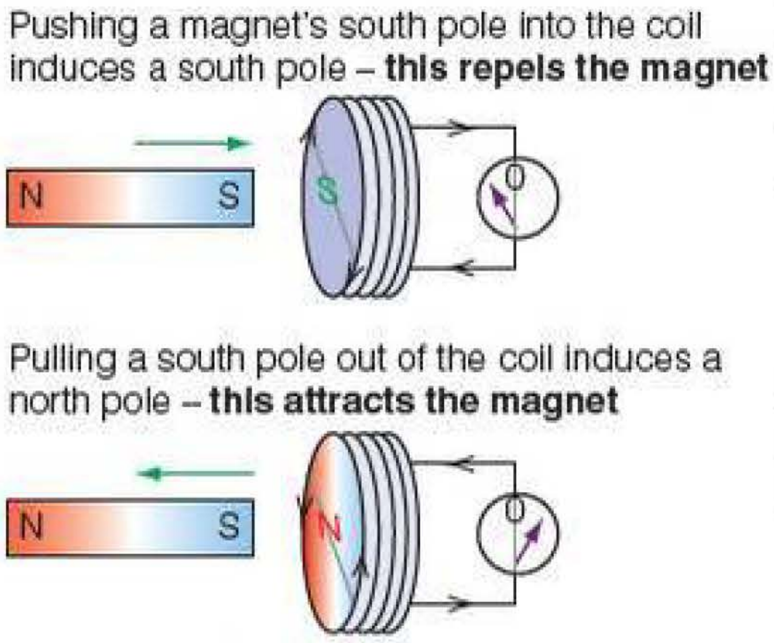


Figure 8.12 A magnet being pushed into, or pulled out of, a coil of wire. Lenz's law determines the direction of the induced e.m.f.

Figure 8.12 shows the south pole of a magnet moving into a coil. This induces an e.m.f. when there is a complete circuit, a current flows and the coil behaves as an electromagnet, with its south pole facing the magnet's south pole, repelling the magnet.

Pulling the magnet out of the coil induces an e.m.f. such that the same end of the coil becomes a north pole, which attracts the magnet.

We can combine Lenz's law with Faraday's law and write

$$\varepsilon = -\frac{\Delta(N\phi)}{\Delta t}$$

where $\Delta(N\phi)$ is the change in flux linkage and Δt is the time over which that changes takes place. Since a coil of wire has a fixed number of turns, this becomes

$$\varepsilon = -N\frac{\Delta\phi}{\Delta t}$$

Lenz's law is the result of conservation of energy. When the south pole of a magnet is pushed into the coil, a current is induced in the wire, which becomes an electromagnet. If the south pole of the electromagnet faces the moving magnet, the poles repel and work must be done to keep pushing the magnet into the coil of wire. If you try this with a very strong magnet in a large coil, you may feel the force you are working against.

If Lenz's law did not apply and, instead, the north pole of the coil faced the magnet's south pole, the magnet would be attracted. This would make the magnet accelerate into the coil, increasing the induced e.m.f. This would start a process in which increasing the e.m.f. increased the acceleration, which increased the e.m.f., and so on. That would imply that energy can be created without doing any work. This, of course, cannot happen.

EXAMPLE

Magnitude of an induced e.m.f.

Calculate the magnitude of the induced e.m.f. when a flat coil of radius 2.0 cm, with 200 turns, is placed at right angles to a varying magnetic field. The field strength is increased from 0 to 3.0 mT in 0.30 s.

Answer

Maximum flux linkage is

$$N\phi = BAN$$

$$= 3 \times 10^{-3}\,\text{T} \times 2\pi \times (0.02)^2\,\text{m}^2 \times 200$$

$$= 7.5 \times 10^{-4}\,\text{Wb turns}$$

Minimum flux linkage = 0 Wb turns

Magnitude of induced e.m.f. is

$$\varepsilon = \frac{\Delta(N\phi)}{\Delta t}$$

$$= \frac{7.5\times10^{-4}\,\text{Wb turns}}{0.30\,\text{s}}$$

$$= 2.5\times10^{-3}\,\text{V}$$

Eddy currents

A metal sheet moving into (or out of) a magnetic field can become very hot. This happens if very large currents, called eddy currents, are set up in the metal sheet. Eddy currents are circulating electric currents flowing in the plane of the metal. They are caused by the change of flux linkage when the metal moves perpendicular to the field, and the currents flow in a direction to oppose the motion creating them.

Eddy currents can become very large because metals have a low resistance. Eddy currents are put to good use in induction cookers. Here a high-frequency alternating current in the cooker produces a rapidly changing magnetic field, which induces a large alternating current in the base of a saucepan causing it to heat up.

Eddy currents can also cause magnetic braking. A pendulum swinging between two magnets slows down quickly because eddy currents are set up in the metal when it enters and leaves the field (Figure 8.13a). Magnetic fields created by these eddy currents interact with the fixed magnetic field, opposing the motion and stopping the pendulum. Cutting slits in the pendulum prevents eddy currents forming and the pendulum continues to swing (Figure 8.13b).

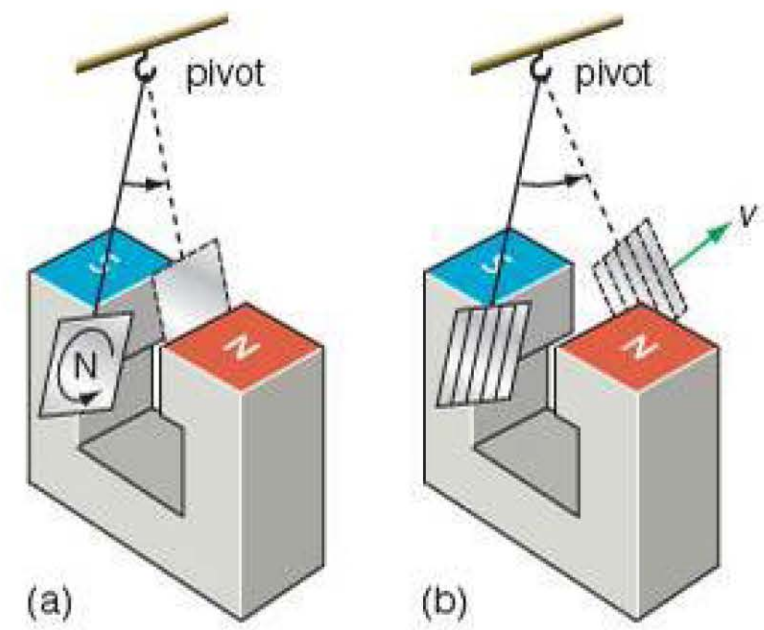


Figure 8.13 (a) Eddy currents form in an aluminium pendulum, slowing it down as it moves in to and out of a fixed magnetic field unless (b) slits are cut in the pendulum.

REQUIRED PRACTICAL 11

Using a search coil to investigate changes in magnetic flux density

Note: This is just one example of how you might tackle this required practical.

A search coil is a small flat coil made from 500–2000 turns of insulated wire mounted on a handle (Figure 8.14). An e.m.f. is induced in the coil when the coil is placed in a magnetic field that varies. The amplitude of induced e.m.f. is directly proportional to the amplitude of the varying flux density of the field.

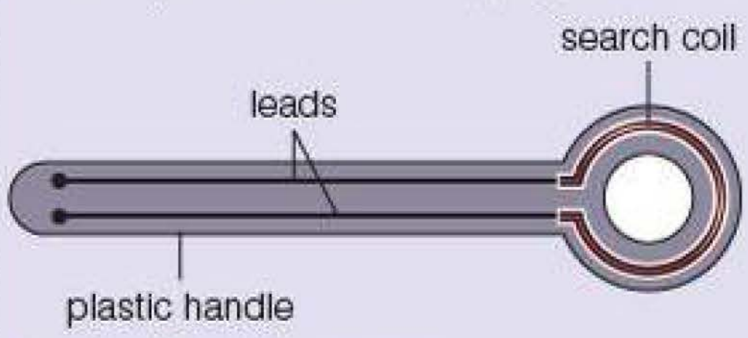


Figure 8.14 A search coil.

You can calibrate the search coil by connecting it to an oscilloscope. When the search coil is placed perpendicular to a known flux density that varies, the trace on the oscilloscope is used to find the amplitude of the e.m.f. induced in the coil.

A calibrated search coil can be used with the oscilloscope to measure the strength of an unknown flux density, or the effect of changing the angle of a coil in a known flux density (Figure 8.15).

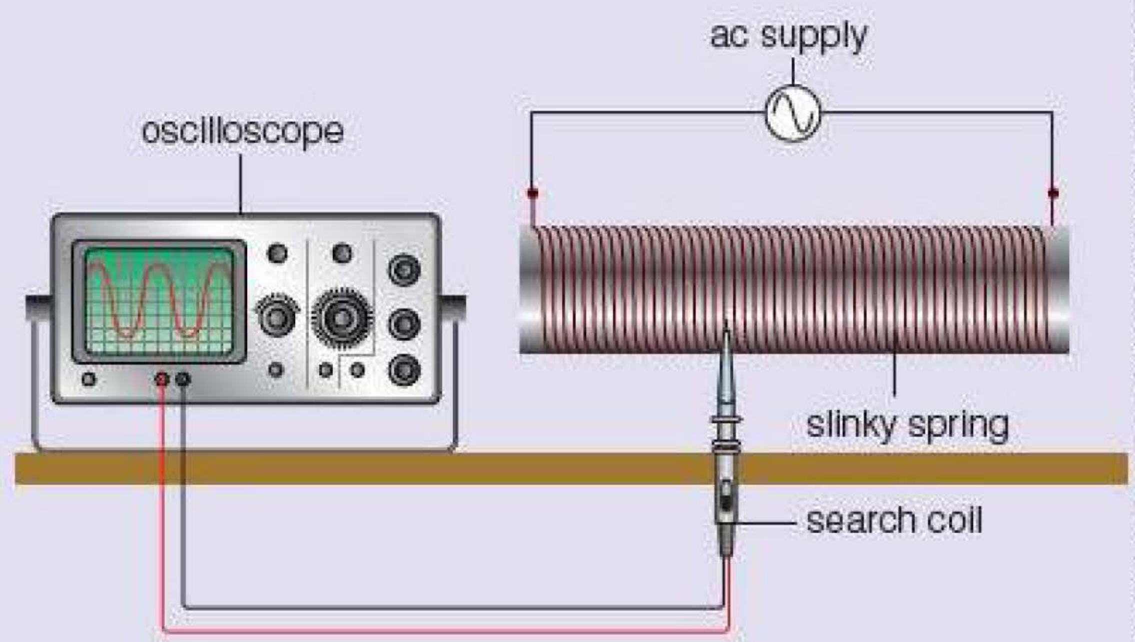


Figure 8.15

To investigate the effect of changing the angle of a coil in a flux density, the calibrated search coil is placed in a known magnetic field that varies. The amplitude of the induced e.m.f. is measured by connecting the search coil to an oscilloscope with the timebase turned off, so the e.m.f. is displayed as a vertical line on the screen. Measurements are taken when the search coil is held at different angles in the magnetic field. The area of the coil is given by $A \cos \theta$, where θ is the angle between the axis of the coil and the flux lines and A is the cross-sectional area of the coil. We find that the amplitude of the induced e.m.f. also varies with $\cos \theta$, which is consistent with flux linkage given by $BAN \cos \theta$.

Note: The slinky spring will heat up – don't leave the current on for only longer than necessary.

EXAMPLE

An experiment with a search coil

A student places a search coil of radius 8 mm and 500 turns in a varying magnetic field of maximum flux density 0.4 T. The search coil is then connected to an oscilloscope and placed in the magnetic field with its face perpendicular to the flux lines. It is turned through 360° taking 0.2 s.

1 Calculate the maximum flux linkage.

Answer

Maximum flux linkage is

$$BAN = 0.4\,\text{T} \times \pi \times (0.008\,\text{m})^2 \times 500$$
$$= 0.040\,\text{Wb turns}$$

2 Sketch a graph showing how the maximum flux linkage varies as the coil is rotated through 360°. Include as much detail as possible.

Answer

The graph is shown in Figure 8.16. Make sure you sketch a cosine curve with the peak values and the *x*-axis labelled.

3 Explain at least two additional steps the student could take to reduce errors.

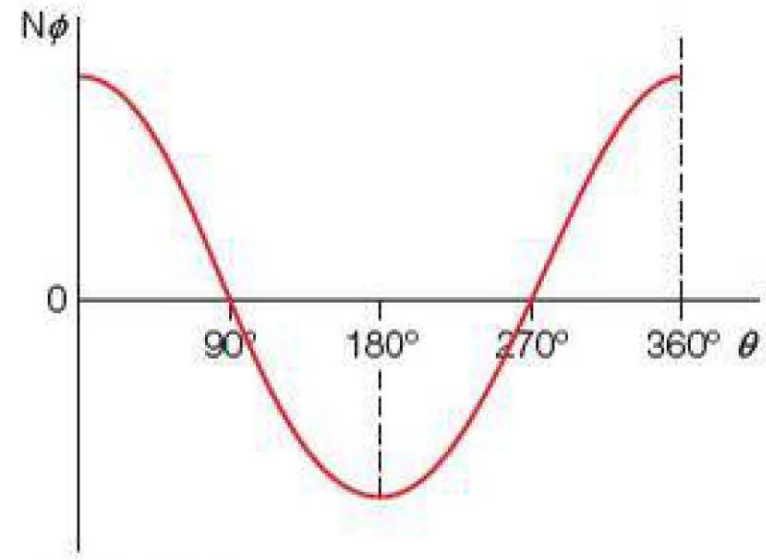


Figure 8.16

Answer

Use a data logger to record changes of e.m.f. with time precisely and permanently (i.e. so the data can be processed).

Use a calibrated motor to rotate the search coil at a steady rate to improve repeatability and accuracy. Repeat measurements of data to prepare mean values for different angles to reduce the impact of random errors.

Reduce systematic errors by calibrating the search coil using a known magnetic field and oscilloscope.

TEST YOURSELF

5 The north pole of a magnet is pushed into a coil of wire.
  a) Describe what happens to the coil as the magnet moves into the coil, rests inside the coil and is pulled back out again.
  b) Sketch a diagram showing how the induced e.m.f. changes with time.

6 The magnetic flux density between the poles of an electromagnet is 0.20 T. A coil, with 500 turns and cross-sectional area $2.0 \times 10^{-4}\,\text{m}^2$, is placed in the field perpendicular to the flux lines. The field increases steadily to 0.60 T in 10 ms.
  a) Calculate the initial magnetic flux linkage, and the flux linkage at 10 ms.
  b) Calculate the e.m.f. induced in the coil.
  c) Calculate the e.m.f. induced in the coil if it was held with its axis at 30° to the field while the field changed.

7 A search coil with a cross-sectional area of $1.0\,\text{cm}^2$ and 2500 turns is placed between the poles of a magnet. If the coil is pulled out of the magnetic field in 5 ms, and the average induced e.m.f. is 0.9 V, calculate the strength of the magnetic field.

8 a) Explain why a copper ring heats up if it is placed in a region with an alternating magnetic field.
  b) You have two coils of insulated wire, both have the same circumference. One coil is made up of just one loop, but the wire in the other coil is three times as long and has been twisted into three loops. Explain why the current induced by passing a magnet through each of the two coils of insulated wire is the same.
  c) Explain why a magnet dropped through a vertical copper pipe falls more slowly than the same magnet falling through a vertical plastic pipe.

9 Figure 8.17 shows a seismometer made from a bar magnet suspended on a spring, which is attached to a metal rod that transmits vibrations from the Earth. Use the diagram to explain how the seismometer detects waves from an earthquake.

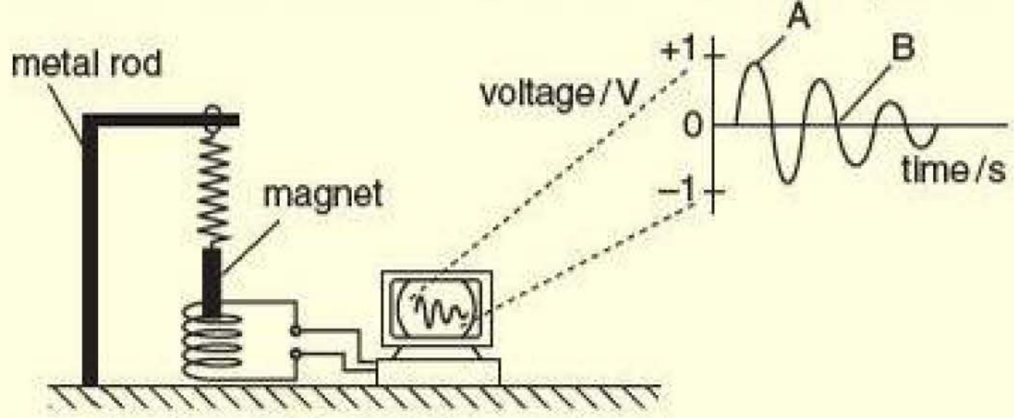


Figure 8.17

Applications of electromagnetic induction

Changing magnetic field

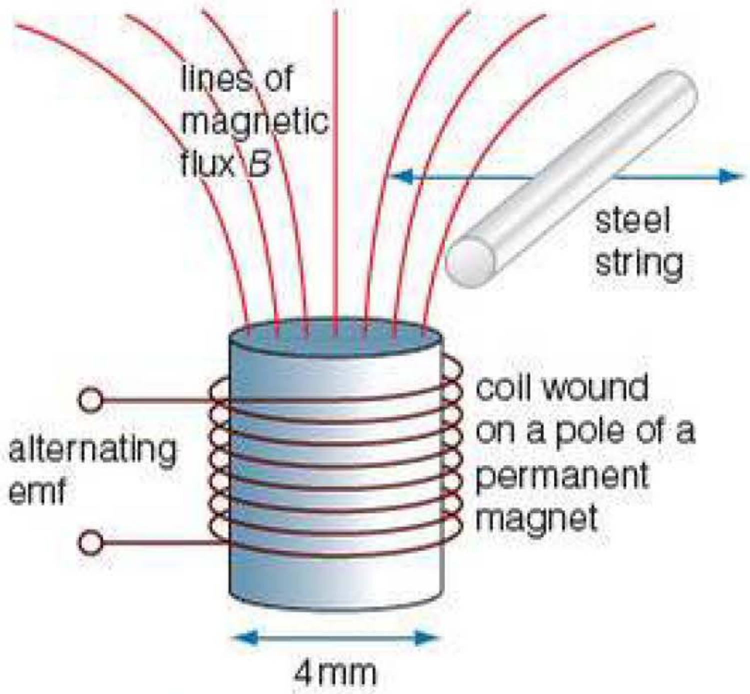


Figure 8.18 A guitar pick-up.

Electric guitar strings are made of magnetised steel. When the strings are plucked, they vibrate directly above pick-ups, which are fixed on the guitar's neck. The pick-ups are bar magnets wrapped in up to 7000 coils of very fine wire (Figure 8.18). The vibrating string causes vibrations in the magnetic field surrounding the coil of the pick-up. This change is converted into an e.m.f. and amplified. The distinctive electric guitar sounds come from deliberate distortion when the note is amplified.

Some microphones uses electromagnetic induction to change a sound wave into an electrical signal. The microphone has a lightweight coil suspended in a circular groove between the poles of a permanent magnet (Figure 8.19). The coil is attached to a diaphragm that vibrates when a sound wave reaches it. Since the coil and magnetic field are perpendicular, an e.m.f. is induced in the vibrating coil, which depends on the frequency and amplitude of sound waves. The induced e.m.f. is amplified, and a loudspeaker changes these signals back to audible sound.

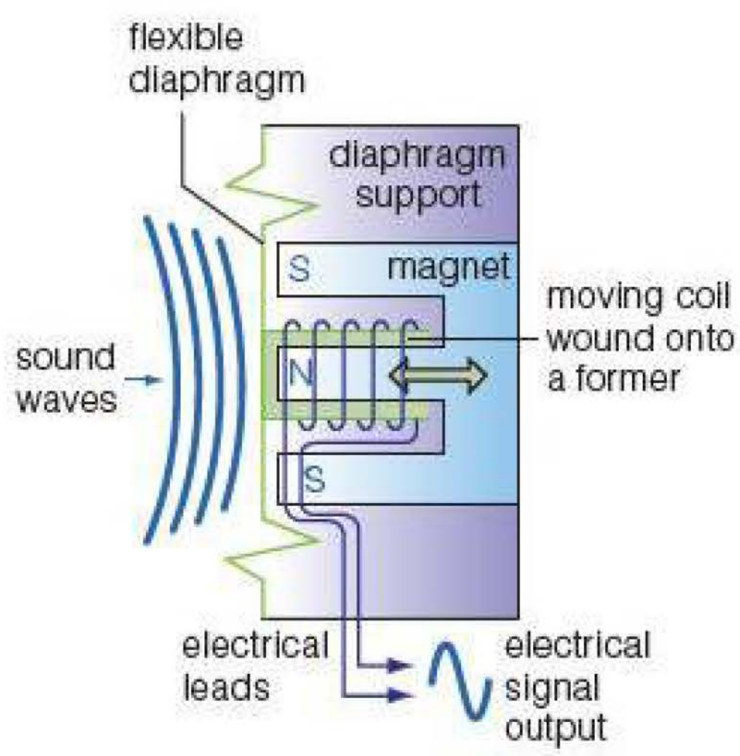


Figure 8.19 A microphone uses electromagnetic induction to change sound into electrical signals.

Conductor moving in a straight line

An induced e.m.f. can be caused by a conductor moving in a magnetic field. For example, a straight wire may be dropped through a uniform magnetic field, or a plane may fly at a constant height and speed in the Earth's magnetic field.

A credit card includes information stored on a magnetic strip. The credit card reader has a small coil in it, and when the credit card is swiped through the reader, an e.m.f. is induced in the coil. It is important to swipe the card quickly enough so that the induced e.m.f. is large enough to be interpreted.

When a conductor moves at a velocity v perpendicular to the flux lines, Faraday's law applies and an e.m.f. is generated. For a conductor of length l travelling in a flux density B, the area swept out per second is length × velocity. The induced e.m.f. equals the rate of change of flux linkage, so

$$\varepsilon = B\frac{dA}{dt}$$

Because the area swept out per second is lv, this becomes

$$\varepsilon = Blv$$

where B is the magnetic flux density (in T), l is the length of the conductor (in m) and v is the velocity of the conductor perpendicular to the field (in $m\,s^{-1}$).

Electrical power

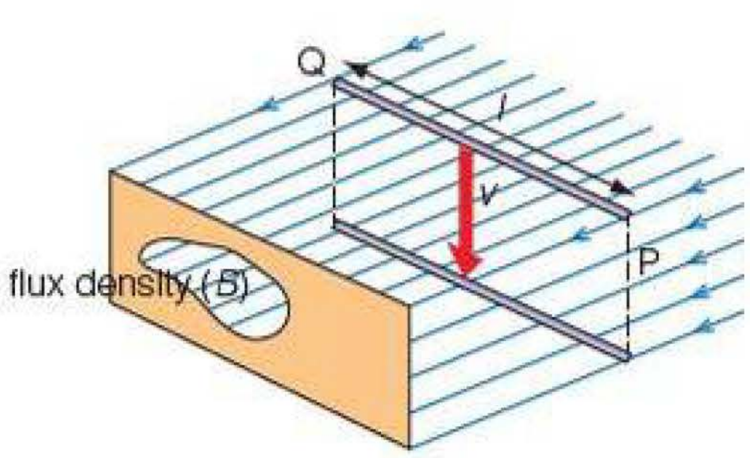


Figure 8.20 A wire moving at right angles to flux lines.

Power is the rate of doing work, and we can show that the expression for electrical power when a wire cuts flux lines (Figure 8.20) is consistent with this.

You already know that $P = VI$ in a circuit, where P is power (W), V is potential difference (V) and I is current (A). When power is generated by electromagnetic induction, we write

$$P = \varepsilon I$$

where ε is the e.m.f. generated (V).

You also know that work done is $F \times d$, where d is the distance travelled in the direction of the force F, so the rate of doing work is $F \times v$, where F is force (N) and v is velocity in the direction of the force ($m\,s^{-1}$).

Each second, the change of flux linkage for a wire moving through a magnetic field is BA, or Blv, where l is the length of the wire perpendicular to the field. Substituting in $P = EI$ gives

$$P = (Blv)I = (BIl)v$$

Since BIl is the force on a conductor in a field, $BIl \times v$ is consistent with the rate of doing work, or power generated, $F \times v$.

EXAMPLE

Induced e.m.f. between wing tips

Calculate the magnitude of the induced e.m.f. generated between the wing tips of an aircraft flying at $220\,m\,s^{-1}$ at a constant height. Assume that the average vertical component of the Earth's magnetic field is 4.1×10^{-5} T. The wing tips measure 30 m from tip to tip.

Answer

The area swept out by the wing tips each second is

$$lv = 30\,m \times 220\,m\,s^{-1} = 6600\,m^2 s^{-1}$$

Then we obtain

$$\frac{\Delta \phi}{\Delta t} = Blv$$
$$= 4.1 \times 10^{-5}\,T \times 6600\,m^2\,s^{-1}$$
$$= 0.27\,Wb\,s^{-1}$$

Since

$$\varepsilon = \frac{\Delta \phi}{\Delta t}$$

the induced e.m.f. is 0.27 V.

Calculating an induced e.m.f. for a rotating coil

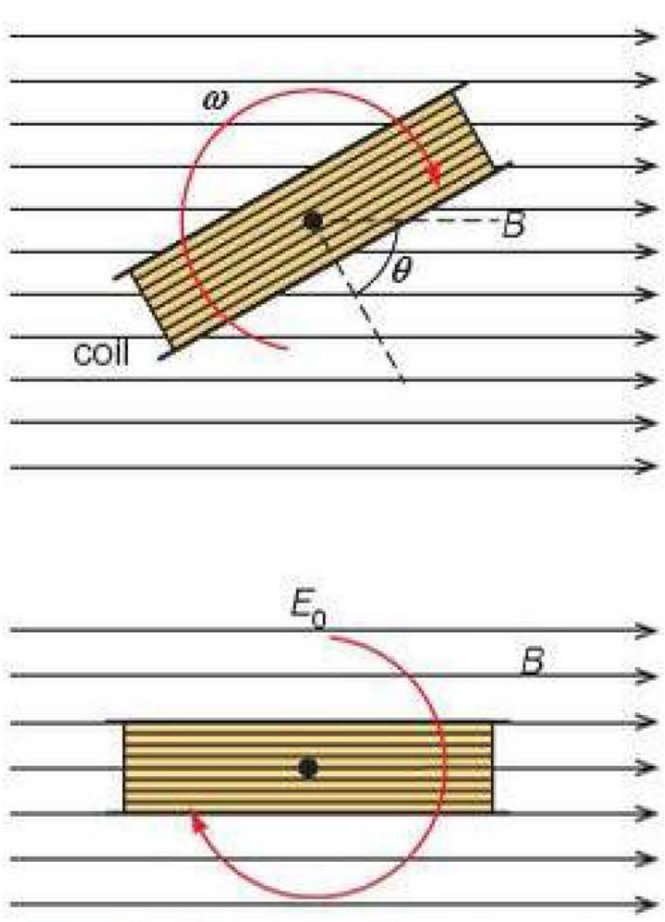


Figure 8.21 Remember to specify the axis of rotation for a coil in a magnetic field, which is shown here as the black dot.

When a coil rotates in a magnetic field (Figure 8.21), an a.c. voltage is induced in the coil.

To calculate the value of the induced e.m.f. at time t, you can use the following equation for a plane coil in a uniform magnetic field so long as the axis of rotation is at right angles to the field:

$$\mathcal{E} = BAN\omega \sin \omega t$$

where $\mathcal{E}$ is the induced e.m.f. (in V), B is the magnetic flux density (in T), A is the cross-sectional are a of the coil (in m^2), N is the number of turns on the coil, ω is the angular speed of the rotating coil (which can also be expressed as $\omega = 2\pi f$, where f is the frequency of rotation current) and t is the time (in s).

Since the maximum value of $\sin \omega t$ is 1, the maximum induced e.m.f. is

$$\mathcal{E}_{max} = BAN$$

Figure 8.22 shows how the magnetic flux linkage and the induced e.m.f. are linked:

- At A, $N\phi$ is a maximum, gradient $\frac{d(N\phi)}{dt}$ is 0, so $\mathcal{E} = 0$.
- At B, $N\phi$ is 0, gradient $\frac{d(N\phi)}{dt}$ is a maximum and negative, so $\mathcal{E}$ is a maximum positive.
- At C, $N\phi$ is a minimum, gradient $\frac{d(N\phi)}{dt}$ is 0, so $\mathcal{E} = 0$.
- At D, $N\phi$ is 0, gradient $\frac{d(N\phi)}{dt}$ is a maximum and positive, so $\mathcal{E}$ is a minimum negative.

MATHS BOX

We can deduce an expression for the induced e.m.f. by substituting

$$\theta = 2\pi ft$$

into the equation

$$N\phi = BAN\cos\theta$$

to give

$$N\phi = BAN\cos 2\pi ft$$

However,

$$\omega = 2\pi f$$

so

$$N\phi = BAN\cos\omega t$$

But since

$$\varepsilon = -N\frac{d\phi}{dt}$$

we find

$$\mathcal{E} = BAN\omega\sin\omega t$$

and the magnitude of the emf is given by

$$\mathcal{E} = BAN\omega\sin\omega t$$

You have come across a similar situation in Chapter 2 (simple harmonic motion).

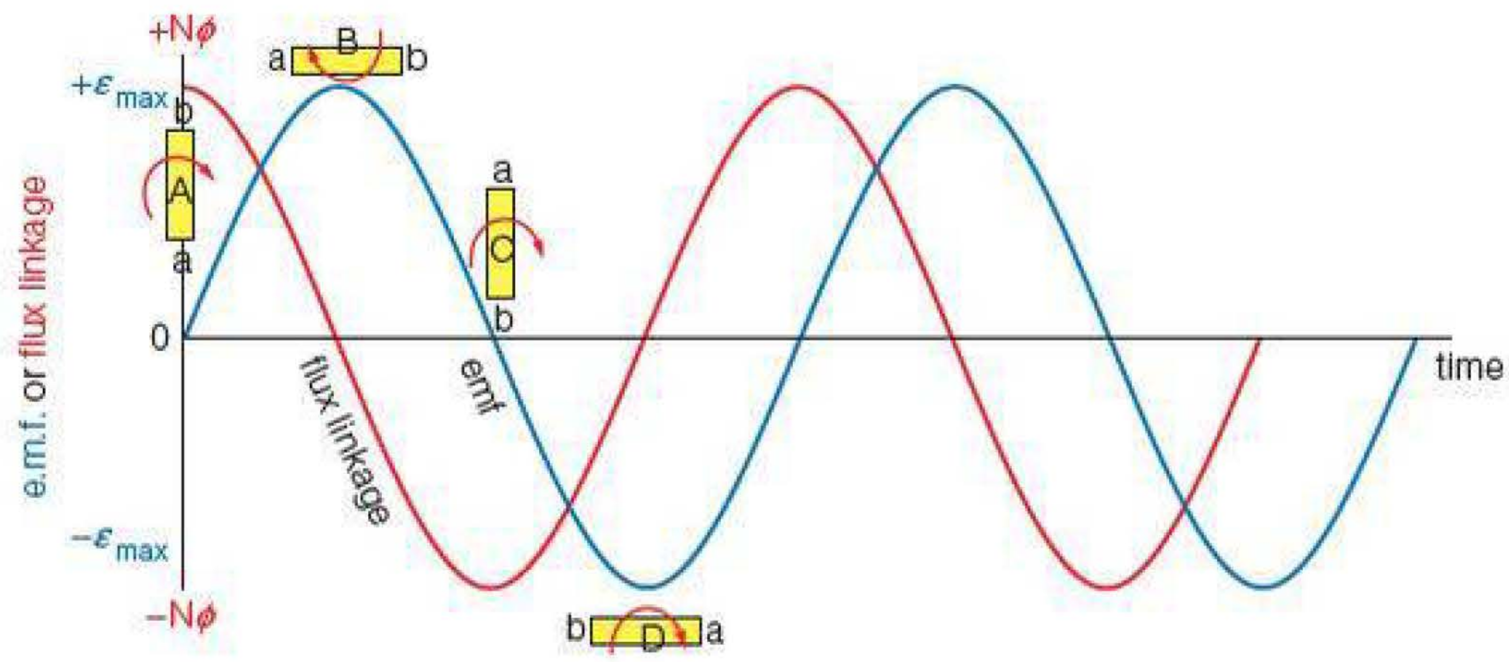


Figure 8.22

EXAMPLE

Induced e.m.f. in a rotating coil

A coil of 500 turns and cross-sectional area $0.18\,m^2$ rotates at a frequency of 5 Hz in a uniform flux density of strength 0.04 T.

1 Calculate the angular frequency of the coil.

Answer

Angular frequency = $2\pi f$
$= 2\pi \times 5\,Hz = 10\pi\,rad\,s^{-1}$.

2 Calculate the maximum value of induced e.m.f. for the coil.

Answer

Since the maximum value of $\sin\omega t$ is 1,
$E = BAN\omega\sin\omega t = BAN\omega$
$= 0.04\,T \times 0.18\,m^2 \times 500 \times 10\pi$
$= 36\pi\,V = 113\,V$

3 Calculate the value of the e.m.f. when $t = 0.20$ s and 0.21 s if the e.m.f. is zero when $t = 0$.

Answer

When t = 0.20 s,
$E = BAN\omega\sin\omega t$
$= 0.04\,T \times 0.18\,m^2 \times 500 \times 10\pi\sin(10\pi \times 0.20)$
$= 36\pi\sin 2\pi$
$= 0\,V$

When t = 0.21 s,
$E = BAN\omega\sin\omega t$
$= 0.04\,T \times 0.18\,m^2 \times 500 \times 10\pi\sin(10\pi \times 0.21)$
$= 36\pi\sin(2.1\pi)$
$= 11.2\pi\,V = 35\,V$

TEST YOURSELF

10 A wire of length 15 cm is moved perpendicular to a magnetic field of flux density 1.2 T. If the wire moves at a speed of $5\,cm\,s^{-1}$, calculate the induced e.m.f. in the wire.

11 A wire of length 8.0 cm, and negligible cross-sectional area, is dropped through a uniform magnetic field of strength 5.0 mT so that it cuts the flux lines.

a) The wire is dropped horizontally. Explain why the e.m.f. induced in the wire increases as it falls.

b) Calculate the induced e.m.f. when the speed of the wire is $3.2\,m\,s^{-1}$.

12 A dynamo has a coil of wire of 800 turns. When it is used, the coil spins three times a second in a region of uniform flux density 2.4 T.

a) Calculate the angular frequency of the coil.

b) The radius of the coil is 5 mm. Calculate the maximum value of the induced e.m.f.

c) The e.m.f. is given by $\mathcal{E} = BAN\omega\sin\omega t$. Calculate the e.m.f. at time 0.36 s.

Practice questions

1 A coil rotates in a plane perpendicular to flux lines in a magnetic field. The flux linkage and induced e.m.f. vary during the cycle. Which one of the following is always true?

A When the flux linkage is a maximum, the induced e.m.f. has a maximum value.

B When the flux linkage is zero, the induced e.m.f. is zero.

C When the flux linkage is a maximum, the induced e.m.f. is zero.

D When the flux linkage is increasing, the induced e.m.f. is increasing.

2 The unit of magnetic flux is

A weber **C** volt metre^2

B weber-turns **D** tesla metre

3 A metal sheet is pulled through a magnetic field, with its plane perpendicular to the flux lines. Once the sheet is moving at a steady speed, the force needed to pull the sheet at a constant speed

A increases **C** is zero

B decreases **D** is constant

4 A coil of wire is moved at right angles into, through and out of a uniform magnetic field at a steady speed. Which diagram in Figure 8.23 shows how the induced e.m.f. varies in the coil as it enters, moves through and leaves the field?

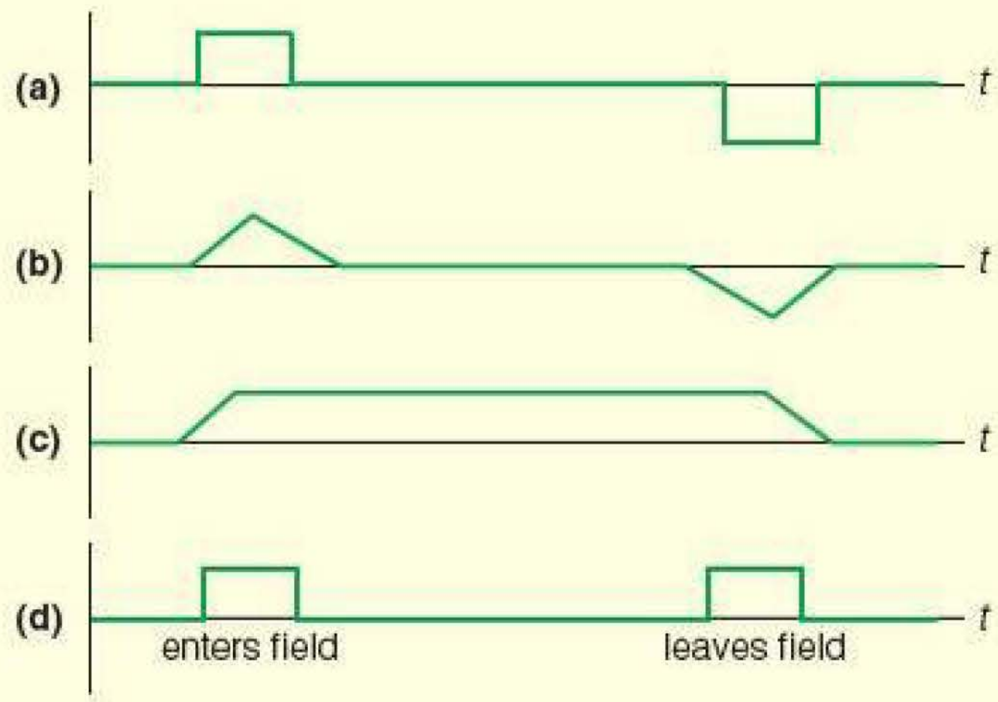


Figure 8.23

5 A large square coil of insulated copper placed in a storeroom has 50 turns. Each of its sides measures 80 cm. The coil is leaning at 45° to the vertical against a wall. The Earth's vertical magnetic flux density, B, at that point is 50 μT. Calculate the magnetic flux linkage of the coil.

A 1.41 mWb **C** 1.13 kWb

B 1.13 mWb **D** 1.13 Wb

6 Figure 8.24 shows how the flux linkage, $N\phi$, changes when a coil moves into a magnetic field.

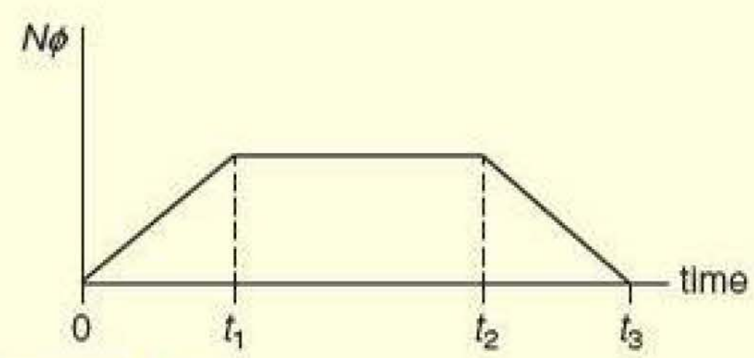


Figure 8.24

The induced e.m.f. in the coil

A increases until t_1 and then is constant between t_1 and t_2

B is constant between t_1 and t_2 and then decreases to zero at t_3

C decreases and then is zero between t_1 and t_2

D is zero between t_1 and t_2 and is constant between t_2 and t_3

7 A small magnet is dropped through a narrow copper tube and then through a plastic tube of the same diameter and length. Which one of the following statements is true?

A The magnet falls at the same speed in both tubes.

B The magnet falls slower in the copper tube because copper is magnetic.

C The magnet falls slower in the copper tube because of eddy currents in the copper tube.

D The magnet falls faster in the copper tube because of eddy currents in the magnet.

8 A coil of 100 turns has a cross-sectional area of $3.5 \times 10^{-3}\,m^2$. It is placed in a uniform magnetic field of flux density 4.9 mT, making an angle of 40° to the flux lines (Figure 8.25).

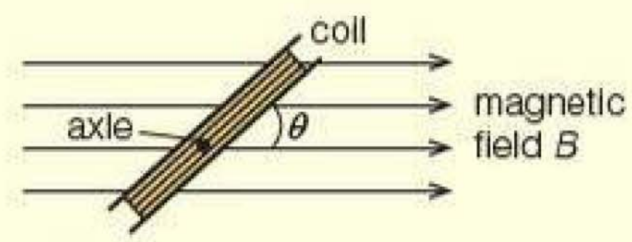


Figure 8.25

The change in flux linkage when the coil is rotated anticlockwise until $\theta = 90°$ is:

A an increase of 0.4 mWb turns

B a decrease of 0.4 mWb turns

C an increase of 0.6 mWb turns

D a decrease of 0.6 mWb turns

9 A dynamo spins with its axis perpendicular to the flux lines in a magnetic field. The period of rotation is 0.01 s. If the period doubles, which of the following changes will occur?

A The maximum e.m.f. and number of cycles per second will double.

B The maximum e.m.f. and number of cycles per second will halve.

C The maximum e.m.f. will double and the number of cycles per second will halve.

D The maximum e.m.f. will halve and the number of cycles per second will double.

10 Calculate the time taken for a search coil to be pulled out of a magnetic field if the maximum e.m.f. generated is 0.6 V. The search coil has an area 0.001 m^2 and 2000 turns and is perpendicular to the magnetic flux. The magnetic flux density is 400 mT.

A 3.0 s | **C** 1.3 s

B 0.48 s | **D** 800 s

11 a) Describe the function of a simple ac generator. (2)

A generator with 600 turns and a cross-sectional area of $3.0 \times 10^{-3}\,m^2$ is placed so it can spin in a horizontal magnetic field of flux density 0.049 T. The coil spins about a vertical axis.

b) Calculate the maximum magnetic flux linkage for the coil. (3)

Figure 8.26 shows how the magnitude of the flux linkage varies as the coil turns.

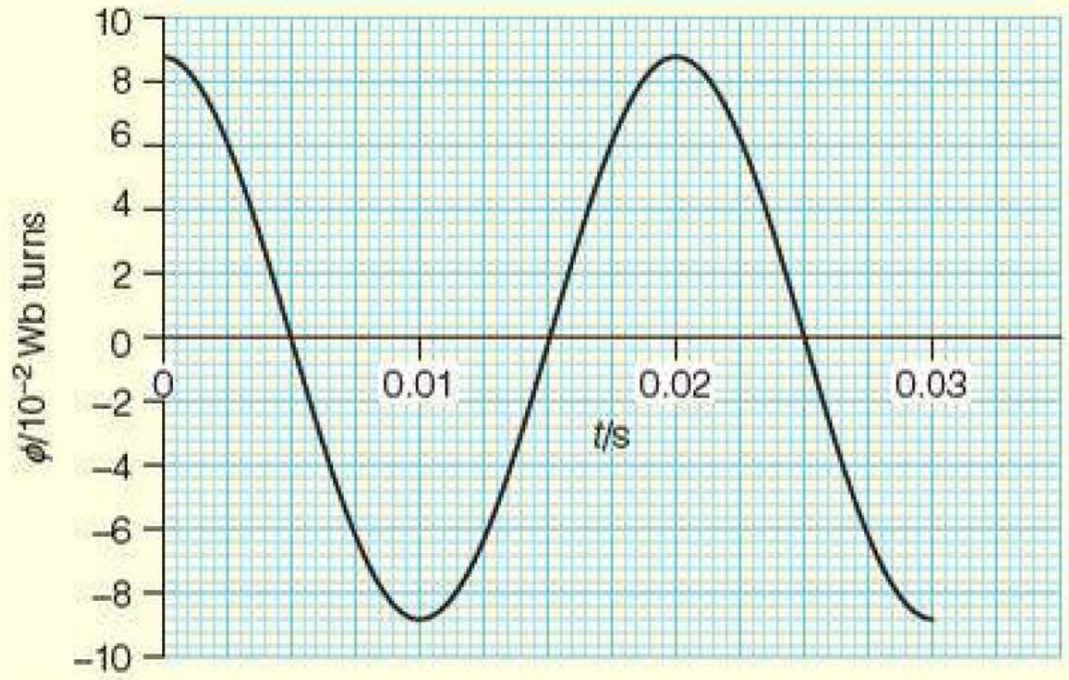


Figure 8.26

c) Explain why the flux linkage changes in this way as the coil turns. (3)

d) Calculate the maximum e.m.f. generated when the coil spins in the field. (4)

e) Use the graph to state when the e.m.f. has its maximum value. (1)

f) Explain how the maximum e.m.f. generated changes when the coil spins at half the speed in the field. (2)

12 a) State the SI unit of magnetic flux. (1)

Two wire coils A and B are placed next to each other (Figure 8.27). Coil A is connected to a switch and a battery. Coil B forms a circuit with a millivoltmeter.

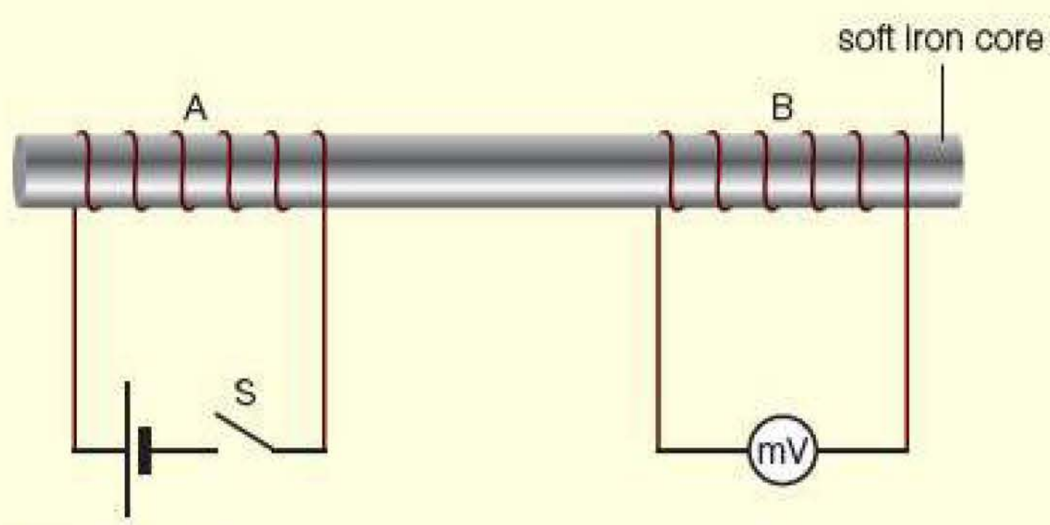


Figure 8.27

b) Describe and explain what is seen on the millivoltmeter when circuit A is switched on and off. (5)

c) Explain how the readings would change if circuit A contained a second cell. (2)

13 a) Explain the difference between magnetic flux and magnetic flux linkage. (1)

b) A straight wire of length 12.4 cm is held horizontally, then released so that it falls through magnetic flux density of 0.3 mT. If the e.m.f. generated across the wire at time t is 14.0 μV, calculate the speed of the wire at this time. (3)

14 A metal rod of length 2.3 m is pivoted at one end. It is moved in a circle making a complete circuit in 4 s.

a) Calculate the area swept out by the rod in 1 s. (1)

b) If the rod is orientated so that it is always perpendicular to a magnetic field of strength 1.2 T, calculate the maximum e.m.f. generated by this movement. (2)

15 A coil of 600 turns rotates at a frequency of 4 Hz perpendicular to a field of flux density 30 mT. The area of the coil is 15 cm^2.

a) Calculate the magnitude of the maximum flux linkage. (4)

b) Calculate the maximum induced e.m.f. (2)

c) If the flux linkage has its maximum value at time $t = 0$, calculate when the induced e.m.f. first has its maximum value. (2)

16 Figure 8.28 shows a way to measure the flow of oil through a pipeline. A small turbine is placed in the pipe so that the oil flow turns the blades around. Some magnets have been placed in the rim of the turbine so that they move past a solenoid.

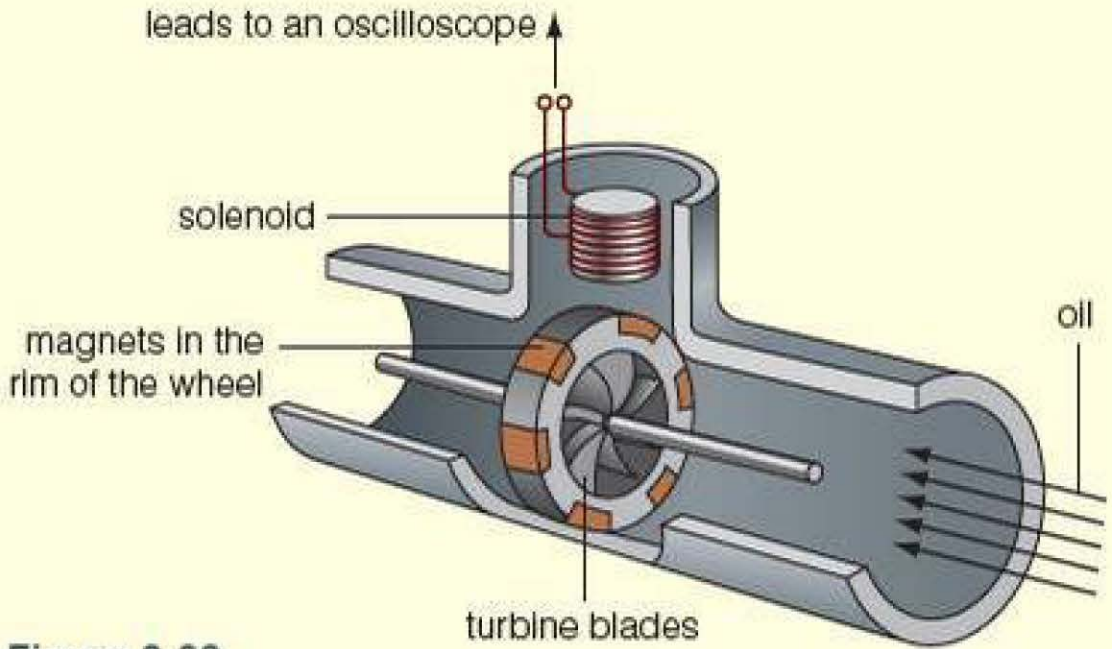


Figure 8.28

These moving magnets induce a voltage in the solenoid, which can be measured using an oscilloscope. Figure 8.29 shows the trace obtained. The faster the turbine rotates, the larger is the voltage induced in the solenoid. By measuring this voltage, an engineer can tell at what rate the oil is flowing.

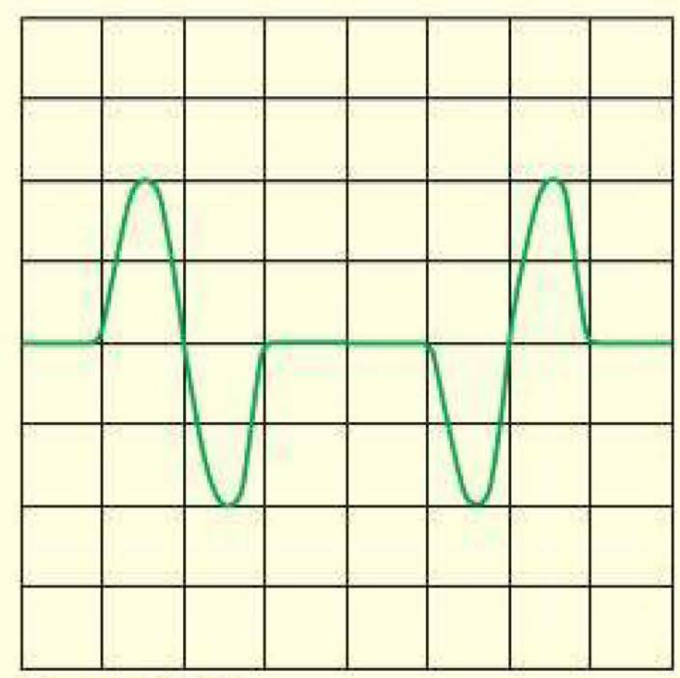


Figure 8.29

a) The poles on the magnet rim are arranged alternately with a north then a south pole facing outwards. Use this fact to explain the oscilloscope trace. *(1)*

b) Sketch the trace on the oscilloscope for the following (separate) changes.

i) The number of turns on the solenoid is made 1.5 times larger. *(1)*

ii) The flow of oil is increased so that the turbine rotates twice as quickly. *(1)*

c) i) Use the trace in Figure 8.29 to show that there is a time of 0.08 s between each magnet passing the solenoid. *(2)*

ii) How long does it take for the turbine to rotate once? *(1)*

iii) How often does the turbine rotate each second? *(1)*

Stretch and challenge

17 An electrodynamic tether is a cable used to control the motion of a satellite as it is taken into orbit. A tether of 20 km in length is used to connect a satellite to a space shuttle. The strength of the Earth's magnetic field at this altitude has a flux density of 50 μT.

a) If the satellite is travelling at an orbital speed of 8.0 km s^{-1} perpendicular to the Earth's field, calculate the e.m.f. generated across the tether.

b) Explain why this figure is likely to be inaccurate, and suggest a more accurate value.

18 A copper ring falls through a region of horizontal magnetic field of flux density *B* (Figure 8.30). Describe how the flux linkage, the induced e.m.f. and the current in the ring change as it enters the field, passes through and leaves it.

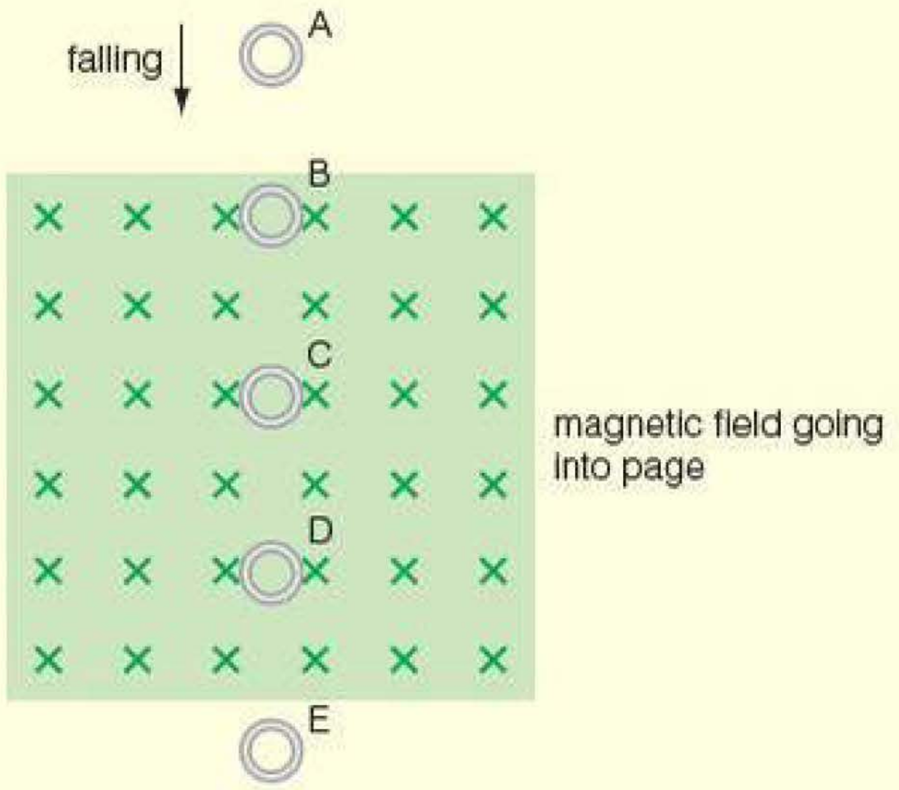


Figure 8.30 A ring falling through a magnetic field.

19 Figure 8.31 shows three coils connected in series to a data logger. A magnet is dropped through the three coils.

The graph in Figure 8.32 shows the voltage measured by the data logger as the magnet falls.

Explain the shape of the graph by commenting on the height of the peaks, the width of the peaks, the gaps between the peaks and the direction of the peaks.

Comment on the area under the peaks.

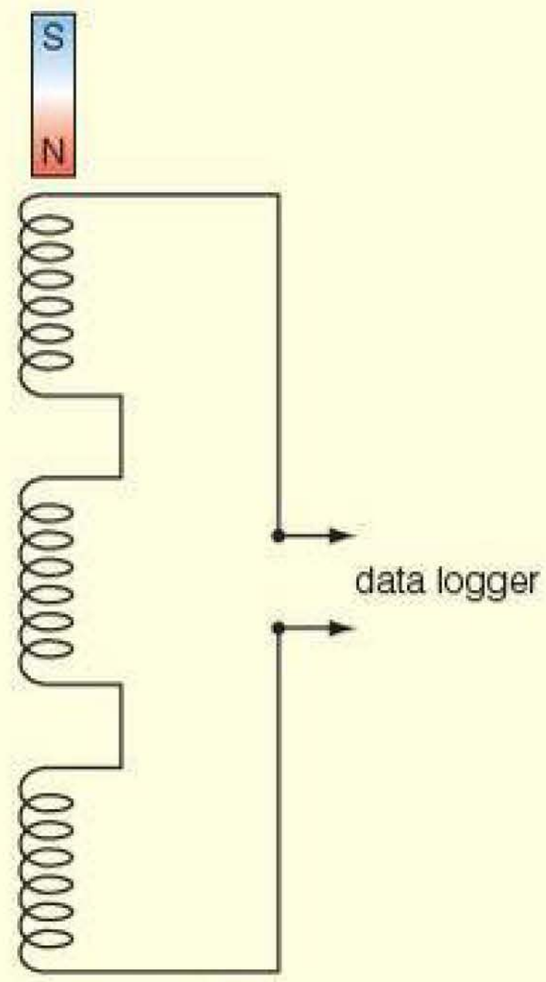


Figure 8.31

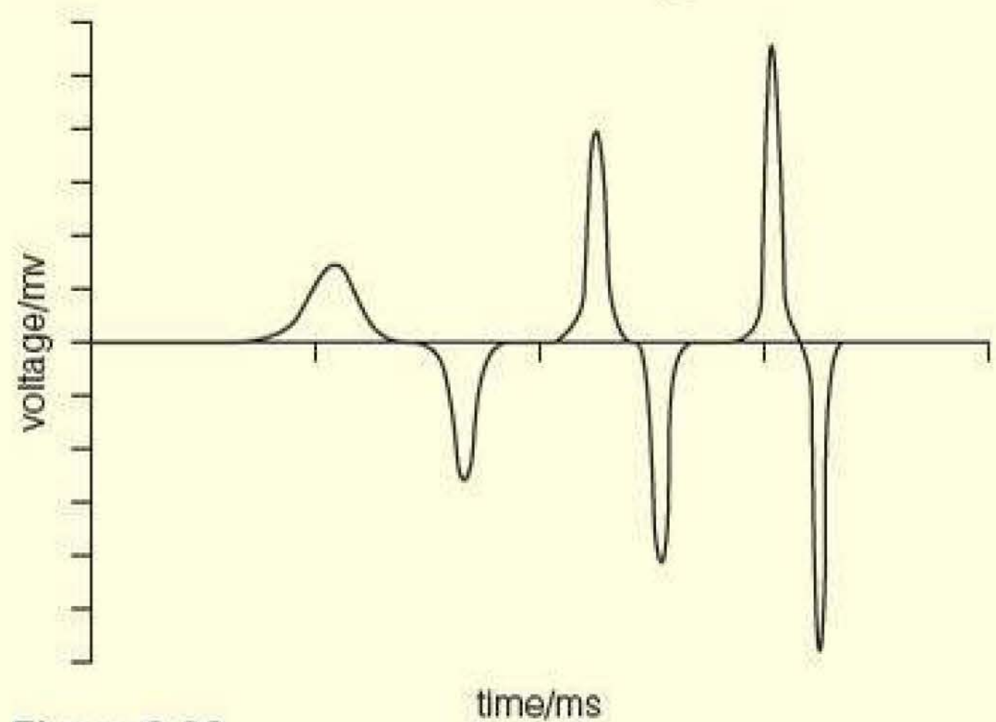
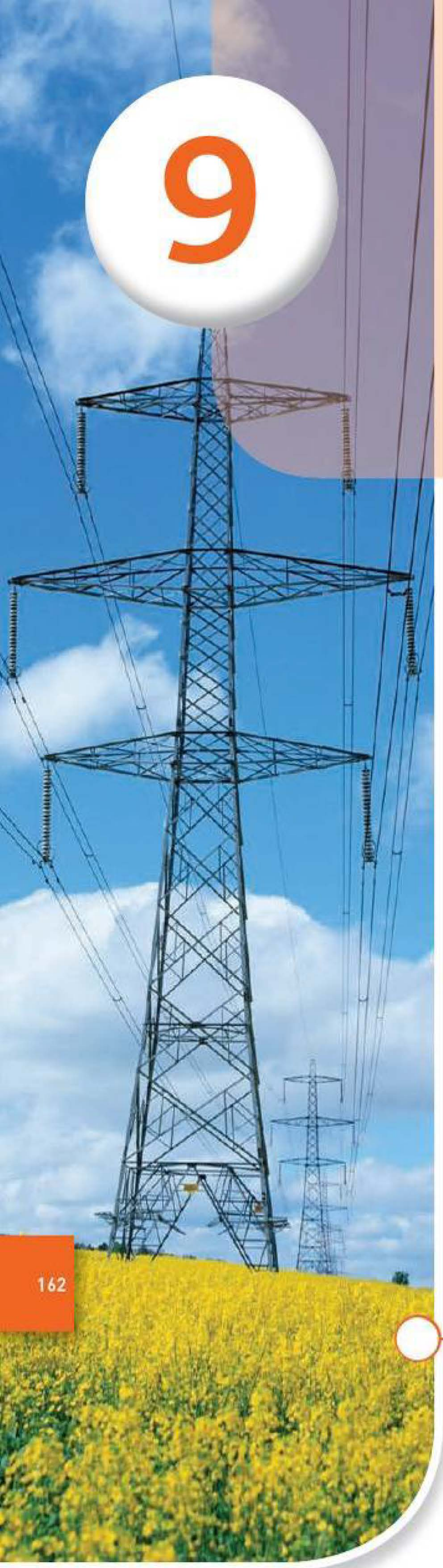


Figure 8.32



9 Alternating currents and transformers

PRIOR KNOWLEDGE

Before you start, make sure that you are confident in your knowledge and understanding of the following points:

- Electric current, I, is the rate of flow of charge, $\frac{\Delta Q}{\Delta t}$, measured in amperes (A).
- Potential difference (voltage), V, is the amount of electrical work done per unit charge, $\frac{W}{Q}$. Potential difference is measured in volts (V).
- Electric current, I, potential difference, V, and resistance, R, in a circuit are related to each other through the equation $V = IR$.
- Electrical power is the rate of doing electrical work, $P = VI = I^2R = \frac{V^2}{R}$
- The frequency, f, of a waveform is the number of complete waves per second and is measured in hertz (Hz).
- The time period, T, of a waveform, measured in seconds (s), is related to the frequency of the waveform, f, by $f = \frac{1}{T}$.
- The e.m.f. induced in a coil is $\varepsilon = -N\,\mathrm{d}\phi/\mathrm{d}t$, where ε is the induced e.m.f. (V), N is the number of turns on the coil, ϕ is the magnetic flux (Wb) and t is time (s).
- Eddy currents are generated in metal sheets by changes in magnetic flux. Eddy currents transfer electrical energy to heat energy.

TEST YOURSELF ON PRIOR KNOWLEDGE

1. An ac voltage has a time period of 0.004 s. What is the frequency of the voltage supply?
2. A current of 13 A flows through an electric fire element, which has a resistance of 14 Ω. Calculate the power dissipated by the fire in kW.
3. A bar magnet, which has poles measuring 1.5 cm × 1.5 cm, is pulled out of a coil in a time of 0.2 s. The coil has 10 000 turns and a resistance of 50 Ω. The average current flowing in the coil while the magnet is moving is about 35 mA. Estimate the flux density near the pole of the magnet.

Mains electricity

Electricity is generated and transmitted around the country in the form of alternating currents (ac) and voltages. These are used because they can be transformed to high voltages and very low currents in order to minimise the thermal energy lost as the current travels through the wires of the National Grid (Figure 9.1). Only about 2–3% of the electrical energy from the generators is lost as heat, saving energy, carbon emissions and money.

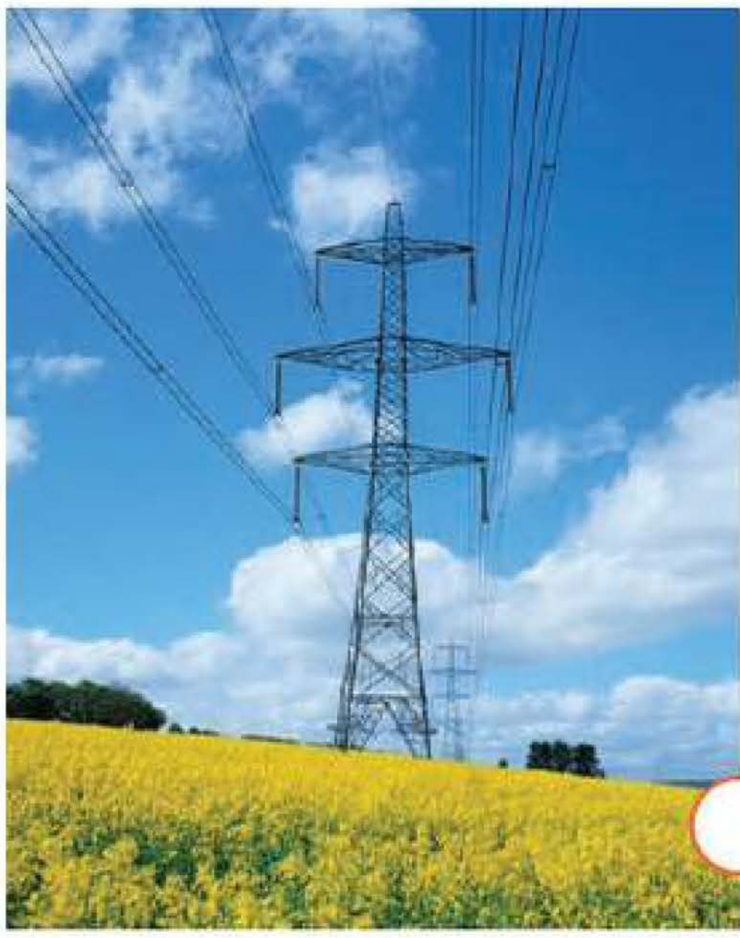


Figure 9.1 Electricity supply pylons – part of the National Grid.

Alternating current is delivered by the National Grid to consumers as a sinusoidally varying supply with a frequency of 50 Hz, and a range of different voltages, depending on the customer. Household mains has a nominal voltage of 230 V, although this value varies throughout the day depending on the demand and supply of electricity. The maximum current that can be drawn by a single domestic supply is about 65 A. The electrical socket ring main in your house has a maximum current of 13 A protected by a fuse or a circuit breaker. However, it is only lamps, heaters, cookers and devices such as vacuum cleaners and mowers, with large electric motors, that use ac directly off the mains. Most other devices work at much lower voltages and as direct currents (dc). This means that devices such as televisions, computers and games consoles all require a separate (or built-in) step-down transformer that converts 230 V ac into (for example) 12 V dc.

Alternating current and voltage

Alternating currents and voltages move in one direction for half of their cycle and in the opposite direction for the other half. Mains electricity comes in a sinusoidally changing pattern, with the magnitude of the current or the voltage continuously varying between maximum positive and negative values. The peak value of the voltage (or potential difference) is the maximum value in either the positive or negative direction, with respect to zero. The peak-to-peak value of the voltage is measured from one peak in the positive direction to the other peak (called a trough) in the negative direction (see Figure 9.2).

Peak voltage is half the peak-to-peak voltage, and is equivalent to the amplitude of the waveform.

The **peak voltage**, V_0, of the alternating waveform is half the peak-to-peak voltage, and is equivalent to the amplitude of the waveform. For a given component such as a resistor, the peak current I_0 and peak voltage V_0 are related to each other through the equation

$$V_0 = I_0 R$$

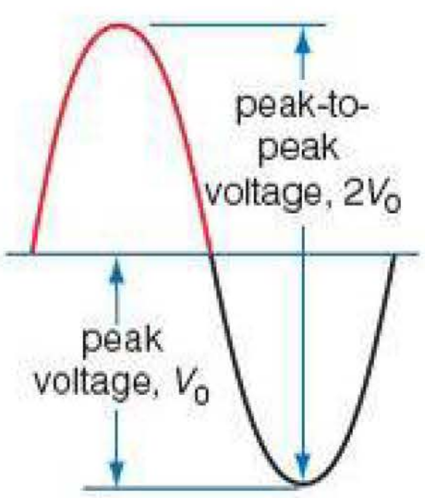


Figure 9.2 An alternating electrical waveform.

Comparing ac and dc equivalents

As alternating currents and voltages vary continuously, what value is used in calculations that gives the same effect as the equivalent direct current or voltage?

The average values cannot be used, because the average values are both zero – there is the same amount of signal above zero as there is below zero. The values chosen are the root mean square (r.m.s.) voltage and current. When multiplied together, these quantities produce the same power in a resistor as would be produced by the same dc values. This can be expressed more easily in the form of the equation:

$$P = V_{dc} I_{dc} = V_{rms} I_{rms}$$

A sinusoidal alternating voltage, V, varying with time, t, can be represented by the equation

$$V = V_0 \sin(2\pi f t)$$

where V_0 is the peak voltage, and f is the frequency of the supply. This is shown on the graph in Figure 9.3.

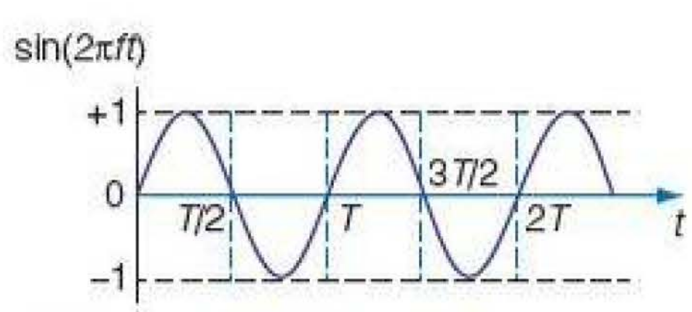


Figure 9.3 Alternating voltage.

If this voltage is applied across a fixed resistor, R, then the power dissipated by the resistor is equal to

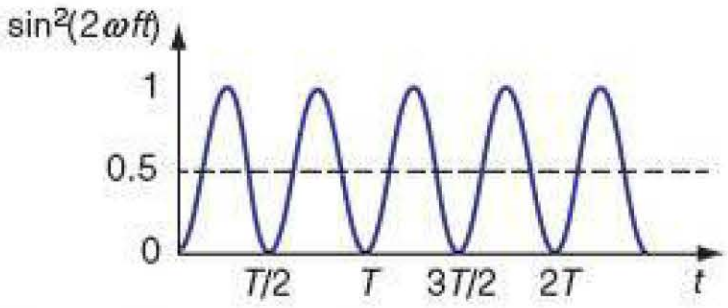


Figure 9.4 The $\sin^2$ graph.

$$P = \frac{V^2}{R} = \frac{V_0^2 \sin^2(2\pi ft)}{R}$$

The average power is the power we need to compare to an equivalent constant dc value, but as $\frac{V_0^2}{R}$ is constant in this equation, we only need to find the average value of $\sin^2(2\pi ft)$. This can be done by analysing the graph of the function in Figure 9.4.

It can be seen from Figure 9.4 that the average value of $\sin^2(2\pi ft) = 0.5$, so

$$P = \frac{\frac{1}{2}V_0^2}{R}$$

This will be the same power as that for an equivalent constant dc value of voltage, V_{dc}:

$$P = \frac{\frac{1}{2}V_0^2}{R} = \frac{V_{dc}^2}{R}$$

Hence we obtain

$$\frac{V_0^2}{2} = V_{dc}^2$$

$$V_{dc} = \sqrt{\frac{V_0^2}{2}}$$

and

$$V_{dc} = V_{rms} = \frac{V_0}{\sqrt{2}}$$

As the alternating current varies in phase with the voltage, using a similar reasoning yields

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

As a result, the mean alternating power, P_{mean}, which is equivalent to the dc power, is given by

$$P_{mean} = V_{rms} I_{rms}$$

and the peak alternating power, P_{peak}, is given by

$$P_{peak} = V_0 I_0$$

Hence, finally we have

$$P_{mean} = \frac{V_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}} = \frac{V_0 I_0}{2} = \frac{P_{peak}}{2}$$

In other words, the mean power dissipated through a fixed resistor by an alternating current and voltage is equal to half the peak power dissipated.

EXAMPLE

Mains power

The given UK mains voltage is 230 V ac – this is an r.m.s. value. The largest power that can usually be drawn from a local grid is 15 kW – this is the mean value, equivalent to a dc supply.

1 What is the r.m.s. current?

Answer

Use the equation for P_{mean} from the text:

$$P_{mean} = V_{rms}I_{rms}$$

$$I_{rms} = \frac{P_{mean}}{V_{rms}} = \frac{15\ \text{kW}}{230\ \text{V}} = 65.2\ \text{A} = 65\ \text{A}\ \ (2\ \text{s.f.})$$

2 What are the peak power, peak voltage and peak current?

Answer

Combine the equations for V_0 and I_0 from the text to give the equation for P_{peak}:

$$V_0 = V_{rms} \times \sqrt{2} = 230\ \text{V} \times \sqrt{2} = 325.3\ \text{V} = 330\ \text{V}\ \ (2\ \text{s.f.})$$

$$I_0 = I_{rms} \times \sqrt{2} = 65.2\ \text{A} \times \sqrt{2} = 92.2\ \text{A} = 92\ \text{A}\ \ (2\ \text{s.f.})$$

$$P_{peak} = V_0I_0 = 325.3\ \text{V} \times 92.2\ \text{A} = 29993\ \text{W} = 30\ \text{kW}\ \ (2\ \text{s.f.})$$

We can then use these values to calculate the peak-to-peak values of p.d. and current:

$$V_{peak\text{-}to\text{-}peak} = 2V_0 = 650\ \text{V}\ (2\ \text{s.f.})$$
$$I_{peak\text{-}to\text{-}peak} = 2I_0 = 180\ \text{A}\ (2\ \text{s.f.})$$

TEST YOURSELF

1 What is meant by the 'root mean square' (r.m.s.) voltage?

2 An ac power supply delivers $V_{rms} = 6.0$ V to a fixed resistor of resistance $R = 2.5\ \Omega$. Calculate:
  a) the r.m.s. current through the resistor
  b) the mean power delivered to the resistor
  c) the peak power delivered to the resistor.

3 In the USA, the nominal r.m.s. voltage is 120 V. If the mean power delivered to a US domestic house is the same as that to a UK house (15 kW), calculate:
  a) the r.m.s. current delivered
  b) the peak voltage delivered
  c) the peak power delivered.

4 The ac motor for a (UK) mains washing machine works with a peak power of 400 W. Calculate:
  a) the mean power drawn by the motor
  b) the r.m.s. current through the motor.

Using an oscilloscope to display waveforms

Analysing alternating waveforms is best done by displaying the waveform using an oscilloscope. Oscilloscopes are a form of visual, calibrated voltmeter, where the operator is able to alter how the waveform is displayed. First, it is possible to control the time taken for the signal to move across the screen by adjusting the *timebase* – often labelled time/div. Secondly, the amplitude of the signal displayed on the calibrated screen can be controlled by adjusting the *y-sensitivity* – this is also known as the vertical sensitivity, *y*-gain or simply volts/div. The timebase (in seconds) provides a scale for the *x*-axis of the screen and indicates the time taken for the signal to move horizontally across one square on the screen. Using the square grid on the screen to measure the number of horizontal squares between two successive peaks (or troughs) allows the period of the waveform to be determined and hence the frequency. The oscilloscope also makes it very easy to measure the peak-to-peak value of the wave by counting vertical squares and then using the *y*-sensitivity (usually calibrated in volts, millivolts or microvolts) to apply a scale.

EXAMPLE

Oscilloscope with a dc signal

The oscilloscope in Figure 9.5 is displaying a dc signal (from a battery for example). Describe the signal.

Answer

The timebase is set to 20 ms/div, so, because there are 10 horizontal divisions on the screen grid, the signal takes 200 ms (0.2 s) to travel from one side of the screen to the other. The y-sensitivity is set to 1 V/div, and the signal is 2.4 divisions vertically up from the centre line. This makes the voltage of the signal 2.4 V.

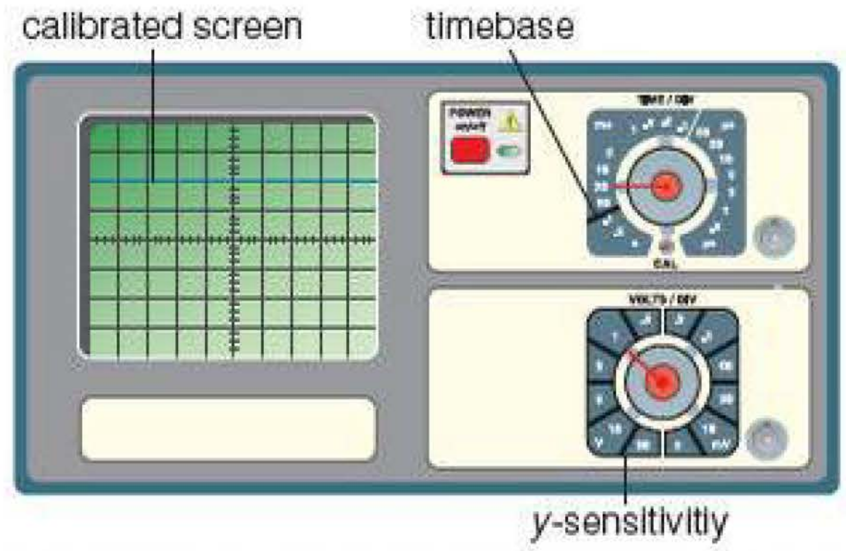


Figure 9.5 Oscilloscope displaying a dc signal.

EXAMPLE

Oscilloscope with an ac waveform

The oscilloscope in Figure 9.6 illustrates an ac waveform from a signal generator. Describe the waveform.

Answer

In this case the timebase and y-sensitivities have not changed. There are five horizontal divisions between the two successive peaks or troughs, and this corresponds to a time period of 100 ms. The frequency of the signal is therefore

$$\text{frequency} = \frac{1}{\text{time period}} = \frac{1}{100 \times 10^{-3}\ \text{s}} = 10\ \text{Hz}$$

The peak-to-peak voltage measured from the bottom of a trough to the top of a peak on the screen is six divisions, corresponding to 6 V. This corresponds to a peak voltage of 3 V and $V_{rms} = \frac{3\ V}{\sqrt{2}} = 2.1 V$.

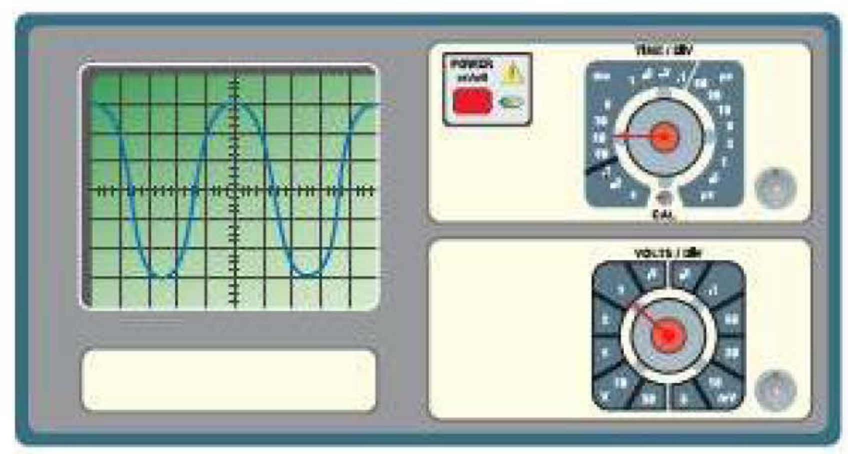


Figure 9.6 Oscilloscope displaying an ac waveform.

ACTIVITY

Virtual oscilloscopes and signal generators

There are many excellent *virtual oscilloscope* and *signal generator simulations* and *apps* available on-line. Using the keywords in italics as search terms in a search engine will take you to a range of different versions, although they all operate using the same principles as the real thing illustrated in Figures 9.5 and 9.6.

Some simulations are just oscilloscopes, and these rely on an external signal being generated and fed through the computer's sound card or microphone. Be careful when doing this – use an external device that does not exceed the sound card's input voltage (a tablet is ideal for this).

Other simulations have a built-in signal generator that allows you to generate an alternating signal directly for display on the oscilloscope screen.

Use one of these simulations to familiarise yourself with the controls of the oscilloscope, so that when you come to use the real thing you will be able to analyse alternating waveforms and extract the relevant key information, such as the frequency and the peak-to-peak values. You could also use your simulation to analyse the voltage signals coming off different music tracks, although the rapidly varying voltages may be tricky to measure unless the simulation has a 'Hold' or 'Freeze' function. Alternatively you could speak or sing directly into the sound card and use the oscilloscope and your voice to analyse some alternating signals.

Transformers

Figure 9.7 Step-up transformers increase the generated voltage from 25 000 V to 400 000 V.

Many devices use transformers to increase or reduce the voltage of an alternating voltage supply. Transformers are used by the National Grid to increase the voltage generated in power stations up to 400 000 V, so that energy can be saved as electricity is transmitted around the country. Then transformers are used to reduce this high voltage for safe use in the home. Figure 9.7 shows the transformers used to step up the voltage in a power station.

Structure of a transformer

The structure of a transformer is simple. It consists of two coils of wire linked by a soft iron core as shown in Figure 9.8.

An alternating current in the primary coil creates a changing magnetic field in the core, which is made of a soft magnetic material such as iron. The secondary coil is also wound round the core. As the magnetic flux in the core changes, the magnetic flux linkage to the secondary coil changes and an e.m.f. is induced in the secondary coil. Because transformers use electromagnetic induction, they only work with an ac supply.

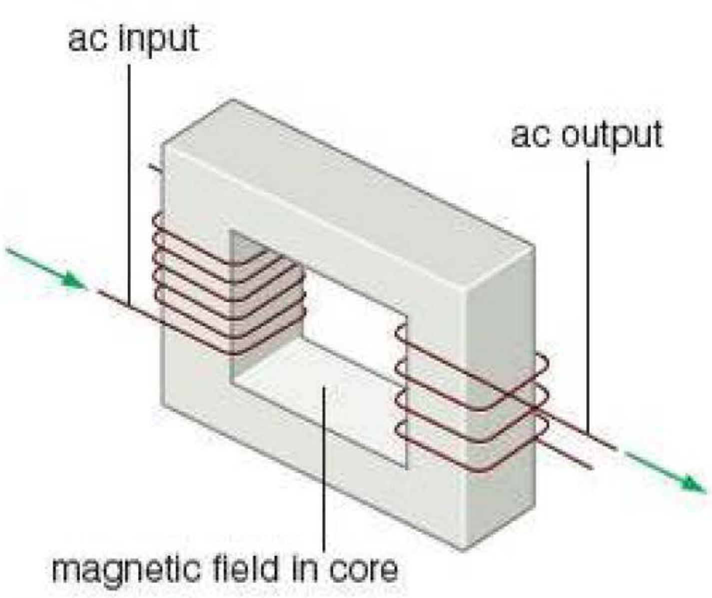


Figure 9.8 The structure of a transformer.

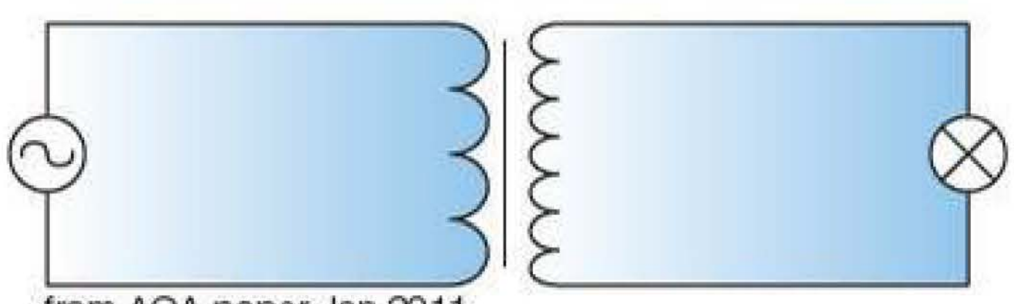
The turns rule

For an ideal transformer, with no power losses, the ratio of the turns on each coil equals the ratio of the primary and secondary voltages. That is

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

where V_s = secondary voltage (V), V_p = primary voltage (V), N_s = turns on the secondary coil and N_p = turns on the primary coil.

A step-up transformer is a transformer that increases voltage, so N_s/N_p is more than 1. A step-down transformer is a transformer that decreases voltage, so N_s/N_p is less than 1. Figure 9.9 shows a simple circuit diagram for a transformer, with the symbols for an ac supply, a step-up transformer and a bulb.



from AQA paper Jan 2011

Figure 9.9

You already know that E depends on the number of turns on the coil. The induced e.m.f. is given by

$$\varepsilon = -N\frac{d\phi}{dt}$$

The rate of flux change $\frac{d\phi}{dt}$ in the core of the transformer is the same for both coils, but the number of turns N is different, so the induced e.m.f. is different in the secondary coil, and depends on the ratio of N_s to N_p.

Transformers cannot increase the power output of the supply. In an ideal transformer, with no power losses, the power input to the transformer must be equal to the power output. Therefore we can write the following equation:

$$V_pI_p = V_sI_s$$

where V_s = secondary voltage (V), V_p = primary voltage (V), I_s = current in the secondary coil (A) and I_p = current in the primary coil (A).

This means that a transformer that reduces the output voltage compared to the input voltage has a larger current in the secondary coil compared to the primary coil.

EXAMPLE

Step-down transformer

A step-down transformer has 2500 turns on the primary coil. It transforms mains voltage, 230 V ac, into a 12 V ac supply.

1 Calculate the number of turns on the secondary coil.

Answer

Rearranging the equation $\frac{V_s}{V_p} = \frac{N_s}{N_p}$ gives

$$N_s = N_p \times \frac{V_s}{V_p}$$

$$= 2500 \times \frac{12\text{ V}}{230\text{ V}}$$

$$= 130\text{ turns}$$

2 When the current in the secondary coil is 1.5 A, what is the current in the primary coil? Assume that the transformer is 100% efficient.

Answer

Rearranging the equation $V_pI_p = V_sI_s$ gives

$$I_p = \frac{V_s}{V_p} \times I_s$$

$$= \frac{12\text{ V}}{230\text{ V}} \times 1.5\text{ A}$$

$$= 0.078\text{ A}$$

Transformer efficiency

Transformers can be very efficient, but they are never 100% efficient. The efficiency of a transformer is calculated using this equation:

$$\text{efficiency} = \frac{V_sI_s}{V_pI_p}$$

where V_s = secondary voltage (V), V_p = primary voltage (V), I_s = current in the secondary coil (A) and I_p = current in the primary coil (A).

EXAMPLE

Efficiency of a transformer

The efficiency of a mains transformer is 90%. The mains supply is 230 V ac and the output of the transformer is 12 V ac. Calculate the current in the secondary coil when the current in the primary coil is 0.5A.

Answer

Use the equation for efficiency and substitute the values known:

$$\text{efficiency} = \frac{V_sI_s}{V_pI_p}$$

$$0.9 = \frac{12\text{ V} \times I_s}{230\text{ V} \times 0.5\text{ A}}$$

Rearranging to make I_s the subject gives

$$I_s = \frac{0.9 \times 230\text{ V} \times 0.5\text{ A}}{12\text{ V}}$$

$$= 8.6\text{ A}$$

Energy losses in transformers

Energy losses in transformers occur because of the following effects:

- Heat is produced in the copper wires of the primary coil and secondary coil when a current flows. Using low-resistance wire reduces these losses. This is particularly important for the secondary coil of a step-down transformer, because the current is larger in the secondary coil compared to the primary coil. A thicker wire is often used in the secondary coil of a step-down transformer.
- Some magnetic flux produced by the primary coil does not pass through the iron core, which means the flux linkage to the secondary coil is not 100%. This can be reduced by designing the transformer with coils close to each other or wound on top of each other, which improves the flux linkage.
- There is an effect called *hysteresis*. Some energy is lost as heat every time the direction of the magnetic field changes because energy is needed to realign the magnetic domains in the core. This is reduced by using a soft magnetic material such as iron, rather than steel which needs more energy to demagnetise and magnetise.
- Eddy currents form in the iron core due to the continuously changing flux. These currents heat the core up, increasing energy losses. Eddy currents are reduced by making the core using laminated sheets separated by thin layers of insulation. Eddy currents are discussed in more detail below.

Eddy currents in transformers

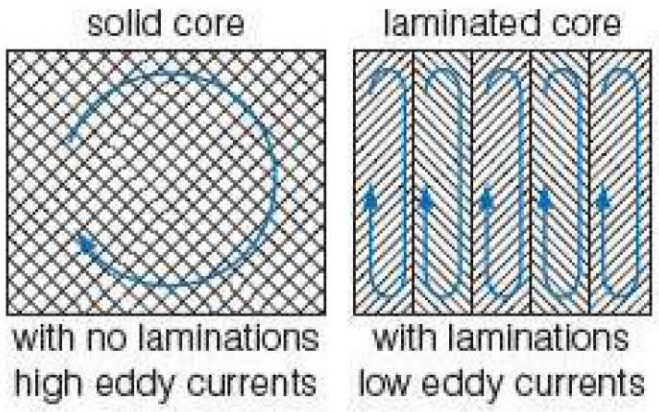


Figure 9.10 Eddy currents are reduced by building the core from thin, insulated layers of iron.

In Chapter 8, you learnt that eddy currents are created in metal sheets when there is a change in magnetic flux. In the core of a transformer, the alternating supply creates alternating magnetic flux changes, and these create eddy currents. Eddy currents flow in loops, in a direction that opposes the magnetic flux changes that cause them. The result is that eddy currents in the iron core will reduce the e.m.f. induced in the secondary coil. In a core made from solid iron, eddy currents could become large enough to melt the core, because the resistance of the iron core is very low. To prevent these problems, the core is built from very thin laminations, or layers, of metal (Figure 9.10). The eddy currents are smaller when there are thin laminations, because the induced voltage drives the current round longer paths – so the resistance to flow increases. The laminations are insulated from each other, for example using layers of insulating varnish.

Transmission of electrical power

Energy losses due to the heating of transmission lines in the National Grid can be very significant because electrical energy can be transmitted very long distances from the power stations to the end users. Electricity is transmitted throughout the UK (Figure 9.11), and also between European countries – for example, between the UK, France and the Netherlands.

Transformers are used to step up the voltage generated in power stations. Since power transmitted is equal to the product $V \times I$, stepping up the voltage in transmission lines reduces the current. Smaller currents have a smaller heating effect on the power lines, so reducing the current in transmission lines reduces energy losses to the surroundings.

Power stations generate electrical energy at a potential of about 25 kV. This voltage is stepped up using transformers shortly after it leaves the power station and is transmitted using transmission lines operating at 275 kV and 400 kV. Overhead transmission lines are supported using the familiar large steel pylons. Transformers in substations step down the voltage for distribution of electricity to the end user. Distribution lines operate at 132 kV, with cables supported on smaller steel pylons. Wooden poles are used to support power lines operating at 11 kV and 33 kV.

Calculating power losses in transmission lines

Power losses in the National Grid total about 3% of demand, and mainly occur in the generator transformers, overhead lines, underground cables and grid supply transformers. Two-thirds of the losses in the National Grid occur in the overhead lines of the transmission system. However, the percentage losses in power lines in the distribution system are bigger than in transmission lines because the voltage is stepped down, so currents in the power lines are larger. Losses in the distribution system can reach as much as 15%.

Power losses are calculated using $P = I^2R$. Because the power losses are proportional to the square of the current, doubling the current quadruples the power losses. Power cables are made from aluminium supported by steel cores, and the low resistance of these cables reduces losses in power lines, since losses are proportional to R.

Step-down transformers in distribution systems are made more efficient by using thicker wire in the secondary coil. The current is higher in the secondary coil of step-down transformers, so I^2R losses due to the heating of the secondary coil can be significant. Reducing the resistance of the secondary coil reduces I^2R losses.

EXAMPLE

Transmission line

A power transmission line in a factory operates at 25 kV. The power input to the cable is 750 kW.

1 Calculate the current in the transmission line.

Answer

Rearranging the equation P = VI for power gives

$$I = \frac{P}{V} = \frac{750 \times 10^3\ \text{W}}{25 \times 10^3\ \text{V}}$$

$$= 30\ \text{A}$$

2 The resistance of the cable is 40 Ω. Calculate the power supplied by the cable.

Answer

Use the equation power supplied = input power − power losses

power losses = $I^2R = (30\ \text{A})^2 \times 40\ \Omega = 36\ \text{kW}$

power supplied = 750 kW − 36 kW = 714 kW (710 kW 2sf)

3 Calculate the efficiency of the transmission line.

Use the equation for efficiency and substitute the values known:

$$\text{efficiency} = \frac{P_{out}}{P_{in}} = \frac{714\ \text{kW}}{750\ \text{kW}} \times 100 = 95.2\%\ (95\%\ 2\text{sf})$$

TEST YOURSELF

5 Explain why:
   a) transformers do not work using dc
   b) the iron core of a transformer is laminated
   c) a thicker wire is used in the secondary coil of a step-down transformer.

6 a) Calculate the turns ratio for a transformer that steps up a 25 kV input to an output of 132 kV.
   b) The transformer in part (a) is 90% efficient, and it has a current of 40A flowing in the primary coil. Calculate the power output in the secondary coil.
   c) Calculate the current in the secondary coil.

7 A step-up transformer transforms the input voltage, 12V ac, into a 48V ac supply.
   a) If the primary coil has 200 turns, calculate the number of turns on the secondary coil.
   b) When the current in the primary coil is 2.4A, what is the current in the secondary coil? Assume that the transformer is 100% efficient.

8 A transformer is 95% efficient. The transformer uses mains voltage, 230V ac, and the output voltage is 6V ac. Calculate the current in the primary coil if the current in the secondary coil is 4.8A.

Practice questions

1 The r.m.s. voltage from a power supply with a peak voltage of 6 V is:

A 3.0 V **C** 4.2 V

B 0.12 V **D** 0.85 V

Use the information in Figure 9.12 about the voltage waveform from an ac power supply to answer questions 2, 3 and 4.

2 The peak-to-peak voltage shown in Figure 9.12 is:

A 128 V **C** 90.5 V

B 64 V **D** 45.3 V

3 The r.m.s. voltage of the signal shown in Figure 9.12 is:

A 128 V **C** 90.5 V

B 64 V **D** 45.3 V

4 The frequency of the ac power supply shown in Figure 9.12 is:

A 0.2 Hz **C** 0.1 Hz

B 200 Hz **D** 100 Hz

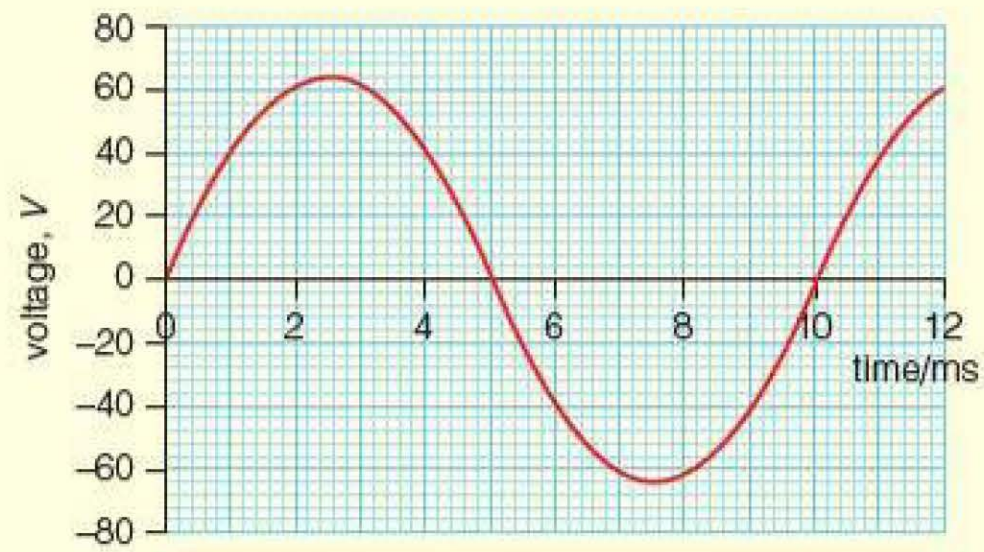


Figure 9.12

5 Which of the waveforms in Figure 9.13 shows a 4.24 V r.m.s. voltage?

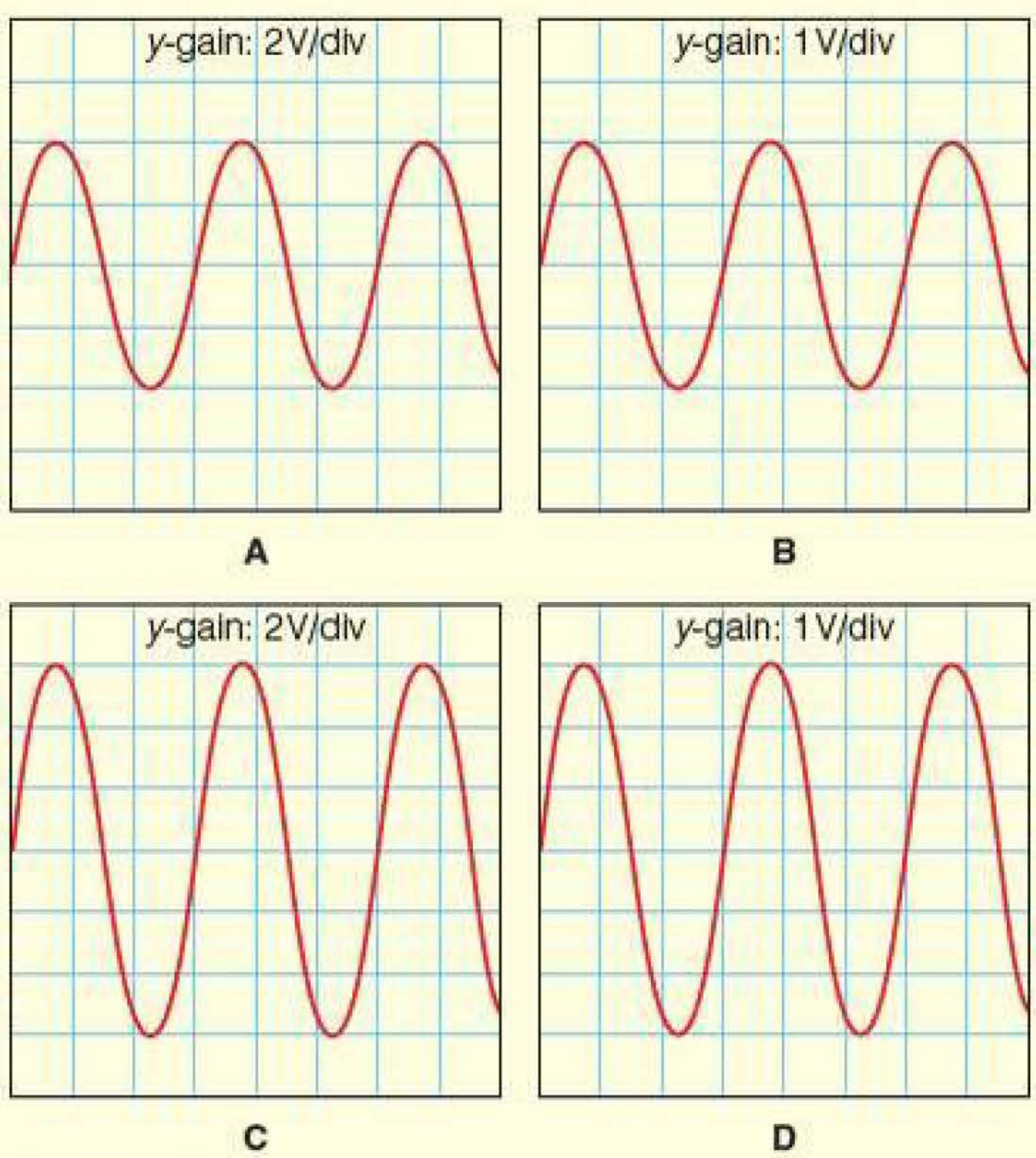


Figure 9.13

6 A transformer has 1500 turns on the primary coil and 600 turns on the secondary coil. The transformer uses 230 V ac mains supply, and draws a current of 0.4 A in normal use. If the efficiency of the transformer is 85%, what is the current in the secondary coil, when the secondary voltage is 92 V?

A 0.65 A **C** 0.85 A

B 1.00 A **D** 0.14 A

7 The primary coil of a step-down transformer uses an ac mains supply. The secondary coil is connected to a phone charger. Which line A–D in the table correctly describes the potential difference and current in the secondary coil in relation to the primary coil?

	Secondary current/primary current	Secondary p.d./primary p.d.
A	>1	>1
B	>1	<1
C	<1	>1
D	<1	<1

8 Which of these does not reduce the efficiency of a transformer?

A heating of the primary and secondary coils

B eddy currents in the iron core

C leakage of magnetic flux from the primary coil

D insulation between the primary and secondary coils

9 The National Grid transmits electrical power from power stations using transmission lines. Substations link transmission lines to distribution systems that distribute electrical power to the final users. Which line A–D in the table correctly describes the arrangement of step-up and step-down transformers in the National Grid?

	Transformers in power stations	Transformers in substations
A	Step-up	Step-down
B	Step-up	Step-up
C	Step-down	Step-down
D	Step-down	Step-up

10 A cable, $4\,cm^2$ in cross-section and of resistivity $5 \times 10^{-8}\,\Omega\,m$, carries a current of 2500 A. The power loss per km is:

A 391 W **C** 391 kW

B 781 W **D** 781 kW

11 An alternating voltage from a signal generator is displayed on an oscilloscope screen with the following settings: timebase, 25 ms per division; and *y*-sensitivity, 3 V per division. The waveform of the voltage signal is shown in Figure 9.14.

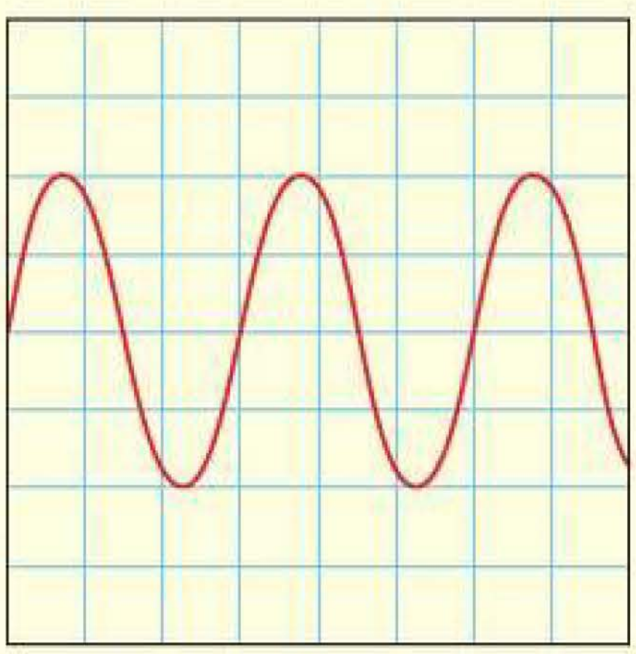


Figure 9.14

Calculate:

a) the peak-to-peak voltage of the signal *(1)*

b) the r.m.s. voltage *(1)*

c) the time period of the signal *(1)*

d) the frequency of the signal. *(2)*

Make a sketch copy of the trace on the oscilloscope screen.

e) On your copy, sketch the dc voltage signal that would produce the same power dissipation in a resistor, of resistance *R*, equivalent to that produced by the signal generator. *(2)*

12 Domestic electricity in the USA is delivered with a peak value of 170 V and a frequency of 60 Hz.

a) State what is meant by the 'peak value' and show how this value is related to the root mean square (r.m.s.) value. *(2)*

b) Calculate the r.m.s. voltage. *(2)*

c) A light bulb is connected to the mains supply in the USA and draws an r.m.s. current of 0.50 A. Calculate the mean power of the bulb. *(1)*

d) Using a suitable set of axes, sketch the voltage waveform of mains electricity in the USA. Include suitable numerical scales on your sketch graph. *(4)*

13 A student is using an oscilloscope to measure the voltage from a range of different voltage sources. She connects the voltage sources to the *y*-input of the oscilloscope. The *y*-gain of the oscilloscope is set to 0.5 V/div, and the timebase is set to 4 ms/div. The screen of the oscilloscope is divided into a 10 × 10 grid as shown in Figure 9.15.

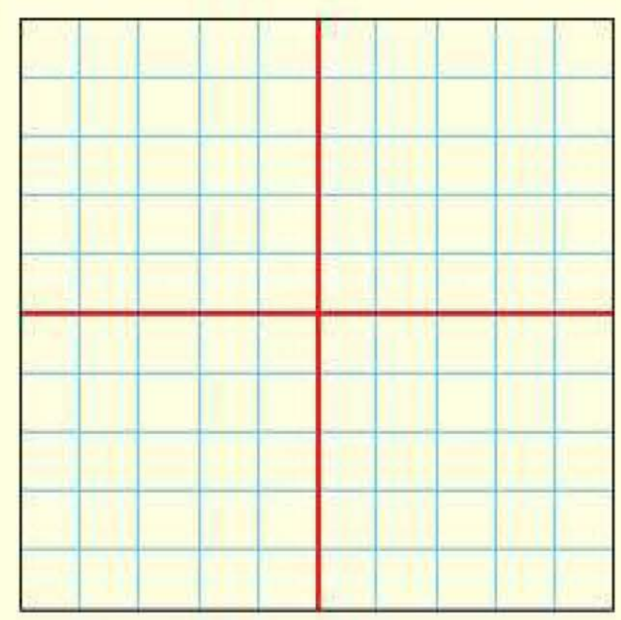


Figure 9.15

a) Copy the diagram twice and draw sketches of the oscilloscope screen illustrating the voltage waveforms of the following sources:

i) 1.5 V cell (battery) *(1)*

ii) UK mains low-voltage ac power supply, 2 V (peak). *(2)*

b) Calculate the r.m.s. voltage of the ac power supply. *(2)*

14 a) Describe how you would use an oscilloscope to compare the output from an ac, 12 V_{rms}, 15 Hz wind turbine and a 12 V dc car battery. You need to consider the quality of your written communication in your answer. *(6)*

b) The car battery is connected to a car headlight bulb and the current is measured to be 2.5 A. Calculate the power of the bulb. *(1)*

c) Calculate the peak power drawn from the wind turbine if it was connected to the same car headlamp, with the same mean power. *(1)*

d) Calculate the peak voltage produced by the wind turbine. *(2)*

15 High-voltage transmission of electrical power in the National Grid can cause large energy losses. Explain how energy losses are minimised when transmitting ac voltage in the National Grid. (6)

16 A transformer is used inside a 12 V, 60 W heater, to step down the mains voltage of 230 V.

a) Calculate the turns ratio for the heater's transformer if the output voltage is 12 V r.m.s. when the heater is connected to a mains supply of 230 V r.m.s. State any assumptions you make. (3)

b) Calculate the current in the supply lead when the heater is connected to the mains supply and turned on. (3)

c) The r.m.s. current flowing in the primary coil is 0.26 A. Calculate the efficiency of the heater's transformer if the r.m.s. output voltage is 11.8 V, and an r.m.s. current of 4.5 A flows in the secondary coil. (3)

17 A factory uses a transformer to step down the voltage from 11 kV to 415 V.

a) Calculate the number of turns on the secondary coil if there are 3000 turns on the primary coil. (3)

b) A crane with maximum power of 60 kW uses the 415 V ac supply. Calculate the current drawn from the 11 kV supply when the crane works at maximum power, at which point the efficiency of the transformer is 85%. (3)

c) State two important causes of energy loss in the transformer and describe how the transformer is designed to reduce these losses. (4)

Stretch and challenge

The first question that follows here is a British Physics Olympiad question.

18 A 20 Ω resistor is connected to an ac power supply with a voltage output that varies from 4 V to −2 V at equal time intervals, as shown in Figure 9.16. What is the mean heating power dissipated in the resistor?

A 0.2 W

B 0.5 W

C 0.8 W

D 1.0 W

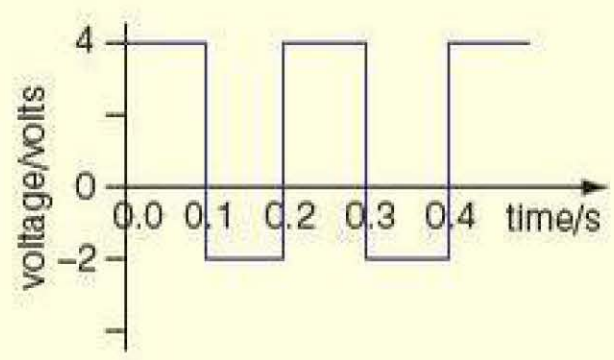
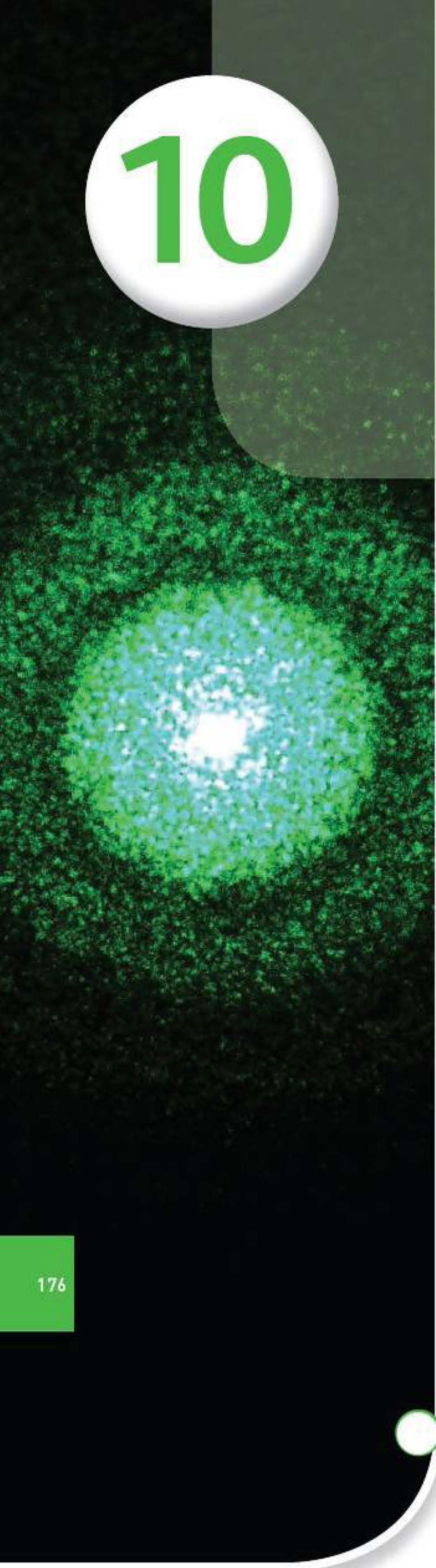


Figure 9.16

(BPhO AS Challenge – 2007 Q4)

19 A 'saw tooth' waveform voltage rises from 0 to a maximum value V_0 in a time t, at which point it immediately falls to 0 again, before rising once more to the value V_0. Show that the power generated by this voltage through a resistance R is the same as would be generated by a dc voltage of $\frac{V_0}{\sqrt{3}}$.



10 The evidence for the nucleus

PRIOR KNOWLEDGE

Before you start, make sure that you are confident in your knowledge and understanding of the following points:

- An atom has a small, massive, positively charged nucleus.
- Rutherford scattering gives evidence for the nuclear model of the atom.
- The atom is neutral; the positively charged nucleus is balanced by negatively charged electrons, which orbit the nucleus.
- Unstable nuclei emit radioactive particles.
- Different isotopes of an element have the same number of protons but different numbers of neutrons.
- Alpha particles are helium nuclei.
- Beta particles are fast-moving electrons.
- Gamma rays are electromagnetic photons, which carry energy away from an unstable nucleus.
- Alpha, beta and gamma radiations may be identified by their differing powers of penetration.

TEST YOURSELF ON PRIOR KNOWLEDGE

1 Describe the nuclear model of the atom.

2 Outline briefly the penetrating powers of alpha, beta and gamma radiations.

3 Nobelium-254, $^{254}_{102}No$, emits an alpha particle to become an isotope of the element fermium, Fm.

  a) Explain the meaning of the word 'isotope'.

  b) Write a balanced equation to describe the alpha decay of nobelium-254.

4 Krypton-85, $^{85}_{36}Kr$, decays by emitting a β-particle to become an isotope of rubidium, Rb. Write a balanced equation to describe this decay.

We use radioactive sources for many purposes in medicine, industry and agriculture. Imagine that a patient is about to receive a dose of gamma radiation to help cure a cancerous tumour. Such doses must be carefully calculated and directed accurately at the cancerous area of the body. The differing ionising and penetrating powers of alpha, beta and gamma rays allow them to be used in various ways to investigate the body and then treat the patient.

Rutherford scattering

Figure 10.1 shows a plan view of the sort of apparatus that Geiger and Marsden used in 1911 to investigate the scattering of alpha particles by a thin foil of gold. Gold was chosen because it can be hammered into very

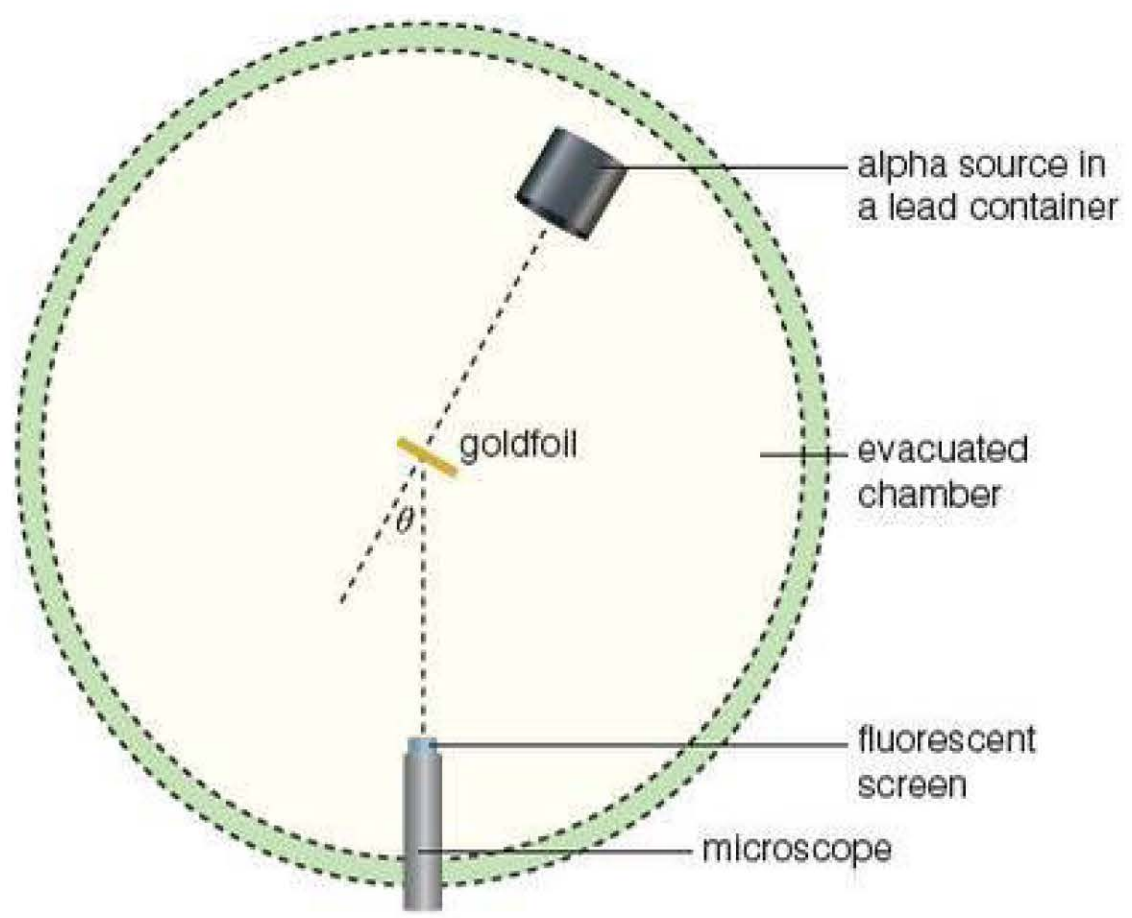


Figure 10.1 A plan view of Geiger and Marsden's apparatus.

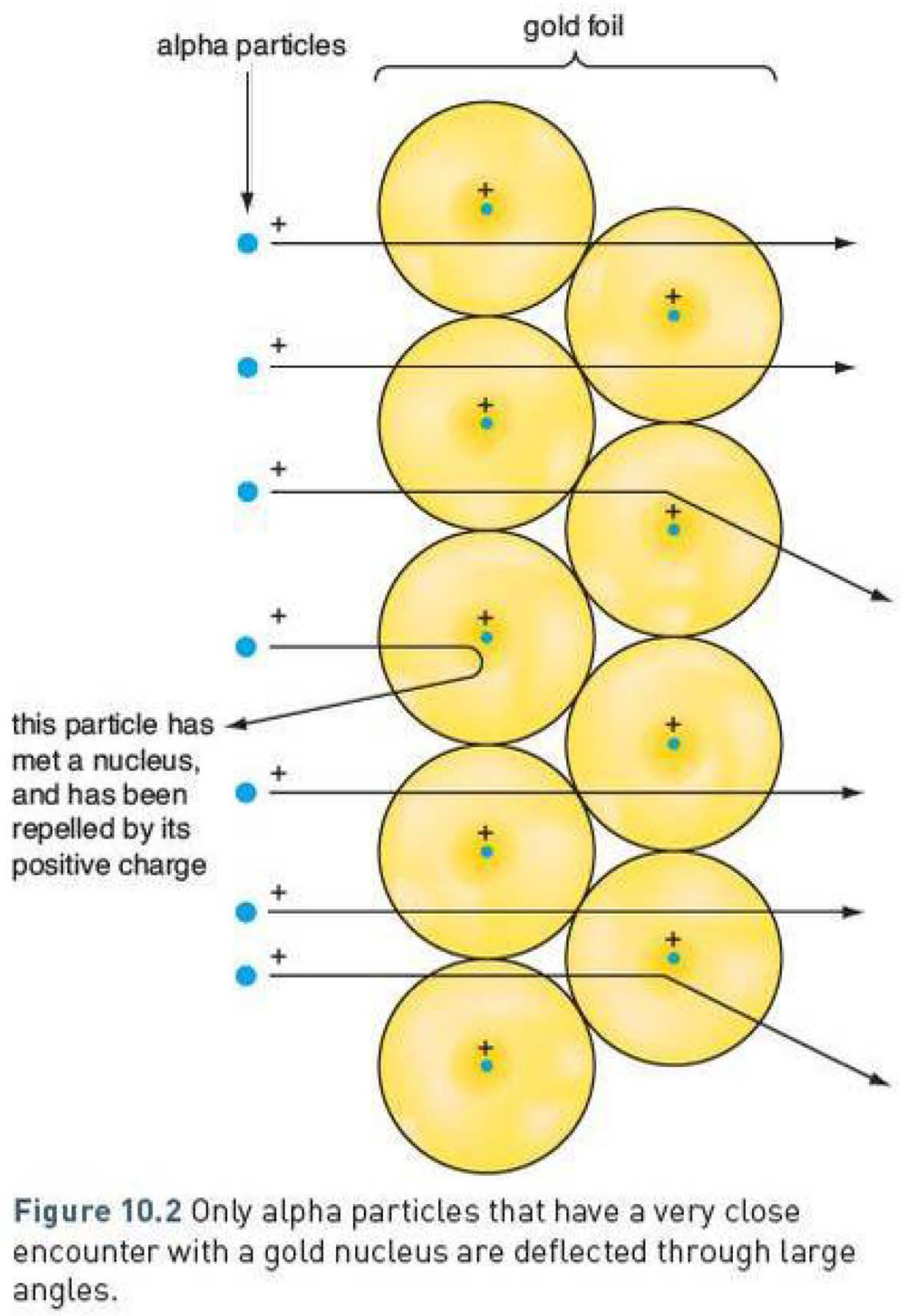


Figure 10.2 Only alpha particles that have a very close encounter with a gold nucleus are deflected through large angles.

thin sheets. An alpha source was placed in a long thin lead container to produce a well-directed beam of alpha particles. The whole apparatus was evacuated so that the alpha particles could travel without being stopped by the air.

Over a period of months, Geiger and Marsden counted the number of alpha particles deflected at different angles θ, shown in Figure 10.2. The alpha particles were detected by a fluorescent screen. Each time an alpha particle hit the screen, a small flash of light was emitted, which was seen through the microscope. Geiger and Marsden counted hundreds of thousands of such flashes of light. The vast majority of the alpha particles were deflected through very small angles. But a very small number of particles were deflected through large angles of about 150° or more. Figure 10.2 illustrates some typical paths of deflected alpha particles.

Rutherford drew the following conclusions from this experiment.

- The atom has a very small positively charged nucleus. Rutherford suggested that the positive charge on the nucleus is responsible for the repulsive force on the positively charged alpha particle, which causes it to change direction. The fact that only a very small number of particles undergo a large deflection tells us that the nucleus is much smaller in diameter than the atom.
- The second important conclusion about the nucleus is that it contains nearly all the mass of the atom. Considerations of the conservation of momentum tell us that the alpha particle would knock a small nucleus out of the way, but that the alpha particle will bounce back after an encounter with a nucleus heavier than itself.

Using our knowledge of electrostatic theory, it is possible to calculate the maximum size of the gold nucleus. If an alpha particle is turned round by 180°, it must have encountered a gold nucleus head-on, and there must have been a moment when the alpha particle stopped moving. Then all of the alpha particle's kinetic energy has been transferred to electrical potential energy.

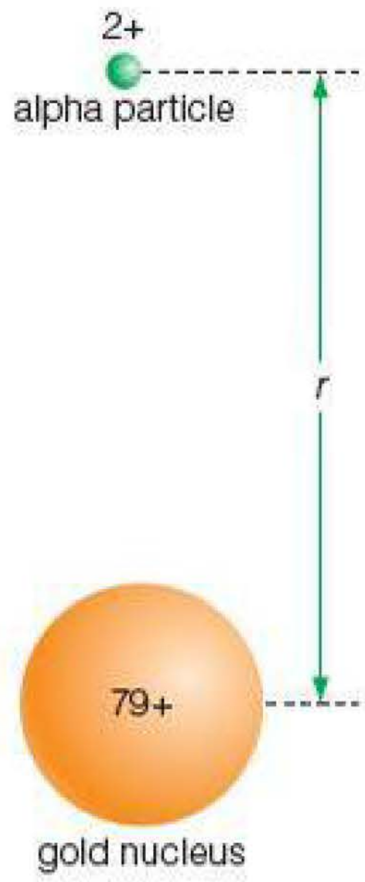


Figure 10.3 An alpha particle at its closest distance of approach to a gold nucleus.

In Figure 10.3 an alpha particle has been stopped by the gold nucleus and reached its closest distance of approach.

The kinetic energy of the alpha particles, used in the original Geiger and Marsden experiment, was about 5 MeV or $5 \times 10^6 \times 1.6 \times 10^{-19}\,\text{J} = 8 \times 10^{-13}\,\text{J}$. So we can write

$$8 \times 10^{-13}\ \text{J} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r}$$

The expression on the right-hand side of the equation gives the electrical potential energy of two charges, Q_1 and Q_2, separated by a distance r. The charge on the gold nucleus is $79e$ and that on the alpha particle is $2e$. The permittivity of free space is $8.85 \times 10^{-12}\,\text{F m}^{-1}$.

$$8 \times 10^{-13}\ \text{J} = \frac{(79 \times 1.6 \times 10^{-19}\ \text{C}) \times (2 \times 1.6 \times 10^{-19}\ \text{C})}{4\pi \times 8.85 \times 10^{-12}\ \text{Fm}^{-1} \times r}$$

Therefore

$$r = \frac{(79 \times 1.6 \times 10^{-19}\ \text{C}) \times (2 \times 1.6 \times 10^{-19}\ \text{C})}{(4\pi \times 8.85 \times 10^{-12}\ \text{Fm}^{-1}) \times (8 \times 10^{-13}\ \text{J})}$$
$$= 4.5 \times 10^{-14}\ \text{m or } 45\ \text{fm}$$

By carrying out scattering experiments on lighter nuclei, Rutherford was able to deduce that the nucleus was even smaller than 45 fm (where fm is the abbreviation for **femtometre**). But he had established the nuclear model of the atom.

Femtometre Nuclear radii and diameters are measured in femtometres, 10^{-15} m. The unit is abbreviated to fm.

MATHS BOX

Another way to make an estimate of the nuclear size is to consider the number of alpha particles scattered through large angles. For example, in a scattering experiment, 1 in 8000 alpha particles is scattered by an angle larger than 150° – this counts as a 'direct hit'. Measurement of the gold foil tells us that it is about 2000 atoms thick.

So, had the foil been only one atom thick, we can deduce that only 1 in 16 000 000 alpha particles would have had a 'direct hit'. Therefore, we deduce that:

$$\frac{\text{cross-sectional area of the atom}}{\text{cross-sectional area of the nucleus}} = \frac{\pi r_a^2}{\pi r_n^2} = 16\,000\,000$$

and

$$\frac{\text{atomic radius}}{\text{nuclear radius}} = \frac{r_a}{r_n} = 4000$$

Because the radius of a gold atom is 1.35×10^{-10} m, the nuclear radius is

$$r_n = \frac{1.35 \times 10^{-10}\ \text{m}}{4000} = 3 \times 10^{-14}\ \text{m or } 30\ \text{fm}$$

TEST YOURSELF

1 Give an account of the evidence that led to Rutherford proposing the nuclear model of the atom. (You should write your answer to secure six marks in an extended writing exercise.)

2 This question refers to the design of the Rutherford scattering experiment illustrated in Figure 10.1.

a) Explain why the apparatus must be evacuated.

b) Explain why the gold foil must be extremely thin, about 10^{-7} m thick.

c) Explain how the design of the holder for the alpha source produces a well-directed beam of radiation.

3 Figure 10.4 shows the path of an alpha particle being deflected by a heavy nucleus with charge $+Ze$.

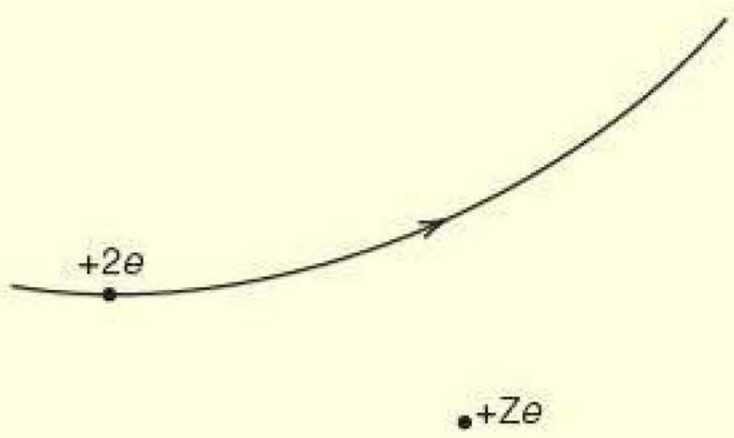


Figure 10.4

a) Sketch diagrams to show possible paths of an alpha particle approaching the same nucleus if the alpha particle has

i) less kinetic energy

ii) more kinetic energy.

b) Sketch diagrams to show possible paths of the alpha particle with its original energy approaching nuclei that have

i) a charge greater than Z

ii) a charge less than Z.

4 An alpha particle with energy 7.7 MeV is scattered back through an angle of 180° by a thin sheet of aluminium foil.

a) Calculate the closest distance of approach of the alpha particle to the aluminium nucleus. The atomic number of aluminium is 13; $\varepsilon_0 = 8.85 \times 10^{-12}\,\text{F m}^{-1}$.

b) i) Calculate the force that the alpha particle and nucleus exert on each other at their closest approach.

ii) Calculate the maximum acceleration of the alpha particle. The mass of the alpha particle is 6.8×10^{-27} kg.

Nuclear radius and density

Figure 10.5 shows a diffraction pattern produced by shining green laser light through a thin film of lycopodium powder, which contains very small particles of about 30 µm in diameter. The photograph shows a series of circular diffraction rings, caused by the scattering and interference of the light off the particles.

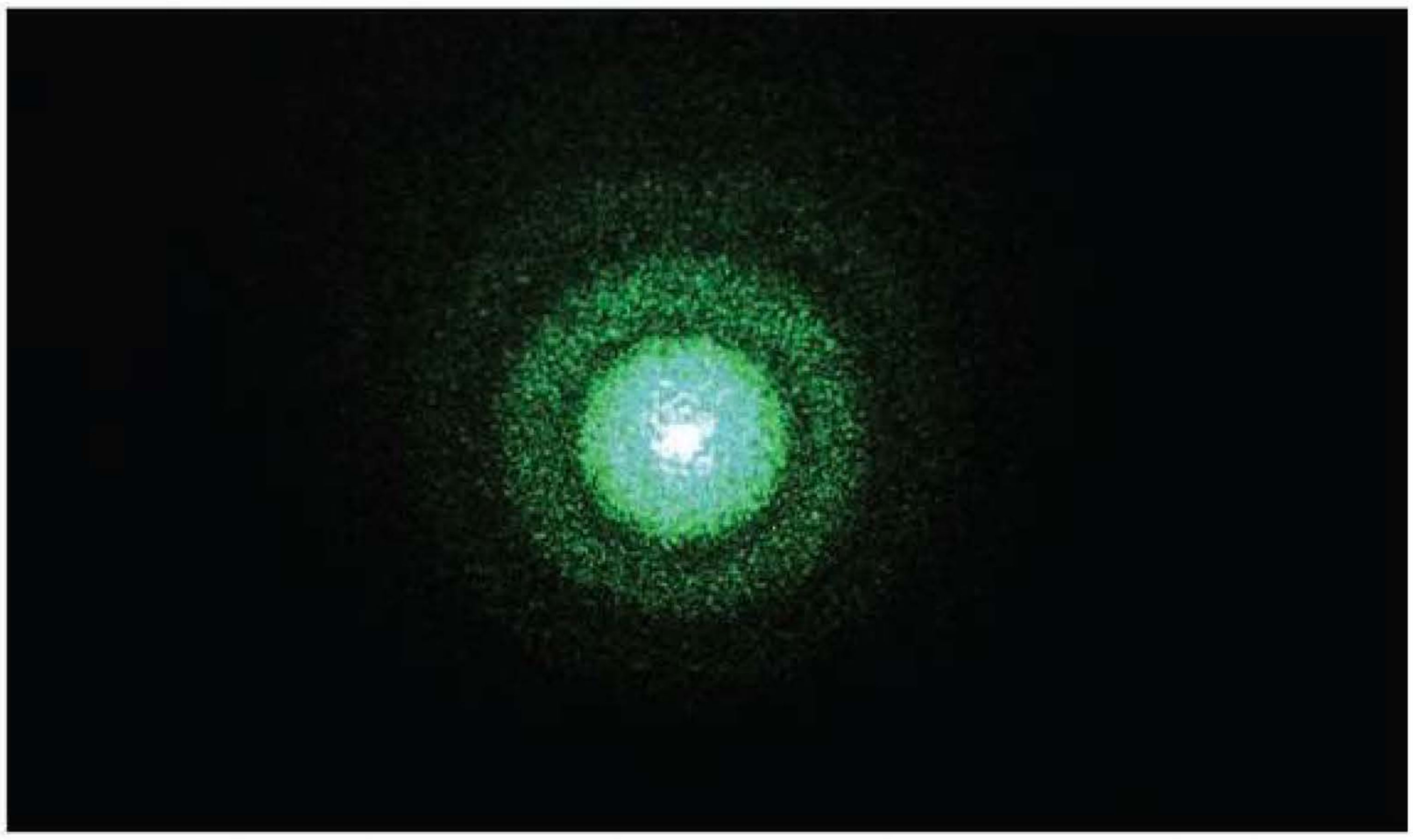


Figure 10.5 Diffraction pattern produced by the scattering of green light off lycopodium powder.

Diffraction theory predicts that the angle, θ, of the first diffraction minimum is given by

$$\sin\theta = 1.22\frac{\lambda}{d}$$

where λ is the wavelength of the light and d is the diameter of the particles.

This diffraction theory enabled nuclear scientists to investigate accurately the diameter of the nucleus of atoms. The principle behind the experiment is illustrated in Figure 10.6.

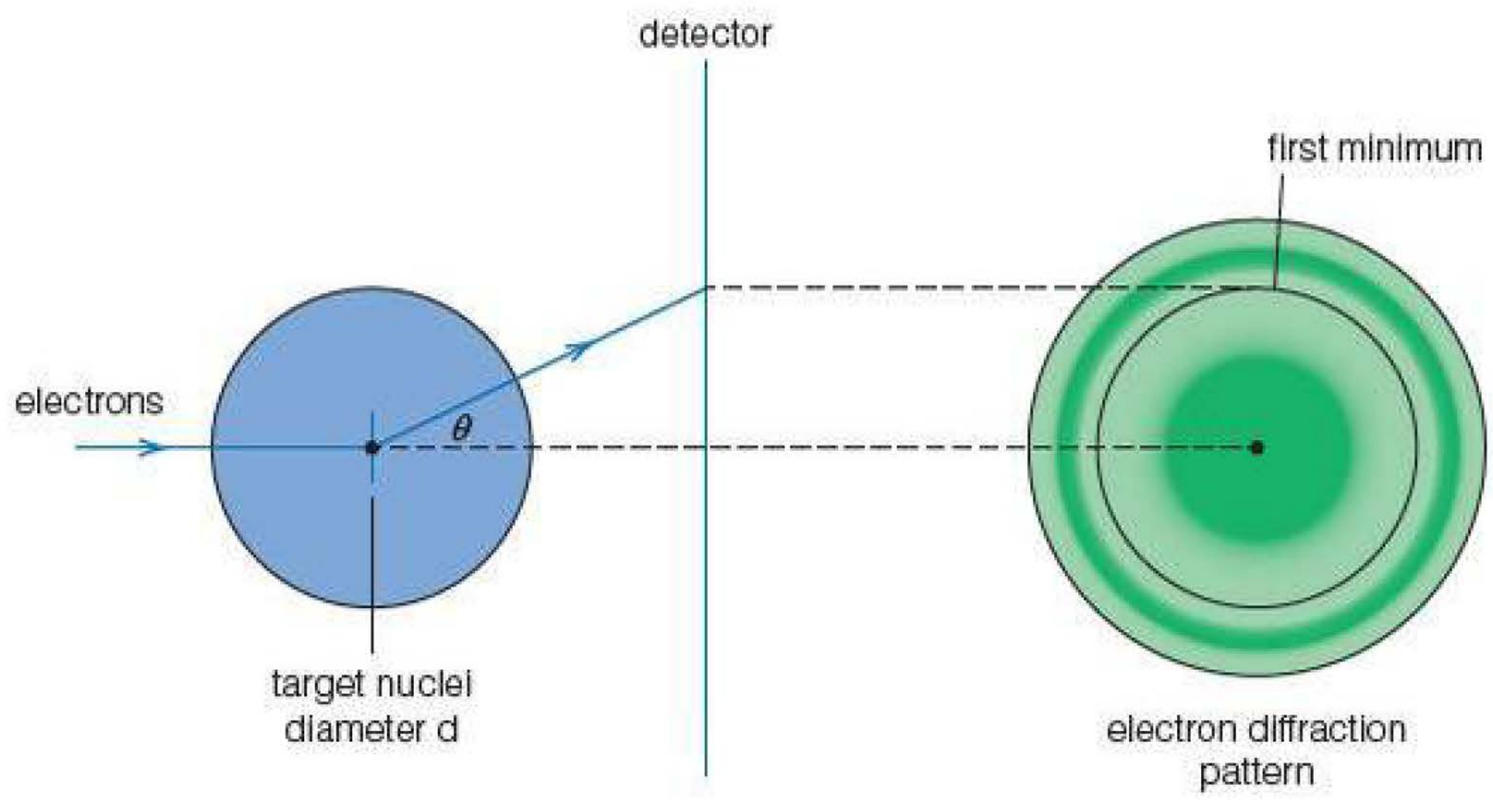


Figure 10.6

High-energy electrons are directed at thin targets of an element, and the nuclei act to scatter electrons in the same way that the lycopodium powder scatters light. You will recall that we can calculate the wavelength, λ, of the electrons using the formula

$$\lambda = \frac{h}{p}$$

where h is the Planck constant, 6.6×10^{-34} J s, and p is the electron's momentum. Such high-energy electrons are travelling close to the speed of light, and we must calculate their momentum using the equation

$$p = \frac{E}{c}$$

where E is the electron energy and c is the speed of light.

EXAMPLE

Electron scattering

A beam of electrons with energy 420 MeV is scattered off a target of carbon. The first diffraction minimum occurs at an angle of 52°.

1 Calculate the momentum of the electrons.

Answer

The momentum of the electrons is given by

$$p = \frac{E}{c}$$

$$= \frac{420 \times 10^6 \times 1.6 \times 10^{-19}\ \text{J}}{3 \times 10^8\ \text{m s}^{-1}}$$

$$= 2.24 \times 10^{-19}\ \text{kg m s}^{-1}$$

2 Calculate the wavelength of the electrons.

Answer

The wavelength of the electrons is

$$\lambda = \frac{h}{p} = \frac{6.6 \times 10^{-34}\ \text{J s}}{2.24 \times 10^{-19}\ \text{kg m s}^{-1}}$$

$$= 2.94 \times 10^{-15}\ \text{m}$$

3 Calculate the radius of a carbon nucleus.

Answer

The diameter of the nucleus is calculated from the equation

$$\sin\theta = 1.22\frac{\lambda}{d}$$

$$d = \frac{1.22\lambda}{\sin\theta}$$

$$= \frac{1.22 \times 2.94 \times 10^{-15}\ \text{m}}{\sin 52°}$$

$$= 4.6 \times 10^{-15}\ \text{m}$$

So the nuclear radius is 2.3×10^{-15} m.

Empirical The equation $r = r_0A^{\frac{1}{3}}$ is an empirical equation. The word 'empirical' means that the equation is based purely on experimental results. It is not exact, but it gives an approximate value for a nuclear radius.

Atomic mass unit One atomic mass unit (1 u) is equal to 1.67×10^{-27} kg.

Experiments to determine the radius of nuclei allowed scientists to produce an approximate **empirical** formula for the radius of a nucleus, which is

$$r = r_0A^{\frac{1}{3}}$$

where r is the radius of the nucleus, r_0 = 1.2 fm and A is the mass number or nucleon number of the nucleus. Figure 10.7 shows how the nuclear radius depends on the nucleon number of the nucleus.

We often use the expression u, which is an abbreviation for **atomic mass unit**. A proton and neutron each have a mass approximately equal to 1.67×10^{-27} kg, and this is 1 u. So, in the example, the mass of a zinc nucleus is 66 u.

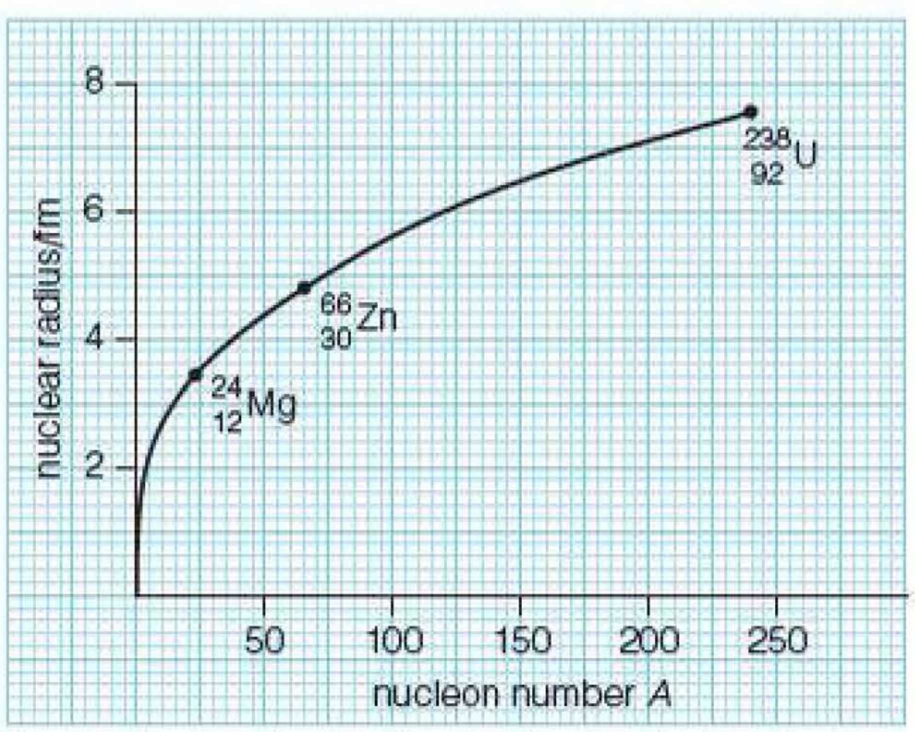


Figure 10.7 This graph shows the relationship between nuclear radius and nucleon number.

EXAMPLE

Empirical formula for nuclear radius

1 Calculate the nuclear radius of the isotope ${}^{66}_{30}Zn$.

$$r = r_0 A^{\frac{1}{3}}$$
$$= 1.2\,\text{fm} \times 66^{\frac{1}{3}}$$
$$= 4.8\,\text{fm}$$

2 Calculate the density of a zinc nucleus, given that 1 u has a mass of 1.67×10^{-27} kg.

Answer

Mass of nucleus = $\frac{4}{3}\pi\rho r^3$, where ρ is the nuclear density. So

$$\rho = \frac{3}{4\pi r^3} \times m$$
$$= \frac{3 \times (66 \times 1.67 \times 10^{-27}\,\text{kg})}{4\pi \times (4.8 \times 10^{-15}\,\text{m})^3}$$
$$= 2.4 \times 10^{17}\,\text{kg m}^{-3}$$

This calculation shows us that the nuclear density is immense – over 100 million million times more dense than water.

TEST YOURSELF

5 a) Use Figure 10.7 to determine the nuclear radius of uranium–238.

b) Calculate the density of a uranium nucleus; 1 u is 1.67×10^{-27} kg.

6 a) Explain the principle behind using electron diffraction to determine the radius of a nucleus.

b) Explain one advantage that electron diffraction has over Rutherford scattering as a means of determining nuclear radius.

7 The element livermorium is a short-lived transuranic element, which has been produced in nuclear reactors.

a) i) Explain what 'transuranic' means.

ii) Why are transuranic elements short-lived?

b) One isotope of livermorium is ${}^{293}_{116}Lv$. Use the empirical formula to predict the radius of a livermorium nucleus.

8 An electron beam with energy 890 MeV is used to investigate the radii of some elements. Figure 10.8 shows how the intensity of scattered electrons varies for two isotopes, gadolinium-160 and calcium-40.

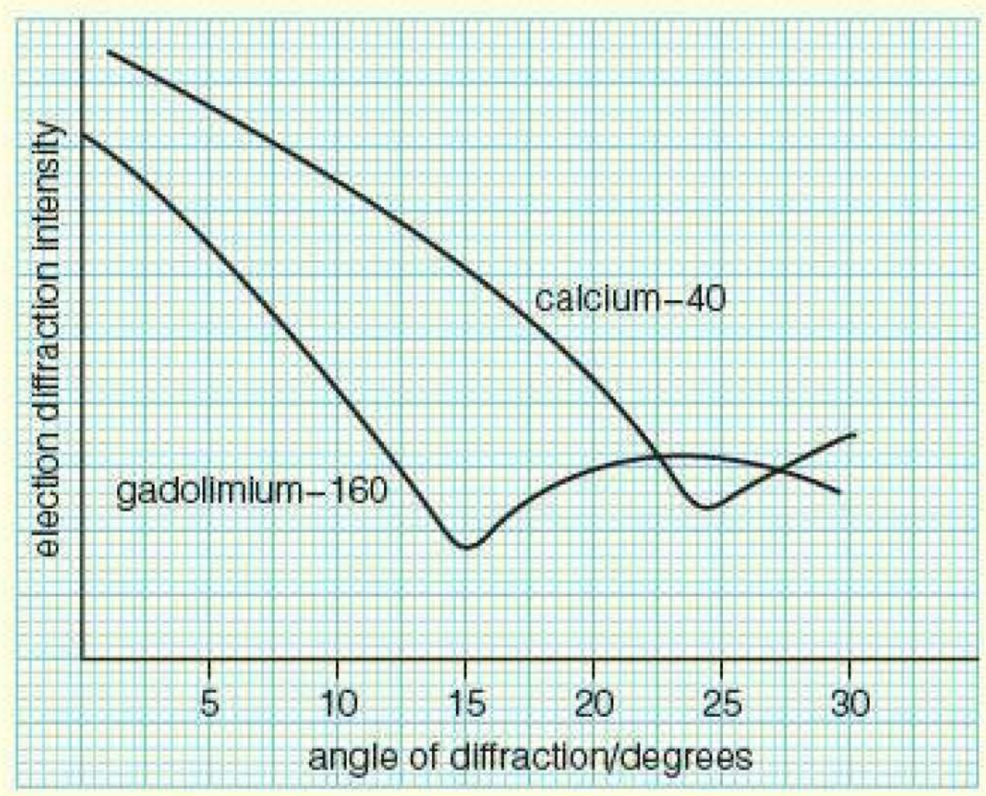


Figure 10.8

a) Use the expression $p = \frac{E}{c}$ to show that the momentum of each electron in the beam is about 4.7×10^{-19} kg m s^{-1}.

b) Calculate the wavelength of the electrons in the beam. Planck's constant = 6.6×10^{-34} J s.

c) Use the information in Figure 10.8, and the information in the text about electron diffraction, to calculate the nuclear radius for

i) gadolinium

ii) calcium.

d) Check your answers for part (c) with the predictions for nuclear radius shown in Figure 10.7.

Radioactive emissions

Henry Becquerel discovered radioactivity in 1896. He placed some uranium salts next to a photographic plate, which had been sealed in a thick black bag to prevent light exposing the plate (Figure 10.9). When the plate was later developed, it had been affected as if it had been exposed to light (Figure 10.10). Becquerel realised that a new form of energy was being emitted from the uranium salts. In his honour, the **activity** of a radioactive source is measured in **becquerels**.

Becquerel The activity of a radioactive source is equal to the number of particles emitted per second. The unit of activity is the becquerel (Bq): 1 becquerel (1 Bq) = an emission of one particle per second.

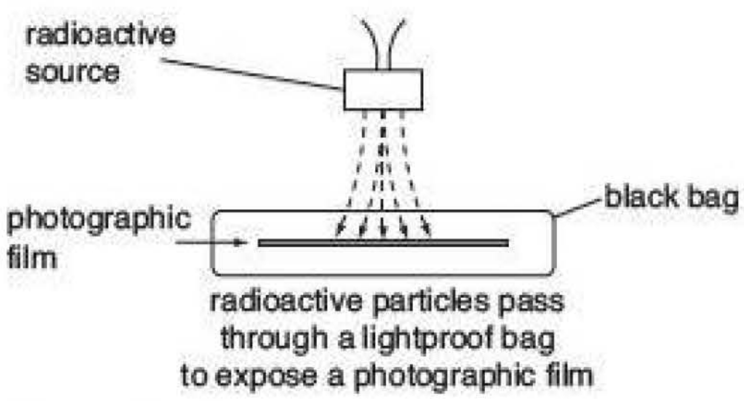


Figure 10.9

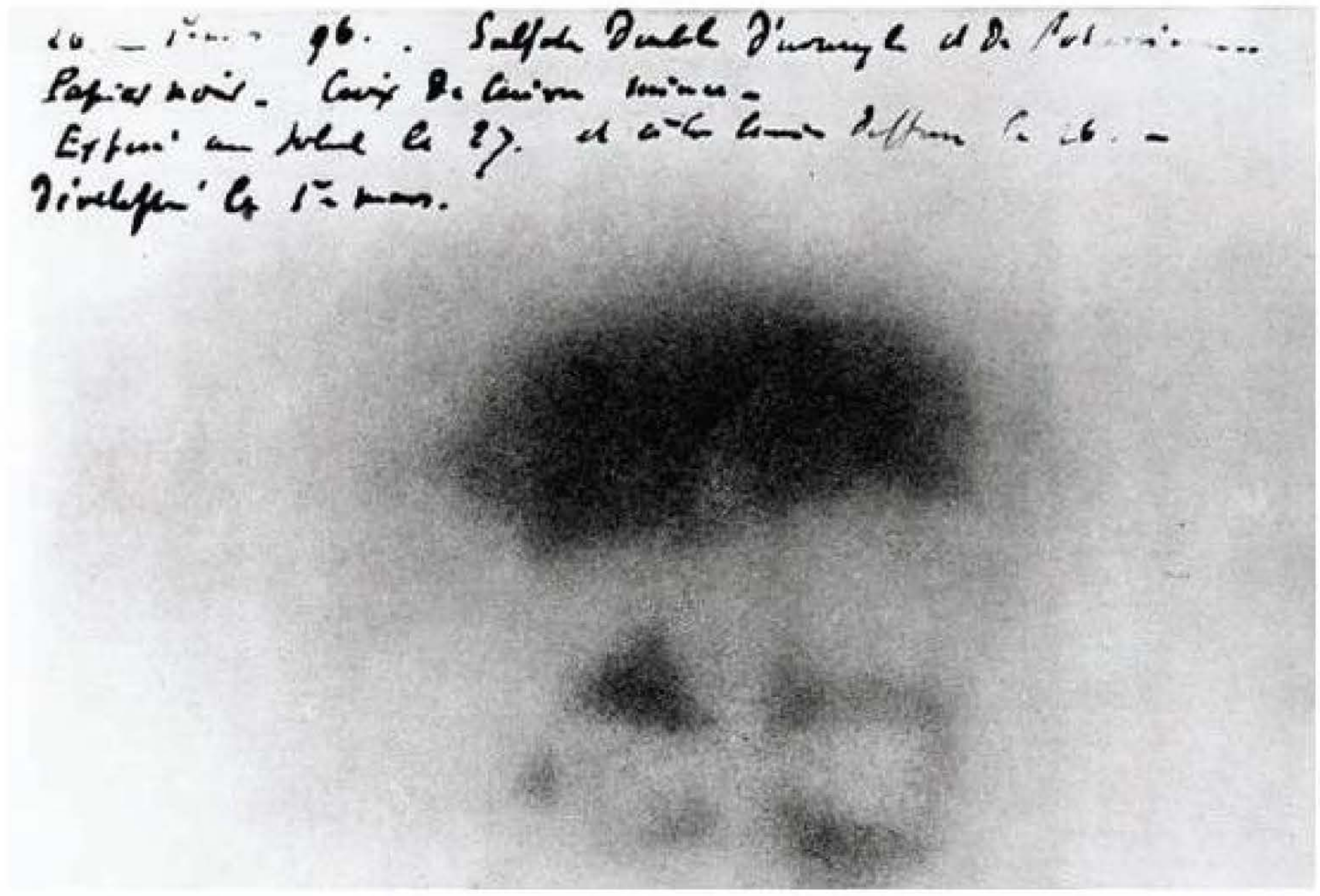


Figure 10.10

The nature of alpha (α), beta (β) and gamma (γ) radiations

Unstable nuclei emit various types of radiation, the most common of which are alpha, beta and gamma radiation. Their nature and properties are summarised below.

Alpha particles

Alpha particles are the nuclei of helium atoms. So they are made up of two protons and two neutrons. They have a mass of 4 u and a charge of $+2e$.

Alpha particles are strongly ionising. The strong charge on the alpha particle pulls electrons out of atoms, creating pairs of positive and negative ions along the particle's path (Figure 10.11). An alpha particle produces about 10 000 ion-pairs per millimetre of path in air.

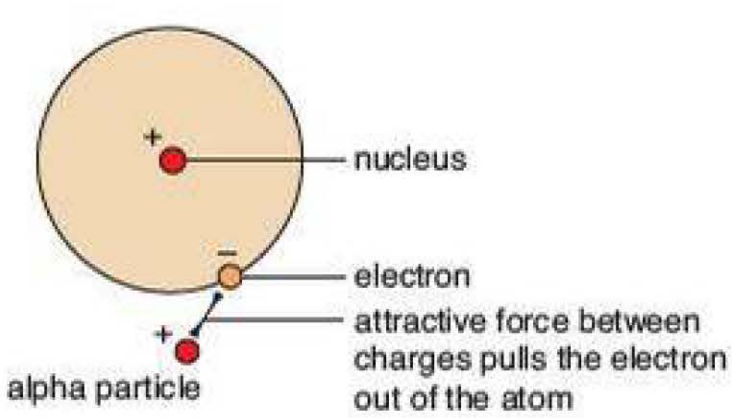


Figure 10.11 The strong electric field of the alpha particles pulls or knocks electrons out of atoms to create positive and negative ions.

Alpha particles travel a few centimetres in air, and can be stopped by a thick piece of paper (Figure 10.12).

Alpha particles are deflected slightly in strong electric and magnetic fields. Typically, alpha particles have kinetic energies of a few MeV as they leave the parent nucleus. An alpha particle with an energy of 5 MeV travels at about 5% of the speed of light (Table 10.1).

Beta particles

Beta particles are fast-moving electrons, which travel at just less than the speed of light. Typically, beta particles have kinetic energies of a few MeV.

Beta particles are much less ionising than alpha particles, producing about 100 ion-pairs per millimetre of path travelled in air.

Beta particles may travel several metres in air, and they are absorbed by aluminium a few millimetres thick (Figure 10.12).

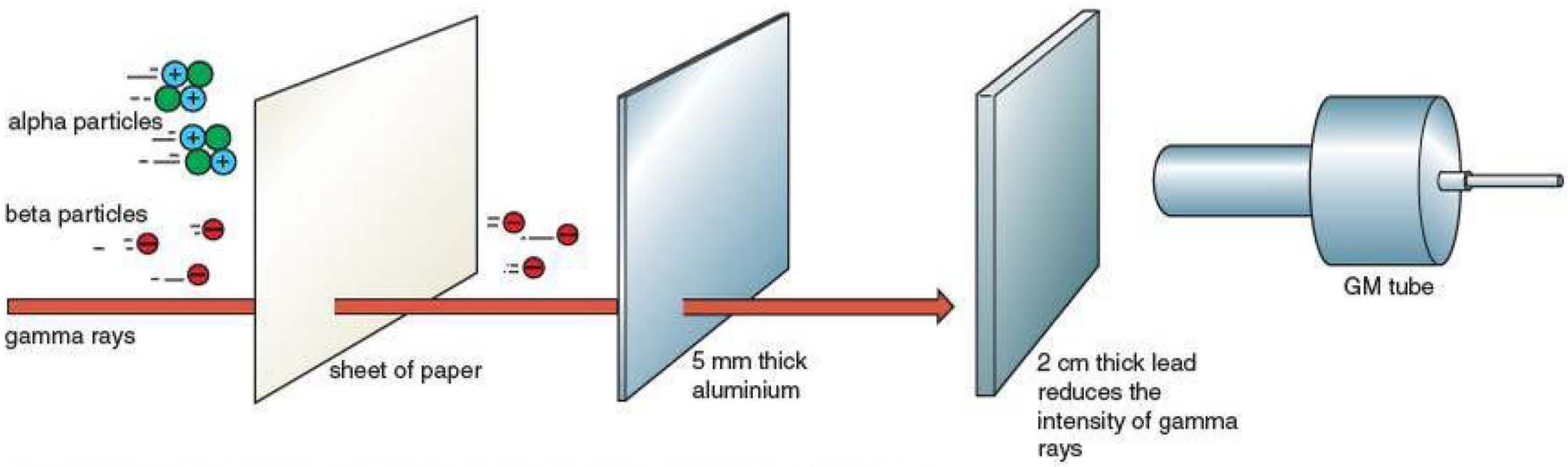


Figure 10.12 The penetrating powers of alpha, beta and gamma radiations.

Beta particles may be deflected through large angles by electric and magnetic fields (Table 10.1).

Gamma rays

Gamma rays are electrically neutral emissions, which are photons (just like any other type of electromagnetic radiation). Typically, a gamma-ray photon might have an energy of about 1 MeV, which corresponds to a wavelength of about 10^{-12} m.

Gamma rays are not deflected in magnetic and electric fields because they are not charged (Table 10.1).

Table 10.1 Properties of alpha, beta and gamma radiations.

Radiation	Alpha particle	Beta particle	Gamma ray
Nature	Helium nucleus	Fast electron	Electromagnetic photon
Charge	+2e	−*e*	0
Mass	6.6×10^{-27} kg (4 amu)	9.1×10^{-31} kg	0
Speed	5% of *c*	98–99% of *c*	*c* (speed of light)
Ions per mm of air for a particle of 3 MeV	10 000	100	1
Detection	Slight deflection in electric and magnetic fields Affects photographic film	Significant deflection in electric and magnetic fields Affects photographic film	No deflection in electric and magnetic fields Affects photographic film

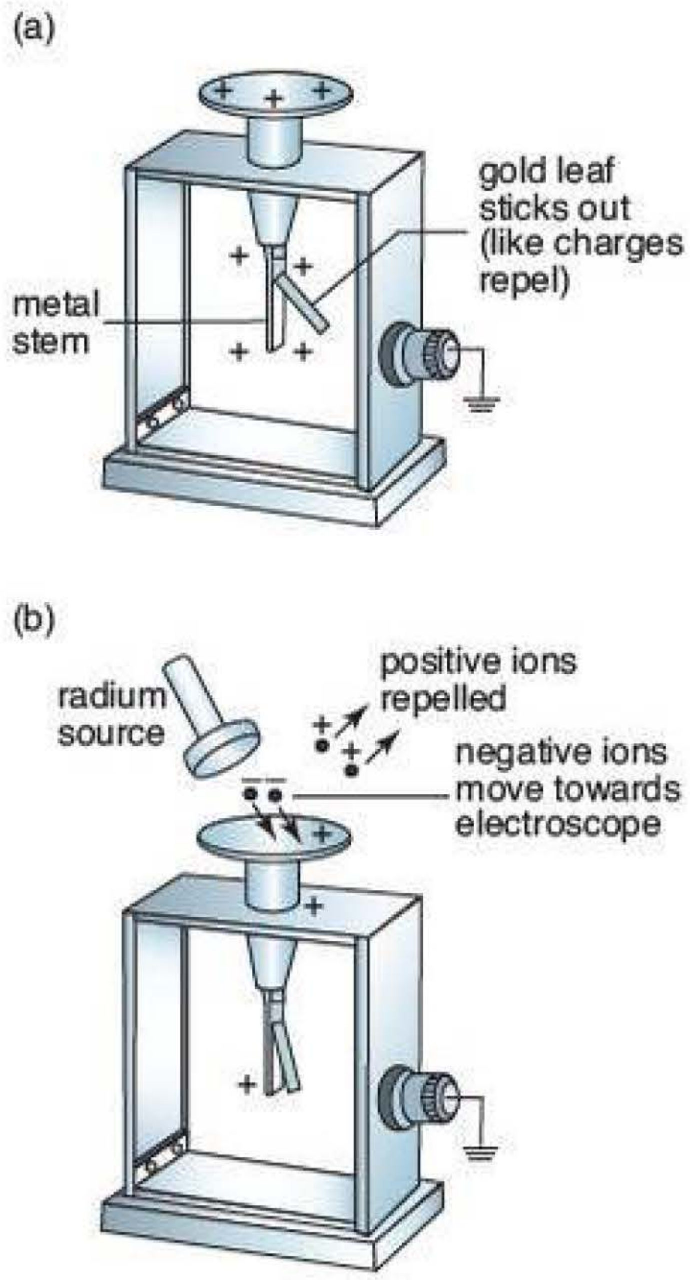


Figure 10.13

Gamma rays are very weakly ionising, producing about one ion-pair per millimetre of path travelled in air. Gamma rays are very penetrating, and their intensity is reduced by a few centimetres thickness of lead (Figure 10.12). Gamma rays can transfer their energy to electrons in metals (rather like a photoelectric effect); then the moving electrons create ion-pairs.

Ionisation

Figure 10.13 shows how a charged gold leaf electroscope can be used to illustrate the strong ionising power of alpha radiation. An alpha source is held above a positively charged electroscope. The alpha particles produce positive and negative ions. The positive ions are repelled from the positively charged electroscope, but the negative ions are attracted to the electroscope and it is discharged.

We make use of the ionising properties of alpha, beta and gamma radiation to detect them. This is done using a Geiger–Muller (GM) tube. Figure 10.14 shows how the tube works. Although GM tubes are still used in schools, solid-state detectors (working on a similar principle) are more widely used elsewhere.

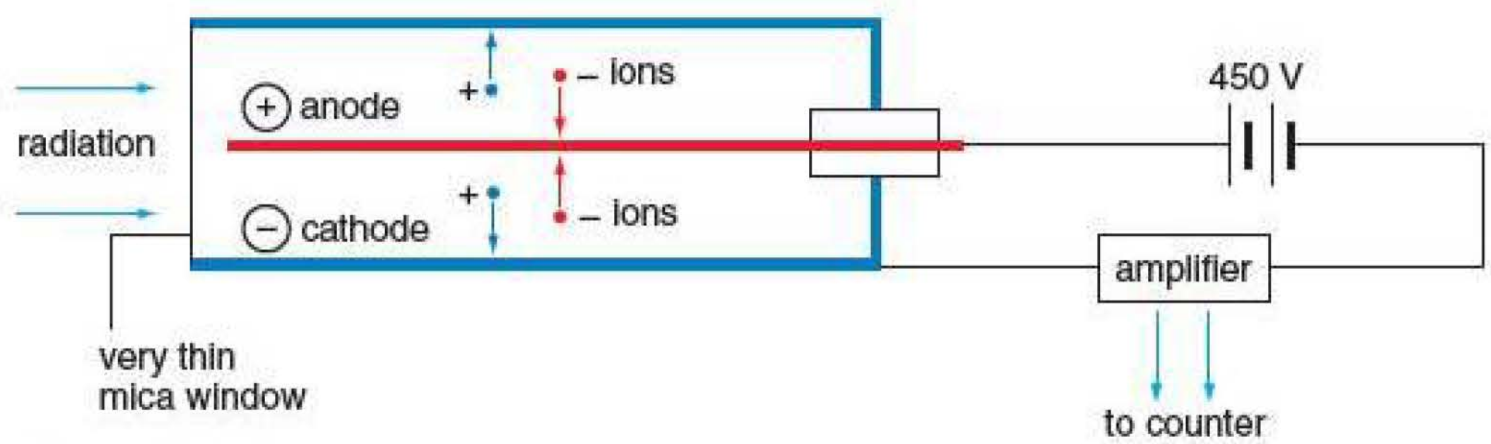


Figure 10.14

A metal tube is filled with argon gas at low pressure. A voltage of about 450 V is applied between a central anode and the outside of the tube. When radiation enters the tube through a narrow window at the front of the tube, atoms are ionised and a small current flows. Each current pulse is amplified and counted, so that we can record the rate at which particles enter the tube.

It is important that we can detect and then understand the effects of ionising radiation. When ionising particles enter the body, the ions that are produced can damage or destroy cells in our bodies, with serious consequences for our health. This is discussed at greater length later.

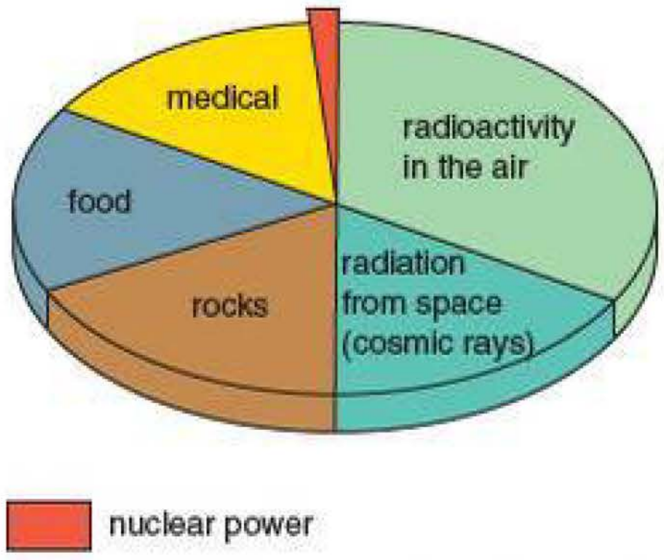


Figure 10.15 Sources of radiation in Britain.

Background radiation

There are a lot of rocks in the Earth that contain radioactive uranium, thorium, radon and potassium, and so we are always exposed to some ionising particles. Radon is a gas that emits alpha particles. Because we can inhale this gas, it is dangerous as radiation can get inside our lungs. In addition, the Sun emits lots of protons, which can also create ions in our atmosphere. These are two of the sources that make up background radiation. Figure 10.15 shows the contribution to the total background radiation from all places in Britain. Fortunately the level of background radiation is quite low, and in most places it does not cause a serious health risk.

In some jobs workers are at a higher risk. X-rays used in hospitals also cause ionisation. Radiographers make sure that their exposure to X-rays is as small as possible. In nuclear power stations, neutrons are produced in nuclear reactors. The damage caused by neutrons is a source of danger for workers in that industry.

TEST YOURSELF

9 Refer to Figure 10.13. The plate on the electroscope is charged negatively. Explain whether or not the gold leaf electroscope would still be discharged by an alpha radiation source.

10 a) Explain what is meant by the term 'background radiation'. What are the sources of background radiation?

b) Design an experiment to investigate what the background count is in your school.

11 To answer this question, you may need to revise the work covered in Chapter 7.

Figure 10.16(a) shows an experiment in which some beta particles are deflected by a magnetic field.

a) Explain how the direction of the beta particle deflection confirms that they are negatively charged.

Figure 10.16(b) shows how the count rate of the deflected beta particles varies with angle.

b) Explain which beta particles have the higher energy, those deflected through 20° or those deflected through 40°.

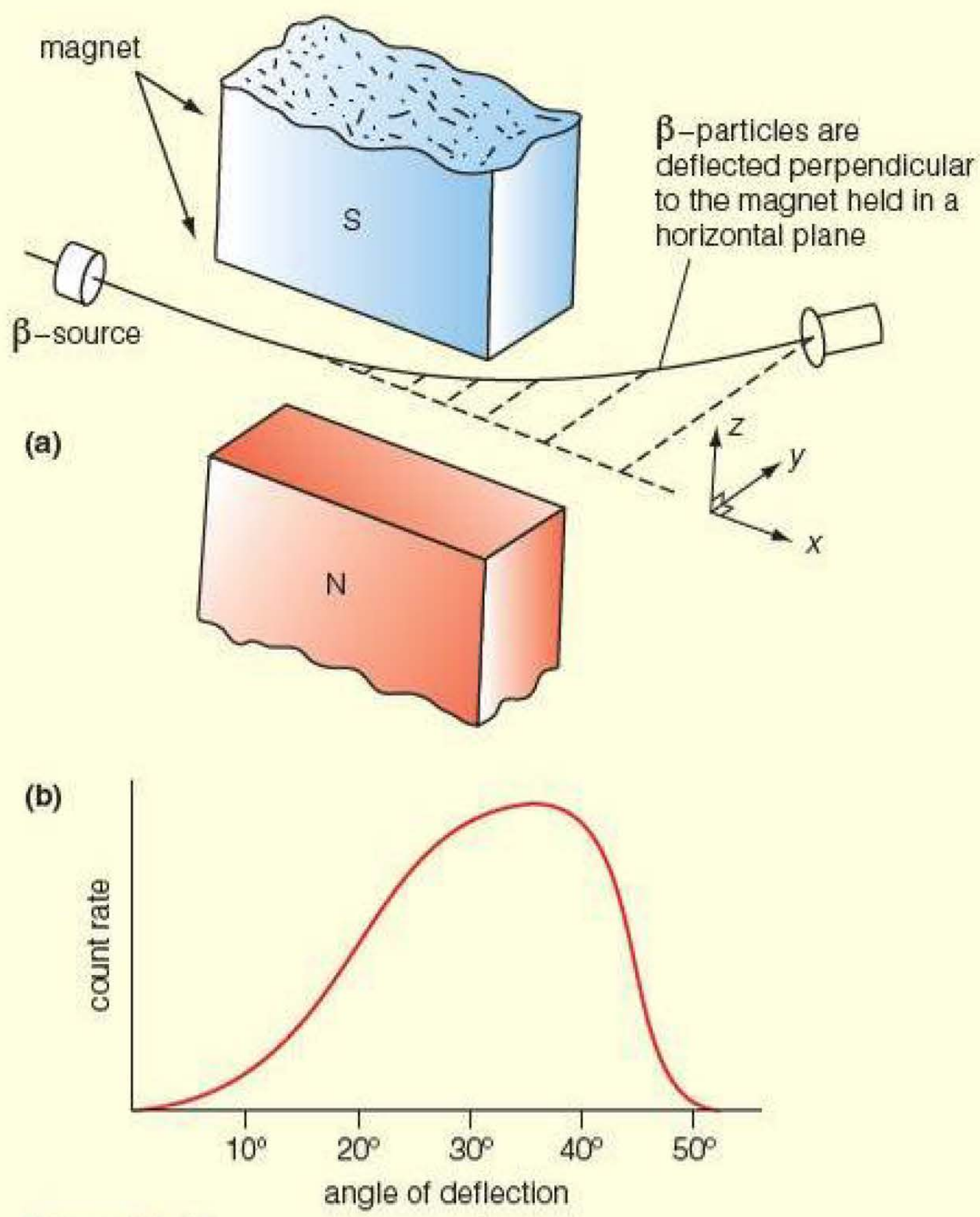


Figure 10.16

ACTIVITY

Identification of radiations

Figure 10.17 shows an experimental arrangement to identify the radiations emitted by a radioactive source.

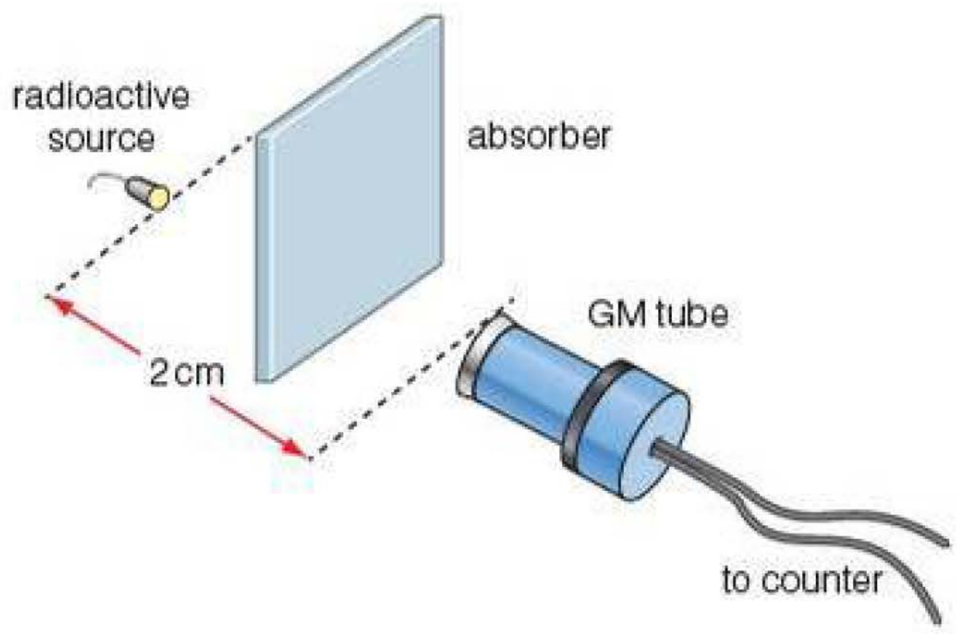


Figure 10.17

1 First, the GM tube is left without a source nearby to establish the background count. Over a period of 15 minutes, the GM tube recorded a background count of 330.

Determine the background count in counts per minute.

2 Then a source A is placed in front of the counter at a fixed distance of 2 cm. Various absorbers are placed between the source and the GM tube. Table 10.2 shows a summary of the results obtained.

Determine the radiation(s) emitted by source A.

Table 10.2

Absorber	Thickness/mm	Count rate/min^{-1}
None		5054
Paper	0.3	3294
Aluminium	0.1	3084
Aluminium	0.5	1954
Aluminium	1.0	244
Lead	1.0	21
Lead	2.0	24

3 A second source B is placed in front of the GM tube at a fixed distance of 2 cm, and the absorbers are placed between the source and the GM tube as before. The results are shown in Table 10.3.

Determine the type(s) of radiation emitted from source B.

Table 10.3

Absorber	Thickness/mm	Count rate/min^{-1}
None		2084
Paper	0.3	2079
Aluminium	0.1	1954
Aluminium	0.5	1251
Aluminium	1.0	1246
Lead	2.0	1068
Lead	5.0	641
Lead	10.0	355

TIP

If the above experiment is actually performed, remember that the appropriate source must be handled safely, in line with agreed standard operating procedures. These should be agreed with a suitable RPA (Radiation Protection Adviser).

Inverse square law for γ-radiation

Gamma radiation behaves like any other electromagnetic radiation, in that it spreads out symmetrically in all directions from its source. The intensity of a light source obeys an inverse square law.

This idea was discussed before in Chapter 3, and Figure 3.2 shows you the reasoning behind the law. So for gamma radiation we can write

$$I = \frac{k}{x^2}$$

where I is the intensity of the radiation (which can be measured in $\text{W}\,\text{m}^{-2}$), x is the distance from the source and k is a constant.v

EXAMPLE

Radioactivity and inverse square law

1 The count rate measured by a GM tube at a distance of 15 cm from a gamma source is 2800 Bq. The source is then moved further away from the GM tube.

a) What will the count rate be at 30 cm?

Answer

This problem is relatively easy to solve, using $I = \frac{k}{x^2}$, because x is doubled, x^2 is quadrupled and $\frac{1}{x^2}$ is a quarter of its previous value.

Thus the corrected count rate is $\frac{1}{4} \times 2800\,\text{Bq} = 700\,\text{Bq}$.

b) What will the corrected count rate be at 50 cm?

Because $I = \frac{k}{x^2}$, it follows that $k = Ix^2$ or $I_1x_1^2 = I_2x_2^2$. So

$$I_2 \times 50^2 = 2800\,\text{Bq} \times 15^2$$

$$I_2 = 2800\,\text{Bq} \times \left(\frac{15}{50}\right)^2$$

$$= 250\,\text{Bq}$$

2 A Geiger–Muller tube window has an area of 2.8 cm^2. A gamma source is placed a distance of 0.1 m from the window and the counter detects a corrected count rate of 150 Bq.

Assuming that the γ-source emits radiation uniformly in all directions, calculate the total number of emissions per second from the source. State any assumptions in the calculation.

First, we assume that all γ-rays are detected at the GM tube window (as you will see later, this is not actually the case).

The γ-radiation is spread over an area of $4\pi R^2$, where R is the distance from the source ($4\pi R^2$ is the surface area of a sphere). So the count detected is:

$$C = C_T \frac{2.8\,\text{cm}^2}{4\pi R^2}$$

where C_T is the total count. So

$$C_T = \frac{C \times 4\pi R^2}{2.8\,\text{cm}^2}$$

$$= \frac{150\,\text{Bq} \times 4\pi \times (10\,\text{cm})^2}{2.8\,\text{cm}^2}$$

$$= 67\,000\,\text{Bq}$$

REQUIRED PRACTICAL 12

Inverse square law for γ-radiation

Note: This is just one example of how you might tackle this required practical.

Figure 10.18 shows an experimental arrangement to investigate the relationship between the intensity of radiation from a gamma source and its distance from a Geiger–Muller tube.

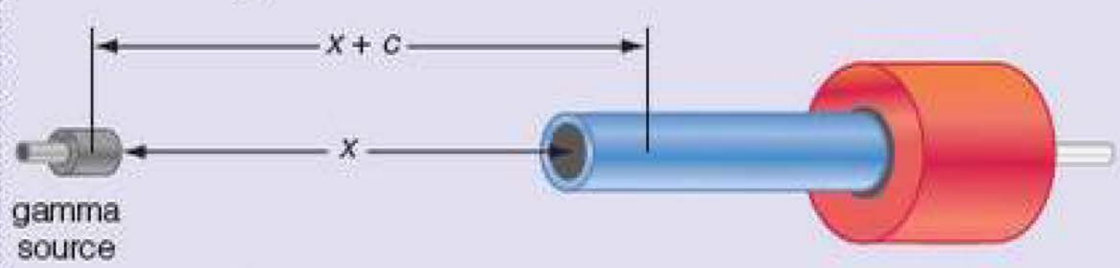


Figure 10.18

In Figure 10.18, x has been defined as the distance between the edge of the source container and the window of the GM tube. However, there is a difficulty with this definition. The source itself is inside the container, and the radiation is not all detected at the window of the tube. So the true distance between the source and the place where the radiation is detected is $x + c$. This is called the corrected distance. So we write

$$I = \frac{k}{(x+c)^2}$$

$$(x+c)^2 = \frac{k}{I}$$

$$x + c = \left(\frac{k}{I}\right)^{\frac{1}{2}}$$

Therefore, if we plot a graph of x against $I^{-\frac{1}{2}}$, we would expect to see a straight line.

In an experiment, the results shown in Table 10.4 were obtained. The background count was determined to be an average of 18 counts per minute.

Table 10.4

x/cm	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0
GM count over 10 s	431	260	195	138	110	81	73	61

1 Copy the table and add two further rows to it.

2 a) Make a suitable correction allowing for background count and add to your table.

b) Add a further row to show (count rate)$^{-\frac{1}{2}}$.

3 a) Plot a suitable graph to investigate whether the intensity of the gamma radiation obeys an inverse square law.

b) Use your graph to estimate the value of c shown in Figure 10.18.

The biological effects of radiation

In the early 1900s, scientists working with radioactive materials did not understand the dangers of radiation, and consequently many people suffered injury and in some cases death.

Ionising radiation is dangerous because ions are produced in our bodies, which damage cells and the functions of enzymes are changed.

Our bodies are made up of many different types of complicated molecules. If an electron is removed or added to a molecule, it has been changed chemically and will therefore behave differently in any interaction with another molecule. Typically it requires a few eV of energy to remove an electron from a molecule. Such energy is carried by photons of ultraviolet light. Alpha and beta particles, and gamma-ray photons all carry energy measured in MeV. Such radiations cause ionisation, and **ionising radiation** is dangerous to us because it can change the chemistry of our bodies. The functions of enzymes can be altered, cells can be damaged and mutations can occur to our DNA, which can lead to cancer.

It is also known that the irradiation of water produces free radicals (see H_2O^+ and H_3O^+ and OH^- below), which are highly reactive. These free radicals change the structure of surrounding molecules with biological implications. Such reactions include

$$H_2O \rightarrow H_2O^+ + e^-$$

$$H_2O^+ + H_2O \rightarrow H_3O^+ + OH$$

Radiation dose

Dose The energy absorbed per kilogram of a body.

Gray The unit of dose is $J\,kg^{-1}$, which is called one gray (Gy).

An absorbed **dose** of radiation is defined as the energy absorbed per kilogram of a body:

$$D = \frac{E}{m}$$

The unit of dose is $J\,kg^{-1}$ and this is given the name **gray** (Gy).

Clearly, if we receive a high dose of radiation, we are at a higher risk of becoming ill due to the damage caused to our bodies. However, the impact of the radiation on our bodies also depends on how the dose is administered, and this depends on the type of radiation we are exposed to.

From our earlier work on the penetration of radiations, you will recall that alpha particles are easily stopped by 5 cm of air, or by a sheet of paper. The lack of penetration of alpha particles is explained by their very high ionising power. Alpha particles lose energy over a shorter range than beta particles and gamma rays, because alpha particles are relatively slow-moving and they carry a high charge. By contrast, beta and gamma rays lose their energy over much longer distances. Consequently, alpha particles are much more damaging to our bodies because many ions are produced in a small volume.

Dose equivalent The measure of the damage done by radiation.

Sievert The unit of dose equivalent is also $J\,kg^{-1}$, but for dose equivalent the unit is called the sievert (Sv).

We measure the damage done by radiation in **dose equivalents**, which is defined as

$$H = W_R D$$

where W_R is the radiation weighting factor, which is a dimensionless number that depends on the type of radiation – see Table 10.5. Dose equivalent has the same units as dose, namely $J\,kg^{-1}$, but to distinguish dose from dose equivalent, the latter is given the unit **sievert** (Sv).

Table 10.5

Type of radiation	W_R
X-rays, gamma rays, β-particles	1
Protons	2
Alpha particles, nuclear fission products	20

When we handle radioactive sources, we must take care to minimise the risk. First, a radioactive source should be enclosed in a container that is leak-proof to avoid the escape of any radioactive liquid or gases. In schools, radioactive sources are of low intensity, but we adopt these precautions:

- Sources are kept in lead-lined boxes and locked away in metal cupboards.
- When in use, sources are used for a short period of time.
- Sources are kept away from our bodies and are handled with long tongs.

There are strict regulations for the handling of radioactive materials in laboratories, hospitals and industry. A leak of a radioactive gas or liquid is particularly hazardous because we can inhale a gas or swallow a liquid, which could then allow radiation to be emitted inside our bodies.

TEST YOURSELF

12 Describe an experiment that would enable you to detect the type of radiations being emitted from a radioactive source. (Remember to describe the apparatus you would use and what measurements you would take.)

13 A source of gamma radiation is placed a distance of 0.2 m away from a small radiation detector. The detector records a corrected count rate of 200 Bq from the gamma source.

Calculate what count rate would be recorded when the detector is moved a distance of 0.5 m away from the source.

14 a) Calculate the energy of an ultraviolet photon with a wavelength of 2×10^{-7} m. Express your answer in electronvolts (eV). (You may need to refer back to the work in Chapter 3 of book 1.)

b) Explain why sunbathing for too long can cause skin cancer.

15 a) Explain what is meant by the term 'ionising radiation'.

b) Why are ionising radiations dangerous to us?

16 A teacher leaves two radioactive sources on a laboratory bench. One source emits γ-rays, and it is kept in a lead-lined box. The second source is an alpha source, which is left out of its box.

a) Explain which source is potentially more dangerous to pupils 2 m away in the class.

b) i) Radon is a radioactive gas that emits alpha particles. The gas is emitted by granite rocks. Explain why radon might be a hazard to health to those who live in an area with granite rocks.

ii) Explain why gamma rays emitted by rocks are less dangerous than radon's alpha particles.

Practice questions

1 A radioactive source is placed 2 cm from the window of a Geiger–Muller (GM) tube. A count rate of 220 Bq is recorded. The table shows the corrected count rate after some sheets of materials are placed between the source and the GM tube.

Count rate/Bq	220	180	10	12
Material	No material	Sheet of paper	Aluminium 1 mm thick	Lead 1 cm thick

The source emits which of the following radiation(s)?

A alpha and gamma

B beta only

C alpha and beta

D alpha, beta and gamma

Use the following information to answer questions 2, 3 and 4.

Below are listed four radioactive sources, together with their emitted radiations.

A americium-241 alpha (α)

B strontium-90 beta minus (β^-)

C cobalt-60 beta minus and gamma (β^- and γ)

D fluorine-18 beta plus (β^+)

2 Which isotope is suitable for the purpose of sterilising hospital equipment sealed inside plastic bags?

3 Which isotope is suitable for the purposes of discharging static electricity that has built up in the manufacture of polythene?

4 Which isotope is suitable for monitoring the thickness of thin metal being produced in a factory?

5 An alpha particle of energy 7.9 MeV is fired towards the nucleus of a gadolinium atom, $^{157}_{64}Gd$. The closest possible distance of approach of the alpha particle to the gadolinium nucleus is

A 23 fm

B 14 fm

C 7 fm

D 2 fm

6 A gamma source is placed 2.0 m away from a Geiger–Muller (GM) tube. A count rate of 150 Bq is recorded. The source is now moved to a distance of 3.0 m from the GM tube. The count rate observed is now

A 225 Bq

B 100 Bq

C 85 Bq

D 67 Bq

7 A gamma ray has an energy of 1.2 MeV. Its wavelength is

A 2×10^{-12} m

B 1×10^{-12} m

C 5×10^{-13} m

D 3×10^{-13} m

Use the following information to answer questions 8, 9 and 10.

The table below contains information about four types of radiation A, B, C and D.

Characteristic	A	B	C	D
Penetrating power	Many cm of lead	A few mm of metal	Many cm of lead	A few cm in air
Mass/kg	1.67×10^{-27}	9.1×10^{-31}	0	6.64×10^{-27}
Deflection in a magnetic field	0	Large	0	Small
Ionisation	Very weak	Weak	Very weak	Strong

8 Which radiation is made up of alpha particles?

9 Which radiation is gamma rays?

10 Which radiation is made up of neutrons?

11 The radius of an atomic nucleus is given by

$$R = 1.2 \times 10^{-15} \times A^{\frac{1}{3}}$$

where A is the mass number of the nucleus.

a) Calculate the density of a nucleus of barium ($^{138}_{56}\mathrm{Ba}$) in $\mathrm{kg\,m^{-3}}$. *(3)*

b) Calculate the radius of a star that has the same density as barium, if the star has a mass of 4×10^{30} kg. *(3)*

12 Describe an experiment you would carry out to investigate whether or not the intensity of gamma radiation emitted from a source obeys an inverse square law. *(6)*

13 A strong source of gamma radiation is used in a hospital to treat a patient suffering from cancer.

Describe precautions that should be taken to safeguard the health of

a) the patient *(2)*

b) the radiographer who is administering the dose of radiation. *(2)*

14 The graph in Figure 10.19 shows how many ion-pairs are produced per millimetre by an alpha particle at each point of its track.

a) Suggest why the alpha particle produces more ions per millimetre towards the end of its track, just before it stops moving. *(2)*

b) Estimate the total number of ion-pairs produced by the alpha particle along its 50 mm track. *(2)*

c) Each ion-pair requires about 30 eV of energy to form. Use this information to estimate the initial energy of the alpha particle. *(1)*

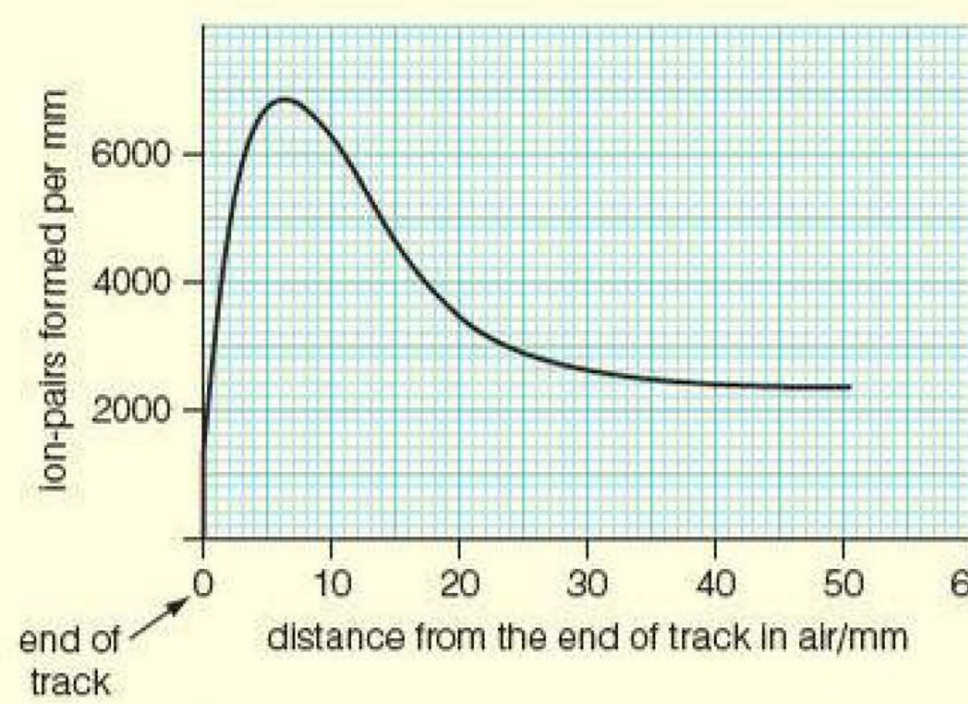


Figure 10.19

15 A source of gamma radiation is placed 0.15 m away from the window of a Geiger–Muller tube, which has an area of $3.2 \times 10^{-4}\,m^2$. The GM tube records a corrected count rate of 38 Bq.

a) Assuming that the gamma rays are emitted uniformly in all directions, estimate the total number of gamma rays emitted per second by the source, if the GM tube only detects 1 in 500 of the gamma rays that enter the tube. *(4)*

b) The gamma rays have an energy of 1.2 MeV. Calculate the energy emitted by the source each second. Express your answer in joules. *(2)*

c) Estimate the count rate measured by the GM tube if the source is moved to a distance 0.10 m away from the window. *(2)*

16 Figure 10.20 shows the paths of two alpha particles that are deflected by a nucleus N. At its closest point, alpha particle A is a distance r away from the nucleus and particle B is a distance $2r$ away from the nucleus.

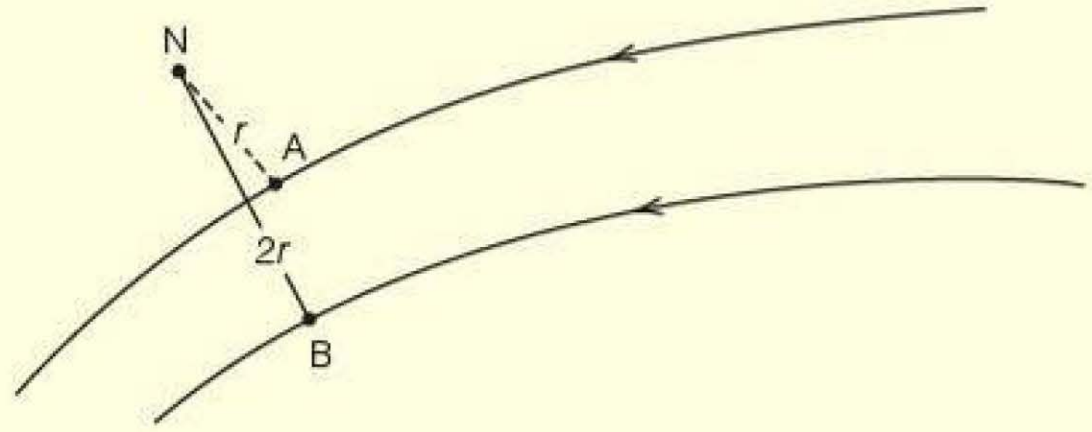


Figure 10.20

a) The forces on the two particles are electrical. How do the size of the two forces compare at positions A and B. *(2)*

b) Explain, by looking at the shape of the tracks, which particle is moving faster. *(2)*

17 The nuclei of atoms may be produced artificially in particle accelerators by firing high-energy alpha particles at a target nucleus. The equation below gives an example of such a reaction:

$$^{27}_{13}\text{Al} + {}^{4}_{2}\text{He} \rightarrow {}^{30}_{15}\text{P} + ?$$

a) Copy and complete the equation to identify a particle that is produced in the reaction. *(1)*

b) It is discovered that the reaction only takes place if the alpha particles have energies in excess of about 10^{-12} J. Use this information to calculate the closest distance of approach of the alpha particle and aluminium nucleus without it reacting. *(3)*

c) Explain why the reaction does not occur for the lower energy alpha particle. *(1)*

Stretch and challenge

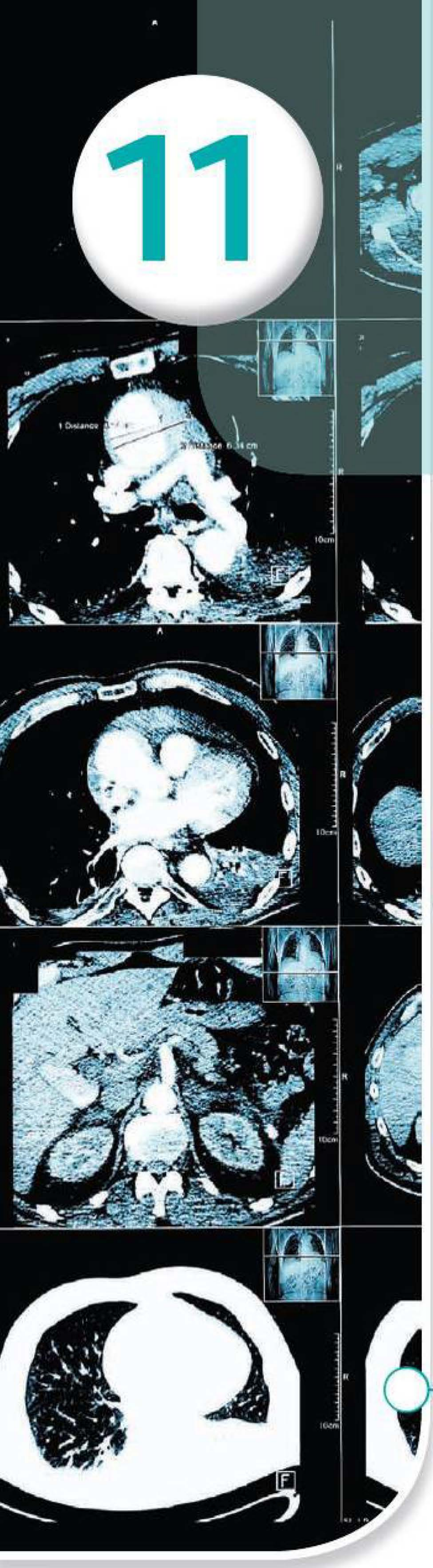
18 An upper limit for the diameter of a carbon nucleus may be obtained from data similar to Geiger and Marsden's. In an alpha-particle scattering experiment, about 1 in 20 000 alpha particles were scattered by more than 150°. This is taken to mean that the particle had scored a 'direct hit' on the nucleus. The thickness of the carbon foil is 2 μm.

a) Given that the diameter of a carbon atom is 1.5×10^{-10} m, calculate how many atoms thick the foil is.

b) Assuming that the probability of an alpha particle making a direct hit on the nucleus is proportional to the thickness of the foil, deduce what fraction of the alpha particles would have been scattered by an angle of 150° or more, had the foil been one atom thick.

c) Now calculate the ratio of the cross-sectional area of the nucleus to the cross-sectional area of the atom.

d) Hence work out the upper limit of the diameter of a carbon nucleus. Explain why the nucleus is likely to be smaller than this estimate.



11 Radioactive decay

PRIOR KNOWLEDGE

Before you start, make sure that you are confident in your knowledge and understanding of the following points:

- Unstable nuclei decay to more stable nuclei by the emission of alpha, beta or gamma radiations.
- An alpha particle is a helium nucleus.
- A beta particle is a fast electron.
- A gamma ray is a high-energy photon of electromagnetic radiation.
- The atomic number of the nucleus is the number of protons in the nucleus. This is also known as the proton number.
- The mass number of a nucleus is the sum of the numbers of protons and neutrons in the nucleus. This is also known as the nucleon number.
- Radioactive decay is a random process. We cannot predict that a particular nucleus will decay, but we can predict that a certain fraction of nuclei will decay in a given time.
- Radioactive decay is described by the term 'half-life'. In one half-life, half of a sample of radioactive nuclei will decay; in another half-life, half of the remaining nuclei will decay.
- Radioactive isotopes have a wide variety of uses in industry, agriculture and medicine.

TEST YOURSELF ON PRIOR KNOWLEDGE

1 Explain the meaning of each of the following terms:
   a) half-life
   b) random
   c) radioactive isotope.

2 The radioactive isotope niobium-89 has a half-life of 2.0 h. At 6 p.m. a scientist has a sample of 1.6 g of niobium-89. How much of this isotope will be left at midnight?

3 Copy and complete the following equations that describe radioactive decays:
   a) $^{208}_{84}\text{Po} \rightarrow {}^{?}_{?}\text{Pb} + {}^{4}_{2}\text{He}$
   b) $^{138}_{56}\text{Ba} \rightarrow {}^{?}_{?}\text{La} + {}^{0}_{-1}\text{e} + ?$

4 Name and explain one practical use of a radioactive isotope.

Spontaneous and random nature of radioactive decay

The emission of radiation from a nucleus is both *spontaneous* and *random*. This means that we cannot tell when a particular nucleus will decay. A nucleus could remain unchanged for millions of years before suddenly it decays by emitting a radioactive particle. Nuclei decay independently of each other, and their behaviour is not affected by the proximity of other nuclei or external factors such as temperature and pressure.

A good model to help us understand radioactive decay is to throw lots of dice. Imagine you have a tray with 600 dice on it and you throw them all on to the floor. Each die has six faces, so there is a probability of $\frac{1}{6}$ for each die to land up as a 6. Therefore, on average, we expect 100 dice to turn up as a 6. However, because it is a random process, that number will rarely be exactly 100. If we repeated the process of throwing 600 dice lots of times, we would see quite a variation around that average of 100 dice turning up with a 6 on top.

Decay constant The decay constant λ is the probability of a nucleus decaying per unit time.

When we consider the decay of a sample of radioactive material, we can apply statistical processes effectively because there is a very large number of nuclei involved in the process. We use a **decay constant** λ to describe nuclear decay: λ is defined as the probability of one nucleus decaying per unit time.

This definition of λ leads to the equation:

λ = fractional change in the number of nuclei, $-\frac{\Delta N}{N}$, per unit time Δt

or

$$\lambda = -\frac{\Delta N/N}{\Delta t}$$

The significance of the minus sign is that the number of radioactive nuclei in a sample of material decreases with time. The unit of λ is s^{-1}. Considering the dice model again helps you to understand the meaning of λ. In the dice 'decay', λ is $\frac{1}{6}$ per throw – every time the dice are thrown, (on average) $\frac{1}{6}$ of them turn up a 6.

The equation above may be written in this form:

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

In words, we can say that:

number of nuclei decaying per second = decay constant × number of nuclei

Activity The activity of a radioactive source is the number of disintegrations per second: $A = \lambda N$

This leads to the definition of the **activity** of a radioactive source, which is the number of emissions per second (of alpha, beta or gamma radiations):

$$A = \lambda N$$

where A is the activity of the source. The unit of activity is the becquerel (Bq), which is a rate of decay of one disintegration per second.

EXAMPLE

The activity of lanthanum

The element lanthanum has two naturally occurring isotopes. Lanthanum-139 is the more abundant isotope and makes up 99.911% of naturally occurring lanthanum. The remaining 0.089% is the radioisotope lanthanum-138, which decays by the emission of beta particles. Lanthanum-138 has a decay constant of $2.0 \times 10^{-19}\,s^{-1}$; and 139 g of lanthanum contain 6×10^{23} atoms.

Use the information above, and your prior knowledge, to answer these questions.

1 What is the difference between a nucleus of lanthanum-139 and a nucleus of lanthanum-138.

Answer

Lanthanum-139 has one more neutron in the nucleus. (Lanthanum-139 has 57 protons and 82 neutrons. Lanthanum-138 has 57 protons and 81 neutrons.)

2 Calculate the activity of a 40 g sample of lanthanum.

Answer

The number of atoms in 40 g of lanthanum is

$$6 \times 10^{23} \times \frac{40}{139} = 1.73 \times 10^{23}$$

However, only 0.089% of these are lanthanum-138. So the number of lanthanum-138 nuclei is

$$N = 1.73 \times 10^{23} \times \frac{0.089}{100}$$
$$= 1.54 \times 10^{20}$$

so

$$A = \lambda N$$
$$= 2.0 \times 10^{-19}\ s^{-1} \times 1.54 \times 10^{20}$$
$$= 31\ Bq$$

TIP

Remember, 1 mole has about 6×10^{23} atoms in it. If the atomic mass of an atom is 30 (for example), then 30 g contains 6×10^{23} atoms.

Decay constant and half-life

Earlier you met the equation

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

where N is the number of nuclei in a radioactive sample, λ is the decay constant for a nucleus, and ΔN is the change in the number of nuclei in time Δt. When both quantities ΔN and Δt tend towards zero, this equation can be written in the differential form:

$$\frac{dN}{dt} = -\lambda N$$

This differential equation has the solution

$$N = N_0 e^{-\lambda t}$$

where N is the number of nuclei in the radioactive sample at time t, and N_0 is the number of nuclei at time $t = 0$, which is the time that we start to observe the sample of nuclei. You do not need to know how to reach the solution to the equation (but the Maths box shows interested mathematicians how to do it). However, you do need to be able to use the equation $N = N_0 e^{-\lambda t}$ to be able to predict the number of nuclei at any time.

MATHS BOX

A differential equation is solved by separating the variables and integrating both sides. So doing this gives

$$\frac{dN}{dt} = -\lambda N$$

$$\int_{N_0}^{N} \frac{dN}{N} = \int_0^t -\lambda\, dt$$

Note that the limits of the integration are from N_0 to N for the nuclei and from 0 to t for the time. Then working through the maths gives

$$[\ln N]_{N_0}^{N} = [-\lambda t]_0^t$$

$$\ln\left(\frac{N}{N_0}\right) = -\lambda t$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$N = N_0 e^{-\lambda t}$$

EXAMPLE

Using the radioactive decay equation

1 A sample of radioactive material contains 100×10^{12} nuclei. The nuclei
have a decay constant of $0.01\,s^{-1}$. Predict the number of nuclei remaining after 10 s.

Answer

$$N = N_0 e^{-\lambda t}$$
$$= 100 \times 10^{12} \times e^{-0.01 \times 10}$$
$$= 10^{14} \times e^{-0.1}$$
$$= 9 \times 10^{13}$$

So there will be about 9×10^{13} nuclei left after 10 s.

2 Draw up a table of the number of nuclei at intervals of 20 s up to a time of 160 s.

Answer

The numbers are shown in Table 11.1. Check them for yourself.

Table 11.1

t/s	0	20	40	60	80	100	120	140	160
$e^{-\lambda t}$	1	0.818	0.670	0.548	0.449	0.367	0.301	0.246	0.201
$N/10^{12}$	100	81.8	67.0	54.8	44.9	36.7	30.1	24.6	20.1

TIP

Summary of useful equations:

Rate of decay $\frac{dN}{dt} = -\lambda N$

Activity $A = \frac{dN}{dt} = \lambda N$

Decay equation $N = N_0 e^{-\lambda t}$

Half-life $T_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$

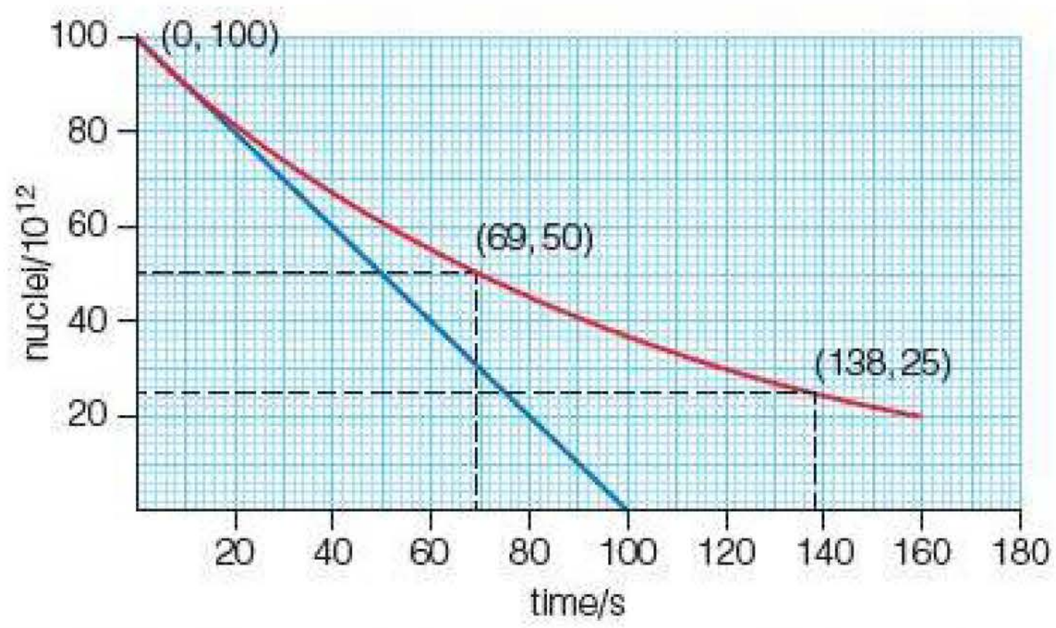


Figure 11.1 The graph shows the exponential decay of a sample of radioactive nuclei.

The numbers in Table 11.1 have been used to plot a graph of the number of nuclei against time; this is shown in Figure 11.1. This graph shows an *exponential decay*, and it has the following important qualities.

Half-life One half-life is the time taken for half of a sample of radioactive nuclei to decay.

- The number of nuclei decreases by the same fraction in the same time interval. In particular, in this case, the **half-life**, $T_{\frac{1}{2}}$, is 69 s. This means that in 69 s the number of radioactive nuclei decreases from 100×10^{12} to 50×10^{12}. In a second half-life of 69 s that number halves again to 25×10^{12}.
- The gradient of the graph at any point is $-\lambda N$. In the graph, the gradient is drawn (blue line) at time $t = 0$:

$$\text{gradient} = -\frac{100 \times 10^{12}}{100\ \text{s}}$$
$$= -10^{12}\ s^{-1}$$

This is the same as

$$-\lambda N = -0.01\,s^{-1} \times 100 \times 10^{12}$$
$$= -10^{12}\ s^{-1} (Bq)$$

or the activity A.

MATHS BOX

We can link $T_{\frac{1}{2}}$ and λ as follows. We have

$$N = N_0\, e^{-\lambda t}$$

After one half-life $T_{\frac{1}{2}}$, there will be $\frac{N_0}{2}$ nuclei left, so

$$\frac{N_0}{2} = N_0\, e^{-\lambda T_{\frac{1}{2}}}$$

$$\frac{1}{2} = e^{-\lambda T_{\frac{1}{2}}}$$

$$\ln\frac{1}{2} = -\lambda T_{\frac{1}{2}}$$

$$\ln 2 = \lambda T_{\frac{1}{2}}$$

$$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

- The third point we can note is that the half-life is connected to the decay constant by

$$T_{\frac{1}{2}} = \frac{0.69}{\lambda}$$

Since $\lambda = 0.01\,s^{-1}$. You can see that

$$T_{\frac{1}{2}} = \frac{0.69}{\lambda} = \frac{0.69}{0.01\,s^{-1}} = 69\,s$$

More generally, we connect $T_{\frac{1}{2}}$ and λ by

$$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

The Maths box shows the theoretical derivation of this result. (You are not expected to be able to derive this, but it shows interested mathematicians how it is done.)

EXAMPLE

The decay of strontium-90

Strontium-90 is a radioactive nuclide that emits beta particles. It has a half-life of about 29 years. A school source of strontium-90 contains about 0.1 μg of this radioisotope.

1 Calculate the decay constant of strontium-90.

Answer

$$\lambda = \frac{\ln 2}{T_{\frac{1}{2}}}$$

$$= \frac{0.693}{29 \times 365 \times 24 \times 3600\,s}$$

$$= 7.6 \times 10^{-10}\,s^{-1}$$

2 How much of this 0.1 μg sample remains after 70 years?

Answer

The fraction left after 70 years is found using

$$\frac{N}{N_0} = e^{-\lambda t}$$

A time of 70 years is $t = 70 \times 365 \times 24 \times 3600 = 2.2 \times 10^9\,s$. So

$$\frac{N}{N_0} = e^{-(7.6 \times 10^{-10} \times 2.2 \times 10^9)}$$

$$= e^{-1.68}$$

$$= 0.19$$

So the mass left after 70 years is 0.19 × 0.1 μg, that is, 0.019 μg.

ACTIVITY

Half-life of protactinium

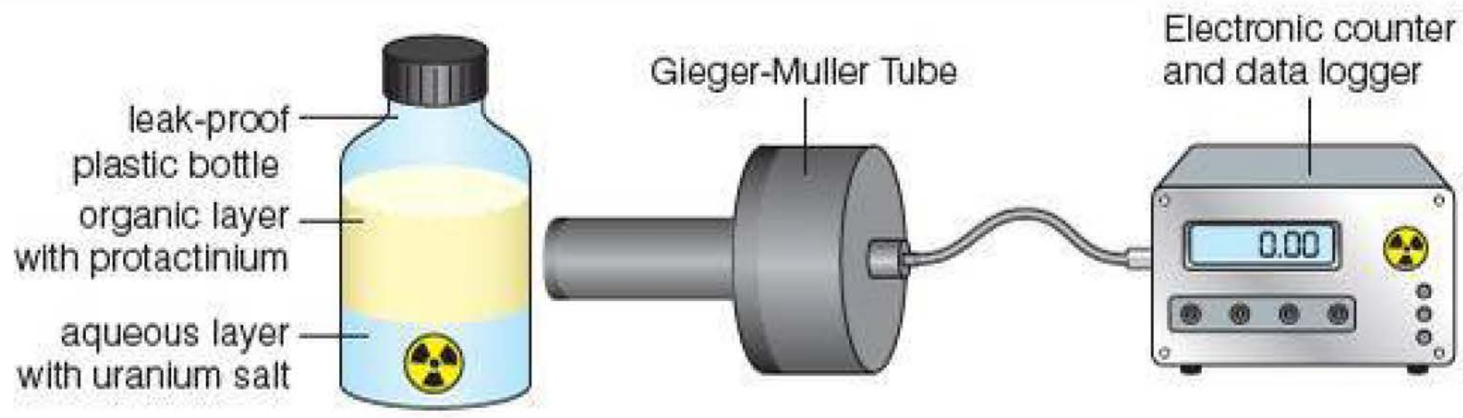


Figure 11.2 The apparatus for determining the half-life of protactinium.

Figure 11.2 shows an experimental arrangement for determining the half-life of a protactinium isotope in a school laboratory. Protactinium is produced in the decay of uranium, which is dissolved into the lower aqueous layer shown in the bottle.

By shaking the bottle vigorously, protactinium is extracted from the aqueous layer and dissolved into the organic layer. The organic layer rapidly separates out as a layer on top of the aqueous layer. The protactinium in the organic layer decays.

The decay of protactinium can then be detected by the Geiger–Muller tube placed near to the organic layer at the top of the bottle.

The results in Table 11.2, for determining the half-life of protactinium, show the count rate measured by the data logger over a period of time. For safety reasons, the count rate from the protactinium is low. This causes us some problems, because a low count rate is subject to random fluctuations, which can make it difficult to determine the half-life accurately. The background count was measured to be 20 counts per minute.

Table 11.2

Time/s	Count rate/Bq
0	8.8
10	7.3
20	6.3
30	6.2
40	5.8
50	4.8
60	4.6
70	4.3
80	4.2
90	4.0
100	3.1
110	2.8
120	2.5
130	2.6
140	2.4
150	1.9
160	2.0
170	1.8

1 Copy the table and add another column of values for the count rate corrected for the background count.

2 Plot a graph of corrected count rate against time and use it to determine the half-life of the protactinium isotope. Comment on the accuracy of your answer.

The activity of protactinium can be written in the form

$$A = A_0 e^{-\lambda t}$$

where A is the activity at time t, and A_0 is the activity at $t = 0$, the start of the experiment. By taking the natural logarithm of both sides of the equation, we get

$$\ln A = \ln A_0 - \lambda t$$

So if we plot a graph with $\ln A$ on the y-axis against t on the x-axis, the gradient of the graph is $-\lambda$.

3 Construct a table of $\ln A$ (ln of the count rate) and time.

4 Plot a graph of $\ln A$ against t, and use it to determine the decay constant and half-life of protactinium. Comment on the accuracy of your answer.

TEST YOURSELF

1 When a quantity decays exponentially, it decreases by a constant fraction in a chosen time interval. Use the data in Table 11.1 to show that the fractional decrease every 20 s is the same.

2 Cadmium-109 is a radioisotope that emits low-energy gamma rays. The differing penetration of these gamma rays through metal alloys allows the metals to be sorted into different types. The half-life of cadmium-109 is 453 days.

  a) Calculate the decay constant for cadmium-109.

  b) A source of cadmium-109 has a mass of 80 μg.

    i) Calculate the number of atoms in the sample.

    ii) Calculate the activity of the sample.

    iii) Calculate the activity of the sample two years after its purchase.

  c) Discuss the safety precautions necessary when this source is used in an industrial site.

3 The activity of a radioactive source falls from 6×10^4 Bq to 2×10^4 Bq in 45 minutes.

  a) i) Calculate the decay constant of this nuclide.

    ii) Calculate the half-life of this nuclide.

  b) How many atoms were there in the original source?

4 A radioactive source, emitting γ-rays, is placed 5 cm from a radiation detector, which has an area of $0.4\ cm^2$. The detector records a count rate of 70 counts per second. The radionuclide contains 4×10^{16} atoms.

  a) Calculate the total number of emissions per second from the source.

  b) Calculate the half-life of the radionuclide.

5 The activity of $2N$ atoms of element P is four times the activity of N atoms of element Q. Element Q has a half-life of 100 years.

  a) Calculate the half-life of element P.

  b) Calculate how much of each element will remain after

    i) 200 years

    ii) 50 years.

6 Describe how you would determine the half-life of a nuclide (which is of the order of a few minutes) in a school laboratory. Credit will be given for the clarity of your explanation and the correct use of English.

Radioisotopes, their half-lives and their uses

Radioisotopes have many applications. They are used widely in the fields of medicine, agriculture and industry. Carbon-14 is used to date archaeological remains, and argon-40 is used to date rocks. Radioisotopes may also be used as a small source of power. You are not expected to know all the possible uses of radioisotopes, but you do need to understand the principles behind them. The Test yourself questions provide some examples.

Importance of half-life and radiations

When choosing a radioisotope for a particular purpose, careful consideration must be given to its half-life and the radiations it emits. Carbon-14, with a half-life of 5700 years, is well matched to dating human settlements a few thousand years old. Argon-40, with a half-life of 1.3 billion years, is well suited to dating igneous rocks. However, many isotopes with short half-lives are also useful because they produce a high activity, from a small quantity of material, over a short period of time.

Because

$$A = \lambda N = \frac{0.693N}{T_{\frac{1}{2}}}$$

the activity A is high if the half-life is short. If the half-life is long, more atoms (larger N) are needed to produce a high activity.

Radiotherapy

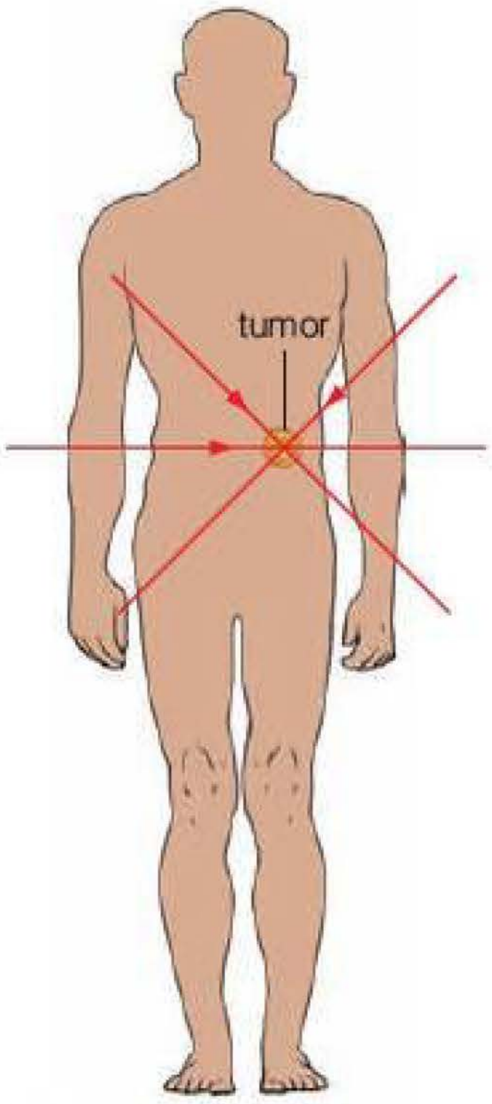


Figure 11.3

In the UK, because we now live longer (our life expectancy is around 80 years), cancer has become the second biggest cause of death. Therefore, considerable research has been carried out into the use of radioisotopes to cure cancer.

Figure 11.3 shows how beams of gamma rays from a source of cobalt-60 can be directed towards a tumour. The hazard of directing gamma rays from outside the body is that the rays also pass through healthy tissues, which could be damaged by the radiation. To reduce the impact of the gamma rays on healthy tissues, the source is rotated around the body, but always directed towards the tumour. In this way the tumour receives a high dose, and healthy tissues receive a low dose, from which the healthy tissues can recover.

Tumours are also treated by short-range internal radiotherapy (Figure 11.4). Under an anaesthetic, a surgeon can place a small needle or wire of a radioisotope into the tumour itself. The radioisotope emits beta particles, which are strongly ionising and short-ranged. Now the radiation is directed straight into the tumour, and the beta particles do not penetrate as far as any healthy tissue. When the correct dose has been administered, the wire is removed. Alpha radiation is also used in targeted alpha therapy (TAT). For example, leukaemia (which is cancer of the blood) can be treated in this way: bismuth-213, an alpha emitter, is attached to an organic compound, which then adheres to cancerous cells.

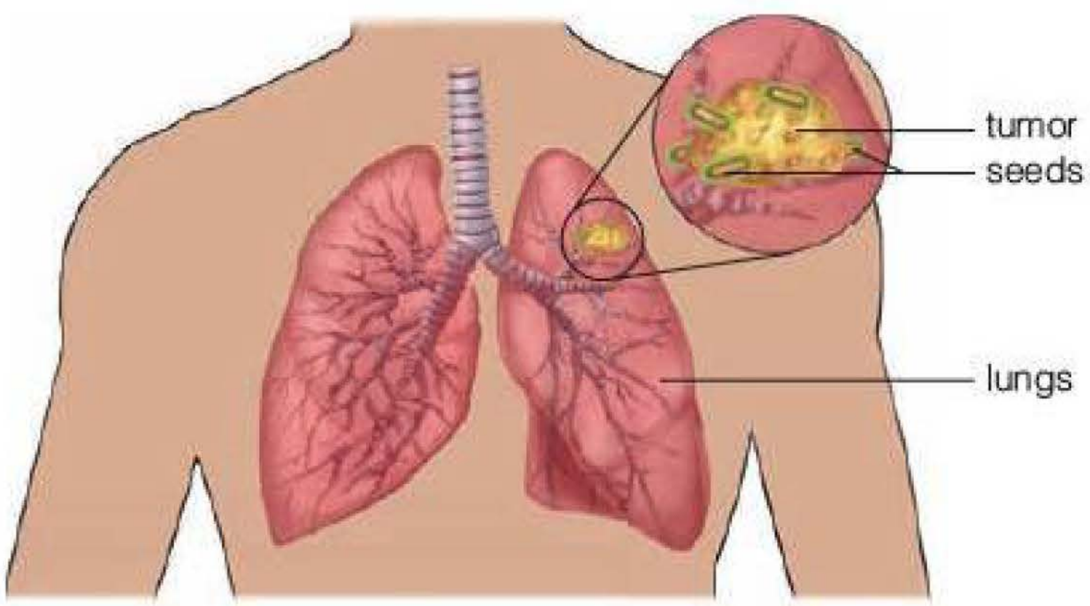


Figure 11.4 These small implants emit beta radiation directly into cancerous tumours.

Short-half-life alpha emitters can be used as a source of energy. Alpha particles that are emitted inside a solid material are self-absorbed – this means that the alpha particles cannot escape through the solid. The particles release their energy to the material and it heats up.

EXAMPLE

The power emitted by polonium-210

Inside a lunar landing vehicle, 1 g of polonium-210 was used as a heat source to keep the components warm. Polonium-210 has a half-life of 139 days and emits alpha particles of energy 5.3 MeV.

1 Calculate the activity of 1 g of polonium-210.

Answer

Number of atoms $= \frac{1}{210} \times 6 \times 10^{23} = 2.86 \times 10^{21}$.

So

$A = \lambda N$

$= \frac{0.693}{139 \times 24 \times 3600} \times 2.86 \times 10^{21}$

$= 1.65 \times 10^{14}\,\text{Bq}$

2 Calculate the power, in watts, emitted by this material.

Answer

Power = A × energy of each particle

$= 1.65 \times 10^{14}\,\text{s}^{-1} \times 5.3 \times 10^{6} \times 1.6 \times 10^{-19}\,\text{J}$

$= 140\,\text{W}$

TEST YOURSELF

7 Table 11.3 gives information about some radioisotopes. Use your knowledge of radioactivity to answer the questions that follow about nuclear medicine.

Table 11.3

Isotope	Radiation emitted	Half-life
Bismuth-213	alpha	45 minutes
Iridium-192	beta, gamma	74 days
Cobalt-60	gamma	5 years
Uranium-233	alpha	150 000 years
Radon-226	beta	6 minutes
Technetium-99	gamma	6 hours

Radiation is used in hospitals for many purposes. Choose an isotope from the table for each of the purposes shown below, explaining your choice. In your answer, explain why the radiation is effective and how your choice protects patients and hospital workers.

a) To sterilise plastic syringes in a sealed plastic bag.
b) To use as a medical tracer that is injected into the body and then detected outside the body.
c) To be used in the form of a wire implant to treat prostate cancer.
d) To be used in a chemical to treat leukaemia.

8 Chromium-51 has a half-life of 27.7 days. It can be used in the form of sodium chromate to measure the volume of blood in a patient. A sample of 10 ml of the patient's blood is 'labelled' with this tracer and injected back into the patient's bloodstream. The activity of the injected sample is 7.40 MBq.

a) Twenty minutes later 10 ml of blood is removed from the patient and its activity is found to be 15.7 kBq. Determine the volume of the patient's blood. State any assumptions you have made.
b) Forty-eight hours later a further 10 ml sample of blood is taken from the patient.
   i) Predict what activity you expect to measure.
   ii) In fact, the doctors measure an activity of 14.5 kBq. What conclusion can you draw?

9 The radioisotope ruthenium-106 is a beta emitter with a half-life of 367 days. It is used in short-range internal radiotherapy.

a) Explain why this isotope is
   i) most effective for the patient
   ii) safe for the surgeon once the isotope is inside the patient's body.
b) A surgeon implants a 0.02 mg sample of ruthenium-106 into a patient. The surgeon has calculated that the patient's tumour must receive a total of 2×10^{12} beta particles from the source. Calculate for how long the implant must be left inside the tumour.

10 The Haraldskaer woman is the body of a woman that was found in an excellent state of preservation in a bog in Jutland, Denmark. Radiocarbon dating revealed that she was buried a long time ago. A 0.1 g sample of modern carbon has an activity of 90 counts per hour. 0.1 g of a sample of carbon from the Haraldskaer woman has a count rate of 66 counts per hour. The half-life of carbon-14 is 5730 years.

a) Calculate the age of the Haraldskaer woman.
b) Explain why radiocarbon dating is only accurate for objects no older than about 60 000 years.

11 Voyager 1 is a spacecraft that was launched to explore the outer Solar System on 5 September 1977. It is powered by three radioisotope thermoelectric generators, which in total produced about 470 W of electrical power when the spacecraft was launched.

Energy from the radioisotope is converted to electricity with an efficiency of 35%. The isotope in use is plutonium-238, which has a half-life of 87.7 years. It emits alpha particles with an energy of 5.5 MeV.

a) Calculate the energy of an alpha particle in joules.
b) i) Calculate the activity of plutonium-238 at the start of Voyager's mission.
   ii) Calculate the mass of plutonium-238 at the start of Voyager's mission.
c) Voyager's instruments will stop working when the electrical power falls to 320 W. Calculate the date when Voyager will no longer be able to contact Earth.
d) Americium-241 is a radioisotope with a half-life of 432 years. Why was this isotope not chosen to power Voyager 1?

Nuclear instability

Every element in the periodic table has many different isotopes. When all these isotopes are added together, they provide a total of several thousand different nuclei. However, most of these isotopes are unstable, and they decay by the emission of radiations to become more stable. In total, there are only 253 stable nuclides. All other nuclides decay, and their half-lives vary from billions of years to fractions of a microsecond.

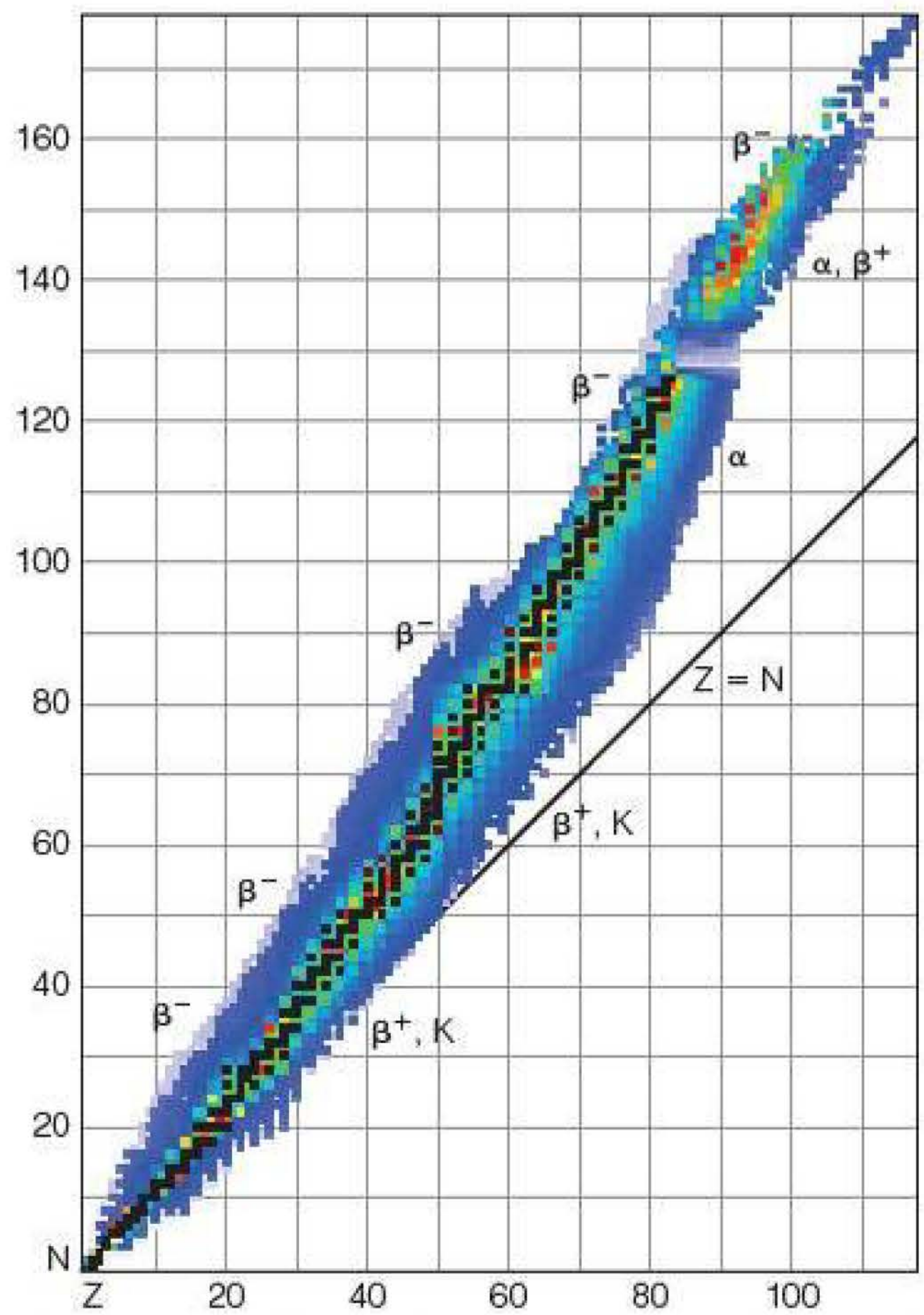


Figure 11.5 A plot of neutron number, *N*, against atomic number, *Z*. The stable nuclei form the black line down the centre of the nuclides. Nuclei above the stable line decay by β^- decay; nuclei below the stable line can decay by α, β^+ decay or K^- capture.

Figure 11.5 shows a chart of nuclei and their stability. The neutron number *N* is plotted on the *y*-axis, against atomic (or proton) number *Z* on the *x*-axis. The least stable nuclei, with very short half-lives, are shown in blue, then the colours green, yellow and red show increasingly stable nuclei. The stable nuclei form the black line down the centre of the chart.

The highest atomic number for a stable nucleus is 82 – this is the element lead. The element above lead in the periodic table is bismuth. Its isotope bismuth-209 is nearly stable, but it decays by alpha emission with a very long half-life of 2×10^{19} years – the age of the Universe is 1.37×10^{10} years. Uranium-238 has the highest naturally occurring atomic number of 92. The isotope uranium-238 decays with a half-life of 4.5 billion years, which is about the same as the age of our Solar System.

The chart in Figure 11.5 shows that, for small values of *N* and *Z*, stable nuclei have roughly equal numbers of protons and neutrons. Examples of nuclei with equal numbers of protons and neutrons include $^{4}_{2}He$, $^{12}_{6}C$, $^{14}_{7}N$, $^{16}_{8}O$, $^{28}_{14}Si$ and $^{40}_{20}Ca$. However, as *Z* increases, the chart shows that the number of neutrons becomes higher than the number of protons. A uranium-238 nucleus has 92 protons and 146 neutrons. The physical reason for this is that the electrostatic repulsion of the protons becomes more significant as the nucleus grows. This repulsive effect is balanced in a stable nucleus by extra neutrons, which provide extra attractive nuclear interactions.

Decay modes of unstable nuclei

You will recall from earlier work that we describe a nucleus in terms of its atomic number *Z*, which is the number of protons in it, and its mass number *A*, which is the sum of the protons and neutrons inside the nucleus. This is written as follows:

$$\begin{matrix}\text{mass or nucleon number} \rightarrow \\ \text{atomic or proton number} \rightarrow\end{matrix} {}^{A}_{Z}X \leftarrow \text{symbol for the element}$$

Daughter nucleus The product of the decay of a radioactive ('parent') nucleus.

When an unstable nucleus decays, it emits radiations, which change the nucleus. So the numbers *A* and *Z* may change. When *Z* changes, the symbol X changes too, as a new element has been formed. This product of the decay is called a **daughter nucleus**.

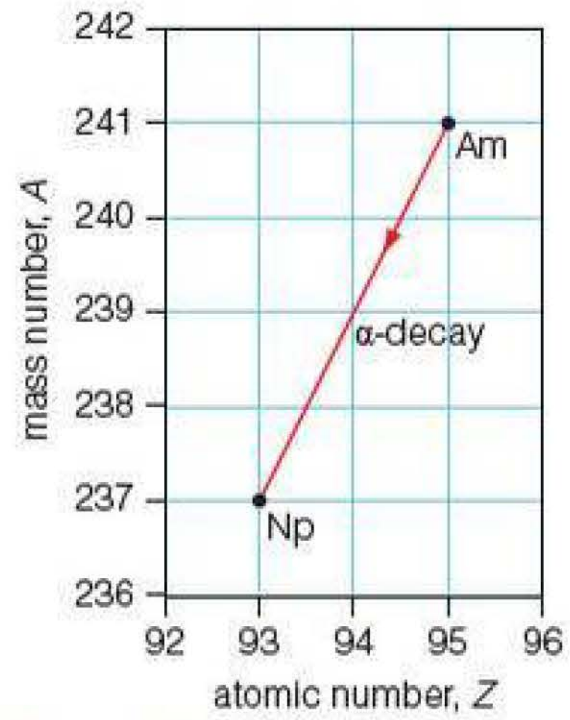


Figure 11.6 A graphical representation of alpha decay: *A* decreases by 4 and *Z* decreases by 2.

Alpha emission

An alpha particle, α, is a helium nucleus, so it has the symbol ${}^{4}_{2}\text{He}$. A typical alpha decay is described below for the α-emitter americium-241:

$${}^{241}_{95}\text{Am} \rightarrow {}^{237}_{93}\text{Np} + {}^{4}_{2}\text{He}$$

Both the mass number and the atomic number must be conserved. The new element formed has 93 protons, which is neptunium. Alpha decay is very rare for elements with *Z* less than 82. If you refer to Figure 11.5, you can see that α decay occurs in heavy nuclei that are rich in protons (which means they are to the right of the curve).

The process of alpha emission may be represented on a plot of nucleon number, *A*, against atomic number, *Z* (see Figure 11.6).

Beta emission

A beta particle, β, is a fast-moving electron. It has a charge of −1, and it is very small in comparison with a proton or neutron. A beta particle is described by the symbol ${}^{0}_{-1}\text{e}$. (The electron mass is about $\frac{1}{2000}$ times that of a proton or neutron, so this has a negligible effect on the mass of an atom.)

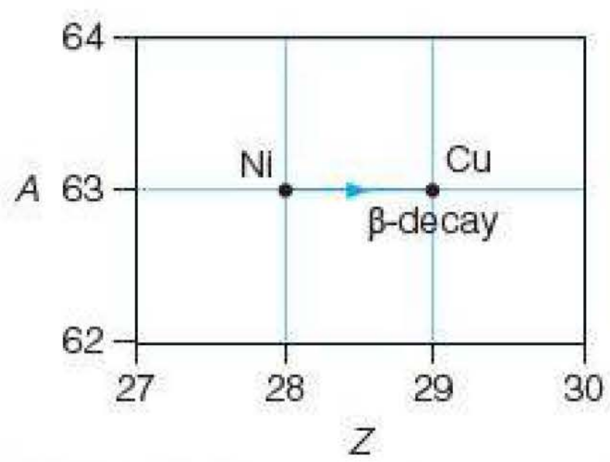


Figure 11.7 A graphical representation of β decay: *A* remains the same and *Z* increases by 1.

The isotope nickel-63 is a beta emitter and its decay is described below. You will recall from last year's work that an antineutrino, $\overline{\nu}$, is also emitted with the beta particle. This particle has virtually no mass but it does have energy:

$${}^{63}_{28}\text{Ni} \rightarrow {}^{63}_{29}\text{Cu} + {}^{0}_{-1}\text{e} + \overline{\nu}$$

When the electron leaves the nucleus, its atomic number increases by 1, and copper-63 is formed. Figure 11.7 shows the change graphically.

If you refer to Figure 11.5, you will see that β decay occurs in elements to the left of the line of stability. These elements have too few protons to be stable. Each of the decays tends to move the nucleus towards the line of stability.

Positron emission and K-capture

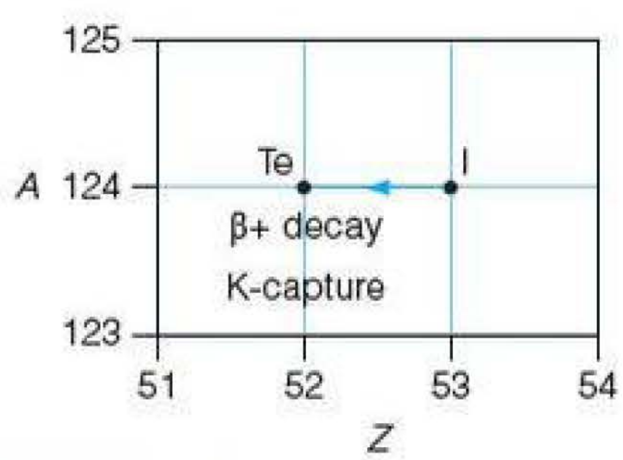


Figure 11.8 A graphical representation of β^+ decay.

Nuclei that are rich in protons, with an atomic number less than 82, tend to decay by positron, β^+, emission or by K-capture. A positron is the antiparticle of an electron. A positron has the same mass as an electron, but has a positive charge. A nucleus that decays by positron decay is iodine-124:

$${}^{124}_{53}\text{I} \rightarrow {}^{124}_{52}\text{Te} + {}^{0}_{1}\text{e} + \nu$$

In positron decay, the atomic number decreases by 1. This is shown graphically in Figure 11.8. A neutrino, which is the antiparticle of the antineutrino, is emitted together with the positron.

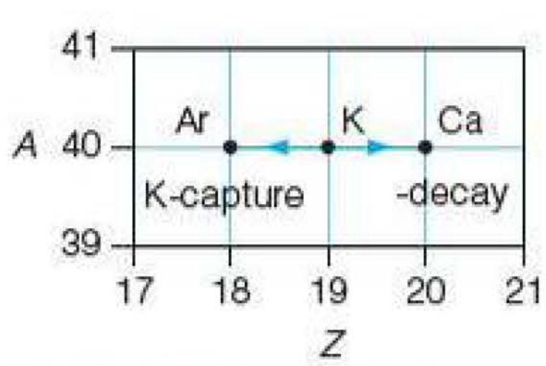


Figure 11.9 A graphical representation of the two modes of decay of potassium-40.

However, iodine-124 also decays by K-capture. In an atom, electrons that orbit the nucleus in the lowest level are in the 'K shell'. These electrons are very tightly bound to the nucleus and actually spend some time inside the nucleus itself. The nucleus can capture such an electron, so that a proton is turned into a neutron. K-capture in iodine-124 can be described as follows:

$${}^{124}_{53}\text{I} + {}^{0}_{-1}\text{e} \rightarrow {}^{124}_{52}\text{Te} + \nu$$

Another nucleus that has two modes of decay is potassium-40; this isotope can decay by *K*-capture or by β-decay (Figure 11.9).

Gamma emission

When a nucleus decays by α, β^- or β^+ emission or by K-capture, it is often left in an excited state. This is similar to an atom being in an excited state, when an electron is in a higher energy level. When an electron drops to a lower energy level, a photon is emitted. Such a photon has an energy of a few eV or perhaps keV in heavy atoms. Protons and neutrons can be left in a higher energy level after a nucleus emits a radioactive particle. When the nucleon drops back to a lower level, a photon is emitted. The energy of these photons is often measured in MeV, and these high-energy photons are the gamma rays that we detect.

Metastable state When an atom or nucleus is in a metastable state, it exists for an extended time in a state other than the system's state of least energy.

In most examples of radioactive decay, the excited nucleus releases its energy very quickly, and gamma rays are emitted shortly after an α, β^- or β^+ particle (with a half-life shorter than 10^{-9} s). When the half-life for gamma emission is much more than 10^{-9} s, we say that the nucleus is left in a **metastable state**. The half-life for metastable states varies from seconds to many years.

To distinguish a metastable state from a stable state, we use the letter 'm'. For example, silver-107m is the metastable state of the common isotope silver-107. These states are also distinguished as shown below, when we use atomic and mass numbers:

- silver-107 in its ground (stable) state $^{107}_{47}\text{Ag}$
- silver-107 in its metastable state $^{107\text{m}}_{47}\text{Ag}$

Metastable technetium-99m

One metastable radioisotope is worth a separate comment. Technetium-99m is a decay product of molybdenum-99, which can be produced in nuclear reactors. The relevant nuclear equations are as follows:

$$^{99}_{42}\text{Mo} \rightarrow {}^{99\text{m}}_{43}\text{Tc} + {}^{0}_{-1}\text{e} + \bar{\nu} \quad \text{(half-life 66 hours)}$$

$$^{99\text{m}}_{43}\text{Tc} \rightarrow {}^{99}_{43}\text{Tc} + \gamma \quad \text{(half-life 6 hours)}$$

The half-life of molybdenum-99 is long enough for it to be transported to hospitals, where it is then put to good use. The technetium-99m is chemically separated from the molybdenum and it is used as a diagnostic tracer. The short half-life of 6 hours makes technetium-99m ideal; it produces a relatively high activity but for a short time.

TEST YOURSELF

12 a) Explain what is meant by a nuclear metastable state.

b) Explain the term 'K-capture'.

13 Complete the following nuclear equations:

a) $^{256}_{?}\text{Lr} \rightarrow {}^{?}_{101}\text{Md} + {}^{4}_{?}\text{He}$

b) $^{?}_{100}\text{Fm} \rightarrow {}^{244}_{?}\text{Cf} + {}^{?}_{2}\text{He}$

c) $^{210}_{85}\text{At} \rightarrow {}^{?}_{?}\text{Po} + {}^{0}_{1}\text{e} + \nu$

d) $^{196}_{77}\text{Ir} \rightarrow {}^{?}_{?}\text{Pt} + {}^{0}_{-1}\text{e} + \bar{\nu}$

e) $^{165}_{68}\text{Er} + {}^{0}_{-1}\text{e} \rightarrow {}^{?}_{?}\text{Ho} + ?$

14 This question refers to the plot of neutron number against proton number shown in Figure 11.5.

a) Explain why heavy nuclei have more neutrons than protons.

b) Explain why some nuclei are β^- emitters whereas others are β^+ emitters.

15 Neptunium-237, $^{237}_{93}\text{Np}$, forms part of a decay series. By a series of α and β^- decays, the stable isotope $^{209}_{83}\text{Bi}$ is produced. In this series of decays, α decay occurs seven times and β^- decay occurs *n* times. Calculate *n*.

Practice questions

1 Samarium-147, $^{147}_{62}Sm$, decays by alpha emission. Into which of the following isotopes does it decay?

A $^{143}_{59}Pr$ **C** $^{145}_{61}Pm$

B $^{143}_{60}Nd$ **D** $^{147}_{63}Eu$

2 The half-life of magnesium-28 is 21 hours. The decay constant for this isotope is

A $4 \times 10^{-3} s^{-1}$ **C** $9 \times 10^{-6} s^{-1}$

B $6 \times 10^{-4} s^{-1}$ **D** $5 \times 10^{-7} s^{-1}$

3 Iodine-123 has a half-life of 13.2 hours. A solution of sodium iodide is prepared for the investigation of a patient's thyroid gland. The solution is to have an initial activity of 200 kBq. The number of iodide ions in the solution is

A 7.9×10^9 **C** 1.4×10^{10}

B 9.6×10^9 **D** 2.3×10^{11}

4 Accurate radiocarbon dating can be done using a mass spectrometer. This allows the ratio $^{14}C/^{12}C$ to be determined with great precision. In a modern sample of wood, the $^{14}C/^{12}C$ ratio is 1.25×10^{-12}. In an ancient piece of wood, the ratio is 0.47×10^{-12}. The half-life of ^{14}C is 5700 years. The age of the wood is

A 650 years **C** 7500 years

B 7000 years **D** 8000 years

Use the following information to answer questions 5 and 6.

Tellurium-128 is a beta emitter, with the longest half-life known, measured at 2.2×10^{24} years.

5 The number of atoms of tellurium-128 in a 1 g sample is

A 9.7×10^{20} **C** 4.7×10^{21}

B 3.4×10^{21} **D** 6.0×10^{22}

6 The activity of a 1 g sample of tellurium-128 is

A 5×10^{-11} Bq **C** 0.2 Bq

B 4×10^{-7} Bq **D** 24 Bq

7 The isotope plutonium-238 is to be used as the energy source for a space satellite. This isotope emits alpha particles of energy 5.5 MeV. The power required for the energy source is 18 W. The minimum activity necessary for the plutonium source is

A 7×10^9 Bq **C** 6×10^{12} Bq

B 4×10^{10} Bq **D** 2×10^{13} Bq

Use the following information to answer questions 8 and 9.

Magnesium-27 is a beta emitter that decays to aluminium-27. Figure 11.10 shows the energy levels within the nucleus for this process. There are two paths possible for this decay, with different energies associated with the beta decay. After the beta decay, the aluminium nucleus is left in an excited state. The nucleus reaches its ground state by the emission of gamma rays. The energy levels in the aluminium nucleus are also shown in the diagram.

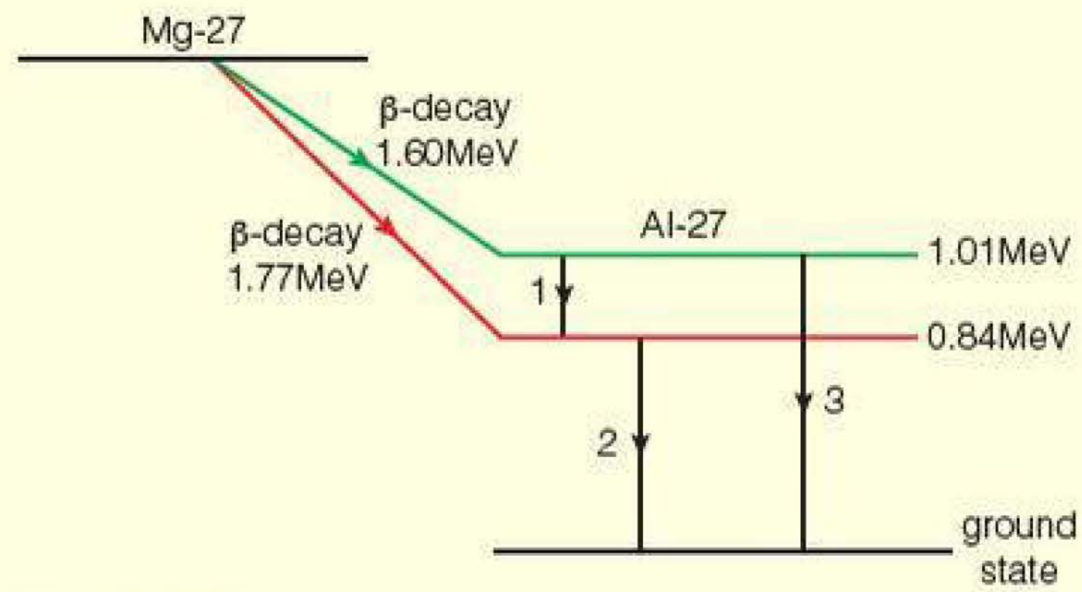


Figure 11.10

8 The magnesium-27 energy level lies above the ground state of aluminium-27 by

A 5.22 MeV **C** 2.61 MeV

B 2.78 MeV **D** 2.44 MeV

9 The wavelength of the gamma ray emitted when the nucleus falls from its second excited level to the first excited level (path 1) is

A 6.6×10^{-13} m **C** 8.4×10^{-11} m

B 7.3×10^{-12} m **D** 7.8×10^{-11} m

10 Molybdenum-99 decays by beta decay to technetium-99 (Figure 11.11). The isotope technetium-99 is a metastable state, which decays to its ground state by the emission of a gamma photon, with a half-life of 6 hours.

The wavelength of the gamma photon is 9×10^{-12} m. The energy difference $E_1 - E_0$ is

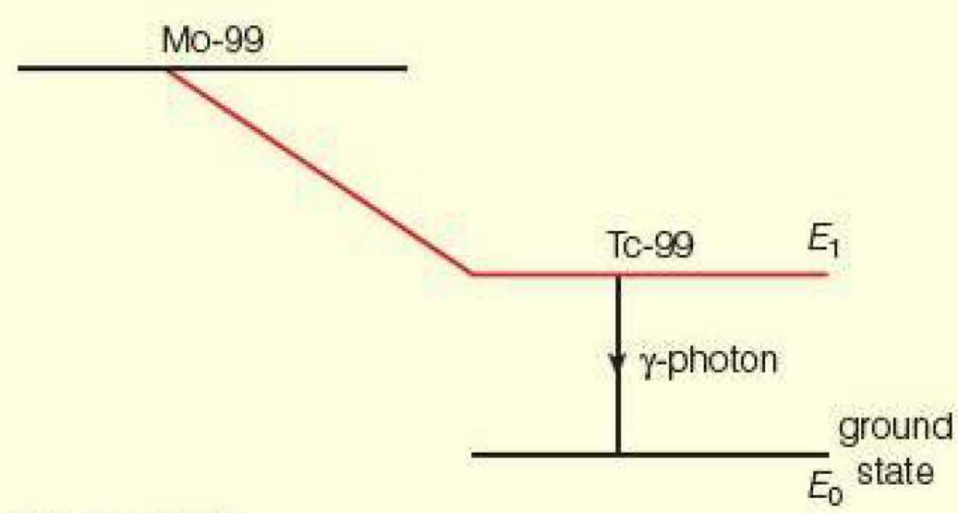


Figure 11.11

A 0.05 MeV **C** 0.18 MeV

B 0.14 MeV **D** 1.2 MeV

11 The isotope of uranium $^{238}_{92}U$, decays into a stable isotope of lead, $^{206}_{82}Pb$, by means of a series of α and β^- decays.

a) In this series, α decay occurs eight times and β^- decay occurs x times. Calculate x. *(1)*

b) The half-life of uranium-238 is 4.5×10^9 years, which is much longer than all other half-lives in the series.

A rock sample when formed contained 6.0×10^{22} atoms of $^{238}_{92}U$ and no $^{206}_{82}Pb$ atoms. At any given time, most of the atoms are either $^{238}_{92}U$ or $^{206}_{82}Pb$, with a negligible number of atoms in other forms in the decay series.

i) Calculate the activity of the uranium when the rock was first formed. *(2)*

ii) Sketch a graph to show how the number of atoms of $^{238}_{92}U$ and the number of $^{206}_{82}Pb$ atoms vary with time over a period of 10×10^9 years after the rock's formation. *(2)*

iii) At a time t, there are three times as many $^{238}_{92}U$ atoms as there are $^{206}_{82}Pb$ atoms. Use this information to calculate the age of the rock. (3)

12 The age of a bone found in a burial site can be calculated by comparing the radioactive decay of $^{14}_{6}C$ from living bone with that of bone from the burial site.

a) $^{14}_{6}C$ decays by β^- emission to an isotope of nitrogen. Write a nuclear equation for this decay. (1)

b) A sample of 2.0×10^{22} atoms of modern bone was prepared for investigation. In modern bone, 1 in 10^{12} of the carbon atoms is the radioactive isotope $^{14}_{6}C$, which has a decay constant of $3.8 \times 10^{-12}\,s^{-1}$.

i) Explain what is meant by 'decay constant'. (1)

ii) Calculate the half-life of $^{14}_{6}C$. (3)

iii) Show that the activity of the $^{14}_{6}C$ in the modern bone is about 0.08 Bq. (3)

c) A sample of 2.0×10^{22} atoms of the bone from the burial site was found to have an activity of 0.051 Bq. Calculate the age of the bone. (3)

13 Radioisotopes are frequently used to treat patients who are ill. Iodine-131 is a β^- and γ emitter, which can be used to treat overactive thyroid glands. When a patient swallows a dose of $^{131}_{53}I$, it is absorbed into the blood, then concentrated in the thyroid gland. The isotope then begins to destroy some cells in the thyroid gland.

a) Write a nuclear equation for the decay of iodine-131 to xenon (Xe). (1)

b) Explain how the iodine destroys cells in the thyroid gland. (2)

c) Radiation from the iodine can be detected outside the body. Explain how this is possible. (2)

d) The half-life of $^{131}_{53}I$ is 8 days. What fraction of the original number of atoms will have decayed after 32 days? (2)

e) A dose is prepared for a patient 48 hours before it is swallowed. The doctors have calculated that an activity of 900 kBq is the correct dose for the patient at the start of the treatment. Calculate the activity of the sample when it is prepared. (3)

14 Chromium-48, $^{48}_{24}Cr$, is a short-lived isotope, which decays by K-capture and the emission of a γ-ray to form an isotope of vanadium, V.

a) Write a nuclear equation for the decay of $^{48}_{24}Cr$. (1)

b) A car manufacturer wants to run tests to measure the wear on a cylinder wall due to the piston movement. A sample of chromium-48, which has a half-life of 22 hours, is used. A very thin layer of the radioactive source is placed on the inside wall of the cylinder and the engine is run continuously. A detector is placed outside the cylinder to measure the count rate.

i) Explain why it is possible to monitor the count rate outside the cylinder. (1)

ii) Explain why an isotope with a short half-life is suitable for this trial. (2)

iii) The count rate was measured to be 450 counts per minute at the start of the trial. Calculate what count rate you would expect after 40 hours. (3)

c) The count rate was actually measured to be 115 counts per minute. Calculate the fraction of the chromium worn away during the trial. (2)

15 Curium-244, $^{244}_{96}Cm$, decays by the emission of an α-particle to plutonium, Pu. The radioisotope has a half-life of 18 years.

a) Write a nuclear equation to describe the decay process. (1)

b) To generate electricity in a submersed vessel, 20 g of $^{244}_{96}Cm$ is to be used as a heat source.

i) Calculate the number of atoms of curium-244 in a sample of 20 g. (2)

ii) Calculate the activity of a 20 g sample of curium-244. (2)

iii) Calculate the maximum power available from 20 g of curium-244. It emits alpha particles with energy 5.8 MeV. (3)

iv) Calculate the maximum power available after 36 years. (1)

16 The table lists some of the isotopes of argon, their half-lives and the mode of radioactive decay.

Isotope	$^{34}_{18}Ar$	$^{35}_{18}Ar$	$^{36}_{18}Ar$	$^{37}_{18}Ar$	$^{37}_{18}Ar$	$^{39}_{18}Ar$	$^{40}_{18}Ar$	$^{41}_{18}Ar$	$^{42}_{18}Ar$
Half-life	0.8 s	1.8 s	stable	35 d	stable	269 y	stable	1.8 h	33 y
Decay	β^+	β^+		K		β^-		β^-	β^-

a) Explain what is meant by the following:

i) isotope (1)

ii) half-life. (1)

b) Calculate the decay constant of $^{34}_{18}Ar$. (1)

c) Suggest why some isotopes decay by β^+ emission and others by β^- emission. (2)

d) Atoms that are close to one another in the periodic table include $_{15}P$, $_{16}S$, $_{17}Cl$, $_{19}K$ and $_{20}Ca$. Write nuclear equations for the following decays. (3)

i) $^{35}_{18}Ar$

ii) $^{37}_{18}Ar$

iii) $^{41}_{18}Ar$

17 The isotope $^{238}_{92}U$ has a half-life of 4.5×10^9 years. Explain how it is possible to calculate the half-life of such an isotope by measuring its activity. *(6)*

Stretch and challenge

18 A nuclear scientist is investigating the activity of two samples of material, P and Q. Sample P has N atoms in it and a half-life of 300 days. Sample Q has $2N$ atoms in it and a half-life of 150 days.

a) Compare the activities of P and Q.

b) Calculate how long it will be before P and Q have the same activity.

19 A radioactive sample is known to contain two different radioactive elements. The sample was placed in front of a GM tube and counter, and the results shown in the table were obtained. The results have been corrected for background radiation.

Time/min	Count rate/s^{-1}
0	800
2	511
4	352
6	261
8	205
10	166
15	109
20	75
25	53
30	37
35	27
40	19

By plotting a graph of ln(count rate) against time, deduce the half-life of each of the elements present in the sample.

20 A radioisotope with a half-life of 24 h is used as a tracer in the human body. The body excretes the tracer with a half-life of 36 h. A doctor monitoring the activity of the tracer at a point close to the body records an activity of 100 Bq when the tracer is first injected. How many hours later does she record an activity of 25 Bq?

12 Nuclear energy

PRIOR KNOWLEDGE

Before you start, make sure that you are confident in your knowledge and understanding of the following points:

- The atom includes a central positively charged nucleus orbited by electrons (Figure 12.1).

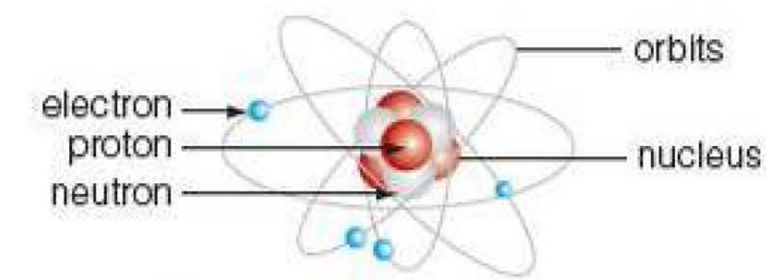


Figure 12.1

- The radius of a nucleus is proportional to $A^{\frac{1}{3}}$, where A is the nucleon number.
- Fission occurs when a nucleus splits into two or more smaller parts.
- Fusion occurs when two nuclei join to form a nucleus of a different element.
- In an elastic collision, kinetic energy is conserved. In a non-elastic collision, kinetic energy is not conserved.
- Momentum is conserved in elastic and non-elastic collisions.
- In a nuclear reaction, mass number and atomic number are conserved.
- One electronvolt $1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$.

TEST YOURSELF ON PRIOR KNOWLEDGE

1. Describe the similarities and differences between the structures of oxygen atoms for the isotopes $^{17}_{8}\text{O}$ and $^{16}_{8}\text{O}$.
2. Write down one example of nuclear fusion, and two examples of nuclear fission.
3. Compare the radii of these nuclei: zinc (64 nucleons) and iodine (127 nucleons).
4. Write down the missing values in this equation: $^{14}_{6}\text{C} \rightarrow {}^{X}_{Y}\text{N} + {}^{0}_{-1}\text{e} + \bar{\nu}$.
5. Write down the numbers of protons and neutrons in $^{238}_{92}\text{U}$.
6. Convert 2.34 MeV into joules.

Since the formation of Solar System, energy from fusion processes in the Sun's core has driven life on Earth. Our understanding of how the Sun generated energy developed only in the last century. When Pierre Curie announced that radium salts release heat continually, physicists at the time suggested that radioactive decay might be a source of energy in the Sun. Calculations and analysis of elements present in the Sun proved this theory was wrong. Albert Einstein proposed his famous equation, $E = mc^2$, at the beginning of the twentieth century, but it was nearly two decades later that Arthur Eddington linked this equation with the generation of energy in the Sun. It took another two decades for the main nuclear cycle in the Sun and the mechanism of nuclear fusion to be described in detail, involving work by many physicists including George Gamow and Hans Bethe.

Einstein, mass and energy

Einstein's work on special relativity led him to publish a paper in 1905. This paper suggested that energy and mass were different ways of expressing the same thing – that energy and mass were interchangeable and linked using the equation, $E = mc^2$, where E is energy (J), m is change in mass (kg) and c is the speed of light, $3 \times 10^8\,\mathrm{m\,s^{-1}}$. By 1932 his ideas were proved experimentally by Cockcroft and Walton.

So what does this equation mean? If we use a helium nucleus as an example, the helium nucleus contains four nucleons – two protons and two neutrons. The mass of the helium nucleus is very slightly smaller that the mass of its separate nucleons.

As the helium nucleus forms, some mass is converted to energy and released. Calculating the energy released when an alpha particle is formed is straightforward:

- mass of a proton is $1.6726 \times 10^{-27}\,\mathrm{kg}$
- mass of a neutron is $1.6749 \times 10^{-27}\,\mathrm{kg}$
- mass of two protons and two neutrons is $6.6950 \times 10^{-27}\,\mathrm{kg}$
- measured mass of a helium nucleus is $6.6337 \times 10^{-27}\,\mathrm{kg}$
- mass difference is $6.13 \times 10^{-29}\,\mathrm{kg}$

Using $E = mc^2$, the energy released when a single alpha particle is formed is $5.5 \times 10^{-12}\,\mathrm{J}$, or 34 MeV.

You can see that, in Einstein's own words, 'a very small amount of mass may be converted into a very large amount of energy and vice versa'.

Work must be done to overcome the very strong nuclear forces that bind the nucleons together and pull a helium nucleus apart. The energy put in to do this creates the extra mass.

A nucleus of Z protons and N neutrons has a mass that is less than the mass of the protons and neutrons that make it up. This difference in mass is called the *mass defect*, where mass defect $\Delta m = Zm_p + Nm_n - M_{nucleus}$ measured in kg or atomic mass units (u).

Since mass and energy are interchangeable, we can also express mass as energy. *Binding energy* is the energy that corresponds to the mass defect, and is related to the mass defect using binding energy = mass defect × c^2.

Binding energy is the energy that would have to be supplied to the nucleus to separate it back into its constituent protons and neutrons. The binding energy can be expressed in J or in MeV.

Atomic mass unit

Single nuclei have such small masses that calculations are simpler if we use a unit of mass called the *atomic mass unit* (u). An atomic mass unit is 1/12 of the mass of an atom of ^{12}C, or $1.661 \times 10^{-27}\,\mathrm{kg}$. Using $E = mc^2$, a mass of $1.661 \times 10^{-27}\,\mathrm{kg}$ is equivalent to $1.495 \times 10^{-10}\,\mathrm{J}$, or 931.5 MeV.

Table 12.1 gives some particle masses in atomic mass units. These are quoted to a large number of significant figures because their differences are small.

Table 12.1

Particle	Mass/u	Mass/10^{-27} kg
Proton	1.00728	1.673
Neutron	1.00867	1.675
Helium nucleus	4.00151	6.647

TIP

Be careful! Mass defect is the difference in mass between individual nucleons, and their mass when they form a nucleus. If you compare two nuclei, you are calculating a mass difference ($M_{\text{original nucleus}} - M_{\text{final nucleus}}$), which is not the same as mass defect.

EXAMPLE

Mass defect for oxygen

Calculate the mass defect (in u) and binding energy (in MeV) for an oxygen nucleus, ^{16}O. The mass of an oxygen nucleus is 15.9949 u.

Answer

The oxygen nucleus has Z = 8 protons and N = 8 neutrons, so the mass of the particles that make up the ^{16}O nucleus is

$$Zm_p + Nm_n = 8 \times 1.00728\,u + (8 \times 1.00867\,u) = 16.1276\,u$$

So the mass defect for oxygen is

$$\Delta m = (Zm_p + Nm_n) - M_{nucleus}$$
$$= 16.1276\,u - 15.9949\,u$$
$$= 0.1327\,u$$

The mass defect is 0.1327 u.

Since 1 u = 931.5 MeV, the binding energy is 123.6 MeV to four significant figures.

EXAMPLE

Mass difference between thorium and radium

Alpha particles are released during the decay of thorium to radium. Calculate the mass difference (in kg) and the difference in binding energy for both nuclei (in J). The atomic mass of thorium is 232.038u and the mass of radium is 228.031 u. Remember that an alpha particle is a helium nucleus.

Answer

The mass difference is

$$232.038\,u - (228.031\,u + 4.00151\,u)$$
$$= 5.49 \times 10^{-3}\,u$$
$$= 9.12 \times 10^{-30}\,kg \text{ (3 s.f.)}$$

The difference in binding energy in J for the two nuclei is given by

$$mc^2 = 9.12 \times 10^{-30}\,kg \times (3 \times 10^8\,m\,s^{-1})^2$$
$$= 8.21 \times 10^{-13}\,J \text{ (3 s.f.)}$$

This energy is transferred in the kinetic energy of the alpha particle and the daughter nucleus. Considerations of momentum conservation show that the small alpha particle has much more kinetic energy than the recoiling nucleus.

Binding energy per nucleon

By measurement, it was found that binding energy is different for different nuclei. The binding energy per nucleon for stable nuclei is shown in Figure 12.2. We use the following equation to calculate the average binding energy per nucleon:

$$\text{binding energy per nucleon} = \frac{\text{total binding energy}}{\text{number of nucleons}}$$

Experimental data shows that nuclei with a high binding energy per nucleon are most stable. More energy per nucleon is needed to pull the nucleons apart. This information allows us to predict the stability of nuclei of different masses.

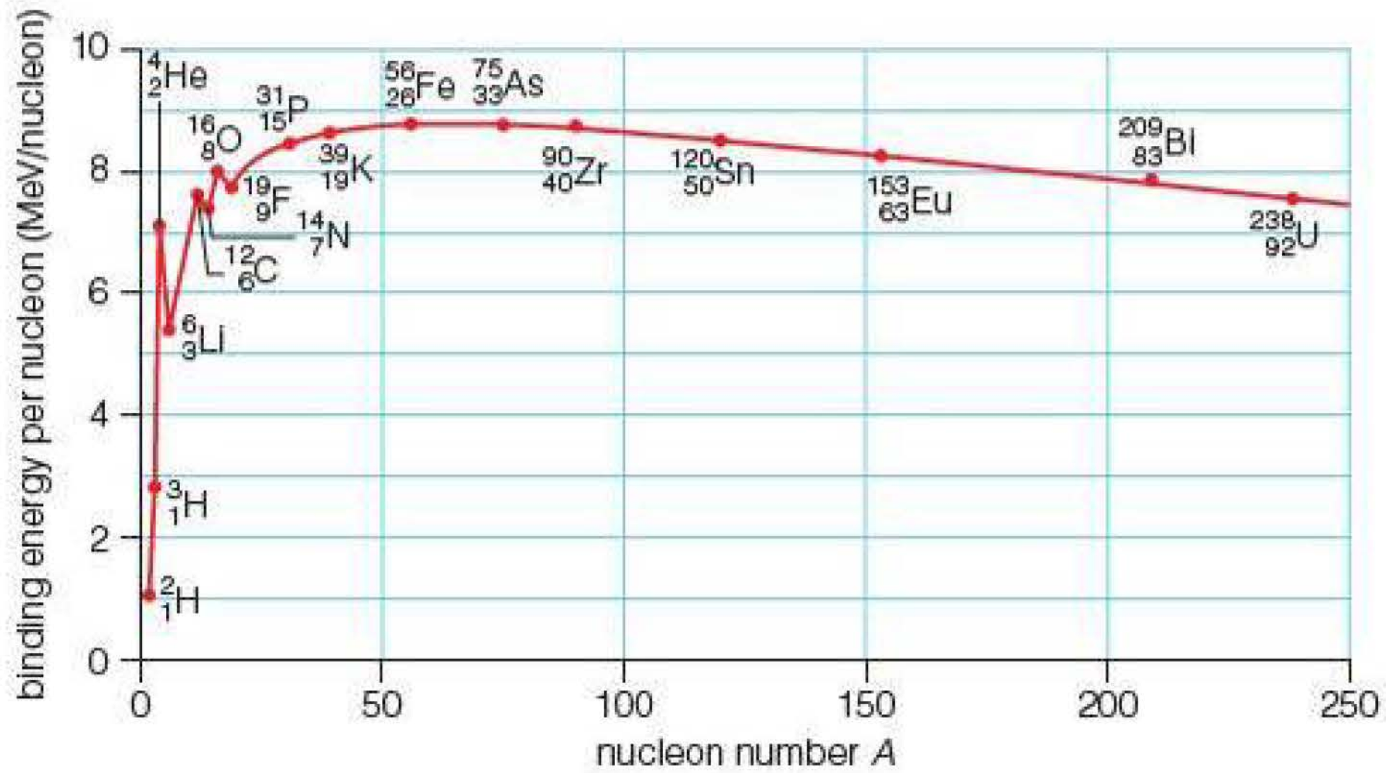


Figure 12.2

The graph in Figure 12.2 shows that:

- Binding energy per nucleon increases rapidly with nucleon number for lighter elements and is about 8 MeV per nucleon for helium and elements heavier than lithium.
- Helium nuclei are very stable relative to other low-mass nuclei, which explains why alpha decay is more common than proton emission.
- Binding energy per nucleon has its highest value for ^{56}Fe, at 8.79 MeV per nucleon, and decreases with increasing nucleon number for any stable nucleus heavier than ^{56}Fe.

Figure 12.2 shows that binding energy has a positive value. However, stable nuclei have less nuclear potential energy than the free nucleons, so you may see binding energy quoted with a negative value. In this chapter, we use the convention that binding energy is the energy that has to be supplied to break the nucleus apart, so it is a positive quantity.

Nuclear reactions

Some nuclei can release energy from nuclear fission or nuclear fusion. Almost all nuclear reactions that occur naturally result in nuclei that are more stable. This increases the binding energy per nucleon compared with the original nuclei. The mass difference between the original nuclei and the nuclei of the

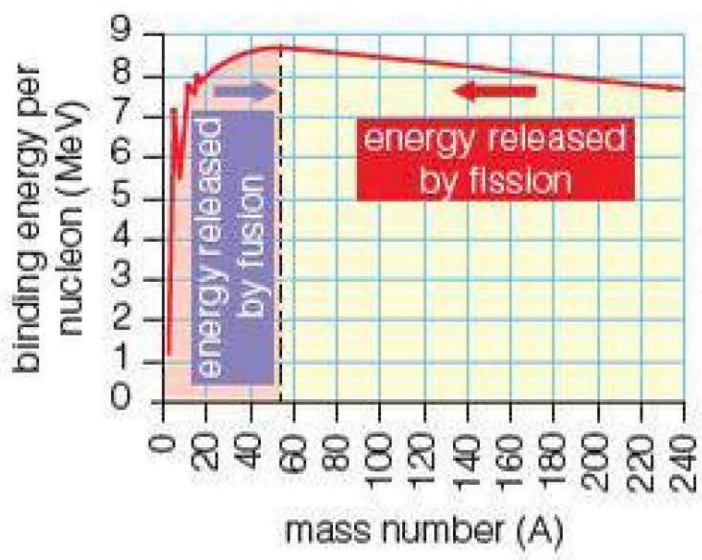


Figure 12.3 Lighter elements release energy by fusion, and heavier elements release energy by fission.

products corresponds to the amount of energy released. Figure 12.3 shows that, to increase the binding energy per nucleon, lighter elements tend to fuse and heavier elements tend to undergo radioactive decay or fission.

In all nuclear reactions, total proton number Z and mass number A are conserved, and the reaction often results in more than one product.

TEST YOURSELF

Use the following data to answer these questions:
$1\,u = 1.661 \times 10^{-27}\,kg = 931.5\,MeV$
mass of proton = 1.00728 u
mass of neutron = 1.00867 u

1 Suggest, with an explanation, how the mass of a $^{12}_{6}C$ nucleus is different from the total mass of its protons and neutrons when separated.

2 ^{56}Fe has 26 protons and 30 neutrons. The mass of an iron nucleus is 55.935 u. Calculate
- **a)** the mass defect in u
- **b)** the binding energy for ^{56}Fe in MeV
- **c)** the binding energy per nucleon in iron-56.

3 Uranium undergoes alpha decay, forming thorium. Calculate the difference in binding energy for the nuclei in MeV. Uranium has 92 protons and 146 neutrons. The mass of a uranium nucleus is 238.0508 u, and that of a thorium nucleus is 234.0426 u.

Fusion

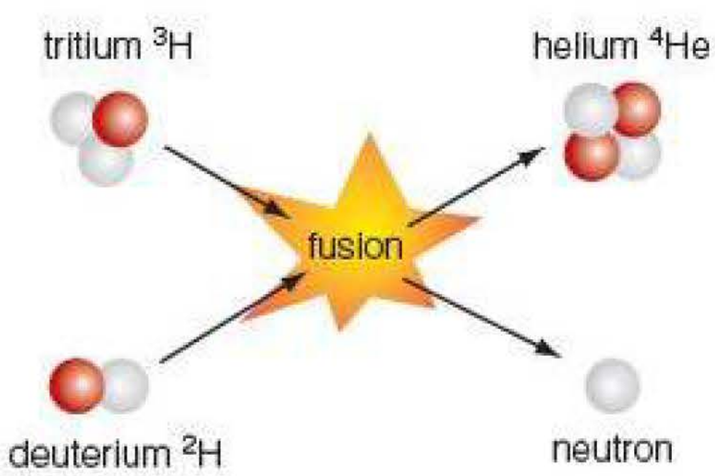


Figure 12.4 The fusion reaction between tritium and deuterium.

Light nuclei can join together by nuclear fusion, form a new element and release energy. Nuclear fusion occurs naturally in stars, which is how stars can release energy for billions of years. Scientists have successfully achieved fusion reactions on Earth too, but very large amounts of energy are needed to create suitable conditions for fusion, similar to the conditions in the cores of stars.

The temperature in a star's core is several million kelvin and the density is in the region of $150\,000\,kg\,m^{-3}$. These very high temperatures give nuclei enough kinetic energy to overcome the electrostatic repulsion between protons in the nucleus. The high density inside the star's core forces nuclei so close together that the strong force becomes involved. This attractive force acts over very short distances.

One fusion reaction that releases energy in stars like the Sun is the fusion of deuterium and tritium to form helium and a neutron. This is shown in Figure 12.4. The equation is

$$^{2}_{1}H + ^{3}_{1}H \rightarrow ^{4}_{2}He + ^{1}_{0}n$$

The mass difference for this reaction is the difference between the mass of the original nuclei (deuterium and tritium) and the mass of the products (helium nucleus and neutron):

$$\text{mass difference} = (2.013553\,u + 3.015500\,u) - (4.001505\,u + 1.008665\,u) = 0.018883\,u$$

The energy released is $0.018883\,u \times 931.5\,MeV = 17.59\,MeV$.

The fusion reactions that occur in different types of stars depend on the star's mass, core temperature and density. A chain of reactions, which may include steps that seem impossible in terms of energy, can happen if conditions are suitable. For example, the triple alpha cycle occurs, but only in red giants and red supergiants, where the core temperatures are greater than 100 million kelvins:

$$^{4}_{2}He + ^{4}_{2}He \rightarrow ^{8}_{4}Be - 93.7\text{ keV}$$

$$^{8}_{4}Be + ^{4}_{2}He \rightarrow ^{12}_{6}C + 7.367\text{ MeV}$$

TEST YOURSELF

Use the following data to answer these questions:

$1\,u = 1.661 \times 10^{-27}\,kg = 931.5\,MeV$

mass of proton $= 1.673 \times 10^{-27}\,kg$

mass of neutron $= 1.675 \times 10^{-27}\,kg$

mass of $^{3}_{2}He = 5.006 \times 10^{-27}\,kg$

mass of $^{4}_{2}He = 6.645 \times 10^{-27}\,kg$

mass of $^{2}_{1}H = 3.343 \times 10^{-27}\,kg$

4 A source of energy in some stars is the reaction $^{2}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{2}He + ^{1}_{0}n$. Calculate the energy released (in J) during this reaction.

5 Explain why a star's core must be at a high temperature for fusion to be possible.

6 The reaction $^{12}_{6}C + ^{4}_{2}He \rightarrow ^{16}_{8}O$ is one that occurs in stars much hotter than the Sun, releasing 7.162 MeV. Compare this with the energy released in one of the fusion reactions that takes place in our Sun: $2^{3}_{2}He \rightarrow ^{4}_{2}He + 2^{1}_{1}p$.

Fission

Nuclear fission is when a nucleus splits into two or more smaller parts, releasing energy.

Nuclear power

Nuclear power stations generate about 20% of the UK's energy using controlled nuclear fission reactions to produce heat used to generate electricity. Nuclear fission can be induced in some isotopes, including those of uranium and plutonium, by making the nucleus unstable when it absorbs a neutron.

The nuclear fuel used in most nuclear power stations contains an isotope of uranium, U-235. The nucleus of U-235 contains 92 protons and 143 neutrons. One problem with using uranium is that U-235 makes up only 0.7% of mined uranium, and most natural uranium is U-238, which does not undergo fission. The mined uranium must be enriched until the U-235 content is about 3% before it can be used as a nuclear fuel – although enriched fuel rods still contain a high proportion of U-238, which is not involved in fission. Some reactors use isotopes of plutonium or thorium instead.

Nuclear fission to generate power

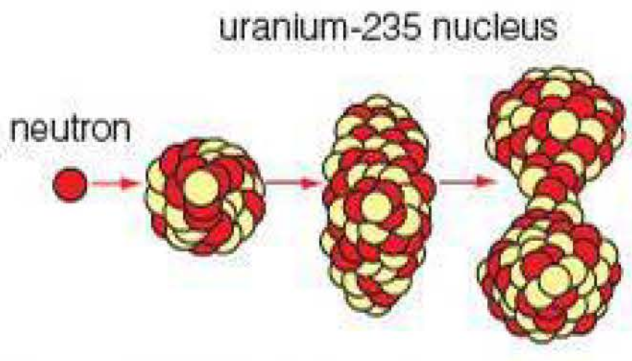


Figure 12.5 U-235 nucleus is very unstable if it absorbs a thermal neutron.

Fission reactions are established in the nuclear fuel using neutrons travelling slowly enough to be captured when they are fired at U-235 nuclei. A U-235 nucleus that captures a neutron becomes very unstable, and splits into two or more smaller pieces, and releases energy in the form of heat.

When it has absorbed the neutron, some people think of the nucleus as being like a wobbly jelly, which splits if it is wobbled too much (Figure 12.5).

Each fission reaction produces two, three or sometimes four neutrons, which may be absorbed by other U-235 nuclei if the neutrons are made to travel slowly enough. There are several possible reactions (Figure 12.6), for example:

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3^{1}_{0}n$$

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{144}_{54}Xe + ^{90}_{38}Sr + 2^{1}_{0}n$$

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{140}_{55}Cs + ^{92}_{37}Rb + 4^{1}_{0}n$$

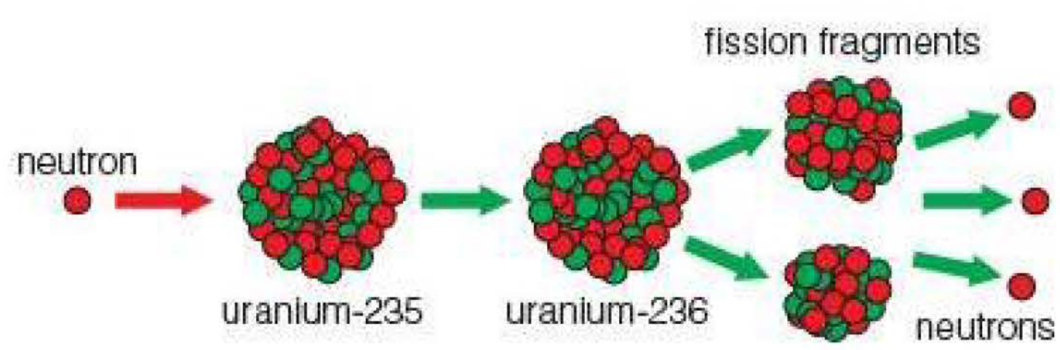
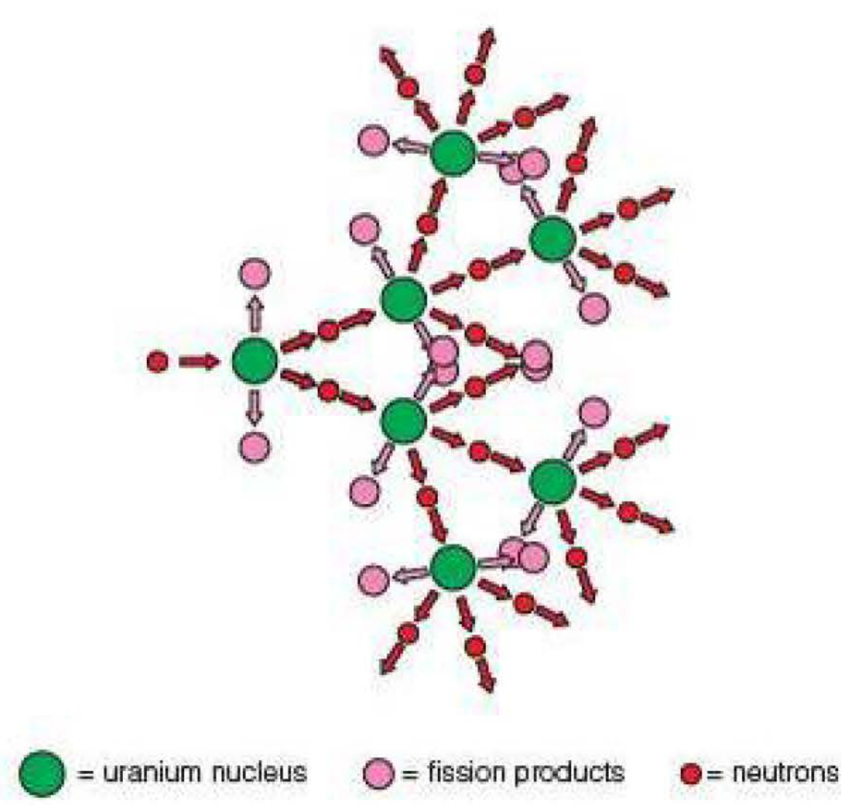


Figure 12.6 Stages in the fission of U-235.

Nuclear fission reactions can only continue in a reactor if the number of nuclei involved in the fission reaction stays constant or increases. This occurs if, on average, one or more neutrons is produced and absorbed per fission reaction.

This type of self-sustaining reaction is called a *chain reaction* (Figure 12.7). Chain reactions are only sustainable with a minimum amount of fuel, called the *critical mass*. This is because neutrons lost from the surface are no longer involved in the chain reactions. The shape as well as the mass of the sample affect the critical mass.

Figure 12.7 A chain reaction grows if more neutrons are produced at each stage than are absorbed.



Role of neutrons in nuclear power stations

Neutrons that induce fission reactions in nuclear reactors are called *thermal neutrons*. Their mean kinetic energy is equivalent to $\frac{3}{2}kT$, where k is the Boltzmann constant and T is the absolute temperature of the reactor core. Typically, thermal neutrons travel at between 2.5 and 3.0 km s^{-1}, relating to a reactor core temperatures of about 290–350 K.

Moderation of neutrons

Neutrons produced by nuclear fission move so fast that they are unlikely to be absorbed in uranium nuclei, so they must be slowed down. The role of the moderator is to slow down fast neutrons as they pass through materials like graphite or water (Figure 12.8). Fast neutrons repeatedly collide with nuclei in the moderator, exciting the nuclei to higher energy levels. The fast neutrons lose energy during these collisions, and further collisions between neutrons and nuclei are elastic, slowing the neutron down even more. The slower neutrons are called thermal neutrons.

The excited nuclei lose their surplus energy as gamma radiation when they return to the ground level.

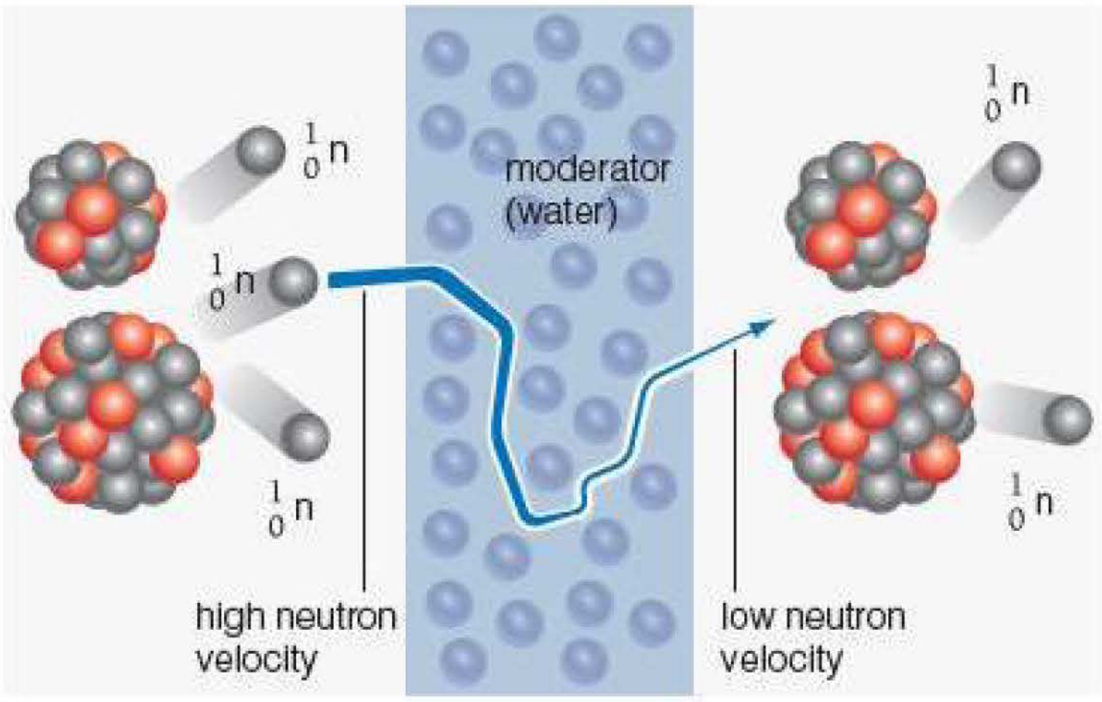


Figure 12.8 Nuclei in the moderator absorb energy from neutrons through elastic collisions, slowing the neutrons down.

Graphite and heavy water are suitable materials for the moderator because they do not absorb neutrons. Also energy is transferred more efficiently during elastic collisions if the mass of the nucleus is close to the mass of a neutron. Consider a snooker ball colliding head-on with a second, stationary ball. One ball stops as its energy is transferred to the other ball, which carries on at the same speed. A much lighter table tennis ball will bounce off a snooker ball, which keeps most of its energy.

Control rods

Control rods control the rate of reactions in the reactor. Materials such as boron steel and cadmium absorb neutrons without undergoing fission. Other materials such as silver, are also suitable but are rare and expensive. Boron is particularly useful because about 20% of the boron in control rods is boron-10, which absorbs neutrons to become boron-11. When a control rod is lowered into the reactor (Figure 12.9), the control rods absorb neutrons, so the rate of the reaction slows down because fewer neutrons are available to trigger fission reactions. The position of the control rods can be adjusted to maintain the chain reaction at a steady rate, or to shut the reactor down completely.

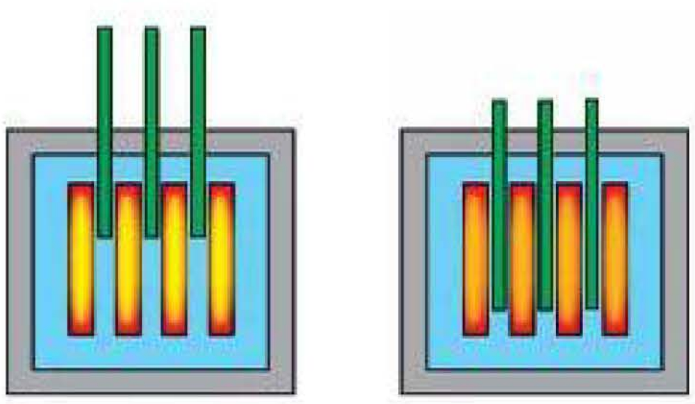


Figure 12.9 Moving control rods deeper into the reactor core absorbs more neutrons and slows down the fission reactions.

The coolant

Coolants are fluids that absorb heat from the reactor, and transfer this heat away to drive the turbines that generate the electricity and to prevent the reactor from overheating. Most of the UK's nuclear reactors use carbon dioxide as a coolant, but some use pressurised water.

The coolant circulates through tubes inside the reactor core, absorbing heat from the reactor. This hot coolant then passes through a heat exchanger or boiler where its heat is transferred to water in a secondary cooling system (Figure 12.10).

As the water in the secondary cooling system heats up, it changes to high-pressure steam and is used to drive the turbines and generator. Any steam remaining in the secondary cooling system is condensed back into water before it circulates through the heat exchanger again. To achieve this, the steam passes through pipes in a condensing unit, which is another heat exchanger that uses cold water-filled pipes. The water in the condensing unit is usually taken from a nearby sea or river.

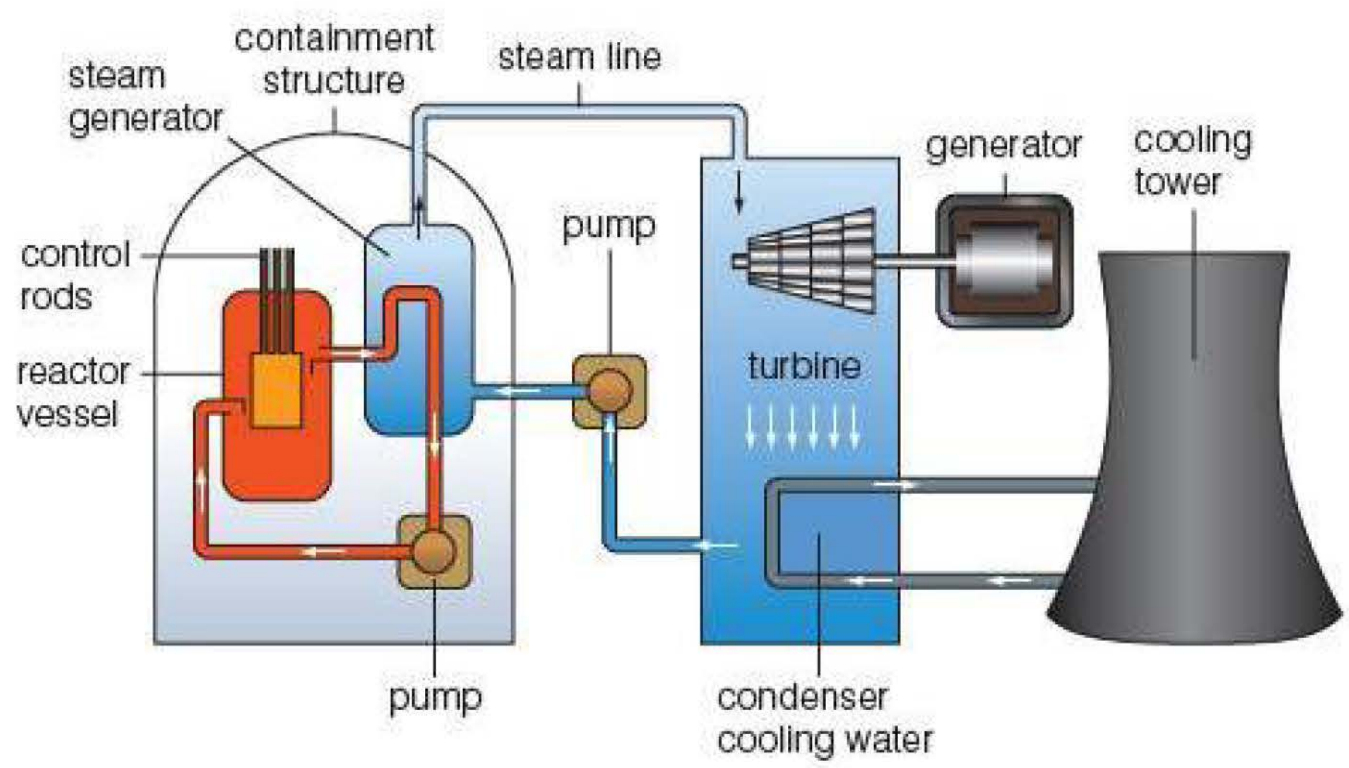


Figure 12.10 Schematic system in a nuclear power station.

Safety aspects of nuclear power

Nuclear fuel

Nuclear fuel, in particular the spent fuel rods, and the nuclear reactor are highly radioactive. Workers and the community must be protected from exposure to radioactive materials to reduce the damage caused by ionising radiation. Exposure to ionising radiation can damage DNA in cells, and increase the long-term risk of cancer. The risk of harm is higher if people are exposed to higher doses of radiation, or if the time or intensity of exposure increase. Workers involved in a nuclear accident may receive very high doses, causing radiation sickness which can be fatal in a few days. Many steps are taken to reduce or prevent exposure.

The reactor is surrounded by shielding, which protects workers from exposure to radiation. In many nuclear power stations, this is a steel pressure container that also contains the high-pressure coolant. This container is surrounded by 5 m of concrete to absorb neutrons and gamma radiation, and this is surrounded by a steel and concrete building, designed to contain radiation even if there is an accident.

Cost and effectiveness are important factors to consider when choosing a material for the shield. Common materials used for shielding are lead, concrete, steel and water. Concrete is one of the most cost-effective materials used in nuclear power stations.

In an emergency, nuclear power stations are designed to shut down automatically. During a shutdown, the control rods drop into the reactor core, absorb the neutrons and slow down or stop the nuclear fission reactions. In many nuclear power stations, the control rods are held vertically above the reactor core using electromagnets. If there is a power failure, the rods drop automatically into the reactor.

Nuclear waste

Nuclear waste is produced from nuclear power stations. It is grouped into three categories – low-, intermediate- and high-level wastes. Nuclear waste is handled remotely to protect workers from exposure to radiation. This includes tele-operation, where workers manipulate equipment remotely, and the use of robotic machinery.

Low-level waste

Low-level waste, including clothing worn by workers, paper and rags, accounts for 90% of the volume of nuclear waste, but only 1% of the radioactivity. Low-level waste is compacted and encased in cement and stored on licensed sites until the radioactivity decays away and it can be disposed of in normal waste. Isotopes in low-level waste have different half-lives and activities, so their exact disposal procedures vary.

Intermediate-level waste

Intermediate-level waste is mainly produced when a nuclear power station is decommissioned, and occurs in chemical sludges and resins. Intermediate-level waste accounts for 7% of the volume of nuclear waste, and 4% of the radioactivity. Intermediate-level waste with long half-lives is encased in cement in steel drums and stored securely underground, for example in caverns or in near-surface facilities. A near-surface facility holds drums containing isotopes with half-lives of less than a few years, which are placed in deep trenches and then covered by several metres of soil.

High-level waste

The main source of high-level waste is spent fuel rods. High-level waste accounts for 3% of the volume of nuclear waste, but 95% of its radioactivity. The spent fuel rods are so radioactive that they continue to emit heat and have to be cooled as well as stored. Initially, spent fuel rods are stored under water which acts as a coolant as well as a shield from ionising radiation. For long-term storage, high-level waste is mixed with molten glass, then solidified inside stainless-steel containers. This process is called vitrification. These stainless-steel cases are stored in specially designed facilities, either above or below ground. The half-life of high-level radioactive waste depends on the isotopes present, but several fission products have half-lives of several thousand years.

Spent fuel rods must be handled and stored much more carefully than unused fuel rods because of the form of the ionising radiation that they emit. The fission reactions that occur inside the spent fuel rods initially emit beta radiation, then gamma and neutron radiation. These forms of ionising radiation are more penetrating than the alpha radiation emitted by unused fuel rods.

Risks and benefits

Nuclear power stations generate electricity using fission reactions. No smoke particles or greenhouse gases are released, so generating electricity by nuclear power does not contribute to acid rain or to global warming. By using nuclear power, many countries have reduced the amount of coal and oil burned to generate electricity, which reduces their greenhouse gas emissions.

The death rate in coal mining and in the oil and gas extraction industries is high, partly because the regulation and safety legislation of mining in different countries varies. For example, many thousands of coal miners have died worldwide since 2000. Oil extraction has one of the highest death rates for workers, even with the improved safety measures introduced in recent decades. Hydroelectricity also kills: when the Banqiao hydroelectric dam (China) collapsed in 1975, the accident killed thousands of people directly, and more also died as a result of the famine and epidemics caused by the resulting displacement of people.

The quantity of waste produced during nuclear power generation is small in comparison to the amounts from other methods of generating electricity, because the energy source, uranium, is very concentrated.

Nuclear power is a very reliable way of generating electricity, and the output from many nuclear power stations can be controlled to match changes in demand.

However, there are significant drawbacks to our use of nuclear power. As with any natural resource, there are limited supplies of uranium, although supplies are likely to last for thousands of years, especially if fast breeder reactors are used to change U-238 into Pu-239, another nuclear fuel.

Although the quantities of uranium mined are small compared to the quantities of coal mined, uranium miners are at increased risk of developing lung cancer from their exposure to the radon gas found in the mines. Uranium ore is considered to be only weakly radioactive.

The radioactive waste products need to be stored securely for many decades or centuries, even though the quantities of waste produced are relatively small and some radioactive waste can be recycled. Storage of radioactive waste underground is considered safe if the geological conditions are suitable. Some evidence for the safety came from studies of rocks that contained U-235 isotopes in the Oklo mine in the Gabon, West Africa. Self-sustaining nuclear fission took place in these rock formations for billions of years. The waste products from this natural fission have remained close to their original site, held in place by the rocks surrounding it.

New nuclear power stations are extremely expensive to build as a result of the safety features that need to be included.

Decommissioning nuclear power stations is also expensive, with the safe disposal of intermediate-level waste adding to ongoing costs.

There is a risk of nuclear accidents, and these have occurred at, for example, Fukushima (Japan, 2011) and Chernobyl (Ukraine, 1986). In both cases, significant safety issues were not addressed when building, maintaining or running the plants. Large-scale nuclear accidents cause massive disruption to the local population, and long-term health concerns. It is likely that the Fukushima nuclear accident (see below) will cause about 200 additional cases of cancer. The Chernobyl nuclear accident killed about 40 people from direct radiation exposure, and potentially 4000 from cancers induced by exposure to the fall-out, although, more than 30 years after the accident, these figures still are unclear. Many of the thyroid cancer cases that developed in Russia after the accident could have been prevented by evacuating residents promptly and issuing iodine tablets so that people could not absorb radioactive iodine isotopes released in the fall-out.

Fukushima nuclear accident

The Fukushima nuclear power plant was hit by a massive tsunami in March 2011, and 14 m waves breached the 10 m high protective walls surrounding the plant. Emergency generators were overcome by flooding, and the electrical supply maintaining the cooling systems stopped working. Although the reactors shut down automatically, fission products in the fuel continued to release heat, so the reactor still needed constant cooling. The cooling systems stopped working, so the reactor started to overheat and after a few days there were several explosions caused by chemical reactions (rather than nuclear reactions). Radioactive material was released to the surrounding environment, including the sea, where it dispersed. Sea water was used to cool the reactors after the cooling systems failed. People living nearby were evacuated quickly and it is thought that they were not exposed to significant amounts of radiation. A more significant health risk was due to the damage caused by the tsunami and the upheaval caused by the evacuation. The surrounding countryside is likely to be sealed off for several decades, although this may change as different techniques in cleaning up contamination are developed.

TEST YOURSELF

7 This question is about nuclear power stations.

a) Explain how control rods are used to reduce the power output from a nuclear reactor.

b) Explain how spent nuclear fuel rods are handled.

c) Explain how the energy of fast neutrons is reduced.

8 a) A fission reaction involving uranium releases 3.2×10^{-11} J. If the power output of the reactor is 6 GW, calculate the number of fission reactions occurring in the reactor each day.

b) Explain why the power output of the reactor is not the same as the output of the power station.

9 A reactor has 1700 fuel rods, each of mass 14 kg, and 3% of each fuel rod is U-235. The electrical power output of the station is 840 MW. The power station converts nuclear energy to electrical energy with an efficiency of 35%.

a) Calculate the power output of the reactor core.

b) Calculate the mass of U-235 available to generate power.

c) Assume that a fission reaction releases 215 MeV. Calculate the mass converted to energy per fission reaction.

d) How long does the fuel last?

(Of course, running the station is a little more complicated than this, because as the uranium depletes the fission process will become less efficient.)

Practice questions

1 The graph in Figure 12.11 shows binding energy per nucleon against atomic mass.

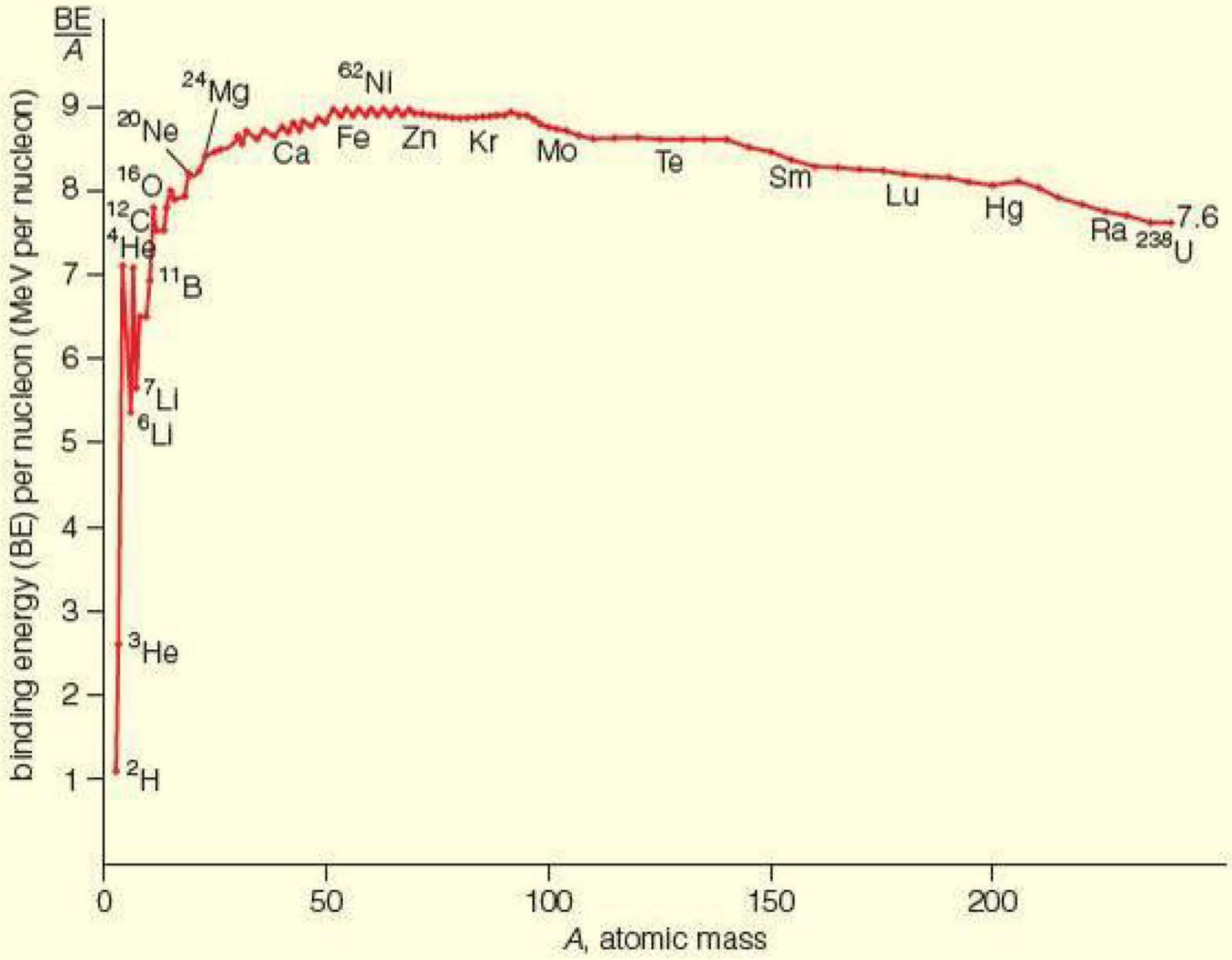


Figure 12.11

Which of the four nuclei shown below is the most stable?

A helium (He) **C** calcium (Ca)

B magnesium (Mg) **D** mercury (Hg)

2 The mass defect for a carbon-12 nucleus is 0.0990 u. The average binding energy per nucleon is

A 8.01 MeV **C** 92.2 MeV

B 1.19 MeV **D** 7.68 MeV

3 What is the energy released in the alpha particle during this reaction?

$$^{226}_{88}\text{Ra} \rightarrow {}^{222}_{86}\text{Rn} + {}^{4}_{2}\alpha$$

The nuclear masses are 225.9771 u for radium-226, 221.9703 u for radon-222 and 4.0015 u for alpha particle.

A 3.23 MeV **C** 4.93 MeV

B 1.23 MeV **D** 4.93 J

4 A deuterium nucleus and a tritium nucleus fuse together, forming helium and releasing a particle, Z, as shown in this equation:

$$^{2}_{1}\text{H} + {}^{3}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + Z$$

What is the particle Z?

A neutron **C** beta particle

B proton **D** alpha particle

5 In a nuclear reactor, fast neutrons are slowed down by

A cooling them using a coolant system

B elastic and inelastic collisions with atoms in the moderator

C lowering control rods

D inelastic collisions only, with atoms in the moderator

6 The purpose of lowering control rods in a nuclear reactor is to

A absorb neutrons and slow down the chain reaction

B slow down fast neutrons, producing thermal neutrons

C produce thermal neutrons and increase the rate of fission

D cool the reactor

7 Heat is generated in a nuclear reactor by

A absorption of neutrons in U-235 atoms

B nuclear fusion processes

C combustion of nuclear fuels such as uranium

D fission of U-235 by neutrons

8 Thermal neutrons travel at a speed of about

A $2.5 \times 10^3 \, \text{m s}^{-1}$

B $2.5 \times 10^7 \, \text{m s}^{-1}$

C $2.5 \, \text{m s}^{-1}$

D $2.5 \times 10^5 \, \text{m s}^{-1}$

9 A suitable material to use as a moderator is

A carbon dioxide

B graphite

C boron steel

D cadmium

10 Critical mass is the

A minimum mass of fissile material used in a reactor

B maximum mass of fissile material that can safely be used in a reactor

C minimum mass of fissile material required for fission to occur

D minimum mass of fissile materials for a chain reaction to occur

11 a) Copy the axes shown in Figure 12.12. On your copy, sketch a graph of binding energy against nucleon number. Add values and a unit to the y-axis. *(3)*

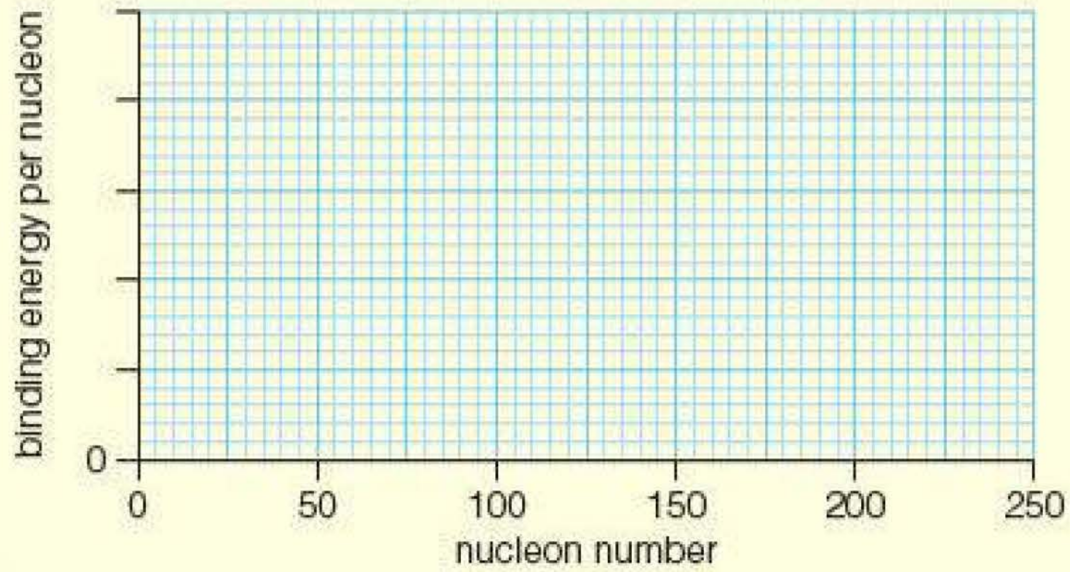


Figure 12.12

b) Use your sketch graph to explain why fission is more likely for heavier nuclei and fusion is more likely for lighter nuclei. *(4)*

12 Each fission reaction inside a thermal nuclear reactor releases two or three neutrons. Explain how a constant rate of fission is maintained in the reactor, describing the nuclear processes that occur. *(6)*

13 The atomic mass of iron, $^{56}_{28}Fe$, is 55.93493. The mass of a neutron is 1.00867 u, the mass of a proton is 1.00728 u and the mass of an electron is 0.000549 u.

a) State what is meant by 'mass defect' for iron. *(1)*

b) Calculate the binding energy per nucleon for iron in MeV. *(4)*

c) The most commonly found isotope of iron is Fe-56. Explain whether the binding energy per nucleon for other isotopes of iron is larger or smaller than that for Fe-56. *(2)*

14 a) Figure 12.13 shows a simplified sketch of a graph of binding energy per nucleon against atomic mass number. Copy the graph. On your copy, add labels stating where fusion is more likely to occur and where fission is more likely. *(2)*

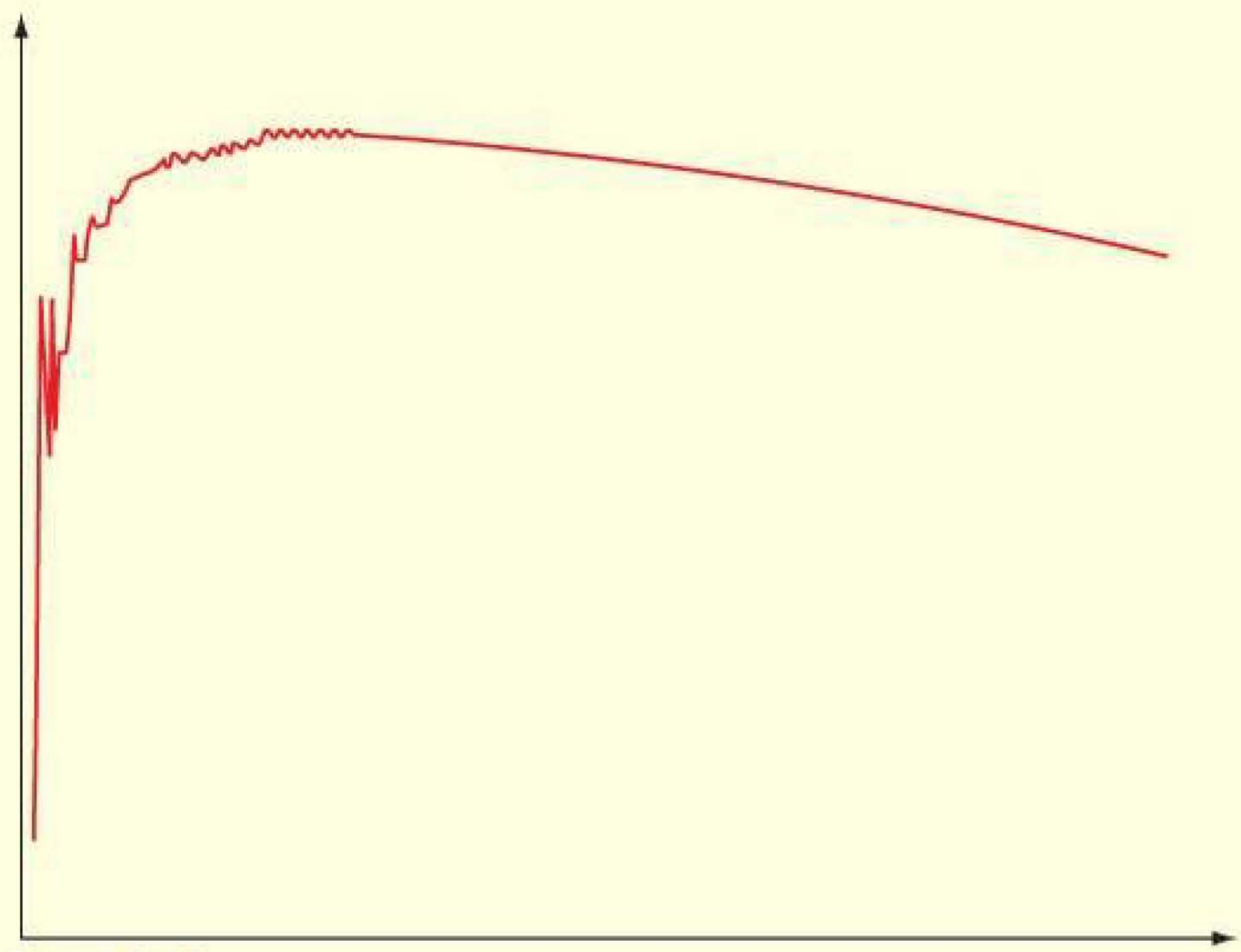


Figure 12.13

b) Explain the conditions required for nuclear fusion to take place. *(4)*

c) Explain why the heaviest element produced during fusion reactions in stars is iron. *(2)*

Stretch and challenge

The question that follows is a British Physics Olympiad question.

15 A uranium atom undergoes fission as shown in this equation:

$${}^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{141}_{56}\text{Ba} + {}^{92}_{36}\text{Kr} + 3\,{}^{1}_{0}\text{n}$$

Use the data in the table below to calculate

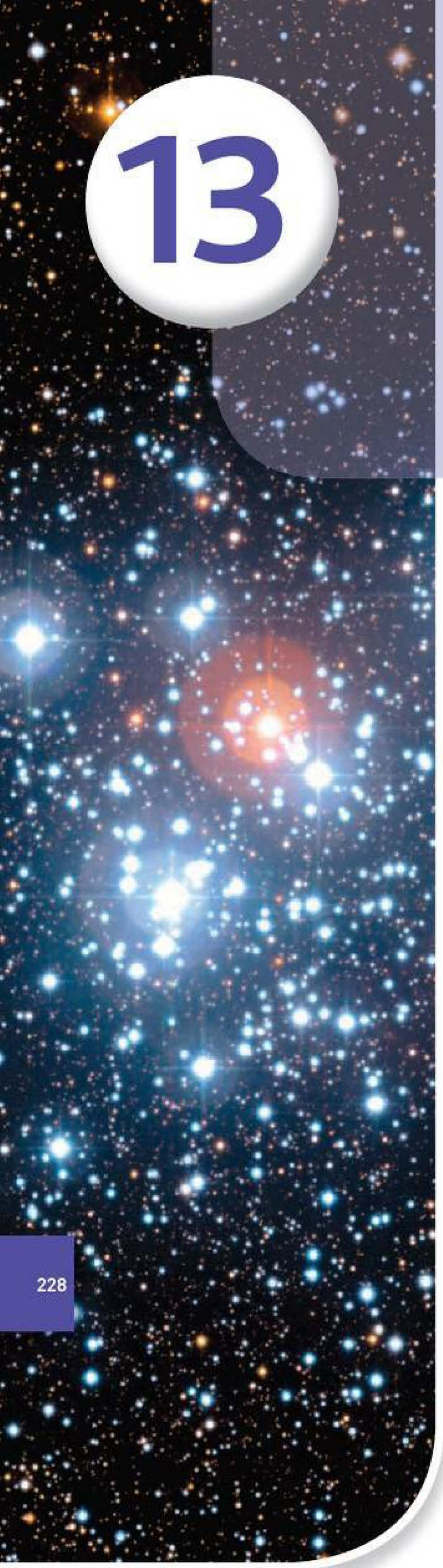
a) the mass difference in u

b) the energy released per fission reaction in MeV

c) the energy released if 10 kg of uranium undergoes this fission reaction in MeV.

Nucleus	Mass/u
U-235	235.04
Ba-141	140.91
Kr-92	91.91
n	1.01

(BPhO R1-2006 Q1)



13 Optional topic: Astrophysics

PRIOR KNOWLEDGE

Before you start, make sure that you are confident in your knowledge and understanding of the following points:

- Light is an electromagnetic wave, which travels at a speed of $3.0 \times 10^8\,m\,s^{-1}$ in a vacuum.
- Light is a wave, which shows the wave properties of reflection, refraction, diffraction and interference.
- A lens can be used to refract light.
- Lenses are used to focus light and to produce images of various objects.
- The Universe is made up of billions of stars and galaxies.
- The distance between galaxies is measured in millions of light years.
- The Universe is about 13.8 billion years old.

TEST YOURSELF ON PRIOR KNOWLEDGE

1 A ray of light is incident on one face of a parallel-sided block of glass, at an angle of 30° to the normal. Draw a sketch to show the path of the ray as it passes into and then out of the block of glass.

2 Describe how you would use a laser and an adjustable small slit to demonstrate the diffraction of light in a laboratory.

3 a) A light year is the distance that light travels in one year. Calculate this distance in metres.

b) A distant galaxy is 2 billion light years from Earth. Calculate this distance in metres.

4 a) Astronomers estimate that our Galaxy, the Milky Way, contains about 300 billion stars. They also estimate that the Universe contains approximately 200 billion galaxies. Calculate the number of stars in the Universe, stating any assumptions you make.

b) Our Sun has a mass of $2 \times 10^{30}\,kg$ and its mass by composition is 75% hydrogen and 25% helium. Make an estimate of the number of hydrogen atoms (or nuclei) in the Universe, assuming that nearly all the Universe's hydrogen is in stars. State any other assumptions you make. The mass of a hydrogen atom is $1.67 \times 10^{-27}\,kg$.

TIP

If you have studied lenses in your GCSE course, you might be able to move on to the section on telescopes. This section is provided as background for those who are unfamiliar with lenses.

The Milky Way is the name we give our Galaxy. Our Sun is one of about 300 billion stars in the Galaxy. On a dark night the Milky Way is an awe-inspiring sight, which has caused people to wonder with amazement at our world. Babylonian astronomers developed geometry and trigonometry, some four thousand years ago, so that they could measure and plot the positions of the stars that they observed. It is an interesting thought that if we lived on a planet that was covered in dense clouds, and where clear skies and stars were never seen, we might not have trigonometry on the school curriculum and we would have little idea about the origin of our Universe.

Lenses

A convex or **converging lens** is designed so that it can focus light rays to a point. For example, you may have used a converging lens to focus the Sun's rays on to a piece of paper, so that it starts to burn. The principle behind a converging lens is illustrated in Figure 13.1. A ray of light is incident on the lens at an angle i to the normal, with an angle of refraction r. As the ray leaves the lens, it bends away from the normal, as shown.

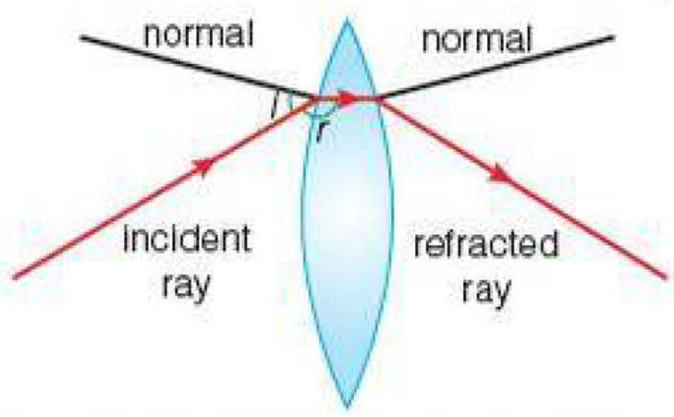


Figure 13.1 The principle behind a converging lens.

Converging lens A converging lens refracts rays of light to a point.

Figure 13.2 shows more about the nature of converging lenses. A lens is constructed to that it is symmetrical about its **principal axis**. A ray that passes along the principal axis passes through the lens undeviated, because it is parallel to the normals on both faces. Rays that are parallel to the principal axis come to a focus at the lens's **focal point**. There are two focal points, one on either side of the lens. The **focal length** of a converging lens is the distance between the centre of the lens and the focal point.

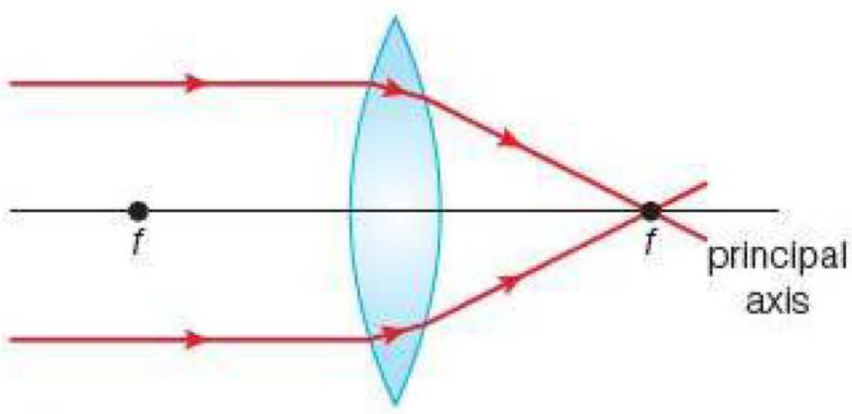


Figure 13.2 Rays parallel to the principal axis meet at the focal point.

The lens in Figure 13.2 has a short focal length because its surfaces have small radii of curvature, and the light is refracted through relatively large angles. The lens in Figure 13.3 is thinner than the lens in Figure 13.2. It is less curved and its focal length is longer.

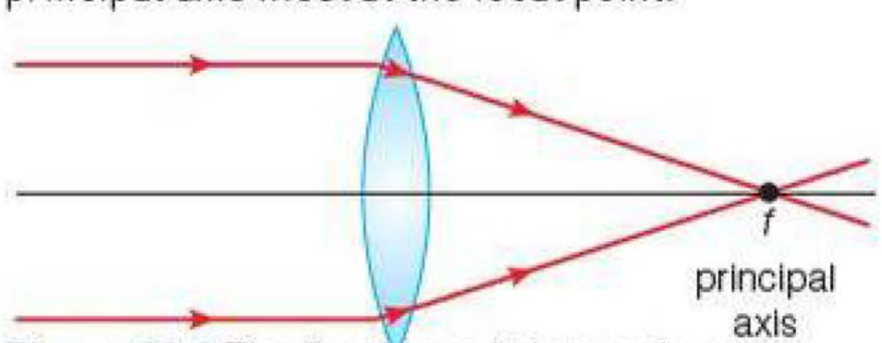


Figure 13.3 The focal lens is longer in a lens that is thinner and less curved.

Construction of ray diagrams

There are three classes of light ray that are used to predict the position of an image formed by a converging lens. These are illustrated in Figure 13.4. (Note that, when we draw a ray diagram for a lens, we simplify the process of refraction by assuming that it happens in just one part of the lens. So the lens is drawn as a thin vertical line. The arrows pointing out from the centre of the lens, at the top and bottom, indicate that this lens is a converging lens. (If the arrows point the other way, it is a diverging lens.)

1 A ray parallel to the principal axis (on the left side of the lens) is refracted so that it passes through the focal point on the right side of the lens.

2 A ray that passes through the optical centre of the lens is undeviated.

3 A ray that passes through the focal point on the left side of the lens is refracted so that it travels on a line parallel to the principal axis on the right side of the lens.

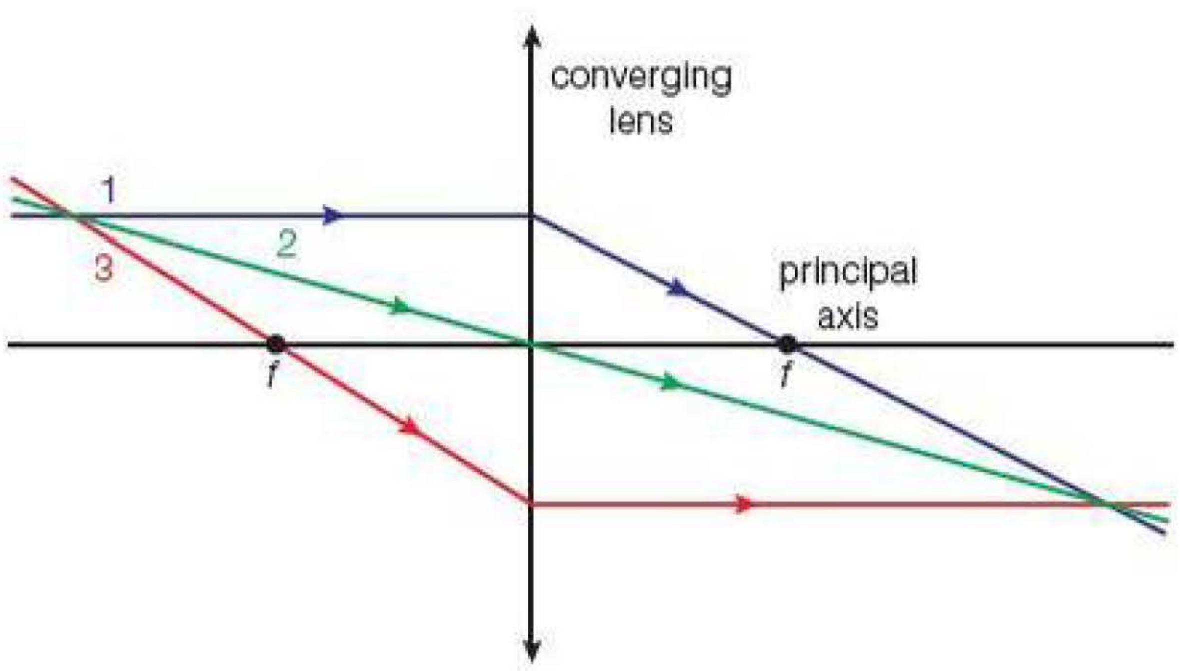
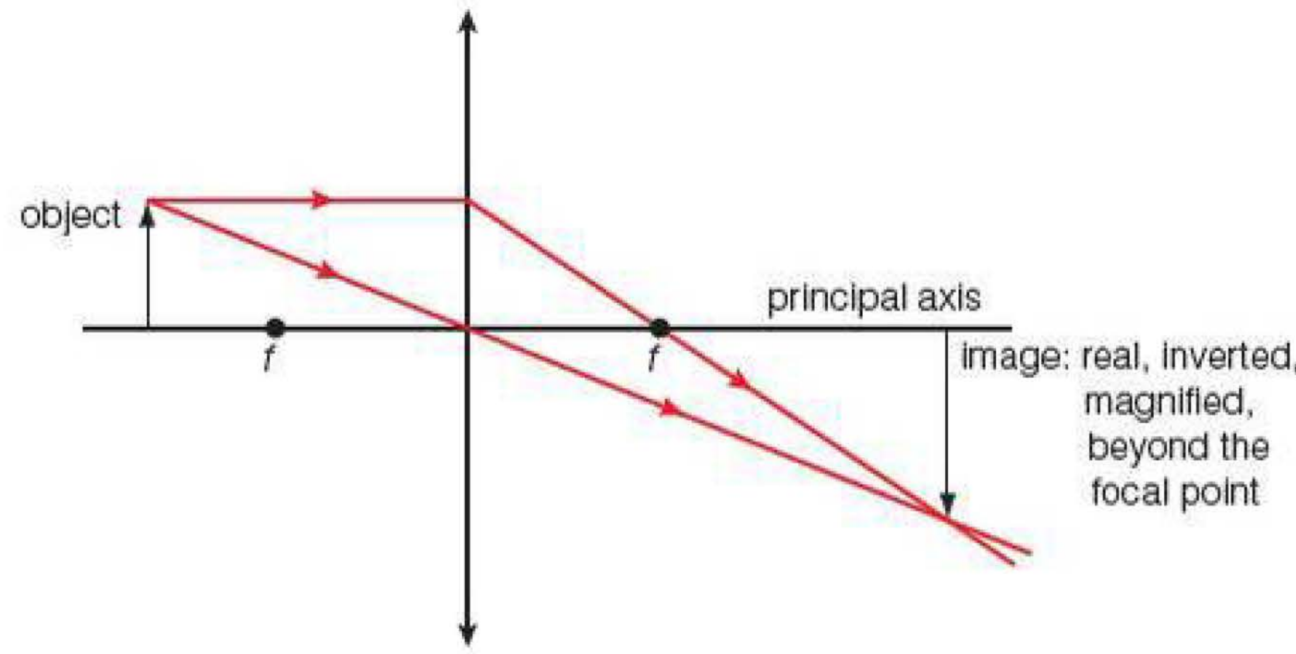


Figure 13.4

Figure 13.5



Principal axis The principal axis of a lens is an imaginary line that passes through the centre of a lens and through the centres of curvature of the faces of the lens.

Focal point The focal point of a lens is the point at which rays parallel to the principal axis of the lens are brought to a focus.

Focal length The focal length of a lens is the distance between the centre of the lens and the point at which rays parallel to the principle axis are brought to a focus.

Projecting an image

Figure 13.5 shows how you can use two of the construction rays to predict where an image will be formed by a converging lens. Provided the object lies outside the focal length of the lens, a real image will be formed. The image is real when the rays converge at a point. This image can be focused on to a screen.

Figures 13.6(a) and (b) show how two different converging lenses can be used to project an image of a distant object. Light rays from the same point on a distant object arrive at the lens very nearly parallel to each other. So, for example, rays from the top of a distant object arrive at the lens parallel to each other and rays from the bottom of the same object also arrive parallel to each other. Lens B produces a larger image than lens A, because it has a longer focal length. This idea will be used later when we consider the design of an astronomical telescope.

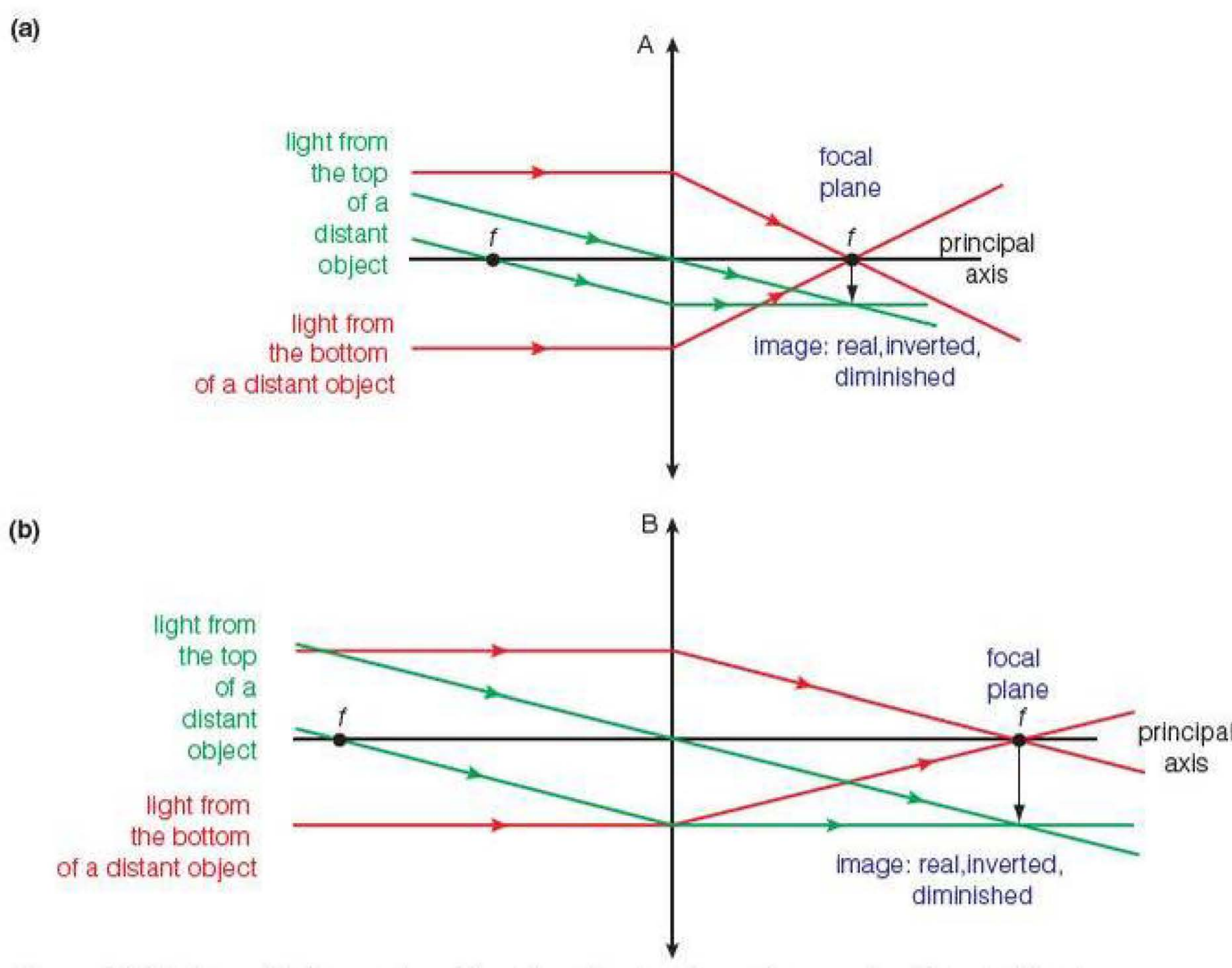


Figure 13.6 A lens with longer focal length projects a larger image of a distant object; the image projected by lens B is larger than the image projected by lens A.

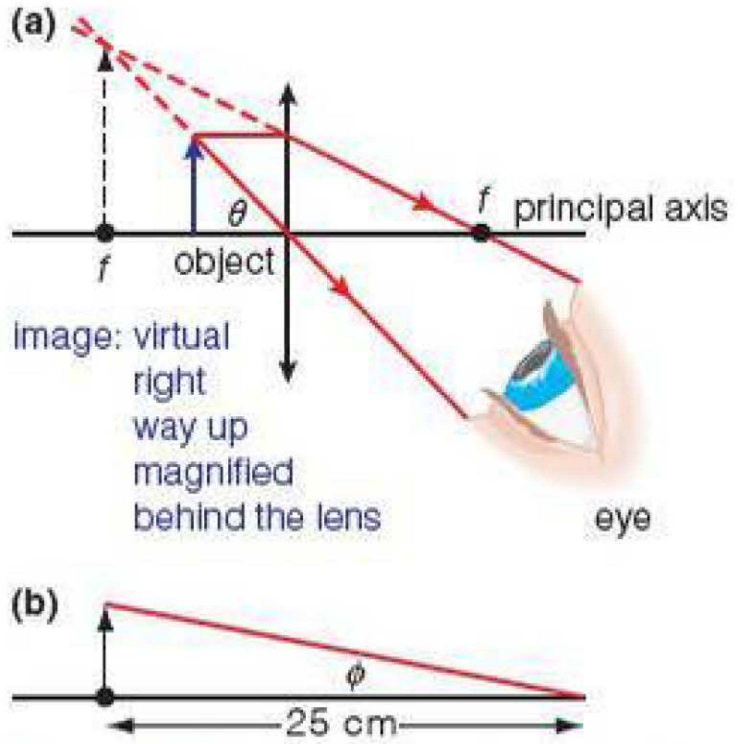


Figure 13.7 (a) An object viewed inside the focal length of a lens produces a virtual magnified image. (b) Without a lens, you can only focus on an object at your near point of vision.

The magnifying glass

Figure 13.7(a) shows what happens when an object is placed inside the focal length of a converging lens. Rays from the top of the object now diverge, and do not come to a focus. If your eye is placed behind the lens, the object appears to be bigger and further behind the lens. This is a virtual image. It cannot be projected on to a screen and it appears only to the eye on the other side of the lens. When the lens is used like this, it is called a magnifying glass. The object appears bigger because the lens produces a magnified image at your near point. Without the lens, you can only focus on the object at your near point of vision – perhaps 25 cm away, as shown in Figure 13.7(b). The lens causes magnification because the angle θ in Figure 13.7(a) is bigger than the angle ϕ in Figure 13.7(b).

Figures 13.8(a) and (b) show how two lenses can be used to view an object situated at the focal point of a lens. In both cases, a virtual image is seen at infinity, behind the lens. However, the magnification of lens D is larger than the magnification of lens C, because angle β is larger than angle α. So a converging lens with a short focal length is a more powerful magnifying glass than a converging lens with a longer focal length. This idea is also important when designing an astronomical telescope.

The astronomical telescope

Figure 13.9 shows the principle behind the astronomical refracting telescope. The objective lens (the lens pointing towards the distant object) projects a real image of a distant object such as the Moon. This image is larger for a longer focal length of the objective lens, f_o. The eyepiece is now used to magnify this image. A short focal length eyepiece produces a larger magnification of the telescope.

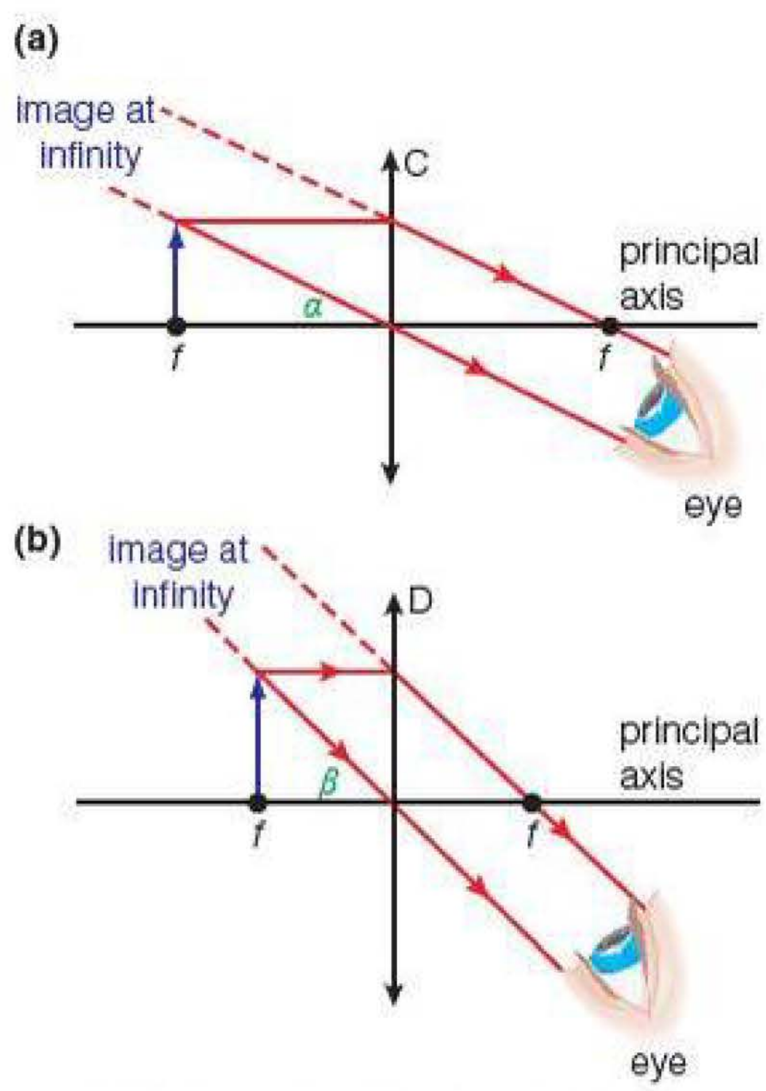


Figure 13.8 A shorter focal length converging lens is a more powerful magnifying glass.

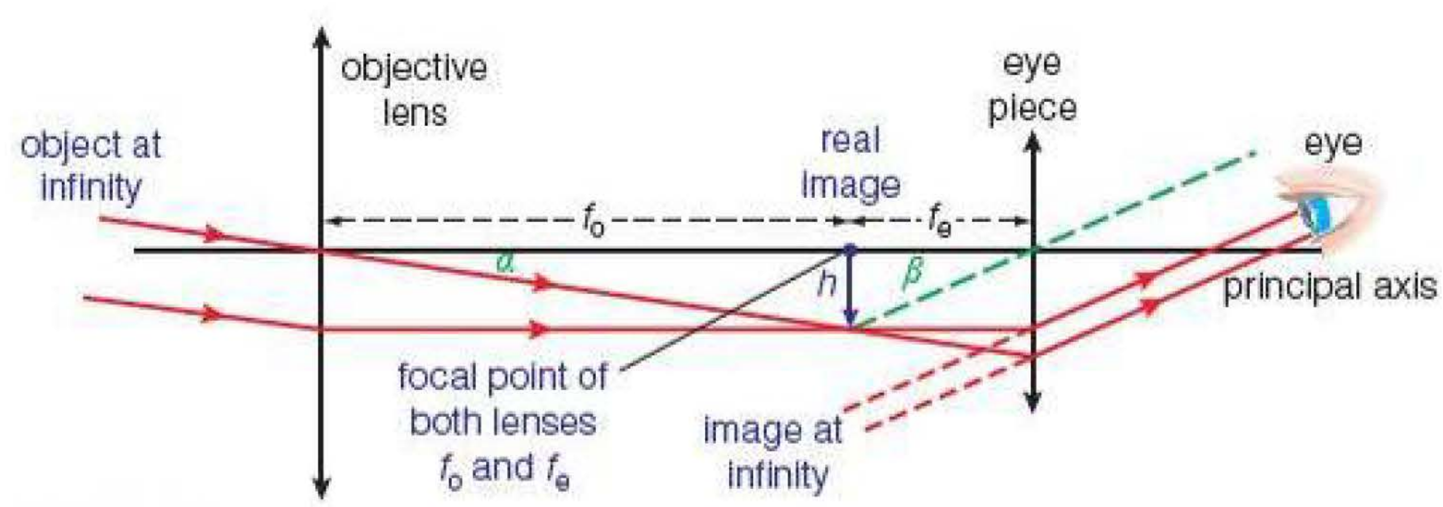


Figure 13.9

Using trigonometry, we can write

$$\tan\alpha = \frac{h}{f_e}$$

$$\tan\beta = \frac{h}{f_e}$$

where h is the height of the real image, f_o is the focal length of the objective lens and f_e is the focal length of the eyepiece lens. But for small angles (expressed in radians)

> **TIP**
> The angle subtended by an object is the angle between the rays coming from the extremities of the object to the eyes or telescope lens.

$$\tan\alpha \approx \alpha \quad \text{and} \quad \tan\beta \approx \beta$$

so

$$\alpha = \frac{h}{f_o} \quad \text{and} \quad \beta = \frac{h}{f_e}$$

The angular magnification, M, of the telescope is defined as:

$$M = \frac{\text{angle subtended by image at eye}}{\text{angle subtended by object at unaided eye}}$$

$$= \frac{\beta}{\alpha} = \frac{h}{f_e} \times \frac{f_o}{h} = \frac{f_o}{f_e}$$

> **TIP**
> The magnification of an astronomical telescope in normal adjustment is $M = \frac{f_o}{f_e}$

A telescope is described as being in normal adjustment when the real image, produced by the objective lens, is viewed at the focal point of the eyepiece. Under these circumstances, a magnified virtual image is viewed at infinity.

Safety: NEVER look directly at the Sun through a telescope – you will burn your eye.

ACTIVITY

A simple model telescope

Select two converging lenses with different focal lengths – for example, 50 cm and 10 cm. Use modelling putty to stick them on to a metre rule 60 cm apart. You have just made a simple model telescope.

1. Look through the 10 cm lens towards the 50 cm lens and describe what you see.
2. Look at a brick wall through your telescope with one eye, and use the other eye to look directly at the wall. Calculate the telescope's angular magnification.
3. Draw a ray diagram to show the passage of light through your telescope.

TEST YOURSELF

1. a) A man of height 1.7 m stands a distance of 10 m away from you. Calculate the angle he subtends at your eye. Give your answer in radians.
   b) The man now moves to a distance of 120 m away. Calculate, in radians, the angle he now subtends at your eye.
   c) Is the small-angle approximation, $\tan\alpha = \alpha$, valid in case (a) or case (b) or both cases?
2. Explain why an astronomical telescope should have
   a) an objective lens of long focal length
   b) an eyepiece with a short focal length.
3. What is the length of an astronomical telescope in normal adjustment, when it has an objective lens of focal length 2.50 m and an eyepiece of focal length 40 mm.
4. The great refractor in the Vienna Observatory has an objective lens with a focal length 10.5 m.
   a) Explain why this telescope has an objective lens with this large focal length.
   b) The telescope is used with an eyepiece of focal length 50 mm. Calculate the angular magnification of the telescope.
5. Two stars are separated by an angle of 0.05° when viewed directly by eye. What angle do the images of the stars subtend when viewed through an astronomical telescope with an objective lens of focal length 2.4 m and an eyepiece of focal length 40 mm?

Lens aberrations

Although refracting astronomical telescopes are very useful instruments, their effectiveness is reduced to some extent by the limitations of their lenses. Glass lenses have two main types of *aberration*, which limit the sharpness of the image that we see.

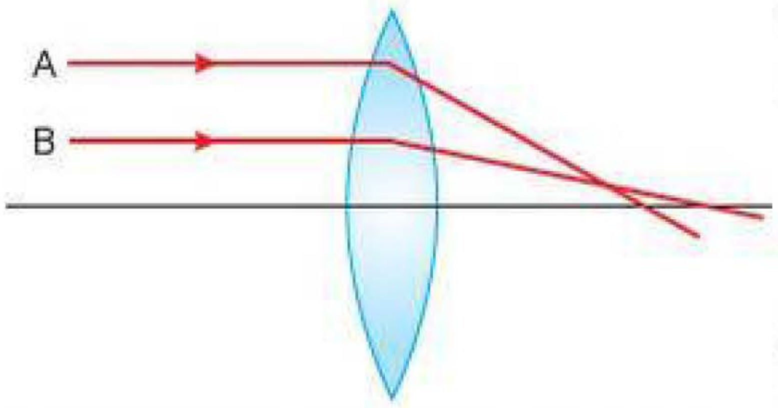


Figure 13.10 Spherical aberration: rays from a distant object are not brought to a focus at a single point.

Spherical aberration

Most lenses are ground into a spherical shape, but this is not quite the ideal shape for a lens. Figure 13.10 shows two rays, parallel to the principal axis of a lens, which come from the same distant object. The two rays refract at different angles, but they do not pass through the same focal point – the ray at the top of the lens, A, comes to a focal point closer to the lens than the lower ray, B. As a result of this there is a slight blurring of the image that we see.

Spherical aberration can be demonstrated easily in the laboratory. A lens is used to project an image of a lamp filament on to a screen. If a card with a small hole is placed in front of the lens, you will see that the image becomes sharper. This is because rays pass through only a small part of the lens.

It is possible to reduce spherical aberration by using a lens with a parabolic shape. However, such lenses are very expensive, and they produce some distortion of the image, except for light exactly parallel to the principal axis.

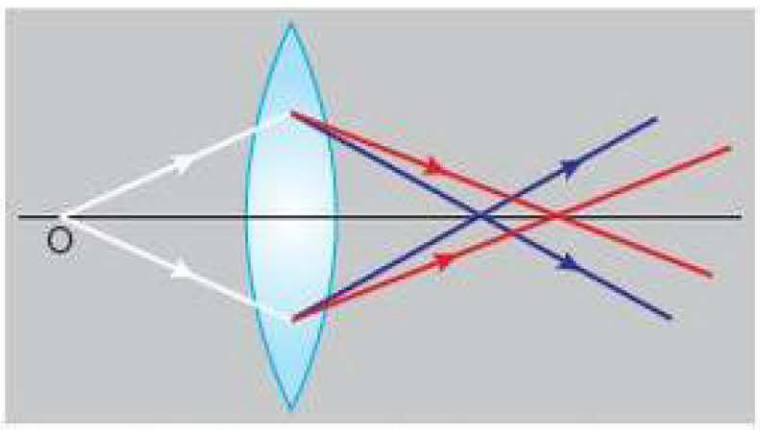


Figure 13.11 Chromatic aberration: different colours of light are refracted by different amounts.

Chromatic aberration

Figure 13.11 shows two rays of white light being refracted by a lens. The speed of light through glass depends on its wavelength. Blue light has a shorter wavelength than red light, and it travels more slowly than red light through glass. Consequently, blue light is refracted more than red light, and there are different points of focus for the two colours. This is called *chromatic aberration*. It is possible to reduce the effects of chromatic aberration, but not to remove it entirely, by constructing a lens using two different types of glass.

Reflecting telescope

Figure 13.12 shows the principle behind the Cassegrain reflecting telescope. Light from a distant object strikes the primary concave mirror, where the light is reflected towards the focal point at F. However, a secondary convex mirror reflects the light again, so that it is focused at F′, where a real image is formed. The observer can then see a magnified image through the eyepiece, which is placed behind a hole in the primary mirror.

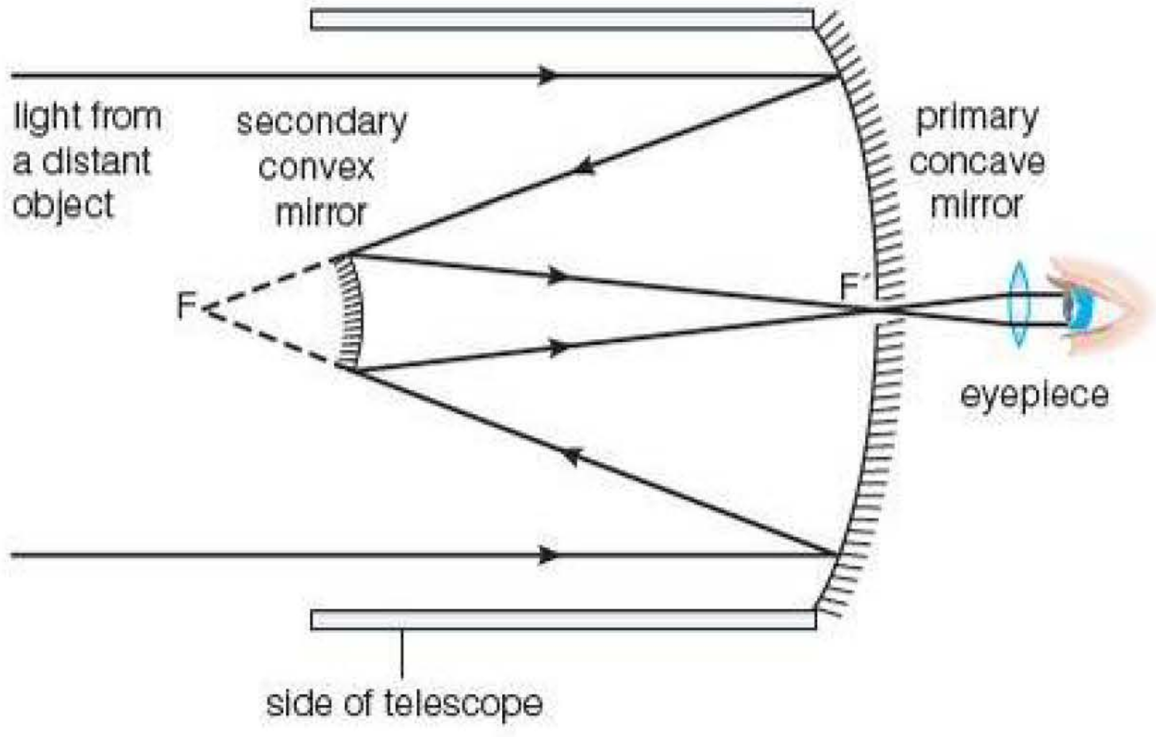


Figure 13.12 Principle of the Cassegrain reflecting telescope.

A reflecting telescope has several advantages over a refracting telescope.

- A good astronomical telescope requires a diameter of about 15 cm or more, so that sufficient light is gathered. It is very difficult to make a high-quality lens of diameter 15 cm, but much easier to make a concave mirror of that size.
- A reflecting mirror has no chromatic aberration, because light is reflected over a metal surface without passing through glass.
- Spherical aberration can be reduced more easily in a reflecting telescope by making the concave mirror parabolic in shape. A parabolic mirror focuses light that is parallel to the principal axis accurately at the focal point.
- It is possible to make reflecting telescopes with larger diameters than refracting telescopes. The world's largest refracting telescope, at the Yerkes Observatory, has a diameter of 1.0 m. There are several reflecting telescopes that have diameters over 8 m – for example, the Subaru Telescope in Hawaii has a mirror of diameter 8.2 m. A glass lens with a diameter of over 1 m begins to sag under its own weight, whereas a mirror can be supported by a strong structure behind it.

The **collecting power** of a telescope is proportional to its area. Since the area of the telescope mirror is $\frac{\pi d^2}{4}$, where d is its diameter, the collecting power is proportional to the diameter squared, d^2. Larger telescopes are able to show fainter objects, because more light is collected. Images in large telescopes are also less affected by diffraction – this is dealt with in detail in the next section.

Collecting power is a measure of the light intensity gathered by a telescope. This is proportional to the square of the telescope's diameter.

EXAMPLE

Comparison of collecting powers

Compare the light gathered by two telescopes – a reflecting telescope that has a mirror with a diameter of 36 cm, and a refracting telescope that has an objective lens with a diameter of 10 cm.

Light gathered by a telescope is measured by the collecting power, which is proportional to the telescope's diameter squared. So:

$$\frac{\text{collecting power of the reflector}}{\text{collecting power of the refractor}} = \frac{(36)^2}{(10)^2}$$

$$= 13 \text{ (2 s.f.)}$$

A refracting telescope does have some advantages over a reflecting telescope.

- The lenses in a refractor are held in place by a metal tube. So little maintenance is required. The mirror in a reflecting telescope is exposed to the air, and might need recoating.
- The mirrors in a small reflector can get out of alignment if the telescope gets knocked. So sometimes the mirrors need adjustment. The strong construction of the refracting telescope makes such misalignment less likely.
- The secondary mirror in a reflecting telescope has the disadvantage of blocking some light from entering the primary mirror.
- The secondary mirror and its supports will cause some diffraction which will degrade the image.

TEST YOURSELF

6 Explain the meaning of the terms:
   a) chromatic aberration
   b) spherical aberration.
7 Explain four advantages that reflecting telescopes have over refracting telescopes.
8 An amateur astronomer uses his 12 cm diameter reflector to take a photograph of Jupiter and its moons. He finds that he needs to expose his photograph for 16 s to get a clear photograph. He visits a friend to take a photograph using her reflecting telescope, which has a diameter of 28 cm.
   What exposure time would you advise for the photograph using the 28 cm reflector? They use the same photographic equipment.

Angular resolution of telescope

You met the idea of the diffraction of light in Chapter 6 of Book 1. To demonstrate the diffraction of light in a laboratory, it is necessary to direct a beam of light through a very narrow slit – then we can see the light spread out. However, the effects of diffraction are apparent when light enters a telescope aperture, even though the telescope has a diameter of many centimetres or even metres.

MATHS BOX

When light passes through a circular aperture of diameter D, the first minimum occurs at angle θ given by

$$\sin\theta = \frac{1.22\lambda}{D}$$

However, we shall work with the approximation that the minimum occurs for small angles at

$$\theta = \frac{\lambda}{D}$$

Figure 13.13 shows how the intensity of light, with wavelength λ, varies after it has been diffracted through a slit of width D. There is an area of high intensity – the central maximum – and the light intensity falls to zero when:

$$\sin\theta = \frac{\lambda}{D}$$

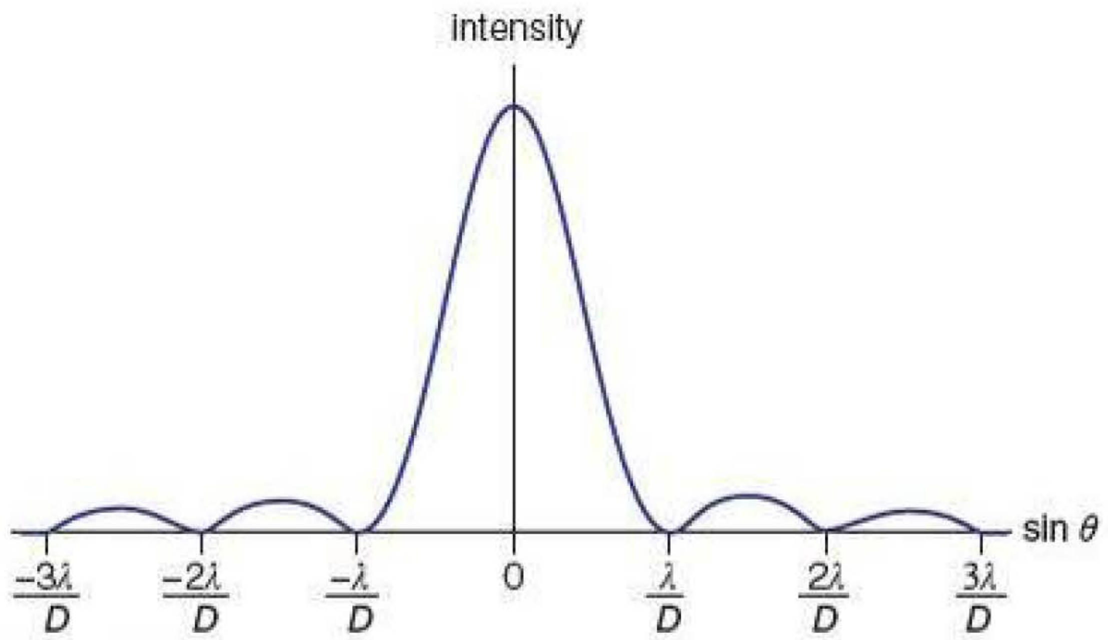


Figure 13.13

Because the angles of diffraction that we shall be dealing with are very small, we can work in the small-angle approximation and say that the first diffraction minimum occurs at an angle of:

$$\theta = \frac{\lambda}{D}$$

So when light from a star passes through a telescope, the image of the star has a measurable width due to diffraction as the light passes through the lens or mirror aperture.

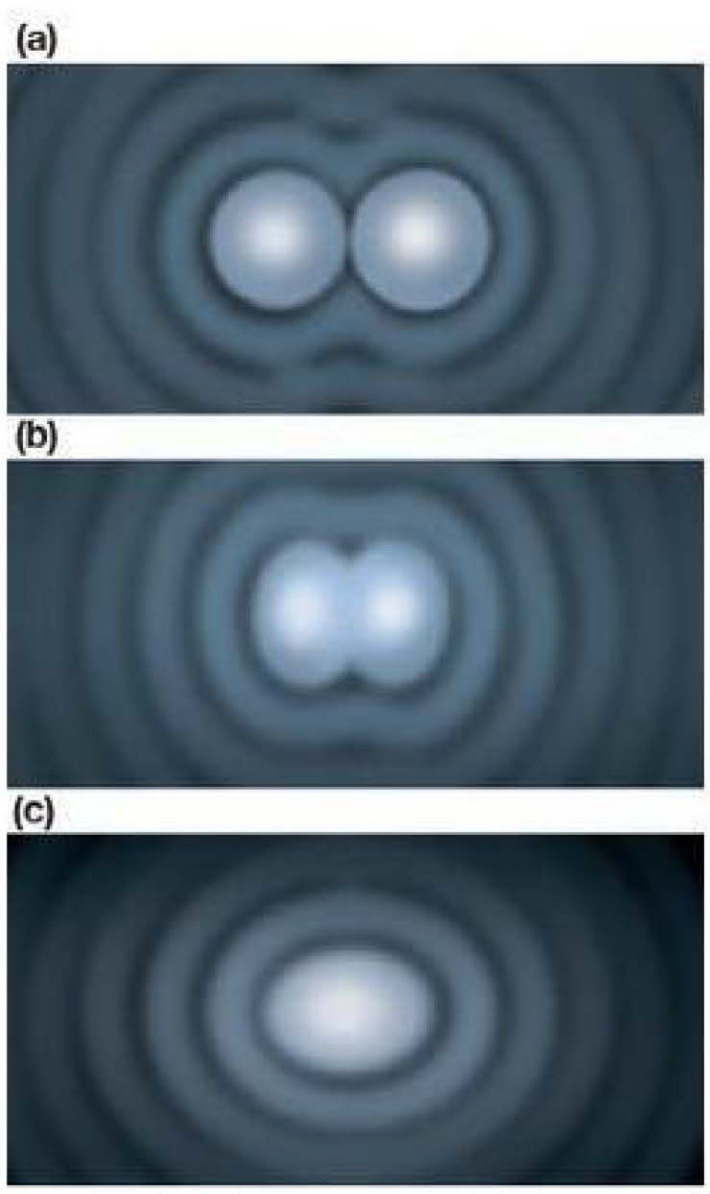


Figure 13.14

Diffraction affects how well a telescope can resolve fine detail. Figure 13.14 shows the idea. Figure 13.14(a) shows the diffraction pattern due to two small sources of light, after passing through a narrow aperture. The patterns

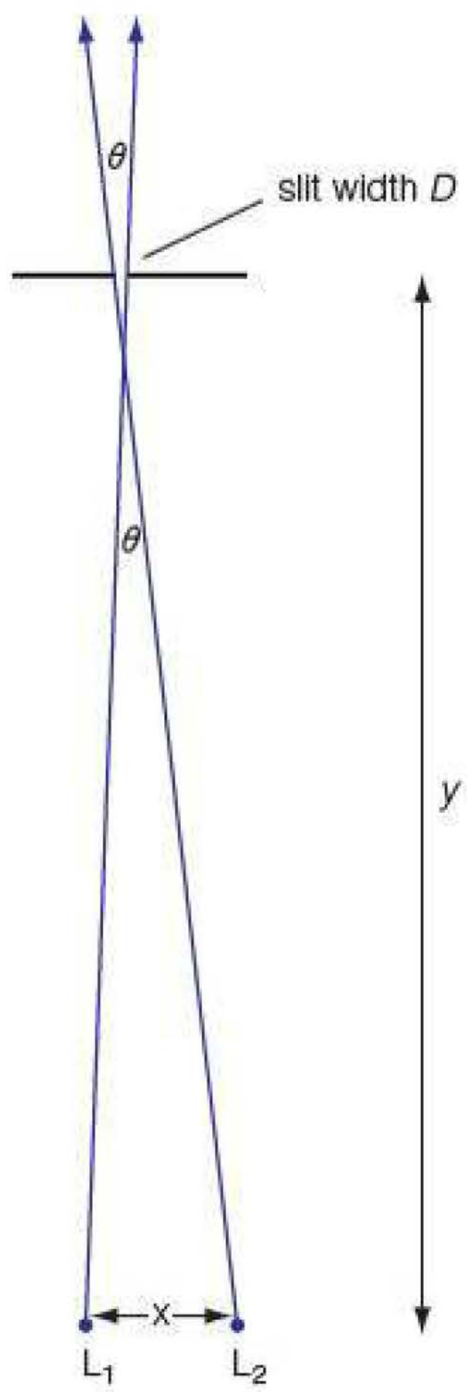


Figure 13.15

overlap, but we can see two separate, distinct patterns. In Figure 13.14(b) the sources have been moved closer together. Now the patterns merge into each other, but we can still see that there are two sources. We say we can just *resolve* the two sources. In Figure 13.14(c) the sources are so close together that we cannot distinguish between them – the sources are not resolved.

Rayleigh's criterion

Figure 13.15 shows an arrangement you can use in the laboratory to investigate the resolution of two small filament lamps. The two filaments are arranged so that they are about 1 cm apart (the distance x in the diagram). They are then viewed through a narrow slit, which can be adjusted to be about 0.2 mm (2×10^{-4} m) wide. What do we see when we look at the lamps as we vary their distance, y, from the slit?

Figure 13.16 shows how the intensity will appear for different values of y. In Figure 13.16(a) the lamps are close to the slit, so their angular separation is relatively large and we see two separate patterns of intensity (this is similar to the photographs in Figure 13.14). In Figure 13.16(b) the lamps are further away, so that they are just resolved, and in Figure 13.16(c) the lamps are so far away that the eye cannot see any small dip in intensity between the lamps – so they cannot be resolved.

Figure 13.16(b) shows the Rayleigh criterion for resolution. When the first minimum of one of the sources coincides with the maximum of the second source, we can just see (resolve) the two separate sources. This rule is only a guide because some people's eyes are better than others.

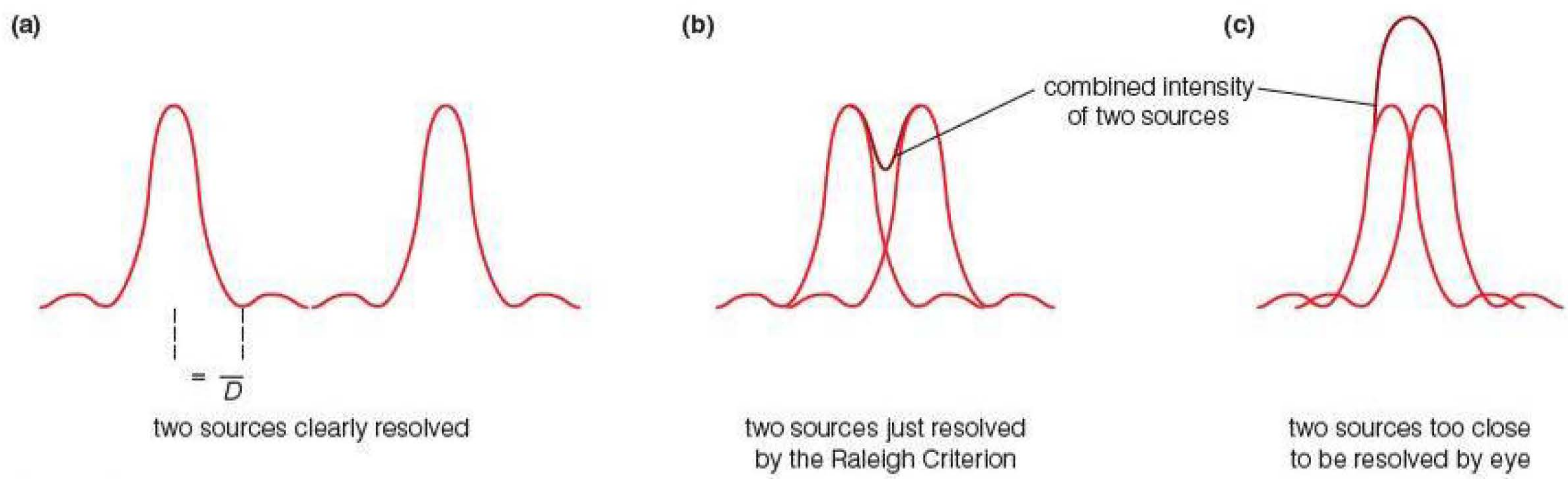


Figure 13.16

Rayleigh's criterion for resolution can be written as follows – when two sources, emitting light of wavelength λ, have an angular separation θ and are viewed through an aperture of diameter D:

- If $\theta > \dfrac{\lambda}{D}$ the sources can be resolved.
- If $\theta \approx \dfrac{\lambda}{D}$ the sources can just be resolved.
- If $\theta < \dfrac{\lambda}{D}$ the sources cannot be resolved.

EXAMPLE

Angular separation of two lamps

Two lamps are separated by a distance of 1.2 cm, and they are placed 4.0 m away from a narrow slit of width 2×10^{-4} m. They are viewed through a blue filter, which allows light of wavelength 4.8×10^{-7} m to pass. Will an observer be able to resolve the two lamps?

Answer

We use the small-angle approximation to calculate the angle between the lamps:

$$\tan\theta \approx \sin\theta \approx \theta = \frac{x}{y}$$

where x is the separation of the lamps, and y is their distance from the slit So the angular separation of the lamps is

$$\theta = \frac{x}{y} = \frac{1.2\text{cm}}{400\text{cm}} = 3 \times 10^{-3} \text{ rad}$$

The smallest angle that the observer will be able to resolve is

$$\frac{\lambda}{D} = \frac{4.8 \times 10^{-7}}{2 \times 10^{-4}} = 2.4 \times 10^{-3} \text{ rad}$$

Because $\theta > \frac{\lambda}{D}$ the lamps may be resolved.

TEST YOURSELF

9 Two small lamps, each with a thin wire filament, are set up with the filaments 1.5 cm apart. They are placed 6.0 m away from a slit of width 0.22 mm. Explain what a student sees when she views the lamps through the slit when the following filters are placed in front of the lamps:

a) a red filter passing light of wavelength 6.5×10^{-7} m

b) a green filter passing light of wavelength 5.4×10^{-7} m

c) a blue filter passing light of wavelength 4.7×10^{-7} m.

10 The presence of turbulence in the atmosphere reduces the resolving power of any telescope by about a factor of 10. What this means is that a large reflecting telescope such as the Subaru Telescope, with a diameter of 8.2 m, is only as effective as a telescope with a diameter of 0.82 m in perfect conditions (in space, for example).

a) The Andromeda galaxy is a distance of 2.2 million light years away from Earth. It is possible to see blue giant stars at this distance, which emit light of wavelength around 4.0×10^{-7} m. What is the minimum separation of two blue giants for the Subaru Telescope to be able to resolve them?

b) The Hubble Space Telescope has the advantage of being above the Earth's atmosphere. It has a mirror diameter of 2.4 m. Repeat the calculation in part (a) for the Hubble Space Telescope.

11 A student draws two black lines 1 mm apart on a piece of paper. She walks away from them until, at a distance of 5 m, she can no longer see them as two separate lines. Another student measures the diameter of the pupil of the eye of the first student, and finds it to be about 3 mm. Make an estimate of the wavelength of light.

Seeing stars

In the days of modern technology, it is easy to think of microscopes, telescopes and cameras, all as excellent optical instruments. However, we must never underestimate the brilliance of our own two eyes. Our eyes and brain process vast amounts of information every second. We can judge depth with binocular vision, and by rapidly looking around we build up an understanding of our surroundings that even the best cameras cannot match. However, optical instruments give us fresh insight into our surroundings, and this is particularly so in the field of astronomy.

The first way we look at stars is to use our eyes, but we see more when we use binoculars or a small telescope. A telescope gathers more light than our eyes, so we see fainter objects, and the larger aperture of the telescope allows us to resolve more detail. However, astronomers realised, around the start of the twentieth century, that even more information could be gathered by using a camera together with a telescope.

By 'driving' a telescope so that it rotates at the same rate as the Earth, it is possible to track stars exactly over a long period of time. Then a very long-exposure photograph can be taken, and the film developed later.

Now, all telescopes used by professional astronomers use cameras with charge-coupled devices (CCDs) to detect the light from stars and galaxies. A CCD is a slice of silicon that stores electrons freed by the energy of incoming photons. The charge on the electrons builds up an image as a pattern of pixels. CCDs are much more sensitive to light than photographic film, and they have the advantage that information can be stored in digital form and processed by computers. Now cameras using CCDs are readily available to us all, and astronomers use high-quality CCDs with hundreds of megapixels to take long-exposure photographs of deep space.

A CCD has a very high quantum efficiency. What this means is that a very high percentage of photons that strike the CCD produce charge carriers, which are then detected. Quantum efficiency is defined:

$$\text{quantum efficiency (QE)} = \frac{\text{number of electrons produced per second}}{\text{number of photons absorbed per second}}$$

The quantum efficiency depends on the frequency of the light incident on the CCD. In Table 13.1 we compare the QEs of our eye, some film and a CCD.

Table 13.1

Device	Quantum efficiency/%
Eye	1–4
Film	4–10
CCD	70–90

As Table 13.1 shows, a CCD has a very high quantum efficiency, so a large telescope equipped with millions of pixels easily outperforms the eye. CCDs can also be designed to be sensitive to other types of electromagnetic radiation, including infrared, ultraviolet and X-rays. So telescopes can be used to investigate waves emitted from stars that lie outside visible wavelengths.

Telescopes beyond the visible range

When astronomers observe the sky, they are not just interested in visible light, because stars and galaxies emit the whole range of electromagnetic radiation from radio waves to X-rays and gamma rays. For example, hot stars emit radiation well into the ultraviolet range, matter close to black holes emits X-rays and colder objects emit infrared radiation and radio waves.

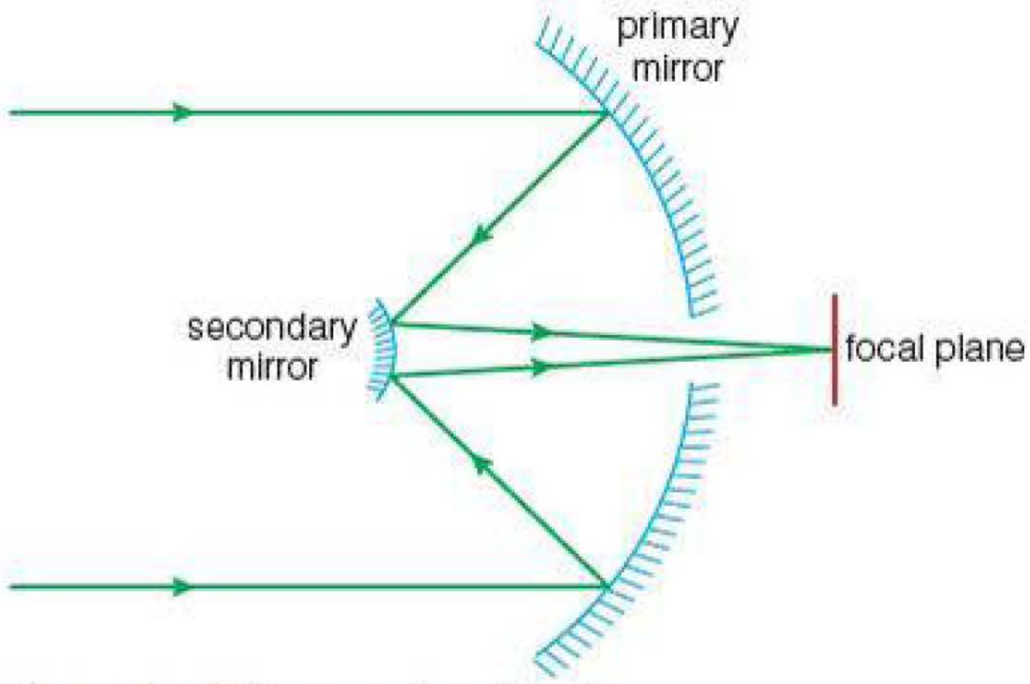


Figure 13.17 Cassegrain reflecting arrangement of mirrors for ultraviolet and infrared radiations, and radio waves.

Telescopes that can detect radiations outside the visible range have many similarities to optical telescopes, but also some important differences. The most obvious difference is that there is no eyepiece because, of course, the eye cannot see infrared, ultraviolet or other radiations. However, for radio waves, infrared and ultraviolet radiations, a Cassegrain reflecting telescope is often used as shown in Figure 13.17. The waves are focused behind the primary mirror: infrared and ultraviolet radiations are detected by CCDs, and aerials can detect radio waves in a radio telescope. Then electrical signals, produced by detectors in the focal plane, are sent to computers which build up colour-coded pictures so that we can 'see' the various intensities of radiations.

Radio telescopes

Figure 13.18 shows a photograph of a radio telescope with its large primary mirror and its secondary mirror, which focuses the waves on to a detector behind the primary mirror. The siting of a radio telescope is not critical because radio waves are not affected by atmospheric conditions – radio waves will still reach the telescope on a cloudy day.

The mirrors or dishes for radio telescopes are very large. To detect radio waves with wavelengths in the range 30 cm to 3 m, dishes are usually larger than 100 m in diameter, but smaller dishes can be effective for shorter-wavelength radio waves. The large-diameter radio dishes mean that the collecting power of the telescope is very high. Often radio telescopes do not have a secondary mirror but position the detector directly at the focal point of the primary mirror.

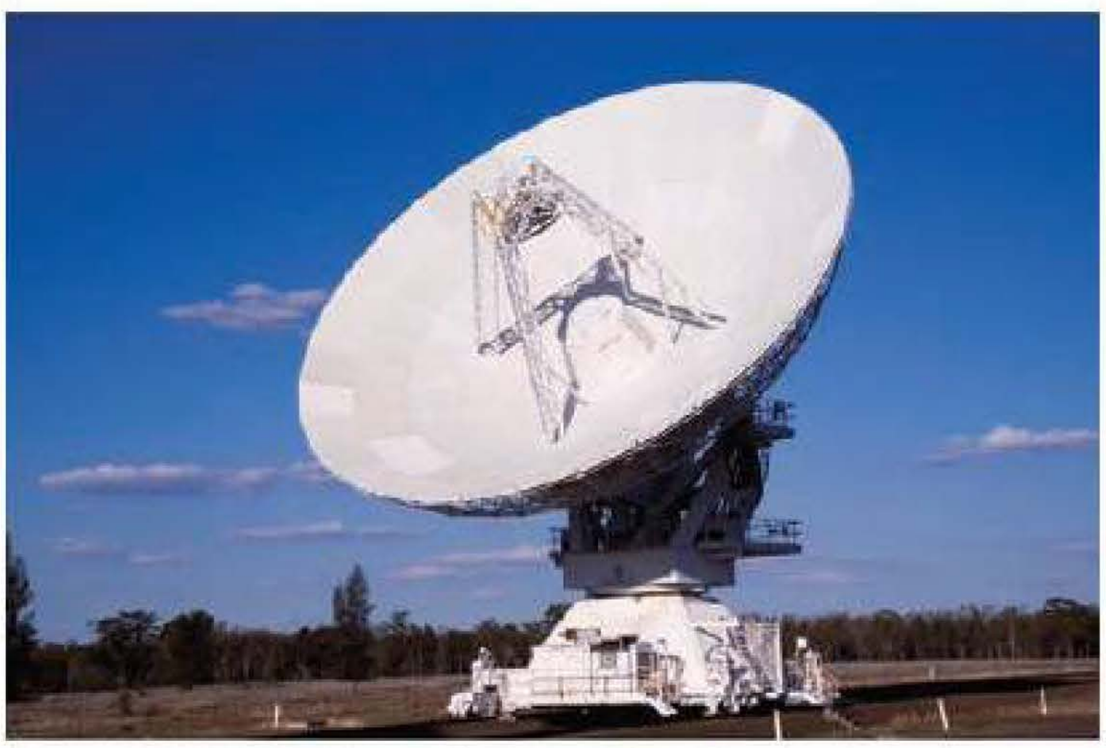


Figure 13.18 A photograph of a radio telescope

The reason for building such large telescopes is to ensure that it is possible to resolve two close radio sources. You will recall from the work on optical telescopes that the criterion for resolving two sources separated by an angle θ is

$$\theta \approx \frac{\lambda}{D}$$

where λ is the wavelength of the radiation, and D is the telescope diameter.

EXAMPLE

Resolving two radio sources

What is the smallest angular separation of two radio sources emitting radio waves of wavelength 0.3 m that can be resolved by a telescope of diameter 60 m?

Answer

Using the expression from the text

$$\theta \approx \frac{\lambda}{D} = \frac{0.3\text{ m}}{60\text{ m}} = 5 \times 10^{-3}\text{ rad}$$

optical telescopes are able to resolve much smaller angle than this.

a) Ultraviolet radiation

b) Visible light

c) X-ray

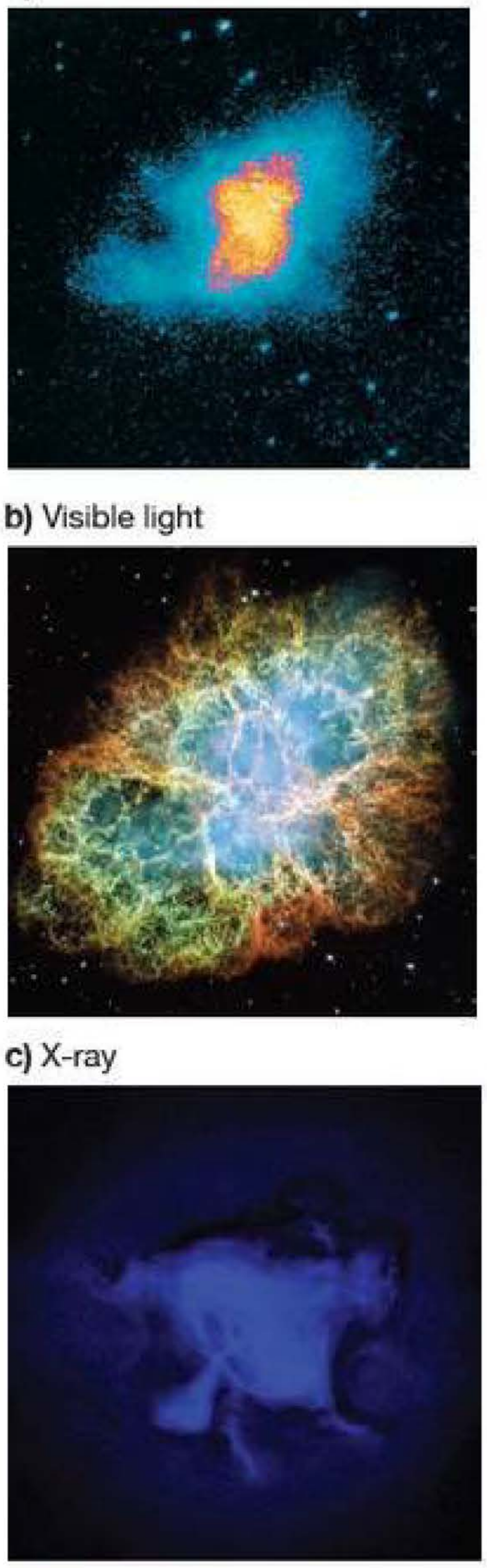


Figure 13.19 Three photographs taken at different wavelengths show the remnants of a supernova explosion seen in 1054. This is known as the Crab nebula.

Ultraviolet and infrared telescopes

The construction of ultraviolet and infrared telescopes is fairly similar to that of an optical telescope, because the wavelengths of the two radiations lie at either end of the visible spectrum. However, careful consideration of the position of these telescopes is essential because of the effect of the atmosphere on ultraviolet and infrared radiations. The majority of ultraviolet radiation is absorbed by the atmosphere, so ultraviolet telescopes are usually in orbit around the Earth in a satellite. Some infrared radiation penetrates the atmosphere, so it is possible to position some infrared telescopes on mountain tops to view specific wavelengths of radiation. Other infrared telescopes are in orbit around the Earth, so that they can detect infrared radiations that do not penetrate the atmosphere.

The collecting power of infrared and ultraviolet telescopes is similar to the that of an optical telescope, because their diameters are similar. However, the resolving power of an ultraviolet telescope is better than for an optical telescope of the same diameter – this is because ultraviolet light has a shorter wavelength than visible light. By contrast, an infrared telescope of the same diameter as an optical telescope does not resolve objects as well as an optical telescope, because the wavelength of infrared radiation is longer than that of visible light. Some telescopes are able to receive near-infrared, visible and near-ultraviolet wavelengths by using a range of CCDs.

Figure 13.19 shows three images of the Crab nebula, taken through different telescopes, detecting three different wavelengths of radiation. The X-ray photograph is able to look through the other layers of the nebula, to detect energy being emitted from a pulsar (a rapidly rotating neutron star) at the centre of the nebula.

X-ray telescopes

X-ray telescopes are also usually situated in space because the atmosphere prevents the majority of X-radiation reaching the Earth's surface. Figure 13.20 shows the design of an X-ray telescope, which is considerably different from the reflecting telescopes discussed above.

X-rays are very penetrating and they are not easily reflected off metal surfaces. You are used to the idea of light being incident on a glass surface. Some light is reflected and some is transmitted by the glass. X-rays behave in this way when incident on a metal surface. However, if X-rays are incident at a very shallow angle, on a highly reflective metal such as iridium, they are all reflected. This is rather like skimming a stone along the surface of water.

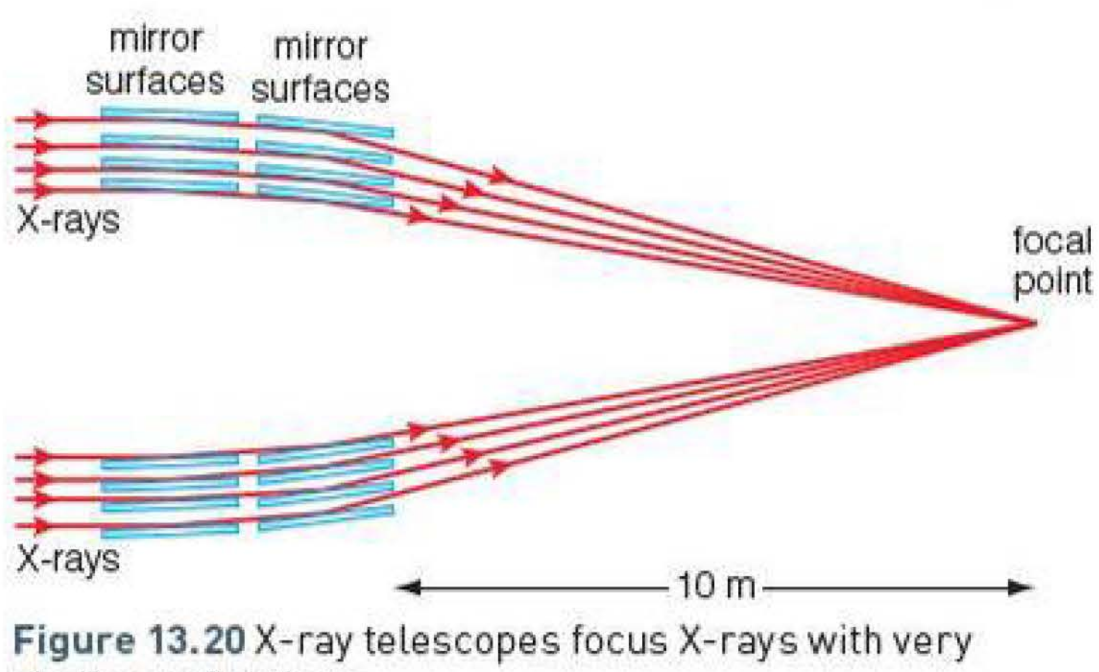


Figure 13.20 X-ray telescopes focus X-rays with very shallow reflections.

In Figure 13.20 X-rays are reflected off a series of mirrors and brought to a focus some 10 m away from the mirrors. Since X-rays have very short wavelengths, 10^{-9} or 10^{-10} m, it is possible to make X-ray telescopes with a small diameter and still produce well-resolved images. The design of telescope shown in Figure 13.20 can also be used to focus some short-wavelength ultraviolet radiations, which are difficult to focus with a conventional telescope.

TEST YOURSELF

12 Explain why radio telescopes have dishes with diameters as large as 100 m.

13 a) Describe how the design of an X-ray reflecting telescope differs from that of an optical reflecting telescope. Account for the differences in design.

b) Explain why X-ray telescopes are in orbit around the Earth rather than on the Earth's surface.

14 The table below shows information about two infrared telescopes.

Telescope	Diameter of primary mirror/cm	Wavelength of radiation detected/μm
Wide-field Infrared Survey Explorer (WISE)	40	3–25
Herschel	350	50–670

a) Explain why the Herschel telescope has a larger diameter than the WISE telescope.

b) Calculate the ratio of the collecting powers of the two telescopes.

c) Calculate the smallest angular separation that each telescope can resolve for

i) the shortest wavelength it detects

ii) the longest wavelength it detects.

Classification of stars

Brightness The brightness of a star is a measure of how much visible light from the star reaches our eyes.

Luminosity The luminosity of a star is the energy it emits per second, in all wavelengths.

When you go outside on a dark moonless night, it is a wonderful sight to see the sky illuminated by thousands of stars. In total, there are about 6000 stars that it is possible to see with the unaided eye. However, with binoculars or a telescope, the number of stars we can see rises into the millions. The **brightness** of the stars we see varies considerably, and this is affected by a star's **luminosity** and how far away it is. The luminosity of a star is the amount of energy it emits per second.

Classification by brightness

Hipparchus was a Greek astronomer who lived some 2200 years ago. He was the first person to begin to categorise stars according to their visual brightness in the sky. Hipparchus began by cataloguing all the brightest stars, and these he called first-magnitude stars. Then he listed the next brightest, and called them second-magnitude stars, and so on until he reached sixth-magnitude stars. The sixth-magnitude stars were the faintest stars that Hipparchus could see by eye.

Two thousand years later modern astronomers looked at the Hipparchus scale of brightness and realised that he had produced a logarithmic scale. A first-magnitude star turns out to be about $2\frac{1}{2}$ times the brightness of a second-magnitude star; and a second-magnitude star is about $2\frac{1}{2}$ times the brightness of a third-magnitude star.

Apparent magnitude A star's apparent magnitude is a measure of its brightness as it appears in the sky.

Astronomers settled on the convention that a first-magnitude star is 100 times brighter than a sixth-magnitude star. This led to a modern, more precise, classification of a star's brightness, or **apparent magnitude**, given the symbol m. The modern scale extends below 1 for the very bright stars, and above 6 for dull stars, which we can see using binoculars or telescopes. Table 13.2 shows a list of some bright stars, seen in the night sky.

Table 13.2 Apparent magnitudes of some bright stars visible in the night sky.

Star	Apparent magnitude (2 s.f.)
Sirius	−1.5
Canopus	−0.7
Vega	0.0
Rigel	0.1
Betelgeuse	0.4
Spica	1.0
Antares	1.1
Bellatrix	1.6
Polaris (Pole Star)	2.0
Acrab	2.5

Comparing brightness of stars

Table 13.2 lists the apparent magnitudes of some bright stars. But what do these magnitudes mean in terms of the brightness (or intensity) of light that we see from different stars?

Earlier, you learnt that the ratio of the brightness of a first-magnitude star to a sixth-magnitude star is 100, and that there is a constant ratio (which we shall call r) between each successive magnitude of brightness (about $2\frac{1}{2}$). This leads to two equations:

$$\frac{I_1}{I_6} = 100$$

defining the ratio in brightness between first- and sixth-magnitude stars, and

$$r^5 = 100$$

Therefore, the ratio of brightness between stars that are one magnitude apart in brightness is

$$r = 100^{\frac{1}{5}} = 2.51$$

Referring to Table 13.2, you can see that Vega has an apparent magnitude of 0.0 and Spica an apparent magnitude of 1.0. This means that Vega is 2.51 times brighter than Spica. Since Polaris has an apparent magnitude of 2.0, it means that Vega is $2.51 \times 2.51 \approx 6.3$ times brighter than Polaris.

It is relatively easy to compare the brightness of stars when their apparent magnitudes are whole numbers apart. It is a little more complicated when their apparent magnitudes are not whole numbers.

EXAMPLE

Comparison of apparent magnitudes

Use Table 13.2 to compare the apparent magnitudes of the following pairs of stars: Sirius and Acrab, Canopus and Bellatrix.

Answer

1 Sirius and Acrab
The difference in apparent magnitudes between Acrab and Sirius is
$2.5 - (-1.5) = 4$. So

$$\frac{\text{brightness of Sirius}}{\text{brightness of Acrab}} = (2.51)^4 = 39.8 \approx 40$$

2 Canopus and Bellatrix
The difference in apparent magnitudes between Bellatrix and Canopus is $1.6 - (-0.7) = 2.3$. So

$$\frac{\text{brightness of Canopus}}{\text{brightness of Bellatrix}} = (2.51)^{2.3} = 8.3$$

TEST YOURSELF

15 a) Explain what is meant by the term 'apparent magnitude'.
b) Star A has an apparent magnitude of 2.0 and star B an apparent magnitude of 8.0. Which star is brighter?
c) Calculate the relative brightness of star A to star B.

16 Use Table 13.2 to calculate the ratio of the brightness of the following pairs of stars:
a) Rigel to Antares
b) Sirius to Bellatrix.

Distance measurement and absolute magnitude

Distance measurement

You are used to using metres or kilometres to measure distances. However, the distances in space are so huge that we use different units to make the numbers easier to handle, and to enable a more straightforward comparison of distances.

Astronomical unit

The average distance from the Earth to the Sun is called an astronomical unit (AU). This distance is 1.5×10^{11} m to two significant figures. Some examples of average distances in astronomical units are:

- the average Earth–Sun distance is 1.0 AU
- the average distance from the Sun to Jupiter is 5.2 AU
- the average distance from the Sun to Sedna (a minor planet) is 532 AU.

Light year (ly)

A light year is the distance travelled by light in one year. So

Parallax Nearby objects appear to move relative to far-away objects, when viewed from a different angle.

1 light year = speed of light × number of seconds in 1 year

$= 3.0 \times 10^8\,\text{m}\,\text{s}^{-1} \times 3.155 \times 10^7\,\text{s}$

$= 9.46 \times 10^{15}\,\text{m}$

Some examples of average distances in light years are:

- the distance to the star Sirius from the Sun is 8.6 light years
- the distance to the Andromeda galaxy from the Sun is 2.5 million light years.

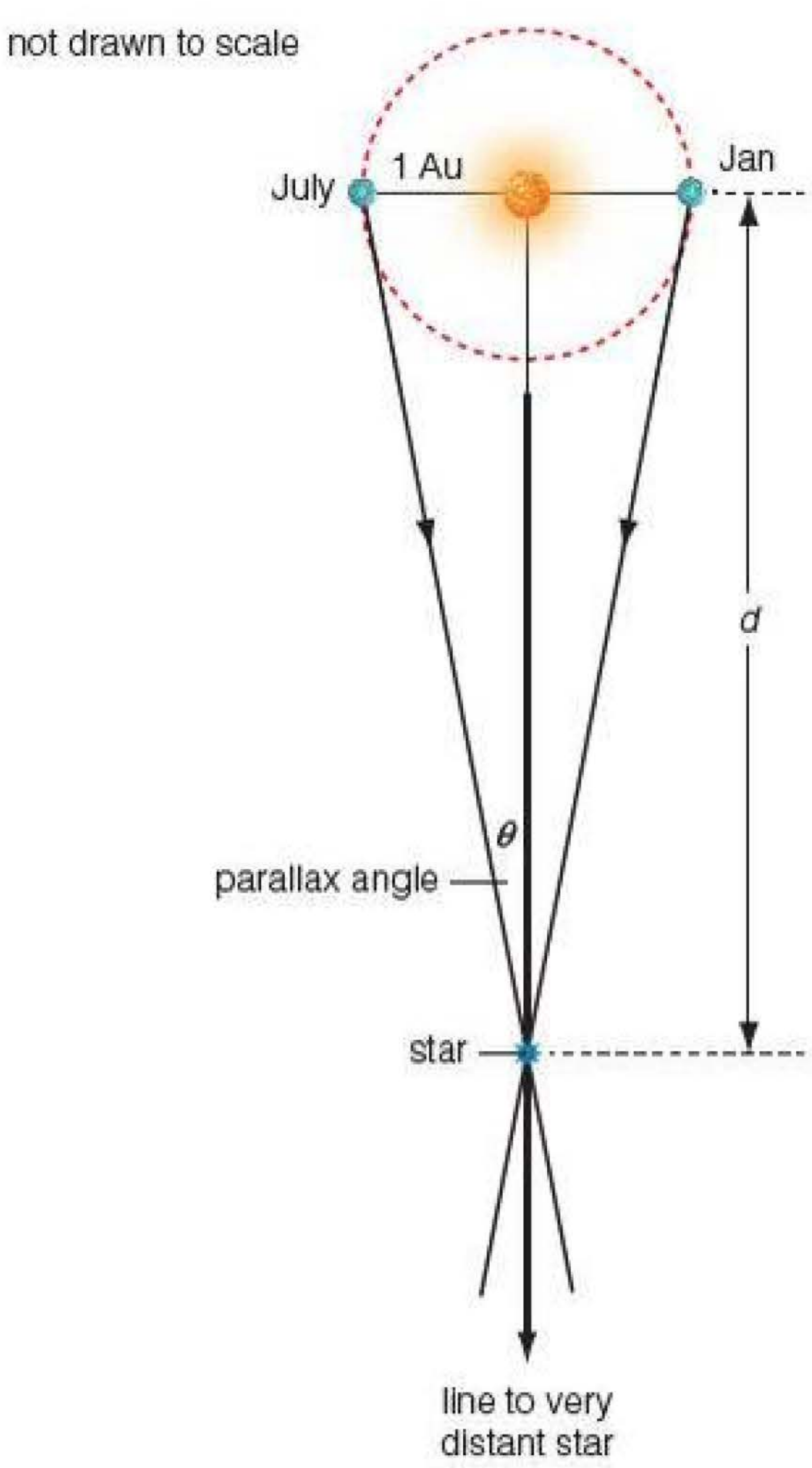


Figure 13.21

Parsec (pc)

When you walk down a street and look at a nearby object such as a lamp post, you will notice that, as you move, the lamp post appears to move relative to more distant objects. This is called **parallax**. We can tell that some stars are closer to us than others because they appear to move slightly as we view them at different times of year. Figure 13.21 (not drawn to scale) shows the idea. In January, for example, we look at a nearby star, then six months later we look at it again. The star appears to have moved relative to more distant stars, which are very far away. The angle shown in the diagram is called the parallax angle. Because even these 'nearby' stars are actually several light years away, this parallax angle is very small.

We can calculate the distance from the Earth to a star:

$$\tan\theta = \frac{1\ \text{AU}}{d}$$

or because θ is very small:

$$\theta = \frac{1\ \text{AU}}{d}$$

$$d = \frac{1\ \text{AU}}{\theta}$$

Remember that θ must be measured in radians. This relationship leads to a new measure of distance, which is directly related to the angle θ. When θ is 1 second of arc, we say that the distance is 1 parsec.

$$1 \text{ second of arc} = \frac{1}{360} \text{degree} = 4.85 \times 10^{-6} \text{ rad}$$

Therefore

$$1 \text{ parsec} = \frac{1 \text{ AU}}{4.85 \times 10^{-6}}$$
$$= \frac{1.5 \times 10^{11} \text{ m}}{4.85 \times 10^{-6}}$$
$$= 3.09 \times 10^{16} \text{ m}$$
$$= 3.26 \text{ light years}$$

If the measured parallax angle is smaller, then the distance to the star is further.

The distances to galaxies are often expressed in megaparsec (Mpc).

MATHS BOX

Here is a summary of the units used in astronomy and their SI units:

1 AU = 1.50×10^{11} m

1 ly = 9.46×10^{15} m

1 pc = 3.26 ly

1 Mpc = 10^6 pc

TEST YOURSELF

17 This question is about converting astronomical distances expressed in metres into light years and parsecs.

a) The distance from Earth to the star Alpha Centauri is 4.13×10^{16} m and the distance to Beta Centauri is 3.31×10^{18} m. Express these distances in

i) light years

ii) parsecs.

b) The distance from Earth to the Virgo cluster of galaxies is 5.0×10^{23} m and the distance to the Corona Borealis cluster of galaxies is 1.1×10^{25} m. Express these distances in megaparsecs.

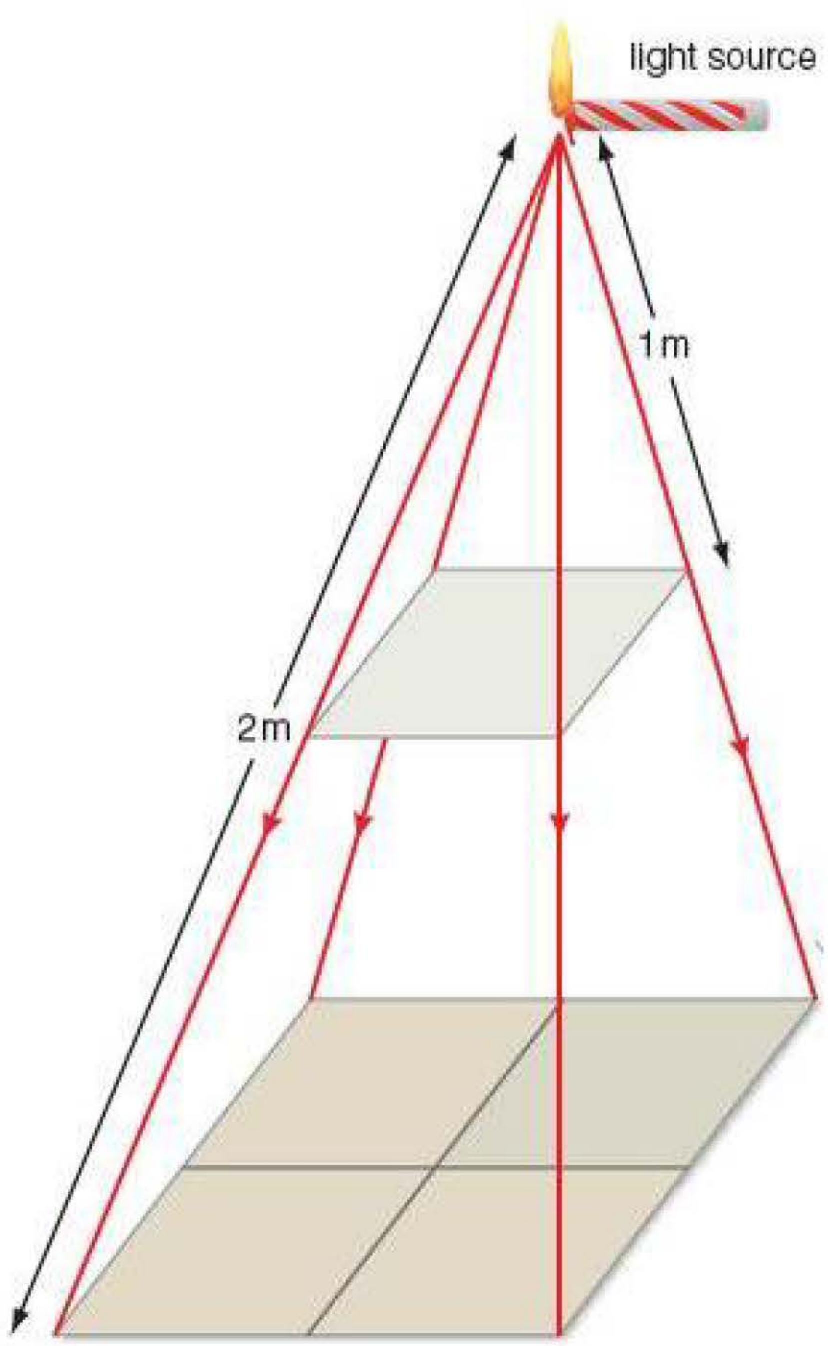


Figure 13.22

Absolute magnitude

Figure 13.22 shows light spreading out from a light source. You can see that, as the light travels further from a source, it spreads over a larger area, so its intensity decreases. When the distance from the source doubles, the intensity of the light reduces by a factor of 4, because the light spreads over four times the area. This is called the inverse square law for intensity:

$$I \propto \frac{1}{d^2}$$

This idea is important when it comes to comparing the brightness of stars. Earlier you met the idea of apparent magnitude – this measures how bright a star appears to be. However, stars appear brighter if they are close to us. So, to compare the brightness of two stars, we need to consider how bright they would appear to be if they were exactly the same distance from us. The distance that is chosen for comparison is 10 parsecs. A star's **absolute magnitude** is the apparent magnitude it would have if it were placed 10 parsecs away from us. In applying the inverse square law for stars, we assume that no light is absorbed by interstellar material such as gas or dust.

Absolute magnitude A star's absolute magnitude is the apparent magnitude the star would have if it were 10 pc away.

The apparent magnitude, m, and the absolute magnitude, M, are linked by the following formula, in which d is the distance of the star from us, measured in parsecs:

$$m - M = 5\log_{10}\left(\frac{d}{10}\right)$$

This formula combines the idea of the inverse square law for light and the standard reference distance of 10 pc. You do *not* need to be able to derive this formula (it is very hard to do), but for interested mathematicians the derivation is shown online with our free resources.

EXAMPLE

Calculation of absolute magnitude

Alpha Centauri has an apparent magnitude of 0.0 and is 1.34 pc from the Sun. Calculate the absolute magnitude of Alpha Centauri.

Answer

Using the formula from the text

$$m - M = 5\log_{10}\left(\frac{d}{10}\right)$$

we obtain

$$M = m - 5\log_{10}\left(\frac{d}{10}\right)$$
$$= 0.0 - 5\log_{10}\left(\frac{1.34}{10}\right)$$
$$= 0.0 - 5 \times (-0.87)$$
$$= +4.4$$

TEST YOURSELF

18 a) P Cygni is a star with an apparent magnitude of 4.8. It is a distance of 1800 pc from Earth. Calculate P Cygni's absolute magnitude.

b) The Sun has an apparent magnitude of −26.7. It is 4.8×10^{-6} pc from Earth. Calculate the Sun's absolute magnitude.

19 Canopus has an absolute magnitude of −5.0 and is a distance of 70 pc from Earth. Calculate the apparent magnitude of Canopus.

20 Capella and Vega are two bright stars, clearly visible in the night sky. Capella has an apparent magnitude of 0.1 and Vega has an apparent magnitude of 0.0. Capella is 42 light years from Earth and Vega 25 is light years from Earth.

a) Calculate the absolute magnitude of each star.

b) Show that Capella emits approximately twice as much visible light as Vega per second.

Classification of stars by temperature

Figure 13.23 This star cluster is called the Jewel Box. Most of the brightest stars you can see are blue, but there are also some bright red stars.

Stars can be put into different categories according to their temperature, colour and the total amount of radiation they emit per second. Blue stars are very hot and bright, so we can see a lot of these by eye (Figure 13.23). Red stars are cooler than blue stars, but some red stars appear bright in the sky because they are very large. These ideas are explained further below.

Black-body radiation

Black-body radiation is the type of electromagnetic radiation that is emitted by a black or a non-reflective body, which is held at a constant uniform temperature.

Black-body radiation has a characteristic wavelength spectrum, which depends only on the absolute temperature of the body. The spectrum peaks

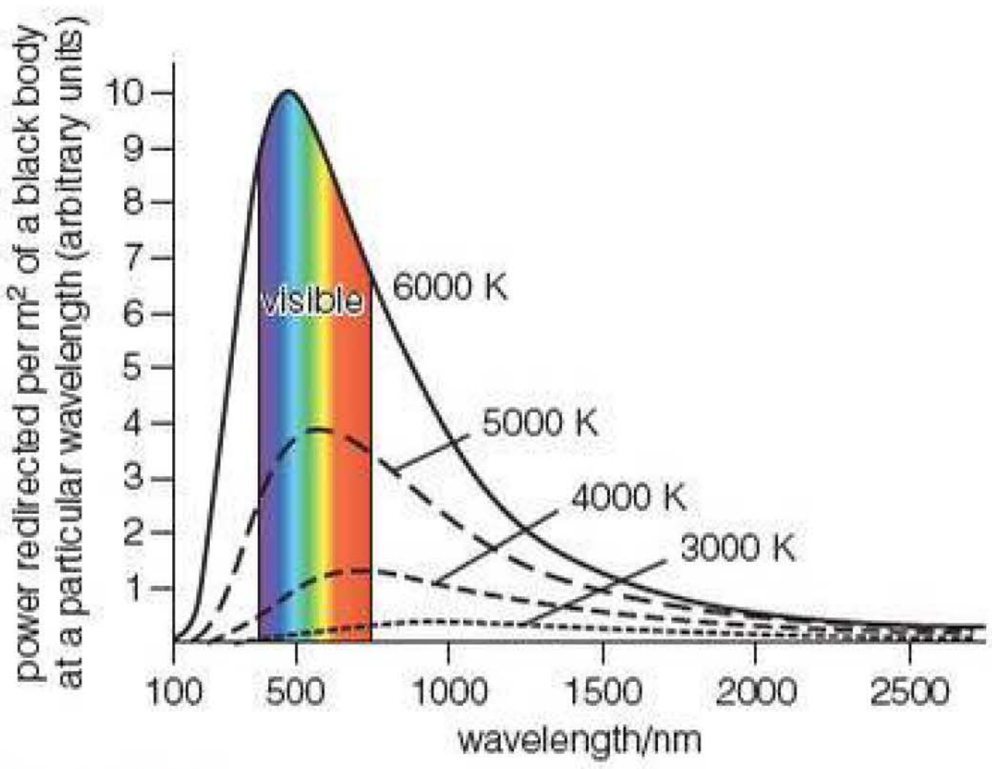


Figure 13.24

at a wavelength that shifts to shorter wavelengths at higher temperatures. Figure 13.24 shows some spectra for black bodies at different temperatures.

The curves in Figure 13.24 show these two important trends.

- As the body gets hotter, more radiation is emitted. The total power emitted by the body is proportional to the area under the graph. So you can see that at 6000 K a black body radiates much more energy than the body at 4000 K.
- The intensity of radiation peaks at a shorter wavelength at higher temperatures. You can see that the peak wavelength corresponds to red light when the temperature of the black body is 4000 K. At 5000 K the peak corresponds to green-yellow light. At 6000 K the peak corresponds to blue-green light.

The term 'black-body radiation' was originally used to describe the spectrum of infrared radiation emitted by hot bodies on the Earth. However, the term also applies to hot bodies such as stars, which emit visible light and also ultraviolet and X-radiation. The shape of the black-body spectra illustrated in Figure 13.24 applies to the stars, and enables us to understand why they have differing absolute magnitudes.

Laws of black-body radiation

There are two laws that summarise the information shown in Figure 13.24.

Stefan's law

Stefan's law states that the total power, P, radiated by a black body of surface A is

$$P = \sigma A T^4$$

where T is the surface temperature of the body (absolute temperature, in K), and σ is the Stefan constant, which is equal to $5.7 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$. The total power radiated by a star is called its *luminosity*, L.

Wien's law

Wien's law states that, for a black-body spectrum, the product of the peak wavelength, λ_{max}, and the absolute temperature of the body, T, is a constant:

$$\lambda_{max} T = \text{constant} = 2.9 \times 10^{-3}\,\mathrm{m\,K}$$

EXAMPLE

Sun's luminosity and peak wavelength

The surface temperature of the Sun is 5780 K and its radius is 7.0×10^5 km.

1 Calculate the Sun's luminosity.

Answer

Luminosity = $\sigma A T^4$

$= 5.7 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}} \times 4\pi \times (7.0 \times 10^8\,\mathrm{m})^2 \times (5780\,\mathrm{K})^4$

$= 3.9 \times 10^{26}\,\mathrm{W}$

$= 4 \times 10^{26}\,\mathrm{W}$ to 1 s.f.

2 Calculate the peak wavelength of the radiations emitted.

Answer

$$\lambda_{max} T = 2.9 \times 10^{-3}\,\mathrm{m\,K}$$

$$\lambda_{max} = \frac{2.9 \times 10^{-3}\,\mathrm{m\,K}}{5780\,\mathrm{K}}$$

$$= 5.0 \times 10^{-7}\,\mathrm{m}$$

$$= 500\,\mathrm{nm}$$

This wavelength is in the blue-green area of the visible spectrum.

TIP
Although the peak wavelength of light from the sun is in the blue-green area of the spectrum, Figure 13.26 shows that all visible wavelengths are emitted and so the light from the sun is a mixture of all colours and is actually white.

Giant stars

The constellation of Orion has two very luminous stars. Rigel is a blue giant with a surface temperature of about 11 800 K and a radius of 54×10^6 km. Betelgeuse is a red giant with a surface temperature of about 3300 K and a radius of 7.7×10^8 km. We can use this information to compare the brightness of these stars with the Sun's brightness.

Rigel

$P = \sigma A T^4$

$= 5.7 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}} \times 4\pi \times (5.4 \times 10^{10}\,\mathrm{m})^2 \times (11\,800\,\mathrm{K})^4$

$= 4.0 \times 10^{31}\,\mathrm{W}$

$\approx 10^5$ times more luminous than the Sun

Betelgeuse

$P = \sigma A T^4$

$= 5.7 \times 10^{-8}\,\mathrm{W\,m^{-2}\,k^{-4}} \times 4\pi \times (7.7 \times 10^{11}\,\mathrm{m})^2 \times (3300\,\mathrm{K})^4$

$= 5.0 \times 10^{31}\,\mathrm{W}$

$\approx 1.3 \times 10^5$ times more luminous than the Sun

Although Betelgeuse is a relatively cool star, its radius is over 1000 times larger than that of the Sun. It is because its surface area is so large that Betelgeuse is one of the most luminous stars in the sky.

Luminosity and brightness

It is important not to confuse *luminosity* and *brightness*.

- Luminosity is the total power emitted by a star in all wavelengths.
- Brightness is a measure of what we can see, and therefore is a measure of the visible light emitted by a star.

For example, a star with a surface temperature of 20 000 K has a peak wavelength, λ_{max}, of about 150 nm, which is well into the ultraviolet spectrum. Such a hot star emits much more of its power outside the visible spectrum.

TEST YOURSELF

21 What are the two factors that affect the luminosity of a star?

22 a) Polaris has a surface temperature of 6015 K, and a radius of 3.2×10^7 km. Calculate its luminosity.

b) Mintaka is a star with a luminosity of 3.6×10^{31} W. Its radius is 1.1×10^7 km. Calculate its surface temperature.

c) 61 Cygni is a star with a surface temperature of 3900 K, and a luminosity of 4×10^{25} W. Calculate the star's radius.

23 A star has a surface temperature three times that of the Sun, and its radius is four times that of the Sun. Calculate how many times bigger the star's luminosity is than the Sun's.

24 Barnard's star is a red dwarf star about 6 light years away from the Sun. The star's surface temperature is 3100 K and its luminosity is 1.4×10^{24} W.

a) Calculate the radius of Barnard's star.

b) Calculate the peak wavelength of the radiation from Barnard's star. In what part of the spectrum does this wavelength lie?

c) Explain why this star has a very low visual brightness.

Stellar spectra

The magnitude of a star and its colour have proved useful for learning about the luminosity and temperature of the star. We can also learn about a star by observing the spectrum of the light that it emits.

Figure 13.25 shows a spectrum of the light emitted from the Sun, when it is viewed through a diffraction grating. The continuous spectrum of light is crossed by dark absorption lines, called Fraunhofer lines. Such absorption lines are produced when light passes through the cooler gases in the outer atmosphere of the Sun.

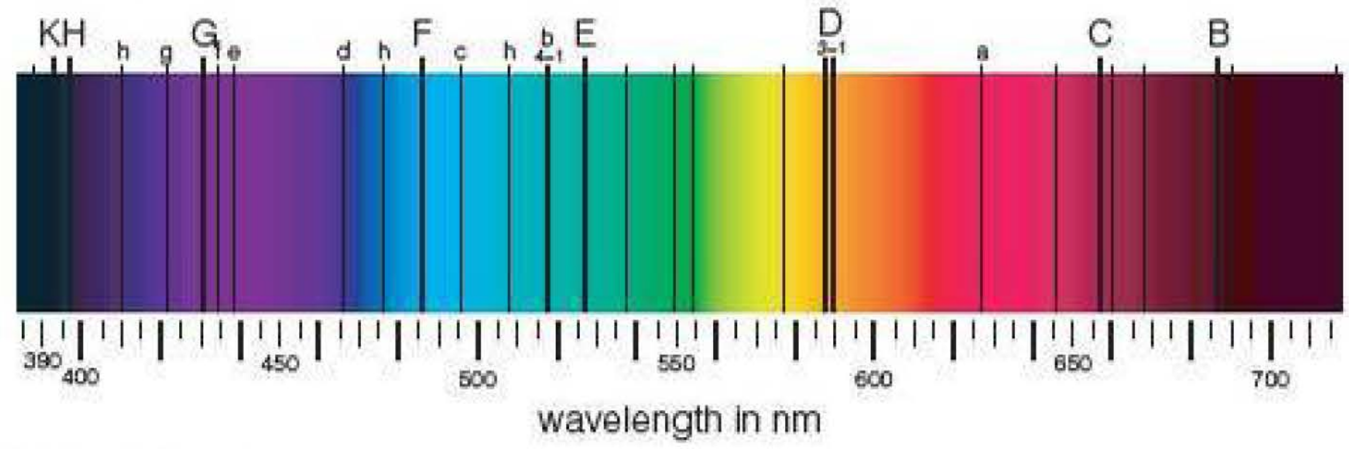


Figure 13.25

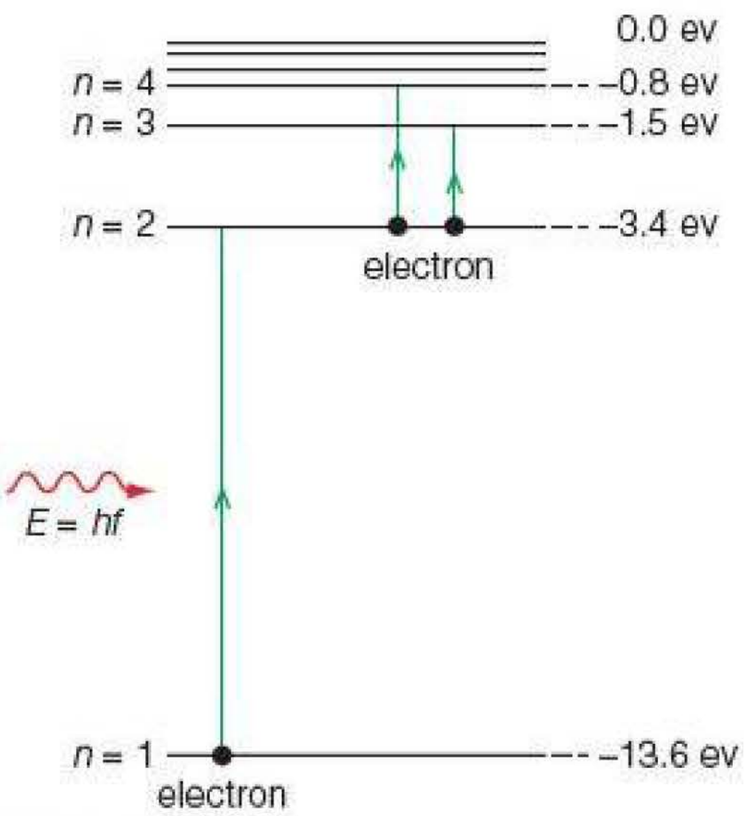


Figure 13.26

Figure 13.26 helps to explain how the process works. The diagram shows an energy level diagram for a hydrogen atom. When a photon has an energy exactly equal to the energy difference between levels 1 and 2, $E_2 - E_1$, the photon can be absorbed by an electron in energy level 1, which is promoted to level 2. Similarly, if there is an electron in level 2, it can move to level 3 if it absorbs a photon of energy $E_3 - E_2$. When light is absorbed in this way, the intensity of these wavelengths is reduced, so black lines appear across the spectrum.

Each element or compound has a unique set of energy levels. These energy levels lead to a unique **absorption spectrum**. Therefore, it is possible to see which elements are present in a star's atmosphere by analysing the absorption lines in its spectrum.

Absorption spectrum This spectrum is seen as a series of dark lines in a continuous spectrum, when some elements absorb specific wavelengths of light.

Spectral classes

When the spectra of a large number of stars were studied, it was realised that stars could be divided into a number of *spectral classes*. These classes were based on which elements were most prominent in the spectra of stars – and these elements varied considerably from star to star.

Originally it was thought that the observation of prominent elements related closely to the chemical composition of the star. However, although there are differences in stellar chemical composition, the most important factor in spectral classes is the star's temperature.

The reason why temperature is very important in determining the spectral class of a star is as follows. For a particular absorption line to be observed, there must be atoms present with an electron in the correct energy level. Hydrogen is the most abundant element in all stars. It is therefore no surprise that we see hydrogen absorption lines in stellar spectra, but we see different patterns of absorption at different temperatures.

When a hydrogen atom is relatively cold, its one electron will lie in its ground state, $n = 1$, nearest the nucleus. Therefore, this electron can be excited to the $n = 2$ level by a photon of the correct energy. Such photons lie in the ultraviolet part of the spectrum, so we do not see these lines when a star is viewed in visible light. These ultraviolet lines are more visible in a star with a surface temperature of about 8000 K than in a star with a lower temperature of, for example, 5000 K, because the hotter star emits more ultraviolet light. However, hydrogen lines are not the most prominent lines seen in cooler stars, because other elements absorb more light than hydrogen.

Table 13.3 lists the various spectral classes of stars, with their most prominent absorption lines.

Table 13.3

Spectral class	Intrinsic colour	Temperature/K	Prominent absorption lines
O	blue	25 000–50 000	He^+, He, H
B	blue	11 000–25 000	He, H
A	blue-white	7500–11 000	H (strongest) ionised metals
F	white	6000–7500	Ionised metals
G	yellow-white	5000–6000	Ionised and neutral metals
K	orange	3500–5000	Neutral metals
M	red	<3500	Neutral atoms, TiO

At higher temperatures, some electrons in atoms move into higher states. At temperatures between about 7500 K and 25 000 K, hydrogen has a significant number of atoms with electrons in the n= 2 state. These temperatures correspond to the A and B spectral types. In these stars, we see prominent hydrogen absorption lines in the visible part of the spectrum. The electron in the $n = 2$ level is able to absorb photons to lift it to the $n = 3$, $n = 4$, $n = 5$ levels and so on. This series of lines is called the Balmer series, after the scientist who discovered them.

In the hottest stars, the most prominent absorption lines come from He and He^+. In the cooler stars, absorption lines are seen from ionised and neutral metals. In the coolest stars, with surface temperatures below 3500 K, titanium oxide produces prominent absorption lines.

TEST YOURSELF

25 What is meant by the 'ground state' of an atom?

26 Explain how an absorption spectrum is produced in a star's continuous spectrum.

27 What is the Balmer series?

28 This question refers to Figure 13.26.

a) Calculate the wavelength of a photon that is absorbed when an electron is excited from the $n = 1$ level to the $n = 2$ level. In what part of the spectrum does this wavelength lie?

b) Calculate the wavelength of a photon that is absorbed when an electron is excited from the $n = 2$ to $n = 4$ level. In what part of the spectrum does this wavelength lie?

(You may need to refer back to Chapter 3 of Book 1 to remind you of how to do these calculations.)

29 Explain why different absorption lines are seen in stars with different temperatures.

The Hertzsprung–Russell diagram

In common with all stars, our Sun was formed from a giant cloud of gas. Figure 13.27 shows part of the Orion nebula, in which stars are being formed.

This nebula is made mostly of cold hydrogen gas. Over millions of years,

Figure 13.27 The Orion nebula is a large cloud of hydrogen gas, which is collapsing to form new stars.

Main sequence star A star in which hydrogen 'burning' takes place. This is the thermonuclear fusion of hydrogen nuclei into helium nuclei.

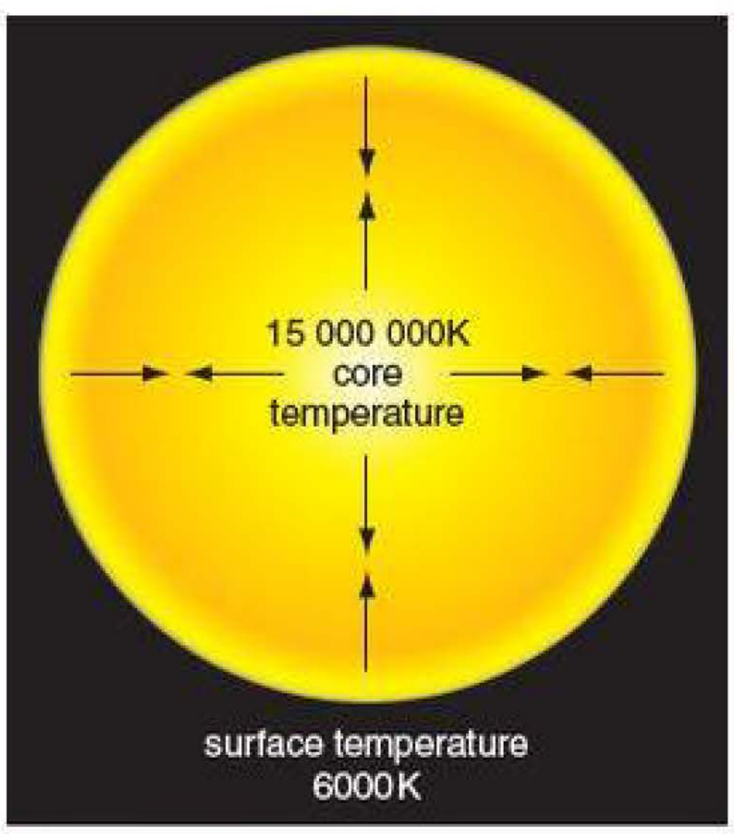


Figure 13.28

gravity acts to coalesce the gas. This collapse warms the gas. As the atoms fall towards each other, potential energy is transferred into kinetic energy, which is then transferred to heat energy as the atoms crash into each other. Because the mass of all the hydrogen atoms is so great, and the distances fallen by the atoms so enormous, the temperature in the middle of such a ball of gas rises to about 15 000 000 K. At this temperature, thermonuclear fusion takes place and hydrogen nuclei (protons) fuse together into helium nuclei, and a star is born. The energy released in the fusion process is emitted as electromagnetic radiation from the star's surface.

A star is a battleground in which competing forces act, as shown in Figure 13.28. The pull of gravity acting inwards is balanced by the outward pressure from the hot core. The pressure at the centre of a star can be billions of times larger than atmospheric pressure on the Earth.

When a cloud of gas collapses, the stars that are formed may be of considerably different masses (Figure 13.29). Stars range in mass from about 100 times the Sun's mass, down to about 0.1 of the Sun's mass. Stars much above 100 solar masses are unstable, and stars below about 0.1 solar masses are too small to start the thermonuclear fusion of hydrogen nuclei.

Most stars are **main sequence stars**, which means that the star is fuelled by the fusion of hydrogen. The more massive stars are much more luminous than the smaller stars. This is because the gravitational forces that tend to collapse a star increase with mass. So for the star to be in in equilibrium, it means that the outward pressure from the core must be larger. Therefore the nuclear reactions must run at a higher rate generating more power, which leads to the star having a higher luminosity.

Stars vary in luminosity from being about 10^6 times more luminous than the Sun (absolute magnitude about −10) to being about 10^4 times less luminous than the Sun (absolute magnitude about +15). The variation in the luminosity of stars is displayed in the Hertzsprung–Russell diagram, as shown in Figure 13.30. The main sequence of stars runs in a diagonal line from the top left-hand corner. At the top left of the diagram are the bright O class stars with absolute magnitudes of −10 and surface temperatures of 50 000 K; at the bottom right of the diagram, are dull M class stars with absolute magnitudes of +15 and surface temperatures of about 2500 K.

Our Sun is a G class star with a surface temperature of about 5780 K and an absolute magnitude of +4.6. The Sun is a significant star in that it is more luminous than 95% of all stars. The best-known stars are the brightest ones, but there are billions of very small, dull stars that cannot be seen by the unaided eye.

The Hertzsprung–Russell diagram also contains further types of stars in the giant and dwarf branches, which will be discussed later on.

The lifetimes of stars

The bright O class stars are very rare because they only live for a short time. Our Sun will exist for a total of about 10^{10} years. It is about 4.6 billion years old, so the Sun is about halfway through its life. A star that is about 100 times more massive than the Sun is about 10^6 times more luminous. So although it has more nuclear fuel, it uses it very quickly. So the brightest stars have lifetimes of the order of a few million years, whereas the dullest stars can live for 10^{12} years or more (which is about 100 times longer than the Universe has been in existence).

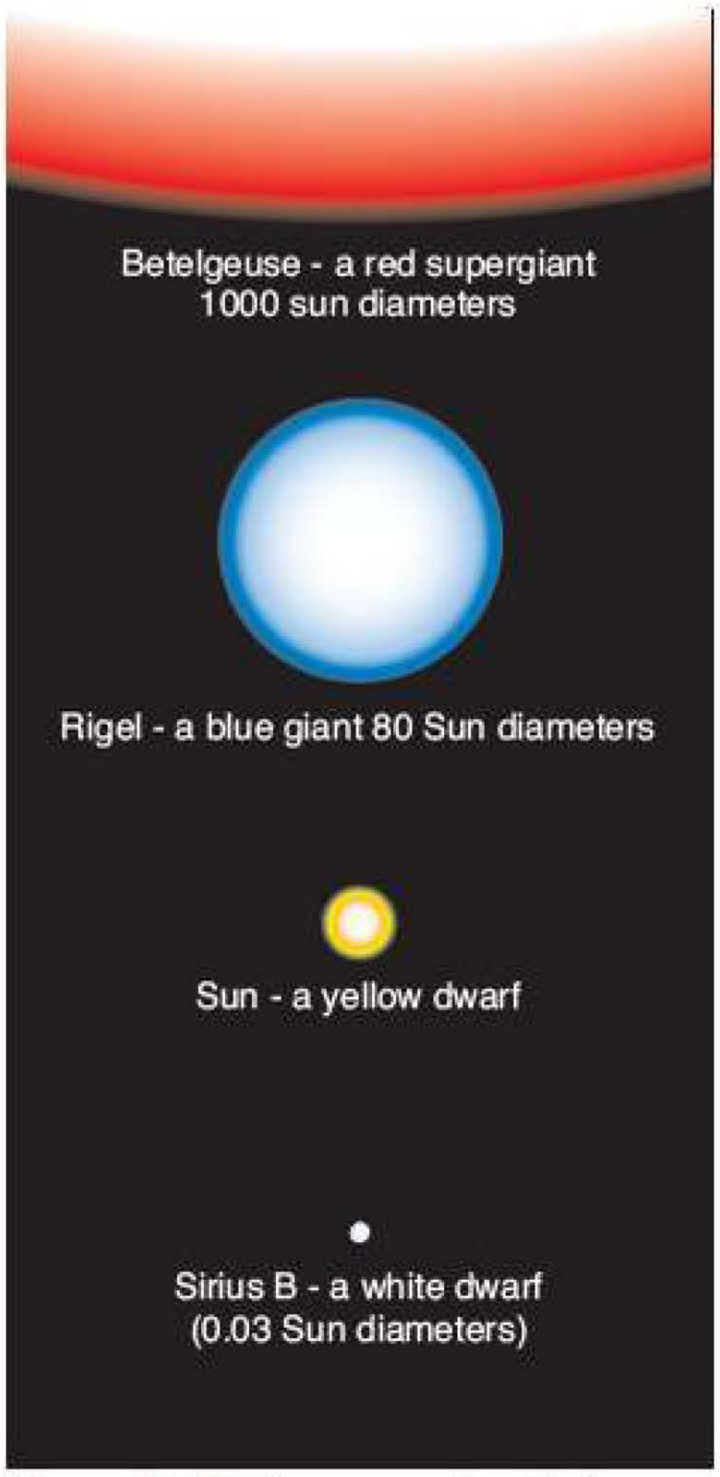


Figure 13.29 Stars come in all sizes.

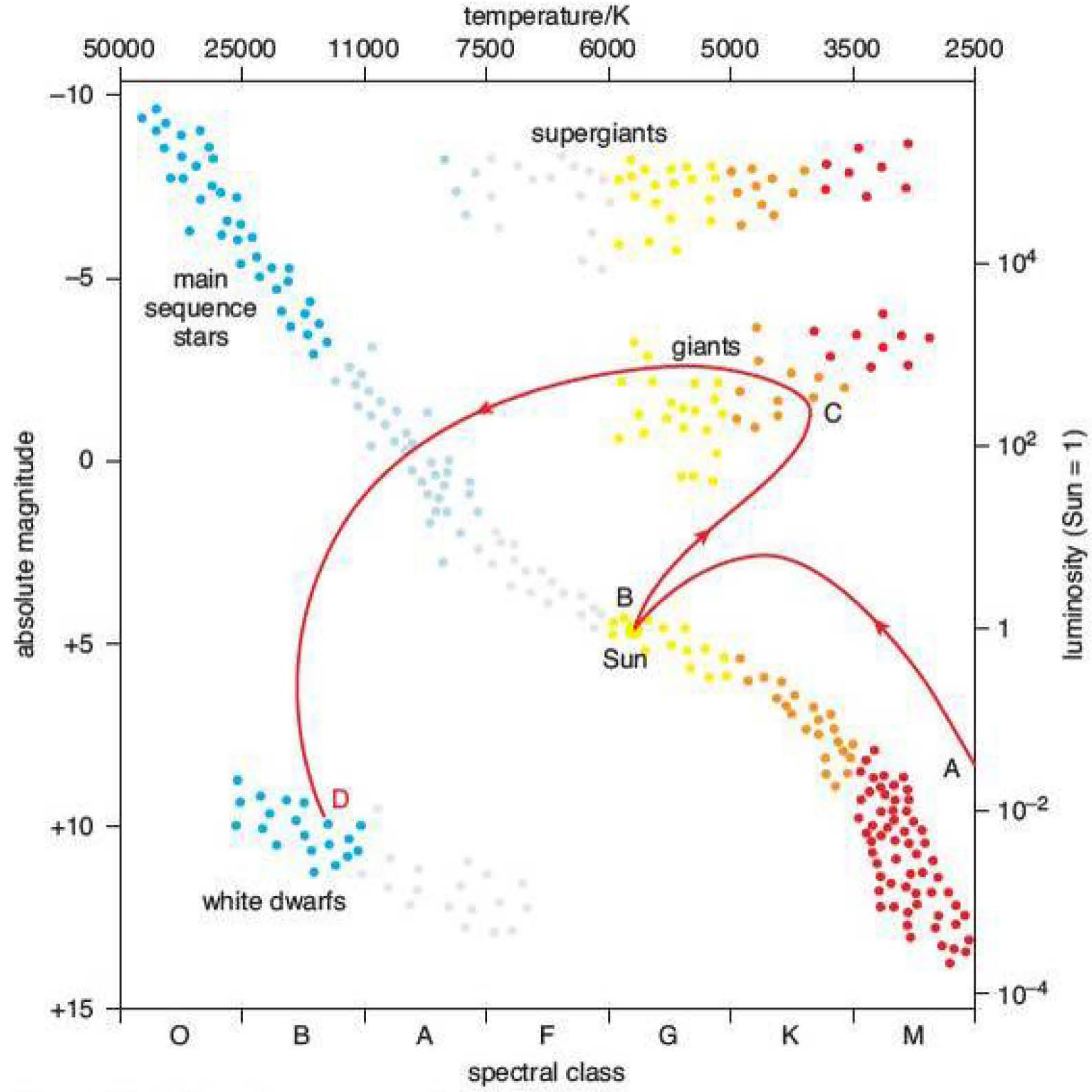


Figure 13.30 The Hertzsprung–Russell diagram.

TEST YOURSELF

30 Explain what is meant by each of these terms:

a) main sequence star

b) red giant star

c) white dwarf star.

31 Explain why stars with very high luminosities are short-lived.

32 Sirius B is the closest white dwarf to us. It has a luminosity of 7.6×10^{23} W, and its surface temperature is 25 200 K.

a) Use Stefan's law to calculate the surface area of the star (the Stefan constant is $5.7 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$).

b) Calculate the radius of Sirius B.

33 Sirius B has approximately the same mass as the Sun, 2×10^{30} kg. Calculate the density of Sirius B, using the result from question 32. Comment on your answer.

The Sun's evolutionary path

The red line ABCD in figure 13.30 shows the evolutionary path of a star, similar to the Sun, on the Hertzsprung–Russell diagram. As described earlier, the star collapses from a cold cloud of gas and reaches its position on the main sequence, where it remains for about 10 billion years, path A to B. After that time the star will have exhausted its supply of hydrogen, which will have been turned into helium. At that point the process of nuclear fusion stops, the pressure inside the core of the star reduces, and the gravitational forces begin to collapse the star. The collapse of the star causes the core to heat up even further, to temperatures in the region of 100 million kelvin (10^8 K).

At that temperature the helium nuclei have enough energy to overcome the repulsive electrostatic forces between them, and to come into contact. Once the helium nuclei get into contact, some of them will fuse into more massive nuclei such as beryllium, carbon and oxygen.

This further nuclear reaction reignites the star. However, the massive temperature causes the star to expand into a red giant, which could be 100 times the current diameter of the Sun. Although the star's surface temperature will be lower, at about 3000 K, the giant's extreme surface area causes it to be much more luminous. The star moves along the path B to C into the giant branch of the stars.

TIP
This section is not required by the specification, so you could skip it. But we hope it provides some background material for the interested reader.

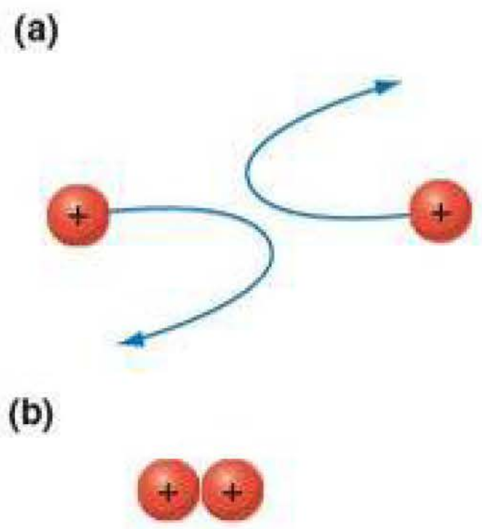


Figure 13.31 (a) At low temperatures (less than 15 million kelvin), two protons repel each other. (b) At high temperatures, two protons have enough kinetic energy to overcome the electrostatic repulsion of their charges, and fusion takes place.

TEST YOURSELF

34 a) Explain why nuclear fusion only occurs at very high temperatures.

b) Why is a higher temperature required to fuse two helium nuclei than two hydrogen nuclei?

c) Explain why the fusion of a helium nucleus with a silicon nucleus is much more likely to happen than the fusion of two silicon nuclei directly.

Further nuclear reactions can occur in stars much larger than the Sun, which takes them into the supergiant branch on the Hertzsprung–Russell diagram. However, the Sun is not massive enough to move into the supergiant branch.

There comes a time when the supply of helium runs out in the star. At this point in a star of the Sun's mass, nuclear fusion stops and the star collapses into a dwarf. Calculations suggest that white dwarfs of the Sun's mass have about the same volume as the Earth. So a white dwarf is extremely dense. The surface temperature of a white dwarf can be 10 000 K, which is much hotter than the Sun's surface. However, because the dwarf star has such a small surface area, it has a low luminosity. The dwarf star is powered by the gravitational potential energy released as it slowly contracts. After a very long time, this energy will run out and the star will become a black dwarf. It is thought that no black dwarfs exist yet because the process takes a longer time than the current age of the Universe.

Nuclear fusion in large stars

Nuclear fusion between nuclei only happens at high temperatures, when the average kinetic energy of particles is very high. Figure 13.31 explains why. In Figure 13.31(a), two protons approach each other at a low temperature and they repel each other and do not collide. In Figure 13.31(b), at a higher temperature, the protons get close enough for the strong nuclear force to act, and the two protons fuse to form deuterium and a positron.

In stars that have cores much hotter than the Sun, the fusion of larger nuclei can take place. Higher temperatures are necessary for such fusions to occur because the larger positive charges on their nuclei result in stronger electrostatic repulsions.

Our Sun will expand into a red giant at the end of its life (in about 5 billion years' time). The Sun is not large enough to progress beyond the helium fusion stage, in which helium fuses to form carbon and oxygen. However, very large stars (about 8 times the mass of the Sun) can progress as far as fusing silicon into larger elements. While a large star lives for millions of years, the silicon fusion stage of its life lasts a matter of only a few days. In a large star there is a lot of helium and larger elements are built up by a process of fusion with helium as follows:

$${}^{28}_{14}\mathrm{Si} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{32}_{16}\mathrm{S}$$

$${}^{32}_{16}\mathrm{S} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{36}_{18}\mathrm{Ar}$$

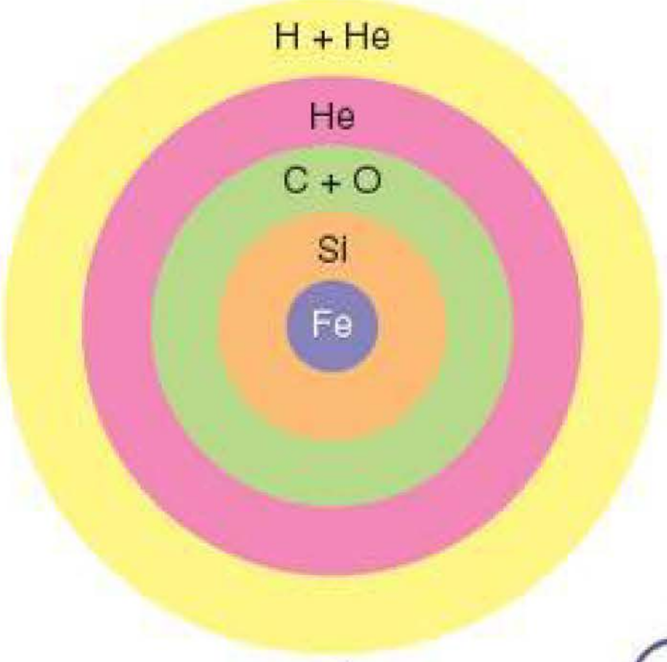
$${}^{36}_{18}\mathrm{Ar} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{40}_{20}\mathrm{Ca}$$

$${}^{40}_{20}\mathrm{Ca} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{44}_{22}\mathrm{Ti}$$

$${}^{44}_{22}\mathrm{Ti} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{48}_{24}\mathrm{Cr}$$

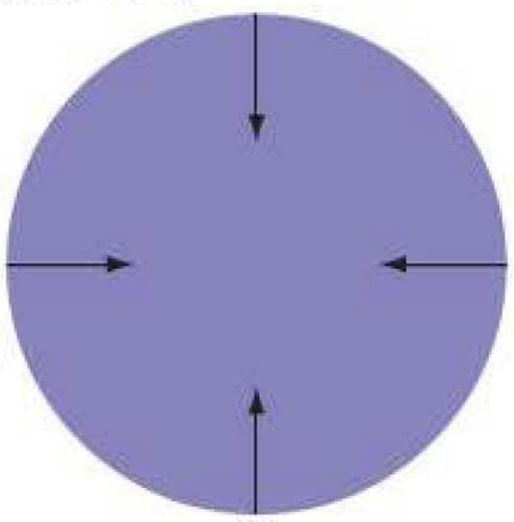
$${}^{48}_{24}\mathrm{Cr} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{52}_{26}\mathrm{Fe}$$

$${}^{52}_{26}\mathrm{Fe} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{56}_{28}\mathrm{Ni}$$



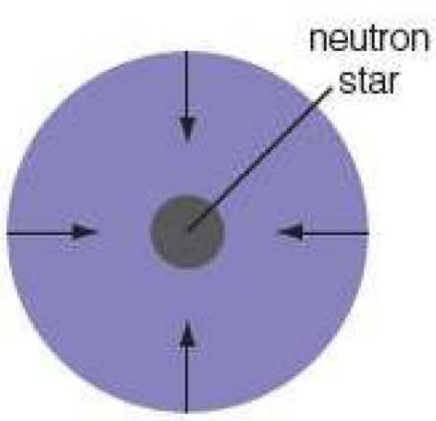
(a)

(a) A large star, more than eight solar masses, has a layered structure, with iron at its core.



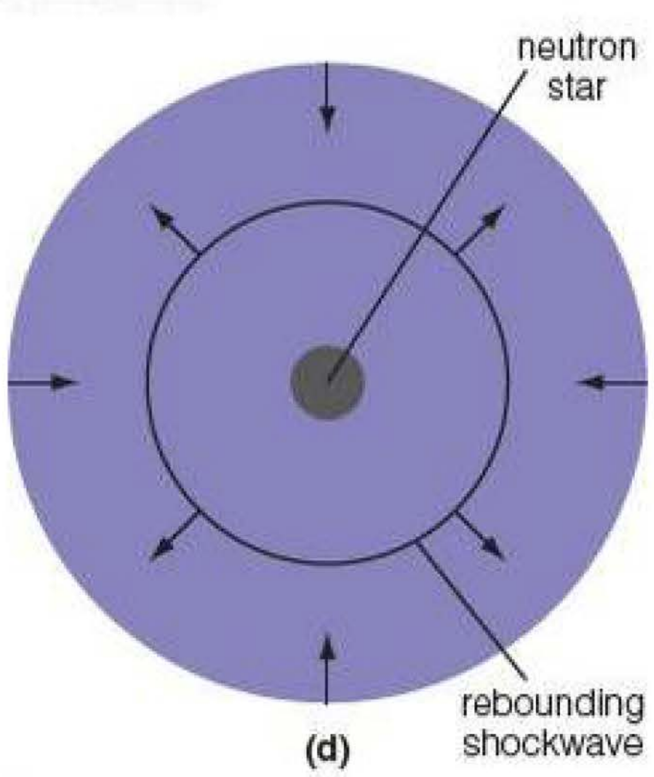
(b)

(b) The core begins to collapse as the nuclear fuel runs out.



(c)

(c) The rapid collapse produces a core of neutrons.



(d)

(d) A shock wave rebounds off the neutron core.

Figure 13.32

The nucleus $^{56}_{28}Ni$ is the end point of nuclear fusion because the fusion into larger nuclei does not release energy. Rather, larger amounts of energy must be supplied to create heavier nuclei. However, $^{56}_{28}Ni$ decays by positron decay as follows:

$$^{56}_{28}Ni \rightarrow\ ^{56}_{27}Co + \ ^{0}_{1}e \qquad \text{half-life 6 days}$$

$$^{56}_{27}Co \rightarrow\ ^{56}_{26}Fe + \ ^{0}_{1}e \qquad \text{half-life 77 days}$$

The nucleus $^{56}_{26}Fe$ is one of the most stable nuclei, which is why it is present inside very large stars.

Supernovae – glorious endings

As explained in the previous section, nuclear fusion does not continue beyond the elements iron and nickel. Figure 13.32(a) shows a large star towards the end of its life. Owing to the various stages of nuclear fusion, the star is layered like an onion with shells of different nuclei – iron in the centre, with helium and hydrogen in the outer layers.

In Figure 13.32(b), which shows the core of the star, the nuclear fuel has just been exhausted, and without the outward pressure from the thermonuclear fusion process, the pull of gravity begins to collapse the star. Under the intense gravitational forces, the core collapses in a matter of seconds. The outer part of the core can reach speeds as high as 20–30% of the speed of light, and the centre of the core rises to temperatures as high as 100 billion kelvin (10^{11} K). At these temperatures, the iron nuclei begin to dissociate into helium nuclei, protons and neutrons.

In such high temperatures and pressures, protons and electrons can combine, in a reverse beta decay, to form neutrons and neutrinos:

$$^{1}_{1}p^{+} + \ ^{0}_{-1}e^{-} \rightarrow\ ^{1}_{0}n + \nu$$

In this way the centre of the core turns into a ball of neutrons, which will become a neutron star, Figure 13.32(c). At this point the core collapses no further and the infalling matter rebounds, producing a shock wave, which spreads outwards as shown in Figure 13.32(d).

The extremely high temperature in the centre of the star restarts the nuclear reactions in the outer layers of the star and a huge amount of energy, perhaps 10^{46} J, is produced in a few seconds. The shock wave moving out from the centre of the star blows the outer layers apart, and energy moves out into space at an enormous rate. This is a supernova (a *type 2 supernova*).

Supernovae are amongst the brightest objects in the sky. They outshine an entire galaxy and in a matter of a few seconds emit more energy than the Sun does in its entire lifetime. Supernovae are colossal events and highly significant for our existence. The energy produced in a supernova explosion produces heavy elements beyond iron, and it is from the remnants of a supernova that our Sun and our Solar System formed.

In 1987 astronomers saw a supernova explosion in the Large Magellanic Cloud, which is a small galaxy (visible from the southern hemisphere) about 170 000 light years from us. Some 20 hours prior to the supernova being seen, scientists detected a burst of neutrinos that had come from the star. These neutrinos were produced in the core as protons and electrons formed neutrons. The neutrinos were able to pass through the outer layers of the star, before the shock wave blew it apart. The supernova was visible to the naked eye, with an apparent magnitude of about +3. Supernovae are characterised by a rapid increase in absolute magnitude, followed by a decay in luminosity over a period of months.

EXAMPLE

Calculation of absolute visual magnitude

Using the information given in the text, calculate the absolute visual magnitude of SN 1987A at its peak brightness.

Answer

SN 1987A is about 170 000 light years from Earth, which is 170 000/3.26 = 52 000 pc. So

$$
\begin{aligned}
M &= m - 5\log\left(\frac{d}{10}\right) \\
&= 3 - 5\log\left(\frac{52000}{10}\right) \\
&= 3 - 5 \times 3.71 \\
&= -15.5
\end{aligned}
$$

The apparent magnitude of a full Moon is about −12.7, so a supernova placed a distance of 10 pc from us would appear about 3 magnitudes brighter than the full Moon, which is about $(2.5)^3$ or about 16 times brighter. So a supernova at that distance would cast very strong shadows at night.

Neutron stars and black holes

Neutron star A collapsed star made of neutrons. It has a very high density.

Black hole A highly condensed state of matter that has an escape velocity higher than the speed of light.

After a massive star has blown itself apart in a supernova explosion, a **neutron star** is often left at the star's core. Neutron stars are even more dense than white dwarfs, as they are made only from highly dense nuclear material. A neutron star of mass about 1.5 times that of the Sun has a radius of only about 12 km.

Some very massive stars (in the region of 20 solar masses) collapse at the end of their lives in an even more spectacular fashion. As their nuclear fuel runs out, the speed of that collapse is so fast that the gravitational tide even manages to collapse the neutrons at its core. Under these circumstances a **black hole** is formed. A black hole is so dense that not even light can escape from it, because its escape velocity is higher than the speed of light.

Gamma-ray bursts

Neutron stars spin very rapidly on their axes. Many such stars spin round several hundred times a second. These rapidly spinning stars are known as pulsars because they emit radiation along their axes of rotation.

Gamma-ray burst A brief intense emission of gamma rays from a collapsing supergiant star.

Figure 13.33 A rapidly collapsing supergiant emits high-powered short bursts of gamma rays.

As supergiant stars collapse into neutron stars or black holes, they emit **gamma-ray bursts**. As matter collapses into the centre of a very massive star, collisions between particles produce very energetic gamma rays, which are emitted along the axis of rotation of the star (Figure 13.33). It is thought that the most energetic gamma-ray bursts are produced when a supermassive star (of some 50 solar masses) collapses into a black hole. The fact that gamma-ray bursts last for a few seconds (or at the most a few minutes) indicates how rapidly larger stars collapse.

A gamma-ray burst produced by a supergiant star, close to the Earth, could have catastrophic consequences. The radiation dose, on the side of the Earth facing the star could be lethal for all animals. The fossil record shows that there was a mass extinction of animals on the Earth some 450 million years ago. One possible explanation is that this was caused by a gamma-ray burst.

Schwarzschild radius

The *event horizon* for a black hole can be described as 'the point of no return', that is the boundary beyond which the gravitational pull becomes so big that escape becomes impossible. So if you are in a spacecraft just outside the event horizon of a black hole you could escape with very powerful rockets. However, once inside the event horizon the escape velocity is higher than the speed of light, and a spacecraft would be trapped (and of course torn apart by the immense gravitational forces).

We can calculate the approximate radius of the event horizon using Newton's law of gravitation. The gravitational potential energy of a spacecraft, of mass m, at a distance R from the centre of a black hole of mass M, is given by

$$E_p = -\frac{GMm}{R}$$

If the spacecraft is to escape, its kinetic energy, $\frac{1}{2}mv^2$, must satisfy the relationship

$$\frac{1}{2}mv^2 - \frac{GMm}{R} > 0$$

or

$$\frac{1}{2}mv^2 > \frac{GMm}{R}$$

Schwarzschild radius The radius of a black hole's event horizon. Light cannot escape from inside a black hole's event horizon.

The radius of the event horizon is known as the **Schwarzschild radius**, R_s, at which point the escape velocity is the speed of light, c. So

$$\frac{1}{2}mc^2 = \frac{GMm}{R_s}$$

and

$$R_s = \frac{2GM}{c^2}$$

EXAMPLE

Radius of event horizon

Calculate the radius of the event horizon for a black hole of mass 20 solar masses. (A solar mass is 2×10^{30} kg.)

Using the formula in the text

$$R_s = \frac{2 \times 6.7 \times 10^{-11}\,\text{N m}^2\,\text{kg}^{-2} \times 40 \times 10^{30}\,\text{kg}}{(3 \times 10^8\,\text{m s}^{-1})^2}$$

$$= 6 \times 10^4\,\text{m}$$

$$= 60\,\text{km}$$

Observations of stars at the centre of our Galaxy, the Milky Way, suggests that millions of stars are contained in a very small volume. Astronomers calculate that there is a supermassive black hole at the galactic centre, with a mass of about four million times that of the Sun.

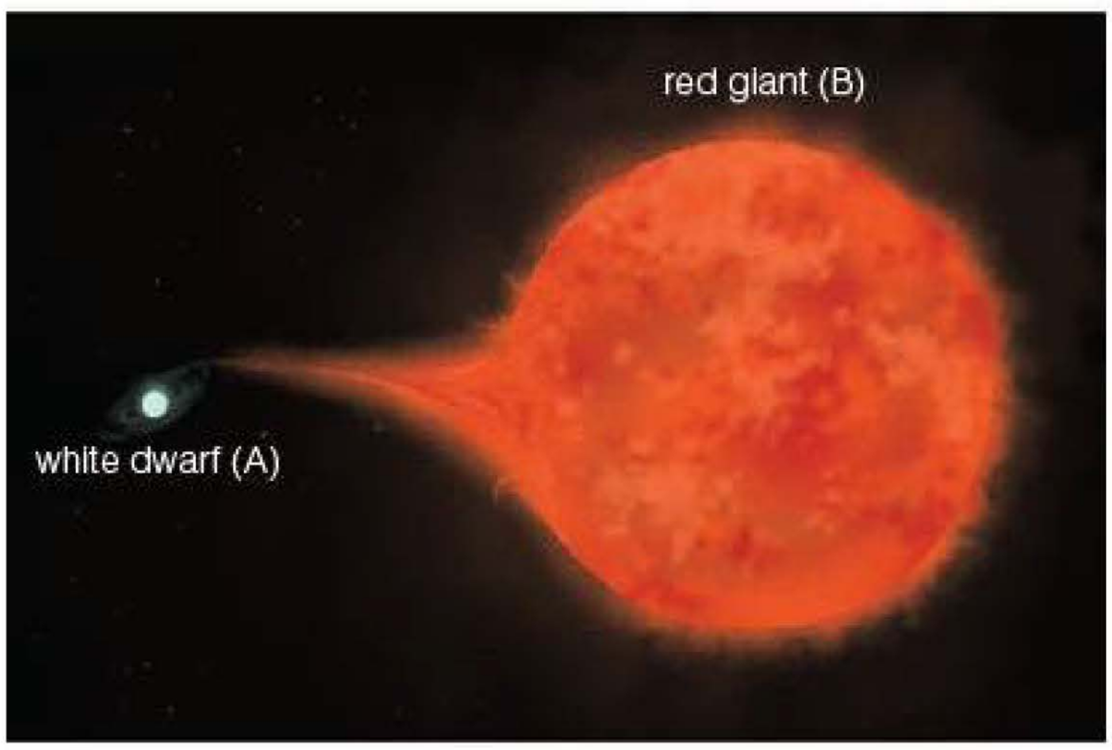


Figure 13.34

Type 1a supernovae

Many stars exist as binary stars, which means that two stars rotate about a common centre of gravity. Such stars can coexist in stable orbits for millions of years. However, if the stars are of different masses, they evolve at different rates.

Figure 13.34 shows a pair of stars that are a little more massive than the Sun. The star A has passed through the main sequence and red giant stages and is now a white dwarf. Later, star B moves into the red giant stage, and as it expands matter is pulled into the white dwarf.

If the mass of the white dwarf grows to be larger than 1.4 solar masses, the star collapses. At this point carbon and oxygen in the white dwarf suddenly begin to undergo nuclear fusion. Such a rapid collapse, followed by the re-ignition of nuclear fusion, can trigger a supernova explosion.

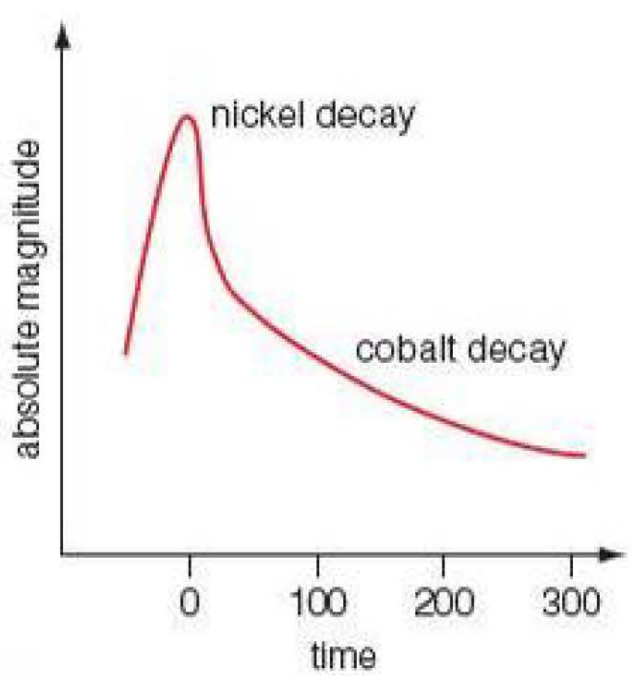


Figure 13.35 All type 1a supernovae produce light curves of a characteristic shape.

Type 1a supernovae are easily identified by astronomers for two reasons. First, they have approximately the same absolute magnitude, because they always occur in stars with about 1.4 solar masses. Secondly, the rapid onset of fusion in the collapsing star produces the nuclear isotope $^{56}_{26}\text{Ni}$. As explained earlier, this isotope decays to cobalt-56 with a half-life of 6 days, and then cobalt-56 decays to iron-56 with a half-life of 77 days. So type 1a supernovae have a characteristic light curve (Figure 13.35), which decays on a time scale governed by the half-lives of the isotopes $^{56}_{26}\text{Ni}$ and $^{56}_{26}\text{Co}$. As the two isotopes decay, massive numbers of high-energy photons are emitted, which power the light emitted by the remnants of the expanding supernova.

Because type 1a supernovae have a characteristic absolute magnitude, they are used as **standard candles**. This means that astronomers can calculate the distance of a galaxy from Earth by measuring the apparent magnitude of a type 1a supernova in the galaxy.

Standard candle A star or supernova of known brightness that can be used to calculate galactic distances.

The use of type 1a supernovae has led to a controversial result. Measuring the distance to very distant galaxies has led cosmologists to the conclusion that the Universe was expanding more slowly in the past. For a long time it was assumed that the action of gravity would cause the expansion of the Universe to slow down. The idea of a Universe with accelerating expansion is a most controversial idea. There is no firm explanation for this theory yet, but cosmologists suggest that some 'dark energy' in the Universe may be responsible for an accelerating expansion.

TEST YOURSELF

35 Calculate the density of a neutron star with mass 4×10^{30} kg and a radius of 14 km.

36 The supermassive black hole at the centre of a galaxy has a mass of 100 million solar masses. Calculate its Schwarzschild radius. The mass of the Sun is 2×10^{30} kg.

37 Explain the meanings of the following terms:
- **a)** neutron star
- **b)** black hole
- **c)** standard candle
- **d)** gamma-ray burst
- **e)** supernova.

38 Explain why type 1a supernovae always have
- **a)** similar shapes of light curves
- **b)** similar absolute magnitudes.

39 A type 1a supernova is seen in a distant galaxy by an astronomer who measures its apparent magnitude to be +11 at its peak of brightness. It is known that a type 1a supernova has an absolute magnitude of about −19, at its peak. Show that the galaxy is about 10 Mpc away

Cosmology

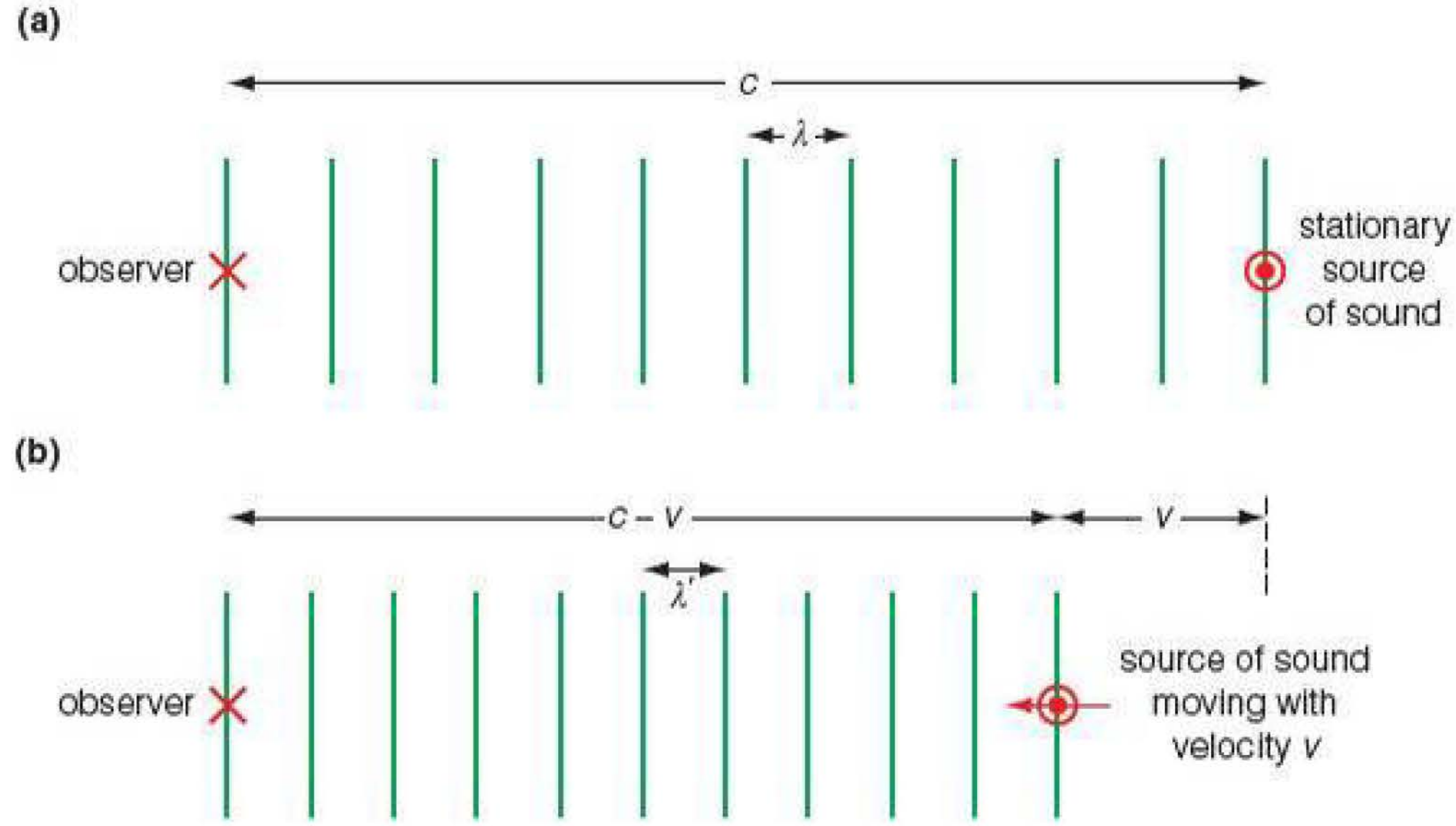
Cosmology is the study of the origin, evolution and eventual fate of the Universe. A detailed understanding of the Universe has been gained by mapping the positions and relative motion of the many groups of galaxies that lie in deep space. You learnt in the previous section that the distance from Earth to galaxies can be estimated using the light seen from type 1a supernovae. Below, you will learn how the Doppler shift in the light seen from galaxies can be used to measure their velocity – and then also deduce the distance of galaxies that are very far away.

Doppler effect

You will be familiar with the Doppler effect. This is the name given to the apparent change in the frequency, or wavelength, of a moving source of sound (or other type of wave). When you hear the siren from a fire engine as you stand in a street, you hear one pitch (frequency) of sound as the fire engine approaches you, and a lower pitch of sound after the fire engine passes you and goes away in the opposite direction. Figure 13.36 helps you to understand why the sound changes pitch.

In Figure 13.36(a) a stationary source of sound is emitting waves, which are heard by the observer, who is a distance c metres away from the source, where c is the speed of sound in $m\,s^{-1}$. So a 1 s burst of sound stretches from the source to the observer. In Figure 13.36(b) the source is moving towards the observer with a velocity v. Now the 1 s burst of sound is squashed into a length $c - v$ metres. This means that the wavelength is reduced (from λ to λ'), and the observer hears a higher frequency. If the source moves away from the observer, the waves are stretched out into a length of $c + v$ metres. The wavelength is increased and the observer hears a lower frequency.

Figure 13.36



From Figure 13.36 you can see that

$$\frac{\lambda'}{\lambda} = \frac{c-v}{c}$$

but $\lambda' = \lambda - \Delta\lambda$, where $\Delta\lambda$ is the change in wavelength. So

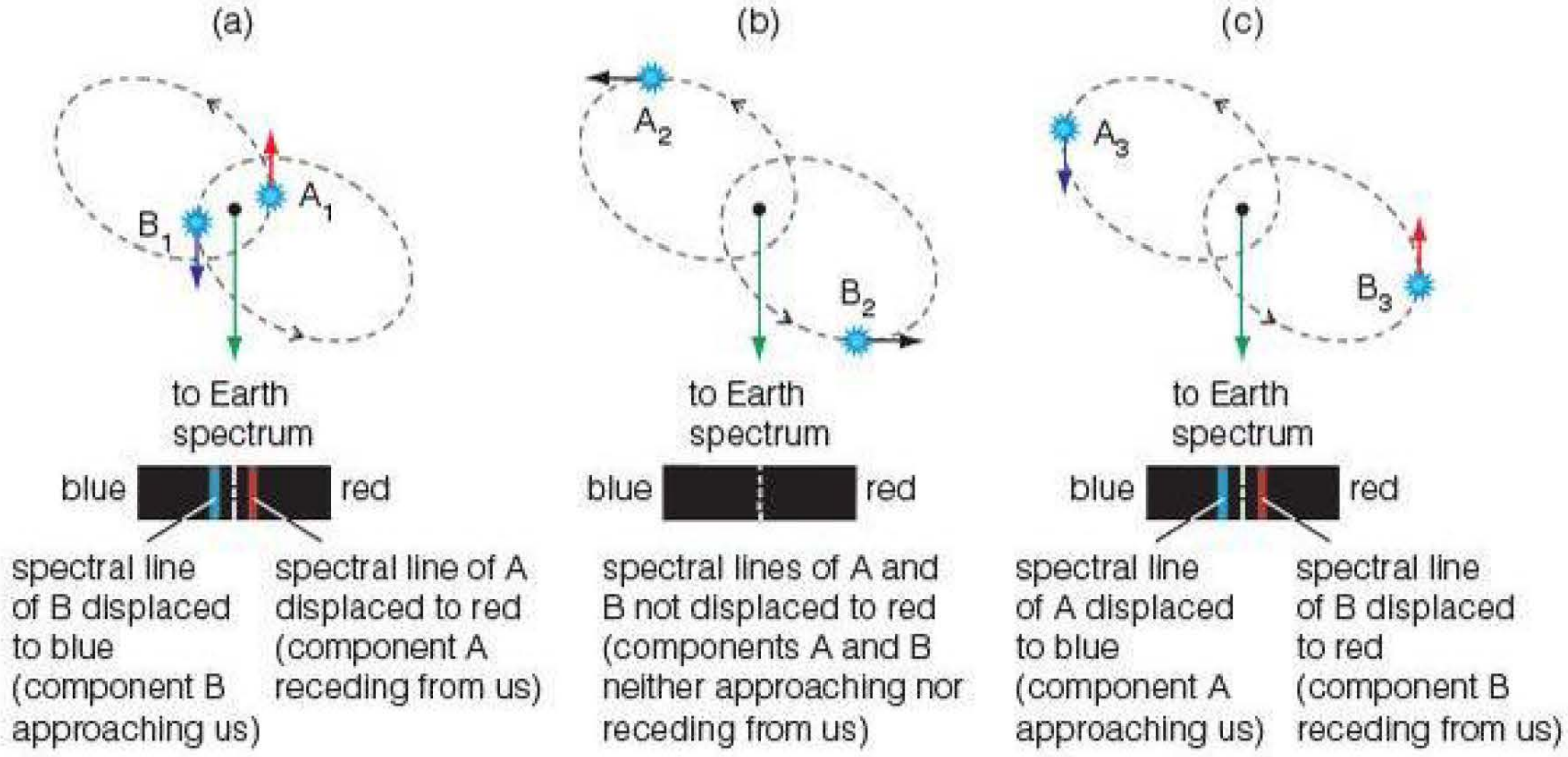
$$\frac{\lambda-\Delta\lambda}{\lambda} = \frac{c-v}{c}$$

$$\frac{\Delta\lambda}{\lambda} = -\frac{\Delta f}{f} = \frac{v}{c}$$

Note that if the source moves away from the observer then the wavelength increases, and the frequency decreases.

Light also shows a Doppler effect or shift when a source is moving. This has proved to be a very successful way of investigating the orbits of binary stars. Figure 13.37 shows a pair of stars that orbit around a common centre of gravity. When a star is moving towards the Earth, the wavelength of the light decreases, and it is shifted towards the blue end of the spectrum. When a star is moving away from the Earth, the light it emits appears to be shifted towards the red end of the spectrum. The spectrum of light emitted from stars are crossed with absorption lines (see Figure 13.25). The shift of these absorption lines towards the red or blue end of the spectrum enables astronomers to calculate the velocity of the stars.

Figure 13.37



The Doppler shift has also shown that galaxies are moving away from us. The *redshift* in the spectral lines emitted from galaxies has proved an invaluable tool in mapping the Universe. The redshift of a spectral line is sometimes expressed as a fraction. For example, if the redshift is 0.1, it means the wavelength has shifted by 10%. Since

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

we can see that the galaxy is receeding at 10% of the speed of light.

TEST YOURSELF

40 An absorption line in the hydrogen spectrum has a wavelength of 656.3 nm. In the spectrum of a star, which is one of a binary pair, the wavelength of the absorption line changes between 655.9 nm and 656.7 nm.

a) Explain why the wavelength of the spectral line appears to change.

b) Calculate the maximum velocity of the star away from the Earth.

41 A spectral line in a galaxy is observed to be shifted from 486 nm to 541 nm. Calculate the velocity of the galaxy.

42 A motorist is in court having been accused of driving through a red light. In his defence he explains to the magistrate that the light looked green as he went past it because he was moving. Discuss whether or not this is a good defence. The wavelength or not of red light is 650 nm, and the wavelength of green light is 530 nm.

Hubble's law

Our Galaxy, the Milky Way, is not alone in space. It is part of a group of some 50 galaxies that we call the Local Group. The largest two galaxies in the Local Group are the Milky Way and the Andromeda galaxy. Our Local Group of galaxies is a very small group and one of billions of such groups. Figure 13.38 shows the distribution of groups or galaxies within 600 million light years of us.

Cepheid variable A bright star whose intensity varies over a matter of days. The period of the variation of intensity is linked directly to the absolute magnitude of the star.

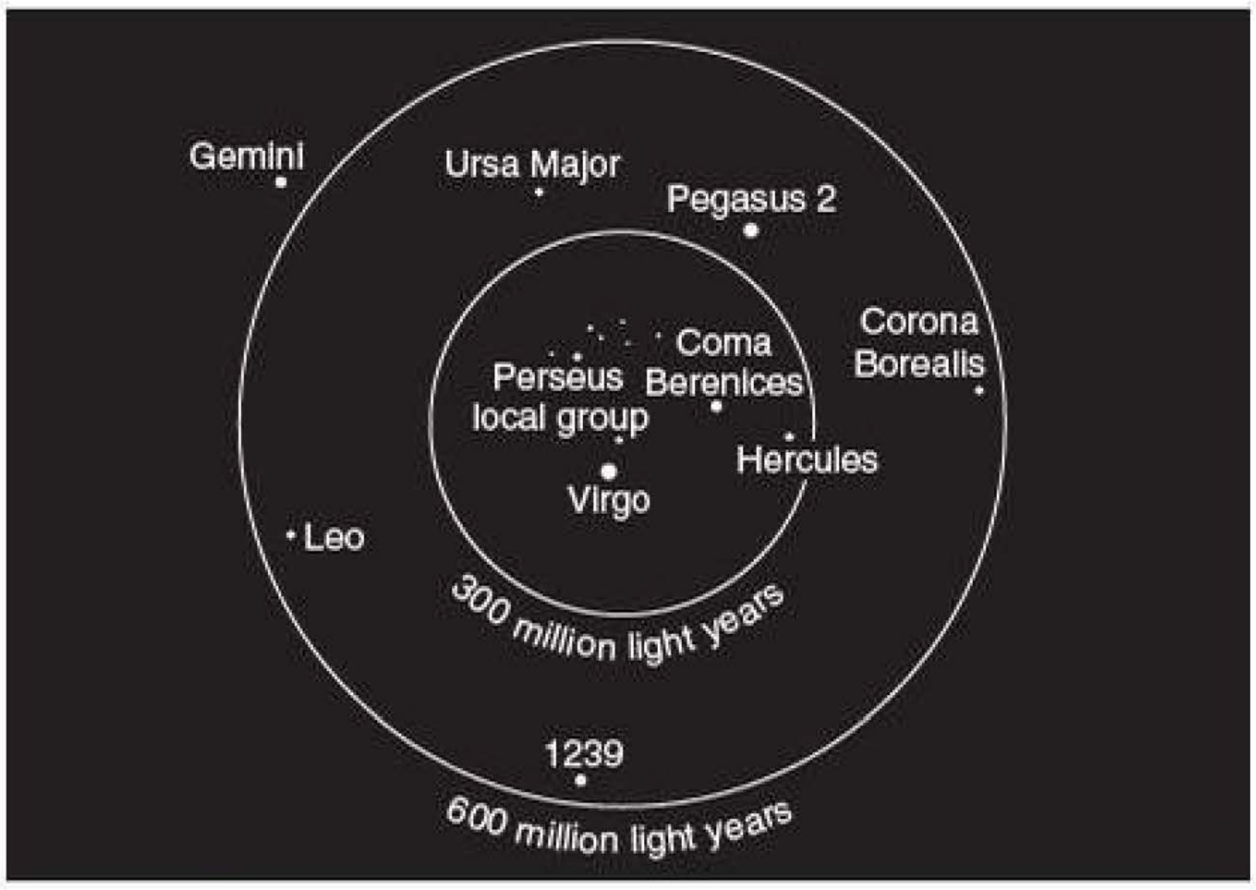


Figure 13.38 This map show groups of galaxies in the vicinity of our Local Group of galaxies.

In the 1920s the American astronomer Edwin Hubble began to plot the positions and distances of galaxies from Earth. He calculated the distance of galaxies using standard candles called **Cepheid variable** stars. Hubble used Cepheids as his standard candles, in the same way as type 1a supernovae are used today. Hubble compared the distances of galaxies with their redshifts, and established that the distance a galaxy is away from us is proportional to its redshift or its velocity of recession. Figure 13.39 shows this linear relationship, which leads to Hubble's law:

$$v = Hd$$

where v is the speed of recession of a galaxy, d is its distance away from us and H is Hubble's constant, which is $67.8\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$ or $20.7\,\text{km}\,\text{s}^{-1}\,\text{Mly}^{-1}$.

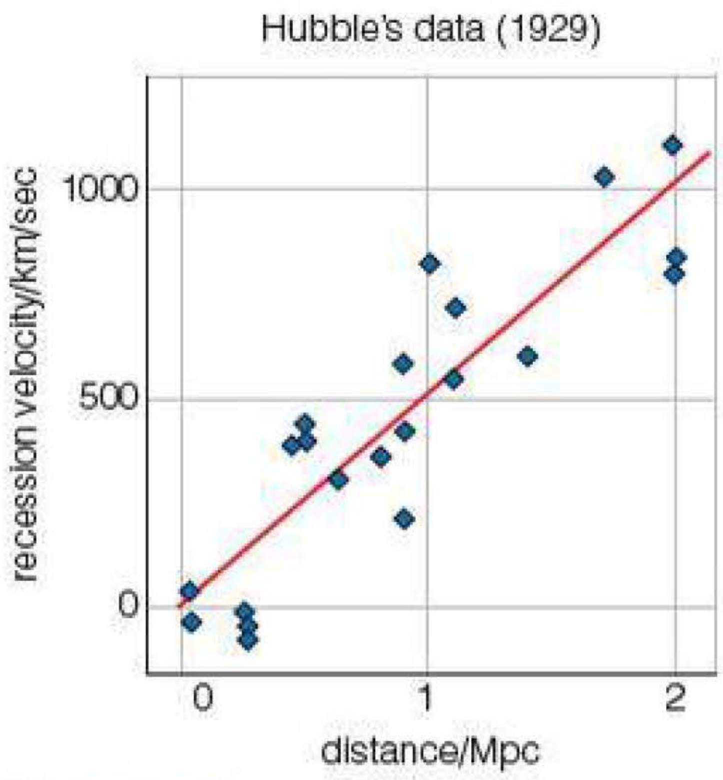


Figure 13.39

TIP

The Hubble constant is not well known and values given in data and questions can vary. Often a figure of 70 kms^{-1} Mpc^{-1} is used.

Hubble's constant tells us that if a galaxy is 1 Mpc away from us its velocity is about 70 $km\,s^{-1}$, and if a galaxy is 10 Mpc away from us then its velocity is about 700 $km\,s^{-1}$.

EXAMPLE

The Hubble constant

Express the Hubble constant in units of s^{-1}.

Answer

From earlier in this chapter, 1 pc = 3.26 ly, so

$$1\,\text{Mpc} = 3.26\,\text{Mly}$$
$$= 3.26 \times 10^{6} \times 3 \times 10^{8}\,\text{m}\,\text{s}^{-1} \times 365 \times 24 \times 3600\,\text{s}$$
$$= 3.1 \times 10^{22}\,\text{m}$$

So

$$H = \frac{67.8 \times 10^{3}\ \text{m s}^{-1}}{3.1 \times 10^{22}\ \text{m}}$$
$$= 2.2 \times 10^{-18}\ \text{s}^{-1}$$

The Big Bang theory

Hubble's law led to the idea of the Big Bang theory. Figure 13.40 shows a region of space, with an observer O at its centre. The observer sees galaxies in every direction. Galaxies that are a distance *r* from O travel with a speed *v*. Galaxies that are a distance 2*r* from O travel with a speed 2*v*. Therefore, it is argued, at some point in the past all the galaxies must have been at the same point.

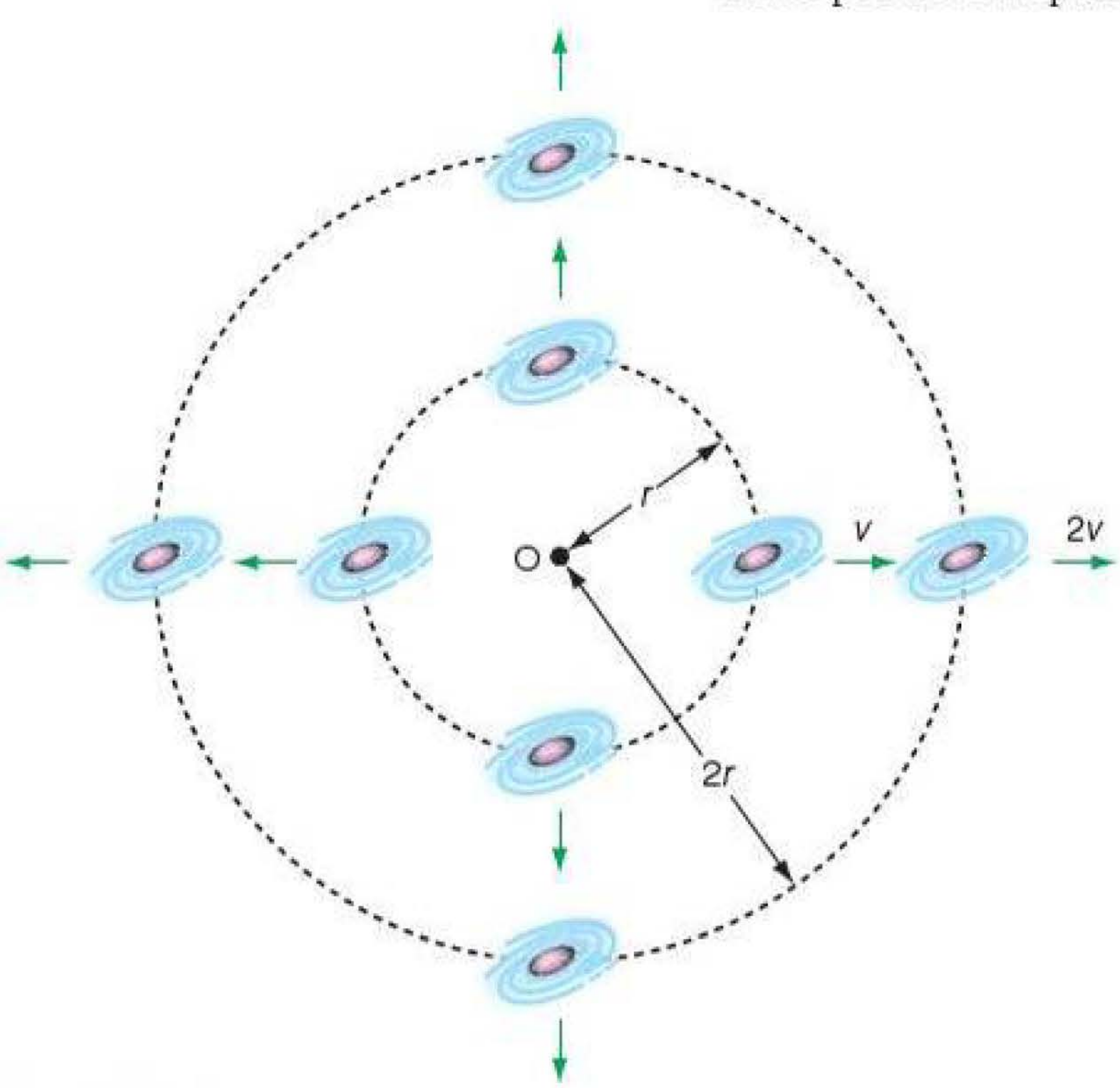


Figure 13.40

Cosmologists now accept the Big Bang theory, which suggests that the Universe originated about 13.8 billion years ago. All the matter we see in the Universe exploded at one point and has been travelling outwards ever since. In the first few seconds after the Big Bang, the Universe was extremely hot, with temperatures in excess of 10^{11} K. As the Universe cooled, atoms of hydrogen and helium were formed. Over billions of years, the force of gravity acted on this matter to pull it together into the stars and galaxies that we see today.

Calculations on the early state of the Universe lead us to think that, in the time between 10 seconds and 20 minutes after the Big Bang, the temperature of the Universe was hot enough to fuse hydrogen into helium, in the same way that fusion takes place in stars. These calculations suggest that the early Universe was composed of about 75% hydrogen and 25% helium, together with traces of other elements, such as deuterium, $^{2}_{1}H$, and lithium, $^{7}_{3}Li$. Observation of some of the Universe's older objects have confirmed that hydrogen and helium are present in the ratio of 75% to 25%, providing support for the Big Bang theory.

Cosmic microwave background radiation

A further piece of evidence to support the Big Bang theory was provided by the discovery of background radiation, which comes uniformly from all directions. After about 350 000 years the Universe had cooled to a temperature of about 3000 K. So the Universe was full of black-body radiation associated with matter at that temperature. As the Universe expanded, it cooled, and the wavelength of that background radiation has shifted to much longer wavelengths. The background radiation peaks at a wavelength of 1.8 mm, which corresponds to a background temperature of space of about 2.7 K.

Age of the Universe

If we assume that the Universe has been expanding at a constant rate, we can use Hubble's constant to estimate its age. The distance a galaxy has travelled since the origin of the Universe is given by

$$\text{distance} = \text{speed} \times \text{time}$$

or

$$d = vt$$

(assuming that the speed has been constant). But Hubble's law says that

$$v = Hd$$

or

$$d = v \times \frac{1}{H}$$

So the age of the Universe is approximately $t = \frac{1}{H}$.
Because $H = 2.2 \times 10^{-18}\,\text{s}^{-1}$, the age of the Universe is

$$t = \frac{1}{H} = \frac{1}{2.2 \times 10^{-18}\,\text{s}}$$
$$= 4.5 \times 10^{17}\,\text{s}$$
$$= 14.46 \text{ billion years}$$

The accepted value of the Universe's age is 13.8 billion years.

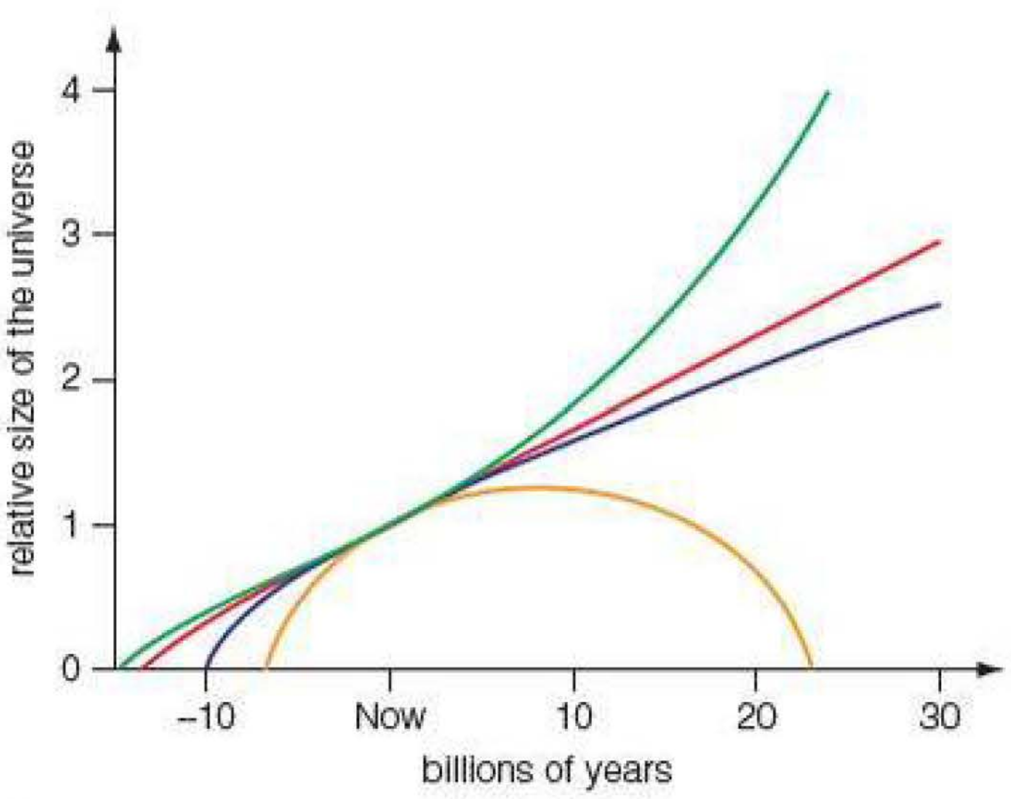


Figure 13.41

The expansion of the Universe is not in question but the rate of expansion is still uncertain and depends on the amount of matter present in the Universe. Observations seem to suggest that while the rate of expansion of the early Universe was slowed because of gravity, the rate of expansion now and in the future is uncertain. If there is enough matter in the Universe, it will reach a maximum size, slow down and reverse, shown by the yellow curve in Figure 13.41. Recent observations indicate this is not the case. If the density of the Universe is a critical density then the rate of expansion will gradually slow down until the expansion stops. In Figure 13.41 the blue curve showing this will gradually get ever closer to horizontal. Slightly less than the critical density and the rate of expansion of the Universe will slow down over a longer period of time and may never stop. This is shown by the red curve in Figure 13.41. However, as suggested previously, some of the most recent measurements show that the rate of expansion of the Universe is increasing and it is suggested that some form of energy, known as Dark Energy, that is part of the fabric of space, is responsible. This is shown by the green curve in Figure 13.41. As yet we do not know the form of this Dark Energy or, indeed, if it exists. Whatever the future expansion of the Universe, the Figure 13.41 shows why the value for the age of the Universe obtained from the Hubble constant is not quite accurate because the calculation using the Hubble constant assumes a steady rate of expansion.

TEST YOURSELF

43 Outline the evidence for the Big Bang theory.

44 a) A group of galaxies lies at a distance of 200 Mpc from the Earth. Calculate the speed of recession of the group. Hubble's constant = $67.8\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$.

b) A group of galaxies is receding from Earth at a speed of $120\,000\,\text{km}\,\text{s}^{-1}$. Calculate the distance of the group from Earth.

45 A group of galaxies has a redshift of 0.22.

a) Calculate the speed of recession of the group.

b) Calculate the distance of the group away from Earth using the value of Hubble's constant = $20.7\,\text{km}\,\text{s}^{-1}\,\text{Mly}^{-1}$.

Quasars

Quasar A small, very distant object, which emits as much power as a large galaxy.

In the 1960s astronomers discovered a new type of object in the sky. It was given the name **quasar**, which is short for quasi-stellar radio sources. The first quasars we were first discovered because they were very intense sources of radio waves. Quasars puzzled astronomers because they appeared to be very luminous indeed and among the most distant objects in the Universe (because large redshifts were measured in their spectra), yet they appeared to be points of light – just like a star. Figure 13.42 shows a photograph of the nearest known quasar, 3C 273, taken through the Hubble Space Telescope. Although the quasar looks like a point when viewed directly through a telescope, its brightness causes a large image to be formed when a photograph is taken.

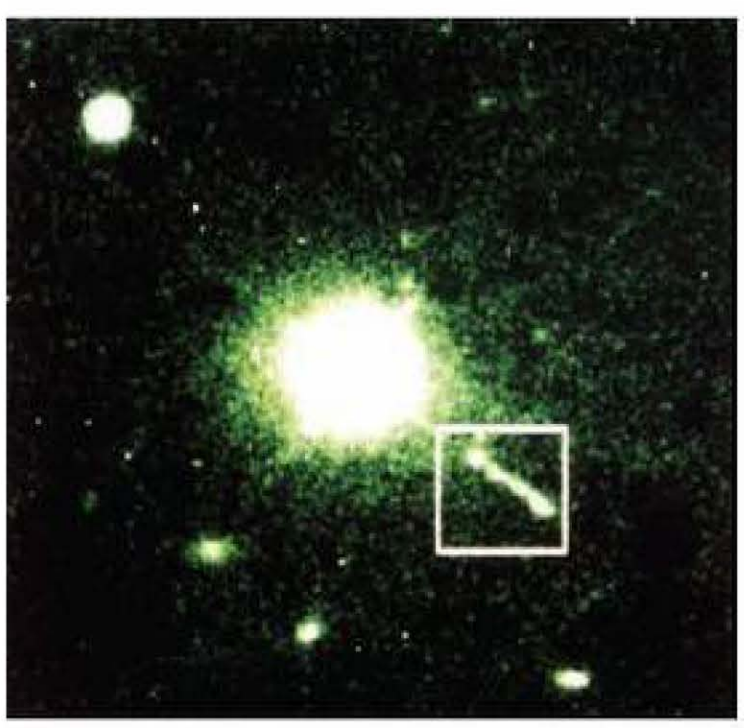


Figure 13.42 Quasar 3C 273 lies a distance of 1000 Mpc away from Earth, yet it outshines galaxies that lie about 20 Mpc from Earth, which you can see in the same photograph.

The lines in the photograph are caused by diffraction effects. The photograph also reveals a jet from the quasar pointing towards the bottom right hand corner.

Quasars are the most luminous objects seen in the sky. The apparent magnitude of 3C 273 is +13, yet its absolute magnitude is −27. This means that if 3C 273 were at a distance of 10 pc from us, it would appear about as bright as the Sun. 3C 273 emits much more light than a large galaxy such as the Milky Way, which contains 200–400 billion stars.

Although the nature of quasars was a mystery for a number of years, astronomers are now convinced that they are caused by massive black holes as large as 10^8 or 10^9 solar masses. The radius of the event horizon of such a massive black hole is the same order of magnitude as our Solar System. Some quasars are so distant (right at the limit of the visible

Universe) that we are seeing them as they were shortly after the Big Bang. They are young galaxies in the making. The density of matter in a quasar is so high that a black hole has formed, and the gravitational pull is so strong that matter is being swallowed up at a great rate. It is calculated that the brightest quasars are swallowing mass equivalent to 110 solar masses per year. As stellar matter falls into the black hole, the gravitational potential energy of the matter is transferred into electromagnetic waves. A black hole tearing up matter releases energy into electromagnetic waves at a much faster rate than thermonuclear fusion does in stars. Quasars are strong emitters of all wavelengths of electromagnetic waves, from radio waves through to X-rays and gamma rays.

Quasars do not live for long – we see quasars as they were billions of years ago. Once a quasar has devoured most of the matter in its vicinity, it then acts as a stable centre of an ordinary galaxy. The Milky Way has an enormous black hole at its centre, which provides a central massive area of gravitational attraction, which helps to keep stars such as our Sun in its stable orbit around the galactic centre.

TEST YOURSELF

46 Explain the origin of the name 'quasar'.

47 List three characteristics of quasars.

48 a) The quasar 3C 273 has an apparent magnitude of +13 and is 1000 Mpc from the Earth. Use the equation

$$M = m - 5\log\left(\frac{d}{10}\right)$$

to confirm that the absolute magnitude of 3C 273 is −27.

b) The absolute magnitude of a large galaxy such as the Andromeda galaxy, which contains over 10^{12} stars, is about −22. Compare the luminosity of 3C 273 with that of the Andromeda galaxy.

49 A large quasar has a mass of 10^{39} kg and an event horizon of radius 3×10^{12} m.

a) i) Show that the gravitational potential close to the event horizon is $-2 \times 10^{16}\,\text{J kg}^{-1}$.

ii) Now show that when a star with the mass of the Sun, 2×10^{30} kg, falls into the quasar from a large distance, the gravitational potential energy lost is about 4×10^{46} J.

b) i) On average, matter equivalent to 20 solar masses falls into the quasar each year. Assuming that 30% of the potential energy of the matter is transferred into electromagnetic waves, calculate the luminosity of the quasar in watts.

ii) The Sun has a luminosity of 4×10^{26} W. Compare the luminosity of the quasar with that of the Sun.

Exoplanets – are we alone?

It is difficult to know exactly how many galaxies there are in the Universe, but current estimates put that number at about 100 to 200 billion. Because each galaxy has hundreds of billions of stars, it is likely that the Universe contains more than 10 000 billion billion, or 10^{22}, stars. There are many stars like our Sun and it is estimated that there are billions of planetary systems similar to ours. Since the laws of physics hold everywhere in the Universe, it is highly probable that somewhere there is another Earth-like planet – but whether there are any life forms there is another question, to which we shall never know the answer.

Discovery of exoplanets

Exoplanet A planet outside our Solar System, in orbit around another star.

Exoplanet is the name given to a planet that lies outside our Solar System. An exoplanet is in orbit around another star. In recent years, new technologies have enabled the discovery of thousands of planets in orbit around other nearby stars. However, only a very small number of giant planets (of Jupiter's size or more) have been observed directly, because planets are much less bright than the stars they orbit. Most exoplanets that have been discovered have been detected by indirect means.

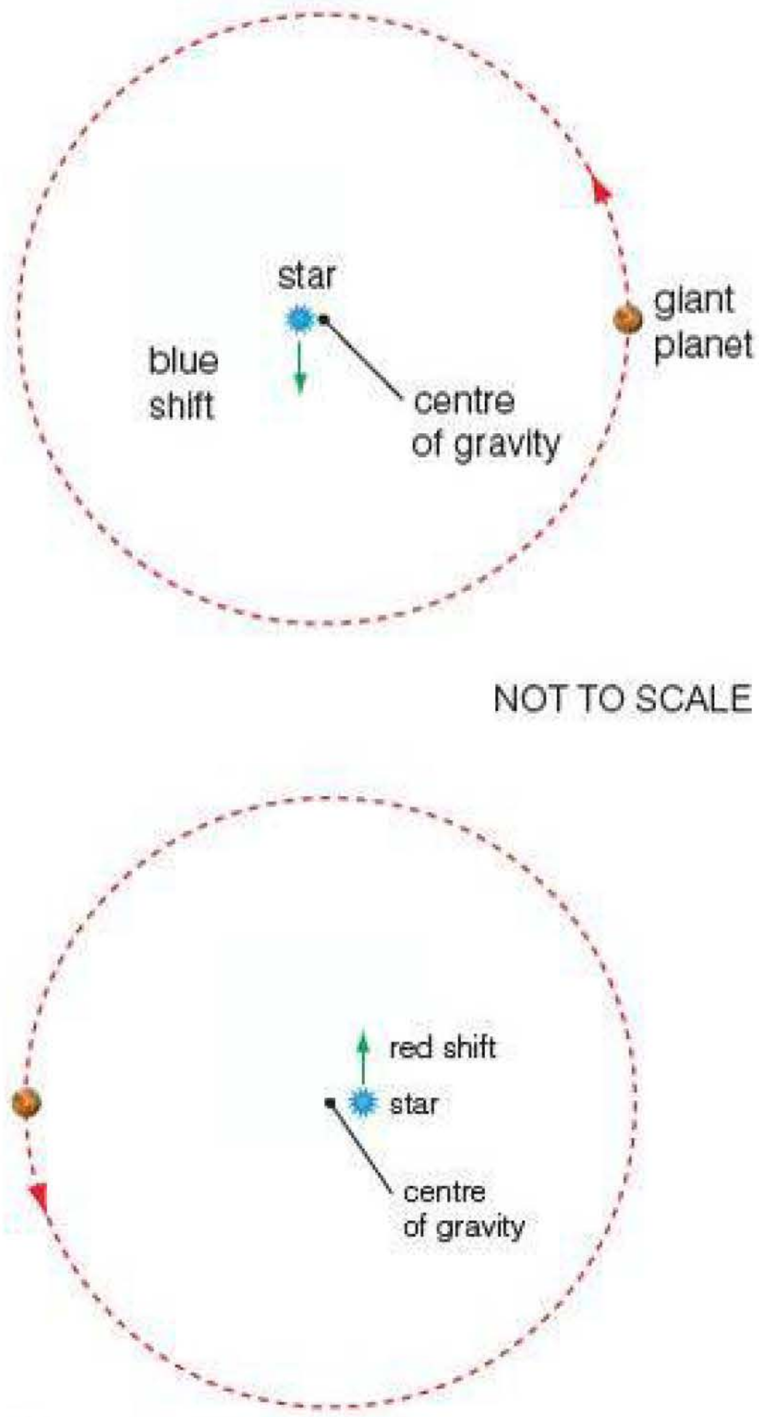


Figure 13.43

Variation in Doppler shift

Figure 13.43 shows (not to scale) a giant planet and a star. We usually say that planets orbit stars, but it is more accurate to say that a giant planet and a star orbit around a common centre of gravity. If a giant planet has a mass of about 0.001 times the mass of the star, the centre of gravity is likely to lie outside the star. Then there are times when the star will be moving towards the Earth, and times when the star will be moving away from the Earth. So there will be small changes to the spectrum of the star, which will be seen as a small redshift or a small blueshift.

This method detects the presence of large planets near to stars, but it does not enable the mass of the planet to be calculated, as we do not know its distance from the star.

Planetary transits

If an exoplanet crosses in front of a star's surface, then the brightness we see will drop by a small amount (Figure 13.44). For example, if a planet covers an area of 5% of the star's disc, then the light intensity would drop by 5%. A planetary transit is the most common method for an astronomer to detect a new exoplanet.

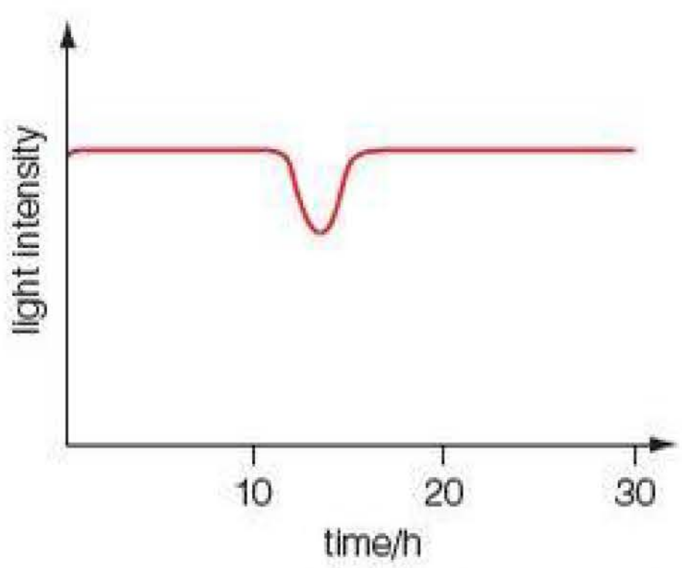


Figure 13.44 A light curve for a typical exoplanet transit across the surface of its parent star.

The light curve allows a rough estimate to be made of the planet's radius, and then the planet's mass – if we make some assumptions about its likely composition and density.

Direct imaging of exoplanets

HR 8799 is a young (30 million years old) main sequence star, located about 39 pc away from the Earth. It is about 1.5 times as massive as the Sun and 5 times as luminous. Figure 13.45 shows a direct image of an exoplanetary system – you can see four planets in orbit around the star. The light from the star has been digitally removed to enhance our view of the planets. All four planets in view are huge gas giants with approximately 10 times the mass of Jupiter. The inner planet, HR 8799e, takes about 45 Earth years to orbit the star and the outer one, HR 8799b, takes about 460 Earth years to orbit the star.

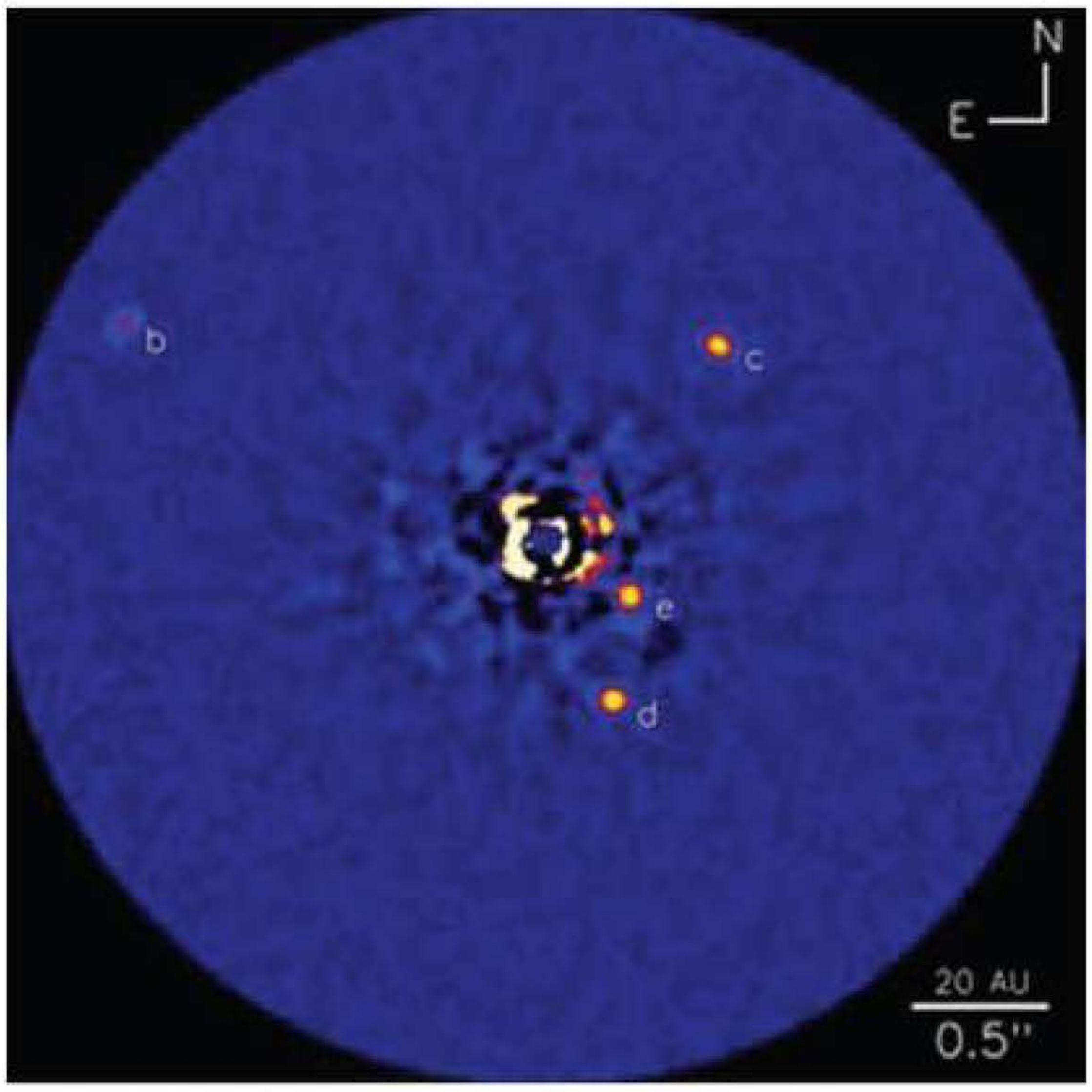


Figure 13.45 A rare direct image of exoplanets.

TEST YOURSELF

50 a) Give an account of three ways in which exoplanets can be detected.

b) Explain why the exoplanets detected are usually larger than the planet Jupiter.

51 When an exoplanet crosses in front of a star, the star's light intensity falls to 96% of its peak value. Calculate the ratio

$$\frac{\text{radius of exoplanet}}{\text{radius of star}}$$

52 The inner planet HR 8799e of the HR 8799 system orbits the star at a distance approximately 15 times the radius of Earth's orbit around the Sun. Discuss what other types of planet may yet be discovered in this system.

Practice questions

1 A reflecting telescope has an objective lens of focal length 120 cm and a diameter of 24 cm. The telescope eyepiece has a focal length of 2.4 cm and a diameter of 1.2 cm. The magnification of the telescope is

A 5 **C** 20

B 10 **D** 50

2 Deneb is a bright star that is 800 pc from Earth. It has an apparent magnitude of 1.2. Its absolute magnitude is

A −13.3 **C** −4.3

B −8.3 **D** −1.3

3 Merak and Ankaa are two stars that have the same black-body luminosities. Ankaa has a surface temperature of 4500 K and a radius $16R_s$, where R_s is the radius of the Sun. Merak has a surface temperature of 9000 K. What is Merak's radius in terms of R_s?

A $2R_s$ **C** $4R_s$

B $3R_s$ **D** $6R_s$

4 A crater on the Moon has a diameter of 500 m. The Moon is 400 000 km distant from Earth. What is the smallest telescope that will be able to resolve this crater, when viewed with light of wavelength 500 nm? Assume perfect viewing conditions.

A 4.0 m **C** 0.40 m

B 2.4 m **D** 0.024 m

5 A star has a surface temperature of 2800 K. The peak intensity of the radiation emitted from the star's surface will be in which part of the spectrum?

A infrared **C** green light

B red light **D** ultraviolet

6 A galaxy has a redshift of 0.185. Hubble's constant is $67.8\,\text{km s}^{-1}\,\text{Mpc}^{-1}$. The distance of the star away from us is

A 1200 Mpc **C** 820 Mpc

B 950 Mpc **D** 570 Mpc

7 Radio telescope A has a diameter of 64 m, and radio telescope B has a diameter of 45 m. The ratio $\dfrac{\text{gathering power of A}}{\text{gathering power of B}}$ is

A 1.2 **C** 2.0

B 1.4 **D** 2.4

Use the following information to answer questions 8, 9 and 10.

The table gives the surface temperature and luminosity of five stars; the luminosity listed is given in units relative to the Sun's luminosity.

	Surface temperature/K	Luminosity $\frac{L_{star}}{L_{Sun}}$
A	22000	0.026
B	40000	2×10^5
C	10000	90
D	3500	4000
E	2500	0.05

8 Which star has the smallest diameter?

9 Which star has the largest diameter?

10 Which star has an O class spectrum?

11 A refracting telescope is made from two lenses, an objective lens and an eyepiece.

a) Figure 13.46(a) shows light arriving at the objective lens of a refracting telescope. Copy and complete the diagram to show how a real image is formed in the focal plane of the lens. (2)

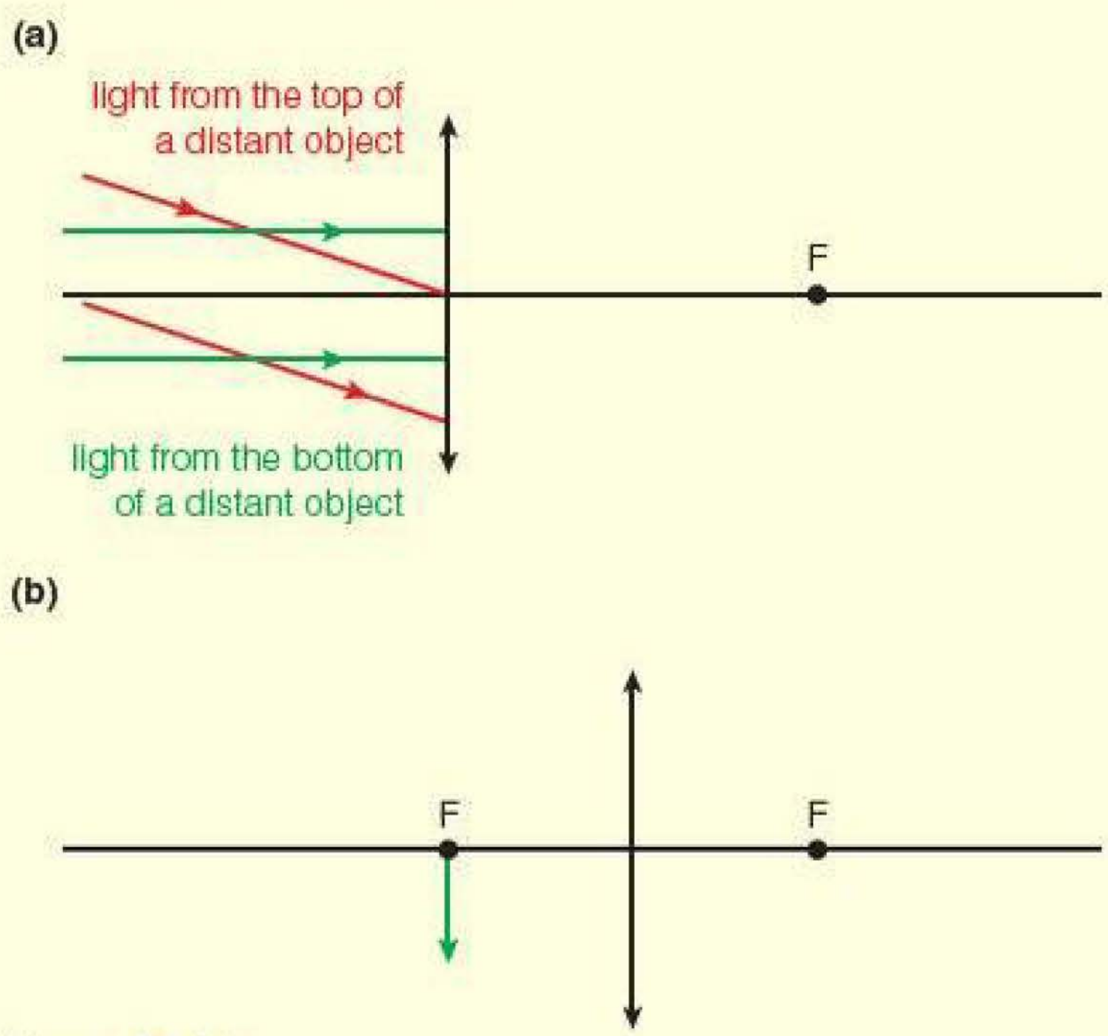


Figure 13.46

b) Figure 13.46(b) shows a real image in front of the eyepiece of the refracting telescope. Copy and complete this ray diagram to show how a virtual image is seen at infinity. Mark in the position of the eye to see this image. (3)

c) The telescope has a length of 2.28 m. Calculate the focal length of the eyepiece that would give the telescope a magnification of 75. (2)

d) Refracting telescopes tend to be affected by chromatic aberration. Explain what causes chromatic aberration. (2)

12 a) Draw the ray diagram for a Cassegrain telescope. Your diagram should show the paths of two rays, initially parallel to the principal axis, as far as the eyepiece. (2)

b) i) Chromatic aberration can be a problem when you use a refracting telescope. Why does a reflecting telescope reduce problems from chromatic aberration? *(1)*

ii) Spherical aberration can be a problem if a reflecting telescope has a concave spherical mirror. Draw a diagram to illustrate spherical aberration caused by a spherical mirror. *(1)*

iii) Explain how telescope makers avoid the problem of spherical aberration. *(1)*

c) A reflecting telescope has a primary mirror with a diameter of 0.30 m. Calculate the minimum angular separation that could be resolved by this telescope when observing point sources of light of wavelength 670 nm. *(2)*

d) This is a gap between the A and B rings in Saturn's ring system. This is called the Cassini division after its discoverer. The division is 4800 km wide, and Saturn is about 1400×10^6 km from Earth. What minimum diameter of telescope do you need to see the Cassini division clearly? *(3)*

13 a) Copy the axes in Figure 13.47 and add to them a sketch of the Hertzsprung–Russell diagram. In your sketch show the main sequence stars, giant stars and white dwarf stars. On the *y*-axis mark in an appropriate scale for the absolute magnitude of stars. *(3)*

b) i) Alioth is a bright star in the constellation Ursa Major. The black-body radiation curve for Alioth shows a peak at a wavelength of 2.7×10^{-7} m. Calculate Alioth's black-body temperature. *(2)*

ii) Alioth has a luminosity 110 times that of the Sun. Calculate the radius of Alioth. The Sun's surface temperature is 5800 K, and the Sun's radius is 6.96×10^8 m. *(3)*

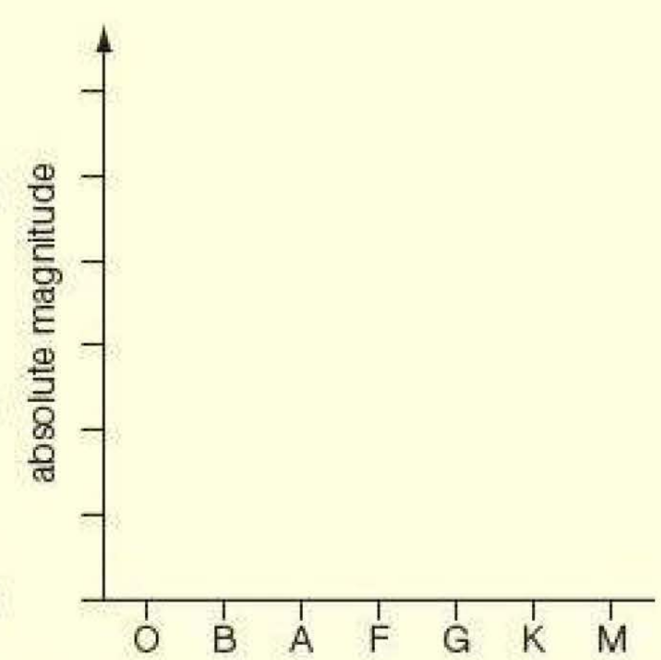


Figure 13.47

c) The spectrum of Alioth contains hydrogen Balmer absorption lines. Describe how hydrogen Balmer lines are produced in the spectrum of a star. *(6)*

14 Some galaxies, known as Seyfert galaxies, have very active centres. These are very similar to quasars, because astronomers think that they have supermassive black holes at their centres.

a) Explain what is meant by the 'event horizon' of a black hole. *(1)*

b) i) A black hole has a mass 8×10^7 times that of the Sun. Calculate the radius of the event horizon. *(2)*

ii) Calculate the average density of matter inside the event horizon. *(2)*

15 Alnilam and Betelgeuse are bright stars in the constellation of Orion. Some properties are summarised in the table below.

Star	Alnilam	Betelgeuse
Absolute magnitude	−6.4	−6.1
Apparent magnitude	1.7	0.4
Black-body temperature/K	26 200	3300

a) Explain what is meant by the terms

i) apparent magnitude (1)

ii) absolute magnitude. (1)

b) Which of the two stars is closer to Earth? Explain your answer. (1)

c) i) Calculate the wavelength of the peak intensity in the black-body radiation curve of Alnilam. (2)

ii) Sketch the black-body curve for Alnilam, using relative intensity on the y-axis and wavelength in nm on the x-axis. Label the x-axis with a suitable scale. (3)

d) Analysis of the light from both stars shows prominent absorption lines in their spectra.

i) To which spectral class does Alnilam belong? (1)

ii) The spectrum of Alnilam shows prominent Balmer absorption lines due to hydrogen. State the other element responsible for prominent absorption lines in the spectrum for Alnilam. (1)

iii) Explain why Betelgeuse does not show Balmer lines in its spectrum. (1)

e) Betelgeuse and Alnilam have very similar absolute visual magnitudes, as shown in the table above. However, Alnilam has a luminosity (power) 375 000 times that of the Sun, in comparison with Betelgeuse's luminosity, which is 120 000 times that of the Sun. Account for the differences between the luminosities and visual magnitudes of the stars. (2)

16 Reflecting telescopes are now more commonly used by professional astronomers than refracting telescopes. Explain what advantages reflecting telescopes have over refracting telescopes. (6)

17 Different types of telescope are used to detect different parts of the electromagnetic spectrum, from radio waves to X-rays. Discuss with reference to three parts of the electromagnetic spectrum the factors that should be taken into account when deciding where to position the telescope and when deciding on the size of the telescope. (6)

18 3C 48 is a quasar that lies in the constellation of Triangulum.

a) 3C 48 has a redshift of 0.367. Calculate the distance of 3C 48 from Earth, stating an appropriate unit. Hubble's constant is $65\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. (4)

b) i) The first quasars were discovered in the 1960s. What property of quasars led to their discovery? (1)

ii) Quasars are the most luminous objects in the Universe. Explain the nature of quasars and why they are so luminous. (3)

19 A group of galaxies seen in the constellation of Hydra shows a redshift of 0.048.

a) i) Explain what is meant by 'redshift'. (1)

ii) Calculate the velocity of the galaxies in Hydra. (2)

iii) Estimate the distance from Earth of the galaxies using Hubble's law. *(1)*

b) A type 1a supernova was detected recently in one of the galaxies. Figure 13.48 shows the typical light curve for a type 1a supernova.

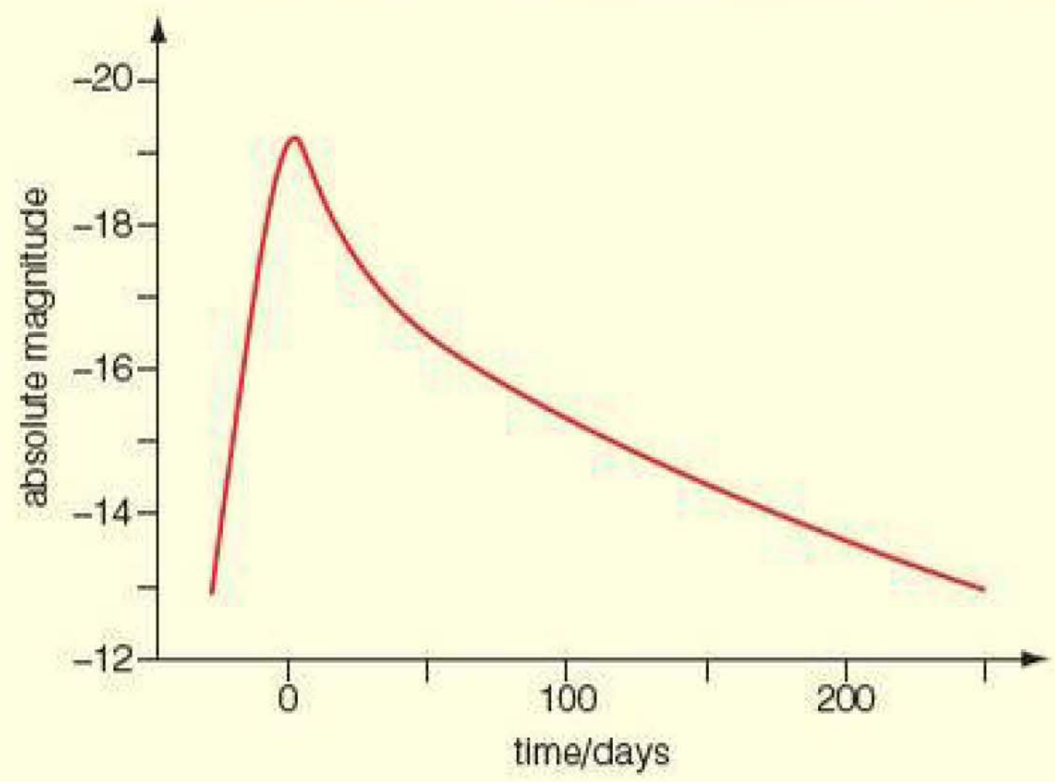


Figure 13.48

i) With reference to Figure 13.48, explain why type 1a supernovae may be used as standard candles to determine distances. *(2)*

ii) The peak value for the apparent magnitude of the supernova was 17.3. Use this information to calculate the distance to the galaxies from Earth. *(3)*

20 **a)** Draw a ray diagram for an astronomical refracting telescope in normal adjustment. Your diagram must show the paths of three non-axial rays through both lenses. *(3)*

b) A refracting telescope has a length of 2.05 m and an angular magnification of 40. Calculate the focal lengths of the eyepiece and of the objective lens. *(2)*

c) Jupiter has a diameter of 7.0×10^4 km and is a distance of 7.8×10^8 km from the Earth. Calculate the angle subtended by Jupiter when viewed through this telescope. *(2)*

d) Refracting telescopes can suffer from chromatic aberration. Draw a ray diagram to show how chromatic aberration can occur when light passes through a lens. *(2)*

21 Albireo A and Allbireo B form a bright double star in the constellation Cygnus. Allbireo A is a red giant star and Albireo B is a green-blue main sequence star. The table below summarises some of the properties of the stars.

Star	Albireo A	Albireo B
Absolute magnitude	−2.5	−0.3
Apparent magnitude	3.1	5.3
Diameter/10^3 km	50000	2000
Black-body temperature/K	4300	12900

a) Explain the terms 'main sequence star' and 'red giant'. (2)

b) Calculate the distance from Earth to Albireo, giving an appropriate unit. (3)

c) By using Stefan's law, show that the ratio below is about 8. (3)

$$\frac{\text{luminosity of Albireo A}}{\text{luminosity of Albireo B}}$$

d) Show that your answer to (c) is consistent with the stated absolute magnitudes of the stars. (2)

22 Figure 13.49 shows a computer-coloured image of radio emissions from Cygnus A, which is one of the strongest radio sources in the sky. Two jets emerge from either side of a giant black hole at the centre of the galaxy. These jets probably extend beyond the width of the host galaxy. When material from the jets is slowed down by the surrounding medium, radio waves are emitted. The strongest areas of emission are seen as the bright lobes on either side of the image. The radio telescopes that recorded this image detected waves with a wavelength of 0.15 m.

a) Use the scale on the diagram to determine the smallest distance that you can resolve in the image. Express your answer in Mly. (1)

b) The galaxy is about 600 Mly from Earth. Use your answer to part (a) to determine the smallest angle that the radio telescope can resolve at this wavelength. (2)

c) The resolution of radio telescopes can be improved by connecting together two or more telescopes separated by a large distance. Use your answer to part (b) to estimate the effective diameter of the radio telescopes used to produce this image. (2)

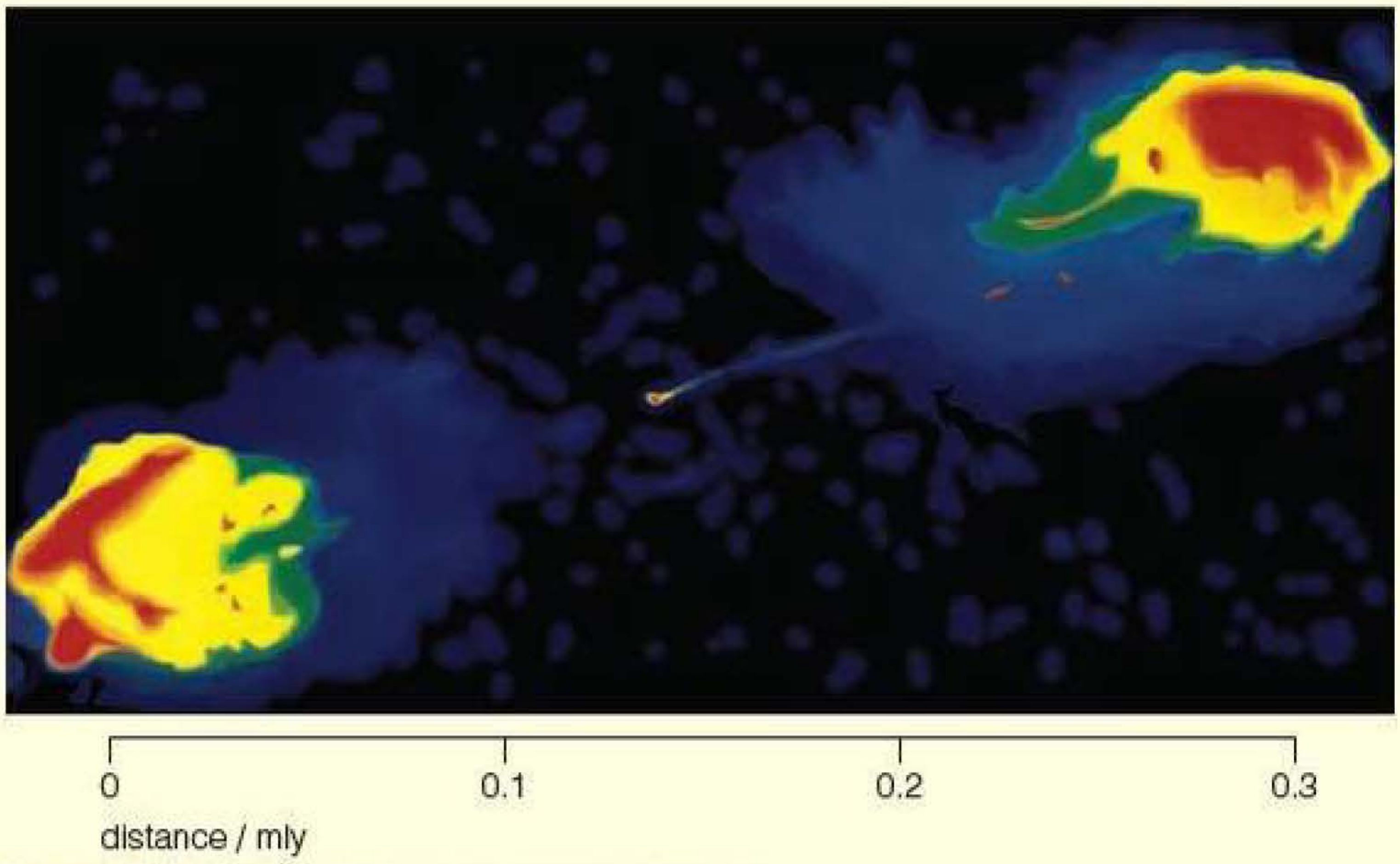


Figure 13.49 Radio emissions from Cygnus A.

Stretch and challenge

23 A red giant has a radius 200 times that of the Sun, and a surface temperature half that of the Sun.

a) Use Stefan's law to show that the luminosity of the red giant is 2500 times that of the Sun.

b) The absolute magnitude of the Sun is +4.6. Calculate the absolute magnitude of the red giant.

24 When a galaxy is moving away from us close to the speed of light, the wavelength of light, λ_0, that we observe is given by

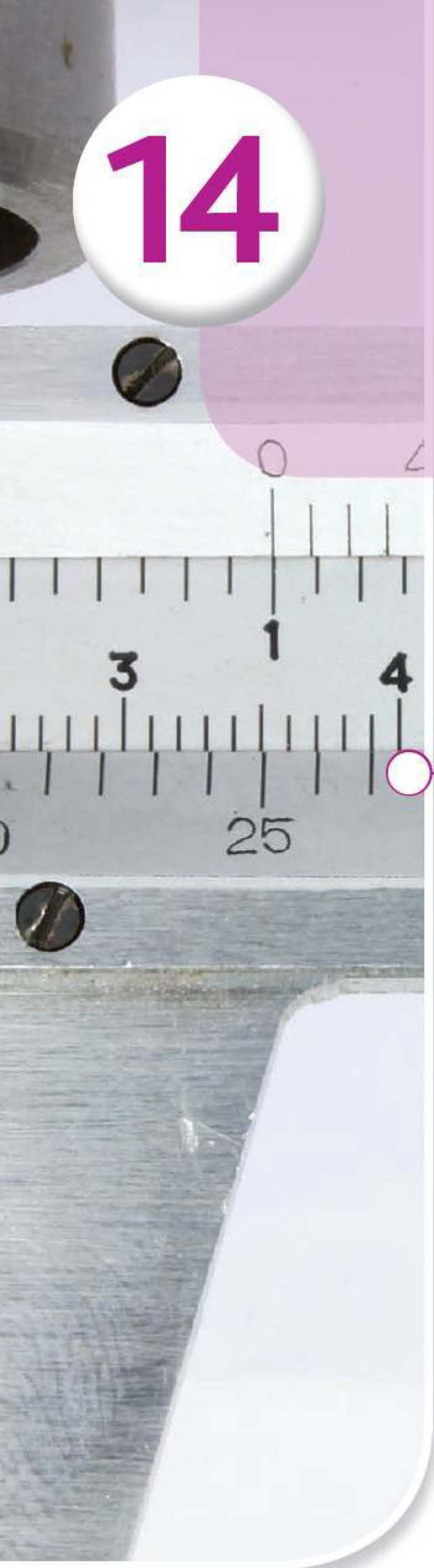
$$\lambda_0 = \lambda_S \frac{\left(1+\frac{v}{c}\right)}{\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}}$$

where λ_s is the wavelength of the light emitted by the galaxy, v is the speed of the galaxy and c is the speed of light.

Show that the redshift $z = \frac{\Delta\lambda}{\lambda_S}$ is given by

$$z = \frac{\left(1+\frac{v}{c}\right)}{\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}} - 1$$

The largest redshift seen for a quasar is about 7. Calculate the speed of recession of such a quasar.



14 Maths in physics

By now you should be familiar with the maths skills you used in Book 1. Some of these are covered here briefly, but most of this chapter concerns maths skills covered in Book 2. We will look again at the following:

- arithmetic skills
- handling data
- exponential and logarithmic functions
- overview of graphs
- trigonometry.

Arithmetic skills

Standard prefixes for units

Prefixes are used for very small or very large measurements. Table 14.1 is a reminder of these standard prefixes.

Table 14.1 Standard prefixes.

Factor of 10	Prefix name	Symbol	Example	Example name
10^{-15}	femto	f	fm	femtometre
10^{-12}	pico	p	ps	picosecond
10^{-9}	nano	n	nm	nanometre
10^{-6}	micro	μ	μg	microgram
10^{-3}	milli	m	mm	millimetre
10^{-2}	centi	c	cl	centilitre
10^{3}	kilo	k	kg	kilogram
10^{6}	mega	M	MJ	megajoule
10^{9}	giga	G	GW	gigawatt
10^{12}	tera	T	TW	terawatt

Ratios and proportion

Ratios compare one quantity with another. Ratios are shown as two quantities separated by a colon. For example, a ratio of 10 neutrons to 12 protons in a nucleus is shown as 10 : 12, or simplified to 5 : 6.

You can use ratios to estimate the effect of changing variables on your measurements or calculations. For example, using an equation you can decide how doubling or halving one factor affects things.

When two variables are in proportion to each other, the ratios of the two quantities are always the same. For example, if y is proportional to x, then if x is doubled, so is y. The ratios are the same, that is $y:x$ and $2y:2x$.

EXAMPLE

Using ratios

1 Calculate the ratio of the period of two pendulums, one of length 1.5 m and one of length 2.5 m.

Answer

The equation linking period and length is $T = 2\pi\sqrt{\frac{l}{g}}$, where T is the period and l is the length of the pendulum. So for the two pendulums:

$$T_{1.5} = 2\pi\sqrt{\frac{1.5}{g}} = 2\pi\sqrt{\frac{1}{g}}\sqrt{1.5} \qquad \text{(i)}$$

and

$$T_{2.5} = 2\pi\sqrt{\frac{2.5}{g}} = 2\pi\sqrt{\frac{1}{g}}\sqrt{2.5} \qquad \text{(ii)}$$

Dividing equation (i) by equation (ii) eliminates $2\pi\sqrt{\frac{1}{g}}$ and gives

$$\frac{T_{1.5}}{T_{2.5}} = \frac{\sqrt{1.5}}{\sqrt{2.5}} = 0.77$$

This can be written as a ratio, $T_{1.5} : T_{2.5} = 0.77 : 1$.

2 Derive an expression relating the orbital period of one planet to its orbital radius, and the orbital period and orbital radius of another planet orbiting the same star.

Answer

Kepler's third law states that (period of orbit)^2 is proportional to (radius of orbit)^3, or $T^2 \propto r^3$. So dividing the relations for planets 1 and 2 we can write

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

where r_1 and r_2 are the radii of the orbits of planets 1 and 2, and T_1 and T_2 are the orbital time periods of the two planets. To calculate the orbital period, you would rearrange this equation, giving

$$T_1^2 = \frac{T_2^2 r_1^3}{r_2^3}$$

Inverse square law

The inverse square law applies when $y = \frac{k}{x^2}$. Inverse square relationships include the following:

- the gravitational force acting on two masses m and M separated by a distance r

$$F = \frac{GMm}{r^2}$$

- the gravitational field strength g at a distance r from a mass M

$$g = \frac{GM}{r^2}$$

- the electrostatic force acting on two charges q and Q separated by a distance r in a vacuum

$$F = \frac{Qq}{4\pi\varepsilon_0 r^2}$$

- the intensity of light at a distance x from a light source

$$I = \frac{k}{x^2}$$

The graph in Figure 14.1 shows the relationship $y = \frac{k}{x^2}$.

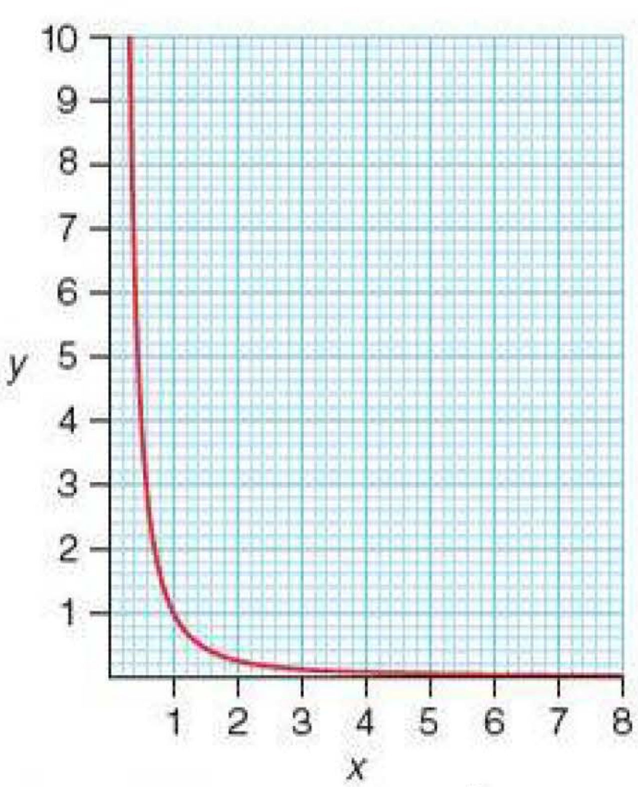


Figure 14.1 Graph of $y = \frac{k}{x^2}$, where k is a constant in this case k = 1 .

EXAMPLE

Gravitational force

What happens to the gravitational force between two masses, M and m, when the distance, r, separating them is doubled?

Answer

Using the equation given in the text, for the two masses

$$F_1 = \frac{GMm}{r^2} \text{ and } F_2 = \frac{GMm}{(2r)^2}$$

So

$$F_2 = \frac{GMm}{4r^2} = \frac{1}{4}F_1$$

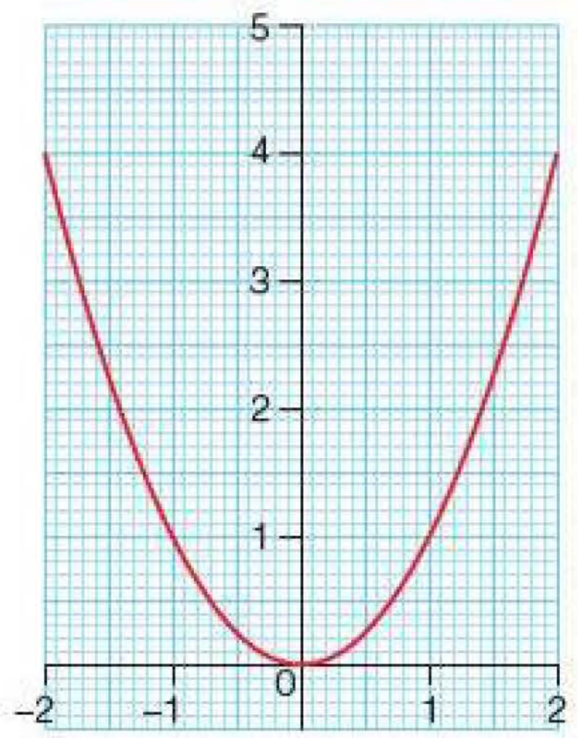


Figure 14.2 Graph of $y = kx^2$, where k is a constant in this case, k = 1.

The relationship $y = kx^2$

Another important relationship is $y = kx^2$. Examples of this relationship include the following:

- the kinetic energy of mass m travelling at speed v

  $$KE = \frac{1}{2}mv^2$$

- the centripetal force on mass m travelling at velocity v in a circular path of radius r

  $$F = \frac{mv^2}{r}$$

The graph in Figure 14.2 shows the relationship $y = kx^2$.

EXAMPLE

Centripetal acceleration

1 Compare the centripetal accelerations experienced by a person on a roundabout, sitting 2.2 m from the centre, when the roundabout increases its rotational speed from 4 to 10 revolutions per minute.

Answer

Because centripetal acceleration is given by $a = (2\pi f)^2 r$, acceleration is proportional to f^2. This means that the person's centripetal acceleration increases by a factor of $10^2/4^2$, or 6.25 times.

2 How does the centripetal acceleration of a person sitting 1.1 m from the centre compare with that of a person sitting 2.2 m from the centre?

Answer

Centripetal acceleration is found using $a = \omega^2 r$, so is directly proportional to r. If r is halved, the acceleration halves. The person sitting at a distance of 1.1 m from the centre experiences half the centripetal acceleration.

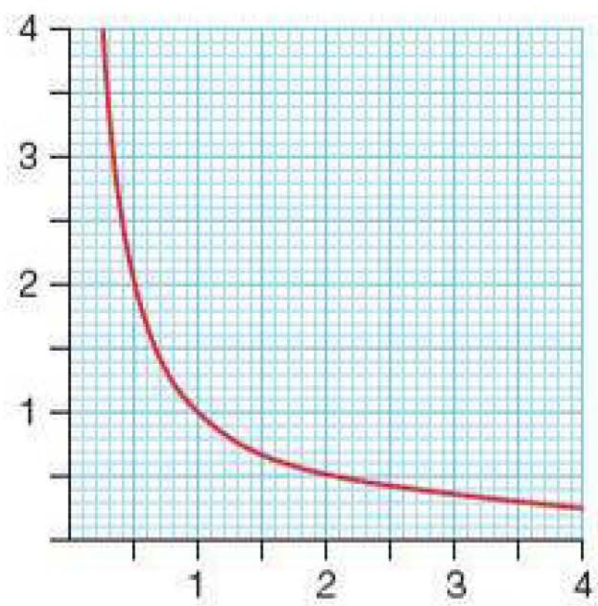


Figure 14.3 Graph of $y = \frac{k}{x}$, where k is a constant.

Inverse proportion

Another important type of relationship is where one factor is inversely proportional to another, $y = \frac{k}{x}$. Examples of this include:

- the gravitational potential in a radial field V is inversely proportional to the distance r from mass M

  $$V = \frac{GM}{r}$$

- the electric potential in a radial field V is inversely proportional to the distance r from charge Q

  $$V = \frac{Q}{4\pi\varepsilon_0 r}$$

- the pressure P is inversely proportional to the volume V for a fixed mass of ideal gas at a constant temperature (Boyle's law)

  PV = constant

The graph in Figure 14.3 shows the relationship $y = \frac{k}{x}$.

TEST YOURSELF

1 Calculate the volume of a sphere, with radius 3.4 mm (the volume of a sphere is $\frac{4}{3}\pi r^3$).

2 A transformer has 60 turns on the primary coil and 240 turns on the secondary coil.
   a) Calculate the ratio of primary turns to secondary turns.
   b) Use your answer to part (a) to estimate the secondary p.d. when the primary p.d. is 3.2 V.
   c) If the primary current is 30 mA, calculate the secondary current assuming no losses in the transformer.
   d) Without calculation, describe how your answers to parts (b) and (c) would change if the transformer had 1000 turns on the secondary coil and 4000 turns on the primary coil.

3 Compare the electrostatic force between two charged particles when
   a) the separation of the charged particles is tripled
   b) the charge of each particle is doubled
   c) the charge of each particle is halved *and* their separation is halved.

4 Compare the ratio of electrostatic force to gravitational force on two protons as their separation doubles.

5 Two planets are orbiting star X. Planet A has a mass of 1×10^{24} kg and is 5×10^{8} km from the centre of the star. Planet B has a mass of 1×10^{22} kg and is 5×10^{6} km from the centre of the star.
   a) Calculate the ratio of the gravitational force experienced between each planet and the star.
   b) Kepler's law states that T^2 is proportional to r^3. Calculate the ratio of the planets' time periods.

6 Sketch a graph, labelling the axes, of
   a) $Q = CV$ (with V on the x-axis and Q on the y-axis)
   b) $F = \frac{GMm}{r^2}$ (with r on the x-axis and F on the y-axis).

Handling data

Order of magnitude

We can make rough predictions or check an answer by using simple tricks. For example, the mass of an electron is about 2000 times smaller than the mass of a proton (or 10^3 times smaller to the nearest order of magnitude). However, for accurate calculations, we must use exact values. For an order-of-magnitude calculation, you should do or note the following:

- Express values in standard form. For example, the radius of a hydrogen atom is about 53 pm, which can be written as 5.3×10^{-11} m (or as 10^{-10} m to the nearest order of magnitude).

- Add (or subtract) powers of 10 when multiplying (or dividing). For example, $10^5 \times 10^9 = 10^{14}$ and $\frac{10^5}{10^9} = 10^{-4}$.
- Some values can be approximated easily. For example, π^2 is about 10.

EXAMPLE

Order-of-magnitude calculation

Use an order-of-magnitude calculation to calculate the radius of a lead nucleus, which has 207 nucleons.

Answer

The radius R of a nucleus can be estimated using the following formula:

$$R = R_0 A^{\frac{1}{3}}$$

where $R_0 = 1.2 \times 10^{-15}$ m. This is 10^{-15} as an order of magnitude and 207 is 10^2 as an order of magnitude.

So using the formula:

$$R = 10^{-15}\,\text{m} \times 100^{\frac{1}{3}}$$

$$= 5 \times 10^{-15}\,\text{m}$$

$$\approx 10^{-14}\,\text{m (to nearest order of magnitude)}$$

The measured radius of a lead nucleus is 6.66 fm ≈ 10 fm or 10^{-14} m to nearest order of magnitude. Thus the order of magnitude calculation gives a value of the same order of magnitude as the measured value.

Significant figures

Significant figures (s.f.) are numbers that tell you something useful. For example, the mass of an electron has been measured to be $9.10938291 \times 10^{-31}$ kg, but is usually quoted as 9.11×10^{-31} kg (3 s.f.). Some values used in nuclear calculations may be given to six significant figures, but most constants you use are quoted to three significant figures.

When you do a calculation, do not round the answer down to a small number of significant figures too early. Complete the calculation, then express the answer to the appropriate number of significant figures.

To work out the number of significant figures in a number, count up the number of digits, and remember the following points:

- Zeros between non-zero numbers are significant (e.g. 3405 has four significant figures).
- Zeros after the decimal point, or with the decimal point shown, are significant (e.g. 34.50, 3450 and 34.05 all have four significant figures).
- Leading zeros are not significant (e.g. 034.5 has three significant figures, and 0.00136 has three significant figures).
- Trailing zeros with no decimal point shown are significant (e.g. 3450 has four significant figures, and 0.04500 has four significant figures).
- It may be unclear if numbers like 6500 have been rounded to the nearest hundred. Use standard form (e.g. 6.5×10^3) or just write 6500 (to 2 s.f.).

Your final answer should have the same number of significant figures as the data values used. For example, $\frac{12}{3.2} = 3.75$. The answer has three significant figures but the data has only two significant figures. So the answer should have two significant figures, 3.8. Too many significant figures overstates the accuracy.

Probability and decay

Probability often measures the chance of something happening in a particular time. It does not mean this will definitely happen in that time.

Nuclear physicists say the probability of a nucleus decaying per unit time is a constant. The decay constant λ is written as

$$\lambda = \frac{\Delta N/N}{\Delta t}$$

where N is the number of nuclei and t is time. The time constant has units of s^{-1}.

Using the decay constant

The activity A is the number of undecayed nuclei N times the probability λ of one nucleus decaying per unit time. This is written as

$$A = \lambda N$$

The number of nuclei decaying per unit time is written as

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

(the minus sign indicates that the number of nuclei reduces), where ΔN is the change in the number of nuclei and Δt is the change in time.

The equation

$$\lambda N = -\frac{\Delta N}{\Delta t}$$

is used to predict a change in the number of nuclei, or the time taken for this change. But, because of random fluctuations, the equation cannot state exactly how many nuclei will decay, or which nucleus will decay. However, when N is very large, the predictions are more accurate because as the errors are of the order of $\pm\sqrt{N}$. For example, if a radiologist gives a dose of 10^{12} particles, the error is $\pm 10^6$, which is very small in percentage terms.

TEST YOURSELF

7 How many significant figures have these numbers got?

a) 5.67×10^{-8}

b) 939.551

c) 0.510 999

d) 3.00×10^{8}

8 Calculate the probability λ for a nuclear decay if the original sample has 1000 undecayed nuclei and after 3 minutes the number of undecayed nuclei is 960.

9 Make order-of-magnitude calculations for:

a) the volume of the Earth, which has a radius of 6378 km (the volume of a sphere is $\left[\frac{4}{3}\pi r^3\right]$

b) the mass of the Earth, which has an average density of $5540\,\text{kg}\,\text{m}^{-3}$.

10 Calculate these quantities to the appropriate number of significant figures:

a) the cross-sectional area of the Earth, radius 6378 km

b) the cross-sectional area of a hair of diameter 90 μm

c) the surface area of a spherical light bulb of diameter 5 cm.

Exponential and logarithmic functions

We usually express numbers using base 10. The logarithm of a number (in base 10) is the number to which we have to raise 10 to get that number.

For example, since $100 = 10^2$, we say that 100 is 10 to the power of 2. The log of 100 is 2 because 10 to the power of 2 is 100. We write this as

$$\log_{10} 100 = 2$$

Logs are rarely integers – for example, $\log_{10}(23.4) = 1.369$.

You will also come across logs in base e, called natural logarithms. The number e is an irrational number approximately equal to 2.718. Logs in base e are written as ln or $\log_e$.

Natural logarithms are used in calculations involving capacitors and radioactive decay.

Logarithm rules

Use these rules when using logarithms:

- $\log(a \times b) = \log a + \log b$
  For example
  $\log 2000 = \log(50 \times 40) = \log 50 + \log 40$
- $\log(a^b) = b \log a$
  For example
  $\log 81 = \log(3^4) = 4 \log 3$
- $\log(a/b) = \log a - \log b$
  For example
  $\log 3 = \log(12/4) = \log 12 - \log 4$

Exponentials or inverse logarithms

Logarithms and exponential terms are connected. The inverse of taking a logarithm is finding the exponential term.

If

$$x = 10^y$$

then

$$\log_{10} x = y$$

For example, $10^3 = 1000$, so $\log_{10} 1000 = 3$. This fits with the rules of logs:

$$\log 10^y = y \log 10$$

Since $\log 10 = 1$, this means that

$$\log 10^y = y$$

If

$$e^y = x$$

then

$$\ln x = y$$

For example, $e^{6.91} = 1000$, so $\ln 1000 = 6.91$.

Exponential terms are used in capacitor equations. For example, the charge Q on a capacitor at time t is given by

$$Q = Q_0 e^{-\frac{t}{RC}}$$

where Q_0 is the original charge, R is the resistance and C is the capacitance.

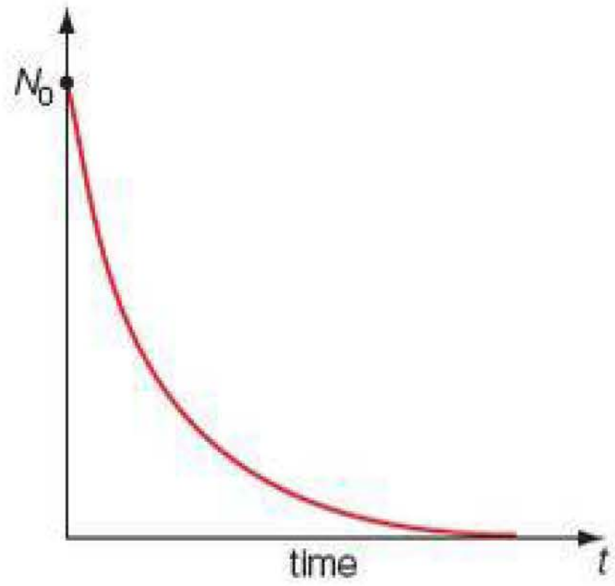


Figure 14.4 Graph of $y = N_o e^{-kt}$, where N and k are constants.

For radioactive decay, the activity can be calculated from

$$A = A_0 e^{-\lambda t}$$

where A_0 is the original activity and λ is the decay constant.

The graph in Figure 14.4 shows the exponential relationship $y = N_o e^{-kt}$.

Using the exponential rules with equations can be a very helpful way to analyse data. Consider the capacitor equation given above:

$$Q = Q_0 e^{\frac{-t}{RC}}$$

If we take natural logs of both sides we get

$$\ln Q = \ln(Q_0 e^{\frac{-t}{RC}})$$

$$= \ln Q_0 + \ln(e^{\frac{-t}{RC}})$$

But $\ln(e^{-\frac{t}{RC}}) = -\frac{t}{RC}$, so

$$\ln Q = \ln Q_0 - \frac{t}{RC}$$

When $\ln Q$ is plotted against t, the values should give a straight line with a gradient of $-1/RC$ and y-intercept of $\ln Q_0$. This can be far more useful than plotting an exponential function because as it is easier to see errors or anomalies on a straight-line graph.

Logarithms using calculators

The log button is for calculations to base 10, and the ln button is for calculations to base e.

To calculate logs using calculators:

- select the log or ln button
- type in the number
- press =

To calculate exponentials using calculators:

- press shift
- select the log (for 10^x) or ln button (for e^x)
- type in the number
- press =

Finding half-life

The half-life, $T_{\frac{1}{2}}$, of a decay process has elapsed when, for example, the activity of (or the number of nuclei in) a radioactive isotope falls to half its original value during that time period $T_{\frac{1}{2}}$.

We can link $T_{\frac{1}{2}}$ and the decay constant λ (defined above) as follows. Since

$$N = N_0 e^{-\lambda t}$$

after one half-life $T_{\frac{1}{2}}$ there will be $\frac{1}{2}N_0$ nuclei left. So

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{\frac{1}{2}}}$$

$$\frac{1}{2} = e^{-\lambda T_{\frac{1}{2}}}$$

which gives

$$\ln\frac{1}{2} = -\lambda T_{\frac{1}{2}}$$

$$\ln 2 = \lambda T_{\frac{1}{2}}$$

$$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

TEST YOURSELF

11 Use your calculator to find

a) log 12

b) ln 45

c) $e^{-3.4}$

d) $10^{2.1}$

12 A radioactive isotope has a decay constant of $0.03\,s^{-1}$. At $t = 0$, its activity is 48 Bq. Calculate the activity 35 s later.

13 Calculate the half-life of a sample if the decay constant $\lambda = 3.4 \times 10^{-2}$ s.

Overview of graphs

Using straight-line graphs

A straight-line graph has the form $y = mx + c$, where m is the gradient of the graph and c is a constant, which is the intercept of the line on the y-axis. If the line goes through the origin, then c is zero.

Where possible, we manipulate equations into the form $y = mx + c$ so that a straight-line graph can be drawn and anomalous points are more obvious.

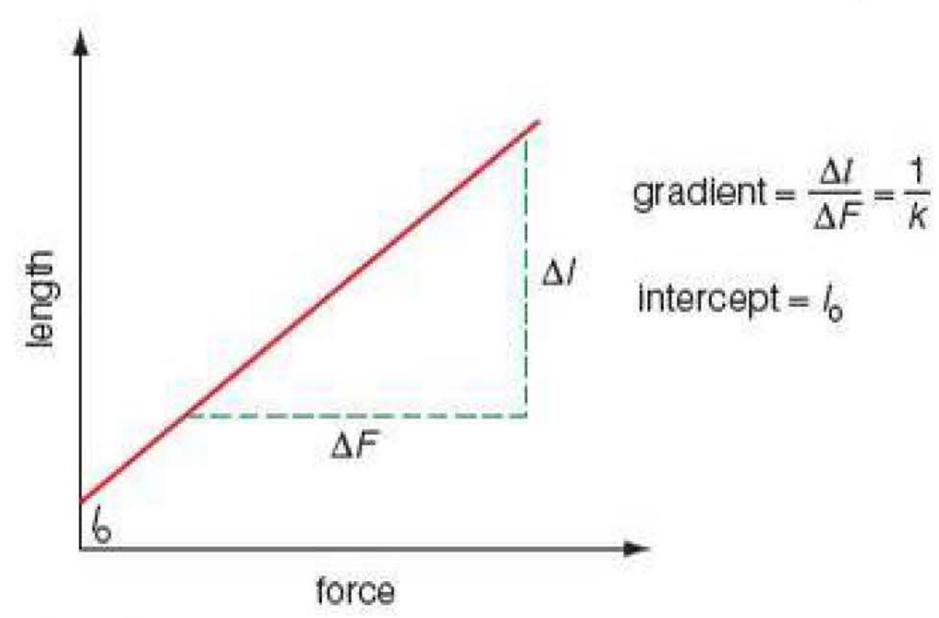


Figure 14.5 You should always use a large triangle when calculating the gradient of a straight-line graph.

For example, in Figure 14.5, the length, l of a spring of unstretched length, l0, and spring constant k is given by

$$l = \frac{1}{k} \times F + l_0$$

which, when compared with

$$y = mx + c$$

shows that a graph of l against F will have a gradient of $\frac{1}{k}$ and an intercept on the y axis at l_0.

Using curved graphs

Examples of curved graphs include a discharge curve for a capacitor. A curve of best fit should be smooth, with equal numbers of points above and below the line.

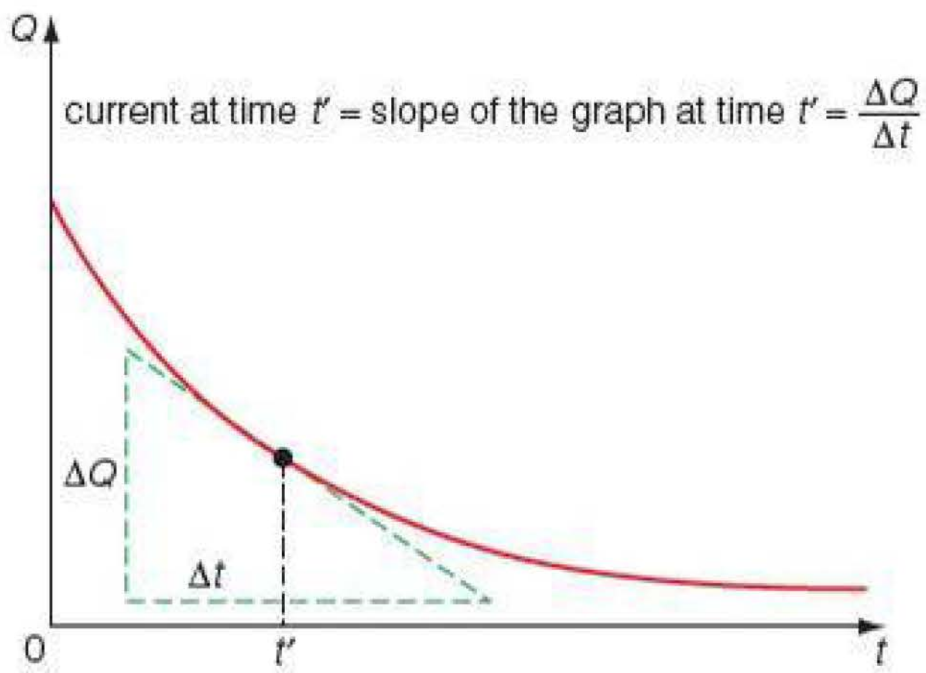


Figure 14.6 The gradient at a point on a curved graph.

To calculate the gradient at a point, draw a large triangle with the hypotenuse as the tangent to the curve at that point. The tangent to a curved line is parallel to the line at that point. The gradient is given by $m = \frac{\Delta y}{\Delta x}$. If time t is plotted on the x-axis, the gradient represents rate of change, with time, at that point.

Figure 14.6 shows an example of how to find the gradient at a particular point of a curved graph. Again, you should always use a large triangle to do this.

Remember that a curved graph shows a non-linear relationship, so the gradient changes. For example, the gradient at time t on a non-linear charge–time graph shows the current at that particular time.

Exponential and logarithmic graphs

Sometimes exponential relationships can be hard to plot on normal graph paper, because they can cover very wide ranges of values. Also, an exponential graph is curved, and therefore it can be more difficult to identify anomalous results. Plotting a log graph of an exponential function allows a straight-line graph to be produced, which shows errors more clearly and enables a linear relationship to be identified.

The two graphs in Figure 14.7 show current readings for a discharging capacitor. When I is plotted against t, as in Figure 14.7(a), the line is an exponential curve because

$$I = I_0 e^{-\frac{t}{RC}}$$

taking natural logs of both sides gives

$$\ln I = \ln I_0 - \frac{t}{RC}$$

When the natural logarithm of I is plotted against t, as in Figure 14.7(b), the result is a straight line with gradient $-1/RC$ (and the y-intercept is $\ell n\, I_0$).

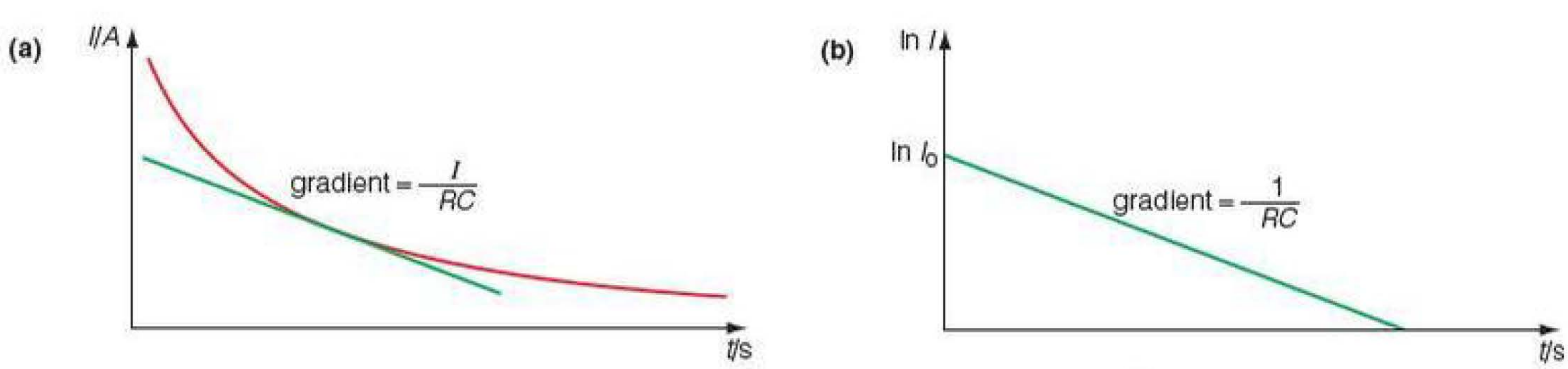


Figure 14.7 (a) The graph for a discharging capacitor is an exponential curve $I = I_0 e^{\frac{-t}{RC}}$. (b) using natural logs gives a straight-line graph $\ln I = \ln I_0 - \frac{t}{RC}$.

You can also plot logs directly onto logarithmic or log–linear graph paper. Lines on logarithmic paper are not evenly spaced because they are proportional to the logarithm of each number. You plot data directly onto the log graph paper without converting to logs first. If a data point lies

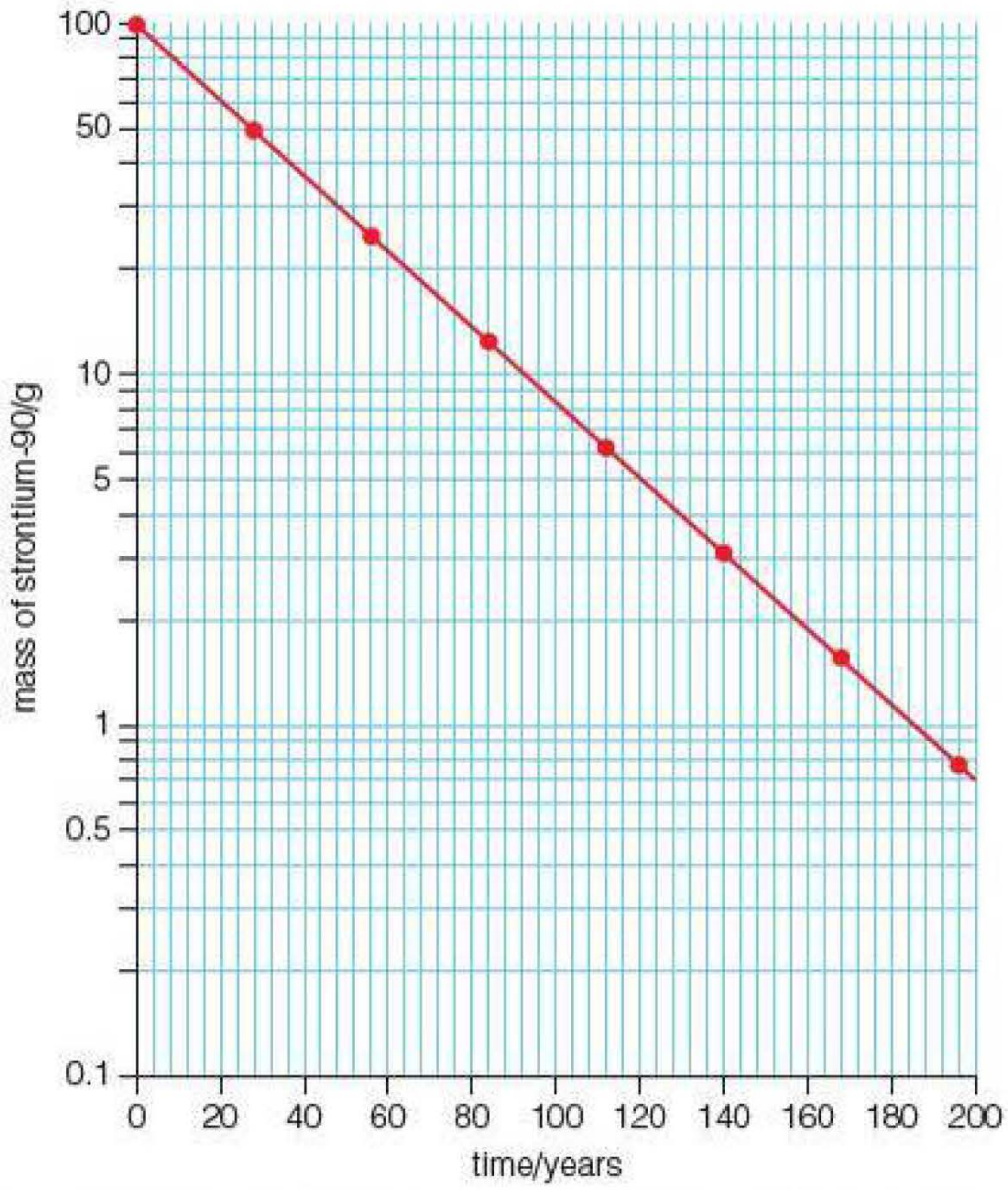


Figure 14.8 Data for radioactive decay plotted on log–linear paper gives a straight-line graph.

between increment 6 and 7, the value of the number you plot will also lie between 6 and 7 on the scale. You should get a straight-line graph if you have been asked to use log paper.

The graph in Figure 14.8 shows an example for radioactive decay, which is exponential, plotted directly onto log–linear graph paper. This gives a straight-line graph.

Calculations using graphs

The area under certain graphs has a specific physical meaning:

- The area under a graph of gravitational field strength, g, against distance from a planet, r, is equal to the change in gravitational potential, ΔV, when moving from one point to another.
- The area under a graph of electric field strength, E, against distance from a planet, r, is equal to the change in electric potential, ΔV, when moving from one point to another.
- The area under the graph of charge, Q, against p.d., V, gives the energy stored in a capacitor, ½QV. The shaded area in Figure 14.9 shows the extra energy stored, ΔQ, when the charge increases from Q to $Q + \Delta Q$.

Remember to use the units shown on the axes, taking special care with prefixes.

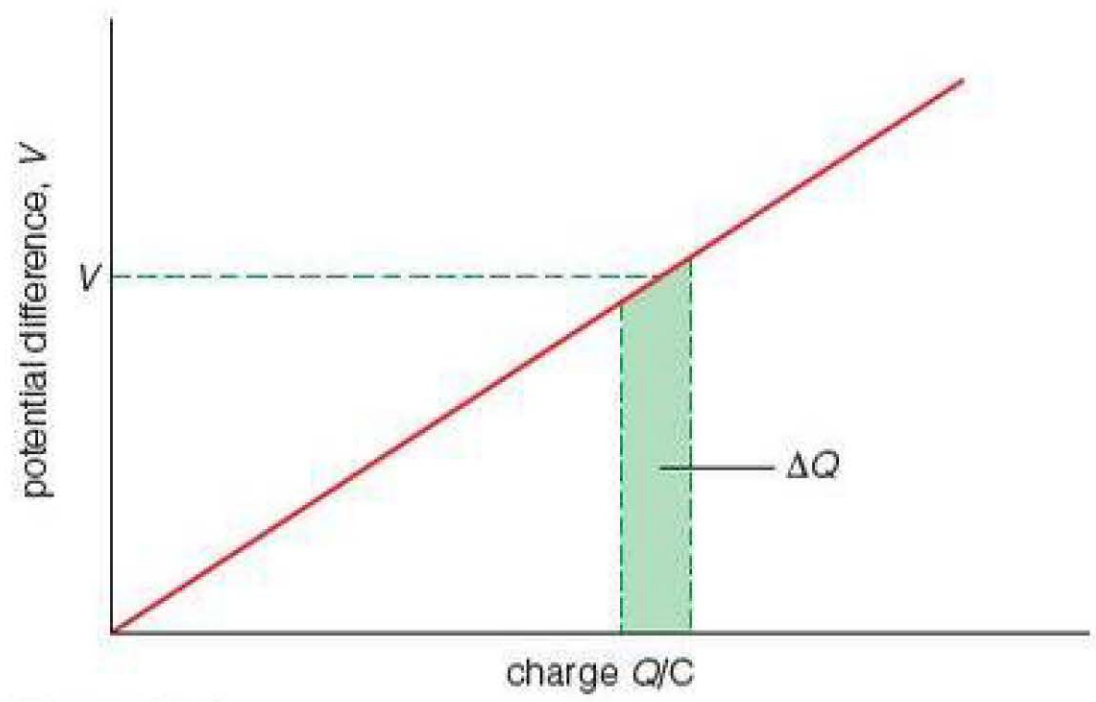


Figure 14.9

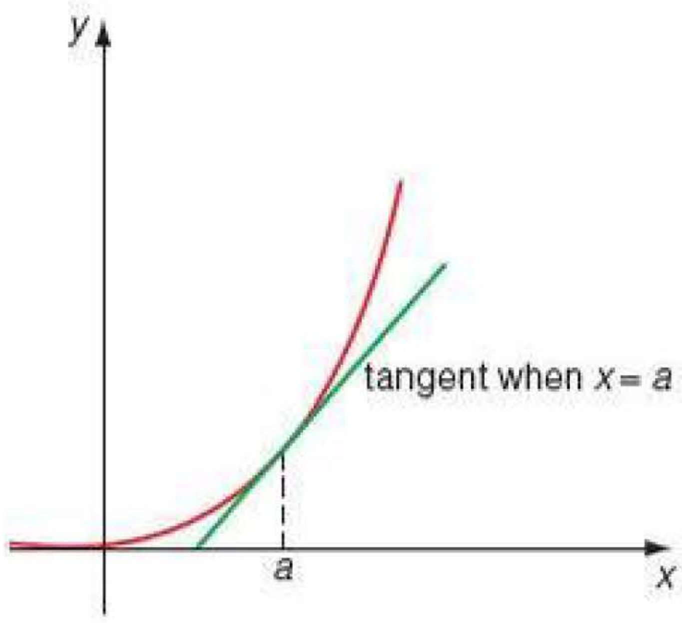


Figure 14.10 For a graph of $y = f(x)$, when $x = a$, then $y = f(a)$ and the gradient at that point is $df(a)/dx$.

Calculus and graphs

Calculus is a branch of mathematics that studies change. Differentiation is a technique used to workout a rate of change, and the main aim is to find the gradient of a graph at a particular point. You do not need to know how to differentiate, but it is useful to know how some functions are linked when differentiated.

The function that describes the shape of the curve can be differentiated. Substituting in a particular value of x allows you to calculate the gradient at that point. Figure 14.10 shows an example.

EXAMPLE

Gradient of graph using calculus

Calculate the gradient of the following graph when $x = 2$:

$$y = 4x^3 + 3x$$

Answer

Differentiating with respect to x gives

$$\frac{dy}{dx} = 12x^2 + 3$$

When x = 2 this gives

$$\frac{dy}{dx} = 12 \times 2^2 + 3 = 51$$

Integral calculus involves calculating the area under curved graphs by treating this area as a series of infinitely narrow strips that are summed together. The function that describes the shape of the curve can be integrated. Substituting two values of x between the limits of the integration calculation allows you to calculate the area under the graph between those points.

EXAMPLE

Area under graph using calculus

Calculate the area under the graph $y = x^2 - 4x + 5$ between the points $x = 2$ and $x = 5$ (Figure 14.11).

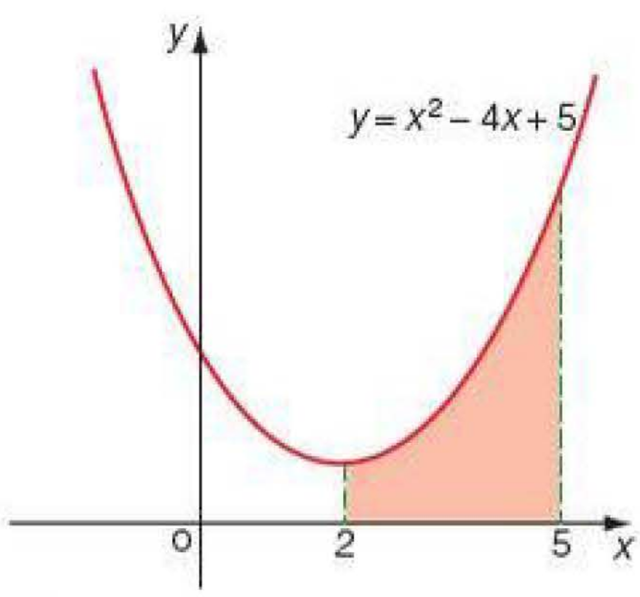


Figure 14.11

Answer

The function should be integrated and evaluated between the limits 2 and 5:

$$\int_2^5 y\,dx = \left[\frac{1}{3}x^3 - 2x^2 + 5x\right]_2^5$$

$$= \left[\frac{1}{3}(5)^3 - 2(5)^2 + 5(5)\right] - \left[\frac{1}{3}(2)^3 - 2(2)^2 + 5(2)\right]$$

$$= [41.67 - 50 + 25] - [2.67 - 8 + 10]$$

$$= 16.67 - 4.67$$

$$= 12.0$$

As an alternative to calculus, you can use a different graphs or spreadsheet modelling to investigate graphs. For example, the tangent to a displacement–time graph is equal to velocity ($v = \Delta s/\Delta t$). So if you have a displacement–time graph for a falling object, you could calculate the velocity at a particular time by measuring the gradient at that time.

TEST YOURSELF

14 Copy the blank table below. Read the *x* and *y* values of the quantities plotted in Figure 14.12, as accurately as possible, and write them in your table.

Period, *T*/s	Length, *L*/m

15 Explain what to plot so that each of the following data sets produces a straight-line graph. Explain the significance of the gradient and *y*-intercept in each case.

a) A graph to show a linear relationship connecting *R* and *A*, where $R = R_0 A^{\frac{1}{3}}$ (r_o is constant).

b) A graph to show a linear relationship connecting *I* and *t*, where $I = I_0 e^{-\frac{t}{RC}}$ (I_o, *R*, *C* are constant).

c) A graph to show a linear relationship connecting *T* and *l*, where $T^2 = 4\pi^2\left(\frac{l}{g}\right)$ (*g* is constant).

16 Labelling the axes, sketch a graph, of $N\phi = BAN\cos\theta$, with θ on the *x*-axis and $N\phi$ on the *y*-axis)

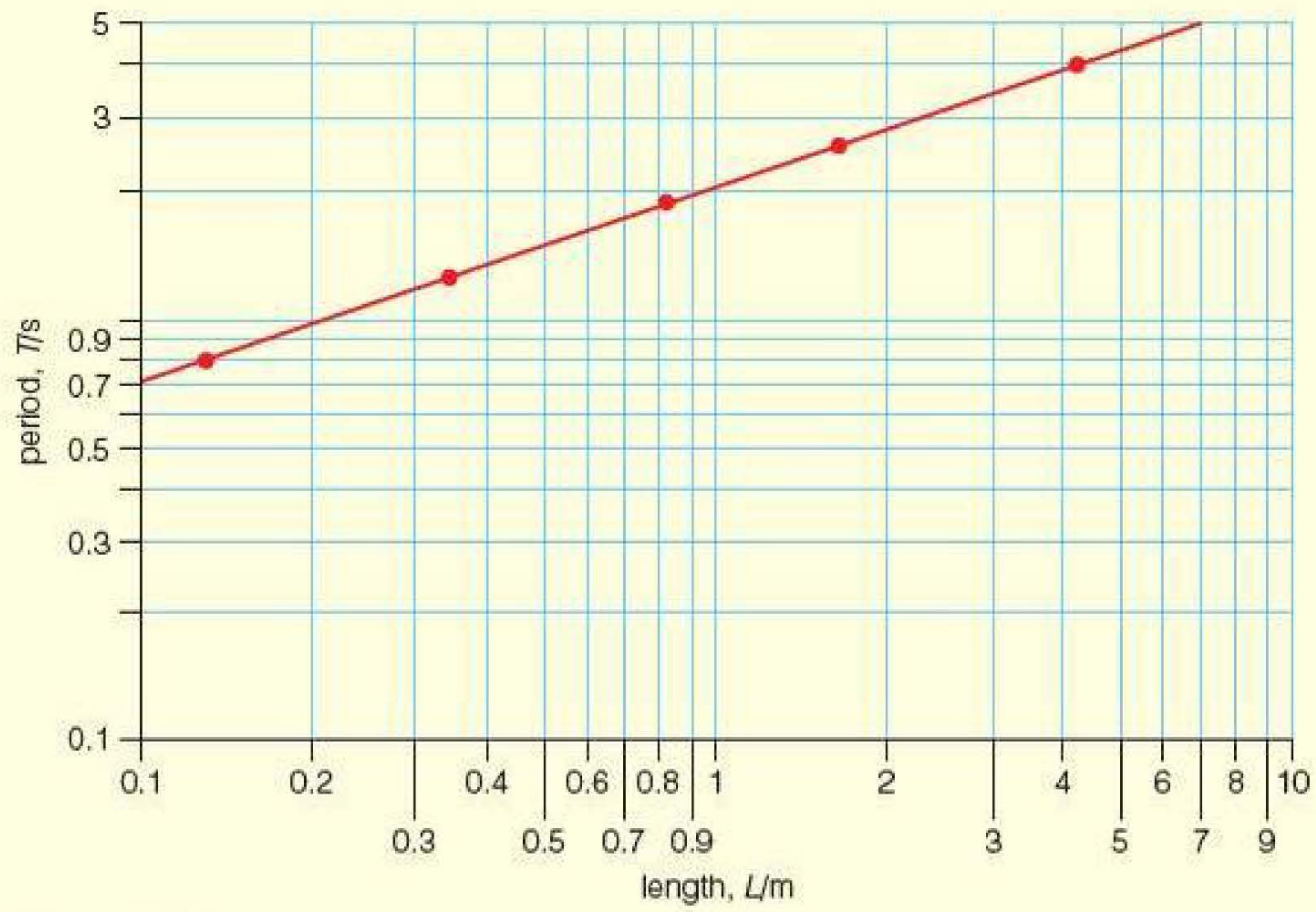


Figure 14.12

Trigonometry

Right-angled triangles are used to define three functions: sine, cosine and tangent. The sides of a right-angled triangle are labelled in relation to the angle as shown in Figure 14.13: opposite (O), adjacent (A) and hypotenuse (H).

You can calculate an unknown value for a right-angled triangle using:

- $\sin\theta = O/H$
- $\cos\theta = A/H$
- $\tan\theta = O/A$

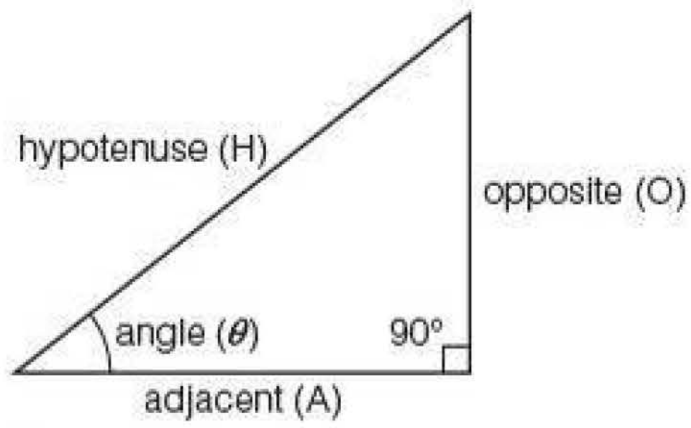


Figure 14.13

A useful rule involving sine and cosine is: $\cos^2\theta + \sin^2\theta = 1$.

For the vector triangle in Figure 14.13, where H is the length of the hypotenuse:

$$H^2 = H^2\cos^2\theta + H^2\sin^2\theta$$

Differentiating sine and cosine functions

If we differentiate a function, f(t), that depends on time t, we write the differential of the function as $\frac{df(t)}{dt}$. This is a quick way of writing the rate of change in function f with time t.

For example, velocity is rate of change of displacement x with time t, or

$$v = \frac{dx(t)}{dt}$$

Acceleration is the rate of change of velocity with time:

$$a = \frac{dv(t)}{dt}$$

When studying simple harmonic motion, you will meet sine and cosine functions that depend on time. We can differentiate the sine and cosine functions to get expressions for displacement, velocity and acceleration.

One solution for the displacement x, at time t is

$$x = A\cos(2\pi ft)$$

This assumes that the initial displacement is A at time $t = 0$. So A is the amplitude of the oscillation, and f is the frequency – these are both constants.

If we differentiate we get

$$v = \frac{dx(t)}{dt} = \frac{d(A\cos(2\pi ft)}{dt}$$
$$= -2\pi fA\sin(2\pi ft)$$

This is a solution for the velocity at time t.

If we differentiate again, we get an expression for the acceleration:

$$a = \frac{dv}{dt} = \frac{d(-2\pi fA\sin(2\pi ft)}{dt}$$
$$= -(2\pi f)^2 A\cos(2\pi ft)$$

$$a = -(2\pi f)^2 x$$

Using calculators: sin, cos and tan functions

If you want to calculate the sine, cosine or tangent function for a known angle, select the sin, cos or tan button:

- type in the angle as a number
- press =

To calculate an angle if you know the sine, cosine or tangent functions:

- press shift
- select the sin, cos or tan button
- type in the number
- press =

To change between radians and degrees:

- press shift set-up
- select DEG for degrees or RAD for radians

If in doubt, read the instructions to your calculator, as there can be small differences between machines.

Sine and cosine graphs

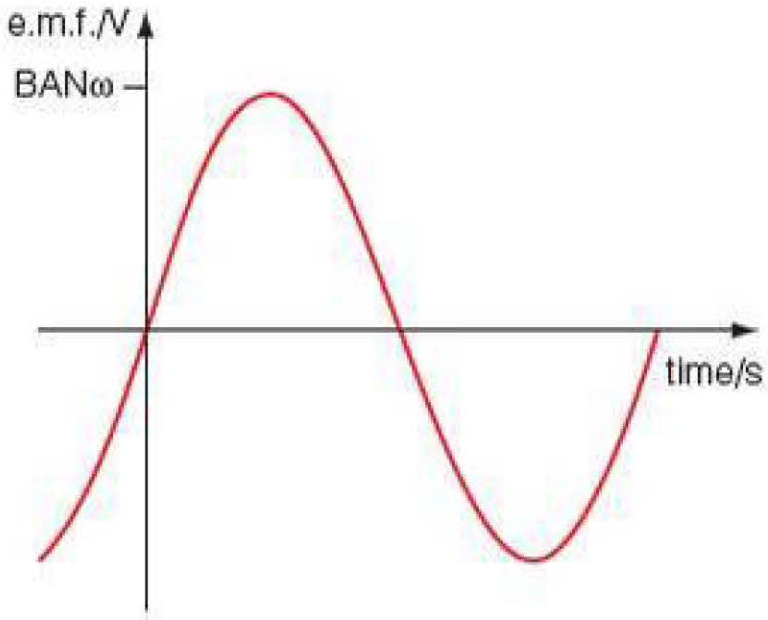


Figure 14.14 Graph of $E = BAN\omega \sin \omega t$ for the induced e.m.f. in a rotating coil.

The graph in Figure 14.14 shows the relationship for the e.m.f. induced in a rotating coil:

$$\varepsilon = BAN\omega \sin \omega t$$

where ω is the angular frequency, t is the time, B is the magnetic flux density (T), A is area (m^2) and N is the number of turns on the coil.

We could also use the relationship

$$\varepsilon = BAN\omega \cos \omega t$$

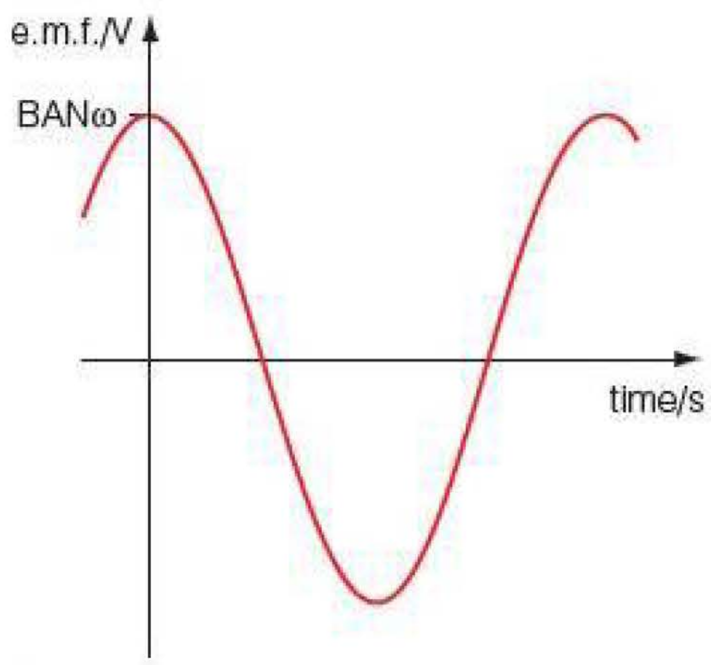


Figure 14.15 Graph of $\varepsilon = BAN\omega \cos \omega t$ for the induced e.m.f. in a rotating coil within its axis initially perpendicular to the field.

to describe the variation of the induced e.m.f. with time – it is really just a case of where we choose to define our starting point in time. The shape of this cosine function is shown in Figure 14.15.

Similar shaped graphs are used to describe other sinusoidal functions, that vary with time, such as the displacement, velocity or acceleration of a simple harmonic oscillator.

Degrees and radians

Degrees and radians are both used to describe angles. Degrees are calculated as 1/360 of the angle turned through a complete revolution. An angle in radians is the length of an arc, s, divided by the radius of the circle, r (see Figure 14.16). This relationship may be expressed by

$$\theta = \frac{s}{r}$$

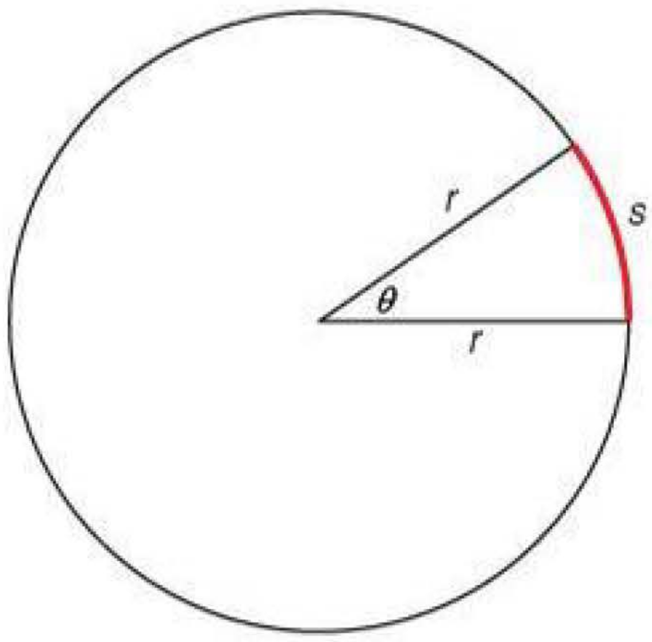


Figure 14.16

In Figure 14.16, the angle θ in radians is $\frac{s}{r}$. To go round the circle once, the arc length is $2\pi r$, so the angle for a complete revolution in radians is $\frac{2\pi r}{r} = 2\pi$.

You must be able to convert between the two units, and to recognise some key values:

- 360 degrees (360°) = 2π radians (2π rad) or one full revolution
- 180 degrees (180°) = π radians (π rad) or half a revolution

Small-angle approximations

For very small angles, you can approximate values for the sine, cosine or tangent. This is useful when calculating the fringe separations in interference patterns. When θ is measured in radians, we can use the following small-angle approximations:

- $\sin\theta \approx \tan\theta \approx \theta$
- $\cos\theta \approx 1$

To convert an angle in radians to an angle in degrees, remember that one radian (1 rad) is $\frac{180°}{\pi} = 57°$.

TEST YOURSELF

17 Calculate the following, in degrees:

a) $\sin 32°$

b) $\tan 163°$

c) $\cos^{-1} 0.92$

18 Calculate the following, in radians:

a) $\sin \pi/8$

b) $\tan \pi/3$

c) $\cos^{-1} 0.92$

19 Convert these angles from degrees to radians:

a) 40°

b) 175°

c) 270°

20 Convert these angles from radians to degrees:

a) $\pi/4$ radians

b) 0.3 radians

c) 1.6 radians

21 Use the small-angle rule to write down close approximations to the values of the following trigonometrical functions:

a) $\tan 0.01$ rad

b) $\cos 0.05$ rad

c) $\sin 0.03$ rad

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