Cambridge International AS and A Level Mathematics Mechanics Practice Book

Answers

1 Motion in a straight line

Exercise 1.1

1 (i) –100 m; 900 m

(ii) +1 m; 9 m

2 (i)

t	0	0.5	1	1.5	2	3	4	5
x	7	6.125	4	1.375	–1	–2	7	32

(ii) 25

(iii) $t = 5$; 32

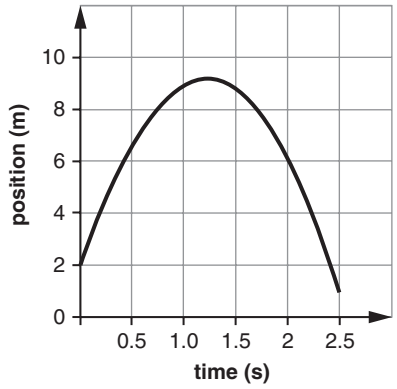
(iv) $43\frac{26}{27}$

3 (i) $t = 1$ s; $t = 3\frac{2}{3}$ s

(ii) t = $2\frac{1}{3}$ s

(iii) $37\frac{2}{3}$ m

4 (i)



(ii) –1.25 m

(iii) 1.2 s; 9.2 m

(iv) 15.65 m

5 (i)

t	0	0.5	1	1.5	2	2.5	3
position	3	8.75	12	12.75	11	6.75	0

(ii)

t	0	0.5	1	1.5	2	2.5	3
displacement	0	5.75	9	9.75	8	3.75	–3

(iii) (a) 9.8 m **(b)** 22.6 m

6 (i)

t	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
x	5	4	1.5	–1.75	–5	–7.5	–8.5	–7.25	–3	5	17.5

(ii)

t	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
displacement	0	–1	–3.5	–6.75	–10	–12.5	–13.5	–12.25	–8	0	12.5

(iii) $t = 3$

(iv) 39.5 m

7 (i) 1 m

(ii) 12 m

(iii) $1 < t < 5$

(iv) 1 s; 5 s

(v) 18 m

Exercise 1.2

1 (i) **(a)** 4, 0

(b) –4

(c) 12

(d) 4, 4; –8, 8

(e) –2

(f) 6

(ii) **(a)** 0, –5

(b) –5

(c) 25

(d) $3\frac{1}{3}$, $3\frac{1}{3}$; 0, 0; –7.5, 7.5

(e) $-\frac{5}{7}$

(f) $3\frac{4}{7}$

(iii) **(a)** 20, 10

(b) –10

(c) 50

(d) –6, 6; 0, 0; $6\frac{2}{3}$, $6\frac{2}{3}$

(e) –1

(f) 5

(iv) **(a)** 0, 20

(b) 20

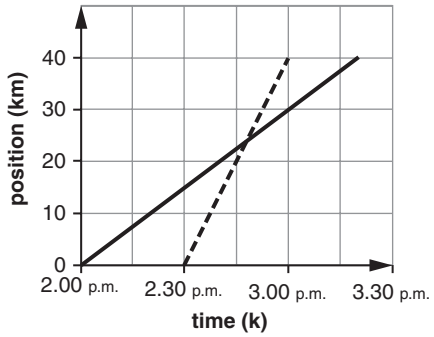
(c) 60

(d) 5, 5; –5, 5; 5, 5

(e) $1\frac{2}{3}$

(f) 5

2 (i)



(ii) 2.48p.m.

(iii) 24 km

3 (i) 1.9583 h = 1 h 57 min 30 s

(ii) 51.06 km h^{-1}

(iii) 0 km h^{-1}

4 (i) 3 m s^{-1}

(ii) 2.25 m s^{-1}

5 (i) 10.38 m s^{-1}; 10.26 m s^{-1}

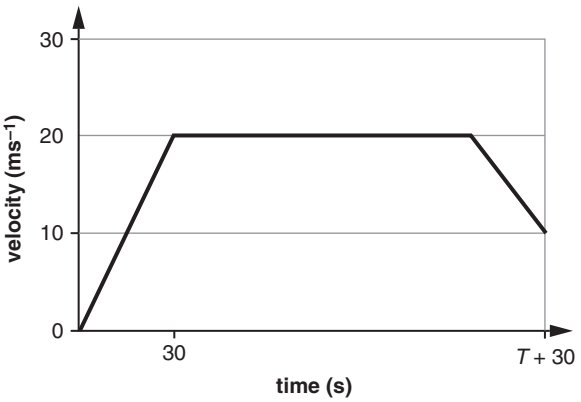
(ii) 1.20 m

6 1020 m

7 (i) 40 km h^{-1}

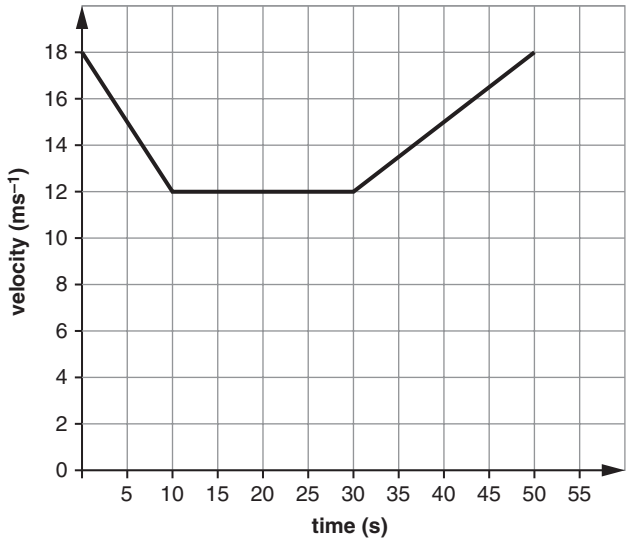
(ii) 0 km h^{-1}

8 (i)



(ii) 4 min 30 s

9 (i)



(ii) 13.8 m s^{-1}

10 (i) 200 m

(ii) 30 s

(iii) 150 m

11 39.928 km h^{-1}; 39.903 km h^{-1}; 2.228 km

12 (i) 28.51 km h^{-1}

(ii) 43.04 km h^{-1}

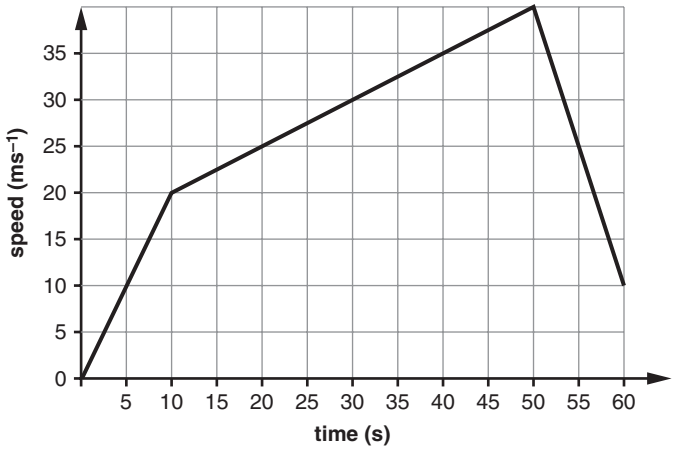
Exercise 1.3

1 (i) 4 m s^{-2}; 0; –4 m s^{-1}

(ii) 500 m

2 (i) 20 m s^{-1}; 40 m s^{-1}; 10 m s^{-1}

(ii)

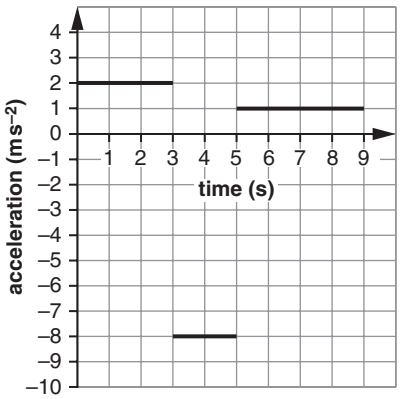


(iii) 1550 m

3 (i) 20 m s^{-1}

(ii) 5.5 s

4 (i) 2 m s^{-2}; –8 m s^{-2}; 1 m s^{-2}



(ii) +15 m; +19 m; +15 m; –9 m

5 (i) 6 m s^{-2}

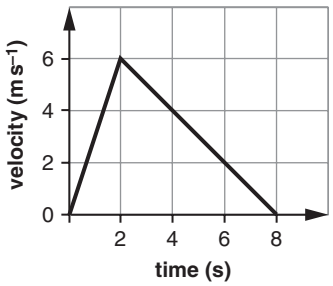
(ii) 6 m s^{-1}

(iii) $a = 3t$

(iv) $t = 5$

(v) 8 m s^{-1} increase

6 (i)



(ii) 24 m

(iii) –1 m s^{-2}

(iv) 3 m s^{-1}

7 (i) 8.76̇ s

(ii) 7.48 s

8 9 min 20 s

Exercise 1.4

1 (i) $-1\frac{2}{3}$ m s^{-2}

(ii) 187.5 m

(iii) 500 m

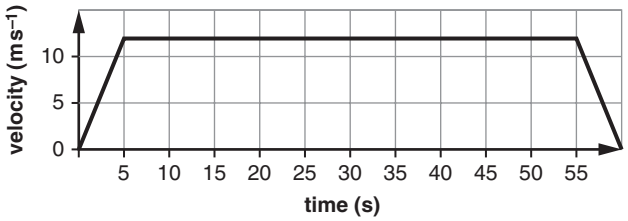
2 (i) 36 m

(ii) 5 s

(iii) –6 m s^{-2}; 3 m s^{-2}

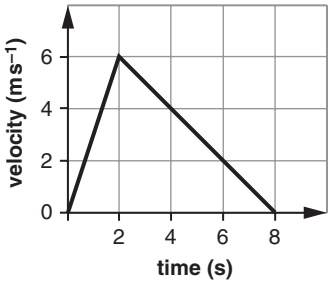
(iv) 18 m s^{-1}; 0.5 m s^{-2}

3 (i)



(ii) 660 m

4 (i)



(ii) 24 m

(iii) $-1\ \text{m s}^{-2}$

(iv) $3\ \text{m s}^{-1}$

5 (i) 500 m; 1500 m; 1500 m; –500 m

(ii) 4000 m

(iii) 180 s

6 (i) 20 s; 60 m

(ii) 45 m

(iii) 75 m

7 $12.5\ \text{m s}^{-1}$

8 (i) $5\ \text{m s}^{-1}$; $3\ \text{m s}^{-2}$

(ii) $17\ \text{m s}^{-1}$, 44 m

(iii) $v = 17 - 2t$

(iv) 116 m

9 (i) 6000 m

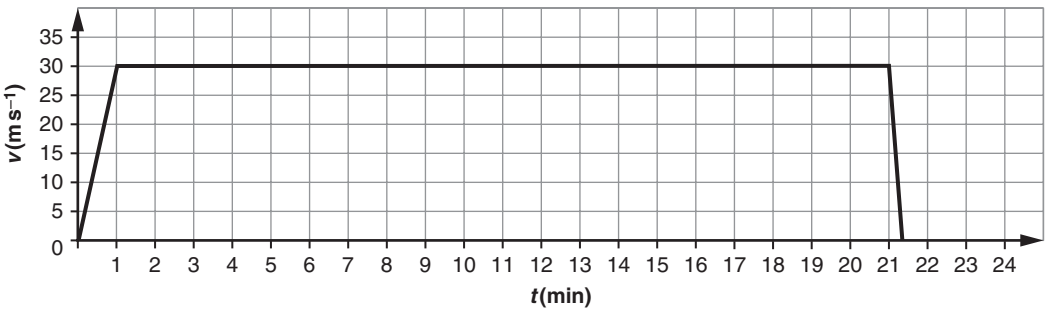
(ii) 125 s

(iii) 24 min 22.5 s

10 5 min 57.5 s

11 6 min 48 s

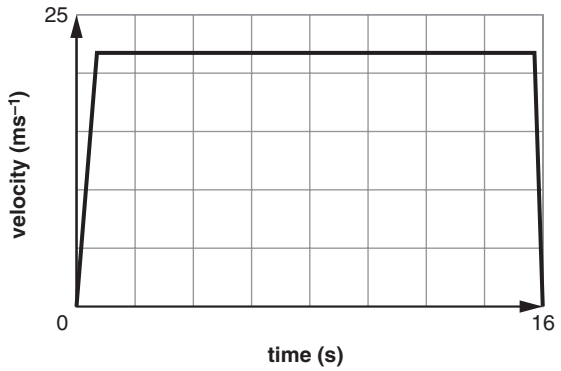
12



37.125 km`

Stretch and challenge

1 (i)



(ii) 21.51 m s^{-1}

(iii) 40 s; 215.05 m

2 180 s

3 t_1 = time to accelerate; t_2 = time to decelerate; V = maximum velocity; $t_1 + t_2 = T$

$\frac{V}{a} + \frac{V}{2a} = T$, therefore V = $\frac{2}{3}aT$, so D = $\frac{1}{3}aT^2$

4 $T = \frac{V}{4a}$; AB = $\frac{3}{2}aT^2$; BC = $\frac{7}{2}aT^2$, so AB : BC = 3 : 7

Exam focus

1 (i) AX = 7.5 m

(ii) V = 1.2 m s^{-1}

(iii) T = 10 s

(iv) 0.4 m s^{-1}

2 The constant acceleration formulae

Exercise 2.1

1 (i) 13 m s^{-1}

(ii) 37.5 m

(iii) –8 m

(iv) 5 m s^{-1}

(v) 6 m s^{-2}

(vi) –1.5 m s^{-2}

(vii) –24 m

(viii) 15 m s^{-1}

2 0.9 m s^{-2}; $2\frac{2}{9}$ s

3 (i) 0.75 s

(ii) 4.06 m

(iii) 1.5 s; 7.5 m s^{-1}

4 (i) $7\frac{1}{9}$ m s^{-2}

(ii) $10\frac{2}{3}$ m s^{-1}

(iii) 10.125 s

(iv) 9.88 m s^{-1}

5 –2 m s^{-2}; 225 m

6 The train stops before the obstacle.

7 8.94 m s^{-2}

8 (i) 4 m s^{-2}

(ii) 22.36 m s^{-1}

9 (i) 21.5 m s^{-1}

(ii) 1.8 s

10 20 m s^{-1}; 20 m

11 2 m s^{-1}

12 0.85 m s^{-2}; 381.94 m

13 3.464 s

14 (i) T = 0.675 s

(ii) a = –6.35 m s^{-2}

(iii) 2.78 s

(iv) 21 m; 76.2 m

(v) 5.58 s

Exercise 2.2

1 u = –1.5 m s^{-1} away from B; a = 0.1 m s^{-2}

2 25 – 10T = 10T so T = 1.25 s; V = 12.5 m s^{-1}; X = 31.25 m

3 (i) u = 18.75 m s^{-1}; a = 0.25 m s^{-2}

(ii) 546.875 m

4 (i) u = 10 m s^{-1}; a = 2.5 m s^{-2}

(ii) 20 m s^{-1}

(iii) 131.25 m

5 (i) 45 m

(ii) 45 – 5t^2 = 18t – 5t^2 so t = 2.5 s; 13.75 m

6 (i) 3.5 s; 2.5 s

(ii) 61.25 m

7 (i) $x_A = 0.4t^2$; $x_B = 10t$

(ii) 40 s; 640 m

8 $a = 0.1125\,\text{m s}^{-2}$; $u = -2\,\text{m s}^{-1}$ away from B

9 **(i)** 3 s

(ii) 15 m

10 $a = 7.5\,\text{m s}^{-2}$; $u = -2.5\,\text{m s}^{-1}$

11 **(i)** $0 = 8^2 + 2 \times -10 \times h$, so $h = 3.2\,\text{m}$

(ii) $6\,\text{m s}^{-1}$

(iii) $v_A = 8 - 7$; $v_B = 6 - 7$

Stretch and challenge

1 $a = 8\,\text{m s}^{-2}$; $v = 50\,\text{m s}^{-1}$

2 30 s; $x_A = 30u + 450a$; $x_B = 10u + 150a$

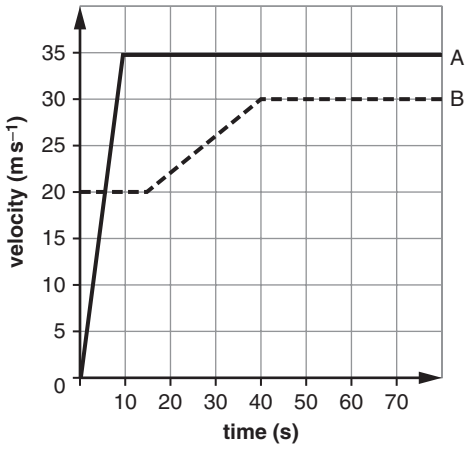
3 109.43 m

4 125 m

5 $20\,\text{m s}^{-1}$

Exam focus

1 **(i)**



(ii) $x_A = 175 + 35 \times 30 = 1225\,\text{m}$;
$x_B = 20 \times 15 + \frac{1}{2}(20 + 30) \times 25 = 925\,\text{m}$

(iii) 80 s

2 **(i)** $16\frac{2}{3}\,\text{m s}^{-1}$

(ii) $\frac{5}{18} = 0.2\dot{7}\,\text{m s}^{-2}$

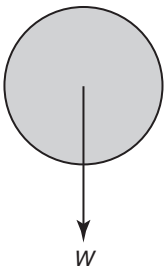
(iii) $50\,\text{m s}^{-1}$

3 Forces and Newton's laws of motion

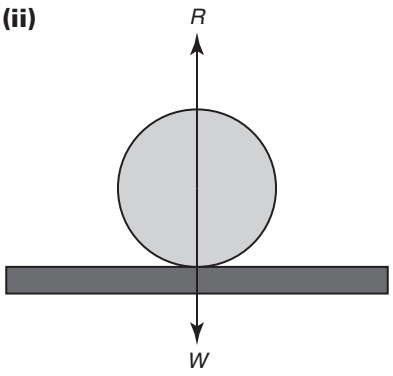
Exercise 3.1

1

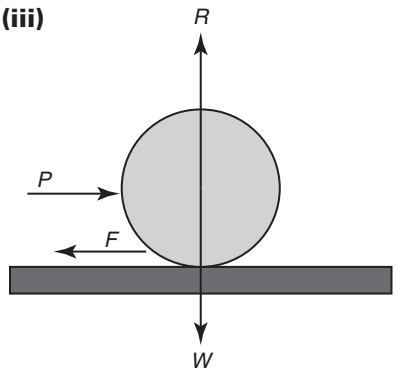
(i)



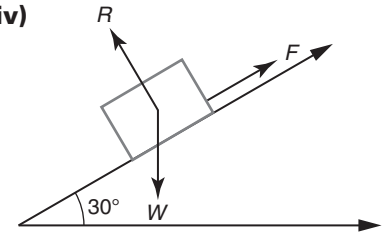
(ii)



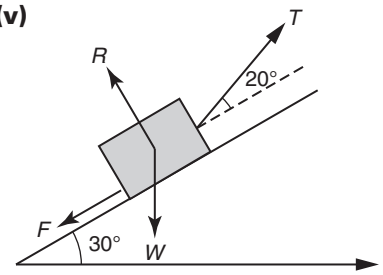
(iii)



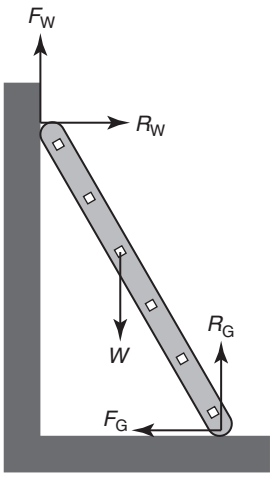
(iv)



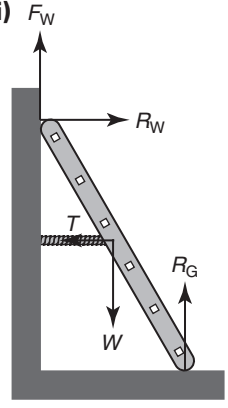
(v)



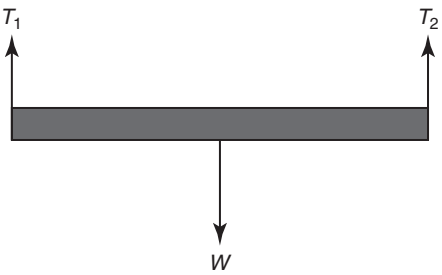
(vi)



(vii)

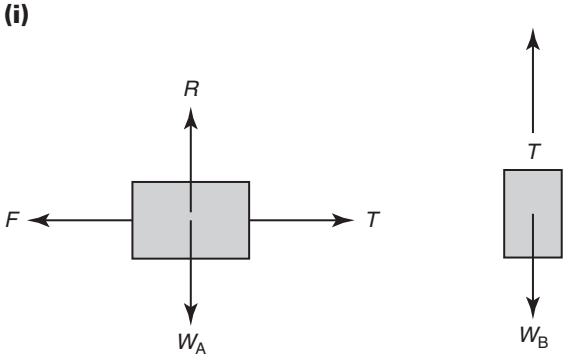


(viii)



Exercise 3.2

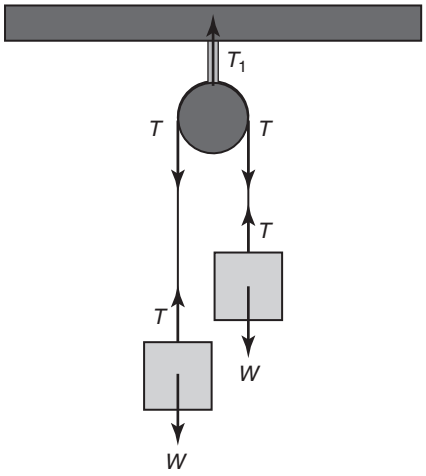
1 (i)



(ii) $T = W_A$; $R = W_B$

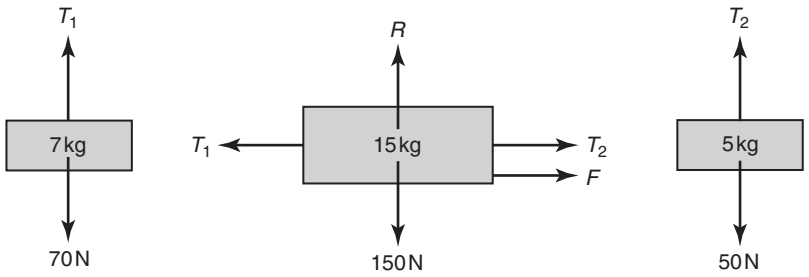
(iii) T

2



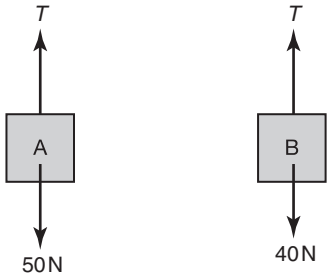
W; $2W$

3 (i)



(ii) $70 - T_1$; $T_1 - T_2 - F$; $T_2 - 50$

4 (i)



(ii) 10 N

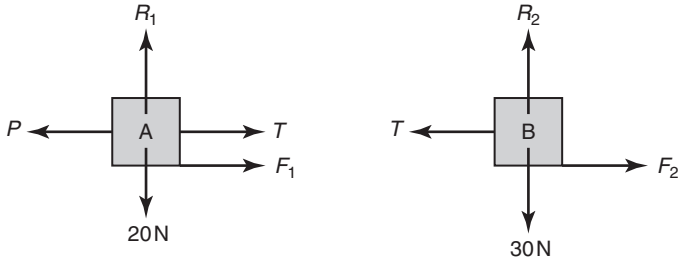
5 (i) $R = W$; net force = 0 N

(ii) $R > W$; net force = $R - W$

(iii) $R < W$; net force = $W - R$

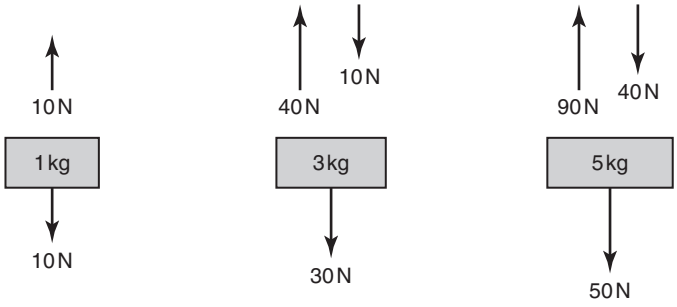
(iv) $R > W$; net force = $R - W$

6 (i)



(ii) $P - F_1 - F_2$

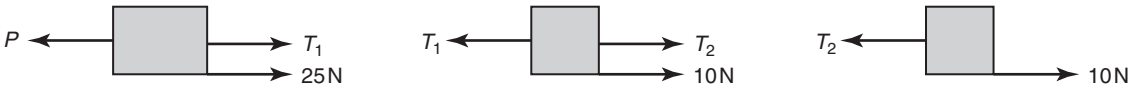
7



8 (i)

$P = 45\,\text{N}$

(ii)

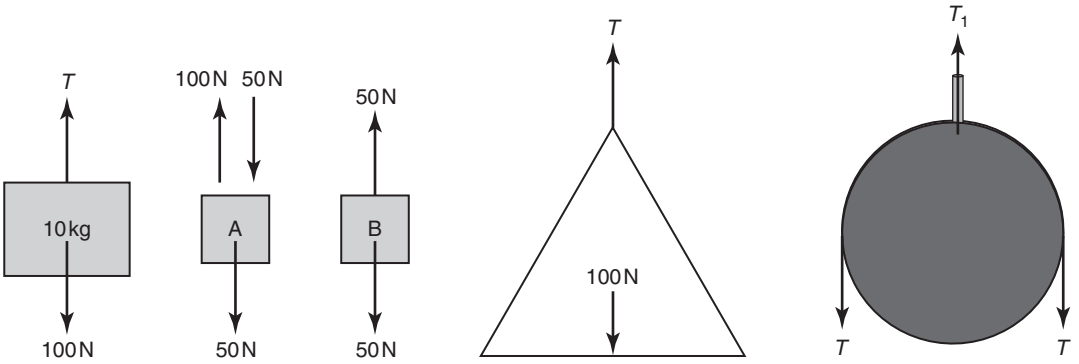


(iii) $T_1 = 20\,\text{N}$

(iv) $T_2 = 10\,\text{N}$

(v) $T_1 = T_2 + 10$

9 (i)

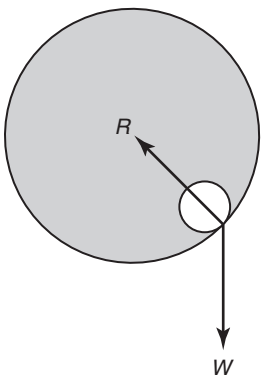


(ii) 100 N

(iii) 200 N

(iv) 50 N

10



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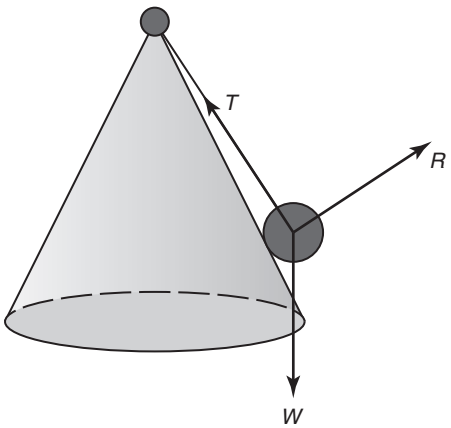


Stretch and challenge

1 (i) 4000 N

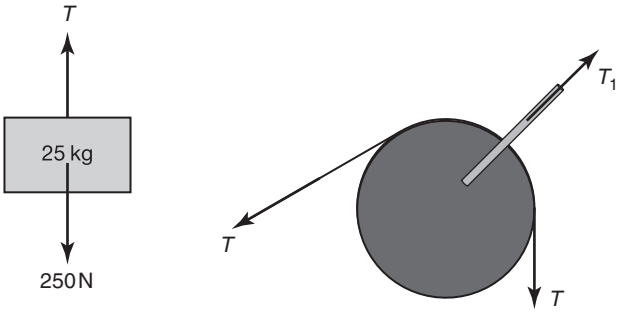
(ii) 100 N

(iii) 200 N

(iv) $T_n = 100 \times n$

(v) 2500 N

2 (i)



(ii) 250 N

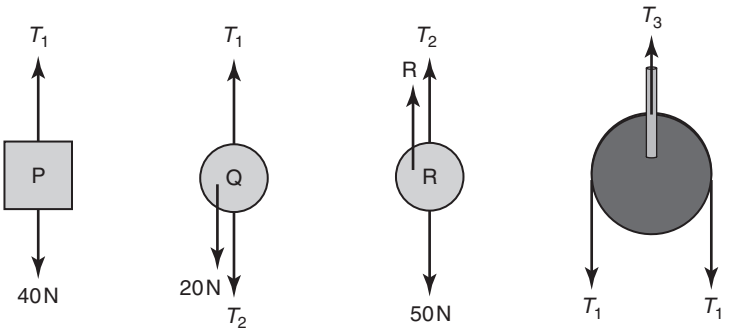
(iii) 433.01 N; 60° below the horizontal

Exam focus

1 (i) 30 N

(ii) 20 N

2 (i)



(ii) The pulley is smooth; 40 N

(iii) 20 N

(iv) 30 N

4 Applying Newton's second law along a line

Exercise 4.1

1 (a) $X = 65$ N

(b) $X = 56$ N

(c) $m = 8$ kg

(d) $M = 4$ kg

(e) $a = 6\,\text{m s}^{-2}$

(f) $a = 8\,\text{m s}^{-2}$

2 10 N

3 (i) 100 N

(ii) 50 N

4 $1.5\,\text{m s}^{-2}$

5 3760 N

6 27.375 N

7 $7\frac{1}{3}\,\text{m s}^{-2}$

8 424 N

9 (i) $-\frac{5}{7}\,\text{m s}^{-2}$

(ii) 696.43 N

10 $2\,\text{m s}^{-2}$

11 $2\,\text{m s}^{-2}$

Exercise 4.2

1 (i) 100 N

(ii) 300 N

2 (i) $400a = -200$

(ii) 50 N

(iii) 350 m

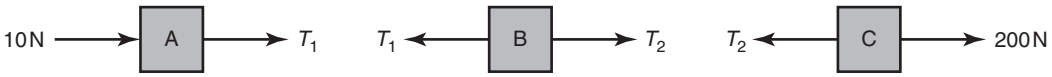
3 (a) (i)

(ii) $10\,\text{m s}^{-2}$; 60 N

(b) (i)

(ii) $5\,\text{m s}^{-2}$; 50 N

4 (i)



(ii) $T_1 + 10 = 4a$; $T_2 - T_1 = 10a$; $200 - T_2 = 16a$

(iii) $7\,\text{m s}^{-2}$; 18 N; 88 N

5 (i) 950 N

(ii) 1950 N

6 (i) $1\frac{1}{9}\,\text{m s}^{-2}$

(ii) $\frac{5}{9}\,\text{m s}^{-1}$

(iii) $\frac{10}{36}$

7 (a) (i) $2.5\,\text{m s}^{-2}$; 37.5 N

(ii) $3.5\,\text{m s}^{-1}$

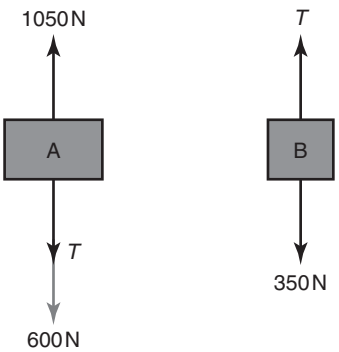
(iii) 0.6125 m

(b) (i) $5\frac{5}{13}\,\text{m s}^{-2}$; $46\frac{2}{13}$ N

(ii) $1.80\,\text{m s}^{-1}$

(iii) 0.16 m

8 (i)



(ii) $60a = 1050 - 600 - T$; $35a = T - 350$

(iii) $a = 1.05\,\text{m s}^{-2}$; $T = 386.84$ N

9 (i) $a = 0.625\,\text{m s}^{-2}$; $T = 93.75$ N

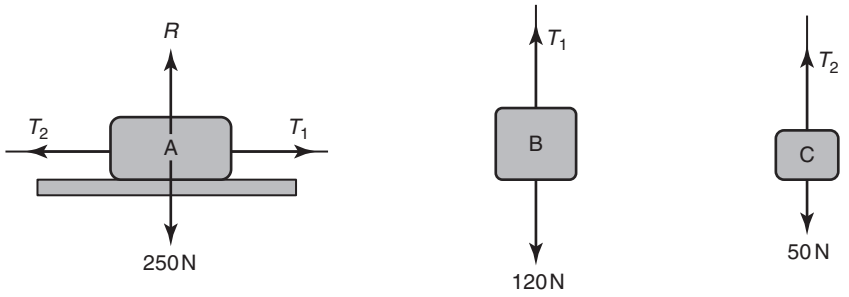
(ii) $15.81\,\text{m s}^{-1}$

(iii) 557.5 N

10 (i) $a = 1.16\dot{6}\,\text{m s}^{-2}$; $T = 1400\,\text{N}$

(ii) $a = -0.5\,\text{m s}^{-2}$; $T = -100\,\text{N}$ (compression)

11 (i)



(ii) $25a = T_1 - T_2$; $12a = 120 - T_1$; $5a = T_2 - 50$

(iii) $a = 1\frac{2}{3}\,\text{m s}^{-2}$; $T_1 = 100\,\text{N}$; $T_2 = 58\frac{1}{3}\,\text{N}$

(iv) 35 N

12 (i) $2.24\,\text{m s}^{-1}$

(ii) 15 N

13 $0.2\,\text{m s}^{-2}$; 2040 N; 2550 N

14 104.76 s

Stretch and challenge

1 (i) $0.125\,\text{m s}^{-2}$

(ii) 31 250 N

(iii) $T_n - (25 - n)250 = (25 - n) \times 8000 \times 0.125$, so $T_n = 31\,250 - 1250n$

(iv) 1250 N

2 (i) $\frac{10}{49}\,\text{m s}^{-2}$

(ii) 0.2 kg

3 (i) $2.5\,\text{m s}^{-2}$; 25 N; 45 N

(ii) 0.125 m

Exam focus

1 (i) $37\,000 - (500 + 300 + 100 + 100) = 120\,000a$, so $a = 0.3\,\text{m s}^{-2}$

(ii) 6100 N

(iii) $-0.1\,\text{m s}^{-2}$

(iv) (a) −1900 N (thrust)

(b) 9100 N (tension)

2 (i) 0.25 m; $1\,\text{m s}^{-1}$

(ii) $a = -10$; $u = 1$; $v = 0$

So $s = \dfrac{v^2 - u^2}{2a} = \dfrac{-1}{-20} = 0.05$ m

Total time = 0.6 s

(iii) $m_A = 0.75\,\text{kg}$; $m_B = 0.5\,\text{kg}$

(iv) (a) Tension before = 17 N

(b) Tension after = 5 N

5 Vectors

Exercise 5.1

1 (i) 13; 67.38°

(ii) 2.24; −63.43°

(iii) 5; −36.87°

(iv) 5.39; 111.80°

2 (i) 1.41; 045°

(ii) 5.83; 149°

(iii) 20.5; 103°

(iv) 30.5; 080°

3 $\mathbf{p} + \mathbf{q}$; $\frac{1}{2}\mathbf{p} + \frac{1}{2}\mathbf{q}$; $\mathbf{q} - \mathbf{p}$; $\frac{1}{2}\mathbf{q} - \frac{1}{2}\mathbf{p}$

4 (i) (a) $\overrightarrow{AM} = \frac{1}{2}\mathbf{b} + \frac{1}{2}\mathbf{c} - \mathbf{a}$

(b) $\overrightarrow{AG} = \frac{1}{3}\mathbf{b} + \frac{1}{3}\mathbf{c} - \frac{2}{3}\mathbf{a}$

(c) $\overrightarrow{BN} = \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{c} - \mathbf{b}$

(d) $\overrightarrow{CP} = \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b} - \mathbf{c}$

(e) $\mathbf{g} = \frac{1}{3}(\mathbf{a} + \mathbf{b} + \mathbf{c})$

(ii) $\overrightarrow{BG} = \frac{2}{3}\overrightarrow{BN}$; $\overrightarrow{CG} = \frac{2}{3}\overrightarrow{CP}$

5 (i) (a) $\overrightarrow{CD} = -\frac{1}{2}\mathbf{p}$

(b) $\overrightarrow{BC} = \mathbf{q} - \frac{1}{2}\mathbf{p}$

(c) $\overrightarrow{AC} = \mathbf{q} + \frac{1}{2}\mathbf{p}$

(d) $\overrightarrow{BD} = -\mathbf{p} + \mathbf{q}$

(ii) $\overrightarrow{AG} = \lambda(\mathbf{q} + \frac{1}{2}\mathbf{p})$; $\overrightarrow{BG} = \lambda(-\mathbf{p} + \mathbf{q})$

$\overrightarrow{AG} = \mathbf{p} + \overrightarrow{BG}$

$\lambda(\mathbf{q} + \frac{1}{2}\mathbf{p}) = \mathbf{p} + \lambda(-\mathbf{p} + \mathbf{q})$

$\frac{3}{2}\lambda\mathbf{p} = \mathbf{p}$

so $\lambda = \frac{2}{3}$

6 5.83; 149°

7 (5, –4)

8 6.40; 6.08

9 $|\mathbf{a} + \mathbf{b}| = |\mathbf{a} - \mathbf{b}| = 13$

10 22

11 5.83; 120.96°

12 (i) 9.43; 148°

(ii) $\mathbf{G} = 1.5\mathbf{F}$

(iii) –11

13 (i) 7.81 N

(ii) 26.33°

14 (i) 9.43; –32°

(ii) 10.77; –21.80°

15 (i) $2\frac{2}{3}$

(ii) $2\frac{2}{3}$

(iii) $5\sqrt{3} - 4$

16 34.37 N; –8.37°

17 $\mathbf{F}_1 = \begin{pmatrix} -5 \\ -1 \end{pmatrix}$; $\mathbf{F}_2 = \begin{pmatrix} 65 \\ 15 \end{pmatrix}$

18 2.05; 52.32°

19 (i) $\theta = 51.32°$

(ii) $F = 6.24$ N

20 (i) $\mathbf{v}(1) = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$; speed $= 2.82\,\text{m s}^{-1}$

(ii) 140°

21 (i) $\begin{pmatrix} 8 \\ -6 \end{pmatrix}$; $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$

(ii) $\begin{pmatrix} -10 \\ 20 \end{pmatrix}$

(iii) 4 s

22 (i) $\mathbf{a} = \begin{pmatrix} -0.1 \\ 0.2 \end{pmatrix}$; $\mathbf{u} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$

(ii) $6\frac{2}{3}$ s

23 (i) 39.05 N; 034°

(ii) 39.05 N; 214°

24 (i) $\mathbf{F}_1 = 204.79\mathbf{i} + 143.39\mathbf{j}$; $\mathbf{F}_2 = 350\mathbf{i}$; $\mathbf{F}_3 = 187.94\mathbf{i} - 68.40\mathbf{j}$

(ii) 746.50 N; 5.77°

(iii) –74.99 N

25 (i) 11.95*P*; 3.51°

(ii) 12.14*P*; 25.15°

Stretch and challenge

1 $P^2 + Q^2 - 2PQ\cos\theta = P^2$, giving $Q^2 = 2PQ\cos\theta$

$4P^2 + Q^2 - 4PQ\cos\theta = 3P^2$, giving $P^2 + Q^2 = 4PQ\cos\theta$

$P^2 + Q^2 = 2Q^2$, hence $Q = P$; $\theta = 60°$

2 3.40 N; 12.90 N

3 $F = 800.48$ kN; resultant = 1800.58 kN

Exam focus

1 (i) $16.40\,\text{m s}^{-1}$; 37.57°

(ii) $5.66\,\text{m s}^{-1}$

2 (i) 36.25 N; –6.73°

(ii) 94.25 N; 60.65°

6 Forces in equilibrium and resultant forces

Exercise 6.1

1 (i) $P = 16.55$ N; $X = 6.99$ N

(ii) $\theta = 60°$; $W = 5.20$ N

(iii) $F = 25$ N; $R = 43.30$ N

(iv) $\theta = 17.66°$; $T = 18.48$ N

(v) $\theta = 29.05°$; $Z = 10.30$ N

(vi) $R = 10.16$ N; $P = 2.40$ N

2 (a) (i) $\begin{pmatrix} 25.43 \\ 5.06 \end{pmatrix}$

(ii) 25.93 N; 11.26°

(b) (i) $\begin{pmatrix} 6.46 \\ 5.27 \end{pmatrix}$

(ii) 8.34 N; 39.18°

3 $T_1 = 64.62$ N; $T_2 = 26.92$ N

4 0.477 N; 80.46°

5 **(i)** 35.63 N

(ii) 39.93 N

6 234.50 N; 168.37 N

7 41.81°; 20 N; 22.36 N

8 68.31 N; 17.90°

9 **(i)** 70.50 N

(ii) 332.87 N

(iii) The horizontal component is unchanged.

(iv) 42.26 N

10 **(i)** 57.13 N

(ii) 6.97 kg

11 **(i)** $T \sin 65° + R = 750$

(ii) 827.53 N; 82.75 kg

(iii) 349.73 N

12 Tension in AB of 220.48 N. Compression of 333.33 N in AC.

13 **(i)** Horizontal components: $T_1 \sin 25° = T_2 \sin 25°$, so $T_1 = T_2$

(ii) 27.58 N

(iii) 13.59 kg

14 $R_A = W$; $R_B = W\sqrt{2}$

15 **(i)** $T_{AB} = 250$ N; $T_{BC} = 250 \times 0.8 = 200$ N

(ii) 30.96°; 233.24 N

Exercise 6.2

1 $a = 0.38\,\text{m s}^{-2}$; $T = 72.63$ N

2 $1.83\,\text{m s}^{-2}$; 34.15 N; 48.30 N

3 **(i)** $0.32\,\text{m s}^{-2}$

(ii) 6.23 N

4 **(i)** 2250 N; 675 N

(ii) 3450 N; 1035 N

5 **(i)** $1.4\,\text{m s}^{-2}$

(ii) $3.74\,\text{m s}^{-1}$

(iii) $1.4\,\text{m s}^{-2}$

(iv) $0\,\text{m s}^{-1}$

6 **(i)** 1800 N

(ii) 400 N; 800 N

(iii) $0.872\,\text{m s}^{-2}$; 400 N; 800 N

7 $3.92\,\text{m s}^{-2}$; 30.19 N in Tension

8 **(i)** 2 kg; 18 N

(ii) $2.59\,\text{m s}^{-2}$; 25.94 N

9 **(i)** 18.75°; 1352.57 N

(ii) 1328.57 N

(iii) $1.73\,\text{m s}^{-1}$

Stretch and challenge

1 $P\cos\theta = \frac{mg}{2}$; $P\sin\theta = \frac{mg}{2\sqrt{3}}$; so $\theta = 30°$; $P = \frac{mg}{\sqrt{3}}$

2 5 m

3 $3.10\,\text{m s}^{-1}$; 0.03 N in tension

Exam focus

1 $\theta = 72.06°$; $W = 9.95$ N

2 $P = X\left(3 - \sqrt{2}\right)$; $W = X\sqrt{2}$

3 58.51 N

4 $0.94\,\text{m s}^{-2}$; 564 N

7 General motion in a straight line

Exercise 7.1

1 **(i)** $v = -9 + 12t - 3t^2$

(ii) $t < 1$, $v < 0$; $1 < t < 3$, $v > 0$

(iii) $a(1) = 6\,\text{m s}^{-2}$; $a(3) = -6\,\text{m s}^{-2}$

2 $76\,\text{m s}^{-1}$; $42\,\text{m s}^{-2}$

3 **(i)** $42\,\text{m s}^{-1}$; $54\,\text{m s}^{-2}$

(ii) $x(0.5) = 4.25$; $x(3) = -27$; so distance = 31.25 m

(iii) $x(0) = 0$; $x(4) = -8$;
so distance = 4.25 + 31.25 + 19 = 54.5 m

4 **(i)** $x = 2.5t^2 - 2t + 3$

(ii) $x = t^3 + 2t^2 - 1$

(iii) $x = -1.5t^2 + t - 2$

5 **(i)** $v = 6t^2 - 8t + 3$; $x = 2t^3 - 4t^2 + 3t + 5$

(ii) $v = 1.4t^{2.5} - 3t - 0.4$; $x = 0.4t^{3.5} - 1.5t^2 - 0.4t + 4.5$

(iii) $v = 0.2t^3 - 0.15t^2 + t - 0.05$;
$x = 0.05t^4 - 0.05t^3 + 0.5t^2 - 0.05t + 0.05$

6 **(i)** $\frac{1}{12}$ m

(ii) $\frac{4}{27}\,\text{m s}^{-1}$

7 **(i)** $a = 3 - \frac{1}{6}t$; $x = 1.5t^2 - \frac{1}{36}t^3 + 5$

(ii) 36 s; 54 s

8 **(i)** 140.625 m s^{-1}

(ii) 1265.625 m

(iii) 20 s; 150 m s^{-1}

9 **(i)** $x = 0.5t^3 - t^2$; $x(2) = 0$; $x(4) = 16$ m

(ii) $1\frac{1}{3}$ s

(iii) $17\frac{5}{27}$ m

10 **(i)** 3.6 m s^{-2}

(ii) 10 s

(iii) $26\frac{2}{3}$ m s^{-1}

(iv) $416\frac{2}{3}$ m

11 3 m

12 **(i)** $t = 0$ or $t = 2$

(ii) $t = 3$

(iii) 6.75 m

(iv) $t = 4$

13 6.25 m

14 16 m

15 **(i)** $a = 0$; $v = 8$ m s^{-1}

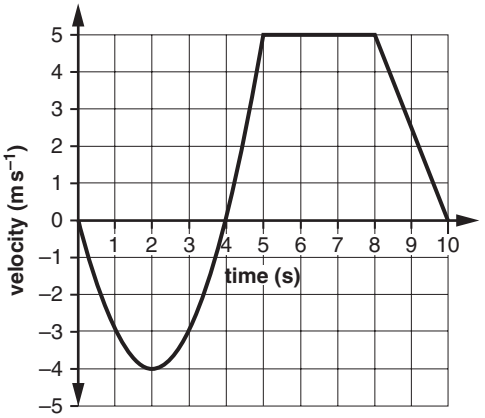
(ii) $10\frac{2}{3}$ m

(iii) $13\frac{1}{6}$ s

16 **(i)** $v(5) = 5$

(ii) $a = -2.5$; $b = 25$; uniform deceleration

(iii)



(iv) 33 m

17 **(i)** 12.15 m

(ii) 13.85 s

(iii) 26.33 s

Stretch and challenge

1 **(i)** Sprinter A: 8.1 m s^{-1}; Sprinter B: 8.64 m s^{-1}

(ii) Sprinter A: 12.15 m; Sprinter B: 17.28 m

(iii) Sprinter A: 13.85 s; Sprinter B: 13.57 s

(iv) Sprinter B wins by 0.27 s or 2.20 m.

2 **(i)** 2.4 m s^{-1}

(ii) $5.86\dot{}$ m

(iii) 23.86 m

3 **(i)** $v = -24 + 18t - 3t^2$; $t = 2$ and $t = 4$

(ii) $x(2) = -2$; $x(4) = 2$

(iii) 28 m

Exam focus

1 **(i)** 15 m

(ii) $v = 12 + 6t - 6t^2$

(iii) $a = 6 - 12t$

(iv) $13\frac{1}{2}$ m s^{-1}

(v) Once

(vi) 214 m

2 **(i)** 1 m s^{-2}

(ii) $x = 18t - 13.5t^2 + 3\frac{1}{3}t^3 - \frac{1}{4}t^4$; 12.916$\dot{}$ m

(iii) Particle goes backwards

(iv) 23.583$\dot{}$ m

8 A model for friction

Exercise 8.1

1 **(a)** **(i)** 50 N

(ii) 0.4

(b) **(i)** 37.5 N

(ii) 0.58

(c) **(i)** 56.3 N

(ii) 0.24

2 **(a)** **(i)** 0.625 m s^{-2}

(ii) 46.875 N

(iii) 37.5 N

(b) **(i)** $2\frac{2}{3}$ m s^{-2}

(ii) $7\frac{1}{3}$ N

(iii) 2 N

(c) **(i)** 0.3125 m s^{-2}

(ii) 77.5 N; 41.25 N

(iii) 30 N

(d) **(i)** $0\,\text{m s}^{-2}$

(ii) 10 N; 30 N

(iii) 20 N

(e) **(i)** $2.83\,\text{m s}^{-2}$

(ii) 28.68 N

(iii) 5.20 N

3 **(i)** $3.75\,\text{m s}^{-2}$

(ii) 0.375

(iii) $83\frac{1}{3}\,\text{m}$

4 0.0625

5 **(i)** $14.39\,\text{m s}^{-2}$

(ii) $11.80\,\text{m s}^{-2}$

6 $3.27\,\text{m s}^{-2}$; $8.86\,\text{m s}^{-1}$

7 679.38 N

8 **(i)** $2.5\,\text{m s}^{-2}$

(ii) 1.095 s

(iii) $2.35\,\text{m s}^{-1}$

9 **(i)** $a = g\left(\frac{m - M\sin\alpha - \mu M\cos\alpha}{m + M}\right)$; $T = \frac{mMg}{m+M}(1 + \sin\alpha + \mu\cos\alpha)$

(ii) $a = g\left(\frac{M\sin\alpha - \mu M\cos\alpha - m}{m + M}\right)$; $T = \frac{mMg}{m+M}(1 + \sin\alpha - \mu\cos\alpha)$

10 **(i)** $3.23\,\text{m s}^{-2}$

(ii) 215 N

11 0.346

12 **(i)** 7 N

(ii) $5.47\,\text{m s}^{-2}$

(iii) 1 s

13 **(i)** 0.10

(ii) 43.4 N

(iii) $5.66\,\text{m s}^{-2}$

14 **(i)** 21.34 N

(ii) 0.19

(iii) 5.86 cm

(iv) Faster

15 **(i)** 67.6 N

(ii) 187.24 N

(iii) 229.93 N

16 0.569

17 0.353

18 $3X = mg\sin\alpha + \mu mg\cos\alpha$
$X = mg\sin\alpha - \mu mg\cos\alpha$
So $\mu = 0.5\tan\alpha$

Stretch and challenge

1 Time to descend distance X on smooth plane: $\sqrt{\frac{X}{3}}$
Time to descend X on rough plane: $\sqrt{\frac{4X}{3}}$

2 $T(\cos\beta + \frac{1}{7}\sin\beta) < 8500(\sin\alpha + \frac{1}{7}\cos\alpha)$
So $T < 1300 + 1200 = 2500$

$T(\cos\beta - \frac{1}{7}\sin\beta) > 8500(\cos\alpha - \frac{1}{7}\sin\alpha)$
So $T > \frac{25}{23}(1300 - 1200) = 109$

3 $4P(\cos\theta - \mu\sin\theta) = mg(\sin\theta + \mu\cos\theta)$
$P(\cos\theta + \mu\sin\theta) = mg(\sin\theta - \mu\cos\theta)$
$4(\sin\theta - \mu\cos\theta)(\cos\theta - \mu\sin\theta) =$
$(\cos\theta + \mu\sin\theta)(\sin\theta + \mu\cos\theta)$
$3\sin\theta\cos\theta(1 + \mu^2) = 5\mu$
So $\mu = 0.6(1 + \mu^2)\sin\theta\cos\theta$

Exam focus

1 (i) $4.5\,\text{m s}^{-2}$

(ii) 2.16 N; 18.79 N; 0.115

(iii) $2.34\,\text{m s}^{-2}$; $4.33\,\text{m s}^{-1}$

2 (i) 20 N

(ii) $4.\dot{4}$ kg

(iii) 23.2 N

3 (i) (a) 50 N

(b) 48.77 N

(c) 58.54 N

(ii) 90.82 N

4 (i) 149.22 N

(ii) 19.81 N

5 0.128

9 Energy, work and power

Exercise 9.1

1 (i) 12.5 J

(ii) 6 m

(iii) 1.25 J

(iv) $0.2\dot{7}$ N

(v) $9.89\,\text{m s}^{-1}$

2 (i) 990 J

(ii) 6875 J

(iii) 10 365 J

3 $31\frac{2}{3}$ N

4 (i) 1159.11 J

(ii) 416.76 J

(iii) $3.77\,\text{m s}^{-1}$

5 1.94 m

6 (i) (a) 20.78 J

(b) 30 J

(c) 50.78 J

(d) $2.16\,\text{m s}^{-1}$

(ii) 1.48 m

7 (i) 25 J

(ii) 20 J

(iii) 5 J

(iv) 0.12

8 $5.48\,\text{m s}^{-1}$

9 (i) 43 750 J

(ii) 875 N

10 (i) 277.84 J

(ii) 22.5 J

(iii) 38.87 N

11 3072 N

12 (i) $4.16\,\text{m s}^{-1}$

(ii) 0.6 m

13 (i) 1.87 J

(ii) $3.86\,\text{m s}^{-1}$

(iii) $4.15\,\text{m s}^{-1}$

14 (i) Loss of 9.14 J

(ii) 0.12 m

Exercise 9.2

1 (i) $20\,\text{m s}^{-1}$

(ii) 2000 N; $1.\dot{1}\,\text{m s}^{-2}$

(iii) 53.29 m

2 (i) 2.9 m

(ii) 5 m s^{-1}

3 (i) 400 000 ÷ 25 000 = 16 m s^{-1}

(ii) $a = 0$; $F = 16\,000$ N; 400 000 ÷ 16 000 = 25 m s^{-1}

(iii) 0.02 m s^{-2}

4 (i) 50.625 kW

(ii) 1.12 m s^{-2}

(iii) 59.2 m s^{-1}

5 (i) $F = \frac{15000}{20} = 750$ N

$F = 1000 \times 10 \times 0.05 + 20k$

$20k = 750 - 500$

$k = 12.5$

(ii) 24.72 m s^{-1}

6 (i) $33\frac{1}{3}$ m s^{-1}

(ii) $18\frac{2}{11}$ m s^{-1}

(iii) 0.8 m s^{-2}

Stretch and challenge

1 320 m

2 Uphill: resistive force = kv^2; driving force = $\frac{60\,000}{25}$ = $1250 \times 10 \times 0.075 + kv^2$; $k = 2.34$

Downhill: $1250a = \frac{40\,000}{20} + 1250 \times 10 \times 0.075 - 2.34 \times 20^2$; $a = 1.60$ m s^{-2}

3 $v = \sqrt{\frac{2gd(M - \mu m)}{m + M}}$ m s^{-1}

4 Driving force = $\frac{P}{V} = mg \sin\theta + R$

$ma = \frac{2P}{V} - mg\sin\theta - \frac{1}{4}\left(\frac{P}{V} - mg\sin\theta\right)$

$a = \frac{7P}{4mV} - \frac{3}{4}g\sin\theta$

Exam focus

1 (i) 3360 J

(ii) 16 000 J

(iii) 17 015 J

(iv) 35.88°

2 25 515 J

3 0.211 m s^{-2}

10 Motion of a projectile

Exercise 10.1

1 (a) (i) $\mathbf{v} = \begin{pmatrix} 3 \\ 4 - 10t \end{pmatrix}$ **(ii)** $\mathbf{r} = \begin{pmatrix} 3t \\ 4t - 5t^2 \end{pmatrix}$

(b) (i) $\mathbf{v} = \begin{pmatrix} 10 \\ -10t \end{pmatrix}$ **(ii)** $\mathbf{r} = \begin{pmatrix} 10t \\ 3 - 5t^2 \end{pmatrix}$

(c) (i) $\mathbf{v} = \begin{pmatrix} 12.99 \\ 7.5 - 10t \end{pmatrix}$ **(ii)** $\mathbf{r} = \begin{pmatrix} 12.99t \\ 7.5t - 5t^2 \end{pmatrix}$

(d) (i) $\mathbf{v} = \begin{pmatrix} 28.98 \\ -7.76 - 10t \end{pmatrix}$ **(ii)** $\mathbf{r} = \begin{pmatrix} 28.98t \\ 20 - 7.76t - 5t^2 \end{pmatrix}$

2 (i) 1.2 s; 7.2 m

(ii) 2.4 s; 30.8 m

(iii) 1.73 s; 15 m

3 (i) 1.6 s; 24 m

(ii) 3.13 s; 46.92 m

(iii) 1.27 s; 17.24 m

4 (i) 2.45 s

(ii) 18.37 m s^{-1}

(iii) 30.62 m s^{-1}

(iv) 53.13°

5 (i) $0 = u^2 + 2 \times -10 \times 45 \Rightarrow u = 30$ m s^{-1}

(ii) 48.6°

(iii) 158.75 m

6 (i) $20 \sin\theta t - 5t^2$

(ii) 0.6

(iii) 12.8 m

(iv) 38.4 m

7 (i) $\frac{ut}{\sqrt{5}}; \frac{2ut}{\sqrt{5}} - 5t^2$

(ii) 38.73 m s^{-1}

(iii) 6.93 s

(iv) 60 m

8 (i) $2.5\sin\theta$

(ii) 23.58°

(iii) 45.83 m

9 11.26 s

10 (i) 29.66 m s^{-1} at 23.34° to the horizontal

(ii) 6.90 m

11 (i) $32 = 0.8ut$ (1); $4 = 0.6ut - 5t^2(2)$; from (1) obtain $t = \frac{40}{u}$, substitute in (2): $4 = 24 - \frac{8000}{u^2}$
$\Rightarrow u = 20$ m s^{-1}

(ii) 17.89 m s^{-1}

12 (i) $u_x = 20$ m s^{-1}; $u_y = 50$ m s^{-1}

(ii) 93.75 m

(iii) 32.02 m s^{-1}; 51.34°

(iv) 150 m

13 (i) 10 m s^{-1} at 36.9° to the horizontal

(ii) 2.8 s; 9.6 m

Exercise 10.2

1 (i) 1 m; (25, 0)

(ii) 10 m; $y = 1 + 0.16 \times 10 - 0.008 \times 10^2 = 1.8$ m

(iii) 0.6 s

(iv) 25 m s^{-1}; 1 s

(v) 25.71 m s^{-1}

2 (i) $x = 10t$; $y = 20t - 5t^2$;
$y = 20 \times \frac{x}{10} - 5 \times \left(\frac{x}{10}\right)^2 = 2x - 0.05x^2$

(ii) 60 m

(iii) 76.0°

3 (i) $x = \frac{ut}{\sqrt{2}}$; $y = 2 + \frac{ut}{\sqrt{2}} - 5t^2$;
$t = \frac{x\sqrt{2}}{u} \Rightarrow y = 2 + \frac{u}{\sqrt{2}} \times \frac{x\sqrt{2}}{u} - 5\left(\frac{2}{u^2}\right)x^2$
$= 2 + x - \frac{10x^2}{u^2}$

(ii) 8 m s^{-1}

4 (i) $\sin\theta = \frac{9}{41}$; $\cos\theta = \frac{40}{41} \Rightarrow x = 40t$, $y = 9t - 5t^2$

(ii) 4.05 m

(iii) 0.264 s; 1.536 s; 50.88 m

(iv) 40.15 m s^{-1}

(v) $y = \frac{9}{40}x - \frac{1}{320}x^2$; 72 m

5 (i) $x = 50\cos\theta\, t$; $y = 50\sin\theta\, t - 5t^2$
$\Rightarrow y = x\tan\theta - \frac{5x^2}{50^2\cos^2\theta}$
$= x\tan\theta - 0.002x^2(1 + \tan^2\theta)$

(ii) 26.6°; 63.4°

6 (i) $x = u_x t$; $y = u_y t - 5t^2 \Rightarrow y = u_y \times \frac{x}{u_x} - 5\left(\frac{x}{u_x}\right)^2$
$\Rightarrow u_x^2 y - u_x u_y x + 5x^2 = 0$

(ii) $2.5u_x^2 - 5u_x u_y + 125 = 0$ (1);
$1.8u_x^2 + 12u_x u_y + 720 = 0$ (2)

Solve (1) and (2) simultaneously to give $u_x = 10$;
$u_y = 7.5 \Rightarrow u = 12.5$ and $\theta = \tan^{-1}(0.75)$

Stretch and challenge

1 (i) $5 = 15\cos\theta\, t$ (1); $3 = 1 + 15\sin\theta\, t - 5t^2$ (2)
Substitute $t = \frac{1}{3\cos\theta}$ in (2)
$\Rightarrow 2 = 5\tan\theta - \frac{5\sec^2\theta}{9}$
$\Rightarrow 2 = 5\tan\theta - \frac{5}{9}(1 + \tan^2\theta)$
$\Rightarrow 5\tan^2\theta - 45\tan\theta + 23 = 0$

(ii) 28.5°; 83.3°

2 133.21 m

3 56.3°; 18.62 m s^{-1}

Exam focus

1 (i) $y = \frac{25}{\sqrt{2}}t - 5t^2$; 3.54 s

(ii) 18.69 m s^{-1}

(iii) 19.0° falling

2 (i) 15.56 m s^{-1}; 1.01 s

(ii) 0.33° below the horizontal

3 (i) $x = 25\cos\theta\, t$; $y = 25\sin\theta\, t - 5t^2$
$\Rightarrow y = x\tan\theta - 0.008(1 + \tan^2\theta)x^2$

(ii) 45°; 76.0°

(iii) 62.5 m; 29.41 m

4 (i) $x_P = ut\cos\theta$; $x_Q = 20 - ut\cos\theta$; $y_P = ut\sin\theta - 5t^2$; $y_Q = 4.5 - ut\sin\theta - 5t^2$

(ii) $uT\cos\theta = 10$; $uT\sin\theta = 2.25$

(iii) $2.25 > 5T^2 \rightarrow 0.45 > T^2 \rightarrow 0.45u^2 > 10.25^2$

11 Moments of forces

Exercise 11.1

1 (a) $T_1 = 33\frac{1}{3}$ N; $T_2 = 16\frac{2}{3}$ N

(b) $T_1 = 35$ N; $T_2 = 55$ N

(c) $T_1 = 54\frac{2}{7}$ N; $T_2 = 45\frac{5}{7}$ N

(d) $T_1 = 35$ N; $T_2 = 75$ N

2 2.31 m

3 (i) $3\frac{1}{3}$ N

(ii) $8\frac{1}{3}$ N

4 (i) $108\frac{1}{3}$ N; $1441\frac{2}{3}$ N

(ii) 0.629 m

(iii) The bench topples.

5 $36\frac{2}{3}$ N; $13\frac{1}{3}$ N

6 $166\frac{2}{3}$ N; $333\frac{1}{3}$ N

7 21 kg

8 86.54 cm from end

9 (a) 21.65 N

(b) 25 N

(c) 194.86 N

(d) $\frac{25\cos\theta}{\sin\phi}$ N

10 $T\sin 60° \times L = W\sin 45° \times \frac{L}{2} \Rightarrow T = \frac{\sqrt{3}}{2} = W\frac{1}{2\sqrt{2}}$
$\Rightarrow T = \frac{W}{\sqrt{6}}$

11 $\theta = 49.1°$; $R = 50\sqrt{3}$ N; $S = 100\sqrt{3}$ N

12 28.2°

13 (i) 233.24 N

(ii) 238.54 N

(iii) 33.02°

14 (i) $66\frac{2}{3}$ N; $133\frac{1}{3}$ N

(ii) 58.11 N

(iii) $25\sqrt{3}$ N

(iv) $\frac{\sqrt{3}}{5} = 0.35$

15 (i) $208\frac{1}{3}$ N

(ii) 1555.72 N

(iii) 1292.39 N at 21.15° below the horizontal

16 (i) 36.46 N

(ii) 7.185 m

Stretch and challenge

1 $F = S$; $S \times 2L\sin\theta = W \times L\cos\theta \Rightarrow 2F\tan\theta = W$

2 (i) 3.20 m

(ii) 0.48

3 (i) MX = 0.2 m; XC = 0.8 m

(ii) 2.86°

(iii) $T_1 = 8739.08$ N; $T_2 = 9987.52$ N

Exam focus

1 68.2°

2 (i) $208\frac{1}{3}$ N

(ii) 186.15 N at 14.25°

3 (i) 625 N

(ii) 548.29 N at 46.85° below the horizontal

4 (i) 1904.76 N

(ii) 1595.36 N at –109.53°

12 Centre of mass

Exercise 12.1

1 5

2 $(4\frac{10}{23}, \frac{1}{23})$

3 (–2, –4)

4 (a) $(2\frac{1}{3}, 1\frac{1}{3})$

(b) (3, 2)

(c) $(1\frac{1}{6}, 1\frac{1}{6})$

(d) $(1\frac{1}{3}, 1\frac{1}{2})$

5 (a) (1.6, 1.6)

(b) $(6, 1\frac{1}{3})$

(c) (0, 1.98)

(d) (–1, 1.27)

6 (a) (2.91, 2.06)

(b) (1.78, –0.39)

(c) (0.875, 0.875)

(d) (1.66, –1.75)

(e) (2, 1.59)

7 $\frac{4(77-6\pi)}{3(40-\pi)} = 2.10$; $\frac{8(61-3\pi)}{3(40-\pi)} = 3.73$

8 25.9°

9 $(\frac{3}{4}L, \frac{3}{4}L)$

10 (i) $(1\frac{3}{8}L, 1\frac{3}{8}L)$

(ii) $(1\frac{5}{12}L, 1\frac{5}{12}L)$

(iii) 41.8°

11 $\frac{1}{3}\pi R^2 H \times \frac{H}{4} + 2 \times \frac{2}{3}\pi R^3 \times \left(-\frac{3}{8}R\right)$
$= \left(\frac{1}{3}\pi R^2 H + 2 \times \frac{2}{3} - \pi R^3\right)\bar{X}$
$\Rightarrow \frac{H^2 - 6R^2}{12} = \frac{1}{3}(H + 4R)\bar{X}$
$\Rightarrow \bar{X} = \frac{H^2 - 6R^2}{4(H + 4R)}$

12 16.18 cm

13 7.86 cm

14 8.78 cm above the base, on the axis of symmetry

15 (i) 5.23 cm

(ii) 33.8°

16 **(i)** 22 cm

(ii) 51.3°

17 **(i)** (0.4, 1.2)

(ii) $(0, \frac{2}{3})$

(iii) (0.25, 1)

(iv) $4m$

18 53.1°

19 31.2 cm

Stretch and challenge

1 L = side of square

50% cut off: $\left(L^2 - \frac{1}{8}L^2\right)\bar{x} = L^2 \times \frac{L}{2} - \frac{1}{8}L^2 \times \frac{5L}{6}$

$\Rightarrow \frac{7}{8}L^2\bar{x} = \frac{19}{48}L^3 \Rightarrow \bar{x} = \frac{19}{42}L$

$= 0.45L < 0.5L$ hence stable

60% cut off : $(L^2 - 0.18L^2)\ \bar{x} = L^2\ x\ \frac{L}{2} - 0.18L^2 \times 0.8L$

$\Rightarrow 0.82L^2\,\bar{x} = 0.356L^3 \Rightarrow \bar{x} = 0.43L > 0.4L$ hence unstable

2 L = length of each rod

$\left(L \times \frac{L}{2} + L + \frac{5L}{4} + L \times \frac{5L}{4} + L \times \frac{L}{2}\right) = 4L\bar{x} \Rightarrow \bar{x} = \frac{7}{8}L$

$L \times 0 + L \times \frac{\sqrt{3}}{4}L + L \times \frac{3\sqrt{3}}{4}L + L \times \sqrt{3L} = 4L\bar{y} \Rightarrow \bar{y} = \frac{\sqrt{3}}{2}L$

$\Rightarrow \tan\theta = \frac{4\sqrt{3}}{7} \Rightarrow \theta = \tan^{-1}\left(\frac{4\sqrt{3}}{7}\right)$

3 **(i)** $(\pi \times 25 + 10\pi \times 10)\bar{y} = 25\pi \times 0 + 100\pi \times 5 \Rightarrow \bar{y} = 4$ cm

(ii) $\frac{1}{2}\ (\sqrt{105 - 5}) = 2.62$ cm

Exam focus

1 **(i)** $\frac{2}{3}$ cm

(ii) $\frac{4}{3}W$

2 **(i)** $\frac{20}{2+\pi} = 3.89$

(ii) 21.3°

3 **(i)** 14.04°

(ii) 38.66°

(iii) Slide

4 **(i)** 7.43 cm

(ii) 53.4°

13 Uniform motion in a circle

Exercise 13.1

1 **(i)** 0.2618 rad h^{-1}

(ii) **(a)** 1668 km h^{-1}

(b) 1536 km h^{-1}

(c) 629 km h^{-1}

(iii) $0.2618^2 \times 6371 \cos(67.85°) = 165$ km h^{-z}

2 **(i)** $\frac{MgL}{\sqrt{L^2 - R^2}}$

(ii) $R\sqrt{\frac{g}{\sqrt{L^2 - R^2}}}$

3 1.549 m s^{-1}

4 **(i)** 56.25°

(ii) 2 m

5 **(i)** 6 N

(ii) 70.53°

6 **(i)** $2.08\dot{3}$

(ii) 0.639 m s^{-1}

7 **(i)** $T\sqrt{3} = S\sqrt{2} + 6$

(ii) $T + S\sqrt{2} = 3.75$; $T = 3.57$ N, $S = 0.13$ N

8 **(i)** $\cos\theta_1 = \frac{0.16}{0.2} = 0.8$; $\cos\theta_2\ \frac{0.09}{0.15} = 0.6$;
$T\cos\theta_1 + T\cos\theta_2 = 2.5 \Rightarrow 8T_1 + 6T_2 = 25$

(ii) $3T_1 + 4T_2 = 15$; $T_1 = 0.71$ N, $T_2 = 3.21$ N

9 **(i)** **(a)** 1.53125 N

(b) 1.58125 N

(ii) 2.31 m s^{-1}

10 **(i)** 35.52 N, 9.48 N

(ii) 32.6 m s^{-1}

11 **(i)** 20 N, 12 N

(ii) $(12 + 20)\ \frac{\sqrt{3}}{2} = 0.4\ \frac{v^2}{0.3 \times \frac{\sqrt{3}}{2}} \Rightarrow v^2 = 18$
$\Rightarrow v = 3\sqrt{2}$ m s^{-1}

(iii) 0.38 s

12 **(i)** 31.28 m s^{-1}

(ii) 0 m s^{-1}

13 17.86 cm

14 **(i)** 56 250 N

(ii) 5.76 cm

15 **(i)** 13.16 rad s^{-1}

(ii) 3.85 N

Stretch and challenge

1 $T\,\text{Cos}\,\theta = 10m$; $T\sin\theta = m \times 25\pi^2 \times L\sin\theta$
$\Rightarrow 25m\pi^2 \times L\,\text{Cos}\,\theta = 10m$
$\Rightarrow h = L\cos\theta = \frac{10}{25\pi^2} = 0.0405...$

2 $R = mg\sin\theta + \frac{mv^2}{L\tan\theta}; T = \frac{mv^2}{L} - mg\cos\theta$

3 (i) $M = 0.6m$

(ii) $5.77\,\text{rad s}^{-1}$

Exam focus

1 (i) $\frac{0.25}{1.24}(1.75\omega^2 + 10)$

(ii) $\frac{\sqrt{2}}{31}(10.5\omega^2 - 17.5)$

(iii) $T_{BP} \geqslant 0 \Rightarrow 10.5w^2 - 17.5 \geqslant 0 \Rightarrow w^2 \geqslant \frac{17.5}{10.5} = \frac{5}{3}$

2 (i) 2.52 N

(ii) 1.98 N

(iii) 40 cm

3 (i) 105 N, 45 N

(ii) $8.94\,\text{m s}^{-1}$

4 (i) 1.73 N

(ii) $2.40\,\text{m s}^{-1}$

14 Hooke's law

Exercise 14.1

1 (i) 0.18 m

(ii) 25 N

(iii) 3 N

(iv) 2.25 m

(v) 1.6 m

(vi) 8 N

2 (i) 80 N

(ii) 48 N

3 (i) 0.35 m, 0.35 m

(ii) 35 N

4 (i) 7.5 N

(ii) $5\,\text{m s}^{-2}$

5 3.69 N

6 (i) $0.41\dot{6}$ m

(ii) $10.41\dot{6}$ J

7 20 N

8 (i) 5 m

(ii) $12.8\,\text{m s}^{-1}$

9 (ii) $4.47\,\text{m s}^{-1}$

(iii) 5.16 m

10 (i) 49.09 N

(ii) $2.93\,\text{m s}^{-1}$

11 (i) 0.24 m

(ii) $0.53\,\text{m s}^{-1}$

12 (i) 0.4 m

(ii) 9.375 J

(iii) $0.5\,\text{m s}^{-1}$

13 0.18 m

14 (i) 25 N, 5 N

(ii) $16\,\text{m s}^{-2}$

(iii) $0.73\dot{}$ m

15 (i) 1.2 m

(ii) 1.6 m

16 (i) 0.65 m

(ii) $100x - 60$; $190 - 100x$

(iii) 42.3125 J

(iv) $2\,\text{m s}^{-1}$

Stretch and challenge

1 (i) $2T\cos\theta = 5$; $\cos\theta = \frac{0.75}{\sqrt{(1.8^2 + 0.75^2)}}$

$= \frac{5}{13}$; $\frac{\lambda}{3.6} \times 0.3 = 6.5$; so that $\lambda = 78$ N

(ii) $14.2\,\text{m s}^{-1}$

2 4 N

3 (i) $\sqrt{40x\left(2.5 - \frac{x}{L}\right)}$

(ii) $\sqrt{62.5L}$

(iii) At 0 the mechanical energy is $10L$. At the maximum height the mechanical energy is $5x + \frac{10}{L}(x - L)^2$. $10L = 5x + \frac{10}{L}(x - L)^2$; $\frac{10x^2}{L} - 15x = 0$ so that $x = \frac{3L}{2}$

Exam focus

1 (i) 20 N

(ii) $6.48\,\text{m s}^{-1}$

2 (i) 0.625 m

(ii) $3.35\,\text{m s}^{-1}$

(iii) 1 m

3 (i) $0.6\dot{3}$ m, 7 N

(ii) $0.81\dot{6}$ J, $1.63\dot{3}$ J

(iii) $1.58\,\text{m s}^{-1}$

4 (i) 24 N, 8 N, 20 m s^{-2}

(ii) 0.24 m

(iii) 1.22 m; At A the mechanical energy is 20 J. At the point where BC becomes slack the elastic potential energy of the system is $30 \times 0.6^2 = 10.8$ J. This is less than 20 J, hence the particle is still in motion. Furthest distance from A is 1.22 m.

15 Linear motion under a variable force

Exercise 15.1

1 (i) $x = -\frac{v}{3} + c$

(ii) $v = -0.5 + Ae^{2t}$

(iii) $v = -2e^{-t} + c$

(iv) $v = \frac{1}{3t + c}$

(v) $x = -1 + Ae^{0.5v^2}$

(vi) $v = \sqrt[3]{15x + c}$

2 (i) 6.5 m s^{-1}

(ii) 1.01 m s^{-1}

(iii) 0.37 m s^{-1}

(iv) 0.275 s

(v) $\frac{\pi}{4} = 0.785$ s

(vi) 0.923 m

3 (i) $v = 16(1 - e^{-\frac{t}{4}})$; $m\frac{dv}{dt} = 4m - 0.25mv$; hence $4\frac{dv}{dt} = 16 - v$

(ii) $4e^{-1}$

4 (i) $0.5v\frac{dv}{dx} = \frac{5}{(x+1)^5} \Rightarrow \int v dv = \int \frac{10}{(x+1)^5} dx \Rightarrow \frac{1}{2}v^2$
$= -\frac{10}{4(x+1)^4} + c$; $v = 0$ when $x = 0$ *hence* $c = 2.5$
$\Rightarrow v^2 = 5\left[1 - \frac{1}{(x+1)^4}\right]$

(ii) $\frac{10}{243}$ m s^{-2}

5 (i) $025\frac{dv}{dt} = -0.05v - 2.5 \Rightarrow \frac{dv}{dt} = -0.2(50 + v)$

(ii) $v = 70e^{-0.2t} - 50$

(iii) $x = 350(1 - e^{-0.2t}) - 50$

(iv) 50 m

6 (i) $v = \sqrt{10t + 9}$

(ii) $\sqrt[3]{102} = 4.67$ m s^{-1}

7 13.9 m s^{-1}

8 (i) $2v\frac{dv}{dt} = -F(x)$; $F(x) = a + bx$; $F(0) = 4$, $F(10) = 9$
$\Rightarrow a = 4$ and $b = 0.5$
$\Rightarrow 2v\frac{dv}{dt} = -(4 + 0.5x) \Rightarrow v\frac{dv}{dx} = -0.25(8 + x)$

(ii) 32.8 m

9 (i) $v\frac{dv}{dx} = \frac{2}{x+1} \Rightarrow \int v dv = \int \frac{2}{x+1} dx = v^2 = 4\ln|x + 1| + k$;
$v = 3$ when $x = 0 \Rightarrow k = 9$
$\Rightarrow v^2 = 4\ln|x + 1| + 9$

(ii) 3.93 m s^{-1}

10 (i) 2.625 N

(ii) $x = 2\left(\frac{1 - e^{-4t}}{1 + e^{-4t}}\right)$

(iii) 2 m

11 (i) $v = \frac{1 - e^{-2t}}{1 + e^{-2t}}$

(ii) 1 m s^{-1}

12 (i) $v = 40(1 - e^{-0.1t})$

(ii) 42.6 m

13 (i) 3.13 m

(ii) 2.39 m s^{-1}

14 (i) $1000\frac{dv}{dt} = -(500 + 2500v) \Rightarrow \frac{dv}{dt} = -2.5(0.5 + v)$

(ii) $v = -0.2 + 25e^{-2.5t}$

(iii) 1.93 s

(iv) 9.54 m

Stretch and challenge

1 (ii) $v = 2000(1 - e^{-0.005t})$

(iii) 124 m

2 (ii) $\frac{2U}{3k}^{1.5}$

3 (i) 1.002 s

(ii) 7.98 m

(iii) $v = 8(1 - e^{-1.25t})$

(iv) $x = 8t - 6.4(1 - e^{-1.25t})$

Exam focus

1 19.5 m s^{-1}

2 (i) 25 m s^{-1}

(ii) 227 $\frac{3}{11}$

3 2.5 m

4 (i) -0.125 m s^{-2}

(ii) 8.75 s

Past examination questions

Mechanics 1 (M1) Paper 1

1 $T = 40\,\text{N}$, $R = 10\,\text{N}$

2 3000 J, 31.1 N

3 8.91 N

4 **(i)** 3N

(ii) 2.4 N

(iii) 1.4 N

5 **(i)** Gain in KE = 3240 J, Loss in PE = 9072.6 J, Work against resistance = 5832.6 J

(ii) 96.1 m

(iii) $0.77\,\text{m s}^{-2}$

6 **(i)** $3.2\,\text{m s}^{-1}$

(ii) $6\,\text{m s}^{-1}$

(iii) 8.5 s

(iv) $1\,\text{m s}^{-2}$

7 **(i)** 20 s

(ii) 80 s

(iii) $4\,\text{m s}^{-1}$

(iv) $1173\frac{1}{3}$ m

Mechanics 2 (M2) Paper 1

1 8.8 cm

2 **(i)** 5.25 cm

(ii) $F > 10\,m \sin 35°$; $R = 10\,m \cos 35° \Rightarrow \mu > \tan 35° = 0.7$

3 **(i)** $x_Q = 48t$, $y_Q = 14t - 5t^2$; $x_P = 24$, $y_P = 7 - 5t^2$.
When $t = 0.5$ $x_Q = x_P = 24$, $y_Q = y_P = 5.75$

4 **(i)** $R \cos\theta = 1.2$; $\cos\theta = \frac{0.3}{0.5} = 0.6 \Rightarrow R = 2\,N$

(ii) $2.31\,\text{m s}^{-1}$

(iii) 1.09 s

5 **(i)** 562.5 N

(ii) 37.9° below the horizontal

6 **(i)** Extension $= 2\sqrt{1.2^2 + 0.5^2} - 2 = 0.6 \Rightarrow T = 0.3\lambda$
$2T \cos\theta = 0.6 \times 10$ and $\cos\theta = \frac{0.5}{1.3} = \frac{5}{13}$
$\Rightarrow 2 \times 0.3\lambda \times \frac{5}{13} = 6 \Rightarrow \lambda = 26\,\text{N}$

(ii) $2.42\,\text{m s}^{-1}$

7 **(i)** $81\,\frac{dv}{dt} = 135 - 9v = 9(15 - v) \Rightarrow \frac{9}{15-v}\,\frac{dv}{dt} = 1$

(ii) $\int \frac{1}{15-v}\,dv = \int \frac{1}{9}\,dt \Rightarrow -ln\,|15 - v| = \frac{1}{9}\,t + c$; $v = 0$ when $t = 0 \Rightarrow c = -ln\,15$
$\frac{1}{9}\,t = ln\left|\frac{15}{15-v}\right| \Rightarrow 15 - v = 15e^{\frac{1}{9}t} \Rightarrow v = 15\,(1 - e^{\frac{1}{9}t})$

(iii) 49.7 m

Mechanics 1 (M1) Paper 2

1 **(i)** 2000 N

(ii) 30 kW

2 **(i)** 25 000 J

(ii) 20 s

3 73 N, 41.1°

4 **(i)** $8.94\,\text{m s}^{-1}$

(ii) $7.07\,\text{m s}^{-1}$

5 **(i)** $x = 2t^3 - \frac{1}{4}\,Kt^4$

(ii) $t = \frac{6}{K}$

(iii) $k = 1$

(iv) $32\,\text{m s}^{-1}$

6 **(i)** $1\,\text{m s}^{-2}$

(ii) **(a)** $hp = 3\,\text{m}$, $hq = 7\,\text{m}$

(b) $2\,\text{m s}^{-1}$

(iii) Time taken for P to reach ground $= t_P$
then $3 = 2t_P + 5t_P^2 \Rightarrow (5t_P - 3)(t_P + 1) = 0 \Rightarrow t_P = 0.6\,\text{s}$
Time taken for Q to reach ground $= t_Q$
then $-7 = 2t_Q - 5t_Q^2 \Rightarrow (5t_Q - 7)(t_Q + 1)$
$\Rightarrow t_Q = 1.4\,\text{s}$
$\Delta t = 1.4 - 0.6 = 0.8\,\text{s}$

7 **(i)** $6\,\text{m s}^{-1}$, $0.6\,\text{m s}^{-2}$

(ii) 13.9 s

(iii) 50 m

Mechanics 2 (M2) Paper 2

1 **(i)** 60°

(ii) 14.4 N, 76.1°

2 **(i)** $2 + 34 + \frac{20 \times 0.8}{Sin^{-1}(0.8)} = 53.25$

(ii) 22.2 cm

3 **(i)** 3 N

(ii) $3\,\text{m s}^{-1}$

4 (i) At *0* the mechanical energy is 0.
At *P* the mechanical energy is
$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{15m}{3}\right)x^2 - 10m\,(x + 3)$
$\Rightarrow v^2 = 20\,(x + 3) - 5x^2$
$= 5\,(12 + 4x - x^2)$

(ii) $20\,\text{m}\,\text{s}^{-2}$ upwards

5 (i) 12 N

(ii) 0.289

6 (i) 76.5 m

(ii) 65.4 m

(iii) $47.7\,\text{m}\,\text{s}^{-1}$ at 61.8°

7 (i) 39.9 N

(ii) 47.5°, 18.5 N