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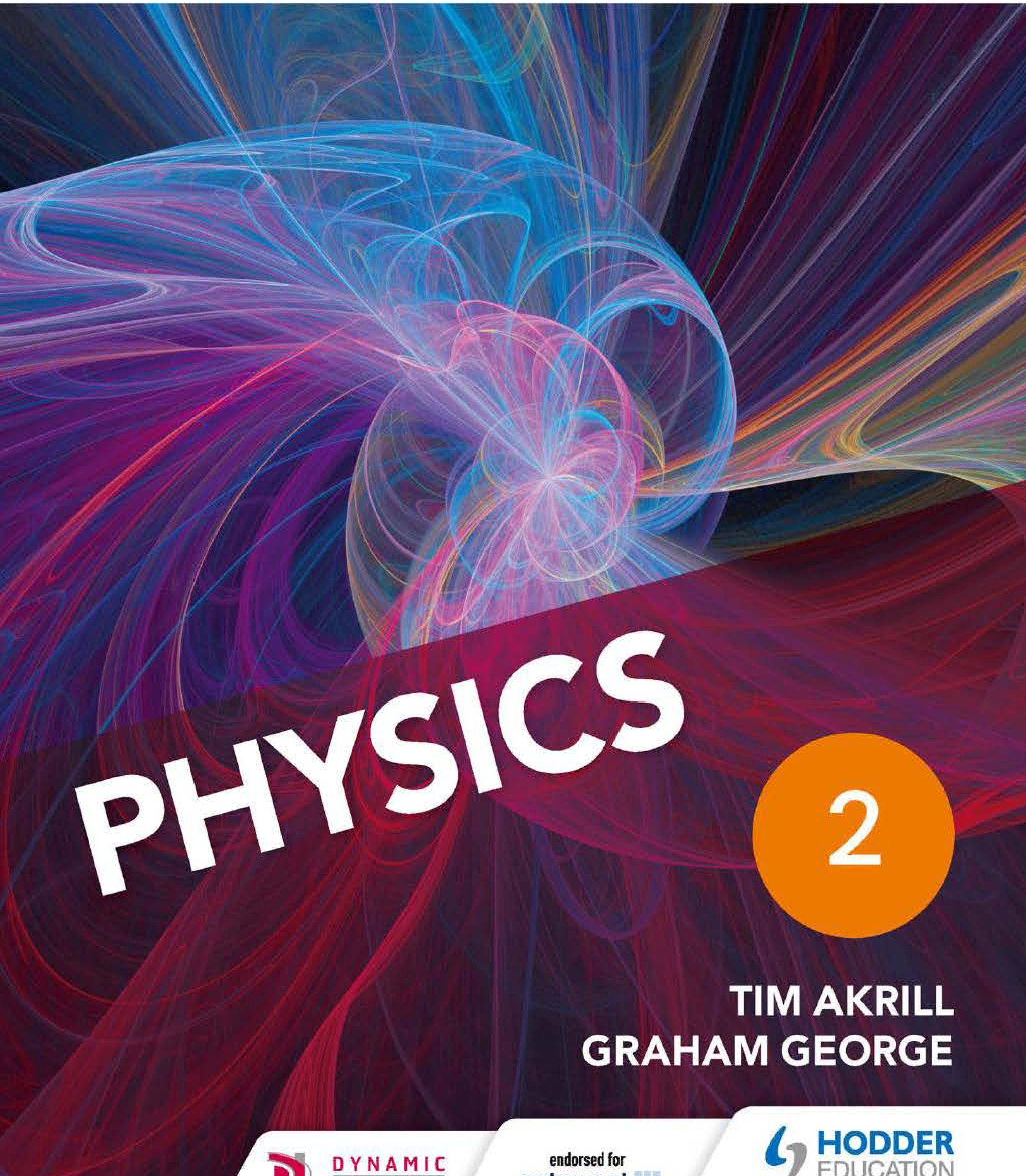

PHYSICS 2

TIM AKRILL
GRAHAM GEORGE

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Contents

Introduction

Welcome to *Edexcel A Level Physics Year 2 Student's Book*. If you used the Year 1 book, you will already be familiar with the approach adopted by the writers – essential text, illustrated with many contextual examples, an emphasis on practical work and lots and lots of questions for you to try. The Edexcel specification has been developed from the best of the Edexcel concept-led and the Salters Horners context-led approaches. Although the book has been specifically written to cover the concept approach to the specification, the contextual examples and practice questions make it a most valuable resource for the context approach as well. The authors both have vast experience of teaching, examining and writing about physics. Both have examined for Edexcel at a senior level for over 30 years and served as Chief Examiners.

Although this book is designed as a resource for the second year of the A-level course, you mustn't forget the work you did in the first year as your A-level examination will test you on the whole of your two years' work. Some of the *Prior Knowledge* questions at the start of each chapter will, hopefully, jog your memory!

A key aspect of the text is the emphasis on practical work. Although you do not have a practical examination as such, questions based on practical work pervade all the A-level papers, particularly **Paper 3**, which comprises practical-based and synoptic questions. There is also an internally assessed Practical Endorsement at A level, for which you should have started a portfolio of work in Year 1. Nine of the *Core Practicals* in the specification were described in Book 1 and the other seven are described here in detail and in such a way that students can carry out the experiments in a laboratory environment. Each experiment has a set of data for you to work through, followed by questions similar to those you will be asked in the examination. Questions within written examination papers will aim to assess the knowledge and understanding that students gain while carrying out practical activities, both within the context of the 16 core practical activities and in novel practical scenarios. In addition, the completion of the 16 core practical activities can provide evidence of competence for the Science Practical Endorsement. The core practicals are also intended to provide students with opportunities to undertake investigative work, therefore the core practical experiments described in this book must be considered as *examples* of the sort of activity that could be undertaken.

Many other experiments – under the heading of *Activities* – are also described, together with data and questions. An additional chapter – *Practical skills* – is available online. It is a reminder of the key practical skills that should be developed through teaching and learning and will form the basis of practical assessment in the written examination.

Before carrying out *any* practical activity, teachers *must* identify any hazards and assess any risks. This can be done by consulting a model (generic) risk assessment such as that provided by CLEAPSS to subscribing authorities.

Emphasis is also placed on practising questions. The text is abundantly illustrated by *Examples*, which are accompanied by answers to enable you to check your progress. There are then *Test Yourself* questions for you to try (Answers for these can be accessed using the QR codes found in the *Free Online Resources* section at the end of this book) and at the end of each chapter there are *Exam practice questions*. These are graded in terms of difficulty (● = A-level Grade E, ● A-level Grade C and ● = A-level Grade A/A*). In the *Exam practice questions* the mark allocation for each part is shown, as it would be in the examination. The answers give an *indication* of how the marks might be awarded but not in the same detail that there would be in an actual mark scheme.

Throughout the book there are *Key Terms* highlighted in the margin that you need to learn. There are also numerous *Tips*. These may be reminders, for example, to use SI units, warnings to avoid common errors or hints about short cuts in performing calculations.

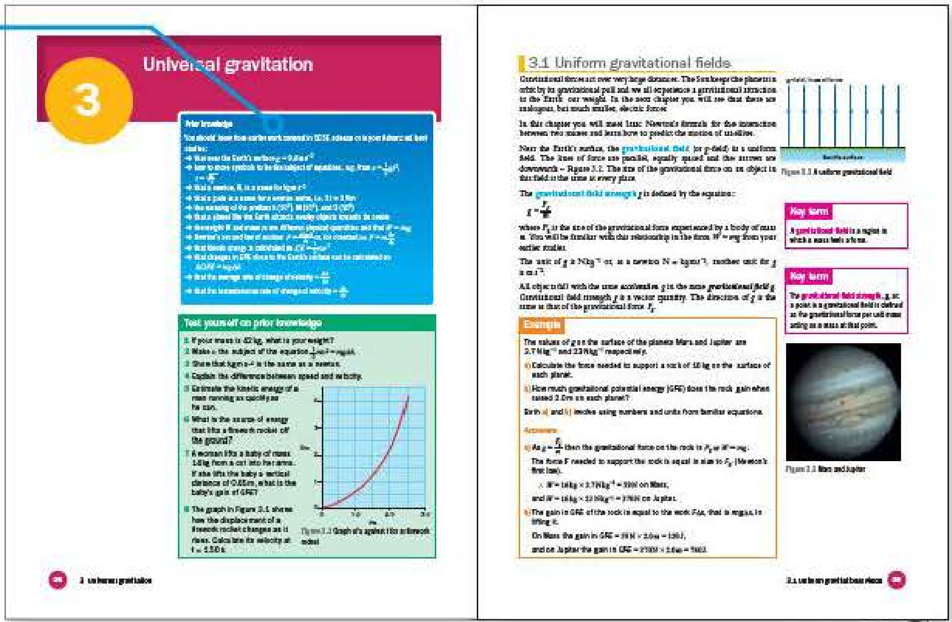
Another additional chapter – *Preparing for the exams* – is available online and is a valuable guide on revision and exam technique. As you need to put these principles into practice from day 1, you are strongly advised to read through this *before* you start your course (although you probably won't be able to attempt the questions). You should then re-visit the chapter from time to time, particularly when prompted in the book to refer back to the questions in that chapter. You may also find it helpful to re-visit Chapter 18 – *Maths in Physics* in the Year 1 book. This provides an outline of the mathematical requirements, together with lots of simple (and not so simple!) examples for you to practise. You should note the extra requirements for A level compared with those for the AS examination.

The authors have enjoyed writing this book – we hope you enjoy reading it and find it, along with the supporting material, a valuable resource to help you with your studies. Good luck!

Get the most from this book

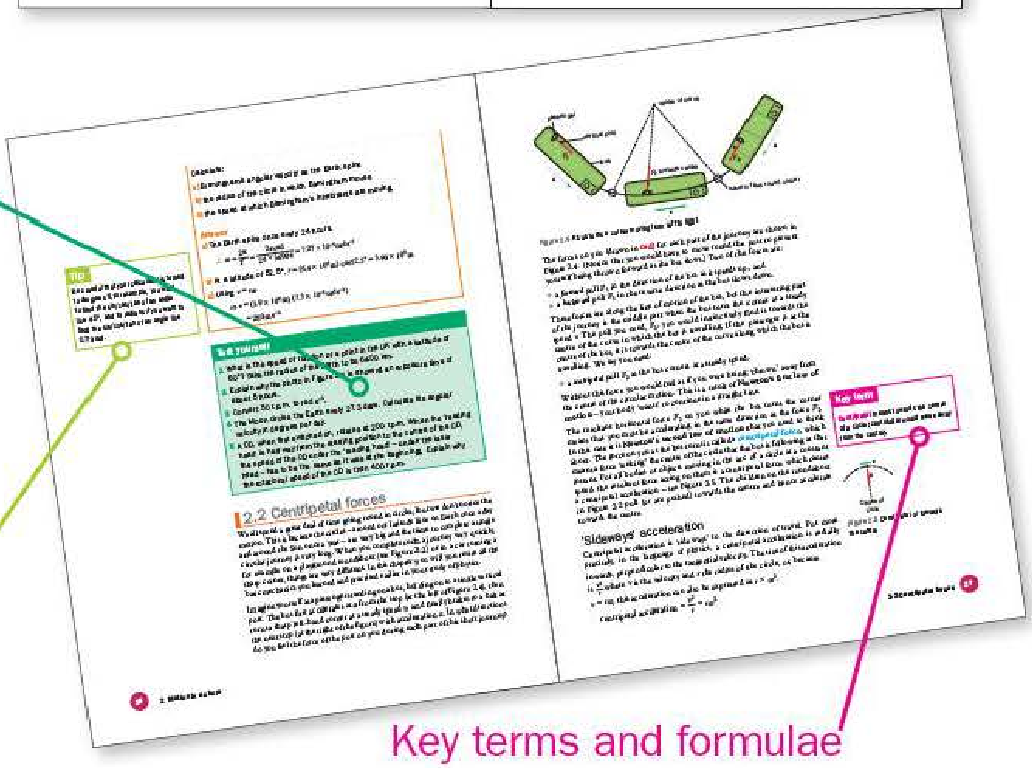
Welcome to the **Edexcel A level Physics Year 2 Student's Book**! This book covers Year 2 of the Edexcel A level Physics specification.

The following features have been included to help you get the most from this book.



Prior knowledge

This is a short list of topics that you should be familiar with before starting a chapter. The questions will help to test your understanding.



Test yourself questions

These short questions, found throughout each chapter, are useful for checking your understanding as you progress through a topic.

Tips

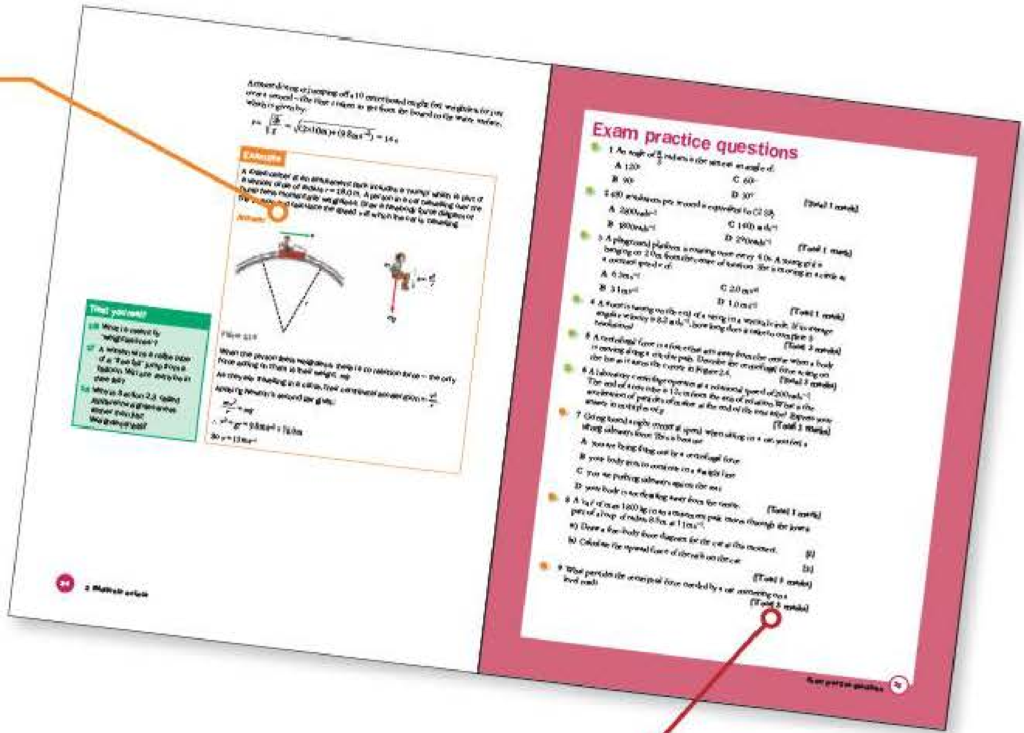
These highlight important facts, common misconceptions and signpost you towards other relevant topics.

Key terms and formulae

These are highlighted in the text and definitions are given in the margin to help you pick out and learn these important concepts.

Examples

Examples of questions and calculations feature full workings and sample answers.

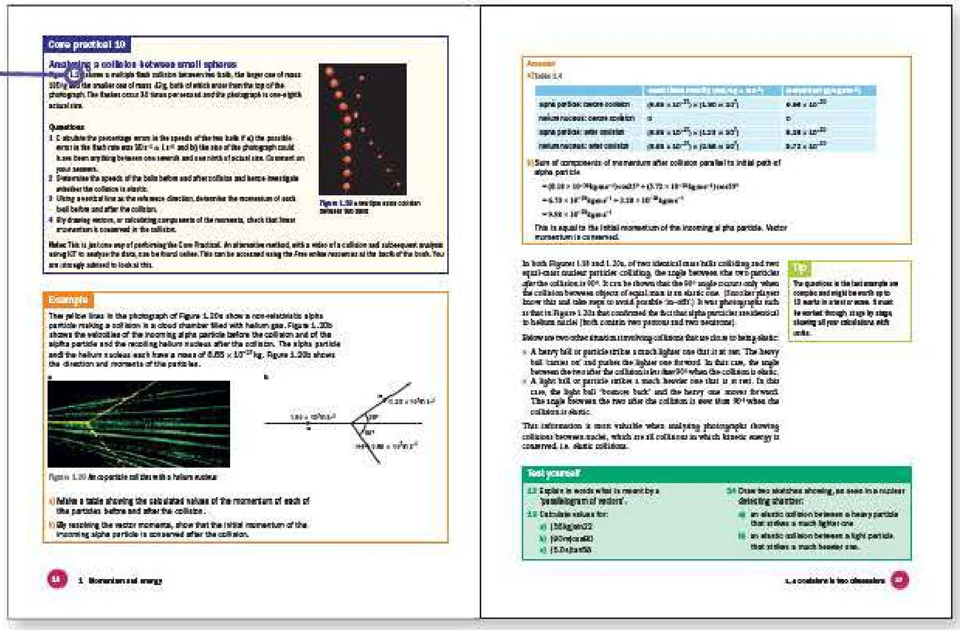


Exam practice questions

You will find Exam practice questions at the end of every chapter. These follow the style of the different types of questions you might see in your examination, including multiple-choice questions, and are colour coded to highlight the level of difficulty. Test your understanding even further, with Maths questions and Stretch and challenge questions.

Activities and Core practicals

These practical-based activities will help consolidate your learning and test your practical skills. Edexcel's Core practicals are clearly highlighted.



Dedicated chapters for developing your **Practical skills** and **Preparing for your exam** are also included with the other *Free online resources*, details of which are available at the end of this book.

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t = top, *b* = bottom, *l* = left, *c* = centre, *r* = right

1 Momentum and energy

Prior knowledge

You should know from earlier work covered in GCSE science or in your Advanced level studies:

- that momentum is defined as $p = mv$ and is a vector quantity
- that vector quantities, e.g. p or v, have a direction, but scalar quantities, e.g. m or t, do not
- how to add vector forces, e.g. 4 N plus 3 N at right angles $\Rightarrow$ a force of size 5 N
- that momentum is measured in N s or kg m s^{-1} as N $\equiv$ kg m s^{-2}
- that kinetic energy is defined as $KE = \frac{1}{2}mv^2$
- that kinetic energy is measured in J or N m or kg m^2 s^{-2}
- how to use algebra to manipulate symbols, e.g. to produce $KE = \frac{p^2}{2m}$
- Newton's second law of motion in the forms $F = m\frac{dv}{dt}$ and $F = v\frac{dm}{dt}$
- Newton's third law: the push or pull of A on B is equal in size to the push or pull of B on A
- that W and m are different quantities and that $W = mg$
- that changes in GPE near the Earth's surface are calculated as $\Delta GPE = mg\Delta h$

Test yourself on prior knowledge

1 State why momentum is a vector quantity and kinetic energy a scalar quantity.

2 State **a)** two vector quantities, and **b)** two scalar quantities other than those quoted in Q1 or Q3.

3 Sketch a diagram to illustrate how to add a displacement of 50 m north to 20 m east.

4 Show that a joule is equivalent to a kilogram metre squared per second squared.

5 Use definitions of momentum p and KE to show that $p^2 = 2m \times KE$.

6 Write down in words a statement of Newton's second law in its most general form.

7 A book of mass 3.2 kg rests on a table. What is the upward push of the table on the book?

8 A crane lowers a 'bag' of aggregate of mass 1600 kg a vertical distance of 2.8 m. Calculate the loss of gravitational potential energy of the bag of aggregate.

1.1 Impulse and momentum

Key term

Impulse = force × time

In *Edexcel A Level Physics Year 1 Student's Book*, we briefly introduced the term **impulse** and saw that it was given by the product of the force acting on a body and the time for which the force acted, i.e.

$$\text{Impulse} = F\Delta t$$

The unit of impulse will therefore be N s

We also know that force = rate of change of momentum (Newton's second law), i.e.

$$F = \frac{\Delta p}{\Delta t} \text{ or, more mathematically, } F = \frac{dp}{dt}$$

We can re-write this as

$$F\Delta t = \Delta p$$

But as $F\Delta t$ is impulse, we can say

Impulse = change in momentum

We can represent impulse graphically as shown in Figure 1.1.

Tip

Don't get confused, remember that:

- Impulse = change in momentum
- Force = *rate* of change of momentum

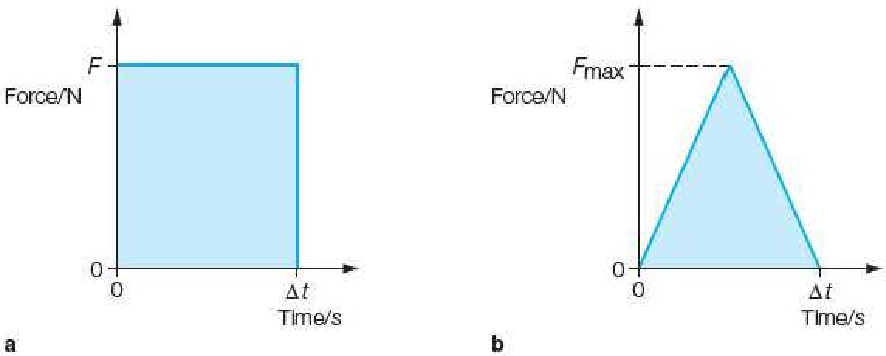


Figure 1.1 Force–time graphs

In Figure 1.1a, the force remains constant and so the impulse is $F\Delta t$. This is, of course, the area under the graph. We can extend this to any shape of graph – the area under a force–time graph is always equal to the impulse. So, in Figure 1.1b, the impulse is the area of the triangle $= \frac{1}{2}F_{max}\Delta t$.

For those of you who are mathematically inclined, we had above that $F = \frac{dp}{dt}$. If we now integrate both sides of the equation we get:

$$\int_0^t F\,dt = \int_{p_1}^{p_2} dp = p_2 - p_1 = \Delta p = \text{impulse}$$

We know that $\int_0^t F\,dt$ means the area under a graph of force against time (even if F is not constant) and so this tells us that, indeed, the area under a F–t graph is the impulse, or change of momentum.

Impulse is an important factor in car safety and plays a major part in all ball games.

Tip

You do not need to have any knowledge of calculus for the examination.

Example

A boy of mass 30 kg jumps off a wall 0.90 m high. On hitting the ground, his impact time is 0.2 s.

a) Determine

i) the speed of impact

ii) his momentum on impact

iii) the impulse of the ground on his feet

iv) the force he experiences.

b) Explain why bending his knees on impact reduces the force exerted on him.

Answer

a) i) Using $\frac{1}{2}mv^2 = mg\Delta h \Rightarrow v = \sqrt{2g\Delta h}$

$$= \sqrt{2 \times 9.8\,\text{m s}^{-2} \times 0.90\,\text{m}} = 4.2\,\text{m s}^{-1}$$

ii) Momentum $p = mv = 30\,\text{kg} \times 4.2\,\text{m s}^{-1}$

$= 126\,\text{kg m s}^{-1}$ or $126\,\text{N s}$

iii) Impulse = Change in momentum = 126 N s (since he comes to rest)

iv) From impulse $= F\Delta t$

$$\Rightarrow F = \frac{\text{impulse}}{\Delta t} = \frac{126\,\text{N s}}{0.2\text{s}} = 630\,\text{N}$$

(This is just over *twice* his weight.)

b) If he bends his knees on impact, he will increase the time of impact as he will come to rest more slowly. If the time of impact is increased, the force of impact will be reduced.

Car safety

Consider a car involved in a head on collision (the major cause of death in car accidents). Before the collision, the car will have momentum $p = mv$. On collision, the car (and the passengers inside) will be brought to rest (zero momentum) very quickly. This change of momentum, Δp, will produce an impulse $F\Delta t$, experienced by the car and its occupants. If we can increase the time of impact, Δt, then the force F experienced by the car and its occupants will be proportionally reduced. This can be achieved by the design of the car (a 'crumple zone' at the front and air bags for the front seats) and by the occupants *all* wearing seat belts.

Figure 1.2 Car undergoing crash test

The safety of European cars is tested in a laboratory and every make of car is given an 'NCAP' (New Car Assessment Programme) safety rating. One of the tests involves driving the car into a concrete block (by driving we of course mean by remote control, with a wired-up dummy in the driver's seat). One such test is shown in Figure 1.2 – here we can see the crumple zone at the front of the car and the driver protection from the seat belt and inflated air bag.

Example

To a good approximation, a force–time graph for a head-on car crash looks like Figure 1.3.

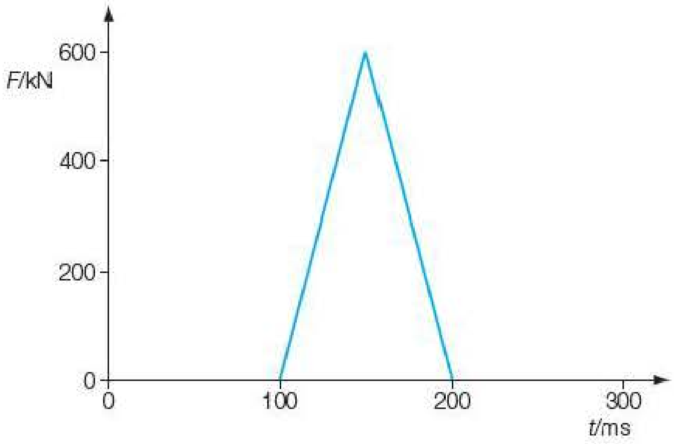


Figure 1.3 Force–time graph for a head-on car crash

a) Calculate the impulse experienced by the car in the crash.

b) If the car has a mass of 1500 kg, what was its speed immediately before collision?

c) To improve the safety of the car, the manufacturer sets out to design a more effective crumple zone. Explain what the manufacturer would try to do to achieve this.

d) Copy Figure 1.3 and add a second graph to show what would happen if the car had an improved crumple zone and collided with the same speed as before. Explain what features of the two graphs are different and what features are the same.

Answer

a) Impulse $= \text{area under curve} = \frac{1}{2}\text{ base} \times \text{height}$

$$= \frac{1}{2} \times 0.10\,\text{s} \times 600 \times 10^3\,\text{N} = 30\,000\,\text{N s or } 30\,000\,\text{kg m s}^{-1}$$

b) From impulse $= \Delta p$ we have $\Delta p = 30\,000\,\text{kg m s}^{-1}$

As $\Delta p = m\Delta v \Rightarrow \Delta v = \frac{\Delta p}{m} = \frac{30000\,\text{kg m s}^{-1}}{1500\,\text{kg}} = 20\,\text{m s}^{-1}$

c) The manufacturer would try to increase the impact time.

As impulse $= m\Delta v = F\Delta t$, for the same car (same m) travelling at the same speed (v) the force of impact will be reduced if the time of impact Δt can be increased.

d) The second graph should start at the same place as the first, have a smaller maximum force and a longer impact time, but the *areas* under each of the two graphs should be the *same*.

A question of sport

Impulse plays a very important part in all ball games. Let's start with football. The longer the foot remains in contact with the ball, the greater the impulse will be for a given kicking force. A greater impulse means a greater change in momentum, so the ball travels with greater speed. A greater time of impact is achieved by the player 'following through'. A good follow through is important in other games too, for example cricket, tennis, golf and hockey. In hockey, players are not allowed to follow through with a high stick if they are close to other players – they would be penalised for dangerous play. Modern hockey players now employ the sweep hit. The player gets low to the ground and 'sweeps' the ball with a horizontal stick. The ball stays on the stick for a longer time, giving a larger impulse and also more control, as shown in Figure 1.4.

Figure 1.4 A sweep hit. The player in yellow is probably thankful that she is wearing shin pads!

Conversely, players wear shin pads in football and hockey. The pads serve to increase the time of impact and therefore reduce the force if a player is kicked playing football or struck with the ball (or stick!) in hockey.

In tennis, the tension in the racket strings is important. If the strings are loosely strung, the ball stays in contact with the racket for a longer time and so the impulse, and therefore the speed with which the ball travels, is greater. However, it is more difficult to control the direction of the ball. Good players exert control by imparting 'top spin'. This causes the ball to rotate in the direction of travel, which increases the air pressure acting on the top of the ball, making the ball 'dip' once it has crossed the net. This is exemplified by Rafael Nadal, who has his rackets strung at low tension and imparts huge top spin to keep the ball in court. On the other hand, Novak Djokovic, who is more of a 'touch' player, has his rackets more tightly strung to give him more control. Table tennis players use top spin and back spin and footballers use side spin to create shots that swerve – 'Bend it like Beckham'.

Figure 1.5 Rafael Nadal plays a top spin forehand – note the follow through

Example

In a dynamic test, a tennis ball of mass 57 g is dropped from a height of 2.54 m onto a steel plate. It rebounds to a height of 1.38 m. A graph of how the force on the ball varied with time was recorded. A simplified version of the graph is shown in Figure 1.6.

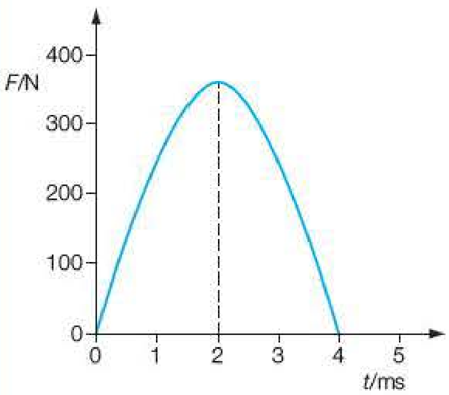


Figure 1.6 Force–time graph for a bouncing tennis ball

a) Show that the ball hits the plate with a speed of about 7 m s^{-1} and rebounds with a speed of about 5 m s^{-1}.

b) Discuss whether the ball meets the specification that 'in the 100 inch test the ratio of the speeds before and after impact must be 0.728 – 0.762'. (1 inch = 2.54 cm).

c) Calculate the impulse of the plate on the ball.

d) Show that the maximum force exerted on the ball calculated from this data is in agreement with that shown on the graph.

e) Comment on the test report statement that 'the maximum force is between 100 and 1000 times the weight of the ball'.

Answer

a) Using $\frac{1}{2}mv^2 = mg\Delta h \Rightarrow v = \sqrt{2g\Delta h}$

$$v_1 = \sqrt{2 \times 9.8\,\text{m s}^{-2} \times 2.54\,\text{m}} = 7.1\,\text{m s}^{-1} \approx 7\,\text{m s}^{-1}$$

$$v_2 = \sqrt{2 \times 9.8\,\text{m s}^{-2} \times 1.43\,\text{m}} = 5.3\,\text{m s}^{-1} \approx 5\,\text{m s}^{-1}$$

b) The ball was dropped from 2.54 m, which is the required '100 inches'.

The ratio of speed after to speed before is $\frac{5.3}{7.1} = 0.75$.

This is within the specified range and so the ball passes the '100 inch test'.

c) Impulse = change of momentum. Taking upwards as positive:

$$\text{Impulse} = m(v_2 - v_1) = 57 \times 10^{-3}\,\text{kg} \times (5.3 - [-7.1])\,\text{m s}^{-1}$$
$$= 57 \times 10^{-3}\,\text{kg} \times 12.4\,\text{m s}^{-1}$$
$$= 0.71\,\text{kg m s}^{-1} \text{ or } 0.71\,\text{N s}$$

(upwards, as its value is +ve)

d) This is represented by the area under the force–time graph. As the graph is almost a triangle, to a good approximation:

$$\text{impulse} = \frac{1}{2} F_{max} \times \Delta t$$

This gives $F_{max} = \frac{2 \times \text{impluse}}{\Delta t} = \frac{2 \times 0.71\,\text{Ns}}{4.0 \times 10^{-3}\,\text{s}} = 360\,\text{N}$

Which is, to 2 SF, in agreement with the maximum value of F on the graph.

e) The weight of the ball is $mg = 57 \times 10^{-3}\,\text{kg} \times 9.8\,\text{N kg}^{-1}$

$$= 0.56\,\text{N}$$

The force of 360 N is 360 N/0.56 N = 640 times greater than the weight of the ball, which is in the stated range of 100–1000.

Tip

In 'show that' questions, always give your answer to one more SF than the values you are asked to show – the examiner then knows that you have actually done the calculations correctly!

You should then use *your* more precise values for the rest of the question. If you can't work out the values for yourself, then just use the 'show that' values for the rest of the question and you will be given full credit.

Tip

Don't forget that force, momentum and velocity are all *vectors* and so their *direction* must be taken into account when doing calculations such as those in c).

Test yourself

1 Calculate:

a) the impulse due to a force of 200 kN acting for 50 ms;

b) the force produced by an impulse of 250 N s acting for 100 ms;

c) the time for a car of mass 1250 kg to be brought to rest from a speed of 24 m s^{-1} in a head-on collision if the average force on the car is 300 kN.

2 Figure 1.7 shows two force–time graphs for passengers in a car crash. Analysis of these graphs can provide us with a lot of information. Passenger A has a mass of 100 kg and passenger B has a mass of 94 kg.

a) Explain why the force experienced by passenger A is much greater than that exerted on passenger B.

b) Suggest where each passenger might have been sitting. Justify your answer.

c) State what the area under each graph represents and explain why the area under each graph is not the same.

d) Calculate the area under graph B and hence estimate the speed with which the car was travelling before collision.

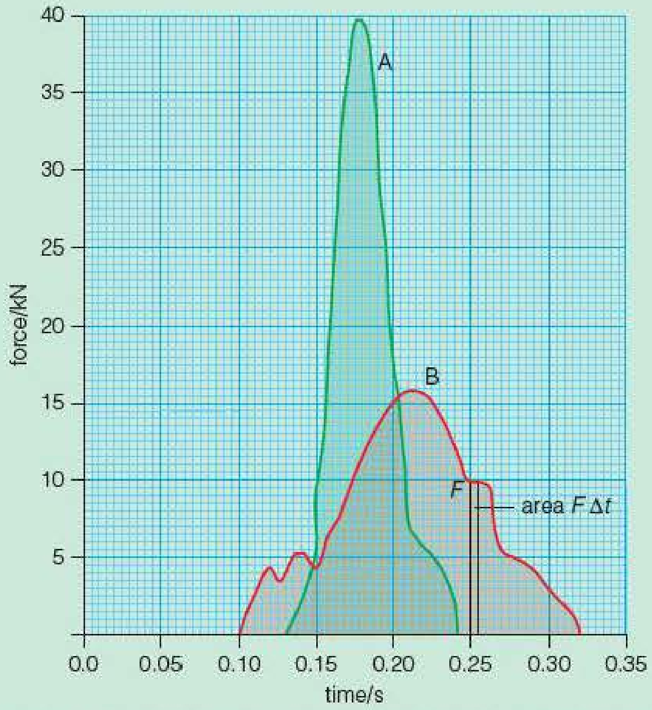


Figure 1.7 Graph of forces acting on passengers in a car accident

Core practical 9

Investigation of the relationship between the force exerted on an object and its change of momentum

This experiment uses the same apparatus and is similar to the experiment described in Book 1 to investigate force and acceleration.

Safety note: Place a padded box beneath the load so that the floor and masses don't get damaged. Also, to avoid injuries to feet, don't stand under falling masses!

The light gates are connected to a suitable data logger and the results can either be interpreted manually or can be fed into a suitable computer programme. The following data is required:

- The length l of the card (200 mm in this experiment)
- The times for the card to pass through each light gate, t_1 and t_2

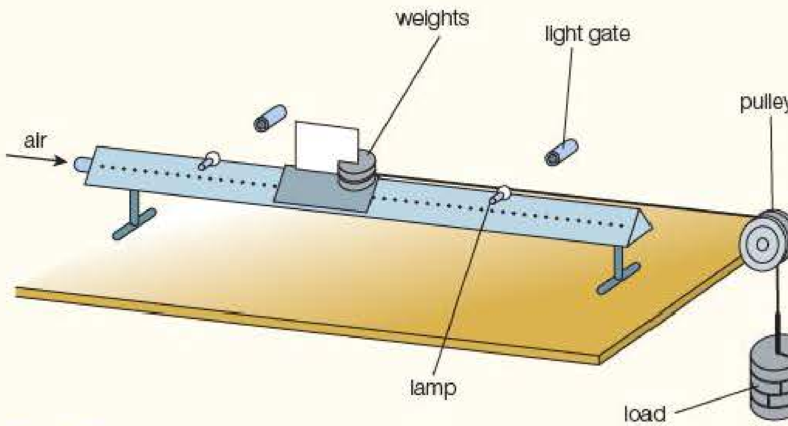


Figure 1.8 Investigating change if momentum

- The time for the car to travel from the first light gate to the second light gate Δt
- The mass of the trolley m_T (450 g in this experiment).

A typical print-out is shown in Figure 1.9.

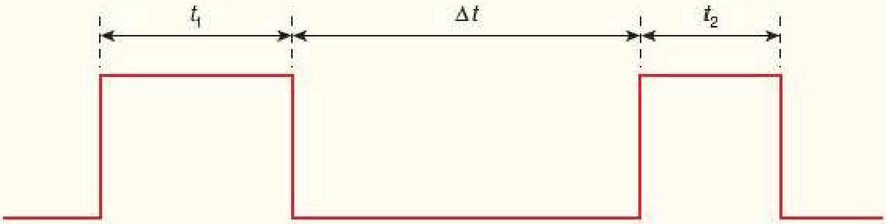


Figure 1.9 Print-out of times

The velocity through the first gate $v_1 = \frac{l}{t_1}$ and the velocity through the second gate $v_2 = \frac{l}{t_2}$.

The change in momentum is then $\Delta p = m(v_2 - v_1) = m\Delta v$ where m is the mass being accelerated.

We need to think carefully about m. The force, which is the weight of the masses hanging over the pulley (labelled 'load'), has to accelerate the combined mass of the trolley, the masses on the trolley *and* the masses hanging over the pulley. In order to vary the force and at the same time keep m constant we have to take one of the masses off the trolley and add it to the load each time.

Times t_1, t_2 and Δt are obtained for a range of values for the load, keeping m constant using the technique as described above, and are tabulated as shown in Table 1.1. The numbers can then be 'crunched' using a spreadsheet.

If a graph of $F\Delta t$ on the y-axis against Δv on the x-axis is plotted, a straight line through the origin would show that $\Delta v \propto F$. If the gradient is found to be m, then we can say that

$$F\Delta t = m\Delta v \text{ or } F = m\Delta v/\Delta t = \text{rate of change of momentum}$$

Questions

1 Copy and complete Table 1.1 – the first row has been done for you. You are recommended to set up a spreadsheet for this.

Table 1.1

F/N	Δt/s	$F\Delta t$/N s	t_1/s	t_2/s	v_1/m s^{-1}	v_2/m s^{-1}	Δv/m s^{-1}
0.10	0.90	0.090	0.51	0.36	0.39	0.56	0.17
0.20	0.64		0.38	0.26			
0.29	0.48		0.29	0.21			
0.39	0.42		0.25	0.18			
0.49	0.37		0.22	0.16			
0.59	0.34		0.19	0.14			

2 By referring to Figure 1.8 and the values for F in the table, determine the mass of each of the weights and hence m, the total mass being accelerated. Explain how you did this.

3 Plot a graph of $F\Delta t$ on the y-axis against Δv on the x-axis.

4 Determine the gradient of your graph.

5 Calculate the percentage difference between the gradient of your graph and the total mass m and comment on your answer.

1.2 Work and energy

This chapter is an opportunity to revise your understanding of momentum and kinetic energy in interactions. Remember that momentum is *always* conserved. You will learn that kinetic energy is usually *not* conserved in collisions. These are known as inelastic collisions. However, elastic collisions in which kinetic energy *is* conserved do occur – for example between molecules and between nuclei, and also sometimes (almost) between larger objects such as snooker balls. Elastic collisions between particles of equal mass result in special outcomes that are important to both particle physicists and snooker players!

Figure 1.10 An easy 'pot'.

You can calculate the work done by a force as

$$\Delta W = F_{av}\Delta x$$

Using this you can calculate how much mechanical energy an object has been given. This energy might be stored as

gravitational potential energy ($\Delta GPE = mg\Delta h$)

or as elastic potential energy ($\Delta EPE = \frac{1}{2}k\Delta x^2$)

or as kinetic energy (KE or $E_k = \frac{1}{2}mv^2$).

Mechanical energy sometimes appears to be lost to the surroundings, often as a result of work done against air resistance or contact friction. When this happens we say that there has been a transfer of mechanical energy to **internal energy** – thermal energy of the particles of the surroundings (see page 12).

The **principle of conservation of energy** states that energy is never created or destroyed, but it can be transferred from one form into another. Other forms of energy include chemical energy and radiant energy. You will meet yet more forms, such as nuclear energy and electrostatic potential energy, later in this book.

Key term

The **principle of conservation of energy**: energy can neither be created nor destroyed, but merely transferred from one form to another.

Some useful equations

Look at the equations for kinetic energy E_k and linear momentum p:

$$E_k = \frac{1}{2}mv^2 \qquad p = mv$$

For an object of mass m we can eliminate the velocity v and get equations linking kinetic energy to momentum. Squaring the second equation and rearranging the first gives:

$$p^2 = m^2v^2 \text{ and } v^2 = \frac{2E_k}{m}$$

$$\Rightarrow p^2 = m^2 \times \frac{2E_k}{m}$$

So, with no v:

$$p = \sqrt{2mE_k} \qquad \text{or} \qquad E_k = \frac{p^2}{2m}$$

Figure 1.11 James Joule: a Manchester brewer after whom the unit for energy was named.

Example

Calculate

a) the momentum of a 58 g tennis ball with kinetic energy of 75 J

b) the kinetic energy of an arrow of mass 0.12 kg with a momentum of $5.2\,\text{kg m s}^{-1}$.

- You will need to use the two equations on the previous page but you are *not* expected to remember them.

Answer

a) Using $p = \sqrt{2mE_k}$ gives the momentum of the ball as

$$p = \sqrt{2 \times 0.058\,\text{kg} \times 75\,\text{J}} = 2.9\,\text{kg m s}^{-1}$$

b) Using $E_k = p^2/2m$ gives the kinetic energy of the arrow as

$$E_k = \frac{(5.2\,\text{kg m s}^{-1})^2}{2 \times 0.12\,\text{kg}} = 113\,\text{J}$$

Tip

Look to see if the equations are homogeneous with respect to units: the key is that $\text{J} = \text{N m} = \text{kg m}^2\,\text{s}^{-2}$, so all is well.

These relationships work in the microscopic world as well as in everyday life. But for fast-moving atomic or nuclear particles the equations only work if the particles are moving at less than about 10% of the speed of light, $c = 3.0 \times 10^8\,\text{m s}^{-1}$. Such particles are called non-relativistic particles. We will consider particles moving at almost the speed of light in Chapter 8.

Key term

The **electron-volt** is the energy gained by a particle of unit electronic charge when accelerated through a p.d. of 1 volt.

The energies of particles such as electrons and protons, and of ions such as alpha particles, are usually quoted in a unit much smaller than the joule; the **electron-volt** (eV).

$$1\,\text{eV} \equiv 1.6 \times 10^{-19}\,\text{J}$$

The energy of *both* an electron *and* a proton that have each been accelerated through 150 V will be 150 eV. However, because they have very different masses, they will have very different momenta.

Example

What is the momentum of:

a) an electron of mass 9.1×10^{-31} kg,

b) a proton of mass 1.7×10^{-27} kg,
each of which has a kinetic energy of 150 eV?

Answer

For both, $150\,\text{eV} = 150 \times (1.6 \times 10^{-19}\,\text{J}) = 2.4 \times 10^{-17}\,\text{J}$.

Using $p = \sqrt{(2mE_k)}$ gives:

a) $p_{\text{electron}} = \sqrt{(2 \times 9.1 \times 10^{-31}\,\text{kg} \times 2.4 \times 10^{-17}\,\text{J})} = 6.6 \times 10^{-24}\,\text{N s}$

b) $p_{\text{proton}} = \sqrt{(2 \times 1.7 \times 10^{-27}\,\text{kg} \times 2.4 \times 10^{-17}\,\text{J})} = 2.9 \times 10^{-22}\,\text{N s}$

Tip

Trying to *remember* formulas like $p = \sqrt{(2mE_k)}$ is not essential - this one is given in the data list as

$$E_K = \frac{p^2}{2m}.$$

Make sure you understand the basic principles.

The same applies to large objects. In Figure 1.12 Sam has a mass of 32 kg, and the tennis ball has a mass of 0.055 kg. Let's work out their momenta if each has a kinetic energy of 100 J.

Using $p = \sqrt{(2mE_k)}$

for Sam $\quad p = \sqrt{(2 \times 32\,\text{kg} \times 100\,\text{J})} = 80\,\text{kg\,m\,s}^{-1}$ or $80\,\text{N\,s}$

for the ball $\quad p = \sqrt{(2 \times 0.055\,\text{kg} \times 100\,\text{J})} = 3.3\,\text{kg\,m\,s}^{-1}$ or $3.3\,\text{N\,s}$

So their momenta are very different, though their kinetic energies are the same.

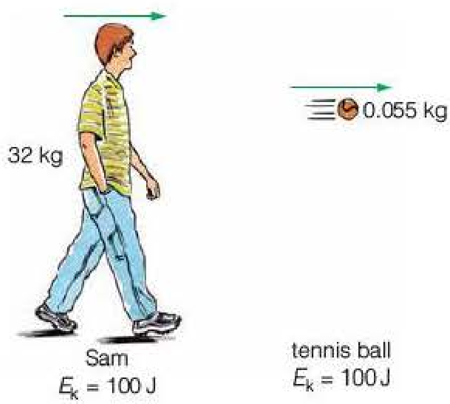


Figure 1.12 Sam and a ball

Test yourself

3 What is the kinetic energy of a sprinter of mass 85 kg running at $10\,\text{m\,s}^{-1}$?

4 Name three forms of energy other than kinetic energy or elastic potential energy.

5 From the equation $\frac{1}{2}mv^2 = \frac{1}{2}k\Delta x^2$, show that $\Delta x = \sqrt{(mv^2/k)}$.

6 Express 6.5 GeV in joules.

7 Show that the two units of momentum, $\text{kg\,m\,s}^{-1}$ and N s, are equivalent.

1.3 Elastic and inelastic collisions

Think of colliding trolleys fitted with a pin and cork so that they stick together when they collide. Figure 1.13 shows the arrangement. What do you think happens to the kinetic energy of the trolleys in such a collision?

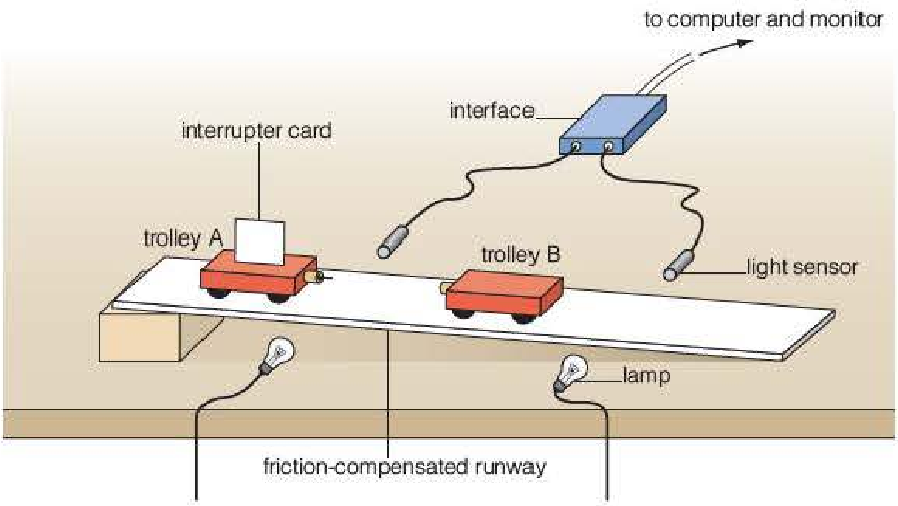


Figure 1.13 A pin-and-cork collision

First suppose the mass of each trolley is m, i.e. $m_A = m_B = m$. The equation expressing the conservation of momentum then becomes:

$mu = 2mv$ that is $v = \frac{1}{2}u$

Total KE before collision $= \frac{1}{2}mu^2 + 0 = \frac{1}{2}mu^2$

KE after collision $= \frac{1}{2}(2m)v^2 = mv^2 = m\left(\frac{1}{2}u\right)^2 = \frac{1}{4}mu^2$

$\therefore$ the total KE has fallen from $\frac{1}{2}mu^2$ to $\frac{1}{4}mu^2$, i.e. KE_{after} is half of KE_{before}.

Momentum has, as always, been conserved but some kinetic energy has been lost – half the initial kinetic energy in this case. This result is the same for all collisions where objects of equal mass form linked collisions. Such a collision is an example of a non-elastic or an inelastic collision.

But remember, total energy must be conserved. The loss of kinetic energy is equal to the gain in internal energy – in this case as the pin enters the cork. The Sankey diagram in Figure 1.14 emphasises this energy conservation.

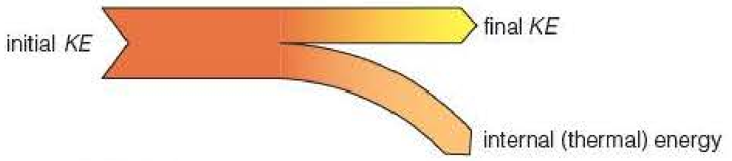


Figure 1.14 Total energy is conserved

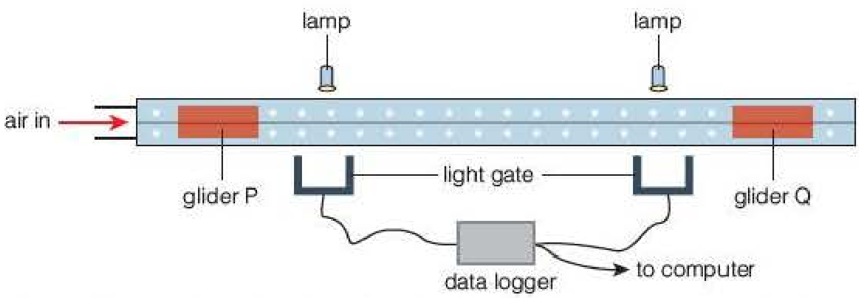


Figure 1.15 Testing conservation of momentum and kinetic energy using gliders on an air track

Table 1.2 shows data for gliders P and Q from Figure 1.15, that move towards each other on an air track, collide and bounce back. The two gliders had been fitted with small magnetic buffers that were set to repel. The change of momentum Δp and the change of kinetic energy ΔE_k have both been calculated and are shown in the last two columns. Left-to-right has been chosen as positive for u, v and Δp. Glider P initially moves to the right and Q moves to the left.

Here are some possible results using this apparatus:

Table 1.2 Positive is to the right for velocity and momentum

	Mass/kg	Initial velocity u/m s^{-1}	Final velocity v/m s^{-1}	Δp/kg m s^{-1}	ΔE_k/J
P	0.20	+0.14	−0.26	−0.080	0.0048 more
Q	0.25	−0.22	+0.10	+0.080	0.0048 less

You can see that the change of momentum of glider P is 0.080 kg m s^{-1} to the left, and that of glider Q is 0.080 kg m s^{-1} to the right, i.e. momentum is conserved. Is kinetic energy conserved?

The data for E_k in Table 1.2 tells you that there was no loss of kinetic energy, as all the 0.0048 J lost by glider Q is gained by glider P. This type of collision, where kinetic energy is conserved, is called an **elastic collision**. In the everyday world almost all collisions are *inelastic*, but in the sub-atomic world elastic collisions are not uncommon. (You could set up a spreadsheet to analyse any two-body collision, if you were provided with the data for the masses and velocities of the two bodies.)

Key term

An **elastic collision** is a collision in which no kinetic energy is lost, i.e. the total kinetic energy of the colliding objects before the collision is equal to the total kinetic energy of the objects after the collision.

Example

A 1200 kg car is stationary on an icy road. The car is hit from behind by a skidding lorry of mass 5600 kg that is moving at 18 km h^{-1} (5.0 m s^{-1}). The two vehicles remain locked together after the crash as shown in Figure 1.16.

a) What type of collision is this crash?

b) What fraction of the lorry's kinetic energy becomes internal energy as a result of the collision?

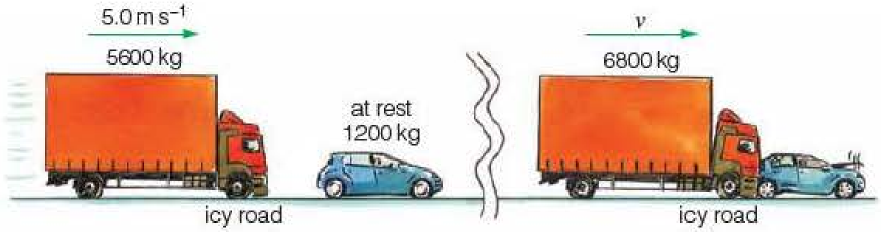


Figure 1.16 A car and lorry collision

Answer

a) The collision is not an elastic collision, it is an inelastic collision.

b) Suppose the velocity after the crash is v. Using the principal of conservation of momentum, which is upheld in all collisions, gives:

$$5600\,\text{kg} \times 5.0\,\text{m s}^{-1} = (5600\,\text{kg} + 1200\,\text{kg}) \times v$$

$$\Rightarrow v = 4.12\,\text{m s}^{-1}$$

$$KE_{\text{before}} - KE_{\text{after}} = \tfrac{1}{2} \times 5600\,\text{kg} \times (5.0\,\text{m s}^{-1})^2 - \tfrac{1}{2} \times 6800\,\text{kg} \times (4.12\,\text{m s}^{-1})^2$$

$$= 70\,000\,\text{J} - 57\,700\,\text{J} = 12\,300\,\text{J}$$

which is 12 300 J/70 000 J = 0.176 or, to 2 SF, 18% of the lorry's original KE.

Fairgrounds with bumper cars also show collisions taking place. Suppose two cars, B and G, are moving along the same line and the boy in B is moving faster than the two girls in G: see Figure 1.17.

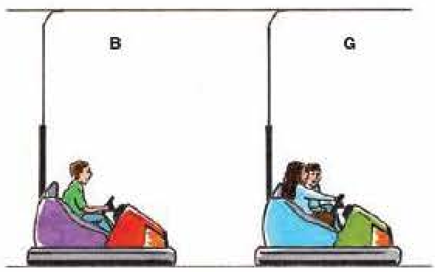


Figure 1.17 Bumper cars at a fair

Table 1.3 gives the masses and speeds of the two cars before and after the collision.

Table 1.3

	Mass/kg	Initial speed/m s^{-1}	Final speed/m s^{-1}
Car B	135	+ 4.0	v
Car G	180	+ 2.5	+ 4.0

When they collide, the car with the two girls is 'bumped' forward at 4.0 m s^{-1}. In order to calculate the resulting speed v of the boy in his car, we use the principle of conservation of momentum (P of C of M):

$$(mu)_B + (mu)_G = (mv)_B + (mv)_G$$

$$(135\,\text{kg} \times 4.0\,\text{m s}^{-1}) + (180\,\text{kg} \times 2.5\,\text{m s}^{-1}) = (135\,\text{kg} \times v) + (180\,\text{kg} \times 4.0\,\text{m s}^{-1})$$

$$\Rightarrow 990\,\text{kg m s}^{-1} = (135\,\text{kg} \times v) + 720\,\text{kg m s}^{-1}$$

$$\therefore v = \frac{270\,\text{kg m s}^{-1}}{135\,\text{kg}} = 2.0\,\text{m s}^{-1}$$

If you work out $KE_{\text{before}} \times KE_{\text{after}}$ for these bumper cars you will find that the collision was, not unexpectedly, inelastic, i.e. there *was* a loss of kinetic energy in the collision. You should check this for yourself.

Tip

'Discuss' here means you have to calculate whether or not any kinetic energy has been lost in the collision.

Tip

When, as in the examples, the problem is not split into separate parts, try to see 'where you are going' before lunging into a solution.

Example

A non-relativistic proton of mass m moving at a speed $10u$ makes a head-on collision with a stationary helium nucleus of mass $4m$. As a result of the collision, the helium nucleus moves forward at $4u$.

Discuss whether this collision is elastic or inelastic.

Answer

Suppose the speed of the proton after the collision is v. As momentum is conserved,

$$m \times 10u = mv + (4m \times 4u)$$

$$\Rightarrow mv = -6mu$$

So the proton bounces backwards at a speed of $6u$.

Kinetic energy of the proton *before* collision:

$$KE_{\text{before}} = \tfrac{1}{2}m \times (10u)^2 = 50mu^2$$

Kinetic energy of the proton and the helium nucleus *after* collision:

$$KE_{\text{after}} = \tfrac{1}{2}m \times (-6u)^2 + \tfrac{1}{2}4m \times (4u)^2 = 18mu^2 + 32mu^2 = 50mu^2$$

As both KE_{before} and KE_{after} are equal to $50mu^2$, the kinetic energy lost is *zero*, so the collision is elastic.

Test yourself

8 Calculate from first principles the kinetic energy of a rowing boat of mass 220 kg having a momentum of 440 N s due south.

9 Sketch a diagram showing the gliders described on page 12 'before' and 'after' their collision.

10 A stationary nucleus of radium-226 decays to radon-222 by emitting an α-particle. Explain whether this is an elastic or a non-elastic event.

11 Confirm that the change of momentum Δp and the change of kinetic energy ΔE_k of glider P in Table 1.2 are correct.

1.4 Collisions in two dimensions

Up to now all collisions we have considered have been between two objects moving in the same straight line: they were all one-dimensional collisions. A very brief look at a game of snooker will show you that collisions on a snooker table are rarely one-dimensional; nor are collisions between fast-moving protons and other nuclear particles.

Example

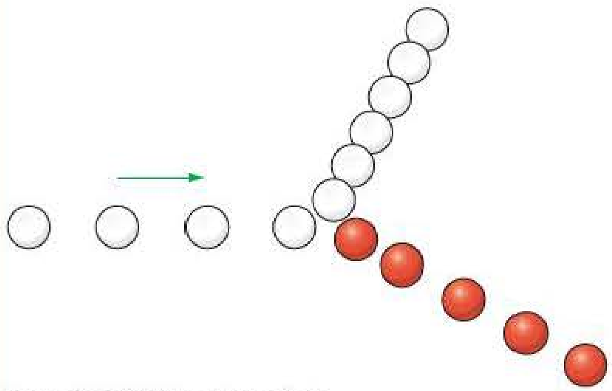


Figure 1.18 A 2-D snooker collision

The balls forming a two-dimensional collision in Figure 1.18 have the same mass, 125 g. The one moving from the left strikes a ball that is stationary. The positions of the balls are taken from a stroboscopic photograph of the collision. The stroboscope flashes every 0.01 s and Figure 1.18 is one-tenth of the true size of the collision.

Questions

a) Calculate the speeds of the balls by measuring the distances they each travel in four camera flashes.

b) Calculate the size of the momenta of each ball before and after the collision.

c) Measure the angles between the initial direction of the white ball and of each of the two balls after the collision.

d) Calculate the component of the momentum of the white ball ⊥ to the initial direction and the component of the momentum of the red ball ⊥ to the initial direction.

e) Comment on the results to part (d) and suggest which measurements are the least precise.

Answers

a) i) The speed of the incoming white ball

$$= \frac{10 \times 34\,\text{mm}}{3 \times 0.01\,\text{s}} = 11\,300\,\text{mm}\,\text{s}^{-1}$$

ii) The speed of the outgoing white ball

$$= \frac{10 \times 15\,\text{mm}}{3 \times 0.01\,\text{s}} = 507\,\text{mm}\,\text{s}^{-1}$$

iii) The speed of the outgoing red ball

$$= \frac{10 \times 27\,\text{mm}}{3 \times 0.01\,\text{s}} = 9000\,\text{mm}\,\text{s}^{-1}$$

b) Dividing the speeds by 1000 mm per m, and the masses by 1000 g per kg gives the size of the momenta in $\text{kg}\,\text{m}\,\text{s}^{-1}$. For example, in (i)

$$p = mv = \left(\frac{125\,\text{g}}{1000\,\text{g}\,\text{kg}^{-1}}\right) \times \left(\frac{1130\,\text{mm}\,\text{s}^{-1}}{1000\,\text{mm}\,\text{m}^{-1}}\right)$$
$$= 0.141\,\text{kg}\,\text{m}\,\text{s}^{-1}$$

Similarly, the size of the two momenta are (ii) $0.063\,\text{kg}\,\text{m}\,\text{s}^{-1}$, (iii) $0.113\,\text{kg}\,\text{m}\,\text{s}^{-1}$.

c) The white ball moves to the left (of the original direction) at an angle of 61° and the red ball moves to the right at an angle of 29°.

d) The component of the momentum of the white ball ⊥ to the initial direction

$= -(0.063\,\text{kg}\,\text{m}\,\text{s}^{-1}) \times \sin 61°$
$= 0.055\,\text{N}\,\text{s}$ (taking 'left' as negative)

The component of the momentum of the red ball ⊥ to the initial direction

$= +(0.113\,\text{kg}\,\text{m}\,\text{s}^{-1}) \times \sin 29°$
$= 0.055\,\text{N}\,\text{s}$ (taking 'right' as positive)

e) The calculated right and left, positive and negative, momenta are equal, i.e. the *sideways* momentum is zero both before (obviously) and after the collision. The least precise measurements are those of the angles.

Another way of using Figure 1.18 to show that momentum is conserved here, is to resolve each of the momentum vectors after the collision along the line of the incoming white ball, and add these components. The sum of the resolved momenta along the direction of the original ball will add up to be (about) the same as the momentum of the incident ball. This will show that the total 'forward' momentum is conserved in the collision. You should check this for yourself.

Tip

When doing calculations like 25 sin 37°, on a calculator, it is best to put in the 37° first, press the 'sin' sign, and *then* multiply by 25. (The result should be 15.)

Core practical 10

Analysing a collision between small spheres

Figure 1.19 shows a multiple flash collision between two balls, the larger one of mass 100 g and the smaller one of mass 43 g, both of which enter from the top of the photograph. The flashes occur 30 times per second and the photograph is one-eighth actual size.

Figure 1.19 A multiple flash collision between two balls

Questions

1 Calculate the percentage errors in the speeds of the two balls if **a)** the possible error in the flash rate was $30\,s^{-1} \pm 1\,s^{-1}$ and **b)** the size of the photograph could have been anything between one seventh and one ninth of actual size. Comment on your answers.

2 Determine the speeds of the balls before and after collision and hence investigate whether the collision is elastic.

3 Using a vertical line as the reference direction, determine the momentum of each ball before and after the collision.

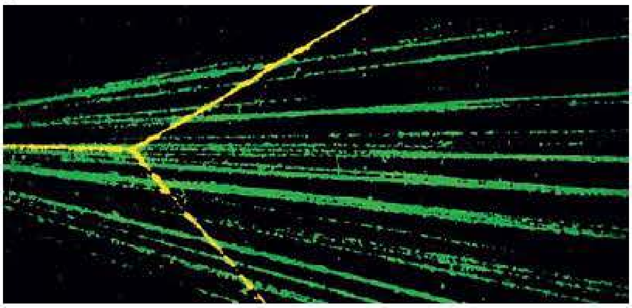
4 By drawing vectors, or calculating components of the momenta, check that linear momentum is conserved in the collision.

Note: This is just one way of performing the Core Practical. An alternative method, with a video of a collision and subsequent analysis using ICT to analyse the data, can be found online. This can be accessed using the *Free online resources* at the back of the book. You are strongly advised to look at this.

Example

The yellow lines in the photograph of Figure 1.20a show a non-relativistic alpha particle making a collision in a cloud chamber filled with helium gas. Figure 1.20b shows the velocities of the incoming alpha particle before the collision and of the alpha particle and the recoiling helium nucleus after the collision. The alpha particle and the helium nucleus each have a mass of 6.65×10^{-27} kg. Figure 1.20b shows the direction and momenta of the particles.

a



b

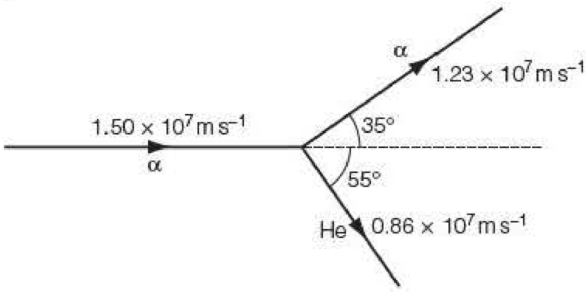


Figure 1.20 An α-particle collides with a helium nucleus

a) Make a table showing the calculated values of the momentum of each of the particles before and after the collision.

b) By resolving the vector momenta, show that the initial momentum of the incoming alpha particle is conserved after the collision.

Answer

a) Table 1.4

	Mass times velocity (mu/kg × m s^{-1})	Momentum (p/kg m s^{-1})
alpha particle: before collision	$(6.65 \times 10^{-27}) \times (1.50 \times 10^7)$	9.98×10^{-20}
helium nucleus: before collision	0	0
alpha particle: after collision	$(6.65 \times 10^{-27}) \times (1.23 \times 10^7)$	8.18×10^{-20}
helium nucleus: after collision	$(6.65 \times 10^{-27}) \times (0.86 \times 10^7)$	5.72×10^{-20}

b) Sum of components of momentum after collision parallel to initial path of alpha particle

$$= (8.18 \times 10^{-20}\,\text{kg m s}^{-1})\cos 35° + (5.72 \times 10^{-20}\,\text{kg m s}^{-1})\cos 55°$$

$$= 6.70 \times 10^{-20}\,\text{kg m s}^{-1} + 3.28 \times 10^{-20}\,\text{kg m s}^{-1}$$

$$= 9.98 \times 10^{-20}\,\text{kg m s}^{-1}$$

This is equal to the initial momentum of the incoming alpha particle. Vector momentum is conserved.

In both Figures 1.18 and 1.20a, of two identical mass balls colliding and two equal-mass nuclear particles colliding, the angle between the two particles *after* the collision is 90°. It can be shown that the 90° angle occurs only when the collision between objects of equal mass is an elastic one. (Snooker players know this and take steps to avoid possible 'in-offs'.) It was photographs such as that in Figure 1.20a that confirmed the fact that alpha particles are identical to helium nuclei (both contain two protons and two neutrons).

Below are two other situations involving collisions that are close to being elastic:

- A heavy ball or particle strikes a much lighter one that is at rest. The heavy ball 'carries on' and pushes the lighter one forward. In this case, the angle between the two after the collision is *less than* 90° when the collision is elastic.
- A light ball or particle strikes a much heavier one that is at rest. In this case, the light ball 'bounces back' and the heavy one moves forward. The angle between the two after the collision is *more than 90°* when the collision is elastic.

This information is most valuable when analysing photographs showing collisions between nuclei, which are all collisions in which kinetic energy is conserved, i.e. elastic collisions.

Tip

The questions is the last example are complex and might be worth up to 10 marks in a test or exam. It must be worked through, stage by stage, showing all your calculations *with units*.

Test yourself

12 Explain in words what is meant by a 'parallelogram of vectors'.

13 Calculate values for:

a) (56 kg)sin22

b) (90 m)cos90

c) (5.0 s)tan58

14 Draw two sketches showing, as seen in a nuclear detecting chamber:

a) an elastic collision between a heavy particle that strikes a much lighter one

b) an elastic collision between a light particle that strikes a much heavier one.

1.5 Rockets and jets

Newton's second law leads to:

Force = rate of change of momentum

i.e. $F = \dfrac{\Delta(mv)}{\Delta t}$

We usually consider situations in which m is constant, so

$F = m\dfrac{\Delta v}{\Delta t}$ or, instantaneously, $F = m\dfrac{dv}{dt}$

Sometimes this can be thought of the other way round:

$F = v\dfrac{\Delta m}{\Delta t}$ or $F = v\dfrac{dm}{dt}$

These equations are needed when answering questions about rockets, see Figure 1.21, or streams of matter such as water being pumped out of pipes. In these situations the stream of matter is ejected with constant velocity, v, at a rate of $\Delta m/\Delta t$ or dm/dt.

Clearly the units of the quantities on the right hand side for each way of 'seeing' Newton's second law are those of mass $\times \dfrac{\text{velocity}}{\text{time}}$, i.e. $\dfrac{\text{kg m s}^{-1}}{\text{s}} = \text{kg m s}^{-2}$ or N (newtons).

Example

A large rocket is shown taking off in Figure 1.21. The rocket ejects gases, the result of exploding a mixture of oxygen and hydrogen, at the rate of 12 000 kg s^{-1} – a rather violent explosion, to say the least! If the exhaust speed of these gases is 4400 m s^{-1}, calculate the upward thrust of the ejected gases on the rocket.

Figure 1.21

Answer

The downward push of the rocket on the exhaust gases, using Newton's second law, is given by

$$F_d = v\frac{dm}{dt}$$
$$= (4400\,\text{m s}^{-1}) \times (12\,000\,\text{kg s}^{-1})$$
$$= 5.3 \times 10^7\,\text{N or } 53\,\text{MN}$$

By Newton's third law, the upward push of the ejected gases on the rocket F_u is equal in size but opposite in direction to this downward push of the rocket on the exhaust gases F_d. It is therefore of size 53 MN (a force big enough to lift a mass of over 5000 tonnes at the Earth's surface).

Tip

The key to most questions like this is to use and remember Newton's third law.

'Water cannons' are widely used in crowd control. The water, of density ρ, that 'hits' an individual exerts a force F on her or him that is equal to $v\frac{\Delta m}{\Delta t}$. In a time Δt the *volume* of water hitting its target is, assuming that the water hits its target full on, $A \times v\Delta t$, where A is the cross-sectional area of the water stream.

Its mass m is therefore

$$\Delta m = A \times v\Delta t \times \rho$$

This gives

$$\frac{\Delta m}{\Delta t} = Av\rho \text{ (measured in kg s}^{-1}\text{)}$$

Therefore the rate of change of momentum is:

$$F = v\frac{\Delta m}{\Delta t} = v^2 A\rho.$$

In this case v might be $8\,\text{m s}^{-1}$ and A about $25\,\text{cm}^2$ or $0.0025\,\text{m}^2$. As the density of water is $1000\,\text{kg m}^{-3}$, the force will be $F = 160\,\text{N}$, a force equal to the weight of a 16 kg mass, which is quite enough to assist the police in controlling a crowd of people.

In fact $F = v\frac{\Delta m}{\Delta t} = v^2 A\rho$ is a general relationship for calculating (approximately) the force of a jet of liquid or gas (perhaps air) that is brought to rest after colliding with something. This 'formula' is, however, *not* one you are expected to remember.

A jet engine: **1** takes in cold air, **2** heats this air, **3** throws out hot air. This, of course, is a very simplified story: in fact aeronautical engineers have been improving and redesigning jet engines for well over sixty years. Some of the earliest successful jet engines were used towards the end of the Second World War to power the 'doodlebugs' that flew to London carrying bombs from German-controlled Europe. These doodlebugs (as the people of London called them) used up all the fuel they carried by the end of their flight, and Londoners could hear the engine cut out – a sudden silence that meant a bomb was about to fall to the ground.

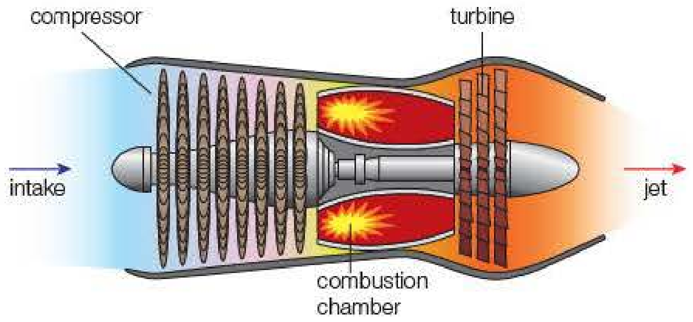


Figure 1.22 A simple jet engine

Tip

When tackling problems about jets of liquids or solids from first principles, it is essential to introduce a time Δt or to quote a given time interval such as 'in 1 second'.

Test yourself

15 Show that $v^2 A\rho$ has the unit of force.

16 Use $F = v^2 A\rho$ to estimate the force of a wind with an average speed of $10\,\text{m s}^{-1}$ (about 16 mph) on a building presenting an area of $200\,\text{m}^2$ to the wind. Take the average density of the air to be $1.2\,\text{kg m}^{-3}$.

17 Explain how a helicopter can 'hover' (remain in one position) in the air.

Exam practice questions

1 Which of the following is always true?

A Kinetic energy is conserved in collisions.

B Recoil is the result of explosive charges.

C Impulsive forces only act for short times.

D Linear momentum is conserved in collisions. **[Total 1 mark]**

2 Explain the difference between an elastic collision and an inelastic collision. **[Total 3 marks]**

3 A loaded railway wagon of mass 21 tonnes collides with an empty stationary wagon of mass 7 tonnes. They couple together and move off at $3.5\,\mathrm{m\,s^{-1}}$.

a) Show that the speed of the loaded wagon before the collision was $4.7\,\mathrm{m\,s^{-1}}$. **[2]**

b) Calculate the percentage loss of kinetic energy in this collision. **[4]**

[Total 6 marks]

4 In Figure 1.23, which diagram represents an elastic collision between a heavy moving body and a light stationary body? **[Total 1 marks]**

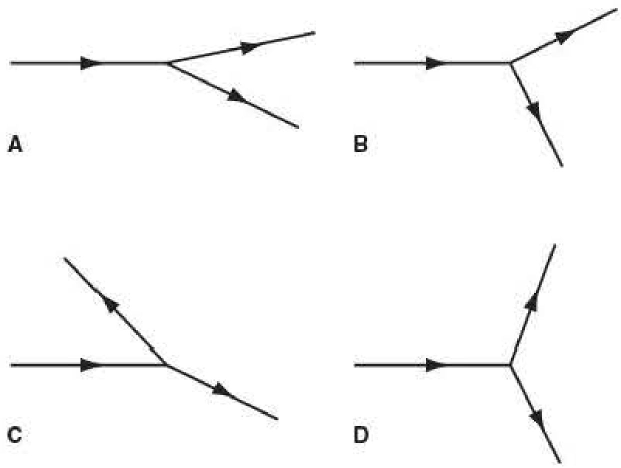


Figure 1.23

5 A compressed spring is placed between two trolleys of mass m and $4m$ and the trolleys are let go. Describe what happens and explain where the kinetic energy of the trolleys comes from. **[Total 3 marks]**

6 Two ice dancers are moving together in a straight line across the ice at $5.8\,\mathrm{m\,s^{-1}}$. The man has a mass of 75 kg and the woman has a mass of 65 kg. They push each other apart. After they separate, they are both still moving in the same direction.

The man's velocity is now $3.88\,\mathrm{m\,s^{-1}}$.

a) Show that the woman's velocity after they separate is about $8\,\mathrm{m\,s^{-1}}$. **[3]**

b) Explain what has happened to their kinetic energy. **[4]**

[Total 7 marks]

7 Taking data from the Example involving an alpha particle colliding with a helium nucleus (page 16), make calculations to decide whether there was any loss of kinetic energy in this two-dimensional collision.

[Total 4 marks]

8 A rocket car can accelerate from 0 to 100 mph in a very short time. Outline the physics principles behind this method of propulsion.

[Total 3 marks]

9 In a laboratory test a tennis ball is served at 180 kph. Figure 1.24 shows a series of photographs of the racket hitting the ball, with the time scale shown at the bottom. The mass of a tennis ball is 57 g.

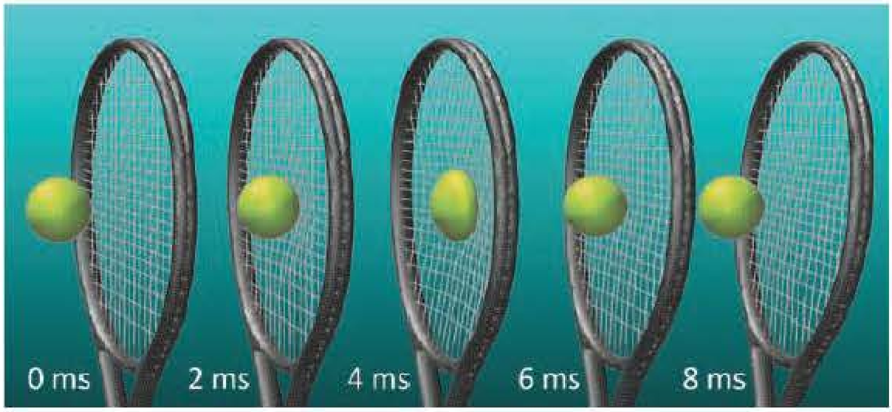


Figure 1.24 Impact of tennis ball on racket

a) Estimate the average force exerted on the ball. **[6]**

b) Suggest why the maximum force is likely to be somewhat greater than this. Justify your answer with reference to Figure 1.24 and by sketching a force–time graph. **[2]**

c) Suggest why it is important for both the ball and the racket strings to be manufactured from elastic materials. **[2]**

[Total 10 marks]

10 A particle of mass $2m$ travelling along a line at a speed of v explodes into two equal parts which separate along the line of travel. The explosion doubles the kinetic energy of the system.

Show that velocities of $2v$ and zero for the two masses m are consistent with the laws of physics. **[Total 5 marks]**

11 A helicopter can 'hover' in the air by pushing the air downwards using its rotating blades.

Suggest why helicopters have a small blades mounted on a horizontal axis near the rear of the helicopter. **[Total 4 marks]**

12 A shell of mass 12 kg moving horizontally at $320\,m\,s^{-1}$ explodes into three fragments A, B and C, which continue to move in a common vertical plane.

Fragment A: mass 2.0 kg continues at $450\,m\,s^{-1}$, 45° above the horizontal.

Fragment B: mass 6.0 kg continues at $400\,m\,s^{-1}$, horizontally.

Fragment C: mass m continues at speed v, in a direction θ to the horizontal.

a) Calculate the size of the momenta of the shell before it explodes and of fragments A and B afterwards. **[3]**

b) Draw a vector diagram showing these momenta and hence deduce values for m, v and θ. **[5]**

[Total 8 marks]

13 A helium nucleus, of mass $4m$, moving at a velocity of $5v$ makes a head-on collision with a stationary oxygen nucleus, of mass $16m$. After the collision the oxygen nucleus moves forward at $2v$.

Figure 1.25

a) Calculate the velocity of the helium nucleus after the collision. **[3]**

b) Discuss whether the collision is an elastic collision. **[4]**

[Total 7 marks]

Stretch and challenge

14 The Nobel Prize for Physics in 1997 was awarded to Steven Chu, who showed that gas atoms could be slowed down by bombarding them with infrared photons. Photons have momentum $p = h/\lambda$, where λ is the wavelength associated with the photon and h is the Planck constant ($6.6 \times 10^{-34}\,J\,s$).

Figure 1.26

a) Show that the change of speed Δu of an atom after absorbing an oncoming photon is $\frac{h}{m\lambda}$. **[3]**

b) For a sodium atom of mass $3.8 \times 10^{-26}\,kg$ absorbing an oncoming photon with a wavelength of 0.025 m, calculate the value of Δu. **[2]**

c) Show that the expression $\frac{h}{m\lambda}$ gives the unit of Δu as $m\,s^{-1}$. **[2]**

[Total 7 marks]

15 (See question 10) Prove that velocities of the two masses in the described explosion *must be* $2v$ and zero. **[Total 6 marks]**

2 Motion in a circle

Prior knowledge

You should know from earlier work covered in GCSE or in your Advanced level studies:

- ➜ that there are 2π radians in a circle (360°)
- ➜ how to use a calculator: working in degrees or radians
- ➜ that the momentum of a body is calculated as $p = mv$
- ➜ the difference between an average and an instantaneous statement e.g. Newton's second law of motion in the forms $F = \frac{\Delta p}{\Delta t}$ or $F = \frac{dp}{dt}$
- ➜ Newton's third law of motion in the form 'the push/pull of A on B is equal in size but opposite in direction to the push/pull of B on A'
- ➜ that the weight on Earth of a mass m is mg where $g = 9.81\,\mathrm{N\,kg^{-1}}$

Test yourself on prior knowledge

1 How many radians are there in 90°?
2 How many degrees are there in π rad?
3 Convert 16.5° to radians.
4 Convert 2.62 rad to degrees.
5 Is momentum a scalar or a vector quantity? Justify your answer.
6 Write down Newton's second law for a small time interval δt.
7 On Earth, what is the weight of a mass of 61 kg?
8 On Earth, what is the mass of a weight of 118 N?
9 Are pushes and pulls scalar quantities? Justify your answer.

2.1 The language of circular motion

Figure 2.1 A long-exposure photograph of stars near the Pole Star

Key term

The **angular displacement** is θ radians where $\theta = \frac{s}{r}$

Over the five hours' exposure of the photograph, each star in the night sky near the Pole Star has rotated through the same angle, the same fraction of a circle. The length of each star track (the arc of length s) divided by its distance from the centre of rotation (the radius, r) is the same – here about 1.3 – try it! This number is the angle θ through which each star has rotated – its **angular displacement.**

This measurement is in **radians**, abbreviation rad, and *not* in degrees. A radian is the angle subtended by the arc of a circle having the same length as the radius of the circle ($s = r$). The circumference of a circle is $2\pi r$, so there are 2π rad in a circle of 360°. This means

$$1 \text{ rad} = 1 \text{ rad} \times \frac{360°}{2\pi} = 57.3°$$

so the 1.3 rad found from the photograph of Figure 2.1 is about 75°.

Remember:

- To convert from radians to degrees, divide by 2π rad and multiply by 360°.
- To convert from degrees to radians, divide by 360° and multiply by 2π rad.

When describing how rapidly an object is rotating in a circle, we could use revolutions per second or revolutions per minute (r.p.m.), but in physics problems we must always use radians per second.

Key term

The **angular velocity** of an object is its rate of change of angle, measured in rad s^{-1}

We call this the **angular velocity** and give it the symbol ω (omega).

Average and instantaneous angular velocities are sometimes represented by the equations:

$$\omega_{av} = \frac{\Delta\theta}{\Delta t}$$

$$\omega_{inst} = \frac{\delta\theta}{\delta t} \text{ or } \frac{d\theta}{dt}$$

Example

Calculate the angular velocity of

a) the London Eye, which rotates once every 30 minutes

b) a CD, which when first switched on rotates at 200 rpm.

Answers

a) $\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \text{ rad}}{30 \times 60 \text{ s}} = 0.0035 \text{ rad s}^{-1}$

b) $\omega = \frac{\Delta\theta}{\Delta t} = \frac{200 \times 2\pi \text{ rad}}{60 \text{ s}} = 21 \text{ rad s}^{-1}$

From $\theta = \frac{s}{r} \Rightarrow s = r\theta$

$$\Rightarrow \frac{\Delta s}{\Delta t} = r\frac{\Delta\theta}{\Delta t}$$

or $v = r\omega$

This is almost common sense. Think of a playground roundabout such as that in Figure 2.2: everyone on it has the same angular velocity ω, but the further you stand away from the centre, r, the faster you are travelling, v.

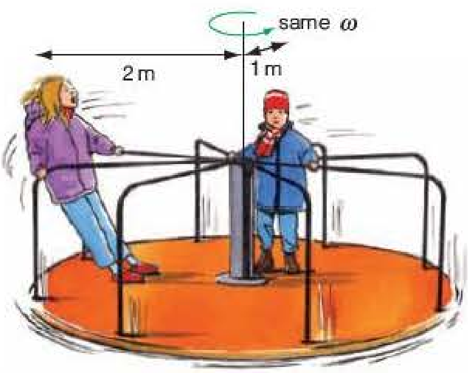


Figure 2.2 Stand near the centre and you'll go more slowly

Example

The information on a CD is recorded as a series of bumps in the tracks on the CD. The information is 'read' by a laser beam. Calculate the speed of the bumps moving past the laser head at 4.0 cm from the centre of a CD which is rotating at 430 rpm.

Answer

$430\,\text{rev}\,\text{min}^{-1} = (430\,\text{rev}\,\text{min}^{-1}) \times (2\pi\,\text{rad}\,\text{rev}^{-1}) \div (60\,\text{s}\,\text{min}^{-1}) = 45\,\text{rad}\,\text{s}^{-1}$

$\therefore\ v = r\omega = 0.040\,\text{m} \times 45\,\text{rad}\,\text{s}^{-1} = 1.8\,\text{m}\,\text{s}^{-1}$

Another useful relationship concerns the time taken for something to complete one circle. If an object is rotating at $1.5\,\text{rad}\,\text{s}^{-1}$, you can see that it will complete one circle in a time

$$T = \frac{2\pi\,\text{rad}}{1.5\,\text{rad}\,\text{s}^{-1}} = 4.2\,\text{s}$$

In general: $T = \frac{2\pi}{\omega}$

Reversing this to $\omega = \frac{2\pi}{T}$ enables you to calculate the angular velocity of the spinning Earth, and then $v = r\omega$ lets you calculate the speed of any point on the Earth's surface provided you know the radius of the circle in which it is moving. (It would be zero at the poles!)

Tip

An angle measured in radians has no units in the SI system. The angle is simply the ratio of two lengths, s and r and their units, the metre, cancel. We only use the radian in order to remember the way in which the angle was measured. Similarly the degree is not an SI unit.

Example

The city of Birmingham is at latitude 52.5° (the equator has a latitude of zero, the north pole 90°). The Earth has a radius of 6400 km. See Figure 2.3.

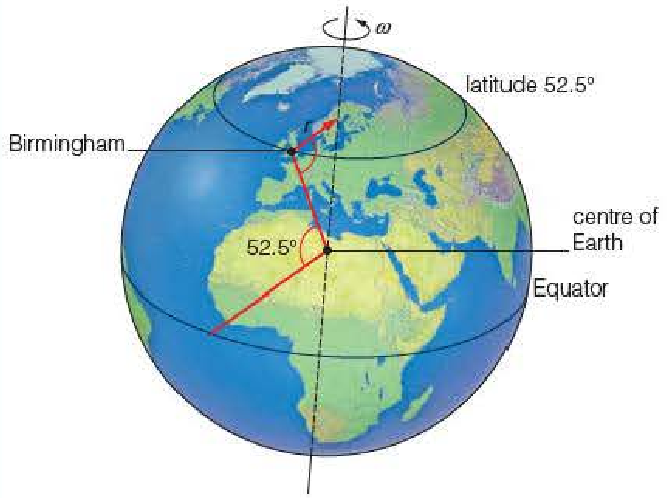


Figure 2.3 Birmingham at a latitude of 52.5°

Calculate:

a) Birmingham's angular velocity as the Earth spins

b) the radius of the circle in which Birmingham moves

c) the speed at which Birmingham's inhabitants are moving.

Answer

a) The Earth spins once every 24 hours.

$$\therefore \omega = \frac{2\pi}{T} = \frac{2\pi \text{rad}}{24 \times 3600\,\text{s}} = 7.27 \times 10^{-5} \text{rad s}^{-1}$$

b) At a latitude of 52.5°, $r = (6.4 \times 10^6\,\text{m}) \cos 52.5° = 3.90 \times 10^6\,\text{m}$

c) Using $v = r\omega$

$$\Rightarrow v = (3.9 \times 10^6\,\text{m})\,(7.3 \times 10^{-5}\,\text{rad s}^{-1})$$

$$= 280\,\text{m s}^{-1}$$

Tip

Be careful that your calculator is turned to degrees if, for example, you want to find the sin/cos/tan of an angle like 45°, and to radians if you want to find the sin/cos/tan of an angle like 0.79 rad.

Test yourself

1 What is the speed of rotation of a point in the UK with a latitude of 60°? Take the radius of the Earth to be 6400 km.

2 Explain why the photo in Figure 2.1 is showing an exposure time of about 5 hours.

3 Convert 50 r.p.m. to rad s^{-1}.

4 The Moon circles the Earth every 27.3 days. Calculate its angular velocity in degrees per day.

5 A CD, when first switched on, rotates at 200 r.p.m. When the 'reading head' is half way from the starting position to the centre of the CD, the speed of the CD under the 'reading head' – under the laser head – has to be the same as it was at the beginning. Explain why the rotational speed of the CD is then 400 r.p.m.

2.2 Centripetal forces

We all spend a great deal of time going round in circles; but we don't notice the motion. This is because the circles – around our latitude line on Earth once a day and around the Sun once a year – are very big and the time to complete a single circular journey is very long. When you complete such a journey very quickly, for example on a playground roundabout (see Figure 2.2) or in a car turning a sharp corner, things are very different. In this chapter you will you revise all the basic mechanics you learned and practised earlier in your study of physics.

Imagine yourself as a passenger standing on a bus, holding on to a single vertical post. The bus first accelerates at a from the stop (at the left of Figure 2.4), then turns a sharp left-hand corner at a steady speed v, and finally brakes to a halt at the next stop (at the right of the figure) with acceleration a. In what directions do you feel the force of the post on you during each part of this short journey?

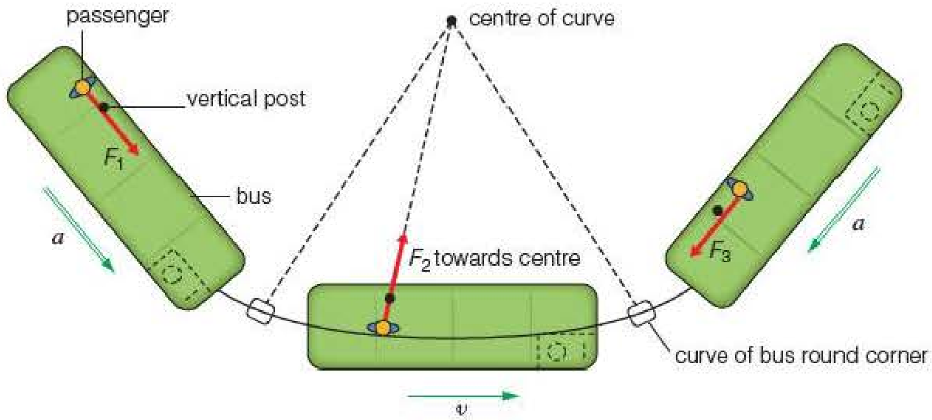


Figure 2.4 A bus turns a corner moving from left to right

The forces on you (drawn in **red**) for each part of the journey are shown in Figure 2.4. (Notice that you would have to move round the post to prevent yourself being thrown forward as the bus slows.) Two of the forces are:

- a *forward* pull F_1 in the direction of the bus as it speeds up, and
- a *backward* pull F_3 in the reverse direction as the bus slows down.

These forces are along the line of motion of the bus, but the interesting part of the journey is the middle part when the bus turns the corner at a steady speed v. The pull you need, F_2, you would instinctively find is towards the centre of the curve in which the bus is travelling. If the passenger is at the centre of the bus, it is towards the centre of the curve along which the bus is travelling. We say you need:

- a *centripetal* pull F_2 as the bus corners at a steady speed.

Without this force you would feel as if you were being 'thrown' away from the centre of the circular motion. This is a result of **Newton's first law of motion** – your body 'wants' to continue in a straight line.

The resultant horizontal force F_2 on you while the bus turns the corner means that you must be accelerating in the same direction as the force F_2. In this case it is **Newton's second law of motion** that you need to think about. The force on you as the bus turns is called a **centripetal force**, which means a force 'seeking' the centre of the circle that the bus is following at that instant. For all bodies or objects moving in the arc of a circle at a constant speed, the resultant force acting on them is a centripetal force, which causes a centripetal acceleration – see Figure 2.5. The children on the roundabout in Figure 2.2 pull (or are pushed) towards the centre and hence accelerate towards the centre.

Key term

Centripetal means towards the centre of a circle (centrifugal would mean away from the centre).

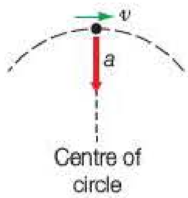


Figure 2.5 Centripetal or towards the centre

'Sideways' acceleration

Centripetal acceleration is 'sideways' to the direction of travel. Put more precisely, in the language of physics, a centripetal acceleration is radially inwards, perpendicular to the tangential velocity. The size of this acceleration is $\frac{v^2}{r}$ where v is the velocity and r the radius of the circle, or, because $v = r\omega$, this acceleration can also be expressed as $r \times \omega^2$.

$$\text{centripetal acceleration} = \frac{v^2}{r} = r\omega^2$$

For example, a runner moving at $8.0\,m\,s^{-1}$ round the end of a track that forms part of a circle of radius 35 m has a centripetal acceleration of $\frac{(8.0\,m\,s^{-1})^2}{35\,m} = 1.8\,m\,s^{-2}$. The unit for acceleration arises naturally.

Example

The centripetal acceleration of the mass on the end of a hammer thrower's wire when it is being rotated in a circle of radius 1.5 m is $500\,m\,s^{-2}$ ($\approx 50\,g$). What is the speed of the mass as it is whirled in this circle?

Answer

Rearranging $a = \frac{v^2}{r}$

gives $v = \sqrt{ar}$

$\therefore v = \sqrt{500\,m\,s^{-2} \times 1.5\,m} = 27\,m\,s^{-1}$

Figure 2.6 A hammer thrower

Activity 2.1

Investigating centripetal force

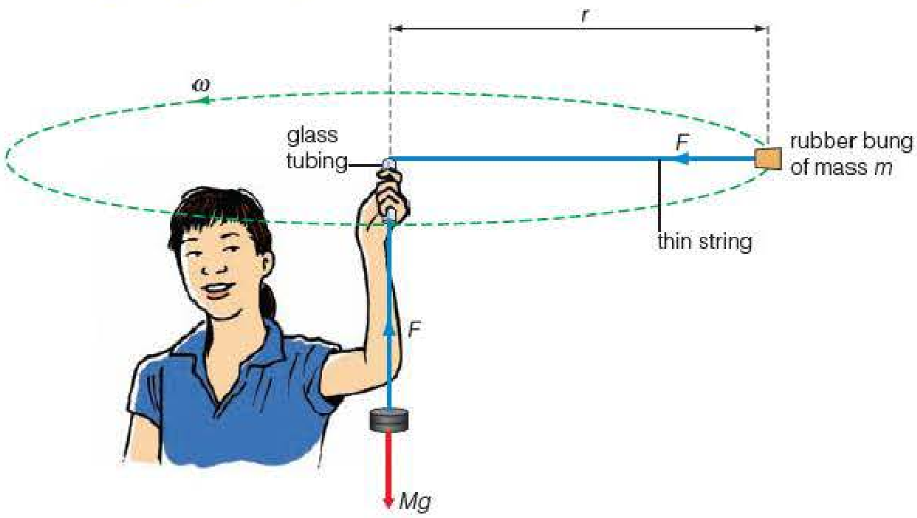


Figure 2.7 Studying circular motion

The centripetal force equation $F = m r\omega^2$ can be investigated using the apparatus shown in Figure 2.7. (The ends of the glass tubing should be smoothed by heat treatment in a Bunsen flame.)

The small rubber bung, of mass m, is made to describe a horizontal circle (as far as possible) while keeping the hanging mass M at a fixed level, so in this case the bung is accelerating (centripetally) but the hanging mass is in equilibrium.

The period of rotation T of the bung is then found by timing a number of rotations. The length r (from the glass tube to the whirling rubber bung) is found once the timing has been completed by pinching the string to stop the rotation and then measuring the length of string from the top of the glass tube to the centre of mass of the bung.

When the supported mass M is in equilibrium, let us assume:

- there is negligible friction between the glass tube and the string
- that the string to the whirling rubber bung is effectively horizontal

With these assumptions: the upward pull F of the string on the mass M will be equal to the horizontal pull F of the string that provides the centripetal force $mr\omega^2$ on the bung, i.e. $Mg = mr\omega^2$. A series of results for different values of M can be used to investigate whether $Mg = mr\omega^2$, i.e. $F = mr\omega^2$.

Example

A teacher, helped by some students, carried out the experiment described in the above Activity.

They obtained the results shown in the Table 2.1 below. The mass m of the bung was 75 g.

Table 2.1

M/g	$F = Mg$/N	$10T$/s	T/s	ω/rad s^{-1}	r/m	$mr\omega^2$/N
50		21.9			0.83	
100		16.2			0.80	
150		13.5			0.93	
200		11.2			0.79	
250		10.7			0.99	

Set up Table 2.1 as a spreadsheet to investigate the extent to which the experimental data confirms that $F = mr\omega^2$. (Remember that $\omega = \frac{2\pi}{T}$ and that 1 kg = 1000 g.)

Answer

(Only the first line is shown.)

M/g	$F = Mg$/N	$10T$/s	T/s	ω/rad s^{-1}	r/m	$mr\omega^2$/N
50	0.49	21.9	2.19	2.87	0.83	0.51

The (completed) table will show that as M gets larger, the agreement between the second and the sixth column becomes weaker, which suggests that this activity is difficult, or rather that it suffers from serious experimental errors.

Tip

Eye protection should be worn, even if the Activity takes place out of doors.

The students' teacher suggested that an alternative way to analyse these results would be to draw a graph, from which they could find a value for the mass m of the bung.

If $F = mr\omega^2$, a graph of F (= Mg) on the y-axis against $r\omega^2$ on the x-axis should give a straight line through the origin. The gradient should be equal to the mass m of the rubber bung. Try this for yourself!

Resultant centripetal force

Newton's second law tells you that the *resultant* force F_{res} on a body of mass m accelerating at $\frac{v^2}{r}$ is:

$$F_{res} = \frac{mv^2}{r}$$

In nearly all applications of the second law for objects moving in a circle of radius r at a uniform speed v, there is only a single external force acting on the object towards the centre of the motion. So the use of the above 'formula' is relatively easy. (Newton's second law can also be used in the form $F_{res} = mr\omega^2$ as in the above Activity.)

Tip

A free-body force diagram shows all the external forces acting on the body you have chosen. Sometimes, as here, you must choose forces in one plane.

Think about water-skiers: they often swing away from the line the towing boat is taking, and move in the arc of a circle. A free-body force diagram of a water-skier as he moves in a circle would look like Figure 2.8a. The water pushes up on the water-skier with a force R (equal and opposite to his weight W), and the water *also* pushes him inwards (centripetally) with a force F_c. (It also pushes him backwards with a force to balance the pull of the tow rope on him. But as we are just looking at forces in the plane containing F_c, the resultant centripetal force, we can disregard this.)

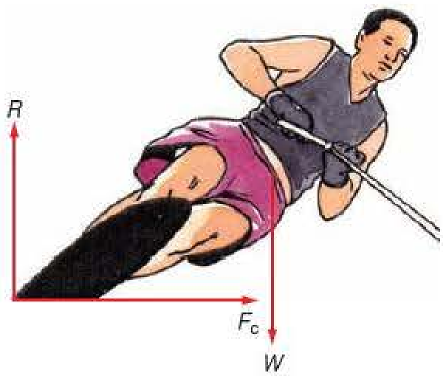


Figure 2.8a Forces on a water skier

Figure 2.8b Photo of a water skier

When you see a water-skier at the extreme of an outward curve, you see a huge plume of water that sprays outwards from the skier's path (Figure 2.8b). This is the result of **Newton's third law**: there is a force on the water equal and opposite to F_c. This force doesn't act on the water skier who is moving in a circle – here it acts on the water.

Table 2.2 lists some more examples of objects moving in circles and the centripetal force(s) acting on them (you will find lots more at a fairground).

Note that the *resultant* centripetal force may involve other forces, such as the weight of the object moving in a circle.

Table 2.2 Examples of circular motion

Object moving in a circle	Centripetal force
Cyclist on a banked velodrome track	Inward component of the force of the track on the cycle's wheels
Single sock in a spin drying machine	Inward force of the drum of the spin dryer on the sock
Fairground car at the bottom of a roller coaster dip	Upward force of the track on the car
The Moon circling the Earth	Pull of the Earth on the Moon
Gymnast on the high bar	Inward force of the bar on the gymnast's hands

Figure 2.9 Racing cyclist on a banked velodrome track

Example

A bobsleigh of total mass 380 kg is travelling at a speed of $31\,\text{m s}^{-1}$ (almost 70 m.p.h.!) along an ice channel. At the moment shown in Figure 2.10 it is turning a corner that is part of a horizontal circle of radius 46 m and the bobsleigh is tilted to the vertical at an angle of 65°.

Assume that friction is negligible.

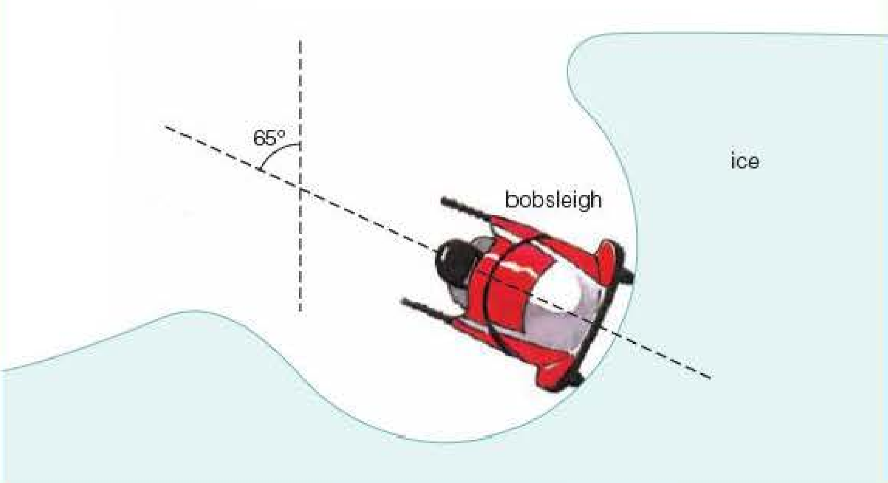


Figure 2.10 A bobsleigh

a) Calculate the centripetal acceleration of the bobsleigh and hence find the horizontal centripetal force acting on it.

b) What force provides this centripetal acceleration?

Answer

a) Centripetal acceleration $a = \frac{v^2}{r} = \frac{(31\,\text{m s}^{-1})^2}{46\,\text{m}} = 21\,\text{m s}^{-2}$

Resultant centripetal force $= ma = 380\,\text{kg} \times 21\,\text{m s}^{-2} = 8000\,\text{N}$

b) This centripetal force is the horizontal component R_h of the normal reaction force R, the push of the ice on the bobsleigh. R acts perpendicular to the bobsleigh's runners. As the angle between R_h and R is 25°

$\therefore R\cos 25° = 8000\,\text{N} \Rightarrow R = 8800\,\text{N}$

(Note that the vertical component R_v of R, $R \sin 25°$, is equal to the weight of the bobsleigh, 3700 N.)

Tip

Whenever possible, use the cosine of the angle between a vector and the direction of its component.

A vector interlude

In order to 'prove' that centripetal acceleration a is equal to $r\omega^2$ or $\frac{v^2}{r}$ consider Figure 2.11.

A body moves at constant speed v in a time Δt between positions 1 and 2. The radius of the circle is r and θ is in radians.

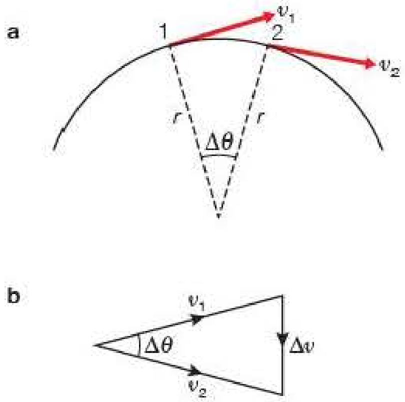


Figure 2.11 Circular motion of a body at constant speed v

Figure 2.11a shows two velocity vectors v_1 and v_2 that are separated by an angle $\Delta\theta$. Figure 2.11b shows that a velocity vector Δv must be added to vector v_1 to produce the vector v_2, i.e. in vectors $v_1 + \Delta v = v_2$. The instantaneous acceleration a of the body is $\frac{\Delta v}{\Delta t}$.

You should know that for very small angles, measured in radians, $\sin\theta \approx \theta$, and that the smaller θ the better the approximation becomes.

We can see from Figure 2.11b that

$$\sin\frac{\Delta\theta}{2} = \frac{\Delta v}{2} \div v$$

For very short time intervals (very small $\Delta\theta$) we cay say

$$\sin\frac{\Delta\theta}{2} = \frac{\Delta\theta}{2} = \frac{\Delta v}{2} \div v = \frac{\Delta v}{2\,v} \text{ (where } \theta \text{ is in radians)}$$

$$\Rightarrow \Delta v = v\Delta\theta.$$

Dividing both sides by Δt gives us

$$\frac{\Delta v}{\Delta t} = v\frac{\Delta\theta}{\Delta t}$$

$$\Rightarrow a = v\omega$$

As $v = r\omega$ (or $\omega = \frac{v}{r}$) the size of the acceleration a is $\omega^2 r$ (or $\frac{v^2}{r}$).

Referring again to Figure 2.11b, we can see that as $\Delta\theta$ gets smaller and smaller, Δv will get closer and closer to being at right angles to v. This tells us that the direction of the acceleration is at right angles to the velocity, i.e. towards the centre of the circle.

This is only an approximate result, but a full calculus analysis also leads to $a = \frac{v^2}{r}$.

Test yourself

6 State Newton's second law **a)** in words, and **b)** as a 'formula'.

7 A car is moving on a level road at 40 mph. ($18\,m\,s^{-1}$). If it is to keep the centripetal acceleration below $9.8\,m\,s^{-2}$, what is the 'tightest' corner it can turn? (By 'tightest' we mean the corner with the least radius.)

8 State Newton's third law in words.

9 How is ω calculated in the fifth column of Table 2.1?

10 In the Example following the Activity, values of Mg and $mr\omega^2$ are given for one set of possible results. Calculate the % difference between these two values.

11 Suggest why the experiment described in the Activity does not produce results that conclusively support the fact that centripetal acceleration is calculated as $r\omega^2$?

12 Describe the forces acting along the line of motion of the water skier shown in Figure 2.8a.

13 Figure 2.12 shows a wet sock in a tumble dryer that is rotating at 790 revolutions per minute.

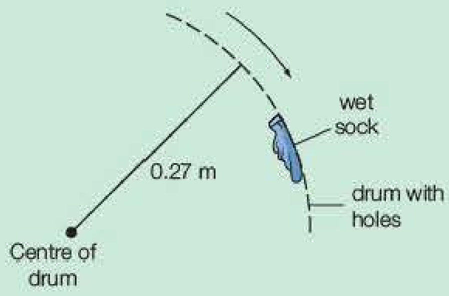


Figure 2.12

a) Show that the speed v of the rim of the drum is $22\,m\,s^{-1}$.

b) What is the centripetal acceleration of the sock?

c) Explain how the sock gets this acceleration.

14 The cyclist shown in Figure 2.9 is moving around a velodrome where the two banked ends together form a circle of radius 25 m. What is the acceleration of a cyclist moving round one end of the velodrome at $20\,m\,s^{-1}$?

15 Figure 2.13 shows two of the forces acting on the cyclist in a plane perpendicular to the cyclist's motion. Is the horizontal component of the force F equal to the centripetal force needed for the cyclist to round the corner?

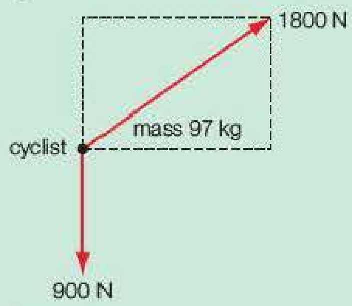


Figure 2.13

2.3 Apparent weightlessness

We don't actually feel the gravitational pull of the Earth on us. What tells us that we have weight is the upward push of a seat or the ground on us. If for a moment we feel no such force, our brain thinks that we have no weight. A person might *feel* weightless when he or she:

- jumps upwards off a trampoline
- treads on a non-existent floor in the dark
- travels in an aeroplane that hits an air pocket.

In each case the person is briefly in 'free fall'; that is, he or she accelerates downwards at $g = 9.8\,m\,s^{-2}$.

Anyone diving or jumping off a 10 metre board might feel weightless for just over a second – the time t taken to get from the board to the water surface, which is given by:

$$t = \sqrt{\frac{2h}{g}} = \sqrt{(2 \times 10\,\text{m}) \div (9.8\,\text{m}\,\text{s}^{-2})} = 1.4\,\text{s}$$

Example

A rollercoaster at an amusement park includes a 'hump' which is part of a vertical circle of radius $r = 18.0\,\text{m}$. A person in a car travelling over the hump feels momentarily weightless. Draw a free-body force diagram of the person and calculate the speed v at which the car is travelling.

Answer

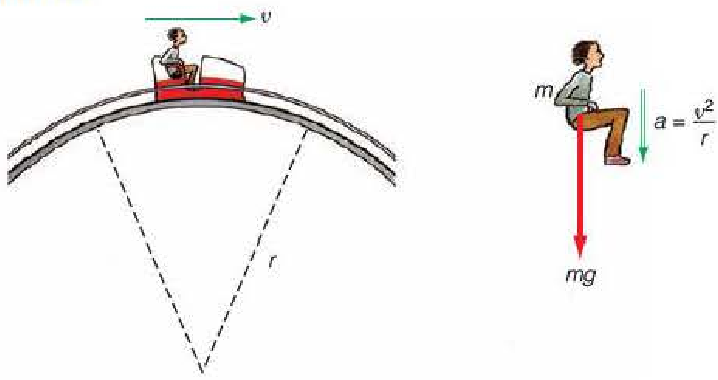


Figure 2.14

When the person feels weightless, there is no reaction force – the only force acting on them is their weight, mg.

As they are travelling in a circle, their centripetal acceleration $= \frac{v^2}{r}$.

Applying Newton's second law gives:

$$\frac{mv^2}{r} = mg$$

$$\therefore v^2 = gr = 9.8\,\text{m}\,\text{s}^{-2} \times 18.0\,\text{m}$$

So $v = 13\,\text{m}\,\text{s}^{-1}$

Test yourself

16 What is meant by 'weightlessnes'?

17 A woman wins a raffle prize of a 'free fall' jump from a balloon. Will she really be in free fall?

18 Why is Section 2.3 called *Apparent* weightlessness rather than just Weightlessness?

Exam practice questions

1 An angle of $\frac{\pi}{3}$ radians is the same as an angle of:

A 120° C 60°

B 90° D 30° **[Total 1 mark]**

2 450 revolutions per second is equivalent to (2 SF)

A $2800\,rad\,s^{-1}$ C $1400\,rad\,s^{-1}$

B $1800\,rad\,s^{-1}$ D $290\,rad\,s^{-1}$ **[Total 1 mark]**

3 A playground platform is rotating once every 4.0 s. A young girl is hanging on 2.0 m from the centre of rotation. She is moving in a circle at a constant speed v of:

A $6.3\,m\,s^{-1}$ C $2.0\,m\,s^{-1}$

B $3.1\,m\,s^{-1}$ D $1.0\,m\,s^{-1}$ **[Total 1 mark]**

4 A stone is swung on the end of a string in a vertical circle. If its average angular velocity is $8.2\,rad\,s^{-1}$, how long does it take to complete 5 revolutions? **[Total 3 marks]**

5 A centrifugal force is a force that acts away from the centre when a body is moving along a circular path. Describe the centrifugal force acting on the bus as it turns the corner in Figure 2.4. **[Total 3 marks]**

6 A laboratory centrifuge operates at a rotational speed of $200\,rad\,s^{-1}$. The end of a test tube is 12 cm from the axis of rotation. What is the acceleration of particles of matter at the end of the test tube? Express your answer in multiples of g. **[Total 3 marks]**

7 Going round a tight corner at speed when sitting in a car, you feel a strong sideways force. This is because

A you are being flung out by a centrifugal force

B your body tries to continue in a straight line

C you are pushing sideways against the seat

D your body is accelerating away from the centre. **[Total 1 mark]**

8 A 'car' of mass 1800 kg in an amusement park moves through the lowest part of a loop of radius 8.5 m at $11\,m\,s^{-1}$.

a) Draw a free-body force diagram for the car at this moment. **[2]**

b) Calculate the upward force of the rails on the car. **[3]**

[Total 5 marks]

9 What provides the centripetal force needed by a car cornering on a level road? **[Total 3 marks]**

10 Figure 2.15 shows a 'conical pendulum' in motion, and a free-body force diagram for the pendulum bob. Use Newton's laws of motion to prove that $\tan\theta = \dfrac{v^2}{gr}$. **[Total 5 marks]**

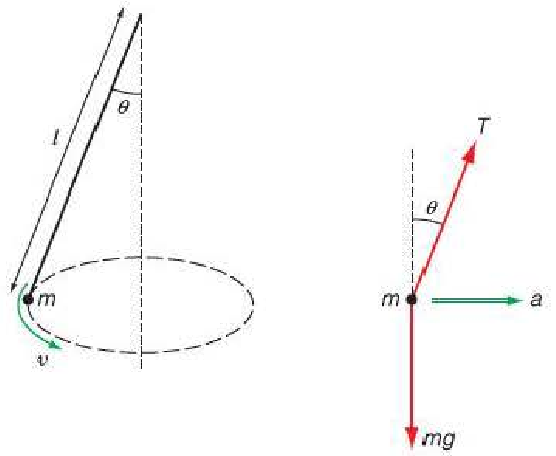


Figure 2.15

11 A proposed space station is designed in the shape of a circular tyre. Figure 2.16 shows a cross-section of the space station.

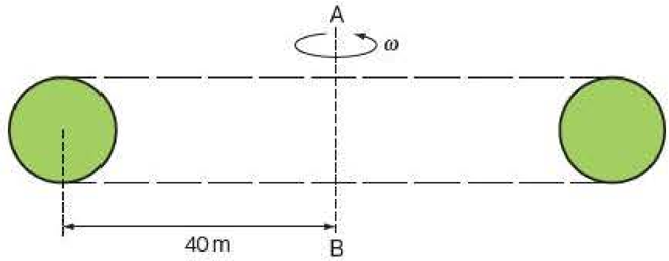


Figure 2.16

a) How often must the space station rotate about its axis AB in order that a person living in it may experience a centripetal acceleration equal to g? **[4]**

b) Suggest two advantages such a space station might have over an existing orbiting space station such as that shown in the film *Gravity*? **[2]**

[Total 6 marks]

12 A teacher whirls a bucket of water in a vertical circle of radius 1.2 m.

a) What is the minimum speed v at which the bucket must be whirled if the teacher is not to get a soaking? **[4]**

b) Suggest a difficulty you might have in putting your solution to the test. **[1]**

[Total 5 marks]

13 A metal ball of mass 2600 kg on the end of a 6.0 m cable is being used to demolish a building. The ball is pulled back until it is 2.5 m above its lowest position and it then swings down in an arc to strike the building.

Calculate the tension in the cable at the moment when the cable is vertical.

[Total 5 marks]

14 The girl in Figure 2.17 has a mass of 56 kg. She is at the Earth's equator.

a) What is her centripetal acceleration? Take the Earth's radius to be 6400 km. **[3]**

b) Show that the force needed to produce this acceleration is about 2 N. **[2]**

[Total 5 marks]

Figure 2.17

Stretch and challenge

15 Suggest why the Earth is an oblate spheroid, i.e. it is a little flattened at the poles and has a slightly larger radius at the equator.

[Total 3 marks]

16 Figure 2.12 shows a wet sock that is being partially dried in a spin-dryer. In the figure the radius of the drum is 0.27 m and the spin-dryer is rotating at 790 revs per minute.

Estimate the acceleration of the sock and describe how the sock becomes dried.

[Total 4 marks]

17 Explain how an aeroplane in horizontal flight can turn in a circular path to right or left.

[Total 4 marks]

3 Universal gravitation

Prior knowledge

You should know from earlier work covered in GCSE science or in your Advanced level studies:

- ➔ that near the Earth's surface $g = 9.8\,\mathrm{m\,s^{-2}}$
- ➔ how to move symbols to be the subject of equations, e.g. from $s = \frac{1}{2}gt^2$, $t = \sqrt{\frac{2s}{g}}$
- ➔ that a newton, N, is a name for $\mathrm{kg\,m\,s^{-2}}$
- ➔ that a joule is a name for a newton metre, i.e. $1\,\mathrm{J} \equiv 1\,\mathrm{N\,m}$
- ➔ the meaning of the prefixes k (10^3), M (10^6), and G (10^9)
- ➔ that a planet like the Earth attracts nearby objects towards its centre
- ➔ the weight *W* and mass *m* are different physical quantities and that $W = mg$
- ➔ Newton's second law of motion: $F = \frac{\mathrm{d}(mv)}{\mathrm{d}t}$ or, for constant m: $F = m\frac{\mathrm{d}v}{\mathrm{d}t}$
- ➔ that kinetic energy is calculated as $KE = \frac{1}{2}mv^2$
- ➔ that changes in *GPE* close to the Earth's surface can be calculated as $\Delta GPE = mg\Delta h$
- ➔ that the average rate of change of velocity $= \frac{\Delta x}{\Delta t}$
- ➔ that the instantaneous rate of change of velocity $= \frac{\mathrm{d}x}{\mathrm{d}t}$

Test yourself on prior knowledge

1 If your mass is 42 kg, what is your weight?

2 Make v the subject of the equation $\frac{1}{2}mv^2 = mg\Delta h$.

3 Show that $\mathrm{kg\,m\,s^{-1}}$ is the same as a newton.

4 Explain the difference between speed and velocity.

5 Estimate the kinetic energy of a man running as quickly as he can.

6 What is the source of energy that lifts a firework rocket off the ground?

7 A woman lifts a baby of mass 14 kg from a cot into her arms. If she lifts the baby a vertical distance of 0.65 m, what is the baby's gain of *GPE*?

8 The graph in Figure 3.1 shows how the displacement of a firework rocket changes as it rises. Calculate its velocity at $t = 1.50\,\mathrm{s}$.

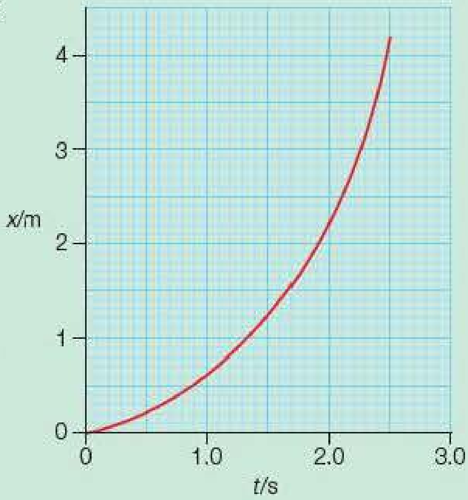


Figure 3.1 Graph of *x* against *t* for a firework rocket

3.1 Uniform gravitational fields

Gravitational forces act over very large distances. The Sun keeps the planets in orbit by its gravitational pull and we all experience a gravitational attraction to the Earth: our weight. In the next chapter you will see that there are analogous, but much smaller, electric forces.

In this chapter you will meet Isaac Newton's formula for the interaction between two masses and learn how to predict the motion of satellites.

Near the Earth's surface, the **gravitational field** (or *g*-field) is a uniform field. The lines of force are parallel, equally spaced and the arrows are downwards – Figure 3.2. The size of the gravitational force on an object in this field is the same at every place.

The **gravitational field strength** g is defined by the equation:

$$g = \frac{F_g}{m}$$

where F_g is the size of the gravitational force experienced by a body of mass m. You will be familiar with this relationship in the form $W = mg$ from your earlier studies.

The unit of g is $\mathrm{N\,kg^{-1}}$ or, as a newton $\mathrm{N} \equiv \mathrm{kg\,m\,s^{-2}}$, another unit for g is $\mathrm{m\,s^{-2}}$.

All objects fall with the same ***acceleration*** g in the same ***gravitational field*** g. Gravitational field strength g is a vector quantity. The direction of g is the same as that of the gravitational force F_g.

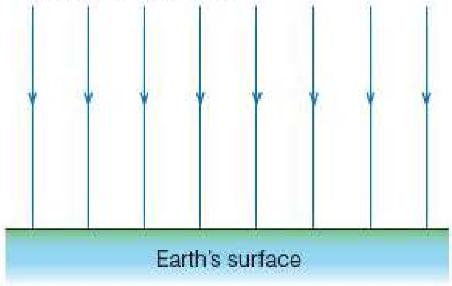


Figure 3.2 A uniform gravitational field

Key term

A **gravitational field** is a region in which a mass feels a force.

Key term

The **gravitational field strength**, g, at a point in a gravitational field is defined as the gravitational force per unit mass acting on a mass at that point.


Figure 3.3 Mars and Jupiter

Example

The values of g on the surface of the planets Mars and Jupiter are $3.7\,\mathrm{N\,kg^{-1}}$ and $23\,\mathrm{N\,kg^{-1}}$ respectively.

a) Calculate the force needed to support a rock of 16 kg on the surface of each planet.

b) How much gravitational potential energy (*GPE*) does the rock gain when raised 2.0 m on each planet?

Both a) and b) involve using numbers and units from familiar equations.

Answers

a) As $g = \frac{F_g}{m}$ then the gravitational force on the rock is F_g or $W = mg$.

The force F needed to support the rock is equal in size to F_g (Newton's first law).

$\therefore W = 16\,\mathrm{kg} \times 3.7\,\mathrm{N\,kg^{-1}} = 59\,\mathrm{N}$ on Mars,

and $W = 16\,\mathrm{kg} \times 23\,\mathrm{N\,kg^{-1}} = 370\,\mathrm{N}$ on Jupiter.

b) The gain in *GPE* of the rock is equal to the work $F\Delta x$, that is $mg\Delta x$, in lifting it.

On Mars the gain in $GPE = 59\,\mathrm{N} \times 2.0\,\mathrm{m} = 120\,\mathrm{J}$,

and on Jupiter the gain in $GPE = 370\,\mathrm{N} \times 2.0\,\mathrm{m} = 740\,\mathrm{J}$.

Variation of g

We have said that the Earth's field is uniform, with parallel field lines near the surface, but this is an approximation. The field is actually a radial field (see Figure 3.11) so the strength decreases with height. The value of g at the top of the Earth's highest mountain is about 0.3% less than its value at sea level.

There is also some variation around the globe. The measured value of the free-fall acceleration varies from $9.83\,\text{m}\,\text{s}^{-2}$ at the poles to $9.78\,\text{m}\,\text{s}^{-2}$ at the equator. Some of this variation is the result of the fact that the Earth is not an exact sphere, and some is the result of the Earth's rotation.

Figure 3.4 Girl on bathroom scales

Example

Figure 3.4 shows a girl standing on a set of bathroom scales. Suppose she is at the Earth's equator.

(This is a Synoptic Example as it uses $a = \frac{v^2}{r}$ from Chapter 2 of this book.)

a) What is her centripetal acceleration? Take the Earth's radius to be 6400 km.

b) Show that the g-field needed to produce this acceleration is less than $0.05\,\text{N}\,\text{kg}^{-1}$ and comment on this result.

Answer

- Remember that g can be thought of in two ways.
- Often when asked to 'comment', there isn't a single correct answer.

a) The girl's speed as the Earth rotates is

$$v = \frac{2\pi \times 6.4 \times 10^6\,\text{m}}{24 \times 3600\,\text{s}} = 465\,\text{m}\,\text{s}^{-1}$$

Her centripetal acceleration is

$$\frac{v^2}{r} = \frac{(465\,\text{m}\,\text{s}^{-1})^2}{64 \times 10^6\,\text{m}} = 0.034\,\text{m}\,\text{s}^{-2}$$

b) To produce this acceleration needs a gravitational field of $0.034\,\text{N}\,\text{kg}^{-1}$.

This g-field is only a small fraction (less that 0.5%) of the Earth's g-field at the equator.

Tip

Alternatively you could solve this by using centripetal acceleration

$$= rw^2$$
$$= 6.4 \times 10^6\,\text{m} \times \left[\frac{2\pi}{(24 \times 3600)\,\text{s}}\right]^2$$
$$= 0.034\,\text{m}\,\text{s}^{-2}$$

If you have time, it is always a good idea to check your calculation like this.

Tip

Look back at Chapter 2 if you need to revise centripetal acceleration.

Tip

When asked to comment, an answer that includes some quantitative statement (numbers) is often better than offering simply words.

On a local scale, tiny variations of g of the order of 1 part in 10^8 can be detected by geologists. Such variations help them to predict what lies below the surface at that point and possibly, for example, to locate oil deposits and places where 'fracking' is most likely to produce oil and gas.

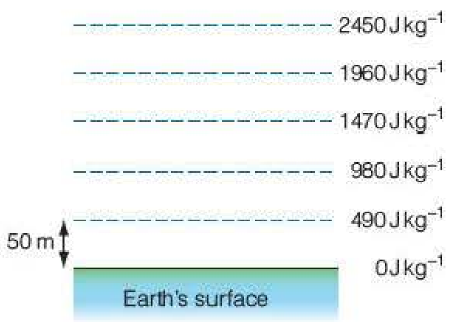


Figure 3.5 Gravitational equipotential surfaces

Equipotential surfaces

The surface of the Earth, the ground, is often taken as a place where objects have zero gravitational potential energy: it is an **equipotential surface**. There is, effectively, no change in the *GPE* of an object when it is moved from one place to another on this surface. Above the ground we can imagine a whole series of flat 'contour' surfaces – the dashed lines in Figure 3.5. Each is an equipotential surface. No work is needed to move an object along such a surface, but to move it upwards between any two adjacent surfaces involves a change in gravitational potential. In the case of Figure 3.5, with 50 m intervals between adjacent surfaces, the change in gravitational potential is

$$g\Delta h = 9.8\,\mathrm{N\,kg^{-1}} \times 50\,\mathrm{m} = 490\,\mathrm{J\,kg^{-1}}$$

These equipotential surfaces can be drawn with any vertical interval; the 50 m is chosen here to reflect the 50 m brown contour lines you see on British Ordnance Survey maps.

Example

A multi-storey car park has six levels each 3.0 m above the other. A car of mass 1600 kg is parked on level 2.

a) Draw a simple labelled sketch of the car park. Label the levels on your sketch with values for the gravitational potential. Take $g = 10\,\mathrm{N\,kg^{-1}}$ and give the first level a potential of $0\,\mathrm{J\,kg^{-1}}$.

b) What is the change in *GPE* of the car as it moves (i) down to level 1 (ii) up to level 5?

Answer

The key to this question is to understand the significance of the 3.0 m in $g\Delta h$.

a) Gravitational potential difference between levels $= 10\,\mathrm{N\,kg^{-1}} \times 3.0\,\mathrm{m} = 30\,\mathrm{J\,kg^{-1}}$ (see Figure 3.6).

b) i) Change in $GPE = (1600\,\mathrm{kg})(-30\,\mathrm{J\,kg^{-1}}) = -48\,000\,\mathrm{J}$ or $-48\,\mathrm{kJ}$

(ii) Change in $GPE = (1600\,\mathrm{kg})(+90\,\mathrm{J\,kg^{-1}}) = +144\,000\,\mathrm{J}$ or $+144\,\mathrm{kJ}$

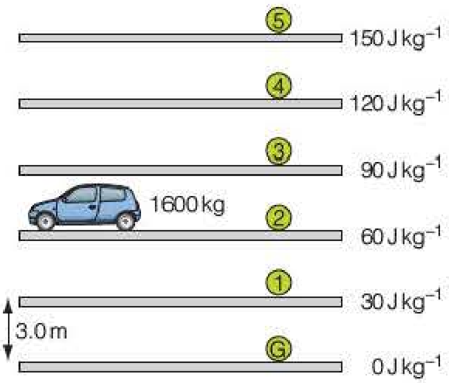


Figure 3.6 A multi-storey car park

Tip

Note the change in *GPE* in **i)** is *negative* because the car has lost *GPE* going from level 2 to level 1.

Test yourself

1 A baby is weighed in a hospital and the birth weight is recorded as 35 N. What is the baby's mass?

If 1.0 kg is equivalent to 2.2 lb, what is the baby's mass in pounds (lb)?

2 What is the weight of one pound (lb) in newtons? Use data from the previous question.

Tip

Remember, if two experimental measurements are made, the % difference is found by dividing the difference between the two values by the *average* value and then multiplyingby 100.

3 Calculate the percentage difference between the measured value of the free-fall acceleration on Earth, which varies from 9.83 m s^{-2} at the poles to 9.78 m s^{-2} at the equator.

4 The value of *g* on the surface of the Moon averages 1.40 N kg^{-1}.

a) What is the weight of an astronaut, together with his clothes and backpack if the total mass is 135 kg?

b) When this astronaut climbs down a ladder from a lunar module to the surface of the Moon, he loses 380 J of *GPE*. Calculate the vertical distance through which he has moved.

5 With what acceleration will a feather fall towards the Moon's surface? Explain your answer.

6 A car and driver of total mass 1800 kg enter a multi-storey car park and rise from level 1 to level 4.

The vertical distance between adjacent levels is 2.8 m.

Calculate the gravitational potential energy (*GPE*) gained by the car and its driver. Express your answer in MJ.

Figure 3.7 The apple and the moon

3.2 Newton's law of gravitation

Isaac Newton knew that bodies are accelerating when they are moving in a circle at a constant speed and hence realised that the Moon is continuously accelerating (falling) towards the Earth. He is said to have conceived his law of gravitation by linking this observation with the fact that a falling apple also accelerates towards the Earth – see Figure 3.7.

This led Newton to propose that every particle in the universe attracts every other particle with a force *F* given by an **inverse square law**

$$F \propto \frac{1}{r^2}$$

The full statement of Newton's Law of gravitation is

$$F = \frac{Gm_1m_2}{r^2}$$

where m_1m_2 is the product of the masses of the two particles, r is their separation and G ('big gee') is a constant called the **gravitational constant**.

Gravitational forces are very small unless one of the 'particles' is a planet or star, and so *G* is very difficult to measure in the laboratory. It has a value, to three significant figures, of 6.67×10^{-11} N m^2 kg^{-2}. (See question 10 in the Exam Practice Questions at the end of this chapter for a way of estimating a value for G.) This inverse square law for gravitational forces, so far as we know, involves only attractive forces.

Key term

Newton's law states that the gravitational force between two particles is proportional to the product of the masses of the particles and inversely proportional to the square of the distance between the particles, i.e. it is an **inverse square law**.

Example

The Moon has a mass of 7×10^{22} kg and is 4×10^{8} m away from us – each to only one significant figure. Estimate the gravitational pull of the Moon on you and comment on your answer.

Answer

For a mass of 70 kg, the force $F = \frac{Gm_1m_2}{r^2}$ is

$$F = \frac{6.67 \times 10^{-11}\,\text{N m kg}^{-2} \times 6 \times 10^{22}\,\text{kg} \times 70\,\text{kg}}{(4 \times 10^{8}\,\text{m})^2}$$

$$= 0.00175\,\text{N} = 0.002\,\text{N to 1 SF}$$

This is a very, very small force compared to your body weight on Earth, which is approximately 700 N.

Tip

Here you are being asked to estimate a value for your mass. Any comment will be better if it involves numbers.

Tip

When putting awkward numbers into calculators to get things like F here, it is often helpful to start with the number on the bottom and square it [x=]. Then use the inverse key [1/x] before multiplying by the numbers on top to get the answer.

Newton's law as previously expressed, applies to *particles* and to *spherical objects* such as the Earth. For such objects, the distance r is to the centre of the object (see example on page 44). If we use it to find the gravitational force between irregular objects a certain distance apart, the calculated force will only be an estimate.

Example

A baby arrives in the world with a birth weight the hospital gives as '3.8 kg'. Calculate the gravitational pull of the Earth on the baby (its weight) using Newton's law.

Take the mass of the Earth to be 6.0×10^{24} kg and its radius to be 6.4×10^{6} m.

Answer

$$F = \frac{Gm_1m_2}{r^2} \approx \frac{(6.7 \times 10^{-11}\,\text{Nm}^2\text{kg}^{-2})(6.0 \times 10^{24}\,\text{kg})(3.8\,\text{kg})}{(6.4 \times 10^{6}\,\text{m})^2}$$

$= 37\,\text{N}$ (exactly as expected, i.e. $3.8\,\text{kg} \times 9.8\,\text{N kg}^{-1}$)

Satellites

A body on which the only force acting is the pull of the Earth on it, its weight, is said to be in a state of **free fall**. Satellites remain in free fall all the time as they circle the Earth. (A free-fall parachutist free falls for only a few seconds before drag forces affect the motion. During most of the fall the parachutist is moving at a steady speed of about 125 m.p.h.)

Tip

You might like to have a look at Section 2.3, *Apparent weightlessness* again.

The Earth has only one natural satellite – the Moon – but there are thousands of 'artificial' satellites placed in orbit above the Earth: communications satellites, weather satellites, military satellites and the International Space Station. (Again, you may have seen the film *Gravity*.) Figure 3.8 represents an artificial satellite orbiting the Earth. A free-body force diagram of the satellite (of mass m moving with speed v) shows there to be only one force acting on it, the gravitational attraction of the Earth towards the Earth's centre – see Figure 3.9.

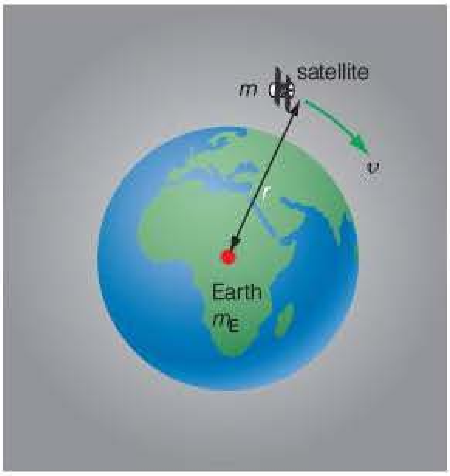


Figure 3.8 A satellite orbiting the Earth

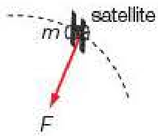


Figure 3.9 A free body force diagram for the satellite

Applying Newton's second law and his law of gravitation to the satellite (of mass m moving with speed v) gives

$$\frac{mv^2}{r} = \frac{Gmm_E}{r^2}$$

and as $v = \dfrac{2\pi r}{T}$

this leads to $r^3 = \dfrac{Gm_E T^2}{4\pi^2}$

As $\frac{Gm_E}{4\pi^2}$ is a constant, the result is that r^3 is proportional to T^2 (this is sometimes known as Kepler's third law), and so the further out the satellite is, the longer it takes to orbit the Earth. For moons orbiting other planets in orbits of known radius, observations of their period of revolution enable us to calculate the mass of the planet as $m_P = \dfrac{4\pi^2 r^3}{GT^2}$.

Tip

One dictionary definition of synchronous 'is occurring at the same time'.

Tip

Apply Newton's second law of motion to the satellite.

Example

Communications satellites are placed in geosynchronous orbits above the Earth's equator.

a) What is the angular velocity of a geosynchronous satellite?

b) Calculate the radius of the satellite's orbit.
(Take $Gm_E = 4.0 \times 10^{14}\,\text{N m}^3\,\text{kg}^{-2}$.)

Answer

a) A geosynchronous satellite must remain above the same place on the Earth as the satellite and the Earth rotate. To do this the satellite must lie in an equatorial plane and complete one revolution once every 24 hours.

Therefore its angular velocity is $\omega = \dfrac{2\pi\,\text{rad}}{24 \times 36000\text{s}} = 7.3 \times 10^{-5}\,\text{rad s}^{-1}$

b) Applying Newton's second law to the satellite:

$$\frac{mv^2}{r} = mr\omega^2 = \frac{Gmm_E}{r^2}$$

$$\Rightarrow r^3\omega^2 = Gm_E = 4.0 \times 10^{14}\,\mathrm{N\,m^2\,kg^{-2}}$$

$$\therefore r^3 = \frac{4.0 \times 10^{14}\,\mathrm{N\,m^2\,kg^{-2}}}{(7.3 \times 10^{-5}\,\mathrm{s^{-1}})}$$

$$\Rightarrow r = 4.2 \times 10^7\,\mathrm{m} \text{ or } 42\,000\,\mathrm{km}\ (36\,000\,\mathrm{km} \text{ above the Earth's surface})$$

Tip

Don't forget that r is the distance between the *centres* of the two masses. In this case it is from the centre of the satellite to the *centre* of the Earth, so the radius of the Earth must be subtracted from r to find the height of the satellite above the Earth's surface.

Continuous free fall

Figure 3.10 shows an astronaut orbiting the Earth. He is in a continuous state of free fall, i.e. he is falling continuously towards the Earth with an acceleration g ($\mathrm{m\,s^{-2}}$) equal to the local gravitational field strength g ($\mathrm{N\,kg^{-1}}$).

From the photograph it is obvious that there is no 'supporting' force acting on the astronaut. When he returns to his space station the only force acting on him will continue to be his weight, and so he can 'float' around, as can other objects in the cabin that are not attached to the walls. The astronaut describes his condition as being one of **weightlessness**, but this is only apparently the case. His weight mg is still acting on him all the time.

Figure 3.10 An astronaut in orbit around the Earth

Test yourself

7 Recent measurements have revealed that the gravitational field on the surface of the Moon varies from place to place by nearly 1%. Suggest possible reasons for this variation.

8 The Moon circles the Earth once every 27.3 days. The distance between the centres of the Earth and Moon is 384×10^3 km. Calculate the centripetal acceleration of the Moon towards the Earth.

9 Sketch a graph to show how the speed of a 'sky diver' varies from when she leaves the plane until she reaches the ground.

10 Use Newton's law of gravitation to calculate the pull of the Earth on an apple of mass 0.10 kg. Take the mass of the Earth to be 6.0×10^{24} kg and its radius to be 6.4×10^6 m.

11 A non-scientist might ask: 'Why doesn't the astronaut in Figure 3.10 fall towards the Earth?' The physicist's answer is: 'He does!' Explain the physicist's answer.

Key term

A body is apparently weightless when the only force acting on it is the pull of the Earth or a nearby planet.

Such a body will therefore – like a high diver – be in a state of free fall, i.e. of **weightlessness.**

3.3 Radial gravitational fields

Figure 3.11 shows in 2-dimensions the lines of force (the blue lines) and some equipotential surfaces (the dashed blue lines) in the gravitational field of the Earth to a distance of about 20×10^3 km from the Earth's surface. (The radius of the Earth r_E is 6.4×10^3 km.) On this scale the lines of force become wider apart as they spread out. Since the gravitational potential increases by equal amounts between the dashed lines, they too become further apart the greater the distance from the Earth. A similar diagram could be drawn for any isolated planet or star.

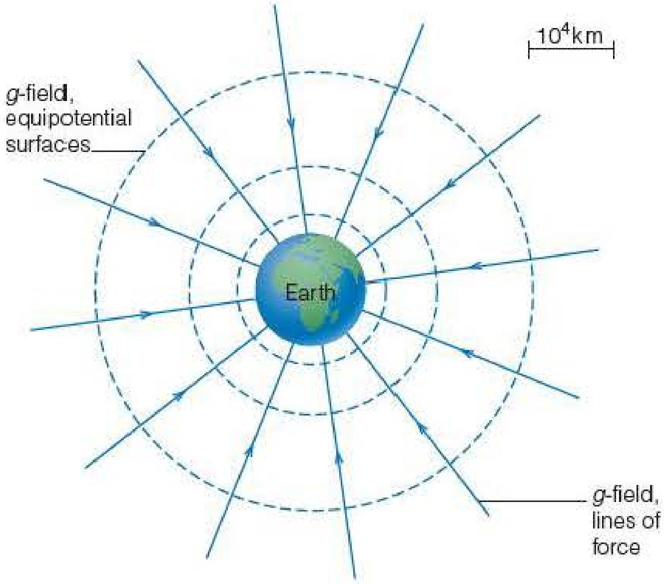


Figure 3.11 Field lines and equipotentials in a radial field

The gravitational force per unit mass, or the gravitational field strength, around the Earth is described by the equation

$$g = \frac{F_g}{m} = \frac{Gmm_E}{r^2} \div m$$

so

$$g = \frac{Gm_E}{r^2} = \frac{4.0 \times 10^{-14}\,\text{N}\,\text{m}^2\,\text{kg}^{-1}}{r^2}$$

where m_E is the mass of the Earth, r is the distance from the centre of the Earth and G is the gravitational constant. In general $g = \frac{Gm}{r^2}$ where m is the mass of the moon, planet, star or other spherical gravitationally attracting body.

Example

The mass of the Earth is 5.98×10^{24} kg and its radius is 6.37×10^3 km.

Calculate the theoretical value of g at the Earth's surface.

Answer

$$g = \frac{Gm_E}{r^2} = \frac{(6.67 \times 10^{-11}\,\text{N}\,\text{m}^2\,\text{kg}^{-2})(5.98 \times 10^{24}\,\text{kg})}{(6.37 \times 10^6\,\text{m})^2}$$

$$= 9.83\,\text{N}\,\text{kg}^{-1}$$

Example

The mass of the Earth is 600×10^{22} kg and the mass of the Moon is 7×10^{22} kg, each to one significant figure. Explain why there is a point P somewhere between the Earth and the Moon where the gravitational field is zero. Give a rough estimate as to where P lies on the line between the two bodies.

- It is crucial to sketch a diagram in answering questions like this.
- 'Give a rough estimate' will use the data and the diagram plus your knowledge of Newtonian gravitation.

Answer

Between the Earth and the Moon the Earth's *g*-field (g_E) is towards E and the Moon's *g*-field (g_M) is towards M.

At some point P the vectors add to zero: $g_E + g_M = 0$.

The point P where g_E is the same size as g_M will be much nearer to the Moon than to the Earth, i.e. EP > PM. This is because the Earth's mass 600×10^{22} kg is almost 100 × bigger than the Moon's mass 7×10^{22} kg and $g \propto m$.

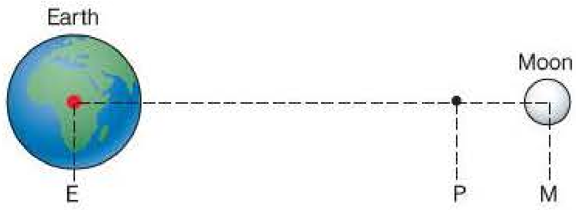


Figure 3.12 Earth and Moon

As $g \propto \frac{1}{r^2}$, EP will only be about $\sqrt{100}$ (= 10) times as big as PM, so the place P where the gravitational field is zero is approximately 90% of the way from the Earth to the Moon.

Earth's *g*-field

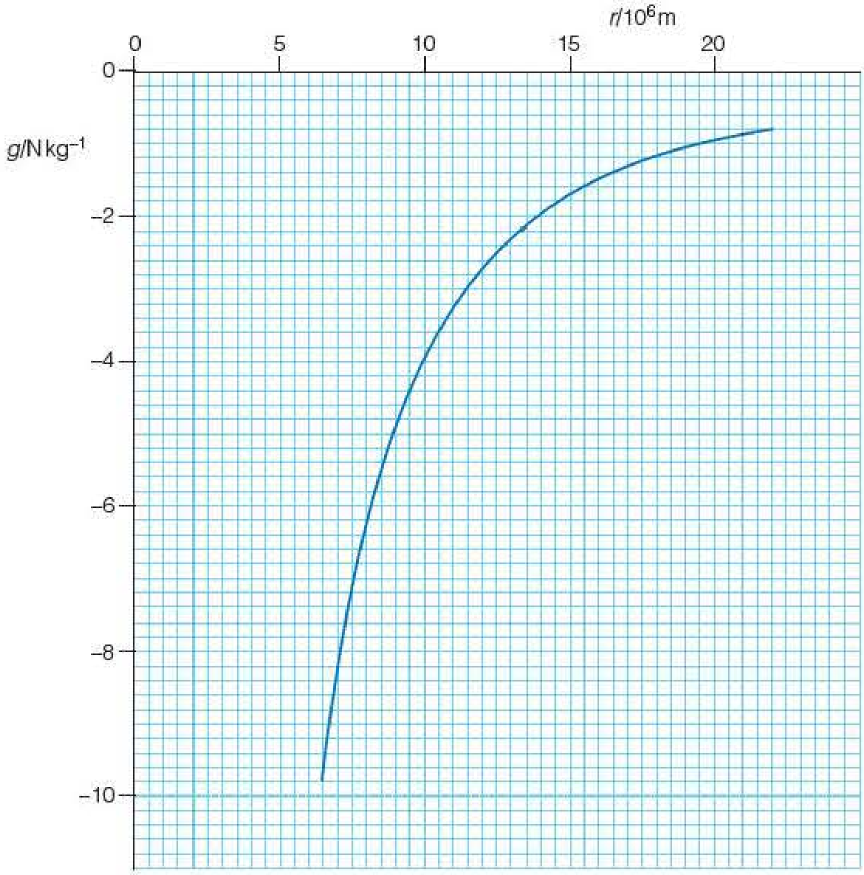


Figure 3.13 Variation of *g* above the Earth's surface

The graph in Figure 3.13 shows the inverse square relationship between g and r for the Earth for values of r greater than 6.4×10^3 km (the radius of the Earth). The values of g are negative because r is measured *away* from the Earth but the g-field is *towards* the Earth. We do not know in detail how the Earth's gravitational field varies below the Earth's surface, but we can be sure it will fall to zero at the centre.

Example

A satellite is in a circular orbit of radius 7.5×10^3 km around the Earth. Use the data in the graph of Figure 3.13 to determine the speed of the satellite in its orbit.

Answer

From the graph, at 7.5×10^6 m the size of g is $7.0\,\mathrm{N\,kg^{-1}}$ or $7.0\,\mathrm{m\,s^{-2}}$

So the centripetal acceleration of the satellite is:

$$\frac{v^2}{r} = 7.0\,\mathrm{m\,s^{-2}}.$$

$$\Rightarrow v^2 = (7.0\,\mathrm{m\,s^{-2}}) \times (7.5 \times 10^6\,\mathrm{m}) = 5.25 \times 10^7\,\mathrm{m^2\,s^{-2}}.$$

So the speed of the satellite is

$$v = 7200\,\mathrm{m\,s^{-1}}$$

Tip

Beware of the units in questions like those above e.g., on this graph's x-axis r is measured in 10^6 m but in the Example the orbit of the satellite is given in 10^3 km.

3.4 Gravitational field and potential

In a *uniform* gravitational field, the relationship between the gravitational field, g, and the change in the gravitational potential difference is

$$\Delta V_g = g\Delta h \text{ (g being constant).}$$

In a *radial* field the relationship between gravitational potential V_g and the distance r from the centre of the Earth (a sphere), for values of $r > r_E$, is:

$$V_{grav} = -\frac{Gm_E}{r} \qquad (1)$$

The relationship between the gravitational potential V_g and r, is related to the way in which the size of the Earth's gravitational field g varies with distance r, again for values of $r > r_E$.

$$g = \frac{Gm_E}{r^2} \qquad (2)$$

Example

Show that the right-hand side of the equation $V_{grav} = -\frac{Gm_E}{r}$ has the unit $\mathrm{J\,kg^{-1}}$.

Answer

G has the unit $\mathrm{N\,m^2\,kg^{-2}}$, m_E has the unit kg and $\frac{1}{r}$ has the unit $\mathrm{m^{-1}}$

So the units of $\frac{Gm_E}{r}$ are $(\mathrm{N\,m^2\,kg^{-2}}) \times \mathrm{kg} \times \mathrm{m^{-1}} = \mathrm{N\,m\,kg^{-1}}$ or $\mathrm{J\,kg^{-1}}$ (as a N m is a J).

A mathematical digression

You will not be asked to show the integration below during an A-level examination, but you do need to recognise and be able to use equation (1) for spherical, gravitationally attracting bodies of mass m:

$$V_g = -\frac{Gm}{r}$$

There is a link between equations (1) and (2):

By definition

$$\Delta V_g = g\Delta r = \frac{Gm_E}{r^2} \times \Delta r$$

Integrating this relationship in the form

$$\int_{V_1}^{V_2} dv = \int_{r_1}^{r_2} -\frac{Gm_E}{r^2} dr$$

(For this more rigorous mathematical treatment we have to insert the minus sign, which means that dr is *away from* the centre and g is *towards* the centre.)

$$V_2 - V_1 = -Gm_E\left[-\frac{1}{r}\right]_{r_1}^{r_2}$$

Hence the difference in the Earth's gravitational potential between distances of r_1 and r_2 from the centre of the Earth is given by

$$\Delta V = -\frac{Gm_E}{r_1} - \left[-\frac{Gm_E}{r_2}\right]$$

$$= -Gm_E\left[\frac{1}{r_1} - \frac{1}{r_2}\right] \qquad (3)$$

The zero of gravitational potential energy is taken to be at infinity; there is a reason for this.

If one moves from a planet's surface, $r_1 = r_P$, to r_2 = infinity, so far away from the Earth that $\frac{1}{r_2} = 0$, then the gravitational potential *at* the surface of a planet is V_g where

$$V_g = -\frac{Gm}{r_P}$$

and so, although the value of V_g is different on the surface of different planets or stars, the value of the gravitational potential is the same for all such objects. Of course what is correct for gravitational fields is also correct for electric fields, which also follow inverse square mathematics. (See the next chapter.)

Tip

You do not need to have any knowledge of calculus for the examination.

Tip

One way of testing this inverse square relationship in Question 12 below is to make a list of values of g and $\frac{1}{r^2}$, and then plot a graph of g against $\frac{1}{r^2}$. Calculating values of gr^2 is easier and quicker.

Tip

Beware! We can't use $\Delta(GPE) = mg\Delta h$ in Question 15 below because g is not constant over the large distance involved. You might like to check for yourself that g at 42 000 km is only 0.23 N kg^{-1}.

Test yourself

12 Refer to the graph in Figure 3.13.

a) Describe how to test that the graph is showing an inverse square relationship between g and r.

b) By taking readings from the graph, use your answer to **a)** to show that $g \propto \frac{1}{r^2}$.

13 State in words the meaning of the relationship $V_{grav} = -\frac{Gm_E}{r_E}$.

14 a) The relationship between the gravitational potential V_g and the distance r from the centre of the Earth (a sphere) is $V_{grav} = -\frac{Gm_E}{r}$.

Make a table of values of V_{grav} and r for values of r from the radius of the Earth r_E to about $3r_E$ (Take $Gm_E = 4.0 \times 10^{14}$ N m^2 kg^{-1} and $r_E = 6.4 \times 10^6$ m.)

b) Using your table, plot a graph of V_g against r.

15 Use equation (3) to calculate the *GPE* needed to raise a satellite, e.g. the Hubble space telescope of mass 11 000 kg, from the Earth's surface into a geostationary orbit.

Take $Gm_E = 4.0 \times 10^{14}$ N m^2 kg^{-1}, the radius of the Earth to be $r_E = 6.4 \times 10^3$ km and the radius of a geostationary orbit to be $r = 42 \times 10^3$ km.

16 Why do we make the gravitational potential an infinite distance from a planet or star zero?

Exam practice questions

1 The unit for g could be each of the following, except:

A $\text{J m}^{-1}\,\text{kg}^{-1}$

B m s^{-2}

C $\text{N s}^{-1}\,\text{kg}^{-1}$

D N kg^{-1}. **[Total 1 mark]**

2 A high jumper of mass 83 kg raises his centre of gravity 1.3 m in crossing a high bar. The increase in GPE needed to achieve this:

A is less than 1000 J

B is about 100 J

C depends on how fast his approach run is

D is independent of his run-up speed. **[Total 1 mark]**

3 A crane lifts a mass of 450 kg from ground level to the top of a building 30 m high.

a) What is its change of GPE? **[2]**

b) What is the change in gravitational potential? **[2]**

[Total 4 marks]

4 The value of the gravitational field g at the surface of a planet of radius r and uniform density ρ is given by the relation $g = \frac{4}{3}\pi G\rho r$, where G has its usual meaning.

Show that the right-hand side of this expression has the correct unit for g.

[Total 3 marks]

5 Show that the value of g hardly changes in the first 100 km above the Earth's surface by calculating g_r and g_{r+10} km. Take $r_E = 6.37 \times 10^6$ m.

[Total 4 marks]

6 A stone of mass 3.0 kg is projected from ground level at a speed of 5 0 m s^{-1}.

a) What will be its height above the ground when it has a speed of 3 0 m s^{-1}? **[4]**

b) Does your answer depend upon the angle at which it is projected? **[2]**

[Total 6 marks]

Tip

Calculations are easier if they only involve scalar quantities, so using energy for this calculation makes it easier than using the equations of motion.

7 The planet Saturn has a radius of 60×10^6 m and spins on its axis once every 3.7×10^4 s (about 10 hours).

a) Calculate the centripetal acceleration of an object at rest on Saturn's equator. **[3]**

b) The gravitational field strength on Saturn's equatorial surface is $10.6\,\text{N kg}^{-1}$. What would an object of mass 100 kg register on bathroom scales at Saturn's equator? **[3]**

[Total 6 marks]

8 The planet Venus has a diameter of 12×10^3 km and an average density of $5200\,\text{kg m}^{-3}$. Calculate the gravitational field at its surface.

[Total 5 marks]

9 (See the formula in question 4.) Isaac Newton 'guessed' that the average density of the Earth was about $5000\,\text{kg m}^{-3}$. He used this guess to find a value for the gravitational constant, G.

What value did he calculate? **[Total 4 marks]**

10 A rifle bullet is fired horizontally on the Moon's equator at a speed v. It makes a full circle of the Moon in a time T.

Calculate values for v and T given the following data:

mass of Moon = 7.3×10^{22} kg,

radius of Moon = 1.7×10^6 m. **[Total 6 marks]**

11 A satellite of mass m is moving at a speed v at a constant height h above the Earth's surface.

The mass and radius of the Earth are m_E and r_E respectively, and G is the gravitational constant.

a) Apply Newton's second law to the motion of this satellite. **[3]**

b) Hence show that v decreases as h increases. **[2]**

c) How does the mass of the satellite affect the speed v at a height h? **[2]**

[Total 7 marks]

12 The diagram shows a body of mass m situated at the distances marked from the centre of the Earth (mass $m_E = 598 \times 10^{22}$ kg) and the centre of the Moon (mass $m_M = 7.3 \times 10^{22}$ kg).

The Earth–Moon distance $R + r = 3.8 \times 10^8$ m.

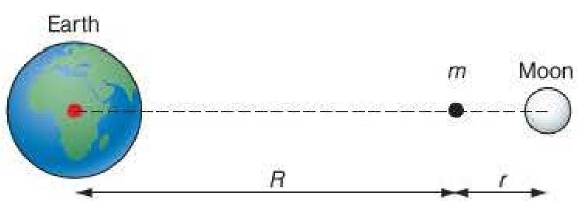


Figure 3.14

If at this point the body experiences zero net gravitational pull towards either body show that

a) $R = \left(\frac{m_E}{m_M}\right)^{\frac{1}{2}} r$ **[4]**

b) $R = 3.4 \times 10^8$ m **[4]**

[Total 8 marks]

13 Physics takes the value of gravitational potential to be zero at infinity, i.e. when $r = \infty$, then $V_{grav} = -\frac{Gm}{r}$. Explain why infinity is a sensible choice for the zero of gravitational potential.

[Total 3 marks]

14 a) Calculate the gravitational potential at
(i) $r = 7500$ km and
(ii) 8000 km from the centre of the Earth.
Take $m_E = 5.98 \times 10^{24}$ kg. **[3]**

b) Hence calculate how much gravitational potential energy must be given to a satellite of mass 11 tonnes (e.g. the Hubble space telescope) in order to lift it between these two positions. **[3]**

[Total 6 marks]

15 An object is to be fired from the surface of the Moon at such a speed that it will never return.

Calculate this speed, v escape, given the radius and mass of the Moon are 7.34×10^{22} kg and 1.64×10^6 m respectively.

[Total 4 marks]

Stretch and challenge

16 The table gives the orbital period T of the four largest satellites of the planet Jupiter. Their mean distance from the centre of Jupiter r is also given.

	Io	Europa	Ganymede	Callisto
T/days	1.77	3.66	7.15	16.7
$r/10^3$ km	422	671	1070	18.0

a) Plot a graph of $\ln(T/\text{days})$ against $\ln(r/10^3\ \text{km})$. **[5]**

b) Calculate its gradient and hence show that these data agree with the relationship $r^3 \propto T^2$. **[3]**

[Total 8 marks]

17 Explain what you get when you differentiate with respect to r both sides of the relationship for the gravitational potential in the Earth's gravitational field: $V_{grav} = -\frac{Gm_E}{r}$.

[Total 3 marks]

4 Electric fields

Prior knowledge

You should know from GCSE or from earlier Advanced level work:

- ➜ how to move symbols to be the subject of equations, e.g. $a = v^2/r \Rightarrow v = \sqrt{ar}$
- ➜ that $g = 9.8\,\mathrm{N\,kg^{-1}}$ or $9.8\,\mathrm{m\,s^{-2}}$
- ➜ that the unit of force, the newton, N, is a name for $\mathrm{kg\,m\,s^{-2}}$
- ➜ that energy is not 'used up': it is conserved
- ➜ that charge is measured in coulombs, C
- ➜ that a volt is a name for a joule per coulomb, $1\,\mathrm{V} \equiv 1\,\mathrm{J\,C^{-1}}$
- ➜ the meaning of k, µ and n as used in, e.g., $\mathrm{kV} \equiv 10^3$ volts
- ➜ that like charges repel and unlike charges attract.

Test yourself on prior knowledge

1. What is the weight of a woman with a mass of 65 kg?
2. Make ρ the subject of the equation $\Delta p = \rho g \Delta h$.
3. Describe a situation in which kinetic energy is *not* conserved.
4. How much charge passes through a cell when it produces a current of 6.0 mA for 60 s?
5. What does 650 MJ stand for?
6. How many electrons make up a charge of 16 nC?
7. Explain what is meant by: 'This is a 6 V battery'?
8. 'The potential difference is 240 J/0.60 C = 4.0 mV.' Why is this wrong?

4.1 Fields in physics

Matter contains atoms and molecules. When you put a very large number of them together to make up a planet like the Earth, their mass produces a *gravitational field*. (Chapter 3 deals with universal gravitation.) When an apple (which also has mass) is released in this field, it falls to the ground because it feels a force – a gravitational force (Figure 4.1a). We do not 'see' what is causing the force, but we know it is there because the apple accelerates towards the Earth.

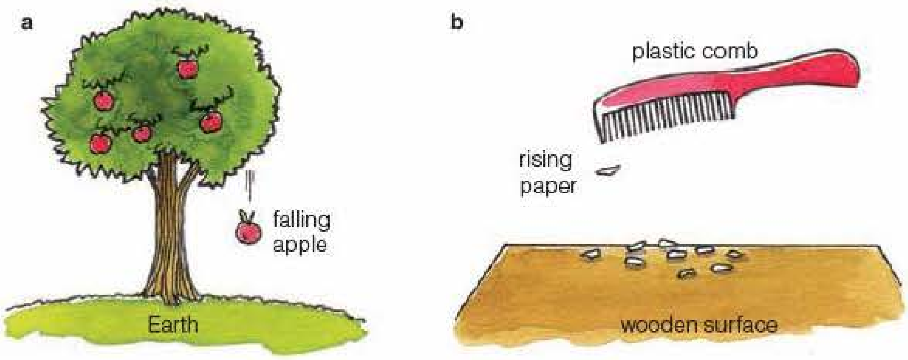


Figure 4.1 Invisible forces

Key term

A **field** is a region of space in which an object experiences a force.

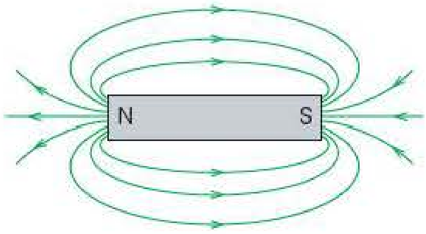


Figure 4.2 Magnetic lines of force

Figure 4.3 Michael Faraday

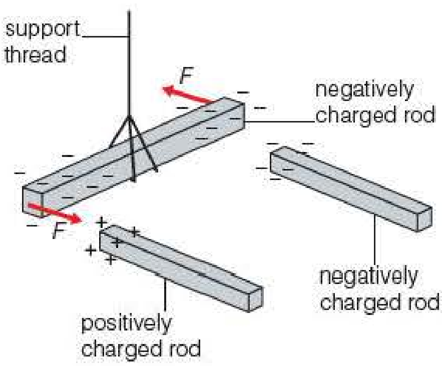


Figure 4.4 Unlike charges attract; like charges repel

The atoms and molecules in matter contain protons and electrons that carry electric charge. Normally the positive and negative charges in an object, like a plastic comb, exactly cancel each other out so that the comb is electrically neutral, i.e. has a total charge of zero. But sometimes a comb that has been rubbed on a cloth has a net charge and can pick up tiny pieces of paper (Figure 4.1b). We do not 'see' the force picking up the pieces of paper, but we know it is there because the paper accelerates towards the comb. The charge on the comb produces what is called an **electric field** and the force on the paper is called an electric or electrostatic force.

Both gravitational force and electric force are mysterious because there is no visible link between the two bodies that accelerate towards one another. The word **field** in physics is a general word used to describe regions in which these 'invisible' forces act.

Another type of field is produced by magnets and by electric currents. These **magnetic fields** are best understood by drawing magnetic **lines of force**, a technique you may have used in your earlier science studies. In exactly the same way, gravitational and electric fields are also described using lines of force. Figure 4.2 is a familiar diagram of a magnetic field, shown by magnetic lines of force with arrows going from N to S on a bar magnet – a magnetic dipole. The arrows show the direction of the force on a N pole. The closeness (density) of the lines shows the strength of the field.

Michael Faraday developed the idea of 'lines of force' to describe magnetic fields. (Chapter 6 develops these ideas.) In this chapter you will learn about the forces between electric charges, and how electric fields can be described using lines of force – another result of Michael Faraday's genius.

4.2 Electric forces

How does the comb pick up the pieces of paper in Figure 4.2b? The plastic comb is electrically charged: if it has extra electrons it is negatively charged. This negative charge repels negatively charged electrons in each tiny piece of paper, electrons that are forced away into the wooden surface on which they lie. You may be surprised that both paper and wood can conduct electricity: they are very poor conductors but are not good insulators unless they are very, very dry. So the paper becomes positively charged when it loses some electrons and then, bingo! – the unlike charges on the comb and the tiny piece of paper attract. To confirm this fact, and to show that like charges repel, we can use charged insulating rods as shown in Figure 4.4.

There is an electric field in the region around a charged rod. If an object carrying a small electric charge Q feels a force F when placed close to the charged rod, we say the **electric field strength** produced there by the rod has a size $\frac{F}{Q}$.

If a piece of paper (see Figure 4.1b) was lifted by an electric force of 1.5×10^{-5} N and carried a charge of 5.0×10^{-8} C, then it must have been in an electric field of 300 N C^{-1}. Notice that a charge of 5.0×10^{-8} C means that (since the electronic charge $e = -1.6 \times 10^{-19}$ C) about 3×10^{11} electrons moved off the tiny piece of paper!

Electric field strength E is a **vector** quantity. The direction of E is the same as the direction of the electric force F, which is defined as the force on a *positive* charge.

Example

A proton of charge $+1.6 \times 10^{-19}$ C is moving in an electric field of strength $500\,\text{N C}^{-1}$.

a) What electric force acts on it?

b) How does this force compare with the weight of a proton (proton mass = 1.7×10^{-27} kg)?

Answer

a) Rearranging $E = \frac{F}{Q}$ to find the force, we get:

$$F = EQ = 500\,\text{N C}^{-1} \times (1.6 \times 10^{-19}\,\text{C}) = 8.0 \times 10^{-17}\,\text{N}$$

b) The weight of the proton = mg

$$= 1.7 \times 10^{-27}\,\text{kg} \times 9.8\,\text{N kg}^{-1}$$
$$= 1.7 \times 10^{-26}\,\text{N}$$

So the electric force here is about 5×10^{9} times bigger (nearly 5 billion times bigger) than the gravitational force. You can always ignore gravitational forces when dealing with electric forces on charged elementary particles.

Key term

$$E = \frac{\text{electric force } F \text{ on a small charge}}{\text{small charge } Q}$$

$$E = \frac{F}{Q} = \text{force per unit charge}$$

The unit for **electric field strength** is the newton per coulomb, N C^{-1}.

4.3 Uniform electric fields

Two oppositely charged plates placed as shown in Figure 4.5 produce a **uniform** electric field between them. In a uniform electric field, the electric lines of force (red in this book) are equally spaced. (At the edges this is not quite true, but we will usually deal with the region of the field in which it *is* true.) When you move a small charged object around in a uniform electric field, the force on it remains constant. Because $E = \frac{F}{Q}$, this means that the value of E is the same everywhere.

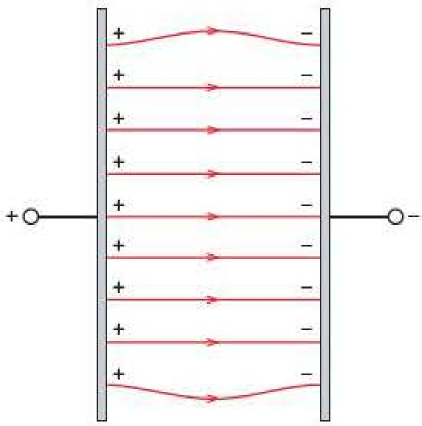


Figure 4.5 A uniform electric field

Activity 4.1

Demonstrating a uniform electric field

To demonstrate that the electric field between two oppositely charged plates is constant, the following procedure can be used:

- Cut a test strip of aluminium foil, about 20 mm by 5 mm.
- Attach it to the bottom of an uncharged insulating rod or plastic ruler.
- Hold the ruler vertical and lower the end with its foil into the space between the two charged plates.
- Move the ruler so that the aluminium foil is at different places between the charged plates.

If the field is uniform, the force on the foil will be constant and the aluminium foil will hang at the same angle to the vertical wherever it is placed in the field. If the charges on the plates are reversed, the test strip swings to the same angle the other way.

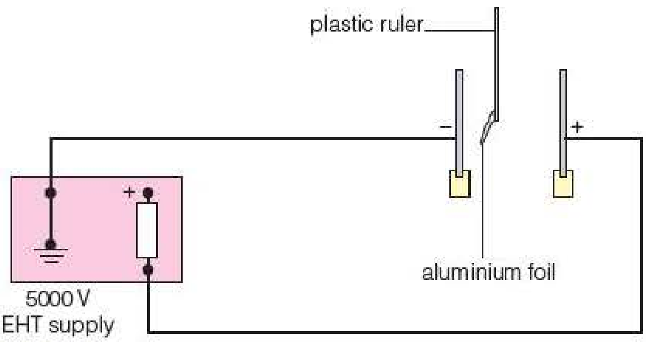


Figure 4.6 For safety the EHT supply should be strictly limited to provide less than 5 mA

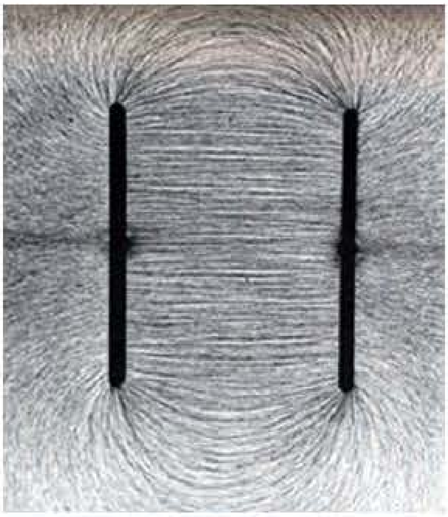


Figure 4.7 Using short threads to show the shape of electric fields

A more direct way of showing the uniform electric field is shown in Figure 4.7. In this photograph a potential difference has been applied to two metal plates that dip into a thin layer of insulating liquid. Lots of short pieces of fine thread are then sprinkled onto the liquid, and they line up end-to-end to show the shape of the field, rather as iron filings do in magnetic fields.

In a uniform electric field, the size of the electric field strength E can also be expressed as

$$E = \frac{V}{d}$$

where V is the (electric) potential difference between the oppositely charged parallel plates or surfaces producing the electric field and d is the separation of the surfaces. V is a scalar quantity so this equation gives only the size of the E vector. The unit for E from this equation will be $\mathrm{V\,m^{-1}}$, apparently different from $\mathrm{N\,C^{-1}}$. However, the two units are equivalent; to show this, starting with a volt:

$$\mathrm{V \equiv J\,C^{-1} \equiv N\,m\,C^{-1}}$$

and dividing both the V and $\mathrm{N\,m\,C^{-1}}$ by the metre tells you that $\mathrm{V\,m^{-1} \equiv N\,C^{-1}}$.

If the plates in Figure 4.7 were 30 mm apart and had a potential difference (p.d. or voltage) of 12 V across them, the strength of the electric field between them would be $\frac{12\,\mathrm{V}}{0.030\,\mathrm{m}} = 400\,\mathrm{V\,m^{-1}}$.

There seem to be two ways of charging objects and so two ways of producing electric fields. One is to use the frictional contact between two bodies, a nylon shirt and a woollen jumper or a Perspex ruler and a cloth; the other way is to use a simple battery or a d.c. supply. In fact, both involve the movement of electric charge, and it is electric charge that generates the electric field.

Tip

If you see the demonstration shown in Figure 4.7 it is worth asking for the charged plates to be replaced by point charges.

Activity 4.2

Investigating the electric field between two charged plates

Two conducting plates are placed parallel to one another in a shallow solution of copper sulphate (Figure 4.8a). A metal probe is connected to the negative terminal of the supply via a digital (very high resistance) voltmeter. The plates are connected to a 12 V d.c. supply via a switch S. When the switch is closed there is a uniform electric field in the liquid between the plates.

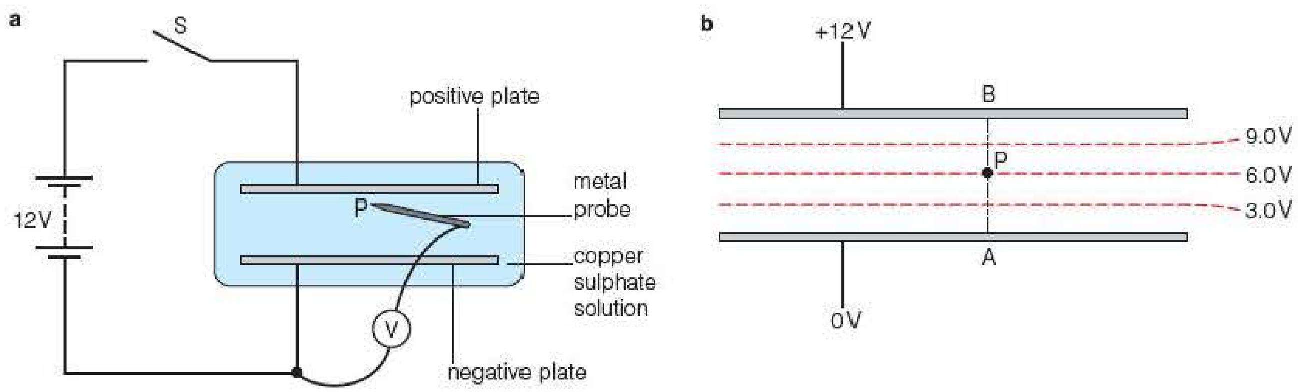


Figure 4.8

When the probe is dipped into the liquid at P, the voltmeter registers the p.d. between the negative plate A and the point in the liquid where the probe is placed P. By dipping the probe at different places in the liquid, a 'map' of p.d. such as that in Figure 4.8b can be produced.

A careful set of voltmeter readings taken along the line AB in the above Activity, recording both the voltmeter reading and the distance AP, might produce the results shown in Table 4.1.

Table 4.1

Distance between A and P x/cm	2.5	5.0	7.5	10.0	12.5
Voltage between A and P V/V	1.9	4.1	5.8	8.0	10.2

Questions

1 What safety precaution should be taken in collecting readings for this Activity?

2 With AB = 15.0 cm and V_{AB} = 12.0 V, plot a graph of V on the y-axis against x on the x-axis.

3 Determine the gradient of your graph.

4 State the size of the electric field between the plates.

In the above Activity, the electric field is perpendicular to the lines along which the voltmeter readings are constant. These lines (as shown in Figure 4.8b) are called **equipotential lines**. The work done in moving a charged object *along* an equipotential line is zero. The work done ΔW in moving an object of charge Q *between* two equipotential lines is ΔW where

$$\Delta W = Q\Delta V$$

and ΔV is the difference in potential or voltage between the equipotential lines.

Tip

Don't forget that fields are *vector* quantities and so are forces, so their direction must be stated.

There is an electric field near to the Earth's surface. The upper atmosphere carries a permanent positive charge and the Earth's surface a permanent negative charge. This electric field is not a constant size day-by-day, but on average, is about $120\,\text{N}\,\text{C}^{-1}$. The field is directed downwards towards the Earth's surface and can on a local (small) scale be considered uniform. In thunderstorms the field builds up to much higher values, and lightning can result.

Figure 4.9 A sudden lightning discharge

Test yourself

1 Write down the mass and charge of **a)** the electron and **b)** the proton.

2 Name three types of 'Field' that you meet in Physics.

3 The Earth's electric field is on average $120\,\text{N}\,\text{C}^{-1}$. Compare the forces on an electron in the Earth's gravitational and electric fields near the Earth's surface.

4 An EHT supply produces a voltage of 5000 V. What resistor in series with the output of the EHT (usually connected internally) will limit the current to 5 mA?

5 What unit is a newton-metre per ampere-second (N m/A s) equivalent to? Show the steps in your answer.

6 During a lightning strike, the Earth's electric field might be $3000\,\text{N}\,\text{C}^{-1}$. Calculate the p.d. between two points on the lightning strike that are 20 m apart.

7 A charge of 3.6 mC is moved in a uniform electric field **a)** 40 m parallel to the field lines, and **b)** 40 m perpendicular to the field lines. Calculate the work done in each case if the strength of the electric field is $300\,\text{N}\,\text{C}^{-1}$.

8 State two different methods of separating electric charges.

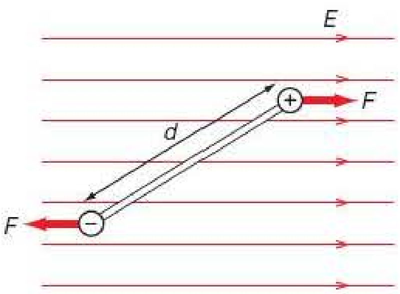


Figure 4.10 An electric dipole in a uniform field

4.4 Making use of electrostatics

An electric dipole consists of two equal but opposite charges $\pm Q$ separated by a distance d. When placed in a uniform electric field E as in Figure 4.10, the two charges experience equal-sized but oppositely directed forces F as shown.

The resultant force on the dipole is therefore zero, but the two forces will exert a twisting action on the dipole, which will try to swing it round until it lies along the electric field. This twisting effect lies at the heart of two widespread modern technologies: the microwave oven and liquid crystal displays (LCDs).

A **microwave oven** operates by generating an electric field that reverses in direction several *billion* times per second. The water molecules in any food or drink placed in the microwave oven are all tiny electric dipoles. (The product of the separate charges on a water molecule and the effective distance between the charges is called the 'dipole moment' of the water molecule and is about 6×10^{-20} C m.) These water molecules respond to this changing electric field by trying to align themselves with it, but the field is reversing so rapidly (usually at about 2500×10^6 Hz) that the water molecules jostle against one another. The energy gained by these molecules from the field is dissipated as internal energy, thus heating the surrounding food material. Have you noticed that food without liquid water in it does not defrost easily when placed in a microwave? The reason is that the 'dipole moment' of other molecules, including ice molecules, is much less than that of water molecules.

You probably use an item containing an **LCD** (**liquid crystal display**) every day. They are all around us – in laptop computers, digital watches, flat TV screens, mobile phones etc. Many electronic devices display alphanumeric information using liquid crystals. How do they work? In a normal liquid such as water, the dipole-like molecules are randomly orientated. Those in a liquid crystal tend to line up. An external electric field can rotate this line-up of liquid crystal dipoles. Figure 4.11 shows how this effect is exploited by influencing the behaviour of the seven small segments used in, for example, alphanumeric displays.

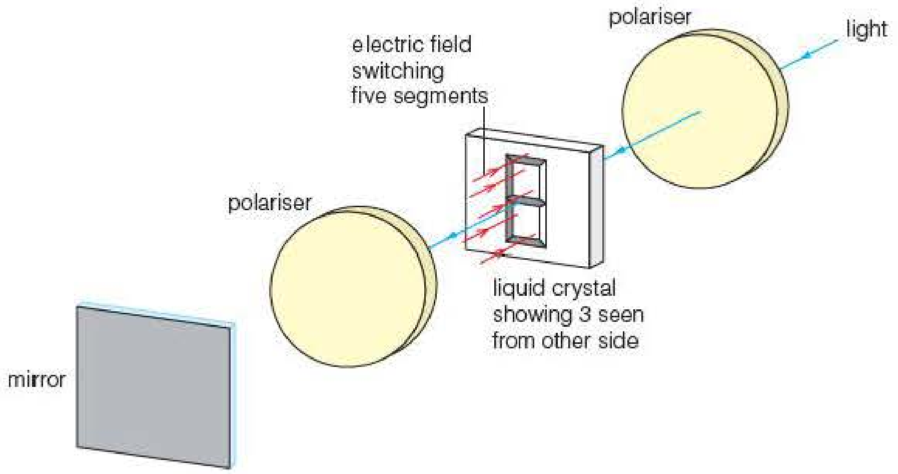


Figure 4.11 How a liquid crystal display works

An electric field can be switched on or off for each segment. This alters the alignment of the dipole-like molecules in a liquid crystal and changes the response of that segment to polarised light. (Polarisation is covered in Section 15.5 of Book 1.) When the electric field is on, the effect is to make the segment black, and we see the number or letter resulting from the pattern of black segments. Liquid crystal displays like this consume very little power, so your calculator can operate quite easily from the power generated by a small solar cell. As the technology has developed, laptops and flat TV displays have used liquid crystals; these can re-form at nearly 100 Hz, displaying moving coloured pictures, but the details are beyond A level.

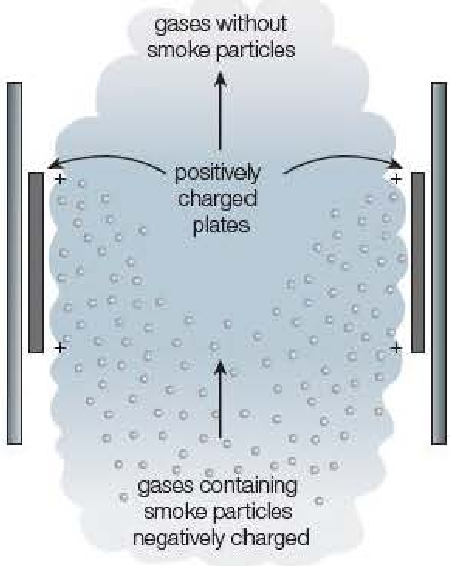


Figure 4.12 An electrostatic precipitator

Electric fields are also used in **electrostatic precipitators**. These devices can remove dust or smoke particles from chimneys. Figure 4.12 illustrates the principle of how they work. Once the smoke or dust particles have been given a negative charge as they pass through a metal grill (not shown), they can be attracted to positively charged plates sited further up the chimney.

After a time, these plates are struck by hammers (you can sometimes hear them) and the layers of smoke or dust fall down the chimney where they can be collected. (You would be correct in thinking that this technology could be used – ought to be used? – to remove the polluting smoke particles that emerge from coal-fired power stations.)

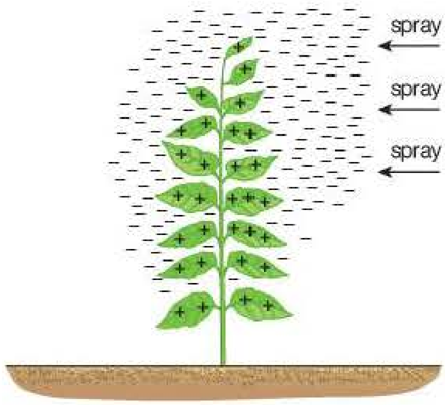


Figure 4.13 Crop spraying

Another use of electric fields is in **crop spraying** with fertiliser or insecticides. The liquid to be sprayed is charged as it exits the nozzle of its container and the sprayed droplets spread out – like charges repel. The spray, perhaps negatively charged, induces an opposite positive charge on the plant or plants to be sprayed and – as unlike charges attract – all parts of the plant(s) are covered by the spray, see Figure 4.13. This technique is widely used in spraying vines all over Europe and is also used to spray-paint metal objects such as car body components.

4.5 Radial electric fields

We have seen that the electric field between two parallel charged plates is a uniform field; what shape of electric field does a small 'point' charge like a single proton produce?

An isolated positive charge P produces an electric field like that shown in Figure 4.14. The radial lines of force show the direction of the field around P. If the charged particle at the centre of the field were negatively charged, the

Tip

Of course, the electric field lines spread out in three dimensions from the charge, Figure 4.14 only shows the electric field lines in two dimensions.

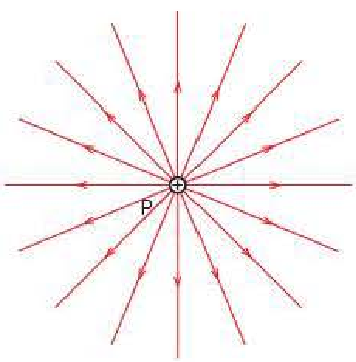


Figure 4.14 A radial electric field

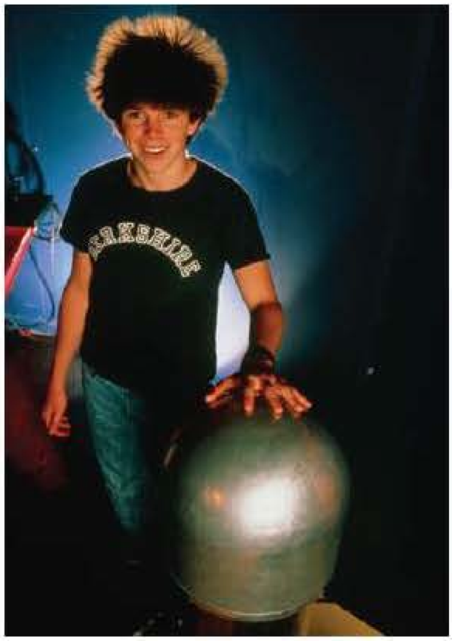


Figure 4.15 It makes your hair stand on end!

Tip

The use of brackets around very large or very small numbers often helps to prevent you forgetting that, as here, you are squaring one of them.

figure would be the same *except* that the arrows on the field lines would all point inwards.

Equipotential lines in this case will be drawn as circles, but in three dimensions they will be spherical surfaces.

A charged sphere, or an almost-spherical charged human head, can also produce a field that is radial. Figure 4.15 confirms this as the boy's hair shows clearly the direction of the electric field around his head.

Mathematically, the size of the electric field E a distance r from a point charge Q is:

$$E = \frac{kQ}{r^2} \quad \text{or} \quad E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

The constant k or $\frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$.

Example

Calculate the size of the electric field 1.0×10^{-10} m from the single proton at the nucleus of a hydrogen atom. (The electron in a neutral hydrogen atom is found on average at this distance from the central proton.)

Answer

The proton charge is $+1.6 \times 10^{-19}$ C, so

$$E = \frac{(8.99 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}) \times (1.6 \times 10^{-19}\,\mathrm{C})}{(1.0 \times 10^{-10}\,\mathrm{m})^2} = 1.44 \times 10^{11}\,\mathrm{N\,C^{-1}}$$

Test yourself

9 What is the net force on an electric dipole placed in a uniform electric field? Explain your answer.

10 What is the dipole moment of the electric dipole shown in Figure 4.10?

11 Why, during crop spraying, does the spray also stick to the *underside* of the leaves?

12 Explain why a rubber balloon can be 'stuck' to a wall after being rubbed on your nylon shirt.

13 Show that the unit of the constant k in the formula $E = \frac{kQ}{r^2}$ is correct.

14 A charged oil drop of mass 8.0×10^{-13} g is found to remain stationary in the space between two horizontal metal plates when there is a p.d. between the plates of 200V. If the vertical distance between the plates is 8.0 mm and the upper plate is positive, calculate the charge on the oil drop and comment on your answer.

4.6 Coulomb's law

The basic law describing the size of the force F between two point charges Q_1 and Q_2 is an **inverse-square law** that depends on the distance r between the charges and the size and sign of the charges Q_1 and Q_2:

$$F = \frac{kQ_1Q_2}{r^2}$$

with the constant k as above. The constant k is often written as $\frac{1}{4\pi\varepsilon_0}$ which makes Coulomb's law

$$F = \frac{Q_1Q_2}{4\pi\varepsilon_0 r^2}$$

The law is named after (Charles) Coulomb who first discovered it, just as the law of gravitation is named after (Isaac) Newton. (See Section 4.8 for a table that compares relationships between the two inverse square laws found in physics.)

You can see that if Q_1 is an isolated charge and Q_2 is a small charge placed near it, then

$$\frac{F}{Q_2} = \frac{kQ_1}{r^2} \quad \text{or} \quad E = \frac{kQ_1}{r^2}$$

That is, the way a radial electric field is described depends on Coulomb's force law between two point charges. The radial field $E = \frac{kQ}{r^2}$ can also be described as $E = \frac{Q}{4\pi\varepsilon_0 r^2}$.

Activity 4.3

Testing Coulomb's inverse-square law for the force between two charges

The forces are going to be very small, so a sensitive measuring device is required to register changes in the force as the distance between the charges is changed. Here an electronic top-pan balance that registers changes in mass to 0.01 g (vertical force changes of 0.0001 N) is used.

Figure 4.16 shows how the top-pan balance is used. Two conducting spheres, shown in blue, are charged, e.g. by flicking the negative lower sphere with a woollen cloth and using the positive terminal of a d.c. supply set at about 30 V to charge the upper sphere. Note that both the charged spheres are insulated so that they do not lose their charge during the Activity.

Readings on the top-pan balance should now be taken for different distances y between the centres of the spheres. One way of recording how this distance changes without touching the charged spheres is to project a shadow of them onto a nearby screen. A set of results like those in the Example on the next page can be collected.

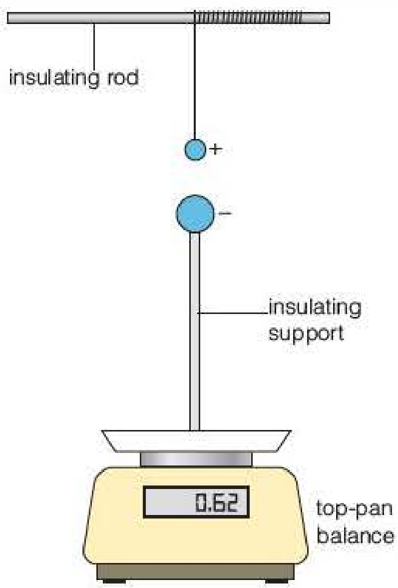


Figure 4.16 Testing Coulomb's law

Example

In an Activity to test Coulomb's law using apparatus like that shown in Figure 4.16, the following data shown in Table 4.2 were produced: *m* is the reading on the electronic balance and *y* is the distance between the centres of the charged spheres.

Table 4.2

m/g	0.11	0.38	0.62	0.83	1.14
y/cm	12.0	6.4	5.0	4.3	3.7

a) Show, by a non-graphical method, that *m* is approximately proportional to $1/y^2$.

b) Suggest difficulties that might arise in this experiment.

- In a) notice the requirement for a method that does *not* involve drawing a graph.
- The 'suggest' in b) means that there is no single correct answer.

Answer

a) If $m \propto 1/y^2$, then the product my^2 should be constant.

Taking values from the table gives the following values of my^2/g cm^2:

16, 16, 16, 15 and 16, each to 2 SF

∴ The product is approximately constant and hence *m* is proportional to $1/y^2$.

b) One difficulty is that some of the charge on one or both of the spheres may leak away during the Activity. Another difficulty is the difficulty in measuring the distance *y* between the centres of the charged spheres.

Tip

In answering questions about the precision of measurements taken during Activities, do *not* try to quantify the *accuracy* of any measuring instruments. You have to assume they are accurate. But, of course, your measurements may not be *precise*.

Tip

When commenting it is often a good idea to be quantitative, i.e. to offer comments that include numbers or even give calculated values.

Example

Two point charges, of +30 nC and −30 nC, form an electric dipole of length 0.25 m.

a) Calculate the size of the force between the charges. Comment on its size.

b) Calculate the electric field strength at the midpoint between the charges.

c) Sketch the shape of the electric field in the region of the dipole.

Answer

a) Substituting in $F = kQ_1Q_2/r^2$:

$$\Rightarrow F = \frac{(9.0 \times 10^9\,\text{N m}^2\,\text{C}^2) \times (30 \times 10^{-9}\,\text{C}) \times (30 \times 10^{-9}\,\text{C})}{(0.25\,\text{m})^2}$$

$$= 1.3 \times 10^{-4}\,\text{N attracting}$$

This is a very small force; it would support a mass of only

$$1.3 \times 10^{-4}\,\text{N}/9.8\,\text{N kg}^{-1} = 1.3 \times 10^{-5}\,\text{kg or } 0.013\,\text{g}.$$

b) Each charge will produce an equal-sized electric field at the midpoint. Both fields act from the positive towards the negative charge.

$$\text{So total } E = \frac{2 \times (9.0 \times 10^9\,\text{N m}^2\text{C}^{-2}) \times (30 \times 10^{-9}\,\text{C})}{(0.25\,\text{m})^2}$$

$$= 8600\,\text{N C}^{-1}$$

c)

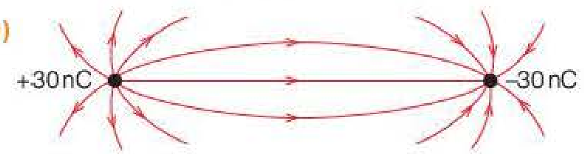


Figure 4.17

4.7 Electric field and potential

We have seen that in a *uniform*, i.e. constant, electric field the relationship between the electric field E and the electric potential difference V is

$$E = \frac{V}{d}$$

Another way of writing this is

$$E = \frac{\Delta V}{\Delta r} \quad \text{or} \quad \Delta V = E\Delta r$$

where ΔV is the difference in potential between two points in the field that are a distance r apart.

In a uniform field E is constant and so $\Delta V/\Delta r$ is constant.

As Δr becomes smaller and smaller, we can write

$$E = -\frac{dV}{dr}$$

The minus sign means that the direction of the electric field is in the opposite sense to the potential difference. In Figure 4.5, the field is from left to right whilst the potential increases from right to left. We can therefore say that the electric field strength is equal to the (negative) potential gradient at any point in the field – see Activity 4.2.

Tip

Don't forget to put arrows on field lines to indicate the *direction* of the field.

What happens in radial electric fields? How far apart would the circles (in two dimensions) be for equal electric potential differences in radial fields? What we need to know is how V varies with r in a field where $E \propto r^{-2}$. The answer is that

$$V \propto \frac{1}{r}$$

Put more fully, for a field $E = \frac{kQ}{r^2}$

$$V = \frac{kQ}{r} \quad \text{or} \quad V = \frac{Q}{4\pi\varepsilon_0 r}$$

This is shown in Figure 4.18 below.

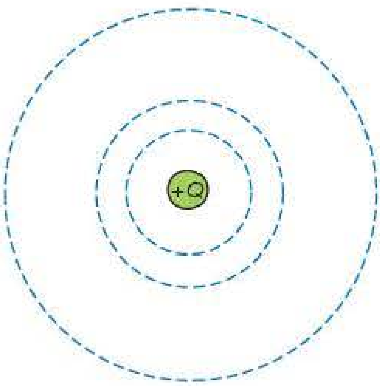


Figure 4.18 Equipotentials around a charged sphere

Tip

Suggest and discuss are both 'difficult' key words in questions like this.

Example

Figure 4.19 shows a graph that relates V to r in an inverse square law electric field.

a) Suggest two methods of showing that V is inversely proportional to r in this graph.

b) Explain which of your methods produces results more quickly.

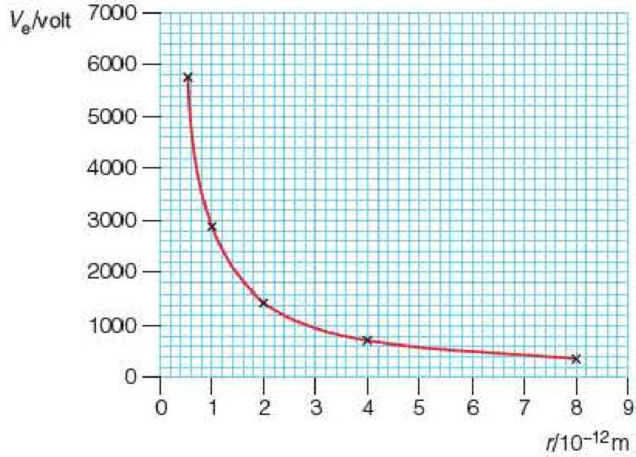


Figure 4.19

c) Show that this graph fits an inverse relationship between V and r.

d) Discuss which of the methods you suggest in **a)** is the more convincing.

Answer

a) Method 1: Make a table of values of V and $1/r$ and draw a graph of V against $1/r$. Method 2: Make a list of values of values of the product $V \times r$.

b) Drawing a graph is laborious whereas a calculator can produce a list of value of Vr quite quickly.

c) The numerical values of Vr are: 5800 × 0.5, 2900 × 1.0, 1500 × 2.0, 700 × 4.0, and 350 × 8.0. These come to: 2900, 2900, 3000, 2800 and 2800.

As these numbers are effectively constant (the average is 2900 to 2 SF), then Vr is constant and hence $V \propto 1/r$.

d) After plotting the graph of V and $1/r$, drawing the best straight line through the points is an averaging process which will lead to a slightly more convincing result that averaging calculated values of Vr.

A mathematical digression

The link between $V = \frac{kQ}{r}$ and $E = \frac{kQ}{r^2}$ involves integration. See also Section 3.4.

We saw above that by definition

$$\Delta V = E\Delta r = \frac{kQ}{r^2}\Delta r \left(\text{or} \frac{Q}{4\pi\varepsilon_0 r^2}\Delta r\right)$$

Integrating this relationship in the form

$$\int_{V_1}^{V_2} \mathrm{d}V = \int_{r_1}^{r_2} \frac{kQ}{r^2}\mathrm{d}r$$

$$V_2 - V_1 = kQ\left[-\frac{1}{r}\right]_{r_1}^{r_2}$$

Tip

You will not be asked to produce this mathematics in examinations, but you must know how to use the relationships for electric field and potential when the field follows an inverse square law.

Hence the difference in the electrical potential between distances r_1 and r_2 from the centre of a positive charge, Q, is given by

$$\Delta V = kQ\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

The zero of electric potential energy is taken to be at infinity. Thus when one moves from a distance *r* from a point positive electric charge to infinity, the *change* in electric potential is $\frac{kQ}{r}$ or $\frac{Q}{4\pi\varepsilon_0 r}$.

So far as the Earth's electric field is concerned, as the Earth has a *negative* charge, we have that $V = -\frac{kQ}{r}$ or $-\frac{Q}{4\pi\varepsilon_0 r}$.

Of course, what is correct for electric fields is also true for gravitational fields. For a gravitational field we showed that $V = -\frac{Gm}{r}$. (This mathematics – see Chapter 6 – *is not relevant for magnetic fields,* where the field does *not* follow an inverse square law.)

4.8 Comparing gravitational and electric fields

The formula, diagrams and graphs of this chapter show remarkable similarities with those of Chapter 3. Table 4.3 summarises the similarities in this **analogy between** *g* **and** *E* **phenomena**.

Table 4.3

	Gravitational effects	Electrostatic effects
Field strength	$g = \frac{F_g}{m}$ unit $N\,kg^{-1}$ or $m\,s^{-2}$	$E = \frac{F_e}{Q}$ unit NC^{-1} or Vm^{-1}
Potential difference in a uniform field	$g\Delta h$, that is $\frac{\Delta(GPE)}{m}$ unit $J\,kg^{-1}$ (no name)	$E\Delta x$, that is $\frac{\Delta(EPE)}{Q}$ unit JC^{-1} or V
Energy conservation in a uniform field	$\Delta(\frac{1}{2}mv^2) = mg\Delta h$	$\Delta(\frac{1}{2}mv^2) = Q\Delta V$
Force laws	Newton: $F = \frac{Gm_1m_2}{r^2}$ $G = 6.67 \times 10^{-11}\,Nm^2kg^{-2}$	Coulomb: $F = \frac{kQ_1Q_2}{r^2}$ k or $\frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9\,Nm^2C^{-2}$
Radial fields	$g = \frac{Gm}{r^2}$	$E = \frac{kQ}{r^2}$ or $\frac{Q}{4\pi\varepsilon_0 r^2}$
Potential difference in a radial field	$-Gm\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$	$kQ\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$
Potential in the Earth's gravitational field or a point electric charge	$V_{grav} = \frac{-Gm_E}{r}$	$V = \frac{kQ}{r}$ or $\frac{Q}{4\pi\varepsilon_0 r}$
Graphs	inverse square, $g \propto \frac{1}{r^2}$ so gr^2 is constant	inverse square, $E \propto \frac{1}{r^2}$ so Er^2 is constant

There are, of course, differences between the two phenomena. One obvious one is that gravity is about masses but electricity is about charges. Other **differences** include:

- Gravitational forces affect all particles with mass, but electrostatic forces affect only particles that carry charge.
- Gravitational forces are always attractive but electrostatic forces can be either attractive or repulsive.
- It is not possible to shield a mass from a gravitational field but it is possible to shield a charge from an electrostatic field.

Example

Two protons in a helium nucleus are each of mass m_p and charge $+e$. Their centres are a distance d apart. The electrical force F_e between them is pushing them apart; the gravitational force F_g between them is pulling them together.

a) Show that the ratio $\frac{F_e}{F_g}$ does not depend on the distance d.

b) Look up values of the relevant physical quantities (see Table 4.3) and show that the ratio $\frac{F_e}{F_g}$ is about 10^{36}.

c) Comment on your answer to part b).

Tip

Both Newton's and Coulomb's law are required here.

Answer

a) $F_e = \frac{kee}{d^2}$ or $\frac{ee}{4\pi\varepsilon_0 d^2}$ and $F_g = \frac{Gm_p m_p}{d^2}$

$\therefore \frac{F_e}{F_g} = \frac{kee}{Gm_p m_p}$ as the two d^2 cancel.

b) $\frac{F_e}{F_g} = \frac{(8.99 \times 10^9\,\text{N m}^2\,\text{C}^{-2}) \times (1.60 \times 10^{-19}\,\text{C})^2}{(6.67 \times 10^{-11}\,\text{N m}^2\,\text{kg}^{-2}) \times (1.67 \times 10^{-27}\,\text{kg})^2}$

$= 1.24 \times 10^{36} \approx 1 \times 10^{36}$

c) The repulsive electric forces are *enormous* compared to the attractive gravitational forces, so there must be some other force holding the protons together. See Chapter 8.

Test yourself

15 a) Express the units of the constant k in base SI units, i.e. in kg, m, s and A.

b) What are the units of ε_0 in base SI units ?

16 Two raindrops falling vertically side by side, 10 mm apart, carry electric charges of +8.0 pC and +12 pC. What is the repulsive force between them?

17 Explain why a piece of dry toast does not warm up noticeably in a microwave oven.

18 What is the electric potential at distance of 1.0×10^{-10} m from a single proton?

19 Calculate the energy needed to remove an electron that is 1.0×10^{-10} m from a proton to infinity, i.e. a long way from the proton.

Exam practice questions

1 Which of the following is not a vector quantity?

A Electric field strength **C** Gravitational field strength

B Electric charge **D** Gravitational force **[Total 1 mark]**

2 The electric field between two charged plates is known to be $650\,\mathrm{N\,C^{-1}}$.

If the distance between the plates is 12 cm, the potential difference between the plates must be:

A 54V **C** 4500V

B 78V **D** 7800V **[Total 1 mark]**

3 The field in Figure 4.20:

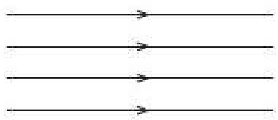


Figure 4.20

A could only be a gravitational field

B could only be a magnetic field

C could only be an electric field

D could be any one of gravitational, magnetic or electric fields.

[Total 1 mark]

4 Write out the full meaning of

a) $F = mg$ **[3]**

b) $F = QE$. **[2]**

[Total 5 marks]

5 a) What do both $\frac{F}{Q}$ and $\frac{V}{d}$ represent? **[1]**

b) Show that their units are equivalent. **[3]**

c) Air 'breaks down' when there is an electric field of more than $3.0 \times 10^6\,\mathrm{N\,C^{-1}}$. Express this electric field in $\mathrm{V\,mm^{-1}}$. **[2]**

[Total 6 marks]

6 In the hydrogen atom, the average distance apart of the proton and the electron is 5.3×10^{-11} m. Calculate the electric force between them. Take the constant k to be $9.0 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$. **[Total 4 marks]**

7 What is the acceleration of a singly charged nitrogen molecule of mass 4.0×10^{-26} kg in an electric field of $360\,\mathrm{N\,C^{-1}}$? **[Total 4 marks]**

8 A negatively charged rod is lowered carefully into a cylindrical metal can without touching it. The can is standing on the ground, a conducting surface. Explain how the can becomes charged and sketch the electric field between the rod and the can. **[Total 5 marks]**

9 a) Show that the graph in Figure 4.21 of E against r in the region of an isolated proton follows an inverse-square law. **[5]**

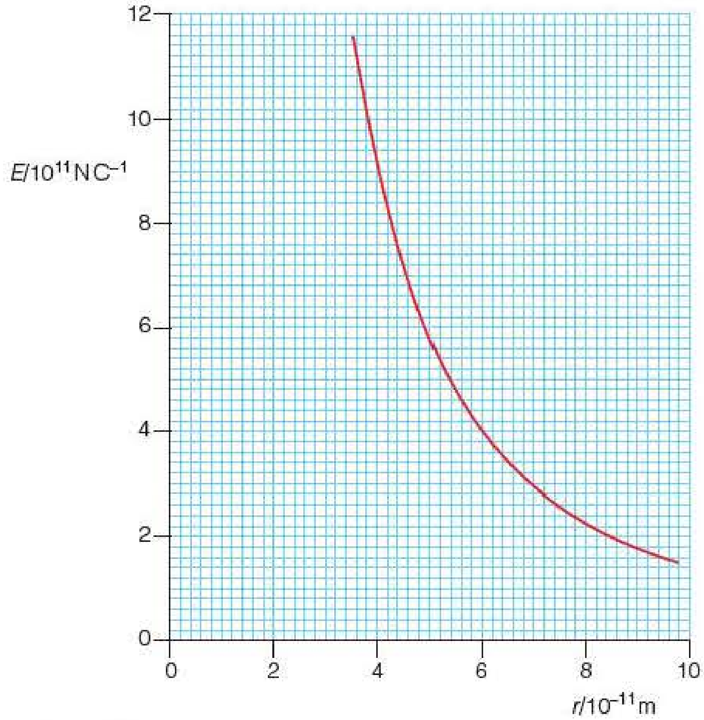


Figure 4.21

b) Calculate a value for the charge producing this field. Take the value of the electric field constant k to be $9.0 \times 10^9\,N\,m^2\,C^{-2}$. **[4]**

[Total 9 marks]

10 a) What is meant by 'ionisation'? **[2]**

b) In the spark plug of a petrol-driven car there are two electrodes separated by a gap of about 0.67 mm. What p.d. must be applied across the electrodes to cause a spark in air? Air begins to ionise when the electric field is about $3 \times 10^6\,V\,m^{-1}$. **[4]**

[Total 6 marks]

11 A speck of dust of mass $2.0 \times 10^{-18}\,g$ carries a negative charge equal to that of one electron. The dust particle is above a flat desert where the Earth's electric field is $150\,N\,C^{-1}$ and acts downwards. Ignoring any forces on the dust particle resulting from air movements, calculate the resultant vertical force on the dust particle. **[Total 5 marks]**

12 A proton of mass $1.7 \times 10^{-27}\,kg$ is moving to the right at a speed of $4.2 \times 10^5\,m\,s^{-1}$. It enters an electric field of $45\,kN\,C^{-1}$ pointing to the left. How far will the proton travel before it comes momentarily to rest?

[Total 6 marks]

13 In one type of ink-jet printer a tiny ink droplet of mass m carrying charge $-Q$ is steered to its place on the paper by a uniform electric field. Figure 4.22 shows the path of this droplet.

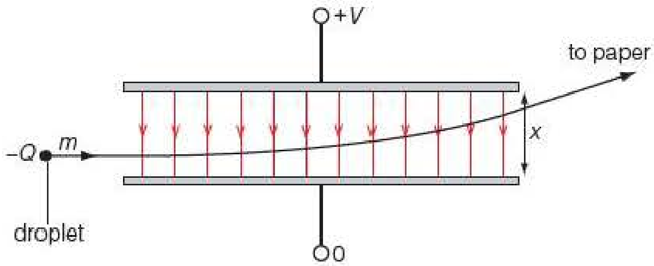


Figure 4.22

a) Give an expression for the acceleration of the droplet caused by this electric field. **[4]**

b) Describe the path of an uncharged droplet in this printer. **[2]**

c) Suggest how different droplets can be steered to different places on the paper. **[3]**

[Total 9 marks]

14 A tiny oil drop of mass 4.0×10^{-15} kg is observed to remain stationary in the space between two horizontal plates when the potential difference between the plates is 490 V and their separation is 8.0 mm.

a) What is the strength of the electric field between the plates? **[2]**

b) Draw a free-body force diagram of the drop showing the gravitational and electric forces acting on it. **[2]**

c) Calculate the size of the forces and, by equating them, deduce the charge Q on the drop. **[3]**

d) Comment on your value for Q. **[2]**

[Total 9 marks]

15 In an attempt to demonstrate the inverse square law relationship between E and r in the region around a point charge, a teacher produced a table of values of E and r. The teacher then:

a) drew a graph of E against $\frac{1}{r^2}$, and

b) made a list of values of values of the product Er^2. Discuss which method is the more reliable in showing that $E \propto \frac{1}{r^2}$.

[Total 4 marks]

16 Two charges, of 40 μC and Q, are placed as shown in Figure 4.23. A third charge placed at X experiences no net force. Show that Q = 10 μC.

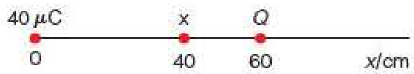


Figure 4.23

[Total 5 marks]

Stretch and challenge

17 In the early 20th century Robert Millikan, a distinguished American physicist, measured the electric charge on many tiny oil drops. He deduced that electric charge was quantised.

a) What does 'quantised' mean in this context? **[2]**

b) In fact Millikan ignored the charge he calculated to exist on over 20% of his experiments. Comment on this treatment of his results. **[4]**

[Total 6 marks]

18 Two vertical plates are placed 20 mm apart in a vacuum. A p.d. of 100 V is produced between the plates.

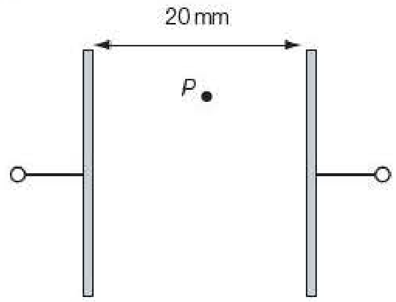


Figure 4.24

A tiny particle of mass 6.0×10^{-15} kg is released at P. The particle carries a charge of 8.0×10^{-18} C.

a) Describe in magnitude and direction the two forces acting on the tiny particle P. **[4]**

b) Deduce the direction in which the tiny particle will move away from P. **[3]**

[Total 7 marks]

19 In 1897 Sir JJ Thomson discovered the electron as a *particle* and was awarded the Nobel Prize for physics in 1906. Some 30 years later his son GP Thomson demonstrated that electrons could be diffracted like a *wave*, for which he was awarded the Nobel Prize for physics in 1937 – it has been said that JJ Thomson got the Nobel Prize for discovering that electrons are particles, and GP Thomson got it for discovering that they aren't!

Comment on these discoveries in terms of the development of our understanding of physics.

[Total 5 marks]

5 Capacitance

Prior knowledge

You should know from earlier Advanced level work or from GCSE:

- that charge is measured in coulombs, C, and that the charge on an electron is -1.60×10^{-19} C
- that current is measured in amperes, A, and that $1\,A \equiv 1\,C\,s^{-1}$
- the difference between a scalar, e.g. mass, and a vector, e.g. weight
- that prefixes such as μ and p mean micro, 10^{-6}, and pico, 10^{-12}, respectively
- that potential difference (p.d.) is often called voltage, and is measured in volts, V
- the equations involving resistance: $R = \frac{V}{I}$ and $V = IR$
- the difference between a series and a parallel circuit
- that in series resistance values add, but in parallel their inverse values add.

Tip

Don't be proud. If you don't recognise something, look it up in the index of this or other books. It may be that you don't remember much about something, and if you do, a bit of revision does you no harm.

Test yourself on prior knowledge

1 How much charge passes a point in a circuit where there is a current of 4.0 mA for 20 s?

2 How many electrons are there in a charge of exactly −1 C?

3 Explain why electric current, I, is a scalar quantity.

4 Complete the sentence: 'A volt is a name for a J____ per C________'.

5 Calculate the voltage across a resistor of 22 kΩ in which there is a current of 0.50 mA.

6 Two resistors, R_1 and R_2, connected in series have a total resistance of $R_1 + R_2$. Write down the equivalent statement if the same two resistors are connected in parallel.

5.1 What are capacitors?

You may not know much about capacitors, yet you probably make use of them quite often without realising it. In this chapter you will learn how capacitors work, how they store charge and how they discharge through a resistor. This discharge is called an 'exponential decay' of charge. Exponential decays are very significant in many real-life situations, as are changes that involve exponential growth, for example in energy use and in population statistics.

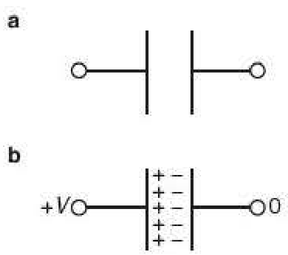


Figure 5.1 a) Circuit symbol for a capacitor, and b) displacement of charge after a p.d. is applied

The electrical circuit symbol for a capacitor is two parallel lines of equal length, which suggests (correctly) that connecting a capacitor in series with other electrical components produces a gap in the circuit.

When you apply a potential difference to the terminals of the capacitor, the sides or plates of the capacitor become electrically charged. There is then an electric field between the plates – see Figure 4.5 on page 55. If the potential

Key term

The **capacitance** of a capacitor is defined as $\frac{Q}{V}$ where $\pm Q$ is the size of the charge on the plates of the capacitor and V is the potential difference between the plates.

Tip

'Queue equals sea vee' may be an easy way to remember this relationship, but make sure you remember what the symbols stand for: charge, capacitance and potential difference.

difference (p.d.) between the plates is V and the charge that this p.d. displaces from one side to the other is Q, then

$$Q \propto V \text{ or } Q = (\text{a constant}) \times V$$

The constant is C is called the **capacitance** C of the capacitor.

$$Q = CV \qquad V = \frac{Q}{C} \qquad C = \frac{Q}{V}$$

For many capacitors the charge displaced or stored for a p.d. of 12 volts is very small, perhaps only a few microcoulomb (μC), so C is often a very small number, of the order of μC per volt. The unit CV^{-1} is called a **farad** (symbol F) – yes after Michael Faraday – so capacitors commonly have capacitances of μF or nF (10^{-9} F) or even pF (10^{-12} F).

Capacitors of more than 1000 μF (1.0 mF) usually have a plus sign marked on one end and a maximum voltage written on them, e.g. 25 V – see Figure 5.8 on page 76. It is very important in any activity to be sure that the + end of such a capacitor is connected to the positive terminal of the supply and that the maximum p.d. is not exceeded.

Example

A potential difference of 30 V displaces 4.1×10^{16} electrons from one plate of a capacitor to the other.

a) How much charge does this represent?

b) Calculate the capacitance of the capacitor.

Answer

a) The charge on an electron is -1.6×10^{-19} C.

$\therefore$ total charge displaced $= (-1.6 \times 10^{-19}\,\text{C}) \times (4.1 \times 10^{16})$

$= -6.6 \times 10^{-3}\,\text{C}$

b) $C = \frac{Q}{V} = 6.6 \times 10^{-3}\,\text{C} \div 30\,\text{V}$

$= 2.2 \times 10^{-4}$ F or 220 μF or 0.22 mF

Activity 5.1

Measuring the capacitance of a capacitor

Measuring C involves measuring both V and Q. The p.d. V is easy – a digital voltmeter does the job. Measuring the charge Q that moves from one plate to the other is more difficult. This charge can be measured using the circuit shown in Figure 5.2, using a combination of a stopclock and a microammeter. If the charging current can be kept constant, a graph of current I against time t will look like Figure 5.3.

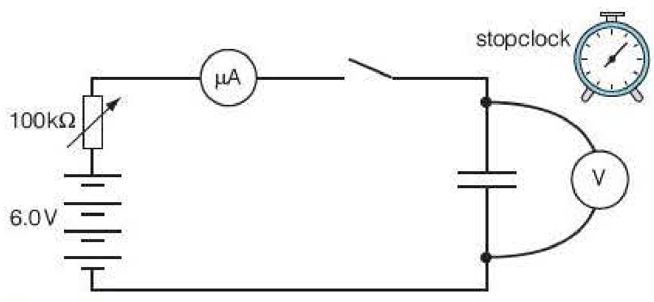


Figure 5.2 Circuit diagram and stopclock

Safety note

If you are using a capacitor with a + sign marked on one end, be sure to connect this to the positive of the cells or battery, and do not exceed the voltage rating of the capacitor.

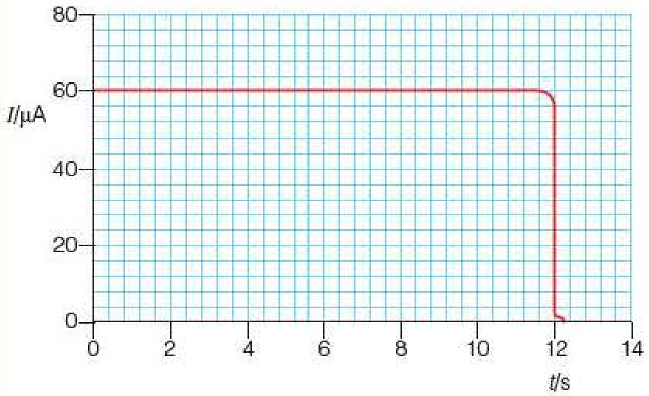


Figure 5.3 A current-time graph

The charge is equal to the area under this current-time graph. Here the current has been constant, so the charge is just the product It. For example, a trickle of 60 μA for 12 s means that 720 μC or 7.2×10^{-4} C of charge has been displaced from one plate to the other.

To keep the current constant, the rheostat should be set at its maximum value of 100 kΩ. As the switch is closed, the clock should be started and the initial reading on the microammeter noted. The current should then be kept at this initial value by gradually reducing the resistance of the rheostat. (The reduction will be slow at first but speeds up as the capacitor becomes charged.) Another way of keeping the current constant would be to use a constant current source, symbol ⦶, in place of the cell and rheostat.

An alternative way of finding the charge on a capacitor is by discharging it through a fixed resistor. This is described on page 84. A coulombmeter is another option, but this can only be used to measure very small charges – see page 77.

5.2 Energy storage by capacitors

Capacitors store energy. The energy transferred, or work done, when a charge ΔQ moves across a potential difference V is given by $\Delta W = V\Delta Q$.

When charging capacitors there is a problem in calculating the energy transferred using this formula because the p.d. changes as the capacitor charges!

Tip

Remember a volt is a joule per coulomb ($1\,\text{V} = 1\,\text{J}\,\text{C}^{-1}$)

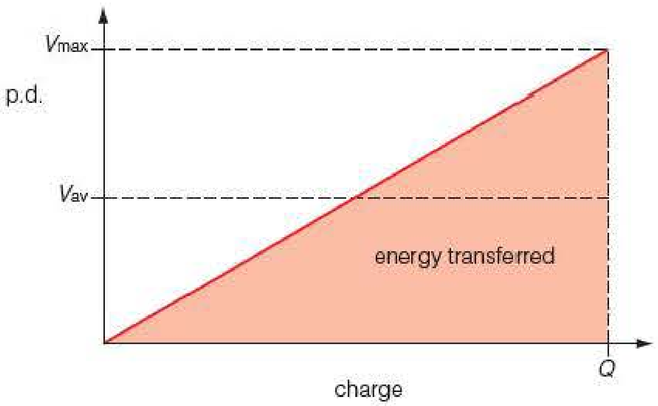


Figure 5.4 A graph of p.d. against charge

Fortunately, V is proportional to Q (as in Figure 5.4), so the *average* p.d. V_{av} is exactly half the final p.d., and we can use $V_{av} = \frac{1}{2}V_{max}$ when calculating the energy transferred.

∴ Energy transferred when charging a capacitor

$$W = V_{av}Q = \tfrac{1}{2}V_{max}Q$$

Tip

Try to remember formulas like $W = V_{av}Q = \frac{1}{2}V_{max}Q$ in words as 'energy transferred equals average potential difference times charge stored' and 'energy stored equals half maximum potential difference times charge stored'. This will help your understanding.

Referring back to Figure 5.4, the energy stored is equal to the area between the graph line and the Q-axis.

Using simply V for the maximum p.d. (it is really V_{max}) then substituting for Q or V using $Q = CV$ gives the energy stored as

$$W = \frac{1}{2}VQ = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$$

The capacitor stores this transferred energy as electric potential energy (*EPE*).

Example

Figure 5.5 shows how the potential difference across a capacitor changes as it is charged.

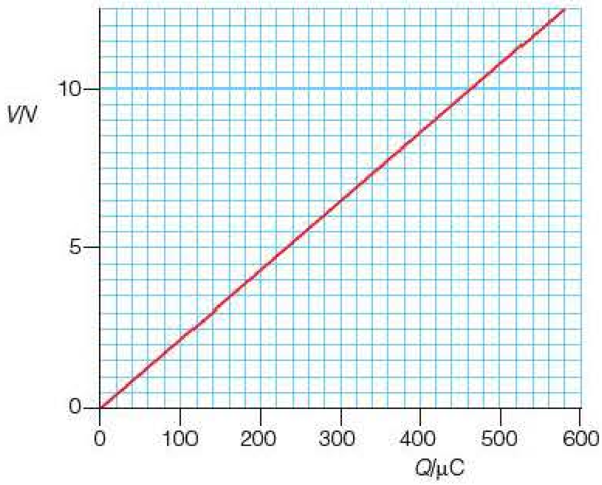


Figure 5.5

a) What is the capacitance of the capacitor?

b) Calculate W, the electrical potential energy (*EPE*) stored in the capacitor when it has the following charges – Q/μC: 100, 200, 300, 400, 500.

c) Sketch a graph of W against Q.

d) What shape would you expect a graph of stored energy against potential difference to have?

Answer

a) $C = \frac{Q}{V} = \frac{300 \times 10^{-6}\,C}{6.5\,V} = 46\,\mu F$

(*Not* the gradient of the graph! It is however, the inverse of the gradient.)

b) Energy stored $W = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$.

For 100 μC the energy stored $= \frac{\frac{1}{2}(100 \times 10^{-6}\,C)^2}{(46 \times 10^{-6}\,F)}$

$= 1.1 \times 10^{-4}\,J$

This and other values are shown in Table 5.1 below.

Table 5.1

Q/μC	100	200	300	400	500
W/mJ	0.11	0.43	0.98	1.74	2.72

c)

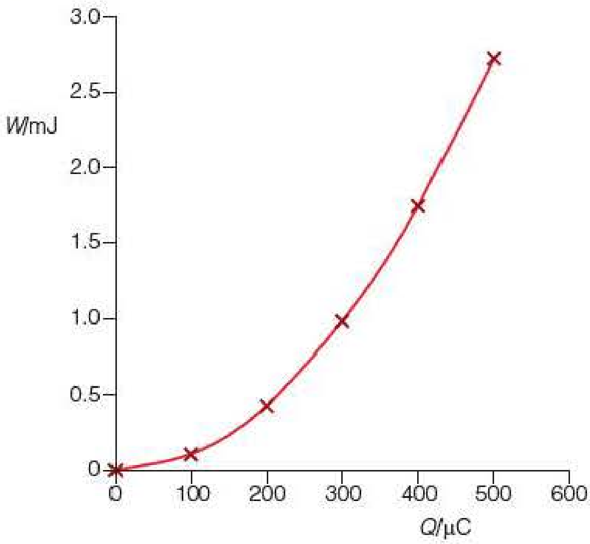


Figure 5.6

d) As W is proportional to Q^2, the graph shows the characteristic shape of a y against x^2 graph. A graph of W against V will show the same shape, as W is also proportional to V^2.

Tip

Don't try to use milli- or micro- *inside* calculations. Always go to standard form.

Activity 5.2

Investigating the efficiency of energy transfer from a capacitor

The electrical potential energy (*EPE*) stored in a 10 000 μF capacitor can be transferred to gravitational potential energy (*GPE*) by discharging the capacitor through a small electric motor. As the capacitor discharges the current in the motor raises a small mass. The set up in Figure 5.7a, with the circuit shown in Figure 5.7b, can be used.

The mass to be lifted could be a small ball of plasticine with a mass *m* of, say, 8.0 g. The cotton is wound round the spindle of the electric motor and a vertical rule arranged to measure how far the plasticine is lifted. The gain of *GPE* by the plasticine is *mgh* so *m* must be small for measurable values of *h* to be obtained. The capacitance of the capacitor *C* must, however, be large. The capacitance shown is 10 000 μF or 0.010 F, so that the loss of *EPE*, $\frac{1}{2}CV^2$, is large enough to lift the plasticine through a measurable height.

A series of readings of *V* the charging voltage and *h* should be taken as the capacitor is first charged from the variable d.c. power supply and then discharged through the motor.

A typical set of observations might be:

Table 5.2

V/V	7.5	9.0	10.5	12.0
h/m	0.27	0.40	0.53	0.71

a) How could these observations be used to test that the gain of *GPE* is proportional to the loss of *EPE*?

b) Why is the gain of *GPE* not equal to the loss of *EPE* in this Activity?

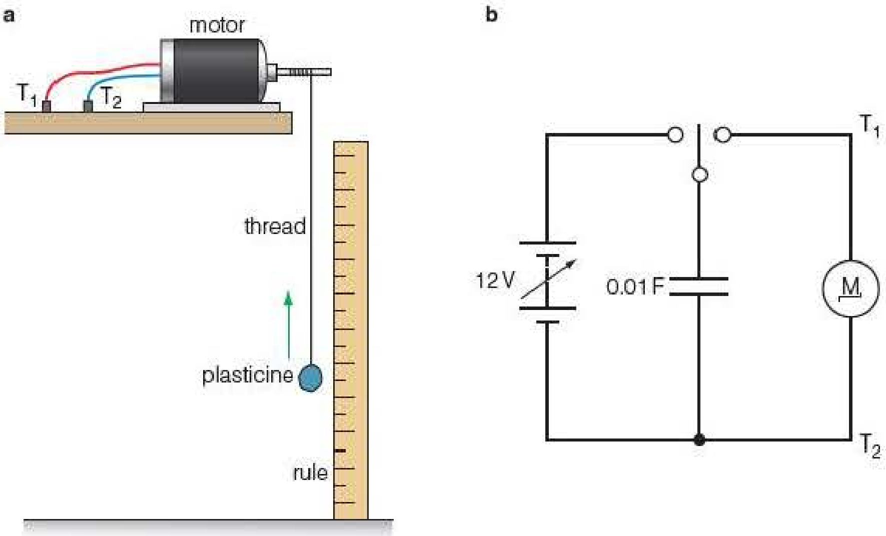


Figure 5.7 a) *EPE* to *GPE* b) Circuit diagram

Test yourself

1 A capacitor carries a charge of ± 20 μC on its plates when charged by a 12 V d.c. battery. Calculate the capacitance of the capacitor.

2 Sketch a graph of Q on the y-axis against V on the x-axis as a capacitor is charged.

3 What does the gradient (slope) of a graph of Q on the y-axis against V on the x-axis represent as a capacitor is charged?

4 A constant current of 3.5 A passes through a resistor for 10 minutes. Calculate how much charge passes through the resistor in this time.

5 The kg m^2 s^{-2} is a base SI unit of what physical quantity?

6 Calculate the electrical potential energy (*EPE*) stored in a 47 μF capacitor after it has been charged to a p.d. of 60 V.

7 Calculate the electrical potential energy (*EPE*) stored in a 47 μF capacitor which has a charge of ± 16 mC on its plates.

8 Draw a circuit diagram showing how to charge a capacitor to 12 V.

Figure 5.8 An electrolytic capacitor

Tip

Part **b)** uses the fact that a volt is a name for a joule *per* coulomb.

Tip

If you need to revise Sankey diagrams: see page 83 of Book 1.

5.3 Capacitors in the real world

The photograph in Figure 5.8 shows an electrolytic 2200 μF capacitor, which *must* be connected the right way into a circuit. Can you see the plus sign (+) to the left of the writing printed on the capacitor, and the minus sign (–) to the right?

Example

A 2200 μF capacitor, such as that shown in Figure 5.8, is fully charged by connecting it to a 6.0 V d.c. supply. The stored energy is then transferred to a discharge tube, creating a flash.

a) Calculate the energy stored in the capacitor.

b) Calculate the energy given to the charge by the 6.0 V battery.

c) Sketch a Sankey diagram to show the operation of the flash unit.

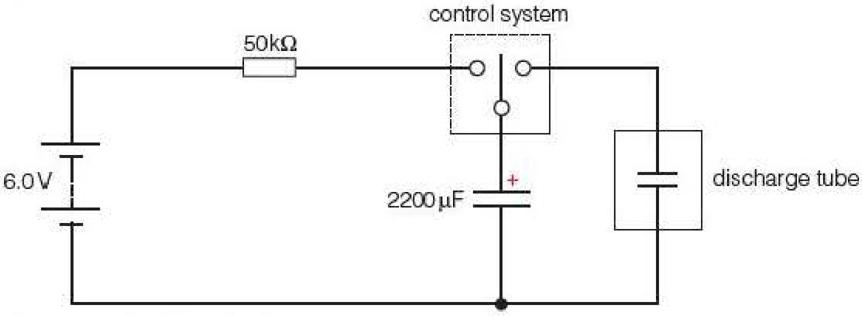


Figure 5.9 Creating a flash

Answer

a) $Q = CV$, so the stored charge is $6.0\,\text{V} \times (2200 \times 10^{-6}\,\text{F}) = 0.0132\,\text{C}$

∴ the energy stored in the capacitor is

$$W = \tfrac{1}{2}VQ = \tfrac{1}{2} \times 6.0\,\text{V} \times 0.0132\,\text{C}$$

$$= 0.0396\,\text{J} = 0.040\,\text{J to 2 SF}$$

b) The energy given to the charge as it passes through the charging cells, the battery, is 6.0 joules for each coulomb of charge (6.0 V or 6.0 J C^{-1}).

∴ The energy transferred to the charge $= 0.0132\,\text{C} \times 6.0\,\text{J C}^{-1}$

$$= 0.0792\,\text{J} = 0.079\,\text{J to 2 SF}$$

c)

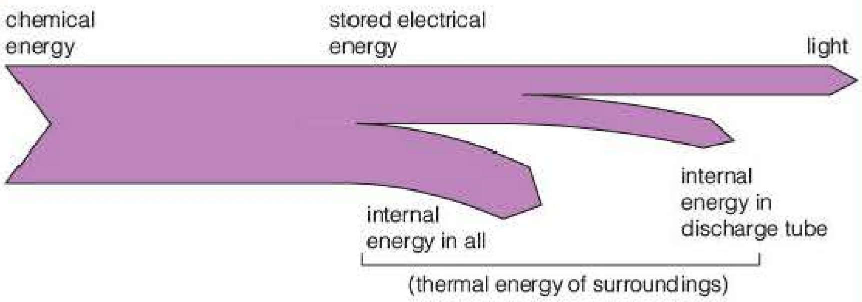


Figure 5.10

Tip

The chemical energy transferred *to* the charge by the battery in this Example is exactly *twice* the energy stored as *EPE* in the capacitor (0.0792 J = 2 × 0.0396 J). The other half of the energy is transferred to internal energy in the connecting wires as work is done in pushing the electrons along.

For the camera flash unit in the previous Example, if the discharge occurs in 0.10 ms, the average power transfer is (0.040 J) ÷ (0.10×10^{-3} s) = 400 J s^{-1} or 400 W. This is a very bright, short flash.

Ordinary capacitors cannot be used to store large quantities of electrical energy: it would take a whole room full of them to supply a hundred watts for an hour. But recent developments have produced *super-capacitors* that can each store as much as 8000 J, and a bank of such super-capacitors can be used, for example, to back up electrical systems in hospitals in the event of a power cut.

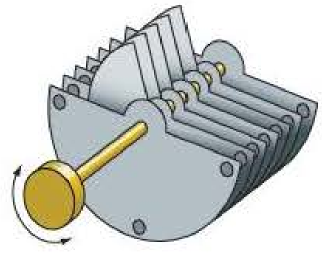


Figure 5.11 A variable capacitor

Figure 5.11 shows a common device – a *variable capacitor*. You turn one like this when you search for a new frequency on an analogue radio.

Two other places where capacitors can be found in the home are a) below the keys on a laptop or computer keyboard and b) inside the cable from a satellite dish or an aerial to a TV set:

- There is a capacitor under each key on the keyboard of any digital numeric device as shown in Figure 5.12. When a key is pressed the two sides or plates of the capacitor become a little closer together. In so doing the capacitance changes slightly (it becomes a little larger); so by scanning all the keys it is possible to deduce which one has been pressed.
- The cable you plug into a fixed TV set in the home is called a coaxial cable. It acts as a cylindrical capacitor. Each metre of the cable has a capacitance of about 100 pF (100 picofarad per metre or 100×10^{-12} F m^{-1}). The total capacitance of the cable can thus be calculated from its length and this enables the installing engineer to reduce reflections of the signal that may otherwise appear as a shadow on the TV screen.

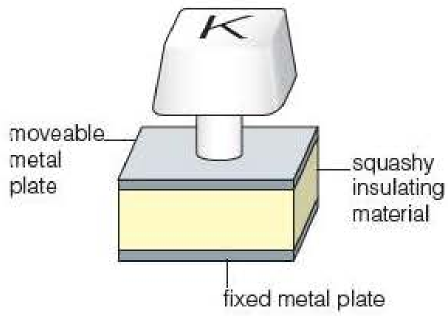


Figure 5.12 A capacitor key

The coulombmeter

The p.d. across a capacitor is proportional to the charge on its sides or plates, so a digital voltmeter can be turned into a charge-measuring meter – a coulombmeter. This is achieved by connecting a capacitor across its terminals (Figure 5.13).

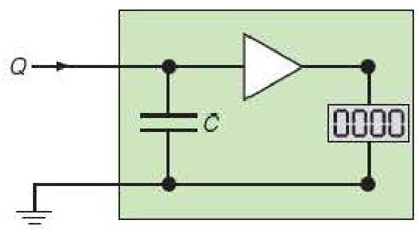


Figure 5.13 Converting a digital voltmeter to a coulombmeter

Assume the capacitance of the added capacitor is 4.7 μF. If this capacitor is initially uncharged, charges from 0 up to 940 nC (940 nanocoulomb or 940×10^{-9} C) delivered to it will produce potential differences from 0 to about 0.20 V. In practice the scale of the meter can, using suitable amplification, be adjusted to read nanocoulombs directly. Such meters, however, can only measure very small charges – up to about 2000 nC or 2 μC.

5.4 Exponential change

What is meant by an exponential change?

Exponential growth is when something gets larger by the *same fraction* or proportion in each (equal) interval of time.

For example the number of bacteria in an opened tin of meat at room temperature doubles about every 6 hours. Graphs illustrating exponential growth such as this show very rapid increase – see Figure 5.14a. In real situations, for example the growth of bacteria, something will intervene to stop the change, i.e. the exponential change does not continue forever.

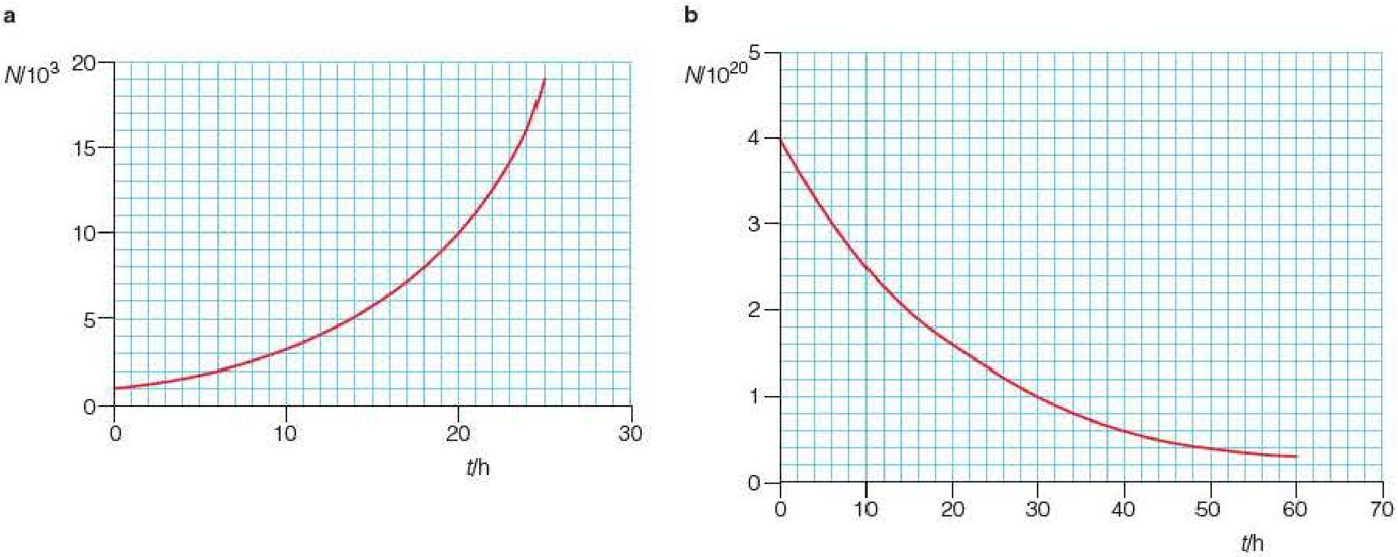


Figure 5.14 a) Exponential growth and b) exponential decay

Tip

It is sometimes tempting to look for successive ratios that are 2 : 1, i.e. 'half-lives', but very often only one or two half-lives are shown. This approach can still be used, however, by measuring the time for the y-value to halve *starting at different initial y-values* and looking to see if the time to halve is always (about) the same.

Exponential decay is when something gets smaller by the *same fraction* or proportion in each (equal) interval of time.

For example the number of radioactive nuclei, N, in a sample of sodium-24 halves every 15 hours. Graphs illustrating true exponential decay get closer and closer to the time axis, but never quite reach it – see Figure 5.14b. In real situations, such as the decay of sodium-24, there will come a time when only one and then no radioactive nuclei remain.

Example

Show that the graph in Figure 5.14b is showing exponential decay.

- The statement about exponential decay above is the key to this question.

Answer

The graph will be showing exponential decay if the ratios of values of N after successive equal time intervals are constant.

From the graph, the values of $N/10^{20}$ at 10 hour intervals are given in Table 5.3.

Table 5.3

t/h	0	10	20	30	40	50
$N/10^{20}$	4.0	2.5	1.6	1.0	0.6	0.4

The ratios of successive values of N in Table 5.3 are (the 10^{20}s cancel out):

$\frac{4.0}{2.5}$ $\frac{2.5}{1.6}$ $\frac{1.6}{1.0}$ $\frac{1.0}{0.6}$ $\frac{0.6}{0.4}$

To 2 SF these are:

1.6, 1.6, 1.6, 1.7, and 1.5

The curve is showing exponential decay as these fractions are, effectively, equal.

Log-lin graphs

A log-lin graph is a graph with a logarithmic y-axis (either $\log_{10}$ or natural logarithms $\ln_e$ – your calculator has both – see Test yourself question 14 below – and a linear x-axis (usually time t). Such graphs can reveal an exponential growth or an exponential decay when the graph turns out to be a straight line (*not* through the origin).

To illustrate this, consider the graph showing exponential growth in Figure 5.14a.

Table 5.4 shows a set of values taken from this graph, including values of $\log_{10}N$ or simply $\log N$.

Table 5.4

t/h	0	5	10	15	20	25
$N \times 10^3$	1.0	1.7	3.0	5.7	10.6	18.9
$\log N$	3.00	3.23	3.48	3.76	4.03	4.28

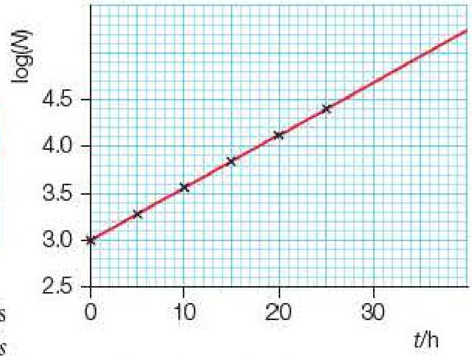


Figure 5.15 A log-lin graph

Figure 5.15 shows the graph that results from plotting $\log N$ against t. As this graph is linear with a positive gradient, the curved graph of Figure 5.14a *is* showing exponential growth.

Test yourself

9 Copy down what is written on the large value electrolytic capacitor in Figure 5.8 on page 76.

10 A current of 30 mA passes in a 1.5 V cell for 2.0 minutes. Calculate

a) the charge passing through the cell in this time

b) the energy given to the charge by the cell in this time.

11 State *two* uses to which a capacitor might be put.

12 A pair of plates of area A separated by a thin sheet of Polythene of thickness d form a capacitor with a capacitance that is proportional to A/d,

i.e. $C \propto \frac{A}{d}$ or $C = \frac{kA}{d}$ where k is a constant.

What are the units of k?

13 Show, by taking readings from the graph of Figure 5.14a and calculating ratios, that this graph is showing exponential growth.

14 Copy the line of numbers below and use your calculator to produce two more lines showing the logarithm to base 10 and the natural logarithm (to base e) for each value of n:

n	1.00	10.0	2.00	2.72	7.39	100
$\log_{10}(n)$						
$\ln_e(n)$						

5.5 The exponential function

Exponential decay graphs are described mathematically using a special 'function'. You have met other functions like $\sin x$ or $\cos x$, and you should know what graphs of $y = \sin x$ or $y = \cos x$ look like – they are wavelike or sinusoidal.

The exponential function is written as e^x, and is pronounced as 'ee to the ex', strictly 'ee to the power ex'.

e is a number equal to 2.718 to 4 SF.

It is, like π, a 'transcendental' number: neither e nor π can be expressed as a fraction.

A graph of $y = e^x$ is called an exponential graph. If x is a positive number the graph shows exponential growth – like Figure 5.14a – and if x is a negative number the graph shows exponential decay like Figure 5.14b.

A graph of $y = y_0 e^{-kt}$ (where k is a constant) is an exponential decay with time t. It starts on the y-axis at $y = y_0$ when $t = 0$. It is a 'concave' curve that approaches but never reaches the t-axis. We will see that for a capacitor of capacitance C discharging through a resistor R, the discharge is exponential and the constant $k = 1/RC$.

5.6 Capacitor discharge

When a capacitor of capacitance C discharges through a resistor of resistance R, the current I in the resistor is proportional to the charge Q remaining on the capacitor, i.e. the rate of flow of charge is proportional to the remaining charge.

Expressed algebraically:

$$I \propto Q \text{ or } -\frac{dQ}{dt} \propto Q$$

$$I = \text{constant} \times Q \text{ or } -\frac{dQ}{dt} = \text{constant} \times Q$$

This is a very special type of relationship: *the less you have the more slowly you lose it.*

The discharge of a capacitor gets slower and slower as the charge left on the capacitor gets less and less, and it is never complete. Graphs of capacitor discharge turn out to be **exponential decay** graphs. The constant in the above relationship is $1/RC$ and has the unit s^{-1}. So that

$$I = \frac{Q}{RC}$$

The product RC is called the **time constant** for the decay of charge from a capacitor C through a resistor R. The larger the time constant, the longer the decay takes. The 'half life' $t_{\frac{1}{2}}$ of the decay, the time for the charge to decrease to one half of its value, is:

$$t_{\frac{1}{2}} = RC\ln 2 = RC \times 0.693$$

(try 2 then ln on your calculator).

Figure 5.16 shows a typical graph of how the charge Q varies with time in a discharge of this kind. Testing the graph to show that it represents exponential decay is easy here, as three half-lives can be read off: 128 to 64; 64 to 32; and 32 to 16, each taking 28 ms. You should try and read off some other half-lives starting at *different* points.

Key term

The k in these equations showing an exponential rise or decay with time, $y = y_0 e^{t/k}$ or $y = y_0 e^{-t/k}$ is the **time constant** for the rise or decay (t/k must have no units).

Tip

Amazingly a tangent drawn at $t = 0$ on a curve like Figure 5.16 cuts the time axis at a time $t = RC$, see exam practice question 22.

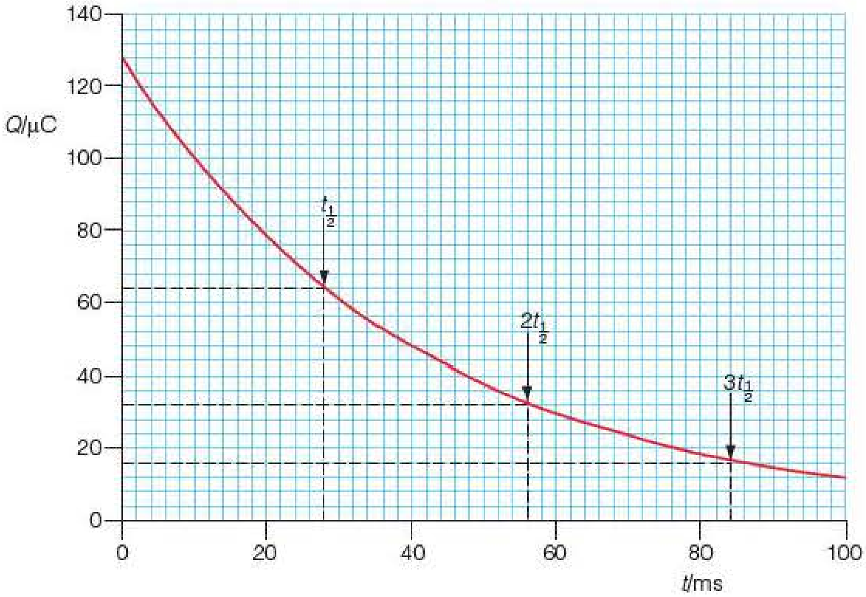


Figure 5.16 Exponential capacitor discharge

$$\text{Here } RC = \frac{t_{\frac{1}{2}}}{0.693} = \frac{28 \times 10^{-3}\,\text{s}}{0.693}$$
$$= 40.4 \times 10^{-3}\,\text{s}$$
$$= 40\,\text{ms to 2 SF}$$

So, for example, if $C = 47\,\mu\text{F}$ then R must be $860\,\Omega$.

Example

Show that the unit of RC – the ohm times the farad – is the second.

Answer

An ohm is a volt per ampere: $\Omega \equiv \text{V A}^{-1}$ from $V = IR$

A farad is a coulomb per volt: $\text{F} \equiv \text{C V}^{-1}$ from $Q = CV$

$\therefore \Omega \times \text{F} \equiv (\text{V A}^{-1}) \times (\text{C V}^{-1}) = \text{A}^{-1} \times \text{C}$

But $\text{C} \equiv \text{A s}$ from $Q = It$, so the unit of $RC = (\text{A s}) \times (\text{A}^{-1}) = \text{s}$, the second.

Activity 5.3

Studying the discharge of a capacitor

A capacitor labelled, for example 470 μF, is charged to 6.0 V and discharged through a resistor and a microammeter of total resistance R.

Readings could be taken every 20 s and a graph plotted of the discharge current I against time t.

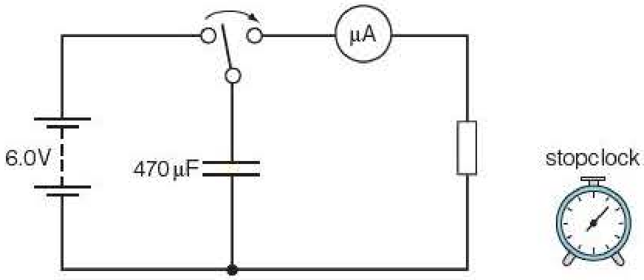


Figure 5.17 Circuit diagram and stopclock

Here is a typical set of readings in this kind of experiment:

Table 5.5

t/s	0	20	40	60	80	100
I/μA	59	39	26	17	11	7

Example

a) Plot a graph of I in μA against t in s for the data in Table 5.5.

b) What is the half life $t_{\frac{1}{2}}$ for this decay process?

c) Use the answer to **(b)** to calculate the time constant for this discharge.

d) Calculate the total resistance R (resistor plus microammeter) in the discharge circuit.

Answers

a)

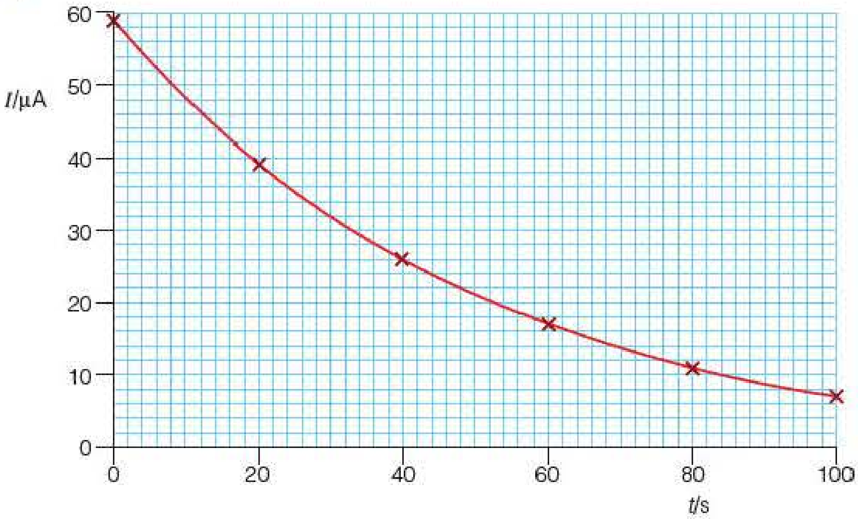


Figure 5.18

b) As the current falls from 50 μA to 25 μA, $t_{\frac{1}{2}} = (42 - 8)\,\text{s} = 34\,\text{s}$

This should be checked at another 2 points:

As the current falls from 30 μA to 15 μA, $t_{\frac{1}{2}} = (66 - 33)\,\text{s} = 33\,\text{s}$

As the current falls from 40 μA to 20 μA, $t_{\frac{1}{2}} = (52 - 19)\,\text{s} = 33\,\text{s}$

c) Time constant $= RC = t_{\frac{1}{2}} \ln 2 = \dfrac{33.3\,\text{s}}{0.693}$

$= 48\,\text{s}$ to 2 SF.

d) As $C = 470\,\mu\text{F}$ then $R = \dfrac{48\,\text{s}}{470 \times 10^{-6}\,\text{F}}$

$= 102\,000\,\Omega = 100\,\text{k}\Omega$ to 2 SF

The resistance of the circuit in the above Activity is probably 100 kΩ as capacitor values are rarely quoted to better than ±5%.

Summarising the various mathematical relationships for capacitor discharge, we have, using natural logarithms, i.e. to base e:

$$y = y_0 e^{-kt}$$

$$I = \frac{Q}{RC}$$

$$Q = Q_0 e^{-t/RC} \Rightarrow \ln Q = \ln Q_0 - \frac{t}{RC}$$

$$I = I_0 e^{-t/RC} \Rightarrow \ln I = \ln I_0 - \frac{t}{RC}$$

$$V = V_0 e^{-t/RC} \Rightarrow \ln V = \ln V_0 - \frac{t}{RC}$$

All the graphs of these logarithmic statements have a gradient = $-\frac{1}{RC}$ and it can be shown that the time for the charge on the capacitor to halve, $t_{\frac{1}{2}} = RC \ln 2$ (= $0.693RC$). Another useful relationship can be found by considering what happens at time $t = RC$. At this point $V = \frac{V_r}{e} \Rightarrow V = 0.368V_O$.

Tip

Another useful relationship can be found by considering what happens at time, $t = RC$. At this point $V = \frac{V_0}{e}$

$\Rightarrow V = 0.368V_0$.

Example

Refer to the graph in Figure 5.18 on page 82.

a) Using the data in Activity 5.3, plot a second graph of the natural logarithm of the current, $\ln(I/\mu A)$, against time t and deduce a second value for R.

b) Discuss whether this second value provides the more reliable value for R than that calculated on page 82.

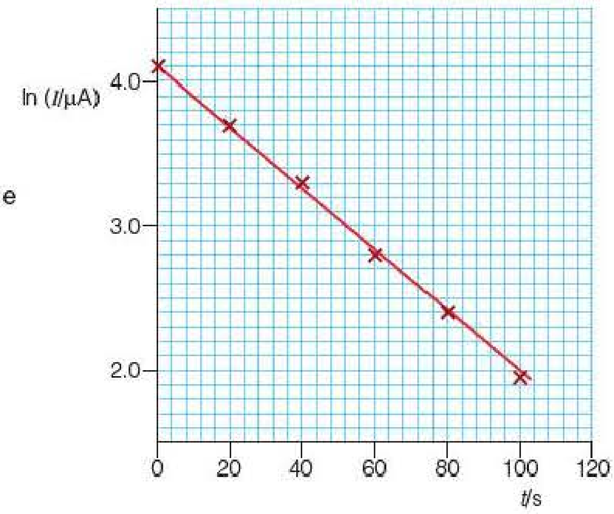


Figure 5.19

Answer

a) The values of $\ln(I/\mu A)$ to 2 SF at the values of t listed in Table 5.6 below, are:

Table 5.6

t/s	0	20	40	60	80	100
ln(I/μA)	4.1	3.7	3.3	2.8	2.4	2.0

Here $\ln I = \ln I_0 - t/RC$ because $I = I_0 e^{-t/RC}$, and so the gradient of the graph = $-\frac{1}{RC}$.

$$\text{Gradient} = \frac{(2.0 - 4.1)}{100\,\text{s}} = -0.021\,\text{s}^{-1}$$

$$\therefore RC = \frac{1}{0.021\,\text{s}^{-1}} = 47.6\,\text{s}$$

Substituting $C = 470 \times 10^{-6}\,\text{F} \Rightarrow R = 101\,000\,\Omega$ or $100\,\text{k}\Omega$ to 2 SF.

b) This value is more reliable than that on page 83, as it has been deduced from a straight line graph using all the experimental results, rather than from only a few points on a curved graph.

Safety note

Ensure that the voltage rating of the capacitor is not exceeded and that the electrolytic capacitor has the correct polarity in the circuit.

Core practical 11

Charging and discharging a capacitor

Using a data logger to display and analyse the potential difference across a capacitor as it charges and discharges through a resistor.

Sometimes you may be asked how to collect charging and discharging data when the overall time is too small for you to collect data as you go. In this case a data logger can be used to display the potential difference. An oscilloscope can also be used.

The circuit shown in Figure 5.20 is set up.

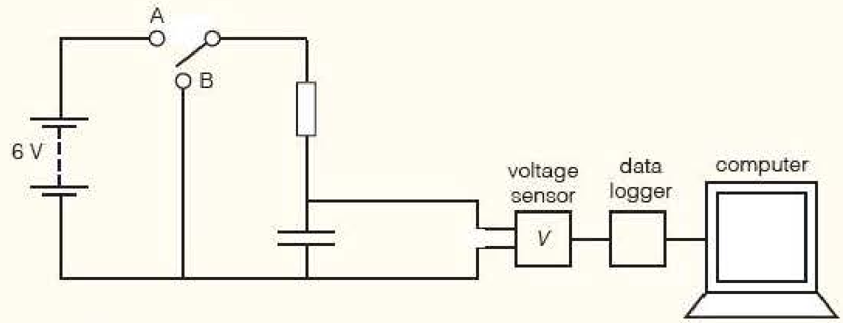


Figure 5.20 Charge-discharge circuit for a capacitor

The capacitor is charged through the resistor by connecting the switch to contact A. The capacitor is then discharges through the same resistor by moving the switch to contact B. The p.d. across the capacitor is measured by the voltage sensor and recorded by the data logger. The data can then be analysed by the computer.

The exact way in which these data are collected and analysed will depend on the apparatus available in your laboratory, but the analysis will be the same however the data have been collected and displayed. A typical display is shown in Figure 5.21.

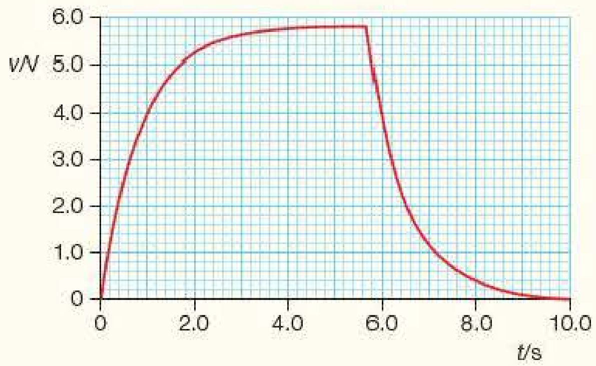


Figure 5.21 Charge-discharge graph for a capacitor

1 Explain what these curves show.

2 Write down the equation for each of the curves.

3 Use Figure 5.21 to estimate the capacitance of the capacitor. Discuss the uncertainty in your value.

4 Explain why your value is only an estimate and suggest how the data could be displayed to give a more accurate value for the capacitance.

Estimating areas

The total initial charge on a capacitor that has been discharged is equal to the total area under the I–t graph – mathematically this is expressed as $Q = \int I dt$. For the decay curve in Figure 5.18 this is best estimated by drawing a triangle of height 60 μA. You now must guess how to draw the hypotenuse so that the area of your triangle is *about* the same as the area under the curve. You have to allow for that part of the curve that is not shown off to the right, but because this is only an estimate, you can't expect to get the area exactly right.

In Figure 5.18 we can estimate, using area of triangle is $\frac{1}{2}$ height × base, that the total charge will be somewhere between $\frac{1}{2}(60 \times 10^{-6}\,\text{A}) \times 90\,\text{s}$ and $\frac{1}{2}(60 \times 10^{-6}\,\text{A}) \times 100\,\text{s}$, i.e. between $2.7 \times 10^{-3}\,\text{C}$ and $3.0 \times 10^{-3}\,\text{C}$.

In Figure 5.18 the capacitor was charged to 6.0 V. We can therefore calculate the initial charge from $Q = CV = 470 \times 10^{-6}\,\text{F} \times 6.0\,\text{V} = 2.8 \times 10^{-3}\,\text{C}$. So our estimate wasn't too bad after all!

Tip

Notice that the two estimates differ by about 10%, which is quite acceptable when *estimating* a quantity. Counting squares takes a lot longer and is usually not worth it during a timed test or examination.

Test yourself

15 A charged capacitor is discharged through a resistor. Explain why a graph of the discharge current I against time t has the *same shape* as a graph of the charge Q remaining on the capacitor against time t.

16 What is the natural antilogarithm of the number 0.693 147, given here to 6 SF?

17 A graph is drawn showing the discharge current I from a capacitor against time t. Describe three methods of checking that this graph is showing an exponential decay.

18 A 47 μF capacitor is connected in series with a 22 kΩ resistor, an open switch and 12 V battery.

- **a)** Draw a labelled circuit diagram of this arrangement.
- **b)** Calculate the *initial* current in the resistor after the switch is closed.

19 An *RC* circuit has a half-life of 25 ms. If the value of *R* is 22 kΩ, calculate the circuit capacitance.

20 Refer to Figure 5.16. *Estimate* the area between the graph line and the time axis.

21 Figure 5.22a shows a circuit for displaying both the current in, and the potential difference across, a capacitor. Figure 5.22b shows the results displayed on a computer screen.

- **a)** Explain why connecting the voltage sensor across the resistor shows the current in the capacitor.
- **b)** Explain the shape of the two graphs.

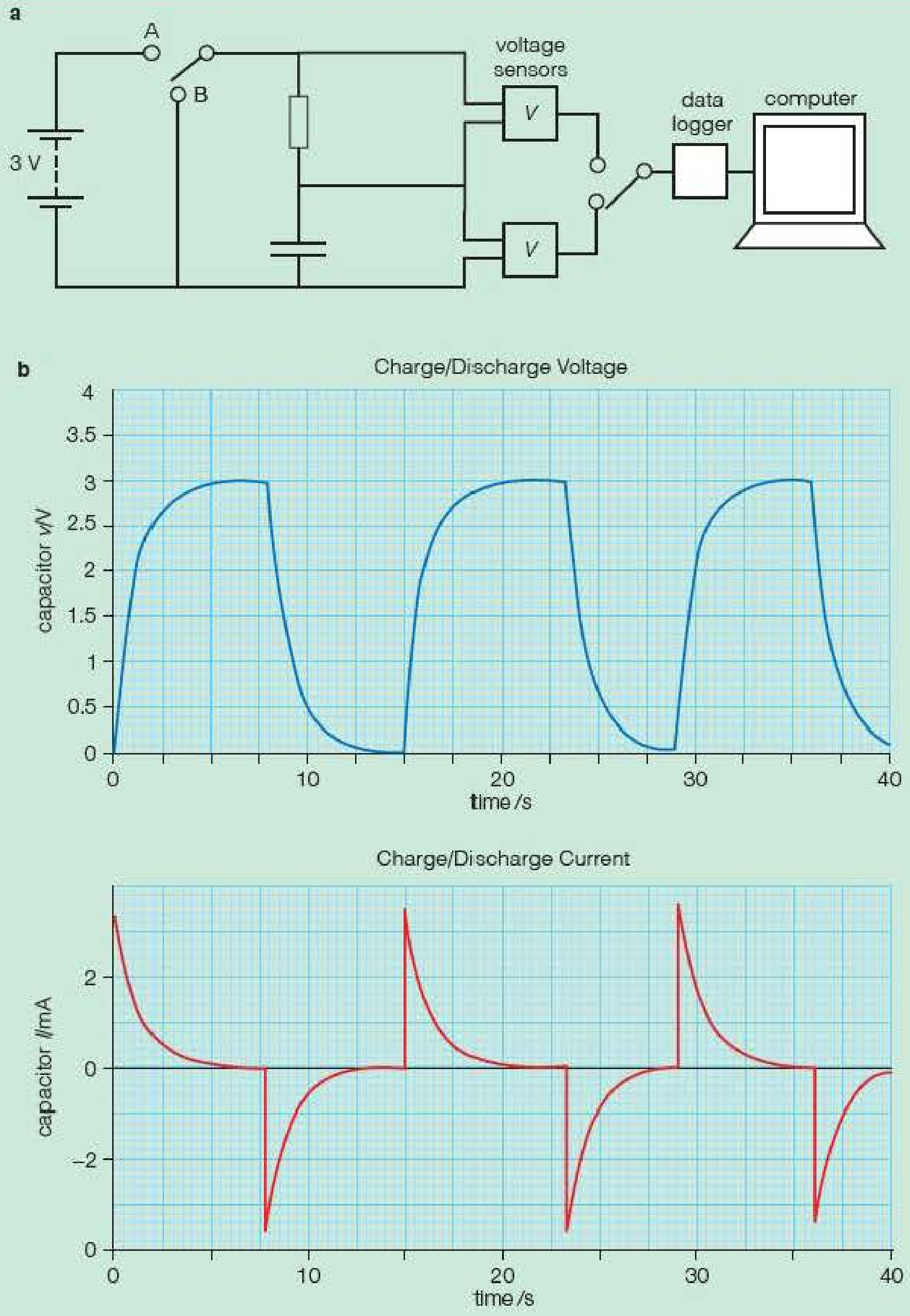


Figure 5.22

c) Use the graphs to estimate:

i) the value of the resistor

ii) the charge stored by the capacitor when it is fully charged

iii) the value of capacitor in the circuit.

Tip

If you have time, it is a good idea to check calculations using a different method if this is possible, as here.

Exam practice questions

1 The unit of capacitance, the farad F, is equivalent to

A VC^{-1}

B CV^{-1}

C JC^{-1}

D CJ^{-1}. **[Total 1 mark]**

2 A capacitor holds a charge of 1.3 μC when charged to 60 V. What is its capacitance? **[Total 2 marks]**

3 Two capacitors of 47 μF and 470 μF are connected to a d.c. supply as shown.

The ratio of the of the charges on plates Q and R in Figure 5.23 is

A 1:1

B 1:10

C 1:11

D not calculable without knowing the e.m.f. of the d.c. supply. **[Total 1 mark]**

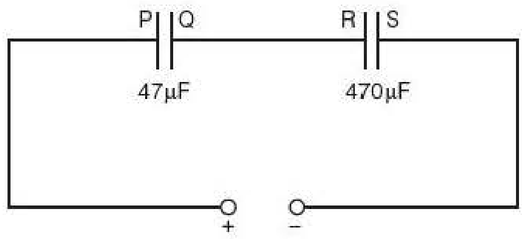


Figure 5.23

4 A 100 μF capacitor is charged at a steady rate of 80 μC s^{-1}. The potential difference across the capacitor will be 12 V after

A 6.7 s

B 9.6 s

C 10.4 s

D 15.0 s. **[Total 1 mark]**

5 A 220 mF capacitor stores 4.0 J of energy.

a) Calculate the p.d. across the capacitor. **[2]**

b) Calculate the charge on each of the capacitor plates. **[2]**

[Total 4 marks]

6 At a certain moment after the switch was closed in the circuit shown in Figure 5.24, the current registered by the ammeter was 8.5 μA. Calculate, for this moment

a) the p.d. across the resistor **[2]**

b) the p.d. across the capacitor (state any assumption you make) **[3]**

c) the charge on each of the capacitor plates. **[2]**

[Total 7 marks]

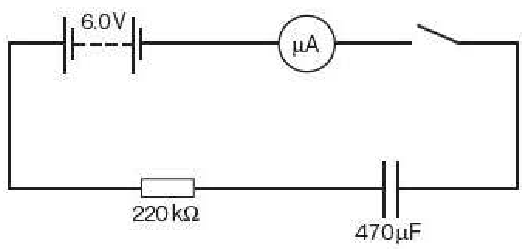


Figure 5.24

7 Two capacitors x and y are storing equal amounts of energy, yet x has twice the capacitance of y. The p.d. across capacitor x is V. The p.d. across capacitor y must be

A 0.50V

B 0.71V

C 1.4V

D 2.0V. **[Total 1 mark]**

8 A 22 mF capacitor is connected across a 10 V d.c. power supply.

a) What is the charge on the capacitor plates, and how much energy is stored? [3]

b) The power supply is then disconnected and an identical capacitor is connected across the 22 mF capacitor in such a way that the charge on the plates is shared equally between the two capacitors.

i) What is the total energy now stored in the two capacitors?

ii) Suggest where the 'lost' energy may have gone. [5]

[Total 8 marks]

9 A 100 μF capacitor can be charged to a maximum p.d. of 20 V, while a 1.0 μF capacitor can be charged to a maximum p.d. of 300 V.

Which column in Table 6.6 correctly shows which capacitor can store the most charge and which can store the most energy?

Table 6.6

	A	B	C	D
Stores most charge	100 μF	100 μF	1.0 μF	1.0 μF
Stores most energy	100 μF	1.0 μF	100 μF	1.0 μF

[Total 2 marks]

10 A pair of plates of area A separated by a thin sheet of Polythene of thickness d form a capacitor with a capacitance that is proportional to A/d, i.e. $C \Rightarrow A/d$ or $C = \frac{kA}{d}$ where k is a constant.

When the value of k is $2.0 \times 10^{-11}\,\mathrm{F\,m^{-1}}$, calculate the distance between the plates for a 4.7 pF capacitor for which the area of the plates is 12 mm by 1 2 mm. **[Total 4 marks]**

11 a) Write down the proportional relationship between F and x for a Hooke's law spring and the proportional relationship between V and Q for a capacitor. [2]

b) Knowing that the energy stored in a capacitor is $\frac{1}{2}QV$, predict the formula for the energy stored in the spring. [2]

[Total 4 marks]

12 Suppose, in Activity 5.3 described on page 81, the circuit contained, first, a resistance, R, and a capacitor, C, in series and, second, $2R$ and C in series.

Sketch, on the same axes, the two graphs of I against t resulting in these two arrangements and explain your graphs. **[Total 5 marks]**

13 A bank of capacitors of total capacitance 0.26 F at the Lawrence Livermore Laboratory in California can deliver an energy pulse of 6×10^{16} W. The pulse lasts for about a nanosecond.

Estimate the p.d. to which the capacitor bank is charged and explain why your answer is only an estimate. **[Total 5 marks]**

14 In circuit 1 below the capacitor is charged to 3.0 V and discharged through the lamp. The brightness and the length of the flash are noted.

The same procedure is followed with circuit 2, and the brightness and the length of the flashes are observed to be exactly the same as those in circuit 1.

All the lamps are identical.

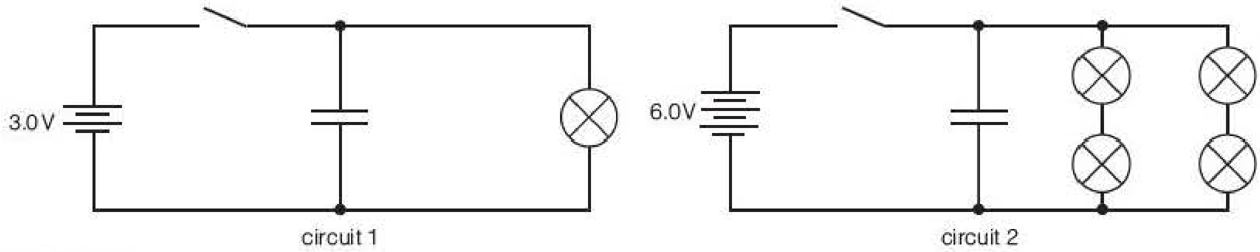


Figure 5.25

a) Explain these observations. **[2]**

b) Describe in words the arrangement of lamps you would set up to provide similar results when the capacitor is charged from a 9.0 V supply. **[3]**

[Total 5 marks]

15 When charging a capacitor of unknown capacitance C through a resistor of 22 kΩ, it is found that the current rises to half its final value in 34 s. Calculate C. **[Total 4 marks]**

16 In the diagram a potential difference of 12 V is applied between P and Q.

a) What is the p.d. across each capacitor? Explain your answer. **[2]**

b) How much charge is there on each capacitor? **[1]**

c) Hence determine the value of the single capacitor placed between P and Q that would be equivalent to the two shown. Comment on your answer. **[3]**

[Total 6 marks]

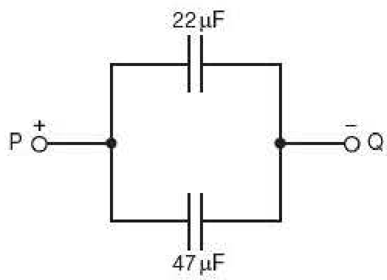


Figure 5.26

17 Refer to Figure 5.14a on page 78.

Without drawing a log-lin graph, show that this curve is exhibiting exponential growth. **[Total 6 marks]**

18 Refer to the graph in figure 5.19 on page 83.

Use the graph to find the currents at $t_{\frac{1}{2}}$ and at $2t_{\frac{1}{2}}$. Comment on your answer. **[Total 7 marks]**

19 The graph shows the current in a circuit as charge from a capacitor decays through a resistor.

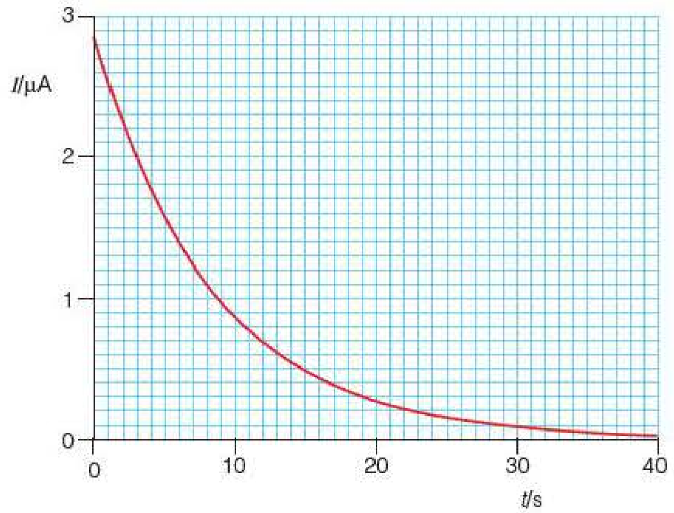


Figure 5.27

Use the graph to find the initial charge on the capacitor.

[Total 5 marks]

Stretch and challenge

20 Often when a car door is opened, a light comes on inside the car and fades away after a few seconds.

Suggest how this might be achieved with an *RC* circuit and offer possible values for *R* and *C*. **[Total 7 marks]**

21 In a period of one time constant, the charge on a capacitor of capacitance *C* discharging through a resistor of resistance *R*, falls to 0.37 of its initial value.

a) Explain the above statement. **[2]**

b) Calculate how many half-lives must elapse before the charge on the capacitor has fallen to *below* 1% of its initial value. **[3]**

c) For a circuit in which $C = 470\,\mu\text{F}$ and $R = 22\,\text{k}\Omega$, calculate the precise time after the start of the discharge at which the charge on the capacitor falls *to* exactly 1.00% of its initial value. **[3]**

[Total 8 marks]

22 A graph of charge Q against time t for an *RC* circuit in which a capacitor, initially carrying a charge Q_0 at $t = 0$, is discharging exponentially through the resistor, is described by the equation $Q = Q_0 e^{-t/RC}$.

a) Sketch a graph Q against t for this discharge, and add a tangent to your graph at $t = 0$, i.e. from the point $(0, Q_0)$. **[3]**

b) Prove that this tangent meets the time axis at $t = t_{\frac{1}{2}}$, that is where $t = 1/RC$. **[4]**

[Total 7 marks]

6 Magnetic fields

Prior knowledge

You should know from work covered in GCSE and in your Advanced level work:

- ➜ that $g = 9.81\ \text{N kg}^{-1}$
- ➜ how to move symbols to be the subject of equations such as $W = mg$
- ➜ that the unit of force, the newton, N, is a name for kg m s^{-2}
- ➜ that energy is not 'used up': it is conserved
- ➜ that a flow of electrical charge is measured in amps, A
- ➜ that the unit of energy is the joule, J
- ➜ that a volt is a name for a joule per coulomb, $1\ \text{V} \equiv 1\ \text{J C}^{-1}$
- ➜ that a volt is the unit of both p.d. and e.m.f.
- ➜ that the unit of power is the watt, W
- ➜ the meaning of μ as used in, e.g. $\mu\text{W} \equiv 10^{-6}$ watts
- ➜ that the unit of resistance is the ohm, Ω
- ➜ that bar magnets (magnetic dipoles) have N and S poles
- ➜ the shape of the magnetic lines of force near a magnetic dipole
- ➜ that like poles repel, unlike poles attract
- ➜ that magnets attract iron objects, e.g. modern 2p coins

Test yourself on prior knowledge

1. What is the mass of a baby who weighs 35.3 N?
2. Make the m subject of the equation $W = mg$.
3. Explain how energy is conserved when a car slows down and stops at traffic lights.
4. What is the current when 0.48×10^{-3} C of charge passes a point in 0.40 s?
5. What is the link between the joule and the watt?
6. What do (i) 3.0 mA and (ii) 98 kJ stand for?
7. Complete the sentence: 'The difference between p.d. and e.m.f. is ...'
8. Explain what is meant by: 'This is a 6 V battery'?
9. 'The resistance is 2.0 V/0.5 A = 4.0 kΩ.' Why is this wrong?
10. Draw a sketch showing iron filings around a bar magnet.
11. Explain how a magnet can pick up a steel safety pin.

Tip

In this chapter you will be using many physical quantities and units that you have met before in your Advanced-level course. It is often useful to use J C^{-1} (joule per coulomb) in place of V (volt), especially when it can help your understanding of what is going on.

6.1 Magnetic field lines

Around a magnet there is a region where objects made of iron or containing iron, a modern 2p coin for example, 'feel' a force pulling them towards the magnet. We say that there is a **magnetic field** around the magnet. It is invisible, just as gravitational and electric fields are invisible – we only know it is there because of the effects it produces.

Key term

A **magnetic field** is a region in which magnetic materials (or moving electrical charges) feel a force. The field can be produced in three dimensions by magnetic poles (or electric currents).

Figure 4.2 on page 54 shows, in two dimensions, the magnetic field around a bar magnet. The green lines are called **magnetic lines of force**. You can investigate the shape of a magnetic field like this using a tiny compass or by sprinkling iron filings on a sheet of paper or card placed over the magnet.

Tip

Remember: magnetic field lines never cross.

There is a simple convention for the N and S labels on the magnets and for the arrows on the lines of force. N and S are used for magnets rather like the + and – that are used for electric charges. Magnetic lines of force then flow from N to S and are drawn in this book in green to distinguish them from gravitational field lines (blue) and electric field lines (red). N and S are called the poles of the magnet. Like electric (and gravitational) fields, the magnetic field is strongest, that is exerts the largest forces, at places where the magnetic field lines are most closely bunched. Unlike positive and negative electric charges, an isolated N or S pole has never been found.

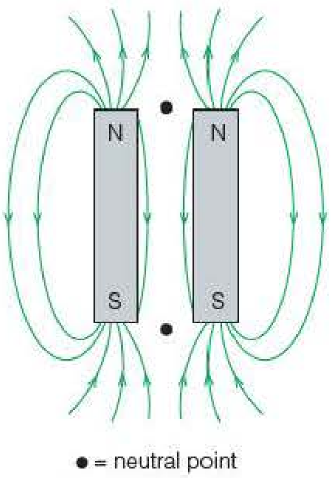


Figure 6.1 Magnets side-by-side

Where the magnetic lines of force from two magnets overlap, their effects add to give a single magnetic field shape. Figure 6.1 shows the resultant field for two bar magnets placed side by side. Somewhere between the two N poles there will be a place where the two magnetic fields cancel out. Such a place is called a **neutral point**. It will show up experimentally as a place where a tiny compass does not know which way to point. There is another neutral point between the two S poles. Composite fields like this show us the vector nature of magnetic fields.

Electric currents also produce magnetic fields. Magnetic lines of force in these magnetic fields are always closed loops. The field with the simplest shape is that produced by a steady current in a long straight wire.

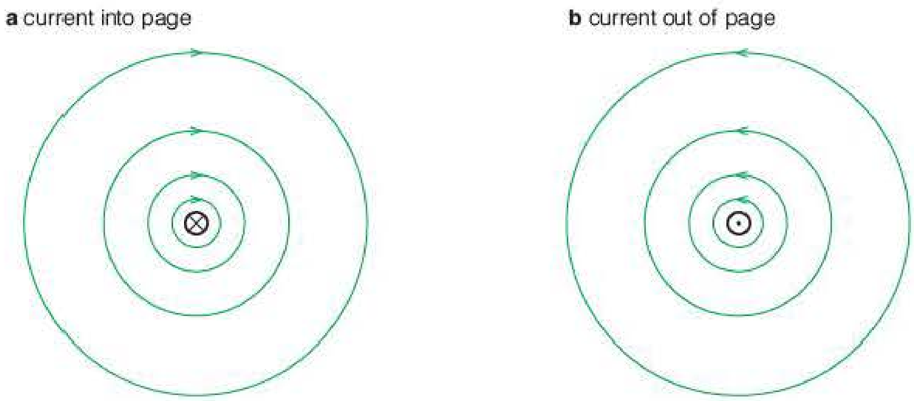


Figure 6.2 Magnetic field of a wire carrying current a) into and b) out of the page

Figure 6.2 shows in two dimensions the magnetic field lines as a series of concentric circles, the field getting weaker further from the wire. The directional arrows on the magnetic lines of force are clockwise when the current is going away from you 'into the paper'.

Tip

To remember, imagine turning a normal right-hand screw in the direction of the current: the way you turn the screw gives the clockwise sense.

The symbols × and • inside the circle of the wire are used to indicate that the current is *into* or *out of* the paper respectively.

When current-carrying wires are twisted into coils or spring-like coils called solenoids, the magnetic fields are like those shown in Figure 6.3. Two ways of representing the magnetic field of a solenoid are shown. Figure 6.3b(i) shows the three-dimensional diagram, and Figure 6.3b(ii) shows a two-dimensional 'cut' through the solenoid.

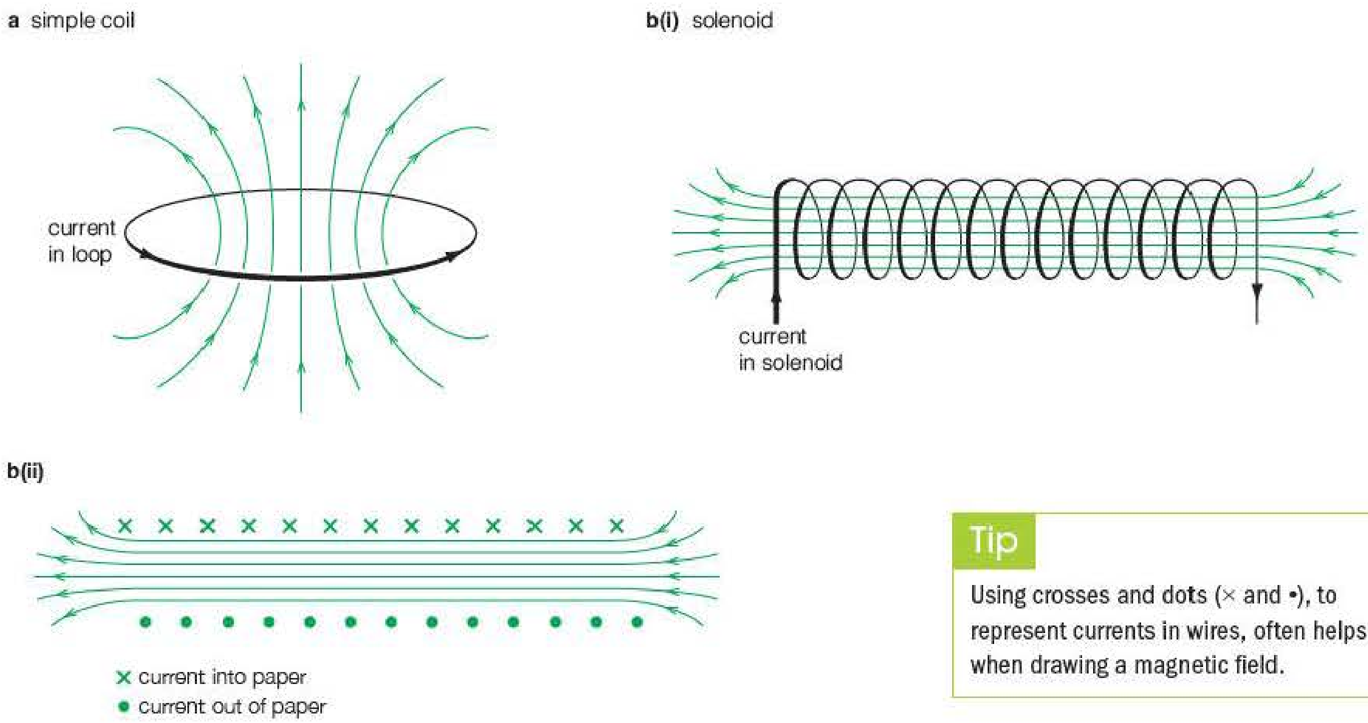


Figure 6.3 Magnetic fields of current-carrying coils

Tip

Using crosses and dots (× and •), to represent currents in wires, often helps when drawing a magnetic field.

6.2 How strong are magnetic fields?

You may have met magnetic fields in your earlier work; but do you know how the strength of a magnetic field is measured? When a current-carrying wire is placed at right angles to a uniform magnetic field, for example the field between the N and S poles of two ceramic magnets (they are the black ones that have their poles across each flat face), the magnetic fields interact, resulting in a force F on the wire. (See Figure 6.6.) This force depends on the current I in the wire and the length ℓ of the wire that lies in the field:

$$F \propto I\ell$$

The strength of the field, which is called the **magnetic flux density** B, is a vector quantity, and is defined as the constant of proportionality in $F \propto I\ell$, so:

$$F = B_{\perp} I\ell$$

The $_{\perp}$ suffix indicates that the wire carrying the current must be perpendicular to the magnetic field. This equation can be written as $F = BI\ell\sin\theta$, where $B_{\perp} = B\sin\theta$ and θ is the angle between B and the wire.

Key term

Magnetic flux density (or magnetic field strength) is B in the equation

$$F = BI\ell\sin\theta$$

The symbols are defined in the text.

Example

Suppose 12 cm of wire in which there is a current of 4.0 A lies across, i.e. is perpendicular to, a uniform magnetic field. The force on the wire is measured and found to be 0.24 N. Calculate the magnetic flux density *B* of the uniform magnetic field.

This is a routine calculation using the equation that defines B.

Answer

Using the equation $F = B_{\perp} I \ell$,

$0.24\,\text{N} = B_{\perp} \times 4.0\,\text{A} \times 0.12\,\text{m}$

$\Rightarrow B_{\perp} = \dfrac{0.24\,\text{N}}{(4.0\,\text{A} \times 0.12\,\text{m})} = 0.50\,\text{N}\,\text{A}^{-1}\,\text{m}^{-1}$

Tip

When using a calculator it is best to put in the angle first and press the sine or cosine button, before multiplying by a number.

Tip

Remember that the left-hand rule applies to the sense of *conventional current*, or to the sense of the motion of positive charge, and *not* to the direction of motion of negatively charged electrons, which will be in the opposite direction.

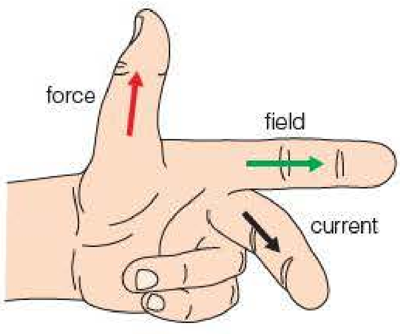


Figure 6.4 The left-hand rule

The unit $\text{N}\,\text{A}^{-1}\,\text{m}^{-1}$ (newton per ampere metre) emerges from the calculation in this example. This unit is usually given the name **tesla**, symbol T. Like the newton, symbol N, the tesla is named after a famous physicist, Nikola Tesla, who was born in Serbia.

A magnetic flux density or magnetic field strength of $0.50\,\text{N}\,\text{A}^{-1}\,\text{m}^{-1}$ or 0.50 T is much stronger than the Earth's magnetic field in the UK. In fact it is 10 000 times stronger. The magnetic flux density of the Earth's magnetic field in London (which varies with time) is about 50 μT and is inclined downwards at about 65° to the horizontal. As *B* is a vector, this means that a horizontal compass needle will respond to the horizontal component of this field. This is a field of only about

$$B_H = (50 \times 10^{-6}\,\text{T}) \cos 65° = 21 \times 10^{-6}\,\text{T or } 21\,\mu\text{T}.$$

The left-hand rule

Like gravitational field strength *g*, and electric field strength *E*, magnetic field strength *B* is a vector quantity. But unlike gravitational and electric fields, where the forces are parallel to the fields, the magnetic force is perpendicular to both the field *and* the current-carrying wire. We are looking at a three-dimensional situation that can be remembered by what is called Fleming's left-hand rule – Figure 6.4. (It must be a *left* hand.)

The thumb and first two fingers of the left hand are set at right angles to each other. With the First finger pointing in the direction of the Field and the seCond finger pointing in the direction of the Current, the Thumb gives the direction of the force or Thrust.

Tip

Because of the three-dimensional (3D) nature of $F = B_{\perp}I\ell$, it is often a great help to draw diagrams with the magnetic field into the paper so that both the current and the force lie in the plane of the paper (2D).

For example, see Figure 6.5, where the magnetic field is represented by a pattern of crosses ×××.

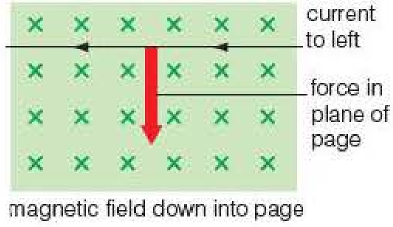


Figure 6.5 A 2-D diagram

Activity 6.1

Studying the force on a current-carrying conductor

The arrangement in Figure 6.6 can be used, with a sensitive electronic balance to measure forces.

The balance is first zeroed and the current is then switched on. Having made sure the horizontal piece of the wire carrying the current lies between the poles of the U-magnet (formed by two ceramic magnets on an iron 'yoke') and that the wire is perpendicular to the magnetic field, a series of balance readings m, in grams, can be taken for a range of currents I.

As a safety precaution, the power supply should be switched off between readings to avoid the copper wire getting too hot.

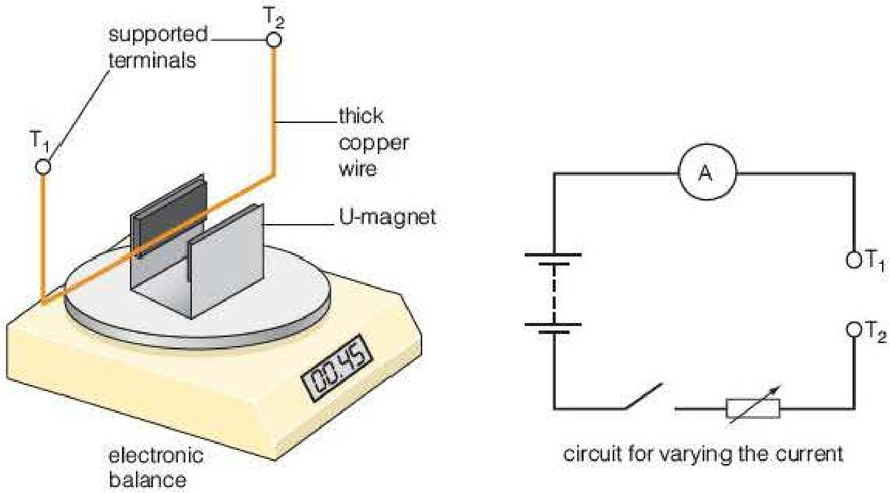


Figure 6.6 Apparatus for the Activity plus a circuit diagram

The magnetic force on the wire is, by Newton's third law, equal in size but opposite in direction to the magnetic force on the magnet – and it is this latter force that is registered by a change in the balance reading. The force can then be calculated using $F = mg$. (Here $g = 9.8\,\text{N kg}^{-1}$, but first the mass in g registered by the electronic balance must be converted to kg.)

A graph plotted of F against the current I enables a value for the magnetic field strength between the poles of the U-magnet to be deduced. (It is also possible to alter the length ℓ of the horizontal wire in the magnetic field by using a second U-magnet, but such an experiment gives only a rough test of how the force varies with ℓ.)

Table 6.1 shows a typical set of observations from this Activity. Assume that the length of current-carrying wire between the magnetic poles to produce these numbers was $\ell = 4.5\,\text{cm}$.

Questions

1. Complete the table to show the values for F (remember that 1 kg = 1000 g).
2. Draw a graph of F against I.
3. Deduce a value for the magnetic flux density between the magnets.
4. Why is the value of B the average value of the magnetic flux density?

Table 6.1

I/A	1.2	2.5	3.4	4.2	5.0
m/g	0.45	0.90	1.25	1.50	1.85
$F/10^{-3}$ N					

Test yourself

1 On Earth the geographical North Pole is *not* a magnetic N pole. Draw a circle to represent the Earth and on it mark the approximate positions of the Earth's magnetic poles. Add the magnetic lines of force, assuming the Earth to be a magnetic dipole with its *south* pole in the region of the magnetic north pole.

2 Explain why the base units of the coulomb, C, are A s, i.e. amps times seconds.

3 a) Where are the magnetic fields the strongest in Figure 6.1?

b) What can you deduce about the magnetic flux density inside a solenoid – Figure 6.3b?

4 Sketch a two-dimensional diagram showing how you would represent the currents and the magnetic field of a current-carrying coil – Figure 6.3a – as seen from *below*.

Use × and • for the magnetic fields into and out of the page.

5 Using the data on page 94, show that the strength of the vertical component of the Earth's magnetic field in London is about 45 μT.

Figure 6.7 Electric mobility

6.3 D.C. electric motors

The force on current-carrying wires in magnetic fields is made use of in electric motors. A motor using direct current (d.c.) enables you to start a car engine. Electric mobility buggies (Figure 6.7), golf buggies and electric vehicles such as electrically assisted bicycles ('e-bikes') run on d.c. motors. Many companies are currently developing electric vehicles to replace the town car. High-powered electric motors, such as those used in trains and pumping stations, make use of the same electromagnetic force but their complex modern technologies are not described here.

Example

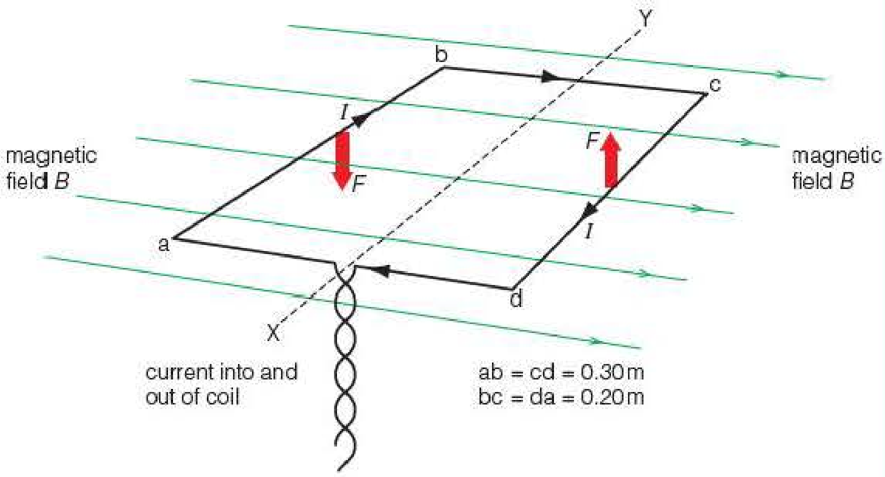


Figure 6.8 Forces on a coil

Figure 6.8 shows a rectangular coil abcd with the dimensions given below the diagram. The coil, which can rotate about the axis XY, carries a current of 40 A. It is placed so that the plane of the coil lies parallel to the magnetic lines of force of a field of 0.65 T.

a) Calculate the forces on each side of the coil: F_{ab}, F_{bc}, F_{cd} and F_{da}.

b) Describe how these forces vary as the coil rotates through 90°.

Answer

a) Using $F = B_{\perp}I\ell$:

F_{ab} and $F_{cd} = 0.65\,\text{NA}^{-1}\text{m}^{-1} \times 40\,\text{A} \times 0.30\,\text{m} = 7.8\,\text{N}$

F_{bc} and $F_{da} = 0$ as there is no component of B perpendicular to I $(\sin\theta = \sin 0° = 0)$

b) Such a pair of forces, down on ab and up on cd, each of 7.8 N, produce a strong twisting effect. (The forces will be multiplied by 10 if there are 10 insulated turns to the coil.)

As the coil rotates, the forces on sides ab and cd remain the same size and act vertically up and down still. By the time it has rotated 90°, however, the two forces are no longer twisting the coil about XY but trying to stretch it – F_{ab} pulling up and F_{cd} pulling down. The forces on sides bc and da grow from zero, but neither of these forces try to twist or turn the coil about the axis XY.

Tip

The 'calculate' here simply means use $F = B_{\perp}I\ell$.

Tip

'Describe' here means concentrate on the direction of the forces – look at how the forces act at the beginning and end of the 90° rotation.

Tip

When asked to discuss magnetic forces, keep your left hand ready with the thumb and first two fingers of the left hand set at right angles to each other.

6.4 Some useful algebra

The current I in a wire is the result of the drift of very large numbers of electrons in the wire. The relationship, which is dealt with in Book 1, linking the current to the drift speed of the charged particles is

$$I = nAvQ$$

where n is the number of charge carriers per unit volume (electrons in a metal wire), A is the cross-sectional area of the wire, Q is the charge on each charge carrier ($Q = -e$ for electrons) and v is the drift speed of the charge carriers.

As the force on a wire is given by $F_{wire} = B_{\perp}I\ell$, inserting $I = nAvQ$ gives $F = B_{\perp}(nAQv)\ell$, which can be rearranged as:

$$F_{wire} = B_{\perp}Qv \times nA\ell$$

In this equation $A\ell$ is the volume of the wire in the magnetic field, so $nA\ell$ is the total number N of charge carriers in that piece of wire.

Hence $\frac{F}{nA\ell}$ is equal to $\frac{F}{N}$, i.e. it is the force on *one* of these charge carriers (usually electrons).

The result is that the force F on *one charged particle* moving at speed v perpendicular to a magnetic field of flux density B is given by:

$$F = B_{\perp}Qv$$

Tip

Do not get confused. Remember that both $F = B_{\perp}Qv$ for a charged particle and $F = B_{\perp}I\ell$ for a wire are usually given the same symbol F.

As the size of the charge on an electron or singly ionised atom is only 1.6×10^{-19} C, this force is very small. But the mass of an electron is of the order of 10^{-30} kg, so the accelerations that these tiny forces produce can be enormous!

Example

In Figure 6.9 the crosses represent a magnetic field into the paper. The long rectangle PQRS is the outline of an enlarged piece of current-carrying wire in which there are many electrons.

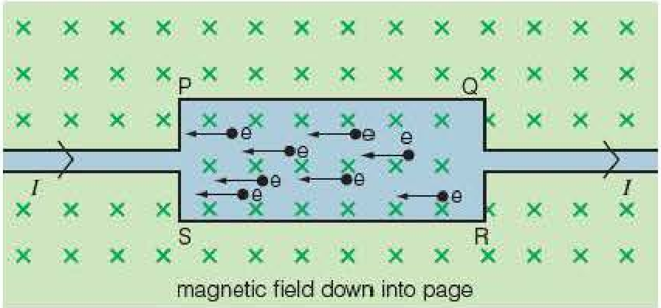


Figure 6.9 Using $B_{\perp}Qv$

Explain why there is a potential difference across the wire, and predict which side of the wire is at the higher potential.

(This potential difference is called a Hall p.d. and forms the basis of the Hall probes used to compare or measure magnetic fields.)

Tip

'Explain' here means use your knowledge and understanding of physics to describe what is happening and hence 'predict' which side of the wire has the positive charge – the top or the bottom.

Answer

Although the electrons are drifting from right to left, the direction of conventional current – the flow of positive charge – is from left to right. With the downward magnetic field into the page, the left-hand rule produces a force on each electron, pushing it *up* towards side PQ as it moves along the wire. There is no magnetic force on any of the protons in the nucleus of the atoms forming the wire, as they are not moving.

Therefore the side PQ of the wire acquires a negative charge and the side RS, which will have lost electrons, acquires a positive charge. These two charged zones mean that there is an electric field between the sides of the wire and thus a potential difference across it. The bottom side is therefore at the higher potential.

Test yourself

6 a) Explain why the unit of n in $nAvQ$ is m^{-3}.

b) Show that the units of $nAvQ$ work out to be $C\,s^{-1}$ or A.

7 What is the force on an α-particle moving at the speed of light perpendicular to a magnetic field of flux density 250 mT?

8 Show that the unit of $B_{\perp}Qv$ is the newton.

9 A typical e-bike ('electric bicycle') contains a 36 V, 9.0 A h battery. This provides current to an electric motor to 'assist' the cyclist.

Suppose a rider and bicycle of total mass 90 kg is moving up a hill, which rises 1.0 m in height for each 15.0 m travelled along the road, at a steady 5.0 m s^{-1}.

a) Calculate how many coulombs the 9.0 Ah battery can deliver before it is 'dead'.

b) Show that the rate at which the bicycle and rider are gaining *GPE* is about 300 W.

c) i) Find the current supplied to the d.c. motor as the bicycle travels up the hill.

ii) For how long the battery will 'last' if it continuously supplies this current?

iii) How far will the cyclist travelled in this time? Comment on your answer if, according to the manufacturer, 'the cycle has a range of about 40 km under average conditions of riding'.

10 Explain what is meant by the Hall effect.

Tip

Remember that a volt is a joule per coulomb, so eV is a product measured in joules.

6.5 Electron beams

Electrons carrying electric currents in wires are drifting very slowly, a fraction of a millimetre per second, but both cathode ray oscilloscopes (C.R.Os) and X-ray tubes produce beams of electrons moving at high speeds in evacuated glass tubes. The electron guns inside such devices, operating at a potential difference V, give each electron a kinetic energy equal to the product eV.

Tip

In the example below, kinetic energy is calculated as $\frac{1}{2}mv^2$ provided very high speeds (approaching the speed of light) are not reached.

Example

Calculate the speed of an electron of mass 9.1×10^{-31} kg after it has been accelerated across a potential difference of 25 V. State any assumption you make.

Answer

The kinetic energy of the electron = $(1.6 \times 10^{-19}\,\text{C}) \times 25\,\text{V}$

$= 4.0 \times 10^{-18}\,\text{J} = \frac{1}{2}mv^2$

$\therefore \frac{1}{2} \times (9.1 \times 10^{-31}\,\text{kg}) \times v^2 = 4.0 \times 10^{-18}\,\text{J}$

$\Rightarrow v = 3.0 \times 10^6\,\text{m s}^{-1}$

Assumption: all the energy becomes kinetic energy – the electron is moving in a vacuum and any change in gravitational potential energy is negligible. As $3.0 \times 10^6\,\text{m s}^{-1}$ is only 1% of the speed of light, it is reasonable to use $\frac{1}{2}mv^2$ for the kinetic energy.

Now suppose that a beam of electrons is fired with a speed v in a vacuum perpendicular to a magnetic field of flux density B. The magnetic force F on each electron is given by Bev, and this force, by the left-hand rule, is perpendicular to both the magnetic field and the direction in which the electron is moving. Figure 6.10a shows the directions of v and F with the magnetic field into the page. For the $3.0 \times 10^6\,\text{m s}^{-1}$ electron in the previous Example, $Bev \approx 10^{-14}$ N for magnetic flux densities of a fraction of a tesla, so the gravitational force $mg \approx 10^{-29}$ N on the electron can be completely ignored.

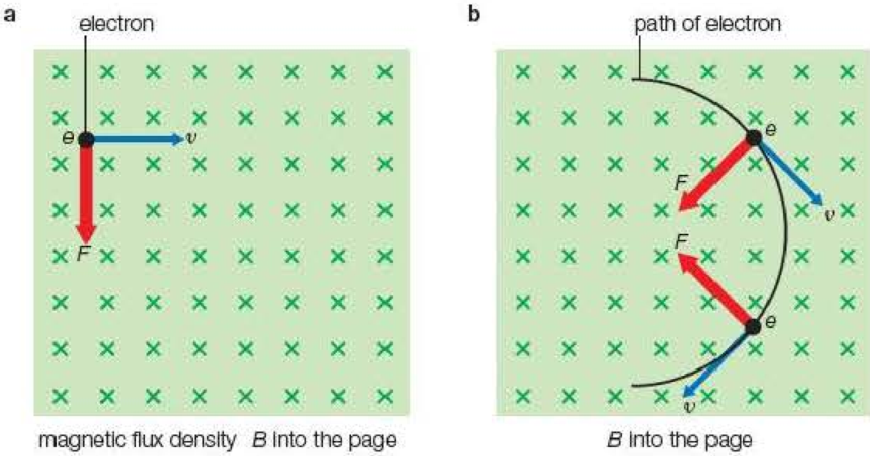


Figure 6.10 Electrons beams in a magnetic field

In Figure 6.10b the effect of the magnetic force on the electron is illustrated: it continues to act perpendicular to both B and v, making the electron move in a *circular path*. In this situation the magnetic force Bev is (effectively) the only force acting on the electron and forms the centripetal force needed for circular motion (see Section 2.2 in Chapter 2). As Bev is always perpendicular to the path of the electrons, it does no work on them. Hence both their kinetic energy and their speed as they move in a circle remain constant.

Tip

Remember that the left-hand rule applies to the sense of conventional current: the opposite direction to the direction of motion of a negative charge, for example, an electron.

Example

A beam of electrons moving to the right in a vacuum enters a region where an electric field E (red) and a magnetic field B (green) act, as shown in Figure 6.11, at right angles to each other. The electrons continue to move in a straight line through the crossed fields.

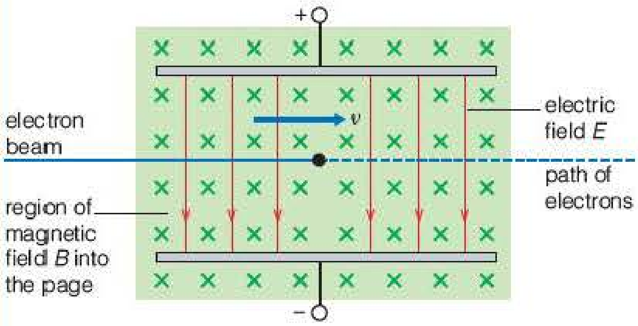


Figure 6.11 An electron beam in 'crossed' fields

a) Write down the electric and magnetic forces acting on an electron in the beam.

b) Explain under what condition the electron beam continues in a straight line.

c) Check that your algebraic expression for this condition 'works' for units.

Answer

a) Electric force on each electron = eE upwards.

Magnetic force on each electron = Bev downwards, using the left-hand rule.

b) For the beam to continue undeviated, $eE = Bev$, or v must equal E/B.

c) The unit for $E/B = NC^{-1}/T = \frac{NC^{-1}}{NA^{-1}m^{-1}} = AC^{-1}m = Cs^{-1}C^{-1}m = ms^{-1}$

So the unit for E/B is (as hoped) ms^{-1}, the unit of v.

Tip

'Explain' here means equate the two forces written down in **a)**.

This is also an opportunity to practise using the base units of V and T.

Tip

This sort of exercise involving units is excellent revision for lots of areas of physics and, even if you are not asked to do it very often in examinations, it is well worth trying from time to time in examples such as this.

Test yourself

11 Explain, in your own words, when an electron can move in a straight line in a uniform magnetic field.

12 What is the unit of a physical quantity measured in Tm^2s^{-1}?

13 A stream of alpha particles each of charge 3.2×10^{-19} C and mass 6.6×10^{-27} kg is travelling with a speed of 1.2×10^7 m s^{-1} perpendicular to a magnetic field of flux density 2.0 T.

a) What is the magnetic force on each α-particle?

b) How does this force affect the path of each α-particle?

c) Calculate the acceleration of each α-particle.

14 Show that the units of (energy) × (time) are the same as the units of (momentum) × (displacement).

6.6 Changing magnetic flux

What is magnetic flux? Magnetic flux *density* B tells us how close together magnetic field lines are; it tells us the strength of the magnetic field. The product of magnetic flux density and the area through which it acts is called the **magnetic flux** through the area, symbol Φ.

The magnetic flux through a window of area 1.0 m^2 (using a value of 18 μT for the horizontal component of the Earth's magnetic field, and assuming that the window faces north or south) will only be

$$18\,\mu T \times 1.0\,m^2 = 18 \times 10^{-6}\,Tm^2 \text{ (weber).}$$

However, the importance of magnetic flux is not simply how large it is but how *quickly it changes*. Suppose you open the window very quickly – say in 0.2 s – and that the magnetic flux through the window after you have opened it is zero – see Figure 6.12.

Key term

Magnetic flux is defined as $B_\perp \times A$ and has the symbol Φ, i.e. $\Phi = B_\perp A$.
The $_\perp$ stresses that the magnetic field must be measured perpendicular to the area through which it is passing.

The meaning of 'density' in calling B the magnetic flux density now becomes apparent, as B is the magnetic flux per unit area. The unit of B is T, the unit of Φ must be T m^2. This unit is given the name **weber** (Wb), but it is safe to use T m^2 as a unit for magnetic flux and important to remember that a tesla T is a name for a N A^{-1} m^{-1}.

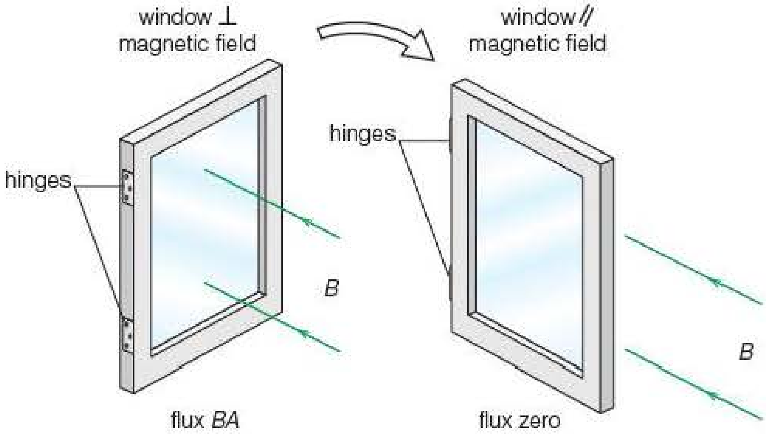


Figure 6.12 The magnetic flux through a window

The *average* rate of change of magnetic flux through the window is given by

$$\frac{\Delta\Phi}{\Delta t} = \frac{18 \times 10^{-6}\,\text{Wb}}{0.2\text{s}} = 90 \times 10^{-6}\,\text{Wb}\,\text{s}^{-1}$$

This looks very uninteresting, until you sort out the units. Let's try it.

$$\text{Wb}\,\text{s}^{-1} \equiv \text{T}\,\text{m}^2\text{s}^{-1} \equiv \text{N}\,\text{A}^{-1}\text{m}^{-1}\text{m}^2\text{s}^{-1} \equiv \text{N}\,\text{m}\,\text{A}^{-1}\text{s}^{-1} \equiv \text{J}\,\text{C}^{-1} = \text{V}$$

WOW – the volt! One $\text{Wb}\,\text{s}^{-1}$ is simply a volt.

So the act of opening the window quickly seems to have produced something that can be calculated as a voltage, here 90 μV.

When a magnetic field passes through a coil of wire that has N turns, the magnetic flux through the coil, or the magnetic flux linking the coil, is $N\Phi$ or NBA. The rate of change of this magnetic flux linkage is the key to understanding where our electricity supplies come from.

6.7 Electromagnetic induction

Michael Faraday, whose name is used for the unit of capacitance in the previous chapter, found out how to produce electric currents from the motion of wires (like metal window frames) in magnetic fields. This is how all the electrical energy in your home is generated. When Michael Faraday first produced an electric current from changing magnetic fields in 1831, the second industrial revolution – the revolution involving electric generators and motors – was born. He called the effect **electromagnetic induction** and gave his name to the basic law governing induced e.m.f.s:

$$\varepsilon = -N\frac{\Delta\Phi}{\Delta t} \text{ or } -N\frac{d\Phi}{dt} \text{ where } N = \text{number of turns in coil}$$

Here ε is the e.m.f. induced in a circuit when the average rate of change of magnetic flux through the circuit is $\Delta\Phi/\Delta t$ (or, instantaneously, $d\Phi/dt$). The e.m.f is N times greater than one when the circuit is a coil with a lot of turns, because the same e.m.f. is induced in each coil. This is rather like having a number of cells in series: if each cell has an e.m.f. of 1.5 V then 20 cells in series will produce 30 V.

Key term

Faraday's law of **electromagnetic induction** states that the induced e.m.f. in a circuit is equal to the rate of change of magnetic flux linkage through the circuit.

Tip

You need to know that the expression $d\Phi/dt$ is a shorthand way of saying 'the rate of change of magnetic flux at a particular instant'.

What about the minus sign? An induced e.m.f. could, if it was connected in a circuit, send an induced current around the circuit, which would set up a magnetic field. The minus sign in the equation for **Faraday's law of electromagnetic induction** above shows that the induced magnetic field would always oppose the change in magnetic flux that is causing it (the induced e.m.f.). This is called **Lenz's law**, and is really an example of the law of conservation of energy.

Let's go back to the example of the window from the previous section. The metal frame around the edge of the window is our circuit, so here $N = 1$. The induced e.m.f. will produce a current around this frame, and the current will itself produce a magnetic field that tries – in this example – to stop the magnetic flux through the frame from getting smaller. The induced e.m.f. is doing work sending the charge round the circuit and so we must provide energy for that by pushing on the window. Of course, for the window we will never 'feel' this force, even though 90 μV could produce a noticeable induced current as the resistance of the metal frame might be only a milliohm (mΩ). Nor will we ever notice that it is harder to open the window quickly than to do it slowly.

Figure 6.13 Lenz's law could be called the law of cussedness, as whatever you try to do produces a result that tries to stop you.

Activity 6.2

Capturing an induced e.m.f. using a data logger

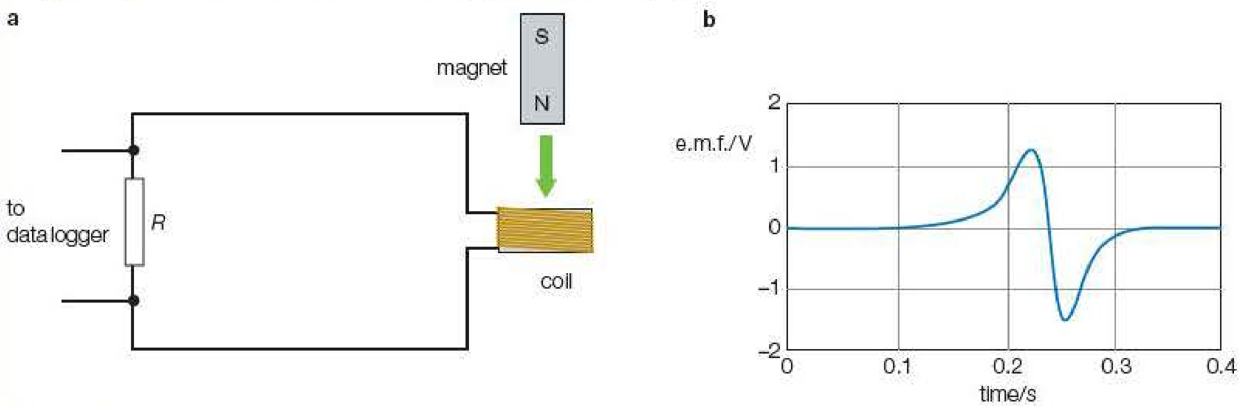


Figure 6.14 Capturing an induced e.m.f.

In this Activity a short bar magnet is dropped through a coil connected in series with a resistor. A data logger (a device that can capture a changing p.d. thousands of times per second) is connected across this resistor – see Figure 6.14a.

The data logger will record the potential difference as it varies across the resistor. The p.d. (V) is equal in size at all times to the e.m.f. (ε) induced in the coil. Accompanying the apparatus is a graph to show how this e.m.f. might vary with time. It will look something like the graph in Figure 6.14b.

Example

a) Explain the shape of the graph produced by the data logger in the above Activity.

b) Explain why the acceleration of the falling magnet is *not quite* 9.8 m s^{-2} (*g*).

c) Describe how the output of the data logger (the graph) would differ from that above if

i) the coil is replaced by a coil with double the number of turns, and

ii) the magnet is dropped from a greater height.

- This is a synoptic exercise. Each part of the example asks you to think about Faraday's law of electromagnetic induction in the context of the falling magnet.
- The rate of change of magnetic flux is the key to most questions like this.

Answer

a) Where the graph is positive, the magnetic flux through the coil is increasing as the magnet approaches the coil. Where the graph is negative, the magnetic flux is decreasing as the magnet exits the coil. The maximum e.m.f. as the magnet exits is a little more than the maximum e.m.f. as it enters because the magnet speeds up (accelerates) as it falls through the coil. Thus the rate of change of magnetic flux is bigger as it exits than as it enters the coil. It can also be seen that the time leaving the coil (< 0.1s) is less than the time entering the coil (> 0.1s) as the magnet accelerates through the coil.

b) The induced e.m.f. produces a current in the coil–resistor circuit. This current in turn produces a magnetic field (see Figure 6.3a, page 93) that will be up out of the coil as the magnet approaches, so repelling the falling magnet – Lenz's law. Similarly, the falling magnet will be attracted back into the coil as the magnet exits the coil. Both these effects are producing upward forces on the magnet so its acceleration is not quite as big as the 'free fall' acceleration *g* (on top of any air resistance, which would be very small).

c) i) Double the number of turns means that the induced magnetic flux through the coil is doubled. Therefore the induced e.m.f. is doubled at each stage of the fall, and so the peaks are each twice as high as they were.

ii) When the magnet is dropped from a greater height the magnetic flux linking the coil changes at a greater rate. Therefore the e.m.f. is greater and the time taken for the magnet to pass through the coil is reduced. The peaks are both higher and narrower.

Figure 6.15 A patient entering an MRI scanner

As we saw earlier, a **solenoid** is a long spring-like coil, of length many times its diameter. It often has lots of turns every centimetre. A current I in a solenoid produces a magnetic field B along its axis (see Figure 6.3b, page 93).

The size of B is uniform over the internal cross-section of the solenoid: $B = \mu_0 nI$ where μ_0 is a constant with a value $1.26 \times 10^{-6}\,\mathrm{N\,A^{-2}}$.

(The constant μ_0 is pronounced mu-nought. You do not need to remember this relationship.)

Huge magnetic field strengths, of the order of 2.0 T (not mT or μT), are produced in the solenoids of nuclear magnetic resonance imaging (MRI) scanners (Figure 6.15). The solenoid is in the 'box' and the patient slides into the solenoid for examination. This solenoid is made up of superconducting coils at liquid helium temperatures and carries very large currents. An MRI scanning system can cost a hospital over £1 million. There are few safety problems for the patient because no ionising radiations are involved, as there are with X-ray CT scans, though some patients do suffer from claustrophobia. Patients are, however, asked to remove any body jewellery – can you think why?

Figure 6.16 shows a simple laboratory solenoid connected to an alternating current supply I that varies sinusoidally ($I = -I_0 \sin\theta$). A small coil of cross-sectional area A with N turns is placed as shown inside the solenoid. Any change in the current in the solenoid will now produce, in accordance with Faraday's law, an induced e.m.f. in the small coil, because there will be a change in the magnetic flux linking the coil.

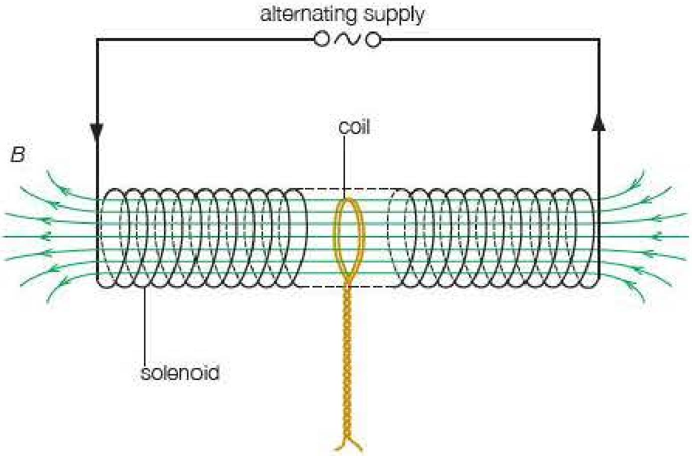


Figure 6.16 A small coil in a solenoid

As the current in the solenoid is alternating, there will be an alternating magnetic field in the solenoid. This will produce an alternating induced e.m.f. in the coil according to Faraday's law of electromagnetic induction.

Figure 6.17 shows the relation between the current I in the solenoid and the induced e.m.f. ε in the small coil.

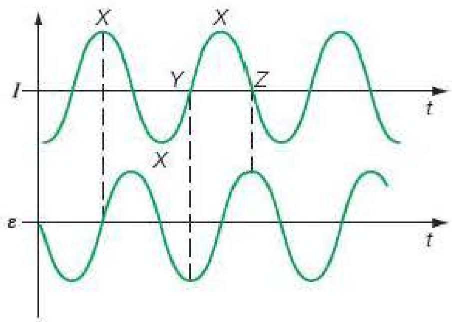


Figure 6.17 How I and ε are related

Example

Explain the phase relationship between I and ε in Figure 6.17.

- 'Explain' is not asking for a mathematical analysis, but a word statement involving your understanding of the relation between the gradient of a sinusoidal graph, I against t here, and how the gradient varies with time, i.e. dI/dt against t.
- Faraday's Law of electromagnetic induction tells you that $dI/dt \propto \varepsilon$.
- A series of separate sentences is probably the best way of picking up the 3 marks that are probably awarded to your answer.

Answer

Faraday's law states that the induced e.m.f. in a circuit is equal in size to the rate of change of magnetic flux linkage through the circuit. Here the 'circuit' is the circuit to which the coil is attached.

The maximum and minimum value of ε will therefore coincide to moments when the current I is changing at the greatest rates, i.e. when the gradient is steepest. This occurs whenever the graph of I v t crosses the time axis.

ε will be zero where the current is instantaneously *not* changing, i.e. whenever the graph of I v t is at a maximum or a minimum.

Tip

Remember that dI/dt means the rate if change of current with time and is the gradient of the current-against-time graph at any particular point.

Tip

Note that from Lenz's law, $\varepsilon = \dfrac{-N d\Phi}{dt}$

This is shown in the graph above; when $\dfrac{dI}{dt}$ is a maximum positive gradient, ε has a maximum negative value, and vice versa.

Test yourself

15 A club tennis player, using a racket with a light metal frame, is playing on a court where the Earth's magnetic flux density is 50 μT and acts at an angle of 65° to the horizontal.

Explain why serving and playing a forehand drive give rise to different induced currents in the frame of her racket. (No calculations are expected.)

16 The formula for the strength of the magnetic field B inside a solenoid of n turns per meter carrying a current I is: $B = \mu_0 n I$.

a) Show that the unit of $\mu_0 n I$ is the tesla. (The value of μ_0 – a constant – is $4\pi \times 10^{-7}\,\mathrm{N\,A^{-2}}$.)

b) What will be the current in a solenoid with 5 turns per millimetre in order to produce a magnetic field of flux density 2.0 T?

6.8 The transformer

Tip

Specific knowledge of transformers is not required for your exams, but it is still very useful to know about them.

Transformers are everywhere: they are in those heavy black plugs used in recharging devices such as that for your mobile; they provide safe voltages for experiments in laboratories; they are the key to the efficient transfer of power over the national grid. What is it that they transform?

Transformers increase or decrease the voltage of alternating current (a.c.) electrical power supplies. They do *not* work with direct current (d.c.) power supplies.

Tip

Explanations of this type are often best laid out as a series of bullet points rather than as an 'essay' of joined-up prose.

- In all transformers a coil – the primary P coil – produces a magnetic field.
- This magnetic field links another coil – the secondary S coil.
- With alternating currents, the P magnetic field varies continuously.
- Therefore the magnetic flux linkage through the S coil also varies continuously.
- The change of this magnetic flux linkage induces a varying e.m.f. in the S coil.

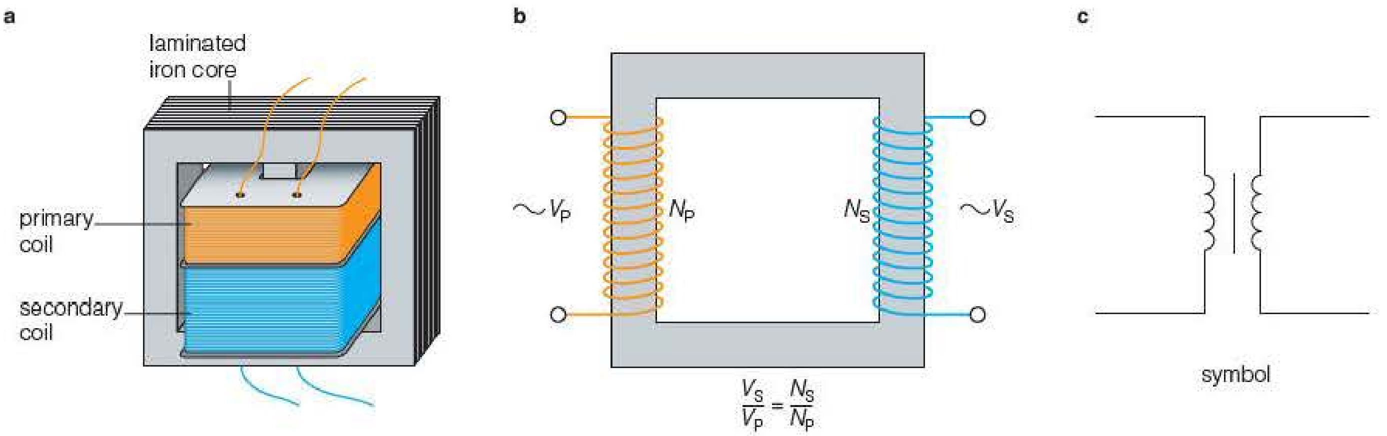


Figure 6.18 Three ways of representing a transformer

Figure 6.18 shows a) the structure of a transformer, b) the principle of the transformer, and c) the circuit symbol for a transformer. The detail of the structure will vary very much with the power that the transformer must pass from the primary P coil to the secondary S coil. It is the principle that mainly concerns us here, but it is worth noting that the coils are wound on an iron core to increase the magnetic flux, and that this core is made up of thin laminated iron sheets which are insulated from one another to reduce the energy losses. (Most transformers become warm, i.e. they waste some electrical energy – to internal energy – when in operation, and large ones need special cooling systems to prevent them overheating.)

The transformer equation, for an **ideal transformer** is:

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

This states that the ratio of the number of turns on the secondary and primary coils determines the ratio of the voltages 'out of' and 'into' the transformer. It may seem that you can get something for nothing, for example get out 2400 V having only put in 240 V. Yes, you can get more volts! But you can't get more *power* out than you put in. The principle of conservation of energy tells us this. For an ideal transformer, the power output is equal to the power input, i.e.

$$I_S V_S = I_P V_P$$

In practice, there are always energy losses and so $I_S V_S < I_P V_P$.

Tip

In the example below, 'explain quantitatively' means use numbers (calculations) to illustrate your answer.

'Suggest values' means give possible values. For this case most values will do as long as they are in the right ratio.

Example

Figure 6.19 gives information about an ideal transformer.

a) Explain quantitatively why this transformer is said to be ideal.

b) Suggest values for the number of turns the transformer might have on its coils.

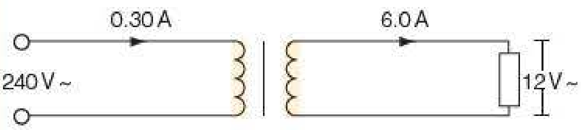


Figure 6.19 A step-down transformer

Answer

a) Input power $= I_P V_P = 0.30\,\text{A} \times 240\,\text{V} = 72\,\text{W}$

Output power $= I_S V_S = 6.0\,\text{A} \times 12\,\text{V} = 72\,\text{W}$

As there is no power loss, the transformer is ideal.

b) The ratio $\frac{V_S}{V_P} = \frac{240\text{V}}{12\,\text{V}} = 20$, so $\frac{N_S}{N_P} = 20$.

There must be 20 times as many turns on the primary coil as on the secondary. For example: N_P might be 2000 and then N_S would be 100.

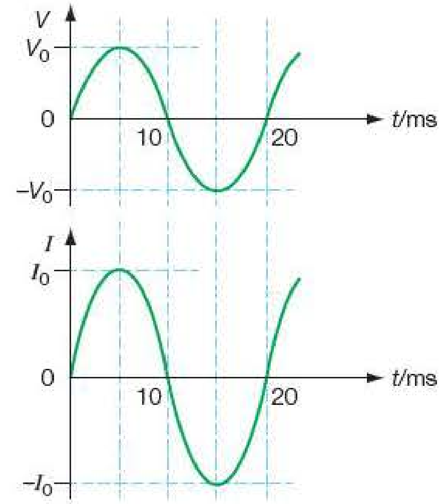
Sinusoidal alternating currents and voltages

An a.c. voltage in a resistor produces an a.c. current in the resistor. In this book the alternation will always be sinusoidal, e.g.

$$V = V_0 \sin 2\pi ft \text{ or } I = I_0 \sin 2\pi ft$$

where f is the alternating frequency.

Chapter 15 deals more fully with oscillations, but here we deal only with frequencies of 50 Hz – the frequency of the 'mains' voltage supply in the UK. Figure 6.20 shows three graphs: first, of a mains a.c. voltage with a



V	+	+	−	−
I	+	+	−	−
P	+	+	+	+

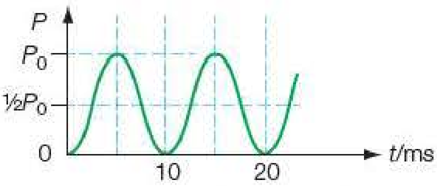


Figure 6.20 $P = IV$ for a 'mains' voltage supply

Key term

The **root mean square** voltage (or current) is the peak voltage (or current) divided by the square root of 2.

Tip

Be careful not to mix up situations involving an alternating V or I with a direct V or I.

maximum of V_0; second, the current with a maximum I_0 such a voltage produces in a resistor; and third, the power P then dissipated in the resistor. (Only one cycle is shown.)

- The first graph shows a cycle of $V = V_0 \sin 2\pi ft$ where $f = 50\,\text{Hz}$.
- The second graph shows a cycle of $I = I_0 \sin 2\pi ft$ again with $f = 50\,\text{Hz}$.
- The third graph shows $P = IV$ with a maximum value $P_0 = I_0 V_0$.

The average value of V is zero and the average value of I is zero. However, the average value of P is *not zero* but is $\frac{1}{2} I_0 V_0$, as shown in Figure 6.20. The shape of this P *v* t graph is another form of sinusoidal graph – a $\sin^2$ graph. However, it is always positive, has twice the frequency of the V and I graphs (i.e. $2f$) and, from its symmetry, averages half its peak value, $P_{av} = \frac{1}{2} P_0$.

Because $P_{av} = \frac{1}{2} I_0 V_0$ and $I_0 = V_0/R$ we can write $P_{av} = \frac{1}{2} V_0^2/R$. A direct current supply of p.d. V will produce a power in a resistor R of V^2/R and hence V^2/R for d.c. is equivalent to $\frac{1}{2} V_0^2/R$ for a.c., so that

$$V = \frac{V_0}{\sqrt{2}}$$

This, the effective (as far as power goes) value of the alternating p.d., is called the **root mean square** (r.m.s.) value of the p.d. as it has been obtained by taking the square root of the mean square p.d. It is the value usually quoted when we refer to alternating voltages, for example, when we say that in the UK the mains p.d. is 230 V, this is the r.m.s. value and the peak value is thus $\sqrt{2} \times 230\,\text{V} \approx 325\,\text{V}$. We refer to the r.m.s. values of alternating currents in a similar way: thus a fuse rated as 5 A would 'melt' if there was a steady current of 5 A or an alternating current of peak value $\sqrt{2} \times 5\,\text{A} \approx 7\,\text{A}$.

Test yourself

17 a) What is the peak value of a sinusoidal 50 Hz laboratory supply of 12 V?

b) What peak current would such a supply produce in a resistor of 220 Ω?

18 What is understood by the terms **i)** frequency, **ii)** period, **iii)** peak value and **iv)** root-mean-square value when applied to mains alternating currents?

19 Sketch a sinusoidal graph of I (current) against t (time) for three complete cycles.

a) Indicate two places where the current is (instantaneously) not changing.

b) Indicate two places where the current is zero.

c) For the case where the current varies with a frequency of 50 Hz, mark the time axis from zero to 0.06 s.

20 An ideal transformer used to recharge a mobile 'phone reduces the 230 V a.c. mains supply to 5.0 V. The current needed to charge the mobile is only 2.0 mA. Calculate

a) the current from the mains

b) the power transferred to the mobile

c) the energy transferred in 5.0 hours.

Exam practice questions

1 What are:

a) the horizontal component, and

b) the vertical component of the Earth's magnetic field at a place where the magnetic flux density is 66 μT and the field dips in to the Earth at an angle of 25° to the vertical? **[Total 3 marks]**

2 Three parallel wires, each carrying equal currents down 'into' the paper, are arranged as shown in Figure 6.21. The resulting magnetic field at P (marked by red blob) will be in the direction of which arrow? **[Total 1 mark]**

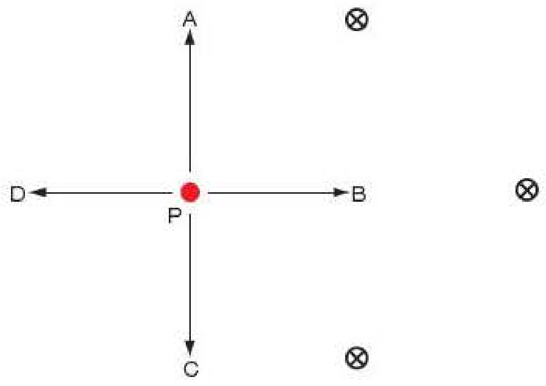


Figure 6.21

3 Two bar magnets are placed side by side with opposite poles facing. The number of neutral points produced will be:

A 0 **C** 2

B 1 **D** 4 **[Total 1 mark]**

4 A charged particle moving in a uniform magnetic field experiences a force. The size of the force depends upon each of the following, except

A the particle's mass

B the particle's velocity

C the particle's charge

D the strength of the magnetic field. **[Total 1 mark]**

5 The magnetic flux density B varies with distance r from a long straight wire carrying a current. Numerically B is inversely proportional to r.

Which graph in Figure 6.22 correctly shows such a relationship between B and r? **[Total 1 mark]**

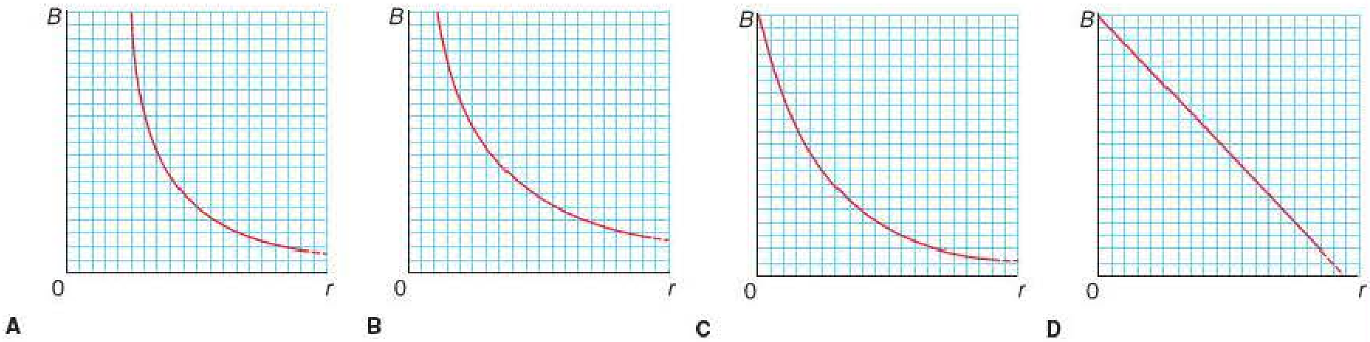


Figure 6.22

6 The power input to an ideal transformer is 60 mW. Calculate the output current I_S if the output voltage is 12 V. **[Total 3 marks]**

7 The unit of magnetic flux Φ is

A the ampere

B the tesla

C the volt

D the weber. **[Total 1 mark]**

8 The cross-channel d.c. cable that brings electricity from France to the UK carries a current of 15 kA (yes kilo). The Earth's magnetic field has a flux density of 66 μT in a region where the cable is horizontal.

Explain why, without further information, you are unable to calculate the size of the force on 1.0 km of cable. What extra information do you need? **[Total 3 marks]**

9 A charged particle moves through a region containing only a uniform magnetic field. Explain what can be deduced if the particle experiences no force? **[Total 3 marks]**

10 There is a current of 2.0 A in each of the conductors OP, OQ and OR shown in Figure 6.23. The conductors are in a magnetic field of flux density 0.25 mT parallel to the plane of the diagram.

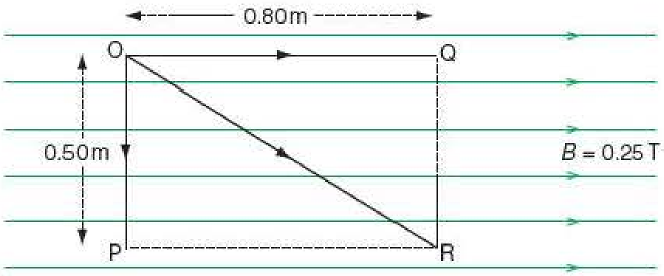


Figure 6.23

What is the size and direction of the force on each conductor? **[Total 5 marks]**

11 The tesla (T) expressed in base SI units is:

A $\mathrm{kg\,m^{-1}A^{-1}}$

B $\mathrm{kg\,s^{-1}C^{-1}}$

C $\mathrm{kg\,s^{-2}A^{-1}}$

D $\mathrm{kg\,m^{2}s^{-2}A^{-1}}$. **[Total 1 mark]**

12 The magnetic flux density in an MRI scanner is 1.5 T. A woman enters the scanner wearing, unnoticed, a gold wedding ring with an internal diameter of 16 mm.

a) What are the maximum and minimum values of the magnetic flux through the ring as she moves her ring finger about? [3]

b) Calculate the e.m.f. induced in her ring when she moves it from maximum to minimum flux linkage in 0.30 s. [3]

[Total 6 marks]

13 An electron moving at right angles to a magnetic field of flux density 0.20 T experiences an acceleration of $3.0 \times 10^{15}\,\mathrm{m\,s^{-2}}$.

a) What extra information is needed in order to calculate the speed of the electron? [2]

b) By how much does its *speed* change in the next 10^{-7} s? [1]

[Total 3 marks]

14 A length of wire is formed into a loop and placed perpendicular to a uniform magnetic field.

The circle is then cut and the wire formed into a double loop as shown in Figure 6.24.

When placed perpendicular to the same magnetic field, the magnetic flux through the double loop is N times that through the single loop, where

A $N = 0.25$ **C** $N = 0.40$

B $N = 0.32$ **D** $N = 0.50$.

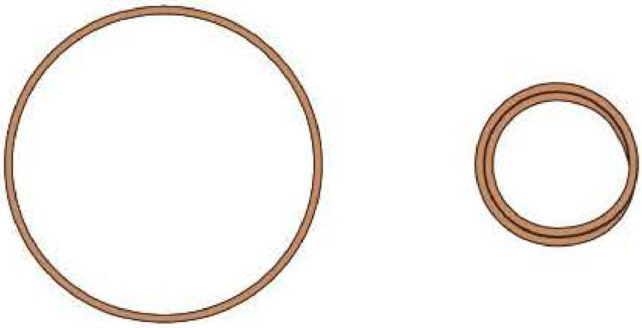


Figure 6.24

[Total 1 mark]

15 A transformer is designed to reduce a 240 V a.c. supply to a laboratory device that operates off a supply of 12 V. There are 200 turns on the secondary coil, and the device takes a current of 1.5 mA.

a) How many turns are there on the primary coil? **[2]**

b) What, assuming it is an ideal transformer, is the current in the primary coil? **[2]**

[Total 4 marks]

16 A horizontal copper wire with a mass per unit length $\mu = 80\,\text{g}\,\text{m}^{-1}$ lies at right angles to a horizontal magnetic field B (not the Earth's field). When there is a current of 5.6 A in the wire it levitates, that is it is supported against the pull of the Earth.

Calculate the size of the magnetic field strength B. **[Total 4 marks]**

17 Why might Lenz's law be called the 'law of cussedness'? **[Total 3 marks]**

18 An alpha particle (charge +2e) moving at a speed of $5.0 \times 10^6\,\text{m}\,\text{s}^{-1}$ enters a region in which there is a uniform magnetic field of 0.15 T.

What is the magnetic force on the alpha particle if the angle between its initial path and the magnetic field is **a)** 90°, **b)** 45°, **c)** 30°?

[Total 5 marks]

19 In Figure 6.25 the switch S is closed in the left circuit.

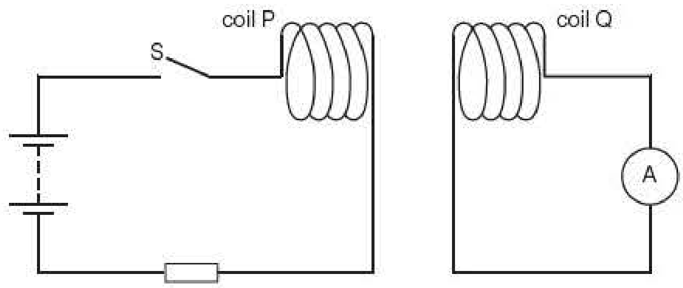


Figure 6.25

a) Explain why the ammeter in the right circuit registers a sudden current surge and then returns to zero. **[4]**

b) Explain what would be observed when the switch is opened again. **[4]**

[Total 6 marks]

20 An aeroplane with a wing span of 36 m is flying horizontally at 240 m s^{-1} in a region where there is a magnetic field with a vertical component of 45 μT.

a) Calculate the value of the product *Blv*. **[2]**

b) Show that the unit of *Blv* is the volt. **[3]**

[Total 5 marks]

21 A metal scaffolding pole, 4.0 m long, falls from the top of a building. The metal pole remains horizontal pointing East–West and the local magnetic field, which points North–South, has a horizontal component of 18 μT.

Calculate the p.d. between the ends of the pole when it has fallen for 1.3 s. (Use the formula given in the previous question.) **[Total 4 marks]**

22 An a.c. supply which provides a sinusoidally varying supply of frequency 50 Hz is connected to a capacitor of capacitance 0.47 μF. If the peak value V_0 of the supply is 17 V, what is the peak value of the current I_0 in the capacitor? **[Total 6 marks]**

23 Comment on the statement that 'an electric car produces no pollution'. **[Total 6 marks]**

Stretch and challenge

24 In a 'hybrid' electric car, regenerative braking is achieved by reversing the electric motor so that it becomes a generator (or dynamo). Discuss how the laws of electromagnetic induction can be used to explain how this both re-charges the car's battery pack and causes a braking effect. **[Total 5 marks]**

25 Compare the size of the forces produced by the Earth's gravitational, electric and magnetic fields on a proton that is moving horizontally across England at a speed of about one-tenth of the speed of light. Look up or estimate any numerical values that you need. **[Total 6 marks]**

26 Explain what the area between the induced e.m.f. and the time axis represents in an experiment demonstrating electromagnetic induction. **[Total 4 marks]**

7 Electrons and nuclei

Prior knowledge

You should know from earlier Advanced level work and from GCSE:

- → that a hydrogen atom has a single proton at its centre
- → that the electronic charge is very small
- → that an electric current is equivalent to a flow of charge
- → that like charges repel and unlike charges attract
- → the relative penetrating power of α- and β-particles
- → what makes up α-particles and β-particles
- → that a volt is a name for a joule per coulomb
- → the unit of force is the newton, symbol N.

Test yourself on prior knowledge

1 What are the Z numbers of hydrogen and helium?
2 What is the unit of charge?
3 What is the charge on an electron?
4 Write down the link between an ampere and a coulomb.
5 Why is an α-particle like a helium nucleus?
6 What is a joule a name for?
7 Is force a vector or a scalar quantity?

7.1 The language of the atom

We have known about electrons since the discovery in the late 19th century of thermionic emission – electrons leaving a hot metal surface. These electrons can be accelerated by electric and magnetic fields, which enables the ratio of their charge to mass, e/m, to be determined. The evidence for the very small positive nucleus at the centre of an atom came from experiments performed at Manchester University in 1913. The experiments involved alpha particles that were fired at gold foil and found to be scattered in a special manner – see Section 7.2.

The **proton number** (or atomic number) Z denotes the number of protons in an atomic nucleus of a given element (and is also the number of electrons in a neutral atom of that element), e.g. for gold $Z = 79$.

The **neutron number** N denotes the number of neutrons in the nucleus of an atom, e.g. for gold $N = 118$.

The **nucleon number** (or mass number) A denotes the total number of protons and neutrons in the nucleus of an atom, e.g. for gold $A = 197$.

Table 7.1 Masses of some sub-atomic particles

mass of a proton	$m_p = 1.67262 \times 10^{-27}$ kg
mass of a neutron	$m_n = 1.67493 \times 10^{-27}$ kg
mass of an election	$m_e = 0.00091 \times 10^{-27}$ kg $= 9.1 \times 10^{-31}$ kg to 2 SF

Atomic nuclei are represented by their symbol, with the proton number at bottom left and the nucleon number at top left, e.g. ${}^{197}_{79}Au$ or ${}^{12}_{6}C$.

nucleon number → A

X ← symbol for element

proton number → Z

These symbols and numbers are also used to represent nuclides – the nucleus plus its electrons. Most elements have at least one stable nuclide plus several unstable nuclides. Nuclides of the same element are called **isotopes**, and have different nucleon numbers A because $A = Z + N$.

For example, for the only stable isotope of gold:

$A(197) = Z(79) + N(118)$

For one of the unstable isotopes of gold:

$A(196) = Z(79) + N(117)$

Unstable isotopes decay to stable ones with characteristic half lives – see Chapter 9 on nuclear decay. In the process they emit alpha (α), beta (β) and gamma (γ) radiations. The properties of these are described in Section 9.4 and summarised in Table 9.1 on page 155. However, from your earlier physics work, you will know what α-particles and β-particles consist of, and that γ rays are part of the electromagnetic spectrum.

Key term

The **isotopes** of an element all have the same Z but different N, i.e. the same number of nuclear protons and surrounding electrons, but different numbers of neutrons in the nucleus.

Test yourself

1 Calculate $\Delta m = m_n - m_p$ and compare Δm with m_e.

2 The symbol for a carbon-12 nucleus is ${}^{12}_{6}C$. Explain what this is telling you.

3 The only stable nucleus of sodium can be described as

A (23) = Z (11) + N ( ).

Explain what number should be put in the N 'box'.

4 Name two regions of the electromagnetic spectrum other than γ-waves.

5 An alpha particle consists of X neutrons and Y protons. What numbers are X and Y?

7.2 Alpha particle scattering

The material of the nucleus is very, very dense – about 10^{16} kg m^{-3}! (Compare this with the density of a metal like gold – about 2×10^{4} kg m^{-3}.) How do we know this?

The fact that the atom is mainly 'empty' was first established experimentally in 1913 when α-particles from a natural radioactive source were fired at a very thin sheet of gold. Most passed through undeflected from their path; others were deflected though small angles. A tiny fraction – fewer than 0.01% of the α-particles – were deflected by more than 90°, and from this unexpected scattering the physicist Ernest Rutherford was able to confirm the **nuclear model** of the atom with its tiny, massive, charged nucleus. (That the nucleus is positively charged comes from the fact that a neutral atom contains electrons, which are negatively charged.) He was further able to calculate a rough value for the diameter of the nucleus. Figure 7.1a shows a modern version of the experiment. The paths of three α-particles that pass just above and very close to a gold nucleus are shown in Figure 7.1b.

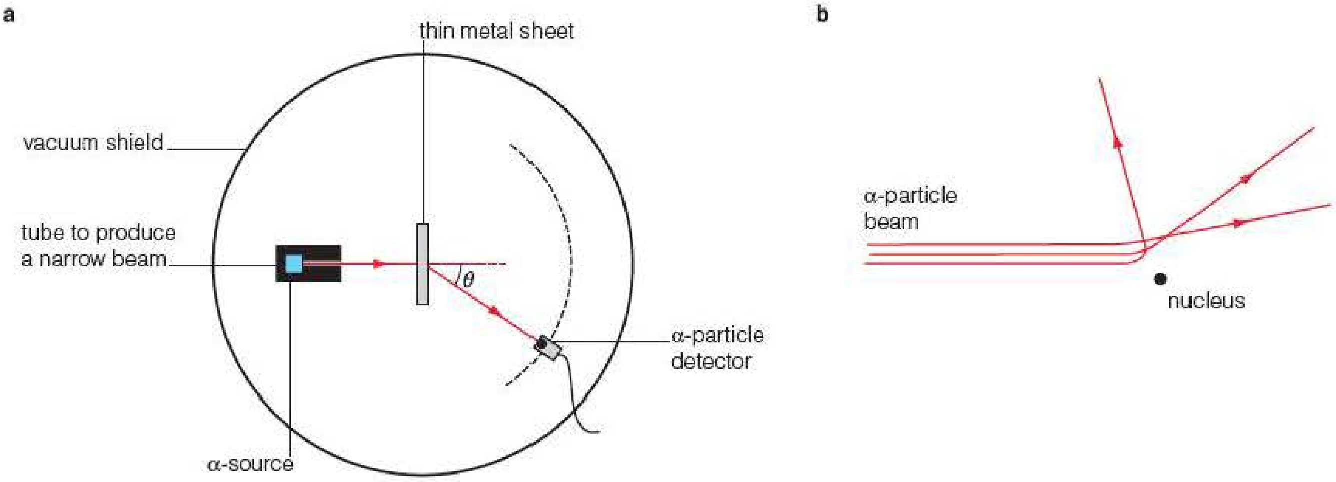


Figure 7.1 α-particle scattering; a) The apparatus b) Some α-particle paths

Example

An α-particle aimed directly towards the nucleus of a gold atom approaches it to within 5.0×10^{-14} m before its motion is reversed.

a) What are the charges on an α-particle and a gold nucleus?

b) Explain why the α-particle rebounds from the gold nucleus.

c) Suggest a value for the diameter of the gold nucleus.

Answer

a) α-particle charge is $+2e = 3.20 \times 10^{-19}$ C;
gold nucleus charge is $+79e = 1.26 \times 10^{-17}$ C

b)
- As the α-particle enters the electric field produced by the positively charged gold nucleus it experiences a repulsive force – Coulomb's law.
- This force gets stronger the closer the α-particle approaches to the nucleus as $F \propto \frac{1}{r^2}$, until all the kinetic energy of the α-particle is 'used up', i.e. stored as electric potential energy *EPE*.
- The α-particle is then repelled by the nucleus, eventually regaining all its initial kinetic energy (i.e. an *elastic* interaction) when it is well away from the gold nucleus.

c) Assuming that the distance of closest approach is approximately equal to the radius of the gold nucleus, then its diameter $\approx 2 \times 5.0 \times 10^{-14}\,\text{m} = 1 \times 10^{-13}\,\text{m}$.

Tip

Although the three bullet points in b) could be written as a single paragraph, it is easier to follow the argument when it is laid out like this. To see where the value for the closest approach comes from, see Exam Practice Question 11 on page 128.

Rutherford was later able to establish conclusively that α-particles, when they gain electrons, are atoms that emit a spectrum exactly like the element helium. For a quantitative treatment of this see the Example below.

Example

Figure 7.2 shows the key stages in an experiment first performed about 100 years ago. The black 'stuff' is liquid mercury, the level of which can be raised and lowered. The gas A in the glass capsule with *very* thin walls is radon, an α-emitter. Helium gas at extremely low pressure is gradually formed in the evacuated capsule B.

Write a bullet-point description of what is happening in this experiment and what it is designed to prove.

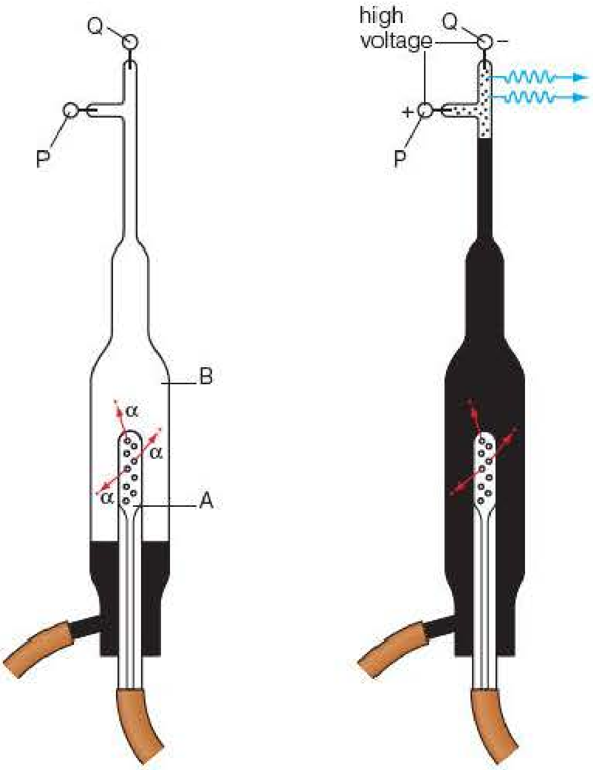


Figure 7.2 The Rutherford and Royds experiment

Answer

- With capsule A filled with radon gas, any α-particles from the decaying radon pass through the very thin walls and emerge into the larger capsule B.
- Here the α-particles gain electrons and become atoms.
- Any gas atoms in capsule B can be compressed, by raising the mercury level, and forced into the top section between points P and Q.
- A high voltage (potential difference) between P and Q then produces the emission of light – a spectrum which is characteristic of any gas between P and Q.
- This gas was identified as helium gas, thus suggesting that if α-particles gain electrons to become gas atoms, i.e. the nucleus of helium is identical to the α-particle.

Tip

This is an example of a 'synoptic' exercise, where you bring an understanding of different areas of physics to focus on an unusual situation. Your examination will contain synoptic exercises.

In the Rutherford and Royds experiment, the wall of capsule A was only 0.01 mm thick, and the compression of the gas in capsule A was only performed after six days! To ensure it was a 'fair test', capsule A was later filled with helium gas, but no helium was found in capsule B, even after several days.

7.3 Thermionic emission

When a piece of metal is heated to a high temperature, negatively charged electrons 'bubble' out of its surface. Of course, they will be attracted back to the surface by the positively charged protons they leave behind. But if a positively charged plate is placed near the piece of metal in a vacuum, the electrons accelerate towards it and can be made into a narrow beam. In this arrangement, called an **electron gun**, the metal is usually heated by a resistor (connected to a 6 V supply) placed behind it and the narrow beam is produced by making a small hole in the positive plate (which is at a potential of about +2000 V). Electron beams such as those found in classroom cathode ray oscilloscopes were described on page 100. Such beams accelerated in a vacuum through 2000 V contain electrons moving at very high speeds.

Summarising from earlier chapters:

- Force on an electron in an electric field $F_E = eE$ parallel to the field.
- Force on an electron moving in a magnetic field $F_B = Bev$ perpendicular to the field.

Figure 6.10a in the previous chapter shows how a beam of electrons can follow a straight path through E and B fields that are perpendicular to one another, and Figure 6.10b helps to explain why electron beams can be made to follow circular paths such as that shown here in Figure 7.3.

In Figure 7.3 the electrons are projected horizontally to the right from an electron gun in a magnetic field that is directed out of the plane of the photo. As electrons are negatively charged, the left-hand rule shows that the Bev force is centripetal and remains so as the electrons move round part of a circle. (The bright curve that enables us to 'see' where they are is caused by having a very low-pressure gas in the tube. A few electrons ionise the gas atoms, which then return to their ground state by emitting visible photons.)

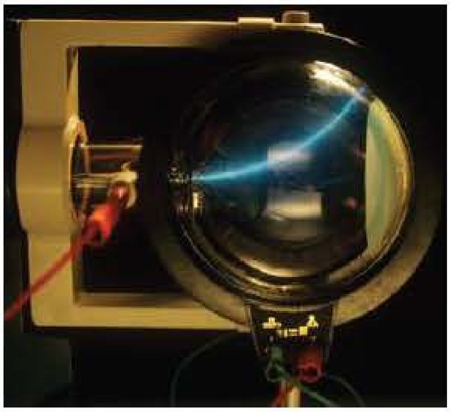


Figure 7.3 An electron beam made visible

Test yourself

6 A pool full of water has a volume of 1200 m^3. The density of water is 1000 kg m^{-3}. What mass of water does the pool contain?

7 Describe, for a non-scientist, what an atom of gold is like.

8 a) Express 0.01% as a fraction
b) Express 90° in radians.

9 Describe an experiment which shows that α-particles cannot penetrate a piece of paper.

10 Draw a sketch of the electron gun described at the beginning of Section 7.3.

11 What are the units of
a) an electric field, and
b) a magnetic field?

7.4 Some useful algebra

Applying Newton's second law to an electron in the beam in Figure 7.3 above gives:

$$\frac{mv^2}{r} = Bev$$

$$\Rightarrow mv = Ber$$

As $mv = p$, the momentum of the electron (even for very large v), this can be rewritten as $p = Ber$, which leads to

$$r = \frac{p}{Be}$$

This tells us that the radius of the circle in which the electron moves is proportional to the momentum with which the electron is fired. (B and e are fixed in a given experiment.)

Example

In Figure 7.3 the magnetic flux density is 0.50 mT and the radius of the circle in which the electrons are moving is 20 cm.

a) Calculate the speed at which the electrons are moving.

b) Deduce through what p.d. the electrons were fired.

Answer

a) As $p = Ber$ we have $mv = Ber$ and therefore $v = \frac{Ber}{m}$

Using $e = 1.6 \times 10^{-19}\,\text{C}$ and $m_e = 9.1 \times 10^{-31}\,\text{kg}$ gives

$$v = \frac{(0.50 \times 10^{-3}\,\text{N A}^{-1}\text{m}^{-1}) \times (1.6 \times 10^{-19}\,\text{C}) \times (20 \times 10^{-2}\,\text{m})}{(9.1 \times 10^{-31}\,\text{kg})}$$

$$= 1.8 \times 10^{7}\,\text{m s}^{-1}$$

b) Kinetic energy of each electron $= \frac{1}{2}mv^2$

$$\frac{1}{2}mv^2 = \frac{1}{2}(9.1 \times 10^{-31}\,\text{kg}) \times (1.8 \times 10^{7}\,\text{m s}^{-1})^2 = 1.5 \times 10^{-16}\,\text{J}$$

$$= \frac{1.5 \times 10^{-16}\,\text{J}}{(1.6 \times 10^{-19}\,\text{J eV}^{-1})}$$

= 940 eV (or 880 eV if numbers held in calculator)

So the electrons were fired through a potential difference of 940 V.

Tip

If you hold numbers in your calculator as you work through a multi-stage calculation such as this, you sometimes get a slightly different answer. Examiners are aware of this.

Referring back to the above algebra:

From $mv = Ber$ we get $\frac{v}{r} = \frac{Be}{m}$

and as $\frac{v}{r} = \omega = 2\pi f$ this tells us that $2\pi f = \frac{Be}{m}$

So the frequency at which the electron circles does not depend on its initial momentum, and hence on its initial kinetic energy, *provided its mass remains constant*. This is the key to the operation of the cyclotron.

7.5 The cyclotron

A cyclotron is a type of **particle accelerator**. The theory that leads to $2\pi f = \frac{Be}{m}$ for the frequency of charged particles circling in a magnetic field enables protons to be accelerated to high energies in a circulating beam.

Figure 7.4 shows the principle of operation of a cyclotron. A source of protons – ionised hydrogen atoms – is placed at the centre of two 'Dees'. These Dees are semi-circular flat metal boxes, open at their diameter, and seen here from above. The Dees are connected to a very high-frequency alternating voltage $V = V_0 \sin 2\pi ft$ and are situated in a strong vertical magnetic field, indicated here by the × × ×. The whole set-up is in a good vacuum.

This is how it works:

- Some charged protons that emerge from the central source moving in a horizontal plane enter D_1,
- where they follow a semi-circular path under the action of the magnetic field.
- When they next arrive at the narrow gap between the Dees they 'see' the opposite Dee to be at a negative potential and are accelerated across the gap by the electric field between the Dees (not shown).
- They now enter D_2 with extra energy eV,
- where they continue in a second semi-circle at a higher speed due to the extra energy (and therefore greater r) and arrive back at the gap only now to 'see' D_1 to be at a negative potential, because the voltage supply is alternating at exactly the right frequency, a frequency equal to twice the frequency of circulation of the protons.
- Extra energy eV is again added and this continues, with the radius of the proton's path increasing each time until they leave D_2 tangentially as shown.

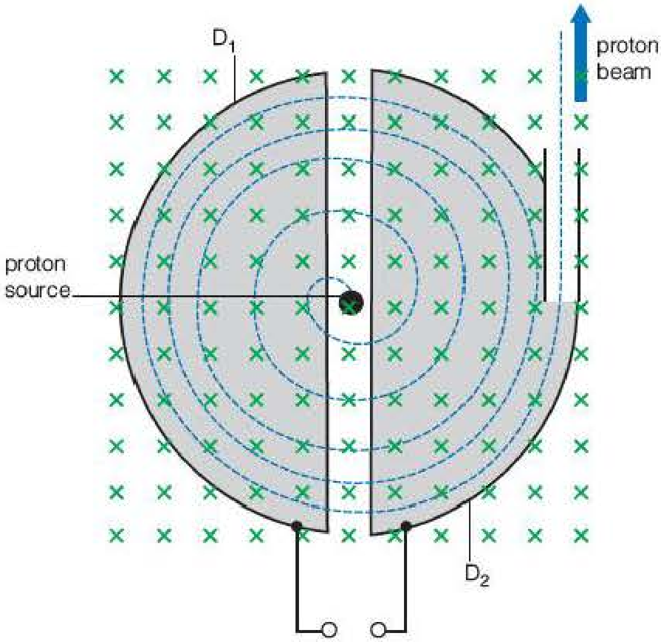


Figure 7.4 The Dees of a proton cyclotron

Tip

Again a detailed explanation is often best set out as a series of bullet points.

Example

Protons are accelerated in a cyclotron in which the magnetic flux density 'through' the Dees is 1.2 T and the voltage between the Dees when the protons cross the gap is 10 kV.

a) Show that the frequency of the voltage supply necessary for it to be synchronised with the arrival of the protons at the gap between the Dees is 36 MHz.

b) How many circles must the protons make in order to reach an energy of 10 MeV?

Answer

a) The mass and charge of a proton are $m_p = 1.7 \times 10^{-27}\,\text{kg}$ and $e = 1.6 \times 10^{-19}\,\text{C}$.

Substituting in $2\pi f_p = \frac{Be}{m_p}$ gives

$$f_p = \frac{1.2\,\text{NA}^{-1}\text{m}^{-1} \times 1.6 \times 10^{-19}\,\text{C}}{2\pi \times 1.7 \times 10^{-27}\,\text{kg}}$$
$$= 1.8 \times 10^7\,\text{Hz} = 18\,\text{MHz}$$

In a cyclotron, the protons only describe half a revolution before they meet the gap again. This means that the frequency of the alternating voltage supply must be twice the frequency of revolution of the protons, i.e. $2 \times 18\,\text{MHz} = 36\,\text{MHz}$.

b) At each crossing the protons gain 10 keV of energy. As they cross twice per circle, they gain 20 keV per circle. Therefore the number of circles needed to reach 10 MeV is $\frac{10\,\text{MeV}}{20\,\text{keV}} = 500$.

Tip

You should not try to remember and quote the formula used in the previous Example. Always start from basic ideas like $\frac{mv^2}{r} = Bev$ and $\omega = \frac{v}{r} = 2\pi f$.

The 10 MeV reached in the previous Example is about the limit for a cyclotron accelerating protons, as the synchronism only lasts so long as m_p remains constant. To overcome this problem, synchro-cyclotrons were developed, in which the frequency of the alternating p.d. was lowered in such a manner as to keep accelerating the protons as their (relativistic) mass increased. Cyclotrons can be used to accelerate other positively charged ions, and are sometimes used to accelerate the protons used in modern medical treatments such as proton beam therapy (see pagr 173).

7.6 Linear accelerators

You have probably seen a small Van de Graaff accelerator during your physics studies (see Figure 4.15). In research laboratories large ones can be used to accelerate charged particles up to energies of around 10 MeV. To accelerate protons (or other charged particles) to energies beyond this, a linear accelerator or **linac**, of a different design detail for each particle, is used. In the electron linac, shown in Figure 7.5, the electrons are given energy as they pass between charged metal tubes. As in a cyclotron, the energy is delivered to the charged particles by the electric field in the small gap between the tubes.

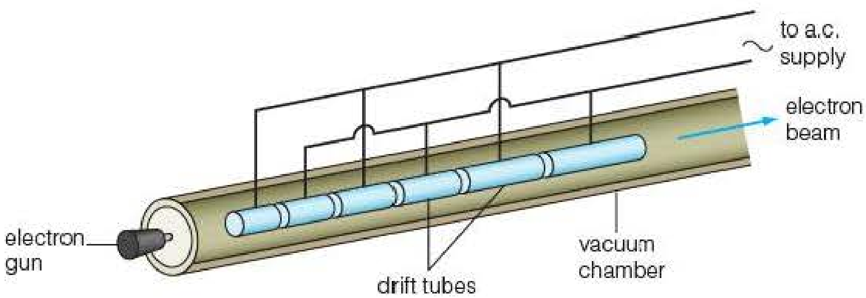


Figure 7.5 An electron linac

The tubes are connected to a high-frequency alternating voltage supply, and the lengths of the tubes are calculated so that there is always a positive charge, as seen by the accelerating electrons, on the 'next' tube. In this way a bunch of electrons fired by the electron gun is attracted to the first tube but while it is in that tube the charge on the next tube again becomes positive, thus attracting the bunch leaving the first tube; and so on. The length of the tubes increases as the speed of the bunch of electrons increases so that the time the electrons spend in each tube is the same.

In any one tube the electrons travel at a steady speed – they drift – there being no electric field inside the metal tubes. This is also the case for charged particles while they are inside a Dee in the cyclotron.

Electron linacs are now routinely used in hospitals to produce beams of high-energy electrons. When the beam hits a tungsten target the result is a beam of X-rays – see Figure 9.4.

Figure 7.6 shows the structure inside the vacuum chamber of the proton linac at Fermilab (Fermi National Accelerator Laboratory) near Chicago in the USA. The drift tubes are clearly seen. This linac is 150 m long and can accelerate protons to 400 MeV.

At Stanford in the USA, an electron linac that is 3 km long can accelerate electrons to 50 GeV (50 000 MeV). (Once the electrons have a very high energy – move at close to the speed of light – the tubes in the linac do not get noticeably longer.)

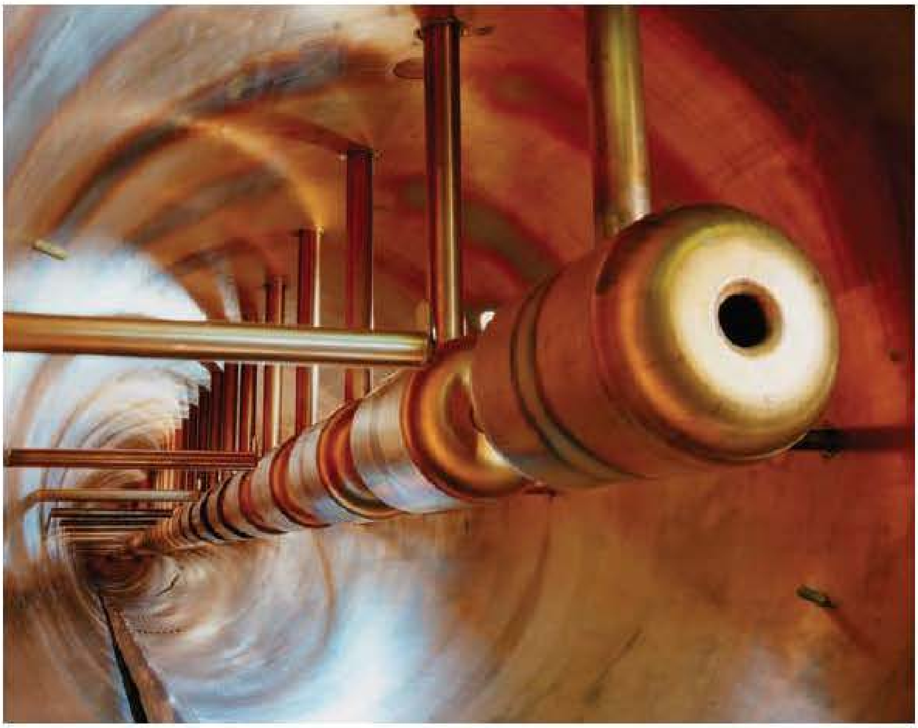


Figure 7.6 Inside the proton linac at Fermilab

Example

A modern proton linac has 420 metal tubes. Its operating voltage frequency is 390 MHz.

Calculate how long it takes a proton to travel along this linac. (Assume the gaps between the tubes are very small.)

Answer

The protons drift for half a cycle in each tube (because every second tube is connected to the same side of the alternating supply).

$\therefore$ time in each tube $= \frac{1}{2} \times \frac{1}{f} = \frac{1}{(2 \times 390 \times 10^6\,\text{Hz})} = 1.28 \times 10^{-9}\,\text{s}$

So the time to travel down 420 tubes $= 420 \times (1.28 \times 10^{-9}\,\text{s}) = 5.4 \times 10^{-7}\,\text{s}$

Test yourself

12 What is the speed of light in a vacuum?

13 Explain why one electron-volt is equivalent to 1.6×10^{-19} joules.

14 Check that the units of $\frac{Be}{m}$ are s^{-1} or hertz.

15 Why is 10 MeV about the limit for the energy of protons accelerated in a cyclotron?

16 Name two features that are common to cyclotrons and linacs.

7.7 Particle detectors

The history of nuclear physics in the 20th century closely followed improvements in the experimental methods available for detecting nuclear particles. Four Nobel Prizes in Physics were awarded in this field: Table 7.2 lists these, including the year (in brackets) of the first use of the detection method. Cecil Powell, the 'father' of Particle physics, sometimes used a team of technicians to search for any interesting feature in his photographic emulsions. In the late 20th century Tim Berners-Lee invented the World Wide Web in order to send data from CERN's experiments to universities all over the planet for them to study. (And – luckily for all of us – he did not make any charge for the use of the web!)

Table 7.2 Nobel Prizes in Physics in the field of particle detection

Date	Prize winner	Detection device
1927	CRT Wilson (Scottish)	Cloud chamber (1911)
1950	CF Powell (English)	Photographic emulsions (1934)
1960	DA Glaser (American)	Bubble chamber (1960)
1992	G Charpak (French)	Drift chamber (1992)

There may be others added to this list in the present century, if the designers of the huge detectors at CERN and other high-energy laboratories are honoured. Figure 7.7 shows such a massive detector being assembled. Note the size of the physicists in their hard hats on the gantry!

The basic principle behind the detection of charged particles has not changed, as energetic charged particles cause ionisation in any material through which they pass.

The cloud chamber made use of ionisation in super-saturated air and the bubble chamber of ionisation in super-saturated liquid hydrogen. The ionised molecules along the particle's path form centres for the formation of tiny liquid water drops and tiny hydrogen gas bubbles respectively. These can be illuminated and photographed: in both cases the 'track' of the ionising particle is thus made visible.

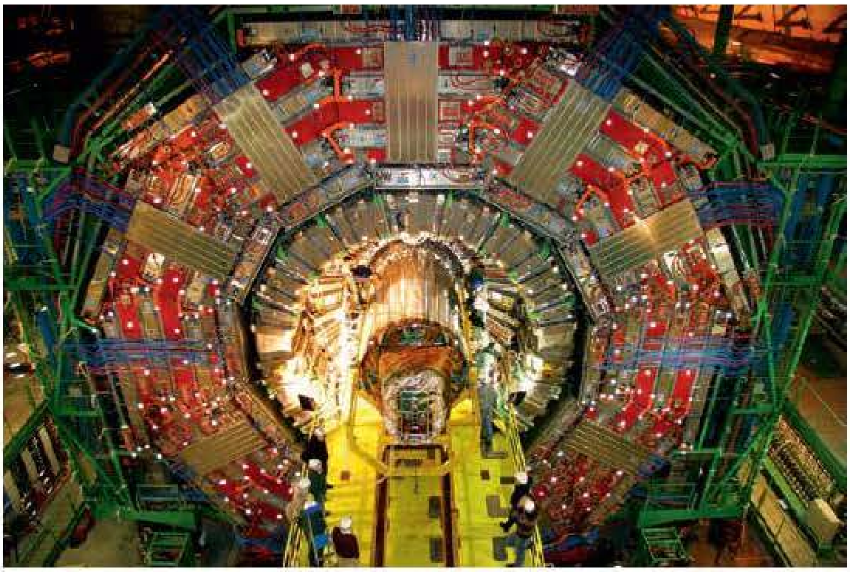


Figure 7.7 One of the huge detectors for the Large Hadron Collider at CERN

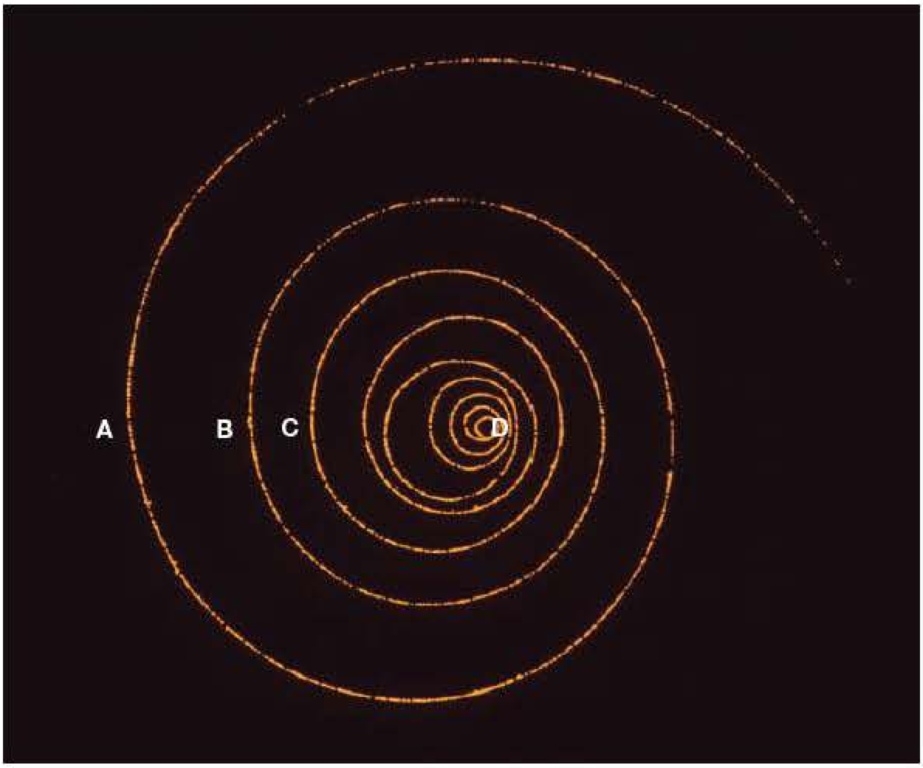


Figure 7.8 A bubble chamber photograph

Figure 7.8 is a photograph of a spiralling curved track in a bubble chamber. This was caused by an energetic electron that entered the chamber at the top right. You may guess (correctly) that there was a magnetic field perpendicular to the electron's spiral, and the left-hand rule will tell you that the field was directed out of the plane of the paper (because an electron is negatively charged). You may also realise that the electron is gradually losing energy. There are two ways of deducing this:

1 Each ionisation will take a few electronvolts from the kinetic energy of the electron, and there are many hundreds if not thousands of ionisations in the spiral.
2 The radius of the spiral is getting smaller, and on page 118 we proved that $r = \frac{p}{Be}$, that is the radius of the electron's path is proportional to its momentum, and thus depends on its kinetic energy.

(The labels A, B, C and D on this photograph are used in Exam Practice Question 21 at the end of this chapter.)

Cloud chamber photographs can be seen in Figures 1.20, 8.3 and 9.11.

High-energy charged particles can also be detected by the sparks they produce between a series of thin sheets of charged metal foil. Such a detector is called a spark chamber and was the forerunner of modern drift chambers.

Figure 7.9 shows how charged particles can be 'tracked' by the sparks they produce. Clearly there was no charged particle in the centre of the photograph – perhaps the incoming particle stopped there, after it had knocked a neutron out of an atom's nucleus.

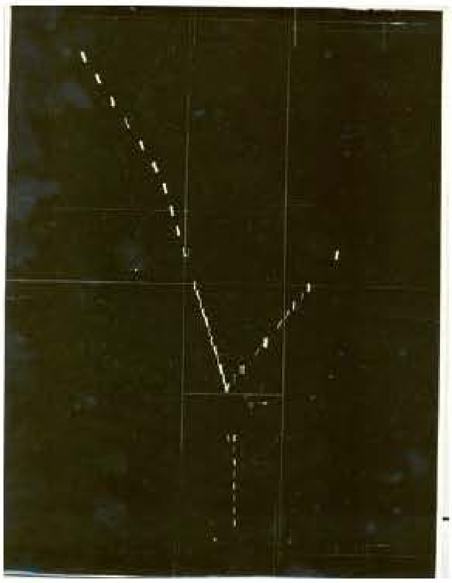


Figure 7.9 A spark chamber photograph

7.8 Einstein's equation

> **Tip**
>
> Be sure to square *c* when using Einstein's equation.

Einstein's famous equation is best written as

$$\Delta E = c^2 \Delta m$$

where $c = 3.00 \times 10^8\,\mathrm{m\,s^{-1}}$ is the speed of light in a vacuum. The Δm tells us that particles have a mass when they are at rest – their **rest mass** m_0 – but a greater mass $m_0 + \Delta m$ when they have extra energy ΔE. It further tells us what the 'rate of exchange' is between mass and energy. An extra energy (perhaps kinetic or internal or elastic) ΔE is equivalent to an extra mass $\frac{\Delta E}{c^2}$.

> **Example**
>
> Calculate how much 'heavier' a 12 V, 50 A h car battery is when fully charged than when totally discharged. State any assumption you make.
>
> **Answer**
>
> Assume that the battery is at the same temperature in both circumstances.
>
> A battery with a capacity of 50 A h will discharge
>
> $$50\,\mathrm{C\,s^{-1}} \times 3600\,\mathrm{s} = 180\,000\,\mathrm{C}$$
>
> The total electrical energy stored in a fully charged 12 V battery is therefore
>
> $$\Delta E = 180\,000\,\mathrm{C} \times 12\,\mathrm{J\,C^{-1}} = 2\,160\,000\,\mathrm{J}$$
>
> This is equivalent to a mass increase of
>
> $$\Delta m = \frac{2\,160\,000\,\mathrm{J}}{(3.00 \times 10^8\,\mathrm{m\,s^{-1}})^2} = 2.4 \times 10^{-11}\,\mathrm{kg}$$
>
> The battery is therefore $2.4 \times 10^{-10}\,\mathrm{N}$ heavier when fully charged. (Negligible!)

The Example above, like many others in our everyday world, shows us that we can normally treat the conservation of mass and the conservation of energy as two separate principles. But in the sub-atomic world, the world of high-energy electrons and nuclear particles, we need to apply the equivalence of mass and energy – Einstein's principle of the **conservation of mass–energy**.

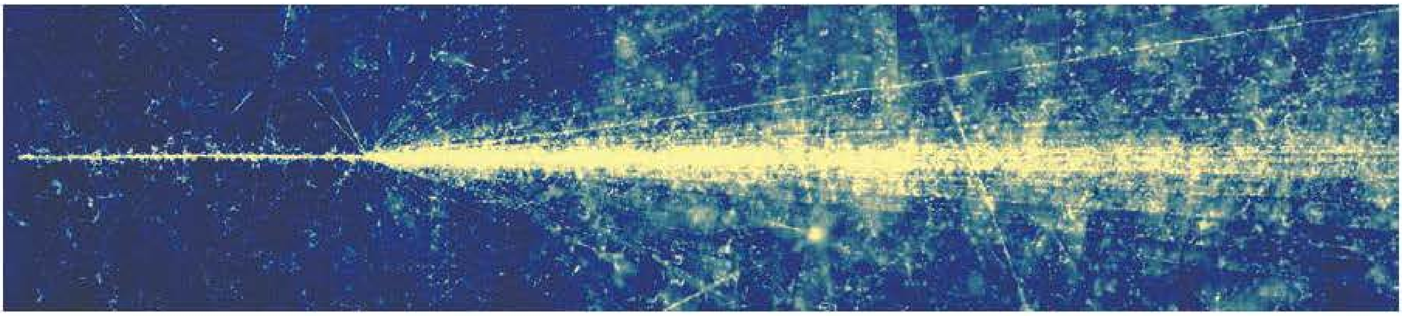


Figure 7.10 Matter from energy!

A dramatic demonstration that kinetic energy and mass are interchangeable is shown in Figure 7.10. Here an iron nucleus approaching the Earth from deep space strikes a silver nucleus in a photographic emulsion raised high into the atmosphere by a balloon. The huge kinetic energy of the iron nucleus is used to create about 750 new ionising particles, the total mass of which multiplied by c^2 equals the loss of kinetic energy in the collision.

Example

Calculate the potential difference through which an electron, of rest mass $m_0 = 9.11 \times 10^{-31}$ kg, must be accelerated in order to double its effective mass.

Answer

Suppose the p.d. is V, then the energy ΔE given to the electron = eV.

In this case the equation $\Delta E = c^2\Delta m$ becomes $eV = c^2\Delta m$, and Δm becomes equal to m_0 because the mass is to be doubled.

Substituting: $(1.6 \times 10^{-19}\,\text{C}) \times V = (3.00 \times 10^8\,\text{m}\,\text{s}^{-1})^2 \times (9.11 \times 10^{-31}\,\text{kg})$

$$\Rightarrow V = 5.1 \times 10^5\,\text{V or } 510\,\text{kV}$$

Let us try to calculate the speed of an electron accelerated through 5000 kV in a linac producing X-rays.

Starting with $\frac{1}{2}m_e v^2 = e\text{V}$

and substituting $m_e = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C and $V = 5000\,000$ V

$$\Rightarrow v = 1.3 \times 10^9\,\text{m}\,\text{s}^{-1} \text{ to 2 SF}$$

This is *not possible* (and we haven't made a mistake in using the calculator!). Nothing can travel faster than the speed of light, $3.0 \times 10^8\,\text{m}\,\text{s}^{-1}$. So what has 'gone wrong'? We used Newton's formula for kinetic energy, and this assumes that objects have the same mass at all speeds. The theory of **special relativity** proposed by Einstein predicted that the faster things go, the heavier they become – as the Example above shows. (The graph in Figure 8.5 on page 134 shows this quantitatively.)

The electrons in the Stanford linac can acquire energies of 50 GeV, which is a factor of 10^4 greater than 5000 keV. The rest mass of the Stanford electrons becomes negligible compared with their total effective mass – quite the opposite result from the example of the charged car battery!

Test yourself

17 What feature is common to most methods used in the detection of sub-atomic particles?

18 What particles do not show up on photographic emulsions used to detect sub-atomic particles?

19 A bill for electricity states that the customer has 'used' 2500 kW h of electricity. Explain this statement.

20 Einstein's equation gives us $\frac{\Delta E}{c^2} = \Delta m$. *Show* that $\text{J}\,\text{s}^2\,\text{m}^{-2}$ is equivalent to 1 kg.

7.9 Particle interactions

Figure 1.20a on page 16 shows an alpha particle striking a helium nucleus in a cloud chamber. As part of the Example on page 16 the *total* vector momenta before and after the collision, in the direction of the incoming α-particle, were calculated and found to be equal. In collisions momentum is *always* conserved; in nuclear collisions charge and mass–energy are also conserved.

In Figure 1.20b kinetic energy, a scalar quantity, is conserved as (try it):

$$\tfrac{1}{2}m_\alpha \times (1.50\times10^7\,\mathrm{m\,s^{-1}})^2 = \tfrac{1}{2}m_\alpha \times (1.23\times10^7\,\mathrm{m\,s^{-1}})^2 + \tfrac{1}{2}m_{He} \times (0.86\times10^7\,\mathrm{m\,s^{-1}})^2$$

After cancelling the halves and the equal masses of m_α and m_{He}, each side is numerically 2.25×10^{14}. Here the speeds are not high enough (less that one tenth the speed of light) for the kinetic energies to represent a noticeable extra mass.

Sometimes collisions occur in which not all the particles are ionising because they are uncharged. Such particles leave no tracks in a bubble chamber.

Example

There is a magnetic field into the plane of the bubble chamber tracks represented in Figure 7.11.

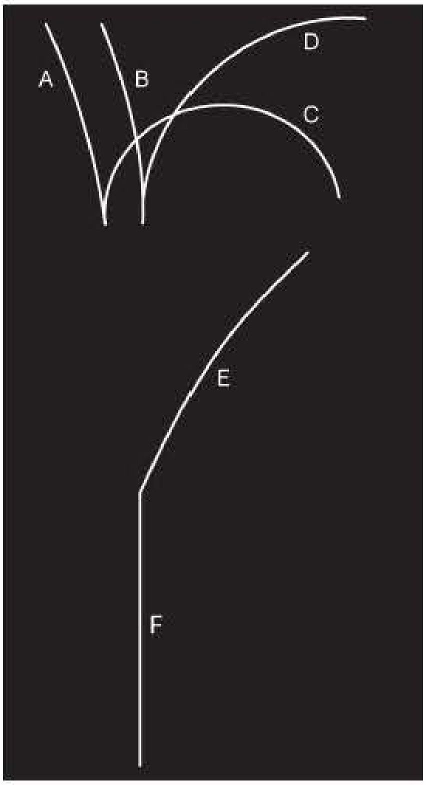


Figure 7.11

a) State and explain any deductions that can be made about the nature of the particles A, B, C, D, E and F.

b) Deduce the existence and nature of any other particles in the interactions between particles E and F.

In a) 'explain any deductions' implies that some deductions will be quantitative.

Answer

a) Each of the particles A, B, C, D, E and F has an electric charge as each provides a track showing ionisation in the bubble chamber.

A and B are positively charged (from the left-hand rule) and the curvature of their tracks are similar. If they have equal charge, then the momentum of each is equal.

C and D must be negatively charged. As $r_D \approx 2r_C$ then perhaps $Q_C = Q_D$ and the momentum of each would then be $p_D = 2p_C$ (since $p = BQr$).

E is a negatively charged particle. F's track is almost straight so it has a very high momentum; it must, however, be negatively charged to conserve charge when F knocks E forward.

b) There must be two neutral particles that do not ionise, moving from the point where F makes a collision producing E. Perhaps they are γ-photons. Each then produces a pair of oppositely charged particles A–C and B–D to conserve charge where they are produced.

There is some guesswork in the answer above, but it illustrates how photographs of particle tracks and interactions can yield information. Nowadays powerful computing techniques are employed to make deductions from the information gathered by vast detecting systems at, for example, the LHC at CERN.

Exam practice questions

1 The mass of a proton is N times the mass of an electron. A sensibly rounded value for N would be about:

A 200 C 2000

B 550 D 5500 **[Total 1 mark]**

2 The proton number of the element uranium is 92. The number of neutrons in an atom of the isotope ^{235}U is:

A 92 C 146

B 143 D 235 **[Total 1 mark]**

3 Which of the following is not a possible unit for momentum?

A $\mathrm{N\,s}$ C $\mathrm{J\,m^{-1}\,s}$

B $\mathrm{kg\,m\,s^{-2}}$ D $\mathrm{kg\,m\,s^{-1}}$ **[Total 1 mark]**

4 A singly ionised helium atom and an ionised hydrogen atom are each accelerated in a vacuum through a potential difference of 200 V. The kinetic energy gained by the helium is:

A a quarter of the energy gained by the hydrogen

B half the energy gained by the hydrogen

C the same as the energy gained by the hydrogen

D twice the energy gained by the hydrogen. **[Total 1 mark]**

5 An energy of 12 nJ is equivalent to an energy of:

A 75 keV C 7.5 GeV

B 75 MeV D 75 GeV. **[Total 1 mark]**

6 According to the Einstein mass-energy relationship, using $c = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}$, a mass difference of 4.0×10^{-15} kg is equivalent to an energy difference of

A 360 J C 120 pJ

B 40 J D 0.36 J. **[Total 1 mark]**

7 The curved path in Figure 7.3 is visible because:

A electrons glow when they travel at high speeds

B electrons leave a trail of tiny water drops along their path

C gas atoms in the tube are attracted to one another and form particles

D gas atoms in the tube are ionised by the electrons. **[Total 1 mark]**

8 Protons being accelerated in a cyclotron move at a constant speed while inside the Dees, because

A they are shielded from the magnetic field while inside a Dee

B there is no force acting on them along their path

C only while in a Dee are they travelling in a vacuum

D the alternating voltage is synchronised to their movement.

[Total 1 mark]

9 Take the density of nuclear matter as $1.0 \times 10^{16}\,\text{kg}\,\text{m}^{-3}$. It can then be deduced that

a) the volume of a gold nucleus $^{197}_{79}\text{Au}$ is:

A $0.2 \times 10^{-41}\,\text{m}^3$ C $1.3 \times 10^{-41}\,\text{m}^3$

B $2.0 \times 10^{-41}\,\text{m}^3$ D $3.3 \times 10^{-41}\,\text{m}^3$. (1)

b) the radius of a gold nucleus is about:

A $2.0 \times 10^{-14}\,\text{m}$ C $3.0 \times 10^{-14}\,\text{m}$

B $5.0 \times 10^{-14}\,\text{m}$ D $6.0 \times 10^{-14}\,\text{m}$. (1)

[Total 2 marks]

10 Figure 7.12 shows the path of an α-particle scattered by a gold nucleus. Copy the diagram.

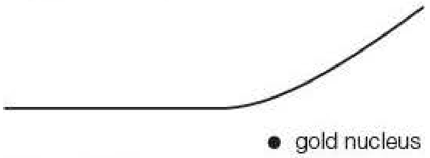


Figure 7.12

a) i) Label with the letter M the point on the α-particle's path where it feels the maximum force from the gold nucle us.

ii) Add the path followed by an α-particle moving initially along the same line but having a smaller energy than that in the diagram.

iii) Add the path followed by the original α-particle but which approaches a nucleus carrying a smaller charge than gold. [3]

b) Identify the two new tracks from **a ii)** and **iii)** with suitable labels. [1]

[Total 4 marks]

11 The electric potential energy (*EPE*) of a particle of charge Q_1 a distance r from a nucleus of charge Q_2 is given by the relationship

$\text{EPE} = \frac{kQ_1Q_2}{r}$ where k or $\frac{1}{4\pi\varepsilon_0} = 9.0 \times 10^9\,\text{N}\,\text{m}^2\,\text{C}^{-2}$

a) Calculate the EPE of an α-particle 5.0×10^{-14} m from the centre of a gold nucleus. (The charge on an α-particle and a gold nucleus is $2e$ and $79e$ respectively.) [2]

b) Express this energy in MeV and explain how an α-particle with this kinetic energy will slow to zero and then reverse as it 'hits' a gold nucleus head on. [4]

[Total 6 marks]

12 The 'radius' of a nucleus and the 'radius' of an atom are about 10^{-14} m and 10^{-10} m respectively. What fraction of a typical atom is therefore 'empty space'? **[Total 3 marks]**

13 In a hydrogen atom the average distance apart of the proton and the electron is 5.3×10^{-11} m. Assuming the electron to be a particle orbiting the proton, explain how the frequency f with which the electron is orbiting can be calculated. Deduce a value for f, and suggest in which part of the electromagnetic spectrum such a frequency lies.

[Total 5 marks]

14 Protons each of momentum p move in a circle of radius r when projected perpendicular to a magnetic field of flux density B. It is shown on page 118 that $r = \frac{p}{Be}$.

For speeds that are not too high, show that the kinetic energy E_k of protons moving in a circle is proportional to r^2, provided B, e and m are constant. **[Total 5 marks]**

15 The gap between two 'tubes' in a proton linac is 25 mm. The alternating voltage has a maximum value of 200 kV. What is the mean value of the accelerating electric field between the tubes? **[Total 4 marks]**

16 Prove that the synchronous frequency in a proton cyclotron is given by $f = \frac{Be}{2\pi m_p}$. **[Total 4 marks]**

17 The track of an α-particle ionising air molecules in its path is a straight line. Each ionisation requires 30 eV. The graph in Figure 7.13 shows the number of ionisations an α-particle makes as it moves from its source to the end of its track.

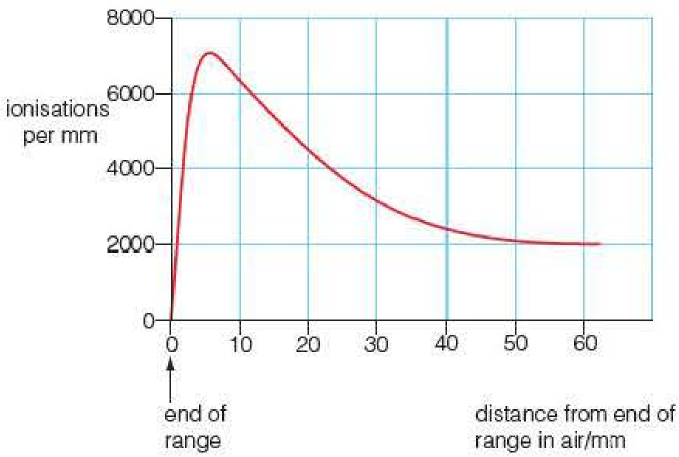


Figure 7.13

Use the graph to estimate the initial energy of an α-particle that travels 60 mm in air. **[Total 5 marks]**

18 Refer to Figure 1.20b on page 16. Show that the momentum at right angles to the initial direction of the α-particle after the collision is zero. What assumption do you make? **[Total 6 marks]**

19 Protons are accelerated to an energy of 20 MeV in a cyclotron. Calculate their relativistic mass increase and express this as a percentage of their rest mass 1.67×10^{-27} kg. **[Total 5 marks]**

Stretch and challenge

20 Figure 7.14 shows three drift tubes that form part of an electron linac – a linear accelerator. The charges on the tubes at one instant are shown.

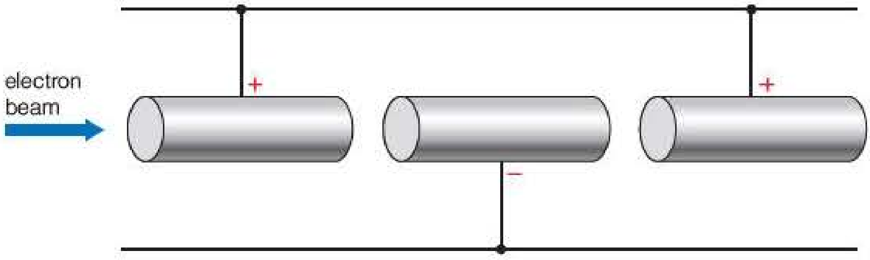


Figure 7.14

a) Copy the diagram. Add electric field lines to your diagram and explain the function of these electric fields on the workings of this electron linac. **[3]**

b) i) Why are the tubes referred to as 'drift tubes'?

ii) Suggest why the tubes appear to be of equal lengths in the diagram. **[6]**

c) Calculate the increase in mass of an electron accelerated through a p.d. of 8.4 GV and comment on the result. **[3]**

d) Show that the units of the calculation in part **c)** are correct. **[3]**

[Total 15 marks]

21 Refer to Figure 7.8. The momentum of an electron when moving perpendicular to a magnetic field of flux density B is given by $p = Ber$, where r is the radius of its path.

a) Measure the radius of the electron's path at A, B and C. Take the centre of its path as D. It is helpful to use the edge of a white sheet of paper under your ruler. (The photograph is ⅔ real size.) **[3]**

b) Calculate the momentum of the electron at A, B and C, given that $B = 1.2$T. **[4]**

c) This high-energy electron is moving at a speed close to $v = 3.0 \times 10^8 \,\mathrm{m\,s^{-1}}$ (to 2 SF) at each point. Calculate values for the mass of the electron at A, B and C. **[3]**

d) How do your answers compare with the rest mass 9.1×10^{-31} kg of an electron? **[1]**

[Total 11 marks]

8 Particle physics

Prior knowledge

You should be familiar with the Prior Knowledge required for the previous chapter 'Electrons and nuclei'. You should also know from earlier work covered in your Advanced level and GCSE:

- ➜ that magnetic flux density or magnetic field strength is measured in teslas, T
- ➜ that a tesla is a name for $N\,A^{-1}\,m^{-1}$
- ➜ that the speed of e-m radiation is $3.0 \times 10^8\,m\,s^{-1}$ in a vacuum
- ➜ that linear momentum ($p = mv$) is a vector and kinetic energy ($E_k = \frac{1}{2}mv^2$) is a scalar
- ➜ how to make things the subject of equations, e.g.

$$E_k = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2E_k}{m}}$$

- ➜ the relationship that centripetal acceleration $a = \frac{v^2}{r}$ or $r\omega^2$
- ➜ that the force on a charged particle moving ⊥ to a magnetic field is $B_{\perp}Qv$
- ➜ the meaning of prefixes such as M (mega = 10^6) and G (giga = 10^9)
- ➜ Einstein's famous equation relating energy and mass $\Delta E = c^2\Delta m$
- ➜ that electromagnetic waves can be described as a flow of 'particles' called photons
- ➜ that the energy of a photon of frequency f is hf, where h is the Planck constant h of numerical value 6.63×10^{-34} J s
- ➜ the de Broglie equation $\lambda = \frac{h}{p}$ relating wave and particle behaviour.

Test yourself on prior knowledge

1 Write down the relationship for the force F on a wire of length ℓ carrying a current I placed ⊥ to a magnetic field of magnetic flux density B.

2 What is the formula if the wire is placed at an angle θ to the magnetic field?

3 What is meant by the statement 'A charged particle with an energy of 6 MeV'?

4 In the form of Einstein's equation $E = m_0c^2$, what is meant by m_0?

5 Show that BQv has the unit N (newton).

6 If a charge Q is moving with speed v in a magnetic field of strength B, under what circumstances will the force on the charge, $F = BQv$, be zero?

7 What is the unit of the Planck constant?

8 What is the energy of a photon that behaves like a wave of wavelength λ?

9 Calculate the de Broglie wavelength associated with an electron moving with a momentum of 2.73×10^{-22} N s.

8.1 The discovery of quarks

The scattering of α-particles by a thin sheet of gold was described in Section 7.2. This famous experiment was performed in Manchester in 1913. In the late 1960s, a similar experiment was performed at Stanford in the USA, but this time the bombarding particles were high-energy electrons and the target was liquid hydrogen in a bubble chamber. At low energies, the negatively charged electrons were deflected by the protons forming the nuclei of hydrogen as if the protons' positive charge was confined in a tiny volume (as in the α-particle scattering experiment). There was no net loss of kinetic energy, i.e. the scattering was elastic (Figure 8.1a).

At high electron energies – above about 6 GeV (6×10^9 electron-volts) – funny things began to happen. Now the electron lost a lot of energy in colliding with the proton and the proton fragmented into a shower of particles rather than recoiling (Figure 8.1b). This energy-to-matter transformation meant that the collision was inelastic, hence the name 'deep inelastic scattering'. The conclusion was that protons were not tiny 'balls' of positive charge but contained *localised charge centres*. The electrons were interacting with these charge centres via the electrostatic force, which is an inverse-square law force – see Section 4.6 Coulomb's law.

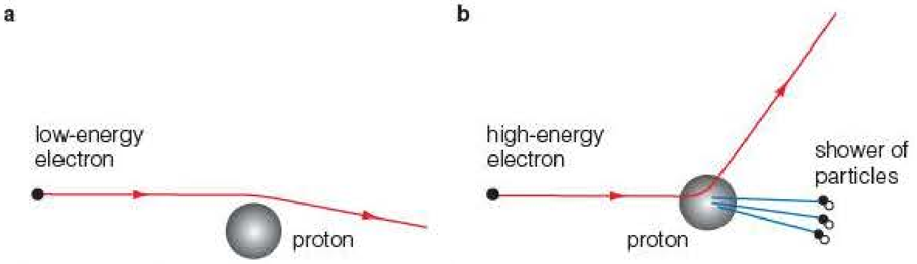


Figure 8.1 a) Elastic and b) inelastic scattering

The charge centres, of which there are three in the proton and also in the neutron, are called **quarks**. Figure 8.2 shows the quarks 'inside' a proton and a neutron. The quarks here are of two varieties:

'up' quarks with a charge $+\frac{2}{3}e$, symbol u

'down' quarks with a charge $-\frac{1}{3}e$, symbol d

The quarks in protons and neutrons are bound together tightly (the springs in the model shown in Figure 8.2 represent this), and in order to break the quarks apart very high-energy bombarding electrons are needed. However, the experimenters did not find any individual quarks in the showers of particles resulting from the inelastic collisions, and no one has yet found a 'free' quark. The showers of particles were mainly 'mesons' consisting of quark–antiquark pairs – see the next page.

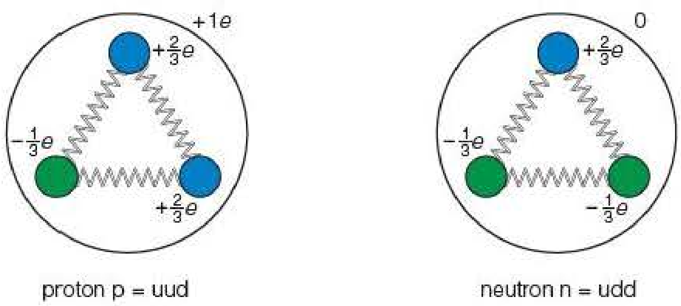


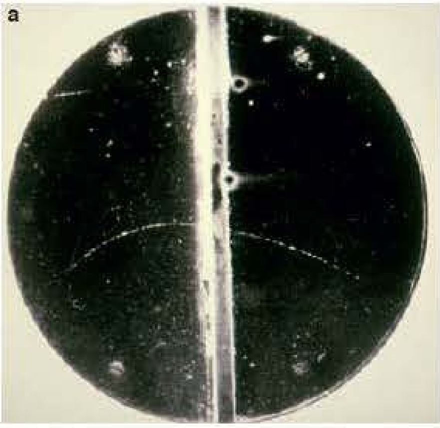
Figure 8.2 The quarks that make up a proton and a neutron

Some of the particles you are learning about here are the building blocks of the matter that we see around us – atoms. This matter that is made of protons, neutrons and electrons (our atoms) is sometimes referred to as hadronic matter. Most importantly it interacts in a manner determined by Newton's law of gravitation – see Chapter 3.

8.2 Matter and antimatter

We now believe that all fundamental particles, like up and down quarks and electrons, have antiparticles with exactly the same mass as their corresponding particles but with opposite charges. The antiparticle to the electron is called a **positron** and has the symbol e^+. The photograph in Figure 8.3a shows a cloud chamber in which a charged particle entering from the right is slowed down as it travels through a 6 mm lead plate across the middle of the chamber. Figure 8.3b shows a diagram of the particle track.

a



b

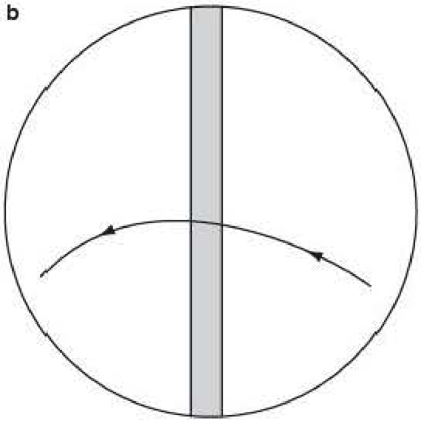


Figure 8.3a) A cloud chamber photograph from 1933
b) diagram of the particle track

On page 126 it was shown that the radius r of such a track if ⊥ to a magnetic field of flux density B was related to the particle's momentum p by the relationship $r = p/BQ$, where Q is the charge on the particle. The remarkable thing about this 1933 photograph is that the magnetic field was directed *downwards*, and so the charge on the particle was, by the left-hand rule, positive. The charge was found to be $+e$; it was a positron. The concept of there being antiparticles like the positron (we can now make anti-atoms of hydrogen) was *predicted* by Paul Dirac in 1928, and for this work he was awarded the Nobel Prize for Physics in 1933 at the early age of 31.

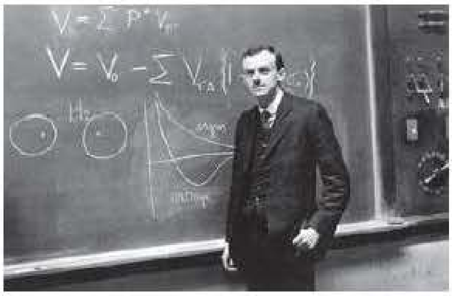


Figure 8.4 Paul Dirac – English physicist

Example

In Figure 8.3, the downward magnetic flux density $B \approx 1.5$ T. Assuming the photograph is 'life-size', calculate the momentum of the positron before and after it penetrates the lead sheet. The measured radii on the photograph are about 60 mm and 30 mm.

Answer

Using $p = BQr$ with $Q = +e$ gives:

$$p_{before} = 1.5\,\text{T} \times (1.6 \times 10^{-19}\,\text{C}) \times 0.06\,\text{m} = 1.4 \times 10^{-20}\,\text{N s}$$

$$p_{after} = 1.5\,\text{T} \times (1.6 \times 10^{-19}\,\text{C}) \times 0.03\,\text{m} = 0.7 \times 10^{-20}\,\text{N s}$$

Tip

When doing or reading Examples like this, it is valuable revision to check the units: here TCm becomes Ns.

The rest mass m_0 of both an electron and a positron is 9.1×10^{-31} kg, so a simple substitution into $p = m_0 v$ would suggest that the positron in Figure 8.3 was travelling *much* faster than the speed of light ($c = 3.0 \times 10^8\,\text{m s}^{-1}$). This is not possible. Einstein's theory of special relativity predicted that the faster things go, the heavier they become. The graph in Figure 8.5 puts this quantitatively and describes the mass m of an object increasing as a fraction of its rest mass m_0 as its speed increases. For example: $m = 2.5m_0$ when a positron is moving at about $0.925c$ or at 92.5% of the speed of light.

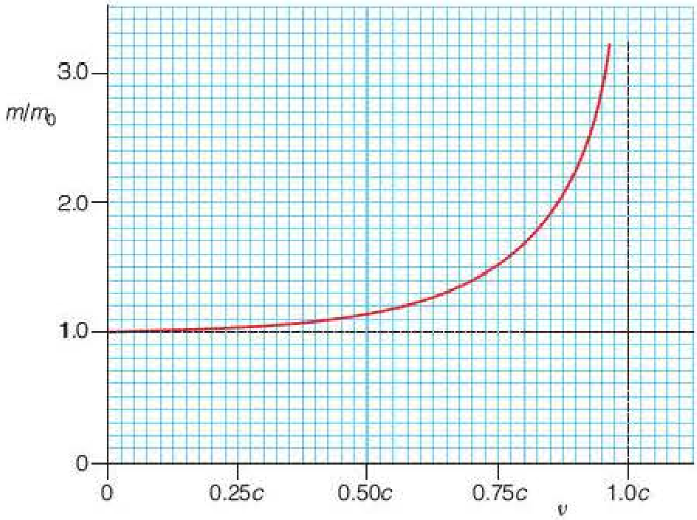


Figure 8.5 The relativistic increase of mass with speed

Test yourself

1 Explain the difference between elastic and inelastic collisions.
2 Express 6 GeV in joules and nanojoules.
3 State Coulomb's law for the force F between charged particles.
4 State the structure of a neutron using u for an up and d for a down quark.
5 What is the electric charge, in coulombs, on a positron?
6 Show that T C m is equivalent to N s.
7 Use Figure 8.5 to find
   a) the fractional increase in mass of a particle moving at 75% the speed of light
   b) the speed at which a particle must be moving to increase its mass to 3 times its rest mass.

8.3 Other mass units

So far in this book we have used the SI unit kilogram for mass and the SI unit joule for energy. We have already used a non-SI unit for energy, the electron-volt (eV). Of course, J and eV can be used with multiples of ten, e.g. kilo-, milli-, as can the gram, g. Another unit for mass, the **unified atomic mass unit**, symbol **u**, is useful in nuclear and particle physics. It measures masses on a scale where an atom of the isotope of carbon $^{12}_{6}C$ is given a mass of exactly 12 u.

Key term

$m_{\text{carbon-12}} = 12\,\text{u}$ (exactly)

One **unified atomic mass unit** (1 u) $= 1.66 \times 10^{-27}$ kg to 3 SF.

Another non-SI unit for mass or extra mass used in high-energy particle physics is MeV/c^2 or GeV/c^2. This results from Einstein's equation $\Delta E = c^2 \Delta m$. So the rest mass of a particle might be given as 250 MeV/c^2 which, after multiplying by 1.6×10^{-13} J MeV^{-1} and dividing by $(3.0 \times 10^8\,\mathrm{m\,s^{-1}})^2$, equates to 4.4×10^{-28} kg.

Table 8.1 summarises these different mass units.

Table 8.1 Mass units used in particle physics

	kg	u	MeV/c^2
1 kg =	1	6.02×10^{26}	5.62×10^{29}
1 u =	1.66×10^{-27}	1	1.07×10^{-3}
1 MeV/c^2 =	1.78×10^{-30}	934	1

Tip

Familiarity with these non-SI units can only be achieved by practising changing from one to another. A common error is to forget to square c. (Remember that $c^2 = 9.0 \times 10^{16}\,\mathrm{m^2\,s^{-2}}$.)

Example

Express the mass of an isotope of a uranium atom $^{238}_{92}\mathrm{U}$ **a)** in kg and **b)** in GeV/c^2. Take the rest mass of $^{238}_{92}\mathrm{U}$ as being 238 u.

Answer

a) $238\,\mathrm{u} \equiv 238\,\mathrm{u} \times (1.66 \times 10^{-27}\,\mathrm{kg\,u^{-1}}) = 3.95 \times 10^{-}\ 25\,\mathrm{kg}$

b) $238\,\mathrm{u} \equiv 238\,\mathrm{u} \times (934\ \mathrm{MeV}/c^2\,\mathrm{u^{-1}}) = 222 \times 10^3\ \mathrm{MeV}/c^2 = 0.222\ \mathrm{GeV}/c^2$

8.4 Creation and annihilation of matter

Tip

Remember that a photon is a quantum (bundle) of electromagnetic energy. Photons behave sometimes like waves and sometimes like particles.

The conservation of mass–energy explains some astonishing events, in particular:

particle production photon → electron + positron

particle annihilation electron + positron → photon(s)

Figure 8.6a shows pair production in a bubble chamber across which there is a magnetic field. Here a γ-ray photon creates an electron–positron pair of charged particles. Figure 8.6b is a corresponding diagram to which a dotted line has been added to show the track of the incoming photon.

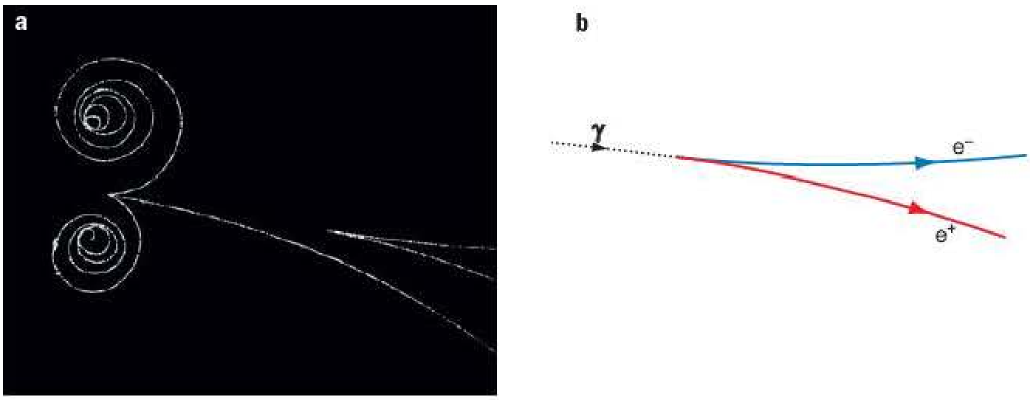


Figure 8.6 Pair production in a bubble chamber

In this event, charge is conserved: zero before and $(-e) + (+e) = 0$ after. Mass–energy is also conserved – see the Example below. (Momentum is conserved, as is shown by the equal but opposite curvature of the two particles' coloured paths in Figure 8.6b.)

Example

Show that the minimum γ-ray energy necessary for electron–positron pair production is 1.02 MeV.

Answer

The rest mass of an electron plus a positron is
$9.1 \times 10^{-31}\,\text{kg} + 9.1 \times 10^{-31}\,\text{kg} = 18.2 \times 10^{-31}\,\text{kg}$.

This total mass has an energy equivalence $\Delta E = c^2 \Delta m$, where $\Delta m = 18.2 \times 10^{-31}\,\text{kg}$. So the *minimum* energy E for the photon must be

$$E = (18.2 \times 10^{-31}\,\text{kg}) \times (3.0 \times 10^{8}\,\text{m s}^{-1})^2 = 1.64 \times 10^{-13}\,\text{J or } 1.02\,\text{MeV}.$$

The annihilation of an electron and a positron obviously conserves charge, but it is more difficult to demonstrate that the resulting electromagnetic radiation conserves energy, unless the resulting photons (often two, for example when the annihilating particles have zero or equal and opposite momenta) subsequently produce detectable charged particles. What we can say is that a pair of photons from such an annihilation will each have an energy of at least half of 1.02 MeV and thus a minimum de Broglie wavelength ($\lambda = hc/E$) of 0.61×10^{-12} m. These are photons in the γ-ray part of the electromagnetic spectrum.

PET scans

Antimatter particles are nowadays in use routinely in hospitals whenever a PET scan (positron emission tomography scan) is used to image activity in a person's brain. Suppose you are asked to help with experiments to see which part of your brain is working when you look at a screen, listen to music, speak to your friends or think about your physics studies. The brain scans for such activities in a normal brain are shown in Figure 8.7.

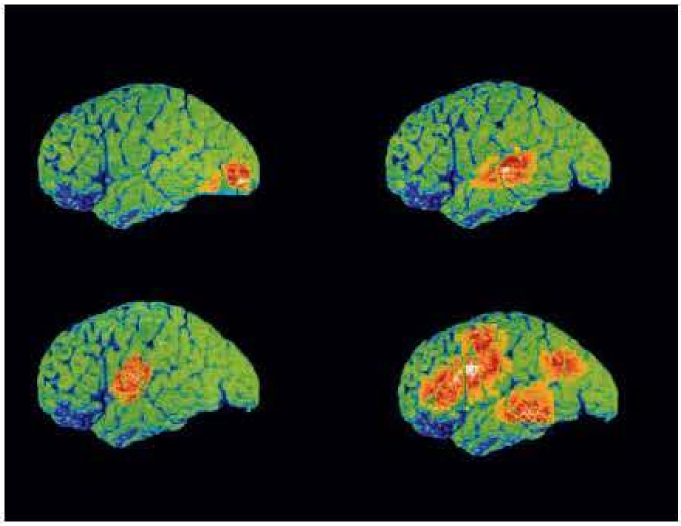


Figure 8.7 Four PET scans of the brain

How is it done? Your blood is injected with a small amount of liquid containing an isotope of oxygen – $^{15}_{8}O$. This is a radioactive isotope with a half life (see Section 9.7) of 122 seconds. In decaying in your brain, each oxygen-15 nucleus emits a positron that immediately annihilates with a local electron to produce two γ-rays, each of energy 511 keV.

$$e^+ + e^- \rightarrow \gamma + \gamma$$

As the positron and electron are effectively stationary, the γ-rays move apart in opposite directions (conserving momentum) and are detected by surrounding devices called scintillators. The part of your brain they come from is deduced from the *difference in time* they take to reach the scintillators. A computer produces the resulting image.

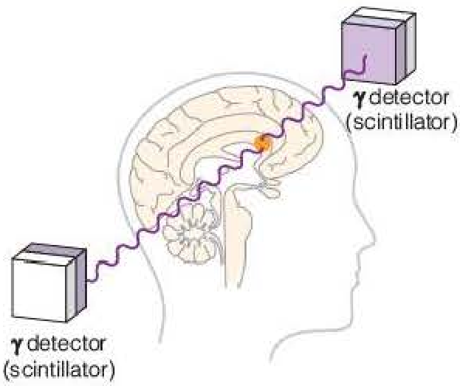


Figure 8.8 Photons from matter-antimatter annihilation enable brain activity to be imaged

Other particle–antiparticle annihilations are possible, for example that resulting from a proton meeting an antiproton. Here the energy is much higher, because the mass loss is much greater. Perhaps in a few years we will see experiments to annihilate a hydrogen atom with an anti-hydrogen atom (these are produced by their thousands in physics laboratories around the world, e.g. at CERN). Nowadays fMRI scans (functional nuclear magnetic resonance scans) are also used to show which part of the brain is responding to different stimuli.

Test yourself

8 Express the joule in base SI units.

9 Express the electron's mass, 9.1×10^{-31} kg, in MeV/c^2.

10 What is the mass in kg of a molecule of an oxygen isotope that is quoted as having a mass of 34 u?

11 Explain how to calculate the energy of a photon of electromagnetic radiation of wavelength λ.

12 How can we tell from a cloud chamber photograph of an electro-positron pair production that linear momentum has been conserved?

13 What is an energy of 1.02 MeV in joules (J)?

14 A proton and an antiproton (each of mass 1.66×10^{-27} kg) annihilate to produce two identical photons. Calculate the frequency of each photon.

8.5 The standard model

In Section 8.1 of this chapter you read that the up and down quarks were discovered using high-energy electrons. All these particles – the u quark, the d quark and the electron – are 'fundamental' particles. The electron and its antiparticle the positron are members of a group of fundamental particles called **leptons**.

There are two other leptons of greater mass (and therefore produced by more energetic events) that we call muons (μ) and taus (τ). Each μ^- and τ^- has a negative charge $-e$, and each antiparticle μ^+ and τ^+ a positive charge $+e$, where $e = 1.6 \times 10^{-19}$ C.

These fundamental leptons form the three 'generations' I, II and III shown in Table 8.2.

Table 8.2 The three generations of leptons

	I	II	III
	electron, e^-	muon, μ^-	tau, τ^-
charge	$-e = -1.6 \times 10^{-19}$ C	$-e = -1.6 \times 10^{-19}$ C	$-e = -1.6 \times 10^{-19}$ C
rest mass	0.511 MeV/c^2	106 MeV/c^2	1780 MeV/c^2

Reactions such as radioactive decay have shown that electrons (and positrons) are associated with the up and down quarks that make up the protons and neutrons of everyday matter. Similarly the muon and tau leptons are associated with other quarks. These quarks are shown in Table 8.3.

Table 8.3 The three generations of quarks

	I	II	III
	up quark, u	**charmed quark, c**	**top quark, t**
charge	$+\frac{2}{3}e$	$+\frac{2}{3}e$	$+\frac{2}{3}e$
mass	a few MeV/c^2	about 1 GeV/c^2	over 100 GeV/c^2
	down quark, d	**strange quark, s**	**bottom quark, b**
charge	$-\frac{1}{3}e$	$-\frac{1}{3}e$	$-\frac{1}{3}e$
mass	a few MeV/c^2	about 0.1 GeV/c^2	a few GeV/c^2

All quarks also have their corresponding antiparticles. These are written using the same symbols but with a bar above them: so, for example, u and $\bar{u}$, d and $\bar{d}$.

Historically, it was the symmetry of the arrangement of the three generations that led physicists to establish the **standard model**. This is summarised in Table 8.4.

Table 8.4 The standard model of fundamental particles

e	μ	τ	leptons, charge $-e$
u	c	t	quarks, charge $+\frac{2}{3}e$
d	s	b	quarks, charge $-\frac{1}{3}e$

Ordinary matter, the stuff the stars and galaxies of stars that we can see are made of, is made entirely from the 'first generation' of particles: the electron plus the up and down quarks. Individual quarks have never been isolated, which is why their masses are not precisely known. They join together in threes to form baryons and twos to form mesons (see below).

Associated with each lepton is a **neutrino**, a mysterious 'particle' which you will learn about in Chapter 9 in relation to radioactive decay. Neutrinos 'may' form part of what is called dark matter – see the Chapter 13 on astrophysics.

Figure 8.9 shows the structure of ordinary matter, starting right to left with quarks, which form baryons, then atoms, which include leptons, and finally crystal structures.

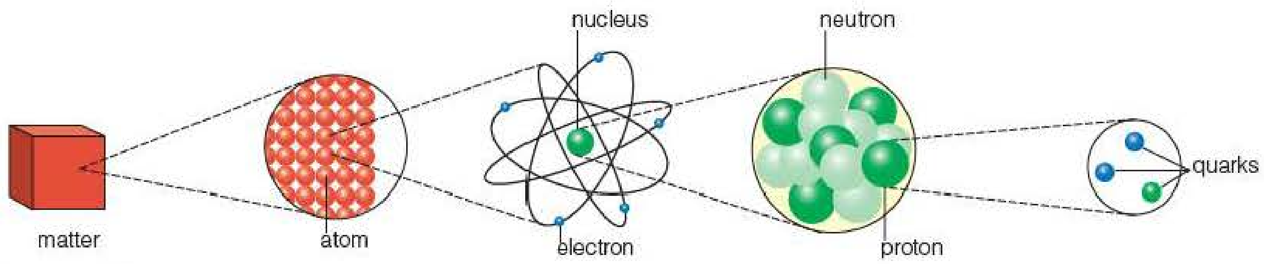


Figure 8.9 The structure of ordinary matter

Time dilation

The lifetime of elementary particles (their half-life) depends on how fast they are moving relative to the observer – the person who tries to measure their speed. This is a result of Einstein's theory of special relativity. In fact we now know that, for an object moving at a speed v, the **time dilation factor** is γ, where:

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

For example, for a particle travelling at nine tenths the speed of light $v/c = 0.9$ and $\gamma = 2.3$ to 2 SF, so the particle will, on average 'last' for more than twice its normal lifetime. It is because of this time dilation factor that high-energy (high speed) particles 'last' much longer than we expect, and leads to strange results such as the twin paradox – see the Example below and exam practice question 20.

Example

Muons are created by protons from the Sun interacting with molecules high up in the Earth's atmosphere. We now know that these muons decay in the laboratory with a very short half-life – about 2.2 μs.

a) How far would a particle moving at the speed of light travel in 2.2 μs?

b) In fact, huge numbers of muons are detected at sea level. Comment on how this is possible.

Answer

a) $d = vt = (3.0 \times 10^8\,\mathrm{m\,s^{-1}}) \times (2.2 \times 10^{-6}\,\mathrm{s}) = 660\,\mathrm{m}$

b) The muons resulting from this reaction are travelling very, very quickly. So quickly that they do not decay in a couple of microseconds but, owing to the time dilation factor, last much longer.

Many of them therefore travel from the upper atmosphere to the Earth's surface before decaying.

8.6 Baryons and mesons

Even before quarks had been discovered, lots of previously unseen, massive particles carrying a charge of zero or $+e$ or $-e$ had been found in experiments using the increasing energies available from various particle accelerators. Assuming these particles are made up of two or three quarks, can you predict, using only u and d quarks plus their antiparticles $\bar{u}$ and $\bar{d}$, what some of these might be?

How about:

ddd: a particle with charge: $\left(-\frac{1}{3}e\right)+\left(-\frac{1}{3}e\right)+\left(-\frac{1}{3}e\right) = -e$

or $d\bar{u}$: a particle with charge: $\left(-\frac{1}{3}e\right)+\left(-\frac{2}{3}e\right) = -e$

If you make use of the other four quarks, many more combinations result. It is even possible to get particles with a charge of $+2e$ or $-2e$; can you see how?

Particles made from **three quarks**, **qqq** (matter) or **three antiquarks** $\bar{q}\bar{q}\bar{q}$ (antimatter), are called **baryons**. Protons and neutrons are baryons; all baryons have their antiparticles, for example the antiproton is $\bar{u}\bar{u}\bar{d}$. Even a baryon made up of three u quarks uuu has been found. It has a charge $+2e$ and is called the delta particle – symbol Δ^{++}.

Tip

Quarks and antiquarks *cannot* be mixed up to make a three-quark baryon; $qq\bar{q}$, for example, is an impossible baryon.

Tip

When listing the charges for a baryon it is safest to bracket the charge on each quark so as not to get confused with +− or −+ situations.

Example

What is the charge on a uds baryon?

Answer

Charge is $\left(+\frac{2}{3}e\right)+\left(-\frac{1}{3}e\right)+\left(-\frac{1}{3}e\right) = 0$, i.e. it is a neutral particle.

Tip

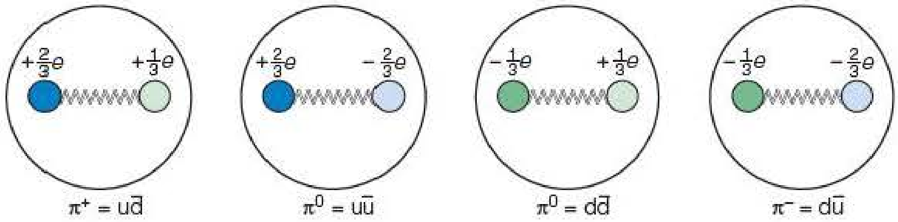
Try to remember that uud is a proton and udd is a neutron.

Other than uud and udd, you do not need to learn the names or structure of any baryons.

The uds in the above Example is called a sigma zero, Σ^0, and is one of a group of Σ particles containing one strange quark – plus, of course, two other quarks. Particles with two strange quarks, e.g. dss, are called xi (Ξ) particles; the dss is xi minus with a charge of $\left(-\frac{1}{3}e\right)+\left(-\frac{1}{3}e\right)+\left(-\frac{1}{3}e\right) = -e$, symbol Ξ^-. The sss baryon, also with a charge of $-e$ (can you see why?), is called omega minus, Ω^-. This particle, the omega, minus was discovered in 1964. Its discovery was an important step in confirming the correctness of the quark model.

Particles made of **two quarks** are called **mesons**. They are all combinations of a quark and an antiquark: $\mathbf{q\bar{q}}$. One group of mesons, known as pions, are $u\bar{d}$, $u\bar{u}$ or $d\bar{d}$, and $d\bar{u}$, as shown in Figure 8.10. These have charges of $+e$, 0, 0 and $-e$, and are designated as π^+, π^0, π^0 and π^-. (Yes, there are two different ways of making a π^0 meson!)

Figure 8.10 The pion group of mesons



You should be able to predict other groups of mesons, but you do not need to remember their names. Those with one strange quark are called kaons (K); the $s\bar{s}$, with two strange quarks, is called the eta meson, symbol η^0. It was showers of mesons that emerged from the deep inelastic scattering of electrons described in Section 8.1 (Figure 8.1b).

It is important to remember that in all reactions

the total number of baryons is conserved.

Thus $p + p \rightarrow p + \pi^+$ is *not* possible (although charge is conserved), but $p + p \rightarrow p + p + \pi^0$ is possible.

Example

One possible outcome when two protons interact is:
$p + p \rightarrow p + n + \pi^+$. Analyse this reaction in terms of the charge and deduce the quark structure of the π^+.

Answer

Left-hand side of equation p + p has charge $(+e) + (+e) = +2e$.

∴ Right-hand side $p + n + \pi^+$ has zero charge from n so must have +2e from p and π^+.

As p has charge +e, π^+ must also have charge +e for charge to be conserved.

Writing the known quarks in brackets: $p + p \rightarrow p + n + \pi^+$ becomes:

$$p\,(uud) + p\,(uud) \rightarrow p\,(uud) + n\,(udd) + \pi^+(q\bar{q})$$

as π^+ is a meson and must consist of a quark and an antiquark. An up quark is needed on the right and an extra down quark seems to have appeared.

∴ The meson must be a $u\bar{d}$ meson, of charge $\left(+\frac{2}{3}e\right)+\left(+\frac{1}{3}e\right)$ = +e, as needed.

The quarks are thus: $uud + uud \rightarrow uud + udd + u\bar{d}$

Tip

'Analyse' is an unusual word at the start of a question. What is means here is 'use your understanding of charge conservation and of the quark structure of protons and neutrons to examine the equation $p + p \rightarrow p + n + \pi^+$.

Tip

In reactions involving baryons and mesons any extra quarks that materialise are always mesons of the form $q\bar{q}$.

What has actually happened in the interaction in this Example is that some energy has gone to producing a $d\bar{d}$ quark–antiquark pair of zero net charge. The d then goes to form the neutron and the $\bar{d}$ to form the π^+ meson.

Some conservation laws

In the standard model, the law of conservation of charge has been assumed. To state it clearly:

- In any particle interaction, charge is conserved.

Other laws, or rules, that apply to interactions between fundamental particles include:

- The number of baryons is conserved.
- Lepton number is conserved.
- Strangeness number is conserved.

To take these laws in sequence:

The number of baryons cannot change in a nuclear reaction. Thus, as mentioned above, a reaction like $p + p = p + \pi$ is not possible, whereas $p + p = p + n + \pi$ is possible as it has two baryons on each side. Each baryon carries a baryon number +1 and each anti-baryon a baryon number −1. Strictly speaking it is baryon number that is conserved, but in this book you can look at equations and see that the number of baryons on each side of the equation is the same (remember, you only need to remember the quark structure of protons and neutrons).

You will meet electron neutrinos in Chapter 9 when beta-decay is described. All neutrinos are given the symbol ν (*'nu'*) and each lepton e, μ and τ has associated with it a neutrino ν_e, ν_μ and ν_τ. Each of e, μ and τ has a charge e^- and, of course, the antiparticle of each lepton has a charge of e^+; the antiparticles themselves have a 'hat' on them. The same 'hat' is found on antineutrinos, but neither neutrinos nor antineutrinos carry any charge. Our Sun emits vast numbers of neutrinos, and many millions of ν_e neutrinos pass through each of your eyes *every second*. Fortunately neutrinos do not interact with ordinary matter except very, very rarely – so you are quite safe being bombarded by them.

All leptons and neutrinos carry a lepton number of 1, while all their antiparticles carry a lepton number of −1. You will find the equations in Chapter 9 all balance in the sense that the total of the lepton numbers on each side of a nuclear equation balance. This is the law of the conservation of lepton number.

Finally, strangeness is conserved in nuclear reactions. To 'see' this it is necessary to look into the quark structure of the interaction and to identify the strange s and anti-strange $\bar{s}$ quarks. The strange quarks have strangeness number +1, the strange anti-quarks a strangeness number of −1, and it is this strangeness number that is conserved, as in $uud + uud = uud + uus + d\bar{s}$. In this book you will always be given the quark structure when the conservation of strangeness number is involved.

Test yourself

15 Write down the quark structure of a) the proton, and b) the neutron

16 What is the key difference between baryons and mesons?

17 What is an sss baryon called, and what is its charge?

18 Is $p + n = p + \pi^0 + \pi^0$ a possible interaction? Explain.

19 Why *might* this equation be correct: $Y \rightarrow Zr + {}^{0}_{-1}e + \bar{\nu}_e$

20 In the decay of Ω^- to a Ξ^- particle and a $\overline{K}^0$ meson, the quark structure of the Ω^- is sss and of the $\Xi-$ is ssd. Predict the quark structure of the meson.

8.7 Wave–particle duality

Wave–particle duality is a physicist's way of saying that waves have particle-like properties and particles behave in a wave-like manner. You have already met photons – 'particles' of electromagnetic radiation. The answer, then, to the question: 'What is light?' is that sometimes it behaves like a wave of wavelength λ and sometimes as a 'particle' or photon of energy hc/λ, where h is the **Planck constant** ($h = 6.63 \times 10^{-34}\,\text{J s}$).

The reverse, that particles with mass, such as electrons, can behave like waves seems astonishing. What should we call them – wavicles perhaps? In 1924 a young French physicist, Prince Louis de Broglie, was awarded his PhD for a thesis suggesting that a particle with momentum p had associated with it a wavelength $\lambda = h/p$. It is said that the awarding committee believed the thesis to be nonsense, but asked Einstein (who happened to be in Paris at the time) to look at it. Einstein said that he agreed it was without foundation, but added: 'You will feel very foolish if de Broglie turns out to be right'. And he *was* right! You have met these ideas before in your physics course; they are outlined in Book 1 of this series.

Table 8.5 Wave–particle duality summarised

Waves with particle properties	Particles with wave properties
$E = hf$	$p = \dfrac{h}{\lambda}$

Don't you think it is amazing that the Planck constant h links energy to frequency (and hence to wavelength) for electromagnetic waves *and* linear momentum to wavelength for particles?

Example

Calculate the wavelength associated with a beam of electrons that has been accelerated through 1200 V.

Answer

A voltage of 1200 V gives the electrons an energy of

$$E = 1200\,\text{J C}^{-1} \times (1.6 \times 10^{-19}\,\text{C}) = 1.92 \times 10^{-16}\,\text{J}$$

For this voltage we can use the non-relativistic relationship between kinetic energy and momentum. Using $m = 9.11 \times 10^{-31}\,\text{kg}$:

$$p^2 = 2mE \Rightarrow p = \sqrt{2 \times 9.11 \times 10^{-31}\,\text{kg} \times 1.92 \times 10^{-16}\,\text{J}}$$

$$= 1.87 \times 10^{-23}\,\text{kg m s}^{-1} = 1.87 \times 10^{-23}\,\text{N s}$$

Hence:

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}\,\text{J s}}{1.87 \times 10^{-23}\,\text{N s}} = 3.5 \times 10^{-11}\,\text{m}$$

You have probably come across the fact that electrons can produce diffraction patterns. Diffraction is a key property of waves, because it is the result of wave superposition. The photograph in Figure 8.11 shows the outcome for electrons diffracted by graphite crystals in a thin sheet of graphite.

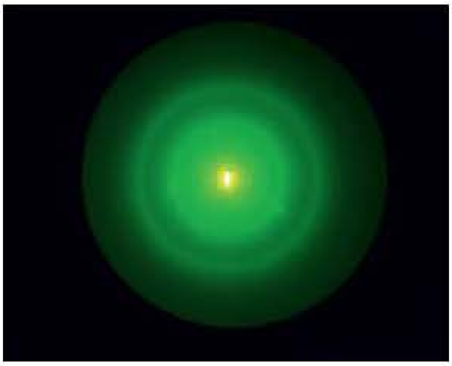


Figure 8.11 Electron diffraction rings

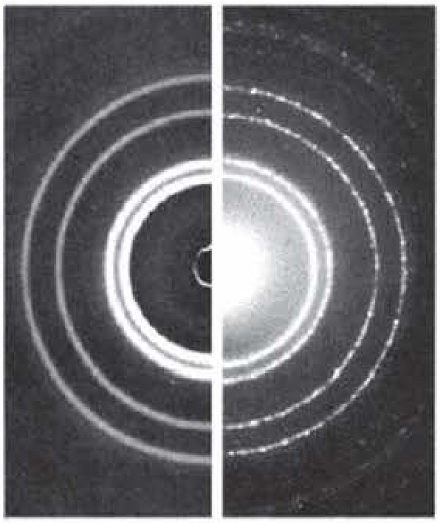


Figure 8.12 Demonstrating wave-particle duality: diffraction patterns for at left X-rays and at right electrons

Tip

That some experimental data support a theoretical prediction does *not* mean that the prediction is necessarily true. Of course it *may be true*, but science moves forward by showing a predicted result to be wrong; an outcome that means a new way of predicting a new theory must be sought.

Other diffracting crystals can be used. Figure 8.12 shows, side by side, the diffraction patterns produced by passing X-rays and electrons through the *same* very thin aluminium foil. It was arranged that the X-rays and the electrons would have the same 'wavelength'. Do you need any further proof of the wave-like behaviour of particles?

Example

The diameters d of the first electron diffraction rings were measured in an experiment involving electrons accelerated through different potential differences, V. The electron beam was diffracted by a graphite crystal in each case.

For energies up to about 5 kV it can be shown that $d \propto \frac{1}{\sqrt{V}}$. Some typical results from the experiment were:

V/V	2500	3000	4000	5000
d/m	0.037	0.034	0.029	0.026

Show that these are results consistent with the above relationship linking d to V.

Answer

If $d \propto \frac{1}{\sqrt{V}}$, then d^2V should be constant.

Values for d^2V, in m^2 V are: 3.4, 3.5, 3.4, 3.4.

The proportional relationship is therefore supported by these figures to 2 SF – the precision of the diameter measurements.

Exam practice questions

1 An elastic collision is one:

A between low-energy particles

B in which electrostatic forces act

C in which mass is conserved

D where no kinetic energy is lost. **[Total 1 mark]**

2 Which of the following is not a unit of mass?

A mg **C** u

B N **D** keV/c^2 **[Total 1 mark]**

3 An energy of 6.4 MeV is equivalent to

A 6.4×10^{-30} J **C** 6.4×10^{-12} J

B 1.0×10^{-15} J **D** 1.0×10^{-12} J **[Total 1 mark]**

4 Which of the following is an impossible baryon?

A ddu **C** $ud\bar{s}$

B $\bar{u}\bar{d}\bar{d}$ **D** $\bar{u}\bar{d}\bar{s}$ **[Total 1 mark]**

5 Which of the following is a possible meson?

A ds **C** $\bar{d}\bar{s}$

B $u\bar{d}$ **D** us **[Total 1 mark]**

6 How many baryons are there in an atom of ^7_3Li?

A zero **C** 4

B 3 **D** 7 **[Total 1 mark]**

7 Which of the following is a possible reaction?

A $p + \pi^+ \rightarrow p + p$

B $p + n \rightarrow \pi^+ + \pi$

C $p + p \rightarrow p + p + n$

D $p + p \rightarrow p + p + \pi^0 + \pi^0$ **[Total 1 mark]**

8 220 GeV/c^2 is equivalent, to 2 SF, to a mass of

A 3.9×10^{-25} kg **C** 2.1×10^5 kg

B 3.9×10^{-22} kg **D** 2.1×10^8 kg **[Total 1 mark]**

9 This drawing of a bubble chamber event showing electron–positron pair production at A is incorrect.

The tracks shown in Figure 8.13 are not possible because

A the particle tracks curve in different senses

B charge is not conserved

C momentum is not conserved

D the magnetic field was upwards. **Total 1 mark]**

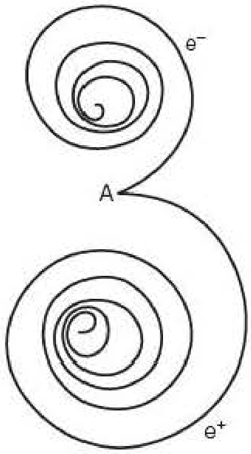


Figure 8.13

10 Calculate the wavelength associated with a tennis ball of mass 57.5 g that is served at $220\,\text{km}\,\text{h}^{-1}$, and comment on the result. **[Total 6 marks]**

11 The Stanford accelerator can accelerate electrons beyond 30 GeV.

a) What is the mass in kg of a particle of mass $30\,\text{GeV}/c^2$? **[3]**

b) Express this energy as a multiple of the rest mass of an electron. **[3]**

[Total 6 marks]

12 Show that the wavelength associated with each of the photons produced in the reaction involving a proton p and an antiproton $\bar{p}$.

$p + \bar{p} \rightarrow \gamma + \gamma$ is about $3 \times 10^{-16}\,\text{m}$

and state any assumption you have made in your calculation.

[Total 6 marks]

13 Show that the unit of $2hc/mv^2$ is the metre. **[Total 5 marks]**

14 The graph in Figure 8.5 can be described by the equation

$$m = \frac{m_0}{\sqrt{\left(1 - v^2/c^2\right)}}$$

By taking at least three readings from the graph, show that it has been correctly drawn. **[Total 5 marks]**

15 The initial (lower) interaction in Figure 7.11 is the decay of a K^- meson into two pi mesons, π^0 and π^-. Express this decay in terms of the quarks involved, indicating how the charge is conserved by your quark equation. **[Total 5 marks]**

16 A K^+ meson can decay into three pions as shown at D in Figure 8.14.

a) State the direction of the magnetic field affecting the paths shown in Figure 8.14. **[1]**

b) Explain why the 'downward' π^+ meson follows a path DA that reduces gradually in radius. **[3]**

[Total 4 marks]

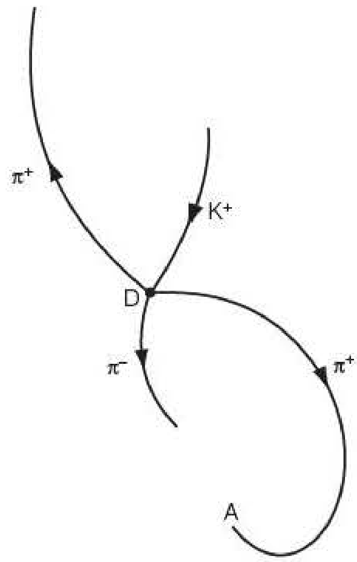


Figure 8.14

Stretch and challenge

17 The famous Ω^- particle, with quark structure sss, was first found as a result of a K^- meson interacting with a proton to give a K^0 and a K^+ plus the Ω^-. Given that the K^- meson is $s\bar{u}$ and the K^0 is $d\bar{s}$:

Write down the quarks involved in this reaction and explain how the conservation laws apply to it. **[Total 8 marks]**

18 Figure 8.15 shows how an electron orbiting a proton in a hydrogen atom might be thought of as forming a standing wave. Suggest how this model might predict discrete or quantised energy levels for hydrogen. **[Total 5 marks]**

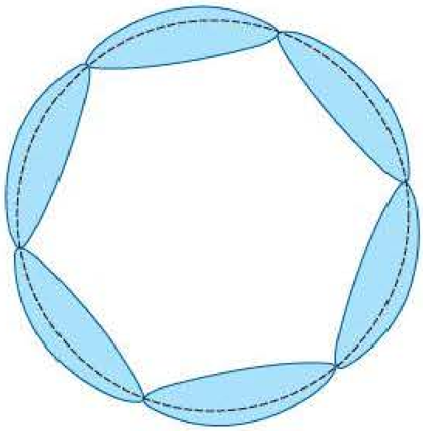


Figure 8.15

19 Figure 8.16 represents tracks in a bubble chamber before and after a particle interaction.

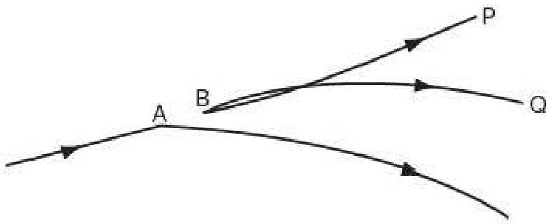


Figure 8.16

a) The quark components of the particles in the reaction at A where a high-energy xi particle decays are:

$$dss \rightarrow uds + \bar{u}s$$

i) State the nature of the particles involved and determine the charge on each.

ii) Hence deduce the sense of the magnetic field across the cloud chamber. **[6]**

b) One of the particles appearing as a result of the decay at B is a proton.

i) Explain which of P or Q is the proton and state the charge on the other particle.

ii) Give a quantitative explanation of which particle has the greater momentum. **[5]**

c) Describe what has happened in the decay of the xi particle. **[6]**

[Total 17 marks]

20 Use a search engine to find out more about 'The twin paradox'. *(You won't come across a question like this in your examination, but it is the sort of thing that you should be able to research for yourself.)*

9 Nuclear decay

Key terms

An **ion** is an atom that has lost or gained one or more electrons and is therefore electrically charged.

The **isotopes** of an element all have the same Z but different N, i.e. the same number of nuclear protons and surrounding electrons, but different numbers of neutrons in the nucleus.

Prior knowledge

You should know from earlier Advanced-level work and from GCSE:

- the simple structure of the atom as having a nucleus of protons and nucleons surrounded by a 'cloud' of electrons.
- that an atom will have the same number of electrons as protons
- atoms can be ionised by gaining or losing one or more electrons. For example, if an atom loses an electron it will became a positive **ion**
- an element can have different **isotopes** (from the Greek '*iso*' (the same) '*topos*' (place)
- that isotopes of a particular element have the same place in the periodic table and therefore the same chemical properties
- the term 'radiation' as meaning the ionising radiation that is emitted all the time by radioactive sources.
- that this includes alpha and beta particles and gamma rays
- that these are randomly emitted by unstable nuclei, causing energy to be transferred
- alpha and beta particles have electric charge and can therefore be deflected by electric and magnetic fields
- gamma rays are very short wavelength (high frequency) electromagnetic waves
- what is meant by radioactive decay and the terms 'half-life' and 'background radiation'
- that radiation has many uses
- that radiation is dangerous and must be treated with care
- what safety precautions need to be taken when dealing with radioactive substances.
- 1 electron-volt (eV) = 1.60×10^{-19} J
- for a photon $E = hf$
- atomic number (proton number): Z = number of protons in the nucleus
- atomic mass number (nucleon number): A = total number of protons + neutrons in the nucleus.

Test yourself on prior knowledge

1 State the nature of:
  a) alpha particles
  b) beta particles
  c) gamma rays.

2 Iodine-131, which is used in radiotherapy, emits radiation of energy 360 keV and has a half-life of 6 days.
  a) What is the energy of the radiation in joules?
  b) What percentage of the iodine-131 will remain after a month (30 days)?

3 Iodine-123, which is used for medical diagnosis, emits gamma radiation of energy 160 keV. Calculate the wavelength of this radiation.

4 Naturally occurring carbon has two isotopes, $^{12}_{6}C$ (98.9%) and $^{13}_{6}C$ (1.1%).

a) For an atom of $^{13}_{6}C$, state the number of:

i) protons in the nucleus

ii) neutrons in the nucleus

iii) electrons.

b) What is meant by 'isotopes'? Illustrate your answer with reference to the two isotopes of carbon.

9.1 Discovery of radioactivity

The word 'nuclear' often conjures up the wrong image – even to the extent that Nuclear Magnetic Resonance (NMR) scanners are now called Magnetic Resonance Imaging (MRI) scanners to avoid the use of the word 'nuclear'. The bad image, of course, is generated by the thought of nuclear weapons and, to a lesser extent, by nuclear power stations. Whilst few would argue against the evil of nuclear weapons, the debate about nuclear energy is finely balanced and one that as a physicist you should be able to discuss in an informed and rational way. The study of radioactivity and its effects and consequences should help you to develop a better understanding of the 'nuclear debate'. Whether we like it or not, the nuclear genie has been released from its bottle and cannot be squeezed back in again.

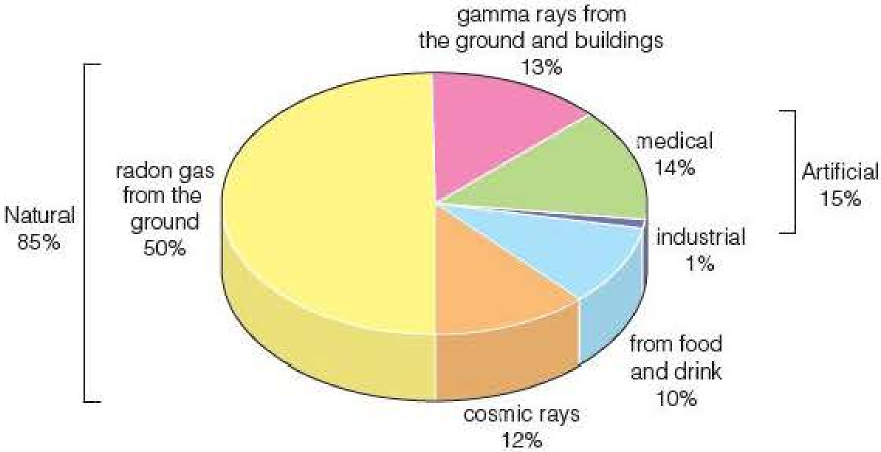
Ionising radiation, or 'radioactivity' as we now call it, was discovered by accident in 1896 by the French scientist Henri Becquerel. He found that certain uranium salts affected a photographic plate even when the plate was covered with black paper. The effect was similar to that of X-rays, which had been discovered by Wilhelm Röntgen a year earlier while working with high-voltage cathode ray tubes.

Becquerel showed that the rays emitted from the uranium caused gases to *ionise* and that they differed from X-rays in that they could be *deflected by a strong magnetic field*. For his discovery of spontaneous radioactivity Becquerel was awarded the Nobel Prize in Physics in 1903, which he shared with Marie and Pierre Curie, who had discovered two further radioactive elements, which they named radium and polonium.

9.2 Background radiation

We live in a radioactive environment. The air you breathe, the ground you walk on, the house you live in, the food you eat and the water you drink all contain radioactive isotopes. This radiation, which is constantly present in our environment, is called **background radiation**.

Figure 9.1 Sources of background radiation



The main natural sources of background radiation are:

- radioactive gases (mainly radon) emitted from the ground, which can be trapped in buildings and build up to potentially dangerous levels – high levels of radon can greatly increase the risk of lung cancer;
- radioactive elements in the Earth's crust – mainly uranium and the isotopes it forms when it decays – these give rise to gamma radiation, which is emitted from the ground and rocks (e.g. granite) and from building materials (e.g. bricks and plaster);
- cosmic rays from outer space which bombard the Earth's atmosphere producing showers of lower-energy particles such as muons, neutrons and electrons and also gamma rays;
- naturally occurring radioactive isotopes present in our food and drink, and in the air we breathe, including carbon-14 and potassium-40.

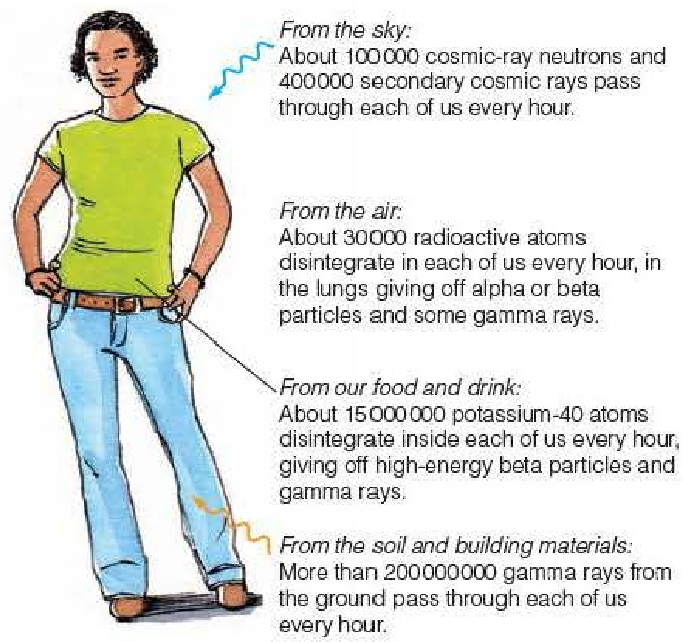
These natural sources make up about 85% of the radiation to which we are exposed (see Figure 9.1). The remaining 15% comes from artificial sources, mainly medical, e.g. X-rays; a tiny fraction comes from nuclear processes in industry.

Figure 9.2 summarises the natural sources of background radiation to which we are exposed.

Tip

The 'background count' must be measured and then subtracted from subsequent readings when undertaking quantitative experiments on radioactivity – see Section 9.8 and Core practical 15.

Figure 9.2 Natural background radiation



Note: The risk of death due to radon-induced cancer at the average radon level in the UK is estimated to be 3 in 1000, or about the same as from a pedestrian traffic accident. This is increased by a factor of 10 for someone who smokes 15 cigarettes a day.

9.3 Dangers of radiation

Figure 9.3 Radiation warning symbol

WARNING – radiation can kill! That's why we have to be very careful when dealing with radiation and must always follow appropriate safety precautions. The dangers of radiation can be caused either by exposure of our bodies to some external source of radiation (such as X-rays) or by breathing or ingesting radioactive matter (such as radon which, as we saw earlier, contributes to the background radiation).

To put things into perspective, a typical dental X-ray dose is equivalent to about 3 days of natural background radiation, while the dose from a chest X-ray is equivalent to about 10 days of background radiation. Having said this, radiation exposure is accumulative and so diagnostic X-rays should always be kept to a minimum.

A detailed discussion of the biological effects of radiation is beyond the scope of this book, except to say that even a very low dose of ionising radiation, whether natural or artificial, has a chance of causing cancer because of the damage it can do to our DNA. So, it is essential to always observe the following precautions:

- keep as far away as possible (at least 30 cm) from all laboratory sources of ionising radiation;
- do not touch radioactive materials – use a handling tool;
- keep sources in their lead storage containers when not in use;
- during an investigation keep the source pointed away from the body, especially the eyes;
- limit the time of use of sources – return to secure storage as soon as possible;
- wash hands after working with a radioactive source.

In addition, people likely to be exposed to larger doses should wear protective clothing, such as lead aprons.

Example

Figure 9.4 shows a radiotherapist using a remote control to operate a linear accelerator, which produces high-energy X-rays.

Explain why the radiotherapist

a) uses a remote control

b) wears a radiation badge.

Figure 9.4

Answer

a) X-rays are particularly dangerous if received in repeated doses because they can inflict biological damage to humans and, in particular, can cause cancer. The radiographer needs to be as far away as possible from the X-rays to minimise the dose she receives, and so she uses a remote control to operate the machine.

b) It is important to monitor on a regular basis the amount of radiation to which workers using X-rays or radioactive materials are exposed. This is done by wearing a 'badge', which can measure the amount of radiation to which the worker has been subjected. If the dose exceeds the safe limit, the worker has to be moved to different tasks.

9.4 Alpha, beta and gamma radiation

Within two years of Becquerel's discovery of radiation, Ernest Rutherford, working at the Cavendish Laboratory in Cambridge, showed that there were two distinct types of radiation, which he called alpha (α) rays and beta (β) rays (α and β being the first two letters of the Greek alphabet). By 1900, a third type of radiation had been identified by the French physicist Paul Villard. This was called gamma (γ) radiation. The property of all these radiations that enables them to be detected and distinguished is their ability to cause **ionisation**. They are often collectively referred to as '**ionising radiation**'.

Demonstrating ionisation

Safety note

This set-up involves the use of a high-voltage supply and a radioactive source. The voltage supply should be limited to give a current of no more than 5 mA. Stringent safety procedures should be followed with the radioactive source.

The set-up is shown in Figure 9.5.

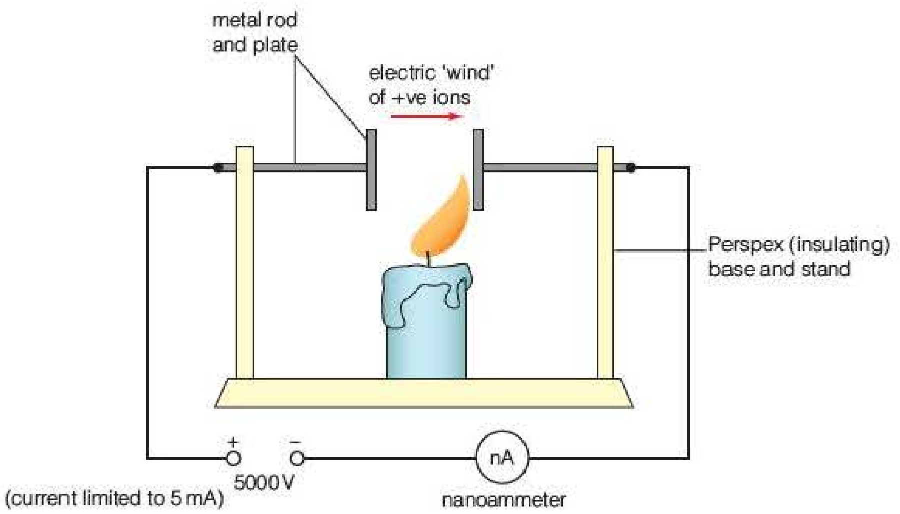


Figure 9.5

The energy from the candle flame gives electrons in the air sufficient kinetic energy to break free from their atoms. This process is called **ionisation**. The negative electrons move towards the left-hand positive plate while the much heavier positive ions of air move towards the right-hand negative plate. The movement of these ions creates a 'wind' that blows the candle towards the right-hand negative plate. The electrons and positive ions act as 'charge carriers', creating a small current in the circuit, which is detected by the nanoammeter.

If the candle is replaced by an α-emitter, such as americium-241, a small current is again observed. This is because the energy from the α-radiation causes the air to become ionised, just like the flame.

Alpha radiation

In the school laboratory, alpha radiation can be detected using either a **spark counter** or an **ionisation chamber**. Details of how these devices work are not needed, except to understand that both depend on the ionising properties of alpha radiation.

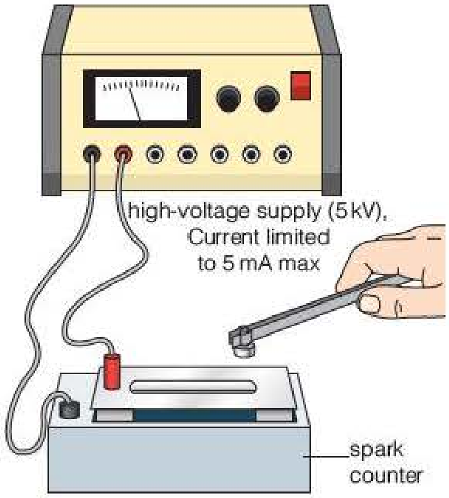


Figure 9.6 Using a spark counter

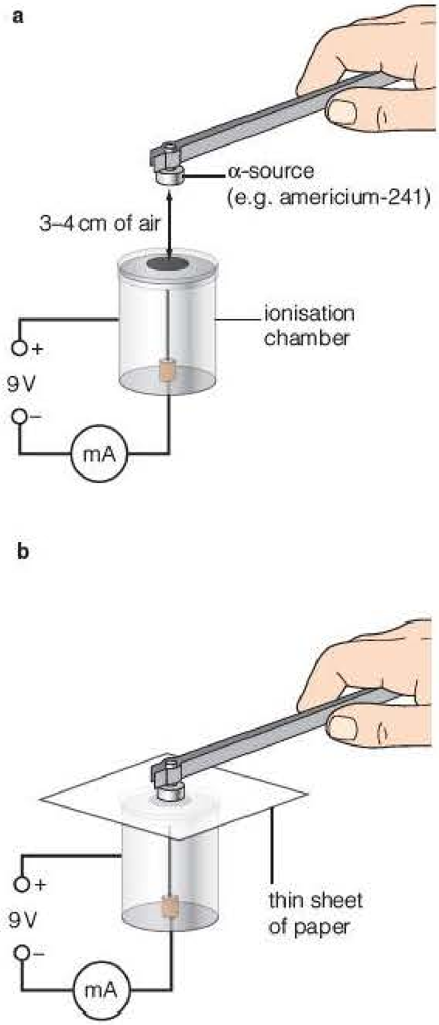


Figure 9.7 Using an ionisation chamber to find the range of alpha radiation

With a spark counter connected to a high-voltage supply or with an ionisation chamber and sensitive current detector arranged as in Figure 9.7, it can be shown that the range of α-radiation is a few centimetres in air (Figure 9.7a) and that α-radiation is completely absorbed by a thin sheet of paper (Figure 9.7b).

Rutherford showed that α-radiation can be deflected by very strong magnetic and electric fields (see Figure 9.11a). From such deflections it can be shown that α-radiation is not actually 'radiation' but consists of positively charged particles. As we saw in Section 7.2, a classic experiment undertaken by Rutherford and Royds in 1909 showed that α-particles were actually **helium nuclei**, i.e. helium atoms without their electrons (Figure 9.8).

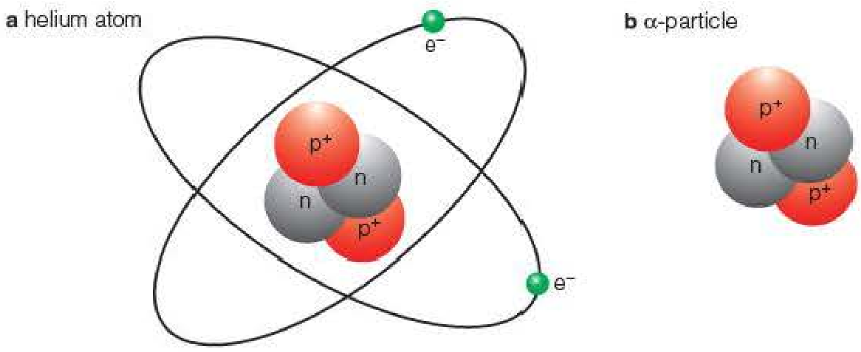


Figure 9.8

Tip

Remember that the essentials for experiments on radioactivity are:

- safety – always follow strict guidelines, e.g. handling sources with tongs
- detector – usually a G-M tube
- counter – either a counter and stopwatch or a rate-meter
- background count – take for a sufficiently long time to get an average.

The fact that they are charged, and have a large mass, means that α-particles readily ionise matter. Thus the energy of an α-particle is rapidly lost and the particle is quickly brought to rest – in other words, the particle is easily absorbed as it passes through matter. (see Exam Practice question **17** – Figure 7.13 in chapter 7.) In the ionisation process the α-particle gains two electrons and becomes an atom of helium gas.

Beta radiation

In order to detect beta radiation we use a **Geiger–Müller tube (G-M tube)** connected to a **scaler** or a **rate-meter**. Such an arrangement cannot be used to detect α-particles because these cannot penetrate the thin mica window of the G-M tube.

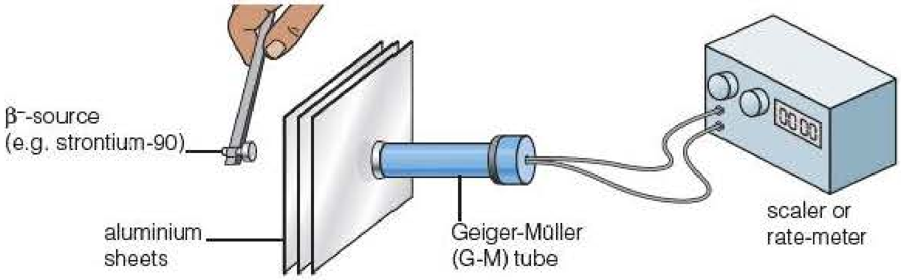


Figure 9.9 Using a G-M tube to find the range of beta radiation

Using the arrangement shown in Figure 9.9, we can show that β-radiation can travel through as much as 50 cm of air and can also pass through a few millimetres of aluminium.

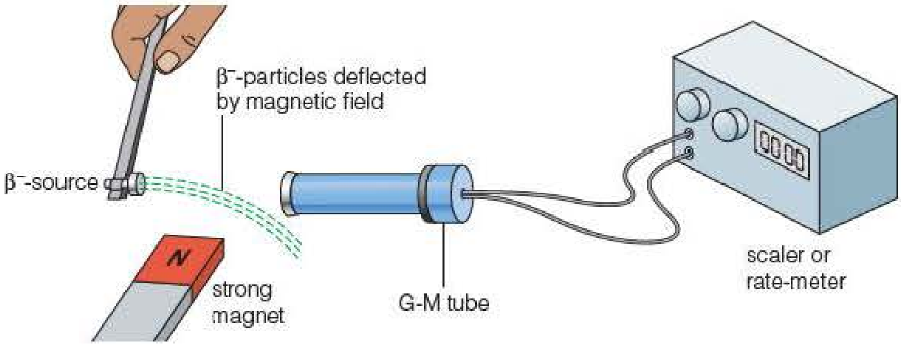


Figure 9.10 Showing the deflection of beta radiation in a magnetic field

If a very strong magnet is brought up as shown in Figure 9.10, the count-rate is observed to drop, showing that the β-radiation has been deflected by the magnetic field (see also Figure 9.11b). Quantitative experiments with magnetic and electric fields confirm that β-radiation consists of fast-moving streams of negatively charged electrons. It is therefore more precisely designated as β^--radiation or β^--particles.

β^--particles, being charged, cause a fair amount of ionisation, but not anywhere near as much as α-particles as they are much less massive (an electron has about 1/8000th of the mass of a helium nucleus). As β^--particles cause far less ionisation than α-particles, they can penetrate matter much further before their energy is used up.

Gamma radiation

All attempts to deflect gamma radiation in the most powerful electric and magnetic fields were to no avail. Eventually Rutherford confirmed that γ-radiation was a high-energy form of electromagnetic radiation, like X-rays, ultraviolet, light, microwaves and radio waves.

Gamma radiation is composed of minute bundles of energy called **photons** (which we talked about in Year 1). Only when one such photon happens to collide directly with an atom of matter does ionisation occur. This means that γ-radiation is only weakly ionising.

Like all electromagnetic radiation, γ-rays obey an inverse-square law, meaning that their intensity falls off as the square of the distance in air. Gamma rays easily pass through aluminium and are not entirely absorbed (or 'attenuated') even by several centimetres of lead (see Core Practical 15 in Section 9.8).

Tip

The energy of ionising particles is usually expressed in electron-volts (eV), where $1\,\text{eV} \equiv 1.60 \times 10^{-19}\,\text{J}$. Thus an energy of 1 pJ is equivalent to

$$\frac{1 \times 10^{-12}\,\text{J}}{1.60 \times 10^{-19}\,\text{J eV}^{-1}} = 6.25 \times 10^{6}\,\text{eV}$$
$$= 6.25\,\text{MeV}$$

As can be seen from Table 9.1, nuclear energies are in the order of MeV. Note that this means that nuclear energies are about a million times greater than energies encountered in chemistry, which are typically in the order of eV.

Summary of the properties of α, β and γ-radiation

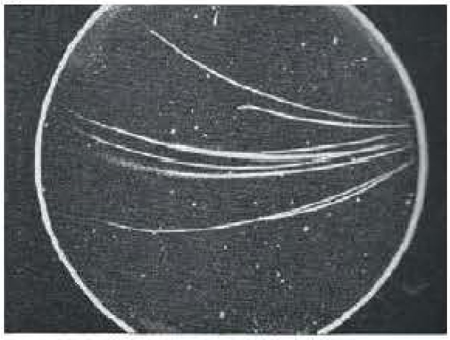
Table 9.1 summarises the properties of the three types of radioactive emissions. We saw in Chapter 8, Section 8.3 that it is common practice in atomic physics to quote masses in **unified mass units**, symbol **u**, where 1u is defined as exactly 1/12th of the mass of the carbon-12 nucleus ($= 1.66 \times 10^{-27}\,\text{kg}$).

Table 9.1

Property	α-particle	β-particle	γ-photon
mass	4 u (6.64×10^{-27} kg)	about $\frac{1}{2000}$ u (9.11×10^{-31} kg)	0
charge	$+2e$	$-e$	0
speed	up to about $\frac{1}{20}c$	up to about $0.99c$	c
typical energy	0.6 to 1.3 pJ (4 to 8 MeV)	0 up to 2.0 pJ (0 to 12 MeV)	0.01 to 1.0 pJ (0.06 to 6 eV)
relative ionising power	10 000	100	1
penetration	few cm of air	few mm of aluminium	few cm of lead
deflection in electric and magnetic fields	small deflection	large deflection	no deflection
nature	helium nucleus	electron	high-frequency electromagnetic radiation

The relative ionising power – and hence penetrating power – of α-particles and β-particles is illustrated in the cloud chamber photographs shown in Figure 9.11.

Figure 9.11a shows the strong, thick tracks produced by the heavily ionising α-particles, while Figure 9.11b shows the much thinner tracks of the less ionising β-particles. In each case, there is a magnetic field directed up out of the diagram (you should check this for yourself from the direction in which the tracks curve using the left hand rule).

a


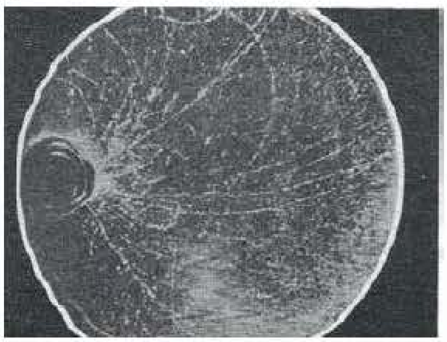
b


Figure 9.11 a) α-particle tracks and b) β-particle tracks

In the case of the α-particles, this field had to be very strong – over 4 teslas. Can you explain why the shorter α-particle tracks are more curved than the longer ones? The large-angle deflection near the end of one track is probably the result of a close encounter with a nucleus.

It can be deduced from the different curvatures of the β-particle tracks in Figure 9.11b that the β-particles must have a range of different energies (or velocities). We will discuss the significance of this on page 159.

We now know that these ionising radiations come from the unstable nuclei of radioactive elements, which, on emission of an alpha particle or a beta particle, decay into another, lower-energy nuclide.

9.5 Disintegration processes

Let's start by revising some basic definitions.

From Chapter 8 you should be familiar with the **proton number** (symbol Z) of an element being the number of protons in one atom. Remember that a neutral atom will also have the same number of electrons, for example an atom of oxygen will contain 8 protons in its nucleus, surrounded by 8 electrons. Note that the proton number is sometimes called the **atomic number** as this defines the element, e.g. $Z = 1$ is hydrogen, $Z = 2$ is helium, ... $Z = 8$ is oxygen, and so on.

The particles making up the nucleus of an atom, namely the protons and neutrons, are called nucleons. The **nucleon number** or **mass number** (symbol A) of an atom is the total number of protons and neutrons in the nucleus of that atom.

Isotopes are different forms of the *same element* (i.e. they have the same proton number, Z) that have a different number of *neutrons* in the nucleus (and therefore a different nucleon number, A). A **nuclide** is the nucleus of a particular isotope and is represented by the following symbol:

nucleon number → A

X ← symbol for element

proton number → Z

Example

Uranium occurs naturally in the form of two isotopes, $^{235}_{92}U$ and $^{238}_{92}U$.

a) Explain what is meant by 'isotopes'.

b) Copy and complete the table to show the proton number, nucleon number and number of neutrons for each isotope.

Table 9.2

Isotope	Proton number	Nucleon number	Number of neutrons
$^{235}_{92}U$			
$^{238}_{92}U$			

Answer

a) Isotopes are different forms of the same element that have a different number of neutrons in the nucleus.

b)

Table 9.3

Isotope	Proton number	Nucleon number	Number of neutrons
$^{235}_{92}U$	92	235	235 − 92 = 143
$^{238}_{92}U$	92	238	238 − 92 = 146

Test yourself

1 a) Explain what is meant by 'background radiation'.

b) List three sources of background radiation.

2 a) Explain why ionising radiation is dangerous for humans.

b) List three precautions that should be observed when working with radiation.

3 Of the three types of radiation, α, β and γ which:

a) causes the most ionisation

b) causes the least ionisation

c) is the most penetrating

d) is the most easily absorbed

e) has a positive charge

f) has a negative charge

g) is most easily deflected in a magnetic field

h) cannot be deflected in an electric field?

4 Sir JJ Thomson used a combination of electric and magnetic fields to investigate the charge/mass ratios for ionised atoms. He was the first to show that the element neon existed in two forms, each with atoms of slightly different masses, in a paper published in 1913. As part of the conclusion of the paper he wrote:

> *There can, therefore, I think, be little doubt that what has been called neon is not a simple gas but a mixture of two gases, one of which has an atomic weight* (we now call it 'mass'!) *of about 20 and the other about 22. The parabola due to the heavier gas is always much fainter than that due to the lighter, so that probably the heavier gas forms only a small percentage of the mixture.*

The two forms of neon were later called isotopes.

a) Explain what is meant by *ionised atoms* and why it is possible to deflect them with electric and magnetic fields.

b) With reference to neon, explain what is meant by *isotopes*.

c) Copy and complete the table to show the proton number, nucleon number and number of neutrons for each isotope of neon.

Table 9.4

Isotope	Proton number	Nucleon number	Number of neutrons
$^{20}_{10}Ne$			
$^{22}_{10}Ne$			

d) Neon has an atomic mass of 20.18 u. Determine the percentage abundance of each of the two isotopes in order to justify Thomson's conclusion that *probably the heavier gas forms only a small percentage of the mixture.*

The proton number determines the number of electrons in each atom, which in turn gives rise to the chemical properties. Isotopes therefore have the *same chemical properties* because they have the same proton number. However, they will have different atomic masses and therefore *different densities*. The latter property of isotopes enables the separation of uranium-235 and uranium-238 by a diffusion process in the manufacture of 'enriched' uranium fuel rods for nuclear reactors.

We can now look at what happens in α-decay and β-decay.

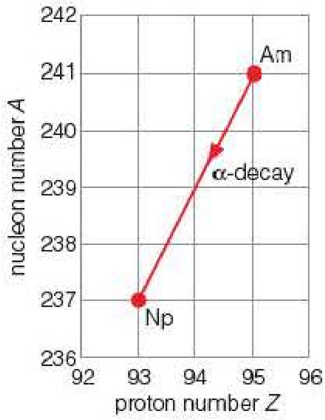


Figure 9.12 Graphical representation of α-decay

α-emission

An α-particle is a helium nucleus, it has the symbol ${}^4_2\text{He}$. A typical α-emitter is americium-241, for which the decay process is

$${}^{241}_{95}\text{Am} \rightarrow {}^{237}_{93}\text{Np} + {}^{4}_{2}\text{He}$$

americium-241 → neptunium-237 + α-particle

Neptunium-237 is formed because conservation laws decree that proton numbers and nucleon numbers must total the same on each side of the equation.

This process can be represented on a plot of nucleon number (mass number), A, against proton number (atomic number), Z. This is shown in Figure 9.12.

Example

Neptunium-237 decays further by α-emission to form an isotope of protactinium (Pa). Write the equation for this decay process.

Answer

Start by writing down what we know:

$${}^{237}_{93}\text{Np} \rightarrow \text{Pa} + {}^{4}_{2}\text{He}$$

Now balance the proton numbers, giving $Z = (93 - 2) = 91$ for protactinium, and then the nucleon numbers, giving $A = (237 - 4) = 233$ for this isotope of protactinium. The equation is thus:

$${}^{237}_{93}\text{Np} \rightarrow {}^{233}_{91}\text{Pa} + {}^{4}_{2}\text{He}$$

β⁻-emission

Since a β^--particle is an electron, we write it as ${}^{0}_{-1}\text{e}$. A typical β^--emitter is strontium-90. In simple terms, the decay process for this is:

$${}^{90}_{38}\text{Sr} \rightarrow {}^{90}_{39}\text{Y} + {}^{0}_{-1}\text{e} + \bar{\nu}_e$$

strontium-90 → yttrium-90 + β^-particle + electron antineutrino

Once again, the proton and nucleon numbers must balance on each side of the equation. In addition, as we saw in Chapter 8, Section 8.6, the **lepton** numbers must also balance, hence the electron antineutrino. We will look at this in more detail a bit later on.

As with α-decay, we can show this on an A–Z plot (Figure 9.13).

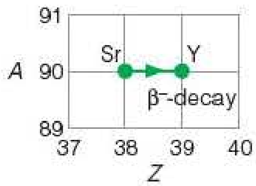


Figure 9.13 Graphical representation of β^--decay

We saw in the cloud chamber photograph in Figure 9.11b that β^--particles are emitted with a range of energies. If a graph of the number of β^--particles having a particular energy is plotted against the energy, a curve like that in Figure 9.14 is obtained.

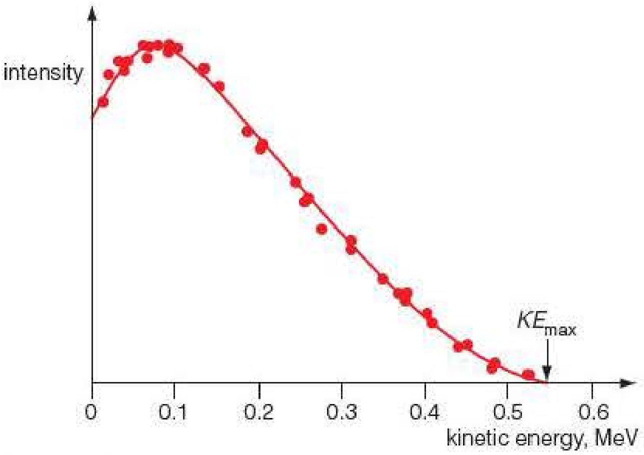


Figure 9.14 Energy spectrum of β^--decay electrons from strontium-90

This continuous distribution of energy could not be explained at first – a particular decay should provide the β^--particle with a definite amount of energy. However, most had a great deal less than this, and where the rest of the energy went to was a mystery. In order to account for this 'missing' energy, Wolfgang Pauli, in 1930, put forward the idea of a new particle, having no mass or charge but a range of possible energies. This imaginary particle, which was necessary for conservation of momentum, as well as energy, in β^--decay, was given the name **neutrino** by Enrico Fermi. By 1934 Fermi had developed a theory of β^--decay to include the neutrino, which some 20 years later led to the concept of the so-called 'weak interaction'. Neutrinos have now been detected and are considered to have no charge but a very, very small mass.

So, β^--decay is accompanied by the emission of a neutrino. In the case of β^--decay, the neutrino is of a particular type called an 'electron antineutrino' (symbol $\bar{\nu}_e$). Why is it an anti-neutrino? We saw in Chapter 8, Section 8.6 that a neutrino was a **lepton** and that by the law of conservation of lepton number the total of lepton numbers on each side of a nuclear equation must balance. Looking at our equation again

$$^{90}_{38}\text{Sr} \rightarrow {}^{90}_{39}\text{Y} + {}^{0}_{-1}\text{e} + \bar{\nu}_e$$

we can see that there are no leptons on the left-hand side. Therefore the lepton number on the right-hand side must also be zero. As an electron has a lepton number of +1, we need an antineutrino (lepton number −1) to balance the nuclear equation.

In the decay of strontium-90, the total energy released from the decay is 0.546 MeV. This is mostly shared by the emitted electron and the antineutrino, with a small amount being taken by the recoiling yttrium nucleus. It follows that the maximum kinetic energy KE_{max} in Figure 9.14 will be about 0.546 MeV.

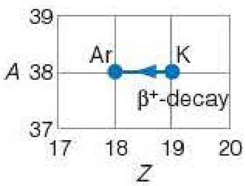


Figure 9.15 Graphical representation of β^+-decay

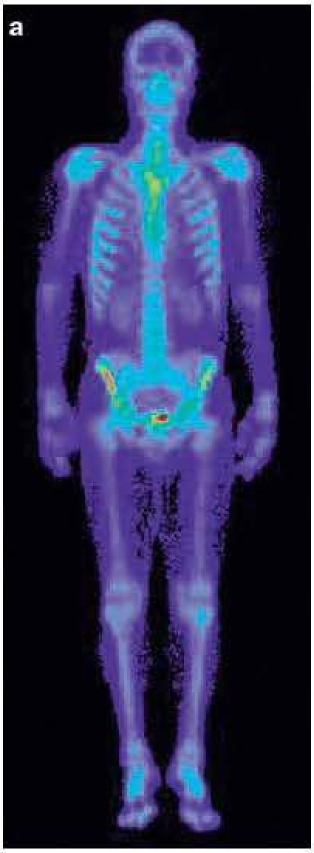


Figure 9.16 a) A modern coloured gamma scan of a human skeleton and b) the first X-ray photograph taken of human bones

Example

Yttrium-90 decays further by another β^--emission to form an isotope of zirconium (Zr). Write down the equation for this process.

Answer

As before, write down the known information:

$$^{90}_{39}\text{Y} \rightarrow \text{Zr} + {}^{0}_{-1}\text{e} + \bar{\nu}_e$$

Balancing the proton numbers gives $Z = (39 - (-1)) = 40$ for zirconium and balancing the nucleon numbers gives $A = (90 - 0) = 90$ for this particular isotope of zirconium. To balance the lepton number we must have an antineutrino. The equation is therefore:

$$^{90}_{39}\text{Y} \rightarrow {}^{90}_{40}\text{Zr} + {}^{0}_{-1}\text{e} + \bar{\nu}_e$$

β^+-emission

We discussed in Chapter 8, Section 8.2 that the electron has an antiparticle, called the **positron**. Some of the lighter elements have isotopes, mostly artificially made, that decay by emitting a positron, or β^+-particle. A typical example is potassium-38, which decays to argon-38 by β^+-emission:

$$^{38}_{19}\text{K} \rightarrow {}^{38}_{18}\text{A} + {}^{0}_{1}\text{e} + \nu_e$$

potassium-38 → argon-38 + β^+-particle + electron neutrino

Note that in this decay process an 'electron neutrino' (ν_e) is also produced. The β^+ is an antiparticle and so has a lepton number of −1. To balance, a neutrino (lepton number +1) is needed. This decay can be represented on an *A–Z* plot as shown in Figure 9.15.

γ-emission

When an α-particle or a β-particle is emitted, the nucleus is usually left with a surplus of energy – it is said to be in an 'excited state'. In order to achieve a stable state the nucleus gives out the excess energy in the form of a quantum of γ-radiation. Thus α- and β-emission is nearly always accompanied by γ-radiation. Gamma rays have identical properties to X-rays; they differ from X-rays only in respect of their *origin*. Gamma rays are emitted from an excited *nucleus*, while X-rays come from the energy released by excited *electrons* returning to lower energy levels outside the nucleus.

Example

The mass of a radium-226 nucleus is 225.9771 u. It decays into radon-222, which has a nuclear mass of 221.9703 u, by emitting an α-particle of mass 4.0015 u and energy 4.8 MeV.

a) Show that the energy released in this decay is about 5.0 MeV.

b) Suggest what happens to the difference between this energy and the 4.8 MeV taken away by the α-particle.

Answer

In Chapter 8, Section 8.4 we used Einstein's equation $\Delta E = c^2\Delta m$ in situations involving the creation and annihilation of matter and antimatter particles. We need to use the same principle here.

a) Adding up the mass of the radon-222 nucleus and the α -particle gives us

$$221.9703\,\text{u} + 4.0015\,\text{u} = 225.9718\,\text{u}$$

Taking this away from the mass of the radium-226 nucleus gives us

$$\Delta m = 225.9771\,\text{u} - 225.9718\,\text{u} = 0.0053\,\text{u}$$

$$\Delta m = 0.0053\,\text{u} = 0.0053 \times 1.66 \times 10^{-27}\,\text{kg} = 8.80 \times 10^{-30}\,\text{kg}$$

So

$$\Delta E = c^2\Delta m = (3.00 \times 10^8\,\text{m}\,\text{s}^{-1})^2 \times 8.80 \times 10^{-30}\,\text{kg} = 7.92 \times 10^{-13}\,\text{J}$$

$$\Delta E = \frac{7.92 \times 10^{-13}\,\text{J}}{1.6 \times 10^{-19}\,\text{J}\,\text{e}\text{V}^{-1}} = 4.95 \times 10^6\,\text{eV} \approx 5\,\text{MeV}$$

b) The α-particle takes away only 4.8 MeV of this energy. Some of the remainder will be taken by the radon-222 nucleus, which recoils in order to conserve momentum. As the radon-222 nucleus is much more massive than the α-particle, its recoil velocity, and hence its kinetic energy, will be very small. The remaining energy is taken away by a quantum of γ-radiation, which is also emitted in the decay.

Test yourself

5 Part of one uranium decay series is shown below:

$${}^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} \rightarrow {}^{234}_{91}\text{Pa} \rightarrow \text{(i)} \rightarrow {}^{230}_{90}\text{Th} \rightarrow {}^{226}_{88}\text{Ra}$$

(ii) (iii) β^- (iv)

a) What is the missing nuclide (i)?

b) What particle is emitted at each of the decays (ii), (iii) and (iv)?

c) Which nuclides constitute a pair of isotopes?

d) The end product of this particular uranium series is a stable isotope of lead, ${}^{206}_{82}\text{Pb}$. How many α-particles and β-particles are emitted in decaying from radon-226 to lead-206? Explain how you worked this out.

6 a) Show that a mass of 1 u has an energy equivalent of 931 MeV (1 u = 1.66×10^{-27} kg).

b) A β-particle has a mass of 9.11×10^{-31} kg. Show that its energy equivalent is about 0.5 MeV.

c) What would be the de Broglie wavelength of a β-particle of energy 0.5 MeV?

9.6 Spontaneous and random nature of radioactive decay

The emission of radiation is both **spontaneous** and **random**. By this we mean that the nuclei disintegrate independently – we cannot tell which nucleus will decay next, nor when it will decay. It is an attempt by an unstable nucleus to become more stable and is unaffected by physical conditions such as temperature and pressure.

As there is usually a *very* large number of nuclei involved, we can employ statistical methods to the process. We can therefore make the assumption that the rate of decay of a particular isotope at any instant will be proportional to the number of nuclei of that isotope present at that instant. In other words, the more radioactive nuclei we have, the more likely it is that one will decay. This is a fundamental concept of radioactive decay and can be expressed in simple mathematical terms as:

$$\text{rate of decay (or activity)} = -\frac{dN}{dt} = \lambda N$$

where N = number of nuclei

and λ = decay constant

Tip

You need to know that the expression $-dN/dt$ is a shorthand way of saying 'the rate of decay at a particular instant' and that the negative sign means that N *decreases* with time t. A full mathematical treatment using calculus is *not* needed for examination purposes.

The unit of **activity** is the **becqerel** (Bq), which is a rate of decay of one disintegration per second. Therefore in base SI units it is equivalent to s^{-1}.

From our equation $-dN/dt = \lambda N$, we can see that, as dN/dt has units of s^{-1} and N is just a number, the decay constant λ must also have units of s^{-1}.

Example

The following is taken from an article about potassium:

Potassium, symbol K, is an element with atomic number 19. Potassium-40 is a naturally occurring radioactive isotope of potassium. Two stable (non-radioactive) isotopes of potassium exist, potassium-39 and potassium-41. Potassium-39 comprises most (about 93%) of naturally occurring potassium, and potassium-41 accounts for essentially all the rest. Radioactive potassium-40 comprises a very small fraction (about 0.012%) of naturally occurring potassium. Potassium-40 decays to an isotope of calcium (Ca) by emitting a beta particle with no attendant gamma radiation (89% of the time) and to the gas argon by electron capture with emission of an energetic gamma ray (11% of the time). Potassium-40 is an important radionuclide in terms of the dose associated with naturally occurring radionuclides.

a) Explain, with reference to potassium, what is meant by *isotopes*.

b) Write down the nuclear equation for the decay of potassium-40 by emission of a beta particle.

c) You have about 130 g of potassium in your body.

i) A mass of 39 g of naturally occurring potassium contains 6.0×10^{23} nuclei. Show that there would be about 2×10^{20} nuclei of potassium-40 in your body.

ii) Given that the decay constant for potassium-40 is $1.7 \times 10^{-17}\,s^{-1}$, calculate the dose (activity) caused by this amount of potassium-40.

iii) Comment on this amount of activity compared with the value for background radiation.

Answer

a) Isotopes are different forms of the same element and so have the same atomic number. For example, potassium has an atomic number of 19, meaning that it has 19 protons in its nucleus. Isotopes have differing numbers of neutrons in the nucleus and therefore a different nucleon number. Potassium-39 has 20 neutrons, potassium-40 has 21 neutrons and potassium-41 has 22 neutrons.

b) ${}^{40}_{19}K \rightarrow {}^{40}_{20}Ca + {}^{0}_{-1}e + \mu_e$

c) i) If 39 g of naturally occurring potassium contains 6.0×10^{23} nuclei, 130 g will contain

$$6.0 \times 10^{23} \times \frac{130}{39} \text{ nuclei} = 2.0 \times 10^{24} \text{ nuclei}$$

But potassium-40 is only 0.012% of naturally occurring potassium, so number of potassium-40 nuclei is

$$2.0 \times 10^{24} \times \frac{0.012}{100} = 2.4 \times 10^{20} \text{ nuclei} \approx 2 \times 10^{20} \text{ nuclei}$$

ii) $-\frac{dN}{dt} = \lambda N \Rightarrow \frac{dN}{dt} - \lambda N$

$$\frac{dN}{dt} = -1.7 \times 10^{17}\,s^{-1} \times 2.4 \times 10^{20} \text{ nuclei}$$
$$= -4.1\,kBq$$

(The minus sign means that the sample is decaying at the rate of 4.1 kBq)

iii) An activity of 4.1 kBq is equivalent to $4.1 \times 10^3\,s^{-1} \times 3600\,s \approx 15\,000\,000$ counts in an hour. From the information given in Figure 9.2, this is of the order of magnitude of background radiation from external sources, so we are being bombarded with radiation from without and within!

9.7 Half-life

We can model radioactive decay with the following experiment using dice.

Activity 9.1

Modelling radioactive decay

Twenty-four dice (or small cubes of wood with one face marked with a cross) are shaken in a beaker and tipped onto the bench. All the dice that have landed with the 'six' uppermost (or the cubes of wood with the 'cross' uppermost) are removed – these are deemed to have 'decayed'. The number N of dice remaining are then counted. This process is repeated until all the dice have 'decayed'. If a graph of N against x is plotted, a curve something like that in Figure 9.17 should be obtained.

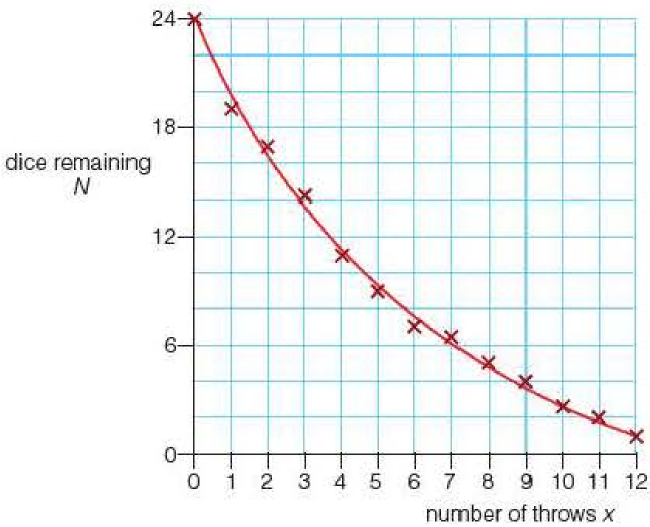


Figure 9.17

As you can see, the curve is not smooth because whether or not a 'six' is thrown is a **random** event, just like radioactivity. It is instructive to repeat the experiment and average the results, or else combine the results with those of one or more classmates. It will be found that the more results that are combined, the smoother the curve will become.

Radioactivity is a random process similar to throwing dice, although it differs from this dice-throwing model insomuch as it is a *continuous* process and the numbers involved are *much, much larger*.

Questions

In a dice-throwing experiment the following data were obtained for 48 dice:

Table 9.5

Number of throws, x	0	1	2	3	4	5	6	7	8
Dice remaining, N	48	41	32	27	23	18	15	14	10

1 Plot a graph of N against x.
2 Use your graph to estimate how many throws it would take for *half* of the initial 48 dice to 'decay' (i.e. 24 left).
3 Explain why the 'decay constant' λ for this model is 1/6. Hence calculate a theoretical value for the 'half-life'. By what percentage does the experimental value differ from the theoretical value?
4 Discuss the extent to which this dice-throwing experiment is a good model for radioactive decay.

Key terms

The **half-life** of a particular isotope is the *average* time for a given number of radioactive nuclei of that isotope to decay to half that number.

As we said before, radioactivity differs from our dice-throwing model as it is *continuous*. We call the time that it takes for a given number of radioactive nuclei to decay to half that number the **half-life**. This is illustrated in Figure 9.18.

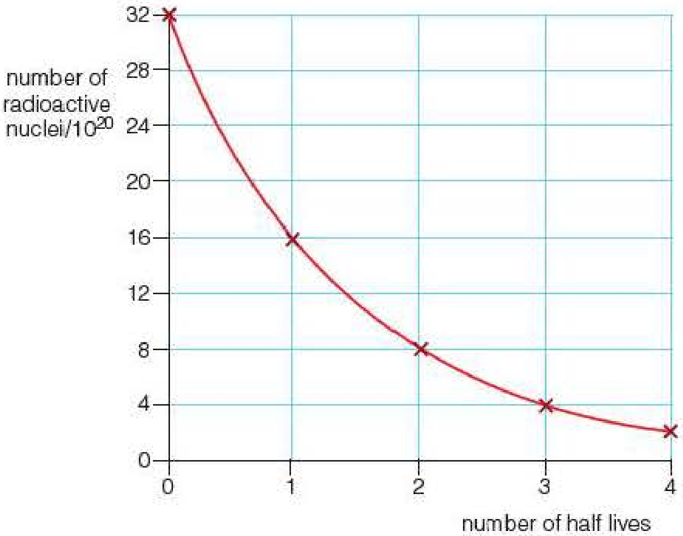


Figure 9.18 Radioactive decay curve

As radioactivity is an entirely *random* process, we can only ever determine an *average* value of the half-life because it will be slightly different each time we measure it (just as you will get slightly different curves each time you do the 'dice' experiment). However, the very large number of nuclei involved in radioactive decay means that our defining equation $-dN/dt = \lambda N$, is statistically valid.

The graph obtained for radioactive decay, such as that in Figure 9.18, is an **exponential** curve. We came across exponential curves in Chapter 5 when we observed the discharge of a capacitor through a resistor – do you remember $Q = Q_0 e^{-t/RC}$? The equivalent equation for radioactive decay is

$$N = N_0 e^{-\lambda t}$$

From this equation it can be deduced mathematically that the half-life, $t_{\frac{1}{2}}$, is related to the decay constant, λ, by the expression

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

If you are studying A-level mathematics, you might like to try to deduce this for yourself, but as far as the physics examination is concerned you only need to recognise and use the expression. You will, of course, need to know which button to press on your calculator to show that $\ln 2 = 0.693$!

Half-lives can vary from the very long to the very short. For example, the half-life of uranium-238 is 4.5×10^9 years, while that of polonium-214, one of its decay products, is a mere 1.6×10^{-4} seconds!

A long half-life means that the radioactive substance has probably been around for a long time if it is a naturally occurring isotope such as uranium-238. More significantly, it will be around for a very long time to come! This is particularly important when considering nuclear reactors; some of the fission products in the nuclear fuel rods have half-lives of millions of years, which means that the re-processing, or safe disposal, of these 'spent' fuel rods is a very dangerous and costly

Tip

Remember that for a graph of any exponential decay the *ratio* of the *y*-values is constant for equal increments in the *x*-values. This is how you can check whether or not a curve is exponential.
The Example on page 78 shows you how to do this.

business. This major disadvantage of nuclear power stations has to be weighed up against the advantages of not using the world's precious supplies of coal and oil and not producing the atmospheric pollution associated with burning fossil fuels.

Example

1 In an earlier example we saw that the isotope potassium-40 has a decay constant of $1.7 \times 10^{-17}\,s^{-1}$. Use this to calculate the half-life, in years, of potassium-40.

2 Calculate the decay constants for uranium-238 and polonium-214 from the values of their half-lives given above.

Answer

1 $$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \frac{0.693}{1.7 \times 10^{-17}\,s^{-1}} = 4.08 \times 10^{16}\,s$$

$$= \frac{4.08 \times 10^{16} s}{(365 \times 24 \times 60 \times 60)\,s\,y^{-1}} = 1.3 \times 10^{9}\,y$$

2 From $t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$ we get, for uranium-238:

$$\lambda = \frac{\ln 2}{t_{\frac{1}{2}}} = \frac{0.693}{4.5 \times 10^{9}\,y \times (365 \times 24 \times 60 \times 60)\,s\,y^{-1}}$$

$$= 4.9 \times 10^{-18}\,s^{-1}$$

and for polonium-214:

$$\lambda = \frac{\ln 2}{t_{\frac{1}{2}}} = \frac{0.693}{1.6 \times 10^{-4}\,s} = 4.3 \times 10^{3}\,s^{-1}$$

Example

The following is taken from an article about iodine:

Iodine-131, which decays by beta and gamma emissions, is used in a number of medical procedures, including monitoring and tracing the flow of thyroxin from the thyroid. With its short half-life of 8 days, it is essentially gone in three months.

a) Calculate the decay constant for iodine-131.

b) Justify the statement that 'it is essentially gone in three months' by showing that the fraction remaining after this time is only about 1 part in 4000.

c) Suggest reasons why iodine-131 is 'used in a number of medical procedures'.

Answer

a) From $t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$

$$\Rightarrow \lambda = \frac{\ln 2}{t_{\frac{1}{2}}} = \frac{0.693}{8.0\,d \times (24 \times 60 \times 60)\,s\,d^{-1}} = 1.0 \times 10^{-6}\,s^{-1}$$

b) The half-life is 8 days, so one month is, to a reasonable approximation, 4 half-lives. Therefore three months will be about 12 half-lives. The fraction of iodine-131 remaining after three months is therefore $(½)^{12}$, which is 1/4096, or about 1 part in 4000.

c) Iodine-131 is a beta and gamma emitter. As these radiations can penetrate the body, they can be detected outside the body, which enables the flow of thyroxin to be monitored easily. The half-life of 8 days is long enough for the patient to be monitored for several days, but, as the calculation shows, short enough that very little remains in the body after two or three months.

Test yourself

7 a) Radium has an isotope $^{226}_{88}Ra$ that has a half-life of approximately 1600 years. What is meant by the term 'half-life'?

b) A sample of the isotope emits both α-particles and γ-rays to form an isotope of radon (Rn). Write down a nuclear equation for this decay.

c) Outline briefly how you could demonstrate experimentally that radium-226 emits both α-particles and γ-rays.

d) How is it possible, that with a half-life of 1600 years, radium-226 is still present in measurable quantities in mineral samples that are over a billion years old?

8 The radon isotope in Question 7 has a half-life of 3.8 days.

a) What is the value of the decay constant for the radon isotope?

b) Show that there are about 4×10^{19} atoms of radon in a sample of mass 15 mg.

c) What would be the activity of this sample?

d) Show that only about 3% would remain after 19 days.

9.8 Experiments involving radioactivity

Safety precautions

Before we look at experiments involving radioactive sources, we need to reiterate that the utmost care must be taken when undertaking such experiments. Safety precautions that must be followed include:

- a sealed source should always be used;
- the source must be kept in a lead-lined box in a special locked metal cupboard in a separate store room, labelled with the radioactivity symbol;
- protective gloves should be worn;
- always handle the source with tongs;
- keep as far away from the source as possible;
- keep exposure time to a minimum;
- as soon as the experiment is finished the source should be locked away again.

Determination of half-life

As the activity of a radioactive isotope, dN/dt, is proportional to the number of nuclei N, we can express the half-life as the time for a given *activity* to reduce to half that rate. If we use the symbol A for activity, our equation becomes

$$A = A_0 e^{-\lambda t}$$

Since we can easily measure activity, or count-rate, this gives us a convenient way of determining half-lives.

You may get the opportunity to see half-life measured in the laboratory but you will not be asked to describe any such experiment in an examination. What

Tip

Make sure you learn the experiment to *model* radioactive decay as described in Activity 9.10.

is important is that you know what safety precautions must be taken and how the results of such an experiment would be analysed to determine the half-life.

Test yourself

9 A typical set of data from an experiment to determine the half-life of protactinium-234, which is a beta-emitter, is shown in Table 9.6.

Background count: 135 counts in 5 minutes

Average: 27min^{-1}

Table 9.6

t/s	0	20	40	60	80	100	120	140	160	180
Count-rate/ min^{-1}	527	437	367	304	262	216	183	157	136	119
Corrected count-rate/ min^{-1}	500	410								

a) Complete the data by calculating the corrected count-rates

b) Plot a graph of this count-rate against time. You should get a characteristic exponential decay curve.

c) Find an average value for the half-life by taking the average of the time to decay from, for example 500 to 250 counts min^{-1} and 250 to 125 counts min^{-1}. You should get a value of about 72 s.

Tip

It doesn't matter at which point you start when finding a half-life. In the example given, you could find the time to decay from 400 to 200 min^{-1}, or from 300 to 150 min^{-1}. Check that these points also give 72s for the half-life.

Taking it further

If we take our equation for radioactive decay, $A = A_0 e^{-\lambda t}$, and take logarithms to base 'e' on both sides of the equation, we get:

$$\ln A = -\lambda t + \ln A_0$$

This is of the form:

$$y = mx + c$$

Therefore, if we plot a graph of ln A (on the y-axis) against t (on the x-axis) we will get a straight line of negative gradient ($-\lambda$) and intercept $\ln A_0$.

You need to be able to analyse an exponential function like this for your Practical Assessment, or in Paper 3, so you should practise plotting 'log' graphs, or more correctly 'ln' graphs

Test yourself

10 a) Using the data from Question 9 above, plot a graph of ln A against t.

b) Determine a value for the decay constant λ from the gradient of your graph and calculate the half-life $t_{\frac{1}{2}}$ from the equation $t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$.

Tip

Remember, if you are plotting a logarithmic graph of an exponential, you must take logs to base 'e' (the 'ln' button on your calculator).

Absorption of gamma radiation by lead

Looking at the properties of the three types of radiation in Section 9.4, we saw that gamma radiation was the most penetrating of the three. Whilst α-particles were absorbed by a sheet of paper and β-particles by a few millimetres of aluminium, γ-rays were not entirely absorbed by even several centimetres of lead.

This can be investigated quantitatively by the experiment described in Core Practical 15.

Core practical 15

Investigation of the absorption of gamma radiation by lead

The experiment involves a closed gamma source, such as 185 kBq (5 micro-Curie) cobalt-60, and 4 or 5 lead discs, ranging in thickness from about 2 mm to 6 mm. The apparatus is set up as shown in Figure 9.19 but *without* the cobalt-60 source, which should not be in the laboratory at this stage.

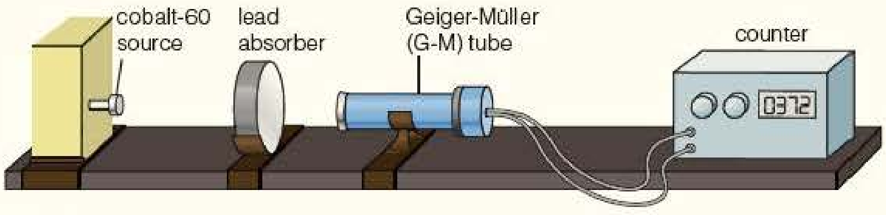


Figure 9.19

Safety note

Remember – radiation is *dangerous*. The safety precautions listed at the start of this section must be strictly adhered to.

The thickness of each of the discs is measured with vernier callipers or a micrometer, and then the background count is recorded for 5 minutes. This is repeated and the average value subtracted from subsequent readings.

Once this has been done, the source is put in position and the count N_0 with no lead discs in place is recorded. The count N is then taken for several thicknesses x of lead by using the discs, either singly or in combination, as absorbers. Care must be taken to keep the distance between the source and the G-M tube constant.

The data are recorded as shown in Tables 9.7 and 9.8 below. As this is an investigation, a graph of the count-rate N against absorber thickness x should initially be plotted. Figure 9.20 shows such a graph for the data in Table 9.8.

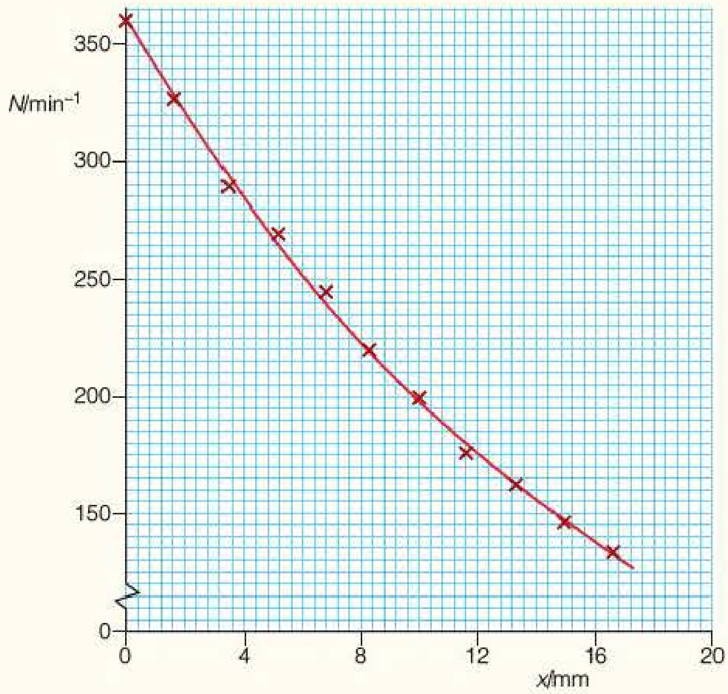


Figure 9.20

Inspection of this graph suggests an *exponential* relationship of the form

$$N = N_0 e^{-\mu x}$$

where μ is a constant – a property of the absorbing material. To test this, a graph of $\ln N$ against x should be plotted.

Questions

1 State what safety precautions you would observe when doing this experiment.

2 The background count was measured for 5 minutes and repeated. The readings were 73 counts and 67 counts. Explain why it is necessary to measure the background count for several minutes and then repeat the measurement to get an average.

3 The measurements in Table 9.7 were recorded for the thickness of 4 lead discs:

Table 9.7

Disc number	1	2	3	4
Thickness/mm	1.7	3.3	5.0	6.6

Explain how an absorber thickness of 14.9 mm could be obtained.

4 The data shown in Table 9.8 were recorded for different thicknesses of lead:

Table 9.8

Total thickness of lead/mm	0.0	1.7	3.3	5.0	6.6	8.3	9.9	11.6	13.3	14.9	16.6
Counts/minute	372	342	304	283	259	234	216	194	178	162	150
Corrected counts/minute	358	328									
ln (N/min^{-1})	5.88	5.79									

Complete the table by adding the rest of the corrected counts and values for ln (N/min^{-1})

5 Use the graph shown in Figure 9.20 to justify the supposition that the absorption of the gamma radiation is an exponential function of the thickness of absorber.

6 Explain why a graph of ln N against x should be plotted in order to test whether N is an exponential function of x of the form:

$$N = N_0 e^{-\mu x}$$

7 Plot a graph of ln (N/min^{-1}) against x/mm.

8 Discuss whether your graph confirms that the absorption of the gamma radiation varies exponentially with the thickness of the absorber as in the proposed relationship.

9 Use your graph to determine a value for the absorption coefficient μ. Explain why the units of μ are mm^{-1}.

9.9 Radioactive dating

One practical application of radioactive decay is radioactive dating. This enables us to determine the age of rocks and other geological features, including the age of the Earth itself, and also of a wide range of natural materials and artefacts made from natural materials.

Carbon-14 dating

Through fixing atmospheric carbon dioxide in the process of photosynthesis, living plants have almost the same ratio of radioactive carbon-14 to stable carbon-12 as in the atmosphere. After plants die, the fraction of carbon-14 in their remains decreases exponentially due to the radioactive decay of the carbon-14. From the measurement of the ratio of carbon-14 to carbon-12 in an organic archaeological artefact – say a wooden ship – the age of the artefact can be estimated. This method is not entirely accurate for a number of reasons, such as the change in composition of the carbon dioxide in the atmosphere that has taken place over the last few thousand years. Also, as the half-life of carbon-14 is 5730 years, the method is not suitable beyond about 60 000 years Nevertheless carbon-14 dating remains a useful tool for archaeologists.

Example

In the 1930s, archaeologists discovered an Anglo-Saxon burial ship at Sutton Hoo, in Suffolk. The many priceless treasures found at the site are now in the British Museum. Carbon-14 dating has helped establish the age of the burial ground. The half-life of carbon-14 is 5730 years.

a) Explain why carbon-14 dating is not suitable for dating things older than about 60 000 years.

b) Show that the decay constant for carbon-14 is about $1.2 \times 10^{-4}\,\text{y}^{-1}$.

c) The count-rate for 1g of carbon in equilibrium with the atmosphere is $15.0\,\text{min}^{-1}$. If samples from the Sutton Hoo burial site were found to have a count-rate for 1g of $12.7\,\text{min}^{-1}$ when measured in 1998, estimate the date of the burial site.

Answer

a) 60 000 years is about 10 half-lives. This means that after 60000 years a fraction of only about $(\frac{1}{2})^{10} = \frac{1}{1024}$, or 1 part in 1000, of the carbo n-14 will remain. This would be too small to measure with any degree of accuracy.

b) From $t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$

$$\Rightarrow \lambda = \frac{\ln 2}{t_{\frac{1}{2}}} = \frac{0.693}{5730\,\text{y}} = 1.21 \times 10^{-4}\,\text{y}^{-1} \approx 1.2 \times 10^{-4}\,\text{y}^{-1}$$

c) From $N = N_0 e^{-\lambda t}$

$$\Rightarrow \frac{N}{N_0} = e^{-\lambda t} \Rightarrow \frac{12.7}{15.0} = e^{-\lambda t} \Rightarrow 0.847 = e^{-\lambda t}$$

Taking natural logs on both sides of the equation gives

$$\ln(0.847) = -0.166 = -\lambda t$$

$$\Rightarrow -0.166 = -1.21 \times 10^{-4}\,\text{y}^{-1} \times t$$

$$\Rightarrow t = \frac{-0.166}{-1.21 \times 10^{-4}\,\text{y}^{-1}} = 1376\,\text{y}$$

The date of the burial site is therefore about (1998 − 1376) = 622. The best we can really say is that it is likely to be early 7th century.

Tip

Science never stands still! As this book is being written (May 2015) proton beam therapy has been very much in the news.

This uses beams of protons accelerated by a particle accelerator to kill cancerous cells. Unlike gamma rays, the protons are stopped once they hit the cancerous cells and so proton beam therapy results in much less damage to surrounding tissue.

Age of the Earth

Long-lived radio-isotopes in minerals, such as potassium-40 which decays to the stable argon-40 with a half-life of 1.3×10^9 years, provide the means for determining long time scales in geological processes. Various methods are used, which provide data for modelling the formation of the Earth and the solar system, and for determining the age of meteorites and of rocks from the Moon. The techniques used are complex and beyond the scope of A level. The average of a large number of independent methods gives the age of the Earth as 4.54 billion years (4.54×10^9 years), with an uncertainty of better than 1%. This is consistent with the age of the universe determined from measurements of the Hubble constant (see Section 14.5), which give a value for the age of the universe of between 13 and 14 billion years.

9.10 Nuclear medicine

No discussion of nuclear decay would be complete without looking more closely at some of its applications in medicine. Ionising radiation is used both for diagnosis (finding out what is wrong) and therapy (trying to put it right). For diagnostic imaging, the radioactive isotope is concentrated in the organ under investigation and the emissions are detected outside the body. For therapy, the emissions are usually designed to destroy cancerous tissue. In both cases, it is important that minimal damage is done to healthy cells.

For diagnosis, the isotope must be absorbed by the organ without affecting the organ's function in any way. The isotope must have a half-life that is long enough to complete the diagnostic examination, but short enough so that the patient is subjected to the minimum dose of radiation.

For therapy, external sources of high-energy gamma rays, for example from cobalt-60, are now being replaced by high-energy (MeV) X-rays, because X-rays are more easily switched off and their energy can be altered as required. Alternatively, high doses of a radio-isotope accumulated in an organ can be used to kill cells from within the body, for example doses of the order of 400 MBq are used to treat overactive thyroids, while doses of several GBq are used for cancer therapy. Since cancerous cells are rapidly dividing, they are more susceptible to radiation damage.

Table 9.9 summarises some of the isotopes used in nuclear medicine.

Table 9.9

Isotope	Emission	Energy	Half-life	Use
Technetium-99m(^{99m}Tc)	γ	140 keV	6.0 h	*Diagnosis:* Localisation of tumours Monitoring blood flow in heart and lungs Kidney investigations
Iodine-123 (^{123}I)	γ	160 keV	13 h	*Diagnosis:* Localisation of tumours Assessing thyroid function
Iodine-131 (^{131}I)	β-, γ	360 keV	8 days	*Therapy:* Thyroid function and tumours

Meta-stable technetium-99m is the most versatile and commonly used radio-isotope and is worth special consideration. It is a decay product of molybdenum-99, which is produced by neutron bombardment in a nuclear reactor. The relevant nuclear equations are:

$^{99}_{42}Mo \rightarrow {}^{99m}_{43}Tc + \beta^- \bar{\nu}_e$ (half-life 67 hours)

$^{99m}_{43}Tc \rightarrow {}^{99}_{43}Tc + \gamma$ (half-life 6.0 hours)

The technetium-99m is chemically separated from the molybdenum by dissolving it in a saline solution and flushing it out. The principle of this process, which is called 'elution', is outlined in Figure 9.21.

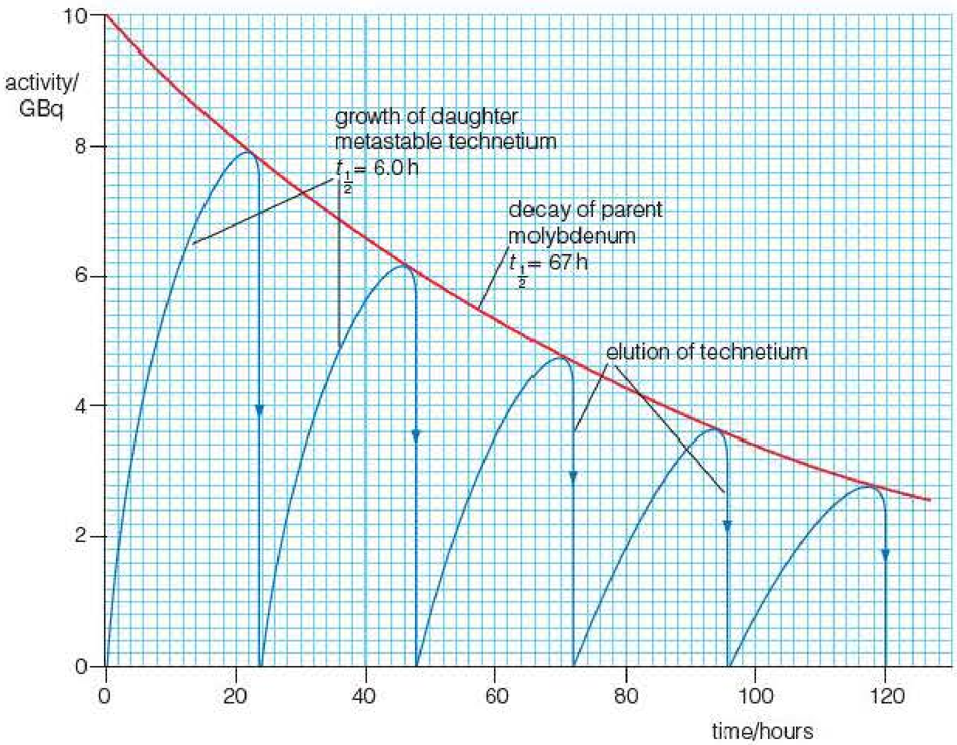


Figure 9.21

Tip

To calculate $e^{-1.73}$, insert 1.73 into your calculator, press +/- to get −1.73, and then press the e^x key. You should get 0.177.

Tip

In calculations involving half-lives, like that in part a), it is a good idea to do a rough check.

A week is (24 × 7) h ÷ 67 h = 2.5 half-lives.

After 1 half-life 50% remains.

After 2 half-lives 25% remains.

After 3 half-lives 12% remains.

So after a week, or 2.5 half-lives, 18% would seem reasonable as it is somewhere between 12% and 25%.

The half-life of the molybdenum is short enough to allow fresh supplies of technetium to be removed daily, and long enough so that the elution cell will last for over a week (2 to 3 half-lives) before it needs to be recharged. The 6-hour half-life of the technetium-99m is ideal for most medical examinations.

Example

a) The half-life of molybdenum-99 is 67 hours. What percentage of a sample will remain after one week?

b) Technetium-99 actually decays by β^-emission, but the half-life is very long (2.2×10^5 years) and its activity is too small to have much effect on the body cells. Explain why its activity is very small.

Answer

a) Using $N = N_0 e^{-\lambda t}$, where $\lambda = \frac{0.693}{t_{\frac{1}{2}}} = \frac{0.693}{67\text{ h}} = 0.0103\text{ h}^{-1}$

$$\Rightarrow N/N_0 = e^{-0.0103 \times 24 \times 7} = e^{-1.73} = 0.177$$

This means that 18% (to the nearest percent) of the sample remains after one week.

b) The half-life is 2.2×10^5 years, so the decay constant λ will be

$$\lambda = \frac{0.693}{(2.2 \times 10^5 \times 365 \times 24 \times 60 \times 60)\text{s}} = 1.0 \times 10^{-13}\text{ s}^{-1}$$

As activity = λN and λ is so small, the activity will also be very small.

Test yourself

11 Iodine-131 ($^{131}_{53}$I) is a β^--emitter and has a half-life of 8.0 days. A sample has an activity of 32 kBq.

a) Explain why the activity will fall to about 2 kBq after one month.

b) Show that its decay constant is approximately $1 \times 10^{-6}\text{ s}^{-1}$.

c) How many nuclei would be needed to give an activity of 32 kBq?

d) Given that 131g of iodine-131 contains 6.0×10^{23} nuclei, what would be the mass of iodine-131 in a source having this activity?

e) This isotope, in the form of a solution of sodium iodide, is used as a radioactive 'tracer' in medicine. Suggest two reasons why it is particularly suitable for this purpose.

f) Iodine-131 decays to an isotope of xenon (Xe). Write down a nuclear equation for this process.

g) Iodine-131 also emits some γ-radiation.

i) In what ways does γ-radiation differ from α- and β-radiation?

ii) Why does the iodine-131 nucleus emit γ-radiation as well as β^--radiation?

Exam practice questions

1 Which of the following types of radiation does not emanate from the nucleus?

A alpha

B beta

C gamma

D X-rays **[Total 1 mark]**

2 Which of the following types of radiation would cause the most ionisation when passing through air?

A alpha

B beta

C gamma

D X-rays **[Total 1 mark]**

3 The range of α-particles in air is typically:

A 0.5 mm

B 5 mm

C 50 mm

D 500 mm **[Total 1 mark]**

4 When an isotope decays by α-emission, which of the following takes place?

A Its mass number and its atomic number both increase.

B Its mass number stays the same and its atomic number increases.

C Its mass number stays the same and its atomic number decreases.

D Its mass number and its atomic number both decrease. **[Total 1 mark]**

5 When an isotope decays by β-emission, which of the following takes place?

A Its mass number and its atomic number both increase.

B Its mass number stays the same and its atomic number increases.

C Its mass number stays the same and its atomic number decreases.

D Its mass number and its atomic number both decrease. **[Total 1 mark]**

6 Bromine-83 has a half-life of 2.4 hours. The fraction of a sample of bromine-83 remaining after one day has elapsed would be about:

A 0.1

B 0.01

C 0.001

D 0.0001. **[Total 1 mark]**

7 a) What is meant by *background radiation*? State two sources of background radiation. **[3]**

b) If you were given a glass beaker of soil that was thought to be radioactively contaminated, how would you determine whether α-, β- or γ-radiation was present in the soil sample? You may assume that you have normal laboratory apparatus available. **[7]**

[Total 10 marks]

8 A particular type of smoke detector uses an ionising chamber containing a radioactive isotope of americium-241 ($^{241}_{95}$Am), which is an alpha-emitter and has a half-life of 460 years. A schematic diagram of the arrangement is shown in Figure 9.22.

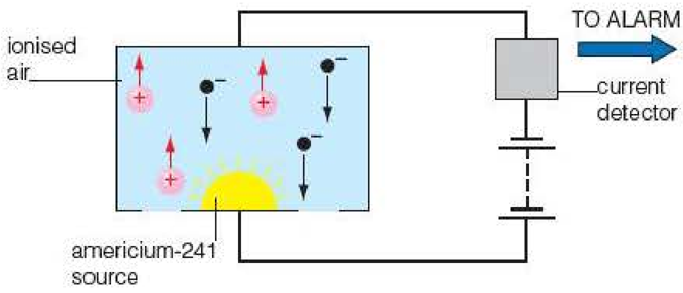


Figure 9.22

The alpha particles ionise the air, thus enabling it to conduct a small current. If smoke particles enter the chamber they reduce the amount of ionisation and the drop in current triggers an alarm.

a) Explain what is meant by ionisation. [2]

b) What is an *alpha particle*? [1]

c) An alpha particle has a range of a few centimetres in air.

i) What happens to its energy as it slows down?

ii) What eventually happens to the alpha particle? [3]

d) i) What is meant by an *isotope*?

ii) How many protons and how many neutrons are there in a nucleus of the isotope americium-241?

iii) The americium-241 decays to an isotope of neptunium (Np). Write down a nuclear equation for this process. [5]

e) i) What is meant by the term *half-life*?

ii) Show that the decay constant for americium-241 is approximately $4.8 \times 10^{-11}\,s^{-1}$.

iii) The mass of americium-241 in a typical source is 1.6×10^{-8} g. Given that 241 g of americium-241 contains 6.0×10^{23} nuclei, calculate the number of radioactive nuclei in the source.

iv) Hence calculate the activity of the source. [6]

f) Suggest three reasons why americium-241 is a suitable source for such a smoke detector. [3]

[Total 20 marks]

9a) Complete these nuclear equations:

i) Plutonium-239 decaying by α-emission to an isotope of uranium (U).

$^{239}_{94}\text{Pu} \rightarrow$ **[2]**

ii) Cobalt-60 decaying by β^--emission to an isotope of nickel (Ni).

$^{60}_{27}\text{Co} \rightarrow$ **[3]**

iii) Magnesium-23 decaying by β^+-emission to an isotope of sodium (Na).

$^{23}_{12}\text{Mg} \rightarrow$ **[2]**

b) Show each of these decays on a plot of nucleon number against proton number. **[3]**

[Total 10 marks]

10 The diagram shows a protactinium generator in which a layer of β-emitting protactinium-234 has been created.

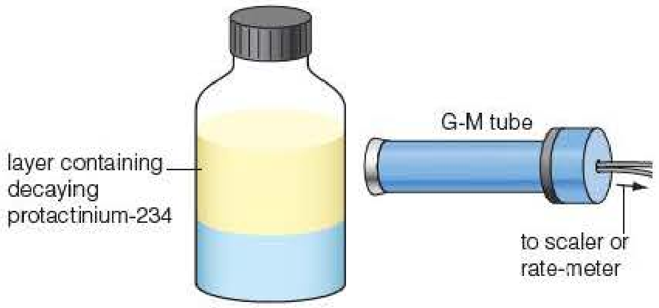


Figure 9.26

a) Define the term *half-life*. **[1]**

b) Explain how you could use the arrangement shown above to find the half-life of protactinium-234, which has a value of about one minute. Your answer should include:

- a description of the readings you would take;
- a sketch of the graph you would plot;
- an explanation of how you would find the half-life from your graph. **[9]**

c) A bottle of milk is contaminated with a small amount of strontium-90, which is also a β^--emitter and has a half-life of 28 years. Give two reasons why the method you have described for the protactinium-234 would not be suitable for determining the half-life of strontium-90. **[2]**

[Total 12 marks]

11 The following was taken from a press article about the suspected murder of a former Russian spy in 2006.

The death of Alexander Litvinenko is linked to a massive dose of polonium-210 found in his body. Polonium-210 decays by emitting high-energy alpha particles that cause terrible internal cell damage, but are difficult to detect outside the body. How the polonium was transported is a mystery, as even a small quantity of it would melt a glass capsule.

a) Explain briefly why alpha particles:

i) 'cause terrible internal cell damage',

ii) 'are difficult to detect outside the body'. **[3]**

b) Polonium-210 has a half-life of 138 days.

i) Calculate the percentage of a given sample remaining after one year.

Sketch a graph of the decay over a 2-year period of a sample that has an initial activity of 2.5×10^8 Bq. **[6]**

ii) Calculate the decay constant for polonium-210.

Hence calculate the activity of 1.0 μg of polonium-210, given that there are 2.9×10^{15} atoms in 1.0 μg. **[3]**

c) The energy of the high-energy alpha particles emitted by polonium-210 is 5.3 MeV. Show that 1 g of polonium would generate a power of about 140 W.

Suggest why 'even a small quantity of it would melt a glass capsule'. **[4]**

[Total 16 marks]

12 The 'Big Bang' was thought to have created mainly hydrogen and helium. In the hearts of stars the formation of carbon is possible through the so-called triple alpha reaction, in which three helium nuclei (alpha particles) fuse to make a nucleus of carbon-12.

Rather than recreate the scorching conditions inside stars, physicists at CERN watched the reaction in reverse. To do this they created nitrogen-12, which is transformed into carbon-12 by beta-plus decay; the carbon-12 then breaks into three alpha particles.

Write down the nuclear equations for the reactions described in each of the paragraphs above.

[Total 6 marks]

Stretch and challenge

13 a) The basic equation in radioactivity is

$$-\frac{dN}{dt} = \lambda N$$

i) Explain the fundamental concept on which this equation is based.

ii) If radioactivity is a random process, how can there be mathematical laws that can be used to calculate what happens?

iii) Use this equation to show that $N = N_0 e^{-\lambda t}$ and hence that $t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$. **[6]**

b) In investigations of the blood flow in the brain a patient is injected with a solution containing the radioactive isotope fluorine-18 ($^{18}_{9}F$) that has a half-life of about 2 hours. The radioisotope undergoes β^+ decay. The emitted positron travels in tissue for a short distance, during which time it loses kinetic energy, until it can interact with an electron producing a pair of annihilation (gamma) photons, each of energy 512 keV moving in approximately opposite directions. These are detected when they reach a scintillator in the scanning device. This is called positron emission tomography (PET) (see Section 8.4).

i) Give two reasons why an isotope having a short half-life is desirable. **[2]**

ii) Suggest why fluorine-18 is a β^+ emitter. **[2]**

iii) Why is it necessary to base PET on the use of positrons rather than electrons? **[2]**

iv) Doctors have to wait for about an hour for the solution to be absorbed into the blood flow. What fraction of the fluorine-18 will have decayed in this time? **[4]**

v) Explain, with reference to conservation laws, why 'a pair of gamma photons, each of energy 512 keV moving in approximately opposite directions' is produced in this electron-positron annihilation. **[4]**

[Total 20 marks]

14 In 1932, James Chadwick experimentally confirmed the existence of neutrons. His conclusion was that

The properties of the penetrating radiation emitted from beryllium, when bombarded by the α-particles of polonium, have been examined. It is concluded that the radiation consists, not of quanta as hitherto supposed, but of neutrons, particles of mass 1 and charge 0. Evidence is given to show that the mass of the neutron is probably between 1·005 and 1·008. This suggests that the neutron consists of a proton and an electron in close combination, the binding energy being about 1 to 2 × 10⁶ electron volts.

In his experiment he produced neutrons by bombarding a sheet of beryllium ($^{9}_{4}Be$) with α-particles of energy 5.3 MeV from polonium-210 ($^{210}_{84}Po$). He then placed a sheet of paraffin wax in the path of the neutrons. Paraffin wax, being a hydrocarbon, has a high density of hydrogen atoms. Some of the neutrons collided head-on with these hydrogen atoms, knocking out protons, which were then detected in an ionisation chamber.

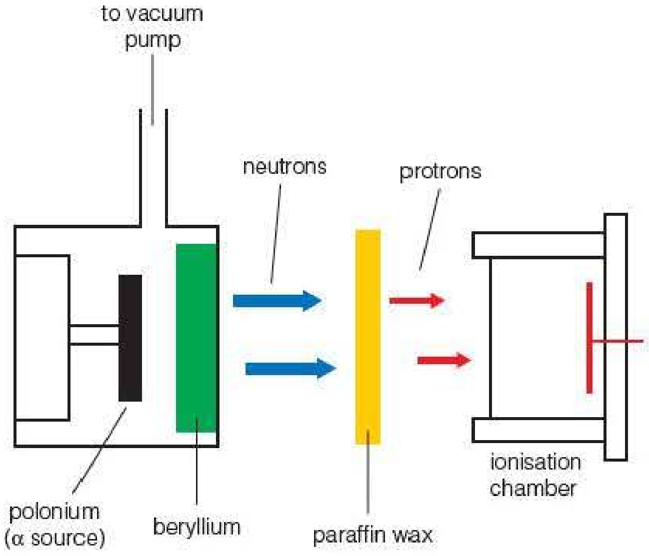


Figure 9.27 Schematic of Chadwick's apparatus

a) Chadwick obtained his polonium from 'Radon D', an isotope of lead ($^{210}_{82}Pb$), which decayed by two β^- decays to form polonium-210. Write down a nuclear equation to show this process and the subsequent decay of the polonium-210. **[3]**

b) Why was it necessary to pump out the air from the chamber containing the polonium source? **[1]**

c) Explain why the neutrons could not be directly detected by an ionisation chamber. **[2]**

d) Assuming that a neutron has approximately the same mass as a proton, explain what happens when a neutron makes a head on, elastic collision with a stationary proton, as in the paraffin wax. **[2]**

e) It can be shown that, to a good approximation, the energy E_γ of γ-radiation needed to knock out protons with energy E_p is $E_\gamma = \sqrt{\frac{1}{2}E_p m_p c^2}$. Show that this would be about 50 MeV for protons of energy 5.7 MeV, as was the case in Chadwick's experiment. What about this suggested to Chadwick that 'the radiation consists, not of quanta, as hitherto supposed' (proton mass = 1.007 267 u)? **[4]**

f) We now know that the mass of a neutron is 1.008 665 u. Show that the binding energy (see Section 14.2) is 'about 1 to 2×10^6 electron volts' as Chadwick had predicted. **[3]**

g) A neutron decays into a proton with a half-life of about 10.3 minutes. Write down the nuclear equation for this decay. Explain the conservation laws involved. **[3]**

h) State what quark change has taken place in this decay and the name of the force involved. **[2]**

[Total 20 marks]

10 Specific heat capacity

Prior knowledge

From earlier work you should by now be very familiar with the concept of conservation of energy. You should know that

- energy can be transferred: thermal ('heat'), electromagnetic waves, sound, kinetic and potential energy, chemical and nuclear
- for a system to remain at a constant temperature, the rate at which it radiates must equal the rate at which it absorbs (or produces) energy. This is called **thermal equilibrium**
- Electrical power $P = IV$
- Electrical energy $\Delta E = IV\Delta t$
- Kinetic energy $E_k = \frac{1}{2}mv^2$
- change in gravitational potential energy $\Delta E_p = mg\Delta h$

Key term

A system is in **thermal equilibrium** when it is radiating energy at the same rate as it is absorbing energy. The system will then have a constant temperature.

Test yourself on prior knowledge

1 The label on an electric heater states that it is rated as 240 V, 3 kW.

a) How much energy, in MJ, does it give out in one hour?

b) How much current does it take?

c) What is the resistance of the element?

2 A laboratory heating coil operates from a 12 V supply and takes a current of 2.5 A.

a) How much power does it produce?

b) How much energy, in kJ, does it convert in 20 minutes?

3 Calculate:

a) The change in gravitational energy per second (i.e. the power) of the water falling over Niagara Falls if the water falls through 51 m at a rate of 2300 $m^3 s^{-1}$.

b) The kinetic energy that has to be transferred when a Airbus 380 of mass 375 tonnes lands at a speed of 80 $m s^{-1}$ and is brought to rest on the runway.

4 A tennis ball is dropped and bounces back at half the speed with which it hit the ground.

a) Draw a velocity–time graph of its motion for *two* bounces.

b) What does the gradient of your graph represent and why is it the same for both bounces?

c) Mark on the graph the points corresponding to the ball hitting the ground.

d) Indicate how you could use the graph to find:

i) the distance the ball drops after it is released

ii) the distance the ball rises after the first bounce.

You may add to your graph if you wish.

e) Outline the energy transfers that take place when the tennis ball is dropped and bounces up again.

10.1 Heating and temperature

In this chapter you will learn about specific heat capacity. This is a property of a *material*. The specific heat capacity of a material (measured in $\mathrm{J\,kg^{-1}\,K^{-1}}$) is an indication of how much energy (in J) would be needed to raise the temperature of 1 kg of the substance by 1 K. Knowledge of the specific heat capacity of different substances is important in the choice of materials for particular applications, for example storage heaters. It is quite easy to determine reasonable values for the specific heat capacity of solids and liquids in a school or college laboratory, so you will be expected to be familiar with such experiments and their limitations. You also need to be clear what is meant by the terms *heating* and *temperature* so that you can use these terms correctly and in the appropriate context.

'Heat' and 'temperature' are two words that we often use in everyday life. They are such common words that you would think it would be easy to explain what we understand by each of them – but it isn't! We know what we mean by 'hot' and 'cold' – we can feel the difference – but ask your friends to tell you what the words 'heat' and 'temperature' mean and see what responses you get!

They might say that temperature is 'how hot something is', but then what does 'hot' mean – it's just another, less scientific, word for 'high temperature'! What we do know from simple observation is that, if two bodies are at different temperatures, *energy* flows from the 'hot' body to the 'cold' body. In physics this flow of energy, from a higher temperature to a lower temperature, due to conduction, convection or radiation, is what is commonly called 'heat' or, more correctly thermal energy. As 'heat' is a form of energy it is measured in the units of energy, i.e. joules (J). Rather than use 'heat' as a noun, it is better to use 'thermal energy' and to use the term heating to describe the process of transferring energy due to a temperature difference. **Heating** is the process by which energy is transferred by conduction, convection or radiation from a higher temperature to a lower temperature. A definition of heating in terms of internal energy is given in Chapter 11.

Key term

The **internal energy** of a body is the sum of potential energy contained within the inter-atomic bonds and the kinetic energy of the vibrations of its atoms.

You should understand that the process of heating is a *flow* of energy. The idea that 'heat' is something contained in a body is wrong. What a body does have is **internal energy**. This is made up of the **potential energy** contained within the inter-atomic bonds and the **kinetic energy** of the vibrations of the atoms. The potential component arises from the energy stored in the inter-atomic bonds that are being continuously stretched and compressed as the atoms vibrate – rather like springs.

Key term

Temperature is directly related to the mean, random, kinetic energy of the vibrating atoms of a body.

The kinetic energy of the atoms depends on the temperature – the hotter a body is, the more rapidly its atoms vibrate and so the greater their kinetic energy becomes. The **temperature** of a body can therefore be defined in terms of the kinetic energy of its vibrating atoms.

We will look at this kinetic theory of matter again in Chapter 12, where we will see that the internal energy of a body (and hence its temperature) can also be increased by doing '**work**' on it – for example, rubbing your hands together makes them hot! Sometimes all the energy transferred just increases the potential energy, for example ice melting, in which case there is no change in temperature until all the ice has melted. This, 'latent' (meaning 'hidden') heat, will be discussed further in Chapter 11. It is called latent heat because energy is being transferred without any change in temperature and so the cause of this energy change is not immediately apparent.

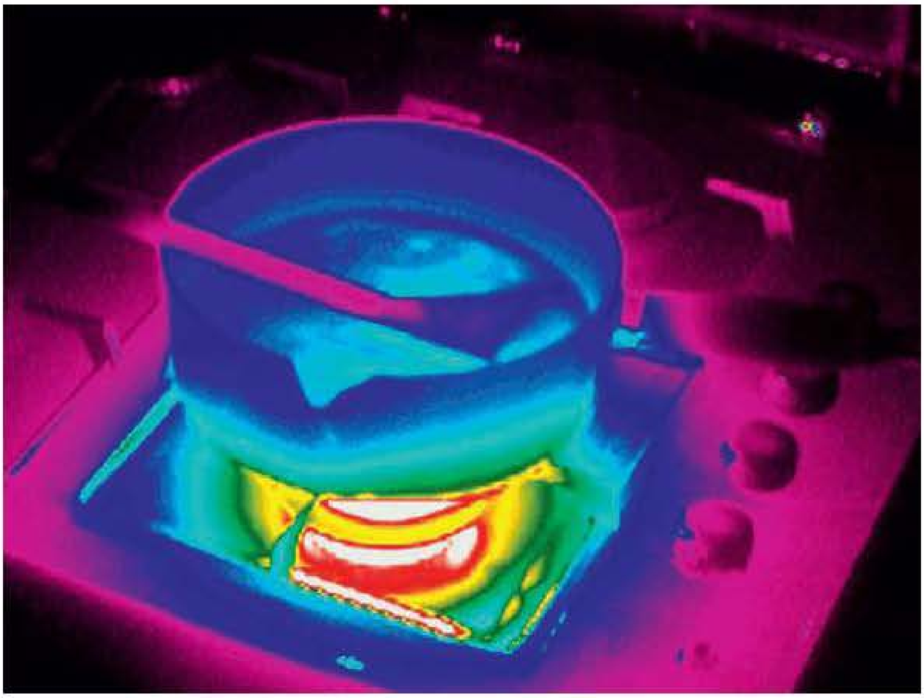


Figure 10.1 A thermogram

Figure 10.1 shows a thermogram of soup being heated in a pan on a gas hob. Thermography records the temperature of surfaces by detecting long-wavelength infrared **radiation**. The colours show variations in temperature, with a scale that runs from white (hottest), through red, yellow, green and blue to pink (coldest). The blue of both the pan and the soup show good thermal **conduction** by the metal pan. The plastic handle of the spoon is a poor conductor and stays pink. The intense temperature of the gas flames is seen under the pan. Thermal energy from the flames to the pan is mainly transferred by **convection**.

10.2 Units of temperature

Temperature is one of the fundamental (base) SI quantities. The base SI unit of temperature is the **kelvin**, symbol K (not °K). This is defined in terms of what is called the absolute thermodynamic scale of temperature, which has **absolute zero** as its zero and *defines* the melting point of ice as 273 K (to 3 SF). The meaning of 'absolute zero' will be explained in Chapter 11.

In practical everyday situations, we measure temperature on the Celsius scale, which defines the melting point of ice as 0°C and the boiling point of water as 100°C under certain specified conditions.

However, a temperature *interval* of 1 K is exactly equivalent to a temperature *interval* of 1°C – for example, a temperature rise of 1°C from 20°C to 21°C is precisely the same as a temperature rise of 1 K from 293 K to 294 K.

We usually give the symbol θ to a temperature recorded in °C and the symbol T if the temperature is in K. From the above, it follows that a change in temperature of $\Delta\theta$ measured in °C is numerically the same as the corresponding *change* of temperature ΔT in K. As the kelvin is the SI unit of temperature we should, strictly speaking, always use 'K' when we are quoting a temperature change.

Absolute zero (0 K) corresponds to a temperature of approximately –273°C on the Celsius scale, therefore temperatures measured in °C can be converted to temperatures in K by using

$$T/\text{K} = \theta/°\text{C} + 273$$

Example

Calculate the missing temperatures in Table 10.1, which shows the boiling points of some common elements.

Table 10.1

Element	°C	K
aluminium	2350	a)
argon	–186	b)
copper	c)	2853
helium	d)	4

Answer

a) 2350°C = (2350 + 273) K = 2623 K

b) –186°C = (–186 + 273) K = 87 K

c) 2853 K = (2853 – 273)°C = 2580°C

d) 4 K = (4 – 273)°C = –269°C

Tip

Remember:

Table 10.2

Temperature	°C	K
absolute zero	–273	0
ice point of water	0	273
boiling point of water	100	373

10.3 Specific heat capacity

Imagine that you were to heat 1 kg of copper, 1 kg of aluminium and 1 kg of water by means of an electric heater, connected to a joule-meter, and measure how much energy was needed to raise the temperature of each by, say, 10°C – whoops, we should say 10 K!

You would find that it took about 3.9 kJ for the copper, 10.1 kJ for the aluminium and 42 kJ for the water. It is clear from this that significantly different quantities of energy are needed to raise the temperature of equal masses of different materials by the same amount.

The property of a material that quantifies this is called its **specific heat capacity**, which is given the symbol c.

If we have a mass m (kg) of the material and we raised its temperature by ΔT (K), the energy needed ΔE (J) would be

$$\Delta E = mc\Delta T$$

Tip

Remember a temperature in K is always 273 greater than the corresponding temperature in °C, so from °C to K always add 273 from K to °C always subtract 273.

Key term

The **specific heat capacity** of a material represents the quantity of energy (in J) per unit mass (in kg) per degree change in temperature (in K). It has units of J kg^{-1} K^{-1}.

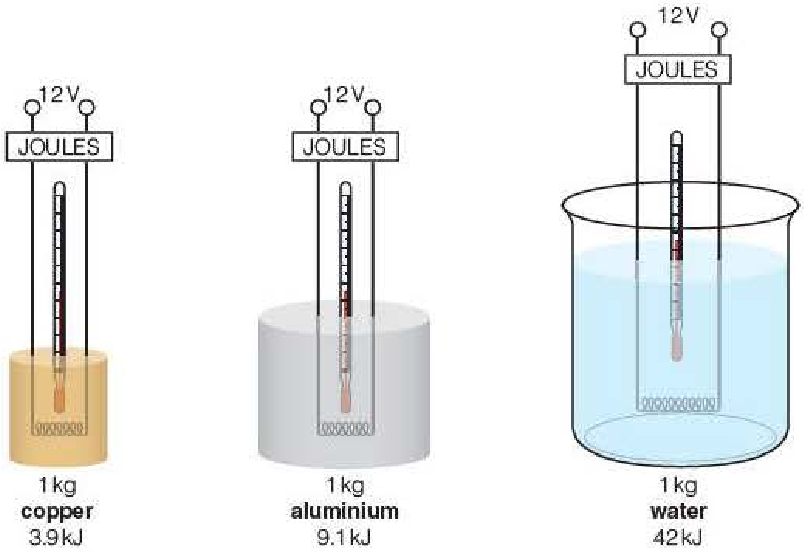


Figure 10.2

Rearranging:

$$c = \frac{\Delta E}{m\Delta T}$$

giving the units of c as $\mathbf{J\,kg^{-1}\,K^{-1}}$.

In Figure 10.2 we had 1 kg of each material and the temperature rise was 10 K in each case. This tells us that the specific heat capacities of copper, aluminium and water are as given in Table 10.3.

Table 10.3

Material	Specific heat capacity/J $kg^{-1}K^{-1}$
copper	390
aluminium	910
water	4200

Example

a) An aluminium saucepan of mass 400 g, containing 750 g of water, is heated on a gas hob. How much energy would be required to bring the water from a room temperature of 18°C to the boil?

b) An electric kettle rated at 2.1 kW contains 1.2 kg of water at 25°C. Calculate how long it will take for the water to come to the boil. Explain why it will actually take longer than you have calculated.

Answer

a) You have to remember that the saucepan will have to be heated to 100°C, as well as the water, so:

$$\Delta E = m_a c_a \Delta T + m_w c_w \Delta T \text{ where } \Delta T = (100 - 18)°\text{C} = 82\,\text{K}$$

$$\Delta E = (0.400\,\text{kg} \times 910\,\text{J}\,\text{kg}^{-1}\text{K}^{-1} \times 82\,\text{K}) + (0.750\,\text{kg} \times 4200\,\text{J}\,\text{kg}^{-1}\text{K}^{-1} \times 82\,\text{K})$$

$$= 30\,\text{kJ} + 260\,\text{kJ} = 290\,\text{kJ}$$

b) Energy to heat water $\Delta E = mc\Delta T = 1.2\,\text{kg} \times 4200\,\text{J kg}^{-1}\,\text{K}^{-1} \times 75\,\text{K}$

$$= 378\,\text{kJ}$$

Energy supplied electrically $= P\Delta t = 378\,\text{kJ}$

$$\Delta t = \frac{378 \times 10^3\,\text{J}}{2.1 \times 10^3\,\text{J s}^{-1}} = 180\,\text{s}$$

In practice, it will take longer than this because some energy will be needed to heat the element of the kettle, and the kettle itself, and some energy will be transferred to the surroundings.

Test yourself

1 Calculate the missing temperatures in Table 10.4, which shows the freezing points of some common elements.

Table 10.4

Element	°C	K
aluminium	660	a)
mercury	–39	b)
copper	c)	1358
oxygen	d)	54

2 Calculate the missing values in Table 10.5. The initial temperature θ_i is 20° C in each case.

Table 10.5

Substance	Mass/g	c/J kg^{-1}K^{1}	θ_{final}/°C	ΔE/kJ
Olive oil	150	2000	30	a)
Aluminium	120	910	b)	5.46
Copper	500	c)	50	5.85
Water	d)	4200	28	8.40

3 In a bathroom there is a bath that holds 250 litres of water and an electric shower rated at 10.5 kW.

a) How much energy must be supplied to heat the water in the bath from 20°C to 40°C?

b) A survey shows that the average time for a student to take a shower is 7 minutes. How much energy will the shower use if it is on for 7.0 minutes?

c) Comment on your two answers.

4 The shower in Question 3 has a flow rate of 7.5 litres of water per minute. If the cold water supply has a temperature of 20°C, what will the temperature of the water at the shower head be?

10.4 Measuring specific heat capacity

The specific heat capacity, of both solids and liquids, can be found by simple electrical methods using the principle:

electrical energy transferred by heater = increase in internal energy of material

$$\Delta E = P\Delta t = IV\Delta t = mc\Delta T$$

Activity 10.1

Measuring the specific heat capacity of aluminium

A block of aluminium is weighed to find its mass m and is then placed in the lagging. A little cooking oil is inserted into the two holes to ensure good thermal contact when the heater and the thermometer are inserted.

The initial temperature θ_i of the block is recorded. The power supply is then switched on and, at the same time, the stopclock started. The current I and the potential difference V are recorded.

After a time Δt of 3.00 minutes, the power is switched off and the highest steady temperature θ_f reached by the block is recorded. Then, from:

$$IV\Delta t = mc(\theta_f - \theta_i)$$

the specific heat capacity c can be calculated:

$$c = \frac{IV\Delta t}{m(\theta_f - \theta_i)}$$

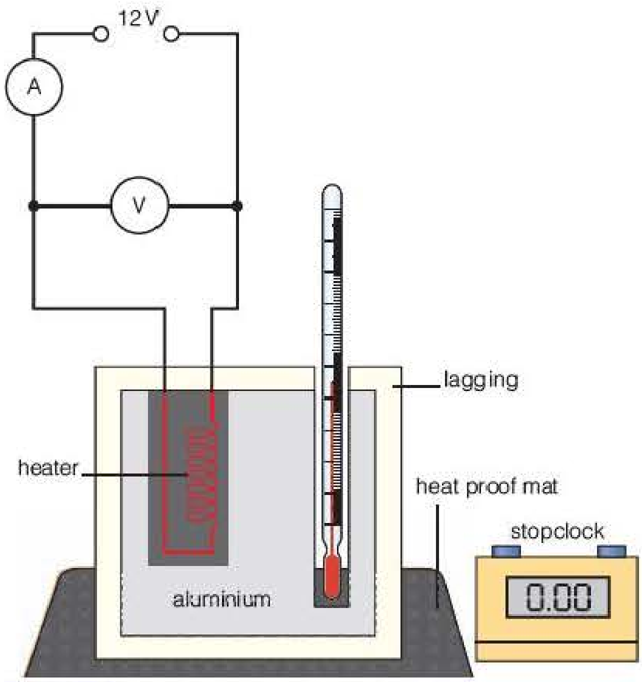


Figure 10.3

Questions

1 The following data were recorded in an electrical method to find the specific heat capacity of aluminium:

$m = 993\,\text{g}$ $\quad V = 10.3\,\text{V}$ $\quad \theta_i = 21.4\,°\text{C}$

$I = 3.10\,\text{A}$ $\quad \Delta t = 3.00\,\text{minutes}$ $\quad \theta_f = 27.3\,°\text{C}$

Use these data to calculate the specific heat capacity of aluminium.

2 Explain which of these measurements is likely to contribute most to the uncertainty in the value obtained for the specific heat capacity.

The experiment described above is quick and easy to perform, and gives quite a goodvalue for the specific heat capacity. There are, however, several sources of error:

- energy is absorbed by the heater itself; this is the main source of error and will make the value of c too large because it means that not all the energy supplied is used to raise the temperature of the block of aluminium;
- energy is transferred to the surroundings, despite the lagging (also making c too large);
- a little energy will be taken by the lagging and the thermometer (again making c too large);
- inaccuracy of the thermometer, especially as $\Delta\theta$ is fairly small (this could make c too large if $\Delta\theta$ were too small or too small if $\Delta\theta$ were too large);
- inaccuracy of the meters (again, this could affect the value of c either way).

This method, nevertheless, is perfectly adequate for examination purposes, providing you are aware of its limitations and understand why the value of c obtained is likely to be too large.

Activity 10.2

Investigating cooling

The same apparatus can be used as in Figure 10.3, but *without* the lagging, to investigate more fully what happens when the block is heated and then allowed to cool.

This time the temperature is recorded during heating at regular intervals (at least every half minute) for 12 minutes, at which point the heater is switched off and the temperature recorded for a further 8 minutes. The current and voltage are recorded at the beginning and end of the heating period in order to get average values.

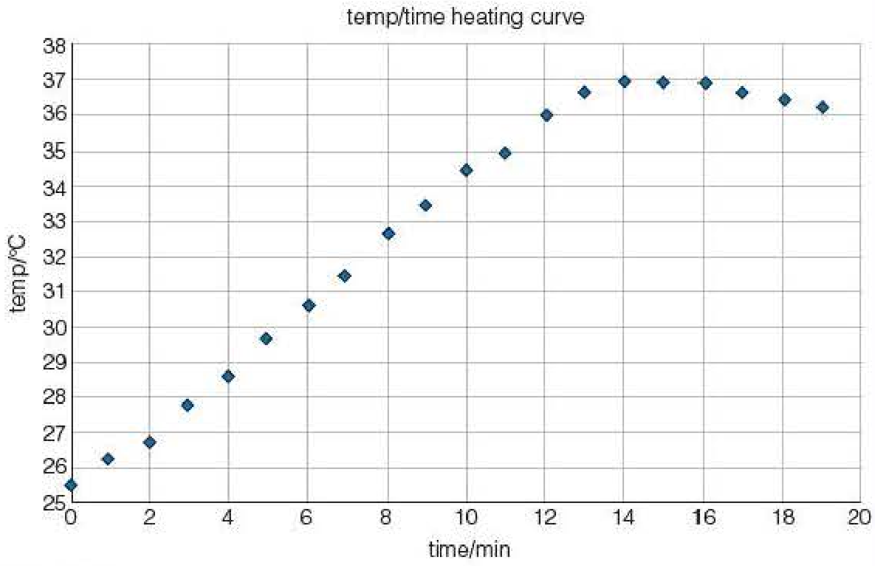


Figure 10.4

A better way of doing this is to record the data electronically using a temperature sensor and data logger; the results can then be recorded and displayed on a computer. A typical such printout is shown in Figure 10.4.

Questions

1 In the experiment that gave rise to the curve in Figure 10.4, the block had a mass of 1.00 kg; the heater was on for 12.0 minutes, with the average current and voltage being 1.65 A and 9.59 V respectively.

An approximate value for the specific heat capacity of the aluminium can be found as before by reading off θ_i as 25.5 °C and θ_f as 37.0 °C from the graph. Check for yourself that this gives a value for c of about 990 J kg^{-1} K^{-1} (don't forget to convert minutes to seconds!).

2 Explain whether this value is likely to be too large or too small.

Tip

Always work in base units when doing specific heat calculations and remember that $\Delta\theta$/°C = ΔT/K (there is no need to add or subtract 273).

Taking account of energy loss to the surroundings

The graph in Figure 10.4 provides us with a lot of information about the transfer of energy that takes place when the block is heated and then allowed to cool. It is therefore worth studying in some detail.

Initially, the rate at which the temperature rises is fairly slow because it takes time for the energy to conduct from the heater, through the block to the thermometer, even though aluminium is a good conductor.

Then the line becomes almost linear when, to a good approximation, the rate of transfer of electrical energy in the heater is equal to the rate at which energy is absorbed by the block. After a few minutes, the rate at which the temperature rises gradually gets less. This is because the rate at which energy is transferred to the surroundings gets greater as the temperature of the block increases.

After the power has been switched off (at 12 minutes), the temperature of the block continues to rise for about a further 2 minutes because it takes a finite time for all the energy to conduct from the heater through the block to the thermometer.

Once all the energy from the heater has been transferred to the block, the transfer of energy to the surroundings becomes clearly apparent as we can now see the temperature of the block starting to fall. The block, of course, has been transferring energy to the surroundings all the time. The value of $990\,\mathrm{J\,kg^{-1}\,K^{-1}}$ that we obtained for the specific heat capacity is likely to be too large because of this energy transferred to the surroundings and the energy absorbed by the heater (and, to a lesser extent, by the thermometer).

One way of making allowance for the energy transferred to the surroundings is to determine the gradient $\mathrm{d}\theta/\mathrm{d}t$ of the graph line at the point where it first becomes almost linear – in this case after about 2 minutes. At this point, the temperature of the block is less than 2 K above room temperature and so the rate of transfer of energy to the surroundings is negligible. We can therefore reasonably assume that:

rate of electrical energy transferred by heater = rate of energy gained by block

$$IV = mc\frac{\mathrm{d}\theta}{\mathrm{d}t}$$

A useful exercise that you might like to undertake is to:

- Determine the gradient of the graph in Figure 10.4 where it is linear – you should find that $\mathrm{d}\theta/\mathrm{d}t$ is about $0.0167\,\mathrm{K\,s^{-1}}$.
- Hence show that that this gives a value for c of $950\,\mathrm{J\,kg^{-1}\,K^{-1}}$ (to 2 SF). You should remember that it is essential to express $\mathrm{d}\theta/\mathrm{d}t$ in units of $\mathrm{K\,s^{-1}}$ and not $\mathrm{K\,min^{-1}}$.

The value of $950\,\mathrm{J\,kg^{-1}\,K^{-1}}$ is much closer to the accepted value of c for aluminium, but does not take into account the energy taken by the heater and thermometer. This energy could be determined from a separate experiment and subtracted from the electrical energy supplied. Alternatively, it could be assumed that the specific heat capacity of the heater material is not significantly different from that of aluminium and so the mass of the heater could be added to that of the block. In practice, this is quite a reasonable approximation to make. It is also reasonable to ignore the energy transferred to the thermometer as this is relatively small compared with the total energy transferred.

Finding the specific heat capacity of a liquid poses a slight problem as the liquid has to be in some form of container, such as a glass beaker. To get a value for the specific heat capacity of the liquid, we must take into account the energy taken by the container as well as by the liquid.

This apparent difficulty can be largely overcome by using an expanded polystyrene cup to contain the liquid. This has the two-fold advantage of having very little mass and also being a very good insulator. This means that it both absorbs very little energy itself and at the same time minimises energy transfer from the liquid to the surroundings.

Activity 10.3

Measuring the specific heat capacity of water

The heater is a 15 Ω, 11 W ceramic body, wire-wound resistor. A 12 V power supply is used, with digital meters set on the 20 V and 2 A ranges respectively.

Using a measuring cylinder, 120 ml (m = 120 g) of water is poured into the expanded polystyrene cup. The set-up is shown in Figure 10.5.

The initial temperature θ_i of the water is taken. The power supply is then switched on and, at the same time, the stopclock is started. The current I and the voltage V are recorded.

After a time Δt of 5.0 minutes has elapsed, the heater is switched off, the water is stirred thoroughly and the highest steady temperature θ_f reached by the water is recorded.

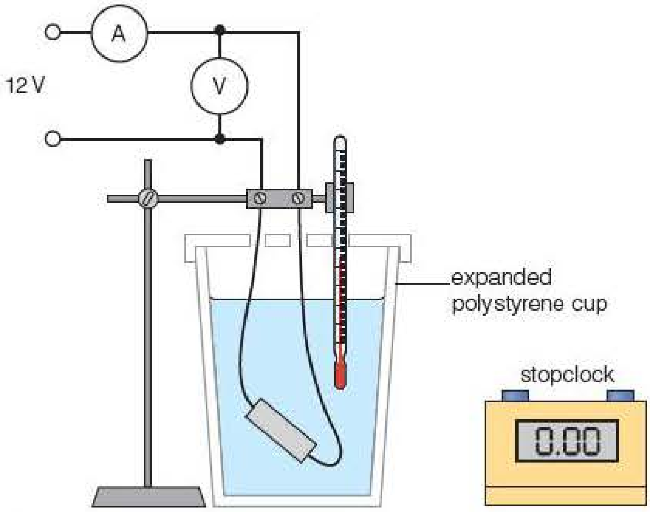


Figure 10.5

The specific heat capacity c of the water can be calculated from:

$$c = \frac{IV\Delta t}{m(\theta_f - \theta_i)}$$

The same method can be used to determine the specific heat capacity of cooking oil, but this can be very messy!

Questions

In an experiment similar to that described above, 120 g of water was used. The water was heated for 5.0 minutes, during which time the average current was 0.77 A and the potential difference was 11.4 V. The temperature of the water rose from 21.2°C to 26.3°C.

1 Explain why a resistor rated at 11 W would be suitable.

2 What value do these data give for the specific heat capacity of water?

3 What are the main sources of error in such an experiment and how would each of them affect this value?

Test yourself

5 A light, expanded polystyrene cup is filled with 150 cm^3 of cooking oil of density 920 $kg\,m^{-3}$. Its temperature is taken and found to be 18.0°C. The oil is then heated with a resistor rated at 15 Ω, 10 W connected to a 12 V power supply.

a) Calculate the current in the resistor and explain why a 10 W rating for the resistor is suitable.

b) After the power has been on for 5.0 minutes, the oil is stirred and the highest steady temperature is found to be 28.4°C. What value does this give for the specific heat capacity of cooking oil?

Tip

Learn the two basic experiments to find the specific heat capacity for a solid and for a liquid and understand the main sources of error in each. Remember to use SI units when doing calculations – particularly kg and seconds.

6 This question also refers to the experiment described in Question 5.

a) In the experiment, it is stated that 'the highest steady temperature is found'. This means that thermal equilibrium has been reached. Explain, in terms of energy, what this means.

b) A mercury thermometer having a heat capacity of 15 J K^{-1} was used (this means that it takes 15 J of energy to raise the temperature of the thermometer by 1 K).

   i) Explain why the thermometer should be left in the cooking oil for a time before taking the initial temperature.

   ii) How much energy would have been taken by the thermometer during the course of the experiment?

   iii) Estimate the percentage error that this would contribute to the value obtained for the specific heat capacity of the cooking oil.

   iv) Explain whether the effect of the thermometer is to give a value for the specific heat capacity that is too large or too small.

7 A student carries out an experiment at home to determine the specific heat capacity of water using an electric kettle. The label on the bottom of the kettle states that it is rated at 230–240 V, 2755–3000 W.

a) Show that this data is consistent with

   i) the plug having a 13 A fuse, and

   ii) the element having a resistance of 19 Ω.

b) As the student does not have a suitable thermometer, she decides to time how long it takes to heat the water from room temperature, which is shown as 20°C on the room thermostat, until it just starts to boil. She finds that with 1.5 litres of water this takes 3.00 minutes. She assumes the local power supply is 240 V. What value would she obtain for the specific heat capacity of water?

c) Noting that the kettle is warm at the end of the experiment, she decides to repeat the experiment with a different volume of water in an attempt to make allowance for the energy needed to heat the kettle. When she uses 1.0 litres of water, it takes 2 minutes and 4 seconds to boil.

   i) Suggest any experimental precautions she should take before starting the second experiment.

   ii) Write down an equation relating the energy supplied by the kettle to the energy needed to heat the water and the kettle.

   iii) Now write down two equations, substituting the data from her two experiments into your equation from ii).

   iv) Hence determine values for the specific heat capacity of water and the energy needed to heat the kettle.

Exam practice questions

The data in questions 1–4 are from an experiment to find the specific heat capacity of cooking oil, in which 139.3 g of the oil is poured into a thin plastic cup. The temperature of the oil is taken and found to be 21.4°C. The oil is then heated for 3.0 minutes by means of an immersion heater, which is rated at 11 W. The maximum steady temperature reached by the oil is 29.3°C.

1 Which of the following base units does *not* occur in the base units of specific heat capacity?

A K **C** m

B kg **D** s **[Total 1 mark]**

2 The value obtained for the specific heat capacity of the oil, in units of $J\,kg^{-1}\,K^{-1}$, is approximately:

A 0.03 **C** 30

B 1.8 **D** 1800. **[Total 1 mark]**

3 The experimental value obtained for the specific heat capacity of the oil should be quoted to:

A 1 significant figure

B 2 significant figures

C 3 significant figures

D 4 significant figures. **[Total 1 mark]**

4 Which of the following could be a reason for the value obtained being too small?

A Energy has been lost to the surroundings.

B The mass of the cup has been ignored.

C The thermometer is reading systematically low.

D The heater is operating at less than its stated value. **[Total 1 mark]**

5 Copy and complete the table, which shows the temperature of the melting points of some common elements. **[Total 4 marks]**

Table 10.6

Element	°C	K
hydrogen	−259	
Iron	1540	
nitrogen		63
sulphur		388

6 A freezer has an internal volume of $0.23\,m^3$ and operates at an internal temperature of −18°C. How much energy must be removed to cool the air inside from 20°C to the operating temperature? You may assume that the density of air is $1.3\,kg\,m^{-3}$ and that the specific heat capacity of air is $1.0 \times 10^3\,J\,kg^{-1}\,K^{-1}$. **[Total 3 marks]**

7 A storage heater consists of a concrete block of mass 45 kg and specific heat capacity 800 $J\,kg^{-1}\,K^{-1}$.
It is warmed up overnight when electricity is cheaper. [2]

a) Show that it needs about 2 MJ of energy to raise its temperature from 10°C to 70°C.

b) How much will it cost to heat it up if 1 unit of electricity (3.6 MJ) costs 10p? [1]

c) What will be the average power given out if it takes 5 hours to cool from 70°C to 10°C? [2]

d) Explain how the power emitted will change as it cools down. [2]

[Total 7 marks]

8 The hot water for a bath is supplied from a hot water tank in which there is a 3.0 kW immersion heater.

a) How much energy would be needed to heat 0.25 m^3 of water for a bath from 15°C to 35°C? The specific heat capacity for water is 4200 $J\,kg^{-1}\,K^{-1}$. [2]

b) Show that it would take nearly 2 hours to heat this amount of water. [2]

c) Why, in practice, would it take longer than this? [2]

d) Suggest why you would probably save energy by taking a shower instead of a bath. [1]

[Total 7 marks]

9 A student pours 117 cm^3 of water (specific heat capacity = 4200 $J\,kg^{-1}\,K^{-1}$) into a thin plastic coffee cup and heats it with a 100 W immersion heater. He records the temperature as the water heats up and then draws a graph as shown in Figure 10.6.

a) Use the graph to show that the rate of rise of the temperature of the water at the beginning of the heating process is about 0.2 $K\,s^{-1}$. [2]

b) Hence determine the rate at which the water is absorbing energy. Comment on your answer. [3]

c) Explain why the rate at which the temperature rises slows down as the heating process continues. [2]

[Total 7 marks]

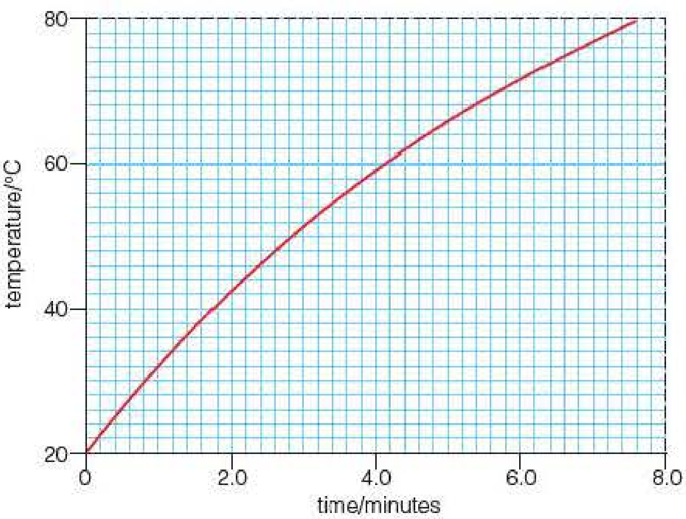


Figure 10.6

10 The following is taken from a manufacturer's advertisement for an electric shower:

The beauty of electric showers is that they draw water directly from a cold water supply and heat it as it is used, so you don't need to have a stored hot water supply. Because they are easy to install, electric showers are extremely versatile. In fact, virtually every home – new and old – can have one.

a) Write a word equation to describe the energy changes that take place in an electric shower.

Rewrite the equation using the appropriate formulae. **[2]**

b) The technical data supplied by the manufacturer states that a particular shower has a 10.8 kW heater and can deliver hot water at a rate of 14 litres per minute.

Calculate the temperature of the hot water delivered by this shower when the temperature of the cold water supply is 16°C. **[3]**

(1 litre of water has a mass of 1 kg). **[Total 5 marks]**

11 A student uses the arrangement shown in Figure 10.7 to determine a value for the specific heat capacity of water.

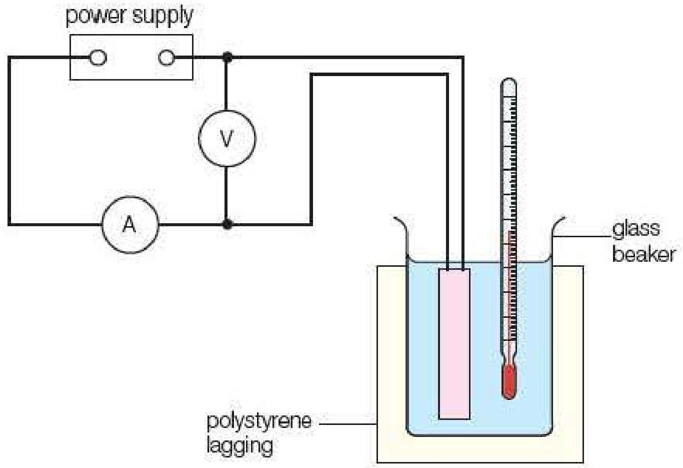


Figure 10.7

He records the following data:

- mass of water $m = 250\,\text{g}$
- potential difference $V = 11.9\,\text{V}$
- current $I = 4.12\,\text{A}$
- time $\Delta t = 4.00$ minutes
- temperature rise $\Delta\theta = 10.2\,\text{K}$

a) Show that the student would obtain a value for the specific heat capacity of water of about $4600\ \text{J kg}^{-1}\,\text{K}^{-1}$ from these data. **[2]**

b) His teacher suggests that this value is significantly higher than the accepted value of 4200 $J\,kg^{-1}\,K^{-1}$ because he has ignored the energy taken by the glass beaker. The student finds out from a data book that the specific heat capacity of glass is 780 $J\,kg^{-1}\,K^{1}$; he then weighs the beaker and finds its mass to be 134 g.

i) What is the percentage difference between the student's value and the accepted value for the specific heat capacity of water?

ii) How much energy is needed to raise the temperature of the water by 1 0.2 K?

iii) How much energy is needed to raise the temperature of the glass beaker by 10.2 K?

iv) Comment on your answers in relation to the teacher's suggestion. **[6]**

c) Explain two advantages of using an expanded polystyrene cup instead of a glass beaker in such an experiment. **[2]**

[Total 10 marks]

Stretch and challenge

12 James Joule is considered to be the person who formulated what we know today as the law of conservation of energy. His approach to this was guided by his religious views, *believing that the power to destroy belongs to the Creator alone.*

In 1840, Joule reported to a meeting of the Royal Society that he had found, using an electric cell, that: the amount of heat produced by a given amount of electric current varied as *the square of the intensity of the current and directly as the resistance, always provided that a given unit of time were used*. This subsequently became known as Joule's Law, but at the time was not received with any great enthusiasm.

In 1847, Joule proposed that the temperature of water at the bottom of a waterfall should be higher than that at the top. This is recalled by Sir William Thomson (later to be Lord Kelvin):

However, he [Joule] *did not tell me he was to be married in a week or so; but about a fortnight later* [after meeting Joule at Oxford] *I was walking down from Chamounix* [Chamonix, in the French Alps] *to commence the tour of Mont Blanc, and whom should I meet walking up but Joule, with a long thermometer in his hand and a carriage with a lady in it not far off. He told me he had been married since we had parted at Oxford; and he was going to try for elevation of temperature in waterfalls. We trysted to meet a few days later at Martigny, and look at the Cascade de Sallanches* [where the main waterfall falls through a height of 270 m], *to see if it might answer. We found it too much broken into spray.*

a) State the law of the conservation of energy. **[2]**

b) Outline the energy changes that gave rise to:

i) Joule's Law

ii) Joule's hypothesis *for elevation of temperature in waterfalls*. **[4]**

c) Calculate the theoretical increase in temperature that Joule would have predicted (specific heat capacity of water = $4200\,J\,kg^{-1}\,K^{-1}$). **[3]**

d) Suggest why Joule had a *long thermometer*. **[2]**

e) Other than the problem of the spray, discuss what other difficulties there would have been in making the required measurements. **[3]**

f) At the bottom of waterfalls such as this, a rainbow can often be seen. Figure 10.8b shows a ray of white light entering a water droplet. The subsequent passage of red light through the droplet is shown.

a)

b)

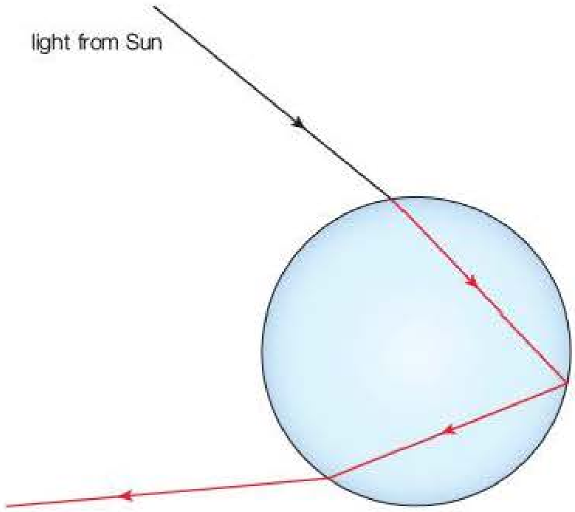


Figure 10.8 a) Rainbow at Niagara Falls b) Passage of light through water droplet

i) Copy Figure 10.8b and add the passage of blue light through the droplet.

ii) Hence suggest how a rainbow is formed. Your explanation should include the terms *refraction, partial reflection, critical angle* and *refractive index*. **[6]**

[Total 20 marks]

11 Internal energy, absolute zero and change of state

Prior knowledge

You need to remember from Chapter 10 that:

- all bodies have internal energy
- internal energy is made up of the potential energy and kinetic energy of the atoms or molecules of the body
- potential energy is due to the inter-atomic or inter-molecular bonds
- kinetic energy is due to the vibration of the atoms or molecules
- we can think of temperature as being a measure of the kinetic energy.

You also need to recall:

- that temperature is one of the base quantities in the SI system
- that the base unit of temperature is the kelvin (symbol K)
- that the kelvin scale has absolute zero as its zero
- the meaning of the terms work and energy
- that a body gains energy if work is done on it
- absolute zero = 0 K = −273°C
- from °C to K always add 273
- from K to °C always subtract 273
- for a *change* in temperature Δ(1 K) = Δ(1°C)
- electrically $\Delta E = IV\Delta t$
- mechanically $\Delta E = F\Delta x$
- power = $\frac{\text{work done}}{\text{time taken}}$

Test yourself on prior knowledge

1 Copy and complete Table 11.1 by adding the missing parts a)–d).

Table 11.1

Temperature of:	T/K	θ/°C
Cosmic microwave background radiation	2.7	a)
Liquid nitrogen	b)	−196
d)	310	c)

2 A cyclist of mass 70 kg is riding a cycle of mass 30 kg at a speed of 8.0 m s^{-1}.

a) Calculate

i) the kinetic energy of the bike and rider

ii) the force that must be applied to stop the cycle in a distance of 10 m.

b) Explain why the brakes get hot.

3 A manufacturer claims its electric car has a power of 125 kW and can accelerate from rest to 72 kph in under 5 seconds. The mass of the car is 1500 kg.

a) Calculate the kinetic energy it has gained when it has reached 72 kph.

b) Calculate the average power it would need if it takes 5.0 s to reach this speed.

c) With reference to your answer to part b), discuss whether the manufacturer's claim is reasonable.

d) Calculate the size of the average force that would have to be applied to bring it to rest in a distance of 30 m.

e) Outline the energy changes that take place when braking from 72 kph to rest.

11.1 Historical background

In 1783 Lavoisier proposed a 'subtle fluid' called caloric as the 'substance of heat'. According to this theory, the quantity of this hypothetical, weightless fluid is constant throughout the universe and it flows from warmer to colder bodies. The theory was refuted by Rumford, who was among the first to show that work could be converted to heat during the process of boring cannons. However, his idea that that heat was not a substance but had something to do with motion was largely ignored. The caloric theory was influential until the mid-19th century, by which time numerous experiments, primarily concerned with the mechanical equivalent of heat, forced a general recognition that heat is a form of energy transfer.

In 1845, Joule read his paper *On the mechanical equivalent of heat* to the British Association meeting in Cambridge. He reported his best-known experiment, involving the use of a falling weight, in which gravitational forces do the mechanical work to spin a paddle wheel in an insulated barrel of water, thereby increasing its temperature. In 1850, Joule published a refined measurement of 772.692 foot pound force (the unit of 'work' at that time) to raise the temperature of 1 pound of water by 1 °F, or 3.066 foot pound force per calorie (a calorie was the unit of 'heat' and was defined as the 'heat' needed to raise the temperature of 1 g of water by 1°C). In today's units, Joule's value would be 4.159 joules per calorie.

We now know that the thermal energy in a body is the total of the kinetic energies of its constituent particles and so has the units of energy – joules! Of course we still use the calorie (well, actually the kilocalorie) to measure the energy from food. A kilocalorie is now defined as 4185 J.

In this chapter we will look at kinetic theory again and see how the internal energy of a body can be increased by heating the body or by doing work on the body. We'll also see how the idea of internal energy leads to the concept of an absolute zero of temperature. Experiments at temperatures close to absolute zero have led to the discovery of superconductors and superfluids. Superconductors have far-reaching applications – from MRI scanners to high-speed trains.

11.2 Internal energy of an ideal gas

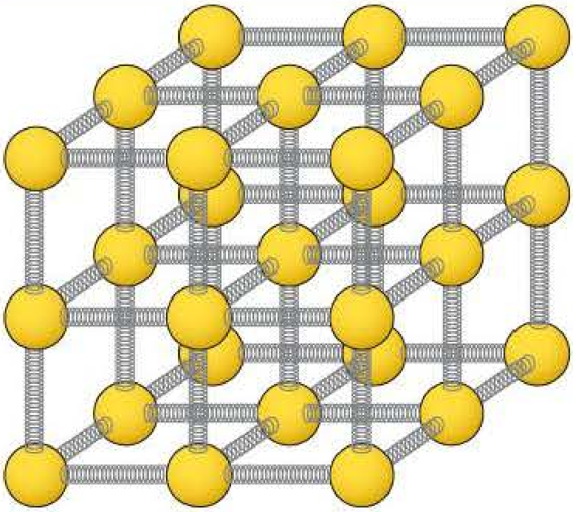


Figure 11.1 Atomic model of a solid. Polystyrene spheres represent the atoms and springs represent the bonds. If the model is held by one sphere, which is then shaken, the springs (bonds) transmit this vibration to the other spheres (atoms) and the whole model vibrates. The model then has potential energy stored in the springs (bonds) and kinetic energy of the vibrating spheres (atoms).

Tip

Remember that the total internal energy of a body is the sum of the potential energy and kinetic energy of its atoms.

In a gas the molecules have kinetic energy due to their random motion. In addition, molecules that are made up of two or more atoms (e.g. O_2 or CO_2) can have kinetic energy due to rotation and vibration.

In an **ideal gas** we assume that the inter-molecular forces are negligible, except during collision (see Chapter 12). Furthermore, we assume that the collisions of the gas molecules with one another and with the walls of any container are *elastic*. Imagine two molecules approaching each other head-on as in Figure 11.2a.

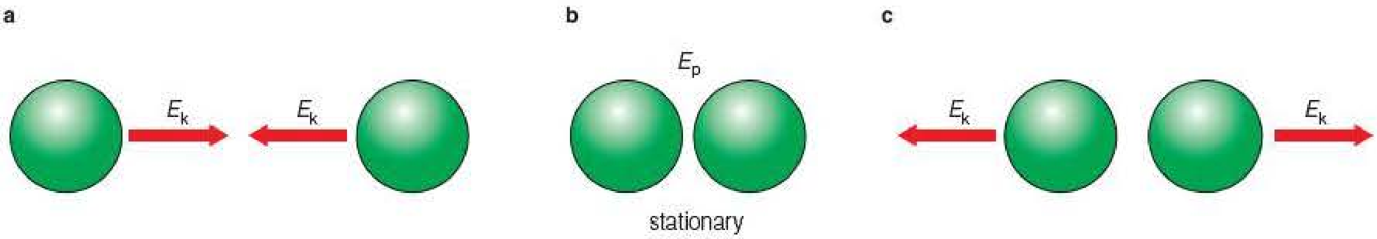


Figure 11.2

Just before collision each molecule has kinetic energy. As the molecules get closer, there are repulsive forces between them, which slow them down until momentarily both are stationary (Figure 11.2b). At this point all the kinetic energy has been converted into potential energy as a result of the work done against the repulsive forces. These repulsive forces become extremely large when the molecules become very close and so do work pushing the molecules apart again. As the collision is **elastic**, all the potential energy is converted back to kinetic energy once more (Figure 11.2c). As collisions between gas molecules take place randomly, there is a continuous interchange of kinetic and potential energy in this way.

In the case of an ideal gas, another assumption is that the duration of collisions is negligible compared with the time spent in between collisions, and so we consider the internal energy to be entirely kinetic. Since temperature is a measure of the kinetic energy of the molecules, this means that for an ideal gas the temperature is a measure of the total internal energy. We'll look at this in more detail in Chapter 12.

Tip

Remember, the internal energy of an *ideal gas* is entirely the random kinetic energy of its molecules.

11.3 Heating and working

Key term

Heating is defined as the *random* interchange of energy between two bodies in thermal contact, resulting in *energy flowing from hot to cold.*

By considering internal energy, we can get a better understanding of what is meant by 'hot' and 'cold'. A hot body has a much greater *concentration of internal energy* compared with a cold body, so in the process of **heating** a body we are increasing its internal energy.

As we saw in Chapter 10, this transfer of energy by heating may be by means of conduction, convection or radiation. The bodies may not necessarily be in *physical* contact; for example the Earth receives energy from the Sun by means of radiation travelling through nearly 150 million kilometres of space!

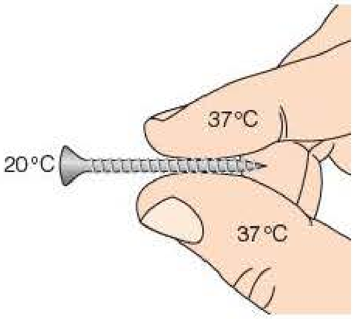


Figure 11.3 A temperature difference causes a flow of energy

For heating to take place there must be a *temperature difference*. For example, let's imagine that you hold a metal screw, which is at room temperature, say 20°C, between your fingers, which are at your body temperature of 37°C (Figure 11.3). Energy will flow from your fingers to the screw.

This transfer of thermal energy increases the internal energy of the molecules of the screw until its temperature is also 37°C. Energy has been taken from your finger, which gives the sensation of the screw feeling 'cold'. The amount of energy ΔE taken from your finger will be given by:

$$\Delta E = mc\Delta\theta$$

where m is the mass of the screw, c is its specific heat capacity and $\Delta\theta$ is the temperature difference between your fingers and the initial temperature of the screw.

Energy can also be transferred between two bodies in the form of **work**, irrespective of any temperature difference. This work can either be *mechanical* or *electrical*.

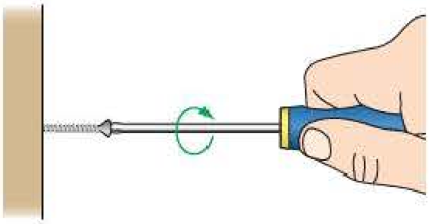


Figure 11.4 Doing mechanical work

Consider screwing the same screw into a piece of hard wood (Figure 11.4). After a time you will notice that the screw gets hot and you get tired! Energy has been transferred *mechanically* from you to the screw by means of the work done against the frictional force between the screw and the wood. The work done, or energy transferred, is

$$\Delta E = F\Delta x$$

where Δx is the distance moved by the screw against the frictional force F.

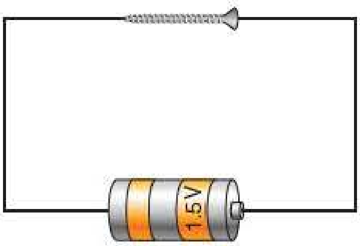


Figure 11.5 Doing electrical work

Alternatively, you could connect a battery across the screw as shown in Figure 11.5. After a short while you will observe that the screw gets warm. In this case energy has been transferred *electrically* by the cell exerting a force on the electron charge carriers in the metal screw. This force, multiplied by the distance moved by the electrons, will give the amount of energy transferred to the internal energy of the screw. From our knowledge of electricity we have that this energy is:

$$\Delta E = IV\Delta t$$

Energy transfer by working is an **ordered** process and is independent of any temperature difference. By 'ordered' we mean that the force, and therefore the energy transfer, is in a defined direction, not random as in the case of heating.

Example

A car of mass 1600 kg has four brake discs, each of mass 1.3 kg. The discs are made from an iron alloy of specific heat capacity 480 J kg^{-1} K^{-1}.

a) Calculate the rise in temperature of the discs when the car brakes from a speed of 90 kph (25 m s^{-1}) to rest.

b) State any assumptions that you make.

c) Explain the energy changes that have taken place.

d) Suggest why manufacturers supply discs with holes drilled in them for high-performance cars.

e) In order to reduce the weight of racing cars, and therefore increase performance, aluminium brakes are being developed. The specific heat capacity of aluminium is 910 J kg^{-1} K^{-1}. Suggest why aluminium is a suitable material for this purpose.

Answer

a) Using $\Delta E = mc\Delta\theta$, where $\Delta E = \frac{1}{2}mv^2$ is the kinetic energy of the car, we have

$$\Delta E = \frac{1}{2} \times 1600\,\text{kg} \times (25\,\text{m}\,\text{s}^{-1})^2 = 5.0 \times 10^5\,\text{J}$$

$$\Delta\theta = \frac{\Delta E}{mc} = \frac{5.0 \times 10^5\,\text{J}}{(4 \times 1.3)\,\text{kg} \times 480\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}} = 200\,\text{K}$$

b) We have to make a number of assumptions:

- the energy is shared equally between the four discs;
- all the energy is given to the discs – in practice some will be given to the pads that press against the discs;
- the discs do not dissipate any of the energy into the air or conduct any energy away during the braking time.

c) The ordered kinetic energy of the car has been converted into the disordered, random internal energy of the brake discs.

d) The holes enable air to pass through the discs. This ventilation dissipates the energy from the discs to the surrounding air more quickly and so prevents the discs from over-heating.

e) The crucial factor is that the specific heat capacity of aluminium is almost twice that of the iron alloy. This means that aluminium discs having half the mass of iron discs can be used without them getting any hotter than iron discs.

Test yourself

1 An electrically assisted cycle ('e-bike') of mass 25 kg has a 36 V battery of capacity 10 A h. It has the maximum permissible power rating for the UK, which is 200 W. A cyclist of mass 75 kg uses the electric motor to accelerate from rest to a speed of $7.2\,\mathrm{m\,s^{-1}}$.

a) Calculate the work done by the motor.

b) Show that the shortest theoretical time in which the cyclist could achieve this speed is about 13 s.

c) Calculate the current when the battery is working at full power.

d) Calculate the charge taken by the battery in order to reach $7.2\,\mathrm{m\,s^{-1}}$.

e) Show that this is only about 0.2% of the charge available from the battery.

2 A powerful food liquidiser like that shown in Figure 11.6 can be used to make soup.

The vegetables are liquidised by four sharp metal blades that rotate at high speed. This soup can then be heated in one of two ways – either by keeping it in the liquidiser for a further 6.0 minutes or by transferring it to a microwave oven and heating it at a power rating of 600 W for 9.0 minutes.

a) The manufacturer states that the 'tips of the blades rotate at 386 km per hour'. If the blades have a length (from the centre) of 35 mm, at what speed, in revolutions per minute, do the blades rotate?

Figure 11.6 Food liquidiser

b) The blender is rated at 240 V/840 W. Show that a 5 A fuse would be suitable.

c) How much energy will be needed to heat 1.0 kg of soup from 20°C to 80°C if its specific heat capacity is $4000\,\mathrm{J\,kg^{-1}\,K^{-1}}$?

d) Discuss the relative efficiency of the two ways of heating the soup.

e) Explain how the soup is heated in each of the two methods.

11.4 Change of state

We have just seen that heating or doing work on a body increases the internal energy of its molecules and usually this raises the temperature of the body. But what happens when a substance melts or vaporises? This is what we call a **change of state**.

We usually think of there being three 'states of matter' – solid, liquid and gas – although there is in fact a fourth state, called a **plasma**. Plasma, consisting of ionised gas, usually at a very high temperature, makes up over 99% of the visible universe and perhaps most of that which is not visible. In a 'plasma TV', with which you may be familiar, collisions excite (energise) the xenon and neon atoms in the plasma, causing them to release **photons** of light energy.

The physics of plasmas has come to be very important – and not just because of TV screens! **Nuclear fusion** is the mechanism that powers our main source of energy, the Sun (see Chapter 14). If we can develop a technique for controlled nuclear fusion (as opposed to the uncontrolled fusion energy of a thermonuclear bomb), we will create a very, very large of 'clean' energy and go a long way to solving the world's energy crisis and environmental problems. At the temperature required for fusion, the reacting material must be in the plasma state. Because plasma is ionised, i.e. made up of charged particles, it can be controlled by magnetic fields. In some modern fusion experiments, the plasma is confined in a doughnut-shaped vessel with magnetic coils called a '*tokamak*'.

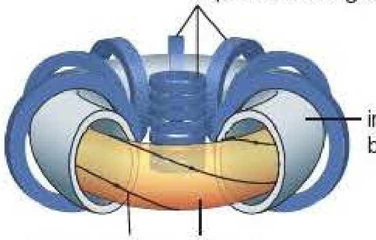


Figure 11.7 A tokamak

Figure 11.7 shows the principle of a tokamak. The plasma is contained in the doughnut-shaped vessel, called a 'torus'. Using superconducting coils (blue) a magnetic field is generated, which causes the extremely high-temperature plasma particles to run around in circles, without touching the vessel wall. In reality, a number of other coils are present, which produce subtle changes to the magnetic field.

Let's get back to what happens when a substance changes state. Imagine taking a cube of ice out of the freezer and leaving it on the laboratory bench to melt (preferably in a beaker!). Its initial temperature will be about −18°C, the normal temperature inside a freezer. The laboratory is probably about 22°C, so we can say that the ice will warm up as energy is transferred from the room to the ice because of a temperature difference. The molecules of the ice will gain internal energy, both potential and kinetic. Because the molecules gain kinetic energy, the temperature of the ice will increase. This continues until the ice reaches a temperature of 0°C and starts to melt. At this point *all* the energy it receives is used to do work, increasing the potential energy and overcoming the bonds that keep the ice as a solid. The solid changes into a liquid. During this process there is *no* increase in the kinetic energy of the molecules and so the ice remains at 0°C until it has all melted. Although the ice is still receiving thermal energy from the warmer surroundings, the effect of this 'heat' is not observed as a rise in temperature of the ice. It is called **latent heat**, 'latent' meaning 'hidden'.

Once the ice has melted, the water thus formed continues to receive thermal energy from the room, but the kinetic energy of the molecules now increases, as well as the potential energy, and the temperature of this water rises until it reaches room temperature and **thermal equilibrium** is established.

If this water were now to be poured into an electric kettle and heated, electrical energy would be converted into thermal energy – internal energy of the water molecules – and the temperature of the water would increase until it reached 100°C. All the electrical energy supplied by the element of the kettle then goes into increasing the potential energy of the molecules and there is *no* increase in their kinetic energy. Once again, the temperature remains constant while the water is boiling and changing state from a liquid into a vapour. The energy required to change the water into vapour is again called the latent heat.

Specific latent heat

Key terms

The **specific latent heat** of fusion is the energy per unit mass (in joules per kilogram) to change a solid into a liquid without change in temperature.

The **specific latent heat** of vapourisation is the energy per unit mass (in joules per kilogram) to change a liquid into a vapour *without change in temperature*.

Tip

Don't forget to include *without change in temperature* if you are asked to define specific latent heat.

In Chapter 10 we found that equal masses of different substances required different amounts of energy to raise their temperature – we called this the *specific heat capacity* of the material, symbol c. Similarly, different substances require different amounts of energy to change state, or phase. We call this their **specific latent heat** and we give it the symbol L. The word *fusion* is used to describe the change from a solid into a liquid (e.g. ice at its melting point into water) and *vapourisation* is used for the change from a liquid into a vapour (e.g. boiling water into steam). The units of L are $\mathrm{J\,kg^{-1}}$.

The energy needed to change the state of Δm of a substance is $\Delta E = L\Delta m$

The specific latent heat is also the energy per unit mass that has to be removed from a substance when it changes from a vapour to a liquid (e.g. steam condensing to water at 100°C) or from a liquid to a solid (e.g. water freezing to become ice at 0°C). By the conservation of energy, the value is the same whether melting from solid to liquid or freezing from liquid to solid. However, the specific latent heat of vapourisation of a substance is always significantly greater than that for fusion as much more work has to be done increasing the internal potential energy in pulling apart and breaking the molecular bonds when a liquid changes into a vapour. Some values are shown in Table 11.2.

Table 11.2

	$L_{fusion}/\mathrm{J\,kg^{-1}}$	$L_{vapourisation}/\mathrm{J\,kg^{-1}}$
Water	3.34×10^5	2.26×10^6
Lead	2.24×10^4	8.55×10^5
Oxygen	1.39×10^4	2.13×10^5

Activity 11.1

Investigating the cooling of stearic acid

You are recommended to look at an excellent demonstration of this experiment on:

http://www.youtube.com/watch?v=YUWJ2jGhEOM

This is a good example of using IT to collect and process data as the stearic acid takes about 40 minutes to cool. You can therefore set the experiment going and then do something else (preferably useful!) whilst the data are being recorded. Figure 11.8 is a typical graph of a stearic acid cooling curve obtained in such an experiment.

Questions

1 Draw a diagram of the apparatus you would set up to obtain a graph such as that shown in Figure 11.8.

2 What value does the graph in Figure 11.8 give for the melting point of stearic acid?

3 Explain the three distinct phases of the curve in terms of the energy of the stearic acid molecules.

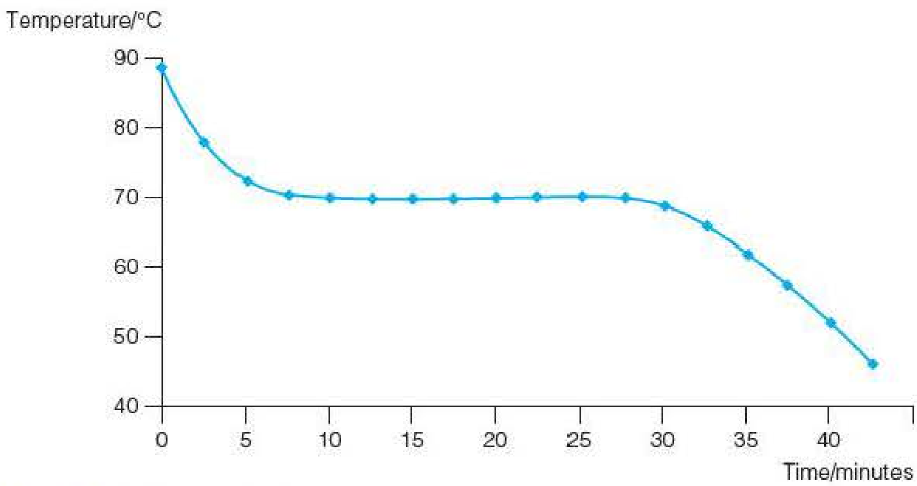


Figure 11.8 Cooling curve for steric acid

Example

Use the data in table 11.2 and take the specific heat capacity of water as 4200 J kg^{-1} K^{-1}.

1 Calculate

a) the energy needed to heat 250 cm^3 of water from 18°C up to its boiling point (100°C) and then change it into steam at 100°C.

b) the energy that must be removed from 200 cm^3 of water to cool from 22°C to 0°C and then freeze it at 0°C.

2 A kettle is rated at 2.4 kW and filled to the 1.5 litre mark with water at 20°C. Sketch a temperature against time graph from the time the kettle is switched on until 0.5 litres have been boiled away. You may assume that the energy taken by the kettle itself is negligible.

Answer

1 a) Mass m of water $= 250\,\text{g} = 0.250\,\text{kg}$

Energy to heat water from 18°C to 100°C $= mc\Delta\theta$

$= 0.250\,\text{kg} \times 4200\,\text{J\,kg}^{-1}\text{K}^{-1} \times (100 - 18)\,\text{K}$

$= 7.56 \times 10^4\,\text{J}$

Energy to change water at 100°C into steam $= mL$

$= 0.250\,\text{kg} \times 2.26 \times 10^6\,\text{J\,kg}^{-1}$

$= 5.65 \times 10^5\,\text{J}$

Total energy needed $= 7.56 \times 10^4\,\text{J} + 5.65 \times 10^5\,\text{J}$
$= 6.41 \times 10^5\,\text{J}$

b) Mass m of water $= 200\,\text{g} = 0.200\,\text{kg}$

Energy removed to cool water from 22°C to 0°C $= mc\Delta\theta$

$= 0.200\,\text{kg} \times 4200\,\text{J kg}^{-1}\text{K}^{-1} \times (22 - 0)\,\text{K}$

$= 1.85 \times 10^4\,\text{J}$

Energy removed to freeze water at 0°C into ice $= mL$
$= 0.200\,\text{kg} \times 3.34 \times 10^5\,\text{J kg}^{-1}$

$= 6.68 \times 10^4\,\text{J}$

Total energy that must be removed
$= 1.85 \times 10^4\,\text{J} + 6.68 \times 10^4\,\text{J}$

$= 8.53 \times 10^4\,\text{J}$

2 Before we can sketch the graph we must do some sums.

Energy supplied by kettle $= 2.4\,\text{kW} = 2400\,\text{J s}^{-1}$

We also need to know that mass of 1 litre $= 1\,\text{kg}$

Energy to heat 1.5 litres of water from 20°C to 100°C $= mc\Delta\theta$

$= 1.5\,\text{kg} \times 4200\,\text{J kg}^{-1}\text{K}^{-1} \times (100 - 20)\,\text{K}$

$= 5.04 \times 10^5\,\text{J}$

Time to heat this water $= \dfrac{5.04 \times 10^5\,\text{J}}{2400\,\text{J s}^{-1}} = 210\,\text{s}$

$= 3.5$ minutes

Energy to boil off 0.5 litres $= mL$
$= 0.5\,\text{kg} \times 2.26 \times 10^6\,\text{J kg}^{-1} = 1.13 \times 10^6\,\text{J}$

Time taken to do this $= \dfrac{1.13 \times 10^6\,\text{J}}{2400\,\text{J s}^{-1}} = 471\,\text{s} = 7.8$ minutes

We can now draw the graph as shown in Figure 11.9.

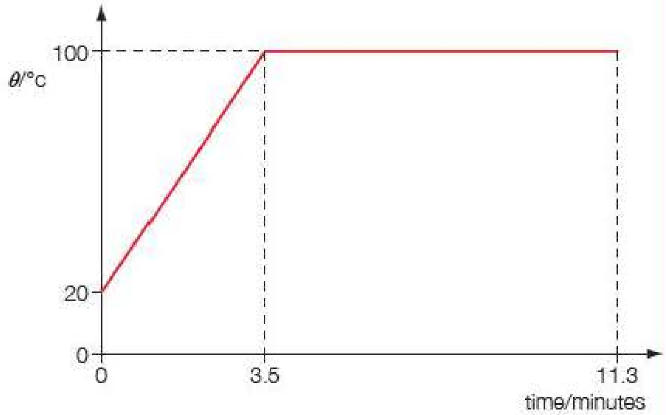


Figure 11.9

As you can see, the temperature:

- rises at a *constant rate* (as the kettle is supplying energy at a constant rate) from 20°C to 100°C in 210 s
- and then *remains constant* at 100°C for a further 7.8 minutes whilst 0.5 litres of water is boiled off.

Test yourself

3 A cube of ice of mass 10 g is taken from a freezer, in which the temperature is – 18°C, and is placed in a beaker on a laboratory bench, where the temperature is 18°C. It melts and, after a little while, thermal equilibrium with the room is attained. The latent heat of fusion of ice is $330\,\text{kJ kg}^{-1}$.

a) Explain what is meant by

i) the latent heat of fusion of ice is $330\,\text{kJ kg}^{-1}$

ii) thermal equilibrium is attained.

b) Describe the changes in the energy of the ice/water molecules during the process described.

c) Calculate the energy taken from the room during this process given:

specific heat capacity of ice $= 2100\,\text{J kg}^{-1}\text{K}^{-1}$
specific heat capacity of water $= 4200\,\text{J kg}^{-1}\text{K}^{-1}$.

4 Assuming, in question 3, that the rate at which the ice/water is taking energy from the room is constant, sketch a graph of temperature against time from the moment that the ice is taken from the freezer until it has reached room temperature.

Core Practical 13 in the A-level specification requires you to '*determine the specific latent heat of a phase change*'. Of the numerous ways of determining specific latent heat experimentally, two are now described.

Core Practical 13A

Measuring the specific latent heat of fusion for water

A beaker of tap water is gently warmed until it is about 10 K above room temperature. A volume of 200 cm^3 (200 g) of this water is then measured into an expanded polystyrene cup and the cup and water weighed.

Meanwhile, some ice is taken from a refrigerator and is crushed with a heavy weight in a paper towel.

Room temperature θ_r and the temperature of the water in the cup θ_i are recorded when the latter is about 5 K above room temperature.

Small pieces of the crushed ice are then quickly dried and added steadily to the water in the cup, gently stirring all the time. This is continued until the temperature of the water is about 5 K below room temperature. The final steady temperature θ_f of the water in the cup is recorded once all the ice added has melted.

The cup of water is then re-weighed to find the mass of ice Δm that has been added.

From the conservation of energy, if we assume that there is no net gain or loss of energy to the surroundings, we can equate the energy required to melt the ice (assumed to be at 0°C), plus the energy to raise its temperature to the final steady temperature, to the energy taken from the warm water, i.e.

$$L\Delta m + \Delta m \times 4200\,\mathrm{J\,kg^{-1}\,K^{-1}} \times (\theta_f - 0)\,\mathrm{K}$$
$$= 0.200\,\mathrm{kg} \times 4200\,\mathrm{J\,kg^{-1}\,K^{-1}} \times (\theta_f - \theta_i)\,\mathrm{K}$$

Questions

1 The following data were recorded for this experiment:

Mass of water in cup $m = 200\,\mathrm{g}$
Room temperature $\theta_r = 19.8°\mathrm{C}$
Initial temperature of this water = $\theta_i = 25.0°\mathrm{C}$
Final temperature of melted ice and water $\theta_f = 15.0°\mathrm{C}$
Mass of ice added $\Delta m = 23.2\,\mathrm{g}$

Use this data to calculate a value for the specific latent heat of fusion of ice.

2 Explain the advantages of using an expanded polystyrene cup.

3 Explain why it is important to use small pieces of melting, but dry, ice.

4 Explain why ice is added until the temperature of the water in the cup is as far below room temperature as its temperature was above room temperature when the ice was first added.

5 Although temperatures have been recorded to a precision of 0.1°C, the realistic uncertainty in recording the temperatures is ± 0.25°C. What does this give for the percentage uncertainty in the value for the specific latent heat?

6 Determine the percentage difference between this experimental value for the specific latent heat and the accepted value of $3.34 \times 10^5\,\mathrm{J\,kg^{-1}}$. Comment on this difference compared with your estimated percentage uncertainty in the experimental value.

7 It is not easy to completely dry the ice before adding it and so the value obtained for the specific latent heat by this method is usually too small. Explain why this is.

Core Practical 13B

Measuring the specific latent heat of vaporisation for water

An insulated glass beaker is about 2/3 filled (~ 400 cm^3) with water at room temperature and placed on an electronic balance. An immersion heater of known power (~ 500 W) is clamped so that it is near the bottom of the water. The balance can either be read directly or else it can be connected to a suitable interface and computer. This is shown in Figure 11.10. Note that as a safety precaution the lead to the heater is supported so that it is kept well clear of the beaker.

Safety note

You should use a 12V low voltage heater, and remember, wet hands and mains electricity can be dangerous!

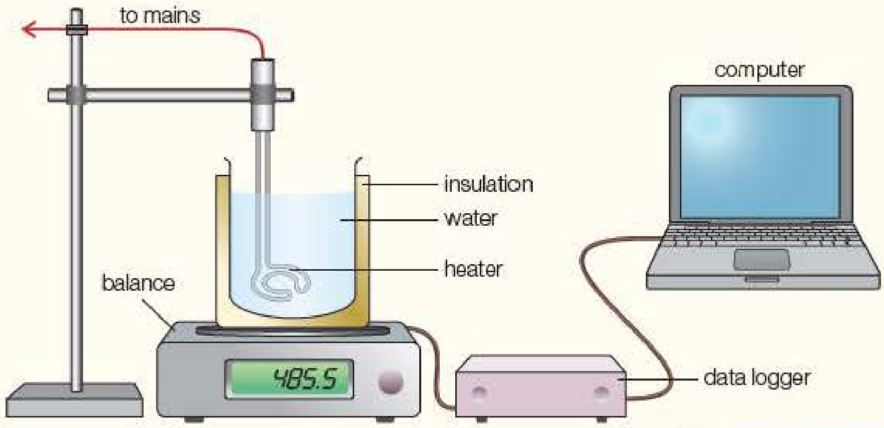


Figure 11.10

The heater is switched and the balance reading is recorded at intervals of 30 s, either using a stopclock or recording it electronically, until about 50 g of the water has boiled off. A graph of mass *m* against time *t* is either drawn manually or printed off from the computer. A typical graph is shown in Figure 11.11.

The specific latent heat of vaporisation can be found by as follows:

From $\Delta E = L\Delta m \longrightarrow \frac{dE}{dt} = -L\frac{dm}{dt}$ (the minus sign indicates that the mass is *decreasing* with time)

where $\frac{dE}{dt}$ = power of heater and $\frac{dm}{dt}$ = rate at which water is being boiled off = gradient of graph

This gives $L = \frac{\text{power of kettle (in W)}}{\text{gradient of graph (in kg s}^{-1})}$

Questions

1. Explain the shape of the graph.
2. Show that the gradient of the linear part of the graph is about -2×10^{-4} kg s^{-1}.
3. Hence show that this gives a value for the specific latent heat of vaporisation for water of about 2.4×10^6 J kg^{-1} (power of heater = 500 W).
4. Calculate the percentage difference between your calculated experimental value and the accepted value of 2.34×10^6 J kg^{-1}.
5. Suggest a reason why the value obtained is larger than the accepted value.

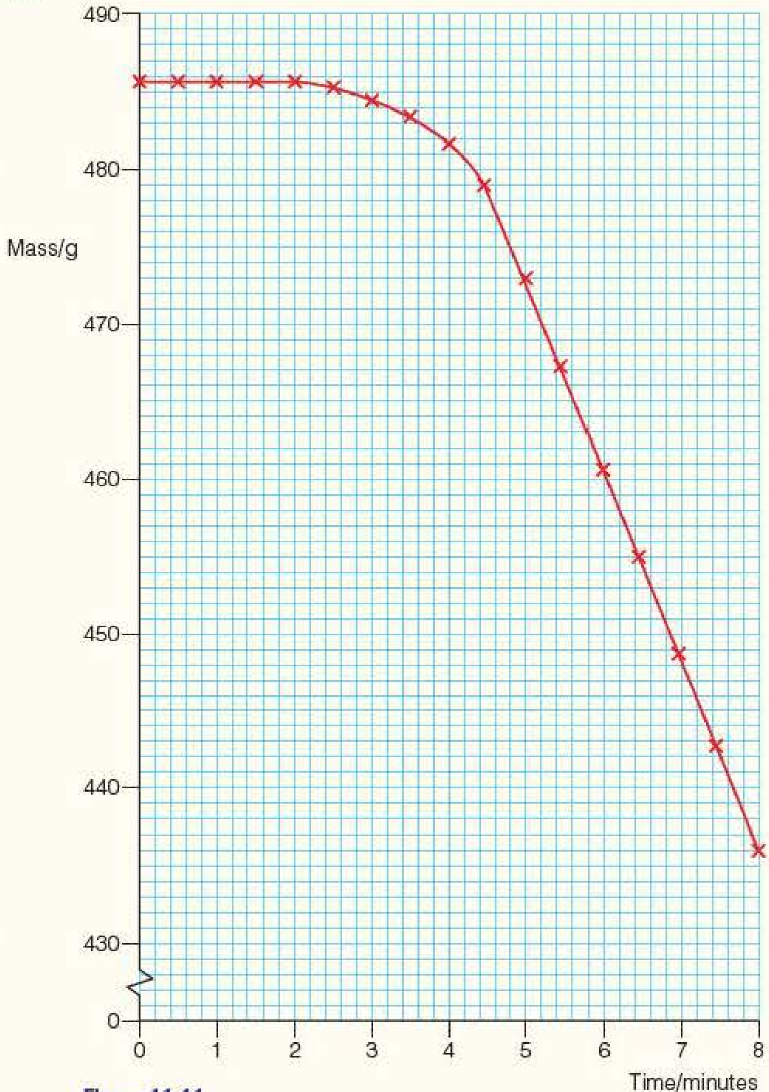


Figure 11.11

Test yourself

5 A student is making a cold drink. He takes 3 ice cubes, each of volume 15 cm^3, from a freezer in which the temperature is −18°C and puts them into a light plastic beaker containing 330 cl of drink at room temperature, 25°C.

a) What is the temperature of the drink when the ice cubes have just melted? You may assume that the beaker takes negligible energy. The specific heat capacity of the drink is 4200 $J kg^{-1} K^{-1}$ and its density is $1.04 \times 10^3 kg m^{-3}$. The density of ice is 920 $kg m^{-3}$, its specific heat capacity is 2100 $J kg^{-1} K^{-1}$ and its specific latent heat is 330 $kJ kg^{-1}$.

b) What other assumptions do you have to make?

6 A student uses an electric heater to heat some water in an expanded polystyrene cup. She knows that the specific heat capacity for water is 4200 $J kg^{-1} K^{-1}$ but she does not have a balance or a thermometer. She finds that it takes 5.0 minutes to bring the water from room temperature, which the room thermostat shows as 20°C, to its boiling point and a further 18.0 minutes for half of it to boil away.

a) Sketch a graph of temperature against time for her experiment.

b) Hence find the value this gives her for the specific latent heat of water.

Tip

Note that, surprisingly, Core Practical 13B enables us to determine a value for specific latent heat *without having to take a temperature!* You should remember that normal laboratory mercury thermometers have a poor resolution, usually 1°C, and are usually inaccurate. Measuring temperatures is therefore always a source of considerable uncertainty.

11.5 Absolute zero

We saw in Chapter 10 that the SI unit of temperature, the kelvin, was defined in terms of what is called the absolute thermodynamic scale of temperature. This has **absolute zero** as its zero. A temperature of 0 K is called absolute zero because it is the lowest temperature that can theoretically be reached. Its value on the Celsius scale is −273.15°C, but we usually round this to −273°C for calculations.

Although it is not possible in practice to cool any substance to absolute zero, scientists have achieved temperatures very close to 0 K, where matter exhibits quantum effects such as superconductivity and superfluidity.

A superconductor is a material that will conduct electricity *without resistance* when cooled below a certain temperature. Once set in motion, the current will flow forever in a closed loop in the superconducting material, making it the closest thing in nature to perpetual motion. There is, however, a catch! The superconductor has to be cooled to a very low temperature by, for example, liquid helium and, unfortunately, energy is needed to maintain this low temperature. The Large Hadron Collider at CERN in Geneva, Switzerland, which is the world's largest and highest-energy particle accelerator, needs 96 tonnes of liquid helium to keep its electromagnets (Figure 11.13) at their operating temperature, 1.9 K (−271°C).

Nevertheless, the uses of superconductors are diverse. For example, they are being used to improve the efficiency of motors and the transmission of electricity, in MRI scanners (see Section 6.7) and in magnetic levitating trains.

Key term

Absolute zero is the lowest temperature that can theoretically exist and is given a temperature of 0 K.

Figure 11.12 A magnet suspended above a superconducting coil

Magnetic levitation ('maglev') is an example of the application of electromagnetic induction (see Chapter 6). When the magnet in Figure 11.12 is dropped and falls towards the superconductor, the change in magnetic flux induces an electric current in the superconductor (Faraday's law). By Lenz's law, the current flows in a direction such that its magnetic field opposes that of the magnet and so the magnet is repelled. As the superconductor has no resistance, the current in it continues to flow, even though the magnet is no longer moving. The magnet is permanently repelled and hovers above the semiconductor.

This principle is used in maglev trains. The Japanese train shown in Figure 11.14 reached a world record speed of 581 km/h in 2003.

Figure 11.13 Superconducting electromagnet for the Large Hadron Collider

Figure 11.14 Maglev train

The lowest temperature recorded in a laboratory is about 100 pK (pico = 10^{-12}). The coldest known region of the universe is in the Boomerang Nebula, 5000 light years away in the constellation of Centaurus. Its temperature is about 1 K above absolute zero.

In terms of the kinetic theory, absolute zero is the temperature at which the molecules of matter have their lowest possible average kinetic energy. In a simplified model, the molecules are considered to have *no* average kinetic energy at absolute zero, in other words they have no random movement. In practice, quantum mechanics requires that they have a *minimum* kinetic energy, called the **zero-point energy**.

Example

The following passage is taken from an article on zero-point energy.

> *In conventional quantum physics, the origin of zero-point energy is the Heisenberg uncertainty principle, which states that, for a moving particle such as an electron, the more precisely one measures the position, the less exact is the best possible measurement of its momentum. The least possible uncertainty of position times momentum is specified by Planck's constant, h. A parallel uncertainty exists between measurements involving time and energy. This leads to the concept of zero-point energy.*

a) The Heisenberg uncertainty principle can be expressed by the equation

$$\Delta p \Delta x = h$$

where p represents momentum in the direction of displacement x. Show that this is consistent with Planck's constant h having units of J s.

b) An electron has a speed of $7.2 \times 10^7\,\mathrm{m\,s^{-1}}$. What is the uncertainty in its position? Comment on your answer.

c) Express the statement 'A parallel [i.e. equivalent] uncertainty exists between measurements involving time and energy' as an equation of similar form to that given in part **a)**. Write down this equation and suggest how this leads to the concept of zero-point energy.

Answer

a) Units of $h = \Delta p \Delta x$ are $\mathrm{kg\,m\,s^{-1}} \times \mathrm{m} = \mathrm{kg\,m^2\,s^{-1}}$

$$\mathrm{J\,s} = \mathrm{N\,m\,s} = \mathrm{kg\,m\,s^{-2}} \times \mathrm{m} \times \mathrm{s} = \mathrm{kg\,m^2\,s^{-1}}$$

So units of h are J s.

b)

$$\Delta x = \frac{h}{\Delta p} = \frac{h}{\Delta(mv)} = \frac{6.63 \times 10^{-34}\,\mathrm{J\,s}}{9.11 \times 10^{-31}\,\mathrm{kg} \times 7.2 \times 10^{7}\,\mathrm{m\,s^{-1}}}$$
$$= 1.0 \times 10^{-11}\,\mathrm{m}$$

This is of the order of the size of an atom, which means that we can only tell where the electron is to within about an atomic diameter.

c) $\Delta E \Delta t$ = a constant value. In fact $\Delta p \Delta x = h$ is equivalent to $\Delta E \Delta t = h$.

ΔE is the minimum uncertainty in, or fluctuation from, a particular measured value of energy, which means that zero energy can never be achieved.

Test yourself

7 Ignoring any quantum effects, describe the behaviour of the molecules, in terms of energy, when:

- **a)** an ideal gas is at absolute zero
- **b)** ice melts into water at 0°C
- **c)** steam condenses into water droplets at 100°C.

8 A student sees on the worldwide weather forecast that the temperature in Hong Kong is 36°C on a day when the temperature in London is 18°C. He says to his friend that it is 'twice as hot in Hong Kong as it is in London'. His friend, a physicist, says 'actually it's only about 6% hotter in Hong Kong'. Explain which of the friends is correct.

Exam practice questions

1 The molecules of an ideal gas do *not* have:

A internal energy **C** potential energy

B kinetic energy **D** random energy. **[Total 1 mark]**

2 When ice is melting, which of the following does not change?

A volume **C** potential energy of the molecules

B internal energy of the molecules **D** kinetic energy of the molecules

[Total 1 mark]

3 Which of the following quantities could *not* be measured in joules?

A energy **C** power

B heat **D** work **[Total 1 mark]**

4 The table below lists the melting points and boiling points for various common substances. Calculate the missing temperatures a) – h).

Table 11.3

Substance	Melting point /°C	Melting point /K	Boiling point /°C	Boiling point /K
water	0	273	100	373
mercury	−39	**a)**	**b)**	630
alcohol	**c)**	156	79	**d)**
oxygen	**e)**	54	−183	**f)**
copper	1083	**g)**	**h)**	2853

[Total 4 marks]

5 A physics textbook states that 'The *internal energy* of a body can be increased by *heating* it or by *doing work* on it.'

How would you explain what this means to a friend? Illustrate your answer with reference to the terms in italics and with examples from everyday life. **[Total 6 marks]**

6 Figure 11.15 shows an arrangement to demonstrate how heating can be achieved by doing mechanical work.

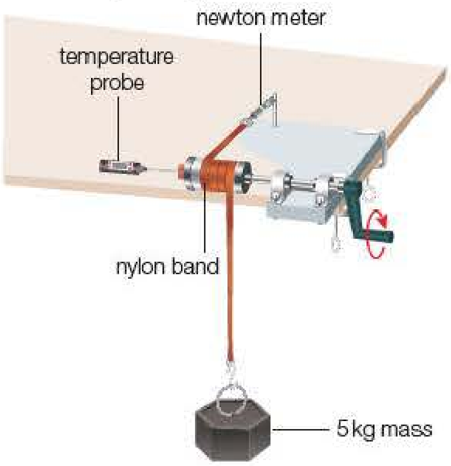


Figure 11.15 Doing mechanical work

The copper cylinder has a diameter of 29 mm and a mass of 197 g. When the cylinder is rotated clockwise as shown, the newton meter reads 16 N. After rotating the cylinder quickly at a steady speed for 400 revolutions, it is found that its temperature has gone up by 14.5 K.

a) Calculate the frictional force exerted by the nylon band on the cylinder. **[2]**

b) Hence show that about 3 J of work is done on the cylinder per revolution. **[2]**

c) Use the data provided to determine a value for the specific heat capacity of copper. **[3]**

d) Discuss whether this value is likely to be higher or lower than the accepted value. **[2]**

[Total 9 marks]

7 a) Explain the molecular energy changes that take place when a tray of water at room temperature is put into a freezer and ice cubes are formed. **[4]**

b) An ice tray containing 200 cm^3 of water at 20°C is put into a freezer. How much energy does the freezer have to remove from the water in order to form ice cubes having a temperature of −18°C? **[4]**

Specific heat capacity of water = $4.2 \times 10^3 \text{J kg}^{-1} \text{K}^{-1}$

Specific heat capacity of ice = $2.1 \times 10^3 \text{J kg}^{-1} \text{K}^{-1}$

Specific latent heat of fusion for water = $3.3 \times 10^5 \text{J kg}^{-1}$

[Total 8 marks]

8 a) An electric kettle is rated at 2.4 kW. It is filled with 1.0 litre of water at 20°C and switched on.

i) If the thermostat were to fail, how long would it take for the kettle to boil dry? **[5]**

ii) Sketch a graph of the volume of water in the kettle as a function of time. **[3]**

b) *You should note that the following question is about using a thermistor in a potential divider circuit as a thermostat. This is Core Practical 12, which you should have done in the first year of your A-level course and which is described in detail in Chapter 11 (Electrical circuits) in the Year 1 book.*

A student sets up a potential divider circuit as shown in Figure 11.16a. He wants the output to be 3.0 V when the temperature of the thermistor is 0°C.

The student looks up the characteristics of the thermistor in the manufacturer's catalogue and finds the graph shown in Figure 11.16b.

i) Sketch a graph of resistance against temperature on a linear temperature scale for temperatures between −20°C and +20°C **[2]**

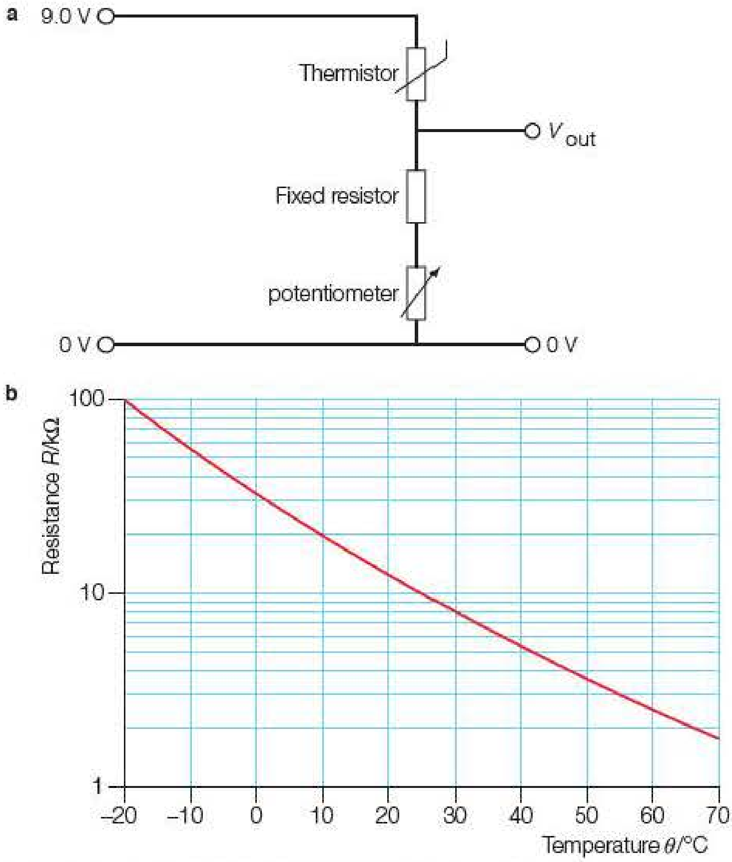


Figure 11.16 a) circuit and b) thermistor resistance as function of temperature

ii) The student has resistors of 10 kΩ, 15 kΩ, 22 kΩ, 33 kΩ and 47 kΩ. Which of these would be the most suitable value for the fixed resistor in the circuit? **[3]**

iii) Explain the purpose of having a potentiometer (variable resistor) in series with the fixed resistor. **[1]**

iv) Describe how the student could calibrate the circuit so that the output is 3.0V when the thermistor is at a temperature of 0°C. **[2]**

[Total 16 marks]

9 In 1913, Heike Kamerlingh Onnes was awarded the Nobel Prize in Physics *'for his investigations on the properties of matter at low temperatures, which led, amongst other things, to the production of liquid helium'*.

The Nobel Prize in Physics in 2003 was awarded jointly to Alexei Abrikosov, Vitaly Ginzburg and Anthony Leggett *'for pioneering contributions to the theory of superconductors and superfluids'*.

a) Helium liquefies at about 4°C above absolute zero. Explain what is meant by the term 'absolute zero'. **[2]**

b) Discuss the significance of Kamerlingh Onnes' achievement of 'the preparation of liquid helium'. **[2]**

c) Explain what is meant by 'superconductors'. [2]

d) Discuss the practical uses now being made of superconductors. [2]

[Total 8 marks]

10 In an article entitled 'Coal Power Station Aims for 50% Efficiency', the manufacturer states that this 'is possible due to a special steam turbine with a steam temperature of 700 instead of 600 degrees Celsius'.

The maximum thermal efficiency of a turbine is given by

$$\text{maximum efficiency} = T_1 - \frac{T_2}{T_1} \times 100\%$$

where T_1 and T_2 are the respective temperatures, in kelvin, of the steam entering and leaving the turbine.

a) The efficiency of a turbine with a steam temperature of 600°C is 46%. Show that the steam leaves the turbine at a temperature of approximately 200°C. [3]

b) Assuming that the temperature at which the steam leaves the turbine remains the same, is the manufacturer's claim that an efficiency of 50% will be achieved with a steam temperature of 700°C valid? [3]

c) Suggest how the remaining 50% of energy could be usefully used. [2]

[Total 8 marks]

Stretch and challenge

11 Due to quantum effects, the specific heat capacity of silver at temperatures approaching absolute zero varies according to the formula:

$$c = \alpha T^3 + \gamma T$$

where α and γ are constants
($\alpha = 1.7 \times 10^{-4}\,\text{J kg}^{-1}\,\text{K}^{-4}$ and $\gamma = 5.8 \times 10^{-3}\,\text{J kg}^{-1}\,\text{K}^{-2}$).

The two terms in the equation arise as a result of energy being used to increase the energy of both the lattice vibrations (T^3 term) and the conduction electrons (T term).

a) Explain **i)** what is meant by *lattice vibrations* and *conduction electrons*, and **ii)** the part played by each in the conduction of electricity in metals. [4]

b) Explain why the units of α are $\text{J kg}^{-1}\,\text{K}^{-4}$. [3]

c) Show that the temperature at which the contribution of the lattice vibrations to the specific heat capacity is equal to that of the conduction electrons is about 6 K. [4]

d) Calculate the specific heat capacity for silver at this temperature. [3]

[Total 14 marks]

12 a) Use the data given in question 11 to determine the energy needed to increase the temperature of 5.0 g of silver from 0 K to 10 K. Do this by means of a suitable graph, using a spreadsheet to process your data (use (0,0) and 10 other data points). [8]

b) If you can integrate, check your answer by using calculus. [4]

[Total 14 marks]

13 Read the article on laser cooling and then answer the questions at the end.

Laser Cooling

The use of lasers to achieve extremely low temperatures has advanced to the point that temperatures of 10^{-9} K have been reached. If an atom is travelling towards a laser beam and absorbs a photon from the laser, it will be slowed by the fact that the photon has momentum. At 300 K, the average velocity of a sodium atom would be about 570 m s^{-1}. Then, if a laser is tuned just below one of the sodium d-lines (589.6 nm, about 2.1 eV), a sodium atom travelling towards the laser and absorbing a laser photon would have its momentum reduced by the amount of the momentum of the photon. It would take a large number of such absorptions to cool the sodium atoms to near 0 K since one absorption would slow a sodium atom by only about 3 cm s^{-1} out of a speed of 570 m s^{-1}. A straight projection requires almost 20 000 photons to reduce the sodium atom momentum to zero. That seems like a lot of photons, but a laser can induce on the order of 10^7 absorptions per second so that an atom could be stopped in a matter of milliseconds.

A conceptual problem is that absorption can also speed up an atom if it hits it from behind, so it is necessary to have more absorptions from head-on photons. This is accomplished in practice by tuning the laser to slightly below the resonance absorption of a stationary sodium atom. From the atom's perspective, the head-on photon is seen as having a Doppler shift upward towards its resonant frequency and it is therefore more strongly absorbed than a photon travelling in the opposite direction, which is Doppler shifted away from the resonance. In the case of the room temperature sodium atom above, the incoming photon would be Doppler shifted up 0.97 GHz.

a) Explain why slowing down atoms enables physicists to 'achieve extremely low temperatures'. **[1]**

b) Show a photon having the wavelength of 589.6 nm has energy of about 2.1 eV. **[3]**

c) Calculate the momentum p of such a photon. **[2]**

d) State the law of conservation of momentum and explain how this leads to a sodium atom slowing down when it absorbs a photon head-on. **[5]**

e) Show that a sodium atom of mass M will slow down by p/M when it absorbs a photon of momentum p head-on. **[5]**

f) Confirm that a sodium atom (atomic mass 23) travelling at 570 m s^{-1} will be slowed down by about 3 cm s^{-1} when it absorbs a photon of wavelength 589.6 nm head-on ($1\,u = 1.66 \times 10^{-27}$ kg). **[2]**

g) Confirm that it will require 'almost 20 000 photons to reduce the sodium atom momentum to zero'. **[2]**

h) Hence show that 'an atom could be stopped in a matter of milliseconds'. **[2]**

i) Explain what is meant by 'its resonant frequency'. **[2]**

j) Explain why the laser is tuned to a frequency slightly lass than that of the resonant frequency. **[2]**

k) Show that this will be a Doppler shift (see Section 14.5) of about 0.97 GHz. **[2]**

[Total 24 marks]

Tip

This is a synoptic question involving knowledge of several areas of Physics and is an extension of Exam Practice Question 14 in Chapter 1.

12 Gas laws and kinetic theory

Prior knowledge

You need to be familiar with:

- ➜ what you (should have!) learned from Chapters 10 and 11
- ➜ using temperatures measured on the kelvin scale, with 0 K the absolute zero of temperature
- ➜ the concept of pressure being defined as force/area and its unit, the pascal, Pa
- ➜ force being the rate of change of momentum
- ➜ the concept of matter being composed of molecules that are in motion
- ➜ internal energy as kinetic energy of molecular motion and potential energy due to inter-molecular bonding
- ➜ absolute zero = 0 K = −273 °C
- ➜ from °C to K always add 273
- ➜ from K to °C always subtract 273
- ➜ pressure $= \frac{\text{Force}}{\text{Area}} \Rightarrow p = \frac{F}{A}$
- ➜ units of pressure $= \frac{\text{N}}{\text{m}^2} = \text{Pa}$

Test yourself on prior knowledge

1 Calculate the pressure on a bicycle seat of area $245\,\text{cm}^2$ caused by a rider of mass 50 kg.

2 A palette (wooden board) holds a tonne of bricks. The palette is carried by the two prongs of a forklift truck, each measuring 7.0 cm by 70 cm. How much pressure is exerted on the prongs?

Comment on your answer.

3 Complete the table by adding the appropriate temperatures a)–d).

Table 12.1

Element	Freezing point		Boiling point	
	θ/°C	T/K	θ/°C	T/K
Mercury	**a)**	234	357	**b)**
Oxygen	−219	**c)**	**d)**	90

4 An oxygen molecule has a mass of 32 u. What is its kinetic energy in

a) joules

b) electron-volts

when it has a speed of $491\,\text{m s}^{-1}$?
($1\,\text{u} = 1.66 \times 10^{-27}\,\text{kg}$).

5 A nitrogen molecule of mass 28 u travelling with a speed of $491\,\text{m s}^{-1}$ makes a head-on elastic collision with the wall of a container.

a) Explain what is meant by:

i) momentum is a vector quantity

ii) an elastic collision.

b) Calculate:

i) the momentum of the nitrogen molecule just before it hits the wall

ii) its momentum immediately after the elastic collision with the wall

iii) the change in momentum of the nitrogen molecule.

12.1 Pressure

Definition and units

In the Year 1 Student's Book, you will remember that we defined pressure as the size of the force per unit area acting at right angles to a surface:

$$p = \frac{F}{A}$$

From the above equation, we can see that the unit of pressure is $N\,m^{-2}$. As pressure is a very commonly used quantity, it is given its own unit, the **pascal** (Pa);

> **Key term**
>
> A **pascal** is the pressure exerted by a force of 1 N acting at right angles to an area of 1 m^2.
>
> $1\,Pa \equiv 1\,N\,m^{-2}$

> **Example**
>
> A brick has dimensions of 200 mm × 100 mm × 50 mm and a mass of 1.80 kg. Calculate:
>
> **a)** the density of the brick
>
> **b)** the pressure it exerts when it **i)** rests on its flat face, and **ii)** stands on one of its ends.
>
> **Answer**
>
> **a)** Density $= \frac{\text{mass}}{\text{volume}} = \frac{1.80\,kg}{0.200\,m \times 0.100\,m \times 0.050\,m} = 1800\,kg\,m^{-3}$
>
> **b) i)** On flat face: $p = \frac{F}{A} = \frac{1.80\,kg \times 9.8\,N\,kg^{-1}}{0.200\,m \times 0.100\,m} = 880\,Pa$
>
> **ii)** On one end: $p = \frac{F}{A} = \frac{1.80\,kg \times 9.8\,N\,kg^{-1}}{0.100\,m \times 0.050\,m} = 3\,500\,Pa\ (3.5\,kPa)$

> **Tip**
>
> Make sure you always use SI units throughout when doing pressure and density calculations, by having
>
> - masses in kg
> - *all* dimensions in m, areas in m^2 and volumes in m^3.

Fluid pressure

We say that a substance is a 'fluid' if it has the ability to flow. Therefore both liquids and gases are fluids. We will look at the particular behaviour of gases in more detail later on.

The pressure at a point in a fluid is defined as the force per unit area acting on a very small area round the point. You are probably familiar with concept that the pressure p at a point of depth Δh in a liquid of density ρ is given by:

$$p = \rho g \Delta h$$

To remind you, with reference to Figure 12.1, the pressure caused by the weight of the column of water of height Δh acting on a small area δA will be:

$$\text{Pressure on base} = \frac{\text{weight of column of water}}{\text{area}} = \frac{mg}{\delta A} = \frac{\rho V g}{\delta A} = \frac{\rho \Delta h \delta A g}{\delta A} = \rho g \Delta h$$

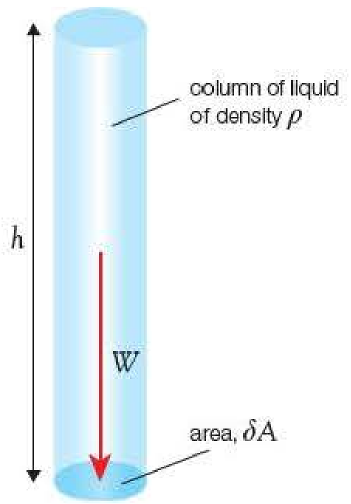


Figure 12.1

Although you do not have to learn this equation, you may be given it to use in the examination. You should note that this is the pressure due to the weight of liquid above the point. There will be *additional* pressure acting at the point because of the pressure p_A of the atmosphere. The total pressure will therefore be $p = p_A + \rho g \Delta h$. Atmospheric pressure is not a constant quantity – for example, we use its changing value to forecast weather – but we take a 'standard atmosphere' to be 1.01×10^5 Pa, or 101 kPa.

If a liquid is stationary, it follows that the pressure at any point in the liquid must act equally *in all directions*. If not, there would be a resultant force, which would cause the liquid to flow.

Tip

Remember that the pressure at a point in a fluid acts *equally in all directions*.

Example

In 2008, a world record for free diving (diving without breathing apparatus) was set at a depth of 122 m. Assuming that the density of sea water has an average value of $1.03 \times 10^3\,\text{kg}\,\text{m}^{-3}$ over this depth and that atmospheric pressure is 101 kPa, calculate:

a) the pressure exerted by the sea water at a depth of 122 m

b) the total pressure, in 'atmospheres', acting on a diver at this depth.

[The pressure p caused by a depth Δh of liquid of density ρ is $p = \rho g \Delta h$.]

Answer

a) Pressure caused by water,

$$\begin{aligned} p_w &= \rho g \Delta h \\ &= 1.03 \times 10^3\,\text{kg}\,\text{m}^{-3} \times 9.81\,\text{N}\,\text{kg}^{-1} \times 122\,\text{m} \\ &= 1.23 \times 10^6\,\text{Pa} \end{aligned}$$

b) The total pressure acting on the diver will be atmospheric pressure (1.01×10^5 Pa) plus the water pressure of 1.23×10^6 Pa, so

$$\begin{aligned} p_{tot} &= 1.01 \times 10^5\,\text{Pa} + 1.23 \times 10^6\,\text{Pa} \\ &= 1.33 \times 10^6\,\text{Pa} \end{aligned}$$

In atmospheres:

$$p_{tot} = \frac{1.33 \times 10^6\,\text{Pa}}{1.01 \times 10^5\,\text{Pa}} = 13 \text{ atmospheres}$$

Tip

Remember that atmospheric pressure must be added to the pressure caused by a depth of liquid in order to determine the total pressure acting.

Test yourself

1 In laboratories, atmospheric pressure can be measured with a mercury barometer. This measures the height of a column of mercury, of density $13.6 \times 10^3\,\text{kg}\,\text{m}^{-3}$, that can be supported by atmospheric pressure.

a) On a particular day, a barometer read 758 mm of mercury. What value does this give for the atmospheric pressure in pascals?

b) For accurate measurements, correction has to be made for the local value of the gravitational field strength and for the temperature. Suggest why this is necessary.

2 Figure 12.2 shows an experiment for finding the density of a liquid, such as oil, that does not mix with water.

a) Explain why points X and Y in the water must be at the same pressure.

b) Why must the pressure exerted by the height Δh_{oil} of oil (WX) be equal to the pressure exerted by the height Δh_{water} of water (ZY)?

c) In the experiment, Δh_{oil} was found to be 200 mm and Δh_{water} was 158 mm. If the density of water is $1000\,kg\,m^{-3}$, what value does the experiment give for the density of oil?

d) Explain the techniques you would use to make your measurements as accurate as possible.

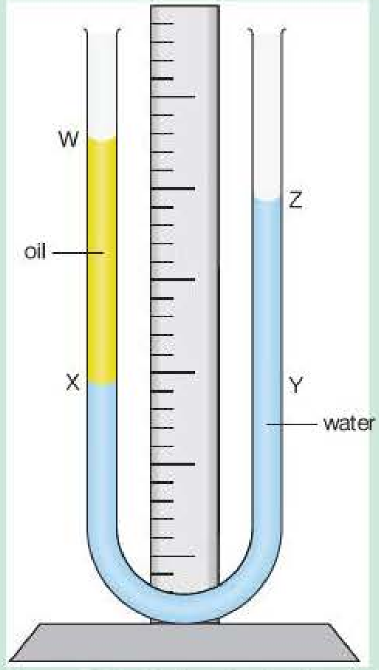


Figure 12.2 Finding the density of oil

12.2 The gas laws

The development of the steam engine in the 18th century revolutionised industry and transport, transforming people's lives. Progress was accelerated in the 19th century by the work of physicists in the field of thermodynamics – the study of how gases behave under various conditions of pressure, volume and temperature. By making some very simple assumptions about the properties of the molecules of a gas, we can explain much about the behaviour of gases, both qualitatively and quantitatively. More recently this has led to practical applications such as the development of more efficient car engines, power stations and refrigerators.

When we are looking at the behaviour of gases we have four variables to consider – the mass, pressure, volume and temperature of the gas. In order to investigate how these quantities are related, we need to keep two constant while we see how the other two vary with one another.

Boyle's law

The relationship between pressure and volume for a fixed mass of gas at constant temperature was discovered by Robert Boyle in the 17th century. This can be investigated by means of a simple experiment as described below.

Core Practical 14

Investigating the relationship between pressure and volume for a fixed mass of gas at constant temperature

The simple apparatus shown in Figure 12.3a can be used. The volume V of the fixed mass of air under test is given by the length l of the column of air multiplied by the area of cross section of the glass tube. If the tube is uniform, this area will be constant, so we can assume that $V \propto l$. (Some tubes may actually be calibrated to give the volume V directly.) The total pressure of the air is read straight off the pressure gauge.

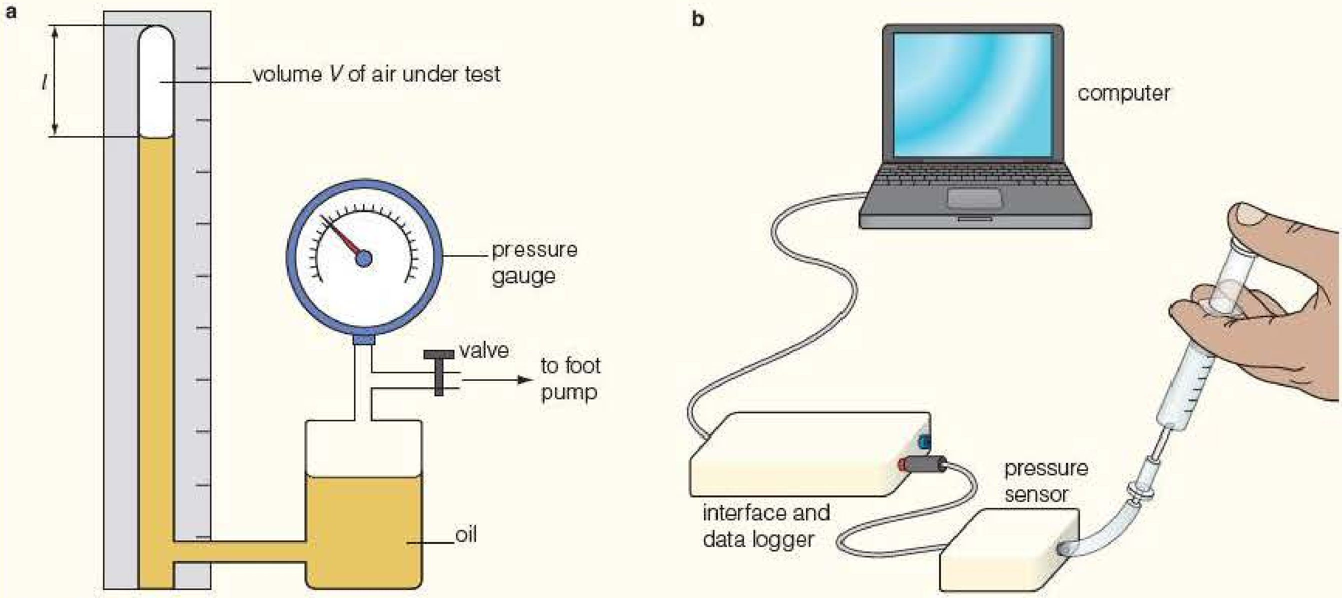


Figure 12.3

The valve is opened so that the air starts at atmospheric pressure and then the pressure is increased by means of the foot pump. This pressure is transmitted through the oil and compresses the air. The pressure p and the corresponding length l (or volume V) of the air column are recorded for as wide a range of values as possible. It is important to leave sufficient time after each change in pressure for the air to reach thermal equilibrium with its surroundings so that its temperature remains constant. The measurements of p and l (or V) are tabulated, together with values of $1/l$ (or $1/V$).

This experiment can also be done using a syringe to contain the air, with a pressure sensor and data logger (Figure 12.3b).

Plotting p against l (or V) and p against $1/l$ (or $1/V$) gives graphs like those shown in Figure 12.4a and 12.4b.

Safety note

It is important to avoid excessive pressure and to make sure connections are secure; otherwise oil may be sprayed at high pressure over observers. A teacher/technician should check equipment before use.

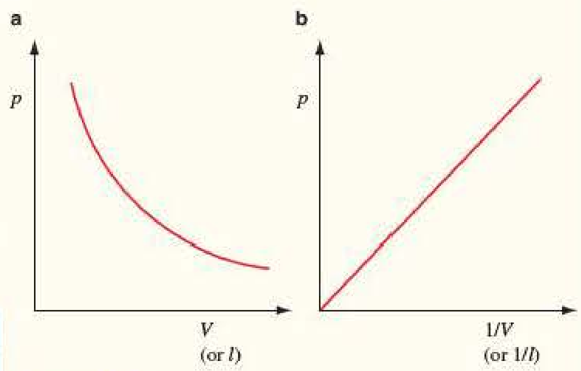


Figure 12.4

Questions

The data shown in Table 12.2 were obtained from a Boyle's law experiment.

Table 12.2

p/kPa	100	120	140	160	180	200	220	240
V/cm^3	20.1	16.6	14.4	12.4	11.2	9.9	9.1	8.4
$(1/V)$/cm^{-3}	0.050							
$(1/V)/10^3$m^{-3}	50							

1 Complete the table by adding values for $1/V$. The first column has already been done for you.
2 Plot a suitable graph to investigate the extent to which Boyle's law is obeyed.
3 Comment on your graph.
4 Explain why it is important to make changes to the pressure slowly and allow time for thermal equilibrium to be reached before taking readings.

Key term

Boyle's law states that the pressure of a fixed mass of gas is inversely proportional to its volume provided that the temperature is kept constant.

Tip

Remember to give the conditions when stating Boyle's law, namely

- a fixed mass of gas
- at constant temperature.

The graph of p against V shown in Figure 12.4a is called an **isothermal** ('iso' means 'the same' and 'thermal' means 'temperature'). An isothermal is a curve that shows the relationship between the pressure and volume of a gas at a particular temperature (see also Figure 12.5a below). The precise nature of this relationship can be determined from Figure 12.4b. As this is a straight line *through the origin* we can deduce that:

$$p \propto \frac{1}{V} \text{ or } p = \text{constant} \times \frac{1}{V}$$

giving $pV = \text{constant}$

This is **Boyle's law**.

You need to be familiar with the three possible graphs that can be plotted to illustrate Boyle's law. These are shown in Figure 12.5.

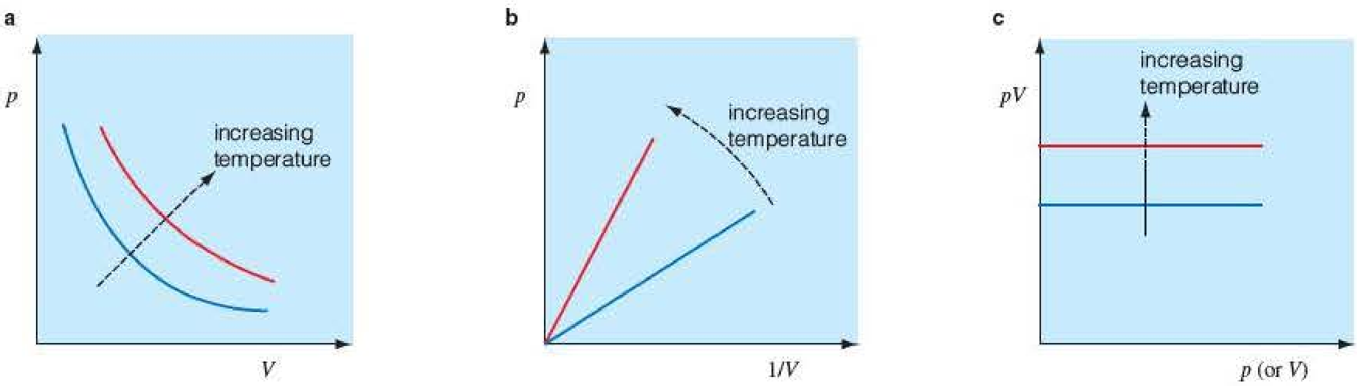


Figure 12.5 Graphs illustrating Boyle's law

Example

A deep-sea diver descends to a depth of 190 m in seawater to investigate a wreck. The seawater has an average density of $1.02 \times 10^3\,\text{kg}\,\text{m}^{-3}$ and atmospheric pressure is $1.01 \times 10^5\,\text{Pa}$.

a) Show that the pressure at this depth is approximately 20 atmospheres.

b) The diver releases a bubble of air having a volume of $3.0\,\text{cm}^3$. Calculate the volume of this bubble when it reaches the surface. What assumption do you have to make in your calculation?

The pressure p caused by a depth Δh of liquid of density ρ is $p = \rho g \Delta h$.

Answer

a) The pressure caused by the depth of water is

$$p_w = \rho g \Delta h = (1.02 \times 10^3)\,\text{kg m}^{-3} \times 9.81\,\text{N kg}^{-1} \times 190\,\text{m} = 1.90 \times 10^6\,\text{Pa}$$

We must now add atmospheric pressure to this, giving a total pressure of

$$p_{tot} = 1.90 \times 10^6\,\text{Pa} + 1.01 \times 10^5\,\text{Pa} = 2.00 \times 10^6\,\text{Pa}$$

In atmospheres this is

$$\frac{2.00 \times 10^6\,\text{Pa}}{1.01 \times 10^5\,\text{Pa}} = 19.8 \approx 20 \text{ atmospheres}$$

b) Using $p_1V_1 = p_2V_2$ for the air bubble, where:

$p_1 = 20$ atmospheres, $V_1 = 3.0\,\text{cm}^3$ and $p_2 = 1$ atmosphere (at the surface),

We get:

$$V_2 = \frac{p_1 V_1}{p_2} = \frac{20 \text{ atmosphere} \times 3.0\,\text{cm}^3}{1 \text{ atmosphere}} = 60\,\text{cm}^3$$

In doing this calculation, we have to assume that the density of the seawater is uniform, which it won't quite be, and, more significantly, that the temperature of the water at the surface is the same as that at a depth of 190 m. In practice, it is likely to be much colder at this depth.

Tip

A convenient way of remembering Boyle's law for calculations is:

$p_1V_1 = p_2V_2$

Tip

When using $p_1V_1 = p_2V_2$, it doesn't matter what the units of p and V are as long as they are the *same on both sides of the equation*. In the worked example we have used non-SI units for both quantities – atmospheres for p and cm^3 for V.

Pressure and temperature

The relationship between the pressure and temperature of a fixed mass of gas at *constant volume* can be investigated as described below.

Activity 12.1

Investigating the relationship between pressure and temperature of a fixed mass of gas at constant volume

This experiment can be performed either using a thermometer and a pressure gauge (Figure 12.6a) or using temperature and pressure sensors with a suitable data logger (Figure 12.6b). Note that the tube connecting the container of air to the pressure gauge/sensor should be as short and of as small an internal diameter as possible to reduce the volume of air inside the tube to a minimum, because this air will not be at the same temperature as that in the flask.

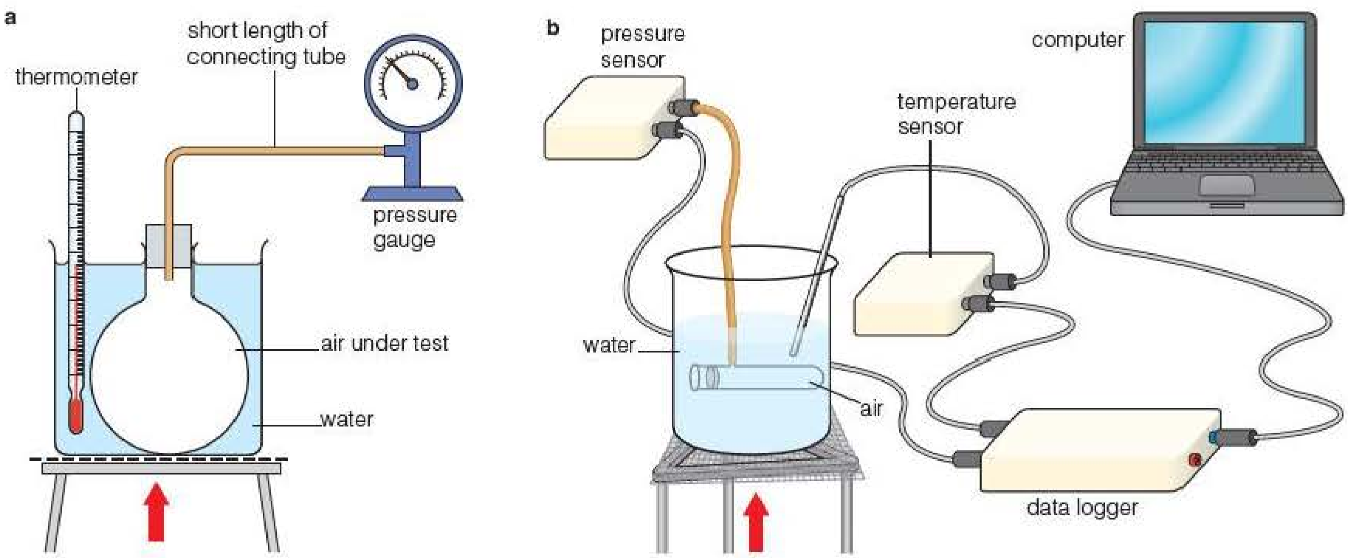


Figure 12.6

Initially the beaker is packed with ice (or ice and salt) to get as low a starting temperature as possible. The pressure p and corresponding temperature θ are recorded for temperatures up to the boiling point of the water. Time must be allowed for the air in the flask to reach the temperature of the water for each reading by turning down the heat and waiting for equilibrium to be attained. (This is not necessary using the data logger.) The water is continuously stirred to ensure an even temperature distribution.

Although the range of p is very limited, it is instructive to plot a graph of p against θ with the pressure scale starting at zero (this gives a very small scale, which would not normally be acceptable) and the temperature scale going from −300 °C to +100 °C. A graph like Figure 12.7 is obtained.

When the graph is extrapolated to zero pressure, the corresponding temperature is found to be −273 °C (or slightly different because of experimental error exaggerated by a long extrapolation). This is, of course, absolute zero (see Section 11.5).

Safety note

The flask chosen as the pressure vessel must to able to withstand the increase in pressure. Eye protection must be worn.

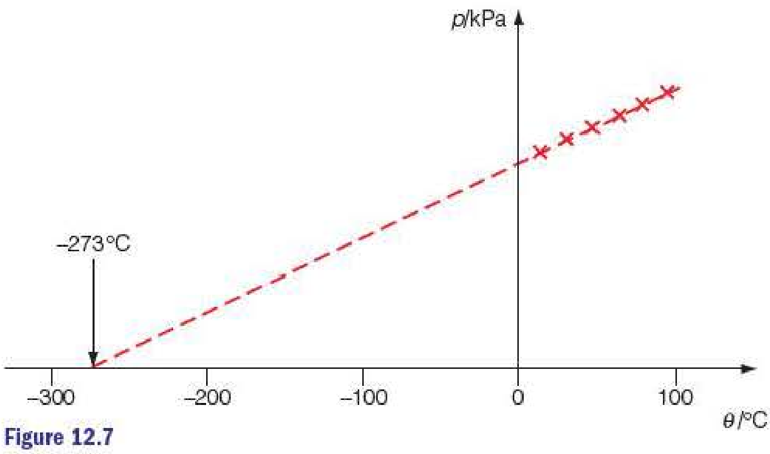


Figure 12.7

Tip

If you are describing a data-capture method in an examination, you must draw *all* the apparatus as in Figure 12.6b and state that you would select 'x-axis = temperature' and 'y-axis = pressure' from the 'Graph' menu.

If the data from the experiment above were re-plotted as a graph of p against T in kelvin, the graph would be like that shown in Figure 12.8.

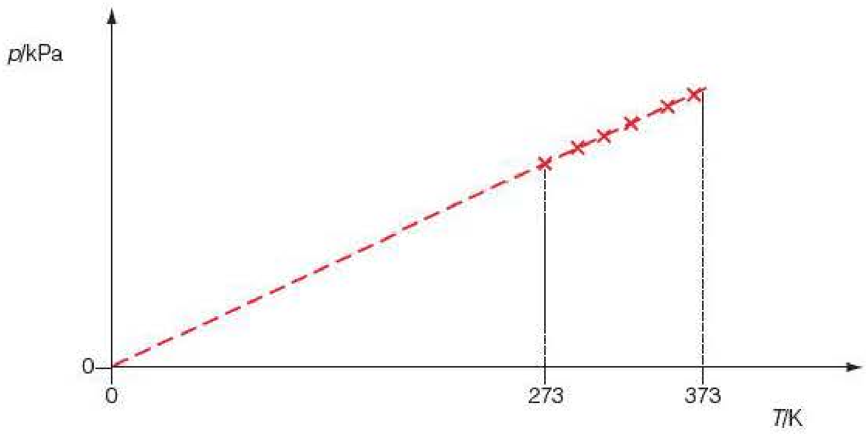


Figure 12.8 Pressure against kelvin temperature for a gas of fixed mass and volume

As the graph is a straight line through the origin, it shows that

$$p \propto T \quad \text{or} \quad \frac{p}{T} = \text{constant}$$

for a fixed mass of gas at constant volume.

Example

A car manual states that the 'tyre pressure must be 36 psi (= 250 kPa)'. (This is the pressure that the air inside the tyre must be *above* atmospheric pressure, which you may assume to be 100 kPa.)

The driver inflates the tyres to 35 psi (pounds per square inch) on a day when the temperature is 14°C.

After a long journey, she checks the pressure again and finds it is now 39 psi.

a) Use the data from the manual to show that 100 kPa is equivalent to 14.4 psi.

b) Show that after the journey the temperature of the air in the tyre would be approximately 37°C.

c) State what assumption you have to make in your calculation.

d) Suggest why the temperature of the air inside the tyre has increased.

Answer

a) If 250 kPa = 36 psi, then

$$100\,\text{kPa} = 36\,\text{psi} \times \frac{100\,\text{kPa}}{250\,\text{kPa}} = 14.4\,\text{psi}$$

b) Using the fact that p/T is constant, and remembering that:

- T must be in kelvin
- p is the *total* pressure of the air in the tyre

$$p/T = \text{constant} \Rightarrow \frac{p_1}{T_1} = \frac{p_2}{T_2} \Rightarrow T_2 = \frac{p_2 T_1}{p_1}$$

where: $p_1 = (35.0 + 14.4)\,\text{psi} = 49.4\,\text{psi}$

$p_2 = (39.0 + 14.4)\,\text{psi} = 53.4\,\text{psi}$

$T_1 = (14 + 273)\,\text{K} = 287\,\text{K}$

$$\text{So } T_2 = \frac{p_2 T_1}{p_1} = \frac{53.4\,\text{psi} \times 287\,\text{K}}{49.4\,\text{psi}} = 310.2\,\text{K} \approx 310\,\text{K} = 37°\text{C}$$

c) You have to assume that the volume of the air remains constant – that is the tyre does not expand.

d) The rubber of the tyres exhibits a property called hysteresis (see page 203 of the Year 1 Book and Figure 12.9). The part of the tyre in contact with the road gets compressed, and as it moves round it relaxes again. This cycle is repeated several times per second. Some of the energy is transferred to the internal energy of the rubber each cycle and so the tyres become warm and conduct some of their internal energy to the air inside, increasing the pressure.

You might like to show for yourself that the energy per unit volume transferred to internal energy for each cycle in Figure 12.9 is about 1.5 MJ m^{-3}.

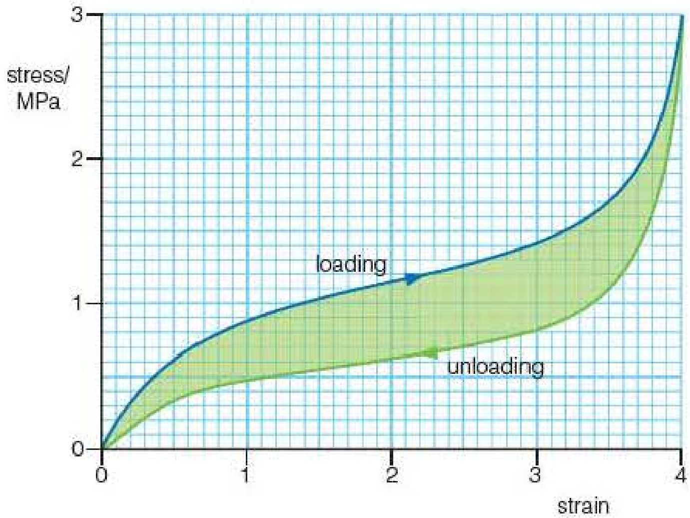


Figure 12.9 Hysteresis loop for rubber: the area of the loop represents the energy per unit volume transferred to internal energy during the cycle

Test yourself

3 A physics student and her friend are playing table tennis when the ball is accidentally dented. The physics student puts the ball into a cup of hot water and the dent pops out. 'That's clever,' says her friend. 'It's just physics,' says the student. Explain the physics of what happens to the ball.

4 A student learns that the pressure of a gas is proportional to its temperature and decides to investigate this by immersing a flask of air, connected to a pressure gauge, in a beaker of water. He starts with the temperature at 27°C and heats the air up to 54°C, expecting the pressure to have doubled. To his consternation, he finds that the pressure has only gone up by 9%.

a) Explain his mistake and why the pressure has only gone up by 9%.

b) To what temperature would the air have to be raised in order to double its pressure?

5 In the game of squash, the ball is hollow and is made of rubber, having a diameter of 40 mm. When the ball is cold it does not bounce very well so, at the beginning of a game, players repeatedly hit the ball hard against the wall to warm it up. It then bounces much better.

a) The ball has a mass of 25 g and is made of rubber having a specific heat capacity of 2000 J kg^{-1} K^{-1}.

i) How much energy must be supplied to raise its temperature from 20°C to 45°C?

ii) Suggest why it is reasonable to ignore the energy needed to heat up the air inside the ball.

iii) Describe what happens to the air molecules inside the ball when the temperature of the ball increases.

b) At 20°C the pressure of the air inside the ball is atmospheric (101 kPa). Assuming that the volume of the ball remains constant, calculate the pressure of the air inside the ball when its temperature is 45°C.

c) From your knowledge of the properties of rubber and air suggest

i) how the air in the ball acquires the energy to increase its temperature.

ii) why a warm ball bounces better than a cold one.

d) In the standard test for the quality of a squash ball, the ball, at a temperature of 45°C, is dropped onto a concrete floor from a height of 2.5 m. It is required to rebound to a height of at least 75 cm. What percentage of the ball's energy is dissipated on hitting the floor if it just reaches this height?

12.3 Equation of state for an ideal gas

We have shown experimentally that p/T = constant. Combining this with Boyle's law, pV = constant, for a fixed mass of gas we get:

$$pV = \text{constant} \times T$$

If we take **one mole** as the 'fixed mass' we find that the constant in the above expression is *the same for all gases*. It is called the **universal molar gas constant**, symbol R, and has a value of $8.31\,\text{J}\,\text{K}^{-1}\,\text{mol}^{-1}$.

If we have n moles of a gas we then have

$$pV = nRT$$

An alternative way of expressing this relationship is in terms of another constant, called the **Boltzmann constant**, symbol k, after Ludwig Boltzmann, who did much to advance the development of thermodynamics in the latter half of the 19th century. The Boltzmann constant is defined as $k = R/N_A$ where N_A is the Avogadro number (i.e. the number of molecules in 1 mole of a substance) and so if we substitute $R = kN_A$ into our equation we get

$$pV = nRT = nN_AkT$$

where nN_A = number of moles × number of molecules in a mole
= total number of molecules

The equation thus becomes

$$\boldsymbol{pV = NkT}$$

where N is the number of *molecules* of the gas and $k = 1.38 \times 10^{-23}\,\text{J}\,\text{K}^{-1}$.

A gas that obeys this equation under *all* conditions of temperature and pressure is called an **ideal gas**, and hence this equation is called the **ideal gas equation** or the **equation of state for an ideal gas**. Of course, there is no such thing as an 'ideal gas', but in practice most gases obey this equation provided that the pressure is not too large and the gas is above a certain temperature, called its critical temperature. This is a very low temperature for most gases, for example −118°C for oxygen and −147°C for nitrogen, the two main constituents of air. This means that under normal laboratory conditions air behaves like an ideal gas.

We showed earlier that experimentally $p \propto T$ for a fixed mass of gas at constant volume. It can also be shown that $V \propto T$ for a fixed mass of gas at constant pressure. It therefore follows that at absolute zero, when $T = 0$, the pressure p and volume V both become zero, which is further confirmation that absolute zero is the lowest temperature theoretically possible. However, this argument is only true for an ideal gas – most real gases liquefy well above absolute zero, and even helium is liquid at 4 K.

Tip

Remember, in the equation $pV = NkT$ you must have:

- p in Pa
- V in m^3
- N as the number of molecules
- T in K.

Key term

An **ideal gas** obeys the equation $pV = NkT$ under all conditions of temperature and pressure.

Example

a) Show that the units of Boltzmann's constant k are $\text{J}\,\text{K}^{-1}$.

b) Show that the volume of one mole (6.02×10^{23} molecules) of an ideal gas at 'standard temperature and pressure' (i.e. = 273.15 K and p = 101.3 kPa) is 22.4 litres.

Answer

a) From $pV = NkT$, we have

$$k = \frac{pV}{NT} = \frac{\text{N m}^{-2} \times \text{m}^3}{\text{K}} = \frac{\text{N m}}{\text{K}} = \text{J K}^{-1}$$

b) From $pV = NkT$, we have

$$V = \frac{NkT}{p} = \frac{6.02 \times 10^{23} \times 1.38 \times 10^{-23}\ \text{J K}^{-1} \times 273.15\ \text{K}}{101.3 \times 10^3\ \text{Pa}}$$

$$= 0.0224\ \text{m}^3 = 22.4\ \text{litres}$$

Rearranging the ideal gas equation $pV = NkT$, we get

$$k = \frac{pV}{NT} = \text{constant}$$

So for a fixed mass of gas (i.e. N = constant) we get

$$\frac{pV}{T} = \text{constant}$$

If, therefore, a fixed mass of gas has initial values p_1, V_1 and T_1, and final values p_2, V_2 and T_2, we can say

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

This equation is a very useful one to remember for working out problems.

Tip

Remember that in the equation T must be in K.

Example

A diver swimming at a depth of 20.0 m in the sea, where the temperature is 7.0°C, expels an air bubble of volume 0.40 cm^3. The bubble then rises to the surface, where the temperature is 24.0°C.

a) Assuming that atmospheric pressure is 101 kPa and that the density of sea water is 1.03×10^3 kg m^{-3}, calculate the pressure inside the bubble at a depth of 20.0 m.

b) Give two reasons why the bubble expands as it rises to the surface.

c) What is the volume of the bubble when it just reaches the surface?

The additional pressure Δp caused by a depth Δh of liquid of density ρ is $\Delta p = \rho g \Delta h$.

Answer

a) $p_{tot} = p_A + \rho g \Delta h$

$$= 1.01 \times 10^5\ \text{Pa} + (1.03 \times 10^3\ \text{kg m}^{-3} \times 9.81\ \text{N kg}^{-1} \times 20\ \text{m})$$
$$= 1.01 \times 10^5\ \text{Pa} + 2.02 \times 10^5\ \text{Pa} = 3.03 \times 10^5\ \text{Pa}$$

b) As the bubble rises to the surface, h gets less and so the pressure acting on the bubble will also get less. As the volume of air is *inversely* proportional to pressure, the volume will *increase*. Additionally, the temperature increases towards the surface and, since $V \propto T$, this will also cause the air to expand.

c) Using $\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2} \Rightarrow V_2 = \frac{p_1V_1T_2}{p_2T_1}$

$$= \frac{3.03 \times 10^5\ \text{Pa} \times 0.40\ \text{cm}^3 \times 297\ \text{K}}{1.01 \times 10^5\ \text{Pa} \times 280\ \text{K}} = 1.3\ \text{cm}^3$$

Tip

It is always a good idea to rearrange the equation first, before putting in the data, particularly when the numbers are complex such as in this example.

Test yourself

6 From a hypothesis put forward by Avogadro in 1811, it can be deduced that a mole of an ideal gas at standard temperature (0°C) and pressure (101 kPa) occupies a volume of 22.4 litres.

a) What is meant by an 'ideal gas'?

b) Under what conditions will a real gas behave like an ideal gas?

c) Use the data given above to show that a mole of an ideal gas contains about 6×10^{23} molecules.

7 A volume of $400\,cm^3$ of air is enclosed in a cylinder by a piston as shown in Figure 12.10a. The initial pressure of the air is 100 kPa and its initial temperature is 25°C. The air is heated by immersing the cylinder in a water bath.

a) If the air is heated to a temperature of 100°C at constant pressure, show that its volume will increase to about $500\,cm^3$.

b) Calculate the pressure of the air if it is now compressed, isothermally, to its original volume.

c) Show that if the air is now cooled at constant volume until its pressure is back to 100 kPa, its temperature will be 25°C again.

d) Show this sequence of events on a copy of the grid shown in Figure 12.10b.

8 The volume of air breathed in and out at rest is known as the tidal volume. This is found to be about 500 ml in an averagely built, healthy, young adult. We can assume that this air is at atmospheric pressure (101 kPa) and at a temperature of 37°C. To a good approximation, we can think of air being made up of 80% nitrogen (molecular mass 28 u) and 20% oxygen (molecular mass 32 u).

a) Explain why:

i) there isn't actually such a thing as an 'air' molecule

ii) we can *think* of an 'air' molecule as having a molecular mass of 29 u.

b) Calculate the number of 'air' molecules taken in per breath.

c) Hence calculate:

i) the number of oxygen molecules taken in per breath

ii) the mass of oxygen taken in per breath.

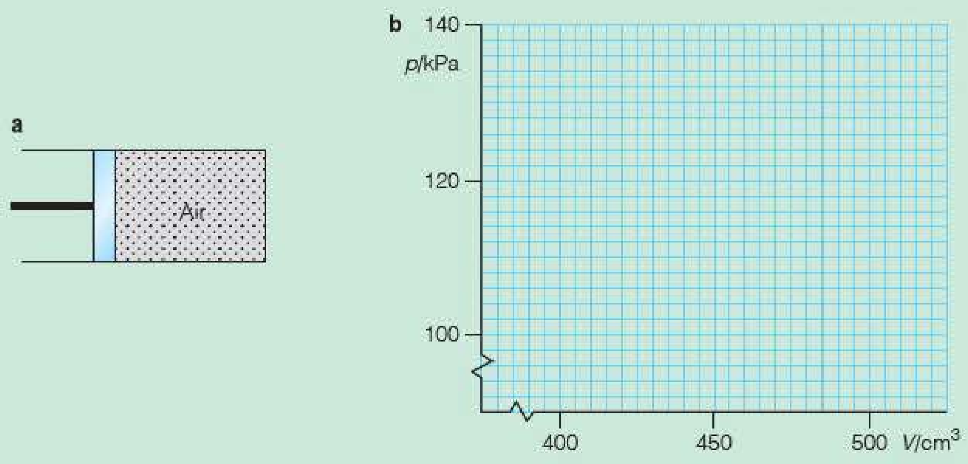


Figure 12.10

12.4 Evidence for the kinetic theory

Direct experimental evidence for the kinetic theory of matter ('kinetic' means 'having motion') was provided by a discovery by a Scottish botanist, Robert Brown, in 1827. He noticed that tiny grains of pollen, when suspended in water and viewed under a microscope, continually moved backwards and forwards with small, random, jerky paths. We now attribute this to unequal bombardment of the very fine grains of pollen by the invisible water molecules, which themselves must therefore be in continuous motion.

Scientific evidence is much stronger if we can quantify it. Einstein did just this – he analysed this 'Brownian motion' mathematically and was able to determine a value for the Avogadro number (the number of particles, 6.02×10^{23}, in one mole of a substance), which agreed very closely with the value obtained by chemical means.

Brownian motion provides strong evidence for particles of matter being in continuous motion. Kinetic theory relates the macroscopic (large-scale) behaviour of an ideal gas, in terms of its pressure, volume and temperature, to the microscopic properties of its molecules.

The assumptions we make in order to establish a kinetic theory for an *ideal gas*, with the corresponding experimental justification for these assumptions, are summarised in Table 12.3.

Table 12.3

Assumption	Experimental evidence
A gas consists of a very large number of molecules	Brownian motion.
These molecules are in continuous, rapid, random motion.	
Collisions between molecules and between molecules and the walls of a container are perfectly elastic (i.e. no kinetic energy is lost).	If not, the molecules would gradually slow down – this cannot be the case as Brownian motion is observed to be continuous. It would also mean that the gas would gradually cool down.
The volume occupied by the molecules themselves is negligible compared with the volume of the container.	It is easy to compress a gas by a large amount.
Intermolecular forces are negligible except during a collision.	From the above it follows that on average the molecules are very far apart relative to their size and so the intermolecular forces become very small.
The duration of collisions is negligible compared with the time spent in between collisions.	This also follows from the fact that the molecules, on average, are very much further apart than their size

12.5 Kinetic model of temperature

The application of kinetic theory, making the assumptions shown in Table 12.3, to the molecules of an ideal gas, together with simple mechanics, enables us to derive an expression for the pressure p of a gas in terms of the density ρ of the gas and speed of its molecules.

Imagine a sample of gas to be contained within a cubic container having sides of length l. Consider a molecule of the gas, having mass m and travelling with velocity c. This velocity can be resolved into three components u, v and w in each of the directions 0x, 0y and 0z as in Figure 12.11a. Consider one such component approaching a wall of the container as shown in Figure 12.11b.

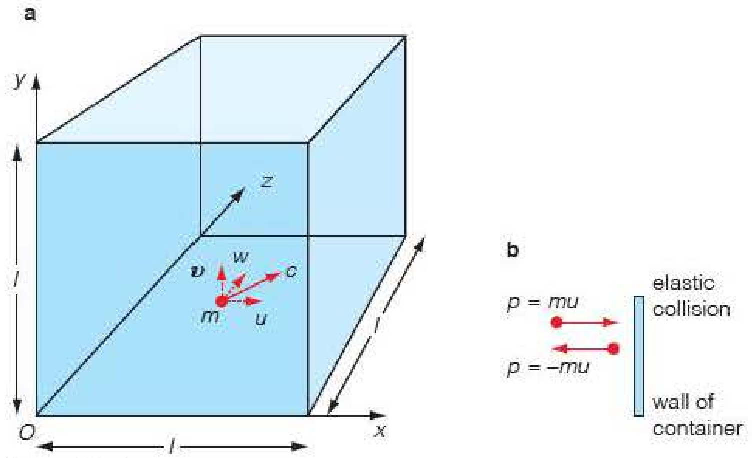


Figure 12.11

The molecule will have momentum mu in the 0x direction. On making an *elastic collision* with the wall of the container it will rebound with momentum $-mu$. The *change* in its momentum will therefore be $-2mu$.

If the speed of the molecule in the 0x direction is u, then it will take a time $t = l/u$ to cross the container. It will therefore make u/l crossings per second. As half of these crossings will be in each direction, it will make $\frac{1}{2}u/l$ collisions with each opposite wall per second.

As force is equal to the rate of change of momentum, the *force acting on the molecule* will be given by the number of collisions per second multiplied by the change in momentum on each collision, i.e.

$$F = \frac{1}{2}\frac{u}{l}\ (-2mu) = -\frac{mu^2}{l}$$

By Newton's third law, the *force of the molecule on the wall* is equal and opposite, namely

$$F = \frac{mu^2}{l}$$

As pressure is force per unit area, the pressure p exerted on the wall (which has area l^2) is

$$p = \frac{\text{force}}{\text{area}} = \frac{mu^2/l}{l^2} = \frac{mu^2}{l^3} = \frac{mu^2}{V} \text{ where } V = l^3 = \text{volume of container}$$

Hence: $pV = mu^2$

Now for the tricky bit! So far we have just considered *one* component (in the 0x direction) of *one* molecule. Let us assume that we have N such molecules with components u_1, u_2, u_3 ....u_N. We then have :

$$pV = m\ (u_1^2, u_2^2, u_3^2 \ldots u_N^2)$$

If we let the symbol $<u^2>$ represent the average or *mean* value of all the squares of the components in the 0x direction, we have

$$<u^2> = \frac{u_1^2 + u_2^2 + u_3^2 + \ldots + u_N^2}{N} \Rightarrow N<u^2> = u_1^2 + u_2^2 + u_3^2 + \ldots + u_N^2$$

$$pV = mN<u^2>$$

If we have a *very large number* of molecules of varying speed in *random* motion, then it is valid to say that

$$<u^2> = <v^2> = <w^2>$$

For each molecule $<c^2> = <u^2> + <v^2> + <w^2>$

From the above, it follows that

$$<u^2> = \tfrac{1}{3}<c^2>$$

Therefore, from $pV = Nm<u^2>$ we get

$$pV = \tfrac{1}{3}Nm<c^2>$$

where $<c^2>$ is the **mean square speed** of the molecules.

The 'mean square speed' is the average of the squares of the speeds of all the individual molecules of the gas. This means we have to square the speed of each molecule, add up all these squared speeds and divide by the total number of molecules to give the average. The 'root mean square speed' is then the square root of the mean square speed, i.e. $\sqrt{<c^2>}$.

Tip

Rather than trying to learn this derivation parrot fashion, try to remember the key steps in the argument and the physics behind them:

- elastic collision of molecule with wall (assumption of kinetic theory)
- molecule undergoes change of momentum
- force on molecule = rate of change of momentum (Newton's second law)
- force on wall equal and opposite to force on molecule (Newton's third law)
- pressure = force/area

Then worry about the maths!

Example

Calculate the mean square speed of gas molecules having the following speeds measured in m s^{-1}:

310, 320, 330, 340 and 350.

(You might like to do this using a spreadsheet to make the number crunching easier).

Answer

Table 12.4

Speed/m s^{-1}	Speed squared/m^2 s^{-2}
310	96 100
320	102 400
330	108 900
340	115 600
350	122 500
Sum of the squared speeds	545 500
Mean square speed (÷ 5)	109 100

Note that, although the mean square value (109 100) is close to, it is *not* the same as, the mean value (330) squared (108 900).

The equation $pV = \frac{1}{3}Nm\langle c^2\rangle$ that we have just derived can be rearranged into a more practical form by dividing each side of the equation by V. We then have:

$$p = \frac{1}{3}\frac{Nm}{V}\langle c^2\rangle$$

As Nm is the total mass of the gas, then Nm/V will be the density ρ of the gas. We therefore have:

$$p = \frac{1}{3}\rho\langle c^2\rangle$$

Example

Air at 15°C and standard pressure (101.325 kPa) has a density of 1.2250 kg m^{-3}. What value does this data give for the root mean square of the air molecules?

Answer

From $p = \frac{1}{3}\rho\langle c^2\rangle \Rightarrow \langle c^2\rangle = \frac{3p}{\rho}$

$$\Rightarrow \sqrt{\langle c^2\rangle} = \sqrt{\frac{3\times 101.325\times 10^3\,\text{Pa}}{1.2250\,\text{kg}\,\text{m}^{-3}}} = 498\,\text{m}\,\text{s}^{-1}$$

Let us now combine the equation $pV = \frac{1}{3}Nm\langle c^2\rangle$, which we derived by using Newton's laws in conjunction with some simple kinetic theory assumptions, with the ideal gas equation $pV = NkT$, which is derived from experimental observation (see Section 12.3):

$$\tfrac{1}{3}Nm\langle c^2\rangle = NkT \Rightarrow \tfrac{1}{3}m\langle c^2\rangle = kT$$

If we now multiply both sides by $\frac{3}{2}$ we get

$$\tfrac{1}{2}m\langle c^2\rangle = \tfrac{3}{2}kT$$

The expression $\frac{1}{2}m\langle c^2\rangle$ is the **average kinetic energy** of the randomly moving molecules of the gas. As k is a constant, it follows that

> **the average kinetic energy of the molecules of a gas is *proportional to the absolute temperature* of the gas.**

It also follows from the above equation that, in this simple, non-quantum model, the average random kinetic energy of the molecules will be zero at absolute zero, i.e. when $T = 0$.

Tip

For examination purposes, you must be able to derive and use the expression

$$\tfrac{1}{2}m\langle c^2\rangle = \tfrac{3}{2}kT$$

remembering that T must be in kelvin.

Example

Calculate the root mean square speed of the oxygen molecules in air at a temperature of 24°C, given that an oxygen molecule has a mass of 5.34×10^{-26} kg.

Answer

Rearranging $\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$ gives us

$$\langle c^2\rangle = \frac{3kT}{m} \Rightarrow \sqrt{\langle c^2\rangle} = \sqrt{\frac{3\times(1.38\times 10^{-23}\,\text{J}\,\text{K}^{-1})\times 297\,\text{K}}{5.34\times 10^{-26}\,\text{kg}}} = 480\,\text{m}\,\text{s}^{-1}$$

Tip

Note that we have used the terms 'root mean square *speed*' and 'average *speed*' for the molecules. The average *velocity* of the molecules would be *zero* because the total volume of air in the laboratory is stationary.

It is worth noting that the 'root mean square speed' – the square root of the mean square speed – is *approximately* equal to the average speed of the molecules. In Year 1 we saw that the speed of sound in air is about $340\,\mathrm{m\,s^{-1}}$. This is about 25% less than the average random speed (about $480\,\mathrm{m\,s^{-1}}$) of the air molecules that carry the sound energy, which would appear to be very reasonable, thereby justifying the assumptions that we have made regarding kinetic theory.

12.6 Historical context

Our study of thermodynamics would not really be complete without a brief look at the historical context in which thermodynamics was developed. The impetus was provided by the Industrial Revolution, which took place from the mid-18th to the mid-19th century. The steam engine allowed improvements in mining coal and in the production of iron, evocatively portrayed in the painting 'Coalbrookdale by Night'. The use of iron and steel in machinery led to major changes in agriculture, manufacturing and transport. This all had a profound effect on the socio-economic and cultural conditions in Britain and around the world, in the same way that electronics and the computer have revolutionised our lives in more recent times.

Figure 12.12 *Coalbrookdale by Night*, painted in 1801 by PJ De Loutherbourg

Figure 12.13 James Watt's steam engine, 1785

Following James Watt's refinements to the design of the steam engine towards the end of the 18th century, engineers sought to improve its efficiency further by the application of the laws of physics. Thus began the intensive study of the behaviour of gases and the development of kinetic theory, thermodynamics and eventually quantum mechanics, which forms the foundation for most theories in physics today.

As we saw in Book 1, the development of quantum mechanics led to wide-ranging experiments on solid materials ('solid state physics') in the 1920s and 1930s. Following the Second World War (when the attention of physicists was diverted elsewhere) Bardeen, Brattain and Shockley invented the transistor in 1947. They were jointly awarded the Nobel Prize in Physics in 1956 '*for their researches on semiconductors and their discovery of the transistor effect*'. Their work on semiconductors led to the development of silicon chips, without which we would not have today's computers, or your iPod!

Test yourself

9 For an ideal gas:

$\frac{1}{2}m<c^2> = \frac{3}{2}kT$

a) Define what is meant by an *ideal gas*.

b) List four assumptions of the kinetic theory for an ideal gas that have to be made in deriving the above equation.

c) Explain what each of the quantities m, $<c^2>$, k and T represents.

d) What does the term $\frac{1}{2}m<c^2>$ represent?

10 The equation of state for an ideal gas is $pV = NkT$.

a) What do the symbols N and k represent in this equation?

b) Show that the product pV has the same units as energy.

c) Hence show that the units of k are $\mathrm{J\,K^{-1}}$.

d) How many molecules of air would there be in a laboratory measuring 10.0 m × 8.0 m × 3.0 m on a day when the temperature is 19°C and the pressure is 101 kPa?

Exam practice questions

1 a) Which of the graphs in Figure 12.14 does *not* represent the behaviour of a gas that obeys Boyle's law? **[1]**

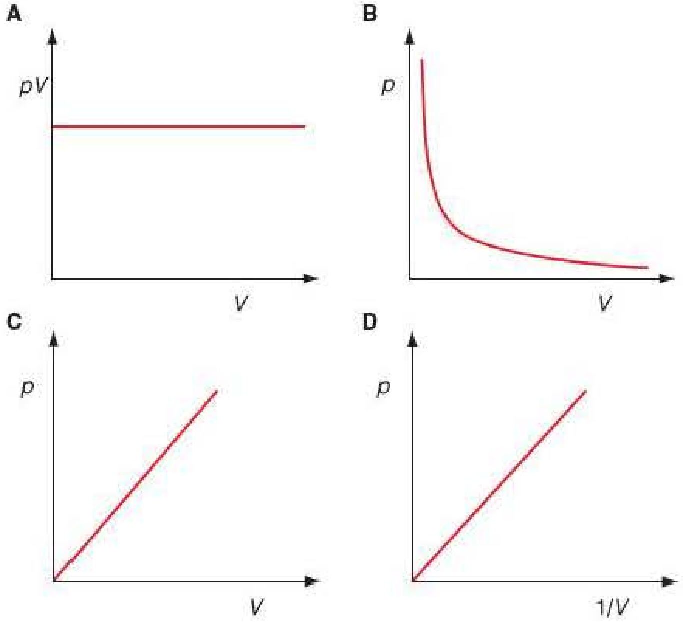


Figure 12.14

b) Which of the graphs in Figure 12.15 does *not* represent the behaviour of an ideal gas? **[1]**

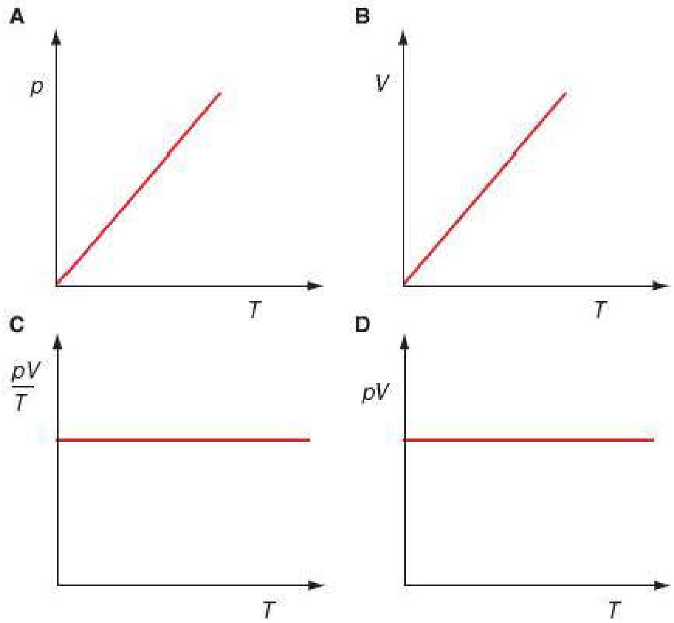


Figure 12.15

[Total 2 marks]

2 The pressure exerted on the ground by

a) the heel of a shoe having an area of $0.80\,cm^2$ when a girl of mass 57 kg puts all her weight on it is:

A 0.07 MPa **C** 7.0 MPa

B 0.70 MPa **D** 70 MPa. [1]

b) the wheel of a car of mass 1600 kg, assuming that each tyre has an area of 5 6 cm^2 in contact with the road and that the weight of the car is evenly distributed, is:

A 0.07 MPa **C** 7.0 MPa

B 0.70 MPa **D** 70 MPa. [1]

[Total 2 marks]

3 In an experiment to investigate how the pressure of air at constant volume depends on the temperature of the air, a flask contains air at a pressure of 101 kPa and a temperature of 17°C. When the air is heated to a temperature of 100°C, the pressure will become about:

A 17 kPa **C** 130 kPa

B 79 kPa **D** 594 kPa. **[Total 1 mark]**

4 a) Show that the equation for the increase of pressure in a liquid, $\Delta p = \rho g \Delta h$, is homogeneous with respect to units. [2]

b) Calculate the pressure difference between offices at the top and bottom of the Canary Wharf building in London, which is 240 m high. Assume that the density of air is $1.29\,kg\,m^{-3}$. What other assumption do you have to make? [4]

c) Discuss whether office workers would notice such a pressure difference on a day when atmospheric pressure was 101 kPa. [2]

d) Explain why a calculation like you performed in part **b)** could not be used to calculate the pressure experienced by an aeroplane flying at a height of 10 000 m. [2]

[Total 10 marks]

5 The laws of Association Football state that the pressure of the ball must be 'between 0.6 and 1.1 atmospheres (600–1100 g/cm^2) above atmospheric pressure at sea level'.

a) Show that the above data gives a value for atmospheric pressure of approximately 100 k Pa. [3]

b) A football is pumped up to a pressure of 0.75 atmospheres above atmospheric pressure in the warm dressing room, where the temperature is 21°C. It is then taken out onto the pitch, where the temperature is only 7°C. Determine whether or not the pressure will still be within the legal limit. [3]

c) State what assumption you have made in arriving at your answer. [1]

[Total 7 marks]

6 A 60W light bulb contains $110\,cm^3$ of argon at a pressure of 87 kPa when it is at a room temperature of 17°C. When the lamp has been switched on for some time, the temperature of the argon becomes 77°C.

a) Calculate the pressure of the argon when the lamp is on. **[2]**

b) Calculate the number of molecules of argon in the bulb, assuming that $k = 1.38 \times 10^{-23}\,J\,K^{-1}$. **[3]**

c) If 6.0×10^{23} molecules of argon have a mass of 40 g, what is the mass of argon in the bulb? **[2]**

[Total 7 marks]

7 The speed of sound in helium at 20°C is about $1000\,m\,s^{-1}$, compared with $340\,m\,s^{-1}$ in air. The high speed of sound in helium is responsible for the amusing 'Donald Duck' voice that occurs when someone has breathed in helium from a balloon.

a) Show that the equation $\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$ is homogeneous with respect to units. **[3]**

b) Explain what is meant by the term $\langle c^2\rangle$. **[2]**

c) What is the mean square speed of five air molecules that have speeds of 300, 400, 500, 600 and $700\,m\,s^{-1}$? **[3]**

d) Calculate the mean square speed for i) air molecules and ii) helium molecules at a temperature of 20°C. Assume that an 'air molecule' has a mass of $4.8 \times 10^{-26}\,kg$ and a helium molecule has a mass of $6.7 \times 10^{-27}\,kg$. **[4]**

e) Comment on your answer in relation to the speed of sound in helium compared with that in air. **[3]**

[Total 15 marks]

8 The graph shows a p–V curve for a gas at a temperature of 300 K. Using the plotted points A, B, C as a guide, sketch the graph on a sheet of graph paper.

a) State Boyle's law. **[2]**

b) By considering the points A, B and C, show that the gas obeys Boyle's law. **[2]**

c) The temperature is now increased to 400 K. What will now be:

i) the pressure for a volume of $8.0 \times 10^{-3}\,m^3$

ii) the volume at a pressure of 400 kPa

iii) the volume at a pressure of 640 kPa? **[6]**

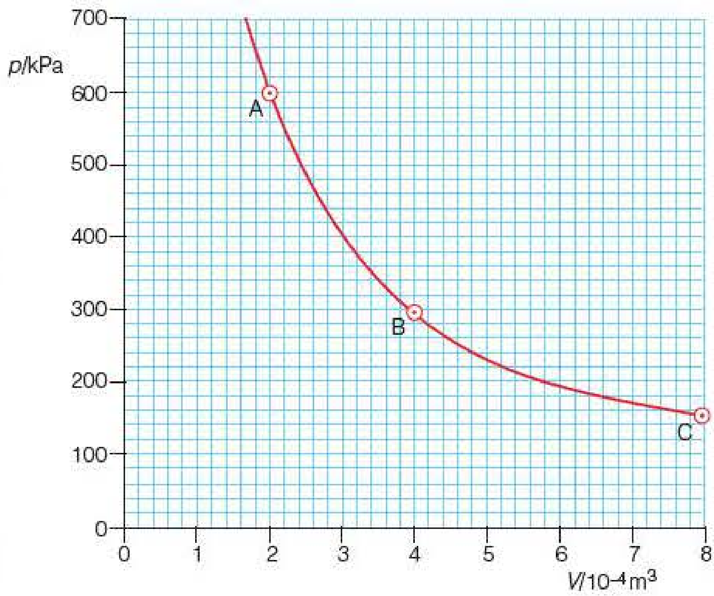


Figure 12.16

d) Use your answers to sketch the p–V curve for 400 K on the same axes as your first sketch. **[4]**

[Total 14 marks]

9 A laboratory measures 10.0 m by 10.0 m and is 3.0 m high.

a) What is the volume of air in the laboratory? [1]

b) When the air pressure p is 102 kPa and the temperature is 18°C, show that there will be about 8×10^{27} air molecules in the laboratory. [3]

c) If the mass of an air molecule is 29 u, what is the mass of air in the laboratory? [2]

d) Use your answers to **a)** and **c)** to calculate the density ρ of the air. [2]

e) Calculate the mean kinetic energy of each of these molecules. [2]

f) Hence show that the root mean square speed of these molecules is about 5 00 $\mathrm{m\,s^{-1}}$. [3]

g) The speed, v, of sound in air is given by the formula

$$v = \sqrt{\frac{1.40p}{\rho}}$$

Calculate what the speed of sound will be in the laboratory. [2]

h) Comment on your answers to **f)** and **g)**. [2]

(Total 17 marks)

Stretch and challenge

10 This question is an extension of Question 9.

a) To a good approximation, the average speed of the molecules can be taken as the root mean square speed (500 $\mathrm{m\,s^{-1}}$). How long will it take for an air molecule to travel from one side of the laboratory to the other? [1]

b) If the molecule makes elastic collisions with the walls, how many times will it cross the laboratory in 1 second? [1]

c) If we consider that, on average, one-third of the molecules (as calculated in Question 9b) are travelling back and forth in each of the horizontal directions parallel to the walls, and one-third are travelling vertically up and down, how many collisions will the molecules make on any one of the walls in 1 second? [2]

d) Calculate the change in momentum of a molecule when it makes an elastic collision with a wall. [3]

e) Determine the force exerted by the molecules on the wall and hence the pressure in the laboratory. [4]

f) List the assumptions that have been made in your calculation of this pressure and comment on their validity in the light of your answer. [4]

[Total 15 marks]

13 Astrophysics

Prior knowledge

You should be familiar with the Prior Knowledge listed at the beginning of Chapter 3 on universal gravitation.

The further key facts you should know or be able to apply are:

- that e-m radiation travels at $3.0 \times 10^8\,\mathrm{m\,s^{-1}}$ in a vacuum
- the link between degrees and radians (360° is equal to 2π radians)
- that 1 arcsec (one arc second) = $(1/60)^2$ of a degree of arc
- that ⊙ is a symbol that represents the Sun, often used as a suffix $_\odot$
- Einstein's equation $\Delta E = c^2 \Delta m$

Test yourself on prior knowledge

1 How far does light travel in one minute?

2 What, in radians, is equivalent to an angle of 90°?

3 Make λ the subject of the equation $\lambda T = k$.

4 What is the force that attracts the Moon to the Earth?

5 Express the unit watt (W) in base SI units.

6 Explain the symbols in Einstein's equation.

13.1 How far to the stars?

'Astronomy' has been practised since very ancient times. Observations of the night sky were detailed and, since the early 17th century, increasingly powerful telescopes have been used. Observations and measurements were analysed and a mathematical picture of the 'Heavens' was developed. Physics has added a new dimension to this knowledge, and in this chapter you will learn about the life and death of stars and of our appreciation of the size of our universe as well as what we do *not* know about 95% of the mass of the universe! .

'Astrophysics' is now a part of most university physics courses and a major field of research.

Figure 13.1 The Milky Way

Many of us who live in towns or cities rarely see the night sky as our ancestors did. Figure 13.1 shows the Milky Way – the billions of stars in the middle of our own galaxy. Have you ever seen the Milky Way like this? Our own star – the Sun – is near the edge of the galaxy, so when we are away from city lights on a clear night and look towards the centre of the galaxy we have that amazing view. The whole galaxy is a swirling disc of several hundred billion ($>10^{11}$) stars.

A similar spiral galaxy is called M51 or the 'Whirlpool'. M51 is roughly 80000 light years across and has a total mass about 150 billion times larger than the mass of the Sun!

The **light year** is a very, very large distance. Professional astronomers use other units such as 'parsecs' or 'astronomical units' for distance, but here we will only use metres and light years.

1 light year = $(3.00 \times 10^8 \mathrm{m s^{-1}})(365 \times 24 \times 3600 \mathrm{s}) = 9.5 \times 10^{15} \mathrm{m}$

Key term

A **light year** is the distance travelled by light in one year: 1 light year = 9.5×10^{15} m or about 10^{16} m.

Trigonometric parallax

'Nearby' stars – by near, we mean up to about 300 light years away – can be seen from ground-based observations to exhibit an annual 'wobble' relative to the 'fixed' distant stars in the background. From satellite observations, for example from the Hubble space telescope, the wobble of more distant stars can be detected. This wobble follows an annual pattern, and is the result of the Earth's movement round the Sun. Figure 13.2 illustrates the phenomenon that is known as **stellar** or **trigonometric parallax**.

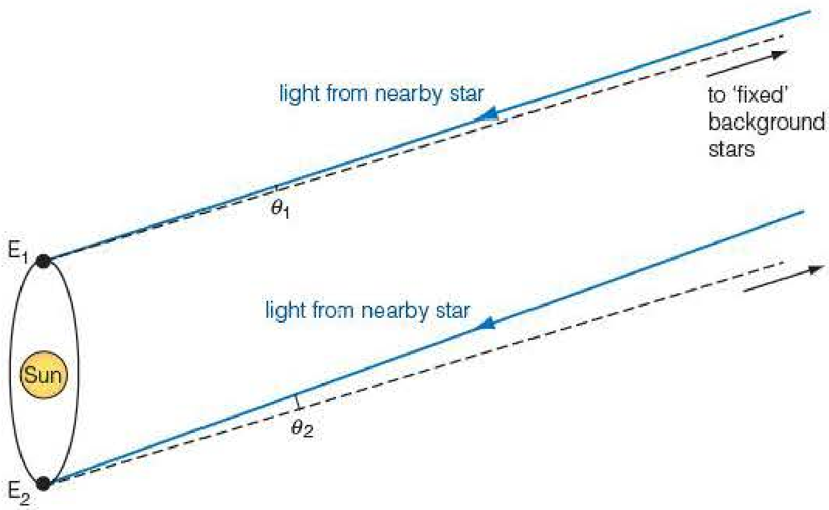


Figure 13.2 Trigonometric parallax

E_1 and E_2 are the positions of the Earth six months apart. The angles θ_1 and θ_2 to the nearby star measured from E_1 and E_2 are different, and the size of $\theta_2 - \theta_1 = \Delta\theta$ measures the size of its 'wobble'. A knowledge of the diameter of the Earth's orbit round the Sun (E_1 to E_2) plus some trigonometry enables us to calculate the distance from the Sun to the nearby star. Using different positions on the Earth's orbit allows the parallax angles to all the nearby stars to be measured. However, the distance to the *nearest* star is over 100000 times the diameter of the Earth's orbit (E_1E_2), so $\Delta\theta$ is tiny. It can be measured to a few millionths of a degree, or 0.01 second of arc. There are only 50 stars for which $\Delta\theta > 0.25$ seconds of arc, so ground-based measurements of parallax only work for a few thousand stars, but measurements from satellites such as Hubble and Hipparcos, that measure from above the atmosphere, have greatly increased this number.

Tip

There are 360° in a circle.

$$1 \text{ minute of arc} = \frac{1}{60^\circ}$$

$$1 \text{ second of arc} = \frac{1}{3600^\circ}$$

A second of arc is also called an arcsec. One arcsec is a very small angle – about the angle between two car headlights seen (if you could) from a distance of 200 km!

Example

Look out of the window and identify something like a chimney or a lamp post that is at least 50 m away. Extend one arm and hold your thumb so that your thumb is between your eyes and the chimney. Now close first one eye and then the other.

a) Describe what you see.

b) Sketch a diagram to explain what you see, giving lengths and angles where possible.

Answer

a) As one eye is closed and then the other one is closed, the chimney swaps over from lying on the left of the thumb when seen by the right eye, to lying on the right of the thumb when seen by the left eye. For some people the movement may be much more in one direction than the other, as one eye may be more dominant.

b) For example:

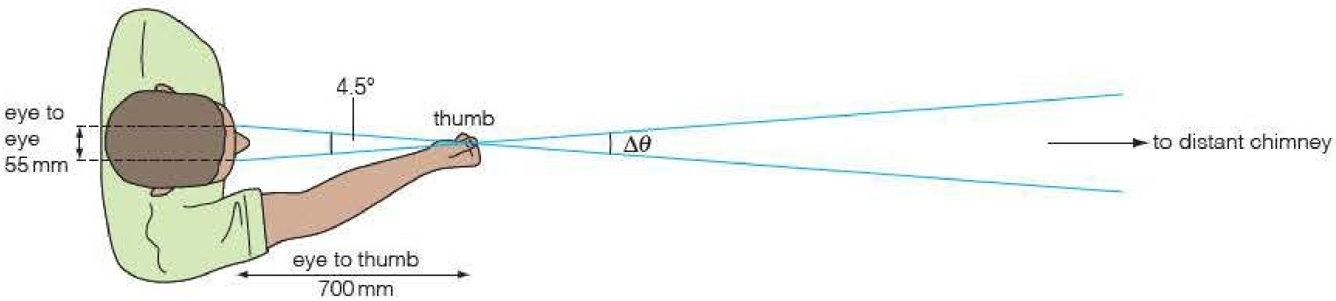


Figure 13.3

Using the lengths in the diagram, the parallax angle can be calculated as

$$\Delta\theta \approx \frac{55\,\text{mm}}{700\,\text{mm}} = 0.079\,\text{rad} = 4.5°$$

or, as $\tan\frac{1}{2}\Delta\theta = \frac{27.5\,\text{mm}}{700\,\text{mm}} \Rightarrow \Delta\theta = 4.5°$

13.2 Luminosity and flux

Luminosity, symbol L, is the word astrophysicists use to describe the **total output power** of a star, unit W.

For example, the luminosity of the Sun is $L_{\odot} = 3.90 \times 10^{26}$ W – an incredible power output of 3.90×10^{26} joules every second. All we know about stars and our universe involves huge numbers.

Example

The Sun's output power of $3.90 \times 10^{26}\,\mathrm{J\,s^{-1}}$ is the result of mass–energy transfer within the Sun. Calculate the rate at which the Sun is transforming matter into electromagnetic wave energy.

Answer

Using $\Delta E = c^2 \Delta m$ tells us that the mass loss needed to produce 3.90×10^{26} J of energy is

$$\Delta m = \frac{\Delta E}{c^2} \times \frac{3.90 \times 10^{26}\,\mathrm{J}}{(3.00 \times 10^8\,\mathrm{m\,s^{-1}})^2} = 4.33 \times 10^9\,\mathrm{kg}$$

which is over a million tonnes of matter – and this happens *each second*. The mechanism for this power production is described in Section 14.1.

The electromagnetic wave energy per second per unit area from a star reaching us on Earth is called the **radiation flux** from the star, symbol F, unit $\mathrm{W\,m^{-2}}$ (i.e. $\mathrm{J\,s^{-1}\,m^{-2}}$).

In non-astronomical situations radiation flux is usually referred to as the intensity of the light. You have probably met light intensity, perhaps in considering a surface illuminated by a 100 W light bulb, in your earlier physics course.

The radiation flux received from the Sun at the Earth's surface is about $1000\,\mathrm{W\,m^{-2}}$, depending on the state of the atmosphere and the cloud cover. Above the atmosphere the radiation flux from the Sun, $F_{\odot}$ is $1350\,\mathrm{W\,m^{-2}}$. For more distant stars the flux is, of course, very, very much smaller. Most importantly, F (or I) and L are linked by the **inverse-square law**:

$$F = \frac{L}{4\pi d^2} \text{ or } I = \frac{L}{4\pi d^2}$$

where d is the distance from Earth to the star.

Tip

The flux, or intensity, may be given the symbol I in your examination papers. You should therefore become familiar with both terms and symbols.

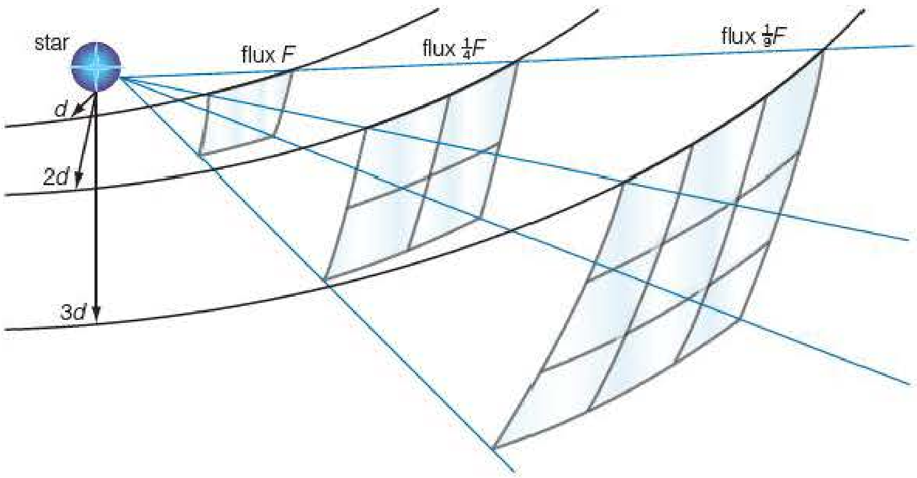


Figure 13.4 Radiation flux and distance: the inverse-square law

Referring to Figure 13.4:

- the radiation flux at $2d$ is one-quarter $(\frac{1}{2})^2$ that at d
- the radiation flux at $3d$ is one-ninth $(\frac{1}{3})^2$ that at d, etc.

When the radiation flux L of a given star is known and F is measured on Earth, the distance from Earth to the star can be determined, as

$$4\pi d^2 = \frac{L}{F} = \frac{\text{luminosity}}{\text{radiation flux}}$$

Example

Use the values of $L_\odot$ and $F_\odot$ given above to confirm that the Sun is 8.3 light minutes from the Earth.

Answer

$$4\pi d^2 = \frac{L_\odot}{F_\odot} = \frac{3.90 \times 10^{26}\,\text{W}}{1350\,\text{W m}^{-2}}$$

$$\Rightarrow d = 1.52 \times 10^{11}\,\text{m}$$

$$8.3 \text{ light minutes} = (3.0 \times 10^8\,\text{m s}^{-1})(8.3 \times 60\,\text{s})$$

$$= 1.49 \times 10^{11}\,\text{m}$$

Hence, to 2 SF, the values are consistent.

Test yourself

1 Describe, in your own words, what is meant by a galaxy.

2 'Nearby' stars are said to be within 300 light years of Earth. What is 300 light years in km?

3 What is meant by the 'fixed background stars' in Figure 13.2?

4 What is meant by $L_\odot$ in the statement $L_\odot \approx 4 \times 10^{26}\,\text{W}$?

5 Distinguish between *luminosity* and *radiation flux*.

6 How could you check that the radiation flux, F, from a bright circular lamp is inversely proportional to the distance r from the centre of the lamp?

13.3 Standard candles

The problem in using $4\pi d^2 = \frac{L}{F}$ to measure how far it is to a star (that is too far away to exhibit parallax) is how to determine the star's full power output – its luminosity L.

In the early 20th century an American astrophysicist, Henrietta Leavitt, working at Harvard, discovered that a type of star now called a **Cepheid** has a luminosity that varies with time. Such stars appear more bright and less bright with periods of the order of days. Further, she was able to establish that the maximum luminosity L of a Cepheid star was related to the period T of its luminosity variation. For the first time it was thus possible, by measuring

T for a Cepheid star that is too far away to show any parallax wobble, to know the star's luminosity. Such stars are valuable **standard candles**, which means that we can determine their absolute luminosity. The pole star Polaris is a Cepheid with a period of about 4 days.

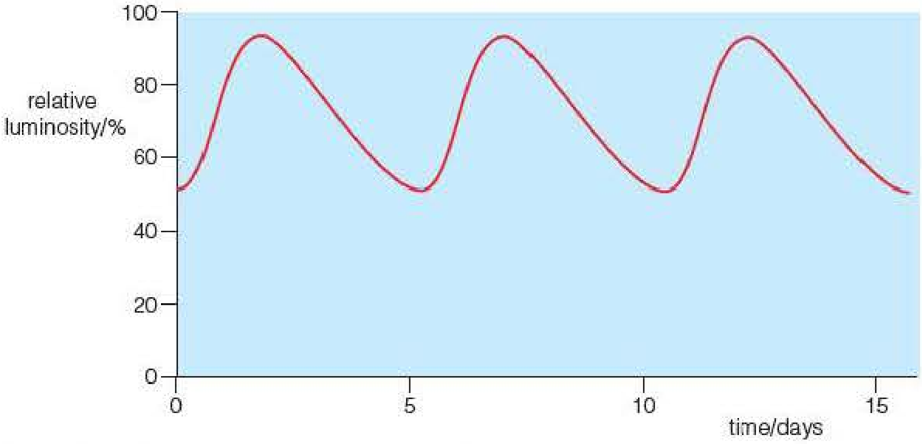


Figure 13.5 Typical luminosity variation for a Cepheid star

> **Key term**
>
> In astrophysics, a **standard candle** is a distant star of known (maximum) luminosity.

The process to find the distance *d* from Earth to a star is therefore:

- locate a Cepheid variable star
- measure its period *T* (may be up to tens of days)
- find the star's luminosity *L* using Leavitt's *T*–*L* data
- measure the radiation flux *F* from the star at Earth
- calculate *d* using $d = \sqrt{(L/4\pi F)}$.

This last measurement is not easy, but is what astronomers have spent hundreds of years perfecting using telescopes that track a chosen star at night as the Earth rotates.

Using this method the true scale of our galaxy became known: the Milky Way is about 150 thousand light years across. Furthermore, as Cepheid stars can be observed in galaxies beyond ours, this led to the earliest indication that our universe was huge. The distance to Andromeda, our nearest galaxy, is about 2.5 million light years. And there are millions of galaxies much, much further away. **Supernova** explosions (type Ia) can be used as standard candles to find the distance to these more distant galaxies, as we believe that the maximum luminosity of these exploding stars is the same all over the universe.

On 1 February 2014, as this book was being written, *The Times* reported in a headline: 'SUPERNOVA IS CLOSEST FOR 150 YEARS' and went on to add, 'Supernovae are discovered frequently, but this one is special. It belongs to a class known as type Ia supernovae … used to measure the rate of expansion of the universe.'

Figure 13.6 summarises the methods of measuring distance in the universe (read downward). (See Section 14.5 for the red shift method.) For all this to work reliably it is important that there are stars that overlap the methods, for example a Cepheid that shows a parallax wobble, and a Ia supernova close to a Cepheid.

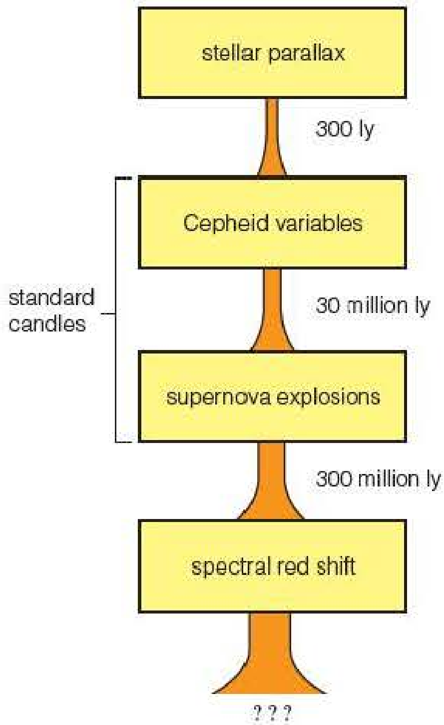


Figure 13.6 Measuring our universe

The 5% universe

At the time of writing, attempts to measure distances in our universe has unearthed (an unfortunate pun) two problems:

1 Nearby galaxies do not have enough visible gravitationally attracting (hadronic) matter to keep the outermost stars moving in circles around the galaxy's centre.

2 Distant galaxies appear to be moving away from the Solar system at an accelerating rate, rather than slowing down as predicted by Newton's (attractive) law of gravitation.

The first 'problem' suggests that about 24% of the mass of nearby galaxies consists of … we do not know what. It is called **dark matter**. The second 'problem' suggests that about 71% of our universe consists of a mysterious 'antigravity' material known as **dark energy**. As 24% plus 71% add up to 95%, this means that, at the moment, *we know nothing about 95% of our universe*. A further issue that is concerning astrophysicists and cosmologists is that, possibly, our universe is only one of many **multiverses**, but any knowledge of other universes is denied to us.

Whatever dark matter consists of, it exerts a gravitational pull on ordinary matter (hadronic matter like you – see page 134). Because of this, the total mass of gravitationally attracting matter within the orbit of a star as it moves around the edge of a galaxy can be calculated. There are a number of suggestions about what dark matter might consist of, but so far no scientific theory to enable such suggestions to be tested. The position about dark energy is even more mysterious: theories relating to what happened in the very early universe and to Einstein's geometrical theory of gravitation – see Section 14.5 – are proposed, but as yet the acceleration remains unexplained.

Tip

Science develops by making theoretical predictions and then testing them. If they 'survive' the test, there is no change to the current state of knowledge, but if they 'fail' the test then a new theory is required and science moves forward.

13.4 The Hertzsprung–Russell diagram

The **Hertzsprung–Russell** or **H–R diagram** is a plot of stellar luminosity against surface temperature. The diagram in Figure 13.7 is a simplified version which indicates clearly the regions in which 'main sequence' stars, white dwarf stars and red giant stars are located. Note it is a diagram not a graph – each dot represents a single star. To understand the diagram you need to remember what the two axes are telling you.

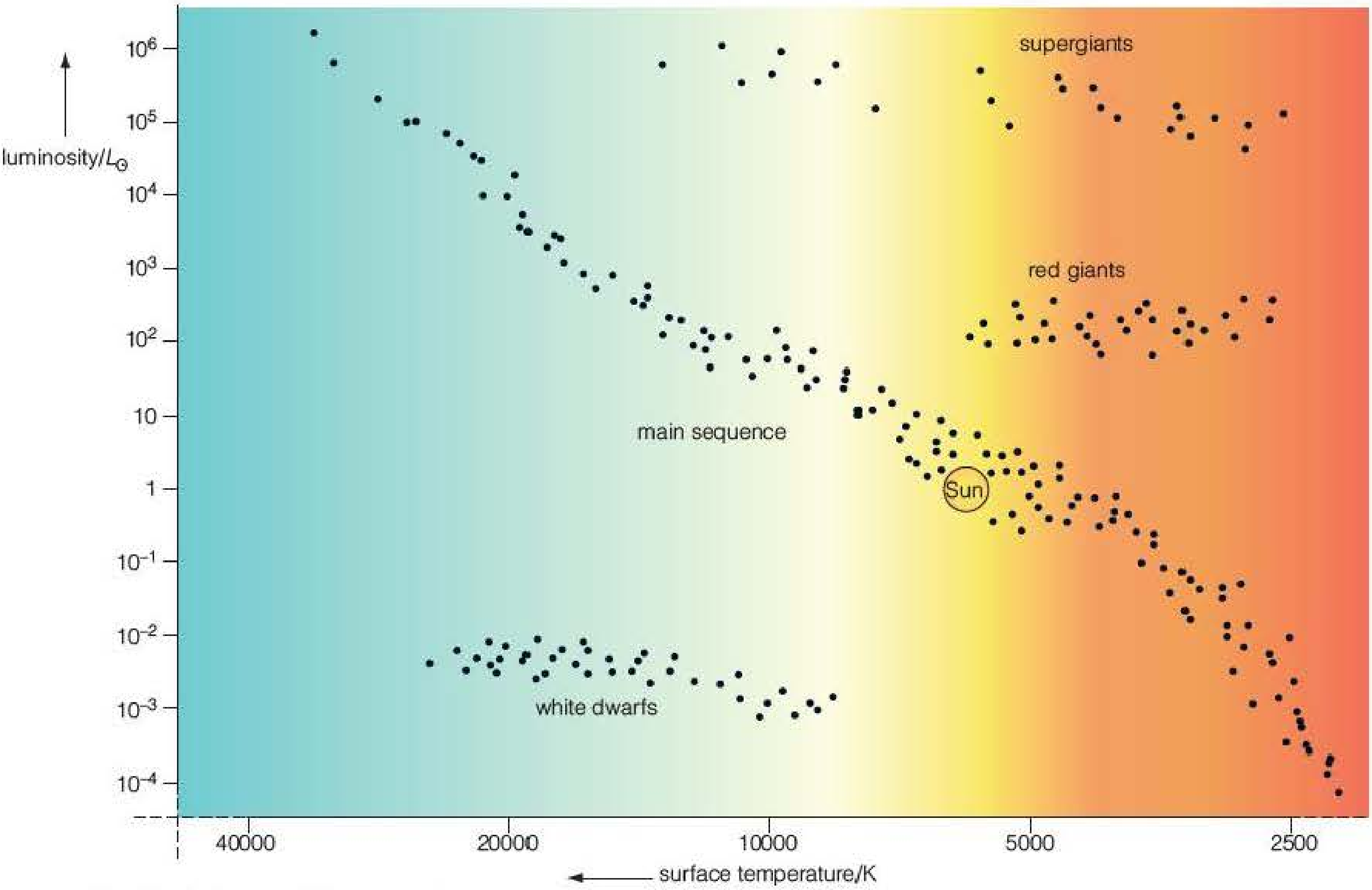


Figure 13.7 The Hertzsprung–Russell diagram

The vertical axis is luminosity, scaled in 'Sun-powers', i.e. multiples of $L_\odot$ (no units). The positions of the stars go up and down from 1 (where the Sun sits) in powers of 10, i.e. the scale is logarithmic. The Sun's luminosity $L_\odot = 3.90 \times 10^{26}$ W.

The horizontal axis is the surface temperature T of the star in kelvin. This scale is also logarithmic and (an unfortunate historical blip) goes from high temperatures on the left to low temperatures on the right. The colours, blue-white-yellow-brown, give you an indication of what a star in our milky way will look like. The Sun has a surface temperature of 5800 K and looks yellowish.

Tip

Copying a diagram such as this will help you to remember the detail it holds.

Tip

In the next Example 'estimate' means that an exact answer cannot be given.

Example

a) Use the H–R diagram in Figure 13.7 to estimate:

i) the luminosity of a main sequence star with a surface temperature of 5000 K

ii) the surface temperature of a main sequence star of luminosity $L_\odot/1000$.

b) Explain why 'main sequence' stars were specified in a).

Answer

a) i) Just less than $L_\odot$, i.e. about 3.5×10^{26} W.

ii) $L_\odot/1000 = L_\odot \times 10^{-3}$, so T is about 2500 K.

b) There are red giants and supergiants with temperatures of 5000 K and there are white dwarfs with luminosities of $L_\odot/10^3$. But neither of these stars lies on the main sequence.

The life cycle of a star

Tip

Remember that an individual star does *not* progress *along* the main sequence band.

The H–R diagram is a snapshot of stars *in our galaxy* at this moment; the vast majority of stars that are now visible lie on the **main sequence**. Once a star is formed (see Section 14.1) it adopts its position on the main sequence and spends most of its life with a fairly constant surface temperature and luminosity (although it does get a little bit brighter during this very long time). More massive stars will have shorter stays on the main sequence, and smaller stars longer stays.

Astrophysicists can predict different stages in the life of an 'average' star like our Sun. They test their predictions by observing the properties of stars of different mass and age that exist in clusters. (Stars in clusters are all the same distance from Earth.) The **stages for our Sun** are believed to be as follows (see also the lower part of Figure 13.10, where these stages are shown diagrammatically):

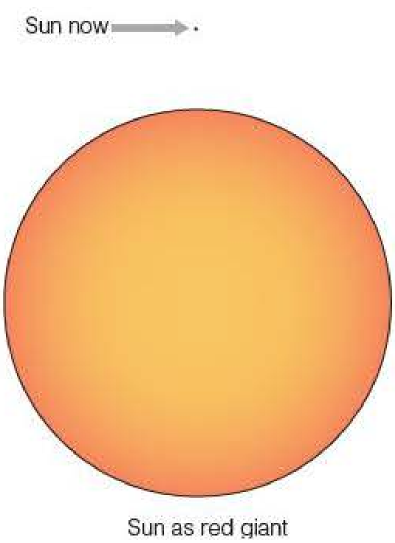


Figure 13.8 Relative sizes of an average main sequence star and a red giant

Stage 1: The Sun was formed or 'born' from a cloud of hydrogen and helium (see Section 14.1).

Stage 2: Our Sun joined the main sequence about 5 billion years ago and will leave the main sequence about 5 billion years from now.

Stage 3: After leaving the main sequence our Sun will expand and become a red giant (see Figure 13.8) and, after losing about 50% of its mass, it will then become a white dwarf.

Stage 4: It will then slowly cool for billions years and effectively 'die'.

Stage 1 takes about 10 million years. The predicted lifetime of our Sun on the main sequence – stage 2 – throughout which time it pours energy to the Earth at more than $1\,\mathrm{kW\,m^{-2}}$, is some 10 billion years! The processes in stage 3 will take place over a billion or so years.

For stars on the main sequence that have a mass greater than $m_{\odot}$ ($m_{\odot}$ is the mass of the Sun), there are two possible fates.

- Stars between $1.4m_{\odot}$ and $3.0m_{\odot}$ end up as spinning **neutron stars** or 'pulsars' – stars that were discovered in 1967 by Jocelyn Bell, shown in Figure 13.9, when she was a graduate student at Cambridge.
- Stars heavier than $3.0m_{\odot}$ finish up as black holes.

Figure 13.9 Jocelyn Bell Burnell

Figure 13.10 illustrates (top) the lives of massive stars, and (bottom) stars like the Sun. Note that the time spent on the main sequence is not drawn to scale.

After stars leave the main sequence, nuclear reactions in their cores produce new elements, with atomic masses up to carbon (12). Further **nucleosynthesis** of heavier elements (up to iron) takes place in the cores of stars of mass $1.4\,m_{\odot}$ to $3.0\,m_{\odot}$. Elements beyond iron up to uranium are produced in supernova explosions.

All these elements are spread through the universe, so we can honestly say that we are all made of star dust!

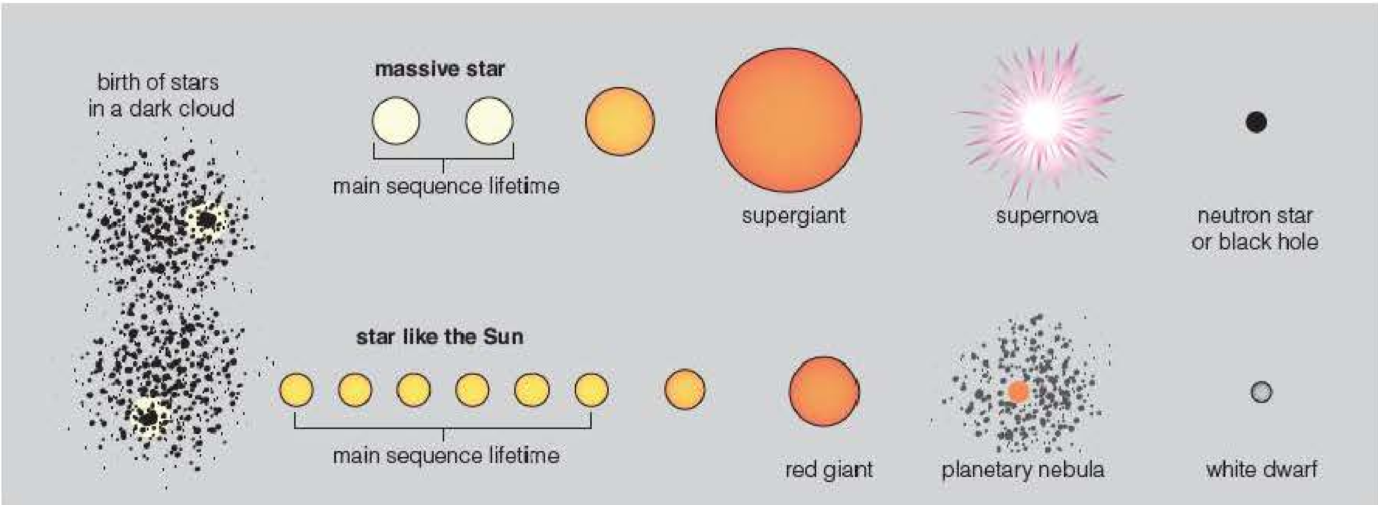


Figure 13.10 The lives of stars

Test yourself

7 Name two female astrophysicists who made significant contributions to our knowledge of the universe during the 20th century.

8 What type of star is Polaris, the pole star?

9 The nearest galaxy, Andromeda, is about 2.5 million light years away. How far is this in km?

10 Draw a sketch, showing the main sequence stars, on the axes used in the H–R diagram.

Indicate the position of the Sun, but do not attempt to show individual stars. You should, however, label and put rough values on the axes.

11 Use the H–R diagram in Figure 13.7 to give a rough value for

a) the average luminosity of red giant stars

b) the temperature of the brightest main sequence star.

12 What sort of stars, and by what process do these stars, produce elements beyond iron and up to uranium?

13 State why, at the time of writing (2014), we sometimes refer to 'The 5% universe'.

Tip

Even if not prompted in the question,' sketches' such as that of an H-R diagram should have the axes labelled with the appropriate units and, where possible, numerical scales.

13.5 Light from the stars

The colouring of the H–R diagram in Figure 13.7 suggests that very hot stars are blue and cool stars are red. The Sun is a yellow star and those a bit hotter than the Sun look white. There is, however, a quantitative link between the peak wavelength λ_m (lambda maximum) in the radiation spectrum emitted by a star and its surface temperature T. This is **Wien's law**:

$$\lambda_m T = 2.90 \times 10^{-3}\,\mathrm{m\,K}$$

To 4 SF the constant is $2.898 \times 10^{-3}\,\mathrm{m\,K}$ (beware – the unit is metre-kelvin, *not* millikelvin, mK) but few constants are given to more than 3 SF, and so we usually use $2.90 \times 10^{-3}\,\mathrm{m\,K}$.

Tip

Explain, with an example, is an unusual question, as you must invent some values – here for λ_m.

Tip

Any 'calculation' that involves reading off values from *T* to *L* or from *L* to *T* on an H–R diagram will only give results to 1 SF, as the main sequence is a narrow band of stars and not a line. It is also difficult reading off values from a logarithmic graph.

Wien's law tells us that λ_m and *T* are inversely proportional and that *you*, at about 310 K, therefore emit electromagnetic waves with a peak somewhere in the infrared ('somewhere' because you are not a 'black body radiator' – discussed in a moment).

Example

Explain, with an example, how a knowledge of λ_m for a given star can lead to a value for its luminosity *L*. State any assumption you make.

Answer

If $\lambda_m = 270\,\text{nm}$, then, using Wien's law:

$T = (2.90 \times 10^{-3}\,\text{m K})/(270 \times 10^{-9}\,\text{m}) = 10\,700\,\text{K}$

On a Hertzsprung–Russell diagram, a star with a temperature of 10 700 K will have a luminosity about 100 times that of the Sun, that is about $4 \times 10^{28}\,\text{W}$.

The assumption is that the star lies on the main sequence.

The Stefan–Boltzmann relationship

The electromagnetic spectrum goes from the smaller γ-rays and X-rays to the longer microwaves and radio waves. A star emits a continuous spectrum for which the total power output, its luminosity *L*, is proportional not to its temperature *T*, but to T^4 (*T* to the power four!). Think what this means: as you move from 6000 K to 12 000 K, the luminosity of a star would increase by a factor 2^4, that is 16 times.

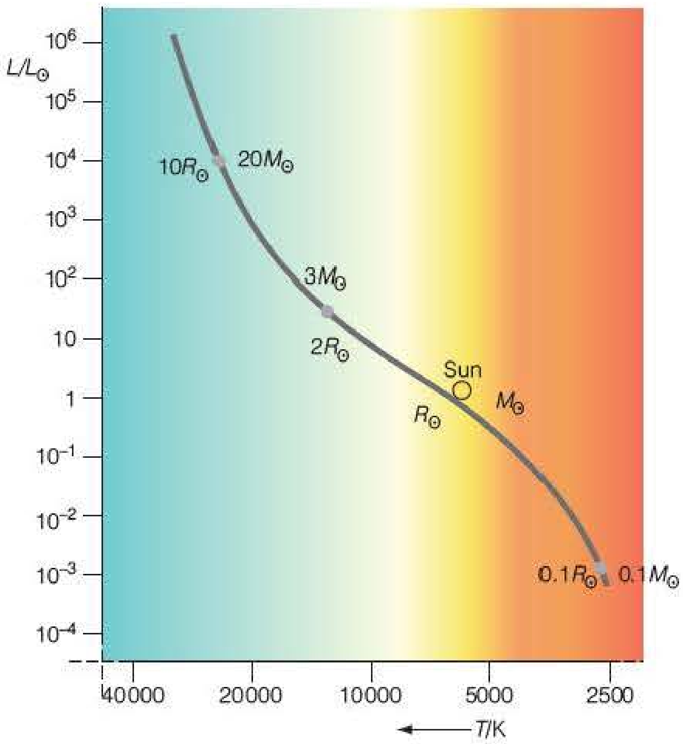


Figure 13.11 H–R diagram showing masses and radii of some main sequence stars

That this does not match up with the *L*–*T* relationship shown for main sequence stars on an H–R diagram is because the high-luminosity stars on the H–R diagram are larger than stars like the Sun, and the Sun is larger than stars at the bottom right of the H–R diagram.

The full relationship is called the **Stefan–Boltzmann law**:

$L = \sigma A T^4$ where A is the surface area

For a sphere $A = 4\pi r^2$, so

$L = 4\pi\sigma r^2 T^4$

The constant, called sigma σ (the Stefan–Boltzmann constant), is $5.67 \times 10^{-8}\,\text{W m}^{-2}\,\text{K}^{-4}$.

We have given the luminosity of the Sun, $3.9 \times 10^{26}\,\text{W}$, but not the radius, so you could work backwards using this expression to find the radius, knowing that the surface temperature of the Sun is 5800 K. You should get $7.0 \times 10^8\,\text{m}$.

The importance of this calculation is that it can be done for *any* star for which both L and T are known. A high-radius star on the main sequence will be a high-mass star. Figure 13.11 illustrates this.

Example

A main sequence star like the Sun becomes a red giant when it moves off the main sequence. Explain why the red giant star is more luminous than the Sun.

Answer

Consider the Stefan–Boltzmann law $L = 4\pi\sigma r^2 T^4$.

The radius of a red giant is much greater than that of the Sun, perhaps 80 times greater, so r^2 increases by a factor of $80^2 = 6400$.

But the red giant is cooler, perhaps half the temperature of the Sun, so T^4 changes by a factor of $(1/2)^4 = 1/16$.

Ignoring the constant $4\pi\sigma$, we are left with $L \propto r^2T^4$. The luminosity L of the red giant is therefore $6400 \times 1/16 = 400$ (between 10^2 and 10^3) times as luminous as the Sun $L_\odot$.

Both Wien's law and the Stefan–Boltzmann law strictly apply only to what are known as **black body radiators**. In fact all stars behave as 'black bodies' – meaning they would completely absorb any electromagnetic radiation falling on them - but this is not really an issue in the physics of stars. The three curves in Figure 13.12 show both proportional relationships:

- the $\lambda_m \propto 1/T$ of Wien's law
- the $L \propto T^4$ of the Stefan–Boltzmann law.

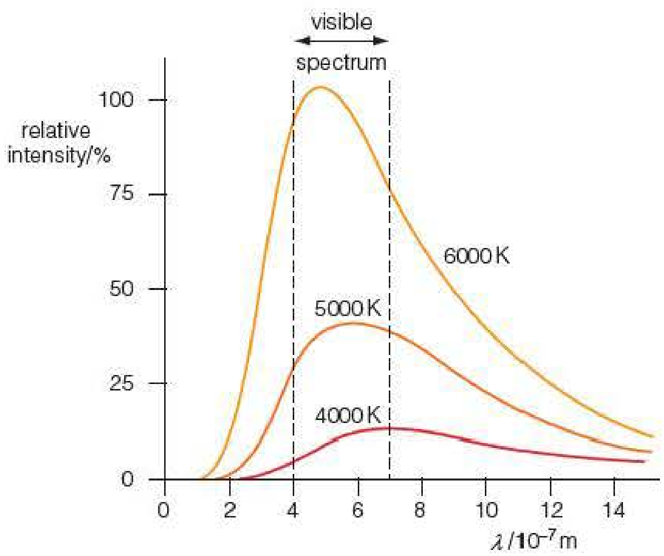


Figure 13.12 Black body spectra at three different temperatures

Tip

Beware of confusing two expressions for luminosity L.

From the inverse-square law you can get $L = 4\pi d^2F$ and from the Stefan–Boltzmann law $L = 4\pi\sigma r^2T^4$.

They 'look' a bit the same, but remember that d is the distance to a star and r is the radius of a star.

The first is quite easy to check. The second says that the total power output is proportional to T^4 and here the total power or luminosity is represented by the area under the curve. As the ratios of 4^4, 5^4 and 6^4 are 256, 625 and 1296, the 6000 K curve should cover about twice the area of the 5000 K curve, and the 5000 K curve should cover an area a bit more than twice the 4000 K curve. They do, more or less!

Stars in the Milky Way

Two measurements and one deduction enable the radius *r* of main sequence stars *and* their distance *d* from us to be determined:

1. Ensure that it is a main sequence star (the spectrum of light from the star tells you that).
2. Measure the peak wavelength λ_m in the star's spectrum.
3. Measure the radiation flux *F* from the star.

Figure 13.13 summarises the route to finding *r* and *d*.

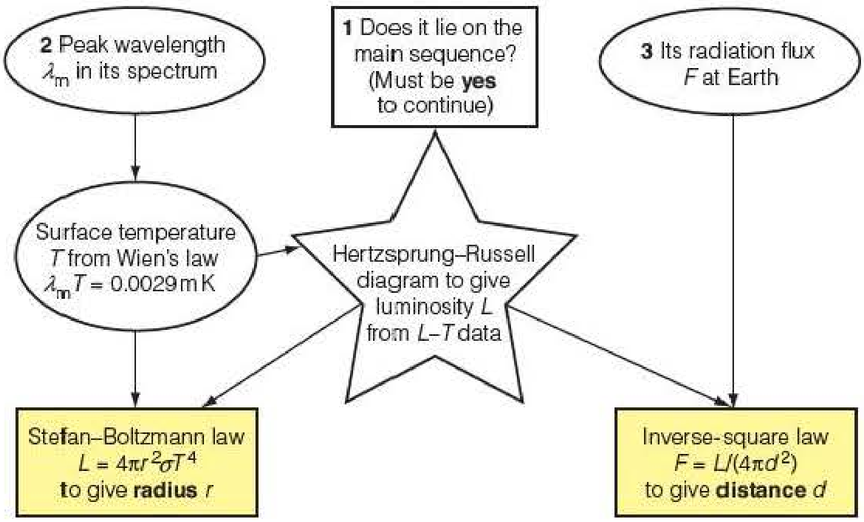


Figure 13.13 How to find the radius *r* and the distance from Earth *d* of a main sequence star

13.6 Why pay to study the stars?

In the UK all tax payers are contributing to research that has produced the details described in this chapter. Are they getting value for money? They also pay for research into the origins of the universe (Chapter 14), another area of what is called 'pure' research. When you repay your student loan, will you be happy for the government to spend some of this money on pure science?

It would be very easy to decide that the money is wasted, but perhaps the answer is to 'wait and see'. Research in astrophysics may surprise us all and turn up some really useful prizes. After all, when the optical laser was first developed, physicists themselves saw it as a solution searching for a problem! Yet all supermarkets now have checkouts using lasers to register your purchases.

Another response is to argue that any civilised society will and should spend money on grand schemes that do not appear to 'pay'. In the Middle

Ages, for example, we built great temples – cathedrals and mosques – for religious purposes. The spirituality behind this and the natural curiosity that drives space exploration and astrophysical research are, it can be argued, what makes us human. You may have found the part of this chapter entitled 'The 5% universe' rather disturbing; the efforts that went into establishing its conclusions were certainly expensive. The discovery of dark energy, incidentally, won the Nobel Prize for Physics in 2011.

There is, of course, no clear and universally agreed answer to the question heading this short section. But it may be worth arguing about.

Test yourself

14 What assumption is being made when the text tells you (on page 251) that '*you*, at about 310 K, emit electromagnetic waves with a peak in the infrared'?

15 The area of the surface of a sphere $A = 4\pi r^2$. Express the Stefan–Boltzmann relation ship using A and hence show that, for a star, $L \propto r^2T^4$.

16 (See the previous question.) Given that our Sun has a surface temperature of 5800 K and a luminosity of 3.9×10^{26} W, calculate a value for the radius of the Sun.

17 Explain what is meant by a 'black body'.

Exam practice questions

1 4.1×10^7 light years is equivalent to:

A 1.3×10^{15} m

B 1.6×10^{22} m

C 3.9×10^{23} m

D 7.8×10^{24} m **[Total 1 mark]**

2 Nearby stars might exhibit trigonometric parallax because:

A the Sun (our star) is moving around the Milky Way

B light from distant stars reaches Earth at different angles in March and September

C light from nearby stars reaches Earth at the same angle in September and March

D the Earth is moving around the Sun (our star). **[Total 1 mark]**

3 A 'standard candle' in astrophysics is:

A a name for the power output or luminosity of a star

B an alternative name for a Cepheid variable star

C an alternative name for a supernova explosion

D a name for a star of predictable luminosity. **[Total 1 mark]**

4 Which of the following graphs shows the relationship between the radiation flux F at a distance d from a star?

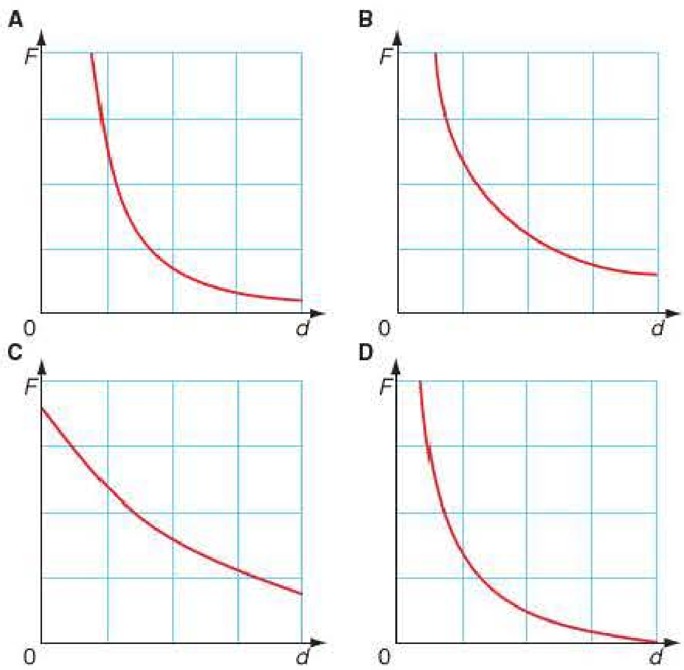


Figure 13.14

[Total 1 mark]

5 Convert 0.01 arcsec (seconds of arc) to degrees and to radians.

[Total 4 marks]

6 The solar radiation flux is reduced by about a factor X between reaching the Earth's atmosphere and reaching the Earth's surface. A value for X to 2 SF might be:

A 0.95

B 0.85

C 0.75

D 0.65. **[Total 1 mark]**

7 Figure 13.15 shows how the distance to a nearby star X can be determined using trigonometric parallax. E_1 and E_2 are the positions of the Earth in its orbit around the Sun S in March and September, i.e. six months apart.

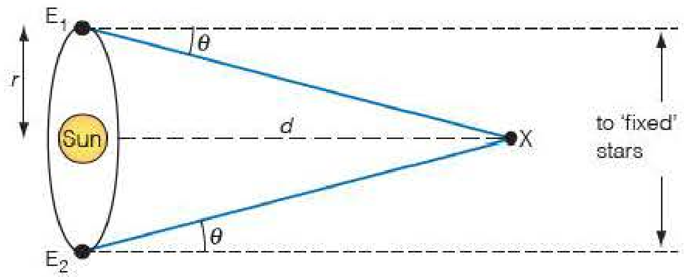


Figure 13.15

a) How are the radius r of the Earth's orbit round the Sun, the measured parallax angle θ and the distance from the Sun to the star d related? **[1]**

b) Taking r to be 1.5×10^{11} m, calculate

i) the distance to the star for which the parallax angle is found to be 4.5×10^{-5} degrees (0.16 seconds of arc),

ii) the parallax angle in degrees and arcsec for a star like X that is 240 light years away. **[5]**

[Total 6 marks]

8 Establish the base SI unit for radiation flux. **[Total 4 marks]**

9 The bright star Rigel has a luminosity of 3.8×10^{31} W. The intensity of radiation from Rigel measured on Earth – its radiation flux – is $5.4 \times 10^{-8}\,\mathrm{W\,m^{-2}}$. Calculate the distance from Earth to Rigel.

[Total 4 marks]

10 The graph shows how the period of what are called type II Cepheid stars varies with their luminosity.

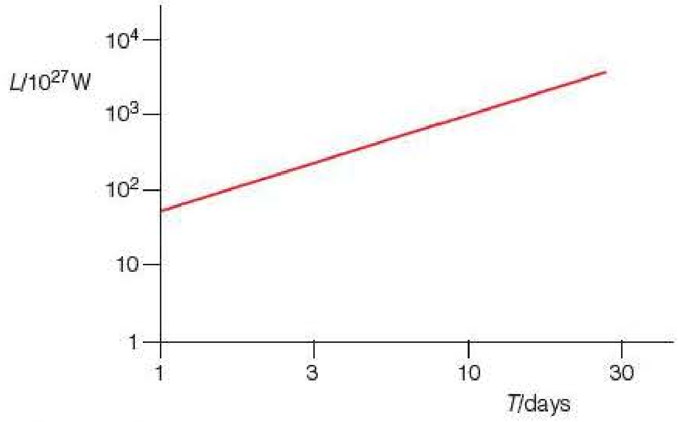


Figure 13.16

Estimate the average luminosity of such a star with a period of 4 days. Justify your answer and explain why it is only an estimate. **[Total 4 marks]**

11 A neutron star has a density of $4 \times 10^{17}\,\mathrm{kg\,m^{-3}}$. What would be the approximate mass of a piece of neutron star the size of a grain of rice?

[Total 5 marks]

12 A white dwarf star has a luminosity of $0.002\,L_\odot$ and a surface temperature of 20 kK, i.e. 20 kilo-kelvin. Calculate its radius. **[Total 4 marks]**

13 The path SXY in Figure 13.17 describes the movement of the Sun on a Hertzsprung–Russell diagram, from the time of its arrival on the main sequence PQ to its becoming a white dwarf at Y.

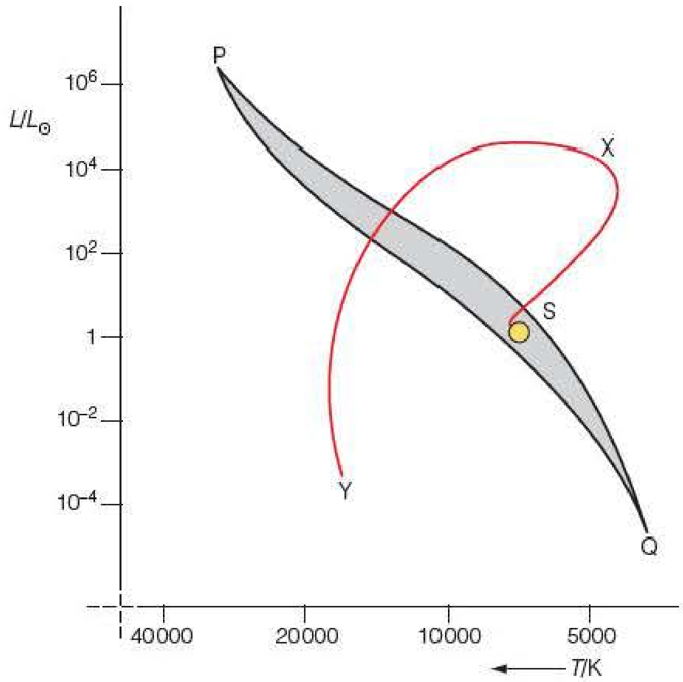


Figure 13.17 A Hertzsprung-Russell diagram

Use the diagram to write a description of the Sun's temperature and luminosity on its journey from S to Y. (Do not attempt to put a timescale on the journey.) **[Total 6 marks]**

14 A main sequence star is observed to have a maximum wavelength λ_m at 440 nm in its spectrum. On Earth its radiation flux F is measured to be $5.8 \times 10^{-10}\,W\,m^{-2}$.

a) Calculate its distance d from Earth in light years. **[8]**

b) Explain the least reliable step in your calculation. **[2]**

[Total 10 marks]

Stretch and challenge

15 Justify the statement: 'we are all made of star dust'. **[Total 4 marks]**

16 List useful websites where you can find information about dark matter. (You will obviously not get a question like this in your examinations, but it is the sort of thing that you should be able to research for yourself.)

14 Cosmology

Prior knowledge

You should be familiar with the Prior Knowledge listed at the beginning of Chapter 3 on universal gravitation and Chapter 13 on astrophysics.

The further key facts you should know or be able to apply are:

- ➜ the structure of alpha (α) and beta (β) particles and the nature of electromagnetic waves including gamma (γ) photons
- ➜ that alpha particles carry charge +2*e* and beta particles, electrons, carry a charge −*e* and are leptons
- ➜ that the electron neutrino (ν_e) is also a lepton and that lepton number is conserved in nuclear decays
- ➜ that photons, including γ-photons, carry zero charge and have energy *hf* where *h* is the Planck constant = 6.63×10^{-34} J s
- ➜ that nuclear decay processes involve exponential decay and are often characterized by their half lives, $t_{\frac{1}{2}}$
- ➜ that 1 eV is equivalent to an energy of 1.6×10^{-19} J

Test yourself on prior knowledge

1. State the nature of alpha-particles.
2. Name a nucleus that is identical in mass to an alpha-particle.
3. What is the mass of a stationary electron?
4. Why, in question 3, does the mass refer to that of a *stationary* electron?
5. State the lepton number of a) an electron, and b) an electron anti neutrino.
6. Calculate the energy of a γ-photon of frequency 4.9×10^{19} Hz.
7. What is the energy calculated in question 6 if expressed in eV?

14.1 How stars begin

In this chapter you will develop your knowledge of energy in nuclear reactions – including fusion and fission – and will consider the ultimate fate of our universe. Many questions – especially the 'why' questions – will, however, remain unanswered.

What we call 'outer space' contains a very tenuous gas consisting mainly of hydrogen atoms. There is also a little helium and some tiny dust particles. In some places the gas clumps together to form 'clouds'. The particles and atoms in such a cloud attract one another gravitationally, the rate of contraction increasing as the size of the cloud decreases and its density increases.

We now believe that this 'ordinary' matter (baryons and mesons made from quarks) represents only about 5% of the total mass of the universe. The rest is called dark matter and dark energy (see Section 13.3 in the previous chapter).

Tip

Discovering more about dark matter and dark energy shows how scientists are driven to explore the unknown.

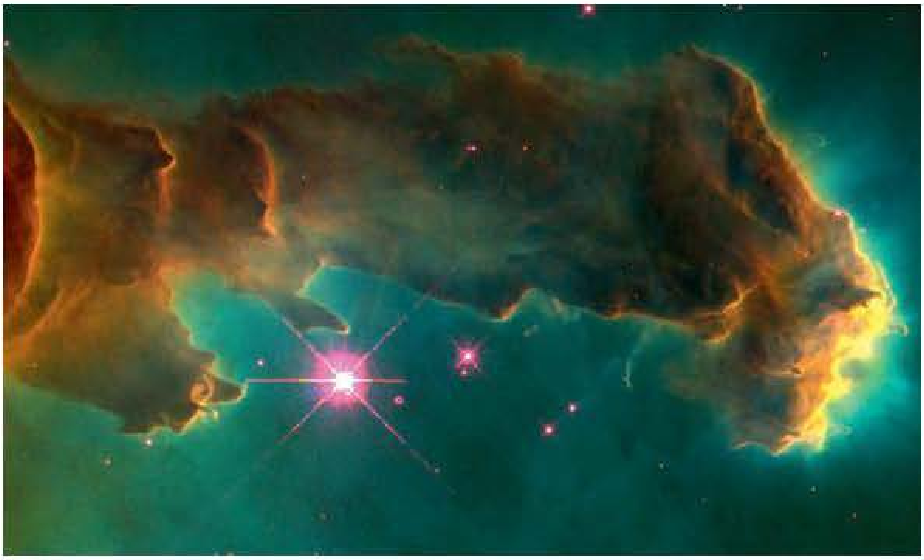


Figure 14.1 A cloud of matter in deep space from which stars are formed

Figure 14.1 shows part of a huge cloud, on the right hand edge of which can be seen 'fingers' of matter each about the size of our solar system. Each of these dense blobs will collapse to form a star. As it collapses it loses gravitational potential energy, energy that is transferred to random kinetic energy of the particles. The particles collide more and more violently. The gas gets hotter until ionised hydrogen atoms – protons – undergo **nuclear fusion** at very high temperatures.

> **Key term**
>
> **Nuclear fusion** occurs when the nuclei of atoms at very high temperatures and pressures approach one another at such a high speed, and therefore with sufficient energy, to overcome the electrical repulsion between them.

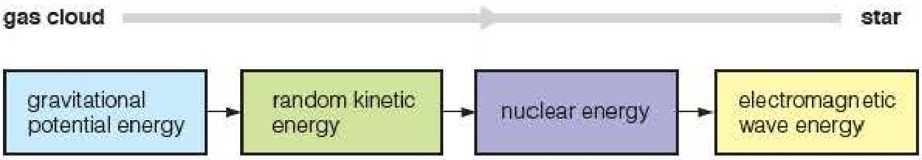


Figure 14.2 Stages of star formation

The fusion process inside our Sun involves a series of steps that converts four hydrogen nuclei (^1_1H) to make one helium nucleus (^4_2He). Figure 14.3 shows the steps involved, and the Example that follows develops the nuclear equations.

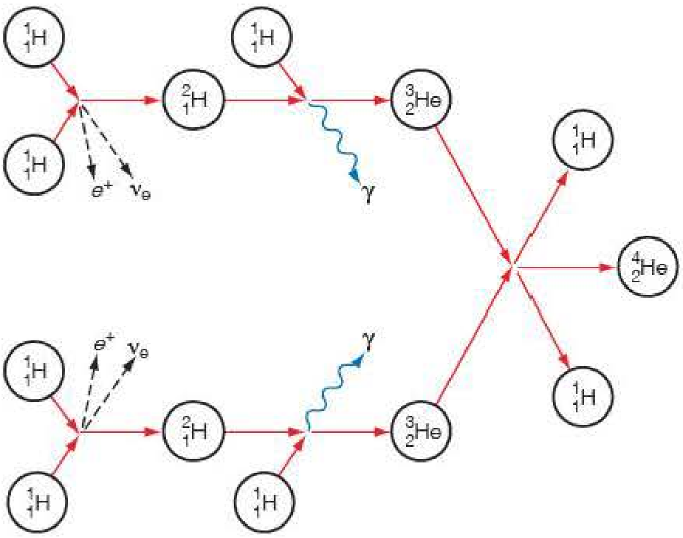


Figure 14.3 Nuclear reactions in the Sun

Example

Write the nuclear equations of the reactions shown in Figure 14.3.

(You would not be expected to remember these reactions.)

Answer

$^{1}_{1}H + ^{1}_{1}H \rightarrow ^{2}_{1}H + ^{0}_{1}e + \nu_e$ (0.42 MeV twice)

$^{2}_{1}H + ^{1}_{1}H \rightarrow ^{3}_{2}He + \gamma$ (5.49 MeV twice)

$^{3}_{2}He + ^{3}_{2}He \rightarrow ^{4}_{2}He + 2^{1}_{1}H$ (12.86 MeV once)

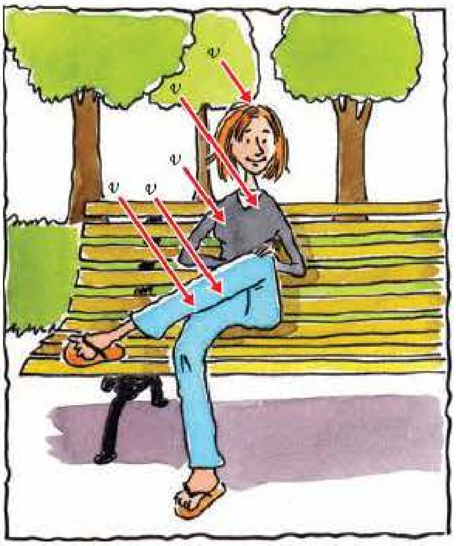


Figure 14.4 Neutrinos interact with matter very, very rarely

The net result of this process is the release of 24.68 MeV of energy. The overall outcome is the conversion of four hydrogen nuclei to make one helium nucleus:

$$4^{1}_{1}H \rightarrow ^{4}_{2}He + 2^{0}_{1}e + 2\nu_e + 2\gamma$$

The two positrons ($^{0}_{1}e$) produced annihilate with two electrons ($^{0}_{-1}e$) from the surrounding plasma, forming four more gamma photons (1.02 MeV twice). The overall result of this **nuclear synthesis**, which occurs in the Sun and in all main sequence stars, is that hydrogen atoms fuse to form helium. At the same time lots of gamma-photons γ and electron-neutrinos ν_e are produced. The neutrinos pour rapidly out of the Sun; tens of millions pass through you every second! The photons take thousands of years to 'fight' their way to the surface of the Sun, but then escape into space as visible and near-visible photons moving at the speed of light.

14.2 Nuclear binding energy

To learn more about energy conservation in these fusion processes, we need to use the relationship $\Delta E = c^2\Delta m$ and/or the equivalences:

$$1\,u = 1.66 \times 10^{-27}\,kg \equiv 930\,MeV, \text{ from Table 8.1 (page 136).}$$

The masses of a proton (m_p), a neutron (m_n) and a helium nucleus ($^{4}_{2}He$) or α-particle (m_α) are:

$$m_p = 1.0073\,u = 1.673 \times 10^{-27}\,kg$$

$$m_n = 1.0087\,u = 1.675 \times 10^{-27}\,kg$$

$$m_\alpha = 4.0015\,u = 6.645 \times 10^{-27}\,kg$$

If you put these data into your calculator you will see that m_α is *not* equal to $2m_p + 2m_n$, but less.

The difference $(2m_p + 2m_n - m_\alpha) = \Delta m = 0.051 \times 10^{-27}\,kg$ or 0.0305 u.

This mass loss or **mass deficit** must be something to do with energy. The energy equivalent of 0.0305 u is:

$$0.0305\,u \times 930\,MeV\,u^{-1} = 28\,MeV$$

(Or you can use $\Delta E = c^2\Delta m$ and convert from joules to get eV).

This is the energy that needs to be given to an α-particle, or helium nucleus, to break it up into its four components: two protons and two neutrons. We can write this as:

$${}^{4}_{2}\text{He} + \text{energy} \rightarrow 2{}^{1}_{1}\text{p} + 2{}^{1}_{0}\text{n}$$

Running this equation backwards, this means that if you can join two protons and two neutrons together, you can create an α-particle and have **lots of energy** to spare.

As all nuclei are built from protons and neutrons, you can similarly add up the total mass of the building blocks and compare this total with the mass of the resulting nucleus. In every case the nucleus is lighter than the constituent protons and neutrons. Figure 14.5 illustrates this idea for carbon-12.

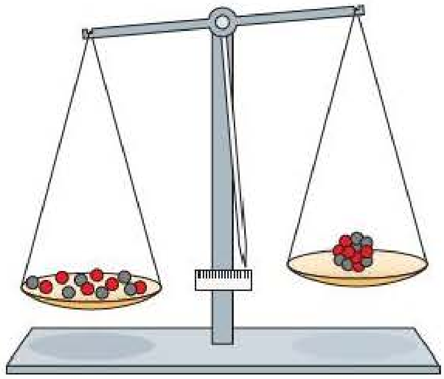


Figure 14.5 Where has the mass gone?

The missing mass has become nuclear energy or **nuclear binding energy**, and, as shown above for an α-particle, you need that energy if you want to tear the nucleus apart. For carbon-12 the mass deficit is 1.61×10^{-28} kg, so the binding energy is:

$$(1.61 \times 10^{-28}\,\text{kg}) \times (3.00 \times 10^{8}\,\text{m}\,\text{s}^{-1})^2 = 1.45 \times 10^{-11}\,\text{J (about 90 MeV)}$$

Table 14.1 lists some atomic rest masses

Table 14.1

Nucleus	Symbol	Mass/u
neutron	${}^{1}_{0}\text{n}$	1.0087
hydrogen-1	${}^{1}_{1}\text{H}$	1.0078
helium-4	${}^{4}_{2}\text{He}$	4.0026
carbon-12	${}^{12}_{6}\text{C}$	12.0000
iron-56	${}^{56}_{26}\text{Fe}$	55.9349
uranium-235	${}^{235}_{92}\text{U}$	235.0439
uranium-238	${}^{238}_{92}\text{U}$	238.0508

Example

Use appropriate data from Table 14.1 to calculate:

a) the binding energy B of the ${}^{56}\text{Fe}$ nucleus,

b) the binding energy per nucleon B/A for ${}^{56}\text{Fe}$.

Answer

a) Iron-56 has 26 protons and 30 neutrons. Using atomic masses,

$$B = 26 \times 1.0078\,\text{u} + 30 \times 1.0087\,\text{u} - 55.9349\,\text{u}$$

$$= 0.5289\,\text{u or } 492\,\text{MeV}$$

b) B/A for this isotope of iron is therefore $492\,\text{MeV} \div 56 = 8.79\,\text{MeV}$ per nucleon.

Figure 14.6 shows how the binding energy per nucleon, B/A, varies with nucleon number, A, for different elements. The main feature of the plot is that B/A peaks at or near iron-56, and this means that this isotope of iron is the most stable nucleus, i.e. the one requiring most energy per nucleon to tear it apart.

Figure 14.6 has much more information to offer:

1. If a nucleus with a very high mass number (A = 230 to 250) can be made to *break up* into smaller nuclei, there will be a large release of energy.
2. If four very light nuclei (e.g. the $4{}^{1}_{1}\text{H}$ in Section 14.1) can be made to *join together* there will be a very large release of energy.

Both these processes are possible; they are called **fission** and **fusion** respectively.

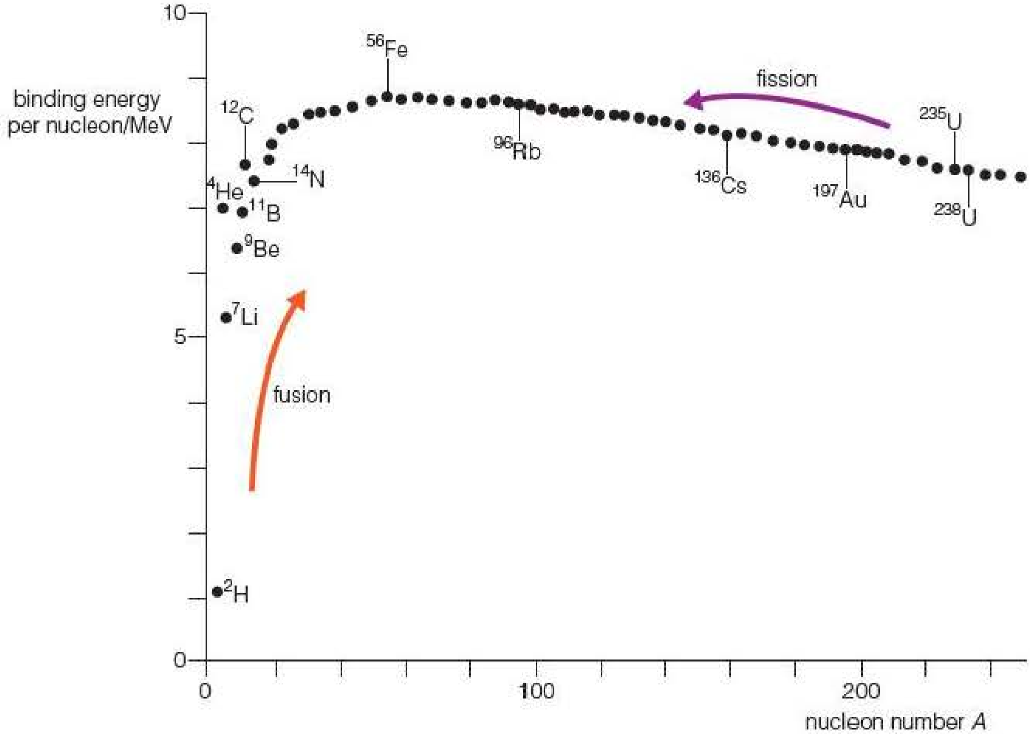


Figure 14.6 Variation of binding energy per nucleon

Test yourself

1 Is 'outer space' empty space?

2 Check that the units are the same on each side of the equation $\Delta E = c^2 \Delta m$.

3 What are represented by ν and γ in the equation $4^1_1\text{H} \rightarrow {}^4_2\text{He} + 2^0_1\text{e} + 2\nu + 2\gamma$?

4 Use Einstein's equation to find the energy liberated when an electron annihilates with a positron.

5 How long does a photon emerging from the surface of our Sun take to travel to the Earth, a distance of about 150×10^6 km?

6 Calculate the value of the fraction $(m_n - m_p) \div m_p$.

7 How is a value in keV converted to joules?

8 Sketch the *shape* of a curve showing 'The binding energy per nucleon, *B/A*' on the *y*-axis and 'the nucleon number, *A*' for nuclei from hydrogen to uranium on the *x*-axis (omit helium).

Label on your curve where a) fission is likely to occur and b) fusion is likely to occur.

14.3 Uranium fission

Nuclei that have high *A* are rich in neutrons, i.e. they contain many more neutrons than protons. When an extra neutron is absorbed, they can immediately break up into two fragments. A common result of such fission is that, as well as the two fragments, two or three neutrons are released. For example:

$${}^{235}_{92}\text{U} + {}^1_0\text{n} \rightarrow \text{fission fragments} + 2 \text{ or } 3 \text{ neutrons} + \text{energy}$$

One likely outcome of the fission of uranium-235 is that the two fission fragments (sometimes called 'daughter nuclei') together have a kinetic energy of 168 MeV. The equation for a typical fission is:

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{144}_{56}Ba + ^{89}_{36}Kr + 3^{1}_{0}n$$

Figure 14.7 illustrates this possible fission of $^{235}_{92}U$.

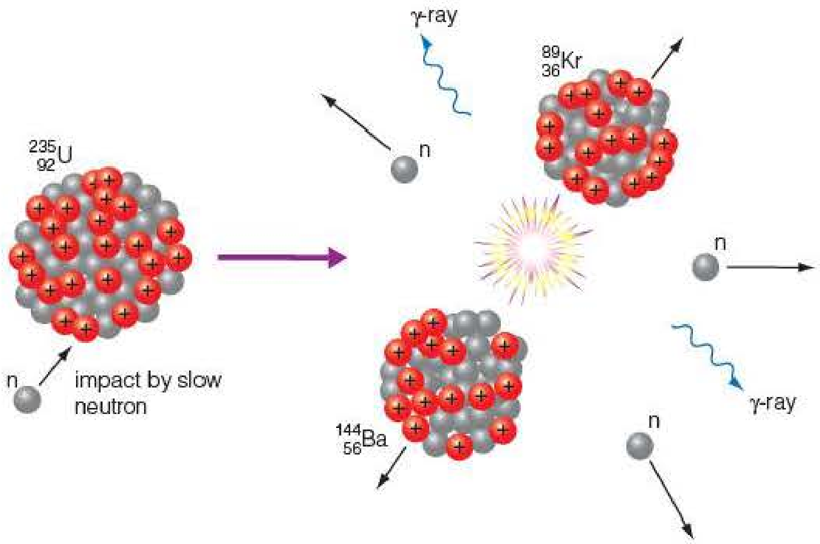


Figure 14.7 Uranium-235 fission

When a neutron enters a uranium-235 nucleus, and it splits into a barium-144 and a krypton-89 nucleus, the graph in Figure 14.6 would suggest that the average *B/A* of the fragments is about 1 MeV greater than the *B/A* of uranium. As *A* for uranium is 235, a single fission will therefore produce over 200 MeV – more than 20×10^7 eV. The average energy available from a single *chemical* reaction is only 10–20 eV, so the energy available from uranium fission is vast compared with that available from burning coal, gas or oil (10 million times more!).

Example

Assuming 200 MeV per fission, calculate the number of fission events occurring each second in a nuclear reactor whose thermal power output is 2400 MW, and comment on your answer.

(A numerical comment is required here.)

Answer

$200\,MeV \equiv (200 \times 10^6\,eV) \times (1.6 \times 10^{-19}\,J\,eV^{-1}) = 3.2 \times 10^{-11}\,J$

∴ From each fission there is 3.2×10^{-11} J of energy.

As $2400\,MW \equiv 2.4 \times 10^9\,J\,s^{-1}$:

Number of fission events per second is

$$\frac{2.4 \times 10^9\,J\,s^{-1}}{3.2 \times 10^{-11}\,J} = 7.5 \times 10^{19}\,s^{-1}$$

Comment: As the power station will (inevitably) be less than 50% efficient, the real number of fission events will be at least twice this, perhaps $20 \times 10^{19}\,s^{-1}$.

The number of fission events in the above Example looks to be a very large number, but 1 kg of uranium will contain more than 10^{24} atoms (a mole of Uranium contains more than 10^{23} atoms), and a nuclear reactor will contain several tonnes of uranium – enough fuel for a year, i.e. for $365 \times 24 \times 3600\,s \approx 3 \times 10^7\,s$. By comparison, a 2400 MW coal- or gas-burning power station can process upwards of 40 tonnes of coal an *hour*!

Example

Another possible fission of uranium-235 is into caesium-138 and rubidium-96 plus *X* neutrons. Write the nuclear equation for this fission and deduce *X*. The element caesium has 55 protons in its nucleus.

Answer

Neutrons carry no charge, therefore to conserve charge rubidium must have: (92 protons from U) minus (55 protons from Cs) = 37 protons in its nucleus.

The equation is therefore

$$^{235}_{92}\text{U} + ^{1}_{0}\text{n} \rightarrow ^{138}_{55}\text{Cs} + ^{96}_{37}\text{Rb} + X\,^{1}_{0}\text{n}$$

To conserve the nucleon number

$$X = (235 + 1) - (138 + 96) = 236 - 234 = 2$$

so there are 2 neutrons 'produced' per fission.

Splitting the atom

The first success in the race to 'split the atom' took place in Cambridge in 1932. Two researchers, John Cockcroft and Ernest Walton, working under the direction of Lord Rutherford (all three are shown in Figure 14.8), fired a proton of a few hundred keV to strike a lithium nucleus ($^{7}_{3}\text{Li}$). The remarkable and unexpected result was that the lithium nucleus 'disappeared' and two alpha particles were detected. The energy of the two resulting α-particles – helium nuclei – was estimated by the length of their tracks in a cloud chamber. The incident protons needed a lot of energy because the positively charged lithium nucleus would repel the incoming positively charged proton according to Coulomb's law. This need for the incoming protons to have a high energy helped to forward the development of the

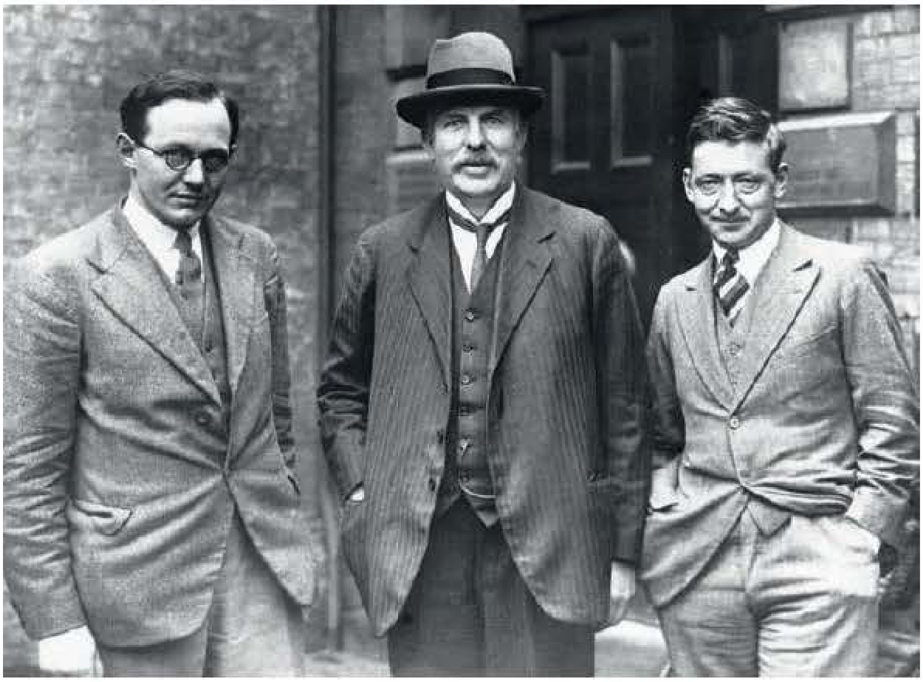


Figure 14.8 Walton, Rutherford and Cockcroft

high-energy particle accelerators we have today, e.g. at CERN. The energy of the emerging helium nuclei was the result of the mass loss in the fission reaction and by estimating this mass loss Δm.

Fission in war and peace

Two neutrons are produced by each fission in the U → Cs + Rb reaction, and three neutrons result from each fission in the U → Ba + Kr reaction. There are many other ways for a uranium-235 to break up after swallowing a neutron, but the average number of neutrons per fission is about 2.5. These neutrons can then cause other nuclei to break up, which can trigger the fission of an even larger number of nuclei, and so on. This **chain reaction** is illustrated in Figure 14.9.

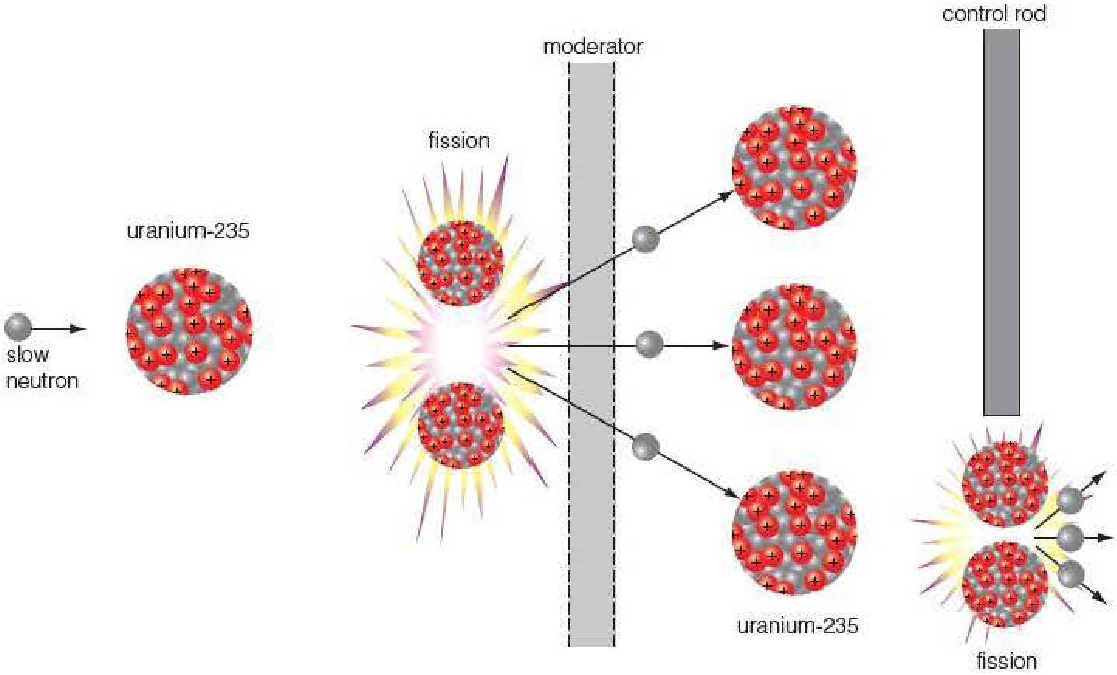


Figure 14.9 A useful chain reaction

The neutrons resulting from such fissions have a lot of energy. It is necessary to slow these neutrons down before they can induce further fissions, and this is shown by the presence in Figure 14.9 of a 'moderator' such as graphite.

When a chain reaction such as this is uncontrolled, you have an explosive situation – a **nuclear bomb** (or atomic bomb as it was and is sometimes described). Fortunately the world has only twice seen the use of an atomic bomb in war, both in Japan in 1945.

If, using control rods to absorb some of the neutrons, on average only one of these neutrons induces a new fission, the outcome is a controlled **nuclear reactor** for use, for example, in an electricity generating power station.

There are two main problems with nuclear power generation:

- First, mined uranium is 99.28% U-238 and only 0.72% U-235, and the U-238 absorbs neutrons but does not undergo fission. This means that the percentage of U-235 must be increased for use in power generation – a difficult 'enrichment' process that cannot be achieved chemically as there is no chemical difference between the two isotopes of uranium.

- Second, the U-238 reaction is ${}^{238}_{92}U + {}^{1}_{0}n \rightarrow {}^{239}_{92}U$, and the ${}^{239}_{92}U$ then decays naturally to ${}^{239}_{94}Pu$ after two beta decays. This isotope of plutonium has a half-life (see Section 9.7) of 24 000 years, and poses a severe problem when it comes to disposing of the radioactive waste – spent fuel – from nuclear power stations. One solution is to use Pu-239 in the core of other nuclear reactors –plutonium reactors. An alternative 'solution' is to use the plutonium-239 in the production of nuclear weapons – a political 'hot potato'.

Test yourself

9 What is the electrostatic force between a slow neutron and a uranium nucleus when the distance between the two is 10^{-15} m? Explain your answer.

10 How many chemical reactions averaging 15 eV per reaction are needed to produce 200 MeV?

11 In the fission of uranium-235 into two fragments, barium-144 and krypton-89, the uranium absorbs a neutron and the fission results in the production of three neutrons. Explain this process.

12 Another possible uranium-235 fission results in nuclei of zirconium-94 and tellurium-139.

a) Assuming that the fission is initiated by a single neutron, predict the proton number of tellurium, given that zirconium has a proton number of 40.

b) Explain why it is not necessary to find how many neutrons result from this fission.

13 Suggest, with a reason, what property of uranium could be used to separate the isotopes found in mined uranium.

14.4 Stellar fusion

The fusion process that generates energy in stars was described in the first section of this chapter. In order for fusion to occur, the temperature T and density ρ of the hydrogen atoms in the gas clouds needs to rise to extremely high values. This is because two positively charged protons will repel each other with a force that increases as they get closer – Coulomb's law: $F \propto 1/r^2$. You have met this Coulomb force in Chapter 4. Figure 14.10 gives some 'order of magnitude' values for T and ρ for fusion to occur. A star is a continuously exploding hydrogen bomb!

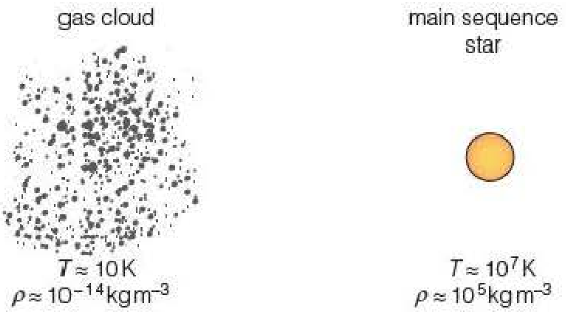


Figure 14.10 Temperature and density in a gas cloud and in a main sequence star

The energy resulting from just one set of nuclear reactions that convert hydrogen into helium in a main sequence stars is about 27 MeV. This is not as great as the result of one uranium fission (about 200 MeV), but in the Sun there are, to an order of magnitude, 10^{38} of these reactions occurring in the Sun's core *every second*!

Example

Show that $27 \times 10^{38}\,\text{MeV s}^{-1}$ is approximately equal to $L_\odot$, the output power of the Sun.

Answer

A power of

$$27 \times 10^{38}\,\text{MeV s}^{-1} = (27 \times 10^{38}\,\text{MeV s}^{-1})(1.6 \times 10^{-13}\,\text{J MeV}^{-1}) \approx 4.3 \times 10^{26}\,\text{W}$$

This agrees approximately with the power output of the Sun ($L_\odot = 3.9 \times 10^{26}\,\text{W}$).

Fusion on Earth

The hot dense material involved in stellar fusion is held together in space by gravitational forces. On Earth, magnetic fields can be used to 'hold' the hot plasma consisting of ionised hydrogen atoms away from the containing walls – see Figure 11.7 on page 204. But controlled fusion has so far only been achieved for *very* short periods, usually taking in more energy to heat the plasma than can be extracted. The possibility of useful nuclear fusion reactors is still a long way off. It is said that successful nuclear fusion is *always* 30 years in the future! Hydrogen or 'thermonuclear' bombs, by contrast, have been with us for over 50 years, but thankfully have never been used in warfare.

14.5 The expanding universe

Stars have a gaseous, low pressure 'atmosphere' around them that contains atoms of the same elements as the star itself. Atoms under high pressure in the outer layers of the star's 'body' emit a **continuous spectrum** of electromagnetic radiation (remember, stars act as black bodies). Atoms in the low-pressure atmosphere selectively absorb photons at wavelengths that excite their electrons to a higher energy level. When the electrons drop back to the lower energy level the resulting photons are emitted as an **emission spectrum** *in all directions*, but the emission spectrum is not observed by us. What we see is the **continuous spectrum** crossed by dark lines that match the atoms' emission spectrum: it is called an absorption spectrum. Figure 14.11 illustrates this.

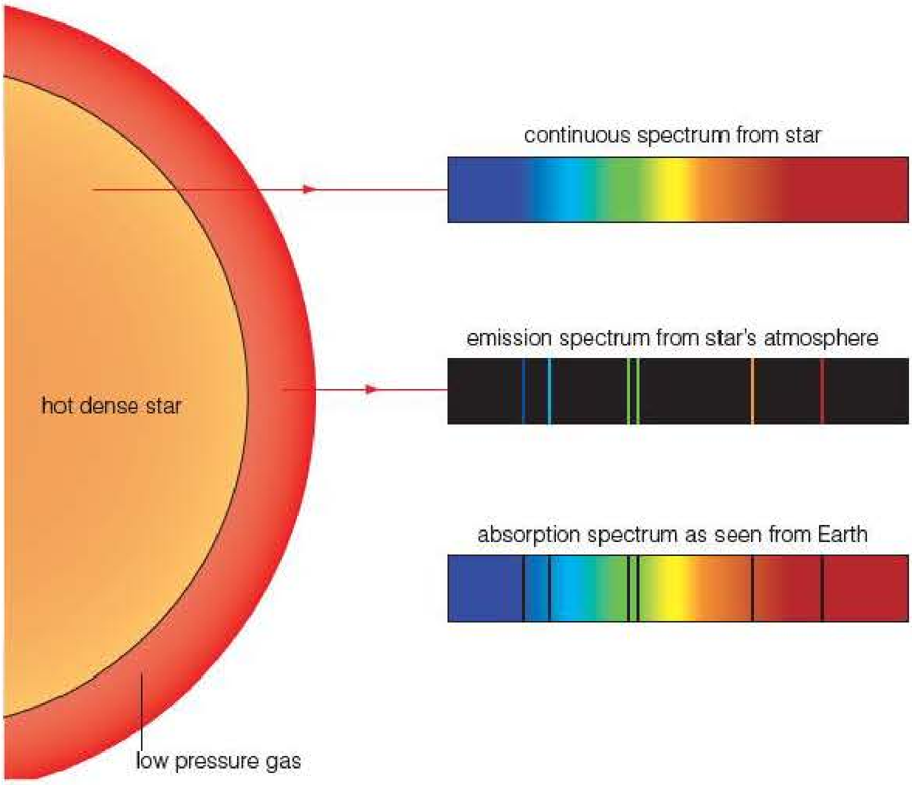


Figure 14.11 What we see from Earth

The Doppler shift

You may have met the idea that the frequency of sound from a source moving towards or away from you changes. This is called the Doppler shift. Figure 14.12 shows how the sound wavefronts become squashed or spread out.

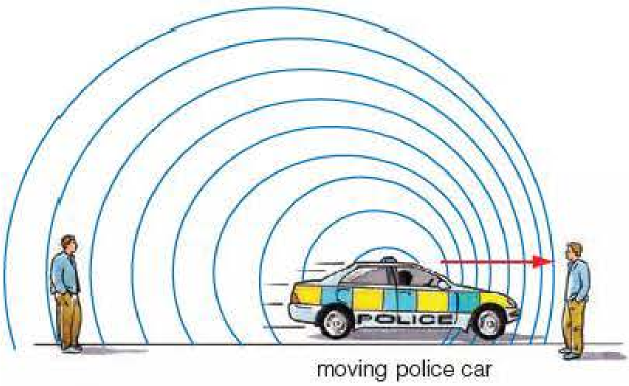


Figure 14.12 Doppler change of wavelength

Example

Describe how an understanding of the Doppler effect can explain what you hear when a police car passes you at high speed with its siren blaring.

(Again this type of question is best answered by a series of points).

Answer

- As the car approaches you, the frequency of the note you hear from the siren is higher than the frequency f_0 heard by the police in the car (wavelength is shorter).
- As the car passes and moves away from you this frequency becomes lower than f_0 (wavelength is longer).
- Therefore the effect is to hear a note that drops in frequency as the police car passes you.

Tip

A full answer to this question would be supported by a diagram similar to Figure 14.12.

The same change in frequency occurs for the electromagnetic waves received on Earth from stars and galaxies. For distant galaxies the frequency is found to be lower – the measured wavelengths of absorption lines in the star's spectrum are moved towards the red – meaning that these stars are moving away from us.

The link between the *change* in wavelength $\Delta\lambda$ and the relative velocity of source and observer v is:

$$z = \frac{\Delta\lambda}{\lambda} \approx \frac{v}{c}$$

for a source emitting electromagnetic radiation of wavelength λ and where c is the speed of light in a vacuum. The size of $\Delta\lambda$, and hence the size of the **red shift** z, is usually very small. For example, for a speed of $60\,\mathrm{km\,s^{-1}}$, $z = 0.0002$ and $\Delta\lambda = 0.1\,\mathrm{nm}$ for an absorption line at $\lambda = 500\,\mathrm{nm}$, i.e. $\Delta\lambda$ is *very* small.

Sometimes, for a star within our own galaxy, or occasionally a galaxy such as our near neighbour Andromeda, $\Delta\lambda$ is found to be negative. This tells us that the star or galaxy is moving *towards* Earth.

Figure 14.13 shows diagrammatically four (invented) absorption spectra, B C D E from distant galaxies. Above them is a laboratory emission spectrum, A, showing the same two lines. The absorption spectra are from four galaxies which are moving away from the Earth. To tackle the following Example you will need to read off the wavelengths of the left hand line in Figure 14.13, using the wavelength scale, λ/nm, below the spectra.

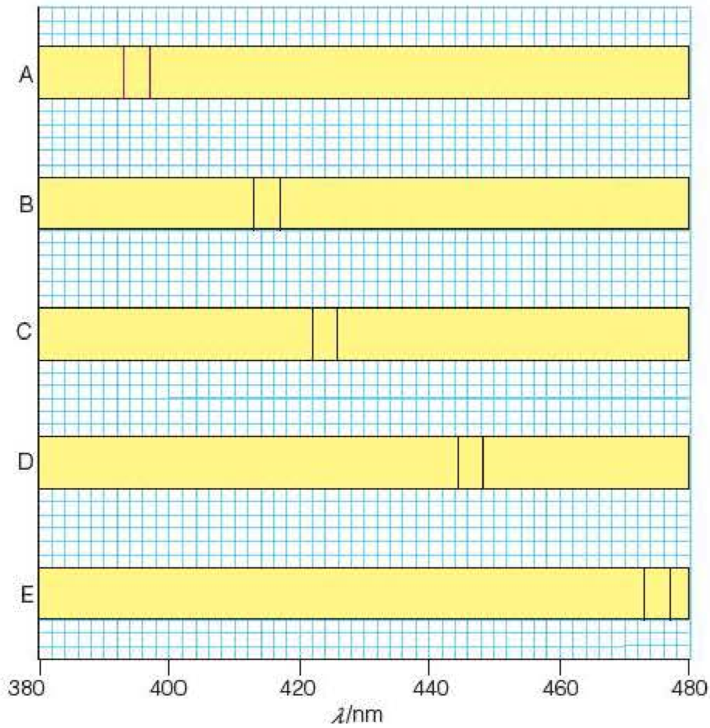


Figure 14.13 Four sets of absorption lines, B, C, D, E and a laboratory emission spectrum, A

Example

Table 14.2 shows the wavelength λ of the first (left-hand) line as measured in the laboratory (spectrum A) plus a list of values for the left-hand line of the absorption spectrum labeled C.

Table 14.2

	A	B	C	D	E
λ/nm of left line	393		422		
$\Delta\lambda$/nm			29		
$z = \dfrac{\Delta\lambda}{\lambda}$			0.0738		
$zc = v/\text{km s}^{-1}$			22 000		

a) Show how the values in column C are calculated.

b) Complete Table 14.2. (Only the last line is given in the answer.)

Answer

a) The wavelength λ is simply read off from Figure 14.13

$$\Delta\lambda = (422 - 393)\text{ nm} = 29\text{ nm}$$

$$z = \Delta\lambda/\lambda = 29\text{ nm}/393\text{ nm} = 0.0738$$

$$v = zc = 0.0738 \times (3.00 \times 10^8\text{ m s}^{-1}) = 22 \times 10^6\text{ m s}^{-1} = 22\,000\text{ km s}^{-1}$$

b) The answers to this are found by following how the answers in a) are produced. Here are the numbers for the last line. Each represents a velocity v of recession in km s^{-1}.

B: 15 000 C: 22 000 D: 40000 E: 61 000

The distances d from Earth to the four galaxies B C D E in the above Example are known to be:

B: 6.0×10^{24} m C: 9.7×10^{24} m D: 17×10^{24} m
E: 27×10^{24} m

The graph in Figure 14.14 shows the relationship between v, the speed of recession of these galaxies and their distance d from Earth. The distance d becomes increasingly difficult to establish as d gets bigger.

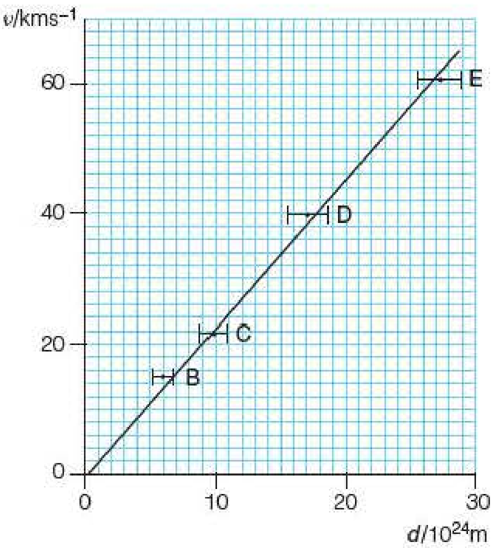


Figure 14.14 Do distant galaxies move away faster?

The Hubble constant

Uncertainty in the measured values of d, indicated by the error bars in Figure 14.14, means that the gradient of the graph is not reliable to better than ±5%. However, it is clear that v is proportional to d. This fact is known as **Hubble's law** after Edwin Hubble, an American astronomer working in the 1920s and 1930s.

The constant linking v and d is called the Hubble constant H_0. Its value to 2 SF is $H_0 \approx 2.3 \times 10^{-18}\text{ s}^{-1}$, which is the value you should get if you work out H_0 from the gradient of the graph shown in Figure 14.14.

$$v = H_0 d \qquad \text{and} \qquad z = \frac{v}{c} = \frac{H_0 d}{c}$$

Figure 13.6 in Section 13.3 summarises methods for measuring d. (Note that for these to work as d gets bigger and bigger there must be overlap between adjacent methods.) Having a value for the Hubble constant, however, we now have a method of finding d for any distant galaxy, provided we can measure its red shift, the value of z. We will need to assume that the straight line in the graph in Figure 14.14 remains linear as the red shift becomes bigger and bigger, i.e. that Hubble's law is universally valid.

Example

How far away is a galaxy that shows a red shift of $z = 0.20$?

Answer

$$z = \frac{v}{c} = \frac{H_0 d}{c} \Rightarrow d = \frac{zc}{H_0}$$

$$\therefore d = \frac{(0.20)(3.0 \times 10^8\,\mathrm{m\,s^{-1}})}{2.3 \times 10^{-18}\,\mathrm{s^{-1}}}$$

$$= 2.6 \times 10^{25}\,\mathrm{m}$$

Physicists believe that the universe started with a **Big Bang** about 14 billion years ago, that is 14 thousand million years ago. Evidence for such a Big Bang depends on the observed expansion of the nearby universe as established by Hubble's law, on predictions such as the existence of a 'cosmic background' microwave radiation and on the proportion of helium to hydrogen in the early universe (about one He atom for every four H atoms).

When cosmologists (people who study the origin and the nature of our universe) talk of the universe expanding, they do not mean that matter flew outwards into space in all directions. They prefer to think of space itself expanding. Think of all the galaxies painted at random on the surface of a part blown-up balloon – Figure 14.15. Now think of time passing as the balloon is gradually blown up more and more. You can try it for yourself.

Figure 14.15 A balloon model of an expanding universe

As Figure 14.15 shows, the distance between any two galaxies increases with time. A red shift would be measured by an observer in *any* galaxy and the calculated value of the Hubble constant would be the same for all observers. You may think that the surface of a balloon does not seem like a three-dimensional universe. In fact the model is quite good although, like all analogies, limited in usefulness. You should try to keep in mind both a Big Bang and expanding space when thinking about our universe.

Example

Rearranging Hubble's law, $v = H_0 d$, gives:

$$\frac{1}{H_0} = \frac{\text{distance to a galaxy}}{\text{its speed of recession}}$$

It is suggested that this reciprocal of the Hubble constant tells us how long has elapsed since the Big Bang.

a) Discuss the ideas behind this suggestion.

b) Calculate $\frac{1}{H_0}$.

- Discuss' in a) may require more than words. Here a bit of algebra is needed.

Answer

a) Suppose the universe was created a time t_u ago. For a galaxy that has been moving away from us at a steady rate v for a time t_u, its distance d from us will now be vt_u.

Hubble's law tells us that $v = H_0 d$, so substituting for d gives us:

$v = H_0 v t_u$, or simply, $H_0 t_u = 1$ and hence that the age of the universe is

$$t_u = \frac{1}{H_0}.$$

b) $\frac{1}{H_0}$

$$\frac{1}{H_0} = \frac{1}{2.3 \times 10^{-18} \text{s}^{-1}} = 4.3 \times 10^{17} \text{ s}$$

$$= 1.4 \times 10^{10} \text{ years or 14 billion years (to 2 SF).}$$

Attractive gravitational forces would mean that the present rate of expansion is *less* than that in the past, and hence t_u will be bigger than $1/H_0$. Unfortunately for those who believe that Newton's law of gravitation is universally applicable, two teams of cosmologists have recently shown by studying very distant supernovae (type Ia), that the distant universe is *accelerating* away from us, casting doubt on our guess that the age of our universe $t_u = 1/H_0$ or casting doubt on our current value of H_0. The three cosmologists whose work led to this conclusion won the Nobel Prize for Physics in 2011.

Test yourself

14 Estimate the repulsion force between two protons a distance 1.0 μm apart, and comment on the result.

15 Show, using data from Table 14.1 that, in the process

$$4{}^1_1\text{H} \rightarrow {}^4_2\text{He} + 2\nu + 6\gamma$$

(ignoring any energy tied up in the neutrinos and gammas) the difference in mass–energy between the four hydrogen nuclei and the helium nucleus is about 27 MeV.

16 Explain the difference between an emission spectrum and an absorption spectrum.

14.6 How will the universe end?

What the fate of the universe will be depends on the average density of matter it now contains. Figure 14.16 shows some possible pasts and futures. With a high density the universe implodes into a Big Crunch (don't panic – this would be billions of years away!); with a small density it will go on expanding forever. The dashed line represents what might have happened had gravity (or some other accelerating force – see above) not interfered with the expansion. Notice that the reciprocal of the Hubble constant is bigger than the age of the universe suggested by both the 'open' and 'closed' scenarios described by this 'graph'.

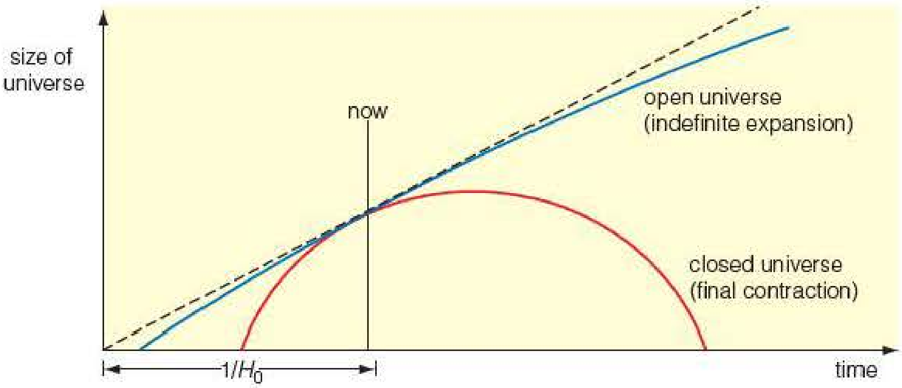


Figure 14.16 Towards a Big Crunch?

When cosmologists try to estimate the average density of matter in the universe, they discover problems. Spiral galaxies are spinning as if they contain much 'more stuff with mass' than the luminosity of the galaxy would predict. The expansion of the 'near' universe appears to be slower than predicted by the gravitational effect of observable mass. Cosmologists estimate that only about 5% of the mass of the universe consists of atoms as we know them. What is the other 95%? We don't know, so we call it **dark matter** and **dark energy**. Unobserved neutrinos and WIMPS might contribute to some of this mass. There are lots of them about – see Figure 14.4. (See Section 13.3 and the 'The 5% universe').

One recent intriguing suggestion is that dark matter and dark energy plus ordinary matter together make the average density of the universe just right for it to end up as neither an open nor a closed universe. Such a 'flat' universe, one that stops expanding after an infinite time, satisfies many cosmologists' gut feelings. Perhaps you will live to see these mysteries resolved.

Exam practice questions

1 In the fission reaction:

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{144}_{56}Ba + ^{89}_{36}Kr + 3^{1}_{0}n$$

the number of neutrons in Ba and Kr are respectively:

A 88 and 53

B 89 and 53

C 143 and 88

D 144 and 89.

[Total 1 mark]

2 All nuclei other than hydrogen are built from

A protons and electrons

B protons and neutrons

C protons only

D electrons and neutrons.

[Total 1 mark]

3 In the controlled chain reaction of uranium-235, as in an atomic (nuclear) power station, what is the purpose of the control rods?

A To absorb some uranium.

B To absorb some neutrons.

C To slow down some neutrons.

D To slow down some U-235 nuclei.

[Total 1 mark]

4 A galaxy shows a red shift of 0.20. Its speed of recession from Earth is:

A $6.0 \times 10^{7}\,m\,s^{-1}$

B $1.5 \times 10^{7}\,m\,s^{-1}$

C $1.5 \times 10^{9}\,m\,s^{-1}$

D $6.0 \times 10^{6}\,m\,s^{-1}$.

[Total 1 mark]

5 Which of the following is *not* a reasonable description of what is meant by the binding energy of a nucleus?

A It is the energy released when the protons and neutrons forming a nucleus join together.

B It is the energy it would take to tear a nucleus apart into its protons and neutrons.

C It is the electric potential energy released when the particles forming a nucleus join together.

D It is the work done in separating the nucleus into its constituent protons and neutrons.

[Total 1 mark]

6 The nuclei of the first three isotopes of hydrogen are:

- ^1_1H with a mass of 1.00728 u
- ^2_1H with a mass of 2.01355 u
- ^3_1H with a mass of 3.01550 u

a) Describe the structure of these three nuclei. **[3]**

b) Calculate the mass deficit in the nuclear reaction **[3]**

$^2_1H + ^2_1H \rightarrow ^3_1H + ^1_1H$

c) How much energy, in MeV, is released in such a reaction? **[2]**

[Total 8 marks]

7 Figure 14.17 represents a cloud chamber photograph showing the first successful 'splitting the atom' in 1932.

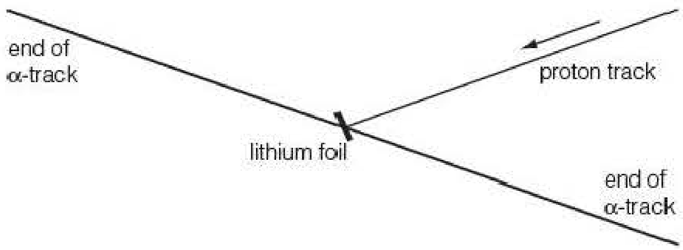


Figure 14.17

a) Cockcroft and Walton deduced that the two α-particles had the same initial energy. Explain how they made this deduction. **[1]**

b) Write a nuclear equation for this event. **[3]**

c) The energy released in the reaction is more than 17 MeV. Calculate the mass of the lithium nucleus, given that the rest masses of a proton and an α-particle are 1.0073 u and 4.0015 u respectively. **[4]**

[Total 8 marks]

8 Calculate the distance from Earth of the galaxy described in question 4 above in

a) metres **[4]**

b) light years. **[4]**

Take the value of the Hubble constant to be $2.3 \times 10^{-18}\,s^{-1}$.

[Total 6 marks]

9 During fission a uranium nucleus splits into two parts of roughly equal sizes. Show that the ratio of the radius r of each fission fragment to the radius R of the uranium nucleus R, i.e. the ratio r/R, is about 0.8.

[Total 3 marks]

10 Figure 14.18 illustrates the Doppler effect. Explain what the figure is telling two observers O_1 and O_2 about the waves from a moving source S.

[Total 6 marks]

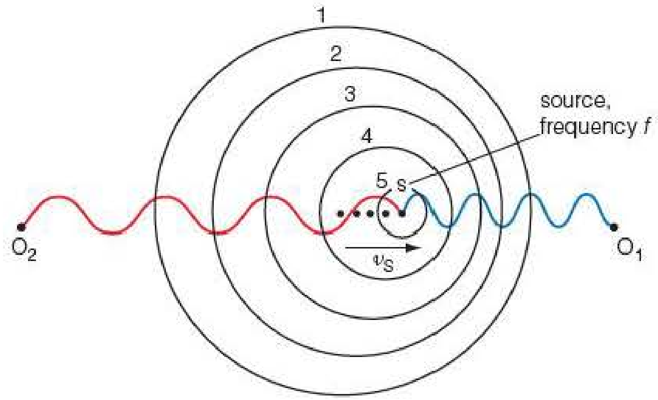


Figure 14.18

11 The Sun spins about its own axis with a period of rotation of 2.2×10^6 s. The Sun's mean diameter is 1.40×10^9 m.

Calculate the change in wavelength for light of wavelength 520 nm coming from the edge of the Sun's equator that is moving away from us.

[Total 5 marks]

12 Consider a small star S that orbits a much more massive star M with a period T of 2.7×10^8 s (between 8 and 9 years): see Figure 14.19. A hydrogen absorption line in the spectrum of S, viewed from Earth, shows a variation from 656.323 nm to 656.241 nm as S moves in its circular orbit around M.

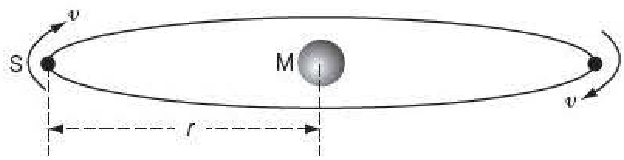


Figure 14.19

a) Explain this variation and show that the speed of S in its orbit is about 20 km s^{-1}. **[5]**

b) Calculate: the radius r of the orbit of S, and hence deduce the mass m_M of M. **[5]**

[Total 10 marks]

13 The spectrum of the star Arcturus was photographed six months apart, in M (March) and S (September). Figure 14.20 shows drawings of some of the most prominent lines in the two absorption spectra. Above and below are laboratory emission spectra drawn to the same scale. The spectra cover wavelengths from 425 nm to 430 nm from left to right. In Figure 14.20 the spectra are 74 mm wide.

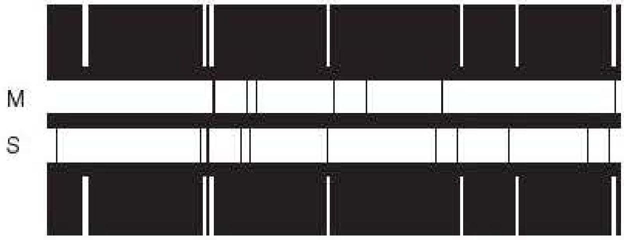


Figure 14.20

Careful measurements of the positions of the spectral lines drawn in the figure show a red shift in March of 0.38 mm and a red shift in September of 0.67 mm.

a) Calculate the velocities with which Arcturus appears to be receding from Earth at these times. **[5]**

b) What can you deduce from your results? **[3]**

[Total 8 marks]

14 The average density of matter in the universe ρ_0 can be calculated from the relationship

$$\rho_0 = \frac{3H_0^2}{8\pi G}$$

where H_0 is the Hubble constant and G is the gravitational constant.

a) Calculate a value for ρ_0, add approximate ± values and show that ρ_0 has the unit kg m^{-3}. **[7]**

b) Approximately how many hydrogen atoms would you find on average in each cubic metre of the present universe? **[2]**

c) Explain why cosmologists want a precise knowledge of the Hubble constant. **[3]**

[Total 12 marks]

Stretch and challenge

15 Describe how cosmologists determine the distance to galaxies that show very large red shifts. **[Total 5 marks]**

16 Discuss why sharp lines in the Sun's absorption spectrum, when they are examined *very* closely, are found to be split into two lines.

[Total 3 marks]

17 Figure 14.22 tells the story of a typical supernova explosion – type Ia. The vertical scale is logarithmic, so we can deduce from the graph that the maximum luminosity of the supernova was about 1.7×10^9 times $L_\odot$ (the luminosity of the Sun), and that this occurs within a few days of the supernova explosion.

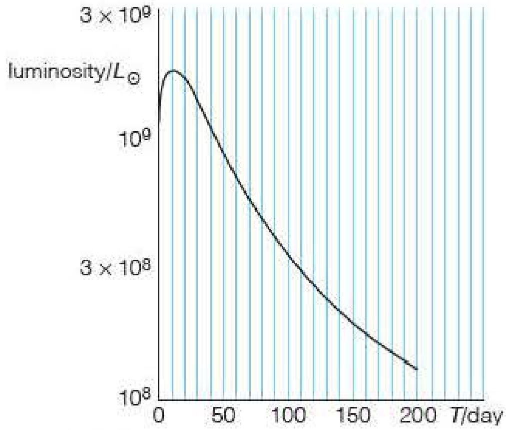


Figure 14.22

a) Assuming that 1.7×10^9 times $L_\odot$ is a realistic estimate of the maximum luminosity of the explosion, calculate its value in W. **[3]**

b) Explain why we do not use other types of supernovae explosions to estimate the distance of a galaxy from Earth. You may need to look up data about supernovae explosions. **[3]**

[Total 6 marks]

15 Oscillations

Prior knowledge

- It would be a good idea for you to spend a few minutes revising your Year 1 work on oscillations and waves, because we will be building on some of the ideas in this chapter.
- In particular, you should be familiar with terms such as displacement, amplitude, wavelength, frequency, time period and phase.
- You should understand that displacement is a vector.
- You should be able to sketch and understand displacement-time and displacement-distance graphs for a body oscillating sinusoidally, such as a mass on a spring.
- You should recall that velocity is the gradient of a displacement-time graph.
- You will also need to use some basic mechanics, such as the relationship between force and acceleration and the interchange of potential and kinetic energy.
- $F = ma$
- $E_k = \frac{1}{2}mv^2$
- $f = \frac{1}{T}$
- $\mathrm{Hz} \equiv \mathrm{s}^{-1}$

Test yourself on prior knowledge

1 A student times a mass oscillating vertically on a spring. She finds the time for 25 oscillations is 31.25 s. What is a) the period and b) the frequency of these oscillations?

2 Copy the displacement-time graph shown in Figure 15.1.

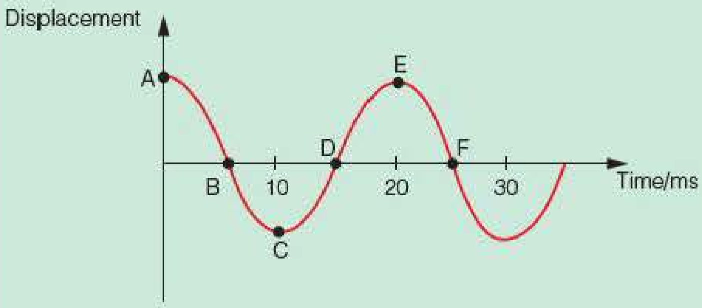


Figure 15.1

a) Add to your graph to show i) the amplitude and ii) the time period of the motion.

b) What is i) the time period and ii) the frequency of the motion?

c) State which points are in phase with each other.

d) What is the phase difference between i) A and B and ii) B and D?

15.1 Simple harmonic motion

As you learnt in Year 1 of your A-level course, waves occur in a variety of forms – from shock waves generated by earthquakes to γ-radiation emanating from the nuclei of atoms. The study of wave motion is therefore very important, as it has so many applications in everyday life. In particular, we will be looking in detail at a special form of oscillatory motion, called simple harmonic motion. Many of the waves you looked at (or heard!) in Year 1 were caused by oscillations approximating very closely to simple harmonic motion. Its study is of great importance in a wide range of areas of physics and engineering – from the design of bridges to the development of MRI scanners.

We saw the importance of mechanical oscillations when we studied waves in Year 1. Oscillating springs, vibrating rules, pendulums, sound waves, water waves and shock waves from earthquakes are all examples of mechanical oscillations caused by some vibrating source.

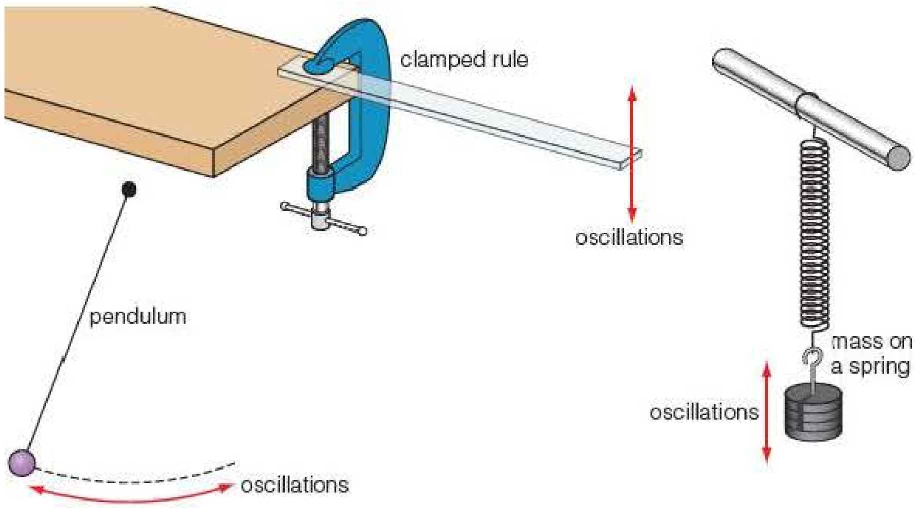


Figure 15.2 Oscillating systems

Key terms

Amplitude, A, is the maximum displacement from the mean (equilibrium) position (in metres, m).

Period, T, is the time taken to complete one oscillation (in seconds, s).

Frequency, f, is the number of complete oscillations per second (in hertz, Hz).

In oscillations, we observe that the motion is repetitive about a fixed point, with the object at rest at the extremes of the motion and moving with maximum speed in either direction at the midpoint. Three properties can be used to describe an oscillation. They are: the **amplitude**, A, the **period**, T, and the **frequency**, f. Remember that period and frequency are related by $f = \frac{1}{T}$ or, conversely, $T = \frac{1}{f}$, and that we measure frequency in Hz ($\equiv s^{-1}$).

Example

1 a) What is the period of the mains alternating current supply, which has a frequency of 50 Hz? Give your answer in milliseconds.

b) The vibrations from a guitar string are observed on an oscilloscope. The period of the oscillations is measured to be 3.9 ms. What value does this give for the frequency of the sound?

2 A clamped rule is found to complete 20 oscillations in 13.3 s. Calculate

a) the period

b) the frequency of the oscillations.

Answer

1 a) $T = \frac{1}{f} = \frac{1}{50\,s^{-1}} = 0.02\,s = 20\,ms$

b) $f = \frac{1}{T} = \frac{1}{3.9 \times 10^{-3}\,s} = 256\,Hz$

2 a) $T = \frac{13.3\,s}{20} = 0.665\,s$

b) $f = \frac{1}{T} = \frac{1}{0.665\,s} = 1.50\,Hz$

To a good approximation, all the oscillations described above show two common characteristics:

- the resultant force acting on the oscillating body, and therefore its acceleration, is *proportional to the displacement* of the body (i.e. its distance from the mean, or equilibrium, position);
- the resultant force, and therefore the acceleration, always acts in a direction *towards the equilibrium position*.

An oscillating body that satisfies these conditions is said to have **simple harmonic motion**, or s.h.m. for short. We can combine these conditions into a simple equation:

$$F = -kx$$

where k is a constant, and the minus sign means that the resultant force, F, is always opposite in direction to the displacement, x, that is, *towards* the equilibrium position (see Figure 15.3). Remember, force, acceleration and displacement are *vectors*, so their direction is important.

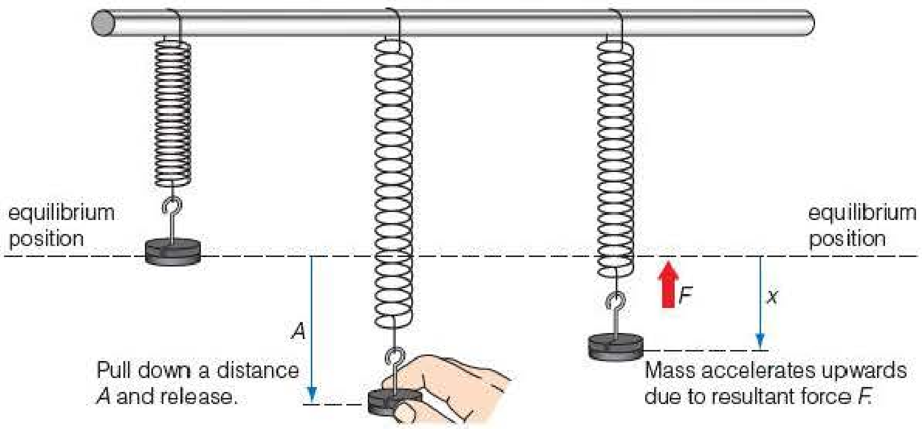


Figure 15.3

Since force is proportional to acceleration, we can express the equation in terms of acceleration. By convention we call the constant ω^2 so the simple harmonic equation becomes:

$$a = -\omega^2 x$$

Now let's look at some examples of simple harmonic motion.

(**Note**: Later on we will see that $\omega = 2\pi f$.)

Key term

Simple harmonic motion is periodic motion about an equilibrium position such that the acceleration is

- proportional to the displacement, and
- always directed towards the fixed point.

Tip

F and x are vectors. As can be seen in Figure 15.3, F (upwards) is in the opposite direction to x (downwards).

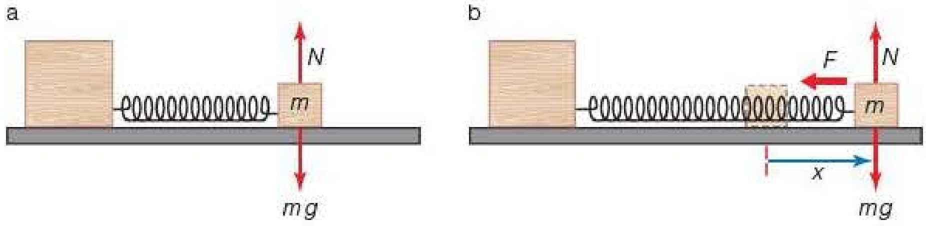
15.2 Spring

In the Year 1 Student's Book we saw that the equation for a spring that obeys Hooke's law is $F = kx$ where k is the spring constant, or 'stiffness' of the spring, and x is the extension.

Figure 15.4a shows the forces acting on a mass m when there is no extension of the spring. As m is at rest, $\Sigma F = 0$ and so $mg = -N$.

If the mass is now given a displacement x, as in Figure 15.4b, the resultant force F on the mass m is simply the pull of the spring, which is $-kx$. So:

Figure 15.4 The forces acting on a mass m on a frictionless surface (e.g. an air-track)



From $F = ma \quad \Rightarrow a = \frac{F}{m}$, where $F = -kx$

$$\Rightarrow a = -\frac{kx}{m} = -\frac{k}{m}x$$

Note the introduction of the minus sign is to show the difference in direction of F and x.

This equation is of the form

$$a = -\omega^2 x$$

where $\omega = \sqrt{\frac{m}{k}}$.

As shown later, $\omega = 2\pi f$, and so $T = 2\pi/\omega$. Therefore a mass m oscillating horizontally on a spring that obeys Hooke's law will execute s.h.m. with an oscillation period given by the equation

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Analysis of a spring oscillating *vertically* is more complex (see Exam Practice Question 15). However, the result turns out to be the same! A mass oscillating on a **light vertical** spring that obeys Hooke's law also execute s.h.m with a period given by

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Activity 15.1

Finding the spring constant of a spring from Hooke's law

A 100 g mass hanger is suspended from a vertical spring as shown in Figure 15.5

A set-square is used against the vertical rule to determine the initial position, h_0, of the bottom of the spring. Extra masses, Δm, in 50 g increments up to about 300 g, are now carefully added. After each mass has been added, the position h of the bottom of the spring is recorded. The extension of the spring for each mass is then $\Delta x = h - h_0$.

The results are tabulated (as in Table 15.1 in the question below) and a graph of F (in newtons) on the y-axis against Δx (in metres) on the x-axis is plotted. A straight line through the origin should be obtained if the spring obeys Hooke's law, but the graph may curve a bit at the bottom end, as some springs need a small force to separate the coils before the spring starts to stretch. The gradient of the straight part of the graph gives the spring constant, k.

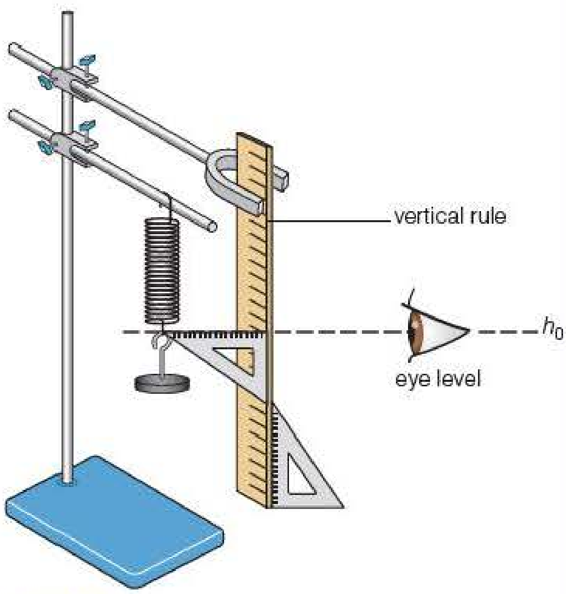


Figure 15.5

Safety note

Safety goggles should be worn.

Questions

The following data were obtained in an experiment to stretch a spring.

1 Copy and complete Table 15.1 by adding the rest of the Δx values ($h_0 = 412\,\text{mm}$).
2 Plot a graph of F (in newtons) on the y-axis against Δx (in metres) on the x-axis.
3 Draw a large triangle to determine the gradient. This should give a value for the spring constant, k,of about $27\,\text{N}\,\text{m}^{-1}$.

Table 15.1

Δm/kg	0.000	0.050	0.100	0.150	0.200	0.250	0.300
F/ N(=gΔm)	0.00	0.49	0.98	1.47	1.96	2.45	2.94
h/mm	412	430	449	468	486	504	523
Δx/ mm (=h–h_0)	0	18					

Activity 15.2

Finding the spring constant of a spring from simple harmonic motion

Using the same arrangement and the same spring as in the previous experiment, 20 small vertical oscillations are timed for masses in the range 100 g to 400 g, in 50 g increments. Note that in this experiment the mass, m, is the *total* mass, including that of the mass hanger. Each timing should be repeated and an average taken.

To help judge the start and stop positions, a marker is put at the *centre* of the oscillations, for example a pin secured to the vertical rule with Blu-Tack as shown in Figure 15.6. This called a 'fiducial' mark – a mark by which the position of the mass can be 'judged'.

The results are tabulated, together with values of T^2 as in Table 15.2 in the question below.

Why do we need values of T^2? Our equation is $T = 2\pi\sqrt{\frac{m}{k}}$ so squaring both sides of the equation gives:

$$T^2 = \frac{4\pi^2 m}{k} \quad \Rightarrow \quad T^2 = \frac{4\pi^2}{k} \times m$$

This means that if we plot a graph of T^2 on the y-axis against m on the x-axis we should get a straight line through the origin of gradient $\frac{4\pi^2}{k}$. If the gradient is determined, then $k = \frac{4\pi^2}{\text{gradient}}$

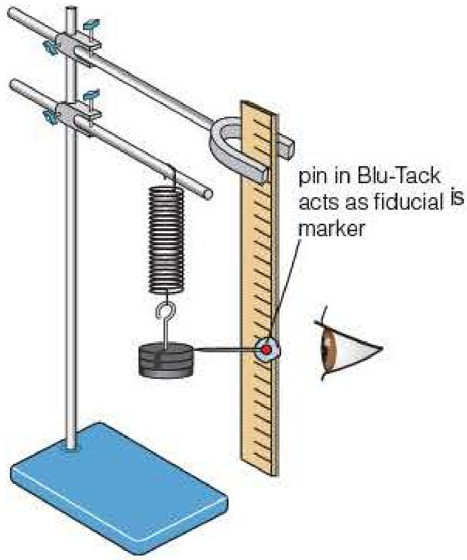


Figure 15.6

Questions

The data in Table 15.2 were obtained by timing small vertical oscillations of masses on a spring.

Table 15.2

m/kg	20T/s		Mean T/s	T^2/s^2
0.100	7.79	7.83	0.390	0.152
0.150	9.41	9.35	0.469	0.220
0.200	11.01	11.07		
0.250	12.41	12.33		
0.300	13.38	13.42		
0.350	14.52	14.59		
0.400	15.68	15.60		

Tip

Remember that the spring must be light (i.e. a few grams) so that we can ignore the weight.

1 Copy and complete the table by adding the rest of the values for T and T^2.
2 Plot a graph of T^2 against m, which should be a straight line through the origin.
3 Draw a large triangle to determine the gradient, which you should find to be about 1.5 ($\text{s}^2\,\text{kg}^{-1}$).
4 Use your value of the gradient to determine a value for the spring constant k.
5 Calculate the percentage difference between your values for k found by the two different methods in Activities 15.1 and 15.2. Comment on which method you think gives the more reliable value for k. Justify your answer with appropriate data.

Tip

Always remember to divide by the number of oscillations when finding the period.

The quickest way here to find the average value for T is to add the two values for $20T$ and then divide by 40.

Note

An alternative, core practical 16b, is described in Section 15.7.

Core practical 16a

Determination of an unknown mass using the natural frequency of oscillations of known masses

The apparatus is the same as for the previous Activity, as shown in Figure 15.6. A suitable 'unknown mass' could be a cube of wood having sides of length about 75 mm with a small hook in the centre of one of its faces.

The periods of oscillation T of different masses m are found at intervals of 50 g for a range of masses from 100 g to 400 g. The period is found for each mass by timing 20 oscillations, repeating and then determining an average value for T. As before, a marker is put at the *centre* of the oscillations to facilitate counting the oscillations. In a similar way, the period T_w for the block of wood is determined.

A graph of T^2 against m is then plotted, which should be a straight line through the origin. The mass of the block of wood m_w can then be determined from the graph by reading off the value of mass corresponding to T_w^2.

Questions

1. A student planning such an experiment decides to plot a graph of T against m. Explain why it is better to plot a graph of T^2 against m.
2. Use the data from Table 15.2 to plot a graph of T^2 against m for the spring – this will be the same graph as in the previous Activity, so you can use this graph if you wish.
3. Two times were measured for 20 oscillations of the block of wood: 11.63 s and 11.69 s. Use your graph to determine the value these data give for the mass m_w of the block of wood.
4. Describe how you would determine as precisely as possible the dimensions of the block of wood in order to find a value for its volume.
5. The block is found to have dimensions of 75 mm × 75 mm × 72 mm. Calculate the volume of the block.
6. Hence determine the density of the wood.
7. Estimate the uncertainty in your value for the density.

15.3 Simple pendulum

In this context, 'simple' means

- a small, dense, pendulum bob, e.g. lead or brass sphere, and
- a light, inextensible string.

In the case of the simple pendulum, the force causing oscillation is provided by a component of the weight of the pendulum bob as shown in Figure 15.7.

The required force is the component $mg\sin\theta$. If θ is small, and in radians, then $\sin\theta \approx \theta$ and so the force is proportional to the displacement. It can then be shown, for oscillations of small amplitude (less angular than about 10°), that the period, to a good approximation, is given by:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where l is the distance from the point of suspension to the centre of gravity of the bob.

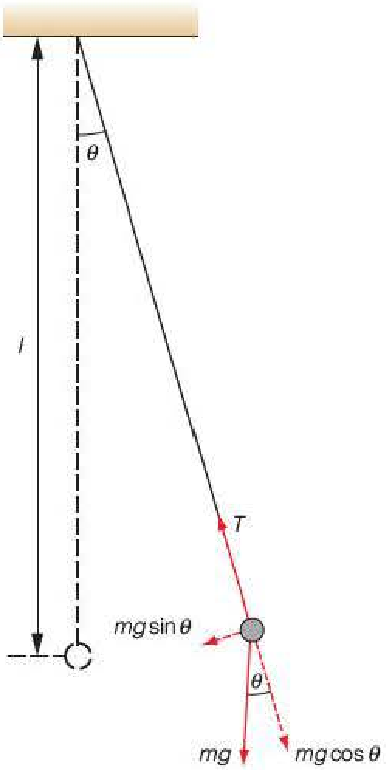


Figure 15.7

Tip

For investigating powers, you could use ln *or* log. For exponential equations you *must* use ln. If you use ln for *both*, you can't go wrong!

Example

We can use measurements of the period of a simple pendulum to practise plotting a logarithmic graph to test a proposed relationship. We can see from simple observation that the period depends on the length of the pendulum, so let's assume that they are related by an equation of the form

$$T = a\ell^b$$

where *a* is a constant and *b* is a power.

If we take logarithms (to base 'e') on both sides of the equation we get

$$\ln(T/s) = b\ln(\ell/m) + \ln a$$

Note that we show the quantities divided by their units (T/s) and (ℓ/m) as we can only take the logarithm of a number, not a physical quantity.

The data in Table 15.3 were obtained for the period *T* of a simple pendulum. Note that if you were doing this yourself, which you might like to do, you should find *T* from the average of two lots of 20 oscillations, as in Activity 15.2.

Copy and complete Table 15.3 by adding the rest of the values of ln(ℓ/m) and ln(T/s), and then plot a graph of ln (T/s) against ln(ℓ/m). Think carefully when choosing your axes – the ln(ℓ/m) values are negative!

You should get a graph like Figure 15.8.

Draw a large triangle on your graph to determine the gradient. This should be close to 0.50, or $\frac{1}{2}$.

This suggests that $T = a\ell^{\frac{1}{2}}$ or $T = a\sqrt{\ell}$ for a simple pendulum.

The intercept on the ln *T* axis is 0.700. So $\ln a = 0.700$, giving $a = 2.01$. As *a* is a physical quantity, it must have units. What are they?

Looking at the equation $T = a\sqrt{\ell}$ that we have just deduced, $a = \frac{T}{\sqrt{\ell}}$ and so has units of $\mathrm{s\,m^{-\frac{1}{2}}}$.

We can now write $a = 2.01\,\mathrm{s\,m^{-\frac{1}{2}}}$.

From the equation for a pendulum *a* should be equal to $\frac{2\pi}{\sqrt{g}}$. Is it?

Table 15.3

ℓ/m	T/s	ln(ℓ/m)	ln(T/s)
0.500	1.42	−0.693	0.351
0.600	1.55		
0.700	1.68		
0.800	1.80		
0.900	1.90		
1.000	2.01		

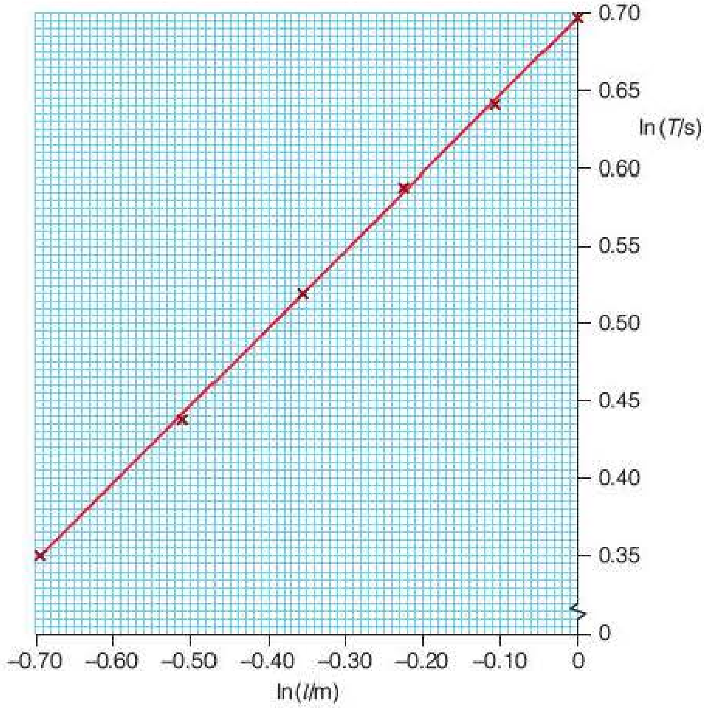


Figure 15.8

Tip

To get a large scale, it may be necessary to choose a graph scale that does not start at the origin. This is quite often the case when plotting a 'log' graph.

Tip

This is typical of the sort of analysis you will need to be familiar with for the Practical Assessment and in Paper 3.

Test yourself

1 A student sets up a simple pendulum to determine a value for the gravitational field constant. The student records the length as 75 cm and counts 5 oscillations in 8.8 s.

a) Calculate the value that these data give for the gravitational field constant.

b) Determine the percentage difference between this value and the accepted value for the gravitational field constant.

c) Make an estimate of the percentage uncertainty in the value obtained for the gravitational field constant.

d) Draw a diagram to show precisely the length that should be measured.

e) Criticise the student's results.

2 A teacher sets up a pendulum that is attached to a beam in the ceiling. The pendulum bob is initially a few centimetres from the floor. The arrangement is shown in Figure 15.9

a) Explain why $T = 2\pi\sqrt{\frac{(H-h)}{g}}$

b) Show that $T^2 = \frac{4\pi^2 H}{g} - \frac{4\pi^2 h}{g}$

c) Explain how you would determine values for the height H of the point of suspension (which you can't reach!) and the gravitational field constant g by taking suitable measurements and then plotting a graph of T^2 against the height h of the pendulum bob above the floor. Sketch the graph you would expect to get.

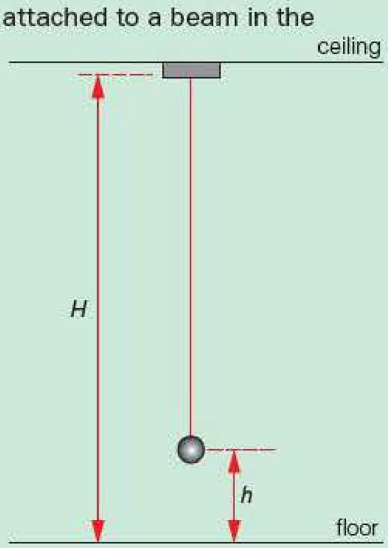


Figure 15.9

15.4 Equations of simple harmonic motion

Before we go any further, we should mention that in perfect simple harmonic motion the frequency of the oscillations *does not depend on the amplitude of oscillation*. We say the motion is **isochronous** – from the Greek 'iso' (the same) 'chronos' (time). In real situations, the motion only approximates to simple harmonic motion, but the following equations will still be more or less valid.

The basic defining equation for s.h.m. is

$$a = -\omega^2 x$$

The graph of this equation is shown in Figure 15.10. Note that the graph line is of finite length, determined by the maximum displacement, that is the amplitude $\pm A$. The corresponding maximum acceleration is $\pm\omega^2 A$.

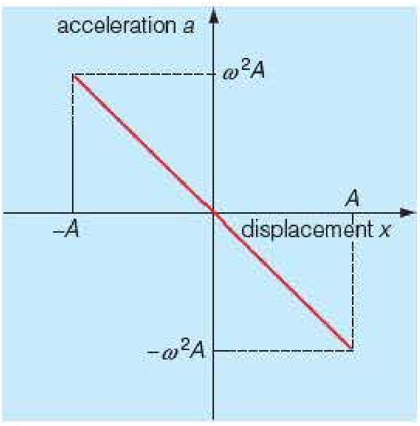


Figure 15.10

This equation has an infinite number of solutions for x, depending on the exact point in the motion at which we decide to start timing. An obvious place to start is at one extreme of the motion – after all, this is how we start a pendulum or spring oscillating, by displacing it and then letting go.

If $t = 0$ at one end of the motion, x will at that time be equal to the amplitude, A. The solution of the equation is then

$$x = A\cos\omega t$$

For the equation $x = A\cos\omega t$, a graph of the displacement x as a function of the time t will be a cosine graph of amplitude A, like Figure 15.11a.

From Year 1 you should recall that velocity is the gradient of a displacement–time graph. The velocity at any point of the s.h.m. will therefore be equal to the gradient of the cosine curve at that point. If you are doing A-level mathematics, you will know that this can be found by differentiating the above equation, which gives

$$v = \frac{d}{dt}(A\cos\omega t)$$

$$v = -\omega A\sin\omega t$$

For your A-level physics examination you just need to be able to recognise and use this expression. The velocity–time graph therefore looks like Figure 15.11b, which is a negative sine wave.

Tip

You do not need to have any knowledge of calculus for the examination.

Why is there a negative sign in the above equation? Well, think of pulling a pendulum to your right and letting go at $t = 0$, as in Figure 15.12. During the first half-cycle of its motion, the velocity of the pendulum will be back towards your left – in other words in the opposite sense to the displacement you have given it. Hence the negative sign. You can also see from the graph for x that the gradient during the first half-cycle is negative.

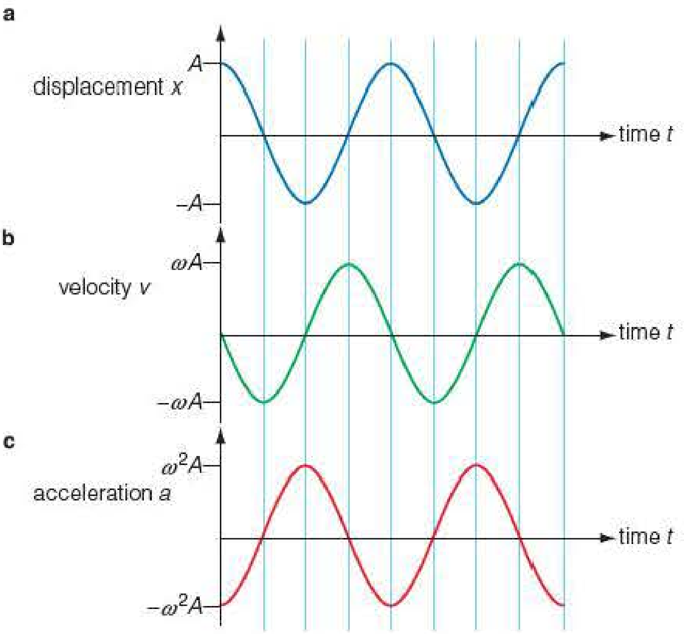


Figure 15.11 Graphical representation of s.h.m

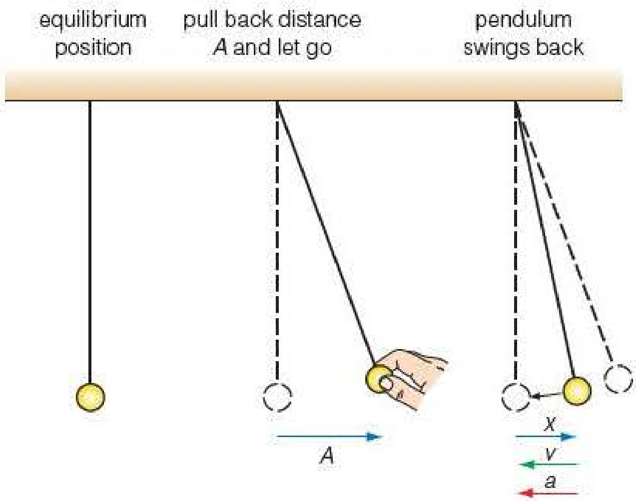


Figure 15.12 If the pendulum is pulled back through a small angle, the distances A and x are proportional to the angular displacement.

Tip

When sketching graphs such as these, draw vertical lines every $\frac{1}{4}$ cycle. Mark where each line has its maximum, minimum and zero values, and then sketch the sine or cosine wave between these points.

There are two key points in the motion:

- at each end of the oscillation, when the velocity is momentarily zero, the displacement curve is a maximum, positive or negative (A or $-A$), and its gradient = 0;
- at the centre of the motion, corresponding to $x = 0$, the velocity has its maximum value – you can see from the graph for x that the gradient, and hence the velocity, is a maximum at each point where the curve crosses the t-axis; furthermore, the gradient alternates between being positive and negative as the body moves first one way, and then back again, through the midpoint of its oscillations.

We have seen how the displacement and velocity vary with time. What about the *acceleration*? As acceleration is defined as the rate of change of velocity, the acceleration will be given by the gradient of the velocity–time graph at any instant. Again, for the mathematicians, this is given by differentiation and yields

$$a = \frac{dv}{dt} = \frac{d}{dt}(-\omega A\sin\omega t)$$

$$a = -\omega^2 A\cos\omega t$$

Don't worry if you can't differentiate – we can arrive at this expression in another way. If we substitute $x = A\cos\omega t$ into $a = -\omega^2 x$, we get $a = -\omega^2 A\cos\omega t$ directly! The graph of this expression is shown in Figure 15.11c.

Tip

Remember:

- $v_{max} = \pm\,\omega A$ as the maximum value that $\sin\omega t$ can have is 1
- $a_{max} = -\omega^2 A$ when x has its maximum value, which is the amplitude A

Activity 15.3

Obtaining the graphs for simple harmonic motion using a motion sensor

Figure 15.13a shows the arrangement for a spring. A card is attached to the masses to give a good reflective surface, but may not be needed if the base of the masses is large enough. Typically the data logger might be set to record for 10 seconds at a sampling rate of 100 per second. The computer can be programmed to give displays of displacement, velocity and acceleration against time, as shown in Figure 15.14.

In Figure 15.13b, a pendulum is shown connected to a rotary sensor. Such a sensor is useful for investigations of rotational motion – in this case it will measure the *angular* displacement of the pendulum.

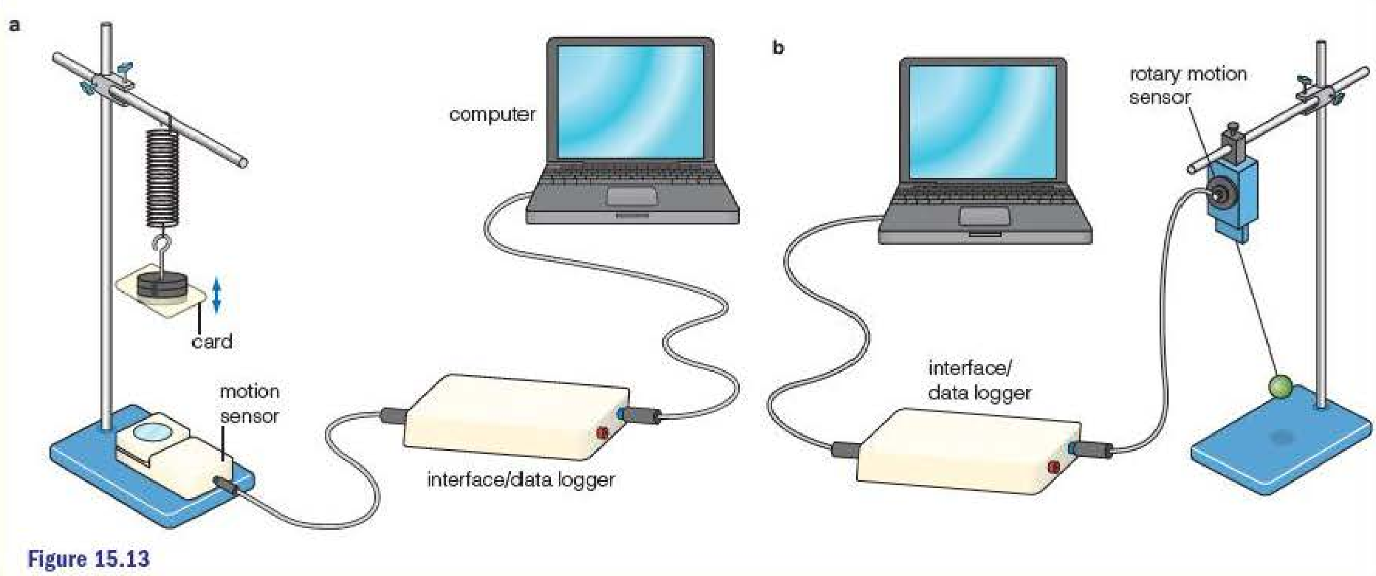


Figure 15.13

Example

A printout from an experiment to investigate the motion of a mass oscillating vertically on a spring is shown in Figure 15.14.

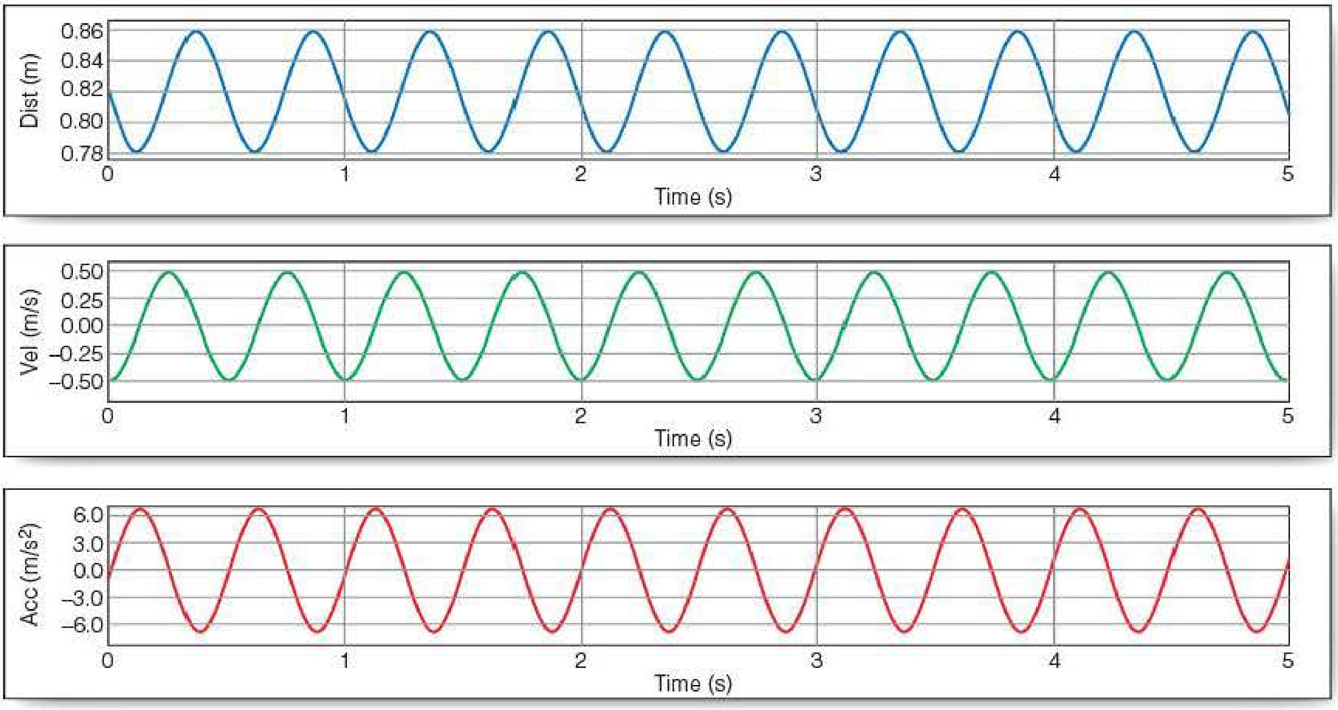


Figure 15.14

a) Use the plot of displacement against time to determine the amplitude A, the period T and the frequency f of the motion.

b) Calculate the spring constant (stiffness) k, given that $T = 2\pi\sqrt{\frac{m}{k}}$ and $m = 200\,\text{g}$.

c) The equation for velocity is $v = -\omega A \sin\omega t$. Use your answers from part **a)** to show that the constant ω is $4\pi\,\text{s}^{-1}$, given that $\omega = 2\pi f$.

d) Use the equation $v = -\omega A \sin\omega t$ to calculate the maximum value of the velocity and compare your value with that from the velocity–time plot.

e) Use the equation $a = -\omega^2 x$ to calculate the maximum acceleration and compare your value with that from the acceleration–time plot.

f) Sketch a graph of acceleration against displacement for the motion.

Answer

a) The amplitude is the maximum displacement in either direction from the centre of the oscillations, (0.86 – 0.82) m and (0.78 – 0.82) m, so $A = \pm 0.040\,\text{m}$.

The period is the time for one complete oscillation. There are two complete oscillations in 1 s,

so $T = 0.50\,\text{s}$.

From $f = \frac{1}{T} \Rightarrow f = \frac{1}{0.50\,s} = 2.0\,\text{Hz}$

b) From $T = 2\pi\sqrt{\frac{m}{k}}$

$\Rightarrow T^2 = 4\pi^2 \frac{m}{k}$

$\Rightarrow k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2 \quad 0.200\,\text{kg}}{(0.50\,\text{s})^2} = 32\,\text{N}\,\text{m}^{-1}$

c) $\omega = 2\pi f = 2\pi \times 2.0\,\text{s}^{-1} = 4\pi\ \text{s}^{-1}$

d) From the equation $v = -\omega A \sin\omega t$, the maximum velocity will be when $\sin\omega t$ has its maximum value, as ω and A are constants. The maximum value that a sine can have is 1, giving:

$v_{max} = \pm\,\omega A = \pm\,4\pi\text{s}^{-1} \times 0.040\,\text{m} = \pm\,0.50\,\text{m}\,\text{s}^{-1}$

From the velocity–time plot we can read off that the maximum velocity is, indeed, 0.50 m s^{-1}.

e) From the equation $a = -\omega^2 x$, the maximum acceleration will be when the displacement x is a maximum, that is when $x = A$:

$\Rightarrow a_{max} = \omega^2 A = (4\pi\ \text{s}^{-1})^2 \times 0.040\,\text{m} = 6.3\,\text{m}\,\text{s}^{-2}$

From the acceleration–time plot we can see that the maximum acceleration is just over 6.0 m s^{-2}, which agrees with the calculated value.

f) The graph of acceleration against displacement is the graph of $a = -\omega^2 x$. It is therefore a straight line of negative slope passing through the origin, as shown in Figure 14.16.

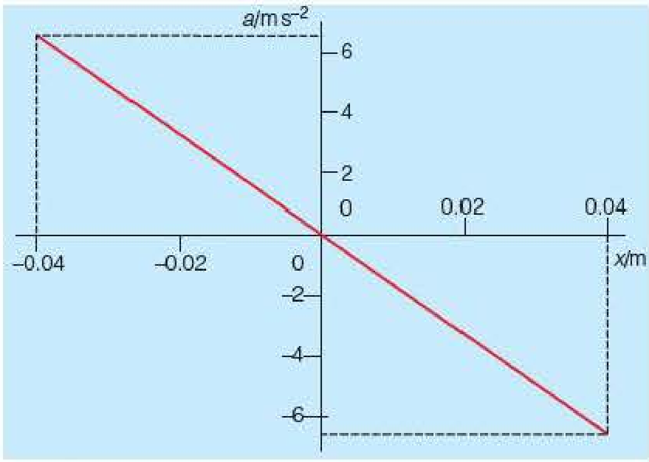


Figure 15.15

Tip

If you are asked to *sketch* a graph, the axes must always be labelled, with units, and also given a scale if you know any numerical values. Your line should then be related to the scale. You do *not* however need to draw the graph accurately on graph paper.

In this example, your scales should be labelled to show the limits of the line, in this case $a = \pm\,6.3\,\text{m}\,\text{s}^{-2}$ when $x = \pm\,0.040\,\text{m}$.

The meaning of ω

We have used ω^2 as the constant in the equation that defines s.h.m, and we said that we would explain later what ω was. So, let's start by plotting a graph of $\cos\omega t$ against ωt: Figure 15.16.

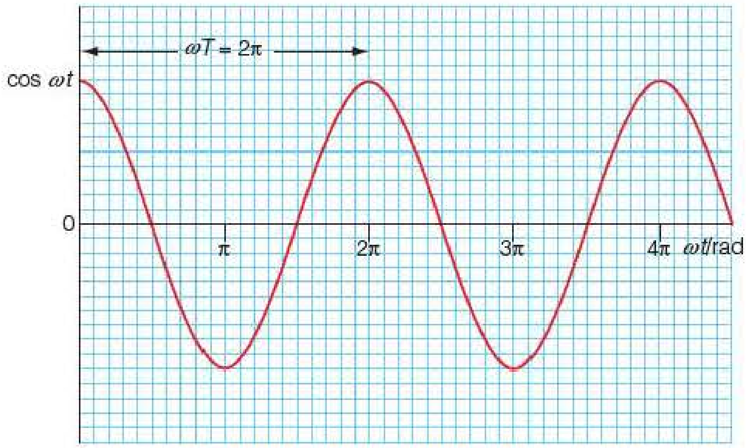


Figure 15.16

Note that ωt is in *radians*. For one complete cosine wave, $\omega t = 2\pi$ radians. One complete cosine wave represents one oscillation, which takes a time T, the period. This means that $t = T$. We therefore have that $\omega T = 2\pi$, or

$$\omega = \frac{2\pi}{T}$$

This means that ω has units of rad s^{-1}, which are the units for angular velocity. As the frequency $f = \frac{1}{T}$ we also have

$$\omega = 2\pi f \quad \text{and} \quad f = \frac{\omega}{2\pi}$$

Example

Figure 15.17 shows one of the cylinders of a car engine. To a good approximation, the piston oscillates up and down with simple harmonic motion. In this engine, the length of the stroke is 80 mm and the piston has a mass of 600 g.

a) What is the amplitude of the piston's motion?

b) What is the frequency of the motion when the engine is running at 3000 revolutions per minute?

c) Show that the maximum acceleration experienced by the piston is about 4000 m s^{-2}. At which point in its motion does this occur?

d) Calculate the maximum force exerted on the piston.

e) Aluminium alloys, which are both light and strong, are being developed for manufacturing pistons. Suggest why.

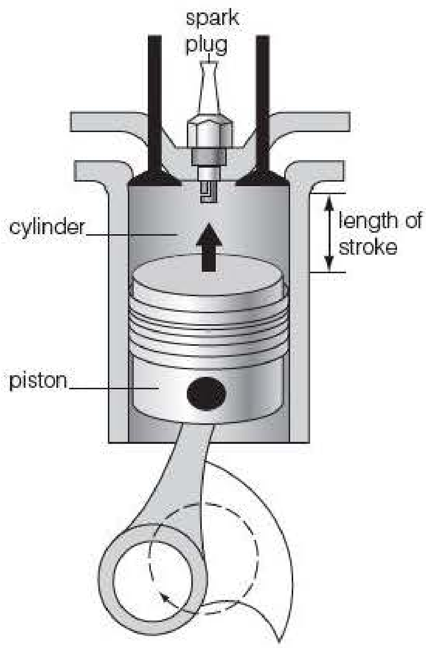


Figure 15.17

Answer

a) The stroke is the distance from the top to the bottom of the piston's motion. The amplitude is the maximum displacement, in either direction, from the midpoint of the motion, so the amplitude is *half* the stroke. The amplitude is therefore 40 mm.

b) The frequency will be 3000/(60 s) = 50 s^{-1} (or Hz).

c) From the s.h.m. defining equation
$a = -\omega^2 x \Rightarrow a = -(2\pi f)^2 x.$

The piston will have maximum acceleration when x has its maximum value, that is when x is equal to the amplitude, A. Its magnitude will be

$$a_{max} = (2\pi f)^2 A = (2\pi \times 50\,s^{-1})^2 \times 0.040\,m$$
$$= 3948\,m\,s^{-2} \approx 4000\,m\,s^{-2} \text{ (or } 400g\text{)}$$

This will occur at the top and bottom of the piston's motion.

d) The maximum force acting on the piston will be given by $F = ma$.

$$F_{max} = 0.600\,kg \times 4000\,m\,s^{-2} = 2400\,N$$

e) A light piston reduces the forces acting on the connecting rod, which enables higher revolutions per minute. It needs to be strong to withstand the very large forces acting on it without distorting.

Tip

There are two points to remember in a calculation like this:

- the amplitude is *half* the distance between the two extremes of the motion, and
- care must be taken to put brackets round $2\pi f$ when squaring it, $(2\pi f)^2$, to ensure that each term is squared.

Example

In a harbour, the water is 4.0 m deep at low tide and 10.0 m deep at high tide. The variation in water level with time is, to a good approximation, simple harmonic motion, with two high tides per day.

a) What is the period, in hours, of this motion?

b) What is the amplitude of the motion?

c) Sketch a graph of the depth of water against time for one day, beginning at high tide.

d) Calculate the depth of water **i)** 2.0 hours after high tide, and **ii)** 1.5 hours after low tide. Show these points on your graph.

Answer

a) If there are two tides per day (24 hours), the period will be 12 hours.

b) The difference in depth between high tide (10.0 m) and low tide (4.0 m) is 6.0 m. The amplitude is the maximum displacement from the midpoint, in either direction, that is ± 3.0 m from the mid-depth of 7.0 m.

c)

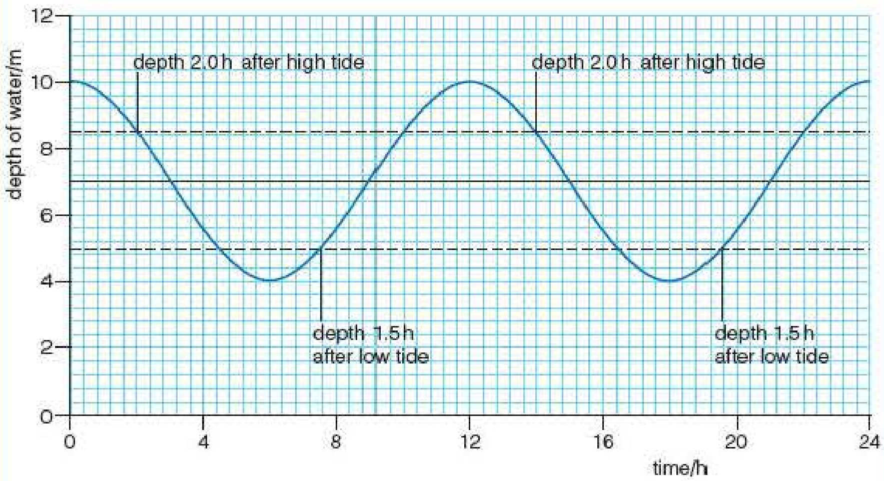


Figure 15.18

d) It might be helpful to draw the two lines on the graph first, as shown, so that you can check that you get sensible answers for the depths of the water.

i) We know that $x = A\cos\omega t$, where $A = 3.0\,\text{m}$ and $t = 2.0\,\text{h}$. Therefore to find x, we must first of all find ω.

$$\omega = \frac{2\pi}{T} = \frac{2\pi\,\text{rad}}{12.0\,\text{h}} = \frac{\pi}{6}\ \text{rad}\,\text{h}^{-1}$$

$$x = A\cos\omega t \Rightarrow x = 3.0\,\text{m} \times \cos\left(\frac{\pi}{6}\text{rad h}^{-1} \times 2.0\,\text{h}\right)$$

$$= 3.0\,\text{m} \times \cos\left(\frac{\pi}{3}\text{rad}\right) = 3.0\,\text{m} \times 0.50 = 1.5\,\text{m}$$

This means that the water level is 1.5 m *above* its midpoint, so the water is $7.0\,\text{m} + 1.5\,\text{m} = 8.5\,\text{m}$ deep.

ii) Low tide will occur 6.0 hours after high tide (taken as $t = 0$), so 1.5 hours after low tide means that $t = 7.5$ h.

$$x = A\cos\omega t \Rightarrow x = 3.0\,\text{m} \times \cos\left(\frac{\pi}{6}\text{rad h}^{-1} \times 7.5\,\text{h}\right)$$

$$= 3.0\,\text{m} \times \cos\left(\frac{5\pi}{4}\text{rad}\right) = 3.0\,\text{m} \times -0.707$$

$$= -2.1\,\text{m}$$

This means that the water level is 2.1 m *below* the midpoint, so the water is $7.0\,\text{m} - 2.1\,\text{m} = 4.9\,\text{m}$ deep.

Tip

Three things to remember:

- x is the *displacement*, which means it is a *vector* – so its sign represents its *direction* relative to the equilibrium position;
- ωt must be in *radians* – you must therefore make sure you know how to put your calculator into radian mode and remember to do so; and
- ω and t must have *consistent* units – in this example there was no need to convert ω from rad h^{-1} into rad s^{-1} because t was in hours.

Alternating current

As we saw in Chapter 6, our electricity supply is generated and transmitted as alternating current, the waveform of which is sinusoidal. An a.c. generator consists of a cylindrical electromagnet, driven by a turbine, rotating at high speed inside large conducting coils. Every half cycle the orientation of the magnet with respect to the coils reverses and so the induced current in the coils changes direction. The geometry of the generator is such that the rate of change of flux linking the coils, and hence the induced e.m.f., is sinusoidal. This can be shown by connecting the a.c. output of a low voltage power supply unit to an oscilloscope as shown in Figure 15.19.

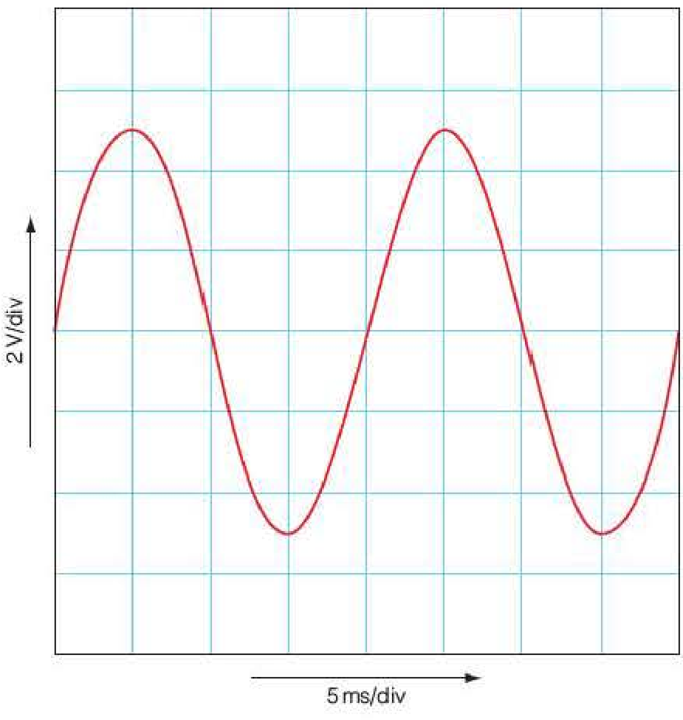


Figure 15.19 Sinusoidal alternating voltage displayed on an oscilloscope

The oscilloscope is set on 2 V per division and 5 ms per division.

The amplitude of the trace is 2.5 divisions, so the peak voltage is

$$V_0 = 2.5 \text{ divisions} \times 2\,\text{V division}^{-1} = 5.0\,\text{V}$$

One wavelength occupies 4 divisions, so the period is

$$T = 4 \text{ divisions} \times 5\,\text{ms division}^{-1} = 20\,\text{ms}$$

The frequency is therefore

$$f = \frac{1}{T} = \frac{1}{20}\,\text{ms} = 50\,\text{s}^{-1} = 50\,\text{Hz}.$$

We can therefore write the equation of the alternating current as

$$x = A\cos(2\pi ft) \Rightarrow V = V_0\cos(2\pi ft) = 5\cos(100\pi t)$$

The s.h.m. 'constant' ω is therefore 100π rad s^{-1}, which is the angular velocity of the turbine.

Tip

You do not need to have any knowledge of calculus for the examination.

Root mean square voltage and power

As an alternating voltage is not constant, varying from V_0 to 0 each half cycle, the power developed in a component having resistance R is **not** $\frac{V_0^2}{R}$.

At a time t, power $P = \frac{V^2}{R} = \frac{V_0^2\cos^2(\omega t)}{R}$

Averaging over one cycle $P = \frac{\int_0^T V_0^2\cos^2(\omega t)\,dt}{R} = \frac{V_0^2}{2R}$ (You can check this for yourself if you can integrate!)

The **average** power is therefore equal to **half** the peak power as we saw in Section 6.8.

Test yourself

3 A 200 g mass is suspended on a vertical spring. When it is pulled down a distance of 50 mm and released it makes 20 oscillations in 14.1 s.

a) The maximum acceleration of the mass is approximately

A: 0.12g C: 0.41g

B: 0.20g D: 0.56g

b) At which point, or points, does the mass have maximum acceleration?

A: middle C: top only

B: bottom only D: top and bottom

c) At which point, or points, does the mass have maximum velocity?

A: middle C: top only

B: bottom only D: top and bottom

d) The spring constant k is approximately

A: 2.5 N m^{-1} C: 16.0 N m^{-1}

B: 8.0 N m^{-1} D: 40.0 N m^{-1}

4 The needle of a sewing machine moves up and down with simple harmonic motion. The total vertical motion of the needle is 16 mm and it makes 720 stitches per minute. What is the maximum speed of the tip of the needle?

5 A Foucault pendulum is a very long pendulum that can demonstrate the rotation of the Earth. Physics students visiting the Science Museum in London time that it takes 95 seconds for 10 oscillations of the museum's Foucault pendulum. The amplitude of the oscillations is 1.20 m.

a) Calculate the length of the pendulum.

b) What angle does the pendulum swing through? Explain why the pendulum equation

$$T = 2\pi\sqrt{\frac{l}{g}}$$

is valid in this situation.

c) Calculate the maximum acceleration of the pendulum bob. At which point of its motion will this occur?

d) If the pendulum is at one end of its swing at $t = 0$, where will it be 3.0 seconds later in relation to its starting point?

(Remember to set your calculator to 'rad' mode!)

15.5 Energy in simple harmonic motion

As a pendulum swings to and fro there is a continuous interchange of kinetic and (gravitational) potential energy. At one end of its swing, the pendulum momentarily comes to rest and so its kinetic energy is zero. At this point it will have maximum potential energy because the bob is at its highest point. As the bob swings down, it loses gravitational potential energy and gains kinetic energy. At the bottom of the swing, the midpoint of the motion, the bob will have maximum velocity and thus maximum kinetic energy. As this is the lowest point of its motion, its gravitational potential energy will have its minimum value. See Figure 15.20. This cyclic interchange of energy is repeated twice every oscillation.

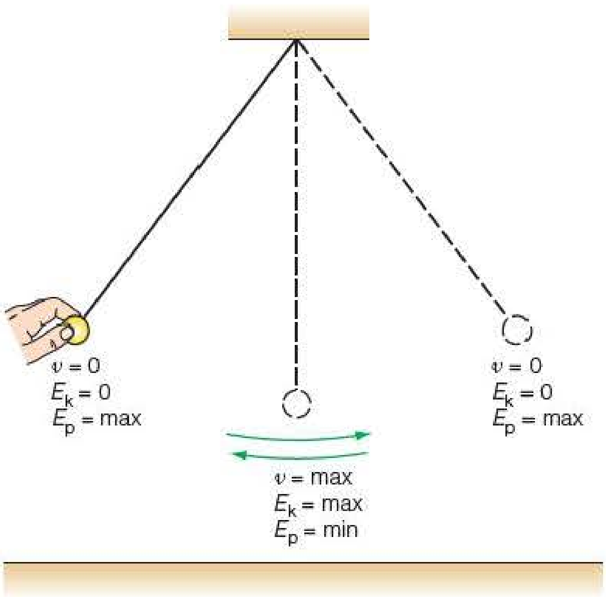


Figure 15.20

We saw in the Example on page 291 that, from the equation $v = -\omega A\sin\omega t$, the maximum velocity will be when $\sin\omega t = \pm1$, giving

$$v_{max} = \pm\omega A.$$

Kinetic energy $E_k = \frac{1}{2}mv^2 \Rightarrow (E_k)_{max} = \frac{1}{2}m(v_{max})^2 = \frac{1}{2}m\omega^2A^2$

This occurs at the equilibrium position, where the potential energy has its minimum value. If we define that the potential energy is zero at the equilibrium position, then the total energy of the system at the equilibrium position is just the kinetic energy, $\frac{1}{2}m\omega^2A^2$. If there is no damping (that is, no energy is transferred to the surroundings), by the law of conservation of energy the *total* energy ($E_k + E_p$) of the system must be constant and equal to $\frac{1}{2}m\omega^2A^2$. This is shown in Figure 15.21.

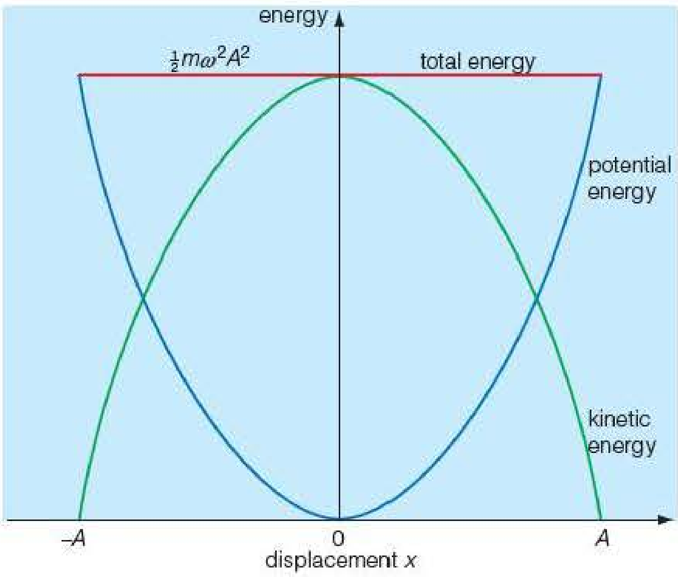


Figure 15.21 Energy interchange in s.h.m.

Tip

Note that the E_k curve is the mirror image of the E_p curve.

Example

The original pendulum used by Foucault to demonstrate the rotation of the Earth consisted of a 67 m wire supporting a 28 kg cannon ball. Imagine this cannon ball is pulled back through a distance of 3.0 m and released.

a) Show that its period of oscillation is approximately 16 s.

b) Through how many degrees will the Earth have rotated in this time?

c) Calculate **i)** the maximum velocity, and **ii)** the maximum kinetic energy of the cannon ball.

d) Discuss the energy changes that take place **i)** during the first half oscillation, and **ii)** over a period of several oscillations.

Answer

a) $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{67\,\text{m}}{9.8\,\text{m s}^{-2}}} = 16.4\,\text{s} \approx 16\,\text{s}$

b) The Earth rotates through 360° in one day
$= (24 \times 60 \times 60)\,\text{s} = 864\,000\,\text{s}$

In 16 s it will have rotated

$$\frac{16\,\text{s}}{864\,000\,\text{s}} \times 360° = 6.7 \times 10^{-3} \text{ degrees}$$

c) i) From $v = -\omega A \sin\omega t \Rightarrow v_{max} = \pm\, \omega A$

$$\omega = 2\pi f = \frac{2\pi}{T} = \frac{2\pi}{16.4\,\text{s}} = 0.383\,\text{s}^{-1}$$

$$v_{max} = \pm\, \omega A = \pm\, 0.383\,\text{s}^{-1} \times 3.0\,\text{m} = \pm\, 1.15\,\text{m}\,\text{s}^{-1}$$

ii) $(E_k)_{max} = \frac{1}{2}m(v_{max})^2 = 0.5 \times 28\,\text{kg} \times (1.15\,\text{m}\,\text{s}^{-1})^2 = 19\,\text{J}$

d) i) At the point of release, the cannon ball will have 19 J of gravitational potential energy. As it starts swinging, this potential energy will be gradually changed into kinetic energy as the ball begins to fall and gain velocity. At the bottom of the swing, the ball will have maximum velocity and all the potential energy will have been converted into 19 J of kinetic energy. As the ball starts to rise again, its kinetic energy will gradually be converted back into potential energy. When it comes to rest at the end of its swing, its kinetic energy will be zero and the potential energy will once again be 19 J.

ii) In practice, some energy will be transferred to the surroundings due to air resistance acting on the bob and vibration at the point of suspension. After several oscillations there will be a considerable reduction in the energy of the oscillating system and the amplitude of the motion will be noticeably less. We say that the oscillations have been **damped**.

15.6 Free, damped and forced oscillations

These are terms that you must know and understand. If an oscillating body is displaced and then released it will begin to vibrate. If no more external forces are applied to the system the oscillations are said to be **free oscillations**. In practice, this is virtually impossible to achieve as the oscillating body will invariably experience air resistance and other frictional forces, for example at the point of suspension of a pendulum or spring. If a small, dense pendulum bob on a thin thread is pulled back and released, the ensuing oscillations will approximate to free oscillations.

An oscillating system does work against the external forces acting on it, such as air resistance, and so uses up some of its energy. This transfer of energy from the oscillating system to internal energy of the surrounding air causes the oscillations to slow down and eventually die away – the oscillations are **damped**, as in the previous Example on Foucault's pendulum. In a pendulum clock, this energy is gradually restored to the oscillating pendulum by means of a coiled spring, which has to be re-wound from time to time.

Key terms

A **free oscillation** is one in which no external force acts on the oscillating system except the force that gives rise to the oscillation.

A **damped** oscillation is one in which energy is being transferred to the surroundings, resulting in oscillations of reduced amplitude and energy.

Key term

Forced oscillations occur if a force is continually or repeatedly applied to keep the oscillation going so that the system is made to vibrate at the frequency of the vibrating source and not at its own natural frequency of vibration.

When you set your mobile phone on 'vibration mode' and it rings, the part of your body in contact with the phone 'feels' the vibrations. This is because your body has been made to vibrate by the phone. You are experiencing **forced oscillations**, which have the *same frequency* as the vibrating source – in this case your mobile phone. Similarly, you hear sound because the mechanical oscillations of the air particles forming the sound wave force your eardrum to vibrate at the frequency of the sound wave. Your ear converts these mechanical vibrations into an electrical signal, which your brain interprets as a sound.

Activity 15.4

Investigating damped oscillations

Damped oscillations can be investigated by means of a long pendulum with a paper cone attached to the bob to increase air resistance.

This experiment is best carried out in pairs, with one person observing the amplitude at one end of the oscillation and the second person at the other end, as shown in Figure 15.22. Alternatively a motion sensor can be used to record the displacement of the oscillations. It works best if the pendulum can be suspended from the ceiling to give a longer period, although reaching the ceiling can be hazardous!

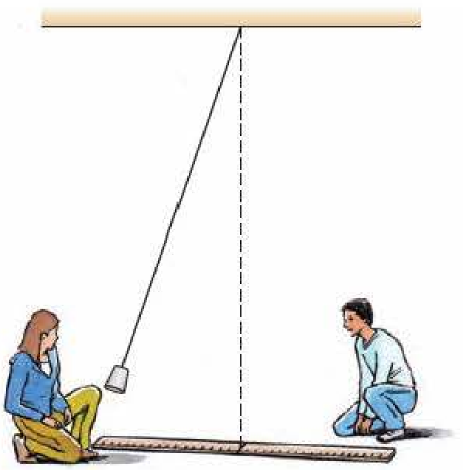


Figure 15.22

Initially the period T of the pendulum *without* the cone is found by timing 10 oscillations twice and finding the average. Then the cone is attached.

The pendulum is now pulled back so that the cone is level with the end of one of the metre rules – that is an initial amplitude of 1.00 m. After letting go, the amplitude at the end of each swing is recorded until it is too small to measure. It may be necessary to experiment with the size of the cone so that it is possible to get about four or five complete oscillations.

The results are tabulated as in Table 15.4 in the question below and a sketch graph of displacement against time is plotted.

Question

The following data were obtained for a damped pendulum.

$10T$/s	$10T$/s	Average T/s
29.81	30.17	3.00

Table 15.4

No. of swings	0	1	2	3	4	5	6	7	8
Amplitude/m	1.00	−080	0.64	−0.51	0.41	−0.33	0.26	−0.21	0.17

Plot a sketch graph of displacement against time. You can plot the points corresponding to the amplitude at the end of each swing, remembering that each swing is half a period and that the displacement will be alternately positive and negative. You now have to use some skill in joining the points with a cosine-like curve. Your graph should look like Figure 15.23.

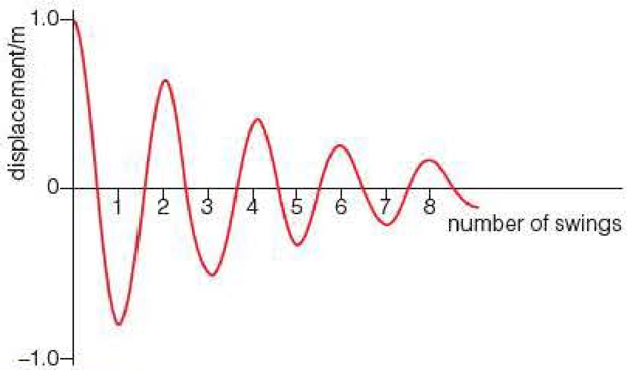


Figure 15.23

The damping looks as though it could be exponential. Can you remember how to test this? If not, you will have to look at the Example that follows.

Example

To check whether the damping is exponential, we have to look at successive *ratios* of the amplitude. We can do this by processing the data in Table 15.4 as shown in Table 15.5 below:

Table 15.5

Swings	0→1	1→2	2→3	3→4
Ratio of amplitudes	$\frac{1.00}{0.80} = 1.25$	$\frac{0.80}{0.64} = 1.25$	$\frac{0.64}{0.51} = 1.25$	$\frac{0.51}{0.41} = 1.24$
Swings	4→5	5→6	6→7	7→8
Ratio of amplitudes	$\frac{0.41}{0.33} = 1.24$	$\frac{0.33}{0.26} = 1.27$	$\frac{0.26}{0.21} = 1.24$	$\frac{0.20}{0.17} = 1.24$

We can see that the ratio is very nearly constant, which shows that the damping is exponential.

Test yourself

6 a) Copy and complete the data in Table 15.6 below to show the ratios of successive positive amplitudes, using data from the Activity question above.

Table 15.6

Swings	0→2	2→4	4→6	6→8
Ratio of positive amplitudes	$\frac{1.00}{0.64} = 1.56$			

b) Discuss the extent to which the data shows that the damping is exponential.

Tip

Remember that the 'number of swings' means from one extreme to the other and so is the number of *half* periods.

15.7 Resonance

A simple way of demonstrating, and understanding, what is meant by 'resonance' is to use the arrangement shown in Figure 15.24, which is known as Barton's pendulums.

Figure 15.24a shows the initial set-up. The heavy, dense pendulum bob, at the right-hand end, is pulled back a few centimetres and released. This pendulum will oscillate at its **natural frequency** f_0, determined by its length and given by the equation $T = 2\pi\sqrt{\frac{l}{g}}$. All the other, light, pendulums are coupled to this driver pendulum by the string to which they are tied. They experience **forced oscillations**, equal in frequency to that of the driving pendulum, as some of the energy of the driving pendulum will be transferred to each of the other pendulums. Although they will all be set in motion, the pendulums of length nearest to the length of the driving pendulum will absorb more energy because their natural frequency of vibration is close to that of the driving pendulum. These pendulums oscillate with larger amplitude as shown in Figure 15.24b. A pendulum having the *same* natural frequency (same length) as the driving pendulum will absorb by far the most energy and will be forced to oscillate with very large amplitude. This is called **resonance**.

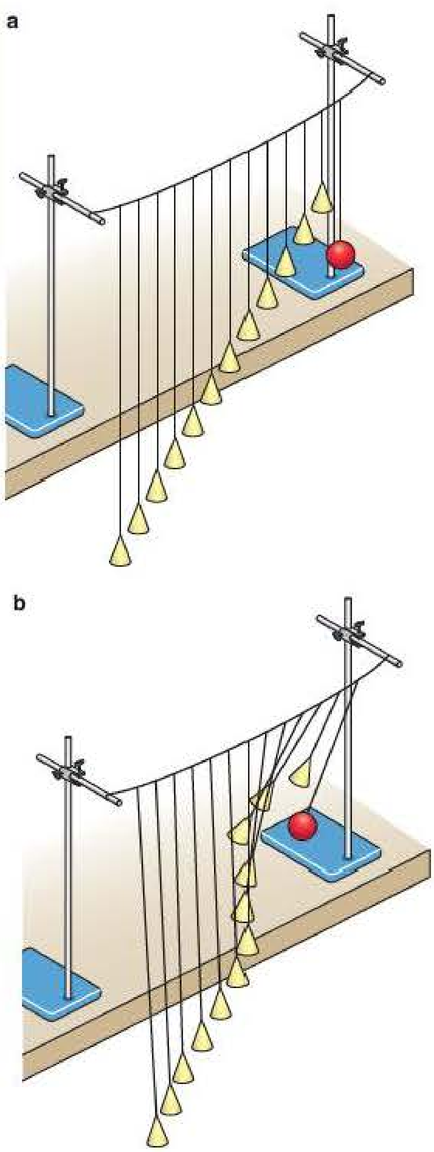


Figure 15.24 Barton's pendulums

Key term

Resonance occurs when an oscillating system is forced to oscillate by an outside source at a frequency that is the same as its own natural frequency.

Activity 15.5

Investigating resonance

Resonance can be investigated using the arrangement shown in Figure 15.25.

The mass is given a small vertical displacement and the period of oscillations, T_0, is determined by timing 20 oscillations and repeating. Hence the natural frequency of oscillation, f_0, can be calculated

The signal generator should be set to a frequency f of about $\frac{1}{2}f_0$ and then switched on. The frequency is gradually increased until the mass vibrates with maximum amplitude – this should happen when $f \approx f_0$. The experiment is repeated, this time starting with $f \approx 2f_0$ and *reducing* the frequency until resonance occurs. An average value for the resonant frequency can then be found. This form of averaging is a technique that you should develop for doing an experiment like this.

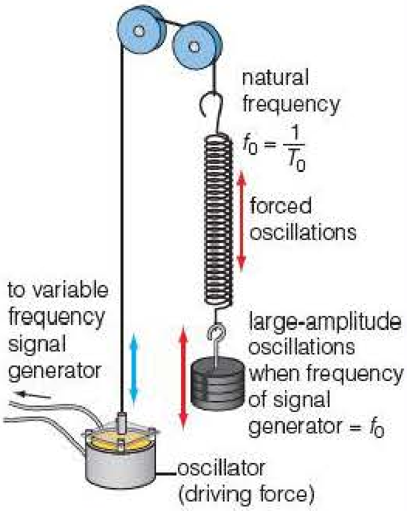


Figure 15.25

The experiment can then be repeated using different masses.

Note

This is an alternative to Core Practical 16a in Section 15.3.

Core practical 16b

Determination of an unknown mass using the resonant frequencies of the oscillations of known masses

The same arrangement is set up as shown in Figure 15.25. The resonant frequency of the oscillations for a range of different masses is found, e.g 100 g – 400 g at 50 g intervals. A graph is then drawn of the resonant frequency f against the mass m.

The resonant frequency f_u for an unknown mass, e.g. a cube of wood of side 75 mm, is then found. The mass m_u of the wood can be determined by reading off the value of m corresponding to f_u from the graph.

Questions

1 A student collected the following data for an experiment such as that described above:

m/kg	0.100	0.150	0.200	0.250	0.300	0.350	0.400	m_u
f/Hz	2.60	2.15	1.85	1.65	1.50	1.40	1.30	1.60

Plot a graph of f on the y-axis against m on the x-axis.

2 Describe the technique you would use to try to get a precise value of the resonant frequency for each mass.

3 Use your graph to get a value for the mass m_u of the wooden block.

4 Estimate the uncertainty in your value for the mass of the block.

5 The student's teacher suggests that it might be better to plot a graph of f^2 against $\frac{1}{m}$. Explain why this could be advantageous.

A graph of the amplitude of oscillation of a particular mass as a function of the driving frequency would look like Figure 15.27.

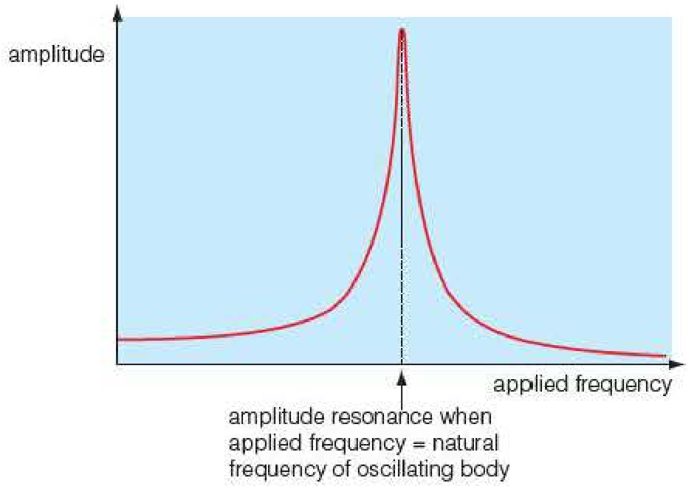


Figure 15.26 For a system with little or no damping, resonance occurs when the applied frequency equals the natural frequency

In Figure 15.26 we can easily see that the maximum amplitude, i.e. *amplitude resonance*, occurs when the driving frequency f is equal to the natural frequency of vibration of the mass, f_0. This is only the case if there is very little damping. As the mass oscillates up and down with large amplitude, there is clearly *energy transfer* from the signal generator to the mass.

For a damped system, the situation is somewhat complex. All you need to know is that, if there is damping, then the resonant frequency at which the *amplitude* is a maximum is lower than the natural frequency, and that this difference increases as the degree of damping increases. This is shown in Figure 15.27. However, the maximum energy transfer, or *energy resonance*, always occurs at the natural frequency.

Figure 15.27 also shows two other features of damped resonance – as the amount of damping increases, the resonant peak is much lower, and the resonance curve broadens out.

Tip

When you explain what happens in resonance always consider the *energy transfer* that takes place.

Tip

Be careful to distinguish between *amplitude* resonance and *energy* resonance.

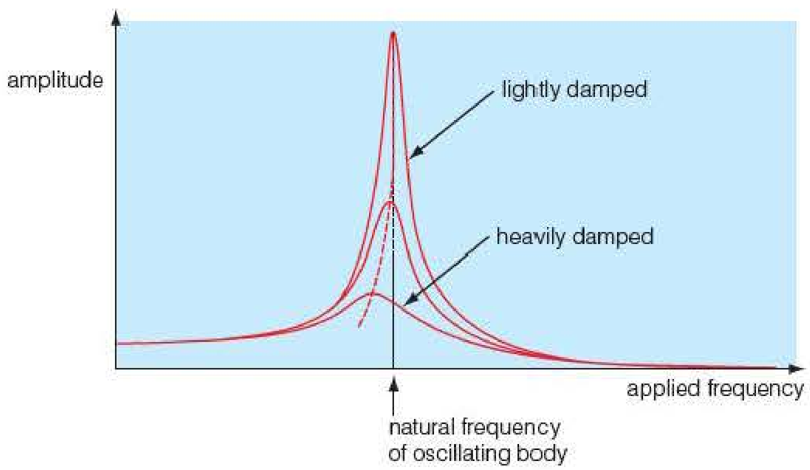


Figure 15.27 For a damped system, amplitude resonance occurs at a frequency that is lower than the natural frequency

Activity 15.6

Investigating a damped system

Damped oscillations can be investigated using a mass on a spring as described in the Activity 15.3 'Obtaining the graphs for simple harmonic motion using a motion sensor'. The apparatus is arranged as shown in Figure 15.13. Damping is provided by a cardboard disc held in between the masses, as shown in Figure 15.28.

Printouts for a small disc (light damping) and for a large disc (heavier damping as more air resistance) are shown in Figure 15.29. The experiment can be extended to investigate how the damping depends on the area of the disc.

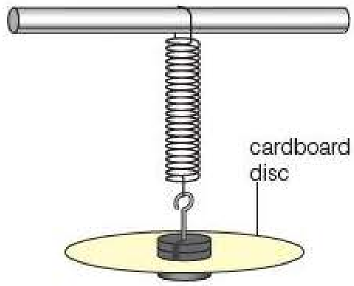


Figure 15.28

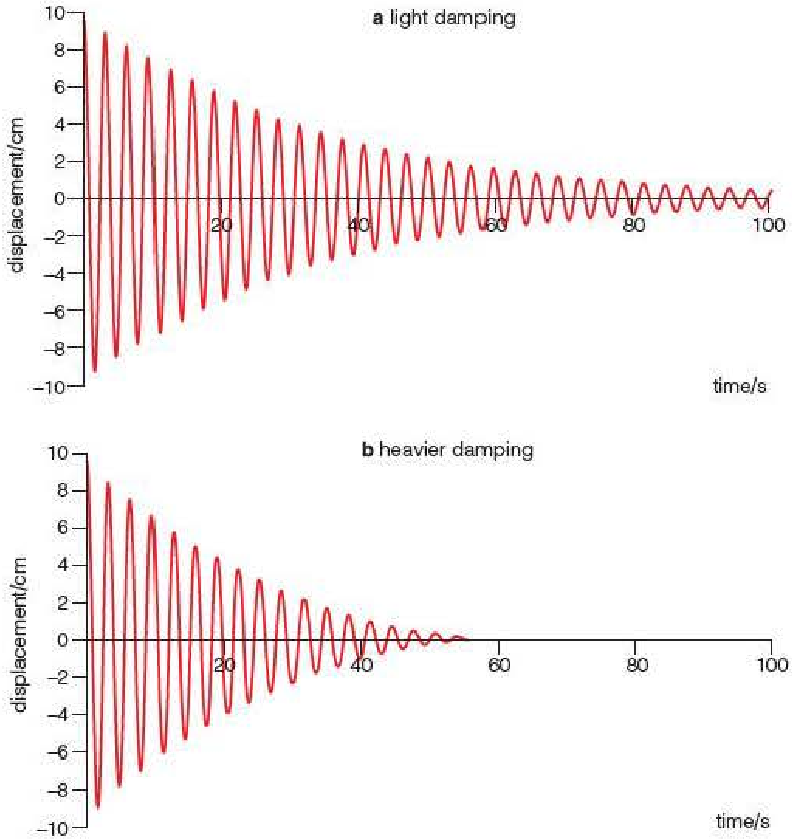


Figure 15.29

Questions

1 Determine the period for both the light and the more heavily damped oscillations, explaining how you did this. Discuss whether there is any significant difference in the two values.

2 The amplitude a after a time t for an exponential decay with an initial amplitude a_0 is given by

$$a = a_0 e^{-kt} \text{ where } k \text{ is a constant}$$

a) Show that

$$\ln (a/\text{mm}) = -kt + \ln(a_0/\text{mm})$$

b) Tabulate values of a for the first 7 oscillations of the more heavily damped motion (the displacement scale is reduced by a factor of 4, but for this exercise there is no need to scale-up the displacement values – you can take a_0 as 25 mm, etc.)

c) Plot a suitable graph to test whether the damping is exponential.

d) Comment on your result.

Activity 15.7

Demonstration of electrical resonance

Resonance can be demonstrated very elegantly with an electrical circuit containing a capacitor, coil and resistor connected in series with a signal generator giving an alternating current. This is shown in Figure 15.30.

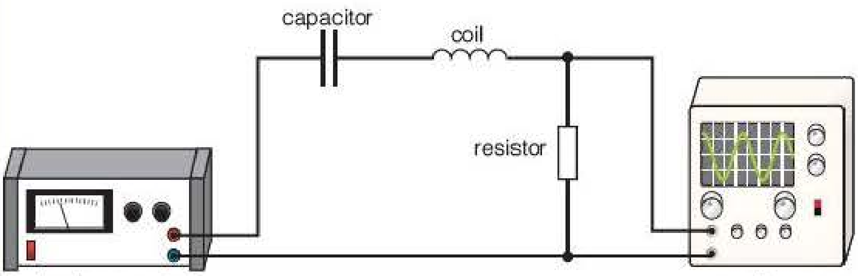


Figure 15.30

In one quarter-cycle of the alternating current, the capacitor charges up and stores energy in the electric field between its plates. When the capacitor is fully charged, the current in the circuit (and therefore in the coil) is zero and so there is no magnetic field in the coil. In the next quarter-cycle, the capacitor discharges, charge flows in the circuit and the current in the coil causes energy to be stored in the magnetic field it creates.

This process is repeated during the next half-cycle, but in the opposite sense. There is thus a continuous interchange of energy stored in the electric field between the plates of the capacitor and energy stored in the magnetic field of the coil.

The amount of energy stored in each component depends on the frequency of the alternating current. At a certain frequency, the energy stored in the capacitor is exactly equal to that stored in the coil, and the energy drawn from the power supply (i.e. the current) is a maximum – in a word, resonance.

In Figure 15.30, the oscilloscope measures the potential difference across a resistor of known value, from which the circuit current can be calculated. The oscilloscope can also be used to check the frequency calibration of the signal generator. The resonance is 'damped' by the resistor. If the value of the resistor is increased, the damping is greater.

Questions

Data from such an experiment are shown in Table 15.7.

Table 15.7

F/kHz	3.0	4.0	5.0	5.5	6.0	6.5	7.0	8.0	9.0
I/mA (R=10 Ω)	20	33	83	125	143	91	63	40	30
I/mA (R=22 Ω)	19	28	60	80	80	56	45	31	24

1 Plot a graph of I against f for each resistance value.
2 Comment on the shape of the curves that you obtain.
3 What is the value of the resonant frequency?

You should obtain graphs like those shown in Figure 15.31.

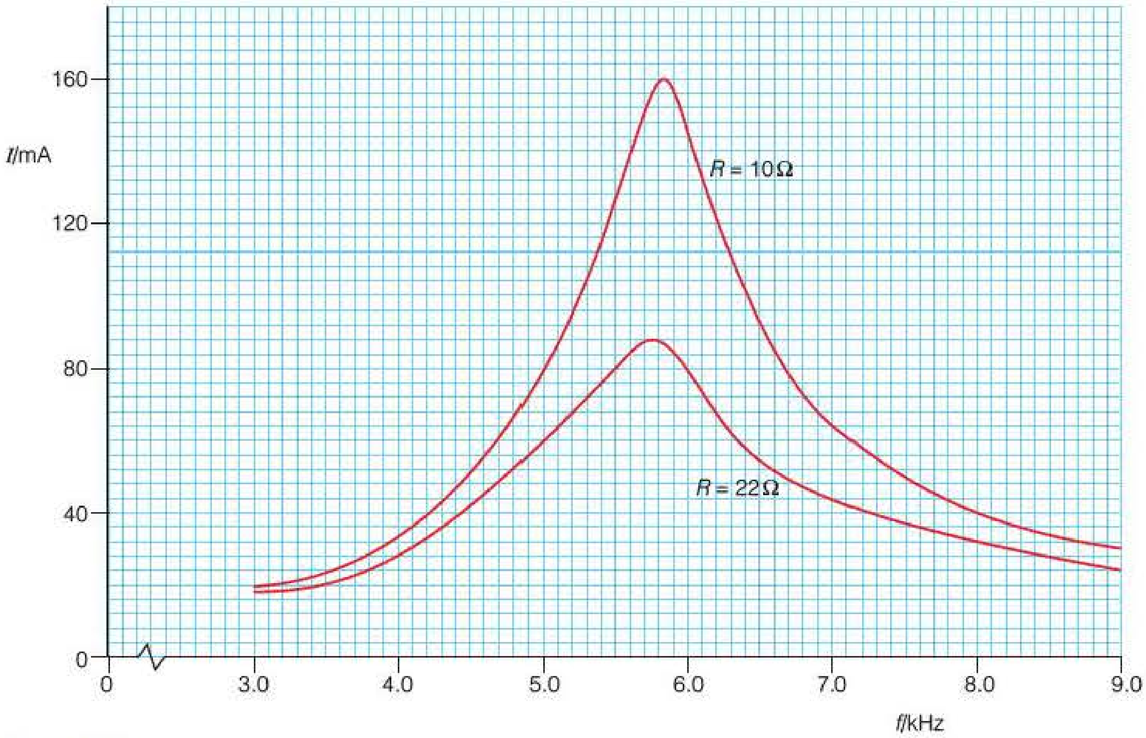


Figure 15.31

Note that a little bit of imaginative, careful drawing needs to be done to get the best curves – the peak of the 10 Ω curve is somewhere between 5.5 kHz and 6.0 kHz and *not* at 6.0 kHz, although the maximum *recorded* current is at 6.0 kHz. The resonant frequency is about 5.8 kHz.

The graphs are characteristic resonance curves, with the flatter 22 Ω curve showing a greater degree of damping and a slightly lower resonant frequency.

Reducing the adverse effects of resonance

Damping is important in the design of machines and buildings to prevent unwanted vibrations, which if they built up to large amplitude through resonance could cause severe damage, witness the famous Tacoma Narrows Bridge collapse in the USA in 1940. Video clips of this spectacular event can be found on numerous sites by putting 'Tacoma Bridge collapse' into a search engine – well worth the effort!

Sixty years later, on 10 June 2000, the Millennium Bridge crossing the River Thames in London was opened. It was nicknamed the Wobbly Bridge as pedestrians felt an unexpected swaying motion on the first two days after the bridge opened. The natural sway motion of people walking caused small sideways oscillations in the bridge, which in turn caused people on the bridge to sway in step, increasing the amplitude of the oscillatious and producing resonance. The bridge had to be closed and the problem was tackled by retrofitting dampers to dissipate the energy. The bridge re-opened on 22 February 2002 and has not been subject to any significant vibration since.

Figure 15.32 a) The Millenium Bridge, London, and b) one of the dampers that had to be fitted

Machines, such as cars, lathes and turbines, produce vibrations because of their moving parts. To prevent these vibrations building up and damaging the machine, various techniques are employed. For example, the shape of a lathe is designed so that its resonant frequency is nowhere near the frequency of rotation of the lathe, and a car engine is mounted on special dampers to absorb the energy of the vibrations.

The latest technology is to coat turbine blades with a special ductile material. You may remember from Year 1 that a ductile material is one that can be deformed plastically without fracture, which means that ductile materials can absorb a lot of energy. If vibrations occur in a ductile material, the material goes through a hysteresis loop each vibration. This absorbs the energy and prevents vibrations of large amplitude building up.

Buildings in earthquake zones are now being designed using ductile construction materials as well as different types of damping mechanisms. Recent research shows that ductility may be a more important factor in the absorption of the energy from earthquake shock waves than damping.

The Burj Dubai, at 828 m, was is the world's tallest building when it opened in 2010. It has a structural system designed and engineered for seismic conditions and extensive wind-tunnel testing has enabled the tower to resist high wind loads while minimising vibration.

Making use of resonance

It's not all bad news. We do actually make use of resonance – every day if you have a radio, television or microwave oven! When you 'tune in' a radio (or a TV, but you don't have to do this very often once the channels are set), you alter the capacitance of a variable capacitor in the tuning circuit (see Figure 5.11 on page 77). When the natural frequency of oscillations in the tuning circuit matches the frequency of the incoming radio signal, resonance occurs and the tuning circuit absorbs energy strongly from the radio waves.

We saw in Chapter 4, Section 4.3 that the oscillating electric field in a microwave oven excites the water molecules in the food, and that energy transferred from the electric field is dissipated as internal energy in the food so that it heats up. The frequency of the microwaves is critical if the energy is to transfer effectively – the microwave frequency must be close to the natural frequency of vibration of the water molecules, and then resonance will take place and the water molecules will strongly absorb energy from the electric field. A frequency of 2.45 GHz is used, because this means the time it takes for the electromagnetic wave to change the electric field from positive to negative is just the right amount of time for the water molecules to rotate. Hence the water molecules can rotate at the fastest possible rate. In addition, this frequency is not used for communications, so microwave ovens won't interfere with mobile phones, televisions, and so forth.

So why do rotating water molecules heat food? The answer has to do with the nature of internal energy and temperature. As we saw in Chapter 10, internal energy is the random kinetic energy of the individual atoms and molecules. As the water molecules rotate, they bump other molecules causing them to begin moving randomly. The process is like frictional heating. Microwave energy converts to internal energy by causing the molecules in food to increase the average speed of their random motions – your meal gets nice and hot!


Figure 15.33 MRI scan of the brain and spinal cord

MRI (magnetic resonance imaging) is a medical diagnostic technique that has been used since the beginning of the 1980s. One advantage of an MRI scan is that it uses magnetic and radio waves, so that there is no exposure to X-rays or any other damaging forms of radiation.

How does an MRI scanner work? The patient lies inside a large, cylinder-shaped magnet (see Figure 6.16 on page 105). The strong magnetic field, 10000 to 30000 times stronger than the magnetic field of the Earth, exerts a force on the protons within the hydrogen atoms of the patient's body. All the protons, which normally lie in random directions, line up parallel to the magnetic field. Then, short bursts of radio waves, of frequency between 1 MHz and 100 MHz, are sent from the scanner into the patient's body. The protons absorb energy from the radio waves (this is the 'resonance' bit of MRI) and are knocked out of alignment. When the burst of radio waves stops, the protons re-align parallel to the magnetic field. As the protons re-align, they emit tiny radio signals. These are detected by a receiving device in the scanner, which transmits the signals to a computer. Most of the hydrogen atoms in our bodies are in the form of water molecules. As each type of tissue has a different water content, the strength of the signal emitted from different body tissues varies. The computer creates a picture based on the strength and location of the radio signals emitted from the body, with a different colour or shade corresponding to the different strength of signal.

MRI magnetic fields are incredibly strong – typically 0.5 to 2.0 T. A watch flying off an arm and into a MRI machine is entirely possible and it has been known for a vacuum cleaner to be sucked into a scanner – it needed a winch to pull it out! How do we get such a strong magnetic field? In a word – superconductivity – superconducting electromagnets, cooled to 4 K by liquid helium (see Section 11.5). What would we do without physicists?

Test yourself

Fun in the playground!

a) Swing

b) Spring

Figure 15.34

7 a) When we walk our legs swing naturally, like a pendulum, for which the period squared may be taken as being proportional to the length of our leg. A father is taking his young daughter to the playground. His legs swing with a period of 1.0 s and he walks with a pace length of 0.70 m. Show that he walks at a speed of about 5 km per hour.

b) The little girl's legs are only half as long as her father's. What is the ratio of the following for the girl compared to that of her father:

- **i)** the period of her leg's free swing
- **ii)** the frequency of her leg's free swing
- **iii)** the length of her stride
- **iv)** her natural speed of walking.

c) We walk most efficiency at a pace when our legs are moving at their natural frequency of oscillation.

- **i)** Suggest why this is.
- **ii)** Explain why the little girl struggles to keep up with her father.

8 When they get to the playground, the little girl goes on a swing. Her father gives the seat a push each time it comes to the end of its swing. Before long his daughter is delighted to be swinging with large amplitude.

a) State what is meant by the *amplitude* of an oscillation.

b) Outline the energy changes that take place each swing.

c) Explain how she gets a *large amplitude*.

9 The little girl, who has a mass of 20 kg, now goes on a spring bouncer. The spring has a spring constant of 3200 N m^{-1}.

a) Show that, in theory, she would oscillate vertically with a frequency of about 2 Hz.

b) Explain why she would find this virtually impossible without some help from her father.

Exam practice questions

1 In simple harmonic motion, which of the following does *not* depend on the amplitude of oscillation?

A acceleration   **C** frequency

B energy   **D** velocity   **[Total 1 mark]**

2 When a pendulum passes through its midpoint it has maximum:

A acceleration   **C** frequency

B energy   **D** velocity.   **[Total 1 mark]**

3 At the end of a swing a pendulum has zero:

A acceleration   **C** frequency

B energy   **D** velocity.   **[Total 1 mark]**

4 A pendulum makes 20 oscillations in 16.0 s. Its equation of motion is of the form $x = A\cos\omega t$, where ω is equal to:

A 0.80π **C** 1.60π

B 1.25π **D** 2.50π.   **[Total 1 mark]**

5 The ratio of the average power to the peak power for a sinusoidal alternating current supply is:

A 1:1   **C** 1:2

B $1{:}\sqrt{2}$ **D** 1:4.   **[Total 1 mark]**

6 You are asked to display a 12 V alternating current of frequency 50 Hz on an oscilloscope.

a) Suggest suitable settings for the input voltage and for the time base. [2]

b) Sketch the trace you would see on the screen with the settings you have chosen. [2]

You should justify your answer with suitable calculations. [2]

[Total 6 marks]

7 A well-designed suspension system in a car can help prevent unwanted resonance in various parts of the car such as the bodywork and the exhaust. Each part has its own particular frequency of vibration.

a) What name is given to this frequency? [1]

b) Explain what is meant by resonance. [2]

c) Sketch a graph to show how the amplitude varies when different frequencies of vibration are applied to a system. Mark the resonant frequency on your graph. [3]

d) A car has shock absorbers to dampen the vibrations. Add a second curve to your graph to show the effect of damping on the system. [2]

e) With reference to your two curves, explain how 'a well designed suspension system in a car can help prevent unwanted resonance'. **[2]**

[Total 10 marks]

8

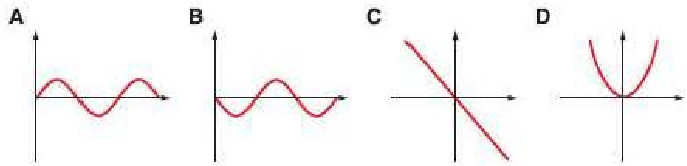


Figure 15.35

Which of the above graphs associated with simple harmonic motion could be a plot of

a) acceleration against displacement,

b) potential energy against displacement? **[Total 2 marks]**

9 A child has a bouncy ball attached to a length of rubber cord.

a) Sketch velocity–time graphs for the ball when it

i) makes vertical oscillations when suspended from the rubber cord

ii) bounces up and down on a hard surface. **[5]**

b) Explain, with reference to your graphs, why the *bouncing* ball does not have simple harmonic motion. **[3]**

[Total 8 marks]

10 The piston in a motorcycle engine moves up and down with simple harmonic motion. The distance from the bottom of the piston's motion to the top of its motion, called the stroke, is 80 mm. The engine is running at 6000 rpm.

a) Explain why the motion of the piston can be represented by the equation

$x = (4.0 \times 10^{-2}\,\text{m}) \cos(628\,\text{s}^{-1} t)$ **[3]**

b) Calculate

i) the maximum acceleration, and

ii) the maximum speed of the piston. **[4]**

c) Draw sketch graphs of

i) the displacement against time, and

ii) the velocity against time for two cycles of the motion.

Draw your graphs under each other with the same time scale and add suitable numerical values to your scales. **[5]**

[Total 12 marks]

11 A spring is suspended vertically with a mass of 400 g attached to its lower end. The mass is pulled down a distance of 60 mm and released. It is then found to make 20 oscillations in 11.4 s. The displacement x of the spring varies with the time t according to the equation $x = A\cos\omega t$.

a) What is the value of A in this equation? [1]

b) Show that ω is approximately equal to 11 rad s^{-1}. [3]

c) What is i) the maximum acceleration, and ii) the maximum speed of the mass? [4]

d) Show that the maximum kinetic energy of the mass is about 0.09 J. [2]

e) Show that the maximum resultant force acting on the mass is about 3 N. [2]

f) Hence calculate the spring constant (stiffness) k for the spring. [2]

g) Calculate the maximum potential energy stored in the spring from the relationship $E_p = \frac{1}{2}kx^2$. Comment on your answer. [2]

h) Sketch a graph showing the kinetic, potential and total energy of the mass as a function of time for one cycle of the motion. [2]

[Total 20 marks]

12 In an earthquake, waves radiate outwards from the epicentre through the Earth, causing the particles of the Earth to vibrate with simple harmonic motion and energy to be transmitted.

a) State what conditions must occur for the motion of the particles to be simple harmonic. [2]

b) Figure 15.36 shows the variation of potential energy E_p with displacement x of a particle at a distance of 100 km from the epicentre.

Sketch a copy of the graph and add labelled lines to show the variation with displacement of

i) the kinetic energy E_k, and

ii) the total energy E_T of the particle. [2]

c) Calculate the stiffness k of the 'bonds' between particles vibrating within the Earth. [4]

d) Explain two ways in which buildings in earthquake zones can be designed to minimise damage in the event of an earthquake. [4]

[Total 12 marks]

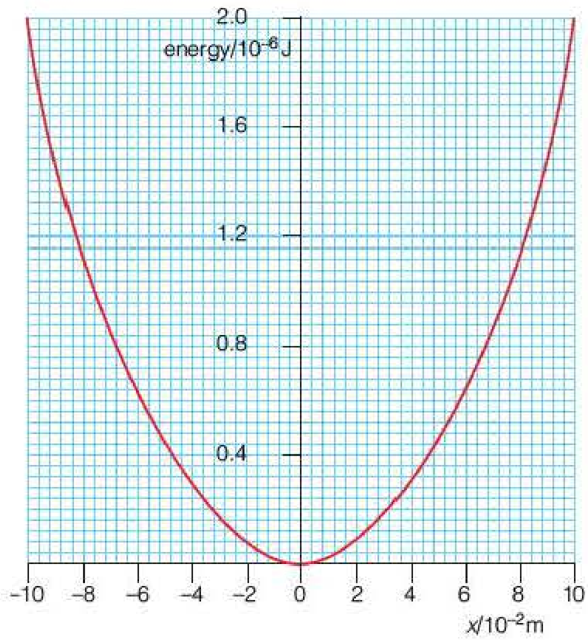


Figure 15.36

13 Some physics students go to their local park to investigate the motion of a swing.

One student sits on the swing. Another student pulls her back a distance of 1.2 m and lets go, simultaneously starting a stopwatch. Two students then measure her displacement at the end of each swing. The students sketch a graph of the motion (Figure 15.37)

a) Draw free-body force diagrams for the student sitting on the swing as she goes through the mid point of the motion for the first time and as she just reaches the end of her first complete oscillation. **[4]**

b) i) In sketching the graph, the students have assumed that the motion is simple harmonic. Suggest why this may not be the case.

ii) Use the graph to estimate the velocity of the student as she goes through the mid point of the motion for the first time. **[5]**

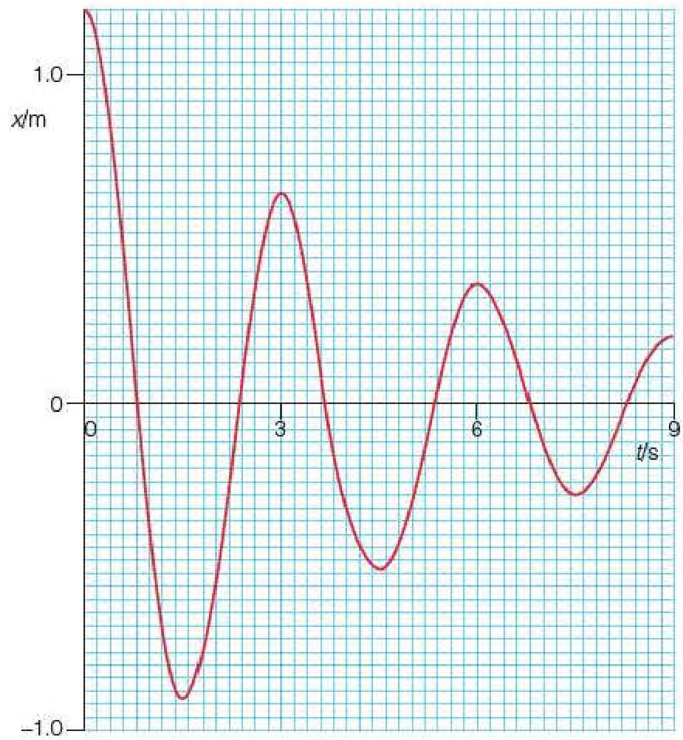


Figure 15.37

c) If the motion of the swing was simple harmonic its velocity would be given by

$v = -2\pi f A \sin 2\pi f t$

Use this formula to calculate a theoretical maximum value for the velocity. **[2]**

d) Explain how the students could use the swing to demonstrate resonance. **[3]**

[Total 14 marks]

Stretch and challenge

14 a) State the conditions necessary for simple harmonic motion. **[2]**

b) The equation governing simple harmonic motion is

$a = -(2\pi f)^2 x$

Sketch a graph of this equation and hence explain how the equation reflects the necessary conditions for s.h.m. **[4]**

c) This equation has an infinite number of solutions, two of which are

$x = A\cos(2\pi f)t$ and $x = A\sin(2\pi f)t$

Show that these equations are both possible solutions of the SHM equation. **[3]**

Sketch a graph of these two solutions on the same axis and hence explain why these are just two of an infinite number of possible solutions of the s.h.m. equation. **[5]**

d) Show that the maximum velocity for the motion is $\pm(2\pi f)A$. **[2]**

[Total 16 marks]

15 A mass of 400 g hangs vertically from a light spring of spring constant 25 N m^{-1}. The mass is given is given a downward displacement of 10 cm, at which point it is 5 cm above the bench. The mass is then released.

a) What is the necessary property of the spring for the mass to oscillate with s.h.m.? **[2]**

b) Calculate the maximum kinetic energy of the mass. **[4]**

c) Calculate the elastic potential energy in the spring when the mass is at the centre, top and bottom of an oscillation. **[8]**

d) Calculate the gravitational potential energy of the mass at the bottom and top of an oscillation. **[4]**

Take the bench as being the zero level of gravitational potential and $g = 10\,\text{N kg}^{-1}$.

e) Assuming that there is no damping, on the same axes draw graphs of

i) the kinetic energy of the mass,

ii) the gravitational potential energy of the mass,

iii) the elastic potential energy of the spring, and

iv) the total energy of the system as a function of the displacement of the mass. **[6]**

[Total 24 marks]

Index

E

F

G

H

I

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